Université de Montréal

The role of investment specific technological shocks in explaining macroeconomic fluctuations

(Version finale)

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Rapport présenté à la Faculté des études supérieures en vue de l'obtention du grade de maîtrise en sciences économiques

Décembre 2004

Acknowledgments

I would like to thank Professor Castro for his support throughout all the stages that led to the drafting of this research paper. He provided literature and MATLAB programs, as well as detailed and helpful comments that allowed me to bring my research project to a successful conclusion. I would also like to thank Professor Riboni for reviewing the final draft of this paper.

Summary

This paper is intended to show the possible impact of shocks on the capacity of an economy to produce new capital goods on the cyclical properties of macroeconomic aggregates. Using a modified version of the basic RBC model that includes these types of shocks, simulations are conducted to evaluate the capacity of the model to replicate macroeconomic fluctuations as they are observed. This exercise will show if the introduction of these shocks can improve the basic RBC model. The methodology followed is the same as the one reviewed in King and Rebelo (1999). That is, a neoclassical stochastic growth model is developed and solved using numerical methods. The choice of parameters is the same as in the King and Rebelo paper, based on empirical studies of the U.S. economy and on the long run properties of the model.

After generating artificial times series for major macroeconomic variables, sample moments are calculated from them and compared to moments calculated from actual data. The result of the simulations is that including shocks on the capacity of an economy to produce capital goods does improve the predictions of the basic model. The principal improvement is additional volatility for output, investment, consumption and work effort. In terms of relative volatility, there are improvements for consumption and work effort. As for correlation with output, adding q to the basic model improves the degree of pro-cyclicality of work effort and the wage rate, but worsens consumption's relation to output.

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Introduction

Business cycles have attracted the attention of economists since the 19th century but it has only been 20 years since economists have used neoclassical Real Business Cycle (RBC) theory to analyze this important issue. Although important progress have been made in explaining macroeconomic fluctuations through this type of model, there are still ways in which the model can be improved. Recent research has pointed to the potential importance of investment specific technological shocks for explaining macroeconomic fluctuations. This paper will investigate this possibility.

The goal of this paper is to see whether a modified version of the standard RBC model with investment specific technological shocks is capable of generating time series for macroeconomic variables that resemble their empirical counterparts. More precisely, following much of the RBC literature, this paper will focus on volatility, co-movement and persistence of the series. The model used in this paper differs from the one reviewed by King and Rebelo (1999), in that it features a technological index representing the capacity of an economy to produce capital, following the work of Greenwood and al. (2000). Changes in that capacity represent investment specific technological shocks. This addition modifies the RBC model by changing the law of motion for capital while leaving other equations unchanged. It modifies how agents in the economy will behave following the shock. Contrary to the basic RBC model, there is little income effect coming from a shift in the production function.

To show the impact of shocks on the variables in the model, the system of equation that composes the model is log-linearized, and solved using numerical methods. The model's properties are then analyzed using impulse response functions. To complete the analysis, sample moments calculated through simulations of the model are compared to similar sample moments calculated from actual data.

Literature Review

This research paper builds on the Real Business Cycle literature, initiated by Kydland and Prescott in 1982. Since then, this strand of literature has expanded in various branches, as the basic RBC methodology was used to study aggregate fluctuations resulting from different types of random shocks. The basic RBC model is concerned with the impact of neutral shocks, which affect the production function to raise the productivity of all factors (TFP). It is assumed that these shocks reflect improvements in the capacity of an economy to produce output from a given level of inputs.

King and Rebelo (2000) review this type of model. They use the neoclassical stochastic growth model, focusing their attention on the fluctuations that result from TFP shocks. By choosing parameter values consistent with the US post war data, their simulation clearly reproduce key properties of U.S. macroeconomic time series. Namely, this type of model performs well in explaining both variability and persistence of most macro economic variables, with the exception of work effort and the wage rate. Indeed, work effort lack variability compared to output while the wage appears to be strongly pro cyclical while it is not the case in observed series.

Neutral shocks are not the only possible type of technological shock. They may also be modeled as improvements in the capacity of an economy to produce capital goods. This is how Greenwood, Hercowitz, and Krussel (2000) (GHK hereafter) proceed to study the impact of non neutral technological shocks on macroeconomic variables. They consider variations in the relative price of consumption goods to equipment investment goods, which reflect the capacity of an economy to produce capital goods and which they denote by q, on macroeconomic variables fluctuations. They adapt the model they developed in their 1997 article, which studied the impact of investment specific technological change on economic growth. This model includes adjustment costs and variable capacity utilization.

Considering only shocks on q, they found that their model explained about 30 of output fluctuations. Investment and consumption are both pro cyclical, but the later one is not sufficiently so. The variability they find for hours is too low as well. They conclude that investment specific technological change is an important factor in explaining macroeconomic fluctuations, although not the most important one. It can only account for roughly 30 percent of output fluctuations (which is still large considering that equipment investment accounts for less than 10 percent of output).

Even though these results suggest a modest role for investment specific technological change, Fisher (2003) found using a macroeconometric model applied to post war U.S. data that investment specific technological change could explain up to half of macroeconomic fluctuations. His model is based the one by GHK but it aggregates all investment (while GHK separated structures and equipment in order to reflect that productivity growth arises mostly from better equipment).

The purpose of the present research paper is to analyze the impact of a combination of neutral and investment specific shocks on macroeconomic fluctuations, in order to try to better reproduce stylized macroeconomic facts. The simplified setting on which the Fisher macroeconometric model is based will be adopted in this paper, which means adjustment costs, capacity utilization will be ignored, as well as the distinction between equipment and structures. This will lead to a straightforward modification of the King and Rebelo basic RBC model.

