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The Canadian-U.S. dollar exchange rate:
Validating an improved version of
the stock-flow model*

by

Robert Lafrance and Daniel Racette

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Abstract

Three views of exchange rate determination are investigated in order to explain the Canadian-U.S. dollar exchange rate in the 1970's. Our empirical results reject the monetary approach and the sticky price asset market model but concur with the stock-flow model of the exchange rate. We establish an original reduced-form equation for the exchange rate which is consistent in all respects with the theory, stable, and shown to be superior to acceptable alternative in non-nested tests.

Résumé

Nous examinons trois théories qui ont été proposées pour expliquer le taux de change : l'approche monétaire, le modèle des marchés des actifs avec rigidité des prix et le modèle dit de stock et flux. Nos estimations des formes réduites de ces modèles effectuées sur le taux de change du dollar canadien en termes du dollar américain sur la période 1971-1980, nous indiquent que seul le modèle stock-flux est acceptable. Nous proposons une équation du taux de change qui s'avère être conforme à la théorie, stable et supérieure aux équations concurrentes, selon des tests d'hypothèses non-emboîtés.
1- Introduction

In the 1970s flexible exchange rates became predominant in foreign exchange markets for the main currencies. The accumulating experience generated an impressive research effort as theorists conceived new explanations and practionners tested their relevance. The research program aimed at determining the essential factors which would explain the volatility and the level of the exchange rate. Looking back at the main contributions of the last decade¹ we observe a rapid evolution from the quiet certainty of the bed-rock neoclassical monetary approach², which links the exchange rate to the evolving conditions in the domestic and foreign money markets, to the modest proposals of the rational expectations-efficient market models, which conclude that the exchange rate is essentially unpredictable (its movements being explained by unforeseen, and unforeseeable, events or "news" impacting on the foreign exchange markets)³.

The neoclassical purity of the monetary approach [flexible prices, and full employment, market equilibrium, absence of intervention in the foreign exchange markets or of sterilization in fixed rate regimes, attention focussed solely on money market conditions and the overall balance of payments position or official reserves] was on the one hand weakened by price rigidities in the goods markets and risk premia in the capital markets, and on the other, strengthened by the rational expectations approach [which brings the long run monetary
model into a short run perspective\textsuperscript{4}. Furthermore the narrow focus of the monetary approach, concentrating on money market conditions and their determinants, has been broadened to consideration of other asset markets (the portfolio approach\textsuperscript{5}), and reconsideration of the balance of payments components as the current account reappears\textsuperscript{6}.

In the context of the aforementioned literature, our purpose is twofold. First, we evaluate, in the context of the Canadian-U.S. dollar exchange rate in the 1970s\textsuperscript{7}, the relative merits of the three basic theories of exchange rate determination: (1) the strict monetary model, (2) the monetary model incorporating price rigidities in the goods markets, (3) the stock-flow model which considers imperfect capital markets and price rigidities. Thus the models proposed by Bilson (1978), Frenkel (1976), Dornbusch (1976), Frankel (1979) and Driskill (1981) are tested.

Secondly, we propose an equation, in the stock-flow tradition, which focuses on the basic macro variables (prices, interest rates, money stocks and expectations). Our empirical results indicate that this equation has high explanatory power, is stable and has significant determinants. Furthermore, the empirical results show that the competing 'empirical' models which are examined are either rejected by the data or dominated by our equation in non-nested tests. Thus our results indicate that the pessimism with regards to the empirical relevance of (reduced form) exchange rate models in the literature, as stated by Meese and Rogoff (1981, 1982) and Backus (1982), may be premature, at least for the Canadian-U.S. dollar exchange rate.
The outline of the paper is as follows: in section 2, a general model incorporating all investigated variations is presented, solved and the various reduced form equations of the exchange rate are established. In section 3, we evaluate the results. Conclusions follow in section 4.

2- Exchange rate models

We examine three basic models of exchange rate determination. The first is the basic monetary model which assumes flexible prices and continuous market clearing in a world market. Fully integrated national goods markets in the international economy imply the law of one price or purchasing power parity and perfect capital mobility is assumed as national assets are perfect substitutes. The exchange rate is the relative value of the domestic and foreign currencies which are established in the respective money markets.

The second view is in the Keynesian world of fix-prices. The lack of confirmation of purchasing power parity in the short (and medium) run was a stimulant to the development of this approach. The main assumption is that asset markets adjust rather quickly (instantaneously) but that sticky price phenomena are pervasive in the goods (and labor) markets. This leads to greater variability in the exchange rate with short-run "overshooting" of its equilibrium level.

The third approach generalizes the second by relaxing the perfect capital mobility assumption. The exchange rate responds to portfolio allocation decisions relating to internationally traded assets and also to trade flows. This framework corresponds to a stock-flow model of the exchange rate.
In each case, the structural models are reduced to a single equation for the exchange rate which will be the basis for empirical evaluation. Notation and structural equations for all models are grouped in Table 1. By tradition and in order to simplify the exposition, we assume that behavioral and adjustment parameters are identical in both countries.

2.1. The monetary approach

The exchange rate is viewed as the relative price of the two currencies. The purchasing power parity condition is assumed to hold by equation [4]. The relative price levels ($\tilde{p}$) are given by the money market equilibrium conditions in equation [1] where money supplies ($m, m'$), outputs ($y, y'$) and interest rates ($r, r'$) are exogenous. The right-hand-side of [1] corresponds to the familiar money demand equation in semi-logarithmic form, expressed in relative terms with identical income elasticities and interest rate semi-elasticities for both countries. The relative price levels ($\tilde{p}$) are solved out in [1] and substituted into [4]. The exchange rate equation is thus:

\[ e = \beta_{11} \tilde{m} + \beta_{12} \tilde{y} + \beta_{13} \tilde{r} \]

