Université de Montréal

ESSAYS IN MACORECONOMICS AND INTERNATIONAL FINANCE

par
Louphou Coulibaly

Département de Sciences Économiques
Faculté des Arts et Sciences

Thèse présentée en vue de l'obtention du grade
Philosophiæ Doctor (Ph.D.) en Sciences Économiques

Juin, 2019

Université de Montréal
Faculté des Études Supérieures et Postdoctorales

Cette thèse intitulée :

Essays in Macoreconomics and International Finance

présentée par :

Louphou Coulibaly

a été évaluée par un jury composé des personnes suivantes:

Emanuela Cardia	Présidente-rapporteur
Julien Bengui	Directeur de recherche
Javier Bianchi	Co-directeur de recherche
Guillaume Sublet	Membre du jury
Luca Fornaro	Examinateur externe

Thèse acceptée le 24 Juillet 2019

À mes parents, Haoua et Moussa

Remerciements

Acknowledgments

J'ai une profonde gratitude envers mon directeur de recherche Julien Bengui qui a cru en moi et m'a soutenu et guidé tout au long de ce cheminement de doctorat, ainsi qu'à mon co-directeur de recherche, Javier Bianchi, pour sa disponibilité, son enthousiasme et nos discussions lors de ma visite à la Federal Reserve Bank de Minneapolis. Je suis très heureux d'avoir bénéficié de leur conseils multiformes et leur supervision. La rigueur et la gentillesse dont ils font preuve sont des valeurs qui continueront à me guider tant dans ma vie professionnelle que personnelle.

Je remercie Anton Korinek pour sa confiance et sa générosité, Guillaume Sublet pour son soutien et son aide précieuse, Emanuela Cardia et Baris Kaymak pour leur disponibilité, ainsi que l'ensemble des professeurs du département de sciences économiques de l'université de Montréal, particulièrement Christopher Rauh et Mathieu Macroux. Je souhaiterais également remercier mon co-auteur Sushant Acharya (Chapitre 3). J'ai énormement appris de cette collaboration. Que Luca Fornaro reçoive aussi l'expression de ma profonde gratitude pour avoir accepté de siéger à titre d'examinateur externe pour évaluer cette thèse

J'exprime également ma reconnaissance à mes amis qui ont contribué à faire de ces six dèrnières années une expérience merveilleuse: Hervé Zéida, Marius Pondi, George Nkwinkeum, Kokouvi Tewou, Souleymane Zerbo, Awé Jean, N'Golo Koné, Mariam Djibril, Laurent Kemoe, Ada Nayihouba, William Gbohoui, Samuel Gingras, David Benatia, Jonathan Crechet, Gabriela Galassi, Ismael Mourifié et Désiré Kédagni. Merci à vous, ainsi qu'à tous les autres avec qui j'ai partagé des moments inoubliables, pour votre amitié et votre support. Mes pensées et ma reconnaissance vont particulièrement à mes parents Haoua et Moussa, mes frères et sœurs pour le soutien moral inlassable et inconditionnel que vous continuez de m'apporter. Pour finir, je tiens à remercier le département de recherche de la Federal Reserve Bank of Minneapolis ainsi que département d'économie de University of Virginia pour leur hospitalité.

Résumé

Comment concevoir de manière optimale des politiques macroéconomiques visant à réduire la fragilité du système financier et à améliorer le bien-être social? Ma recherche développe des théories normatives et utilise l'analyse quantitative pour améliorer notre compréhension des avantages potentiels et des implications pour le bien-être des mesures réglementaires lorsqu'il y a une prise de risque excessive sur les marchés financiers. Cette thèse est un recueil de trois essais en macroéconomie et finance internationale qui oriente le débat sur la conception optimale des politiques macroéconomiques et macroprudentielles à la fois en prévision et pendant les crises financières. Le premier chapitre est une introduction générale, tandis que les chapitres restants constituent le cœur de la thèse. Le deuxième chapitre porte sur la conception optimale des plans de sauvetage financier. Il conclut que lorsque gouvernement est incapable de s'engager sur ses politiques futures, un plan de sauvetage financier concu de manière optimale ne devrait couvrir qu'un sous-ensemble d'investisseurs. Un résultat important pouvant rationaliser la couverture limitée des plans de sauvetage, tels que l'accès aux liquidité du prêteurs de dernier recours. Le troisième chapitre étudie la politique monétaire dans les économies émergentes. Ce chapitre propose d'abord une théorie sur les raisons pour lesquelles la politique monétaire est procyclique dans les économies émergentes. Il conclut que cette politique est en fait optimale dans une économie sujette aux crises financières associées au phénomène de retournement brusques des flux de capitaux et dans laquelle le gouvernement manque de crédibilité. L'analyse quantitative montre ensuite que les politiques macroprudentielles de contrôles des capitaux ont des implications sur la conduite de la politique monétaire et sont très efficaces pour réduire à la fois la fréquence et la sévérité des crises financières. Enfin, le quatrième chapitre décrit et analyse l'efficacité des réglementations macroprudentielles spécifiques à chaque pays en tant qu'outil de stabilisation macroéconomique dans les unions monétaires. L'analyse soutient l'idée selon laquelle les politiques macroprudentielles spécifiques à chaque pays devraient être définies par une autorité centralisée à l'échelle de l'Union, plutôt qu'au niveau national.
Mots-clés: Plans de sauvetage, incohérences temporelles, banques fantômes, crises financières, flux de capitaux, politique monétaire optimale, politique macroprudentielle, contrôle des capitaux, union monétaire, coordination des politiques.

Abstract

How can we best design macroeconomic policies to reduce financial fragility and improve social welfare? My research develops normative theories and uses quantitative analysis to improve our understanding of the potential benefits and welfare implications from regulatory measures in face of excessive risk taking in financial markets. This thesis is a collection of three essays in macroeconomics and international finance that helps guide the debate on the optimal design of macroeconomic and macro-prudential policies ahead of potential financial crises and during crises. The first chapter is a general introduction, while the remaining chapters form the core of the thesis. The second chapter studies the optimal design of financial safety nets. It shows that, in an economy where the government lacks commitment, an optimally designed financial safety net should cover only a subset of investors. An important result that can rationalize the prevalent limited coverage of safety nets, such as the lender of last resort facilities. The third chapter focuses on monetary policy in emerging market economies. This chapter first proposes a theory of why monetary policy is pro-cyclical in emerging economies. It shows that pro-cyclical monetary policy is in fact an optimal policy in an economy subject to the risk of financial crises associated with sudden stops of international capital inflows, and in which the government lacks commitment. The quantitative analysis then shows that macro-prudential policies in the form of capital controls have radical implications for the conduct of monetary policy, and are very effective at reducing both the occurrence and magnitude of financial crises. Finally, the fourth chapter design and analyze the effectiveness of country-specific macroprudential regulations as a macroeconomic stabilization tool in currency unions. The analysis lends support to the view that country-specific macroprudential policies should be set by a union-wide centralized authority, rather than at the national level.

Keywords: Bailouts, time inconsistency, shadow banks, financial crises, capital flows, optimal monetary policy, macroprudential policy, capital controls, currency union, policy coordination.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

How can we best design macroeconomic policies to reduce financial fragility and improve social welfare? This thesis is a collection of three essays in macroeconomics of international finance that develops normative theories and uses quantitative analysis to improve our understanding of the potential benefits and welfare implications from regulatory measures in face of excessive risk taking in financial markets. This thesis thus helps guide the debate on the optimal design of macroeconomic and macro-prudential policies both ahead of potential financial crises and during episodes of financial crises.

The 2008 financial crisis has raised fundamental questions on the scope and reach of financial safety nets. While safety nets can prevent a complete meltdown of the financial sector by relieving the liquidity strains of financially distressed entities, they may lead to excessive risk taking in financial markets. Therefore, in line with Greenspan (2001), policymakers must be "very cautious about purposefully or inadvertently extending the scope and reach of the safety net". Chapter 2 studies the optimal design of financial safety nets under limited private credit by asking whether and when it is optimal to restrict ex ante the set of investors that can receive public liquidity support. The design of financial safety nets is tackled using a stylized model of liquidity demand in which the unique source of market incompleteness is that private contract are not enforceable which in turn limits borrowing between investors to smooth liquidity shocks and creates role for public liquidity provision. When the government can commit, the optimal safety net covers all investors. Introducing a wedge between identical investors is inefficient. Without commitment, however, an optimally designed financial safety net covers only a subset of investors. Covering only a subset of investors is a way of balancing the stability gained from the safety net against the moral hazard problem. Compared to an economy where all investors are protected, this results in more liquid portfolios, better social insurance, and higher welfare. These results can rationalize the prevalence of limited safety nets, as well as the coexistence of traditional and shadow banks.

The focus of Chapter 3 is on monetary policy in emerging market economies. This chapter builds on the notion that a key characteristic of emerging economies is that they are prone to financial crises associated with sudden stops of international capital inflows. While countercyclical stabilization policy constitutes a central tenet of macroeconomics, monetary policy procyclicality appears to be a pervasive feature of emerging economies (Kaminsky et al., 2004). This chapter proposes a parsimonious theory explaining this fact in a small open economy where access to foreign financing depends on the real exchange rate and the government lacks commitment. The discretionary monetary policy is pro-cyclical to mitigate the adverse balance sheet effects originating from exchange rate depreciations. The mechanism described in this chapter corresponds closely to what policymakers argue when they engage in such policy. This chapter also performs a quantitative analysis of the described setup. It shows that relinquishing the ability to perform pro-cyclical monetary policy and committing to an inflation target raises welfare and reduces the frequency of financial crises, although crises are more severe when they do occur. Intuitively, overly active monetary policy to improve access to foreign credit in crisis times induces the economy to borrow more ex ante, largely offsetting the welfare benefits while also imposing welfare costs in terms of deviating from the optimal inflation target. Allowing the policymaker to also use capital controls significantly ameliorates the trade-off between the adverse ex-ante incentive effects and the positive ex-post effects of pro-cyclical policy: with capital controls, a policymaker can directly control how much debt the economy accumulates and mitigate financial constraints so that there is less need for pro-cyclical monetary policy and the associated distortions to the price-setting process. As a result, an economy with capital controls and discretionary monetary policy achieves higher welfare and feature both less frequent and less severe financial crises than an economy with commitment to an inflation target.

Finally, Chapter 4 propose a simple theoretical framework for the analysis of the effectiveness and design of country-specific macroprudential regulations as a macroeconomic stabilization tool in currency unions. The challenges of conducting macroeconomic stabilization policy in a currency union whose member countries face diverging economic prospects has been outlined by the Eurozone's experience of the last decade. Because in the context of a currency union, conventional macroeconomic stabilization tools are lacking, we ask whether country-specific macroprudential policies could also be assigned a macroeconomic stabilization function. While common monetary policy in a currency union is doomed to affect aggregate demand in both countries symmetrically, country-specific macroprudential policy is not. As a result, this chapter shows that such tools can be highly effective at dealing with the rigidities in adjustment mechanisms inherent to a currency union when the degree of trade integration is low. Our analysis also lends support to the view that country-specific macroprudential policies should be set by a union-wide centralized authority, rather than at the national level, except in knife-edge cases. Policy coordination is found to be particularly relevant under high trade integration and in response to shocks leading to trade imbalances within the currency union.

Chapter 2

Financial Safety Nets[1]

2.1 Introduction

Safety nets are a central pillar of modern financial architectures. By granting liquidity support to a collection of institutions, a safety net can relieve the strains of eligible members in financial distress. A long-standing concern about safety nets, however, is that they can lead to excessive risk taking.[2] Accordingly, a key question regarding the design of safety nets is: How should the stability gained from a financial safety net be balanced against the moral hazard problem? Despite extensive discussions, the literature lacks a theoretical framework that can be used to address this question.

In this paper, we tackle the design of financial safety nets using a stylized model of liquidity demand under limited private credit. As in Holmström and Tirole (1998), the government can relax credit constraints by providing public liquidity. The question we address is whether the government should restrict ex ante the set of investors to whom it provides liquidity support ex post. In a nutshell, how wide should the financial safety net be?

The importance of defining the scope of financial safety nets was underscored during the 2008 financial crisis, especially surrounding the run on the shadow banking system. Because only depository institutions are granted access to discount window facilities, the Federal Reserve found it challenging to provide a backstop to those non-depository institutions in financial distress. Invoking legal constraints, the US Treasury and the Federal Reserve let Lehman Brothers fail, despite mounting pressures to provide a rescue package.[3] At the time, many observers

[1] This chapter is co-authored with Julien Bengui (Université de Montréal), and Javier Bianchi (Federal Reserve Bank of Minneapolis). It has been published in International Economic Review.

[2] Greenspan (2001) notably warned that policymakers must be "very cautious about purposefully or inadvertently extending the scope and reach of the safety net."

[3] In his testimony at the Financial Crisis Inquiry Commission in 2009, then Chairman Ben Bernanke stated: "I will maintain to my deathbed that we made every effort to save Lehman, but we were just unable to do so because of a lack of legal authority."

interpreted the failure to rescue Lehman as a manifestation of a line in the sand between depository institutions and shadow banks, or, through the lens of our model, a definition of the scope of the safety net.[4]

We study a model in which investors can save in short-term and long-term assets. These investors are subject to private idiosyncratic liquidity shocks, which occur before the long-term asset's payoffs are realized, as in Diamond and Dybvig (1983). A lack of enforceability of private contracts limits investors' ability to borrow to smooth liquidity shocks. This introduces a role for public liquidity provision. The government issues bonds to finance a liquidity facility, and investors can anonymously trade these bonds. The new feature we introduce in this model is a government's choice about the share of investors that are eligible for public liquidity support. Specifically, we consider a government that chooses at time 0 the share of investors that will be eligible for liquidity support in the interim period and lacks commitment to the magnitude of the liquidity support provided to each eligible investor. The assumption that the government can commit to the safety net captures in a stylized manner that once a safety net is defined, it is more difficult for the government to bypass the scope of the safety net ex post, as vividly illustrated by the example of Lehman's bankruptcy. We label the set of investors eligible for ex post public support the *protected sphere* and the set of investors excluded from it the *unprotected sphere*. Having access to a public liquidity facility, protected agents choose higher yield, longer-term portfolios than unprotected agents, who have to rely on the short-term asset to self-insure.

Our analysis of financial safety nets delivers several results, on both the positive and normative fronts. We first show that if the government can commit to a future liquidity provision policy, the optimal safety net covers the entire set of investors. With commitment, the government can provide an amount of public liquidity that induces the efficient amount of investment in long-term assets and thereby leads to the efficient allocation.[5] Offering a differential treatment to identical investors is inefficient if the government has commitment. In this case, the optimal size of the unprotected sphere is zero.

We then analyze the optimal safety net when the government lacks commitment. Specifically, we study a time-consistent equilibrium in which the government chooses without commitment the liquidity support in the interim period. We can characterize in closed form three distinct regions depending on the width of the safety net (i.e., the size of the protected sphere chosen by the government at time 0). In an economy with a small protected sphere, only a

[4] Other examples of safety nets arise naturally in international credit markets, involving, for example, the IMF or other multilateral organizations.

[5] As in Yared (2013), the optimal amount of liquidity provision under commitment introduces a wedge between the technological rate of return on the long asset and the rate of return on government bonds.

small subset of agents invest in the higher yield, long-term asset, resulting in low output. The fact that few agents can access the liquidity facility results in a low level of public debt and a low interest rate. A low interest rate benefits protected agents because it results in a low cost of accessing the liquidity facility in the event of a liquidity shock. Conversely, unprotected agents who save in the short-term assets are hurt by the low return on their savings. As a result, there is a large welfare gap between protected and unprotected agents. An economy with a larger protected sphere features more agents investing in the higher yield, long-term asset and therefore results in higher output. Further, the fact that more agents resort to the liquidity facility leads to a higher level of public debt and a higher interest rate, which reduces the welfare gap between protected and unprotected agents.

Our main normative result is that in a time-consistent equilibrium, the optimal ex ante government's choice implies an intermediate-size protected sphere. Unlike in the commitment case, it is optimal to leave a fraction of investors, strictly between 0 and 1, without liquidity support. A safety net covering all investors is undesirable because, under lack of commitment, the government provides too much liquidity support to protected investors ex post. Anticipating access to the public liquidity facility, protected investors free ride on others' investment in short-term assets and choose excessively illiquid portfolios. In order to finance the liquidity facility, the government needs to issue a large amount of public debt. This in turn yields an interest rate on government bonds that is too high from a social point of view. A high interest rate redistributes resources away from investors that have liquidity shortfalls and hurts ex ante welfare. Because of incomplete markets, the costs of this higher interest rate for borrowing investors that have a shortfall of liquidity outweigh the benefits to lending investors that have a surplus of liquidity. In addition, a midsize protected sphere also dominates a small protected sphere because it features less socially costly liquidity hoarding. A safety net with a midsize protected sphere is thus desirable from an ex ante welfare perspective.

Related literature This paper is related to a vast literature on public liquidity provision. Woodford (1990) and Holmström and Tirole (1998) are classic papers showing how public liquidity provision may relax private borrowing limits. In our model, the government also has a special role as a liquidity provider, but we address a distinct issue—the design of financial safety nets. In particular, our model rationalizes a key feature of prevailing safety nets, where some financial institutions have access to a discount window while other institutions performing essentially the same activities do not.

Jacklin (1987) argues that full trading opportunities in the Diamond and Dybvig (1983) model generate a free-rider problem that leads to an inefficient equalization of the marginal rate of transformation and the interest rate. Farhi et al. (2009) study a mechanism design problem and show how liquidity regulation can achieve a constrained efficient outcome in this context. In Grochulski and Zhang (2015), the ability of banks to bypass liquidity regulation puts an additional constraint on the mechanism design problem and reduces the magnitude of the intervention.[6] We abstract from ex ante regulation and focus instead on public liquidity provision and the design of the safety net.

Our environment is closest to Yared (2013). He studies optimal liquidity provision under commitment and shows that it entails introducing a wedge between the technological rate of return on the long asset and the rate of return on government bonds. The government restricts transfers and bond issuances, so that the return on government bonds remains low, which leads to superior risk sharing.[7] We study instead the optimal liquidity provision policy when the government lacks commitment. In particular, we show that the government provides too much liquidity ex post for investors within the safety net, and hence the optimal liquidity provision plan under commitment is not implementable. We characterize aggregate investment and risk sharing as a function of the size of the safety net and show that it is strictly optimal to leave a share of investors outside the safety net.

A related literature highlights how bailouts can increase financial fragility when the government lacks commitment. Farhi and Tirole (2012) show that discretionary interest rate policy makes private leverage decisions strategic complements and generates multiple equilibria. Lack of commitment also plays an important role in the analysis of bailouts by Acharya and Yorulmazer (2007), Diamond and Rajan (2012), and Chari and Kehoe (2016). Nosal and Ordoñez (2016) show that a government's uncertainty about whether failed institutions were affected by idiosyncratic or systemic shocks creates strategic restraint in leverage decisions and supports government commitment. Freixas (1999) shows that randomizing between bailing out banks in distress or not can create "constructive ambiguity" and reduce risk taking. Keister (2016) presents an environment in which a commitment to a no-bailout policy is undesirable because it can increase the likelihood of bank runs, and Keister and Narasiman (2016) show that these policy conclusions emerge regardless of whether bank runs are driven by expectations or fundamentals. Bianchi (2016) finds that bailouts are desirable despite the moral hazard effects if conducted only during systemic crises. None of these papers, however, study the optimal design of financial safety nets.

Our paper also relates to a growing literature on shadow banking. Existing work emphasizes

[6]In a setting with systemic risk externalities, Bengui and Bianchi (2018) analyze prudential policy under imperfect regulation enforcement and find that increased risk taking by the unregulated sphere may call instead for a stronger intervention on the protected sphere.

[7]In a different environment, Yared (2015) and Bhandari et al. (2015) study the effects of government debt on inequality.

regulatory arbitrage as the *raison d'être* of shadow banks (see, for instance, Acharya et al. (2013); Gorton and Metrick (2012); and Pozsar et al. (2010)). In this spirit, Plantin (2015) develops a model in which capital requirements lead banks to bypass regulation through a shadow banking sector. Ordoñez (2013) shows that the bluntness of capital requirements can make shadow banks desirable as a way to build reputation and better align the interests of banks and bondholders. In contrast, our analysis shows that the very existence of these institutions could be the result of the optimal plan of a government that is subject to a classic time-inconsistency problem.

The paper proceeds as follows. Section 3.2 describes the model. Section 3.3 describes the main results, and Section 2.4 concludes. The Appendix includes all of the proofs.

2.2 The Model

2.2.1 Technology and Preferences

The environment is based on the Diamond and Dybvig (1983) model of consumer liquidity demand that has been a workhorse in the study of financial intermediation. It is closest to the model presented by Yared (2013). The economy lasts for three dates: $t = 0, 1, 2$. There is a single consumption good and there are two technologies, which we label the short asset and the long asset. The short asset pays one unit of the good at $t + 1$ for each unit invested at t. The long asset pays $\hat{R} > 1$ units at date 2 for each unit invested at date 0. For simplicity, we assume that the long asset cannot be liquidated at date 1.[8]

The economy is populated by a unit continuum of ex-ante identical agents. These agents are endowed with e units of the good at $t = 0$. The type space has two dimensions. At date 0, each individual draws the first dimension of his type: $s = \{P, U\}$. A fraction $\gamma \in [0, 1]$ of individuals is of type $s = P$, and the complementary fraction $1-\gamma$ is of type $s = U$. P stands for *protected*, while U stands for *unprotected*. As we will explain below, protected agents have access to public liquidity and unprotected agents do not. The type dimension s is public information, and the parameter γ is a policy choice on which we elaborate more in Section 2.2.2. At date 1, an agent draws the second dimension of his type, $\theta = \{0, 1\}$, which determines the preference for early consumption. With probability $\pi \in (0, 1)$, an individual is of type $\theta = 0$, while with probability $1 - \pi$, he is of type $\theta = 1$. The distribution parameter π is a fundamental of the economy. Agents have Diamond-Dybvig preferences: the utility of an individual of type (s, θ) is given by

$$U\left(c_1^s, c_2^s, \theta\right) = (1-\theta)u\left(c_1^s\right) + \theta\rho u\left(c_1^s + c_2^s\right), \tag{2.1}$$

[8]All results carry through qualitatively as long as the date 1 liquidation value is strictly smaller than one.

where c_1^s and c_2^s represent the respective date 1 and date 2 consumption levels, while ρ is a discount factor. The utility function $u(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave, and satisfies the Inada conditions $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$.

The type dimension θ refers to liquidity shocks. Agents of type $\theta = 0$ are hit by liquidity shocks and only value consumption at date 1, whereas agents of type $\theta = 1$ are not hit by liquidity shocks and are indifferent between consumption at date 1 and date 2. As is standard in the literature, we assume that the type dimension θ is private and cannot be observed by other agents or the government. We will often refer to agents hit by a liquidity shock as *impatient* agents and to agents not hit by a liquidity shock as *patient* agents.

The type dimension s determines the eligibility for public support at $t = 1$. Agents of type $s = U$ are unprotected and are not entitled to public liquidity provision, whereas agents of type $s = P$ are protected and can receive public liquidity at date 1. Unlike the type dimension θ, the type dimension s is public. In what follows, we will denote an allocation of consumption across consumers by $\{c_1^s(\theta), c_2^s(\theta)\}_{\theta\in\{0,1\},s\in\{U,P\}}$.

We define $\ell^s \in [0, 1]$ as the fraction of the date 0 endowment invested in the short asset by a type s individual. Accordingly, we denote by $L^s \in [0, 1]$ the aggregate choice of type s individuals. In equilibrium, consistency will require that aggregate and individual choices coincide, that is, $L^s = \ell^s$ for $s \in \{U, P\}$.

We make some parametric assumptions to ensure that the equilibria we consider fall within economically interesting regions.

Assumption 1. *As in Diamond and Dybvig (1983), the relative risk aversion is weakly larger than 1:*

$$-\frac{u'(c)}{cu''(c)} \le 1 \quad \textit{for all} \quad c > 0, \tag{2.2}$$

and $\hat{R}^{-1} < \rho < 1$.

Assumption 1 notably implies that efficient risk sharing requires impatient agents to consume more than e and patient agents to consume less than $\hat{R}e$.

Assumption 2. *The probability of being hit by a liquidity shock is not too small:*

$$\pi \ge \frac{\rho(\hat{R} - 1)}{1 + \rho(\hat{R} - 1)}.$$

This assumption ensures that in all equilibria we consider, agents make investment choices such that they never consume positive amounts at date 2 when they turn out to be impatient.[9]

2.2.2 Public Liquidity Provision and Markets

We assume that private contracts are not enforceable. The assumption of unobservability of liquidity shocks implies that contracts cannot be written at date 0 contingent upon their realization at date 1, and the lack of enforceability implies that agents cannot borrow privately either at date 0 or at date 1. The assumption of imperfect private contract enforceability motivates the analysis of public liquidity provision and the design of an optimal safety net.

The government makes two distinct choices at date 0 and date 1. At date 0, it sets the share of protected agents γ, and hence sets the respective probabilities with which an agent draws a type $s = U$ or $s = P$ at date 0. At date 1, it provides a liquidity facility to protected agents, and finances it by issuing public debt. We hereafter provide details on the government's policy at date 1 and postpone our exposition of the government's date 0 safety net decision to Section 3.3.

At date 1, the government issues public debt and extends credit to protected agents. At date 2, it uses the proceeds from protected private investors' repayments to pay back public debt holders. In the background, we assume that the government has a superior technology to enforce repayment, which is a standard assumption in the literature on public liquidity provision. As we will show below, this access to a better enforcement technology allows the government to reach the efficient allocations under commitment, but not under discretion.

We assume that the credit facility is contingent on protected agents' portfolio position.[10] Because the liquidity shock realization is private information, the credit facility cannot be made contingent on θ. Accordingly, we denote the quantity of credit extended by the government to agents with short asset position ℓ by $B(\ell)$ and the aggregate amount of public debt by $\mathcal{B}$. The government demands the same interest rate $1/q$ on the credit it extends to protected agents as the one it pays on its own public debt, it has zero initial public debt, and it does not finance any public expenditures. Its budget constraints thus require that

$$\int_0^\gamma B(\ell_j)dj = \mathcal{B}. \tag{2.3}$$

[9]Agents who turn out to be impatient do not value date 2 consumption, but if liquidity shocks occur with a sufficiently low probability, they might find it optimal to make investment decisions at date 0 that result in an ex post wasting of date 2 resources in the contingency where they are hit by these shocks. Assumption 2 rules out this case.

[10]By making the government liquidity provision contingent on individual variables as opposed to aggregate variables, we are able to abstract from issues of multiplicity that would arise in this model when we turn to the optimal time-consistent equilibria (see, e.g., Farhi and Tirole, 2012).

We denote by $b^s(\theta)$ the individual holdings of government bonds and assume that government bonds cannot be shortened (i.e., $b^s(\theta) \geq 0$). The budget constraints of an unprotected agent are represented by

$$\begin{aligned} \ell^U &\in [0,1], && (2.4) \\ c_1^U(\theta) &= \ell^U e - q b^U(\theta), && (2.5) \\ c_2^U(\theta) &= \hat{R}(1-\ell^U)e + b^U(\theta), && (2.6) \end{aligned}$$

while those of a protected agent are represented by

$$\begin{aligned} \ell^P &\in [0,1], && (2.7) \\ c_1^P(\theta) &= \ell^P e - q b^P(\theta) + qB(\ell^P), && (2.8) \\ c_2^P(\theta) &= \hat{R}(1-\ell^P)e + b^P(\theta) - B(\ell^P), && (2.9) \end{aligned}$$

for $\theta \in \{0,1\}$. We have used in (2.5) and (2.8) that in an equilibrium with $q \leq 1$, agents weakly prefer to save using government bonds between date 1 and date 2 rather than use the short asset.[11] Combining the government's budget constraint (2.3) with (2.5), (2.6), (2.8), (2.9) and the public debt market clearing condition

$$\mathcal{B} = (1-\gamma)[\pi b^U(0) + (1-\pi)b^U(1)] + \gamma[\pi b^P(0) + (1-\pi)b^P(1)], \quad (2.10)$$

we obtain that a feasible allocation needs to satisfy the economy's resource constraint

$$\begin{aligned} &\pi\left[(1-\gamma)\left(c_1^U(0) + \frac{c_2^U(0)}{\hat{R}}\right) + \gamma\left(c_1^P(0) + \frac{c_2^P(0)}{\hat{R}}\right)\right] \\ &+(1-\pi)\left[(1-\gamma)\left(c_1^U(1) + \frac{c_2^U(1)}{\hat{R}}\right) + \gamma\left(c_1^P(1) + \frac{c_2^P(1)}{\hat{R}}\right)\right] = e. \quad (2.11) \end{aligned}$$

An alternative representation of the agents' constraint set is given by

$$\begin{aligned} c_1^s(\theta) + q c_2^s(\theta) &= \ell^s e + q\hat{R}(1-\ell^s)e \\ c_1^s(\theta) &\leq \ell^s e + \mathbb{I}_{\{s=P\}} qB(\ell^s). \end{aligned}$$

These substitutions show that the protected agent's problem induced by a government debt policy is equivalent to that of an agent who faces an exogenous borrowing limit $b^P(\theta) \geq -B(\ell^P)$ in the absence of public liquidity provision. On the other hand, because they do not benefit

[11] An equilibrium with $q > 1$ implies $b^s(\theta) = 0$. That is, if the return on government bonds is lower than the return on short assets, government bonds would be strictly dominated assets.

from public liquidity provision, unprotected agents always face an effective borrowing limit $b^U(\theta) \geq 0$. The government's date 0 choice about the size of the protected sphere will determine the respective fractions of agents facing a relaxed borrowing limit $-B(\ell)$ and of those facing an unrelaxed limit at 0. One might think that the government would like to maximize the fraction of agents who face a relaxed borrowing limit ex post, but as our analysis of Section 3.3 reveals, this not the case when the government cannot commit ex ante about its debt issuance policy. The timeline is summarized in Figure 2.1.

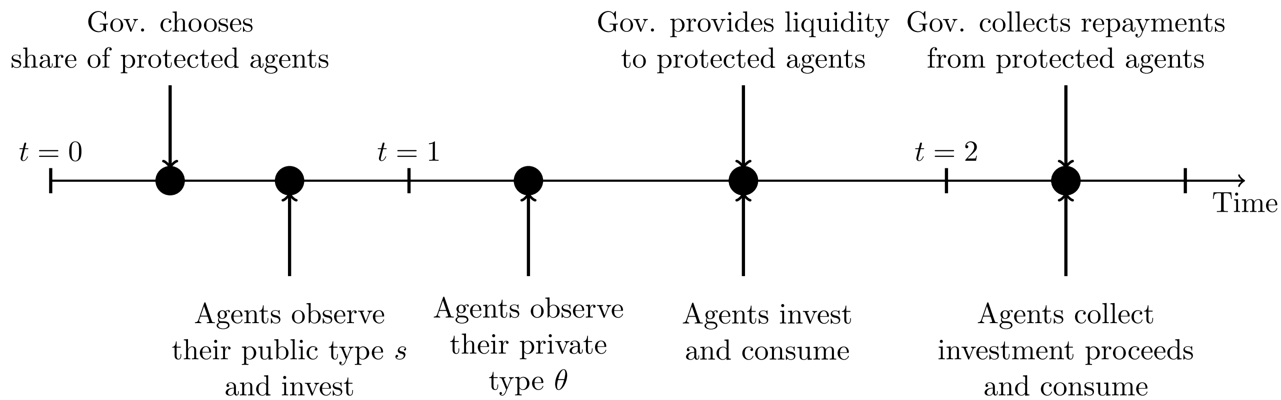

Figure 2.1: Timeline of the model.

We let (s, ℓ, θ) denote an agent's individual state and $X \equiv (\gamma, L^U, L^P)$ denote the aggregate state. We also let $B(\ell)$ denote a public liquidity provision policy, $q(X)$ denote the bond pricing function, and $\mathcal{C}_1(s, \ell, \theta, X), \mathcal{C}_2(s, \ell, \theta, X)$ represent the date 1 decision rules of an agent, whose problem is

$$\mathcal{V}_1(s, \ell, \theta, X) = \max_{c_1, c_2} U(c_1, c_2, \theta) \tag{2.12}$$

subject to

$$c_1 + q(X)c_2 = \ell e + q(X)\hat{R}(1-\ell)e, \qquad \text{and} \qquad c_1 \leq \ell e + \kappa(s, \ell, X),$$

where

$$\kappa(s, \ell, X) \equiv \begin{cases} 0 & \text{for } s = U \\ q(X)B(\ell) & \text{for } s = P \end{cases} \tag{2.13}$$

is a type- and agent-specific effective borrowing limit. This problem is defined for any policy $B(\ell)$. We can then define a date 1 continuation equilibrium.

Definition 1 (Continuation equilibrium)**.** *Given a government policy $B(\ell)$, a continuation equilibrium is a value function $\mathcal{V}_1(s, \ell, \theta, X)$ with associated decision rules $\mathcal{C}_1(s, \ell, \theta, X), \mathcal{C}_2(s, \ell, \theta, X)$, and a bond pricing function $q(X)$ such that*

1. *given $q(X)$ and $B(\ell)$, $\mathcal{V}_1(s,\ell,\theta,X)$ solves the agent's date 1 problem (2.12), and*

2. *the markets for date 1 and 2 consumption clear:*[12]

$$\sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_1(s, L^s, \theta, X) \leq \sum_s \gamma_s L^s e, \tag{2.14}$$

$$\sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_2(s, L^s, \theta, X) = \sum_s \gamma_s \hat{R}(1-L^s)e + \left[\sum_s \gamma_s L^s e - \sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_1(s, L^s, \theta, X)\right]. \tag{2.15}$$

This is a standard definition of a competitive equilibrium, adapted to accommodate the dependence of the government's liquidity provision policy upon the ex-ante choices of agents. Condition (2.14) requires that aggregate date 1 consumption does not exceed the aggregate payoff of the short asset at date 1. Condition (2.16) requires that aggregate date 2 consumption does not exceed the aggregate payoff of the long asset, plus the payoff of the short asset invested in between date 1 and date 2. By Walras' law, the market clearing condition on government bonds is satisfied. The following lemma characterizes a continuation equilibrium. This characterization will be useful when we turn to analyze the optimal government policy and highlight the role of commitment.

Lemma 1 (Continuation equilibrium)**.** *A continuation equilibrium features*[13]

1. *a bond price function satisfying*

$$q(X) = \min\left\{ e\frac{1-\pi}{\pi}\frac{\gamma L^P + (1-\gamma)L^U}{\gamma \min\{\hat{R}(1-L^P)e, B(L^P)\}}, 1\right\}, \tag{2.16}$$

2. *consumption allocations satisfying*

$$\mathcal{C}_1(s,\ell,0,X) = \ell e + \min\left\{q(X)\hat{R}(1-\ell)e, \kappa(s,\ell,X)\right\}, \tag{2.17}$$

$$\mathcal{C}_2(s,\ell,0,X) = \max\left\{\hat{R}(1-\ell)e - \frac{\kappa(s,\ell,X)}{q(X)}, 0\right\}, \tag{2.18}$$

$$\mathcal{C}_1(s,\ell,1,X) = 0, \tag{2.19}$$

$$\mathcal{C}_2(s,\ell,1,X) = \hat{R}(1-\ell)e + \frac{\ell e}{q(X)}. \tag{2.20}$$

[12] To simplify notation, we define $\pi_0 \equiv \pi$ and $\pi_1 \equiv 1-\pi$, as well as $\gamma_P \equiv \gamma$ and $\gamma_U \equiv 1-\gamma$.

[13] When an equilibrium features $q=1$, any other allocation such that $c_1 + c_2 = \hat{R}(1-\ell)e + \ell e$ (and $c_1, c_2 \geq 0$), together with the price $q=1$, also constitutes an equilibrium, but we can focus without loss of generality on the one featuring $c_1 = 0$ and $c_2 = \hat{R}(1-\ell)e + \ell e$.

Proof. See Appendix A.1.

According to this lemma, in the absence of public liquidity provision, all impatient agents consume the proceeds of their short asset at date 1 and consume the payoff of their long asset at date 2. The latter is wasteful because these agents do not value date 2 consumption, but they have no choice since credit constraints prevent them from transferring resources from date 2 to date 1. By relaxing protected agents' effective credit constraint, the extension of public liquidity allows this set of agents to transfer some or all of their date 2 illiquid wealth stemming from the payoff of their long asset back into date 1. Patient agents, on the other hand, consume only at date 2. These agents are natural savers at date 1, and therefore public debt issuance does not generate an asymmetry between the protected and unprotected patient agents' consumption. However, patient agents (at least weakly) benefit from a higher level of public debt to the extent that it (weakly) increases the interest rate they earn on date 1 bond purchases (the bond price q is weakly decreasing in public debt issuance $\mathcal{B}$ since the demand for government bonds by patient agents is decreasing in the price).

For a given liquidity provision policy $B(\ell)$, an agent's date 0 problem can be represented as

$$\mathcal{V}_0(s, X) = \max_{\ell \in [0,1]} \pi \mathcal{V}_1(s, \ell, 0, X) + (1 - \pi)\mathcal{V}_1(s, \ell, 1, X). \tag{2.21}$$

Given this date 0 problem and Definition 1 of a continuation equilibrium, we have the following definition of a competitive equilibrium:

Definition 2 (Competitive equilibrium)**.** *Given government policies $\{\gamma, B(\ell), \mathcal{B}\}$, a competitive equilibrium is a vector of aggregate variables X, a date 0 value function $\mathcal{V}_0(s, X)$ with associated policy function $\ell(s, X)$, a date 1 value function $\mathcal{V}_1(s, \ell, \theta, X)$ with associated decision rules $\mathcal{C}_1(s, \ell, \theta, X), \mathcal{C}_2(s, \ell, \theta, X)$, and a bond price function $q(X)$ such that*

1. *$\mathcal{V}_1(s, \ell, \theta, X), \mathcal{C}_1(s, \ell, \theta, X), \mathcal{C}_2(s, \ell, \theta, X)$, and $q(X)$ are induced by a continuation equilibrium according to Definition 1,*

2. *$\mathcal{V}_0(s, X), \ell(s, X)$ solve the agent's problem (2.21),*

3. *aggregate variables are consistent with individual choices: $X = (\gamma, \ell(P, X), \ell(U, X))$,*

4. *the government's budget constraint (2.3) is satisfied.*

Discussion of assumptions. Before we turn to the normative analysis of safety nets, it is useful to discuss some assumptions we have made. A first set of assumptions regard the imperfect enforceability of private contracts, which are standard in the literature to motivate a role for public liquidity provision. We also assume that there is no private credit market at either date 0 or date 1, and that there are no secondary markets for long assets in the interim period.[14] Both of these assumptions can be relaxed to some extent, as discussed in Section 2.3.3. As long as the constraints on either borrowing or sales of assets are sufficiently tight, our qualitative results remain unchanged.

