Université de Montréal

On Abstraction in a Carnapian System

Par

Parzhad Torfehnezhad

Département de philosophie

Faculté des arts et des sciences

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Résumé

Rudolf Carnap (1891-1970) rejette deux distinctions philosophiques conçues par Gottlob Frege (1848-1925) : la distinction objet-concept et la distinction sens-référence. Dans la tradition analytique et parmi ces distinctions, une famille de systèmes analytiques a été construite et développée (appelée les « systèmes frégéen »), dans lesquels plusieurs notions ont été employées, incluant la notion d'abstraction. En fait, les néo- frégéen ont déclaré que la notion d'abstraction de Frege est capturée par ce qu'on appelle le « principe d'abstraction ». Le but de cette dissertation est de présenter la notion d'abstraction de Carnap en particulier et le système de Carnap en général, en comparaison aux notions de Frege. Nous allons argumenter que l'admission et le rejet de ces distinctions entraîneront des systèmes analytiques fondamentalement différents. Ainsi, nous allons démontrer comment chaque système utilise différentes notions d'abstraction. L'abstraction dans un système frégéen sera caractérisée comme un processus indépendant qui est confiné à ses propres règles, tandis que dans un système carnapien, l'abstraction sera caractérisée comme un processus défini d'éloignement du sens. Nous arriverons à la conclusion que le système carnapien a plus d'avantages que celui de Frege (comme la simplicité du système) et que son aspect technique a besoin d'être développé davantage.

Mots-clés : philosophie analytique, philosophie de la langue, philosophie de la science, logique, Frege, Carnap, abstraction, distinction objet-concept, distinction sens-référence.

Abstract

Rudolf Carnap (1891-1970) rejects two philosophical distinctions that have been made and admitted by Gottlob Frege (1848-1925), namely the object-concept and the sense-reference distinctions. In the analytic tradition and upon these distinctions, a family of analytic systems have been constructed and developed (which we call Fregean systems), within which a number of notions have been employed including the notion of abstraction. It has been claimed (by Neo-Fregeans) that the Fregean notion of abstraction has been captured by what is commonly known as the "principle of abstraction". The goal of this dissertation is to present the notion of Carnapian abstraction, in particular, and the Carnapian system, in general, in distinction to the Fregean counterparts. We will argue that the admission and rejection of these distinctions will entail fundamentally different analytic systems. Hence, we will show how each system undertakes a different notion of abstraction. Abstraction in a Fregean system will be characterized as a mind-independent process subject to its own rules, whereas in a Carnapian system, abstraction will be characterized as a defined process of distancing from meaning in a linguistic framework. We will conclude that the Carnapian system has advantages over the Fregean one (among which is its simplicity), and that its technical aspect is yet to be developed.

Keywords: analytic philosophy, philosophy of language, philosophy of science, logic, Frege, Carnap, abstraction, linguistic framework, object-concept distinction, sense-reference distinction.

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Abbreviations

ASD	Analytic-Synthetic Distinction
ESO	"Empiricism, Semantics and Ontology" (Carnap, 1950)
HP	Hume's Principle
LF	Linguistic Framework
LD	Linguistic Doctrine of logical truths: logical truths are true by linguistic convention
PC	Propositional Calculus
NTT	Normal Truth Table
SCT	Strong Completeness Theorem

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Introduction

Rudolf Carnap (1891-1970) is a major figure in the history of analytic philosophy who famously took a linguistic approach towards philosophical problems. Despite the vast literature on Carnap it is still unclear whether Carnap ever succeeded in proposing a unified general framework for philosophical analysis. Under the banner of "logical empiricism", Carnap's technical contributions in the fields of philosophy of language, of logic, of mathematics, and philosophy of science seemingly have been, internalized, digested, and either superseded or thrown out; thus, "Carnap seemed, like the other major figures of logical empiricism, to have been of only ephemeral importance" (Gabriel, 2004, p. 3). This led to the loss of interest in searching for "Carnapian system" or "Carnapian analysis" as independent systems. Researchers, in general, are used to seeing Carnap's philosophical work as an annex or a version of Frege's conceptual analysis, not as an alternative to it. We intend to advocate the latter view here, since the former perception has begun to change recently. "It has come to be realized that there was a good deal more to Carnap than his particular technical contributions to various specialized fields. There was also a vision that held all these parts together and motivated them, a vision whose importance transcends and outlasts the parts" (*Ibid.*). Although we focus on the accommodation of the notion of abstraction in a Carnapian system, we intend to present the Carnapian system, in general, as an independent analytic system which gives us a different vision altogether.

The main goal of this dissertation is to present the Carnapian notion of abstraction, in particular, and the Carnapian system, in general, in distinction to the Fregean ones. As it is well known, there is a notion of abstraction in the analytic tradition that is supposed to be governed by the "principle of abstraction". Abstraction is a vague term that has never been clearly defined in philosophy despite being widely in use. In the analytic tradition, perhaps the closest call for precision has been proposed by the neo-Fregeans in the form of a principle, namely the *principle of abstraction*, based on a universal understanding of the equivalency relationship (as we will see). In the family of analytic methods using the abstraction principle on the basis of the universal equivalency relationship (we call them Fregean methods), the extensions of concepts and the extensionality of logical systems are the key factors in their methods of conceptual analysis and of referring to objects. These factors stem from two fundamental philosophical distinctions that have been made and admitted by Gottlob Frege (1848-1925), namely the object-concept distinction[1] and the sense-reference distinction[2]. Admitting these distinctions evidently puts us in a specific philosophical framework, within which certain rules would be applied for identifying concepts and thereby considering notions such as abstraction, according to which the behaviour of objects would be realized. In the course of this dissertation, the term "Fregean system" (or method) refers to the systems (or methods) that are constructed using these distinctions, whether or not they have anything to do with Frege himself. As we will see, abstraction in a Fregean system could be characterized as a mind-independent process subjected to its own rules. This characterization is, of course, the result of admitting the two distinctions mentioned above.

[1] Note that the object-concept distinction obviously goes back to Aristotle while Kant modifies the origin of concepts in terms of intuitions. For Kant, mathematical knowledge relies on both object and concept. Concepts have to be schematized in intuition and it is in this way that one gets mathematical objects in Kant. For Frege, on the other hand, everything follows from concepts. At least at the beginning (before the basic law V), he was aware that he can do mathematics, at least arithmetic, without objects. Frege then reintroduces objects (as extensions) and brings them into the picture in a way in which objects become subordinates of concepts. This is how Frege's distinction is different than that of Kant. What we mean here by object-concept distinction is exactly the Fregean one.

[2] Just to remind you about this distinction, for example, the phrases "morning star" and "evening star" have the same reference, but their senses are different.

What if we do not admit to the object-concept and the sense-reference distinctions? What kind of framework, and hence analytic method or system, would we have if we reject these distinctions? How would we accommodate notions like abstraction in such a framework? These are the type of questions we would like to address in our exposition of Carnap's philosophy. Carnap is one of the philosophers who reject both of the above-mentioned distinctions; thus, he must propose an alternative framework, and he does, in my view. In short, we may summarize the differences between a Fregean and a Carnapian frameworks in Table I.

	Ontological Status	**Analytic System**	**Equivalency Relation**	**Meta-language**	**Abstraction**
Fregean	Independent	System of concepts	Universal	Extensional	Mind-independent
Carnapian	Linguistic (language-dependent)	System of objects	Non-universal	Non-extensional	Voluntary action

Table I Differences between Fregean and Carnapian analytic frameworks

Despite the vast literature on Carnap and Frege, the literature on comparing the two is quite limited[3], almost none of which is mainly concerned with comparing abstraction between the two. Nonetheless, there is some literature in which the notion of Carnapian abstraction in distinction to the Fregean one has been acknowledged and discussed, e.g., (Beaney, 2004; Leitgeb, 2007); they consider the Carnapian abstraction as a version of the Fregean one that is essentially the same with minor differences, not as an alternative or a rival version. Concentrating on the method of quasi-analysis in Carnap's *Aufbau*, Leitgeb proposes that if one wants to reconstruct quasi-analysis in a Fregean framework (using our terminology), one is able

[3] See for examples: Gabriel (2007); Hanzel (2006); Holland (1978); Lavers (2013); Reck (2004); Ricketts (2004); Steinberger (2017); Stoenescu (2014); Wilholt (2006)

to do so adequately and consistently. As valuable as Leitgeb's work is, it differs from the point we are trying to make, in that the Carnapian framework is fundamentally different from the Fregean one, so that one can no longer hold on to the "abstraction principle" and to the universality of the equivalency relationship in the same way one would in the Fregean philosophical framework. Beaney correctly points out some of the important differences between Fregean analysis and Carnapian quasi-analysis as well as Carnap's tendency towards Russell's program. With regard to the vagueness of Russell's logicism, Beaney asks this question: "Does it entail a program of *ontological eliminativism*, or just of *epistemological reductionism*?" (Beaney, 2004, p. 125). And he rightly believes that "it is clear that Carnap himself interpreted Russell's maxim epistemologically rather than ontologically" (*Ibid.*). Nonetheless, Beaney also concludes that "in essence, Carnap's method of quasi-analysis is just that method of contextual definition or logical abstraction that Frege had introduced in the *Grundlagen*" (*Ibid.*). This conclusion might be true, if we limit ourselves just to *Aufbau*, and overlook Carnap's overall linguistic approach and minimize the meaning of the term "abstraction" in Carnap's other works, especially in (1939) and (1962).

The lack of literature on appreciating a separate notion of abstraction for Carnap is understandable for the following reasons. In *Aufbau*, where Carnap uses the term "abstraction" extensively, he explicitly says that his concept of abstraction is directly derived from Frege and Russell (as we will see), there also are several places (other than *Aufbau*) where he considers himself as a loyalist to Frege's project and declares that his differences with Frege's method is not theoretical (meaning his goal still is providing a systematic analysis) but methodic or practical (we will see that in chapter three, section 3.1, in more detail). Furthermore, we see a sharp decline in the use of the term "abstraction" in Carnap's subsequent works, with the exception of his *Foundations of Logic and Mathematics* (1939), within which there is no serious reference to "abstraction" (or so we might have thought). It is not up until *Meaning and Necessity* (1947) that "abstraction" reappears in Carnap works, and even then, it is meant to refer to the "abstraction operator" (lambda operator). The only place we could see the return of "abstraction" in Carnap's work almost as serious as in the *Aufbau* is in his *Logical Foundations of Probability* (1962). Therefore, it is natural to think that Carnap's conception of abstraction is basically the same as that of Frege's.

Aufbau	1928
Logical Syntax of Language	1937
Foundations of Logic and Mathematics	1939
Introduction to Semantics	1942
Formalization of Logic	1943
Meaning and Necessity	1947
Logical Foundations of Probability	1962

Table II Carnap's major works

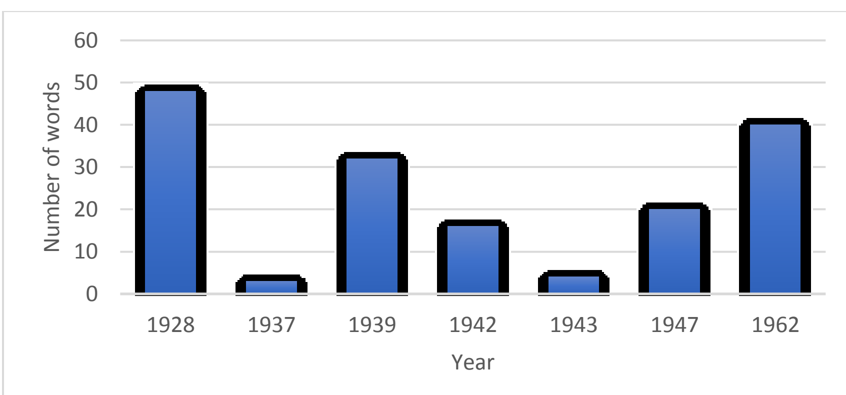

Figure 1 The usage of the word "abstraction" or "abstract" in Carnap's works

With a comprehensive look at Carnap's work (from 1928 to 1962), on the other hand, one may realize that, on the basis of his linguistic approach, we may have a whole other meaning for "abstraction". In a Fregean system, the sign "≡" designated an equivalency relation, which could be universally applied to the objects or concepts (of the same type). According to the principle of abstraction, holding an equivalency relation between two entities necessarily leads to more abstract concepts. In a Carnapian system, on the other hand, the sign is a function of the type (or category) of the designators of the both sides of the sign so that the resulting equivalency sentence "... ≡ ⋯" in each case has a different meaning altogether.

In Carnap's understanding of abstraction, abstraction could be done in different ways; that is why he uses "abstractions" (plural) in (Carnap, 1962). To picture how Carnap considers different ways of abstracting, let us consider the following analogy he gives in the support of his linguistic approach.

> Suppose a circular area is given, and we want to cover some of it with quadrangles which we draw within the circle and which do not overlap. This can be done in many different ways; but, whichever way we do it and however far we go with the (finite) procedure, we shall never succeed in covering the whole circular area. […] In any construction of a system of logic or, in other words, of a language system with exact rules, something is sacrificed, is not grasped, because of the abstraction or schematization involved. However, it is not true that there is anything that cannot be grasped by a language system and hence escape logic. For any single fact in the world, a language system can be constructed which is capable of representing that fact while others are not covered. (Carnap, 1962, p. 210)

Using this analogy, we may say that the Fregean system would be just one of the possible quadrangles. We can clearly see here that the scope of what Carnap means by "abstraction(s)" is much larger than what one means by this term in a Fregean system. In the Carnapian system, scientific and philosophical abstractions work hand-in-hand in a complementary fashion.

Carnap values both scientific and philosophical abstractions, and pictures them using an analogy of personality traits.

> One of the factors contributing to the origin of the controversy about abstractions is a psychological one; it is the difference between two constitutional types. Persons of the one type (extroverts) are attentive to and have a liking for nature with all its complexities and its inexhaustible richness of qualities; consequently, they dislike to see any of these qualities overlooked or neglected in a description or a scientific theory. Persons of the other type (introverts) like the neatness and exactness of formal structures more than the richness of qualities; consequently, they are inclined to replace in their thinking the full picture of reality by a simplified schema. In the field of science and of theoretical investigation in general, both types do valuable work; their functions complement each other, and both are indispensable. (Carnap, 1962, p. 218)

Carnap then warns us about the major weakness of those who limit themselves just to formal constructions (including himself).

> Their chief weakness is the ever present temptation to over-schematize and oversimplify and hence to overlook important factors in the actual situation; the result may be a theory which is wonderful to look at in its exactness, symmetry, and formal elegance, and yet woefully inadequate for the task of application for which it is intended. (This is a warning directed at the author of this book by his critical super-ego). (*Ibid.*)

Despite my intuitive suggestions for formalizing the Carnapian notion of abstraction, I have no intention on or claim about the relation between the Carnapian notion and the Fregean one, or, about whether the Carnapian notion could be formalized at all. My main claim is that the Fregean and Carnapian analytic systems are fundamentally different on the basis of admitting and rejecting the above-mentioned philosophical distinctions (object-concept and sense-reference), and that that would entail the employment of different notions of abstraction in each case.

As mentioned above, the Carnapian approach to philosophical problems is a linguistic approach. Hence, with this approach the notion of *linguistic framework* becomes important and a key factor in the general understanding of a Carnapian system. In the first chapter, we set aside *Aufbau*, in which "abstraction" was used seemingly too close to Frege's notion and take other works of Carnap into account. In this chapter, we want to give a general sense of the linguistic framework, its components, and the role abstraction plays in it. After establishing the general notion of the linguistic framework, we move on to the second chapter, in which we return to *Aufbau* to see what kind of a framework (or system) we would have if we reject the object-concept distinction; knowing that in this Carnapian system of objects (instead of a system of concepts) the first linguistic step for acknowledging the existence of objects is to name them. In the third chapter, we will see how Carnap rejects Frege's sense-reference distinction, clearly characterizes abstraction as a voluntary purposeful act of distancing from meaning, and once again how this rejection would cash out in the framework in question. Putting all three chapters together, we wish to present a Carnapian framework as an alternative to the Fregean one.

In the first chapter, we will show that, according to Carnap, there are basically two methods for constructing a linguistic framework; given the hierarchical setting of the framework, the first method is a bottom-up construction, while the second method is top-down. In this presentation we would like to consider our approach to abstraction as the main contributor to the first method of constructing a framework. In this chapter, we will also find that many of Carnap's distinctions, such as his analytic-synthetic distinction, are to be taken as internally established and *relative* distinctions, as opposed to externally fundamental and absolute ones.

In the second chapter, assuming abstraction as a move from objects (or object-expressions) to concepts (or concept-expressions), we will distinguish two philosophical frameworks in which the notion of object could be employed; we want to see what the general philosophical requirements and properties are for each setting. Considering this, we will realize that some basic notions, e.g., the notion of object ought to be different in each setting, which follows that the notion of abstraction is different in each case. With *Aufbau* being our main source in this chapter, we want to show how Carnap (who does not agree with the object-concept distinction) proposes a theory of construction. In other words, the main question is how Carnap extends Russell's type theory in order to include concrete objects. We will also present a description of the frameworks in which the Fregean object-concept distinction is adopted. Consequently, we will assess the Fregean claim that concepts generally could be treated irrespective of objects, and hint to some problems associated with that claim. Upon all these considerations we will realize that the notion of the abstraction, employed in a typical Carnapian framework, ought to be different than the one employed in a Fregean framework.