The model

Following the standard procedure in the RBC literature, a neoclassical stochastic growth model is used to investigate the impact of random shocks on aggregate output and other macroeconomic variables. A simple version of the model is used, one that ignores government and taxation, which makes it possible to exploit the equivalence between a centralized and decentralized economy. Some of the refinements introduced by GHK to study shocks on the capacity of an economy to produce capital goods, such as adjustments costs and variable capital utilization, are also left aside to simplify the analysis.

The neoclassical stochastic growth model is composed of an inter-temporal utility function, whose arguments are consumption and leisure, which is maximized by a central planer subject to a number of constraints. The expected utility function takes the usual form:

$$E_0 \sum_{t=0}^{\infty} b^t u(C_t, l_t)$$

where b is positive and represents the subjective discount factor. Following King and Rebelo(1999: 26), we use a type of constant elasticity utility function of the form:

$$u(C_t, l_t) = \log(C_t) + \frac{\theta}{1-\eta}(l_t^{1-\eta} - 1)$$

For the computational experiment conducted later in the paper, we use a value of $\eta = 1$, which amounts to using a log-log utility function.

There are four constraints in this optimization problem. The first one is the time constraint:

$$n_t + l_t = 1$$

One cannot spend more time on work or leisure than one has in a day, n_t and l_t therefore represent the fraction of a day allocated to work and leisure respectively. The second constraint is the production function:

$$Y_t = Z_t K_t^{1-\alpha} n_t^{\alpha}$$

An economy cannot produce more output than what its technological level allows given input levels. We chose the Cobb-Douglas form as it is standard procedure in the RBC literature. The third constraint is the aggregate resource constraint:

$$Y_t = C_t + I_t$$

Aggregate output must be the sum of aggregate consumption and aggregate investment. Finally, the last constraint is the capital law of motion:

$$K_{t+1} = Q_t I_t + (1-\delta)K_t$$

Capital stock in the next period is equal to the sum of today's un-depreciated capital stock and gross investment, expressed in terms of units of capital, where Q_t represents the amount of capital that can be produced with one unit of output. So if Q_t rises over time, this means that capital is becoming cheaper relative to output (one unit of output can be transformed into more units of capital than before) and accumulation will accelerate if everything else remains the same.

The first step in studying a neoclassical stochastic growth model is to transform the variables so as to make them stationary at the steady state. To do so, King and Rebelo (1999: 13) divide each variable that grows in the steady state by the deterministic component of aggregate productivity. Since the variables grow at the same rate as this component in the steady state, the ratios of variables to the deterministic component doesn't grow. The model studied here is a bit different. There are two sources of productivity growth. One source is the total factor productivity variable Z_t and the other one is the Q_t variable, which embodies investment specific technological change. We assume here that both Z_t and Q_t variables are equal to the product of a deterministic and random component. That is:

$$Z_t = \overline{Z}_t \cdot z_t, Q_t = \overline{Q}_t \cdot q_t$$

where $\overline{Z}_t$ and $\overline{Q}_t$ are the deterministic components and respectively grow at rates γ_z and γ_q. To make the variables stationary in the steady state, they have to be divided by a combination of $\overline{Z}_t$ and $\overline{Q}_t$, which we label X_t.

To derive X_t, we follow Greenwood et al. (2000: 98) and posit that variables in the model grow at the same rate γ_x, with the exception of capital, which grows faster at rate γ_k. Using the Cobb-Douglas production function, we know that γ_x must be equal to $\gamma_z \, \gamma_k^{1-\alpha}$ since Y_t grows at γ_x, Z_t grows at γ_z, and capital grows at γ_k. Employment (or work effort) does not grow over time. We also know from the law of motion of capital that $\gamma_k = \gamma_x \, \gamma_q$. With these two equations, we are able to solve for γ_x and γ_k:

$$\gamma_x = \gamma_z^{1/\alpha} \gamma_q^{1-\alpha/\alpha}$$

$$\gamma_k = \gamma_z^{1/\alpha} \gamma_q^{1/\alpha}$$

The equations for γ_x and γ_k tell us how to construct X_t in order to make variables stationary. Variables that grow at γ_x, that is output (Y_t), consumption (C_t), investment (I_t) should be divided by $\overline{Z}_t^{1/\alpha} \overline{Q}_t^{1-\alpha/\alpha}$. Since capital ($K_t$) grows gamma times faster than the previous variables, it has to be divided by $X_t \overline{Q}_t$ or $\overline{Z}_t^{1/\alpha} \overline{Q}_t^{1/\alpha}$. The variable Z_t only needs to be divided by its deterministic component. Using the definition of X_t, it is now possible to transform the problem so as to make it stationary. To indicate a stationary variable, we follow King and Rebelo and use lowercase letters: Yt / Xt = y_t.

After the transformation, the optimization problem is very similar to the original one but with slight modifications. The production function becomes:

$$y_t = z_t k_t^{1-\alpha} n_t^{\alpha}$$

and the capital law of motion becomes:

$$\gamma_x \gamma_q k_{t+1} = q_t i_t + (1-\delta) i_t$$

while the aggregate accounting identity becomes:

$$y_t = c_t + i_t$$

The subjective discount factor is also modified according to $\beta = b\gamma_x^{1-\sigma}$, but since we use a log utility function (σ = 1), it is not affected.

The optimization problem can by solved using the Lagrange multipliers method. The Lagrangean function is:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \frac{\theta}{1-\eta}(l_t^{1-\eta} - 1) \right]$$

$$- E_0 \sum_{t=0}^{\infty} \lambda_t \beta^t \left[c_t + q_t^{-1} \gamma_x \gamma_q k_{t+1} - q_t^{-1}(1-\delta) k_t - z_t k_t^{1-\alpha} n_t^{\alpha} \right]$$

$$- E_0 \sum_{t=0}^{\infty} \omega_t \beta^t \left[n_t + l_t - 1 \right]$$

Note that the multipliers are discounted, which means they cannot be interpreted in the usual way. They represent shadow prices in terms of present value instead of current value.