Our second set of assumptions regard our modeling of safety nets. We assume that the government can credibly commit to excluding a set of investors from financial assistance. In line with our motivation, we are interested in developing an environment in which the government is able to implement such a policy. In the background, we want to capture a variety of situations in which explicitly defining a safety net makes it more difficult for the government to rescue ex post those institutions that are not eligible to receive assistance. While in practice there are certainly circumstances under which the government bypasses the scope of the safety net defined earlier in time, doing so brings up reputational costs for the government.[15] Incorporating these reputational costs would require a dynamic environment, which is beyond the scope of this paper. Still, Section 2.3.3 shows that when investors anticipate that the safety net announced by the government will be implemented, the government does not find it optimal to deviate from the announcement ex post.

In addition, we also assume that investors have no choice regarding which sphere they belong to. The government chooses the size of the protected and unprotected spheres, and investors are assigned randomly to either of the two spheres. As it will turn out, the equilibrium welfare of protected investors will be higher than that of unprotected investors. A potentially interesting issue that we abstract from is investors' entry decision into the two spheres in the presence of other costs associated with being in the protected sphere (e.g., there could be a tax on protected investors so that in equilibrium, investors are indifferent between belonging to either of the two spheres).[16]

[14] Allowing for a secondary market for assets is equivalent to allowing for private credit markets. The basic idea is that a patient investor would buy the assets from an impatient investor, in the same way as a patient investor would lend to an impatient investor in the credit market. A standard argument for restrictions on secondary markets is asymmetric information. If the quality of the asset cannot be verified by the buyer, the market could break down or restrictions could be imposed on the amount of assets that can be sold (e.g., Akerlof (1970); Kiyotaki and Moore, 2012).

[15] For example, the Federal Reserve ended up providing emergency liquidity assistance to non-depository primary dealers, through programs such as the Primary Dealer Credit Facility (PDCF) and the Term Securities Lending Facility (TSLF).

[16] In an interesting paper, Grochulski and Zhang (2015) analyze optimal regulation in an environment in which banks can choose to shift activity to an unregulated sector to avoid liquidity regulation by incurring an exogenous cost. One possible foundation for this cost could be a loss of safety net coverage associated with shifting activity away from the regulated sector that we model in this paper.

2.3 Optimal Policy Analysis

2.3.1 Efficient Allocation

We start by characterizing the efficient allocation. This allocation will serve as a benchmark for our normative analysis. In presenting this allocation, we abstract from the type dimension s of agents, since it is unrelated to their preferences. The efficient allocation solves the following problem:

$$\max_{c_1(0),c_2(0),c_1(1),c_1(2)} \pi U\left(c_1(0),c_2(0),0\right) + (1-\pi)U\left(c_1(1),c_2(1),1\right) \tag{2.22}$$

subject to

$$\pi\left[c_1(0)+\frac{c_2(0)}{\hat{R}}\right] + (1-\pi)\left[c_1(1)+\frac{c_2(1)}{\hat{R}}\right] \leq e,$$

and $c_1(0), c_2(0), c_1(1), c_2(1) \geq 0.$

The solution to this problem is described in the lemma below.

Lemma 2 (Efficient Allocation)**.** *The solution to the planning problem satisfies* $e < c_1^*(0) < c_2^*(1) < \hat{R}e$ *and* $c_2^*(0) = c_1^*(1) = 0$*, with* $u'(c_1^*(0)) = \rho\hat{R}u'(c_2^*(1))$*.*

Proof. See Appendix A.2.

Thus, as is standard under Diamond-Dybvig preferences, the allocation features zero date 2 consumption of impatient agents, zero date 1 consumption of patient agents, and risk sharing between patient and impatient agents that is consistent with an equalization of the social marginal rate of transformation $1/\hat{R}$ and the marginal rate of substitution $\rho u'(c_2^*(1))/u'(c_1^*(0))$.

2.3.2 Optimal Safety Net under Commitment

We now turn to analyzing decentralized equilibria. We start by assuming that the government can commit at date 0 about its date 1 liquidity provision policy. This will be helpful to highlight the role of the government's inability to commit in our analysis of the design of the optimal safety net.

In this case, after the government sets γ and credibly announces a future liquidity provision policy $B^c(\ell)$ at date 0, private agents make investment choices. Recall that agents know whether they are protected at the time of making their date 0 investment choice. As discussed earlier, households draw their type randomly and thus cannot choose to become protected or unprotected.

Under commitment, the government chooses the policy $\{\gamma^c, B^c(\ell), \mathcal{B}^c\}$ to attain the competitive equilibrium with highest time 0 utility.

Proposition 1 (Optimal policy under commitment)**.** *A safety net architecture covering all agents ($\gamma^c = 1$), together with a commitment to provide an amount of public liquidity $B^c(\ell) = \mathcal{B}^c = (1-\pi)c_2^*(1)$, achieves the efficient allocation described in Lemma 2.*

Proof. See Appendix A.3.

This proposition shows that it is optimal to cover all agents and that the appropriate amount of liquidity provision achieves the efficient allocations. The latter result is related to Yared (2013), who established that under a weaker version of Assumption 2, a fiscal policy scheme equivalent to our credit facility can achieve the efficient allocations when the government has commitment.[17] Proposition 1 indicates that if the government were able to commit to a future liquidity provision policy, it would not leave any agent outside the safety net. In fact, setting a boundary between protected agents and unprotected agents not only is redundant but also would deliver strictly lower ex ante welfare.[18]

Below, we relax the assumption that the government can commit to its liquidity provision policy and show that having a smaller safety net becomes strictly optimal. To put the results below into perspective, it is useful to note that under commitment, the amount of liquidity provision that the government commits to provide puts a lower bound on the amount of short assets that agents choose to invest in. If agents were to invest less in short assets than the level associated with the efficient allocation and were to end up being impatient, they would become credit constrained. This will contrast with the outcome that prevails when the government lacks commitment. As we show below, under discretion the government will ex post relax agents' credit constraints *unconditionally* (i.e., for any values of their investment choice). Anticipating the reaction of the government, agents will invest too little in short assets in the initial period. The inability of the government to commit to the extent of an ex post public liquidity provision will create a rationale for optimal management of the safety net.

2.3.3 Optimal Safety Net under Discretion

To analyze optimal policy under discretion, we proceed by backward induction. We start by solving for the government's optimal liquidity provision policy at date 1 when it is not

[17]Without this assumption, Yared (2013) finds that the government, despite not reaching the efficient allocation, should still restrict public debt issuances to prevent underinvestment in liquid assets.

[18]To see this, note that an unprotected impatient agent's consumption cannot exceed e, which is strictly lower than the impatient agents' consumption in the efficient allocation.

bound by past commitments. We then move back to date 0 choices and characterize time-consistent equilibria, for a given safety net architecture represented by the value of γ. Finally, we characterize the optimal ex ante choice of γ.

Ex post Policy: Liquidity Provision

After date 0 choices have been made, the government chooses the liquidity provision policy rule $B^d(\ell)$ to maximize the average welfare of unprotected and protected agents subject to the private sector's date 1 response to its actions. The government solves

$$\max_{\{B_j\}_{j\in[0,\gamma]}} \quad \int_0^\gamma \left[\pi \mathcal{V}_1\left(P,\ell_i,0,X\right)+(1-\pi)\,\mathcal{V}_1\left(P,\ell_i,1,X\right)\right]di \qquad (2.23)$$

$$+\int_\gamma^1 \left[\pi \mathcal{V}_1\left(U,\ell_i,0,X\right)+(1-\pi)\,\mathcal{V}_1\left(U,\ell_i,1,X\right)\right]di,$$

where $\mathcal{V}_1(\cdot)$ is given by our definition of a continuation equilibrium.

The following proposition establishes that an optimal ex post policy always features a full relaxation of impatient protected agents' effective borrowing constraints at date 1.

Proposition 2 (Optimal ex post bailout)**.** *An equilibrium with an optimal ex post policy features a full relaxation of impatient protected agents' effective credit constraints,* $B^d(\ell) = \hat{R}(1-\ell)e$*. Further, the equilibrium bond price is given by*

$$q(X) = \min\left\{\frac{1}{\hat{R}}\frac{1-\pi}{\pi}\frac{\gamma L^P+(1-\gamma)L^U}{\gamma(1-L^P)},1\right\}, \qquad (2.24)$$

and the equilibrium consumption of protected agents is given by

$$\mathcal{C}_1(P,\ell,0,X) \;=\; \ell e + q(X)\hat{R}(1-\ell)e, \qquad (2.25)$$

$$\mathcal{C}_2(P,\ell,1,X) \;=\; \hat{R}(1-\ell)e + \frac{\ell e}{q(X)}. \qquad (2.26)$$

Proof. See Appendix A.4.

Proposition 2 establishes that it is always optimal for the government to provide an amount of public liquidity that makes a protected agent unconstrained in a date 1 continuation equilibrium. The intuition for the ex post optimality of fully relaxing constraints is as follows. For $B_i < \hat{R}(1-\ell_i)e$, increasing B_i always increases the equilibrium consumption of some agent without decreasing the equilibrium consumption of another agent. To see this, we distinguish the situations in which $q = 1$ from the ones in which $q < 1$. When $q = 1$, an increase in B_i increases the equilibrium consumption of the protected impatient agent i while leaving the equilibrium consumption of other agents unchanged. The increase in agent i's consumption is the result of a borrowing constraint relaxation at a locally unchanged interest rate. When $q < 1$, on the other hand, an increase in B_i increases the equilibrium consumption of all patient agents while leaving the equilibrium consumption of impatient agents unchanged. The increase in patient agents' consumption results from the upward pressure on the return on government debt from date 1 to date 2 (i.e., q is decreasing in B_i). It follows that the government's value function is strictly increasing in B_i for $B_i < \hat{R}(1 - \ell_i)e$. For $B_i \geq \hat{R}(1 - \ell_i)e$, on the other hand, equilibrium consumption allocations do not locally depend on B_i. It follows that the debt issuance policy $B^d(\ell) = \hat{R}(1 - \ell)e$ is optimal.

A higher level of public liquidity provision is thus always desirable ex post up to the point where protected agents' effective credit constraints are fully relaxed. This is true *for any level of private investment*. In the absence of commitment, an optimal public liquidity provision policy hence works as insurance provided freely to protected agents. This contrasts with the optimal policy under commitment, which offers a limited amount of insurance. This extra layer of insurance present under discretion will distort ex ante incentives.

As we will see below, the moral hazard costs induced by discretion in public liquidity provision depend on the size of the protected sphere. In the next sections, we provide a sharp analytical characterization of this relationship and analyze the key trade-offs involved in the optimal setting of the size of the safety net.

Time-Consistent Equilibrium

After the government has set γ at date 0, private agents make investment choices. Agents know γ and forecast aggregate actions L^P, L^U to form beliefs about $q(X)$. They also rationally anticipate the ex post public liquidity provision policy rule $B^d(\ell)$. We can define a time-consistent equilibrium as follows:

Definition 3 (Time-consistent equilibrium for given safety net γ)**.** *For a given γ, a time-consistent equilibrium is a liquidity provision policy $B^d(\ell)$, a bond price $q(X)$, consumption policies $\mathcal{C}_1(s, \ell, \theta, X)$, $\mathcal{C}_2(s, \ell, \theta, X)$ and investment portfolio ℓ such that:*

1. *$B^d(\ell)$ solves (2.23),*
2. *ℓ solves (2.21),*
3. *$\mathcal{V}_1(s, \ell, \theta, X)$, $\mathcal{C}_1(s, \ell, \theta, X)$, $\mathcal{C}_2(s, \ell, \theta, X)$, $q(X)$, and $B^d(\ell)$ are a continuation equilibrium according to Definition 1.*

Using Proposition 2's result that $B^d(\ell) = \hat{R}(1-\ell)e$, the time-consistent equilibrium for given γ can be conveniently solved for in closed form, as summarized in the following proposition.

Proposition 3 (Characterization of time-consistent equilibria for given safety net γ)**.** *In a time-consistent equilibrium, unprotected agents always invest all of their endowment into the short asset:* $L^U = 1$*. For other variables, we can distinguish between three mutually exclusive regions, characterized by boundaries* $0 < \underline{\gamma} < \overline{\gamma} < 1$*, with* $\underline{\gamma} \equiv \frac{1-\pi}{1-\pi+\hat{R}\pi}$ *and* $\overline{\gamma} \equiv 1-\pi$*:*

- *Region I (*$0 \leq \gamma < \underline{\gamma}$*): protected agents invest* $L^P = 0$*, the date 1 bond price is* $q = 1$*, and the consumption allocations are* $c_1^U(0) = c_2^U(1) = e$ *and* $c_1^P(0) = c_2^P(1) = \hat{R}e$*.*
- *Region II (*$\underline{\gamma} \leq \gamma \leq \overline{\gamma}$*): protected agents invest* $L^P = 0$*, the date 1 bond price is* $q = \frac{1}{\hat{R}}\frac{(1-\pi)(1-\gamma)}{\pi\gamma}$*, and the consumption allocations are* $c_1^U(0) = e$*,* $c_2^U(1) = \hat{R}\frac{\pi\gamma}{(1-\pi)(1-\gamma)}e$*,* $c_1^P(0) = \frac{(1-\pi)(1-\gamma)}{\pi\gamma}e$*, and* $c_2^P(1) = \hat{R}e$*.*
- *Region III (*$\overline{\gamma} < \gamma \leq 1$*): protected agents invest* $L^P = \frac{\pi+\gamma-1}{\gamma}$*, the date 1 bond price is* $q = 1/\hat{R}$*, and the consumption allocations are* $c_1^U(0) = c_1^P(0) = e$ *and* $c_2^U(1) = c_2^P(1) = \hat{R}e$*.*

Proof. See Appendix A.5.

The equilibrium of the model takes different forms depending on the size of the protected sphere γ. In all of the cases that arise, unprotected agents always invest all of their endowment in the short asset at date 0. We note that in the laissez-faire benchmark where all agents are unprotected ($\gamma = 0$), everyone invests his entire endowment into the short asset ($L^U = 1$) and consumes an amount equal to that endowment whether hit by a liquidity shock or not at date 1 ($c^U(0) = c^U(1) = e$). Thus, the laissez-faire benchmark features an extreme form of self-insurance that results in clear efficiency losses relative to the efficient allocation. We now discuss equilibrium properties in the different regions.

Region I When the fraction of protected agents is smaller than a threshold $\gamma < \frac{1-\pi}{1-\pi+\hat{R}\pi}$, we say that there is a *small protected sphere.* In this case, the demand for government bonds by patient unprotected agents at date 1 is large enough to push the interest rate down to its lower bound $1/q = 1$. In this region, impatient protected agents benefit from fully relaxed credit constraints *and* a low interest rate at date 1, which allow them to transfer the date 2 proceeds of their long investment back into date 1 one for one. On the other side of the trade, patient unprotected agents are not able to earn a return higher than the storage technology between date 1 and date 2 on the proceeds of their date 0 short investment. Thus, in equilibrium, protected agents always end up consuming $\hat{R}e$, and unprotected agents always end up consuming e, whatever the realization of their liquidity shocks. This region features an extreme form of redistribution between the two spheres. A large unprotected sphere has the same consumption profile as in the laissez-faire benchmark (i.e., when $\gamma = 0$) and implicitly subsidizes a small set of protected agents.

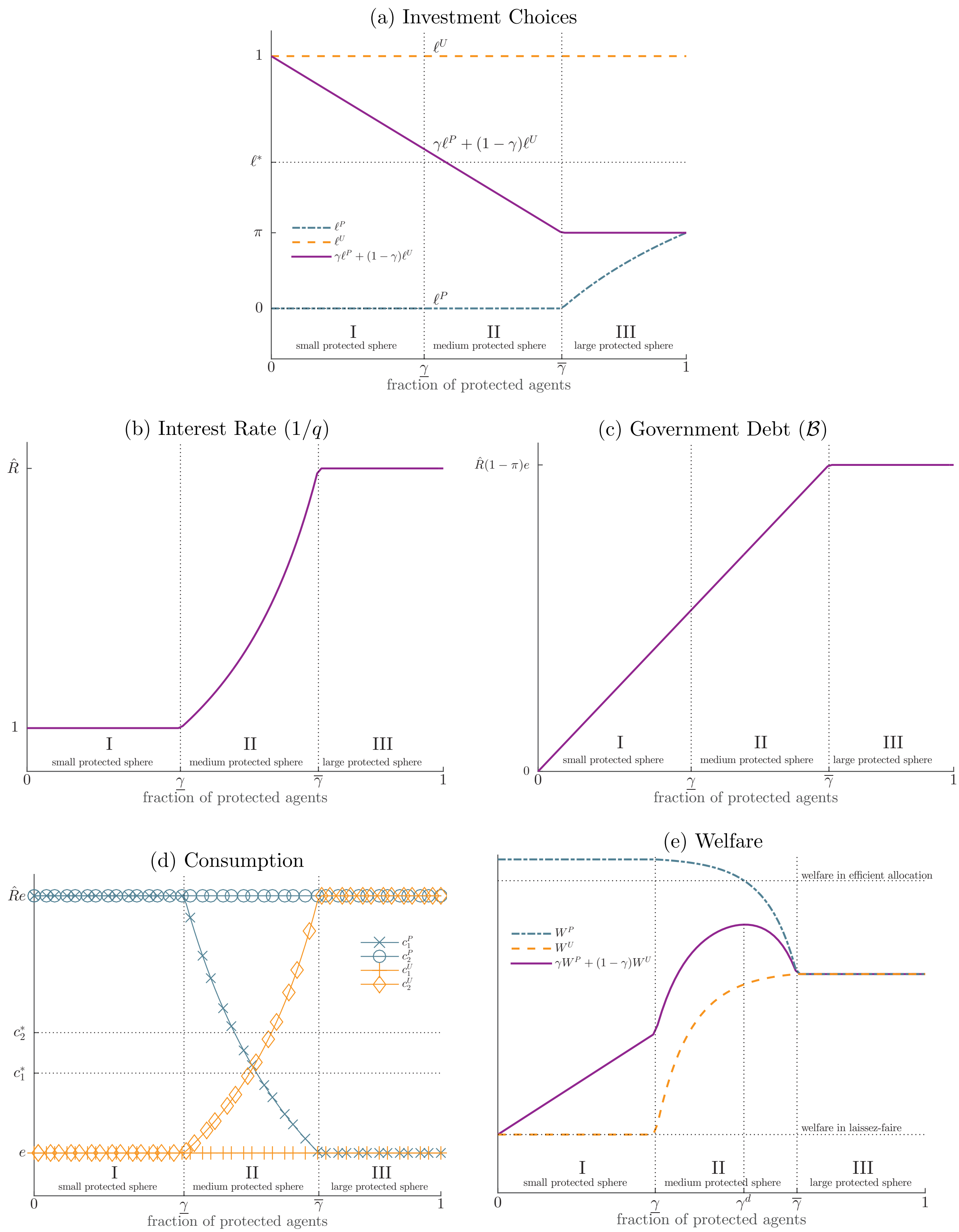

Figure 2.2: Policies, Welfare, Prices, and the Safety Net.

Region II When the fraction of protected agents is between two thresholds $\frac{1-\pi}{1-\pi+\hat{R}\pi} \leq \gamma \leq 1-\pi$, we say that there is a *medium protected sphere*. In this case, the mass of unprotected agents is still large enough for protected agents to completely rely on the short asset investment made by unprotected agents. However, the aggregate amount of debt issued by the government is not high enough relative to the supply of funds to push the date 1 interest rate to $\hat{R}$, so impatient protected agents, whose credit constraints are fully relaxed by the bailout, can enjoy a consumption level higher than e (the date 0 payoff on their long investment, $\hat{R}e$, is worth more than e when discounted into date 1 at the prevailing interest rate). Patient unprotected agents, on the other hand, earn a positive return, albeit lower than $\hat{R}$, between date 1 and date 2 on the proceeds from their date 0 short investment. Their date 2 consumption is therefore higher than the laissez-faire level of e, but it falls short of $\hat{R}e$. This discussion, together with Panel (d) of Figure 2.2, makes it clear that in this region, government bailouts induce a redistribution of resources from unprotected to protected agents, whose strength decreases with the share of protected agents γ. As γ increases, the gap between the equilibrium consumption of protected and unprotected agents narrows in both liquidity risk contingencies ($\theta = 0$ and $\theta = 1$). The fact that this gap is decreasing in γ reflects the fact that as γ increases, there are fewer and fewer unprotected agents who self-insure by investing in the short asset, which puts an increasing upward pressure on the date 1 interest rate.

Region III When the fraction of protected agents is greater than a threshold $\gamma > 1-\pi$, we say that there is a *large protected sphere*. Protected agents invest only a fraction $L^P = \frac{\pi+\gamma-1}{\gamma}$ of their date 0 endowment in the short asset. This fraction is equal to zero when $\gamma = 1-\pi$, is increasing in γ, and reaches π when $\gamma = 1$. Panel (e) of Figure 2.2 represents the date 0 investment choice of agents as a function of the size of the protected sphere γ. Protected agents anticipate being bailed out by the government at date 1. This a priori eliminates their incentive to self-insure by investing in the short asset. However, someone needs to invest in the short asset to support the consumption of impatient agents at date 1, and when the number of unprotected agents in the economy is small, protected agents need to do their share of short investment at date 0. Given a full relaxation of credit constraints by the government ex post, for there to be an incentive to invest in the short asset for protected agents, the return on government bonds between date 1 and date 2, $1/q$, needs to equal the return on the long asset $\hat{R}$. Panel (d) of Figure 2.2 represents the consumption allocations of agents as a function of the size of the protected sphere γ. In this region, all impatient agents consume e, and all patient agents consume $\hat{R}e$. This consumption allocation strictly dominates the allocation achieved by the benchmark economy without government intervention ($\gamma = 0$). Perhaps surprisingly, protected agents are not better off than unprotected agents in this case, even though the former benefit from a public liquidity provision and the latter do not. Despite not benefiting directly from a public liquidity provision, unprotected agents benefit from it indirectly through the price system. In this region, the government issues aggregate amounts of public debt that are sufficiently high to push the date 1 interest rate on government bonds to its upper bound $\hat{R}$. Unprotected agents who turn out to be patient are thus able to earn a return of $\hat{R}$ between date 1 and date 2 on the proceeds from their date 0 short investment. This enables them to achieve the same equilibrium consumption profile as protected agents. Panel (b) of Figure 2.2 displays the interest rate as a function of the size of the protected sphere, and Panel (c) represents aggregate public debt issuance.

To see more clearly why the allocations under commitment are not an equilibrium outcome under discretion, consider the date 0 decision of a protected agent in a fully protected economy, when all other agents hypothetically make the same investment choice as under commitment. Note that this collective investment choice leads to an interest rate such that $1/q < \hat{R}$ ex post. And given that the government will always relax the protected individual's credit constraint ex post, it is strictly optimal for this agent to invest all if his endowment into the long asset at date 0. By doing so, he is better off in any contingency. If he turns out patient, he enjoys a strictly higher date 2 consumption of $\hat{R}e > c_2^*(1)$. If he turns out impatient, he can freely borrow against his date 2 investment income $\hat{R}e$ at an interest rate $1/q = c_2^*(1)/c_1^*(0)$ and hence will also enjoy a strictly higher date 2 consumption of $q\hat{R}e = c_1^*(0)\,\hat{R}e/c_2^*(1) > c_1^*(0)$. Since there is an incentive for individual deviations, this, of course, cannot constitute an equilibrium. The fundamental problem is that agents free ride on other agents' short investments when the government lacks commitment about its liquidity provision policy, exactly as they would if side trades were available (Jacklin, 1987).[19]

Now that we have fully characterized time-consistent equilibria conditional on the size of the protected sphere γ, we can finally turn to the analysis of the optimal choice of this parameter by a welfare benevolent government at date 0.

Ex ante Policy: Size of the Protected Sphere

We now consider the date 0 choice of a welfare benevolent government that sets the size of the protected sphere γ while anticipating the response of agents in a time-consistent equilibrium.[20]

[19] This free rider problem is distinct from the coordination problem typically present in the literature on bailouts (e.g., Farhi and Tirole, 2012, Keister, 2016).

[20] We deliberately abstract from prudential policies that influence agents' portfolio choices at time 0.

The government solves

$$\mathcal{W}_0 = \max_{\gamma \in [0,1]} \gamma \mathcal{V}_0(P, X) + (1 - \gamma)\mathcal{V}_0(U, X) \tag{2.27}$$

subject to

$$X = (\gamma, \ell(P, X), \ell(U, X)).$$

The following proposition contains our main result.

Proposition 4 (Optimal ex ante size of protected sphere)**.** *The optimal size of the protected sphere is interior, satisfying* $\underline{\gamma} < \gamma^d < \overline{\gamma}$.

Proof. See Appendix A.6.

Proposition 4 establishes that the optimal size of the protected sphere is not a corner solution. The optimal safety net architecture from an ex ante perspective features positive masses of protected and unprotected agents. The intuition for this result can most easily be built by considering how welfare depends on γ within each of the three regions defined in Section 2.3.3.

Panel (e) of Figure 2.2 represents ex ante average welfare as a function of the size of the protected sphere γ. We first note that ex ante average welfare is continuous in γ since all equilibrium consumption allocations are continuous in γ. In region I, protected agents always consume $\hat{R}e$, whereas unprotected agents always consume e. Hence in that region, the welfare of protected agents is strictly higher than that of unprotected agents. It follows that ex ante average welfare is strictly increasing in γ in that region, with a maximum of $\underline{\gamma}u(e)+(1-\underline{\gamma})u(\hat{R}e)$ at the upper bound $\gamma = \underline{\gamma}$. Safety net architectures with small protected sectors strictly dominate the laissez-faire benchmark ($\gamma = 0$) because protected agents are strictly better off than in the laissez-faire benchmark and unprotected agents are no worse off. In region III, protected and unprotected agents consume the same amounts in equilibrium in the contingency in which they are patient. Likewise, they consume the same in the contingency in which they are impatient. It follows that within this region, ex ante average welfare is constant with respect to γ and given by $\pi u(e) + (1 - \pi)\rho u(\hat{R}e)$. We also note that since $\underline{\gamma} < 1 - \pi$, ex ante average welfare is strictly higher in region III than in region I. By the continuity of ex ante average welfare with respect to γ, it must therefore be that the optimal size of the protected sphere falls in region II. But a key feature of our first result in Proposition 4 is that the optimal size of the protected sphere lies in the interior of region II rather than at its right boundary, so that the optimal safety net architecture strictly dominates a fully protected economy.

The ex ante optimality of restricting the scope of protection in the economy can be drawn from the fact that the left derivative of ex ante average welfare is strictly negative at $\gamma = \overline{\gamma} =$

$1-\pi$. The intuition has to do with the improvement in risk sharing induced by the splitting of agents between a protected and an unprotected sphere. In region II, a marginal decrease in γ causes a decrease in the equilibrium date 1 interest rate. Since in this region impatient protected agents borrow in equilibrium from patient unprotected agents, this decrease in the interest rate redistributes wealth from the latter to the former.[21] At $\gamma = \overline{\gamma} = 1-\pi$, such a wealth redistribution is necessarily socially desirable given that (i) the masses of patient unprotected and impatient protected agents are equal (i.e., to $\pi(1-\pi)$), and (ii) relative to the efficient allocation, impatient agents consume too little and patient agents consume too much (a consequence of Assumption 1). Hence, a marginal decrease in the share of protected agents from $\gamma = 1-\pi$ unambiguously increases the ex ante average welfare criterion. It directly follows that the optimal size of the protected sphere γ^d must lie strictly between $\underline{\gamma}$ and $\overline{\gamma}$. This intuition is illustrated in Panel (e) of Figure 2.2. There, it is apparent that directly to the left of $\gamma = \overline{\gamma} = 1-\pi$, the welfare increase for protected agents more than offsets the welfare decrease for unprotected agents, so that ex ante average welfare is strictly decreasing in γ.

The optimal size of the safety net γ^d then balances the risk-sharing benefits just described with the costs induced by a distortion resulting from a dispersion in the consumption of protected and unprotected agents. In region III, this distortion is absent, since all impatient agents consume e and all patient agents consume $\hat{R}e$. But in region II, when γ is lowered starting from $\overline{\gamma}$, consumption gaps of $c_1^P(0) - c_1^U(0) = (1-\pi-\gamma)e/(\pi\gamma)$ and $c_2^P(1) - c_2^U(1) = (1-\pi-\gamma)\hat{R}e/[(1-\pi)(1-\gamma)]$ emerge between protected and unprotected agents. These gaps are decreasing in γ. Therefore, the optimal safety net γ^d can be seen as the point where, from the perspective of the government, the benefits arising from the (pecuniary externality induced) redistribution between unprotected patient agents and protected impatient agents just offset the costs arising from the distortion between protected and unprotected agents.

How does the safety net vary with fundamental parameters of the economy? The following proposition characterizes how the size of the safety net varies with the probability of experiencing a liquidity shock and the return on the long asset.

Proposition 5 (Comparative statics)**.** *The optimal size of the protected sphere γ is strictly decreasing in the probability of experiencing a liquidity shock (i.e., $\partial\gamma^d/\partial\pi < 0$) and weakly decreasing in the long asset return (i.e., $\partial\gamma^d/\partial\hat{R} \leq 0$).*

Proof. See Appendix A.7.

The interpretation of these comparative statics results can be framed in terms of the trade-off described above. We start by discussing the partial effect of π. Consider a marginal increase in

[21] The effect of this redistribution on equilibrium consumption can be inferred from Panel (d) of Figure 2.2.

π from π_0 to π_1, and label the optimal safety nets associated with π_0 and π_1 by γ_0^d and γ_1^d, respectively. At γ_0^d, the risk-sharing benefits of lowering γ further are larger for π_1 than for π_0, both because the interest rate becomes more sensitive to γ as π grows (i.e., $\partial(1/q)/(\partial\gamma\partial\pi) > 0$) and because the gap in consumption between patient unprotected and impatient protected agents increases with a higher π (i.e., $c_2^U(1) - c_1^P(0)$ is increasing in π). Meanwhile, the distortionary costs of lowering γ further diminish, as the consumption gaps between protected and unprotected agents get smaller as π grows (i.e., $c_1^P(0) - c_1^U(0)$ and $c_2^P(1) - c_2^U(1)$ are both decreasing in π). In a nutshell, as the probability of liquidity shocks increases, manipulating the size of the safety net to improve social insurance becomes simultaneously more powerful, more beneficial, and less costly. As a result, γ_1^d must be strictly lower than γ_0^d.

The interpretation of the partial effect of $\hat{R}$ is more subtle because it involves counteracting effects. Consider a marginal increase in $\hat{R}$ from $\hat{R}_0$ to $\hat{R}_1$, and label the optimal safety nets associated with $\hat{R}_0$ and $\hat{R}_1$ by γ_0^d and γ_1^d, respectively. At γ_0^d, the risk-sharing benefits of lowering γ further are larger for $\hat{R}_1$ than for $\hat{R}_0$, both because the interest rate becomes more sensitive to γ as $\hat{R}$ grows (i.e., $\partial(1/q)/(\partial\gamma\partial\hat{R}) > 0$) and because the gap in consumption between patient unprotected and impatient protected agents increases with a higher $\hat{R}$ (i.e., $c_2^U(1) - c_1^P(0)$ is increasing in $\hat{R}$). However, this time the distortionary costs of lowering γ further increase with $\hat{R}$ as well, as the consumption gap between protected and unprotected agents who are patient gets larger as $\hat{R}$ grows (i.e., $c_1^P(0) - c_1^U(0)$ does not vary with $\hat{R}$ but $c_2^P(1) - c_2^U(1)$ is increasing in $\hat{R}$). In a nutshell, as the return on the long asset increases, manipulating the size of the safety net to improve social insurance becomes more powerful and more beneficial, but also more costly. When risk aversion is exactly one, these two effects happen to cancel out, whereas when it is larger than one, the first effect dominates. As a result, we have $\gamma_1^d \leq \gamma_0^d$, with $=$ only if $-cu''(c)/u'(c) = 1 \;\forall c > 0$.

Extension and Robustness

In this section, we consider the relaxation of two previously maintained assumption and explain why our main result is robust to these changes.

Absence of commitment over γ. We have assumed throughout that the government could commit not to bail out a measure $1-\gamma$ of agents. Here, we show that even if the government has the possibility of changing the value of γ ex post, it has no incentive to deviate from γ^d when the market anticipated that the government would implement γ^d. To be clear, we maintain the assumption that if an unprotected agent individually deviates at time 0 by investing in long assets, he would not receive a bailout from the government.[22]

It is straightforward to show that the value γ^d derived in Section 2.3.3 under the assumption that the government could commit to its choice of γ remains implementable when the government can renege on its announcement. For a given value of γ anticipated by agents at date 0, the government's date 1 problem is now

$$\max_{\tilde{\gamma},\{s_j\}_{j\in[0,1]},\{B_j\}_{j\in[0,\tilde{\gamma}]}} \int_0^1 \gamma\left[\pi\mathcal{V}_1\left(s_i, L^P, 0, X\right) + (1-\pi)\mathcal{V}_1\left(s_i, L^P, 1, X\right)\right] + (1-\gamma)\left[\pi\mathcal{V}_1\left(s_i, L^U, 0, X\right) + (1-\pi)\mathcal{V}_1\left(s_i, L^U, 1, X\right)\right] di \tag{2.28}$$

subject to $\int_0^1 \mathbb{I}_{\{s_i=P\}} di = \tilde{\gamma}$, for $X = (\gamma, L^P, L^U)$.

To see that the value of γ chosen in problem (2.27) remains optimal ex post, we observe that substituting $X = (\gamma^d, 0, 1)$ into (2.29) gives a government's ex post payoff of[23]

$$\tilde{\mathcal{W}} = \begin{cases} \mathcal{W}_0 - (\gamma^d - \tilde{\gamma})\pi \underbrace{\left[u\left(q(X)\hat{R}e\right) - u(0)\right]}_{>0} & \text{for } \tilde{\gamma} < \gamma^d \\ \mathcal{W}_0 & \text{for } \tilde{\gamma} \geq \gamma^d, \end{cases}$$

where $\mathcal{W}_0$ is the government's payoff under commitment over γ in (2.27). Hence, reneging upon its date 0 announcement to choose a lower γ is not optimal for the government. The reason is that in doing so, the government would leave some impatient agents who anticipated to be protected, and hence invested all of their endowment in the long asset, with zero consumption. Similarly, the government cannot increase its payoff by choosing a higher γ ex post. This time the reason is that reneging on the announcement would extend the safety net to impatient agents who do not need it anyway because they anticipated being unprotected and hence invested all of their endowment in the short asset.

Private market. We have assumed throughout that enforcement problems are such that private borrowing is fully precluded. Under a milder assumption on enforcement, we could have instead assumed that private credit is possible but constrained. In that case, (2.13) would be replaced by

$$\kappa(s, \ell, X) \equiv \begin{cases} \bar{d} & \text{for } s = U \\ q(X)[B(\ell) + \bar{d}] & \text{for } s = P, \end{cases} \tag{2.29}$$

[22] That is, the government is able to commit to not bail out an agent who, following the government's ex post choice of γ, ends up belonging to the unprotected sphere.

[23] It is straightforward to see that it is optimal to choose $\{s_j\}_{j\in[0,1]}$ such that $\mathbb{I}_{\{s_i=P\}}$ is weakly decreasing in i, for it would not be optimal to "unprotect" more than the minimum measure of agents who expected to be protected.

for $\bar{d} > 0$. For a sufficiently tight private borrowing constraint $\bar{d}$,[24] all of our qualitative results would apply. In particular, despite impatient unprotected agents being hurt by higher interest rate, it would still be ex post optimal for the government to fully relax impatient protected agents' credit constraint so that the bailout policy of Proposition 2 would become $B^d(\ell) = \hat{R}(1-\ell)e - \bar{d}$. Given the possibility of limited private credit, unprotected agents' equilibrium investment in the short asset would be lowered to $L^U = 1 - \bar{d}/(\hat{R}e)$, as would the thresholds of Proposition 3, which would generalize to $\underline{\gamma}(\bar{d}) \equiv [1 - \bar{d}/(\hat{R}e)]^{-1}[\underline{\gamma} - \bar{d}/(\hat{R}e)]$ and $\bar{\gamma}(\bar{d}) \equiv [1 - \bar{d}/(\hat{R}e)]^{-1}[\bar{\gamma} - \bar{d}/(\hat{R}e)]$.[25] Our main result on the optimality of an interior safety net would hence be preserved.

2.4 Conclusion

In this paper, we presented a model of financial safety net. We study a workhorse model of liquidity demand with limited private credit, in which the government chooses the share of investors that will be eligible for liquidity support. Our analysis delivers the following key results. First, if the government can commit about future policies, the optimal financial safety net covers all agents. Second, when the government lacks commitment, the government provides excessive liquidity to agents protected by the safety net. Third, in the absence of commitment, the optimal financial safety net includes only a subset of agents. Compared with an economy where all agents are protected, this results in more liquid asset portfolios, lower interest rates, and superior social insurance.

Our analysis underscores the importance of the institutional design of central banks' framework for liquidity provision. Following the financial crisis, there have been calls to expand the safety net to include shadow banks. Our paper presents a cautionary note to this view and highlights that expanding the safety net could lead to underinvestment in liquid assets and too little risk sharing. We do abstract, however, from important elements, such as liquidity regulation and issues of regulatory arbitrage. In Grochulski and Zhang (2015), for example, liquidity regulation improves risk sharing, but investors can bypass regulation at a cost, by engaging in shadow banking activities. An interesting approach would be to investigate the interaction between liquidity regulation, regulatory arbitrage, and financial safety nets, and how this interaction affects risk sharing and moral hazard.[26]

[24] More precisely, for $\bar{d} \leq e(1-\pi)\left\{u'(e)/[\rho u'(\hat{R}e)] - 1\right\}^{-1}$.

[25] Details are available from the authors upon request.

[26] In a very recent paper, Farhi and Tirole (2017) make progress in this direction.

Chapter 3

Monetary Policy in Sudden Stop-prone Economies

3.1 Introduction

Countercyclical stabilization policy constitutes a central tenet of macroeconomics. Yet, unlike in advanced economies, monetary policy appears to be procyclical in many emerging markets, meaning that it is often expansionary in booms and contractionary in recessions (Kaminsky et al., 2004). In this paper, I propose a theory to explain this contrasting empirical regularity based on the premise that governments lack the ability to commit to future policies and that emerging markets economies frequently experience capital account reversals, often referred to as Sudden Stops (Calvo, 1998), associated with adverse balance sheet effects originating from real exchange rate depreciations.