In chapter three, the goal is to make more precise the notion of abstraction in a Carnapian framework; once we reject the sense-reference distinction. We will see how some key concepts, such as "extension" and "intension", are used differently in the Carnapian system versus the Fregean one. We will also see how Carnap shows that the meta-language, used for propositional analysis, ought to be non-extensional, and how one may encounter some awkwardness in a fully-extensional analysis. In this chapter Frege's sense and reference distinction is under scrutiny, which, according to Carnap, leads to some unnecessary multiplication of objects and thus a complicated system. In a Carnapian system we regard "designation" and "interpretation" as the major top-down one-to-many relations, while abstraction is viewed as the major bottom-up many-to-one relation.

Finally, on the basis of the simplicity of the Carnapian analytic system and what we presented from the contemporary literature that could be considered as some sort of support for this system, we would like to conclude that investing on updating the Carnapian system is not only a worthy endeavor, but also an urgent need.

Chapter 1. A General View of Abstraction in Carnap's Philosophy[4]

In this chapter, we want to show how Carnap envisions the role of abstraction in the construction of a linguistic framework generally. It is famously known that Carnap, along with other members of Vienna Circle, take a linguistic approach towards philosophical problems. They privileged this approach because in their view "the issue in philosophical problems concerned the language, not the world, [and] these problems should be formulated, not in the object language, but in the meta-language" (Schilpp, 1963, p. 54). The adoption of the linguistic doctrine of logical truth[5] (i.e., logical truths are true by linguistic conventions; henceforth LD) and considering language as an object of study on its own[6] are all indications of how central language and the concept of language was to them. In fact, accepting the ability of speaking about language in isolation is the departing point of Carnap from Wittgenstein. According to Carnap (Schilpp, 1963, p. 52), it is not possible for Wittgenstein to talk about language in isolation. By referring to what he calls a "linguistic framework" (henceforth LF), Carnap derives many philosophical dichotomies (such as analytic- synthetic, internal-external, and theoretical-factual) that, he believes, are all relative to the construction of LF. In this chapter, as I will present the LF briefly, we will see Carnap's constructive view and how he essentially considers

[4] The paper version of this chapter has been already published (Torfehnezhad, 2016), so one may find some the content of this chapter exactly similar to the published version.

[5] The adoption of this doctrine was, of course, an established point of consensus among Carnap and other members of Vienna Circle, although Carnap was not completely in agreement with this formulation of the doctrine (Schilpp, 1963, p. 914).

[6] In agreement with Neurath, along with other members of Vienna Circle, Carnap admits the possibility of speaking *about* language in isolation (as an object of its own) (Schilpp, 1963, p. 52).

LF as a construction which, to some extent, is subjected to our conventional decisions. Most importantly, we will see the role and place of abstraction in constructing LF and how Carnap pictures abstraction as a mean for losing factual content and gaining conventional theoretical elements at the same time. I should be clear that I do not claim, by no means, that Carnap shares our belief in the cognitive nature of abstraction at all; quite the opposite, he wanted to show that, regardless of our conviction of the nature of abstraction (or its mechanism), there are some basic concepts and/or properties such as "being analytic" or "being synthetic" that, upon investigating the nature of language, are revealed to be essentially *relative* to and hinge upon the construction of LF, which we use to study language. As scientists, primarily, may be motivated to start constructing their abstraction hierarchy for explaining some perceptual observations such as falling, heating, growing, etc., and mathematicians may start constructing theirs to have a system for counting, measuring, classifying, etc. each might have started their project from a perceptually available topic of study. In what follows the topic of study, in a similar fashion, is language (as an object; historically given and perceived) in its broadest sense or, to use Carnap's word, language as a vast array of any set of "communicative signs" (Carnap, 1994, pp. 291-294).

What I am trying to show (in the end) is that not only our description and characterization of abstraction fits perfectly with the method of constructing LF as described by Carnap, but also some of the conclusions already made by Carnap might be easier to understand by looking at them from the angle of abstraction. In the end, just for the sake of mentioning the elephant in the room, I will briefly talk about Quine's objection to Carnap's philosophy, which might be understood as a topic that is not directly relevant to the topic of our discussion at large.

1.1 Historical Background

In this section I will gloss over some historical background in order to elucidate why the notion of language is such a central point in Carnap's philosophy and why LD (the linguistic doctrine of logical truth) becomes such an important doctrine among the neo-empiricists of the Vienna Circle.

In the following quotes, Carnap speaks about his general view on the world-language relationship and his view on the specific position of logic with regard to language. He speaks of both in connection with the ideas of two important figures, Wittgenstein and Neurath[7]:

> For me personally, Wittgenstein was perhaps the philosopher who, besides Russell and Frege, had the greatest influence on my thinking. The most important insight I gained from his work was the conception that the truth of logical statements is based only on their logical structure and on the meaning of the terms. (Schilpp, 1963, p. 24)
>
> We [in Vienna Circle] read in Wittgenstein's book that certain things show themselves but cannot be said; for example the logical structure of sentences and the relation between the language and the world. In opposition to this view, first tentatively, then more and more clearly, our conception developed that it is possible to talk meaningfully about language and about the relation between a sentence and the fact described. Neurath emphasized these facts in order to reject the view that there is something "higher", something mysterious, "spiritual", in language, a view which was prominent in German philosophy. I agreed with him, but pointed out that only the structural pattern, not the physical properties of the ink marks, were relevant for the function of language. (Schilpp, 1963, p. 28)

According to Carnap (Schilpp, 1963, p. 52) it is not possible for Wittgenstein to talk about language in isolation. It is also apparent from the last couple of verses of *Tractatus* that speaking

[7] It is well-known among Carnap scholars that Carnap's thoughts, in general, were influenced by many figures such as Frege, Hilbert, Russell, Tarski, Gödel, and others. Yet, the ideas of Wittgenstein and Neurath were more directly concerned with the concept of language than Carnap's more significant influences.

of propositions and rules of language in total separation from where they are being employed is meaningless.

This instrumental role of language, which brings about logic as a representative system, seems to regard language with a different ontological status than that of the rest of the actual world. This seems to be the problem with this view. On this view, language is something by which we, for instance, explain the world. Language is one thing and the world is another. Language is a tool we use to satisfy a purpose. The question, then, is whether or not the two are ontologically distinct. The problem gets worse when we start thinking *about* logic. On one hand, we start off our search for logic and get to the "essence of language" from accidental linguistic statements. Therefore, we have to acknowledge some sort of dependency between logic and language. On the other hand, we have to say logic or, as Wittgenstein put it, "the rules of possibilities", is totally independent of all language forms. Accordingly, one has to accept a very mysterious status for logic and language with respect to the rest of the world.

Carnap departs from Wittgenstein at exactly this point; unlike Wittgenstein, talking about language in isolation is possible for Carnap because language itself is a worldly object. In agreement with Neurath, along with other members of Vienna Circle, Carnap admits the possibility of speaking *about* language in isolation (Schilpp, 1963, p. 52). Unlike Wittgenstein, Neurath considers language as something within the world, not something that refers to the world from the outside (Schilpp, 1963, p. 28). This view of language is one of the most important turns in Carnap's philosophy (*Ibid.*). Language can still preserve its instrumental role, but now it is a tool that works within a system and not outside of it. To give an analogy, although we may consider red blood cells as instruments or tools for transporting oxygen across the body, yet they are still parts of the human body and have their own properties. The case is different when we consider instruments for constructing buildings, for example. They are tools that are no longer part of the building after its construction. Tools, in this latter sense, have an ontological status over and above the building (just like language and logic in Wittgenstein's view, which have a distinct status over and above the world). In the former case, red blood cells do not bear such a status. Similarly, we may still consider language as an instrument to talk about the world, but, at the same time, language itself is an object of the world that bears a special relationship to other objects.

According to Carnap (*Ibid.*), it was this idea that led him to consider what he later called the "logical syntax of language". Centrality of language also helped Carnap take more radical positions against traditional metaphysics, and adopt a more neutral attitude toward "the various philosophical forms of language", e.g., realism, idealism and the like (Schilpp, 1963, pp. 17-18 & 24). Carnap formulated this neutral attitude[8] in the form of a "principle of tolerance" in his "Logical Syntax of Language" (Carnap, 1937). Now, in settling the mentioned philosophical controversies such as the realist-nominalist debate (which was caused by the diverse use of language), our concerns are to first look at the syntactical properties of the various forms of language, and secondly, the "practical reasons for preferring one or the other form for given purposes" (Schilpp, 1963, p. 54). Construing philosophical problems as metalinguistic problems as opposed to linguistic ones is obvious when Carnap explains his major motivation for adopting the syntactic method:

[8] For a discussion on the neutral formulation see chapter three, section three.

> In our discussions in the Vienna Circle it had turned out that any attempt at formulating more precisely the philosophical problems in which we were interested ended up with problems of the logical analysis of language. Since in our view the issue in philosophical problems concerned the language, not the world, these problems should be formulated, not in the object language, but in the meta-language. (Schilpp, 1963, p. 54)

It might be fair to say that the idea of considering language as an object within the world and, hence, the possibility of talking about language in isolation, are the main motives in formulating LD; logical truths are true by linguistic convention. The adoption of this doctrine was, of course, an established point of consensus among Carnap and other members of Vienna Circle, although Carnap was not completely in agreement with this formulation of the doctrine (Schilpp, 1963, p. 914). The acceptance of the doctrine immediately implies a linguistic-based and conventional[9] nature of the logical structure that can be revealed via a complete analysis of language. Any theory that provides descriptions of the steps involved in completing such an analysis, as well as explaining all properties, features, and rules involved in taking these steps eventually (and inevitably), proposes or describes the characterizations of a framework according to which one makes assertions. Carnap's attempt to propose such a theory is the subject matter of the following section.

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[9] Carnap himself would rather not use the term "convention" or "conventional" for fear of giving the impression that there is too much liberty and arbitrariness involved in the process of identifying logical truths. Since this concept becomes clearer in the following section, I use the term as-is and skip the controversy about "convention" or "conventional". In Carnap's own words:

> Among the various formulations […] there are some which today I would no longer regard as psychologically helpful and would therefore avoid. One of them is the characterization of logical truth as based on "linguistic fiat" or "linguistic conventions". […] The term "linguistic convention" is usually understood in the sense of a more or less arbitrary decision concerning language, such as the choice of either centimeter or inch as a unit of length. (Schilpp, 1963, pp. 914-915)

A philosophical linguistic analysis, in general, is concerned with methods of clarifying concepts behind the terms of the ordinary language with respect to the structures in which the terms are being used; one may simply call the methods of this sort an "explication". The notion of a linguistic framework, evidently, is not only of great importance in his linguistic analysis but also is directly related to the subject matter of Carnap's overall philosophy.

1.2 Linguistic Framework and its Components

So far, we may summarize the implications of adopting LD as follows:

1. Language has a (logical) structure.
2. In the very *first* attempt of investigating such a structure there has to be a language in place (as an object).
3. Conventionality is part and parcel of such an investigation.

The main question now is how we can investigate the mentioned structure of the language. How does logic, or science for that matter, emerge? How is it differentiated from the rest of ordinary language? Carnap provides us with a detailed answer (Carnap, 1939), which I will summarize in this section. For Carnap, language is inclusive of a vast array of "communicative signs" (Carnap, 1994, pp. 291-294). The major purpose of Carnap's project, from now on, is to show the ways in which a so-called "scientific language" differs from our ordinary use of language. To put it differently: by what mechanism does a system of scientific statements (in general, science) start to emerge from the context of ordinary statements? It was the work of people like W. C. Morris (e.g., "Foundations of the Theory of Signs") that helped Carnap develop a complete theory of language (Carnap, 1994, pp. 291-294), so that it is inclusive of the entire spectrum of human assertions. Therefore, a complete theory of language should cover the whole range; from the assertions in ordinary discourses to mathematical and logical assertions. Carnap recognizes three major components of a comprehensive linguistic survey, all interrelated albeit different in their subject matter and focus. He frequently refers to these three parts in nearly all of his works after 1939 (Carnap, 1939, 1942, 1959, 1994). These three components are: Pragmatics, Semantics, and Syntax[10].

> Therefore, an analysis of theoretical procedures in science must concern itself with language and its applications. […] we shall outline an analysis of language and explain the chief factors involved. Three points of view will be distinguished, and accordingly three disciplines applying them, called pragmatics, semantics, and syntax. […] The complete theory of language has to study all these three components. (Carnap, 1939, pp. 3-4)

These three components have different focuses of attention and, consequently, they lead to different types of research or activity. In pragmatics, the focus is on the speaker and how she produces signs. In semantics, what is under investigation is the relation of designation regardless of the speaker (where we expand or limit the meaning of a term or phrase in our ordinary use of language). Syntax is where we begin to investigate the logical structure of language regardless of the designation relation. Carnap considers language systems as hierarchical systems. They consist of three parts; respectively, from the bottom to the top, these parts are pragmatics,

[10] I should note here that the terms "pragmatics", "semantics", and "syntax" have been originally borrowed from the terminology of linguistics, but for Carnap the scope of these terms is broader; they don't have the exact same referent as they do in linguistics (linguistics, as a pragmatic scientific discipline). One should not confuse the scientific, or rather, the more pragmatic uses of the terms, which bear a descriptive nature, with the more theoretical applications of them. The latter is the manner in which Carnap intend to employ them. Linguistics, at the scientific level, is concerned with the study of actualities about an actual language like English. Therefore, semantics and syntax are to be considered as descriptive semantics and descriptive syntax. They eventually yield an English dictionary or an English grammar. At this level of abstraction, we are still at the level of pragmatics (in the Carnapian use of this term). That is why Carnap sometimes has to emphasize the distinction by using expressions such as "logical syntax" or "pure semantics" as opposed to "descriptive syntax" and "descriptive semantics" in order to avoid the confusion (see Carnap, 1942, p. 240).

semantics, and syntax. One should keep in mind that the world under investigation in pragmatics is strictly the actual world (see below). Thus, it consists of a finite number of objects. One other important point in the subsequent sections, which deals with the methods of constructing a framework for language, is that the language in question is considered to be an instance of actual historical natural languages. Later on, when we talk about the second method of construction, we will consider this topic in light of artificial languages as well.

1.2.1 Pragmatics

According to Carnap, the central subject matter in pragmatics (the first part of our investigation) is the speaker of the language (Carnap, 1994). In this part, the subject of study is the action, state, and environment of the person who speaks, hears, or writes the expressions of the language. The method that one may employ in this field is entirely empirical (Carnap, 1939, p. 6). "The study of the activities of observation in their relation to observation sentences belongs to pragmatics" (Carnap, 1994).

In pragmatics, speakers of the language generate signs for objects, events, relations, properties, etc., in order to communicate inside the language community, understand actual events, construct theories about the world, etc. Carnap considers problems of a factual and empirical nature, which deal with gaining and communicating knowledge, as problems that belong to pragmatics (Carnap, 1942, p. 250). These problems have to do with the speaker's activities of perception, observation, comparison, registration, confirmation, etc., as far as they lead to (or refer to) knowledge formulated in a language (*Ibid.*, p. 245). Carnap is explicit that "pragmatics is the basis of all linguistics" (*Ibid.*, p. 13). The descriptive nature of the pragmatic concepts is what distinguishes them from other concepts, which are of a more theoretical nature.

If, for example, we consider naming as one of the primary activities of the speaker at the pragmatic level, we should also consider the descriptive characteristic of this process that is specific to pragmatics, and which might be subjected to change at the higher levels. Naming, at this stage, is primarily of an indexical or ostensive nature (or simply observational), and the truths regarding linguistic phrases of these sorts are to be considered as special kinds of truth called "factual truths" (*F*-truth). This means it has to be established via observation, empirical factors, and immediate confirmation of the language community. As mentioned by Carnap, pragmatics is where we test our scientific theories about the actual world or where we start to make new ones (Carnap, 1994). According to Carnap, the central subject matter of pragmatics is the speaker of the language. In pragmatics the subject of study is the action, state, and environment of the person who speaks, hears, or writes the expressions of the language, and the method that one may employ in this field is entirely empirical (Carnap, 1939, pp. 4-9).

In pragmatics, we study methods of testing hypotheses and theories by deriving predictions from them in the form of "observation sentences", and then comparing these predicted results with new observation sentences: "The outcome of such a procedure of testing an hypothesis is either a confirmation or an infirmation of that hypothesis, or, rather, either an increase or a decrease of its *degree of confirmation*." (Carnap, 1994).

Due to the immediate relation between signs and objects, in pragmatics, the speaker (or, in specific cases, the scientist) essentially maintain one criterion for making assertions with regard to the truth, which is the criterion of accordance with observation. One could imagine that a complete research in pragmatics will provide the speaker a finite number of signs (names for objects, relations, and/or other forms of expressions) that supposedly almost mirror the actual phenomena within which the language community exists and/or intends to talk about. On the basis of pragmatic investigations, the meaning of one notion, at least, becomes clear to the speaker, and that is the meaning of "being the case". As it will be explained below, this immediate speaker-object relationship and the criterion of accordance with observation are what would be missing in the next level, once the speaker gets ready to close her eyes, and starts to speculate about the world regardless of "being the case" or "not being the case" by assuming that everything exists in the same fashion that has been observed and described the last time.