The first order conditions of this problem are the following:

$$\frac{\partial L}{\partial c_t} = 0 \Rightarrow c_t^{-1} = \lambda_t$$

$$\frac{\partial L}{\partial l_t} = 0 \Rightarrow \theta l_t^{-1} = \omega_t$$

$$\frac{\partial L}{\partial n_t} = 0 \Rightarrow \lambda_t z_t k_t^{1-\alpha} \alpha n_t^{\alpha} = \omega_t$$

$$\frac{\partial L}{\partial k_{t+1}} = 0 \Rightarrow \lambda_t q_t^{-1} \gamma_x \gamma_q = \beta \lambda_{t+1} \left[q_{t+1}^{-1}(1-\delta) + z_{t+1}(1-\alpha) k_{t+1}^{-\alpha} n_{t+1}^{\alpha} \right]$$

These conditions, combined with the four constraints and the definition of factor prices under competitive equilibrium

$$MPK_t = R_t - 1 + \delta$$

$$MPL_t = w_t$$

form a nonlinear system of dynamic equations. To complete the model, we need to define the processes that generate the stochastic components of Z_t and Q_t. We assume that these components follow AR(1) processes with autocorrelation parameters ρ_z and ρ_q, and standard deviations σ_z and σ_q.

Instead of solving the above system of non linear equations, the standard procedure in the RBC literature is to log-linearize each of the equations around the steady state to yield the following system of linear equations:

$$-\hat{c}_t = \hat{\lambda}_t$$

$$\left[\frac{\eta \bar{n}}{1-\bar{n}} + (1-\alpha) \right] \hat{n}_t = \hat{\lambda}_t + \hat{z}_t + (1-\alpha)\hat{k}_t$$

$$\hat{y}_t \bar{y} = \hat{c}_t \bar{c} + \hat{i}_t \bar{i}$$

$$\gamma_x \gamma_q \hat{k}_{t+1} = (1-\delta)\hat{k}_t + \frac{\bar{i}\bar{q}}{\bar{k}}(\hat{q}_t + \hat{i}_t)$$

$$\hat{y}_t = \hat{z}_t + (1-\alpha)\hat{k}_t + \alpha \hat{n}_t$$

$$\frac{\gamma_x \gamma_q}{\beta \bar{q}}(\hat{\lambda}_t - \hat{q}_t - \hat{\lambda}_{t+1}) + \frac{(1-\delta)}{\bar{q}} \hat{q}_{t+1} = (\bar{z}(1-\alpha)\bar{k}^{-\alpha}\bar{n}^{\alpha})(\hat{z}_{t+1} - \alpha \hat{k}_{t+1} + \alpha \hat{n}_{t+1})$$

$$\bar{R}\hat{R}_t = (\bar{R} - 1 + \delta)(\hat{z}_t - \alpha \hat{k}_t + \alpha \hat{l}_t)$$

$$\hat{z}_t + (1-\alpha)\hat{k}_t - (1-\alpha)\hat{n}_t = \hat{w}_t$$

The RBC model developed can be viewed as an analog of a decentralized competitive economy even though behavioral equations were derived assuming the presence of planner maximizing the life-time utility of an infinitely lived representative agent. But in order for the model to be viewed as a competitive general equilibrium outcome, the assumption of rational expectations is required. As King and Rebelo (1999, 81) show, this assumption assures that optimal conditions from the centralized economy will coincide with those resulting from firms and household maximizing profits and utility. Prices for the labour and capital markets are added to the 8 equations that compose the central planner model so as to allow agents to decide how much labour to supply and how much capital to accumulate. Since the equilibrium prices depend on technological change parameters A_t and q_t, shocks on these will affect the evolution of prices.

Solving the model

The King and Watson[1] (1998) MATLAB program is used to obtain a state space representation of the model's solution, from which simulated series and impulse response functions are generated. But in order to conduct the simulation, model parameters have to be chosen. Since the model used here is the basic RBC model by King and Rebelo, the same parameter values are used. Using the same parameter values will allow the comparison between the original model and the modified one. The only additional parameters that need to be specified are the growth rate of q and the variance and autocorrelation parameters of the stochastic component of q.

Parameter selection

The parameter selection associated with the q variable in GHK (2000) is based on annual estimates while this paper is based on a quarterly model[2]. It is therefore not possible to use GHK parameter values for our simulation exercise. But the Cummins-Violante (2002) dataset on relative prices of capital and consumption was used instead, although it was modified using the Denton interpolation method. This method allows the generation of a quarterly time series using a combination of annual series and quarterly series. The idea is to expand the annual time series which is of good quality with the information on quarterly variation contained in the quarterly

[1] See King and Kurmann (2003) for a simple introduction.

[2] The authors based their econometric analysis on the Gordon annual equipment relative price index, which covers the 1954-1983 period, and expanded it to 1990 with NIPA data. The authors regressed the log of the price series on time and assumed AR(1) errors. They found the deterministic parameter to be 0.032, meaning that the price index grew 3.2 percent per year on average over the sample period. The stochastic component of q follows an AR(1) process with rho=0.64, and variance of 0.035.

series of lesser quality. In this case, the quarterly index prices for private investment and personal consumption (NIPA tables 1.1.4 and 5.3.4) were used in conjunction with the Gordon-Cummins-Violante series.