In my model, in the midst of a Sudden Stop, the government is inclined to pursue a contractionary monetary policy to appreciate the real exchange rate and support the value of the collateral accepted by foreign lenders, in an effort to relax binding financial constraints faced by domestic agents. Ex ante, however, the expectation of such monetary policy interventions during Sudden Stops aggravates overborrowing problems. As a result, the government finds it attractive during tranquil times to pursue an expansionary monetary policy that shifts demand towards the non-tradable sector and away from tradable goods, so as to mitigate overborrowing by private agents. In contrast, absent credit frictions, the government would aim for a perfect stabilization of output and prices at all times, in line with the standard prescription of the New Keynesian literature.

In addition to offering a theory that explains monetary policy procyclicality in emerging markets, I conduct an evaluation of the performance of alternative policy regimes. This analysis suggests that the procyclical discretionary monetary policy regime is dominated by several alternative candidates. First, I find that, relative to the procyclical discretionary monetary regime, committing to an inflation targeting regime generates welfare gains and reduces the frequency of financial crises, despite increasing their severity. Second, I also find that absent commitment, the ability to resort to capital controls to deal with overborrowing reduces the degree of procyclicality of discretionary monetary policy, and brings welfare gains associated with a drop in both the frequency and severity of financial crises. My analysis thus supports the view that procyclical monetary policy, despite being optimal under discretion, may be costly for emerging market economies.

My model builds on the small open economy real model of Sudden Stops of Mendoza (2002), in which domestic households consume tradable and non-tradable goods, and face a borrowing constraint linked to the real exchange rate. In this model, Sudden Stops result from an adverse feedback loop between tightening credit constraints and real exchange rate depreciations following a sequence of bad shocks. It is well understood that this class of models feature overborrowing, as private agents fail to internalize their contribution to systemic risk stemming from the influence of current borrowing decisions on the economy's future borrowing capacity via the real exchange rate (Korinek, 2009, Bianchi, 2011). To study monetary policy in this environment, I introduce monopolistically competitive firms and nominal rigidities in the non-tradable production sector.

I start by studying the optimal monetary policy problem of a government that cannot commit to future policies in an economy into which capital flows freely (i.e., in the absence of capital controls). The discretionary monetary policy in this environment can easily be related to the standard targeting rules developed in the New Keynesian literature (see Woodford, 2003). It expresses the path of the inflation rate as a function of a measure of the output gap (i.e., the labor wedge) and variables that account for the relevant financial conditions before and during financial crisis episodes.

This characterization of the discretionary monetary policy highlights the role of financial frictions in the design of monetary policy. In the absence of credit frictions, a price stability policy is always optimal. This finding is in line with the standard result in the New Keynesian literature suggesting that a policy that offsets distortion stemming from nominal rigidities is welfare-dominant when nominal rigidities and monopolistic competition are the only sources of frictions (see, e.g., Kollmann, 2002 and Schmitt-Grohé and Uribe, 2007). Credit frictions, however, create policy trade-offs between price stability and financial stability. The government only finds it optimal and time-consistent to follow a price stability policy in the knife-edge case where the inter-temporal elasticity of substitution is equal to the intra-temporal elasticity of substitution between tradable and non-tradable goods.

The manner in which financial conditions influence monetary policy crucially depends on the values of the elasticities of substitution. For standard values, the discretionary monetary policy involves a form of *pro*-cyclicality, being contractionary when a financial crisis occurs and expansionary ahead of a crisis. During financial crises, the real exchange rate falls and reduces significantly foreign borrowing capacity. Private agents, however, fail to internalize the general equilibrium benefits of a decrease in their demands for goods (i.e., *an aggregate demand externality*). A contractionary monetary policy reducing activity in the non-tradable sector serves to prevents too large a real exchange rate depreciation to sustain the value of collateral and tame balance sheet effects. Ahead of a potential crisis, private agents tend to overborrow because they fail to internalize how their private borrowing decisions affect the real exchange rate in the future, which in turn may lead to financial amplification and systemic risk during financial distress (i.e., *a pecuniary externality*). This overborrowing decision tends to make the real exchange rate overvalued from a financial fragility perspective. Generating a boom in the non-tradable sector reduces households' marginal propensity to consume the tradable good, discourages foreign borrowing and contains the rise in the current real exchange rate, which in turn mitigates future real exchange depreciations and prevents a large drops in borrowing capacity during future episodes of financial distress. This characterization helps account for the observation of Kaminsky et al. (2004) that monetary policy appears to be procyclical in emerging economies.

The availability of capital flow taxes modifies the features of the discretionary monetary policy. When capital flow taxes are used optimally, the conditions under which a price stability policy is desirable are less restrictive than when they cannot be used. Ahead of potential crises, capital inflow taxes are effective at correcting the pecuniary externality, and thus stabilizing prices is optimal. A policy trade-off between price stability and financial stability only arises during periods when the credit constraint is binding since capital controls are then of little help.

A quantitative analysis of the model delivers several results that contribute to the current discussion on the optimal design of monetary policy in emerging market economies, and the dilemma/trilemma debate in international finance (see, Farhi and Werning, 2014 and Rey, 2015). First, when capital controls cannot be used, relative to the discretionary monetary policy regime, there are substantial gains from committing over a simple targeting rule that stabilizes the producer price index, hereafter an inflation targeting regime. Such a commitment results in a lower unconditional probability of crises (4.3 percent versus 5.5 percent under the procyclical discretionary regime), but more severe crises (output drops by 29.7 percent versus 21.9 percent under the procyclical discretionary regime). Overall, committing to inflation targeting regime delivers welfare gains relative to the procyclical discretionary regime.

Second, capital controls are always useful under both an inflation targeting policy and the discretionary monetary policy. This is because capital controls prevent excessive risk exposure (see Bianchi, 2011; Jeanne and Korinek, 2012; Bianchi and Mendoza, 2018), which reduces the volatility of the external accounts. Moreover, when the government has access to capital controls, relative to an inflation targeting regime, the discretionary monetary policy regime, which is no longer procyclical during tranquil times, delivers larger welfare gains and reduces both the incidence of crises (1.1 percent versus 1.3 percent under an inflation targeting regime with capital controls) and the severity of crises (total output drops by 17.2 percent versus 17.8 percent under an inflation targeting regime with capital controls).

This paper is related to the literature on the design of monetary policy in economies with financial frictions.[1] Christiano et al. (2004), Curdia (2007) Gertler et al. (2007) and Braggion et al. (2009) study monetary policy in times of crisis in frameworks where financial markets are incomplete and crises are unexpected one-shot events. Caballero and Krishnamurthy (2003), Aghion et al. (2004) and Benigno et al. (2011) examined the design of monetary policy in economies that last two or three periods and in which credit constraints become binding unexpectedly and remains binding afterwards. My contribution to this literature is to provide a characterization of the optimal monetary policy in normal times when financial crises are expected and endogenous.

Several other recent studies have explored monetary policy in dynamic environments featuring both nominal rigidities and financial frictions. Fornaro (2015) compares the performance of different monetary regimes and shows that a monetary regime that responds to financial conditions in times of crisis delivers higher welfare than an inflation targeting regime. In contrast, my main focus is on the analysis of discretionary monetary policy in the absence of capital controls. Ottonello (2015) studies exchange rate policy with optimal capital flow taxes in an economy featuring downward nominal wage rigidity and a collateral constraint. In the present paper, instead, the theoretical analysis mainly focus on the design of the optimal monetary policy in an environment in which the government does not have access to capital-controls and faces a time-inconsistency problem. Finally, Devereux et al. (2015) study optimal monetary and capital control policy in a related model where collateral constraints depend on the expected future value of assets and prudential capital flow management is never desirable, under flexible prices. In contrast, and in line with most of the literature, I consider a model where current prices matter for credit access and where, as a result, prudential capital flow management is generally beneficial.

[1] An earlier literature compared the performance of different monetary regimes in an environment with financial market imperfections (see, e.g., Clarida et al., 2002; Céspedes et al., 2004 and Moron and Winkelried, 2005).

This paper also relates to literature studying pecuniary externalities and inefficiencies resulting from the presence of market prices in borrowing constraints in real environments (see, e.g., Lorenzoni, 2008). Bianchi (2011), Korinek (2011) Benigno et al. (2013) and Bengui and Bianchi (2018) studied how this externality leads to overborrowing and showed to what extent capital control can restore constrained efficiency and reduce vulnerability to financial crises. Focusing on asset prices as a key factor driving debt dynamics and pecuniary externalities, Bianchi and Mendoza (2018) point out the time-inconsistency issues in macroprudential policies originating from the forward-looking nature of asset prices. Finally, my paper also relates to the work of Schmitt-Grohé and Uribe (2016), Farhi and Werning (2016) and Acharya and Bengui (2018) examining the use of taxes on financial transactions as a tool for managing aggregate demand in the presence of nominal rigidities and constraints on monetary policy. My paper draws upon to both strands of the literature and stands out by analyzing how monetary policy may be designed to simultaneously address both aggregate demand and pecuniary externalities.

The remainder of the paper is organized as follows: Section 3.2 describes the analytical framework. Section 3.3 presents the optimal policy analysis. Section 3.4 conducts the quantitative analysis. Section 3.5 concludes, and is followed by an extended appendix.

3.2 Model

Consider a dynamic model of a small open economy. Households in this economy consume two goods (a tradable good and a non-tradable good) and can also engage in borrowing from foreign investors. The tradable good can be exchange with the rest of the world and the non-tradable good is consumed by domestic agents only.

3.2.1 Households

The economy is inhabited by a continuum of mass one of identical households with preferences described by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \tag{3.1}$$

where $\beta \in (0,1)$ is a subjective discount factor, ℓ_t is labor supply and c_t denotes consumption. The period utility function takes the standard constant relative-risk-aversion form, with a relative-risk-aversion coefficient σ. The consumption good is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to an Armington-type CES aggregator:

$$c_t = \left[a(c_t^T)^{-\eta} + (1-a)(c_t^N)^{-\eta}\right]^{-\frac{1}{\eta}}, \quad \eta > -1, a \in (0,1) \tag{3.2}$$

The elasticity of substitution between tradable and non-tradable goods, which I will refer to as the intra-temporal elasticity, is $\gamma = 1/(\eta + 1)$. Households receive a stochastic endowment of tradable goods y_t^T, profits Φ_t from the ownership of firms producing the non-tradable good, and labor income in each period t. Households can trade internationally with foreign investors in one period non state-contingent bonds denominated in units of the foreign currency. The bond pays an interest rate R_t, determined exogenously in the world market. The tradable endowment and the gross interest rate are the only sources of uncertainty, and the vector of shocks $s_t \equiv (y_t^T, R_t) \subseteq \mathbb{R}_+^2$ is assumed to follow a first-order Markov process. The sequential budget constraint of the household in terms of the domestic currency is given by:

$$P_t^T c_t^T + P_t^N c_t^N + \frac{\mathcal{E}_t b_{t+1}}{R_t} = P_t^T y_t^T + W_t \ell_t + \Phi_t + \mathcal{E}_t b_t \tag{3.3}$$

where b_{t+1} denotes bond holdings that households choose at the beginning of time t. $\mathcal{E}_t$ is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency. In terms of the domestic currency, P_t^T is the nominal price of the tradable good, P_t^N the nominal price of the non-tradable good and W_t the nominal wage rate. The law of one price holds for the tradable good, which implies that $P_t^T = \mathcal{E}_t P_t^{T*}$ where P^{T*} denotes the foreign currency price of the tradable good. For simplicity, the foreign-currency price of the tradable good is assumed to be constant and normalized to one. It follows that the domestic currency price of the tradable good is equal to the nominal exchange rate (i.e. $P_t^T = \mathcal{E}_t$).

Households' borrowing capacity denominated in units of tradables is limited by a fraction κ of total income composed of tradable income and non-tradable income

$$\frac{\mathcal{E}_t b_{t+1}}{R_t} \geq -\kappa \Big[P_t^T y_t^T + W_t \ell_t + \Phi_t \Big] \tag{3.4}$$

This credit constraint captures balance sheet effects and financial amplification via exchange rate depreciations described as one of the main vulnerability of emerging market economies during financial crises (see Korinek, 2011). This formulation of the borrowing constraint was introduced by Mendoza (2002). One motivation for this formulation of the credit constraint is that it can result from institutional features of credit markets or financial-market frictions (such as monitoring costs, bankruptcy risk or imperfections in the judicial system) and captures the willingness of foreign lenders in such an environment to not allow borrowing beyond a certain limit tied to the borrower's income (see e.g., Bianchi, 2011 and Bianchi and Mendoza, 2018).[2]

The household's problem is to choose stochastic processes $\{c_t^T, c_t^N, b_{t+1}\}$ to maximize the

[2]As shown by Bianchi and Mendoza (2018), the credit constraint depends on current prices (rather than future prices) when the possibility of default arises at the end of the current period.

expected utility (3.1) subject to (3.2)-(3.4), taking as given the sequence of prices, profits, exchange rates, stochastic tradable endowments and interest rates, as well as the initial debt level b_0. Letting λ_t/P_t^T denotes the multiplier associated with the budget constraint (3.3) and μ_t/P_t^T the multiplier associated with the credit constraint (3.4), the household's optimal decision between tradable consumption and non-tradable consumption is given by:

$$p_t^N = \frac{1-a}{a}\left(\frac{c_t^T}{c_t^N}\right)^{1/\gamma} \tag{3.5}$$

where $p_t^N \equiv P_t^N/\mathcal{E}_t$ denotes the relative price of non-tradables in terms of tradables. Similarly in what follows, $w_t \equiv W_t/\mathcal{E}_t$ represents the wage in terms of tradables and $\phi_t \equiv \Phi_t/\mathcal{E}_t$ is the firm profits in terms of tradables. The optimality condition (3.5) equates the marginal rate of substitution between the two goods, the tradable and the non-tradable, to their relative price. This condition describes the demand for the non-tradable good as a function of their relative price and the level of tradable absorption, and can be re-written as:

$$c_t^N = \left[\frac{a}{1-a}p_t^N\right]^{-\gamma} c_t^T \equiv \alpha(p_t^N)c_t^T.$$

Letting u_T and u_N denote the marginal utility of tradable consumption and non-tradable consumption respectively, the remaining household's first-order conditions are:

$$\frac{-u_\ell(t)}{u_N(t)} = \underbrace{\left[1+\frac{\kappa\mu_t}{u_T(t)}\right]}_{z_t}\frac{w_t}{p_t^N} \tag{3.6}$$

$$\lambda_t = u_T(t) \tag{3.7}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t \tag{3.8}$$

$$\mu_t \geq 0,\ \mu_t\left[b_{t+1} + \kappa\left(y_t^T + w_t\ell_t + \phi_t\right)\right] = 0. \tag{3.9}$$

The optimality condition for labor supply (3.6) equates marginal cost in terms of non-tradable consumption from working one additional unit with the marginal benefit, which includes the relative wage w_t/p_t^N and the relaxation effect on the credit constraint. The variable z_t indicates the wage multiplier of an increase in labor supply. In particular, when the credit constraint binds, an increase of one unit of labor relaxes the constraint by $\kappa\mu_t{\cdot}w_t$, and the wage multiplier of this increase in labor is greater than 1 ($z_t > 1$). The Euler equation for debt (3.8) states that the current shadow value of wealth equals the expected value of reallocating wealth to the next period plus an additional term that represents the shadow price of relaxing the credit constraint. Thus, conditions (3.6) and (3.8) show that the credit constraint introduces two distortions: an inter-temporal distortion arising from the presence of a credit constraint and an intra-temporal distortion that hinges on the wage income entering the credit constraint.

3.2.2 Production Sector

Non-tradable goods are supplied by firms, and denoted y_t^N. I introduce nominal rigidities in the non-tradable goods market by separating the sector into monopolistically competitive intermediate producers and perfectly competitive retailers. The non-tradable final good is produced by competitive firms that combine a continuum of non-tradable varieties indexed by $j \in [0, 1]$ using the constant returns to scale CES technology

$$y_t^N = \left(\int_0^1 {y_{j,t}^N}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$ is the elasticity of substitution between any two varieties. Each variety $y_{j,t}^N$ is produced by a monopolistically competitive producer using labor $h_{j,t}$ according to a linear production function $y_{j,t}^N = Ah_{j,t}$. These producers hire labor in a competitive market with wage W_t, but pays $(1 - \tau^h)W_t$ net of a labor subsidy. Cost minimization implies that each firm faces the same real marginal cost (or unitary cost): $mc_t = \frac{1-\tau^h}{A} \frac{W_t}{P_t^N}$.

Price setting The intermediate goods firms face sticky price setting à la Rotemberg (1982). Accordingly, each firm j faces a cost of adjusting prices which, when measured in terms of the final non-tradable good, is given by:

$$\frac{\varphi}{2} \left(\frac{P_{j,t}^N}{P_{j,t-1}^N} - 1 \right)^2 y_t^N$$

where φ is an adjustment cost parameter which determines the degree of nominal price rigidity and $P_{j,t}^N$ is the nominal price of variety j. Taking as given the sequence for mc_t, y_t^N and $\mathcal{E}_t$, the monopolistically competitive firm j optimally chooses the sequence of prices of its own variety, $P_{j,t}^N$, to maximize the stream of its expected discounted profit given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[\left(\frac{P_{j,t}^N}{P_t^N} - mc_t \right) y_{j,t}^N - \frac{\varphi}{2} \left(\frac{P_{j,t}^N}{P_{j,t-1}^N} - 1 \right)^2 y_t^N + T_t \right]$$

where $\Lambda_{i,t+i} \equiv \beta^t u_N(t+i)/u_N(i)$ is the household's stochastic discount factor for the non-tradable good between date t and date $t+i$. T_t represents the lump sum tax that firms pay to the government at date t. Each period, the intermediate goods firm faces a demand curve for its product arising from competitive final good firms' production function:

$$y_{j,t}^N = \left(\frac{P_{j,t}^N}{P_t^N} \right)^{-\varepsilon} y_t^N.$$

In a symmetric equilibrium, all firms choose the same price ($P_{j,t}^N = P_t^N$ for all j), and firms' optimal pricing rule is described by the following condition:

$$\pi_t^N(1+\pi_t^N) = \frac{\varepsilon}{\varphi}\left[mc_t - \frac{\varepsilon-1}{\varepsilon}\right] + \mathbb{E}_t\Lambda_{t,t+1}\left[\frac{y_{t+1}^N}{y_t^N}\pi_{t+1}^N(1+\pi_{t+1}^N)\right] \tag{3.10}$$

where $1+\pi_t^N \equiv P_t^N/P_{t-1}^N$ is the inflation rate of the non-tradable good. Condition (3.10) is the Rotemberg version of the non-linear New-Keynesian Phillips curve that relates the current inflation to the current deviation of marginal cost from marginal revenue and to the expected future inflation. It states that, given marginal costs, firms expecting higher inflation in the future would already raise prices in the current period to smooth out the necessary price adjustments over the time. According to this condition, firms would optimally set prices to equate the cost of adjusting prices today to a weighted average of current and future expected deviation of marginal cost from marginal revenue. Therefore, under full price flexibility (i.e. $\varphi = 0$), firms would always set prices to equate marginal revenue to marginal cost. At the other extreme, when prices are fully rigid (i.e. $\varphi \to \infty$) firms would set prices once and for all to equate an average of current and future expected marginal revenues to an average of current and future expected marginal costs.

3.2.3 Government

The government (or central bank) in this economy sets the path of the nominal exchange rate as its monetary policy instrument.[3] By controlling for the exchange-rate level $\mathcal{E}_t$, the government influences the relative price of non-tradables and is thus able to set the path for the inflation rate in the production sector, which can be seen as representing the government's monetary policy rule in this environment. To see more clearly how monetary policy operates, notice that any change in the relative price of non-tradables has an expenditure switching effect, and the demand for the non-tradable good is thus affected. This in turn requires a change in employment, which necessitates a change in the equilibrium wage through households' labor

[3]There exists a nominal domestic interest rate that would implement this policy. This interest rate can be found by introducing into the model a domestic bond that can be traded only among domestic households (its net supply is equal to zero in equilibrium). Then, given an equilibrium allocation, the Uncovered Interest Parity can be used to back out the nominal interest rate of domestic bond. This is the standard cashless approach (Woodford, 2003). Appendix B.2.1 provides further details.

supply condition (3.6). Therefore, with its action on the exchange rate, the government affects firms' current marginal costs and thus their price-setting decisions. It then follows that by setting the level of the exchange rate, the government implicitly determines the path of the inflation rate.

The government also sets, once and for all, a constant labor subsidy τ^h which is financed through a lump sum tax on firms such that the government budget is balanced:

$$\tau^h W_t h_t = T_t$$

I set this constant labor subsidy to a level that would fully offset the monopoly distortion under flexible prices. This level is given by $\tau^h = 1/\varepsilon$.

3.2.4 Recursive Competitive Equilibrium

Given a constant labor tax τ^h and an exchange rate path $\{\mathcal{E}_t\}_{t=0}^{\infty}$, a competitive equilibrium is defined by stochastic sequences of allocations $\{c_t^T, c_t^N, b_{t+1}, \ell_t, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t\}_{t=0}^{\infty}$ such that: (a) agents maximize their lifetime utility (3.1) subject to the sequence of budget and credit constraints given by (3.3) and (3.4) for $t = 0, \ldots, \infty$, taking as given $\{P_t^N, W_t\}_{t=0}^{\infty}$; (b) the markets for labor, non-tradable goods and tradable goods clear at each date t. The market clearing condition in the labor market, non-tradable goods market and tradable goods market are respectively given by

$$h_t \equiv \int_0^1 h_{j,t} dj = \ell_t$$

$$y_t^N = c_t^N + \frac{\varphi}{2}(\pi_t^N)^2 y_t^N$$

$$c_t^T + \frac{b_{t+1}}{R_t} = y_t^T + b_t \tag{3.11}$$

I now turn to describing a competitive equilibrium in recursive form. The aggregate state vector of the economy is (B, s) where B is the aggregate bond holdings and s is the exogenous shocks realization. The state variables for an household's optimization problem are the individual bond holdings b, and the aggregate state (B, s). Denoting by $\Gamma(B, s)$ the perceived law of motion for aggregate bonds that households need to form expectations of future prices, and by $w(B, s)$ and $p^N(B, s)$ the respective pricing functions for labor and the non-tradable good, the optimization problem of households in recursive form is given by:

$$\begin{aligned} V(b,B,s) &= \max_{c^T,c^N,\ell,b'} u\big(c(c^T,c^N),\ell\big)+\beta\mathbb{E}_{s'|s}V(b',B',s') \\ s.t.\quad c^T+p^N(B,s)c^N+\frac{b'}{R} &= y^T+w(B,s)\ell+\phi(B,s)+b \\ \frac{b'}{R} &\geq -\kappa\Big[y^T+w(B,s)\ell+\phi(B,s)\Big] \\ B' &= \Gamma(B,s) \end{aligned}$$

The solution to this problem yields decision rules for individual bond holdings $\hat{b}'(b,B,s)$, labor supply $\hat{\ell}(b,B,s)$, tradable consumption $\hat{c}^T(b,B,s)$ and non-tradable consumption $\hat{c}^N(b,B,s)$. In a recursive rational expectations equilibrium, the law of motion for aggregate bonds must coincide with the actual law of motion for aggregate bonds induced by the decision rule for bond holdings, and given by $\hat{b}'(B,B,s)$. The firm j's optimal pricing rule satisfies

$$(1+\pi_j^N)\pi_j^N=\frac{\varepsilon-1}{\varphi}\left[\frac{1}{A}\frac{w(B,s)}{p^N(B,s)}-1\right]+\mathbb{E}_{s'|s}\Lambda\left[\frac{\hat{h}'(B',s')}{\hat{h}_j}(1+\pi^N(B',s'))\pi^N(B',s')\right] \quad (3.12)$$

In a symmetric equilibrium, the decision rules satisfy $\hat{\pi}_j^N(B,s)=\hat{\pi}^N(B,s)$ and $\hat{h}_j(B,s)=\hat{h}(B,s)$ for all firms. A recursive rational expectations equilibrium is defined below.

Definition 4 (Recursive Competitive Equilibrium)**.** *For a given government's policy rule $\pi_t^N(B,s)$, a recursive competitive equilibrium is defined by pricing functions $\{w(B,s),p^N(B,s)\}$, a perceived law of motion for aggregate bond holdings $\Gamma(B,s)$, firms' policies $\{\hat{\pi}^N(B,s),\hat{h}(B,s)\}$, and households' decision rules $\{\hat{b}'(b,B,s),\hat{c}^T(b,B,s),\hat{c}^N(b,B,s),\hat{\ell}(b,B,s)\}$ with associated value function $V(b,B,s)$ such that:*

1. *$\Big\{\hat{b}'(b,B,s),\hat{c}^T(b,B,s),\hat{c}^N(b,B,s),\hat{\ell}(b,B,s)\Big\}$ and $V(b,B,s)$ solve households' recursive optimization problem, taking as given $w(B,s)$, $p^N(B,s)$ and $\Gamma(B,s)$.*

2. *$\{\hat{\pi}^N(B,s),\hat{h}(B,s)\}$ satisfies (3.12), taking as given $w(B,s),p^N(B,s)$ and $\pi^N(B,s)$.*

3. *The perceived law of motion for aggregate bonds and the government's policy rule are consistent with the actual law of motion for individual bonds and actual inflation policy, respectively: $\Gamma(B,s)=\hat{b}'(B,B,s)$ and $\pi(B,s)=\hat{\pi}(B,s)$.*

4. *The labor market and the tradable good market clear*

$$\begin{aligned} \hat{h}(B,s) &= \hat{\ell}(B,B,s) \\ \hat{c}^T(B,B,s)+\frac{\Gamma(B,s)}{R} &= y^T+B \end{aligned}$$

3.3 Monetary Policy Analysis

This section discusses the trade-off between macroeconomic stabilization and financial stabilization that a monetary policymaker faces in the model economy presented in the previous section and formally characterizes the optimal time-consistent monetary policy, that is the discretionary monetary policy.

I assume that the government (or central bank) lacks the ability to commit to future policies, and chooses its policy instruments subject to the credit constraint and all others competitive equilibrium conditions. Since the set of restrictions on the optimal policy includes forward-looking constraints, namely the Euler equation for households (3.4) and an inter-temporal pricing rule for firms (3.10), the optimal policy setup amounts to a dynamic game between successive governments. Thus, following Klein et al. (2008) and Bianchi and Mendoza (2018), I focus on Markov-stationary policy rules that are expressed as functions of the payoff-relevant state variables (B, s).[4] To simplify the analysis of the discretionary policy in this section, I focus on additively separable preferences, that is preferences satisfying $u_{c,\ell} = 0$. This specification of preferences is common in analytical studies of monetary policy open economy models (see for example Benigno and Benigno, 2003 and Gali and Monacelli, 2005). Appendix B.2.2 shows that the results are robust to alternatives specifications where preferences satisfies the form introduced by Greenwood et al. (1988), often referred to as GHH preferences.[5]

3.3.1 Government's Optimization Problem

Problem The government sets its policy to maximize the agents' welfare subject to the resource, credit and implementability constraints. Unlike private agents, the government internalizes the general equilibrium effects of borrowing decisions on market prices. To simplify the description of the government's optimal policy problem, I introduce the concept of a labor wedge, defined as the gap between firms' marginal product of labor in the non-tradable production sector and households' marginal rate of substitution between leisure and non-tradable consumption:

$$\omega_t \equiv 1 + \frac{1}{A}\frac{u_\ell(t)}{u_N(t)} \tag{3.13}$$

[4] A Markov-perfect equilibrium is a fixed point in the policy rule chosen by the government in any given period, taking as given the policy rule that represent the future governments' decisions. The key property of this fixed point is that the policy rule of the current government matches the policy rules of future governments that the current government takes as given to solve its optimization problem.

[5] This formulation of preferences is widely used in the growing macro-finance literature (see for example, Fornaro, 2015, Bianchi and Mendoza, 2018) for the purpose of the quantitative analysis. I provide further details on the properties of GHH preferences in section 3.4 and use this formulation for the quantitative analysis.

The labor wedge is zero at efficient allocation. Let $\mathcal{M}(b,s) = (1+\hat{\pi}^N(b,s))\hat{\pi}^N(b,s)$ be the monetary policy rule of future governments that the current government takes as given, and $\{\mathcal{C}(b,s), \mathcal{L}(b,s), \mathcal{B}(b,s), \mathcal{V}(b,s)\}$ the equilibrium functions that return the values of the corresponding variables under that policy rule. Taking these functions as given, the government's time-consistent problem in recursive form is:

$$\mathcal{V}(b,s) = \max_{c^T,c^N,\ell,b',p^N,\pi^N,\mu} u\left[c\left(c^T,c^N\right),\ell\right] + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',s') \quad (3.14)$$

$$s.t. \quad c^N = \alpha(p^N)c^T \quad (3.15)$$

$$c^N = \left[1-\frac{\varphi}{2}(\pi^N)^2\right]A\ell \quad (3.16)$$

$$c^T = y^T + b - \frac{b'}{R} \quad (3.17)$$

$$\frac{b'}{R} \geq -\kappa\left(y^T + p^N c^N\right) \quad (3.18)$$

$$\mu = u_T(c,\ell) - \beta R\mathbb{E}_{s'|s}u_T\left(\mathcal{C}(b',s'),\mathcal{L}(b',s')\right) \quad (3.19)$$

$$\mu \times \left[b' + \kappa\left(y^T + p^N c^N\right)\right] = 0,\ \mu \geq 0 \quad (3.20)$$

$$0 = \varphi\pi^N(1+\pi^N) + (\varepsilon-1)\big[1 - z^{-1}(1-\omega)\big] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right] \quad (3.21)$$

where the variables z_t and ω_t are respectively defined in (3.6) and (3.13). (3.15) is an intra-temporal implementability constraint, (3.16) is the resource constraint for the non-tradable good, (3.17) is the resource constraint for the tradable good, (3.18) is the credit constraint, (3.19)-(3.20) are the inter-temporal implementability constraints associated with households' borrowing choices, and (3.21) is the inter-temporal implementability constraint associated with firms' pricing decision (i.e., the non-linear New Keynesian Philipps Curve). I denote the multiplier on (3.17), (3.18), (3.19) by $\lambda^* \geq 0$, $\mu^* \geq 0$ and υ. Note that λ^* and μ^* differ from λ and μ.

Aggregate demand externality Factoring the non-tradable good's demand (3.15) and resource constraint (3.16) into the period utility function, an indirect utility function can be defined as $\mathcal{S}(t) \equiv u\left[c\left(c^T,\alpha(p^N)c^T\right),\left(1-\frac{\varphi}{2}(\pi^N)^2\right)^{-1}\alpha(p^N)c^T\right]$. The social marginal value of the tradable good consumption in equilibrium is given by:

$$\frac{\partial\mathcal{S}(t)}{\partial c_t^T} = \left[1 + \left(\frac{p_t^N y_t^N}{c_t^T}\right)\left(\omega_t - \frac{\varphi}{2}(\pi_t^N)^2\right)\right]u_T(t) \quad (3.22)$$

There is thus a wedge between households' private marginal value of the tradable good and this social marginal value in equilibrium, due to the presence of an aggregate demand externality. This wedge is proportional to the labor wedge net of the cost of inflation, weighted by the relative expenditure share of the non-tradable good relative to the tradable good.[6] To understand the source of this wedge, consider a marginal increase in households' tradable good consumption. Households only value this increase according to their private marginal utility $u_T(t)$. But as they increase their consumption of tradables, households also demand more non-tradables. And since price adjustment is costly, this larger demand translates into a partial price adjustment as well as into more non-tradable good production. These price and production adjustments have non-internalized welfare ramifications. The relevant non-tradable output multiplier is given by the relative expenditure share of the non-tradable to the tradable good $p_t^N y_t^N / c_t^T$, and the price and employment adjustment detailed above create a net benefit of $\omega_t - (\varphi/2)(\pi_t^N)^2$. Therefore, the non-internalized benefit is precisely $(p_t^N y_t^N / c_t^T)(\omega_t - (\varphi/2)(\pi_t^N)^2)$, which has a marginal utility $u_T(t)$.

Pecuniary externality The second externality present in this environment is a pecuniary externality originating from the credit constraint. To describe this externality, I compare the government's and households' bond choices in the absence of nominal rigidities (i.e. for $\varphi = 0$).[7] The government's optimal decision for bonds in sequential form can be described by the two equations:[8]

$$\lambda_t^* = u_T(t) + \gamma^{-1} \kappa \frac{p_t^N c_t^N}{c_t^T} \mu_t^* \tag{3.23}$$

$$\lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1}^* + \mu_t^* \tag{3.24}$$

The pecuniary externality can be understood by comparing (3.24) with the corresponding households' optimality conditions (3.7). The government's shadow value of tradable consumption λ_t^* is equal to the marginal utility of the tradable consumption, $u_T(t)$, plus an additional term that represents the relaxation effect on the credit constraint induced by the rise in the relative price of non-tradables associated with a marginally higher tradable consumption. The credit constraint relaxation term is absent from the private condition (3.7). As emphasized in the normative macro-finance literature (see Korinek, 2011 and Bianchi, 2011), this wedge between social and private valuations of tradable consumption, when the credit constraint is binding, generates overborrowing ex ante. Indeed, when the credit constraint is not currently

[6]This result is in line with Farhi and Werning (2016). In a similar environment where firms sets their prices once and for all, they highlight the existence of an aggregate demand externality and show that the wedge between the social and private marginal values equals the *weighted labor wedge*; that is, the labor wedge weighted by the relative expenditure share of non-tradable goods relative to tradable goods.

[7]When prices are flexible ($\varphi = 0$), there is no role for the monetary policy due to the dichotomy between nominal and real variables (see Lucas, 1972 and Caplin and Spulber, 1987).

[8]Because if the government decisions for bonds coincide with households decisions for bonds, the implementability constraint (3.19) would be always satisfied (that is, the associated multiplier would be equal to zero), the multiplier on this constraint is set to zero in order to discuss the presence of the externality.

binding, households equate the benefits $u_T(t)$ of an additional unit of borrowing to its private costs $\beta(1+r)\mathbb{E}_t u_T(t+1)$. However, the government has a higher marginal cost given by $\beta R_t \mathbb{E}_t[u_T(t+1) + \mu^*_{t+1}\Theta_{t+1}]$ with $\Theta_t = (1/\gamma)\kappa p^N_t c^N_t / c^T_t$. The additional cost, $R_t\Theta_{t+1}$, incurred by the government represents how a one unit increase in borrowing at date t tightens the ability to borrow at date $t+1$, which has a marginal utility cost of μ^*_{t+1}.

3.3.2 Discretionary Monetary Policy in Absence of Credit Frictions

I start the normative analysis by considering a case in which the economy features unconstrained access to the international credit market. This economy can be regarded as a financially robust economy in which either there is no credit constraint, or the credit coefficient κ is large enough that the credit constraint never binds.[9] In this case, the only constraint faced by agents on their borrowing is their natural borrowing limit. The allocation of the competitive equilibrium in this economy is the same as the one described in Section 3.2, but with $\mu_t = 0$ for all t. This case will serve as a benchmark.

The discretionary monetary policy solves (B.2.2) subject to (3.15)-(3.17), (3.19) with equality $\mu_t = 0$ and (3.21). The discretionary policy problem amounts to choosing a path for the inflation rate of the non-tradable good to maximize private agents' welfare. The following lemma describes the solution to this problem.

Proposition 6 (Discretionary Monetary Policy without Credit Frictions)**.** *With no credit frictions, a price stability policy (i.e. $\pi^N_t = 0$ for all t) is the discretionary monetary policy. It perfectly stabilizes the economy (i.e. $\omega_t = 0$), and the allocation satisfies (3.5), (3.11), $-u_\ell(t) = Au_N(t)$ along with $c^N_t = y^N_t = A\ell_t$.*

Proof. See Appendix B.1.1

This result can be intuited from the observation that when there are no financial market imperfections (no credit frictions), the only distortions in this economy arise from price stickiness. Hence, as is standard in the New Keynesian literature, the optimal monetary policy eliminates the inefficiency stemming from sticky prices by making price adjustments unnecessary and production supply determined. Specifically, when the economy experiences a negative tradable endowment shock, private agents feel poor and want to reduce their demand for both goods. Firms in turn aim to adjust their price downward in response to the resulting decrease in the demand for non-tradables. The discretionary monetary policy thus features an exchange rate depreciation that generates expenditure switching toward the non-tradable good and renders an adjustment of the price of the non-tradable good unnecessary. Because the labor tax

[9] Absence of a credit constraint or higher values of κ can be interpreted as representing a highly-developed financial system in which households are able to borrow against a large share of their total income.

is assumed to be set to completely offset monopolistic distortions, the resulting allocation is efficient.

3.3.3 Discretionary Monetary Policy in Presence of Credit Frictions

I now turn to analyzing the discretionary monetary policy in an economy featuring limited access to the international credit market. The discretionary policy in this environment solves (B.2.2) subject to (3.15)-(3.21). The two aforementioned externalities create policy trade-offs in the presence of credit frictions, and a price stability policy is generally not optimal.

In what follows, a boom (a recession) in the non-tradable production sector corresponds to a situation in which the labor wedge is negative (positive), in line with the general class of New Keynesian models.[10] The proposition below characterizes the discretionary monetary policy in the presence of credit frictions.

Proposition 7 (Discretionary Monetary Policy)**.** *In the presence of credit frictions, the path of the inflation rate under the discretionary monetary policy satisfies:*

$$\begin{aligned}\varphi\left(\Delta_{0,t}+\frac{\pi_t^N}{2}\right)y_t^N\pi_t^N &= y_t^N\omega_t+\varphi\mathbb{E}_t\left[\Delta_{1,t+1}\pi_{t+1}^N\right]\\ &\quad+(\sigma-\gamma^{-1})\frac{c_T(t)}{c_t}c_t^N\upsilon_t+(1-\gamma^{-1})\kappa p_t^N c_t^N\frac{\mu_t^*}{u_N(t)} \qquad (3.25)\end{aligned}$$

where $\Delta_{0,t}>0$. As a result, price stability is always optimal in the knife-edge case where $\gamma=\sigma=1$.