In general, Carnap considers pragmatics as the realm in which we form explicanda. Later on, in pure semantics, we are to provide explicata for them (Carnap, 1955a, p. 34). Therefore, the construction of the meaning or *intension* of the terms should start at the pragmatic level. The following is an example.

The explicandum "belief" is considered to be the relationship *T*, between a person and a sentence (not a proposition); because the relationship *B*, between a person and a proposition is nonpragmatical and "characterizes a state of a person not necessarily involving language" (Carnap, 1955b). A sentence of the form

$$B\ (X, t, p)$$

would say that the person X at the time t believes that p. On the other hand, a sentence of the form

$$T(X, t, S, L)$$

would say that the person X at the time t takes the sentence S of the language L to be true (consciously or not). "Now the pragmatical concept of *intension* serves as a connecting link between B and T. Let a sentence of the form

$$Int(p, S, L, X, t)$$

say that the proposition p is the intension of the sentence S in the language L for X at t" (Carnap, 1955b).

Once a natural language becomes actualized or activated at the pragmatic level, we may disregard the speaker-world relationship, and go up to the semantics where the designation relationship is our central focus. "If we abstract from the user of the language and analyze only the expressions and their designata, we are in the field of semantics" (Carnap, 1942, p. 9).

1.2.2 Semantics

In semantics we disregard the speaker of the language and we will only consider the relation of designation that is the relation between a term and its "designatum". Here is where we assign names, properties, relations, etc. to objects, and indirectly determine the truth conditions of the sentences. The more precise the rules we set up for designation, the more accurate the results (or way of speaking). This accuracy, in turn, leads to less controversy in discourses within the language community. Although we ourselves set up the rules for deciding what is right or wrong according to the system (since we are making the conventions), the rules are not arbitrary. They are bound to the empirical node mentioned above. This is explicitly clear from the following quotation where Carnap is talking about an imaginary language "B" which belongs to the world of facts, and our own established semantics for this language, "B-S", and which has all and only the properties that we have constructed by our rules.

> Nevertheless, we construct B-S *not arbitrarily* but with regard to the facts about B. Then we may make the empirical statement that the language B is to a certain degree in accordance with the system B-S. The previously mentioned *pragmatical facts* are *the basis* […] of some of the rules to be given later. (Carnap, 1939, p. 7) (emphasis mine)

Since the main goal of setting semantic rules is to achieve the highest degree of accordance with facts, we are bound to this accordance, and preferring one semantic system over another is not a mere matter of terminological choice but rather a matter of degree of confirmation with respect to the facts. Here is, in semantics, where we define synonymy and where we form our theories of meaning.

Semantics would ideally give us an "interpretation" of the language by which we would be able to understand expressions of the language. According to Carnap (Carnap, 1939, p. 11), understanding a language, a sign, an expression, or a sentence are all due to the semantic rules of the language system.

Let us not forget that we are not entirely unconcerned with empirical observations (at least as far as it concerns descriptive semantics). But at a certain point when setting up semantic rules of designation, we are no longer concerned with non-linguistic objects. Once a natural language becomes actualized or activated at the pragmatic level, we may disregard the speaker-world relationship, and go up to the semantics where the designation relationship is at the center of attention. Here, naming, for example, has a referential characteristic as opposed to an observational or ostensive characteristic, which it has at the level of pragmatics. For example, in pragmatics, the speaker may point to the snow while verbalizing the word “snow” or make the claim that “the snow is white”, which may be followed by the immediate confirmation of the language community, whereas, in semantics, the utilization of the word “snow” only rests on the presumption that there is an object called “snow”, i.e., the referent of the word is assumed regardless of any observational consideration regarding its actual existence. Semantics, according to Carnap, is the lowest level of abstraction. Semantics may begin by simply switching our observational concern (closing our eyes) from the whole (already built up) vocabulary at pragmatics to referential concerns about the occurrences of signs. This switch of attention means nothing more than disregarding empirical factors involved in observation and just focusing on the designation relation between the signs and their designata regardless of their actual existence. At this point we are ready to study the inherited language, built up at pragmatics, as an object by itself; we may call it the “object language”. So, the mark for entering into the realm of abstraction is just switching our attention from observation to designation by presupposing the existence of the involved objects (events, relations, etc.). Just as we disregarded empirical factors in observation to focus on the designation relation, we may continue disregarding the factual content of the statements even further in order to ascend to higher abstract levels. This is done via a process normally called “purification”. In semantics, the sentence “the apple is red”, for instance, is just a claim, regardless of its factual truth. We may further purify this claim from its dependency to “the apple” by using the variable x as a placeholder for any object-names such as “apple”. This purification, therefore, enables the sentence "x is red" to talk about red objects other than just the apple. In the same way, we may purify the same sentence from its dependency to the predicate "red" by using φ as a placeholder for any other predicates of the sort. Again, in the same way, "the apple is φ" could talk about properties of the apple other than just being red. Or, we may purify the whole sentence from both the object and the predicate at the same time by using the letter "*P*" as a placeholder for "x is φ" to talk about any sentence of this form. Each purification procedure would bring us to a different level of abstraction.

It is fairly obvious that emptying observational expressions from their factual content simultaneously requires the introduction of new terms (new conventions), such as x, φ, and P, along with new conventional rules regarding their use. This means that parallel to studying our object language we invent a new "meta-language"[11] at the same time that may have its own properties (more on this topic later). Following each line of purification, depending on our research interests, we will end up in a different branch of logic (modal logic, temporal logic, relevance logic, etc.[12]). Regardless of the type of logic, they all result in a pure semantics, or the set of rules for truth-value assignments, and syntax, or the set of rules for constructing valid logical structures. In propositional logic, for example, in which the goal is to study the relationship among atomic declarative sentences, the meaning of logical connectives is introduced by conventional truth-value assignments to each and every possible situation, which may occur among the propositions relative to one another. Here, too, in order to introduce logical connectives, "object language" connectives will undergo some sort of purification. This means

[11] This is different from an "artificial language", which is the result of a change in syntax. See section 1.4.

[12] These examples are given in light of modern developments in logic. This is not to claim that Carnap was aware of them, at least not to the present extent of the matter.

that logical connectives would be considered as pure relationships (or operations) and we would disregard the actual forms they may adopt in the object language. Now, we are at the level that is called "pure semantics" in which both object language sentences and their connectives have been purified. The truth about atomic and molecular sentences at pure semantics (*L*-truth) can solely be investigated via the rules of our conventional truth-value assignments regarding the logical connectives.

To determine a truth-value for statements such as "there is an apple in the fridge" the application of semantic rules is absolutely necessary, i.e., (in this example) the truth-value of the statement depends on the meaning of "apple", "fridge", the role of "in", "an", etc. On the other hand, in statements such as "there is an apple in the fridge or not" we may replace "apple" by any other object-name, as well as "fridge", and the truth value of the sentence will remain the same. Thus, in this kind of statement the truth-value of the sentence is independent of normal semantic rules of designation. In the first case, the sentence might be true or false based on an observation (in the case of being true, for example, it is called factually true or *F*-true). But, in the second case, if the sentence is true it is necessarily *F*-true regardless of any observation and that is why we call it logically true (or *L*-true). Although "true", "false", "*F*-true", "*F*-false", "*L*-true" and "*L*-false" are all still semantic terms, if we symbolize the form of such sentences of our object-language in a meta-language by using *P* as a meta-variable standing in for any sentences, and $\sim P$ standing in for the negative form of the same sentence, then we may establish a general form "P or $\sim P$", and regard it as the general form of its instances, which are *L*-true. Thus, we regard any sentence that comes in this form as true regardless of their factual content. As you may notice, in this process of formalization there is a special kind of semantics involved. For example, we said *P* stands for any sentence, and $\sim P$ stands for the negative form of the same sentence, and we assigned a truth value (*L*-true) to the combination of the two with "or". This special semantics in which designata of the signs (sentences, names, connectives and the like) are not outside of the language system is what Carnap calls "*L*-semantics" (short for logical semantics) or "pure semantics" (Carnap, 1939).

Investigating the rules that would allow us to make such truth-value assignments in *L*-semantics (assigning *L*-true or *L*-false) is the goal of the final part of our language analysis, i.e., the syntax. Now we have passed the skin (pragmatics) and the muscles (semantics) and have reached the skeleton of the language (syntax).

1.2.3 Syntax

In syntax, the relation of designation will be completely disregarded. Here, by formalizing in a meta-language we determine and set up the rules according to which we may assign semantic terms such as *L*-true, *L*-false, and the like, to sentences. Syntactical rules would serve two purposes: constructing proofs and making derivations[13]. Carnap defines *C*-true sentences (*C* for calculus) as "the sentences to which the proofs lead" (Carnap, 1939, p. 17). Logic is a discipline that takes care of this purpose, and Carnap sees it as a system that has been established and developed by thinkers like Aristotle and Euclid, grown up in the hands of philosophers like Leibniz and Boole, and became more comprehensive by mathematicians and philosophers like

[13] Proofs could be construed as a special sub-class of derivations, namely ones that proceeded from truths, whereas derivations are any move in the proof system, which might proceed from false premises. The conclusion of a proof is a truth. The conclusion of a derivation is indeterminate.

Schroeder, Frege, Peano, Whitehead, and Russell, and benefitted a good deal from Hilbert's axiomatic method (Carnap, 1939, p. 17)[14].

At the syntactic level our concerns are no longer the objects but the validity of the structure (or sequentiality) of the objects (or signs). "The syntax of a language, or of any other calculus, is concerned, in general, with the structures of possible serial orders (of a definite kind) of any elements whatsoever" (Carnap, 1937, p. 6). In propositional logic, we call these structures "rules of inference". With modus ponens, for example, successive true appearances of a material conditional and its antecedent guarantee the true appearance of its consequent. It is obvious that, in the search for valid structures, the truth of the objects should be presumed so that the truth of the last sentence in the sequence (the consequent, in this example) solely depends on the structure that has been taken to be valid. Here, again, we are facing yet another kind of truth (more precisely, another explicatum of truth) that is called *C*-truth (*C* for calculus). The truth of *L*-true sentences solely relies on the semantic rules (or truth-value assignments), whereas the truth of the *C*-true sentences solely relies on the rules of inference (or valid structures). In the case of true semantic interpretation of *C* (see Carnap, 1939, p. 21, for the conditions of true interpretation), all *C*-true sentences become *L*-true sentences but not vice versa, in the same way that the all interpretations of all *L*-true sentences become *F*-true. One important characteristic of the syntactic level is that the structures in question have no factual content at all; they are purely conventional.

[14] I should notify that I intentionally limited the discussion here to the first-order propositional logic to make my point. One of the major objectives of this chapter is to give a general schematic view of Carnap's LF in order to provide a basis for further discussion on the same topic. Consequently, I will avoid getting into more detailed and technical discussions about analyticity or syntactical rules.

As in the case of semantics, in the case of syntax, too, Carnap distinguishes descriptive syntax from pure syntax. "Descriptive syntax is related to pure syntax as physical geometry to pure mathematical geometry; it is concerned with the syntactical properties and relations of empirically given expressions (for example, with the sentences of a particular book)" (Carnap, 1937, p. 7). Therefore, pure syntax inherits at least some of the properties of the descriptive syntax (if we consider a bottom-up move). Or, pure syntax should be respectful (or loyal) to some descriptive properties by making it possible to provide a useful interpretation (if we consider a top-down move). The relation between descriptive and pure syntax can be defined by introducing "correlative definitions" by means of which "the kinds of objects corresponding to the different kinds of syntactical elements are determined (for instance, material bodies consisting of printers' ink of the form' ∨ ' shall serve as disjunction symbols)" (*Ibid.*). For instance, sentences like "the second and forth sentences of a particular series of sentences (or a passage) contradict one another" or "the third sentence is not syntactically correct (let's say according to English grammar)", are sentences of descriptive syntax. But, sentences like "the sequence $\varphi \supset \psi$ has a general form of *Var*(*x*) *Con*(*x'*) *Var*(*x''*)", where *Var* stands for variable and *Con* for constant, belong to pure syntax. At the same time *Var*(*a*) *Con*(*a'*) *Var*(*a''*) still have a descriptive nature. "Pure syntax is thus wholly analytic, and is nothing more than *combinatorial analysis*, or, in other words, the geometry of finite, discrete, serial structures of a particular kind" (*Ibid.*).

> When we say that pure syntax is concerned with the forms of sentences, this 'concerned with' is intended in the figurative sense. An analytic sentence is not actually 'concerned with' anything, in the way that an empirical sentence is; for the analytic sentence is without content. The figurative 'concerned with' is intended here in the same sense in which arithmetic is said to be concerned with numbers, or pure geometry to be concerned with geometrical constructions. (Carnap, 1937, p. 7)

As we saw, pure syntax is the level that completely disregards factual content, and so is maximally conventional. According to this schematic, abstraction could be construed as a bottom-up process of simultaneously disregarding factual content and becoming increasingly conventional. From this point of view, one could see, in general, how abstraction could be subjected to degradation and how it could be correlated with some sort of gradual disengagement process at each step. In order to go from a lower level of abstraction to a higher one, we would disregard a relationship, an object or a predicate of some sort, and make some presuppositions at each step. We also saw in this disengagement process that there is a voluntary element of choice or switch of attention involved (that can be justified pragmatically). This choice may be considered either positively, as to which relationship we want to preserve, or, negatively, as to which relationship we no longer want to be engaged with. One noteworthy observation to make in the picture that Carnap draws of abstraction is to note where the major steps of abstraction are taking place, i.e., from pragmatics to semantics and from semantics to syntax. In both cases, there is a single relationship that is being disregarded. Simultaneously, there are presuppositions to be made regarding the relationship on which we want to concentrate. For example, in the case of moving from pragmatics to semantics, the relationship we wanted to concentrate on was the designation relationship between the signs and their designata, and the relationship that we wanted to disregard was the speaker-world relationship or the relationship between the sign and the actual object; therefore, we presupposed the existence of all designata. In the next major shift in abstraction from semantics to syntax, we wanted to find valid structures regardless of the designation of their elements; therefore, we presupposed the semantical truth of those elements (i.e., we presuppose the designation relationship holds for all the elements).

In the abstraction model just described, we started the construction of our language system from pragmatics all the way to syntax. According to Carnap, as we will see in the next section, this is only one of the two possible ways of constructing a language system, which we may call a bottom-up method (or an abstractive method). The inverse top-down method (or interpretive method) is also possible, which will be explained in the following section.

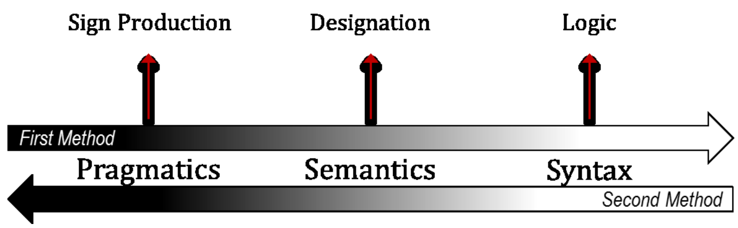

Figure 2 Components of a complete language analysis

1.3 LF and the two methods

Carnap acknowledges that the difference between these three parts is their level of abstraction.

> We distinguished three factors in the functioning of language: the activities of the speaking and listening persons, the designata, and the expressions of the language. We abstracted from the first factor and thereby came from pragmatics to semantics. Now we shall abstract from the second factor also and thus proceed from semantics to syntax. (Carnap, 1939, p. 16)

One may realize that what is interesting here is that Carnap, by establishing the ladder of gradual abstraction (i.e., the gradual loss of factual content), is indirectly suggesting the possibility of a systematic way for dealing with the concept of abstraction. Carnap is clear that if we are to construct a language for science we ought to give up absolute verifiability and consider "gradual confirmation” (Carnap, 1938). He recognizes two methods for constructing a language for science (or basically any sort of language):

> Let us suppose we are going to construct an empirical language for the whole of science, […] At which point in the system of terms shall we begin with the construction? At the one end of the system there are the elementary, concrete terms like 'blue' and 'hard', which can be applied on the basis of simple observations. On the other end there are the abstract terms as they occur in the most general laws of theoretical physics, e.g. ‘electric field’. There are now two possible ways open to us, each of them having certain advantages. (Carnap, 1938)

Before we get into the descriptions of these methods let’s once again consider LF in the following presentation, but this time with respect to the levels of abstraction:

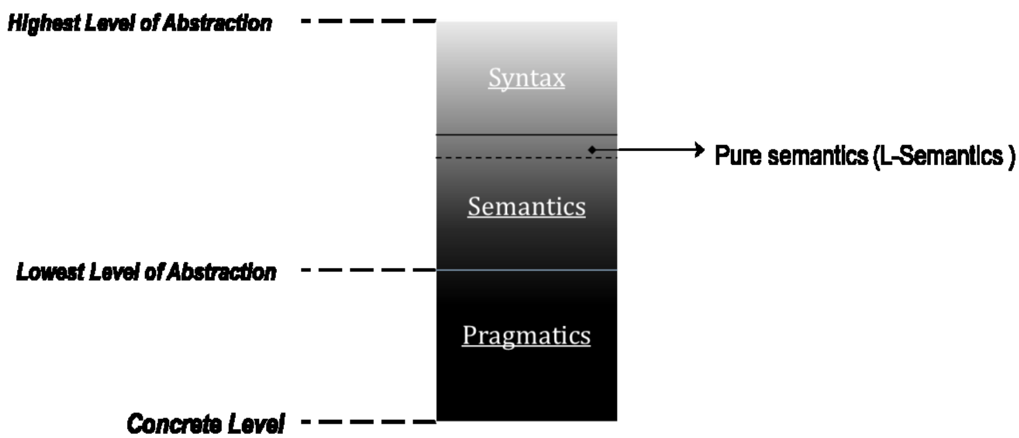

Figure 3 Levels of abstraction in a LF

One important point is that, in terms of the factual contents of the sentences, there is some sort of heterogeneity (or factual-conventional duality, if you wish) involved in constructing languages according to this model. That is, the statements in the middle of the factual-conventional spectrum are neither completely factual nor completely conventional. As we have noticed, sentences formed at the lowest level have maximum factual content, and as we go up the abstraction ladder, they lose factual content and become more and more conventional. Consider, for example, how the following set of sentences become more conventional as we go up the abstraction ladder. Looking at the following example gives us a sense of how the statements gradually lose their factual content.