where $\beta_{11} = 1$, $\beta_{12} = -\alpha_1$ $\beta_{13} = \alpha_2$

The model predicts that relatively expansionary domestic monetary policies (as measured by the money stock) lead to depreciation of the currency, relatively higher domestic growth to appreciation and higher domestic interest rates depreciate the currency. Equation [12] corresponds to Bilson's (1978) model and, in an inflationary context, to Frenkel's (1976) equation.
<table>
<thead>
<tr>
<th>Equations</th>
<th>Notation and definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money and goods markets:</td>
<td>m = log of stock of money</td>
</tr>
<tr>
<td>[1] ( \dot{m} = \dot{p} + \alpha_1 \dot{y} - \alpha_2 \dot{r} )</td>
<td>y = log of output</td>
</tr>
<tr>
<td>[2] ( \ddot{p}(+1) - \ddot{p} = \delta \frac{1}{\gamma} [\ddot{d} - \ddot{y}] + \gamma \ddot{\pi} )</td>
<td>p = log of the price level</td>
</tr>
<tr>
<td>[3] ( \ddot{d} = \alpha_3 (e - \ddot{p}) + \alpha_4 \ddot{y} - \alpha_5 (\ddot{r} - \gamma \ddot{\pi}) )</td>
<td>d = log of aggregate demand</td>
</tr>
<tr>
<td>Monetary:</td>
<td>r = interest rate</td>
</tr>
<tr>
<td>[4] ( e = \ddot{p} )</td>
<td>e = log of the exchange rate (domestic price of foreign currency)</td>
</tr>
<tr>
<td>Sticky-prices:</td>
<td>( \pi = ) expected rate of inflation (in the long run)</td>
</tr>
<tr>
<td>[5] ( x = \ddot{r} )</td>
<td>x = expected rate of depreciation : ( \delta (+1) - e )</td>
</tr>
<tr>
<td>[6] ( x = \delta_2 (e_e - e) + \gamma \ddot{\pi} )</td>
<td>w = log of real wealth</td>
</tr>
<tr>
<td>Stock-flow:</td>
<td>F = net stock of foreign assets</td>
</tr>
<tr>
<td>[7] ( F = \alpha_6 (x - \ddot{r}) + \alpha_7 \ddot{w} )</td>
<td>T = trade balance</td>
</tr>
<tr>
<td>[8] ( T = \alpha_8 (e - \ddot{p}) - \alpha_9 \ddot{y} )</td>
<td>A = other net autonomous flows.</td>
</tr>
<tr>
<td>[9] ( F - F(-1) - T - A = 0 )</td>
<td>The following conventions are used, for any variable Z</td>
</tr>
<tr>
<td>Stochastic assumptions:</td>
<td>Z : refers to the domestic variable</td>
</tr>
<tr>
<td>[10] ( \ddot{m} = \ddot{m}(-1) + \gamma \ddot{\pi} + v )</td>
<td>Z' : is the foreign corresponding variable</td>
</tr>
<tr>
<td>( \ddot{\pi} = \ddot{\pi}(-1) + v_2 )</td>
<td>( \ddot{Z} = Z - Z', ) is the difference between domestic and foreign values</td>
</tr>
<tr>
<td>[11] ( \ddot{y} = \ddot{y}(-1) + v_3 )</td>
<td>Z* : refers to an equilibrium value</td>
</tr>
<tr>
<td>where ( v_1, v_2 ) and ( v_3 ) are zero-mean,</td>
<td>( \ddot{Z} ) : is the expected value of Z</td>
</tr>
<tr>
<td>serially uncorrelated random variables.</td>
<td>Note that time subscripts are omitted. For time periods other than the contemporaneous one, the notation is as follows:</td>
</tr>
<tr>
<td></td>
<td>( Z(+k) = Z_{t+k} ); ( Z(-k) = Z_{t-k} )</td>
</tr>
<tr>
<td></td>
<td>The ( \alpha ) parameters (( \alpha_i, i=1,...9 )) are behavioral.</td>
</tr>
<tr>
<td></td>
<td>The ( \delta )'s (( \delta_j; j=1,2 )) are adjustment parameters.</td>
</tr>
<tr>
<td></td>
<td>All equations are specified such that all the parameters (( \alpha, \delta )) are assumed to be positive. Note that ( \gamma ) is a</td>
</tr>
</tbody>
</table>
2.2. The sticky-price asset market model

Prices adjust according to an expectations augmented Phillips curve [2]. Aggregate demands respond to the deviations from purchasing power parity or the real exchange rate, output or real income, and real interest rates as specified in equation [3]. The one period expected depreciation of the currency is a function of the gap between the current level of the exchange rate (e) and its long run equilibrium value (e*) and of the relative domestic and foreign long run expected rates of inflation. This is given by equation [6]. If foreign and domestic assets are perfect substitutes (no risk premia) and there is perfect capital mobility then uncovered interest rate parity holds and the expected depreciation is equal to the interest rate difference in equation [5]. Note that in equilibrium (e = e*) this assures real interest rate parity (\(r_\pi - \pi = 0\)). Combining [5] and [6] yields the short-run exchange rate as a function of its long run equilibrium level and relative real interest rates:

\[
[13] \quad e = e_\star - \delta_2^{-1}(\tilde{r} - \gamma\tilde{\pi}).
\]

The equilibrium exchange rate is obtained by solving the goods market equations [2], [3] when they clear (\(\tilde{d} = \tilde{y}\)) where prices are given by the money markets [1]:

\[
[14] \quad e_\star = \tilde{m}_\star + (\alpha_3^{-1}(1 - \alpha_4) - \alpha_1)\tilde{y}_\star + \alpha_2\gamma\tilde{\pi}
\]