Proof. See Appendix B.1.2

Proposition 7 states that the inflation path under discretionary monetary policy is given by a targeting rule that is also a function of the labor wedge, the government's Lagrange multipliers on the credit constraint (3.18) and households' Euler equation (3.19). The first term accounts for the output stability motive of the government. It shows that the government engineers an expansionary monetary policy when there is a positive labor wedge, and a contractionary monetary policy when there is a negative labor wedge. The second term accounts for the price stability motive. It shows that if the government expects higher future inflation, it will already promote inflation in the current period to smooth out the necessary price adjustments. The last two terms account for monetary policy's response to the adverse effects stemming from the presence of credit constraints. How the monetary authority responds to financial conditions

[10] In this class of models (see, among others, Gali and Monacelli, 2005 and Schmitt-Grohé and Uribe, 2007), there is a boom when the level of output is above its natural level, which is defined as the output level under flexible price allocation.

when the credit constraint is not binding ($\upsilon_t \neq 0$) and when it is binding ($\mu_t \neq 0$) is respectively embedded into the third and the fourth term, and depends on substitution elasticities.

When the constraint is not binding, the effectiveness of the monetary policy in correcting the pecuniary externality and preventing overborrowing depends on how an increase in the non-tradable output and consumption affects households' private marginal utility of tradables (that is, their cost of borrowing), which is captured by the difference between the intra-temporal and the inter-temporal elasticity of substitution. Likewise, when the constraint binds, the effectiveness of the monetary policy in correcting the aggregate demand externality and tames the overheating induced by the excessive labor supply of private agents depends on how an increase in the non-tradable output and consumption affects the value of the collateral, which is captured by the intra-temporal elasticity of substitution.

The discretionary monetary policy thus strikes a compromise between price stability and financial stability. Notice that, this characterization of the discretionary monetary policy, given by equation (3.25), confirms the previous result that in absence of credit frictions (that is, for $\upsilon_t = \mu_t = 0$) a price stability policy is optimal. The remarks below provide further details on the characterization of the discretionary policy with credit frictions.

Remark 1 (Prudential Monetary Policy)**.** *When the credit constraint does not currently bind, a price stability policy is optimal if and only if $\gamma = 1/\sigma$. Further, for $\gamma > 1/\sigma$ the current government optimally deviates from price stability policy to perform an expansionary monetary policy.*

This remark adds further insights and describes how monetary policy is used as a prudential tool. First, it is important to notice that the shadow of the implementability constraint (3.19), υ_t, can be seen as a measure of the difference between socially desirable level of borrowing and private agents' level of borrowing. Because under a price stability policy private agents overborrow ex ante, $\upsilon_t > 0$,[11] the government does not use monetary policy solely to make any adjustments in the prices of non-tradables unnecessary, but also to address the pecuniary externality. In the knife-edge case where $\gamma = 1/\sigma$, the intra-temporal and the inter-temporal effects of the monetary policy stance on tradable consumption cancel each other out, thus generating either a boom or a bust in the non-tradable sector is impotent to address overborrowing. Therefore, in this case, the government only focuses on price stability and replicates flexible price allocation. When the intra-temporal elasticity of substitution is greater than the inter-temporal elasticity of substitution (i.e., when $\gamma > 1/\sigma$), an expansionary monetary policy generating a boom in the non-tradable sector reduces households' marginal propensity to con-

[11] The analytical expression is $\upsilon_t = \sum_{j=1}^{\infty} \beta^j \mathbb{E}_t \left[\left(\Pi_{k=t}^{j} \frac{R_{t+k}}{\chi_{t+j}} \right) \Theta_{t+j} \mu^*_{t+j} \right]$, with $\chi_{t+j} = 1 + \frac{\beta R_{t+j} u_{TT}(t+j+1)}{u_{TT}(t+j)} > 1$.

sume the tradable good, contains the rise in the relative price of non-tradables and addresses overborrowing.

Remark 2 (Ex Post Policy). *When the credit constraint binds, a price stability policy is optimal if and only if $\gamma = 1$. Further, for $\gamma < 1$ the current government optimally deviates from a price stability policy to perform a contractionary monetary policy.*

When the credit constraint binds, the social marginal value of tradable consumption is $\mathcal{S}_T(t) = \left[1 + (p_t^N c_t^N / c_t^T)\omega_t\right] u_T(t) < u_T(t)$, with $\omega_t = -\kappa\mu_t / u_T(t) < 0$, and households overvalue the tradable good. The government then faces a trade-off between price stability and the relaxation of the credit constraint to address the aggregate demand externality. When the intra-temporal elasticity of substitution equals 1 ($\gamma = 1$), the effect on the relative price of non-tradable cancels out the quantity effect when the economy is experiencing a boom or a bust in the non-tradable sector ($\partial p_t^N c_t^N / \partial c_t^N = 0$). Therefore, neither an expansionary policy nor a contractionary policy will have an effect on the value of the collateral. The government then has no incentive to deviate from a price stability policy. In contrast, when the intra-temporal elasticity of substitution is less than 1 ($\gamma < 1$), the price effect dominates the quantity effect ($\partial p_t^N c_t^N / \partial c_t^N < 0$). Thus, by performing a contractionary monetary policy and creating a recession, the current government sustains the value of the collateral, limits the fall in private agents' borrowing capacity and alleviates the effects of the aggregate demand externality.

Discussion Kaminsky et al. (2004) document that monetary policy is countercyclical in almost all the OECD countries while it is mostly procyclical in emerging market ones.[12] This section highlights conditions under which procyclical monetary policy can be optimal in an economy in which financial markets are imperfect, capital flows freely and the government lacks commitment. Remarks 1 and 2 together show that the discretionary monetary policy is procyclical when the intra-temporal elasticity is greater than the inter-temporal elasticity and the former is less than 1 ($1/\sigma < \gamma < 1$). The basic idea here is that, in an economy featuring an endogenous sudden stop mechanism, lowering the degree of nominal rigidity is not always welfare increasing due to the tradeoff between price and financial stability documented in this environment. When the economy is experiencing bad shocks that lead to capital outflows (that is periods in which the constraint binds), there are gains from moving away from a price stability policy to perform a contractionary monetary policy when goods are complements ($\gamma < 1$). In these circumstances, a contractionary monetary policy prevents large real exchange

[12] Kaminsky et al. (2004) use a sample of 21 OECD countries and 83 developing countries (which include 18 middle-high income countries). The procyclicality of the monetary policy is measured by the correlation between the cyclical components of the real GDP and the cyclical components of short-term interest rates (the interbank rate, the T-bill or the discount rate).

depreciation (fall of the relative price of non-tradables), slackens the credit constraint, restrain capital outflows and mitigates balance sheets effects. Furthermore, in advance of crisis periods (that is periods in which the credit constraint does not bind), good shocks lead to excessive capital inflows (overborrowing), and there are gains from moving away from a price stability policy to perform an expansionary monetary policy. This procyclical monetary policy increases the cost of borrowing, contains the current rise of the real exchange rate, and mitigate balance sheet effects from future depreciation.

3.3.4 Discretionary Monetary Policy with Capital Flow Taxes

I now consider the possibility of the government having an additional policy instrument consisting in capital flow taxes and look at how the availability of this additional tool affects the optimal design of monetary policy. The budget constraint of households is:

$$c_t^T + p_t^N c_t^N + \frac{b_{t+1}}{R_t(1+\tau_t^b)} = y_t^T + w_t \ell_t + \phi_t + b_t + \mathcal{T}_t$$

where $\mathcal{T}_t$ represents a lump-sum transfer that households receive from the government. The households' Euler equation for bonds becomes:

$$u_T(t) = \beta R_t (1+\tau_t^b) \mathbb{E}_t u_T(t+1) + \mu_t \tag{3.26}$$

Given quantities, the tax on debt can be used to back out households' Euler equation. It turns out that allowing the government to use capital taxes is equivalent to assuming that it controls the credit operations of households and rebates back the proceeds of the transactions in a lump-sum fashion. Hence, the government's optimal policy problem amounts to choosing the path for the inflation rate and making debt choices for households, while allowing them to choose their labor supply and their allocation of consumption between tradable goods and non-tradable goods in a competitive way. The government's optimization problem thus reduces to solving (B.2.2) subject to (3.15)-(3.18) and (3.21).[13] The following proposition characterizes the discretionary monetary policy in an economy with credit frictions when capital flow taxes are used optimally.

Proposition 8 (Discretionary Policy under Availability of Capital Flow Taxes)**.** *When capital flow taxes are available, the path of the inflation rate under the discretionary monetary policy*

[13] Appendix B.1.3 shows that any allocation that satisfies (3.15)-(3.18) and (3.21) also satisfies the general equilibrium.

satisfies:

$$\varphi\left(\Delta_{0,t}+\frac{\pi_t^N}{2}\right)y_t^N\pi_t^N = y_t^N\omega_t+\varphi\mathbb{E}_t\left[\Delta_{1,t+1}\pi_{t+1}^N\right]+(1-\gamma^{-1})\frac{\kappa}{u_T(t)}\mu_t^*,$$

As a result, price stability is always optimal when $\gamma = 1$.

Proof. See Appendix B.1.3

This proposition provides insight into how the availability of capital flow taxes changes the discretionary monetary policy. When the credit constraint binds, the design of the discretionary monetary policy is quite similar to that in an economy where capital flow taxes cannot be used. In that case, the discretionary monetary policy is a compromise between two objectives: correcting nominal rigidities (macroeconomic stability) and relaxing the credit constraint to correct aggregate demand externalities (financial stability). The government optimally stabilizes prices if the intra-temporal elasticity of substitution equals 1 ($\gamma = 1$), and deviates from a price stability policy to create a recession if $\gamma < 1$. However, when the credit constraint does not currently bind, the discretionary monetary policy is qualitatively different from the case where capital flow taxes are not available. In this case, capital flow taxes appear to be the preferred tool for correcting the pecuniary externality, and the monetary policy focuses on granting price stability.

The proposition also shows that the conditions under which a price stability policy is optimal in the absence of capital flow taxes are a subset of those when capital flow taxes are available and used optimally. In the latter, the discretionary monetary policy is a price stability policy when $\gamma = 1$, while in the former a price stability policy is optimal only under the more restrictive aforementioned conditions ($1/\sigma = \gamma = 1$).

3.4 Quantitative Analysis

This section evaluates the quantitative implications of the model. I solve numerically for the problem of the government, under both free capital mobility and availability of capital flow taxes, using global non-linear methods. I also solve for the competitive equilibrium in which the monetary policy is characterized by a price stability policy.

3.4.1 Calibration

The model is calibrated to annual data from Argentina. Preferences in this section are specified following Greenwood et al. (1988) where utility is defined in terms of the excess of consumption over the disutility of labor $u\left(c_t - g(\ell_t)\right)$, and $g(\ell_t)$ is a function that measures the disutility of the labor supply. This formulation of preferences allows international real business cycle models to explain observed business cycle facts, and delivers realistic/empirically plausible, dynamics for employment in emerging economies. The functional forms for preferences are:

$$\begin{aligned} u(c-g(\ell)) &= \frac{(c-g(\ell))^{1-\sigma}}{1-\sigma}, \text{ and} \\ g(\ell) &= \chi\frac{\ell^{1+\theta}}{1+\theta}, \end{aligned}$$

The coefficient of relative risk aversion is set to $\sigma = 2$, a standard value in the real business cycle literature for small open economies. In line with evidence by Kimball and Shapiro (2008), the Frisch elasticity of labor supply $1/\theta$ is set to 1. The labor disutility parameter χ is set so that mean employment in the non-tradable sector is equal to 1, which requires $\chi = 0.69$. The elasticity of substitution among differentiated intermediate goods ε is set to 7.66, corresponding to a 15% net markup that is in the range found by Diewert and Fox (2008). I also set the Rotemberg price adjustment cost parameter to ensure that nominal prices are sticky for three quarters on average, which requires $\varphi = 62$.[14] The value of the total factor productivity (TFP) in the non-tradable sector is normalized to 1. The process for the exogenous driving forces $s_t = (y_t^T, R_t)$ is taken to be a first-order bivariate autoregressive process

$$\begin{bmatrix} \ln\left(y_t^T\right) \\ \ln\left(\frac{R_t}{\bar{R}}\right) \end{bmatrix} = \rho_s \begin{bmatrix} \ln\left(y_{t-1}^T\right) \\ \ln\left(\frac{R_{t-1}}{\bar{R}}\right) \end{bmatrix} + \begin{bmatrix} \epsilon_t^T \\ \epsilon_t^R \end{bmatrix} \quad \text{where} \quad [\epsilon_t^T, \epsilon_t^R] \sim i.i.d.\ N(0, \Sigma_\epsilon^2)$$

I estimate this process by OLS with the risk-free rate and the cyclical component of tradable GDP from the World Development Indicators for the 1965-2014 period. The risk-free rate is measured by a U.S. real interest rate (Treasury-bill rate, deflated with expected U.S. CPI inflation). The tradable endowment is measured with the cyclical component of value added in agriculture and manufacturing. The OLS estimates of ρ_s and Σ_ϵ are respectively

$$\hat{\rho}_s = \begin{bmatrix} 0.6663 & -0.5238 \\ 0.0842 & 0.7861 \end{bmatrix}, \quad \hat{\Sigma}_\epsilon = \begin{bmatrix} 0.0028889 & -0.0001182 \\ -0.0001182 & 0.0001648 \end{bmatrix} \quad \text{and} \quad \bar{R} = 1.0219$$

The vector of shocks is discretized into a first-order Markov process, with seventeen points, using the quadrature-based procedure of Tauchen and Hussey (1991). The mean of the endowment is set to 1 without loss of generality.

[14] The parameter φ is set to ensure that the first-order approximation of the New-Keynesian Phillips curve (NKPC) in the current Rotemberg model, equation (3.10), is equivalent to the one in a Calvo model where firms keep their price unchanged with a probability δ. The log-linear version of the NKPC is: $\hat{\pi}_t^N = \beta\hat{\pi}_{t+1}^N + \bar{\kappa}\hat{y}_t^N$, where $\hat{y}_t^N$ represents the output gap, $\bar{\kappa} = (\varepsilon - 1)/\varphi$ in this model and $\bar{\kappa} = (1-\delta)(1-\beta\delta)/\delta$ in the Calvo model.

Table 3.1: Calibration

Description	Parameter-Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Frisch elasticity parameter	$\theta = 1$	Kimball and Shapiro (2008)
Elasticity of substitution	$\gamma = 0.83$	Conservative value
Monopoly power	$\varepsilon = 7.66$	15% net markup
Adjustment cost parameter	$\varphi = 62$	Three quarter of price stickiness
Discount factor	$\beta = 0.905$	Average NFA-GDP ratio = -29%
Weight on tradables in CES	$a = 0.315$	Share of tradable output = 32%
Collateral coefficient	$\kappa = 0.319$	Frequency of crisis = 5.5%
Labor disutility coefficient	$\chi = 0.686$	Mean labor = 1
TFP in non-tradable sector	$A = 1$	Normalization

The parameters $\{\gamma, \beta, a, \kappa\}$ are calibrated following the baseline calibration of Bianchi (2011). The calibration strategy consists in choosing values for the parameters so that the model economy under the procyclical discretionary monetary policy matches some key aspects of the Argentina data. The intra-temporal elasticity of substitution between tradable and non-tradable goods, γ, is set to a conservative value of 0.83. The three parameters $\{\beta, a, \kappa\}$ are respectively set so that the long-run moments of the equilibrium under the procyclical monetary policy match the following three historical moments of the data: (i) an average net foreign asset position to GDP of -29 percent, (ii) a share of tradable goods in production of 32 percent and (iii) an observed frequency of 5.5 percent of "Sudden Stops". Sudden Stops are defined as events in which the credit constraint binds, and this leads to an increase in net capital outflows that exceeds one standard deviation. This approach leads to $\beta = 0.905$, $a = 0.315$ and $\kappa = 0.319$.

3.4.2 Policy Functions

I start by analyzing the policy functions under different monetary regimes. I consider an *inflation targeting regime* in which the policy rule is set to stabilize the producer price index by offsetting all of the distortions originating from nominal rigidities, and there is no tax on capital. This regime captures the price stability objective of central banks. I also consider *discretionary monetary policy regimes* with and without capital flow taxes, under which the monetary policy is characterized by the discretionary monetary policy rules derived in section 3.3.3.

Figure 3.1 plots the decision rules as a function of the current holdings of bonds for a negative one-standard deviation shock. The presence of the endogenous borrowing constraint generates a kink in the policy function, and the bond decision rule is non-monotonic. I distinguish two

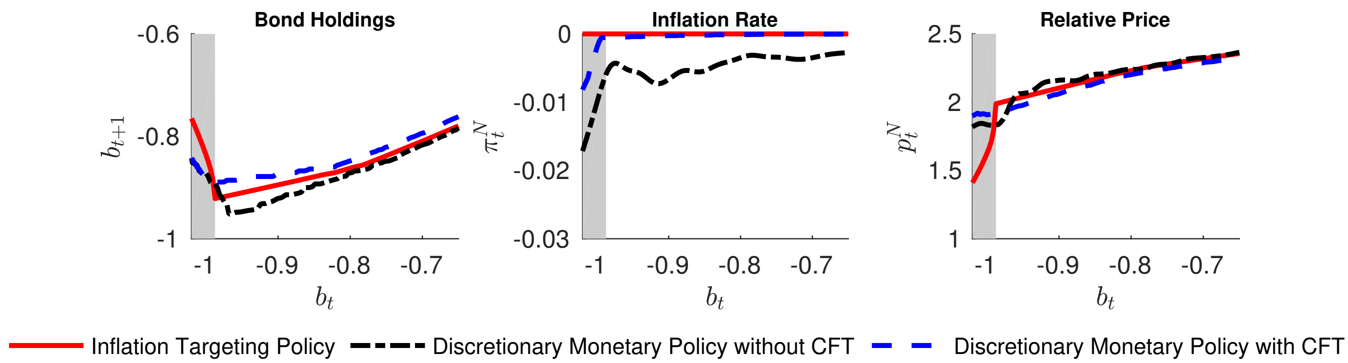

Figure 3.1: Decision rules for negative one-standard-deviation shocks

different regions in each panel below focusing on the change in the slope of the borrowing decision rule under an inflation targeting regime, which occurs at the point where the credit constraint binds. The solid red line corresponds to the bond decision rule under an inflation targeting regime. The dashed blue line and the dashed-dotted black line corresponds to the bond decision rule under a discretionary monetary policy regime, with and without capital flow taxes, respectively.

The constrained region (shaded region) represents the situations in which the current debt level is sufficiently high such that the credit constraint binds under an inflation targeting regime. In this region, under an inflation targeting regime, by the market clearing condition for the tradable good (3.11) for a given choice of next-period level of debt, an increase in the current level of debt would imply a decrease in tradable consumption. Notice that because the intratemporal elasticity of substitution is less than 1 ($\gamma < 1$), the price effect dominates the quantity effect. Thus, the decrease of the relative price of non-tradables dominates the increase in the demand of non-tradables that accompanies the decrease in tradable consumption, which means that the next-period level of debt must be decreased further to satisfy the credit constraint.

Under both discretionary monetary policy regimes, the government uses its policy instrument (here, the exchange rate) to increase the relative price of non-tradables and sustain the value of the collateral, which means that households can borrow more. Moreover, the increase in the relative price of non-tradables implies a lower marginal cost. Thus, for a given future government policy, monopolistically competitive firms reduce prices of the non-tradable good by equation (3.10), which in turn generates a lower level of inflation when compared to its level under the inflation targeting regime.

In the unconstrained region, which corresponds to the state-space where the credit constraint does not bind, under a monetary regime that replicates flexible price allocation (an inflation targeting regime), households do not accumulate sufficient precautionary savings, due to the presence of the pecuniary externality. Under discretionary monetary policy regimes, the government also uses its policy instruments to increase the cost of borrowing and prevent a larger drop in households' borrowing ability if the credit constraint becomes binding in the next period. Without capital flow taxes (CFT), given future policies lowering the exchange rate level to increase the relative price of non-tradables has two effects: an inter-temporal effect that reduces the cost of borrowing since it is relatively cheaper to purchase debt, and an intra-temporal effect that increases the cost of borrowing since the non-tradable good is relatively more expensive. As shown in Proposition 7, since the intra-temporal effect dominates ($\gamma > 1/\sigma$), the optimal policy is expansionary and thus generates a higher level of debt. Furthermore, a higher relative price of non-tradables implies that firms face lower marginal costs and then adjust their nominal prices downward according to their optimal pricing decision given by equation (3.10). The inflation rate under an optimal discretionary monetary policy regime without capital flow taxes thus turns out to be lower than under an inflation targeting regime. Under the discretionary monetary policy regime with capital flow taxes, the government uses taxes to control for households' credit operations and the monetary policy focuses only on stabilizing prices, as implied by Proposition 8. Thus, households accumulate uniformly lower levels of debt, which in turn help contain the rise in the relative price of non-tradables under a flexible price allocation and prevent a larger drop in households' borrowing ability if the credit constraint becomes binding in the next period.

3.4.3 Monetary Regimes and the Dynamics of Financial Crises

This section analyzes the costs and gains associated with the adoption of an inflation targeting regime as opposed to a discretionary monetary policy regime, in an economy in which capital flows are free (i.e. in the absence of capital controls).

Economic Behavior During Crises

To describe the effectiveness of a discretionary monetary policy regime in reducing the severity of crises, I construct an event analysis using simulated data and analyze the dynamics of the economy during financial crises. A financial crisis is defined as a period in which the credit constraint is binding, and in which the current account is one standard deviation above its mean in the ergodic distribution corresponding to the economy under each monetary regime.

The construction of the event analysis follows the procedure proposed by Bianchi and Mendoza (2018). First, the model economy under an inflation targeting regime is simulated for 500,000 periods. After dropping the first 1,000 periods and identifying all of the crisis events under an inflation targeting regime, I construct five-year event windows centered in the period in which the crisis takes place. Then at each period, I compute averages for each simulated

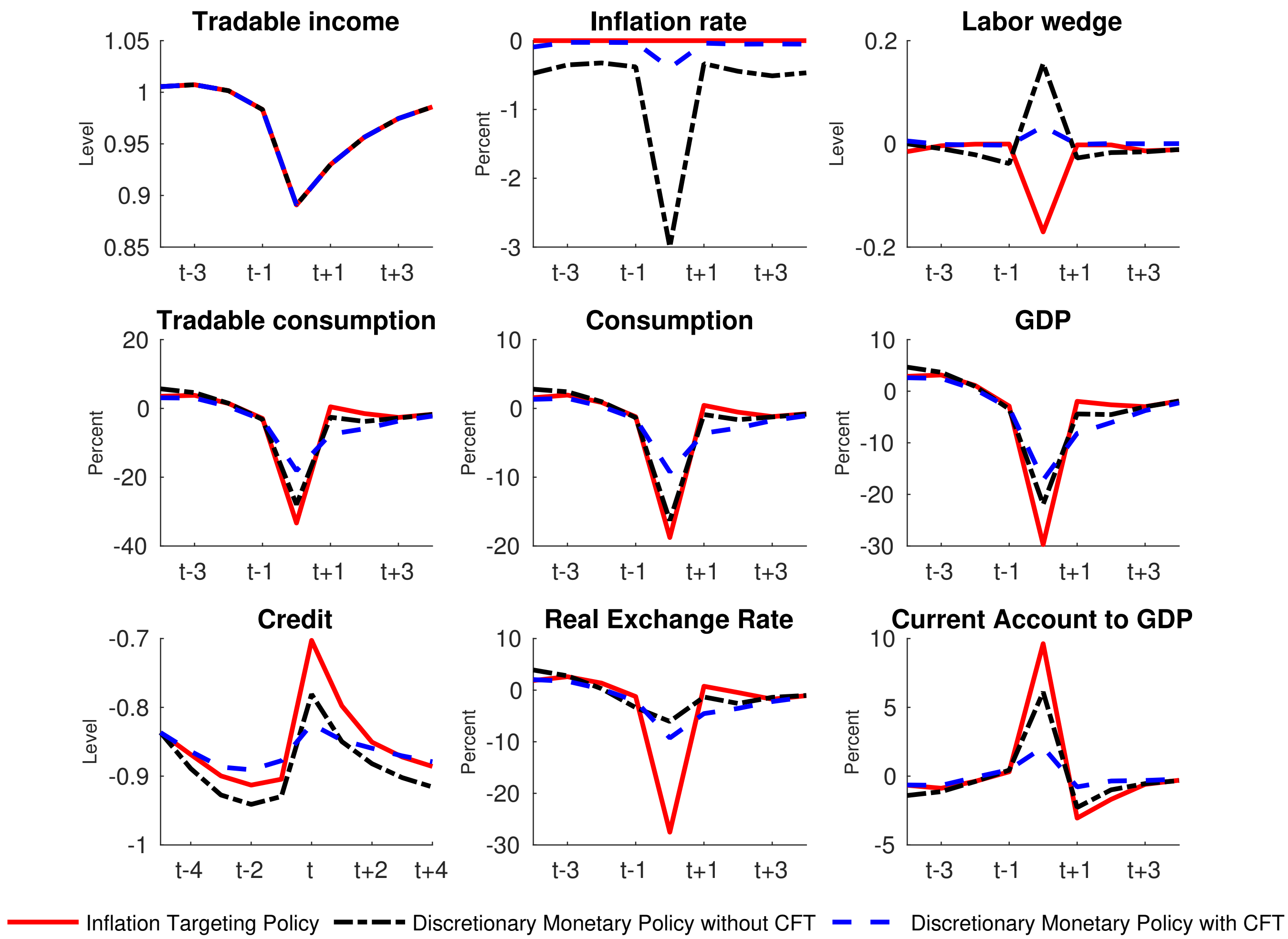

Figure 3.2: Comparison of crises dynamics

Note: The real exchange rate, tradable consumption, consumption and GDP are expressed in percentage deviations from averages in the ergodic distribution.

variable across the event windows in each year $t-2$ to $t+2$, and produce the economic dynamic under an inflation targeting regime. An initial value for bonds and a five-year sequence of tradable realizations is determined by calculating the median initial debt at $t-3$, and the median tradable shock across a cross-section of crisis events. Finally, I feed this sequence of shocks and initial value of bonds into the decision rules of the model economy under discretionary monetary policy regimes and compute the corresponding endogenous variables.[15] The model's predictions during financial crises for both monetary regimes is depicted by Figure 3.2.

The top middle panel shows that, under the discretionary monetary policy regime without capital flow taxes, there is a negative inflation rate in the run-up to crises. The reduction of the inflation level (-3.0 percent) is more important in the year of the crisis. In this way, the

[15]This procedure ensures that the dynamics under each model economy are simulated using the same initial state and the same sequence of shocks.

government allows for more credit access, and the economy under this monetary regime features more debt than the economy under an inflation targeting regime during crises but also in the years before and after the occurrence of the crises (see the bottom right panel). As a result, consumption of the tradable good falls by a much smaller percentage during crises than it does the under the inflation targeting regime (-28.1 vs. -33.3 percent). This relatively large fall of tradable consumption under the inflation targeting regime arises because the binding credit constraint forces households to reduce their next period level of debt, as captured by the sharp reversal of the current account-to-GDP ratio. Under the discretionary monetary policy regime, the government generates a boom ahead of financial crises and a recession during crises, as measured by the negative labor wedge, to sustain the value of the collateral. As a consequence, the fall in the real exchange rate is smaller during crises (-6.0 vs. -27.5 percent under the inflation targeting regime), which in turn mitigates the drop-in output and absorption during crises. The middle panel of Figure 3.2 shows that, in contrast to a discretionary monetary policy regime, decline in the total output and consumption is larger under inflation targeting: total output and consumption drop by 21.9 and 18.0 percent, respectively, under the discretionary monetary policy regime (vs. 29.7 percent for total output and 18.8 percent for consumption under the inflation targeting regime).

Economic Behavior ahead of Potential Crises

I now turn to analyzing the impact of monetary policy on debt accumulation and the frequency of crises in sudden stop-prone economies. In the absence of capital flow taxes, because households in an economy under an inflation targeting regime fail to accumulate sufficient precautionary savings due to the pecuniary externality, the government optimally deviates from a price stability policy to sustain the value of the collateral ahead of a potential financial crisis. Therefore, as shown in figure 3.3, the economy under the discretionary monetary policy regime is likely to have higher debt than the economy under an inflation targeting regime.

Formally, there is a 7.6 percent chance that households in the economy under the discretionary monetary policy regime carry an amount of debt larger than -1.0, which corresponds to the maximum amount of debt that households in the economy under an inflation targeting regime can hold. It is then apparent that the long-run probability of financial crises is 5.5 percent under the discretionary monetary policy regime versus 4.3 percent under an inflation targeting regime.

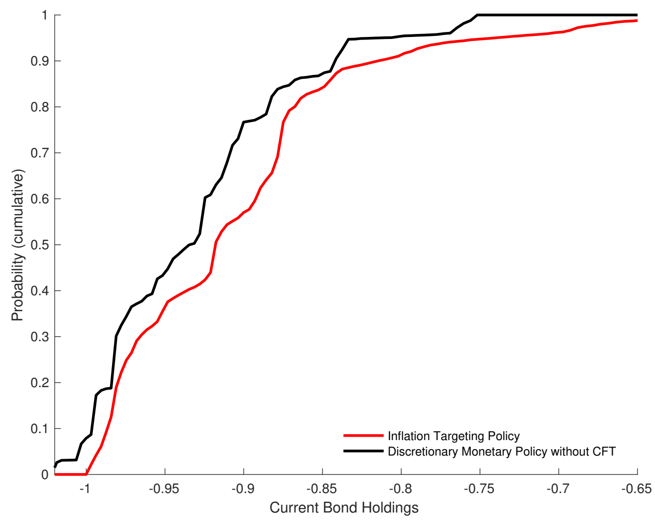

Figure 3.3: Distribution of bond holdings.

Monetary Policy and Capital Controls

The use of capital controls in the form of capital flow taxes along with a time-consistent monetary policy appears to be very effective in reducing the magnitude of crises as shown in Figure 3.2. Ahead of potential financial crises, taxes are used to diminish households' debt and restrain the boom in tradable consumption. This in turn prevents a larger drop in households' borrowing ability during crises, as captured by the smaller reversal of the current account-to-GDP ratio. The decline in tradable consumption, aggregate consumption and total output are thus smaller than their decline under both the discretionary monetary policy regime without capital flow taxes and the inflation targeting regime. Moreover, since the government uses taxes on debt to encourage households to accumulate sufficient precautionary savings, the economy under the discretionary monetary policy regime is less vulnerable to financial crises.

Table 3.2 highlights the importance of capital flow taxes in reducing the long-run frequency of crises regardless of the monetary regime. The long-run probability of crises is 1.1 percent under a discretionary monetary policy regime with capital flow taxes, and 1.3 percent under an inflation targeting regime with capital flow taxes (vs. 4.3 percent an under inflation targeting regime, and 5.5 percent under a discretionary monetary policy regime). Table 3.2 also points out the role of capital flow taxes in reducing the severity of crises. Because monetary policy is less effective in correcting the pecuniary externality, the drop in capital inflows is more pronounced when capital flows taxes are not available. The probability that the decline in total output will exceed 20 percent is 80.3 percent under a discretionary monetary policy regime, and 14.8 percent under a discretionary monetary policy regime with capital flow taxes (87.7 percent under inflation targeting vs. 24.4 percent under inflation targeting with capital flow

Table 3.2: Probability and severity of crises

	Inflation Targeting		Time-Consistent	
	no CFT	with CFT	no CFT	with CFT
Probability of crises	4.3	1.3	5.5	1.1
Current-Account to GDP	9.6	2.4	6.2	2.1
Real exchange rate depreciation	27.5	11.4	6.1	8.3
Output				
average	-29.7	-17.8	-21.9	-17.2
$\mathbb{P}(\hat{y} < -20\%)$	87.7	24.4	80.3	14.8
Consumption				
average	-18.8	-9.6	-18.0	-9.2
$\mathbb{P}(\hat{c} < -15\%)$	83.3	8.1	78.9	7.2

Note: Consumption, output and real exchange rate are expressed in percentage deviations from averages in the corresponding ergodic distribution.

taxes). Further, the probability that the consumption will drop by more than 15 percent is 78.9 percent under a discretionary monetary policy regime, and 7.2 percent under a discretionary monetary policy regime with capital flow taxes (83.3 percent under inflation targeting vs. 8.1 percent under inflation targeting with capital flow taxes). These results from Table 3.2 suggests that monetary policy should be supplemented with capital flows taxes.

Another important result is that the design of the monetary policy also affects the optimal tax rate on debt. It is apparent that the optimal tax rate under the discretionary monetary policy regime is higher than under the inflation targeting regime. On average, tax on debt is about 6.1 percent under a discretionary monetary policy regime and 5.0 percent under an inflation targeting regime. This finding can be inferred from the fact that under a discretionary monetary policy regime, the government uses its monetary policy to allow for more credit access during crises. Further, because future choices of bond holdings affect current optimal choices, the government needs to raise (relatively) more taxes in order to cause households to accumulate a socially desirable level of debt. This additional result suggests that monetary policy and macroprudential policy are complementary.

3.4.4 Long-run moments

The table below depicts unconditional second moments computed using the ergodic distribution for the economy under each monetary regime considered. In line with the macro-finance literature, this model, which incorporates an occasionally binding credit constraints, accounts for some key regularities of the business cycles of emerging countries: the variability in GDP is higher than the variability in consumption and the strong procyclicality of capital flows.

Furthermore, Table 3.3 points toward strong effects of monetary regimes on the volatility of the macroeconomic indicators, especially GDP, consumption, unemployment, and real exchange rate.

Table 3.3: Second Moments

	Inflation Targeting		Time-Consistent		Data
	no CFT	with CFT	no CFT	with CFT	
Standard Deviations					
Consumption	5.4	5.2	7.3	5.7	6.2
Employment	2.3	2.8	5.3	3.4	2.9
Real Exchange Rate	8.5	6.7	6.5	5.9	8.2
Current Account-GDP	2.9	1.3	2.0	1.2	3.0
Trade Balance-GDP	3.1	1.4	2.1	1.2	2.8
Correlation with GDP					
Consumption	0.93	0.98	0.88	0.96	0.88
Employment	0.80	0.99	0.69	0.95	0.74
Current Account-GDP	-0.65	-0.54	-0.59	-0.54	-0.63

Notes: Data are annual from WDI and Global Financial Data (GFD). Data period covers 1965-2014.

The first result from Table 3.3 enhances the existence of a trade-off between the variability in GDP and employment. Indeed, compared to the economy under a discretionary monetary policy regime, the economy under an inflation targeting regime features lower business cycle variability in employment and consumption, pointing to the importance of price stability policy in absorbing shocks and stabilizing employment. By stimulating employment and reducing the variability in the relative price of non-tradables during episodes of financial crises, a discretionary monetary policy regime lowers the volatility of the real exchange rate, the current account-to-GDP ratio, and the trade balance-to-GDP ratio. Table 3.3 further highlights the strong role of capital flow taxes in stabilizing the economy regardless the monetary regime considered. As discussed by Bianchi (2011) and Fornaro (2015), there are two main reasons for this to happen: the probability of crises is much larger in an economy where capital flow taxes are not available, and the externalities interfere with households' desire to smooth consumption over time. Clearly, when there is a bad shock and the credit constraint binds, households are forced to reduce their debt, which in turn generates a countercyclical current account balance-to-GDP ratio. Thus, the correlation of the current account-to-GDP ratio with the GDP is key to explaining the role played by capital flow taxes in smoothing consumption. From table 3.3, the lowest countercyclicality of the current account-to-GDP ratio is obtained when capital flow taxes are used optimally. The consumption smoothing therefore works better under a monetary regime with capital flow taxes.

3.4.5 Welfare Effects

The welfare implications of a monetary regime for each initial state, denoted $\lambda(b, s)$, is defined as the percent variation in the lifetime consumption stream that equalizes the expected utility of an household living in the economy under the discretionary monetary policy regime without capital flow taxes (TC) to the expected utility of an household living in an economy under the alternative monetary regime (AP) considered. Formally, for each initial state (b, s), the welfare implications of the alternative monetary regime are computed as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^{TC}(1+\lambda) - g(\ell_t^{TC})\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^{AP} - g(\ell_t^{AP})\right)$$

where c_t^s and ℓ_t^s denote consumption and labor supply allocations under the monetary regime $s \in \{TC, AP\}$. Figure 3.4 depicts the welfare gains of moving away from the discretionary monetary policy regime without capital flow taxes to an alternative monetary regime as a function of the current level of bond holdings, and for negative, one standard deviation shocks.

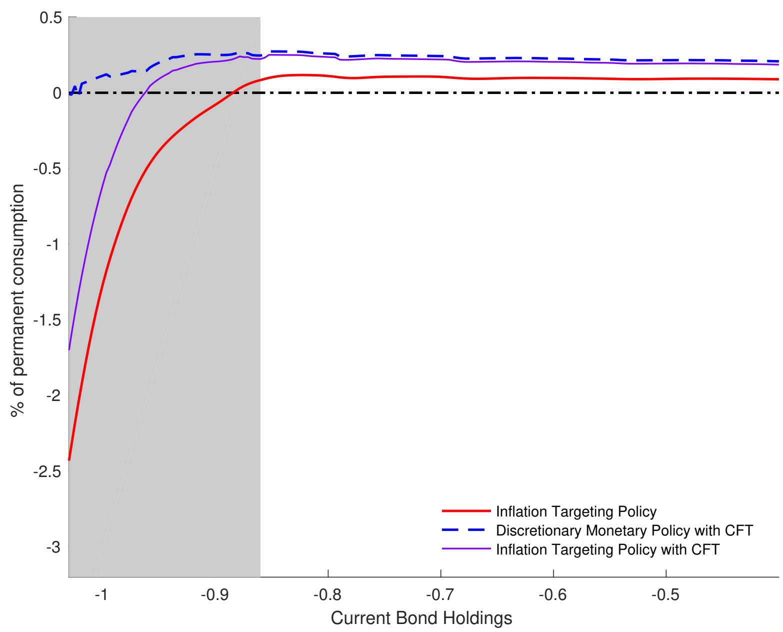

Figure 3.4: Welfare gains/costs of moving away from the procyclical discretionary monetary policy regime.

First, in the unconstrained region, the monetary regimes with capital flows taxes deliver larger welfare gains. This is because with capital flow taxes used by the government to address the pecuniary externality, households living in an economy under one of the monetary regimes with capital flow taxes act in a more precautionary way and the welfare increases. Furthermore, Figure 3.4 shows that, when capital flow taxes cannot be used, there are welfare gains from adopting an inflation targeting regime rather than the discretionary monetary policy regime. The rationale is that under the discretionary monetary policy regime, the government selects the best action given the current situation, which does not result in the social objective function being maximized. Rather, by relying on the inflation targeting policy rules, economic performance is improved (Kydland and Prescott, 1977).