- This is an apple. (Factual)
- The apple is red.
- Red is a color.
- Color is a concept.
- Concept is $F(x)$.
- $F(x)$ is P.
- P is F-determinate.
- P is F-determinate if and only if "$P \wedge \sim P$" is L-determinate.
- "$P \wedge \sim P$" is L-determinate if and only if "$P \vee \sim P$" is C-true. (Conventional)

As we may realize, the construction of a calculus upon which we consider $P \wedge \sim P$ as false (or more specifically, L-false) is purely conventional without any participating factual component. "Now consider the predicator $H \bullet \sim H$. No factual knowledge is needed for recognizing that this predicator cannot possibly be exemplified" (Carnap, 1956, p. 21). In the same way, deciding whether $P \vee \sim P$ is L-determinate (hence analytic) or L-indeterminate (hence synthetic) is entirely based upon constructor of the framework, and could be done regardless of any fact[15]. Carnap acknowledges the heterogeneity of LF with respect to the factual content in ESO as well as in

[15] Not that whether $P \vee \sim P$ is L-determinate or L-indeterminate is a semantical (pure semantic) decision and each choice would entail different set of syntactic rules.

other places (e.g., Carnap, 1936, 1965). Now we can easily see how we may continue losing factual content up to the syntactical level, where the realm of pure conventions begins.

1.3.1 The first method

In the first method, we start constructing our language system (LF) by taking elementary terms (such as "blue", "hot", "hard") as primitive terms and then introducing them to higher levels of abstraction. "If a suitable set of elementary terms is chosen as a basis, every other term of the language […] is either definable or at least reducible to them" (Carnap, 1938). The advantage of the first method, according to Carnap, is that "it allows a closer check-up with respect to the empirical character of the language of science. By beginning our construction at the bottom, we see more easily whether and how each term proposed for introduction is connected with possible observations" (*Ibid.*).

One of the points to which we should pay special attention to, again, is that in the first method of constructing a LF, we are not completely arbitrary precisely because we are empirically constrained. Not paying attention to this point has led to some confusion in the literature. For example, some philosophers, e.g., (Maddy, 2007, p. 86), hold the idea that making scientific theories is just a mere terminological choice or just a matter of language, for Carnap. As we saw in section 1.2.2, semantical rules cannot be chosen arbitrarily, and Carnap is clear that they are empirically constrained by factual observations in pragmatics. Since the same relationship that holds between pragmatics and semantics also holds between semantics and syntax (semantics is an abstraction of pragmatics and syntax is an abstraction of semantics), we may say that by the first method of construction, the entire LF is committed to factual observations, and therefore constructing a LF by the first method is not completely arbitrary. Carnap is fairly clear that, in the first method, pragmatic and empirical criteria can be regarded as "practical guides" (or constraints) in setting up rules or making conventions (Carnap, 1939, p. 6). So, in constructing a language system, our choices of rules for an already-interpreted language (a natural language) are not completely arbitrary. Nevertheless, "nobody doubts that the rules of a pure calculus, *without* regard to any interpretation, can be chosen arbitrarily" (*Ibid.*, p. 27) (emphasis mine).

In sections 11 and 12 of (Carnap, 1939), Carnap is quite clear that in the case of constructing a syntax (or a calculus) for an existing language, which is an instance of employing the first method, we are not completely free and we do bring some commitments to bear. Indeed, we are limited in "some essential respects", because the syntax must be constructed in such way that it gives us a true interpretation of the existing semantics. The only freedom one may have in this regard would be limited to minor choices in classifying the signs and formulating the rules[16]:

> If a semantical system S is given and a calculus C is to be constructed in accordance with S, we are bound in some respects and free in others. The rules of formation of C are given by S. And in the construction of the rules of transformation we are restricted by the condition that C must be such that S is a true interpretation of C […]. But this still leaves some range of choice. We may, for instance, decide that the class of C-true sentences is to be only a proper subclass of the class of L-true sentences, or that it is to coincide with that class or that it is to go beyond that class and comprehend some factual sentences, e.g., some physical laws. […] This choice, however, is not of essential importance, as it concerns more the form of presentation than the result. If we are concerned with a historically given language, the pragmatical description comes first, and then we may go by abstraction to semantics and to syntax. (*Ibid.*)

[16] If we need an example of the choices between different formulations (amongst others), e.g., for propositional logic, we may think of the choices between Łukasiewicz's system of notations or the notational system of Whitehead and Russell. Both cases, no matter how different they may be, are still committed to satisfying the main condition, which is to provide a true interpretation for the existing semantics.

Therefore, in the first method of construction we are not only limited to a true interpretation of the existing semantics, but also committed to the facts of the matter. Carnap also reminds us that the order of the methods is of essential importance because "if we have chosen some rules arbitrarily, we are no longer free in the choice of others" (*Ibid.*). Then, the first method has an essential priority compared to the second one.

1.3.2 The second method

Traditionally, being used to the application and rules of one sort of logic might make us prejudiced in favor of that logic; we may even go so far as to construe the system we are familiar with as "obvious". Carnap, on the other hand, sees the possible range of assertions as far more diverse and versatile:

> It is important to be aware of the conventional components in the construction of a language system. This view leads to an unprejudiced investigation of the various forms of new logical systems which differ more or less from the customary form (e.g., the intuitionist logic constructed by Brouwer and Heyting, the systems of logic of modalities as constructed by Lewis and others, the systems of plurivalued logic as constructed by Lukasiewicz and Tarski, etc.), and it encourages the construction of further new forms. (Carnap, 1939, p. 28)

The second method is when we take abstract terms of the highest levels of abstraction or syntax and introduce them to lower levels all the way to the elementary terms. "If a suitable set is chosen, here again every other term, down to the elementary ones, can be introduced. And here, it seems, explicit definitions will do." (Carnap, 1938). The advantage of this method is that "it represents the systematic procedure as it is applied in the most advanced fields of science, especially in physics" (*Ibid.*). If it is to be somewhere, here is precisely where creativity and language planning come to play an essential role.

When using the second method, we are basically free to use whatever calculus (set of syntactical rules) we wish to satisfy our purpose. One of our options is, of course, to stay with the same resulting calculus (let's say classical logic) of the first method and make our changes at lower levels to what Carnap calls "indeterminate statements" (Schilpp, 1963, p. 920). This might be the most common philosophical/scientific practice, and the result would be LFs sharing the same logic[17]. This fact, of course, does not rule out the other possibility of the adoption of totally different calculi (e.g., intuitionistic logic). If the readjustment[18] has to be done at highest levels, it will result in a different language. One should keep in mind that even in the case of adopting different calculi, our final interpreted language should ultimately be accountable to the empirical facts of the matter, but the choice of the adoption is only pragmatically, not principally, constrained. There is no logic in choosing logics; one should notice that, in the case of adopting different calculi, we are no longer in the same LF. In the case of changing the language from L_n to L_{n+1}, the concept of "being syntactic", for example, is totally different in each language. That is, "… is syntactic" in L_n is a different concept than "… is syntactic" in L_{n+1}; the same is true for "being analytic" (Schilpp, 1963, p. 920). Therefore, since the property of "being syntactic" (or "being analytic") is totally dependent on our choice of syntax (which follows no logic and is only justifiable pragmatically), then, the concept of "… is syntactic" is only decidable upon our purely arbitrary chosen calculus. "With respect to a calculus to be constructed there is only a question of expedience or fitness to purposes chosen, but not of correctness" (Carnap, 1939, p. 25).

[17] We may think of pure non-Euclidean geometries, which share the same logic as the Euclidean geometry, as an example of this.

[18] In the case of conflict with experience, Carnap distinguishes between two kinds of readjustments (in LF), namely between changing truth-value assignments to the "indeterminate statements" (i.e., statements whose truth value are not fixed by the rules of language, say by the postulates of logic, mathematics, and physics) and changing the language (Schilpp, 1963, pp. 920-921).

The second method of constructing a language system, then, is first to construct a calculus *C* and then a corresponding semantics *S* accordingly. And here is how Carnap describes this process:

> We begin again with a classification of signs and a system F of syntactical rules of formation, defining 'sentence in C' in a formal way. Then we set up the system C of syntactical rules of transformation, in other words, a formal definition of 'C-true' and 'C-implicate'. Since so far nothing has been determined concerning the single signs, we may choose these definitions, i.e., the rules of formation and of transformation, in any way we wish. […] Then we add to the un-interpreted calculus C an interpretation S. Its function is to determine truth conditions for the sentences of C and thereby to change them from formulas to propositions. […] Finally we establish the rules for the descriptive sign (*Ibid.*).

The relevance and effectiveness of our choice of *C* will finally be determined by the richness of the language it yields. Here is where, once again, empirical data will determine how rich and effective the language is for the purpose of communicating among the targeted community.

Now, the question of the conventionality of logic may become clearer. The question, as Carnap puts it (Carnap, 1939, p. 27), is as follows: are the rules on which logical deduction is based to be chosen at will, and consequently judged only with respect to convenience but not to correctness? Or, is there a distinction between objectively right and objectively wrong systems, so that in constructing a system of rules we are free only in relatively minor respects (as, e.g., the way of formulation) but bound in all essential respects? One may see, by now, that Carnap's answers to both questions are affirmative. On one hand, in the unobjectionable possibility of constructing a language system from a calculus *C* to its corresponding semantics *S* (the second method), we are free in choosing the rules of *C* and the choice is simply a matter of convenience. On the other hand, in constructing a language system from the point at which the "meaning" of logical signs are given before the rules of deduction are formulated (the first method), the statements might be considered objectively right or wrong on the basis of the presupposed "meaning" of the signs. Carnap summarizes his response to the question of conventionality of logic in the following passage:

> Logic or the rules of deduction (in our terminology, the syntactical rules of transformation) can be chosen arbitrarily and hence are conventional if they are taken as the basis of the construction of the language system and if the interpretation of the system is later superimposed. On the other hand, a system of logic is not a matter of choice, but either right or wrong, if an interpretation of the logical signs is given in advance. But even here, conventions are of fundamental importance; for the basis on which logic is constructed, namely, the interpretation of the logical signs (e.g., by a determination of truth conditions) can be freely chosen[19]. (Carnap, 1939, p. 28)

It is worth emphasizing again that, up to this point, it is fairly evident that the process of losing factual content is a gradual process that coincides with a corresponding gain in conventionality, and that this eventually leads to the pure conventionality of syntax. This point will be of special importance later on where we talk about analytic-synthetic distinctions.

1.4 Confirmation and Changes in LFs

The main question in this section is how do LFs differ from one another? When we are to talk about the difference between LFs, one should pay special attention to the essential differences they may have. According to what has been explained so far, the difference between LFs could be construed at two different levels: the difference could be at the syntactic (or abstractive) level or it could be at the semantic (or interpretive) level. When we are considering a syntactic

[19] Compare a two-valued logic with a many-valued logic, for example.

difference, then we are taking about adopting different logical systems (different syntaxes). Hence, one expects a dramatic change in the framework. In that case, we can no longer talk about the concepts of “right” or “wrong”, since they are internal concepts to each framework. On the other hand, keeping the syntax intact, we may talk about semantic differences between two LFs, and then we may talk about right or wrong *interpretations* (provided our explicandum is unique[20]).

If we decide to keep the syntax intact, then what is at stake might be the *F*-truth of the statements that are to be established by confirmation. We should keep in mind that Carnap does not see any fundamental difference between particular and universal sentences regarding confirmation:

> Thus, instead of verification, we may speak here of gradually increasing confirmation of the law. Now a little reflection will lead us to the result that there is no fundamental difference between a universal sentence and a particular sentence with regard to verifiability but only a difference in degree. (Carnap, 1936, p. 425)

In agreement with Reichenbach, Carnap sees every sentence as a probabilistic sentence subjected to gradual confirmation (Carnap, 1936, pp. 425-427); the higher the level of abstraction, the higher the degree of confirmation. For example, confirming the sentence “the apple in my lunch box is red” requires a lower frequency of supporting instances than “all apples are red”.

> The facts do not determine whether the use of a certain expression is right or wrong but only how often it occurs and how often it leads to the effect intended, and the like. A question of right or wrong must always refer to a system of rules. (Carnap, 1939, p. 6)

[20] In the case that explicandum is not unique we may have equally right, yet different, interpretations. According to Carnap, this is the case in dealing with the concept of probability: “There are two explicanda, both called 'probability': (1) logical or inductive probability (probability$_1$), (2) statistical probability (probability$_2$)”. (Carnap, 1973)

I do not intend to talk about Carnap's position on universals and particulars here; what I would like to shed light on is Carnap's avoidance of the terms "right" or "wrong", generally, in the context of these kinds of changes in LF. Although, using his own vocabulary, one should be allowed to use "*F*-true" (in the case of confirmation) and "*F*-false" (in the case of infirmation), the essential points here are two-fold: one is that in this kind of change, where the syntax is intact, the changes are to be implemented at the lower levels of abstraction, and what is at stake is the subject of confirmation and/or the confirmation method. The second point is the concept of gradual confirmation in accordance with the level of abstraction that may or may not lead to the change of the second kind in the LF.

We have to pay attention to the fact that, considering Carnap's LF, what we refer to as language is slightly different than the ordinary or traditional sense of the word "language". According to what we have seen so far, as long as LFs share the same syntax they are not to be considered as different languages but rather different ways of speaking. In this sense, we no longer refer to English and Persian as different languages, as long as we establish our arguments in both English and Persian according to the same set of rules (e.g., the rules of elementary logic). For Carnap, the same is true for different theories (expressed in the same language) using quantification over two sorts of variables, or only one to cover both ranges, as long as they follow the same logical rules:

> Thus our present acceptance of the two more explicit forms of translation is merely an introduction of two ways of speaking; it does by no mean imply the recognition of two separate kinds of entities-properties, on the one hand; classes, on the other. (Carnap, 1956, p. 17)

What makes a confirmation possible, in a LF, is the part of the LF that makes it possible to drive our predictions (and then test them against the facts). This part, of course, is the syntactical rules of the LF. As long as we keep the logical syntax of a LF intact, we may talk about which theory (or which way of speaking) is *F*-right/confirmed or *F*-wrong/infirmed. For, the general concept of wrong or right would be decidable only according to the same syntactical rules.

Changing the syntactic rules is, in principle, possible. In this case, what would the resulting LF look like? By changing syntactical rules, we are making a radical change in the logical fabric of the LF, and this is the very structure that holds everything together in a LF. The first things to lose as a result of this kind of change would be the concept of "right" or "wrong". "Now, the task is not to decide which of the different systems is "the right logic", but to examine their formal properties and the possibilities for their interpretation and application in science" (Carnap, 1939, p. 28). The only things left to decide are going to be pragmatic considerations such as simplicity, fruitfulness, and the like, assuming the new syntax could generate a new and fully interpreted language (an artificial language). Again, that it is only in the case of syntactical changes where we refer to different LFs as different languages; as mentioned earlier, in other cases we consider different LFs as different ways of speaking the same language.

To sum up, changing our LF in response to resolving a conflict with experience (or otherwise) can be done in two different ways: one in which the new LF is communicable to the old LF which shares the same logical fabric (and where the statements are sortable according to their degrees of confirmation); and the second in which the new LF is incommunicable to the old one since it does not share the same logical fabric.

1.5 Analyticity and Quine's objections[21]

Before getting into the more detailed discussion, I will present a general picture of how Quine and Carnap construe our belief system, and how they envision the changes in this system.

Quine's proposal: our belief system has a web-like structure that encompasses all our theories, including our theories of logic and mathematics that constitute the core of the web. The periphery of the web is more susceptible to change according to actual facts than the core is. Any changes to this system ought to be initiated from outside of the web even if the readjustments require some changes at the core. Subsequently, any change in our mathematical or logical theories should be essentially in response to some change in our empirical data.