Using the newly minted quarterly series for capital/consumption relative prices, the GHK econometric model for the q process was estimated:

$$\log q_t = \alpha + \beta t + e_t$$

where e_t follows an AR(1) process. The model was estimated over the same time interval as for the parameter used by King and Rebelo, that is 1947 to 1996. Since the series for q has auto correlated errors, we use the Prais-Winsten method to transform the model to take this into account. The estimated parameter values found are $\beta = 0.0058$, $\rho = 0.998$ and $\sigma = 0.0083$, which are quite different to the ones used by GHK in their simulation. The quarterly series shows very strong persistence but less volatility. This is the result of using a linear trend with data that exhibits non linear growth in logs (see Figure 1). To try and produce more sensible estimates of the persistence and volatility parameters, the HP filter is used to detrend the log of q series. The difference between the actual data and the filtered data was modelized as an AR(1) and the following estimates were found: $\rho = 0.87$ and $\sigma = 0.0069$, which are both lower than the ones derived using a linear trend. The growth rate value of 0.0058 was used in combination with $\rho = 0.87$ and $\sigma = 0.0069$ to simulate the model.

Analysis of impulse response functions

Impact of a shock on the productivity of all factors (z)

We will use the parameter values used by King and Rebelo (KR) to generate a benchmark for our analysis of the impact of investment specific shocks. By setting the variability of q to zero, our results are almost identical to the ones from KR. The difference comes from the use of a slightly different growth rate for the model economy. Although KR already analyze response functions derived from their model, we will repeat the exercise in a little more detail, focusing on the various wealth and substitution effects that the shocks generate.

Because of the production function Cobb-Douglas form, the productivity shocks will induce a shift in the production function, affecting both its level and slope. Both labour and capital marginal products rise as a result of a positive shock on the z parameter. The impulse response functions on Figure 2 are the same as the ones presented on figure 10 in the KR paper. They show the impact of a one percent rise in the stochastic component of the Z variable over one hundred quarters.

The response function of output has a similar shape as the z one. This is not surprising considering that a shock on z leads to a direct effect on output through the production function equation. As the shock on z weakens, the effect on output also does. Note however that a one percent shock produces a 1.5 percent rise in output over its steady state value. This is a result of the rise in work effort that follows the shock. Because of the shock, it is possible to produce more with the same amount of inputs and since the shock is transitory, it is efficient to produce even more to take advantage of the new but yet temporary opportunities.

Since the marginal product of labour is higher, firms have an incentive to use more labour, and even more so since the rise is not permanent, which constitutes an inter temporal substitution effect. As a result, output grows even more than by the direct effect. Labour grows even more as the interest rate rises to limit consumption and induce more work effort. So there is as well another inter temporal substitution effect of consumption and leisure following the rise in the interest rate. But over time, as the marginal product of labour declines, the additional labour use is reduced and output goes back to its steady state value. The fall in the interest rate also induces consumers to favor more leisure over work.

The consumption and investment response functions show how the structure of output is affected by a neutral shock. Investment demand surges following the rise in the marginal product of capital. The increased profitability of existing and future capital provides the incentive to firms to increase their capital stock. But as the shock fades, this incentive vanishes and investment demand returns to its steady state value.

As a result of the wealth effect, consumption demand rises although not as much as if the shock was permanent. Since consumers wish to even out their consumption trough time, they will wish to save a large portion of their additional income. But this desire to save will not be as intense as when the shock lasts one period because of the high persistence of the shock (ρ = 0.979). So consumption rises as the result of the shock. It continues to rise even as the shock becomes weaker because of the abnormally high capital stock. This stock is used to produce more consumption goods as investment demand falls. But eventually, consumption has to decrease because the extra capital stock has been run down.

Prices are affected as well by a transitory shock. The interest rate goes up as investment demand and to a lesser extend consumption demand go up following the shock. The rise in the rate serves to reduce demand pressures that cannot be met by production. But immediately after the rise, the interest rate starts to go down as the capital stock grows, which reduces the marginal product of capital and at the same time, investment demand. At one point, the rate is reduced below its steady state value to induce consumers to consume more and let the capital stock deplete and eventually return to its equilibrium value.

The wage rate jumps 0.75 percent following the shock as demand for labour rises at the same time as labour supply grows because of the jump in the interest rate. The wage rate continues to grow after the shock, even though work effort falls. On one hand, because of the growing level of capital in the economy, the marginal product of labour continues to rise and so does labour demand. On the other hand, supplied labour decreases because the interest rate is falling, inducing agent to substitute work for leisure. But eventually, the declining capital stock has a negative impact on labour productivity and inevitably, on the wage rate.

Impact of a shock on the capacity of an economy to convert output into capital (q)

A shock on q is different from a shock on the TFP parameter because it does not affect the production function, but rather the ability of an economy to convert output into capital. From the law of motion of capital, we see that as q rises, the economy will be able to make more capital out of a unit of output. The product i_t*q_t is the amount of capital derived from i_t units of output. So a growing q should provide the economy with more capacity for production and therefore more consumption in the future. But contrary to the case of a rising TFP, consumption and investment will initially evolve in opposite direction since agents have to sacrifice consumption in order to invest more.

To see intuitively why this is so, consider the Euler equation from our model and taking q as a constant. It shows how optimal consumption should evolve over time given a value for q, ignoring uncertainty and labour. By rearranging the equation, we see that the MRS is a function of q:

$$MRS = \frac{u'(c_t)}{u'(c_{t+1})} = \beta q MPK_{t+1} + \beta(1-\delta)$$

As q rises, so does the MRS. For the MRS to rise, it must be that the marginal utility of future consumption is becoming relatively smaller than marginal utility of present consumption. Because the marginal utility of consumption is decreasing in c_t, this means that it is optimal for consumers to postpone their consumption to a future date and save more in the present, since output is fixed when the shock occurs. The additional saving will be used to invest in capital and expand future consumption with the additional capacity.