Secondly, in the region where the credit constraint binds, comparing the welfare effects under both the inflation targeting regime and the effect under the discretionary monetary policy regime with capital flow taxes, it is apparent that monetary policy is more effective in correcting demand externalities than capital flow taxes. In this region, under the time-consistent monetary policy, the government sustains the value of the collateral, allows for more credit access and significantly improve social welfare. It is also important to notice that there are benefits from using capital controls under the inflation targeting regime, in contrast to the previous studies (e.g. Bianchi, 2011 and Ottonello, 2015) where the welfare gain from using capital flow taxes under a policy that replicates the flexible price allocation arises only in relation to how future allocations will differ. The reason is that, in this environment, when the credit constraint binds, capital flow taxes are used to offset the intra-temporal distortion in the labor supply decision and stabilize the economy.

I also calculate the average welfare gain $\bar{\lambda}$ as the average $\lambda(b, s)$ computed with the ergodic distribution in the economy under the discretionary monetary policy regime without capital flow taxes. Because when capital flow taxes cannot be used, an inflation targeting policy only delivers welfare losses in the constrained region, and the economy under this monetary regime spends less than 16 percent of the time in this region, on average there are welfare gains of adopting an inflation targeting regime rather than the discretionary monetary policy regime, which corresponds to 0.04 percent of permanent consumption. Another key result of this welfare analysis indicates the importance of capital flows taxes in changing the desirable property of the discretionary monetary policy regime. The discretionary monetary policy regime with capital flow taxes delivers the largest welfare gain (0.16 percent of permanent consumption). In comparison, the welfare gain from using capital flow taxes under the inflation targeting regime is only 0.14 percent of permanent consumption.

3.5 Conclusion

This paper provides an explanation to why monetary policy is procyclical in emerging economies using a small open economy model in which domestic agents face a credit constraint that limits borrowing to a fraction of their current income. The procyclicality of monetary policy is the result of the lack of commitment and the risk of capital account reversals (i.e. Sudden Stops). The discretionary monetary policy is contractionary during Sudden Stops to appreciate the real exchange rate, sustain the value of the collateral and relax binding financial constraints faced by domestic agents. During tranquil times, the discretionary monetary policy is procyclical because the expectation of monetary policy interventions ex post exacerbates the overborrowing problem. This policy then reduces domestic agents' marginal propensity to consume the tradable good, so as to mitigate overborrowing by private agents. Quantitatively, committing to an inflation targeting regime dominates the discretionary monetary policy regime in term of welfare. In addition, relative to the discretionary monetary regime, it reduces the occurence of financial crises although there are more severe when they occur. The quantitative analysis also suggests that there is much to be gained when monetary policy and macroprudential regulation such as capital controls are conducted jointly. Prudential capital controls are found to be very effective in correcting the externality stemming from financial constraints and make the discretionary monetary policy less procyclical, which as a result reduce excessive risk exposure of the economy, and deliver higher social welfare.

Chapter 4

Macroprudential Arrangements for Currency Unions[1]

4.1 Introduction

Eurozone's experience of the last decade has outlined the challenges of conducting macroeconomic stabilization policy in a currency union whose member countries face diverging economic prospects. During the same period, macroprudential regulation has established itself as a new cornerstone of bank regulators' and central banks' toolkits. Most of the policy discussion surrounding macroprudential regulation has revolved around its role of promoting financial stability. But in contexts, such as in a currency union, where conventional macroeconomic stabilization tools are lacking, it appears legitimate to question whether macroprudential policy could (and should) also be assigned a macroeconomic stabilization function.

The objective of this paper is to propose a simple theoretical framework for the analysis of the effectiveness and design of country-specific macroprudential policy as a macroeconomic stabilization tool in currency unions. In particular, we develop the idea that country-specific macroprudential policies carry both costs and benefits in a currency union. On the one hand, the benefits take the form of the possibility of tailor-made stimulus of aggregate demand in a context where a single policy rate and a fixed exchange rate put major constraints on macroeconomic adjustment to asymmetric shocks. On the other hand, the costs take the form of distortions arising from diverging consumption patterns across countries, or violations of so-called "international efficiency."

To formalize these insights, we provide an analytically tractable New Keynesian model of a currency union featuring an interaction between macroprudential policy and aggregate demand.

[1]This chapter is co-authored with Sushant Acharya (Federal Reserve Bank of New York) and Julien Bengui (Université de Montréal).

The framework builds upon the standard two-country general equilibrium model that constitutes the workhorse of New Open Economy Macroeconomics and is commonly used to study positive and normative issues pertaining to international shock transmission, exchange rate regimes, and policy spillovers. It has a single currency, and features extra pigouvian taxes akin to country-specific macroprudential policy regulation. Besides allowing for a rigorous analysis of the effectiveness of these country-specific instruments and the desirability of their use, the framework naturally lends itself to the study of the optimal institutional design of the currency union. Should the country-specific macroprudential instruments be set in a decentralized way by national authorities, such as bank regulators or national central banks? Or should they instead be set centrally by the union's central bank?

Our model features two countries (or regions), which for illustrative purposes can be labeled Core and Periphery.[2] These countries are populated by households and firms. Households consume goods produced in both countries, but we capture the possibility of less than perfect trade integration by allowing for home bias (i.e., an over-representation of home goods in agents' consumption baskets). As a result, our model nests any configuration between the polar cases of full trade integration (no home bias) and autarky (full home bias). Firms produce differentiated goods and face nominal rigidities, as is standard in the New Keynesian literature. The two countries share a common currency and have surrendered their monetary sovereignty to the union's central bank, whose task is to maximize union-wide welfare. A novelty of our analysis resides in the introduction of pigouvian taxes, which we label a country-specific macroprudential regulation, that alter the effective cost of borrowing for households. For instance, for a given Core nominal interest rate set by the central bank, an expansionary country-specific macroprudential regulation in the Periphery leads to a lower effective interest rate there than in the Core.

The key limitation of monetary policy is this environment, as it is in practice, is that it performs well as long as the member countries experience similar shocks but is poorly equipped to deal with asymmetric shocks. To make matters interesting, we thus explicitly allow for asymmetric productivity shocks. But while common monetary policy is doomed to affect aggregate demand in both countries symmetrically, country-specific macroprudential policy is not. In particular, to the extent that there is some home bias in consumption, an expansionary macroprudential policy in the Periphery will stimulate demand for Periphery goods more strongly than for Core goods. This provides the background to our first main insight: country-specific macroprudential regulation is effective at stabilizing asymmetric shocks when the degree of openness is not too high (there is a decent amount of home bias).

[2]Our insights naturally carry over to situations with a larger number of countries.

Accordingly, a normative analysis of the model confirms that unless trade integration is perfect (no home bias), it is desirable to combine monetary policy with country-specific macroprudential regulation to perform stabilization policy. The general prescription that emerges is that expansionary country-specific macroprudential regulation should always be used to give an extra kick to aggregate demand in the country experiencing the most severe recession. As a result, a superior stabilization is achieved union-wide, albeit at the cost of a mild deviation from international consumption risk-sharing. We find that the benefits of country-specific macroprudential regulation decrease with labor supply elasticity and trade openness.

We also use our framework to investigate the optimal institutional design of country-specific macroprudential regulation. Here, a game theoretic analysis suggests that there is a strong case for coordinating country-specific regulation via a central agency, such as the union's central bank, as a decentralized macroprudential policy design may give rise to dynamic terms of trade wars of the kind featured in Costinot et al. (2014). This is because following shocks that lead to within-union trade imbalances, national authorities face incentives to use country-specific macroprudential regulation to limit trade flows in an effort to exert market power in export and import markets. These actions amount to rent seeking unrelated to the macroeconomic stabilization benefits outlined above, and, we show, may be powerful enough to overwhelm these.

Related literature The paper is related to several strands of the literature. First, it contributes to the optimum currency area literature, which has traditionally emphasized the need for alternative adjustment mechanisms or variables to make up for the rigidity of a fixed exchange rate system, such as factor mobility (Mundell 1961), trade integration (McKinnon 1963) or fiscal transfers (Kenen 1969). Recent work has revisited these mechanisms and explored new ones (see, e.g., Beetsma and Jensen 2005; Gali and Monacelli 2008; Farhi and Werning 2014; Schmitt-Grohé and Uribe 2016; Sergeyev 2016; Farhi and Werning 2017). Closest to our work in this area is the work of Farhi and Werning (2014) and Sergeyev (2016). Farhi and Werning (2014) analyze the stabilization benefits of capital controls in a fixed exchange rate regime in a small open economy, but without regard to international spillover effects and associated coordination issues that are at the core of our analysis. Sergeyev (2016) studies monetary and macroprudential policy in a hybrid model featuring both nominal rigidities and financial market imperfections, but our framework is simpler, more tractable and easier to relate to the traditional two-country general equilibrium model that is the workhorse of the New Open Economy Macroeconomics literature. In this vein, we explore how an appropriate use of taxes on financial transactions can reduce the losses from adopting a common currency. Second, our project also relates to a recent normative literature that views macroprudential policy interventions as a way to correct aggregate demand externalities arising in environments with nominal rigidities and constraints on monetary policy (Acharya and Bengui 2018; Farhi and Werning 2016; Korinek and Simsek 2016). For theories motivating macroprudential regulation through pecuniary externalities arising from financial market imperfections, see, e.g., Gromb and Vayanos 2002; Lorenzoni 2008; Bianchi 2011; Davila and Korinek 2017.

4.2 Model

The currency union consists of two equally sized countries, designated "Home" and "Foreign." In each country, households consume goods and supply labor, while firms hire labor to produce output. Foreign variables are denoted with asterisks.

4.2.1 Households

In each country, there is a representative household. We focus on Home households. By symmetry, the same applies to Foreign households. In Home, the preferences of the representative household are represented by the utility functional

$$\int_0^\infty e^{-\rho t} \left[\log C_t - \frac{(N_t)^{1+\phi}}{1+\phi} \right] dt,$$

where C_t is consumption, N_t is labor supply, ϕ is the inverse Frisch elasticity of labor supply, and ρ is the discount rate. The consumption index C_t is defined as

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(l)^{\frac{\varepsilon-1}{\varepsilon}} dl \right]^{\frac{\varepsilon}{\varepsilon-1}}$ denotes an index of domestically produced varieties, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(l)^{\frac{\varepsilon-1}{\varepsilon}} dl \right]^{\frac{\varepsilon}{\varepsilon-1}}$ is an index of foreign produced varieties. Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods, while $\varepsilon > 1$ is the elasticity of substitution between varieties produced within any given country. The home biais parameter $\alpha \in (0, 0.5]$ captures the degree of openness. In the extreme case where $\alpha = 0.5$, households in Home and Foreign will have the same consumption baskets, and countries belonging to the currency union will be described as very open economies without home bias.

There is a single currency and a single bond market. Households have access to the market for bonds. Since the model does not feature uncertainty, these bonds trivially span the space of states of nature. We explicitly allow for taxes and subsidies on capital flows to the extent that bonds acquired at a given period are potentially taxed or subsidized. The home household's budget constraint is then given by

$$\dot{D}_t = (i_t + \tau_t) D_t + W_t N_t + T_t - \int_0^1 P_{H,t}(l) C_{H,t}(l) \, dl - \int_0^1 P_{F,t}(l) C_{F,t}(l) \, dl \tag{4.1}$$

where i_t is the interest rate on the home bond, D_t denotes holdings of the home bond, W_t is the nominal wage and T_t denotes lump-sum transfers and Π_t denotes the payout of domestic firms. τ_t is a tax on capital inflows (or a subsidy on capital outflows) in the home country. The proceeds of these taxes are rebated lump sum to the domestic households.

The household's demand for a differentiated good l is given by $C_{j,t}(l) = (P_{j,t}(l)/P_{j,t})^{-\varepsilon} C_{j,t}$, for $j = H, F$. Expenditure minimization leads to home's consumer price index (CPI) definition

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

where $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(l)^{1-\varepsilon} dl\right]^{\frac{1}{1-\varepsilon}}$ is home's produce price index (PPI), $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(l)^{1-\varepsilon} dl\right]^{\frac{1}{1-\varepsilon}}$ is home's price index of imported goods.[3] We assumed that the law of one price holds, which implies $P_{j,t}(l) = P^*_{j,t}(l)$ for $j = H, F$. At the final good level, it implies $P_{j,t} = P^*_{j,t}$ for $j = H, F$. The terms of trade between Home and Foreign are defined as the ratio of PPIs, $S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{P^*_{F,t}}{P_{H,t}}$, while the real exchange rate is defined as the ratio of CPIs, $Q_t \equiv \frac{P^*_t}{P_t}$.

Using the above price index definitions, the home household's budget constraint (4.1) can be expressed as

$$\dot{D}_t = (i_t + \tau_t) D_t + W_t N_t + T_t - P_t C_t.$$

The home household chooses consumption, labor supply and bond holdings to maximize utility. His optimal labor supply condition, and Euler equations are respectively given by

$$\begin{aligned} \frac{W_t}{P_t} &= N_t^{\phi} C_t^{\sigma} \\ \frac{\dot{C}_t}{C_t} &= \frac{1}{\sigma}\left[i_t + \tau_t - \pi_t - \rho\right]. \end{aligned}$$

where $\pi_t \equiv \dot{P}_t / P_t$ is CPI inflation in Home.

Foreign households are symmetric. Regarding initial conditions, we assume that countries have symmetric net foreign asset positions (i.e., equal to 0) at time 0.

[3] Similarly, $P^*_t \equiv \left[(1-\alpha)\left(P^*_{F,t}\right)^{1-\eta} + \alpha \left(P^*_{H,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is foreign's CPI, with $P^*_{F,t} \equiv \left[\int_0^1 P^*_{F,t}(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ being foreign's PPI, and $P^*_{H,t} \equiv \left[\int_0^1 P^*_{H,t}(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ being foreign's price index of imported goods.

4.2.2 Firms

Technology Firms in Home and Foreign produce differentiated goods $l \in [0,1]$ with a linear technology: $Y_t(l) = A_t N_t(l)$, respectively $Y_t^*(l) = A_t^* N_t(l)$, where A_t and A_t^* are time-varying productivity levels.

Price setting We assume that the monopolistically competitive firms in each country engage in infrequent price setting à la Calvo. In this setup, each firm is allowed to reset its prices only at the time when a price-change signal is received following a Poisson process with intensity $\delta \geq 0$. In the case where $\delta = 0$, prices are entirely rigid.[4] As a result, per unit of time a fraction δ of firms in Home receive the price-change signal and set their prices, $P_{H,t}^r(j)$, to maximize the expected discounted profits

$$\int_t^{\infty} \delta e^{-\delta(k-t)} \frac{\lambda_k}{\lambda_t} \left(P_{H,t}^r(j) Y_{k|t} - P_{H,k} MC_k Y_{k|t} \right) dk$$

subject to the demand for their own good, $Y_{k|t} = \left(\frac{P_{H,t}^r}{P_{H,t+k}} \right)^{-\varepsilon} Y_k$, and taking as given the sequences Y_t, $P_{H,t}$ and the real marginal cost MC_t. The real marginal cost, defined as $MC_t \equiv \frac{1-\tau^L}{A_t} \frac{W_t}{P_{H,t}}$, is the marginal cost deflated by Home PPI, where τ^L denotes a constant labor subsidy.[5] λ_k is the time t home households' marginal utility of consumption and λ_k / λ_t represents the stochastic discount factor between time t and time k.

4.2.3 Policy Authorities

Central monetary authority The central monetary authority (central bank of the currency union) sets the nominal rate as its monetary policy instrument.[6]

Macroprudential authorities The macroprudential authority sets the capital flow taxes in a country and transfers lump-sum the proceeds from taxes on inflows to the Household in their country. The macroprudential authority budget is balanced. Thus, for the home economy, we can write: $\mathcal{T}_t = \tau_t D_t$. Similarly, for the foreign economy, we have $\mathcal{T}_t^* = \tau_t^* D_t^*$.

[4]This case is an extreme one, particularly in an environment where the exchange rate is unable to adjust. The joint assumption of fully rigid prices and fixed exchange rate means that absolute and relative prices are fixed forever: $P_t = P_t^* = P_{F,t} = P_{H,t} = P_{F,t}^* = P_{H,t}^* = Q_t = S_t = 1$. As a result, PPI and CPI inflations are always zero.

[5]As is standard in the New Keynesian literature, we assume that the constant labor subsidy τ^L and τ^{L*} are set optimally considering a symmetric steady state with flexible prices.

[6]This is the standard cashless approach (Woodford, 2003).

4.2.4 Equilibrium

International "risk"-sharing An international consumption smoothing condition relating the ratio of marginal utility in both countries to the real exchange rate can be derived by combining the home and foreign households' Euler equations for the bond [7]

$$C_t = \Theta_t Q_t C_t^*, \tag{4.2}$$

where $\Theta_t \equiv \Theta_0 \exp\left[\int_0^t (\tau_s - \tau_s^*)\, ds\right]$. Θ_0 is a constant related to initial relative wealth positions. Absent capital flow taxes, (4.2) indicates a constant ratio of marginal utilities out of nominal income in both countries. With capital flow taxes, however, the ratio of marginal utilities become *time-varying*. Notice that the evolution of this ratio is given by

$$\frac{\dot{\Theta}_t}{\Theta_t} = \tau_t - \tau_t^* \tag{4.3}$$

Market clearing In equilibrium, bond markets, goods markets and labor markets all have to clear. Bond market clearing requires $D_t + D_t^* = 0$. Equilibrium in the market for good l in home requires

$$Y_t(l) = \underbrace{(1-\alpha)\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t}_{C_{H,t}(l)\text{: Home demand for Home variety } l} + \underbrace{\alpha\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*}_{C_{H,t}^*(l)\text{: Foreign demand for Home variety } l}. \tag{4.4}$$

Thus, at the level of Home's aggregate output, defined as $Y_t \equiv \left[\int_0^1 Y_t(l)^{\frac{\varepsilon-1}{\varepsilon}} dl\right]^{\frac{\varepsilon}{\varepsilon-1}}$, market clearing simply requires

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)\, C_t + \alpha Q_t^{\eta} C_t^*\right]. \tag{4.5}$$

Similarly, aggregate market clearing in Foreign requires

$$Y_t^* = \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta}\left[(1-\alpha)\, C_t^* + \alpha Q_t^{-\eta} C_t\right]. \tag{4.6}$$

Finally, for aggregate employment defined as $N_t \equiv \int_0^1 N_t(l)dl$ and $N_t^* \equiv \int_0^1 N_t^*(l)dl$, equilibrium in Home's and Foreign's labor markets require $N_t = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon} dl$ and $N_t^* = \frac{Y_t^*}{A_t^*}\int_0^1 \left(\frac{P_{F,t}^*(l)}{P_{F,t}^*}\right)^{-\varepsilon} dl$.

[7] In models featuring complete markets, this condition is often labeled as an international risk sharing condition. Notice that (4.2) bears similarity to what is commonly referred to as the Backus-Smith condition (see Kollmann 1991 and Backus and Smith 1993) in which Θ_t would represent a Pareto weight in a planning problem.

4.3 LQ approximation and policy tradeoffs

This section derives a local approximation of the non-linear optimal policy problems. The approximation is taken around a symmetric steady state. We provide a log-linearized version of the equilibrium conditions, and following Benigno and Benigno (2006) we derive a second order approximation of the welfare function under commitment from a "timeless perspective" (see also Benigno and Woodford, 2005, 2012).[8,9]

4.3.1 Equilibrium conditions

The above equilibrium conditions can be combined in a way that greatly simplifies the structure of the optimal policy problems we consider in the next sections. We describe a parsimonious representation of the equilibrium conditions.

The demand side Substituting the "risk"-sharing condition (4.2) into the market clearing conditions (4.5) and (4.6) and linearizing around the steady state yields

$$\tilde{c}_t = y_t + \alpha\tilde{\theta}_t - (\alpha+\omega)\tilde{s}_t, \qquad \text{and} \qquad \tilde{c}^*_t = \tilde{y}^*_t - \alpha\tilde{\theta}_t + (\alpha+\omega)\tilde{s}_t, \tag{4.7}$$

with $\omega \equiv 2\alpha(1-\alpha)(\eta-1)$, and where $\tilde{x}_t$ denotes the log-deviation of a variable X_t from its flexible price allocation. For instance, $\tilde{\theta}_t$ is the log-deviation of the consumption ratio Θ_t. Differentiating these equations with respect to time, and substituting the law of motion for the consumption ratio (4.3) leads to Home's and Foreign's New Keynesian IS equation

$$\dot{\tilde{y}}_t = \left[i_t - \pi_{H,t} - r^n_t\right] + \omega\dot{\tilde{s}}_t + \left[(1-\alpha)\tau_t + \alpha\tau^*_t\right] \tag{4.8}$$

$$\dot{\tilde{y}}^*_t = \left[i_t - \pi^*_{F,t} - r^{n*}_t\right] - \omega\dot{\tilde{s}}_t + \left[\alpha\tau_t + (1-\alpha)\tau^*_t\right], \tag{4.9}$$

the terms of trade evolves according to $\dot{\tilde{s}}_t = \left(\pi^*_{F,t} - \pi_{H,t}\right) - (r^n_t - r^{n*}_t)$, where the home and foreign natural real interest rates, r^n_t and r^{*n}_t, are respectively given by[10]

$$r^n_t = \rho + \left[(1-\psi_a)\,\dot{a}_t + \psi_a\dot{a}^*_t\right], \qquad \text{and} \qquad r^{n*}_t = \rho + \left[\psi_a\dot{a}_t + (1-\psi_a)\,\dot{a}^*_t\right], \tag{4.10}$$

[8] The "timeless perspective" refers to the commitment of the policymaker to its policy plan, including the initial policy, that is assumed to have been in place in some infinite past leading up to date 0.

[9] In order to derive the second order approximation of the welfare function under the "timeless perspective", we first consider the stochastic version of problem and consider the welfare for a specific path of the shocks.

[10] These natural rates are the implicit real interest rates that would prevail in the first-best allocation, or equivalently, the real interest rates that would prevail under flexible prices provided the monopolistic competition distortions are corrected by an appropriate labor subsidy.

where $\psi_a \equiv \frac{\omega\phi}{1+\phi(1+2\omega)}$. The constraint on monetary policy associated with a currency union is reflected in the fact that the same interest rate i_t appears in both IS equations (4.8) and (4.9), with an identical coefficient (of unity). Absent macroprudential policy, because the exchange rate is pegged and there is a common interest rate, monetary policy is doomed to affect aggregate demand symmetrically in both countries. Aligning the real interest rate ($r_t \equiv i_t - \pi_{H,t}$ and $r_t^* \equiv i_t - \pi_{F,t}^*$) with the natural rate in each country would be required for perfect stabilization (i.e., $\tilde{y}_t = \tilde{y}_t^* = 0\ \forall t$) only if shocks are symmetric or in the knife edge case where where the elasticity of substitution between home and foreign goods is equal to unity. As our analysis of Section 4.4 transparently confirms, monetary policy alone can thus at best be expected to stabilize the currency unions' economies *on average*.

The IS equations (4.8) and (4.9) indicate that a tax on inflows into Home (resp., Foreign) is contractionary in both countries, but more so in Home (resp., Foreign) due to the home bias. The interpretation is as follows. The implicit rate on home bonds is given by $i_t + \tau_t$, so that $\tau_t > 0$ effectively discourages consumption by home agents for a given consumption of foreign agents and level of prices. Since home agents consume both home and foreign goods, this ends up discouraging demand in both countries. However, under home bias home agents consume more of their own good, so the taxes end up being more contractionary for the home good than for the foreign good. The effect of macroprudential policy on aggregate demand is thus symmetric in the absence of home bias ($\alpha = 0.5$), but asymmetric in the presence of home bias ($\alpha < 0.5$), and the more so, the higher the home bias (the lower α). Now, since the use of capital flow taxes is associated with distortions of its own, their use only makes sense if their effect on aggregate demand in the two countries is sufficiently asymmetric. Our analysis of Section 4.5 confirms this intuition.

The supply side The dynamics of the PPI inflation in terms of real marginal cost in each country under our price setting assumption can be described by Home's and Foreign's New Keynesian Phillips curve equation

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \kappa \underbrace{\left[(1+\phi)\,\tilde{y}_t - \omega\tilde{s}_t + \alpha\tilde{\theta}_t\right]}_{\text{Home firms' marginal cost}}, \quad \text{and} \quad \dot{\pi}_{F,t}^* = \rho\pi_{F,t}^* - \kappa \underbrace{\left[(1+\phi)\,\tilde{y}_t^* + \omega\tilde{s}_t - \alpha\tilde{\theta}_t\right]}_{\text{Foreign firms' marginal cost}} \tag{4.11}$$

where $\kappa \equiv \theta(\rho + \theta)$. The interpretation of these equations is as follows. For given output gaps, a lower consumption ratio following an increase of tax on capital inflows into the home country is associated with a higher marginal cost for Home firms and a smaller marginal cost for Foreign firms. The intuition is that for given levels of output, increase of tax on capital inflows into the home country translates into a temporary decline in home consumption, and therefore higher home marginal utility and a lower home marginal rate of substitution of consumption for leisure, thus reducing the marginal cost of Home firm. Likewise, in Foreign, consumption is relatively higher which in turn translates into a lower marginal utility, a higher marginal rate of substitution, and thus a lower marginal cost of firms.

Moreover, the dynamics of the PPIs inflation show that except in the knife-edge case, where the elasticity of substitution between home and foreign goods, η, is equal to unity (the latter also corresponds in this setup to the intertemporal elasticity of substitution between current and future consumption), there is no direct exposition of inflation in any given country to the static terms of trade $\tilde{s}_t$.[11] When goods are complements (i.e. $\eta < 1$), an increase in the term of trade $\tilde{s}_t$ lowers home firms' marginal cost and as a consequence home CPI inflation. However, such increase in $\tilde{s}_t$ raises foreign CPI inflation by raising Foreign firms' marginal cost. Therefore, it follows that an increase in the term of trade mimic a negative markup shock on home CPI inflation and positive markup shock on foreign CPI inflation. The opposite happens when goods are substitutes (i.e. $\eta < 1$).

4.3.2 The welfare objective

The welfare objective from the perspective of a union-wide centralized authority that maximizes welfare for the currency union as a whole can be written as

$$\mathcal{U}_G = \mathcal{U}_H + \mathcal{U}_F^*.$$

$\mathcal{U}_H$ and $\mathcal{U}_F^*$ are the welfare objectives of home and foreign policy authorities respectively. The second order approximation of the welfare objective in each member country is then

$$\mathcal{U}_H \equiv -\frac{1}{2}\int_0^\infty e^{-\rho t}\left[\mathbb{L}_{H,t} - 2g_t\right] dt, \qquad \text{and} \qquad \mathcal{U}_F^* \equiv -\frac{1}{2}\int_0^\infty e^{-\rho t}\left[\mathbb{L}_{F,t}^* + 2g_t\right] dt, \tag{4.12}$$

where the per-period loss function of national authorities in both Home and Foreign, $L_{H,t}$ and $L_{F,t}^*$, are respectively given by

$$\begin{aligned}
\mathbb{L}_{H,t} &\equiv \frac{\varepsilon}{\kappa}(\pi_{H,t})^2 + (1+\phi)(\tilde{y}_t)^2 + \alpha(1-\alpha)\left(\tilde{\theta}_t - (\eta-1)(1-2\alpha)\tilde{s}_t\right)^2, \\
\mathbb{L}_{F,t}^* &\equiv \frac{\varepsilon}{\kappa}(\pi_{F,t}^*)^2 + (1+\phi)(\tilde{y}_t^*)^2 + \alpha(1-\alpha)\left(\tilde{\theta}_t - (\eta-1)(1-2\alpha)\tilde{s}_t\right)^2,
\end{aligned}$$

[11] A rise in the static term of trade, $\tilde{s}_t$ has two opposite effects. It reduces Home demand and increases Foreign demand for Home goods via good market clearing conditions (4.4), and increases the purchase power of foreign households relative to home households. These two effects explain why η determines the exposition of any given country to the static term of trade $\tilde{s}_t$.

In the per-period loss functions, the first two terms are familiar in the New-Keynesian models (see for example Gali and Monacelli, 2005) and accounts for the incentive to reduce price and output fluctuations. The third term captures one the one hand the distortions introduced by capital flow taxes, and on the other hand the external distortion that arises because domestically produced goods and imported goods are not perfect substitutes as described in Corsetti and Pesenti (2001).

g_t collects "terms specific to the national authority" and is defined as

$$g_t \equiv \alpha\tilde{\theta}_t - (\alpha+\omega)\tilde{s}_t$$

where $\tilde{s}_t$ is the log-deviation of the static term of trade and $\tilde{\theta}_t$ is the consumption ratio (or dynamic term of trade) in terms of log. g_t then represents the member countries' incentives of self-interested terms of trade manipulations. National authorities can reduce their own losses originating from the fluctuations of domestic output and inflation by manipulating the terms of trade. It is important to notice that under cooperation there terms of trade externality are internalized, and the terms specific to the national authorities, g_t, canceled each other out in the welfare objective the union-wide centralized authority.

4.4 Optimal monetary policy

We start by analyzing a case where monetary policy is conducted optimally at the currency union level, absent taxes on capital flows. In that case, the planner chooses an interest rate path in the common currency to maximizes a symmetrically weighted average of the two countries' welfare. This planner's problem amounts to[12]

$$\min_{\{i_t,\tau_t\}} \int_0^\infty e^{-\rho t}\Big\{\mathbb{L}_{H,t}+\mathbb{L}^*_{F,t}\Big\}dt \tag{4.13}$$

subject to

$$\begin{aligned}
\dot{\pi}_{H,t} &= \rho\pi_{H,t} - \kappa(1+\phi)\tilde{y}_t + \kappa\omega\tilde{s}_t,\\
\dot{\pi}^*_{F,t} &= \rho\pi^*_{F,t} - \kappa(1+\phi)\tilde{y}^*_t - \kappa\omega\tilde{s}_t,\\
\dot{\tilde{y}}_t &= i_t - \pi_{H,t} - r^n_t + \omega\tilde{s}_t,\\
\dot{\tilde{y}}^*_t &= i_t - \pi^*_{F,t} - r^{n*}_t - \omega\tilde{s}_t,\\
\tilde{y}_t &= \tilde{y}^*_t + (1+2\omega)\,\tilde{s}_t,
\end{aligned}$$

[12]We follow the literature in normative open-economy macroeconomics in assuming that the planner has access to a date 0 transfer across the two countries. This assumption allows us to drop the country resource constraints from the planning problem.

The following proposition characterizes the solution to this problem.

Proposition 9 (Optimal monetary policy in currency union)**.** *Optimal monetary policy in the currency union is characterized by the following linearized targeting rules*

$$\tilde{y}_t + \tilde{y}^*_t = 0, \qquad \textit{and} \qquad \pi_{H,t} + \pi^*_{F,t} = 0. \tag{4.14}$$

Proof. See Appendix C.1.

As is standard with targeting rule in New Keynesian models (see, e.g., Woodford (2003), Gali (2015)), this rule does not directly describe what optimal policy should be, but rather what it should target. It indicates that the planner aims for a balance between distortions (labor wedge distortions and distortions arising from price stickiness) experienced by Home and Foreign. In the case of symmetric shocks ($A_t = A^*_t \ \forall t$), output gaps and PPI inflations in both countries are zero and perfect stabilization is achieved. With asymmetric shocks, however, the first-best cannot be achieved and monetary policy can at best stabilize the currency union members' economies in an average sense, hence the *average* zero output gap and PPI inflations condition (4.14).

In order to derive the characteristics of the interest rate and output dynamics under the optimal monetary policy, we differentiate condition (4.14) with respect to time and substitute the New Keynesian IS equations (4.8) and (4.9) which leads to

$$i_t = \frac{1}{2}\left(r^n_t + r^{*n}_t\right), \text{ and } \dot{\tilde{y}}_t = -\dot{\tilde{y}}^*_t = -\frac{1}{2}\left(r^n_t - r^{*n}_t\right) + \kappa[1+\phi(1+2\omega)] \int_0^t \frac{e^{(t-s)\nu^+} - e^{(t-s)\nu^-}}{\nu^+ - \nu^-}\left(r^n_s - r^{*n}_s\right) ds. \tag{4.15}$$

with $\nu^{\pm} = \rho \pm \left[\rho^2 + 4\kappa(1+\phi(1+2\omega))\right]^{1/2}$. The regime with optimal monetary policy alone is straightforward to characterize: the union-wide average of both the PPI inflations and output gaps are zero, the common policy rate is equal to the average of natural rates. The growth rate of the home output gaps is equal to half of the gap between the foreign and home natural rates when firms cannot reset their prices ($\kappa = 0$). When firms are allowed to adjust their prices, the growth rate of the home output gaps is relatively lower due to the cost of nominal rigidities.

4.5 Cooperative macroprudential policy

We now augment the centralized policy maker's toolbox with a tax on capital flows between the two countries. In that case, the planner chooses an interest rate path and in the common currency to maximizes a weighted average of the two countries' welfare. This planner's problem amounts to

$$\min_{\{i_t,\tau_t\}} \quad \int_0^\infty e^{-\rho t}\Big\{\mathbb{L}_{H,t} + \mathbb{L}^*_{F,t}\Big\}dt \tag{4.16}$$

subject to

$$\begin{aligned}
\dot{\tilde{\theta}}_t &= \tau_t - \tau_t^*,\\
\dot{\pi}_{H,t} &= \rho\pi_{H,t} - \kappa(1+\phi)\tilde{y}_t + \kappa\omega\tilde{s}_t + \kappa\alpha\tilde{\theta}_t,\\
\dot{\pi}^*_{F,t} &= \rho\pi^*_{F,t} - \kappa(1+\phi)\tilde{y}^*_t - \kappa\omega\tilde{s}_t - \kappa\alpha\tilde{\theta}_t,\\
\dot{\tilde{y}}_t &= i_t - \pi_{H,t} - r^n_t + \omega\tilde{s}_t + [(1-\alpha)\tau_t + \alpha\tau^*_t]\,,\\
\dot{\tilde{y}}^*_t &= i_t - \pi^*_{F,t} - r^{n*}_t - \omega\tilde{s}_t + [\alpha\tau_t + (1-\alpha)\tau^*_t]\,,\\
\tilde{y}_t &= \tilde{y}^*_t + (1-2\alpha)\,\tilde{\theta}_t + (1+2\omega)\,\tilde{s}_t,
\end{aligned}$$

The first constraint not present in the optimal monetary policy problem (C.1), reflects the dependence of the consumption ratio upon the capital flow taxes. Without loss of generality, we assume that the planner sets the foreign capital flow tax τ^*_t to zero.[13]

The following proposition characterizes the solution to this problem.

Proposition 10 (Optimal monetary and cooperative macroprudential policy)**.** *Optimal monetary and cooperative macroprudentital policy in the currency union is characterized by the linearized targeting rules*

$$\tilde{y}_t + \tilde{y}^*_t = 0, \qquad \textit{and} \qquad \pi_{H,t} + \pi^*_{F,t} = 0 \tag{4.17}$$

and

$$\tilde{\theta}_t = \frac{(1-2\alpha)\,(1+\phi)}{4\alpha(1-\alpha)}\Big[(\tilde{y}^*_t - \tilde{y}_t) + \omega\dot{\tilde{s}}_t + \kappa(\varphi_t - \varphi^*_t)\Big] + \frac{\kappa}{2(1-\alpha)}(\varphi_t - \varphi^*_t) \tag{4.18}$$

where the multiplier on the Philipps curves φ_t and φ^*_t satisfies: $\kappa(\dot{\varphi}_t + \dot{\varphi}^*_t) = -\varepsilon(\pi_{H,t} + \pi^*_{F,t})$.

The first targeting rule, (4.17), pertaining to monetary policy is analogous to the relevant rule when monetary policy is the only available instrument (see (4.14)). The second targeting rule, (4.18), pertains to macroprudential policy (taxes on capital flows). This rule states that, given prices in both countries, the planner wants to distort the allocation of consumption in favor of the country with the lowest, or equivalently the negative (since $\tilde{y}^*_t - \tilde{y}_t = 2\tilde{y}^*_t = -2\tilde{y}_t$),

[13]The (standard) implicit assumption that the planner has access to a date 0 transfer across the two countries allows us to drop the country resource constraints from the planning problem and makes the tax differential $\tau_t - \tau^*_t$ (rather the individual taxes τ_t, τ^*_t) the only relevant instrument. Normalizing $\tau^*_t = 0$ is thus without loss of generality.

output gap i.e., experiencing the more severe recession, more so the higher the degree of home bias. In the absence of home bias (i.e., $\alpha = 0.5$), faced with asymmetric shocks across countries, capital flow taxes are only used by the planner to distort the allocation of consumption in favor of the country facing the highest cost of inflation.

Differentiating (4.18) with respect to time and combining the resulting expressions with the New Keynesian IS curves (4.8) and (4.9) yields a system of linear equations whose solutions reveals closed form expressions for the optimal policy instrument

$$\tau_t = \Psi \left(r_t^n - r_t^{*n} \right) + \kappa \int_0^t \xi_s^{coop} \left(r_s^n - r_s^{*n} \right) ds, \qquad \text{and} \qquad i_t = \frac{1}{2} \left(r_t^n + r_t^{*n} \right) + \frac{1}{2} \tau_t, \tag{4.19}$$

where ξ_s^{coop} is a term independent of policy instruments and shocks (see appendix **??** for further details) and for

$$\Psi \equiv \frac{(1-2\alpha)(1+\phi)}{4\alpha \left(1-\alpha\right) + \left(1-2\alpha\right)^2 \left(1+\phi\right)} \geq 0.$$

Several comments are in order. First, the optimal regime involves using macroprudential taxes except in the case where there is both no home bias ($\alpha = 0.5$) for which $\Psi = 0$, and firms in both countries never have the ability to reset their prices ($\kappa = 0$). This lesson is consistent with our discussion of Section 4.3.1. It emphasizes that as soon as home bias was present, or when the real interest rate is different in both countries (which the case with asymmetric shocks and sticky prices), the macroprudential tax can act as a complementary instrument to monetary policy. Second, the expression for τ_t in (4.19) indicates that (unless there is no home-bias), for a given level of prices, it is optimal to subsidize capital flows into the country with the lowest natural rate, consistent with the idea that macroprudential policy allows an extra stimulus of aggregate demand in the country with the largest need for it.[14] Third, the expression for τ_t in (4.19) also indicates that with no home bias (unless prices cannot be adjust), each member country faces a non-zero inflation ($\pi_{H,t} = -\pi_{F,t}^* \neq 0$) and it is optimal to macroprudential policy to reduce welfare loss associated with inflation fluctuations. And fourth, the expression for i_t (4.19) indicates that it is then optimal to partially offset the effects of macroprudential policy by adjusting the monetary policy stance to make it neutral *on average*.