Carnap's proposal: all our beliefs about the world that are expressible in the form of communicable assertions are subjected to a structured system, which provides them meaning. This system has a hierarchical structure that is more susceptible to change, according to the facts of the matter at the bottom and is less susceptible at the top. Since the susceptibility of the structure is inversely proportional to the factual content of the statements, at some point in the structure, the statements have no factual content. The conflict between the system and the facts can be resolved in two ways: (1) implementing changes from the bottom to the top, or (2) making changes in the none-susceptible part of the hierarchy to the desired effect.

[21] It may seem that I have not been charitable enough to Quine in this chapter as I am citing Quine much less than Carnap. There are two reasons for this: first, since I am defending Carnap's position, it is obvious that I tend to clarify his position by citing his own works. The second reason is that the core of almost all of Quine's arguments against Carnap's points and positions seem to be similar and turn on proving the centrality and fundamentality of analytic-synthetic distinction. Since I tend to argue against this centrality and fundamentality, citing various versions of the same claim would be redundant.

So far, we have established the following:

1. The first method of construction is essentially dependent on and is bound to empirical observations (§1.3.1). Therefore, as far as the first method is concerned, LF is entirely committed to the facts and empirical considerations (§1.2.1). (reserving our minor conventional liberties in notations, classifications of the signs, and formulating the rules)
2. The possibility of using the second method with total disregard to the empirical data is an unobjectionable possibility (§1.3.2).
3. Carnap admits that resolving a conflict with experience may or may not require syntactical changes (§1.4, first quote).
4. Changing the LF is possible in two different ways (§1.5): by making new ways of speaking (keeping the syntax intact) or making new languages (changing the syntax).
5. The first method is practically prior to the second one.
6. Syntax is purely conventional as it stands at one end of a factual-conventional spectrum or assertion without any reference to the outside objects. (§1.4, pp7)

In his terminology, Carnap makes use of the terms "factual", "*L*-indeterminate", and "synthetic" to refer to the lower levels of abstraction in a LF. "A sentence is called *L*-determinate if it is either *L*-true or *L*-false; otherwise it is called *L*-indeterminate or *factual*." (Carnap, 1956, p. 7). Accordingly, the terms "theoretical", "*L*-determinate", "syntactic", and "analytic" are being used to refer to the higher levels of abstraction. It is fairly obvious that these terms are intended to use as directional guides. The terms "synthetic" or "analytic" should be considered as indications of a place in a hierarchy, and not a property of an object. To say "all LFs have synthetic statements and analytic ones" is like saying "all geographical regions have an east part and a west part"; no one objects to the east-west distinction, and, for the same reason, the analytic-synthetic distinction is not objectionable, if one considers it this way. Quine, according to the evidence given below, clearly does not share the idea that the terms "synthetic", "factual", "analytic", and "theoretical" are supposed to be considered as relative terms pointing to some location rather than absolute ones pointing to some objects. Quine's confusion is understandable because it is easy to see how a person's view would have been considered dogmatic and nonsensical if the person thinks of the east-west distinction as an absolute and fundamental one when distinguishing western provinces from eastern ones, for instance.

The analytic-synthetic distinction (henceforth ASD) is by no means an absolute distinction for Carnap for the following reasons: first, the ASD is a distinction that depends solely upon our decision on where we separate semantics from syntax (simply on our choice of logic). Carnap is fairly clear about this, as I noted earlier. Considering "$P \vee \sim P$" as an L-determinate sentence (or not) is principally based upon our decision, and what to do with the interpretations of P. It is not the case that the actual world (under investigation in pragmatics) dictates and forces us to consider "$P \vee \sim P$" as an L-determinate sentence, no matter how this principle is inspired by the actual world phenomena. Second, if the ASD was fundamental for Carnap, one could not see any inter-changeability between analytic to synthetic and vice versa. However, in the following letter to Quine, Carnap clearly acknowledges the possibility of such a change, from "being analytic" to "being synthetic" and vice versa:

> The difference between analytic and synthetic is a difference *internal* to two kinds of statements *inside* a given language structure; it has nothing to do with the transition from one language to another. "Analytic" means rather much the same as true in virtue of meaning. Since in changing the logical structure of language everything can be changed, even the meaning assigned to the '.' sign, naturally the same sentence (i.e., the same sequence of words or symbols) can be *analytic in one system and synthetic in another*, which replaces the first at some time (Creath, 1991, p. 431) (emphasis mine).

In the previous sections you may have noticed that, in introducing and characterizing a LF, we did not make any reference to the ASD, for we did not have to. We saw that, by accepting LD, a LF becomes immediate and that there are good reasons for adopting LD. Then, as Carnap mentions in the above quotation, the ASD becomes an internal difference directly decidable upon the set of rules we prefer to take as our set of syntactical rules. Quine, on the other hand, apparently does see this the other way around. Quine holds the idea that the ASD is a fundamental and absolute distinction for Carnap, and without which neither LF, nor the external-internal distinction, nor other terms such as “artificial language” or “meaning postulates”, and the like, would be possible to use:

> Carnap has recognized that he is able to preserve a double standard for ontological questions and scientific hypotheses only by assuming an *absolute* distinction between the analytic and the synthetic; and I need not say again that this is a distinction which I reject. (Quine, 1951b, p. 43) (emphasis mine)

> Modern empiricism has been conditioned in large part by two dogmas. One is a belief in some *fundamental* cleavage between truths which are analytic, or grounded in meanings independently of matters of fact, and truth which are synthetic, or grounded in fact. (*Ibid.*, p. 20) (emphasis mine)

In the following quotes, it is even more apparent that Quine takes the ASD as a dogmatic belief that stems from an unnecessary (and perhaps wrong) ontological difference between the two. For him, the ASD refers to a differentiation among objects and entities rather than relative terms in classification:

> One conspicuous consequence of Carnap's belief in this dichotomy may be seen in his attitude toward philosophical issues, e.g. as to what there is. It is only by assuming the cleavage between analytic and synthetic truths that he is able e.g. to declare the problem of universals to be a matter not of theory but of linguistic decision. (Quine, 1960)

> Now to determine what entities a given theory presupposes is one thing, and to determine what entities a theory should be allowed to presuppose, what entities there really are, is another. It is especially in the latter connection that Carnap urges the dichotomy which I said I would talk about. (Quine, 1951b)

Quine also sees Carnap's external-internal distinction regarding existential questions as on a par with, or rather, as based upon the ASD. Quine holds that both distinctions would disappear by our trivial choice of the types of variables involved in our scientific theories:

> No more than the distinction between analytic and synthetic is needed in support of Carnap's doctrine that the statements commonly thought of as ontological, viz. statements such as 'There are physical objects,' 'There are classes,' 'There are numbers,' are analytic or contradictory given the language. No more than the distinction between analytic and synthetic is needed in support of his doctrine that the statements commonly thought of as ontological are proper matters of contention only in the form of linguistic proposals. (Quine, 1951a, p. 71)

Quine fails to acknowledge what we explained above concerning the gradual loss of factual content as we move toward more general laws. Because he thinks of the ASD as such a profound and absolute distinction, everything in Carnap's model seems to fall into some sort of black-or-white schema. For Carnap, on Quine's account, statements are either analytic or synthetic; there is no middle ground. And, as we saw above, that is not the case for Carnap at all:

> Whether the statement that there are physical objects and the statement that there are black swans should be put on the same side of [Carnap's] dichotomy, or on opposite sides, comes to depend on the rather trivial consideration of whether we use one style of variables or two for physical objects and classes. (Quine, 1951a, p. 69)

In §1.4 we saw that Carnap already (in 1949) admits of the possibility of choosing one or two types of variables (see §1.4, the third quote, Carnap, 1956, p. 17), and we saw that Carnap refers to these choices as two different ways of speaking of the same language. It should be clear that

Quine is missing Carnap's main point. It is true that we can change our quantification variables, but in both cases, we still keep the syntax intact. Still, this is really not the crucial point. Quine goes on to construe Carnap's external-internal questions as category-subclass questions:

> The external questions are the category questions conceived as propounded before the adoption of a given language; and they are, Carnap holds, properly to be construed as questions of the desirability of a given language form. The internal questions comprise the subclass questions and, in addition, the category questions when these are construed as treated within an adopted language as questions having trivially analytic or contradictory answers. (Quine, 1951a, p. 69)

According to our explanations so far, we may agree with Alspector-Kelly (2001) when he says that "Quine's interpretation has Carnap claiming that a sentence turns analytic when the sortal's scope widens far enough for it to count as a universal word. But Quine was wrong" (*Ibid.*, p. 106). Nevertheless, Quine insists, again, that Carnap's external-internal distinction (as well as his other distinctions, such as ontological-empirical or theoretical-factual) is constructed upon the meaningless ASD. "If there is no proper distinction between analytic and synthetic, then no basis at all remains for the contrast which Carnap urges between ontological statements and empirical statements of existence" (Quine, 1951a, p. 71).

We discussed that all these distinctions can be predicated upon the conception of a LF, and that a LF is immediate after accepting LD. So, if we want to reject the distinction, all we have to do is to reject LD and LF. One simply cannot accept LD and reject LF. Emptiness of analytic truths from factual content at the syntactic level was very clear to Carnap as well as to other members of Vienna Circle. Carnap is even surprised why Quine finds it is necessary to elaborate on this point, given the prior agreements in Vienna:

> The main point of his [Quine's] criticism seems rather to be that the doctrine is "empty" and "without experimental meaning". With this remark I would certainly agree, and I am surprised that Quine deems it necessary to support this view by detailed arguments. In line with Wittgenstein's basic conception [LD], we agreed in Vienna that one of the main tasks of philosophy is clarification and explication. (Schilpp, 1963, p. 216)

The centrality and importance of LD, for Carnap, is even more evident where, in a reply to one of Quine's criticisms against his view on logical truth (Quine, 1960), Carnap hopes Quine would not regard LD as a false statement, because it is only then that Carnap is in a difficult situation:

> He [Quine] himself says soon afterwards: "I do not suggest that the linguistic doctrine is false". I presume that he wants to say that the doctrine is not false. (If so, I wish he had said so) He nowhere says that the doctrine is meaningless […]. (Schilpp, 1963, p. 916)

Carnap again returns to LD, where Quine regards elementary logic as "obvious", when he notes that: "Every truth of elementary logic is obvious (whatever this really means), or can be made so by some series of individually obvious steps." (Quine, 1960, p. 353). First, Carnap is not sure whether Quine is talking about factual obviousness or theoretical obviousness. In fact, we may never know what Quine meant because he does not distinguish the two:

> I shall sometimes be compelled to discuss Quine's views hypothetically, that is to say, on the basis of presumptions about the meanings of his formulations, because I have not been able to determine their meanings with sufficient clarity. […] I presume that he does not understand the word "obvious" here in the sense in which someone might say: "it is obvious that I have five fingers on my right hand", but rather in the sense in which the word is used in: "it is obvious that, if there is no righteous man in Sodom, then all men in Sodom are non-righteous". […] If Quine has this meaning in mind, we are in agreement. (Schilpp, 1963, p. 915)

Given that Quine is in agreement with the second sense of the word "obvious", and since Quine adds later on that LD "seems to imply nothing that is not already implied by the fact that elementary logic is obvious or can be resolved into obvious steps." (Quine, 1960, p. 355) Carnap shows that Quine's argument against his view on logical truth can actually be regarded as a proof of LD (*Ibid.*, p. 916):

1. Elementary logic is obvious.
2. LD "seems to imply nothing that is not already implied by the fact that elementary logic is obvious".
3. Whatever is implied by LD is implied by (1).

Hence, since LD is implied by LD:

4. LD is implied by (1).

Again, we can clearly see the importance of LD for Carnap. Thus, and in accordance with what I have explained so far, the assumption of LF comes to us naturally, and from there one may impose their theory about the LF's properties, functions, and the like. It seems obvious that we may only talk about all the different distinctions, such as factual-conventional, etc., once we already accept there is such a thing called LF. It might be quite clear by now that none of Quine's presented objections can be construed as objections against Carnap's main points.

1.6 Conclusion

As we saw, abstraction is the main participatory factor in shifting from pragmatics to semantics and then syntax. These shifts are correlated with loosing factual content of the statements such as syntactical statements bear no factual content. One may say the conceptual privilege that Carnap considers for the first method stems from his empirical stance toward abstraction. In short, I may summarize my points as follows:

1. If the first method of construction (or making changes) is the one and only possible method, then:
    a. LF, as a whole, is essentially committed to the facts of the matter, and
    b. There is only one direction (bottom-up) for change. And,
    c. In that case, the ASD is useless and redundant.
2. If the second method is possible, in addition to the first one, then:
    a. LF, as a whole, is only committed to the facts essentially in one direction and pragmatically in the other direction. And,
    b. There are two possible ways for changing LF. And,
    c. In that case, the ASD is a useful labelling convention.
3. The second method is possible.

Therefore, the ASD is a valid distinction, and it should be regarded as a relative distinction with respect to a LF.

As it may be seen, one may find the Carnapian LF's structure, built by the first method, quite similar to the Quineian "web of belief" (and, in my view, it is). As described, Carnapian LF's structure holds the same commitments to the facts as the Quineian model does. We saw that Carnap acknowledges the possibility of a bottom-up change in syntax, and he refers to such changes as "radical alterations". For Quine, as well, syntactical changes play the same essential and radical role, and that is why he puts them at the center of his web of belief to keep them safe from immediate changes (Quine & Ullian, 1978, p. 134). Quine takes syntactical rules to be on a par with other rules, and, when the time comes, they are not immune to change. The same can be said for Carnap. The only thing that Carnap points out, and that Quine dismisses, is that in the event that such a change has occurred, we are no longer speaking the same language. Consequently, the major difference between the two is that, for Quine, the only legitimate move for readjusting and modifying the structure of our language system is from the boundary to the core of the web (in the Quineian model) or from the bottom of the LF to the top (in the Carnapian model). For Carnap, on the other hand, the move in the other direction is equally legitimate. Quine's justification for taking this position, according to the above discussion, is the obviousness of elementary logic (whatever this might mean). On the other hand, the obviousness of elementary logic, for Carnap, is a theoretical obviousness and belongs to the most conventional part of our language. Therefore, if we admit our principal ability to change whatever we accept conventionally, then change at the syntactical level is both possible in principle and legitimate.

Another interesting conclusion that we may draw from our discussion is that, according to Carnap, coexisting theories in different languages (adopting radically different frameworks) is possible. But, for Quine, there is only one valid theory, i.e. "the theory". It is the theory that encompasses all our explanations about the world. This is the reason that I find Quine's position rather conservative and more akin to traditional ways of thinking.

Chapter 2. System of Objects versus of System of Concepts

“Abstraction” is a word that has been used by many philosophers throughout the history of philosophy despite the fact that no one has given a precise definition of the word. Perhaps the closest definition, so far, is what has been proposed by the neo-Fregeans in the logical form of “principle of abstraction”. What does this principle mean for philosophers? Or, what are the philosophical requirements and consequences of admitting it as a principle? Obviously, to answer these questions, one should consider the whole philosophical framework that would include presuppositions, assumptions, distinctions, and the like, upon which the framework is built or constructed, and within which a conception of abstraction is employed.

In a general sense, abstraction might be referred to as a process of arriving at concepts. Here we are going to consider a narrower but vague conception of abstraction, the way in which it is simply construed as an intellectual move from the understanding of some expressions to generating others in such a way that they could be presented in a hierarchy; with objects[22] (or object-names) constituting the level zero, the concepts of the first-order predication (using Frege’s terminology) constituting the first level, and the concepts of the second-order predication constituting the second level, and so on. Considering the just-described depiction of abstraction, our goal in this chapter is to see what kind of construction system would result from Carnap’s rejection of object-concept distinction. What would be the properties of such a system? And where would abstraction stand in such a system? We wish to show and confirm the following:

[22] Our reference to object does not necessarily include “abstract object”.

1. Abstraction is one of the main features in Carnap's overall constructional approach, considering that it is the major factor in constructing a linguistic framework, as we saw in the last chapter (the specific meaning of abstraction could account for the difference between Carnap's logicism and that of Frege).
2. The ways in which Carnap's abstraction is different from abstraction in the Fregean tradition, given Carnap's assumptions.