Note that in [14] we have used the real interest rate parity condition (\(\tilde{r}_\star = \tilde{\pi}\)). The equilibrium exchange rate depends on expectations with respect to future money supplies, outputs, and inflation levels. In order
to obtain an empirically relevant model additional assumptions pertaining to the stochastic processes which govern the evolution of the exogenous variables $\bar{m}$ and $\bar{y}$ are required. Following Frankel (1979) we assume that $\bar{y}$ follows a random walk [equation 11] and that $\bar{m}$ follows a random walk around a trend ($\bar{\bar{m}}$) which also follows a random walk [equation 10]. Under these assumptions $\bar{m}_*$ and $\bar{y}_*$ may be substituted by their current levels $\bar{m}$ and $\bar{y}$. Using this result, [14] and [13] the exchange rate equation is:

$$[15] \quad e = \beta_{21} \bar{m} + \beta_{22} \bar{y} + \beta_{23} \bar{\bar{m}} + \beta_{24} \bar{\bar{y}}$$

where $\beta_{21} = 1$, $\beta_{22} = \alpha_3^{-1} (1 - \alpha_4) - \alpha_1$; $\beta_{23} = -\delta_2^{-1}$; $\beta_{24} = \alpha_2 + \delta_2^{-1}$

This model predicts that relatively expansionary domestic monetary policies, and higher expected domestic inflation lead to a depreciation of the currency. However relatively higher domestic interest rates and (presumably) output lead to an appreciation. Equation [15] is Frankel’s (1979) real interest rate differential model. If we assume that $\gamma = 0$, that is inflation is not important (or $\bar{\bar{m}} = 0$, expected inflation is the same in both countries), then [15] corresponds to Dornbusch’s (1976) model. Note that in these models the expected depreciation adjustment parameter $\delta_2$ can be shown to depend in a certain way on the other parameters of the model (rational expectations being at the core of the model).

Driskill and Sheffrin (1981) pointed out that [15] is not a true reduced form as interest rates ($\bar{\bar{r}}$) are the endogenous variables in the money markets [1]. Note that prices ($\bar{\bar{p}}$) are exogenous in this model.
as well as $\bar{m}$, $\bar{y}$ and $\bar{p}$. Driskill and Shefrin use [1] to solve out for $\bar{r}$ and substitute in [15] to obtain:

$$e = \beta_{31} \bar{m} + \beta_{32} \bar{y} + \beta_{33} \bar{p} + \beta_{34} \bar{y} \bar{p}$$

where $\beta_{31} = 1 - \beta_{23} \alpha_{2}^{-1}$; $\beta_{32} = \beta_{22} + \beta_{23} \alpha_{1} \alpha_{2}^{-1}$; $\beta_{33} = \beta_{23} \alpha_{2}^{-1}$; $\beta_{34} = \beta_{24}$

The equation predicts that $\beta_{31} > 1$, $\beta_{33} < 0$ and $\beta_{31} + \beta_{33} = 1$. Note that $\beta_{31} > 1$ implies overshooting of the exchange rate in the short-run in relation to its equilibrium value in response to monetary shocks (as $\beta_{23} = -\delta_{2}^{-1}$ and $\delta_{2} < \infty$).

Setting $\gamma = 0$, Driskill (1981) suggests an alternative solution to Dornbusch's model. Thus relative money supplies follow a simple random walk [equation 10] and Driskill assumes that all relative income changes are transitory ($\bar{y} = 0$).

Thus equation [14] reduces to:

$$e = \bar{m} = \bar{m}$$

Then equations [1], [2] and [3] are solved for $\bar{p}(+1)$:

$$\bar{p}(+1) = [1 - \delta_{1} \alpha_{5} \alpha_{2}^{-1} - \delta_{2} \alpha_{3}] \bar{p} + \{\delta_{1} [(\alpha_{4} - 1) - \alpha_{5} \alpha_{1} \alpha_{2}^{-1}]\} \bar{y}$$

$$+ [\delta_{1} \alpha_{5} \alpha_{2}^{-1}] \bar{m} + \delta_{1} \alpha_{3} e$$

Equation [18] is lagged one period and [1], [6] and [17] are used to solve for the exchange rate:
\[ e = \beta_{41} \hat{m} + \beta_{42} \hat{y} + \beta_{43} e(-1) + \beta_{44} \hat{m}(-1) + \beta_{45} \hat{p}(-1) + \beta_{46} \hat{y}(-1) \]

where, \( \beta_{41} = 1 + (\delta \alpha)^{-1} \); \( \beta_{42} = -\alpha \left( \alpha \delta \right)^{-1} \); \( \beta_{43} = -\alpha \delta \left( \delta \alpha \right)^{-1} \)

\( \beta_{44} = -\delta_1 \alpha_5 (\delta \alpha)^{-1} \); \( \beta_{45} = -\left( 1 - \delta_1 \alpha_5 \alpha_2^{1/2} - \delta_1 \alpha_3 \right) (\delta \alpha)^{-1} \)

\( \beta_{46} = + \delta_1 \left[ (1 - \alpha_4) + \alpha_1 \alpha_5 \alpha_2^{1/2} \right] (\delta \alpha)^{-1} \)

This equation predicts: \( \beta_{41} > 1, \beta_{42} < 0, \beta_{43} < 0, \beta_{44} < 0, \beta_{45} < 0, \beta_{46} < 0 \) and \( \beta_{41} + \beta_{43} + \beta_{44} + \beta_{45} = 1 \). Note that \( \beta_{41} = \beta_{31} \).

### 2.3. Stock-flow models

The monetary and asset market models neglect trade-flows and the traditional elasticities approach. The stock-flow model reintroduces these elements and provides a general synthesis of various theories of exchange rate determination. The major contributors in this area are Branson (1976), Niehans (1977), Henderson (1980) and Driskill (1981).

It is a generalization of the sticky-price asset market model which relaxes the assumption of perfect capital mobility, as internationally traded assets are imperfect substitutes. A major theoretical advantage is that it provides a rich menu of short-run exchange rate dynamics.