A shock in q also affects the behavior of firms since it changes the profitability of additional capital. Consider the optimality condition for firms and notice that a rise in q directly improves the marginal product of capital[3]:

$$q_{t+1} MPK_t - \delta - \gamma_q \approx r_t$$

Firms therefore have an incentive to expand their stock of capital until capital decreasing returns brings investment back to its steady state value. The firms will use the additional savings from consumers to finance their purchases of capital and later provide consumers with additional goods and services. Based on this analysis, one should expect to see consumption decline and investment rise immediately after a positive shock on q.

[3] The return on capital is the following : $(P_{t+1}^{y} MPK_t + (1-\delta)P_{t+1}^{k} - P_t^{k}) / P_t^{k}$ By noting that $P_t^{y} / q_t = P_t^{k}$, the equation can be rearranged to yield the above equation.

This is indeed the result the model provides as Figure 3 shows. Impulse response functions on figure 3 were generated with the modified model calibrated with the same persistence and volatility parameters as in the basic RBC model. Figure 4 is based on persistence and volatility parameters as they were estimated using the quarterly relative price data. The response functions are different because of the very strong persistence in Figure 3, which leads to responses close to what they would be in the event of a permanent shock in q.

Although the production function is not affected by a rise in q, the surge in demand for capital more than compensates for the decline in demand for consumption as can be seen from Figure 3. The excess demand pushes the interest rate up, which provides an incentive for people to postpone consumption and leisure, and supply more work effort through an inter-temporal substitution effect. The economy is therefore able to produce more output as people work more hours, even though the economy is not more productive in producing output (only capital). Because of above steady state investment, the capital stock grows and allows for more production, and for some time, output expands. But with the combination of declining work effort as the interest rate falls and declining capital stock, output eventually returns to its steady state level.

Following the shock, the interest rate goes up and so individuals supply more work effort. Since labour demand has not changed, the wage rate drops temporarily. But as soon as the interest rate starts going down, so does the supply of labour. But at the same time, the rising capital stock has the effect of raising the marginal productivity of labour, which induces firms to demand more labour since it is now profitable to do so. Consequently, hours worked still decline but less dramatically and the wage rate goes up after dropping initially. Eventually, labour returns to its steady state value.

The wage rate takes more time to return to its steady state value because, even though labour used is almost the same as before the shock, labour demand is higher than before the shock (because the capital stock is above its steady state value and therefore, so is the marginal product of labour) and labour supply is lower than before the shock (the interest rate is below its steady state value, inducing people to consume more leisure today than in the future) and so the wage rate remains above its steady state value. As the interest rate goes up back to its steady state value and capital continues to depreciate, labour supply and demand go back to their initial values and so does the wage rate.

The interest rate is strongly related to the evolution of the capital stock, because by definition in the RBC model, the interest rate is equal to the marginal product of capital minus the depreciation rate. Initially, the capital stock is fixed and since output grows (because of additional work effort), the marginal product of capital rises. But because of declining marginal returns, as soon as the capital stock starts growing following investment, the marginal product of capital starts declining. The decline in the interest rate works as an incentive for people to save less (consume more and work less) because further investment is not as profitable as it was following the shock. The interest rate eventually falls below its steady state value in order to induce people in consuming without replacing the superfluous capital stock.

Impulse response functions in Figure 4 are qualitatively the same as the ones on Figure 3 but the transition towards the steady state takes less time. Recall that the persistence parameter used to calibrate the model in Figure 3 is equal to 0.87 compared to 0.979, which means agents and firms will behave more as if the shock on q was a temporary one. Since the effects of a heightened capacity of the economy to transform output into capital will lasts for less time, agent are more pressed to take advantage of the temporary opportunity to accumulate. This is why investment, work effort and the interest rate rise more when the shock on q is less persistent.

Analysis of moments

The RBC model leads to the generation of times series for each variable in the model, from which moments can be calculated. A good model should produce times series with moments as close as possible to those calculated from observed data. Table 1 provides such calculations for three different models as well as for actual data. The first panel of moments is from the basic RBC model as it is reviewed in King and Rebelo (1999). From now on, it will be called model I. The other two panels of moments are those derived from the modified RBC model presented in this paper. Model II will be the modified model in which only shocks on q are allowed to operate. Model III will be the modified model in which both shocks on q and z are allowed to operate. These moments will show if the addition of q in the basic RBC model has led to improvements in the ability of the model to reproduce actual fluctuations.

There are two moments than can help assess the ability of the model RBC model to reproduce actual fluctuations. The first one is the degree of volatility (standard deviation) of the time series it generates. A good RBC model would reproduce the variability observed in the actual data. The basic RBC model does fairly well with regards to this requirement. Simulated output, investment and wage rate are almost as volatile as their observed counterparts. Simulated consumption, work and the interest rate however, are not sufficiently volatile.

Model II does not do much better. Consumption and investment volatilities are close to the ones from model I while work effort is more volatile, closer to what is observed in actual data. But output volatility is much too weak. It's less than half of what it is actual data. This is a consequence of the absence of a shift in the production function following a shock on q. Since consumption and investment move in opposite directions, output does not vary as much as its two components. Since a shock on q does not affect the production function, it is less capable to generate fluctuations. When shocks on z and q are mixed in model III, the results are slightly better than when shocks on z alone are allowed in model I. Adding shocks on q adds volatility in the basic model but still not enough to completely reproduce actual volatility, especially for consumption and work effort.

Relative volatility (with respect to output) can also help assess the quality of a RBC model. Certain variables should be more volatile, says investment, than other ones, consumption for example, as is the case in actual data. To evaluate the performance of the model, relative standard deviation is calculated. Table 1 shows how the degree of volatility of each variable relative to output. The basic model does well for investment but consumption and work effort should be relatively more volatile. In fact, work effort should be as volatile as output. Only the wage rate shows too much volatility compared to actual data.