There is a lot of well-known information on Carnap's philosophical tendencies. On the one hand, we know him as a hard-core empiricist, whose project was to overthrow metaphysics entirely. On the other hand, we know he was fascinated by Frege's and Russell's developments in logic and was loyal to the main idea of logicism to the end. Considering these tendencies and by looking at the case of Carnap from the outside, it looks like the main question for Carnap is how to make a proper use of logical analysis (which seems, in nature, independent of the facts) as his main tool while basing everything on the facts; basically, where, and how, to cut-off the reality part in the sense that is useful for scientists yet maintain its relation to his project philosophically. Frege's strategy in this regard, which was to make an absolute distinction between objects and concepts, obviously was not leading to this desired effect; thus, Carnap had to think otherwise. In my belief, Carnap's strategy was to bridge "objects" and "concepts" (in the traditional sense) by abstraction in a specific way so that these terms become relativistic terms. Carnap would employ this strategy; first, by not considering the possibility of having an absolute analysis but by a quasi-analysis, due to the assumption of the essential inaccessibility of the entirety of reality of even simple things like tables and chairs. Second, assuming that our interaction with reality eventually produces some expressions; among them, terms for object and "elementary terms" as abstractions from reality. The abstraction can go further on, as we will see, to produce "hetero-psychological objects". Now all we have are some terms and expressions that have been given to us, and we analyze their relations to one another; therefore, we could, temporarily, cut our attention off from where and how they came from and focus on what they mean (or how they produce meaning), as if we were to take a sample of language (in a specific time and circumstance) as a physical object and bring it to an analytic lab to find out more about its structure, mechanism, form, etc. Thus, although philosophical, from now on, our investigation, primarily, has a linguistic tone; and, we should never forget that since our disagreements are probably more based on facts about language rather than on facts about other realities. In this way, Carnap can manage to cut-off the reality part and make a proper use of logical analysis while everything is essentially based on facts. If one adopts this method, the first thing that one should throw out is the absolute sense of universality. In a sense, Carnap's methodology is more radical than both Frege's and Russell's (as he says so himself (Carnap, 1967, p.8)), since it not only includes concepts but objects as well. With this introduction, we are now going to discuss the relevant parts of Carnap's philosophy to make our comparison based on the criteria mentioned below.

In the following, we should keep in mind the difference between language as the object that we theorize about, and language as the theoretical model by which we express our theory about the former. The former, of course, is an object of empirical inquiry, and can be approached differently by various theorists with different theoretical and pre-theoretical assumptions and goals. The latter, on the other hand, we may call "language-model", as it is called by Bielik (2012), and is "a theoretical entity that results from applying certain scientific procedures to language, such as *idealisation*, *abstraction*, *projection*, *explication* etc." (*Ibid.*, p. 326). Bielik (2012, pp. 235-) reminds us that the term "language-model" is meant to refer to those theoretical aspects of language that result from applying scientific methods; "Thus, 'language-model' is used in a much wider sense than 'model of language', which usually occurs in the set-theoretical sense"; and we will also keep that in mind when we speak of language with regard to abstraction in our discussion. Many theories of meaning do not explicitly state what is their underlying language-model (i.e., what theoretical aspects of language they analyze, what principles they take as constitutive for language, etc.). As noted by Bielik (2012, p. 328), many theories of meaning are often "silent about the relation of a model of meaning to the modelled meaning of expressions". According to Bielik, there are theories that assume that "the model of meaning is the same as the meaning", and others that assume that "the model of meaning represents fundamental aspects of meaning (of some category of expressions, e.g., definite descriptions)", and yet others that assume that "their model of meaning represents only some aspects of meaning"[23]. These could be considered as examples of our hypothesis on the theoretical assumptions of these theories. To show what may go wrong (or be missing) in appealing to models of language instead of model-language, Bielik explains what might be missing due to different theoretical assumptions about the language-model:

> […] If the model of a natural language assumes that language is a set of functions from expressions to meanings (for now, put aside the question what are meanings), the model of meaning has to ignore those aspects of meaning that are dependent on the language user, e.g., what she really meant by saying something or what kind of

[23] Bielik (2012, p. 328), in the footnote says this about the importance of the classification of theories of meaning:

> The classification of theories of meaning according to the character of the relation between the model of meaning and meaning (of natural language expressions) is in fact a nontrivial and difficult interpretative business. It rests on the evaluation of different implicit or explicit claims made by the theories. Nevertheless, some examples may be offered: I suppose the theorists such as Tichý (1986, 1988) or Duží, Jespersen, and Materna (2010) take it that the model of meaning elaborated by them is almost the same as meaning (in the semantic language-model). In developing his theory of meaning, Tichý proposes the concept of construction as a theoretical explicans of meaning. He claims: "To *understand* the expression '9 – 2' is clearly to know which particular construction it expresses" (Tichý 1986, p. 515). On the other hand, Carnap's model of meaning developed for his language model *S*1 (Carnap, 1947) seems to be (according to his theoretical aims) a relatively weak representation of the natural language meaning.

> communication (speech) act she performed by saying those words, etc. Or, if the language-model is aimed at representing a language conceived synchronically, the meanings are probably modelled as invariant entities, or more precisely, the connection of meanings with linguistic entities are assumed as invariant, etc. (Bielik, 2012, p. 329)

In order to show that the relationship between the language-model and the model of meaning is not so trivial, Bielik (2012, pp. 329-332) focuses on methodological notes made by some semantic theorists such as Russell (1905) and Church (1951/2001). Bielik believes that these thinkers share a common methodology but not a common language model. He concludes that they differ in ontological and epistemological principles prescribed by the particular features of their models of meaning. "They both fare well in fulfilling their own theoretical aims, but their models have different groundings and can be thus evaluated differently according to their (hypothetically reconstructed) assumptions" (*Ibid.*). In the following discussion, in agreement with Bielik, when we speak of language and abstraction what we actually have in mind is the language-model, as described above; thus, our discussion primarily focuses on the basic philosophical assumptions of linguistic analysis rather than on its technical aspects.

Regardless of the role of abstraction in language construction or language analysis, one of the important philosophical issues about abstraction is its relationship with psychology (not exactly the scientific discipline, but rather its philosophical notion). As it is famously-known, as we will see here again, for Frege abstraction is a process completely independent of psychology (meaning that abstraction is not, and cannot be, psychological). Is this the same for Carnap? In many ways, the epistemological influence of human psychology is an old philosophical question that has its roots in the rationalist-empiricist debate: is our knowledge about logic and mathematics (in general what we may call abstract knowledge) ultimately the result of the interaction between our intellectual faculty with the actual world, or does it have its own genuine source? Are we living in a fundamentally dualistic world or is dualism just an intrinsic characteristic of acquiring knowledge through our intellectual faculty? The question one may ask is that, if Carnap includes psychology, how or at what point could he switch his concentration from psychological issues to purely logical issues (which he does)?

In the following sections, we will discuss some parts of Carnap's philosophy that are directly or indirectly related to a general understanding of abstraction and its philosophical status. First, we want to emphasize that, unlike some other abstraction considerations (e.g., Fregean), Carnap reserves a role for psychology in considering abstraction. In our discussion, we will highlight some points of philosophical significance in understanding the conceptions of abstraction according to which one might be able to cognize the ways in which one appeals to abstraction in constructing meaning or language analysis, in a specific philosophical setting, and to compare a Carnapian framework with others regarding the notion of abstraction. These points are:

(1) interpretability of language system;
(2) the conception of "object";
(3) knowability of existential claims; and
(4) the empirical dependency or independency of the conceptual hierarchy.

One of the crucial and most significant points of interest would be the relations between level zero and the first level, which could be established in two ways: from the first level, downward, to the level zero (by reference), or from level zero upward to the first level (by abstraction). This point is of particular philosophical significance for many reasons; it could be construed as the borderline between real-world phenomena and the linguistic world, or between reality and intellectuality. Obviously, related to this point and for the sake of comparison, the philosophy of language, along with the ontology and epistemology of objects and concepts, becomes relevant and significant during our discussion. Thus, our analysis of abstraction would be in the context of linguistic analysis, primarily focused on the conception of object and first-order predication. To give a general view of our discussion, so that it also satisfies the above-mentioned hierarchical structure, we may say that abstraction in this context could be generally referred to as a process of introducing new concepts/terms on the basis of either objects or the old concepts/terms (lower level concepts/terms).

2.1 Quasi-analysis

Carnap introduces the abstractive method of "quasi-analysis" in *Der Logische Aufbau der Welt* (Carnap, 1967) (henceforth, *Aufbau*), and in unpublished manuscripts before the *Aufbau* (Carnap, unpublished manuscript RC-081-05-01 [1922]; RC-081-04-01 [1923], as cited by Leitgeb (2007, p. 181)). The method was derived from the Frege-Russell abstraction principle (Carnap, 1967, p. 69), and formulated on the basis of only one basic element, "*elementary experience*", and one basic relation (concept), "*similarity*", which is a certain relation between elementary experiences (Carnap, 1967, p. vii). As Leitgeb (2007, p. 181) points out, Carnap needed to appeal to a similarity relation for his project, "since the empirical domain seemed to demand descriptions in terms of similarity relations rather than in terms of the more restrictive equivalence relations, the standard method of abstraction had to be adapted in order to enable also the logical (re-)construction of empirical entities". Leitgeb (2007, pp. 183-184) reformulates, summarizes, and modifies Carnap's main points and terminology, regarding the similarity relation and quasi-analysis via the following definitions, in order to show that quasi-analysis could yield adequate results under certain conditions, and that it is "definitely to remain in the philosopher's—and perhaps also the scientist's—toolbox" (Leitgeb, 2007, p. 221).

Leitgeb starts by defining similarity and property structures.

1. **Definition of similarity structure:** a pair $\langle S, \sim \rangle$ is a *similarity structure* on S if
   (1) S is a non-empty set.
   (2) $\sim \subseteq S \times S$ is a reflexive and symmetric relation on S.

If we have such similarity structure, then members of S are called "individuals"[24].

2. **Definition of property structure:** a pair $\langle S, P \rangle$ is a *property structure* on S if
   (1) S is a non-empty set.
   (2) P is a set of subsets of S, $\emptyset \notin P$, and $\forall x \in S, \exists X \in P$ such that $x \in X$

In this case, we can still call the members of *S* "individuals", while the members of *P* are called "properties" (according to $\langle S, P \rangle$). According to Leitgeb, "the typical *Aufbau* set S of individuals would be a set of so-called elementary experiences, i.e., total momentary slices through a subject's stream of experience in a specified interval of time" (*Ibid.*, p. 189).

Leitgeb (2007, pp. 185-188) uses the following definitions to show that every property structure determines a *unique* similarity structure, which means $\langle S, \sim \rangle$ is determined by $\langle S, P \rangle$.

3. **Definition of determined similarity structure:** $\langle S, \sim^P \rangle$ is a *determined similarity structure* by $\langle S, P \rangle$ if $\forall x, y \in S: x \sim^P y$ iff $\exists X \in P$such that $x, y \in X$

[24] As Leitgeb (2007, p. 222) notifies, for Carnap, the set S of elementary experiences is considered to be finite (Carnap, 1967, p. 180).

According to the definition, while $\langle S, P\rangle$ determines $\langle S, \sim^P\rangle$ every property structure on S determines a *unique* similarity structure on S. "Thus, we are entitled to refer to *the* similarity structure determined by $\langle S, P\rangle$"[25] (*Ibid.*, p. 185). Under definition 3, we may see that every similarity structure $\langle S, \sim\rangle$ on S can be determined by at least one property structure $\langle S, P\rangle$ on S. Leitgeb, then, points out:

> Under certain conditions there is a particularly nice isomorphic copy of $\langle S, \sim^P\rangle$ as follows: Replace each individual x in S by the class of properties in P that include x as a member; define these 'new' individuals to be similar if and only if they have non-empty intersection. (Leitgeb, 2007, p. 186)

And that is exactly Carnap's intention (see section 2.1.1 below). Thus, according to Leitgeb (*Ibid.*), we may consider a similarity structure such as $\langle S', \sim'\rangle$ where $S' = \{A | \exists x \in S\ if A = \{X \in P | x \in X\}\}$ such that $A \sim' B$ iff $A \cap B \neq \emptyset$. Now, we may consider the members of S' as "the *individual concepts* of the original individuals in S (relative to the properties of the given property structure $\langle S, P\rangle$)" (*Ibid.*, p. 168, emphasis mine). We may see here that "individuals" could be replaced by "individual concepts". This replacement means that the notion of *individual* could be enhanced in certain ways relative to properties. We may wonder if this enhancement of individual (i.e., object) could include properties as well so that it does not seem necessary to postulate a distinct ontology for concepts? It seems that Carnap gives a positive answer to this question, as we will see below.

Now, if we wish to see in what sense a similarity structure may be said to determine a property structure, that is where quasi-analysis enters the picture, according to Leitgeb.

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[25] *Example*. Consider a property structure $\langle S_1, P_1\rangle$with only four individuals and two properties: $S_1 = \{a, b, c, d\}$, $P_1 = \{\{a, b, c\}, \{c, d\}\}$. In that case, the similarity structure $\langle S_1, \sim_1\rangle = \langle S_1, \sim^{P_1}\rangle$, in which:

$$\sim_1 = \{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle, \langle d, d\rangle, \langle a, b\rangle, \langle b, a\rangle, \langle a, c\rangle, \langle c, a\rangle, \langle b, c\rangle, \langle c, b\rangle, \langle c, d\rangle, \langle d, c\rangle\}$$

> The main idea of quasi-analysis can be explained easily in terms of an informal example: Think of a room with coloured objects, where *sharing a colour* is used as a similarity relation. Each colour may be supposed to embrace a certain range of hue, brightness, and intensity, and colours are permitted to 'overlap'. A set *X* of individuals which are brown (partially or completely) will then certainly be a clique with respect to similarity, since every two members of *X* share a colour. In order to turn from a set such as X to *the* set of brown individuals in this room, and accordingly for the other colours, one might take *maximal* cliques rather than just cliques simpliciter in order to constitute the colour properties. That is essentially the core of the method of quasi-analysis, a procedure by which Carnap's so-called 'similarity circles' (see *Aufbau*, Sections 70–73, 80–81, 97, 104), i.e., our maximal cliques, are constituted. (Leitgeb, 2007, p. 187)

In order to give a definition of quasi-analysis, Leitgeb defines a *clique* in saying $X \subseteq S$ is a *clique* of $\langle S, P \rangle$ if and only if: $\forall x, y \in X: x \sim y$. *X* would be a *maximal clique* of $\langle S, P \rangle$ if and only if *X* is a *clique* and there is no $Y \subseteq S$ such that $X \neq Y$, $X \subset Y$, and *Y* is a *clique*. Thus, Leitgeb (2007, p. 188) gives the definition of quasi-analysis as follows:

4. **Definition of Determined Property Structure; Quasi-analysis**: $\langle S, P^{\sim} \rangle$ is determined by $\langle S, \sim \rangle$ if $P^{\sim} = \{X \subseteq S | X \textit{ is a maximal clique of} \langle S, \sim \rangle\}$.

Accordingly, every similarity structure $\langle S, \sim \rangle$ determines a *unique* property structure $\langle S, P \rangle$. Leitgeb (2007) shows that not every property structure on S can be determined by a similarity structure on S in the manner of definition 4 because "there may be maximal nonempty intersections of similarity circles that do not coincide with *any* of the original 'pointlike' properties" (*Ibid.*, p. 192). Leitgeb defines "pointlike" properties as follows:

> Let S be the set of elementary experiences of a particular subject at a particular interval of time: If P is 'induced' by quality points in the sense that for every property X in P there is a point p in one of the quality spaces (visual, auditory,), such that *X* is the set of elementary experiences which realize p, then the determined

similarity relation is one of part identity; call the corresponding properties 'pointlike'. (Leitgeb, 2007, p. 190)

Leitgeb distinguishes two versions of Carnap's quasi-analysis and then goes on to show both versions could be accommodated based upon his suggestion, which we do not intend to discuss here. As we saw, in Leitgeb's the philosophical notion of object or individual (or elementary experiences) could be extended to the properties and this is one of our main points of discussion here.

As we said, one of the points of our discussion in this section is to enhance our understanding regarding Carnap's conception of "object" (or "individual"). Thus, it might be useful to remind ourselves some of the Carnap's terminology in this regard from the Table III (Carnap, 1972, p. 9):

Designator	Intension	Extension
Individuator	individual concept	individual
One-place predicator	property	class
***n*-place predicator**	*n*-adic relation	class of n-tuples
Sentence	proposition	truth-value (*T*, *F*; or 0, 1)

Table III Designators

In his vocabulary, Carnap calls *a*, *b*, *c*, … (values of the variables *x*, *y*, *z*, …) individual signs of objects (individual constants) that belong to the level-zero[26]. "Individual signs designate the

[26] An attribute (Table IV) that is attributed to something at the level n, and the predicate designating it as belonging to the level $n + 1$.

individuals of the realm in question (objects); they belong to the zero level. Their properties and relations, and the predicates by which these are designated, belong to the first level" (Carnap, 1942, pp. 16-17). In order to distinguish the application of a "sign" when it is meant for a single object or event, as opposed to the kind to which many objects belong, Carnap distinguishes sign-events from sign-designs (as well as expression-events and expression-designs)[27], and then introduces the basic signs (and their designata) to his semantics in the Table IV (*Ibid.*, p. 18; modified). Table IV is the list of Carnap's semantic entities. I should notify that the use of the term "entity" does not have any bearing on ontology for Carnap:

> The term 'Entity' is frequently used in this book. I am aware of the metaphysical connotations associated with it, but I hope that the reader will be able to leave them aside and to take the word in the simple sense in which it is meant here, as a common designation for properties, propositions, and other intensions, on the one hand, and for classes, individuals, and other extensions, on the other. It seems to me that there is no other suitable term in English with this very wide range. (Carnap, 1947, pp. 22-23)

[27]Carnap explains expression-events and expression-designs:

> An expression-event consists of (one or more) sign-events, and an expression-design consists of sign-designs. However, the relation is not the same in the two cases. In an expression-event all elements are different (i.e., non-identical); there is no repetition of sign-events, because an event (e.g., a physical object) can only be at one place at a time. On the other hand, in an expression-design a certain sign-design may occupy several positions, in this case we speak of the several *occurrences* of the sign (-design) within the expression (-design). (Carnap, 1942, p. 7)

<table>
<tr><th>Sign or Expression</th><th>Designata</th><th></th><th></th><th></th></tr>
<tr><td>Constant</td><td>Individual (object)</td><td></td><td></td><td rowspan="5">Entity</td></tr>
<tr><td>One-place predicate</td><td>Property</td><td rowspan="2">Attribute</td><td rowspan="3">Concept</td></tr>
<tr><td>n-place predicate ($n \geq 2$)</td><td>Relation</td></tr>
<tr><td>Functor</td><td>Function</td><td></td></tr>
<tr><td>Sentence</td><td>Proposition</td><td></td><td></td></tr>
</table>

Table IV Semantic Entities

The purpose of Carnap's "*quasi-analysis*" method is to overcome the difficulty that results from the fact that elementary experiences are unanalyzable (Carnap, 1967, p. 110). Carnap introduces a quasi-analytic relation *R* (whose extension, in general, is neither transitive nor intransitive) that only presupposes symmetricity and reflexivity for its extensions (*Ibid.*, p. 73), and then adds the *presumption* of transitivity to this relation: "for precisely this case obtains frequently in the formation of concepts in various different fields, and, moreover, it is of a special formal simplicity" (*Ibid.*, p. 119), which then makes *R*, an equivalence relation, a special case and not *a priori*. We may see that considering transitivity as *an assumption* speaks to Carnap's empiricist position; since this consideration does not accord with a rationalist position about the identity relation (or equivalency relation).