Our closed form results have the benefit of offering precise insights into the mechanisms and trade-offs at work in the optimal policy regime. One category of insights concerns how aggressively the macroprudential tax should be used in response to a gap in natural rates across countries. A second one concerns how effective the optimal policy is at stabilizing both

[14]This is either achieved by taxing inflows into the country with the highest natural rate, or by subsidizing inflows into the country with the lowest natural rate.

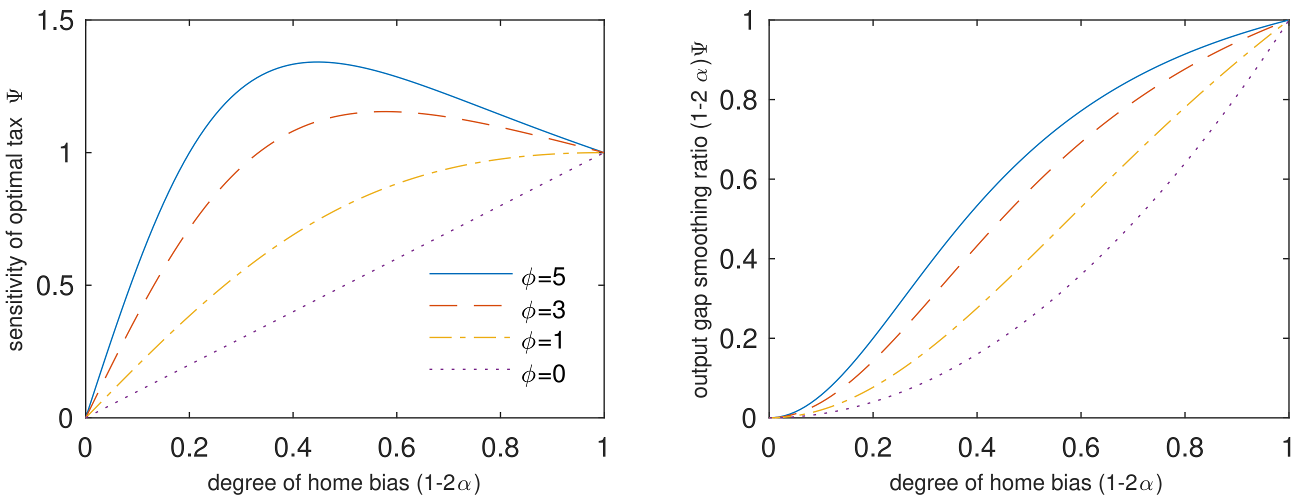

Figure 4.1: Sensitivity of optimal tax and stabilizing effect on output (for $\kappa = 0$).

countries.

The parameter Ψ_r represents the sensitivity of the optimal tax to the natural rate gap $r_t^n - r_t^{*n}$. It is easy to see that $\Psi \gtreqless 1$ for $(1-2\alpha) \gtreqless \phi^{-1}$. In other words, the optimal tax overreacts (underreacts) to the natural rate gap if the degree of home bias $(1-2\alpha)$ is larger (smaller) than the Frisch elasticity of labor supply. When the Frisch elasticity is above unity, the optimal tax underreacts for low degrees of home bias, and overreacts for high ones. When the Frisch elasticity is below unity, the optimal tax always underreacts. Intuitively, overreaction occurs when the macroprudential policy is both highly effective (high degree of home bias) and highly desirable (low Frisch elasticity). Furthermore, the sensitivity of the optimal tax with respect to the natural rate gap is decreasing in the Frisch elasticity ($\partial\Psi/\partial\phi > 0$), since a higher Frisch elasticity reduces the benefits of output gap stabilization relative to the distortionary costs of macroprudential policy (arising from deviations from international "risk"-sharing). Finally, the dependence of the sensitivity of the optimal tax on the degree of home bias is more subtle and depends on the value of the Frisch elasticity as $\frac{\partial\Psi}{\partial\alpha} \gtreqless 0$ for $(1-2\alpha)^2 \gtreqless \phi^{-1}$. As already mentionned, more home bias makes macroprudential more effective because it increases the asymmetric incidence of capital flow taxes on aggregate demand. For low degrees of home bias, for which the use of the macroprudential policy instrument is not very aggressive to start with (Ψ is low), increasing home bias necessarily increases the sensitivity Ψ: the increasing effectiveness makes an increasingly aggressive use of the instrument more attractive. For high degrees of home bias, provided the Frisch elasticity is sufficiently low that the use of the instrument is aggressive enough to start with (Ψ is large enough), increasing home bias decreases the sensitivity Ψ: the increasing effectiveness makes a decreasingly aggressive use of the instrument desirable. These patterns are represented in the left panel of Figure 4.1.

Turning to the degree of effectiveness of optimal macroprudential policy, the comparative statics is more straightforward assuming firms cannot adjust their prices ($\kappa = 0$). Thus, the growth rates of the output gaps can be written as

$$\dot{\tilde{y}}_t = -\dot{\tilde{y}}_t^* = -\left[1 - (1 - 2\alpha)\,\Psi\right] \frac{1}{2} \left(r_t^n - r_t^{*n}\right) \tag{4.20}$$

(4.20) emphasizes that the growth rates of the output gaps are smoothed out by a factor of $(1-2\alpha)\Psi$ by the optimal policy intervention, relative to the case with optimal monetary policy only.[15] This ratio by which the growth rate of the output gaps in both countries is smoothed out by the addition of the macroprudential tool, $(1 - 2\alpha)\Psi$, is equal to 0 when $\alpha = 0.5$, to 1 in the limit where $\alpha \to 0$, and satisfies $\partial \left[(1 - 2\alpha)\,\Psi\right]/\partial\alpha < 0$ and $\partial \left[(1 - 2\alpha)\,\Psi\right]/\partial\phi > 0$ for $\alpha \in (0, 0.5)$. Thus, the stabilization benefits are zero without home bias, they are infinite in the limit of full home bias (i.e., perfect stabilization is achieved), and they are monotonically increasing in the degree of home bias in between. Furthermore, the lower the Frisch elasticity, consistent with the notion that macroprudential policy is used more aggressively, the larger the stabilization benefits. These patterns are represented in the right panel of Figure 4.1.

4.6 Non-cooperative macroprudential policy

In order to shed light on the role of the institutional design of the currency union with regard to macroprudential policy, we now analyze an alternative non-cooperative setting where monetary policy is set optimally by a union-wide central bank, and macroprudential policy is set optimally by national macroprudential authorities in both countries.

More precisely, we consider a simultaneous move game between the union-wide central bank, a home macroprudential authority, and a foreign macroprudential authority. The union-wide central bank sets monetary policy (as well as a date 0 transfer) optimally to maximize a weighted average of the two countries' welfare. The home macroprudential authority sets taxes on capital inflows (or subsidies to outflows) into (out of) Home, and the foreign macroprudential authority sets taxes on capital inflows (or subsidies to outflows) into (out of) Foreign. Each of the three policy makers takes the actions of the other two policy makers as given. Formal details of the game are given in Appendix C.3.

[15] This can be seen by comparing the expression in (4.20) with the one in (4.15).

4.6.1 Optimal policy problems

The union-wide central bank (monetary policy authority) solves

$$\min_{\{i_t\}} \quad \int_0^\infty e^{-\rho t}\left\{\mathbb{L}_{H,t} + \mathbb{L}^*_{F,t}\right\}dt$$

subject to the New-Keynesian Philips curves (4.11), the IS equations (4.8), (4.9), the law of motion for the consumption ratio (4.3) and the home country budget constraint[16]

$$\int_0^\infty e^{-\rho t}\left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right)dt = T, \tag{4.21}$$

Because in this framework the linear terms in the approximated welfare may induce spurious welfare evaluation, we use a second order approximation of the Philipps curve to substitute out the linear term of $\tilde{s}_t$ and a second order approximation of the country budget constraint to substitute out the linear term of $\tilde{\theta}_t$. The home macroprudential authority solves

$$\begin{aligned}\min_{\{\tau_t\}} \quad \int_0^\infty e^{-\rho t}\Big\{&\mathbb{L}_{H,t} - \alpha\beta\left(\mathbb{L}_{H,t} - \mathbb{L}^*_{F,t}\right) \\ &+ \left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right)\left[(1+\hat{\beta})\left(\tilde{\theta}_t + 2\int_0^t \tau^*_s ds\right) - 2\alpha\beta\left(\tilde{y}_t + \tilde{y}^*_t\right)\right]\Big\}dt\end{aligned}$$

where $\beta \equiv 1 + 2(1-\alpha)(\eta - 1)$ (which implies that $\alpha\beta \equiv \alpha + \omega$), and $\hat{\beta} \equiv \beta\frac{1+\phi(1-2\alpha)}{1+\phi}$, subject to (4.3), (4.8), (4.9), (4.11) and (4.21). Using the same strategy to eliminate linear terms in the loss function, the foreign macroprudential authority solves

$$\begin{aligned}\min_{\{\tau^*_t\}} \quad \int_0^\infty e^{-\rho t}\Big\{&\mathbb{L}^*_{F,t} + \alpha\beta\left(\mathbb{L}_{H,t} - \mathbb{L}^*_{F,t}\right) \\ &+ \left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right)\left[(1+\hat{\beta})\left(\tilde{\theta}_t - 2\int_0^t \tau_s ds\right) + 2\alpha\beta\left(\tilde{y}_t + \tilde{y}^*_t\right)\right]\Big\}dt\end{aligned}$$

subject to (4.3), (4.8), (4.9), (4.11) and (4.21).

A Nash equilibrium of the game is a set of policy actions by the union-wide central bank, the home macroprudential authority and the foreign macroprudential authority $\{i_t, \tau_t, \tau^*_t\}_{t\geq 0}$, T and associated allocations $\left\{\tilde{y}_t, \tilde{y}^*_t, \pi_{H,t}, \pi^*_{F,t}, \tilde{\theta}_t\right\}_{t\geq 0}$ such that: (i) taking $\{\tau_t, \tau^*_t\}_{t\geq 0}$ as given, the actions $\left\{i_t, \tilde{y}_t, \tilde{y}^*_t, \pi_{H,t}, \pi^*_{F,t}, \tilde{\theta}_t\right\}_{t\geq 0}$, T solve the union-wide central bank's problem, (ii) taking $\{i_t, \tau^*_t\}_{t\geq 0}$, T as given, the actions $\left\{\tau_t, \tilde{y}_t, \tilde{y}^*_t, \pi_{H,t}, \pi^*_{F,t}, \tilde{\theta}_t\right\}_{t\geq 0}$ solve the home macroprudential

[16] Simple accounting implies that if the home country's budget constraint is satisfied, so is the foreign country's budget constraint.

authority's problem, and (iii) taking $\{i_t, \tau_t\}_{t\geq 0}$, T as given, the actions $\left\{\tau_t^*, \tilde{y}_t, \tilde{y}_t^*, \pi_{H,t}, \pi_{F,t}^*, \tilde{\theta}_t\right\}_{t\geq 0}$ solve the foreign macroprudential authority's problem.

We proceed by characterizing the optimal choices of the three authorities, before turning to an analysis of the Nash equilibrium of the game they play.

4.6.2 Best responses

We start by describing the optimal decisions of the three authorities. For each authority, the optimal choice is described as his best response functions taken as given the policy instruments of other authorities. The following proposition characterizes authorities' best responses.

Proposition 11 (Best responses under uncoordinated macroprudential policies)**.** *In the uncoordinated macroprudential policy regime, the linearized best responses of the union-wide central bank, home macroprudential authority and foreign macroprudential authority are respectively given by*

$$
\begin{aligned}
i_t &= \frac{1}{2}\left(r_t^n + r_t^{n*}\right) + \frac{1}{2}\left(\tau_t + \tau_t^*\right) \\
\psi\tau_t &= -(1-2\alpha-\omega)\Big[\alpha\phi\tau_t^* + (1+\phi)\left(r_t - r_t^n - \omega\tilde{s}_t\right)\Big] + \frac{\hat{\beta}}{2}\omega\dot{\tilde{s}}_t + \kappa\left(1+\phi(1-2\alpha)\right)\dot{\varphi}_{H,t}, \\
\psi\tau_t^* &= -(1-2\alpha-\omega)\Big[\alpha\phi\tau_t + (1+\phi)\left(r_t^* - r_t^{n*} + \omega\tilde{s}_t\right)\Big] - \frac{\hat{\beta}}{2}\omega\dot{\tilde{s}}_t + \kappa\left(1+\phi(1-2\alpha)\right)\dot{\varphi}_{F,t}^*,
\end{aligned}
$$

with $\psi \equiv \alpha(1+\hat{\beta}) + (1-2\alpha-\omega)\big[1+\phi(1-\alpha)\big]$ *and where* $\dot{\varphi}_{H,t} \equiv \dot{\varphi}_H(i_t, \tau_t, \tau_t^*)$ *and* $\dot{\varphi}_{H,t} \equiv \dot{\varphi}_F^*(i_t, \tau_t, \tau_t^*)$ *are functions of the policy instruments.*

Proof. See Appendix C.3.

This proposition indicates that the central monetary authority, as in the coordinated solution, finds it optimal to adjust the monetary policy stance in order to partially offset the effects of macroprudential policy and make it neutral on average. Furthermore, it indicates that the national macroprudential authority responds to the others authorities policy instruments when there is home bias or all elasticities are not equal to unity. With home bias, for given level of prices, the home (the foreign) macroprudential authority finds it optimal to subsidize capital flows when the iinterest rate is below its natural interest rate level which in turn allows an extra stimulus of aggregate demand in the country when all elasticities are equal to unity. When the elasticities are not equal to unity, because changes in the term of trade resembles the effect of a markup shock on inflation, a national macroprudential authority has an incentive to respond to other policy authorities' instruments (even with no home bias). Finally, it is also important to notice that unlike the cooperative solution, capital flow taxes are not used by national macroprudential authorities when three conditions are satisfied: (i) no home bias for which $\alpha = 0.5$, (ii) firms in all member countries are not allowed to reset their prices for which $\kappa = 0$ and (iii) all elasticities are equal to unity for which $\omega = 0$.

4.6.3 Nash equilibrium

We combine the best responses of the three policy authorities considered and analyze the Nash equilibrium of the game. In the uncoordinated macroprudential policy regime, the equilibrium tax wedge is given by

$$\tau_t - \tau_t^* = \Big[(1-\Phi)\Psi + \omega\Phi\Big]\left(r_t^n - r_t^{n*}\right) + \kappa \int_0^t \xi_s^{nash}\left(r_s^n - r_s^{*n}\right) ds \tag{4.22}$$

where $\xi_s^{nash} = (1-\Phi)\xi_s^{coop}$ is a term independent of policy instruments and shocks (see appendix C.3.4 for further details) and for

$$\Phi = \frac{\alpha(1-\hat{\beta}) - [1+\phi(1-2\alpha)]\,\omega}{4\alpha\left(\frac{3}{2}-\alpha\right) + (1+\phi)(1-2\alpha)^2 - [1+\phi(1-2\alpha)]\,\omega + \alpha(1-\hat{\beta})}$$

The key difference between the non-cooperative solution and the cooperative solution follows from examining the expression for the tax wedge in (4.22) compared with the corresponding equation for the cooperative macroprudential policy (4.19). The sensitivity of the optimal tax wedge to the natural rate gap in the Nash equilibrium (which corresponds to the non-cooperative solution) is $\Big[(1-\Phi)\Psi + \omega\Phi\Big]$. The parameter Φ then represents the non-cooperative bias (or terms of trade externalities bias) originating from terms of trade manipulation.

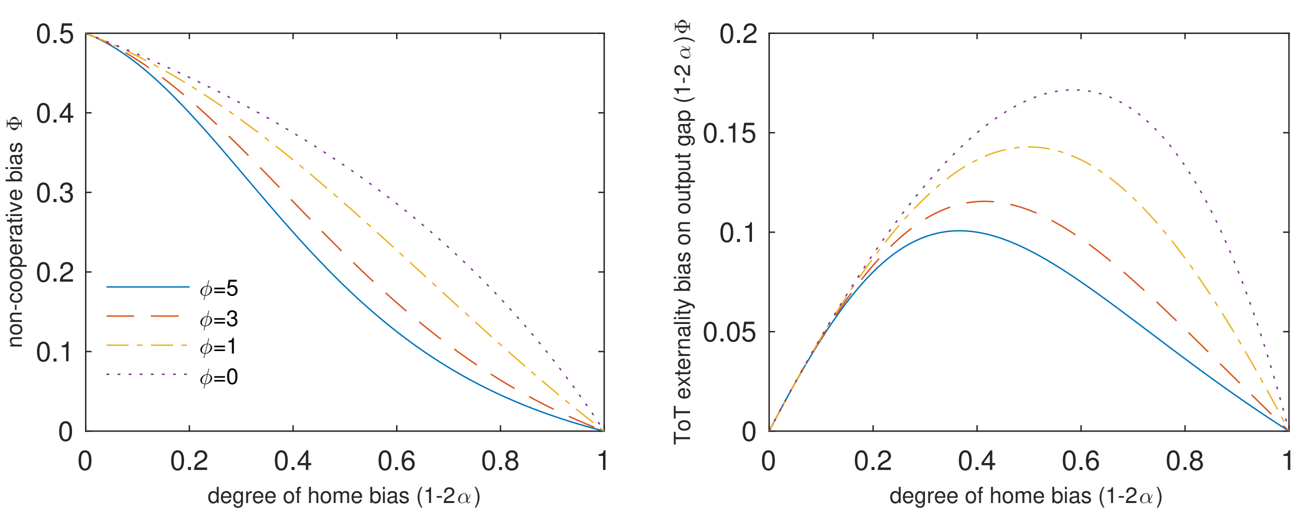

Figure 4.2: Non-cooperative bias parameters (for $\kappa = 0$)

In the closed economy limit ($\alpha \to 0$) in which the currency union members' countries barely trade with each other, there is no non-cooperative bias ($\Phi = 0$) and the sensitivity of the optimal tax wedge to the natural rate gap coincide at the cooperative and non-cooperative solutions. When the currency union members' countries sufficiently trade with each other, even for very low degree of trade integration, the optimal tax underreacts to the natural rate gap relative to the cooperative solution. Furthermore, the non-cooperative bias is decreasing in the degree of home bias ($\partial\Phi/\partial\alpha > 0$). For high degrees of home bias ($(1 - 2\alpha) \to 1$), the non-cooperative bias is low since as already mentioned more home bias makes it less attractive for national macroprudential authority to manipulate the terms of trade. Increasing the degree of trade integration (low degree of home bias) necessarily increases the incentive for self-interested terms of trade manipulations. These patterns are represented in the left panel of Figure 4.2.

We also analyze how the non-cooperative bias affects the effectiveness of optimal macroprudential policy in the Nash equilibrium. As in section 4.5, we only consider the case where firms are not allowed to adjust their prices ($\kappa = 0$). The growth rates of the output gaps in the non-cooperative solution is now given by

$$\dot{\tilde{y}}_t = -\dot{\tilde{y}}_t^* = -\frac{1}{2}\left[1 - (1 - 2\alpha)((1 - \Phi)\Psi + \omega\Phi)\right](r_t^n - r_t^{n*}) \tag{4.23}$$

The ratio by which the growth rate of the output gaps in both countries is smoothed out by the addition of the macroprudential tool, is $(1 - 2\alpha)((1 - \Phi)\Psi + \omega\Phi)$. The pattern is represented in the right panel of Figure 4.1.

4.7 Conclusion

We argue that country-specific macroprudential regulations in the form of tax on capital inflows are highly effective macroeconomic stabilization tools in currency unions. Our results show that it is optimal to subsidize capital flows into the country with the lowest natural rate allowing for a reallocation of demand and expenditures, and therefore stabilizing the currency union members' economies. Due to the existence of markets that insure cross-country consumption risk, each member country has an incentive for self-interested terms of trade manipulations (terms of trade externality). This term of trade externality problem vanishes under cooperation of macroprudential policy in which macoprudential instruments (taxes on capital inflows) are set by union-wide centralized authority. As a result, we show that when macroprudential regulations presents a "non-cooperative bias" when there are set by national planners in the currency union. Moreover, policy coordination is found to be particularly relevant under high trade integration and in response to shocks leading to trade imbalances within the currency union. Consequently, our analysis lends support to the view that country-specific macroprudential policies should be set by a union-wide centralized authority, rather than at the national level.

Bibliography

Acharya, Sushant and Julien Bengui, "Liquidity Traps, Capital Flows," *Journal of International Economics*, 2018, *114*, 276–298.

Acharya, Viral and Tanju Yorulmazer, "Too Many to Fail—An Analysis of Time-Inconsistency in Bank Closure Policies," *Journal of Financial Intermediation*, 2007, *16*, 1–31.

_ , Philipp Schnabl, and Gustavo Suarez, "Securitization without Risk Transfer," *Journal of Financial Economics*, 2013, *107* (3), 515–536.

Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee, "A corporate balance-sheet approach to currency crises," *Journal of Economic Theory*, 2004, *119* (1), 6–30.

Akerlof, George, "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *The quarterly journal of economics*, 1970, pp. 488–500.

Backus, David and Gregor Smith, "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, November 1993, *35* (3-4), 297–316.

Beetsma, Roel and Henrik Jensen, "Monetary and fiscal policy interactions in a micro-founded model of a monetary union," *Journal of International Economics*, 2005, *67* (2), 320–352.

Bengui, Julien and Javier Bianchi, "Macroprudential Policy with Leakakges," 2018. Mimeo.

Benigno, Gianluca and Michael Woodford, "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economic Association*, 2005, *3*, 1185–1236.

_ and _ , "Linear-quadratic Approximation of Optimal Policy Problems," *Journal of Economic Theory*, 2012, *147*, 1–42.

_ **and Pierpaolo Benigno**, "Price Stability in Open Economies," *Review of Economic Studies*, 2003, *70* (4), 743–764.

_ **and _ ,** "Designing targeting rules for international monetary policy cooperation," *Journal of Monetary Economics*, 2006, *53*, 473?506.

_ **, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric Young**, "Monetary and Macro-Prudential Policies: An Integrated Analysis," 2011. Mimeo, London School of Economics.

_ **, _ , _ , _ , and _ ,** "Financial crises and macroprudential policies," *Journal of International Economics*, 2013, *89* (2), 453–470.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J Sargent, "Taxes, Debts, and Redistributions with Aggregate Shocks," 2015. NBER Working Paper No. 19470.

Bianchi, Javier, "Overborrowing and systemic externalities in the business cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

_ , "Efficient Bailouts?," *American Economic Review*, 2016, *106* (12), 3607–3659.

_ **and Enrique Mendoza**, "Optimal Time-Consistent Macroprudential Policy," *Journal of Political Economy*, 2018, *126* (2), 588–634.

Braggion, Fabio, Lawrence Christiano, and Jorge Roldos, "Optimal monetary policy in a 'sudden stop'," *Journal of Monetary Economics*, 2009, *56*, 582–595.

Caballero, Ricardo and Avrind Krishnamurthy, "Inflation targeting and sudden stops," 2003. NBER Working Paper No. 9599.

Calvo, Guillermo, "Capital flows and capital-markets crises: The simple economics of sudden stops," *Journal of Applied Economics*, 1998, *1* (1), 35–54.

Caplin, Andrew and Daniel Spulber, "Menu Costs and the Neutrality of Money," *The Quarterly Journal of Economics*, 1987, *102* (4), 703–726.

Céspedes, Felipe, Roberto Chang, and Andrés Velasco, "Balance Sheets and Exchange Rate Policy," *American Economic Review*, 2004, *94* (4), 1183–1193.

Chari, V.V. and P.J. Kehoe, "Bailouts, Time Inconsistency and Optimal Regulation," *Forthcoming, American Economic Review*, 2016.

Christiano, Lawrence, Christopher Gust, and Jorge Roldos, "Monetary policy in a financial crisis," *Journal of Economics Theory*, 2004, *119* (1), 64–103.

Clarida, Richard, Jordi Gali, and Mark Gertler, "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics*, 2002, *49* (5), 879–904.

Corsetti, Giancarlo and Paolo Pesenti, "Welfare and macroeconomic interdependence," *Quarterly Journal of Economics*, March 2001, pp. 421–445.

Costinot, Arnaud, Guido Lorenzoni, and Ivan Werning, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 2014, *122* (1), 77 – 128.

Curdia, Vasco, "Monetary Policy under Sudden Stops," 2007. Federal Reserve Bank of New York. Staff Reports.

Davila, Eduardo and Anton Korinek, "Pecuniary Externalities in Economies with Financial Frictions," *Review of Economic Studies*, 2017.

Devereux, Michael, Eric Young, and Changhua Yu, "A new dilemma: capital controls and monetary policy in Sudden Stop economies," 2015. NBER Working Paper No. 21791.

Diamond, Douglas and Philip Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 1983, *91* (3), 401–419.

_ **and Raghuram Rajan**, "Illiquid Banks, Financial Stability, and Interest Rate Policy," *Journal of Political Economy*, 2012, *120* (3), 552–591.

Diewert, Erwin and Kevin Fox, "On the estimation of returns to scale, technical progress and monopolistic markups," *Journal of Econometrics*, 2008, *145*, 174–193.

Farhi, Emmanuel and Ivan Werning, "Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows," *IMF Economic Review*, 2014, *62* (4), 569–605.

_ **and** _ , "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," *Econometrica*, 2016, *84*, 1645–1704.

_ **and** _ , "Fiscal Unions," *American Economic Review*, 2017, *107* (12), 3788–3834.

_ **and Jean Tirole**, "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, 2012, *102* (1), 60–93.

— **and —** , "Shadow Banking and the Four Pillars of Traditional Financial Intermediation," 2017. Working Paper.

— **, Mikhail Golosov, and Aleh Tsyvinski**, "A Theory of Liquidity and Regulation of Financial Intermediation," *Review of Economic Studies*, 2009, *76* (3), 973–992.

Fornaro, Luca, "Financial crises and exchange rate policy," *Journal of International Economics*, 2015, *95*, 202–215.

Freixas, Xavier, "Optimal bail out policy, conditionality and constructive ambiguity," 1999. Financial Market Group Discussion Paper 237, LSE.

Gali, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduciton to the New Keynesian Framework* 2015.

— **and Tommaso Monacelli**, "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 2005, *72*, 707–734.

— **and —** , "Optimal Monetary and Fiscal Policy in a Currency Union," *Journal of International Economics*, 2008, *76* (1), 116–132.

Gertler, Mark, Simon Gilchrist, and Fabio Natalucci, "External constraints on monetary policy and the financial accelerator," *Journal of Money, Credit and Banking*, 2007, *39* (2-3), 295–330.

Gorton, Gary and Andrew Metrick, "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, 2012, *104* (3), 425–451.

Greenspan, Alan, "Speech at the 37th Annual Conference on Bank Structure and Competition of the Federal Reserve Bank of Chicago, Chicago, Illinois," 2001.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory Huffman, "Investment, capacity utilization, and the real business cycle," *American Economic Review*, 1988, *78*, 402–417.

Grochulski, Borys and Yuzhe Zhang, "Optimal Liquidity Regulation with Shadow Banking," 2015. Working paper WP 15-12R, Federal Reserve Bank Richmond.

Gromb, Denis and Dimitry Vayanos, "Equilibrium and welfare in markets with financially constrained arbitrageurs," *Journal of Financial Economics*, 2002, *66* (3-2), 361–407.

Holmström, Bengt and Jean Tirole, "Private and Public Supply of Liquidity," *Journal of Political Economy*, 1998, *106* (1), 1–40.

Jacklin, Charles, "Demand Deposits, Trading Restrictions, and Risk Sharing," 1987.

Jeanne, Olivier and Anton Korinek, "Managing credit booms and busts: A pigouvian taxation approach," 2012. manuscript.

Kaminsky, Graciela, Carmen Reinhart, and Carlos Vegh, "When It Rains, It Pours: Procyclical Capital Flows and Policies," 2004. NBER Working Paper No. w10780.

Keister, Todd, "Bailouts and Financial Fragility," *Review of Economic Studies*, 2016, *83* (2), 704–736.

_ **and Vijay Narasiman**, "Expectations vs. Fundamentals-Driven Bank Runs: When Should Bailouts be Permitted?," *Review of Economic Dynamics*, 2016, *21*, 89–104.

Kenen, Peter, "The theory of optimum currency areas: an eclectic view," in R. Mundell and A. Swoboda, eds., *Monetary Problems of the International Economy*, University of Chicago Press, 1969.

Kimball, Miles and Matthew Shapiro, "Labor supply: are the income and substitution effects both large or both small," 2008. NBER Working Paper No. 14208.

Kiyotaki, Nobuhiro and John Moore, "Liquidity, business cycles, and monetary policy," 2012. NBER Working Paper No. 17934.

Klein, Paul, Per Krussel, and Jose-Victor Rios-Rull, "Time-consistent public policy," *Review of Economic Studies*, 2008, *75*, 789–808.

Kollmann, Robert, "Essays on International Business Cycles." PhD dissertation, University of Chicago 1991.

_ , "Monetary policy rules in the open economy: effects on welfare and business cycles," *Journal of Monetary Economy*, 2002, *49* (5), 989–1015.

Korinek, Anton, "Regulating Capital Flows to Emerging Markets: An Externality View," 2009. Unpublished.

_ , "The New Economics of Prudential Capital Controls: A Research Agenda," 2011. IMF Economic Review, International Monetary Fund.

_ **and Alp Simsek**, "Liquidity Trap and Excessive Leverage," *American Economic Review*, 2016, *106* (3), 699–738.

Kydland, Finn and Edward Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 1977, *85* (3), 473–492.

Lorenzoni, Guido, "Inefficient Credit Booms," *Review of Economic Studies*, 2008, *75*, 809–833.

Lucas, Robert, "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 1972, *4*, 103–124.

McKinnon, Ronald, "Optimum Currency Areas," *American Economic Review*, 1963, *53*, 717–725.

Mendoza, Enrique, "Credit, prices, and crashes: business cycles with a sudden stop. Preventing Currency Crises in Emerging Markets," *University of Chicago Press*, 2002, pp. 335–392.

Moron, Edouardo and Diego Winkelried, "Monetary policy rules for financially vulnerable economies," *Journal of Development Economics*, 2005, *76* (1), 23–51.

Mundell, Robert, "A Theory of Optimum Currency Area," *American Economic Review*, 1961, *51*, 657–665.

Nosal, Jaromir and Guillermo Ordoñez, "Uncertainty as commitment," *Journal of Monetary Economics*, 2016, *80*, 124–140.

Ordoñez, Guillermo, "Sustainable Shadow Banking," 2013. NBER Working Paper No. 19022.

Ottonello, Pablo, "Optimal Exchange-Rate Policy Under Collateral Constraints and Wage Rigidity," 2015. manuscript, University of Michigan.

Plantin, Guillaume, "Shadow Banking and Bank Capital Regulation," *Review of Financial Studies*, 2015, *28* (1), 146–175.

Pozsar, Zoltan, Tobias Adrian, Adam Ashcraft, and Hayley Boesky, "Shadow banking," 2010. Staff Report 458, Federal Reserve Bank of New York.

Rey, Hélène, "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence," 2015. NBER Working Paper No. 21162.

Rotemberg, Julio, "Sticky prices in the united states," *Journal of Political Economy*, 1982, *90*, 1187–1211.

Schmitt-Grohé, Stephanie and Martin Uribe, "Optimal simple and implementable monetary and fiscal rules," *Journal of Monetary Economy*, 2007, *54*, 1702–1725.

— **and** — , "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124*, 1466–1514.

Sergeyev, Dmitriy, "Optimal Macroprudential and Monetary Policy in a Currency Union," Working Papers 2016.

Tauchen, George and Robert Hussey, "Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models," *Econometrica*, 1991, *59* (2), 371–396.

Woodford, Michael, "Public Debt as Private Liquidity," *The American Economic Review*, 1990, *80* (2), 382–388.

— , "Interest and Prices: Foundations of a Theory of Monetary Policy," 2003. Princeton University Press.

Yared, Pierre, "Public Debt under Limited Private Credit," *Journal of the European Economic Association*, 2013, *1* (2), 229–245.

— , "Optimal Fiscal Policy in an Economy with Private Borrowing Limits," 2015. Mimeo, Columbia.

Appendix A

Appendix to Chapter 1

A.1 Proof of Lemma 1

Step 1 : Date 1 consumption choice

A type s agent solves the following date 1 problem:

$$\mathcal{V}_1(s,\ell,\theta,X) = \max_{c_1,c_2} U\left(c_1,c_2,\theta\right) \tag{A.1}$$

$$\textit{subject to}$$

$$c_1 + q(X)c_2 = \ell e + q(X)\hat{R}(1-\ell)e \tag{A.2}$$

$$c_1 \leq \ell e + \kappa(s,\ell,X) \tag{A.3}$$

$$c_1, c_2 \geq 0.$$

Note that c_2 cannot be negative. Thus, for an agent who turns out to be impatient (type $\theta = 0$) at date 1, it is optimal to maximize c_1 and minimize c_2. It must therefore be that

$$\mathcal{C}_1(s,\ell,0,X) = \ell e + \min\left\{q(X)\hat{R}(1-\ell)e, \kappa(s,\ell,X)\right\}, \tag{A.4}$$

and

$$\mathcal{C}_2(s,\ell,0,X) = \max\{\hat{R}(1-\ell)e - \frac{\kappa(s,\ell,X)}{q(X)}, 0\}. \tag{A.5}$$

For an agent who turns out to be patient (type $\theta = 1$) at date 1, it is weakly (strongly if $q < 1$) optimal to set $c_1 = 0$ and[1]

$$\mathcal{C}_2(s,\ell,1,X) = \hat{R}(1-\ell)e + \frac{\ell e}{q(X)}. \tag{A.6}$$

[1]When $q \geq 1$, any plan such that $c_1 + c_2 = \hat{R}(1-\ell)e + \ell e$ (and $c_1, c_2 \geq 0$) is also optimal for a patient agent, but we can focus without loss of generality on the one featuring $c_1 = 0$ and $c_2 = \hat{R}(1-\ell)e + \ell e$.

Step 2 : Bonds price

First, note that the opportunity to invest in the short asset at date 1 requires that $q \leq 1$. We now show that conditional on the aggregate state X, if $q(X) < 1$, then it must satisfy

$$q(X) = \frac{\mathcal{C}_1(s, L^s, 0, X)}{\mathcal{C}_2(s, L^s, 1, X) - \mathcal{C}_2(s, L^s, 0, X)} = e\frac{1-\pi}{\pi}\frac{\gamma L^P + (1-\gamma)L^U}{\gamma \min\{\hat{R}(1-L^P)e, B(L^P)\}}. \tag{A.7}$$

To establish this, we use the fact that in equilibrium, we must have $\ell = L^s$ for the agents of type s (consistency). From (A.2) and $\mathcal{C}_1(s, L^s, 1, X) = 0$, we have

$$\begin{aligned} \mathcal{C}_1(s, L^s, 0, X) + q(X)\mathcal{C}_2(s, L^s, 0, X) &= L^s e + q\hat{R}(1 - L^s) \\ q(X)\mathcal{C}_2(s, L^s, 1, X) &= L^s e + q\hat{R}(1 - L^s). \end{aligned}$$

Combining these two equations allows us to obtain the first equality in (A.7):

$$q(X) = \frac{\mathcal{C}_1(s, L^s, 0, X)}{\mathcal{C}_2(s, L^s, 1, X) - \mathcal{C}_2(s, L^s, 0, X)} \quad \text{for } s \in \{U, P\}, \tag{A.8}$$

which itself implies

$$q(X) = \frac{\gamma \mathcal{C}_1(P, L^P, 0, X) + (1-\gamma)\mathcal{C}_1(U, L^U, 0, X)}{\gamma[\mathcal{C}_2(P, L^P, 1, X) - \mathcal{C}_2(P, L^P, 0, X)] + (1-\gamma)[\mathcal{C}_2(U, L^U, 1, X) - (\mathcal{C}_2(U, L^U, 0, X)]}. \tag{A.9}$$

Now, as $q < 1$, agents do not invest in short assets between date 1 and date 2, since they could otherwise make themselves strictly better off by saving in public bonds. Thus, the market clearing condition for date 1 consumption holds with equality:

$$\sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_1(s, L^s, \theta, X) = \sum_s \gamma_s L^s e.$$

Along with the fact that $\mathcal{C}_1(s, L^s, 1, X) = 0$ for $s \in \{P, U\}$, this implies

$$\gamma \mathcal{C}_1(P, L^P, 0, X) + (1-\gamma)\mathcal{C}_1(U, L^U, 0, X) = \frac{e}{\pi}(\gamma L^P + (1-\gamma)L^U). \tag{A.10}$$

Using (A.5) (with (2.13)) and (A.6), the denominator in (A.9) is given by

$$\begin{aligned} &\gamma[\mathcal{C}_2(P, L^P, 1, X) - \mathcal{C}_2(P, L^P, 0, X)] + (1-\gamma)[\mathcal{C}_2(U, L^U, 1, X) - \mathcal{C}_2(U, L^U, 0, X)] \\ &\qquad = \frac{\gamma L^P + (1-\gamma)L^U}{q(X)} e + \gamma \min\left\{\hat{R}(1-L^P)e, B(L^P)\right\}. \end{aligned} \tag{A.11}$$

Substituting (A.10) and (A.11) into (A.9) yields, after some algebraic manipulation, to the second equality in (A.7):

$$q(X) = e\frac{1-\pi}{\pi}\frac{\gamma L^P + (1-\gamma)L^U}{\gamma \min\left\{\hat{R}(1-L^P)e, B(L^P)\right\}}.$$

Since the opportunity to invest in the short asset at date 1 requires that $q \le 1$, the general bond price expression in a continuation equilibrium is given by

$$q(X) = \min\left\{e\frac{1-\pi}{\pi}\frac{\gamma L^P + (1-\gamma)L^U}{\gamma \min\{\hat{R}(1-L^P)e, B(L^P)\}}, 1\right\}. \tag{A.12}$$

A.2 Proof of Lemma 2

We start by establishing that $c_2^*(0) = c_1^*(1) = 0$. First, $c_2(0) > 0$ cannot be optimal, since impatient agents do not value consumption at date 2. Second, if it were the case that $c_1(1) > 0$, then the planner could decrease $c_1(1)$ by some $\epsilon > 0$ arbitrarily small while increasing $c_2(1)$ by $\epsilon\hat{R}$ and while still satisfying the resource constraint (2.11) and strictly increasing welfare.