In this approach, equation [5] is replaced by the balance of payments equation [9] where \( F \) represents the net stock of foreign assets, \( T \) corresponds to net trade flows and \( A \), other net autonomous flows which Driskill assumes to be constant. The net demand for foreign assets (equation [7]) is specified as a linear function of the expected net yield \( (x - \bar{r}) \) and real relative wealth \( (\hat{w}) \) where presumably an increase in wealth has a
positive effect on the demand for foreign assets. As in Driskill (1981),
the trade balance, equation [8], is a linear function of the log of rela-
tive prices and the log of relative real incomes. Furthermore we will
neglect the inflation expectations terms in the model (i.e., $\gamma = 0$).

Assuming that $\alpha_7 = 0$, $\gamma = 0$, and that all relative income
changes are transitory (i.e. $\tilde{\gamma}_w = 0$), equations [1], [2], [3], [6], [7],
[8], [9] and [10] reproduce Driskill's model. The reduced-form exchange
rate equation is identical to [19]; however its interpretation is very
different:

\[ e = \beta_{51} \bar{m} + \beta_{52} \bar{y} + \beta_{53} \bar{e}(-1) + \beta_{54} \bar{m}(-1) + \beta_{55} \bar{p}(-1) + \beta_{56} \bar{y}(-1) \]

where, $\beta_{50} = [\alpha \delta + \alpha]^{-1} \beta_{51} = \beta_{50} [\alpha \delta + \alpha]^{-1}$

$\beta_{52} = \beta_{50} (\alpha - \alpha \alpha^{-1})$; $\beta_{53} = \beta_{50} [\alpha \delta + \alpha \delta (\alpha - \alpha \alpha^{-1})]$

$\beta_{54} = - \beta_{50} (\alpha \delta + \alpha \delta^{-1} - \delta \alpha \alpha^{-1} (\alpha - \alpha \alpha^{-1}))$

$\beta_{55} = \beta_{50} [(\delta + \alpha \alpha^{-1}) (\alpha \alpha^{-1}) + \alpha (1 - \delta + \alpha \alpha^{-1} - \delta \alpha)]$

$\beta_{56} = \beta_{50} [(\delta + \alpha - 1) - \delta \alpha \alpha^{-1}] (\alpha - \alpha \alpha^{-1}) + \alpha \alpha^{-1} \alpha ]$

Our priors on the coefficients are as follows: $\beta_{51} > 0$, $\beta_{52} > 0$,
$\beta_{53} < 1$, $\beta_{54} < 0$, $\beta_{55} > 0$, $\beta_{56} < 0$ and $\beta_{51} + \beta_{53} + \beta_{54} + \beta_{55} = 1$.

We propose the following variant to Driskill's model: wealth terms
are reintroduced in the net demand for foreign assets equation as in
[7], $\bar{y}$ is assumed to follow a random walk as in [11]. We solve equa-
tions [1], [2] and [3] for the equilibrium exchange rate $e_*$ under the
assumption that $\bar{\pi} = 0$:
\[ [21] \quad e_* = \tilde{m}_* + [\alpha_3^{-1}(1 - \alpha_4) - \alpha_1] \tilde{y}_* + (\alpha_2 + \alpha_3^{-1}\alpha_5) \tilde{r}_* \]

Then we solve equations [6], [7], [8] and [9] for \( e \)

\[ [22] \quad e = (\alpha_8 + \alpha_6 \delta_2)^{-1} [\alpha_6 \delta_2 e_* + \alpha_9 \tilde{y} + \alpha_8 \tilde{p} - \alpha_6 \tilde{r} + \alpha_7 \tilde{w} - F(-1) - A] \]

Assuming that \( \tilde{y}_* = \tilde{y}, \tilde{m}_* = \tilde{m}, \tilde{r}_* = 0 \) and \( A = \text{constant} \) and replacing \( e_* \) in [22] by its value in [21], we get

\[ [23] \quad e = \beta_{61} \tilde{m} + \beta_{62} \tilde{y} + \beta_{63} \tilde{r} + \beta_{64} \tilde{p} + \beta_{65} \tilde{w} + \beta_{66} F(-1) + k \]

where \( \beta_{60} = (\alpha_8 + \alpha_6 \delta_2)^{-1} \), \( \beta_{61} = \beta_{60} \delta_2 \)

\[ \beta_{62} = \beta_{61} [\alpha_3^{-1}(1 - \alpha_4) - \alpha_1 + \alpha_9 \delta_2^{-1}] \]

\[ \beta_{63} = -\beta_{60} \alpha \]

\[ \beta_{64} = \beta_{60} \alpha \]

\[ \beta_{65} = \beta_{60} \alpha \]

\[ \beta_{66} = -\beta_{60} \]

\[ k = -\beta_{60} A \]

This equation predicts: \( 0 < \beta_{61} < 1, \beta_{62} > 0, \beta_{63} < 0, 0 < \beta_{64} < 1, \beta_{65} > 0, \beta_{66} < 0 \) and \( \beta_{61} + \beta_{64} = 1 \). This model suggests "undershooting" as \( \beta_{61} < 1 \). The domestic currency will depreciate when domestic monetary policy is relatively expansionary (\( \beta_{61} > 0 \)), domestic prices increase relatively (\( \beta_{64} > 0 \)) and domestic wealth increases relatively (\( \beta_{65} > 0 \)) leading to a greater demand for foreign assets. Relatively higher domestic interest rates lead to an appreciation of the currency (\( \beta_{63} < 0 \)
through capital flows. The effect of relative incomes on the exchange rate is ambiguous a priori. Inspection of $\beta_{62}$ reveals the conflicting predictions of the Keynesian (absorption) approach where higher income leads to import leakages ($\alpha_9 + \alpha_3^{-1}(1 - \alpha_4)$) and currency depreciation ($\beta_{61}^{-1}[\alpha_3^{-1}(1 - \alpha_4) + \alpha_9 (\alpha_6 \delta_2)^{-1}] > 0$) and the asset market (and monetary) theory which predicts the opposite effect ($-\beta_{61} \alpha_1 < 0$) notably as the demand for money ($\alpha_1$) increases. Hence, for instance, the lower the relative income effect on the trade balance ($\alpha_3$), the greater the sensitivity of the demand for internationally traded assets with respect to interest rates ($\alpha_6$) and the quicker the adjustment of exchange rate expectations to the gap between the exchange rate and its equilibrium value ($\delta_2$), then the greater the probability that a relative increase in domestic income will lead to an appreciation of the currency ($\beta_{62} < 0$).