> Of the two conceptions of a relation extension […]—identity and part similarity—only the first can obtain in this case: the similarity circles of *R* must here themselves be considered the quasi constituents; in this case, we shall call them abstraction classes of *R*. It follows, moreover, that the class of elements which stand to any given element in the relation (extension) *R* forms an abstraction class. Hence, the abstraction classes and thus the quasi constituents can here be defined as the (nonempty) classes of elements which are akin to a given element. (Carnap, 1967, p. 119)

We may say that what we observe around us is in similarity, and that it is not transitive but that its abstraction is. Here, we clearly see that the identity relation is yielded (constructively) by the imposition of transitivity (as a voluntary choice for the pragmatic reasons); in other words, if R is not transitive there would be no abstraction classes by definition, because "the class of elements which stand to any given element in the relation (extension) R forms an abstraction class" (*Ibid.*, p. 119). According to Carnap, "the procedure of quasi-analysis in this simplest case of a transitive relation extension corresponds to the 'principle of abstraction', which was first explicitly mentioned by Russell, Frege, and then also by Whitehead and Russell, for the construction of the cardinal numbers […]" (*Ibid.*, p. 119); this quote is quite important because while Carnap makes a link between his version of abstraction and that of his predecessors, he is clear that the Fregean abstraction is the simplest version of his. In his version, analysis is quasi-analysis in disguise (made linguistically by abstraction); since the basic elements, relations, etc. are essentially unanalyzable. Thus, analysis is what we synthesize or what we take it to be the case; not what we discover to be the case.

> If class and relation extension are acknowledged as the only constructional steps, then the methodological unanalyzability of the basic elements follows for any constructional system, and, from the choice of the essentially unanalyzable elementary experiences follows a materially determined unanalyzability. […] We can therefore say: Quasi analysis is a synthesis which wears the linguistic garb of an analysis. (Carnap, 1967, p. 120)

Carnap expresses his agreement with Russell's methodological principle in this way: "The supreme maxim in scientific philosophizing is this: Wherever possible, logical constructions are to be substituted for inferred entities" (*Ibid.*, p. 8). However, he also acknowledges that his interpretation of this principle is more radical than Russell's: "We shall, however, employ this principle in an even more radical way than Russell (for example, through the choice of an auto-psychological basis, in the construction of that which is not seen from that which is seen, and in the construction of hetero-psychological objects)." (Carnap, 1967, p. 8)[28]. Carnap evidently brings psychology into the discussion by describing a move from auto-psychological (subjective) assertions to hetero-psychological (intersubjective) ones via abstraction (see section 2.1.3). Carnap insists in paying attention to the abstractive characteristics of the constituents: "many epistemological systems (especially the positivistic ones) which are otherwise closely related to our constructional system have used, not experiences themselves, but sensory elements or other constituents of experiences as basic elements, without paying heed to their character as abstractions" (Carnap, 1967, p. 120). Before discussing elementary experience, let us see how Carnap speaks about *quasi-objects*. Let us picture Carnap's philosophical framework in the simplest way: the interaction of our intellectual faculty with the rest of the actual world amounts to our elementary experiences in which physical objects themselves are abstractions of the real world and are subjected to quasi-analysis by abstraction (since proper analysis is impossible). Moulines even describes Carnap's quasi-analysis as a step-by-step abstraction procedure.

> Carnap's idea (inspired by the Gestalt theory but formally independent of the particular results of this theory) is that the primary basis of knowledge should be conceived of as a total experience (= "*Erlebnis*") or, still better, as an experience flow (= "*Erlebnisstrom*"), out of which particular phenomenal items like coloured spots should be constructed by a step-by-step abstraction procedure. The latter is what he calls "quasi-analysis". (Moulines, 1991, p. 272) (emphasis mine)

The objects in question (level zero objects) and the abstraction procedure for arriving at the individual concepts, then, could be repeated for constructing the entire conceptual hierarchy.

[28] We will see this quote again in the following subsections.

Here is where the concept of "quasi-object" and "spheres of objects" become quite significant. For now, we just saw that appealing to similarity to arrive at properties is quite possible and legitimate. And, this would allow us to talk about classes of objects as "new objects".

2.1.1 Quasi-objects

One of the participating factors in the enhancement of the notion of object, and eventually setting aside the famous object-concept distinction, is the introduction of "quasi-objects" in the *Aufbau*. If we simply consider abstraction as a move from object to concept, then our understanding of what we mean by "object" (in a philosophical setting) is quite significant with regard to the understanding of how abstraction works. As said above, one of the points that we are trying to establish here is the conception of "object" in Carnap's philosophy, which we claim as having quite a flexible meaning (compared to that of Frege's). Instead of the given-ness of the first order concepts, for Carnap the elementary experience is "given", meaning: "never found in consciousness as mere raw material" (Carnap, 1967, p. 158). With this, it is clear that the cognitive faculty (or epistemological characteristics) plays an important role in shaping Carnap's philosophy.

> The synthesis of cognition, i.e., the formation of entities, or representations of things and of "reality", from the given, does not, for the most part, take place according to a conscious procedure. […] In science, too, synthesis, the formation of objects, and cognition take place, for the most part, intuitively and not in the rational form of logical deductions. […] The fact that the synthesis of cognition, namely, the object formation and the recognition of, or classification into, species, takes place intuitively, has the advantage of ease, speed, and obviousness. (Carnap, 1967, p. 158)

Linguistically speaking, Carnap (1967, p. 27) believes one may basically divide the signs (or strings of signs) into the ones with an independent meaning[29] and those which have meaning only in connection with other signs. Among the latter, proper names are specifically distinguishable since they designate definite concrete individual objects which are, for that reason, relatively more independent than the rest of the sentences made by them. “The traditional view is that the proper names have a relatively independent meaning and are thereby distinguished from the other signs” (*Ibid.*, p. 48); Frege famously considers them as “saturated” signs. Whereas, the rest of the complete sentence (a function in Fregean sense) is called “unsaturated”. Carnap finds the Fregean distinction, between “saturated” and “unsaturated” signs, “not logically precise” (*Ibid.*, p. 48). With no intention to give a more precise definition to the concept of “proper names”, Carnap suggests we may open the boundary so that not only proper names could refer to objects but also general names: “Perhaps there is only a difference in degree and the choice of a boundary line is arbitrary; at least, this seems to be the upshot of the later discussions on individual and general objects (§158)” (*Ibid.*, p. 48). Hence, he considers quasi-objects as well.

> In the original usage of signs, the subject position of a sentence must always be occupied by a proper name. However, it proved advantageous to admit into the subject position also signs for general objects and, finally, also other incomplete [unsaturated] symbols. This improper use, however, is permissible only when a transformation into proper use is possible, i.e., if the sentence can be translated into one or more sentences which have only proper names in their subject positions. […] Thus, in improper use, incomplete symbols are used if they designated an object in the same way as an object name. One even speaks of “their designata”, consciously or unconsciously introducing the fiction that there are such things. We wish to retain this fiction for reasons of utility. But, in order to remain perfectly aware of this fictional character, we will not say that an incomplete symbol designates an “object”, but that it designates a *quasi-object*. (In our view, even the so-called “general objects”, e.g., “a dog” or “dogs” are already quasi objects). (Carnap, 1967, p. 49)

[29] “Strictly speaking, only those (mostly complex) signs which designate a proposition, i.e., sentences, have independent meaning” (*Ibid.*).

Carnap (1967, pp. 49-53) explains that what is important in the sentences such as "Fido is a dog" (in which Fido is a proper name) and "a dog is a mammal" is the structure or the "propositional function", since empty spaces in "… is a dog" or "… is …" can be properly filled by the object-names (objects or quasi-objects). Same thing with "Berlin is a city in Germany"; by eliminating one proper name (Berlin), the resulting propositional function has the structure of *property*; while, by eliminating both, it has the structure *relation* (x is a city in y). Thus, "a propositional function with precisely one argument position we call a *property* or *property concept*" and "a propositional function with two or more argument positions we call a (two-place or many-place) *relation* or a *relational concept*" (*Ibid.*, p. 51). Carnap is quite clear that "the 'objects' of science are almost without exception quasi objects" (*Ibid.*, p. 50). In order to lessen the difference between objects and quasi objects, Carnap then defines the *isogeny* of objects in terms of propositional functions and *object spheres*[30].

> Two objects (and this always includes quasi objects) are said to be *isogenous* if there is an argument position in any propositional function for which the two object names are permissible arguments. If this is the case, then it holds for any argument position of any propositional function either that both names are permissible arguments, or that neither of them is. This is a consequence of the logical theory of types, which we cannot here discuss in detail. If two objects are not isogenous, then they are termed *allogeneous*. (Carnap, 1967, pp. 51-52)
>
> By the *sphere* of an object we mean the class of all objects which are isogenous with the given object. (Since isogeny is transitive, the object spheres are mutually exclusive.) If every object of a given object type is isogenous with every object of another object type, then we call the object types themselves "isogenous". Correspondingly, we also speak of "allogeneous" object types. (*Ibid.*, p. 52)

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[30] By pointing out the complications that may occur in the formation of a conceptual system due to confusing the spheres of objects, Carnap explains what we should avoid in philosophy.

> We are here not concerned with straightforward ambiguity (homonymy) as it occurs, e.g., in such words as "cock", "spring", etc., nor with somewhat more subtle ambiguities as they occur in many expressions of ordinary life, of science and of philosophy, as, for example, in the words "representation", "value", "objective", "idea", etc. In our daily lives, we are well aware of the first type of ambiguity, while in philosophy we concern ourselves with the second, and we can thus avoid at least the more obvious mistakes. (Carnap, 1967, p. 53)

Given that quasi objects are objects, we can clearly see that, for Carnap, the conception of "object" is quite flexible since it may well include the first-order concepts. Carnap is explicit that "extensions, too, are quasi objects" (*Ibid.*, p. 56) and also "classes, since they are extensions, are quasi objects" (*Ibid.*, p. 57). "We must emphasize the fact that classes are quasi objects in relation to their elements, and that they belong to different spheres" (*Ibid.*, p. 58). We may see that the spheres would be determined by their relative levels of abstraction since the elements of the class of chairs, for example, correspond to the level zero while the class itself is an object (quasi-object) of the first level. "Like classes, relation extensions are quasi objects" (*Ibid.*, p. 59). "If an object is logically reducible to others, then we call it a *logical complex* or, in brief, a *complex* of these other objects, which we shall call its elements"; accordingly, "classes and relation extensions are examples of complexes" (*Ibid.*, p. 61). "If an object stands to other objects in a relation such that they are its parts relative to an extensive medium, e.g., space or time, then we call the first object the *extensive whole* or, in brief, the *whole* of the other objects. The whole consists of its parts" (*Ibid.*, pp. 61-62). An independent *logical complex*, on the other hand, "does not have this relation to its elements, but rather, it is characterized by the fact that all statements about it can be transformed into statements about its elements" (*Ibid.*, p. 9). Although, Carnap is clear that "the concepts *whole* and *complex* are not mutually exclusive" (*Ibid.*, p. 62), he is also clear that "construction theory is especially concerned with those complexes which do not consist of their elements, as a whole consists of its parts"; therefore, he calls these complexes (such as classes) "*autonomous complexes*" (*Ibid.*, p. 62). "Thus, we differentiate a whole from an autonomous complex by the fact that in the former the elements are parts in the extensive sense; in the latter, they are not" (*Ibid.*). It is important how Carnap considers quasi-objects and objects (in general) such as classes and sets and the like as *autonomous complexes*. “From the definition of construction and complex, it follows that, if an object is constructed from other objects, then it is a complex of them. *Thus all objects of a constructional system are complexes of the basic objects of the system*” (*Ibid.*). Carnap believes that the same should hold for quasi-objects such as classes and sets[31].

> If we are concerned with a statement about a quasi-object, i.e., a statement which is expressed in the form of a sentence in which an incomplete symbol occurs in a position where the sentence structure originally allows only an object name, then this use of the incomplete symbol must be defined; it must be possible to translate this sentence into another sentence, where we find only proper object names in argument (e.g., subject) positions. From this it follows that a quasi-object which belongs to a certain object domain is always a complex of the objects of this domain; i.e., it is an autonomous complex and not the whole of its elements. For a whole is an object of the same object type as its elements. (Carnap, 1967, pp. 62-63)

As you may see, considering classes and sets as autonomous complexes requires not to consider them of the same object type as their elements. Thus, not only are classes and sets not identical with the corresponding wholes but they also belong to different object spheres, according to Carnap. Philosophically speaking, Carnap considers objects of different spheres as different “modes of being” (*Ibid.*, §42). In the process of construction Carnap distinguishes two different cases: cases in which explicit definition is possible and cases in which appealing to explicit

[31] As Carnap is clear that classes and sets are to be considered as autonomous complexes, he reminds us of the importance of this view since this was not clear even for Cantor. Carnap also points out that Frege explicitly says that “the extension of a concept does not consist of the objects which fall under the concept”, and that Russell is making the same point by calling attention to “unit classes” and “null classes” (*Ibid.*, p. 64); thus, they both agree with this view.

> The same holds for the mathematical concept of a set, which corresponds to the logical concept of a class. A set, too, does not consist of its elements. This is important to notice, since the character of a whole or a collection (or of an “aggregate”) has erroneously been connected with the concept of a set ever since its inception (i.e., ever since Cantor’s definition). In set theory itself, this notion does not generally have any consequences, but it seems to be the reason that the methodologically most advantageous and logically unobjectionable form of definition for the concept of power (or cardinal number), one of the central concepts of set theory, is frequently opposed. (Carnap, 1967, p. 63)

definition is not possible[32]. In the former, "the new symbol is declared to have the same meaning as the compound one" (*Ibid.*, p. 65); for example, if, in set theory, C is introduced for the intersection of A and B, $C = A \cap B$, or, in propositional logic $R \equiv P \wedge Q$. In this case, the new object would not be considered as a quasi-object relative to certain older objects. "Thus, it remains within one of the already formed object spheres, even if we should consider it as a representative of a new object type" (*Ibid.*). Carnap is perfectly aware and clear about the pragmatic aspects of constructing by definitions in saying "differentiation of types, unlike the opposition between spheres, is not logically precise but depends upon practical purposes of classification" (*Ibid.*). While in the second case, where no explicit definition is possible for an object, "its object name, given in isolation, does not designate anything in the manner of already constructed objects; in this case, we are confronted with a quasi-object relative to the already constructed objects" (*Ibid.*)[33].

Now, in terms of abstracting from objects to concepts, we can clearly see that the term "concept" could be used as a temporary label (in a Carnapian framework) for the relatively higher-level objects/quasi-objects (spheres) when speaking of two immediate levels of abstraction; for example, as when speaking of zero level objects and the first-order predicate, the object in

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[32] For the second case, Carnap uses the term "definition in use", and avoids to use "implicit definition for the following reason:

> The expression "implicit definition" is customary for an entirely different determination of objects through axiomatic systems and should be reserved for this purpose. Occasionally, when one is concerned with the contrast between implicit and explicit definitions, both, definitions in use and explicit definitions proper, are called "explicit definitions in the wider sense". (Carnap, 1967, p. 66)

[33] In this case, Carnap points out the need for the new translation rules.

> However, if an object is to be called "constructed on the basis of the previous objects", then it must nevertheless be possible to transform the propositions about it into propositions in which only the previous objects occur, even though there is no symbol for this object which is composed of the symbols of the already constructed objects. Thus we must have a translation rule which generally determines the transformation operation for the statement form in which the new object name is to occur. (Carnap, 1967, pp. 65-66)

question and the corresponding predicate (in a complete sentence) belong to two different spheres of objects. The notions of bottom-up moves (on the levels) are almost evident when Carnap describes "*ascension forms*"[34] (e.g., classes and relation extensions) of construction and "*constructional levels*".