Model II does poorly in that respect. All variable have too much variability compared to output, with the exception of the interest rate. Not only does this model produce too little variability but it is as well not well distributed among variables. Model III, that combines the two types of shocks is the best, although the results are not that much better than the ones for the basic model. Although the relative variability of work effort is significantly closer to what it is in actual data, but the moments for the rest of the variables barely change. So adding q in the basic model does improve the relative standard deviation of variables but this improvement is not very important. That's because de additional volatility generated by including q in the basic model is not sufficient to modify significantly the distribution of volatility among variables generated by shocks on z. The inter-temporal substitution effect induced by a higher capacity to convert output into capital is not important enough.

Correlation with output is the second moment used to evaluate the performance of a RBC model. Variables should be pro or counter cyclical with varying degree. Consumption, investment and work effort are all strongly pro cyclical in the actual data with correlation coefficients above 0.8, as can be seen in table 1. Model II produces a strong counter cyclical consumption and wage rate which is at odds with actual data. This result comes from the inter-temporal substitution of consumption and leisure induced by the temporary change in the economy's capacity to convert output into capital. There could therefore not only be this type of shock in the actual U.S. economy. But work effort and investment are strongly pro-cyclical, slightly more so than in actual data. The interest rate, like in the basic model, is pro cyclical wile it should be the opposite.

The combined shock model produces correlation coefficients that are more closely related to what is observed, with again the exception of the interest rate. The combined model generates better work effort time series but weaker consumption/output correlation. These changes from the basic RBC model are the result of the counter cyclical consumption and strongly pro cyclical work effort the shocks on q produces. The correlation between the wage rate and output is also closer to what is observed in the actual data than it is in the basic model, but it's still too strong.

Conclusion

The goal of this paper was to see whether a modified version of the standard RBC model with investment specific technological shocks was capable of generating time series for macroeconomic variables that resemble their empirical counterparts. More precisely, the objective was to find out if including such shocks would improve the basic RBC model's ability at reproducing observed macroeconomic fluctuations.

The model that allowed only for random shocks on the economy's capacity at producing capital goods did not performed better than the basic RBC model. Given the volatility of the stochastic component of q_t, the transmission mechanisms from q to output was too weak to explain a majority of the observed volatility. Furthermore, the model generated relative volatility that did not reflect what is observed in actual data, again because of too little output volatility. Finally, the model produced a strong counter cyclical consumption and wage rate, which was again at odds with actual data. Although these result imply that there could not be only this type of shock in the actual U.S. economy, the exercise showed what kind of effect adding q in the basic RBC model would have.

The model that combined shocks on q and z performed well compared to the basic model but did not improve in a major way the ability of the basic model at reproducing observed macroeconomic fluctuations. The principal improvement was additional volatility for output, investment, consumption and work effort wile volatility of prices was barely affected. In terms of relative volatility, there were improvements in consumption and work effort. As for correlation with output, adding q to the basic model improved the degree of pro-cyclicality of work effort and the wage rate, but worsened consumption's in an important way. Other correlation coefficients were not significantly affected.

Adding shocks on q to the basic RBC model did not modify much the previous results, mainly because they do not add much volatility, except in investment. So it is difficult to imagine a modification of the model that would significantly improve our results. Within a more sophisticated framework, GHK (2000) already found that shocks on q were not the main factor behind macroeconomic fluctuations. But still, a better calibration might perhaps improve the results. The estimation of the parameters from the q process could be done using a VAR model to try and pick up the effects of q on z and conversely. The construction of the quarterly time series from q could possibly be ameliorated using alternative relative prices series when combined with the Cummins-Violante annual series.

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Tables and Figures

Table 1

Table 1*

	Basic RBC model (I)			Modified model, shocks on q only (II)		
Variables	Standard deviation	Relative standard deviation	Correlation with output	Standard deviation	Relative standard deviation	Correlation with output
Y	1.39	1.00	1.00	0.73	1.00	1.00
C	0.61	0.44	0.94	0.68	0.93	-0.87
I	4.09	2.95	0.99	4.31	5.91	0.98
N	0.67	0.48	0.97	1.09	1.49	0.97
w	0.75	0.54	0.98	0.42	0.58	-0.77
R	0.05	0.04	0.95	0.03	0.05	0.77

	Observed US Series			Modified model, shocks on q and z (III)		
Variables	Standard deviation	Relative standard deviation	Correlation with output	Standard deviation	Relative standard deviation	Correlation with output
Y	1.81	1.00	1.00	1.60	1.00	1.00
C	1.35	0.74	0.88	0.89	0.56	0.25
I	5.30	2.93	0.80	5.72	3.59	0.92
N	1.79	0.99	0.88	1.30	0.81	0.85
w	0.68	0.38	0.12	0.85	0.53	0.58
R	0.30	0.16	-0.35	0.07	0.04	0.91

Source: Data for the Basic RBC model and observed US series are from King and Rebelo (2000) tables 3 and 1 respectively.

*Following King and Rebelo, the series were logged and then detrended using the Hoddrick Prescott filter before moments were calculated.