3- Empirical results

The recent floating exchange rate period began in Canada on June 1, 1970. Allowing for a short transition period our quarterly data covers the period 1971.I to 1980.IV. The money stock definitions are M1B, interest rates are corporate paper (short) and corporate bond rates (long), output is G.N.P. and prices are the implicit G.N.P. deflators. Nominal wealth is nominal income discounted by a long term government interest rate averaged over the last 8 quarters. (All variables except interest rates are in logs). Details and sources are given in the appendix. All estimated equations include a constant term which is not reported.
3.1. The monetary approach

As can be seen from table 2, the monetary approach [equations [12], [12']] is not acceptable. Equation [12] is Bilson's (1978) model with short term interest rates [$\tilde{r} = \tilde{r}_s$] and equation [12'] is Frenkel's (1976) variant where expected inflation is proxied by long term interest rates [$\tilde{\pi} = \tilde{r}_L$] as in Frankel (1979). The OLS estimates reveal high serial correlation of the residuals (symptomatic of an error in specification). Once corrected by the Cochrane-Orcutt procedure the exchange rate appears to be a random walk: the estimated first order autocorrelation coefficient is very close to 1 (at .963 and .960) in both equations and the other coefficients are not significantly different from zero at the conventional 5% level.

3.2. The sticky-price asset model

The asset market model with sticky prices is also questionable. Consider the evidence. First, if we discard inflation expectations ($\gamma = 0$), equation [12] with $\beta_{1.3} < 0$ corresponds to Dornbusch's (1976) model as simplified by Frankel. The signs of the estimated coefficients correspond to a priori expectations but, as we stated, once corrected for autocorrelation, there is little explanatory power in the model. One may object that there is a simultaneity problem as $\tilde{r}$ is an endogenous variable of the structural model. However Driskill's true reduced form of the model, equation [19] in table 3, reveals that Dornbusch's model is not validated. Though the overall statistics are good, the estimates of $\beta_{1.1}, \beta_{1.3}, \beta_{4.5}$ and $\beta_{4.6}$ contradict the theory.
<table>
<thead>
<tr>
<th></th>
<th>[12]</th>
<th>[12']</th>
<th>[15]</th>
<th>[16]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td></td>
<td>$\beta_{11} = 1$</td>
<td>$\beta_{11} = 1$</td>
<td>$\beta_{21} = 1$</td>
<td>$\beta_{31} &gt; 1$</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>1.238</td>
<td>1.00</td>
<td>1.196</td>
<td>1.00</td>
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<td></td>
<td>(0.311)</td>
<td>-</td>
<td>(0.278)</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-1.208</td>
<td>-0.681</td>
<td>-0.757</td>
<td>-0.768</td>
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<td></td>
<td>(0.607)</td>
<td>(0.391)</td>
<td>(0.536)</td>
<td>(0.381)</td>
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<td>$\bar{r}$</td>
<td>$\beta_{13} &gt; 0$</td>
<td>$\beta_{13} &gt; 0$</td>
<td>$\beta_{23} &lt; 0$</td>
<td>$\beta_{33} &lt; 0$</td>
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<tr>
<td>(p in (16))</td>
<td>-4.498</td>
<td>-0.424</td>
<td>-38.38</td>
<td>-2.249</td>
</tr>
<tr>
<td></td>
<td>(3.298)</td>
<td>(1.11)</td>
<td>(11.46)</td>
<td>(4.826)</td>
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<tr>
<td>$\pi$</td>
<td>$\beta_{24} &gt; 0$</td>
<td>$\beta_{24} &gt; 0$</td>
<td>$\beta_{34} &gt; 0$</td>
<td>$\beta_{34} &gt; 0$</td>
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<td>LnL</td>
<td>54.0</td>
<td>89.6</td>
<td>58.4</td>
<td>89.6</td>
</tr>
<tr>
<td>SSRX10²</td>
<td>14.15</td>
<td>2.13</td>
<td>11.34</td>
<td>2.127</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.283</td>
<td>0.893</td>
<td>0.427</td>
<td>0.893</td>
</tr>
<tr>
<td>DW</td>
<td>0.29</td>
<td>2.10</td>
<td>0.58</td>
<td>2.12</td>
</tr>
<tr>
<td>RH0</td>
<td>-</td>
<td>0.963</td>
<td>-</td>
<td>0.960</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.045)</td>
<td></td>
<td>(0.045)</td>
</tr>
</tbody>
</table>