Next, we characterize $c_1^*(0)$ and $c_2^*(1)$. The planner's first-order condition with respect to $c_1(0)$ and $c_2(1)$ is given by

$$u'(c_1^*(0)) = \rho\hat{R}u'(c_2^*(1)).$$

Since $\rho\hat{R} > 1$ by Assumption 1, this implies that $c_1^*(0) < c_2^*(1)$. As shown in Diamond and Dybvig (1983) (footnote 3), condition (2.2) on the relative risk aversion in Assumption 1 further implies that $u'(e) > \rho\hat{R}u'(\hat{R}e)$, and therefore that $c_1^*(0) > e$ and $c_2^*(1) < \hat{R}e$.

A.3 Proof of Proposition 1

We show that $\gamma = 1$ and $B = (1-\pi)c_2^*(1)$ achieve the efficient allocation described in Lemma 2. Since this safety net architecture only features protected agents, we ignore unprotected agents in what follows.

The protected agents' date 0 problem is

$$\max_{\ell\in[0,1]} \pi u\left(\ell e + q\min\left\{\hat{R}\left(1-\ell\right)e, B\right\}\right) + \left(1-\pi\right)\rho u\left(\hat{R}\left(1-\ell\right)e + \frac{\ell e}{q}\right)$$

We consider separately the agent's problem in the two intervals $[0, 1 - B/(\hat{R}e)]$ and $[1 -$

$B/(\hat{R}e), 1]$. In the first interval, the problem is

$$\max_{\ell\in\left[0,1-\frac{B}{\hat{R}e}\right]} \pi u\left(\ell e+qB\right)+\left(1-\pi\right)\rho u\left(\hat{R}\left(1-\ell\right)e+\frac{\ell e}{q}\right)$$

The first-order condition is given by

$$\Psi\left(\ell\right)\equiv e\pi u'\left(\ell e+qB\right)-e\left(1-\pi\right)\rho\left(\hat{R}-\frac{1}{q}\right)u'\left(\hat{R}\left(1-\ell\right)e+\frac{\ell e}{q}\right)\lesseqqgtr 0$$

with "$\leq$" if $\ell=0$, "≥ 0" if $\ell=1-B/(\hat{R}e)$, and "$=$" if $\ell\in(0,1-B/(\hat{R}e))$. Note that the agent's objective function is strictly concave in ℓ, as $\Psi'\left(\cdot\right)<0$ for $\ell\in[0,1-B/(\hat{R}e)]$. In the second interval, the problem is

$$\max_{\ell\in\left[1-\frac{B}{\hat{R}e},1\right]} \pi u\left(\ell e+q\hat{R}\left(1-\ell\right)e\right)+\left(1-\pi\right)\rho u\left(\hat{R}\left(1-\ell\right)e+\frac{\ell e}{q}\right).$$

We conjecture that the policy consisting of $\gamma=1$ and $B=(1-\pi)c_2^*(1)$ leads to an equilibrium collective choice of $L^P=1-B/(\hat{R}e)=1-(1-\pi)c_2^*(1)/(\hat{R}e)=\pi c_1^*(0)/e$, and verify this conjecture. Under the conjecture, the bond price is given by

$$q=\min\left\{e\frac{1-\pi}{\pi}\frac{L^P}{\hat{R}\left(1-L^P\right)e},1\right\}=\frac{1-\pi}{\pi}\frac{L^P}{\hat{R}\left(1-L^P\right)}=\frac{c_1^*(0)}{c_2^*(1)}<1.$$

Starting with the first interval, we have

$$\begin{aligned}\Psi\left(1-\frac{B}{\hat{R}e}\right) &\equiv e\pi u'\left(c_1^*(0)\right)-\left(1-\pi\right)e\left(\hat{R}-\frac{1}{q}\right)\rho u'\left(c_2^*(1)\right)\\ &> e\pi u'\left(c_1^*(0)\right)-\left(1-\pi\right)e\left(\hat{R}-1\right)\rho u'\left(c_2^*(1)\right)\\ &> e\left[\pi-\rho\left(1-\pi\right)e\left(\hat{R}-1\right)\right]u'\left(c_1^*(0)\right)>0,\end{aligned}$$

where the last inequality follows from Assumption 2. The fact that $\Psi(1-B/(\hat{R}e))>0$ implies that within this first interval, $\ell=1-B/(\hat{R}e)$ is optimal. Since the conjecture induces a bond price such that $q=c_1^*(0)/c_2^*(1)>1/\hat{R}$, in the second interval the agent's objective function is strictly decreasing in ℓ, so $\ell=1-B/(\hat{R}e)$ is optimal. We have thus verified that the privately optimal investment choice over $\ell\in[0,1]$ is consistent with the conjectured aggregate investment choice above. The fact that the policy in question achieves the efficient allocation follows from simple algebra.

A.4 Proof of Proposition 2

At date 1, the government chooses a debt issuance policy $B(\ell)$ to maximize the average welfare of agents, subject to the private sector's date 1 response to its action. The government solves

$$\max_{\{B_j\}_{j\in[0,\gamma]}} \int_0^\gamma \left[\pi \mathcal{V}_1(P,\ell_j,0,X) + (1-\pi)\mathcal{V}_1(P,\ell_j,1,X)\right] di + \int_\gamma^1 \left[\pi \mathcal{V}_1(U,\ell_j,0,X) + (1-\pi)\mathcal{V}_1(U,\ell_j,1,X)\right] dj$$

Using Lemma 1, this problem can be written as

$$\begin{aligned}\max_{\{B_j\}_{j\in[0,\gamma]}} \quad & \int_0^\gamma \left[\pi u\left(\ell_j e + q(X)\min\left\{\hat{R}(1-\ell_j)e, B_j\right\}\right) + (1-\pi)\rho u\left(\hat{R}(1-\ell_j)e + \frac{\ell_j e}{q}\right)\right] dj \\ & + \int_\gamma^1 \left[\pi u(\ell_j e) + (1-\pi)\rho u\left(\hat{R}(1-\ell_j)e + \frac{\ell_j e}{q}\right)\right] dj\end{aligned}$$

subject to[2]

$$q = \min\left\{ e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\int_0^\gamma \min\left\{\hat{R}(1-\ell_j)e, B_j\right\} dj}, 1\right\}.$$

The first-order condition for B_i is

$$\begin{aligned} & \mathbb{I}_{\left\{B_i<\hat{R}(1-\ell_i)e\right\}} \pi u'(\ell_i e + qB_i) q + \\ & \mathbb{I}_{\left\{e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\int_0^\gamma \min\left\{\hat{R}\left(1-\ell_j\right)e, B_j\right\}dj}<1\right\}} \times \mathbb{I}_{\left\{B_i<\hat{R}(1-\ell_i)e\right\}} \left[-\frac{q}{\int_0^\gamma \min\left\{\hat{R}(1-\ell_j)e, B_j\right\}dj}\right] \\ & \times \left[\pi \int_0^\gamma u'\left(\ell_j e + q\min\left\{\hat{R}(1-\ell_j)e, B_j\right\}\right)\min\left\{\hat{R}(1-\ell_j)e, B_j\right\}dj \right. \\ & \left. + (1-\pi)\rho \int_0^1 u'\left(\hat{R}(1-\ell_j)e + \frac{\ell_j e}{q}\right)\left(-\frac{\ell_j e}{q^2}\right)dj\right] \quad = \quad 0. \end{aligned}$$

To show that the optimal bailout rule satisfies $B(\ell) \geq \hat{R}(1-\ell)e$, suppose otherwise, seeking a contradiction. Then the first-order condition, evaluated at symmetric date 0 investment choices, becomes

$$\begin{aligned} & \pi u'(\ell_i e + qB_i) q \\ & + \mathbb{I}_{\left\{e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\gamma B_i}<1\right\}} \left[-\frac{q}{\gamma B_i}\right] \times \left[\pi\gamma u'(\ell_i e + qB_i)B_i + (1-\pi)\rho u'\left(\hat{R}(1-\ell_i)e + \frac{\ell_i e}{q}\right)\left(-\frac{\ell_i e}{q^2}\right)\right] = 0. \end{aligned}$$

[2] This equilibrium price expression is obtained by a procedure analogous to that of Lemma 1, but without (yet) imposing the symmetry of date 0 investment choices.

If $e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\gamma B_i} \geq 1$, we are left with $\pi u'(\ell_i e + qB_i) q = 0$, which is a contradiction. If $e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\gamma B_i} < 1$, we have

$$(1-\pi)\rho u'\left(\hat{R}(1-\ell_i)e + \frac{\ell_i e}{q}\right)\frac{\ell_i e}{q\gamma B_i} = 0,$$

which is also a contradiction. It follows that the optimal rule satisfies $B(\ell) \geq \hat{R}(1-\ell)e$. Note that any such rule trivially satisfies the first-order condition, since in that case the indicator variable $\mathbb{I}_{\left\{B_i < \hat{R}(1-\ell_i)e\right\}}$ is zero. Without loss of generality, the solution is thus $B(\ell) = \hat{R}(1-\ell)e$.

A.5 Proof of Proposition 3

Step 1 : Date 0 short asset choice

An agent at date 0 faces this problem:

$$\mathcal{V}_0(s,X) = \max_{\ell\in[0,1]} \pi u\left(\mathcal{C}_1(s,\ell,0,X)\right) + (1-\pi)\rho u\left(\mathcal{C}_1(s,\ell,1,X) + \mathcal{C}_2(s,\ell,1,X)\right) \tag{A.13}$$

subject to (A.4) with $B(\ell) = \hat{R}(1-\ell)e$, (A.6), and (2.24).

First, it is useful to prove that $1/\hat{R} \leq q(X) \leq 1$. We have already argued that the presence of the short asset at date 1 requires $q(X) \leq 1$. We now show that $1/\hat{R} \leq q(X)$. Seeking a contradiction, we suppose that $q(X) < 1/\hat{R}$. In this case, from the perspective of date 0, investing in the short asset strictly dominates investing in the long asset. As a result, all agents invest only in the short asset at date 0, resulting in $L^R = L^U = 1$, and, according to (2.24), in $q(X) = \min\{\infty, 1\} = 1$, a contradiction. It follows that $1/\hat{R} \leq q(X) \leq 1$.

Next, we specialize the problem (A.13) for an unprotected agent as

$$\max_{\ell\in[0,1]} \pi u(\ell e) + (1-\pi)\rho u\left(\hat{R}(1-\ell)e + \frac{\ell e}{q(X)}\right). \tag{A.14}$$

The first-order condition is

$$\psi(\ell) \equiv e\pi u'(\ell e) - e(1-\pi)(\hat{R} - \frac{1}{q(X)})\rho u'\left(\hat{R}(1-\ell)e + \frac{\ell e}{q(X)}\right) = 0.$$

Note that the agent's objective function is strictly concave in ℓ, as $\psi'(\cdot) < 0$ for $\ell \in [0,1]$. Seeking a contradiction, suppose that $\ell = 1$ is not optimal. It must thus be that $\psi(1) < 0$, or

$$\pi u'(e) < (1-\pi)\left(\hat{R} - \frac{1}{q(X)}\right)\rho u'\left(\frac{e}{q(X)}\right) \leq (1-\pi)(\hat{R}-1)\rho u'\left(\frac{e}{q(X)}\right) \leq (1-\pi)(\hat{R}-1)\rho u'(e),$$

which requires $\pi < \frac{\rho(R-1)}{1+\rho(R-1)}$. This contradicts Assumption (2). The solution to (A.14) must thus feature $\ell = 1$.

Problem (A.13) specialized for a protected agent is given by

$$\max_{\ell \in [0,1]} \pi u \left(\ell e + q(X)\hat{R}(1-\ell)e \right) + (1-\pi)\rho u \left(\hat{R}(1-\ell)e + \frac{\ell e}{q(X)} \right). \tag{A.15}$$

We distinguish two cases: $q(X) = 1/\hat{R}$ and $q(X) > 1/\hat{R}$. When $q(X) = 1/\hat{R}$, date 1 and 2 consumption does not depend on ℓ, and therefore protected agents are then indifferent across all levels of $\ell \in [0,1]$. When $q(X) > 1/\hat{R}$, agents optimally choose $\ell = 0$, since in that case the objective function is strictly decreasing in ℓ.

Step 2 : Time-consistent equilibrium (as a function of γ)
The investment decision of unprotected agents always leads to $\ell = L^U = 1$. Regarding protected agents, we consider several cases. When $q = 1/\hat{R}$, protected agents are indifferent across any short-term investment level. Therefore, we can have $L^P \in [0,1]$, but consistency with the equilibrium price expression (2.24) requires

$$L^P = \frac{\pi + \gamma - 1}{\gamma}.$$

And since $L^P \geq 0$, this constellation only prevails when $\gamma \geq 1-\pi$. The equilibrium consumption allocations are then given by $c_2^s(0) = c_1^s(1) = 0$ for $s \in \{U, P\}$ and

$$c_1^U(0) = c_2^P(0) = e, \quad \text{and} \quad c_2^U(1) = c_2^P(1) = \hat{R}e. \tag{A.16}$$

When $q > 1/\hat{R}$, protected agents' short asset decision at date 0 leads to $L^P = 0$. Substituting $L^P = 0$ and $L^U = 1$ into the equilibrium price expression (2.24), we obtain

$$q = \min \left\{ \frac{1}{\hat{R}} \frac{1-\pi}{\pi} \frac{1-\gamma}{\gamma}, 1 \right\}.$$

Consistency thus requires $\gamma < 1 - \pi$, and the equilibrium consumption allocations are given by

$$c_1^U(0) = e, \quad c_2^U(1) = \frac{1}{q}e, \quad c_1^P(0) = q\hat{R}e, \quad \text{and} \quad c_2^P(1) = \hat{R}e. \tag{A.17}$$

A.6 Proof of Proposition 4

Let us define $\underline{\gamma} \equiv \frac{1-\pi}{1-\pi+\hat{R}\pi}$ and $\overline{\gamma} \equiv 1-\pi$. The government chooses γ to maximize the average indirect utility function of private agents. It solves

$$\mathcal{W}_0 = \max_{\gamma\in[0,1]} \gamma\mathcal{V}_0\big(P,(\gamma,L^P(\gamma),1)\big) + (1-\gamma)\mathcal{V}_0\big(U,(\gamma,L^P(\gamma),1)\big). \tag{A.18}$$

To characterize the solution to this problem, it is convenient to separately consider the optimal choice of γ in the three intervals $[0,\underline{\gamma}]$, $[\underline{\gamma},\overline{\gamma}]$, and $[\overline{\gamma},1]$. We note that the objective function is continuous in γ.

First, for $\gamma \in [\overline{\gamma},1]$, the problem reduces to

$$\max_{\gamma\in[\overline{\gamma},1]} \pi u(e) + (1-\pi)\rho u(\hat{R}e).$$

The objective function is constant with respect to γ, and therefore any $\gamma \in [\overline{\gamma},1]$ is optimal.

Next, for $\gamma \in [0,\underline{\gamma}]$, the problem is

$$\max_{\gamma\in[0,\underline{\gamma}]} [\pi + (1-\pi)\rho]\left[\gamma u\left(\hat{R}e\right) + (1-\gamma)u\left(e\right)\right].$$

The objective function is strictly increasing in γ, so the optimal choice is given by $\gamma = \underline{\gamma}$.

Finally, for $\gamma \in [\underline{\gamma},\overline{\gamma}]$, the problem is given by

$$\max_{\gamma\in[\underline{\gamma},\overline{\gamma}]} \gamma\left[\pi u\left(e\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}\right) + (1-\pi)\rho u(\hat{R}e)\right] + (1-\gamma)\left[\pi u(e) + (1-\pi)\rho u\left(\hat{R}e\frac{\pi}{1-\pi}\frac{\gamma}{1-\gamma}\right)\right]. \tag{A.19}$$

Since the overall objective function in (A.18) is strictly increasing over $[0,\underline{\gamma}]$ and constant over $[\overline{\gamma},1]$, it must be that if (A.19) admits a strictly interior solution, then it will be the global solution of (A.18).

The first-order condition for problem (A.19) is

$$\begin{aligned}\phi(\gamma) \;\equiv\; & \left[\pi u\left(e\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}\right) + (1-\pi)\rho u(\hat{R}e)\right] - \left[\pi u(e) + (1-\pi)\rho u\left(\hat{R}e\frac{\pi}{1-\pi}\frac{\gamma}{1-\gamma}\right)\right] \\ & -e\frac{1-\pi}{\gamma}u'\left(e\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}\right) + e\frac{\pi}{1-\gamma}\rho\hat{R}u'\left(\hat{R}e\frac{\pi}{1-\pi}\frac{\gamma}{1-\gamma}\right) \lesseqqgtr 0,\end{aligned}$$

with "$\leq$" if $\gamma^d = \underline{\gamma}$, "$\geq$" if $\gamma^d = \overline{\gamma}$, and "$=$" if $\gamma^d \in (\underline{\gamma},\overline{\gamma})$. Evaluating $\phi(\cdot)$ at the bounds $\underline{\gamma}$ and

$\overline{\gamma}$, we have

$$\begin{aligned}\phi(\underline{\gamma}) &= [\pi + (1-\pi)\rho]\left[u\left(\hat{R}e\right) - u(e)\right] + e\left(1-\pi+\hat{R}\pi\right)\rho\left[u'(e) - \rho^{-1}u'\left(\hat{R}e\right)\right] > 0,\\ \phi(\overline{\gamma}) &= -e\left[u'(e) - \rho\hat{R}u'\left(\hat{R}e\right)\right] < 0.\end{aligned}$$

The global optimum is therefore strictly interior: $\gamma^d \in (\underline{\gamma}, \overline{\gamma})$.

A.7 Proof of Proposition 5

To establish how γ^d changes with π and $\hat{R}$, we use the implicit function theorem:

$$\frac{\partial \gamma^d}{\partial x} = -\frac{\phi'(x)}{\phi'(\gamma)}, \quad \text{for } x \in \{\pi, \hat{R}\}.$$

First, we note that

$$\phi'(\gamma) = \frac{e^2(1-\pi)^2}{\pi\gamma^3}u''\big(q\hat{R}e\big) + \frac{e^2\pi^2}{(1-\pi)(1-\gamma)^3}\hat{R}^2\rho u''\left(\frac{e}{q}\right) < 0.$$

The derivative of the implicit function $\phi(\cdot)$ with respect to π is given by

$$\begin{aligned}\phi'(\pi) &= \left[u\left(q\hat{R}e\right) - u(e)\right] + e\frac{1}{\gamma}\left(1 - \frac{1-\gamma}{\pi}\right)u'\left(q\hat{R}e\right)\\ &\quad +\rho\left[u\left(\frac{e}{q}\right) - u(\hat{R}e)\right] + \rho\hat{R}e\frac{1}{1-\gamma}\left(1 - \frac{\gamma}{1-\pi}\right)u'\left(\frac{e}{q}\right)\\ &\quad +e^2\frac{(1-\pi)(1-\gamma)}{\pi^2\gamma^2}u''\left(q\hat{R}e\right) + e^2\frac{\pi\gamma}{(1-\gamma)^2}\frac{1}{(1-\pi)^2}\rho\hat{R}^2u''\left(\frac{e}{q}\right).\end{aligned}$$

Since $u(y) - u(z) \leq u'(z)(y-z)$ for all $z \geq 0$ and $y \geq 0$ and $u'(\hat{R}e) \leq u'(e/q)$, we have

$$\phi'(\pi) \leq e^2\frac{(1-\pi)(1-\gamma)}{\pi^2\gamma^2}u''\left(q\hat{R}e\right) + e^2\frac{\pi\gamma}{(1-\gamma)^2}\frac{1}{(1-\pi)^2}\rho\hat{R}^2u''\left(\frac{e}{q}\right) < 0,$$

and therefore

$$\frac{\partial \gamma^d}{\partial \pi} = -\frac{\phi'(\pi)}{\phi'(\gamma)} < 0.$$

The derivative of the implicit function $\phi(\cdot)$ with respect to $\hat{R}$ is given by

$$\begin{aligned}
\phi'(\hat{R}) &= \frac{1-\pi}{\hat{R}}\rho\left[\hat{R}eu'(\hat{R}e) - \frac{e}{q}u'\left(\frac{e}{q}\right)\right] + e\frac{\pi}{1-\gamma}\rho u'\left(\frac{e}{q}\right)\left[u'\left(\frac{e}{q}\right) + \frac{e}{q}u''\left(\frac{e}{q}\right)\right] \\
&= \frac{1-\pi}{\hat{R}}\rho\left[\int_{\frac{e}{q}}^{\hat{R}e} \frac{\partial}{\partial z}[zu'(z)]dz\right] + e\frac{\pi}{1-\gamma}\rho u'\left(\frac{e}{q}\right)\left[u'\left(\frac{e}{q}\right) + \frac{e}{q}u''\left(\frac{e}{q}\right)\right] \\
&= \frac{1-\pi}{\hat{R}}\rho\left[\int_{\frac{e}{q}}^{\hat{R}e} [u'(z) + zu''(z)]dz\right] + e\frac{\pi}{1-\gamma}\rho u'\left(\frac{e}{q}\right)\left[u'\left(\frac{e}{q}\right) + \frac{e}{q}u''\left(\frac{e}{q}\right)\right],
\end{aligned}$$

where we note that the both terms in square brackets are necessarily non-positive by Assumption 1. It follows that $\phi'(\hat{R}) \leq 0$, which in turn implies

$$\frac{\partial \gamma^d}{\partial \hat{R}} = -\frac{\phi'(\hat{R})}{\phi'(\gamma)} \leq 0.$$

Appendix B

Appendix to Chapter 2

B.1 Proofs

The government's time-consistent problem in recursive form is rewritten here for convenience:

$$\mathcal{V}(b,s) = \max_{c^T,\ell,b',p^N,\pi^N,\mu} u\left[c\left(c^T,c^N\right),\ell\right] + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',s') \tag{B.1}$$

subject to

$$c^N = \alpha(p^N)c^T \tag{B.2}$$

$$c^N = \left[1-\frac{\varphi}{2}(\pi^N)^2\right]A\ell \tag{B.3}$$

$$c^T = y^T + b - \frac{b'}{R} \tag{B.4}$$

$$\frac{b'}{R} \geq -\kappa\left(y^T + p^N c^N\right) \tag{B.5}$$

$$\mu = u_T(c,\ell) - \beta R\mathbb{E}_{s'|s}u_T\left(\mathcal{C}(b',s'),\mathcal{L}(b',s')\right) \tag{B.6}$$

$$\mu \times \left[b' + \kappa\left(y^T + p^N c^N\right)\right] = 0 \tag{B.7}$$

$$0 = \varphi\pi^N(1+\pi^N) + (\varepsilon-1)\left[1-z^{-1}(1-\omega)\right] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right] \tag{B.8}$$

Let $\iota \geq 0$, $\lambda^* \geq 0$ and $\mu^* \geq 0$ be the multiplier on the resource constraint for non-tradables, tradables and the credit constraint respectively; δ, υ, ν and ξ be the multiplier on (B.2), (B.6)-(B.8). I define an auxiliary variable: $\psi \equiv (\varepsilon-1)\left[1-z^{-1}(1-\omega)\right] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right]$.

B.1.1 Proof of Proposition 6

In the absence of credit frictions, the government's optimisation problem reduces to solving (B.1) subject to (B.3), (B.4), (B.6) with equality $\mu_t = 0$ and (B.8). The proof proceeds by analyzing a relaxed problem where the government is not subject to (B.6) and then showing that this condition is satisfied. Abstracting from the implementability constraint (B.6), the government's optimality conditions, after eliminating the multiplier δ in the key equations, are:

$$c_t^T :: \qquad \lambda_t^* = u_T(t) + \mu_t^* \gamma^{-1} \kappa \frac{p_t^N c_t^N}{c_t^T} + \xi_t \psi_{T,t} \tag{B.9}$$

$$b_{t+1} :: \qquad \lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t^* + \xi_t \psi_{b,t} \tag{B.10}$$

$$\ell_t + c_t^N :: \qquad y_t^N u_N(t) \left(\omega_t - \frac{\varphi}{2} (\pi_t^N)^2 \right) = \xi_t \left(-c_t^N \psi_{N,t} - \ell_t \psi_{\ell,t} \right) \tag{B.11}$$

$$\pi_t^N :: \qquad \xi_t = \frac{\varphi \iota_t y_t^N}{1 + 2\pi_t^N} \pi_t^N \tag{B.12}$$

where the optimality condition (B.11) combines the first order condition with respect to labor and non-tradable consumption to eliminate the multiplier ι.

Consider now a price stability policy, $\pi_t^N = 0$ for all t. Under this policy, by equation (B.12), it follows that $\xi_t = 0$. Substituting into (B.11) implies that $\omega_t = 0$, and into (B.9) implies that $\lambda_t^* = u_T(t)$. Using the latter together with (B.10) leads to:

$$u_T(t) = \beta R_t \mathbb{E}_t u_T(t+1).$$

The implementability constraint (B.6) is then satisfied. Therefore, a price stability policy is optimal and it stabilizes the economy (i.e. $\omega_t = 0$).

B.1.2 Proof of Proposition 7

To characterize the optimal time-consistent monetary policy, I solve for the government's optimization problem (B.1) given that future path of the inflation rate $\mathcal{M}$ are chosen by future government with which are associated with policies $\{\mathcal{C}(b,s), \mathcal{L}(b,s), \mathcal{B}(b,s), \mathcal{V}(b,s)\}$.

The optimality conditions of the government problem (B.1), after eliminating the multiplier δ in the key equations, are given by:

$$c_t^T :: \qquad \lambda_t^* = u_T(t) + \mu_t^* \gamma^{-1} \kappa \frac{p_t^N c_t^N}{c_t^T} - \upsilon_t u_{TT}(t) + \xi_t \psi_{T,t} \tag{B.13}$$

$$b_{t+1} :: \qquad \lambda_t^* = \beta R_t \mathbb{E}_t \left(\lambda_{t+1} + \upsilon_t \frac{\partial u_T(t+1)}{\partial b_{t+1}} \right) + \mu_t^* + \xi_t \psi_{b,t} \tag{B.14}$$

$$\mu_t :: \qquad \upsilon_t + \nu_t \left[b_{t+1} + \kappa \left(p_t^N y_t^N + y_t^T \right) \right] + \xi_t \left(\varepsilon - 1 \right) \left(1 - \omega_t \right) z_t^{-2} \frac{\kappa}{u_T(t)} = 0 \tag{B.15}$$

$$\ell_t + c_t^N :: \quad u_N(t) y_t^N \left(\omega_t - \frac{\varphi}{2} (\pi_t^N)^2 \right) + \left(\sigma - \gamma^{-1} \right) \frac{c_T(t)}{c_t} c_t^N u_N(t) \upsilon_t$$

$$\left(1 - \gamma^{-1} \right) \kappa p_t^N c_t^N \mu_t^* = \xi_t \left(-c_t^N \psi_{N,t} - \ell_t \psi_{\ell,t} \right) \tag{B.16}$$

$$\pi_t^N :: \quad \xi_t = \frac{\varphi \iota_t y_t^N}{1 + 2\pi_t^N} \pi_t^N \tag{B.17}$$

where the optimality condition (B.16) combines the first order condition with respect to labor and non-tradable consumption to eliminate the multiplier ι.

These expressions are obtained by assuming that the policies and value functions are differentiable, with $\tilde{\mu}_t^* = \mu_t^* + \nu_t \mu_t$ that represents the government's effective shadow value on the credit constraint. It can also be shown that $\tilde{\mu}_t^* = \mu_t^*$ for all t. To clearly see this, notice first that when the constraint does not binds (i.e. $\mu_t^* = 0$), the implementability constraint (B.7) is always satisfied. Thus, setting ν_t to zero is optimal and it follows that $\tilde{\mu}_t^* = \mu_t^* = 0$. When the constraint binds, the implementability constraint (B.7) implies that $\mu_t = 0$ and again $\tilde{\mu}_t^* = \mu_t^*$. To simplify the notation, I define $\tilde{\iota}_t \equiv \iota_t / (1 + 2\pi_t^N) u_N(t)$. Combining (B.16) with (B.17), using the definition of ψ_t, and rearranging the expression yields

$$\varphi \left(\Delta_{0,t} + \frac{\pi_t^N}{2} \right) y_t^N \pi_t^N = y_t^N \omega_t + \varphi \mathbb{E}_t \left[\Delta_{1,t+1} \pi_{t+1}^N \right] + (\sigma - \gamma^{-1}) \frac{c_T(t)}{c_t} c_t^N \upsilon_t + (1 - \gamma^{-1}) \kappa p_t^N c_t^N \frac{\mu_t^*}{u_N(t)}$$

where

$$\Delta_{0,t} = (\varepsilon - 1) z_t^{-1} \tilde{\iota}_t \frac{-u_\ell(t)}{A u_N(t)} \left[\frac{\gamma^{-1} \kappa p_t^N \mu_t - c_t^N u_{NN}(t)}{z_t u_N(t)} + \ell_t \frac{u_{\ell\ell}(t)}{u_\ell(t)} \right] > 0 \tag{B.18}$$

and

$$\Delta_{1,t+1} = \left(1 + \frac{c_t^N u_{NN}(t)}{u_N(t)} \right) \frac{\tilde{\iota}_t}{\ell_t} \Lambda_{t,t+1} \ell_{t+1} (1 + \pi_{t+1}^N)$$

The equilibrium under the discretionary monetary policy can be characterized by sequences $\{c_t^T, c_t^N, \ell_t, b_{t+1}, \mu_t, \pi_t^N, p_t^N\}_{t=0}^\infty$ that satisfy (3.4)-(3.8), (3.10), (3.11) along with the complementary slackness condition and (B.1.2).

B.1.3 Proof of Proposition 8

The proof proceeds by first showing that any allocation $\{c_t^T, \ell_t, b_{t+1}, p_t^N, \pi_t^N\}$ that satisfy (3.16)-(3.18) and (3.21) also satisfy the general equilibrium, and then describing the optimal monetary policy and optimal tax rate.

Consider any allocation $\{c_t^T, \ell_t, b_{t+1}, p_t^N, \pi_t^N\}$ that satisfy (3.16)-(3.18) and (3.21). Then, set $c_t^N = \left[1 - \varphi \left(\pi_t^N \right)^2 \right] A \ell_t$ and $\mu_t = 0$ to satisfy (3.15) and (3.20) respectively. Choose $\tau_t^b = 1 - \beta R_t \frac{\mathbb{E}_t u_T(t+1)}{u_(t)}$. By definition, this makes (3.26) hold also. The government's optimisation problem then reduces to

$$\mathcal{V}(b,s) = \max_{c^T,\ell,b',p^N,\pi^N} \; u\left[c\left(c^T, \alpha(p^N)c^T\right), \ell\right] + \beta \mathbb{E}_{s'|s}\mathcal{V}(b',s')$$

subject to

$$\alpha(p^N)c^T = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A\ell$$

$$c^T = y^T + b - \frac{b'}{R}$$

$$\frac{b'}{R} \geq -\kappa\left(p^N A\ell + y^T\right)$$

$$\varphi\pi^N(1+\pi^N) + (\varepsilon-1)\omega - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right] = 0$$

Again $\iota \geq 0$, $\lambda^* \geq 0$ are the multiplier on the resource constraint for the nontradable good and the tradable good respectively, $\mu^* \geq 0$ is the multiplier on the credit constraint, $\xi \geq 0$ is the multiplier the nontradable good pricing implementability constraint. The optimality conditions of the government's problem, when capital flow taxes are available, are given by:

$$c_t^T :: \qquad \lambda_t^* = u_T(t) + \mu_t^* \gamma^{-1}\kappa \frac{p_t^N c_t^N}{c_t^T} + \xi_t \psi_{T,t} \tag{B.19}$$

$$b_{t+1} :: \qquad \lambda_t^* = \beta R_t \mathbb{E}_t \lambda_{t+1}^* + \mu_t^* + \xi_t \psi_{b',t} \tag{B.20}$$

$$c_t^N + \ell_t :: \qquad y_t^N u_N(t)\left(\omega_t - \frac{\varphi}{2}(\pi_t^N)^2\right) + \left(1-\gamma^{-1}\right)\kappa p_t^N c_t^N \mu_t^* = \xi_t\left(-c_t^N\psi_{N,t} - \ell_t\psi_{\ell,t}\right) \tag{B.21}$$

$$\pi_t^N :: \qquad \xi_t = \frac{\varphi \iota_t y_t^N}{1+2\pi_t^N}\pi_t^N \tag{B.22}$$

where the optimality condition (B.11) combines the first order condition with respect to labor and non-tradable consumption to eliminate the multiplier ι.
Combining (B.21) with (B.22) and rearranging the expression yields

$$\varphi\left(\Delta_{0,t} + \frac{\pi_t^N}{2}\right)\pi_t^N = y_t^N\omega_t + \varphi\mathbb{E}_t\left[\Delta_{1,t+1}\pi_{t+1}^N\right] + (1-\gamma^{-1})\kappa p_t^N c_t^N \frac{\mu_t^*}{u_N(t)} \tag{B.23}$$

This complete the proof that when taxes are available and used optimally, the path for the inflation rate when the government cannot commit to future policies satisfies (B.23) as in Proposition 8. The optimal tax rate is given by:

$$\tau_t^b = \frac{\beta R_t \mathbb{E}_t\left[\Theta_{t+1}\mu_{t+1}^* + \xi_{t+1}\psi_{T,t+1}\right] - \Theta_t\mu_t^* - \xi_t\psi_{T,t} + \xi_t\psi_{b',t}}{\beta R_t \mathbb{E}_t u_T(t+1)}$$

with $\Theta_t = (1/\gamma)\kappa p_t^N c_t^N / c_t^T$.

B.2 Alternatives Formulations

B.2.1 Nominal Interest Rate as Monetary Policy Instrument

In this section, I assume that households have access, in addition to a one period non-state-contingent foreign bond, to a one period non-state-contingent domestic bond. The domestic bond is a one period non-state-contingent traded only among domestic households, and pays a net nominal interest rate i_t determined by the central bank (government).
The representative household' problem can be re-written as

$$\max_{c_t^T, c_t^N, \ell_t, b_{t+1}, b_{t+1}^h} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c\left(c_t^T, c_t^N\right), \ell_t\right)$$

subject to

$$\begin{aligned} P_t^T c_t^T + P_t^N c_t^N + \frac{\mathcal{E}_t b_{t+1}}{R_t} + \frac{b_{t+1}^h}{1+i_t} &= P_t^T y_t^T + W_t \ell_t + \Phi_t + \mathcal{E}_t b_t + b_t^h \\ \frac{\mathcal{E}_t b_{t+1}}{R_t} &\geq -\kappa \Big[P_t^T y_t^T + W_t \ell_t + \Phi_t \Big] \end{aligned}$$

where b_{t+1}^h denote the household?s holdings of domestic and foreign bond respectively. Combining the optimality condition for both foreign bond and domestic bond, it follows that

$$\beta \mathbb{E}_t \left[u_T(t+1) \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} (1+i_t) - R_t \right) \right] = \mu_t \tag{B.24}$$

It is important to observe that the net supply of domestic bond is equal to zero in equilibrium. Therefore, for any alloocation under the discretionary monetary policy described in the section 3.3, the equation (B.24) can be used to back out the domestic nominal interest rate.

B.2.2 GHH Preferences

In this section, I derive the discretionary monetary policy in an environment in which households' preferences are specified following Greenwood et al. (1988), where utility is defined in terms of the excess of consumption over the disutility of labor. Formally, I assume

$$u(c, \ell) = u\left(c - g(\ell)\right)$$

The disutility function $g(\cdot)$ is twice-continuously differentiable, strictly increasing and convex.

The discretionary monetary policy, with credit frictions and when capital flow taxes cannot be used, then solves

$$\mathcal{V}(b,s) \;=\; \max_{c^T,c^N,\ell,b',p^N,\pi^N,\mu} \; u\left[c\left(c^T,c^N\right)-g(\ell)\right]+\beta\mathbb{E}_{s'|s}\mathcal{V}(b',s')$$

subject to (B.2), (B.3), (B.4), (B.5), (B.6), (B.7), (B.8)

Solving this problem, and following the same procedure as in appendix B.1.2, it can be shown that the path of the inflation rate under the discretionary monetary policy satisfies:

$$\begin{aligned}\varphi\left(\Delta_{0,t}+\frac{\pi_t^N}{2}\right)\pi_t^N \;&=\; y_t^N\omega_t+\varphi\mathbb{E}_t\left[\Delta_{1,t+1}\pi_{t+1}^N\right]\\ &\quad+\left((\sigma-\gamma^{-1})\frac{c_T(t)}{c_t}c_t^N+\sigma\frac{\ell_t g'(\ell_t)}{p_t^N c_t}\right)\upsilon_t+(1-\gamma^{-1})\kappa p_t^N c_t^N\frac{\mu_t^*}{u_N(t)}\end{aligned}$$

where the expression for $\Delta_{0,t}>0$ is also given by (B.18). It straightforward to see that the discretionary monetary policy is procyclical for $1/\sigma<\gamma<1$. An interesting feature of the GHH preferences is that it amplifies the effect of overborrowing during tranquil times.

B.3 Numerical Solution Method (Algorithm)

B.3.1 For Competitive Equilibrium under a Price Stability Policy

This algorithm is build on Bianchi (2011)'s algorithm that incorporates the occasionally binding endogenous constraint, modified to account for the nominal rigidities. Formally, the computation of the competitive equilibrium operates directly on the first-order conditions and requires solving for functions $\{\mathcal{B}(b,s),\mathcal{L}(b,s),\mathcal{C}^T(b,s),\mathcal{P}^N(b,s),\mu(b,s)\}$ such that:

$$\mathcal{C}^T(b,s)+\frac{\mathcal{B}(b,s)}{R}=y^T+b \tag{B.25}$$

$$\alpha\left(\mathcal{P}^N(b,s)\right)\mathcal{C}^T(b,s)=A\mathcal{L}(b,s) \tag{B.26}$$

$$\frac{\mathcal{B}(b,s)}{R}\geq-\kappa\left(A\mathcal{P}^N(b,s)\mathcal{L}(b,s)+y^T\right) \tag{B.27}$$

$$\begin{aligned}&u_T\left(c(b,s)-g\left(\mathcal{L}(b,s)\right)\right)\\ &\qquad=\beta R\mathbb{E}_{s'|s}\left\{u_T\left(c(\mathcal{B}(b,s),s')-g\left(\mathcal{L}(\mathcal{B}(b,s),s)\right)\right)\right\}+\mu(b,s)\end{aligned} \tag{B.28}$$

$$u_N\left(c(b,s)-g\left(\mathcal{L}(b,s)\right)\right)+\frac{1}{A}u_\ell\left(c(b,s)-g\left(\mathcal{L}(b,s)\right)\right)=-\kappa\mathcal{P}^N(b,s)\mu(b,s) \tag{B.29}$$

where $c(b,s)\equiv c\left(\mathcal{C}^T(b,s),A\mathcal{L}(b,s)\right)$. The steps for the algorithm are the following:

1. Generate discrete grids $G_b=\{b_1,b_2,...,b_M\}$ for the bond position and $G_s=\{s_1,s_2,...,s_N\}$

for the shock state space, and choose an interpolation scheme for evaluating the functions outside the grid of bonds. The piecewise linear approximation is used to interpolate the functions and the grid for bonds contains 200 points.