Figure 1

Persistence and volatility in the log of Q_t time series

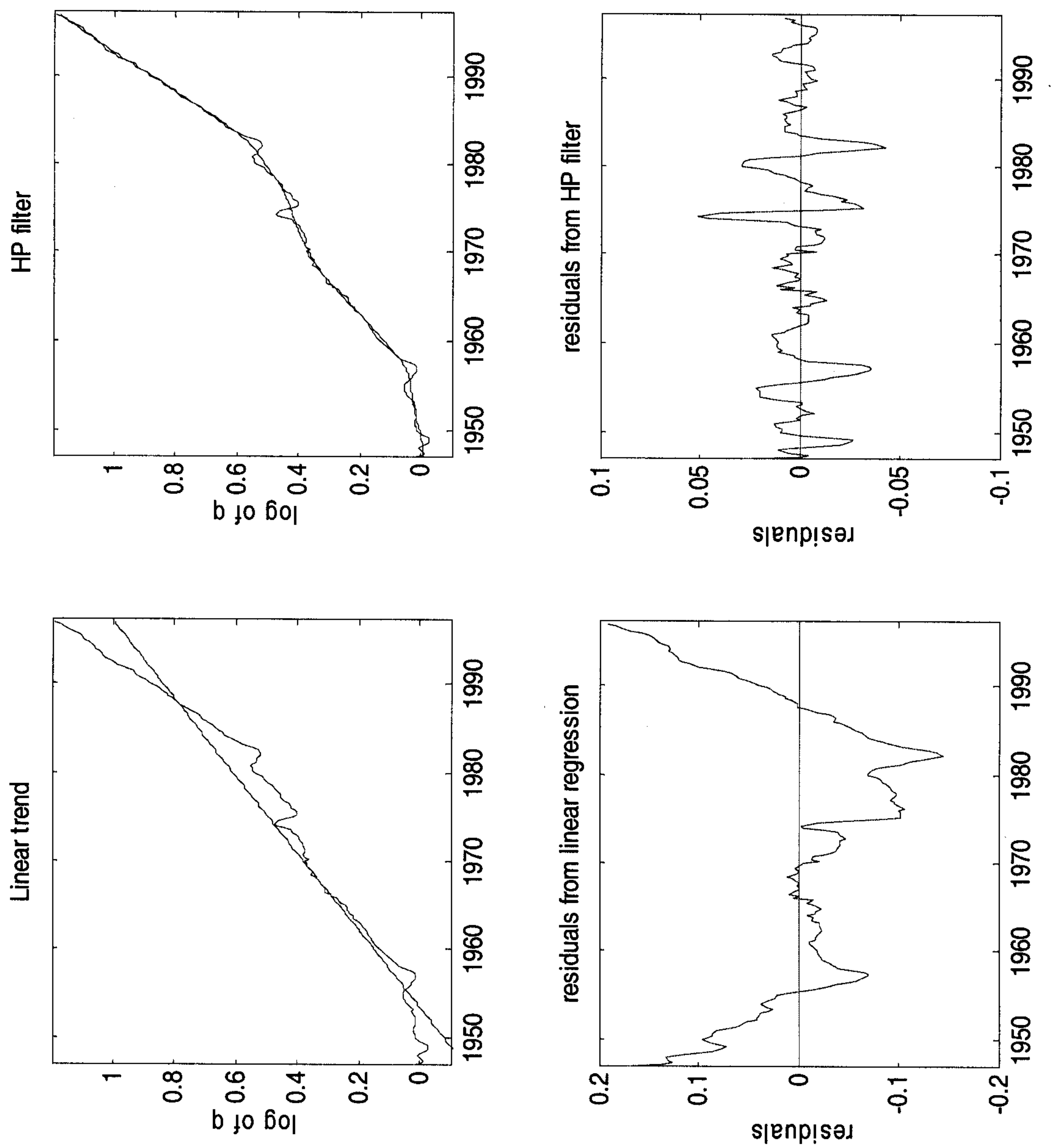

Figure 2

Impact of a 1 percent positive shock on z_t

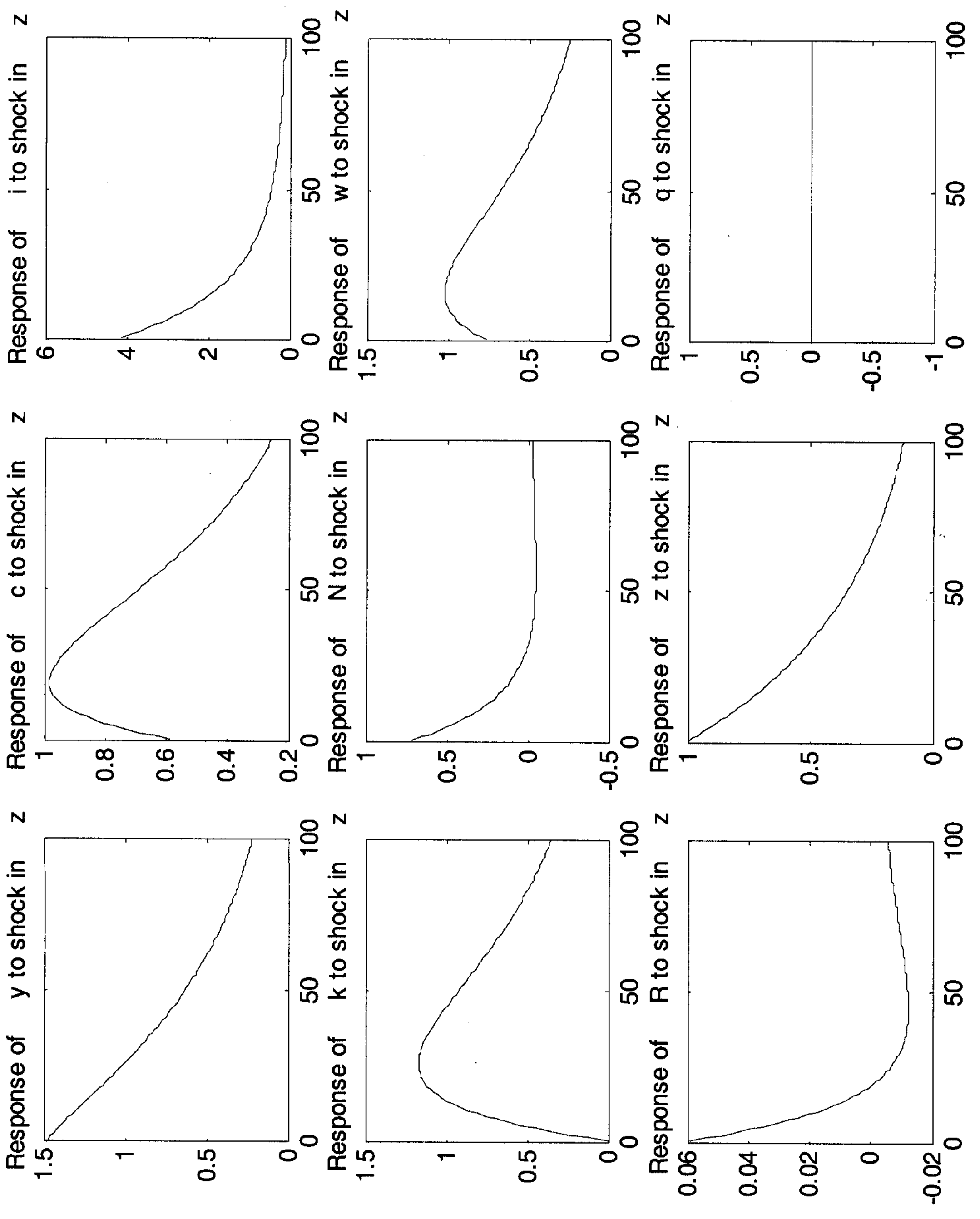

Figure 3

Impact of a 1 percent positive shock on q_t