(a) Ordinary least squares; (b) OLS with Cochrane-Orcutt correction and theoretical constraints imposed
(c) Instrumental variables using Fair's method. Instruments include all lagged endogenous and exogenous variables to insure consistency while correcting for first order serial correlation, $\pi, \bar{m}, \gamma$ and the difference between Canadian and U.S. (3 month) Treasury Bill rates.
<table>
<thead>
<tr>
<th>$\beta_{41} &gt; 1 \ [\bar{m}]$</th>
<th>$\beta_{51} &gt; 0$</th>
<th>$\beta_{42} &lt; 0 \ [\bar{y}]$</th>
<th>$\beta_{52} &lt; 0$</th>
<th>$\beta_{43} &lt; 0 \ [e(-1)]$</th>
<th>$\beta_{53} &lt; 1$</th>
<th>$\beta_{44} &lt; 0 \ [\bar{m}(-1)]$</th>
<th>$\beta_{54} &lt; 0$</th>
<th>$\beta_{45} &lt; 0 \ [\bar{p}(-1)]$</th>
<th>$\beta_{55} &gt; 0$</th>
<th>$\beta_{46} &lt; 0 \ [\bar{y}(-1)]$</th>
<th>$\beta_{56} &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.290 (0.169)</td>
<td>-0.631 (0.277)</td>
<td>0.777 (0.084)</td>
<td>-0.307 (0.156)</td>
<td>0.363 (0.118)</td>
<td>0.200 (0.315)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$[23a]$</th>
<th>$[23b]$</th>
<th>$[23c]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \beta_{61} &lt; 1 \ [\bar{m}]$</td>
<td>0.255 (0.085)</td>
<td>0.229 (0.099)</td>
</tr>
<tr>
<td>$\beta_{62} &lt; 0 \ [\bar{y}]$</td>
<td>-0.967 (0.133)</td>
<td>-1.037 (0.121)</td>
</tr>
<tr>
<td>$\beta_{63} &lt; 0 \ [\bar{r}]$</td>
<td>-7.509 (3.218)</td>
<td>-9.476 (3.158)</td>
</tr>
<tr>
<td>$0 &lt; \beta_{64} &lt; 1 \ [\bar{p}]$</td>
<td>0.539 (0.075)</td>
<td>0.771 -</td>
</tr>
<tr>
<td>$\beta_{65} &gt; 0 \ [\bar{w}]$</td>
<td>1.061 (0.094)</td>
<td>1.180 (0.128)</td>
</tr>
<tr>
<td>$\beta_{67} &gt; 0 \ [\bar{w}']$</td>
<td>0.089 (0.057)</td>
<td>0.092 (0.060)</td>
</tr>
<tr>
<td>$\beta_{68} &gt; 0 \ [\bar{y}']$</td>
<td>-0.142 (0.054)</td>
<td>-0.143 (0.055)</td>
</tr>
</tbody>
</table>

LnL 105.6 115.8 113.8 103.9
SSRX$10^2$ 0.981 0.607 0.599 0.654
$R^2$ 0.944 0.967 0.947 0.938
DW -0.81 (h) 1.727 1.80 1.97

Standard errors in (). a: OLS estimates, additivity constraint not imposed. b: OLS with $\beta_{61} + \beta_{64} = 1$ imposed. c: Instrumental variables estimates with $\beta_{61} + \beta_{64} = 1$. Instruments include all exogenous variables in the structural model.
Secondly, if we consider an inflationary environment as Frankel does the relevant equations are [15] and [16]. The OLS estimate [15a] is dismal with a high SSR and low DW statistic and only two significant coefficients, one of which is of the wrong sign. The instrumental variable approach [15c] only redeems the output variable. The true reduced form [16] fairs slightly better but again three coefficients ($\beta_{31}$, $\beta_{33}$, $\beta_{34}$) do not correspond to prior expectations and first-order serial correlation is very high.

3.3. The stock-flow models

The stock-flow models cannot be dismissed. In Driskill's version (table 3, equation 20), OLS parameter estimates are consistent with the theory and the F statistic with respect to the additivity constraint ($\beta_{51} + \beta_{53} + \beta_{54} + \beta_{55} = 1$) indicates that the constraints cannot be rejected ($F = 1.49 < F_{0.05}(1,33) = 4.14$). Moreover, the sum of squared residuals is much smaller than in previous equations. However the standard errors of the money stock coefficients are slightly large.

Our version of the stock-flow model (table 3, equations 23a, b, c) is even better: In all three equations, the sum of squared residuals is much smaller and the Durbin-Watson statistic does not indicate first-order autocorrelation.

Before commenting on each of the three results separately, it must be noted that the results shown do not include the lagged net stock of foreign asset variable ($F(-1)$) which appears in the theoretical model. In fact, we included this variable in our regressions,
using the data (net private stocks) published by Backus (1982) and calculated with the methodology presented by Branson, Halttunen and Masson (1977). But the results with this variable were not changed significantly from the results presented here and the t statistic on the variable was only .4. Hence this variable was omitted. We note that the result on this variable may be due in part to measurement problems.

Equation 23a gives interesting results but the additivity constraint \( \beta_{61} + \beta_{64} = 1 \) was (barely) rejected in this version of the model. One problem of this specification, as pointed out by Hayes and Stone (1981), might be that it constrains all coefficients on corresponding domestic and foreign explanatory variables to be equal in size. We have assumed that coefficients \( \alpha_7 \) and \( \alpha_9 \) are different across countries. Thus we add coefficients for foreign income \((y')\) and wealth \((w')\) to the specification\(^{11}\). With this specification, the results are very good and, the F statistic with respect to the additivity constraint \( \beta_{61} + \beta_{64} = 1 \) indicates acceptance \(F = .25 < F_{.05} (1,32) = 4.15\) such that the constraint is imposed in 23b as presented.

There remains a question of the endogeneity of the interest rate (note that in the estimation we used long term interest rates on a quarterly basis, \( \tilde{r} = \tilde{r}_q \)). Theoretically, this will bias our parameter estimates. Estimation by the instrumental variable technique [equation 21c] reveals that endogeneity is not a serious problem as the estimated coefficients are quite similar except for the interest rates. Dufour (1980a) suggests an exact test for small samples by which we may evaluate if the included endogenous variable is correlated with the
error term. The computed statistic indicates that simultaneity is not a problem such that \( \hat{r} \) may be considered (statistically) as an exogenous variable.