2. Conjecture $\mathcal{B}_h(b,s)$, $\mathcal{L}_h(b,s)$, $\mathcal{C}_h^T(b,s)$, $\mathcal{P}_h^N(b,s)$, $\mu_h(b,s)$ at time H, $\forall b \in G_b$ and $\forall s \in G_s$.

3. Set $i = 1$

4. Solve for the values of $\mathcal{B}_{h-i}(b,s)$, $\mathcal{L}_{h-i}(b,s)$, $\mathcal{C}_{h-i}^T(b,s)$, $\mathcal{P}_{h-i}^N(b,s)$, $\mu_{h-i}(b,s)$ at time $h-i$ using (B.25)-(B.29) and $\mathcal{B}_{h-i+1}(b,s)$, $\mathcal{L}_{h-i+1}(b,s)$, $\mathcal{C}_{h-i+1}^T(b,s)$, $\forall b \in G_b$ and $\forall s \in G_s$:

   (a) First, assume that the credit constraint (B.27) is not binding. Set $\mu_{h-i}(b,s) = 0$ and using (B.28), (B.29) and a root finding algorithm solve for $\mathcal{C}_{h-i}^T(b,s)$ and $\mathcal{L}_{h-i}(b,s)$. Solve for $\mathcal{B}_{h-i}(b,s)$ and $\mathcal{P}_{h-i}^N(b,s)$ using (B.25) and (B.26).

   (b) Check whether $\frac{\mathcal{B}_{h-i}(b,s)}{R} \geq -\kappa \left(A\mathcal{P}_{h-i}^N(b,s)\mathcal{L}_{h-i}(b,s) + y^T \right)$ holds. If the credit constraint is satisfied, move to the next grid point.

   (c) Otherwise, using (B.25), (B.27), (B.28), (B.29) and a root finding algorithm solve for $\mu_{h-i}(b,s)$, $\mathcal{B}_{h-i}(b,s)$, $\mathcal{C}_{h-i}^T(b,s)$ and $\mathcal{L}_{h-i}(b,s)$ and using (B.26) solve for $\mathcal{P}_{h-i}^N(b,s)$.

5. Convergence. The competitive equilibrium is found if $\left\| \sup_{B,s} x_{h-i}(b,s) - x_{h-i+1}(b,s) < \epsilon \right\|$ for $x \in \{\mathcal{B}, , \mathcal{L}, \mathcal{C}^T\}$. Otherwise, set $x_{h-i}(b,s) = x_{h-i+1}(b,s)$, $i \approx i+1$ and go to step 4.

B.3.2 For Optimal Time-Consistent Monetary Policy

The solution method proposed here uses a nested fixed point algorithm to solve for optimal time-consistent monetary policy and is related to the literature using Markov perfect equilibria (e.g. Klein et al. (2008) and Bianchi and Mendoza (2018)). In the inner loop, using the Bellman equation and value function iteration, solve for value function and policy functions taking as given future policies. Formally, given functions $\{\mathcal{C}^T(b,s), \mathcal{P}^N(b,s), \mathcal{B}(b,s), \mathcal{L}(b,s), \mathcal{M}(b,s)\}$, the Bellman equation is given by:

$$\mathcal{V}(b,s) = \max_{c^T, \ell, b', p^N, \pi^N, \mu} u\left[c\left(c^T, \alpha(p^N)c^T\right) - g(\ell)\right] + \beta \mathbb{E}_{s'|s} \mathcal{V}(b', s') \tag{B.30}$$

subject to

$$\alpha(p^N)c^T = \left[1 - \frac{\varphi}{2}(\pi^N)^2\right] A\ell \tag{B.31}$$

$$c^T = y^T + b - \frac{b'}{R} \tag{B.32}$$

$$\frac{b'}{R} \geq -\kappa\left(p^N A\ell + y^T\right) \tag{B.33}$$

$$\mu = u_T(c,\ell) - \beta R \mathbb{E}_{s'|s} u_T\left(c\left(\mathcal{C}^T(b',s'), \mathcal{P}^N(b',s')\right) - g\left(\mathcal{L}(b',s')\right)\right) \tag{B.34}$$

$$\mu \times \left[b' + \kappa\left(p^N A\ell + y^T\right)\right] = 0 \tag{B.35}$$

$$\varphi\pi^N(1+\pi^N) - (\epsilon-1)\left[z^{-1}(1-\omega)-1\right] - \varphi\ell^{-1}\mathbb{E}_{s'|s}\Lambda\left[\mathcal{L}(b',s')\mathcal{M}(b',s')\right] = 0 \tag{B.36}$$

Given the solution to the Bellman equation, update future policies as the outer loop. The steps for the algorithm are the following:

1. Generate discrete grids $G_b = \{b_1, b_2, ..., b_M\}$ for the bond position and $G_s = \{s_1, s_2, ..., s_N\}$ for the shock state space, and choose an interpolation scheme for evaluating the functions outside the grid of bonds. The piecewise linear approximation is used to interpolate the functions and the grid for bonds contains 200 points.

2. Guess policy functions $\mathcal{B}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ at time H, $\forall b \in G_b$ and $\forall s \in G_s$.

3. For given $\mathcal{L}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ solve the recursive problem using value function iteration to find the value function and policy functions:

    (a) First, assume that the credit constraint (B.33) is not binding. Set $\mu = 0$ – (B.35) is thus satisfied – and solve the optimization problem (B.30) subject to (B.31), (B.32), (B.34), (B.36) using a Newton type algorithm and check whether (B.33) holds.

    (b) Second, assume that the credit constraint (B.33) is binding – (B.35) is thus satisfied. Solve the optimization problem (B.30) subject to (B.31)-(B.34), (B.36) using a Newton type algorithm.

    (c) Compare the solutions in (a) and (b).The optimal choices in each state is the best solution. Denote $\{\nu^i\}_i$, with $i \in \{b', \ell, c^T, p^N, \pi^N\}$, the associated policy functions.

4. Evaluate convergence. Compute the sup distance between $\mathcal{B}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ and $\{\nu^i\}$, with $i \in \{b', c^T, p^N, \pi^N\}$. If the sup distance is not smaller enough (higher than $\epsilon = 1e-7$), update $\mathcal{B}$, $\mathcal{C}^T$, $\mathcal{P}^N$, $\mathcal{M}$ and solve again the recursive problem.

Appendix C

Appendix to Chapter 3

C.1 Optimal monetary policy

The planner's problem is an optimal control problem with control variable i_t, and state variables $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t, \tilde{y}^*_t$. In order to facilitate the derivation of the optimal monetary policy, we define two auxiliary variables

$$\pi^u_t = \frac{1}{2}\left[\pi_{H,t} + \pi^*_{F,t}\right], \qquad \text{and} \qquad \tilde{y}^u_t = \frac{1}{2}\left[\tilde{y}_t + \tilde{y}^*_t\right], \tag{C.1}$$

representing the average union output gap and average CPI inflation in the currency union. We then represent the dynamics of CPI inflation and output gap in a given country in term of deviation from the average CPI inflation and average output gap respectively. The problem of the centralized monetary authority can be rewritten as

$$\min_{\{\pi^u_t, \tilde{y}^u_t\}} \int_0^\infty e^{-\rho t}\Big\{\mathbb{L}_{H,t} + \mathbb{L}^*_{F,t}\Big\}dt$$

subject to (C.1) and

$$\begin{aligned}
\dot{\pi}_{H,t} - \dot{\pi}^u_t &= \rho\left(\pi_{H,t} - \pi^u_t\right) - \kappa(1+\phi)\left(\tilde{y}_t - y^u_t\right) + \kappa\omega\tilde{s}_t, &\text{(C.2)}\\
\dot{\pi}^*_{F,t} - \dot{\pi}^u_t &= \rho\left(\pi^*_{F,t} - \pi^u_t\right) - \kappa(1+\phi)\left(\tilde{y}^*_t - y^u_t\right) - \kappa\omega\tilde{s}_t, &\text{(C.3)}\\
\dot{\tilde{y}}_t - \dot{y}^u_t &= -\left(\pi_{H,t} - \pi^u_t\right) + \omega\dot{\tilde{s}}_t - \frac{1}{2}\left(r^n_t - r^{n*}_t\right), &\text{(C.4)}\\
\dot{\tilde{y}}^*_t - \dot{y}^u_t &= -\left(\pi^*_{F,t} - \pi^u_t\right) - \omega\dot{\tilde{s}}_t + \frac{1}{2}\left(r^n_t - r^{n*}_t\right), &\text{(C.5)}\\
\tilde{y}_t - \tilde{y}^*_t &= (1+2\omega)\,\tilde{s}_t. &\text{(C.6)}
\end{aligned}$$

where φ_t, φ^*_t, λ_t, λ^*_t are the co-state variables associated with $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t$ and $\tilde{y}^*_t$. Let also φ^u_t and λ^u_t denote the multiplier on the constraints in equation (C.1). The planner's optimal choice is characterized by the first-order conditions associated with the CPI inflation and output gap in each country

$$\dot{\varphi}_t = -\frac{\varepsilon}{\kappa}\pi_{H,t} + \lambda_t + \frac{1}{2}\varphi_t^u \tag{C.7}$$

$$\dot{\varphi}_t^* = -\frac{\varepsilon}{\kappa}\pi_{F,t}^* + \lambda_t^* + \frac{1}{2}\varphi_t^u \tag{C.8}$$

$$\dot{\lambda}_t - \rho\lambda_t = -(1+\phi)\tilde{y}_t + \kappa(1+\phi)\varphi_t - \Lambda_t + \frac{1}{2}\lambda_t^u \tag{C.9}$$

$$\dot{\lambda}_t^* - \rho\lambda_t^* = -(1+\phi)\tilde{y}_t^* + \kappa(1+\phi)\varphi_t^* + \Lambda_t + \frac{1}{2}\lambda_t^u \tag{C.10}$$

$$\omega\left[(\dot{\lambda}_t + \dot{\lambda}_t^*) - \rho(\lambda_t + \lambda_t^*)\right] = (\eta-1)(1-2\alpha)^2\omega\tilde{s}_t + \kappa\omega\left(\varphi_t - \varphi_t^*\right) - (1+2\omega)\Lambda_t \tag{C.11}$$

where Λ_t is the multiplier on the resource constraint. The first-order conditions associated with the union-wide CPI inflation and union-wide output gap

$$(\dot{\varphi}_t + \dot{\varphi}_t^*) = (\lambda_t + \lambda_t^*) + \varphi_t^u, \qquad \text{and} \qquad \dot{\lambda}_t + \dot{\lambda}_t^* - \rho(\lambda_t + \lambda_t^*) = \kappa(1+\phi)\left(\varphi_t + \varphi_t^*\right) + \lambda_t^u,$$

the transversality conditions $\lim_{t\to\infty}\varphi_t\pi_{H,t} = \lim_{t\to\infty}\varphi_t^*\pi_{F,t}^* = 0$ and $\lim_{t\to\infty}\lambda_t\tilde{y}_t = \lim_{t\to\infty}\lambda_t^*\tilde{y}_t^* = 0$, and initial conditions. Combining (C.7) and (C.8) and substituting the first equation in (C.1) yield

$$\pi_{H,t} + \pi_{F,t}^* = 0$$

Moreover, combining (C.9) and (C.10) and and substituting the first equation in (C.1) yield

$$\tilde{y}_t + \tilde{y}_t^* = 0$$

C.2 Cooperative macroprudential policy

The planner's problem is an optimal control problem with control variables i_t, τ_t, τ_t^*, and state variables $\pi_{H,t}$, $\pi^*_{F,t}$, $\tilde{y}_t, \tilde{y}^*_t, \tilde{\theta}_t$. The associated present value Hamiltonian is given by

$$\begin{aligned}\mathcal{H} \;=\; & e^{-\rho t}\Big\{\frac{1}{2}\Big[\mathbb{L}_{H,t}+\mathbb{L}^*_{F,t}\Big]\\ & +\varphi_t\Big[\rho\pi_{H,t}-\kappa(1+\phi)\tilde{y}_t+\kappa\omega\tilde{s}_t-\kappa\alpha\tilde{\theta}_t\Big]\\ & +\varphi^*_t\Big[\rho\pi^*_{F,t}-\kappa(1+\phi)\tilde{y}^*_t-\kappa\omega\tilde{s}_t+\kappa\alpha\tilde{\theta}_t\Big]\\ & +\lambda_t\left[(i_t-\pi_{H,t}-r^n_t)+\omega\dot{\tilde{s}}_t+(1-\alpha)\tau_t+\alpha\tau^*_t\right]\\ & +\lambda^*_t\left[\left(i_t-\pi^*_{F,t}-r^{n*}_t\right)-\omega\dot{\tilde{s}}_t+\alpha\tau_t+(1-\alpha)\tau^*_t\right]\\ & +\mu_t(\tau_t-\tau^*_t)+\Lambda_t\Big[(\tilde{y}_t-\tilde{y}^*_t)-(1-2\alpha)\tilde{\theta}_t-(1+2\omega)\tilde{s}_t\Big]\Big\}\end{aligned}$$

where $\varphi_t, \varphi^*_t, \lambda_t, \lambda^*_t, \mu_t$ are the co-state variables associated with $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t, \tilde{y}^*_t, \tilde{\theta}_t$. The planner's optimal choice is characterized by the first-order conditions (the first order condition for τ^*_t can be also be found by combining the first order condition for τ and the co-state equation for $\tilde{\theta}_t$. We then set optimally $\tau^*_t = 0$):

$$\frac{\partial\mathcal{H}}{\partial i_t} = \lambda_t+\lambda^*_t = 0 \tag{C.12}$$

$$\frac{\partial\mathcal{H}}{\partial \tau_t} = \mu_t+(1-\alpha)\,\lambda_t+\alpha\lambda^*_t = 0 \tag{C.13}$$

$$\dot{\varphi}_t = -\frac{\varepsilon}{\kappa}\pi_{H,t}+\lambda_t \tag{C.14}$$

$$\dot{\varphi}^*_t = -\frac{\varepsilon}{\kappa}\pi^*_{F,t}+\lambda^*_t \tag{C.15}$$

$$\dot{\lambda}_t-\rho\lambda_t = -(1+\phi)\tilde{y}_t+\kappa(1+\phi)\varphi_t-\Lambda_t \tag{C.16}$$

$$\dot{\lambda}^*_t-\rho\lambda^*_t = -(1+\phi)\tilde{y}^*_t+\kappa(1+\phi)\varphi^*_t+\Lambda_t \tag{C.17}$$

$$\dot{\mu}_t-\rho\mu_t = -2\alpha(1-\alpha)\tilde{\theta}_t+\omega(1-2\alpha)\tilde{s}_t+\kappa\alpha\left(\varphi_t-\varphi^*_t\right)+(1-2\alpha)\,\Lambda_t \tag{C.18}$$

$$\omega\left[\left(\dot{\lambda}_t-\dot{\lambda}^*_t\right)-\rho\left(\lambda_t-\lambda^*_t\right)\right] = 2\alpha(1-\alpha)(\eta-1)\eta\tilde{s}_t-\omega\left(\tilde{y}_t-\tilde{y}^*_t\right)+\kappa\omega\left(\varphi_t-\varphi^*_t\right)-(1+2\omega)\Lambda_t$$

the transversality conditions $\lim_{t\to\infty}\varphi_t\pi_{H,t} = \lim_{t\to\infty}\varphi^*_t\pi^*_{F,t} = 0$ and $\lim_{t\to\infty}\lambda_t\tilde{y}_t = \lim_{t\to\infty}\lambda^*_t\tilde{y}^*_t = 0$, and initial conditions for the co-state variables. First we combine (C.14) and (C.15). Then we differentiate (C.16) and (C.17) and substitute them into (C.14) and (C.15) to obtain the following first order differential equation

$$\dot{\tilde{y}}_t+\dot{\tilde{y}}^*_t = -\varepsilon(\pi_{H,t}+\pi^*_{F,t})$$

The solution of this first order differential equation describes the optimal monetary policy in target form and is given by

$$\pi_{H,t} + \pi^*_{F,t} = 0, \qquad \text{and} \qquad \tilde{y}_t + \tilde{y}^*_t = 0 \tag{C.19}$$

Note: it is straightforward to see that introducing the union-wide CPI inflation and union-wide output gap, as in section C.1, and expressing the Philipps cruve and IS curve in term of deviation from these union-wide variables leads to the same result. In addition, from the first order condition for τ_t and the costate equations for $\tilde{y}_t$ and $\tilde{y}^*_t$, we get that

$$-2\alpha(1-\alpha)\tilde{\theta}_t - \frac{1}{2}(1-2\alpha)(1+\phi)(\tilde{y}_t - \tilde{y}^*_t) + \frac{\kappa}{2}(1+\phi(1-2\alpha))(\varphi_t - \varphi^*_t) = 0 \tag{C.20}$$

Note that using (C.6) to eliminate $\tilde{s}_t$, (C.14)-(C.17) and (C.2)-(C.5) constitute a system of first-order ordinary differential equations in state variables $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t, \tilde{y}^*_t$ and costate variables whose particular solution satisfying the transversality conditions and initial conditions. Using the optimal monetary in target form (C.19), it straightforward to see that this can be reduced to a system of four first-order ordinary differential equations. We solve this system of differential equation to obtain the tax wedge (derivative of $\tilde{\theta}_t$) as a function of policy instruments only.

C.3 Noncooperative macroprudential policy

C.3.1 Union-wide central bank's problem

The union-wide central bank's problem is an optimal control problem with control variables i_t and state variables $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t, \tilde{y}^*_t, \tilde{\theta}_t$. The associated present value Hamiltonian is given by

$$\begin{aligned}
\mathcal{H} \;=\; & e^{-\rho t}\Big\{\frac{1}{2}\Big[\mathbb{L}_{H,t} + \mathbb{L}^*_{F,t}\Big] \\
& + \varphi_{G,t}\Big[\rho\pi_{H,t} - \kappa(1+\phi)\tilde{y}_t + \kappa\omega\tilde{s}_t - \kappa\alpha\tilde{\theta}_t\Big] \\
& + \varphi^*_{G,t}\Big[\rho\pi^*_{F,t} - \kappa(1+\phi)\tilde{y}^*_t - \kappa\omega\tilde{s}_t + \kappa\alpha\tilde{\theta}_t\Big] \\
& + \lambda_{G,t}\left[(i_t - \pi_{H,t} - r^n_t) + \omega\dot{\tilde{s}}_t + (1-\alpha)\tau_t + \alpha\tau^*_t\right] \\
& + \lambda^*_{G,t}\left[\left(i_t - \pi^*_{F,t} - r^{n*}_t\right) - \omega\dot{\tilde{s}}_t + \alpha\tau_t + (1-\alpha)\tau^*_t\right] + \mu_{G,t}(\tau_t - \tau^*_t) \\
& + \Lambda_t\Big[(\tilde{y}_t - \tilde{y}^*_t) - (1-2\alpha)\tilde{\theta}_t - (1+2\omega)\tilde{s}_t\Big]\Big\} + \Gamma_G\left[\int_0^\infty e^{-\rho t}\left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right)dt - T\right]
\end{aligned}$$

where $\varphi_{G,t}, \varphi^*_{G,t}, \lambda_{G,t}, \lambda^*_{G,t}, \mu_{G,t}$ are the co-state variables associated with $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t, \tilde{y}^*_t, \tilde{\theta}_t$; and Γ_G is the multiplier on the home lifetime budget constraint.

The union-wide central bank's optimal choice is characterized by the first-order conditions

$$\frac{\partial \mathcal{H}_G}{\partial i_t} = \lambda_{G,t} + \lambda^*_{G,t} = 0 \tag{C.21}$$

$$\dot{\varphi}_{G,t} = -\frac{\varepsilon}{\kappa}\pi_{H,t}e^{-\rho t} + \lambda_{G,t} \tag{C.22}$$

$$\dot{\varphi}^*_{G,t} = -\frac{\varepsilon}{\kappa}\pi^*_{F,t}e^{-\rho t} + \lambda^*_{G,t} \tag{C.23}$$

$$\dot{\lambda}_{G,t} - \rho\lambda_{G,t} = -(1+\phi)\tilde{y}_t + \kappa(1+\phi)\varphi_{G,t} - \Lambda_{G,t} \tag{C.24}$$

$$\dot{\lambda}^*_{G,t} - \rho\lambda^*_{G,t} = -(1+\phi)\tilde{y}^*_t + \kappa(1+\phi)\varphi^*_{G,t} + \Lambda_{G,t} \tag{C.25}$$

$$\dot{\mu}_{G,t} - \rho\mu_{G,t} = -2\alpha(1-\alpha)\tilde{\theta}_t + \omega(1-2\alpha)\tilde{s}_t + \kappa\alpha\left(\varphi_{G,t} - \varphi^*_{G,t}\right) + (1-2\alpha)\,\Lambda_{G,t} \tag{C.26}$$

the transversality conditions $\lim_{t\to\infty}\varphi_{G,t}\pi_{H,t} = \lim_{t\to\infty}\varphi^*_{G,t}\pi^*_{F,t} = 0$ and $\lim_{t\to\infty}\lambda_{G,t}\tilde{y}_t = \lim_{t\to\infty}\lambda^*_{G,t}\tilde{y}^*_t = 0$, and initial conditions for the co-state variables. We first combine (C.22) and (C.23). Then differentiating both (C.24) and (C.25) and substituting them into (C.22) and (C.23), we obtain the following first order differential equation

$$\dot{\tilde{y}}_t + \dot{\tilde{y}}^*_t = -\varepsilon(\pi_{H,t} + \pi^*_{F,t}),$$

the solution of this first order differential equation describes the optimal monetary policy in target form and is given by

$$\pi_{H,t} + \pi^*_{F,t} = 0, \qquad \text{and} \qquad \tilde{y}_t + \tilde{y}^*_t = 0 \tag{C.27}$$

C.3.2 Home macroprudential authority's problem

The home macroprudential authority's problem is an optimal control problem with control variable τ_t, and state variables $\pi_{H,t}$, $\pi^*_{F,t}$, $\tilde{y}_t$, $\tilde{y}^*_t$, $\tilde{\theta}_t$. The associated present value Hamiltonian is

$$\begin{aligned}\mathcal{H}_H = e^{-\rho t}\Bigg\{&\frac{1}{2}\left[(1-\alpha\beta)\mathbb{L}_{H,t} + (\alpha\beta)\mathbb{L}^*_{F,t} + \left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right)\left((1+\hat{\beta})\left[\tilde{\theta}_t + 2\int_0^t \tau^*_s ds\right] - 2\alpha\beta\left(\tilde{y}_t + \tilde{y}^*_t\right)\right)\right]\\ &+\varphi_{H,t}\left[\rho\pi_{H,t} - \kappa(1+\phi)\tilde{y}_t + \kappa\omega\tilde{s}_t - \kappa\alpha\tilde{\theta}_t\right]\\ &+\varphi^*_{H,t}\left[\rho\pi^*_{F,t} - \kappa(1+\phi)\tilde{y}^*_t - \kappa\omega\tilde{s}_t + \kappa\alpha\tilde{\theta}_t\right]\\ &+\lambda_{H,t}\left[(i_t - \pi_{H,t} - r^n_t) + \omega\dot{\tilde{s}}_t + (1-\alpha)\tau_t + \alpha\tau^*_t\right]\\ &+\lambda^*_{H,t}\left[\left(i_t - \pi^*_{F,t} - r^{n*}_t\right) - \omega\dot{\tilde{s}}_t + \alpha\tau_t + (1-\alpha)\tau^*_t\right] + \mu_{H,t}(\tau_t - \tau^*_t)\\ &+\Lambda_{H,t}\left[(\tilde{y}_t - \tilde{y}^*_t) - (1-2\alpha)\tilde{\theta}_t - (1+2\omega)\tilde{s}_t\right]\Bigg\} + \Gamma_H\left[\int_0^\infty e^{-\rho t}\left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right)dt - T\right]\end{aligned}$$

where $\varphi_{H,t}, \varphi^*_{H,t}, \lambda_{H,t}, \lambda^*_{H,t}, \mu_{H,t}$ are the co-state variables associated with $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t, \tilde{y}^*_t, \tilde{\theta}_t$; Γ_H is home macroprudential authority's multiplier on the home lifetime budget constraint.

The home macroprudential authority's optimal choice is characterized by the first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{H}_H}{\partial \tau_t} &= (1-\alpha)\lambda_{H,t} + \alpha\lambda^*_{H,t} - \mu_{H,t} = 0 && \text{(C.28)}\\
\dot{\varphi}_{H,t} &= -(1-\alpha\beta)\frac{\varepsilon}{\kappa}\pi_{H,t} + \lambda_{H,t} && \text{(C.29)}\\
\dot{\varphi}^*_{H,t} &= -\alpha\beta\frac{\varepsilon}{\kappa}\pi^*_{F,t} + \lambda^*_{H,t} && \text{(C.30)}\\
\dot{\lambda}_{H,t} - \rho\lambda_{H,t} &= -(1-\alpha\beta)(1+\phi)\tilde{y}_t + \alpha\beta\left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right) + \kappa(1+\phi)\varphi_{H,t} - \Lambda_{H,t} && \text{(C.31)}\\
\dot{\lambda}^*_{H,t} - \rho\lambda^*_{H,t} &= -\alpha\beta(1+\phi)\tilde{y}^*_t + \alpha\beta\left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right) + \kappa(1+\phi)\varphi^*_{H,t} + \Lambda_{H,t} && \text{(C.32)}\\
\dot{\mu}_{H,t} - \rho\mu_{H,t} &= -\alpha(2-\alpha+\hat{\beta})\tilde{\theta}_t - (1+\hat{\beta})\alpha\int_0^t \tau^*_s ds + \left[(1-2\alpha-\omega) + \frac{1}{2}\hat{\beta}\right]\omega\tilde{s}_t\\
&\quad + \alpha^2\beta\left(\tilde{y}_t + \tilde{y}^*_t\right) + \kappa\sigma\alpha\left(\varphi_{H,t} - \varphi^*_{H,t}\right) + (1-2\alpha)\Lambda_{H,t} - \alpha\Gamma_H && \text{(C.33)}
\end{aligned}$$

the transversality conditions $\lim_{t\to\infty}\varphi_{H,t}\pi_{H,t} = \lim_{t\to\infty}\varphi^*_{H,t}\pi^*_{F,t} = 0$ and $\lim_{t\to\infty}\lambda_{H,t}\tilde{y}_t = \lim_{t\to\infty}\lambda^*_{H,t}\tilde{y}^*_t = 0$, and initial conditions for the co-state variables.
From the first order condition for τ_t and the costate equations for $\tilde{y}_t$ and $\tilde{y}^*_t$, we get that

$$\begin{aligned}
0 &= -\alpha\left[2(1-\alpha) + \hat{\beta} - \omega\right]\tilde{\theta}_t - (1+\hat{\beta})\alpha\int_0^t \tau^*_s ds - (1-2\alpha-\omega)(1+\phi)\tilde{y}_t - \alpha\Gamma_H\\
&\quad + \left[(1-2\alpha-\omega) + \frac{1}{2}\hat{\beta}\right]\omega\tilde{s}_t + \kappa(1+\phi(1-2\alpha))\varphi_{H,t} - \alpha^2\beta\phi\left(\tilde{y}_t + \tilde{y}^*_t\right) && \text{(C.34)}
\end{aligned}$$

and differentiating this equation, and rearranging we get

$$\alpha\left[2(1-\alpha) + \hat{\beta} - \omega\right]\tau_t = -(1-2\alpha-\omega)\left[(1+\phi)\dot{\tilde{y}}_t - \alpha\tau^*_t - \omega\dot{\tilde{s}}_t\right] + \frac{\hat{\beta}}{2}\omega\dot{\tilde{s}}_t + \kappa(1+\phi(1-2\alpha))\dot{\varphi}_{H,t}$$

Substituting the law of motion for $\tilde{y}_t$ we obtain the Home macroprudential authority's best response which takes as given the policy instruments of other authorities as in Proposition **??**

C.3.3 Foreign macroprudential authority's problem

The foreign macroprudential authority's problem is an optimal control problem with control variable τ^*_t, and state variables $\pi_{H,t}$, $\pi^*_{F,t}, \tilde{y}_t, \tilde{y}^*_t, \tilde{\theta}_t$. The associated present value Hamiltonian is given by

$$\begin{aligned}
\mathcal{H}_F^* &= e^{-\rho t}\Bigg\{\frac{1}{2}\Bigg[(1-\alpha\beta)\mathbb{L}_{F,t}^* + \left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right)\left((1+\hat{\beta})\left[\tilde{\theta}_t - 2\int_0^t \tau_s ds\right] + 2\alpha\beta\left(\tilde{y}_t + \tilde{y}_t^*\right)\right)\Bigg] \\
&\quad + \varphi_{F,t}^*\left[\rho\pi_{H,t} - \kappa(1+\phi)\tilde{y}_t + \kappa\omega\tilde{s}_t - \kappa\alpha\tilde{\theta}_t\right] \\
&\quad + \varphi_{F,t}^*\left[\rho\pi_{F,t}^* - \kappa(1+\phi)\tilde{y}_t^* - \kappa\omega\tilde{s}_t + \kappa\alpha\tilde{\theta}_t\right] \\
&\quad + \lambda_{F,t}^*\left[(i_t - \pi_{H,t} - r_t^n) + \omega\dot{\tilde{s}}_t + (1-\alpha)\tau_t + \alpha\tau_t^*\right] \\
&\quad + \lambda_{F,t}^*\left[\left(i_t - \pi_{F,t}^* - r_t^{n*}\right) - \omega\dot{\tilde{s}}_t + \alpha\tau_t + (1-\alpha)\tau_t^*\right] + \mu_{F,t}^*(\tau_t - \tau_t^*) \\
&\quad + \Lambda_{F,t}^*\left[(\tilde{y}_t - \tilde{y}_t^*) - (1-2\alpha)\tilde{\theta}_t - (1+2\omega)\tilde{s}_t\right]\Bigg\} + \Gamma_F^*\left[\int_0^\infty e^{-\rho t}\left(-\alpha\tilde{\theta}_t + \omega\tilde{s}_t\right)dt - T\right]
\end{aligned}$$

where $\varphi_{F,t}, \varphi_{F,t}^*, \lambda_{F,t}, \lambda_{F,t}^*, \mu_{F,t}$ are the co-state variables associated with $\pi_{H,t}$, $\pi_{F,t}^*, \tilde{y}_t, \tilde{y}_t^*, \tilde{\theta}_t$; Γ_F^* is foreign macroprudential authority's multiplier on the foreign lifetime budget constraint (expressed in units of the home agent's marginal utility).

The foreign macroprudential authority's optimal choice is characterized by the first-order conditions

$$\begin{aligned}
\frac{\partial\mathcal{H}_F^*}{\partial\tau_t^*} &= \alpha\lambda_{F,t} + (1-\alpha)\lambda_{F,t}^* - \mu_{F,t}^* = 0 &\text{(C.35)}\\
\dot{\varphi}_{F,t} &= -\alpha\beta\frac{\varepsilon}{\kappa}\pi_{H,t} + \lambda_{F,t} &\text{(C.36)}\\
\dot{\varphi}_{F,t}^* &= -(1-\alpha\beta)\frac{\varepsilon}{\kappa}\pi_{F,t}^* + \lambda_{F,t}^* &\text{(C.37)}\\
\dot{\lambda}_{F,t} - \rho\lambda_{F,t} &= -\alpha\beta(1+\phi)\tilde{y}_t - \alpha\beta\left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right) + \kappa(1+\phi)\varphi_{F,t} - \Lambda_{F,t}^* &\text{(C.38)}\\
\dot{\lambda}_{F,t}^* - \rho\lambda_{F,t}^* &= -(1-\alpha\beta)(1+\phi)\tilde{y}_t^* - \alpha\beta\left(\alpha\tilde{\theta}_t - \omega\tilde{s}_t\right) + \kappa(1+\phi)\varphi_{F,t}^* + \Lambda_{F,t}^* &\text{(C.39)}\\
\dot{\mu}_{F,t}^* - \rho\mu_{F,t}^* &= -\alpha(2-\alpha+\hat{\beta})\tilde{\theta}_t + (1+\hat{\beta})\alpha\int_0^t \tau_s ds + \left[(1-2\alpha-\omega) + \frac{1}{2}\hat{\beta}\right]\omega\tilde{s}_t \\
&\quad - \alpha^2\beta\left(\tilde{y}_t + \tilde{y}_t^*\right) + \kappa\sigma\alpha\left(\varphi_{F,t} - \varphi_{F,t}^*\right) + (1-2\alpha)\Lambda_{F,t}^* + \alpha\Gamma_F^* &\text{(C.40)}
\end{aligned}$$

the transversality conditions $\lim_{t\to\infty}\varphi_{F,t}\pi_{H,t} = \lim_{t\to\infty}\varphi_{F,t}^*\pi_{F,t}^* = 0$ and $\lim_{t\to\infty}\lambda_{F,t}\tilde{y}_t = \lim_{t\to\infty}\lambda_{F,t}^*\tilde{y}_t^* = 0$, and initial conditions for the co-state variables.

From the first order condition for τ_t and the costate equations for $\tilde{y}_t$ and $\tilde{y}_t^*$, we get that

$$\begin{aligned}
0 &= -\alpha\left[1+\hat{\beta}+(1-2\alpha-\omega)\right]\tilde{\theta}_t + (1+\hat{\beta})\alpha\int_0^t \tau_s ds + (1-2\alpha-\omega)(1+\phi)\tilde{y}_t^* + \alpha\Gamma_F^* \\
&\quad + \left[(1-2\alpha-\omega) + \frac{1}{2}\hat{\beta}\right]\omega\tilde{s}_t - \kappa(1+\phi(1-2\alpha))\varphi_{F,t}^* + \alpha^2\beta\phi\left(\tilde{y}_t + \tilde{y}_t^*\right) \qquad \text{(C.41)}
\end{aligned}$$

differentiating this equation, and rearranging we get

$$-\alpha\left[1+\hat{\beta}+(1-2\alpha-\omega)\right]\tau_t^*=(1-2\alpha-\omega)\left[(1+\phi)\dot{\tilde{y}}_t^*-\alpha\tau_t+\omega\dot{\tilde{s}}_t\right]+\frac{\hat{\beta}}{2}\omega\dot{\tilde{s}}_t-\kappa(1+\phi(1-2\alpha))\dot{\varphi}_{F,t}^*$$

Substituting the law of motion for $\tilde{y}_t^*$ we obtain the Foreign macroprudential authority's best response which takes as given the policy instruments of other authorities as in Proposition **??**

C.3.4 Characterization of Nash equilibrium

The solution of the game can be described as the particular solution to a system of first-order ordinary differential equations in $\pi_{H,t}$, $\pi_{F,t}^*, \tilde{y}_t, \tilde{y}_t^*, \tilde{\theta}_t$ satisfying the transversality and initial conditions. First, differentiating (C.21) with respect to time and substituting (C.24) and (C.25) into the resulting equation yields the targeting rule (C.27). Second, combining the laws of motion for $\tilde{y}_t$ and $\tilde{y}_t^*$ to eliminate i_t yields

$$\dot{\tilde{y}}_t-\dot{\tilde{y}}_t^*=-\left(r_t^n-r_t^{n*}\right)+(1-2\alpha)\left(\tau_t-\tau_t^*\right) \tag{C.42}$$

Third, differentiating (C.34) with respect to time and substituting (C.27) and (C.42) into the resulting equation yields the targeting rule for the home macroprudential authority

$$\psi\tau_t=-(1-2\alpha-\omega)\Big[\alpha\phi\tau_t^*+(1+\phi)\left(i_t-\pi_{H,t}-r_t^n-\omega\tilde{s}_t\right)\Big]+\frac{\hat{\beta}}{2}\omega\dot{\tilde{s}}_t+\kappa\left(1+\phi(1-2\alpha)\right)\dot{\varphi}_{H,t}. \tag{C.43}$$

where $\psi\equiv\alpha(1+\hat{\beta})+(1+\phi(1-\alpha))(1-2\alpha-\omega)$. Fourth, differentiating (C.34) with respect to time and substituting (C.27) and (C.42) into the resulting equation yields the targeting rule for the foreign macroprudential authority

$$\psi\tau_t^*=-(1-2\alpha-\omega)\Big[\alpha\phi\tau_t+(1+\phi)\left(i_t-\pi_{F,t}^*-r_t^{n*}+\omega\tilde{s}_t\right)\Big]-\frac{\hat{\beta}}{2}\omega\dot{\tilde{s}}_t+\kappa\left(1+\phi(1-2\alpha)\right)\dot{\varphi}_{F,t}^*. \tag{C.44}$$

Substituting (C.41) and (C.44) into the law of motion for $\tilde{\theta}_t$ yields

$$\left[2\left(1-2\alpha\right)+1+\beta\right]\alpha\dot{\tilde{\theta}}_t=(1-2\alpha)\left(1+\phi\right)\left(\dot{\tilde{y}}_t^*-\dot{\tilde{y}}_t\right)+\kappa\left(1+\phi\left(1-2\alpha\right)\right)\left(\dot{\varphi}_{H,t}-\dot{\varphi}_{F,t}^*\right) \tag{C.45}$$

Note that (C.29)-(C.32) and (C.2)-(C.5) on the one hand; and (C.36)-(C.39) and (C.2)-(C.5) on the other hand constitute a system of first-order ordinary differential equations in state variables $\pi_{H,t}$, $\pi_{F,t}^*, \tilde{y}_t, \tilde{y}_t^*$ and costate variables in any given country whose particular solution satisfying the transversality conditions and initial conditions constitutes a solution to the game between the three policy authorities. Using the union-wide central bank's optimal monetary policy in target form (C.27), each system of first-order ordinary differential equations can be reduced to a system of four first-order ordinary differential equations with four unknown variables. The associated interest rate, home capital flow tax and foreign capital flow tax wedge are respectively given by

$$\begin{aligned} i_t &= \frac{1}{2}\left(r_t^n + r_t^{*n}\right) + \frac{1}{2}\left(\tau_t + \tau_t^*\right) \\ \dot{\tilde{\theta}}_t &= (1-\Phi)\Psi\left(r_t^n - r_t^{n*}\right) + \kappa\int_0^t \xi_t^{nash}\left(r_s^n - r_s^{*n}\right)ds \end{aligned}$$