(with parameter values set to $\rho = 0.979$, $\sigma = 0.0072$, $\gamma_q = 1.004$)

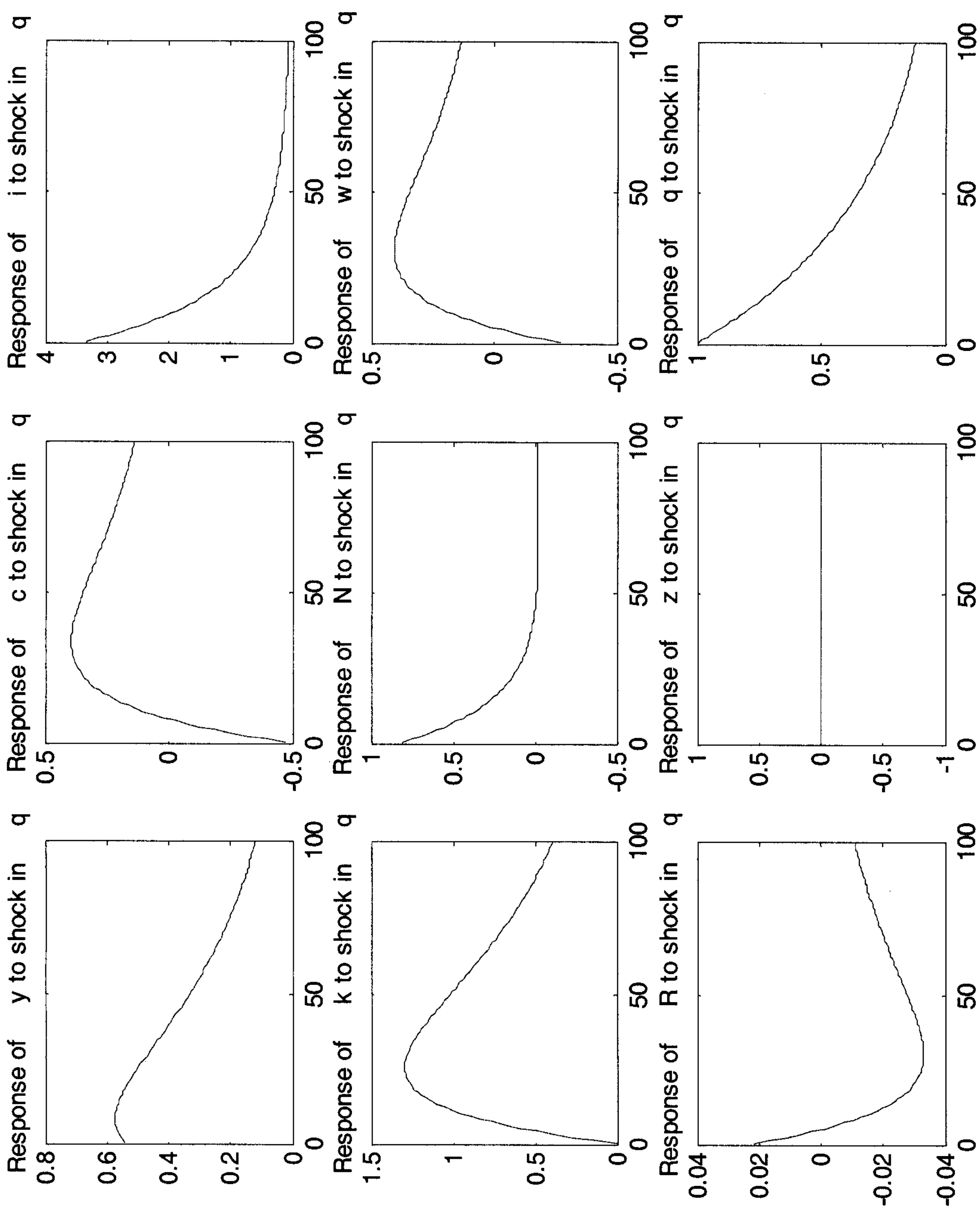

Figure 4

Impact of a 1 percent positive shock on q_t
(with parameter values set to $\rho = 0.87$, $\sigma = 0.0069$, $\gamma_q = 1.0058$)