3.4. Non-nested hypothesis tests

The next question is which is the best model: Driskill's or our version. Non-nested hypothesis tests may be used to rank the two equations. We thought it might be interesting to compare these models to two other simple models of the exchange rate. The first which is linked to the efficient markets literature suggests that the exchange rate might follow a random walk. Meese and Rogoff (1981) found it to be the best equation in post sample forecasting for the U.S. in the 1970's. The second, based on a weak-form test of market efficiency, examines if the forward rate \( (f) \) is an unbiased predictor of the next period exchange rate. Both models predict that the constant term should be zero and that the coefficient on the exogenous variable would be equal to one. Based on the results, it would be absurd to reject these models:

\[
\begin{align*}
[24] & \quad e = 0.0041 + 1.002 \quad e(-1) \\
& \quad (0.0037) \quad (0.045)
\end{align*}
\]

\( \bar{R}^2 = 0.926 \quad \text{SSRX10}^2 = 1.493 \quad \text{Durbin h} = 0.57, \text{standard errors in ( )} \)

\[
\begin{align*}
[25] & \quad e = 0.0029 + 1.015 \quad f(-1) \\
& \quad (0.0037) \quad (0.045)
\end{align*}
\]

\( \bar{R}^2 = 0.928, \text{D.W.} = 1.79, \text{SSRX10}^2 = 1.475, \text{standard errors in ( )}. \)

The non nested test is the C test of Davidson and McKinnon (1981).
We wish to compare two models, which seek to explain the same variable $y$

$$M_1 : y = h_1(\beta_1) + u_1$$
$$M_2 : y = h_2(\beta_2) + u_2$$

where observations on the explanatory variables are implicit in the functions $h_1$ and $h_2$ and $\beta_1$, $\beta_2$ are the parameters to be estimated. If $\hat{h}_1$ and $\hat{h}_2$ are the fitted values of $M_1$ and $M_2$, the test consists in estimating the following regression:

$$y - \hat{h}_1 = \alpha(\hat{h}_2 - \hat{h}_1) + v$$

if $\alpha$ is significantly different from zero then the maintained hypothesis that $M_1$ is the correct model is not true. Basically the test consist in nesting the two models in a compound model:

$$M_c = (1 - \alpha)h_1(\beta_1) + \alpha h_2(\beta_2) + u_3$$

thus if $\alpha = 0$ we get $M_1$ and if $\alpha = 1$ we get $M_2$. Table 4 reports the results$^{13}$. Our version of the stock-flow model beats all comers.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$ is the stock-flow model $^{[23]}$</td>
<td>0.130</td>
<td>0.128</td>
<td>0.111</td>
</tr>
<tr>
<td></td>
<td>(0.873)</td>
<td>(1.21)</td>
<td>(1.02)</td>
</tr>
<tr>
<td>$M_2$ is the stock-flow model $^{[23]}$</td>
<td>0.869</td>
<td>0.871</td>
<td>0.888</td>
</tr>
<tr>
<td></td>
<td>(5.80)</td>
<td>(8.31)</td>
<td>(8.15)</td>
</tr>
</tbody>
</table>

$t$ statistics in ()
3.5. Stability tests

Finally we wanted to see if our equation [23] was stable over the sample period, given the fact that monetary policies were modified and that there were notable instances of intervention in the foreign exchange market in the period\(^{14}\). Two tests were conducted. First a simple Chow-test with an even split of the period into two subperiods (1971I - 1975IV) and (1976I - 1980IV). The split corresponds to a change in the Canadian monetary policy where concern with interest rates was replaced by a money stock target policy. The computed F-statistic for the Chow (1960) test rejects instability \((F = 1.39 < F_{0.05} (7,26) = 2.39)\).

Secondly we tested for instability using a method suggested by Dufour (1980, 1981). In this test a dummy variable for each observation in the period over which stability is tested is included in the equation. When certain auxiliary variables are significant this test detects outliers or extraordinary circumstances in the economy or the exchange rate market and if the coefficients on the dummy variables are jointly significant, the test indicates structural instability. Neither forms of instability are present: t-statistics for the dummy variables (starting in 1978I up to 1980IV) range from \(-0.019\) for 1980II to \(1.62\) for 1978I and the F statistic rejects joint significance of the variables \((F = 0.062 < F_{0.05} (12,21) = 2.25)\). Thus our equation cannot be shown to be unstable on the basis of these tests.
4. Conclusion

Our investigation of the Canadian-U.S. exchange rate has enabled to establish a certain number of results. First, with respect to the various exchange rate theories (monetary, sticky-prices asset markets, stock-flow) we conclude that the most general approach which takes into consideration capital markets, trade flows and rigidities as well as the money markets is clearly the best model of exchange rate determination. This suggests that theories which oversimplify by excluding relevant phenomena may not have much to say about the real world and that their policy prescriptions may be viewed with skepticism.

Secondly, we have discovered a 'reduced form' single equation for the exchange which is consistent with the stock-flow theory, acceptable by standard statistical conventions, stable over the sample period and which dominates the otherwise admissible alternatives. We would therefore reject the pessimism in regarding the empirical relevance of exchange rate theories which certain researchers (e.g. Meese and Rogoff, Backus) have expressed.

Thirdly, our equation shows that the exchange rate is responsive to nominal and real shocks in the short-run and that relative money stocks, output interest rates, prices and wealth all contribute to its determination. Our results do not indicate overshooting ($\beta_{61} < 1$). Though the relative income effect gives credence to the monetary approach, the interest rates effect is compatible with the Keynesian interpretation. The fact is that both approaches are limiting views of the general stock-flow model.
Notes

1 Various surveys of the theoretical and empirical literature are provided by Bilson (1979), Dornbusch (1980), Frankel (1982), Isard (1978), Kreinin and Officer (1978), Mussa (1979) and Schadler (1977).

2 Also termed global monetarism. In this approach integrated international goods and capital markets are the norm and the flexible price - full employment - equilibrium - in-all-markets paradigm is adopted. Its fountainhead is the University of Chicago. Main references are Frenkel [1976], Frenkel and Johnson [1978], Bilson [1978]. In the reduced form version of this approach all shocks to the exchange rate are channelled through the money markets. One result is that relatively inflationary monetary policies lead to a depreciation of the national currency.

3 See for example Levich (1978, 1979) and Longworth (1981). The latter examines the efficiency of the foreign exchange market for the Canadian-U.S. dollar exchange rate. Examination of the data leads to a rejection of the efficient markets hypothesis (under the no risk premium assumption) for various subperiods in the 1970s. The "news" view is discussed by Dornbusch (1980), Frenkel (1981b) and Mussa (1982).

4 With respect to introducing price rigidities, Dornbusch (1976) is the seminal article. Frankel (1979, 1981) extends his model to an inflationary environment. Risk premia in capital markets lead to wider portfolio models and wealth effects. Risk premia are considered by Dornbusch (1980) in his analysis of the dollar/deutch mark exchange rate, and incorporated into various models by Frankel (1982), Isard (1980) and Hooper and Morton (1982).

5 Contributors to this line of research include Dornbusch (1975), Branson, Halttunen and Masson (1977) and Bisignano and Hoover (1982).

6 The current account position is an important factor in the context of exchange rate dynamics and stock-flow equilibria. See Niehans (1977), Kouri (1976), Dornbusch and Fischer (1980), Rodriguez (1980). The point is that the cumulative trade balance position generates changes in wealth which affect the exchange rate through portfolio allocation.

7 This exchange rate has been the subject of several studies. Longworth (1981) rejected market efficiency (if no risk premium is assumed) of the Canadian-U.S. exchange rate market. Backus (1982) finds all existing models of the exchange to be inadequate. Miles (1978) argued that a currency substitution approach was validated but this was found to be untrue (due to specification error) by Bordo and Choudhri (1982). Haas and Alexander (1979) used an asset approach based on internationally traded short term capital which
can be viewed as a sub model of the stock-flow approach (in section 2.3.). They found wealth to be an important factor. Girton and Roper (1977) suggested an exchange market pressure model which favored the monetary approach. However as the dependent variable includes the exchange rate and official reserves adjustment, it is not appropriate for a study of the exchange rate per se. Bisignano and Hoover (1982) estimated the portfolio model. Their preliminary results, in their own opinion, were not very satisfying.


9 Frankel's equation differs slightly because his aggregate demand equation is \( \dddot{d} = \alpha_3 (e - \dddot{p}) \).

10 According to our solution to Driskill's model it seems that some typographical errors were introduced in his J.P.E. article.

11 In this case, \( \beta_{67} = \beta_{60} (\alpha_{7} - \alpha'_{7}) \) and \( \beta_{68} = \beta_{60} (\alpha_{9} - \alpha'_{9}) \) where the \( \alpha_{i} \) are the coefficients for the domestic variables and the \( \alpha'_{i} \) are the coefficients for the foreign variables. Allowing for all coefficients to differ across countries would imply introducing three additional variables in the model: the equilibrium value of foreign real income, of the foreign real interest rate, and of the foreign nominal interest rate.

12 We estimate equation [21b] with an additional variable \( \hat{\nu} = \hat{r} - \hat{\nu} \)

\( \hat{r} \) is the fitted value of \( r \) when regressed on all the instruments (exogenous variables of the model) by OLS. If the coefficient of \( \hat{\nu} \) is not significantly different from zero, according to an F-test then the problem that \( \hat{r} \) and the residual term are correlated is not statistically important. The F-statistic is 1.10 which is less than its critical value (at the 5% level) for 1 and 32 degrees of freedom (4.15).

13 Davidson and McKinnon also suggest an alternative test (P-test) which is based on a linearisation of the compound model \( M_c \) around the point \( (\alpha = 0, \beta_1 = \tilde{\beta}_1, \beta_2 = \tilde{\beta}_2) \). The results were similar in all respects to the C-tests, they are not reported.

Data sources and Definitions

The data are from the Cansim database compiled by Statistics Canada and from the Federal Reserve Bulletin. We report below the symbols used, the definition of each variable, the Cansim retrieval codes and the value of each variable in 1976 III. Note that for estimation purposes, the log of all variables are taken except for interest rates.

\( y = \) Gross National Expenditures \[D40593, 119296.0\]
Millions of 1971 dollars - seasonally adjusted at annual rates

\( y' = \) Gross National Expenditures (U.S.) \[B50301 \times 1000, 1303300.0\]
Millions of 1972 dollars - seasonally adjusted at annual rates

\( p = \) GNE deflator = GNE in millions of current dollars \( \div \) GNE in millions of 1971 dollars \[D40252 \div D40593, 1.616\]

\( p' = \) GNE deflator = GNE in millions of current dollars \( \div \) GNE in millions of 1972 dollars (U.S.) \[B50201 \div B50301, 1.327\]

\( m = \) Canadian M1-B in millions of dollars, seasonally adjusted \[B1620, 25140.0\]

\( m' = \) U.S. M1-B in millions of dollars \[Federal Reserve Bulletin, 300900.0\]

\( r_s = \) 90 day Finance Company Paper Rate (expressed in quarterly terms) \[B14017, 9.47\]
\[ r_s' = \text{Commercial Paper Rate (90 day) (U.S.)} \quad [B54412, 5.39] \]
(expressed in quarterly terms)

\[ r_g = \text{Average Bond Yields, Government} \quad [B14013, 9.16] \]
of Canada Securities, 10 years
and over (expressed in quarterly terms)

\[ r_g' = \text{Average Bond Yields, U.S. Government} \quad [B54403, 6.70] \]
10 years and over (expressed in quarterly terms)

\[ r_{\ell} = \text{Average Bond Yield, 10 industrials} \quad [B14016, 10.33] \]
(McLeod, Young, Weir) (expressed in quarterly terms)

\[ r_{\ell}' = \text{Corporate Bonds Industrial Average} \quad [B54410, 8.63] \]
(Moody's) (expressed in quarterly terms)

\[ E = \text{Rate of exchange: Closing Spot} \quad [B3414, 0.975] \]
Rates, Canadian dollars per unit of U.S. dollar

\[ w \text{ and } w' = \text{Real Wealth} = \frac{1}{8} \ln \left( \frac{\sum_{i=0}^{7} y_{t-i} \cdot p_{t-i}}{r_{g_{t-i}}} \right) - \ln p_t \]
References


Bilson, J. (1979) "Recent Developments in Monetary Models of Exchange Rate Determination", International Monetary Fund Staff Papers, 26, 201-223.