> Now a constructional definition is either explicit or it is a definition in use. In the first case, the object to be constructed is isogenous with some of the preceding objects (i.e., no new constructional level is reached through it). *Thus, the ascension to a new constructional level takes place always through a definition in use*. Now, every definition in use indicates that a propositional function which is expressed with the aid of a new symbol means the same as a propositional function which is expressed only with the older symbols. […] Thus, we can interpret the new propositional function purely extensionally: we introduce the new symbol as an extension symbol. Thus, through a constructional definition which leads to a new constructional level, we always define either a class or a relation extension, depending upon whether the defining propositional function has only one argument position or whether it has several of them. (Carnap, 1967, pp. 67-68)

As you may see, there are no signs of a fundamental division between objects and concepts in Carnap's philosophy (as it is the case of Frege's), and the ways in which we speak about extensions are quite different than that of Frege; in the Carnapian sense, we speak of extension in a constructive mode[35] (bottom-up) with the consideration of pragmatic aspects of classification, as said above. The conception of object is quite relative, for Carnap, as he is quite clear that "the relativity of the concept 'quasi-object', which holds for any object on any constructional level relative to the object on the preceding level, is especially obvious" (*Ibid.*, p. 70). Given, as we will see, that the elementary experiences and quasi-objects are products of abstraction, one may interpret what Carnap means by "constructional levels", in the following quote, as "levels of abstraction".

[34] Carnap says: "the two *ascension forms* of construction which will be used in our system and which will be discussed in the sequel are *forms of quasi objects*" (Carnap, 1967, p. 50).

[35] The case for Frege is in a referential mode, as we will see; top-down.

> If, in a constructional system of any kind, we carry out a step-by-step construction of more and more object domains by proceeding from any set of basic objects by applying in any order the class and relation construction, then these domains, which are all in different spheres and of which each forms a domain of quasi objects relative to the preceding domain, are called *constructional levels*. Hence, constructional levels are object spheres which are brought into a stratified order within the constructional system by constructing some of these objects on the basis of others. (Carnap, 1967, pp. 69-70)

As we see, the relativity of objects/concepts with respect to their constructional level (or level of abstraction) is evident from this quote, for Carnap. Carnap emphasizes the metaphysical neutrality of the constructional language which is primarily intended to express only "epistemic-logical" relations; "the expression 'quasi-object' designates only a certain logical relationship and is not meant as the denial of a metaphysical reality. It must be noted that all real objects (and construction theory considers them as real to the same degree as do the empirical sciences, cf. §170) are quasi-objects" (*Ibid.*, §52).

2.1.2 Objects and Properties

With what we have seen regarding the relativity of objects we now want to show what is Carnap's propositional approach towards properties. In this approach, we will see the same relativity and bottom-up mode of construction from another angle. Replacing individuals by properties reminds us of the possibility of considering objects in terms of concepts, in an old-fashion way. Coniglione (2004) reiterates the Aristotelian notion of object in terms of ordinary set theory by letting *U* be the universe of objects, and *A* as one of its subsets, such that they all share a common property *P* (e.g., whiteness). Thus, we have the following set:

$$A = \{x : P(x)\}$$

where $a \in A$ means that the individual *a* belongs to the set *A* because it possesses the property *P*.

> By so doing we have performed an abstraction in the Aristotelian sense and we give the abstract concept thus obtained the name [*A*] which indicates the property ("whiteness") shared by all the elements belonging to the set *A*. In the light of this, the fact that a given concrete individual is not definable means for Aristotle that it is the conjunction of a (possibly infinite) number of properties [...]. (Coniglione, 2004, p. 61)

$$a = \bigwedge_{i=1}^{n} P_i$$

As we will see, this is exactly how Carnap considers objects, but in an indirect way via state-descriptions. In this case, as Coniglione (2004, p. 62) noted, "we have defined a class intensively by identifying the property possessed by its elements, so identification of this property logically precedes identification of the elements belonging to the class". The other way is to consider things extensionally (Fregean way) in which one would consider $A = \{a, b, c\}$, for instance, and let $[A]$ be the name of *A*. Thus, we can say "*a* is [*A*]", meaning that [*A*] is the name designating the class of individuals *a*, *b*, *c*, which have nothing in common except for the fact that they belong to *A*. In this case, the following is true:

> we have given an extensive (or iterative) definition by simply listing the elements belonging to the class, without identifying any properties they may share; therefore (according to the axiom of extensionality) a set is completely determined by its elements, so the iterative concept of a set is different from the dichotomous concept which allows each set to be obtained by dividing all things into two categories (i.e. things which possess a certain property and things which do not). (Coniglione, 2004, p. 62)

The point here is not to simply point out the intensional and extensional conception of sets; the point is that, if we adopt the Aristotelian notion, then the intensive understanding of the concepts (involved in constructing an object) would be prior to the understanding of the object itself; in other words, we may define object in terms of concept, not the other way around.

Regardless of how we arrive at concepts such as “red”, “cold”, “hard”, and so on (and setting aside the world-language relation), we may still treat them as linguistic entities (belonging to the world of language; here-in we consider language as an object itself), and call them “elementary terms” (Carnap, 1939, p. 61). In this way, we pre-assume a certain world-language relation (undetermined and under investigation) about which we are not going to talk, but rather, we will talk about language in isolation and as an object in the world. This is the position (which is philosophically different than that of Frege’s in the following way) that was adopted by Carnap and perhaps other logical empiricists of the Vienna Circle. Carnap is clear in that “bright”, “dark”, “red”, and other concepts of this sort are elementary terms and “meant as properties of things, not as sense-data” (Carnap, 1939, p. 62). Thus, they have already passed an abstractive stage that converts sense data into linguistically expressible property-words (elementary terms); but, while we may consider elementary terms as having independent values, Carnap still considers them as being abstracted from pragmatics. Carnap considers semantic information, in general, to be an *approximation* to pragmatic information that is achievable by abstraction.

> We shall talk about the information carried by a sentence, both by itself and relative to some other sentence or set of sentences, but not about the information which the sender intended to convey by transmitting a certain message nor about the information a receiver obtained from this message. An explication of these usages is of paramount importance, but it is our conviction that the best approach to this explication is through an analysis of the concept of semantic information which, in addition to its being an approximation by abstraction to the full-blooded concept of

> pragmatic information, may well have its own independent values[36]. (Carnap & Bar-Hillel, 1952, pp. 2-3)

It is evident from this quote alone that the underlying philosophical understanding of abstraction, for Carnap, is empirical in the sense that the first step is based on the information received from pragmatics, which is an absolutely empirical part (as we go forward we will see more evidence that points in this direction). And this is exactly why Carnap sees a framework in which science and mathematics/philosophy could be seen along the same line. As Carnap places the whole enterprise of science (including theory making, testing, adjusting, modeling, etc.) into pragmatics, he considers science the main concept/term-generator that supplies information by abstracting from pragmatics, delivering it to semantics. In the introduction to a

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[36] In this quotation Carnap refers to the works of Donald MacKay (1922-1987), a British physicist, who made important contributions to the question of "meaning" in information theory. Using MacKay's terminology here, Carnap considers the actual world as the sender of information signals, and hence, our cognitive apparatus is considered to be the receiver. This shows that Carnap not only acknowledges the contribution of our cognitive apparatus (via the sender-receiver relation), but also finds it quite important; nevertheless, he chooses not to include this part of abstraction into his project (and leave that part to science, as he embeds natural sciences entirely in pragmatics), but to concentrate on abstraction from semantics to syntax. It could be for the same reason (i.e., not regarding the sender-receiver relation) that Carnap tends to consider existential questions to not have an absolute meaning, but rather, to be internal questions; because beyond the receiver everything ought to be internal.

> […] whether or not the sentence '*P*(*a*)' is true, is not a task of semantics but of empirical science. Semantics has the task not of fact-finding but of interpreting language. Although limited to this task, semantics does and must refer to extra-linguistic entities, e.g., physical objects, their properties and relations. (Carnap, 1945, p. 148)

> In pragmatics we are concerned with the user of scientific language, i.e. the scientist. His activities are studied in so far as they are connected with and hence relevant to his use of the language. Thus, for instance, the study of the activities of observation in their relation to observation sentences belongs to pragmatics. Another example is the study of the methods of testing hypotheses or theories by first deriving predictions from observation sentences with their help, and then comparing these predictions with new observation sentences which report the results of experiments: The outcome of such a procedure of testing an hypothesis is either a confirmation or an infirmation of that hypothesis, or, rather, either an increase or a decrease of its *degree of confirmation*. (Carnap, 1994, p. 292)

> But also for the logician a study of pragmatics may be useful. If he wishes to find out an efficient form for a language system to be used, say, in a branch of empirical science, he might find fruitful suggestions by a study of the natural development of the language of scientists and even of the everyday language. (Carnap, 1955a, p. 34)

theory of semantic information, Carnap and Bar-Hillel say: "It seems desirable that one should be able to say not only what information a message or an experiment has supplied but also how much. Hence, we are going to distinguish between information (or content) and amount of information" (*Ibid.*, p. 1). Thus, in their project, "information" is on par with "content". From semantics, we may continue abstraction as a process of emptying "empirical content" in order to have access to structural information.

As we discussed above, individuation is important regarding abstraction, and we can clearly see that, even though he did not want to deal with it, Carnap preserves a place for individuation in his theory. In his extension-intension method, Carnap considers the "individuator" as one of the main designators (Carnap, 1972, p. 9); see Table III.

Carnap abstracts from descriptive semantics (which is abstracted from pragmatics) and moves on to logical semantics (*L*-semantic), which is limited only to logical concepts or *L*-concepts (e.g., *L*-true, *L*-equivalence, *L*-implication, etc.) (Carnap, 1942, p. 56). In *L*-semantics, with respect to a semantic system *S*, Carnap considers an *L-state* as "a possible state of affairs of all objects dealt with in a system *S*, with respect to all properties and relations dealt with in *S*" (*Ibid.*, p. 95). Thus, "a sentence or sentential class designating an *L-state* is called a *state-description*" (*Ibid.*).

In general, for a language$L_n^\pi$where n is the number of individual constants, and π is the number of primitive monadic predicates the following holds:

a. The number of atomic sentences is $\beta = \pi n$.
b. The number of Q-predicates is $\kappa = 2^{\pi}$.
c. The number of state-descriptions is $2^{\beta} = 2^{\pi n} = (2^{\pi})^{n} = \kappa^{n}$.

where an atomic sentence is in the form of $P = F(x)$, and Q- predicates express and identify structure-descriptions Str_i that are the disjunction of isomorphic *state-descriptions* Z_i in the language L_n^{π}. For example, in a language with only one individual constant (*a*) and three monadic predicates (*F*, *G*, *H*) or L_1^3, we will have the following situation:

P_1, P_2, P_3	Q-predicate Expression	Q-predicates
+, +, +	$P_1 \wedge P_2 \wedge P_3$	Q_1
+, +, -	$P_1 \wedge P_2 \wedge P_3$	Q_2
+, -, +	$P_1 \wedge P_2 \wedge P_3$	Q_3
+, -, -	$P_1 \wedge P_2 \wedge P_3$	Q_4
-, +, +	$P_1 \wedge P_2 \wedge P_3$	Q_5
-, +, -	$P_1 \wedge P_2 \wedge P_3$	Q_6
-, -, +	$P_1 \wedge P_2 \wedge P_3$	Q_7
-, -, -	$P_1 \wedge P_2 \wedge P_3$	Q_8

Table V Q-predicates in L^3

Str_i	**Q-Predicates**	**Expression**	**Logical Nature**
1		$P_i \wedge \sim P_i$	*Empty*
2	Q_1	$P_i \wedge P_j \wedge P_k$	
3	$Q_1 \vee Q_2$	$P_i \wedge P_j$	
4	$Q_1 \vee Q_2 \vee Q_3$	$P_i \wedge (P_j \vee P_k)$	
5	$Q_1 \vee Q_2 \vee Q_3 \vee Q_4$	P_i	*Factual*
6	$Q_1 \vee Q_2 \vee Q_3 \vee Q_4 \vee Q_5$	$P_i \vee (P_j \wedge P_k)$	
7	$Q_1 \vee Q_2 \vee Q_3 \vee Q_4 \vee Q_5 \vee Q_6$	$P_i \vee P_j$	
8	$Q_1 \vee Q_2 \vee Q_3 \vee Q_4 \vee Q_5 \vee Q_6 \vee Q_7$	$P_i \vee P_j \vee P_k$	
9	$Q_1 \vee Q_2 \vee Q_3 \vee Q_4 \vee Q_5 \vee Q_6 \vee Q_7 \vee Q_8$	$P_i \vee \sim P_i$	*Universal*

Table VI Q-predicates for L_1^3

For example, in Table V, if we have only three predicates “blue”, “round” and “sweet” and one object a, Q_1 would be “a is blue and round and sweet” and Q_2 would be “a is blue and round and not sweet” and so on. In Table VI, it is evident that factual Q_1 corresponds to the factual *state description* Z_1 which is $P_1 \wedge P_2 \wedge P_3$ where $P_1 = F(a)$, $P_2 = G(a)$ and $P_3 = H(a)$. In other words:

$$Z_1 = \bigwedge_{i=1}^{3} P_i$$

We can clearly see that this factual *state description* of an object accords with what said above about intensional consideration of objects (introduced as Aristotelian notion), which was:

$$a = \bigwedge_{i=1}^{n} P_i$$

So far, we have seen that, for Carnap, all of our construction is primarily based upon our elementary experiences, with an intensional consideration for objects affected by our psychology. Thus, we can say with certainty that Carnap, unlike Frege, gives psychology a place in his overall philosophical framework, although at some point (when the statements become increasingly empty of empirical content) it will be out of the question (at which point the statements become logical). In fact, we may roughly say that abstraction, for Carnap, is an intellectual journey from psychologically-based statements to logical ones as well as from psychological objects to logical ones (see below).

2.1.3 Elementary Experience and Elementary Terms

In this section, our focus will be on the most basic constituents in Carnap's constructional system, *elementary experiences*, which we arrive at through abstraction, as we will see. Carnap does not speak of "elementary experience" in his later works. The reason is that, as we saw in the previous chapter (§1.2.1), Carnap would rather leave all experiential matters to pragmatics (and consider them as topics of scientific inquiries). Carnap prefers to focus on the outcome of pragmatics ("elementary terms"), which he considers as objects of philosophical interest in making his constructional system. Once we face an elementary experience and then decide to transform that experience into an expressive mode using language, linguistic entities (terms, verbs, expressions, etc.) and the ways in which they should be used become our main concern; while our relationship with the elementary experience still counts as our main source for generating terms (term-generator). Nonetheless, elementary experience is the basic element in *Aufbau*, regarding Carnap's abstractive method of quasi-analysis, and understanding its epistemic value and primacy would help us to have a better understanding of Carnap's philosophical position with regard to abstraction.

As said above, the most basic elements for Carnap's constructional system, which are "objects of the lowest constructional level" (Carnap, 1967, §61), are called "elementary experiences" (*Ibid.*, §67). These objects have an auto-psychological basis (more on this in the subsequent section). Carnap explains that elementary experiences are epistemically secondary and considered to be abstractions from the given ones.

> One could perhaps think of choosing the final constituents of experience at which one arrives through psychological or phenomenological analysis, or, more generally, psychological elements of different types from which experiences can be formed. However, upon closer inspection, we realize that in this case we do not take the given as it is, but abstractions from it (i.e., something that is epistemically secondary) as basic elements. (Carnap, 1967, pp. 107-108)

Carnap is quite clear that finding relations and making comparisons (i.e., arriving at quasi-objects) are the result of abstraction from the given experiences, since elementary experiences "down to the last elements, are derived from these [given] experiences by relating them to one another and comparing them (i.e., through abstraction)" (*Ibid.*, p. 108). Thus, "the basic elements, that is, the experiences of the self as units, we call *elementary experiences*" (*Ibid.*). As we clearly can see, for Carnap, elementary experiences and quasi-objects both are products of abstraction. Carnap believes that: "there is more and more proof that, in perception, the total impression is primary, while sensations and particular feelings, etc., are only the result of an abstracting analysis" (*Ibid.*). Therefore, what we are experiencing is a totality and it is the total impression that is epistemically primary (according to Carnap, modern psychological research has confirmed this more and more, *Ibid.*, p. 109); hence, various sense modalities, individual sensations, and the categories into which we might divide these, visual, auditory, etc., are derived only through abstraction (even though one may say afterward that the perception is composed of them).

> The more simple steps of this abstraction are carried out intuitively in prescientific thought already, so that we quite commonly speak, for example, of visual perceptions and simultaneous auditory perceptions, as if they were two different constituents of the same experience. The familiarity of such divisions which are carried out in daily life should not deceive us about the fact that abstraction is already involved in the procedure. This applies a fortiori to elements which are discovered only through scientific analysis. (Carnap, 1967, p. 108)

As one may see, abstraction has a profound meaning and application in Carnap's philosophy. It should be evident by now that dividing the totality of the surrounding physical phenomenon into different objects, senses, etc., requires the same procedure as it requires for arriving at quasi-objects, explained above, and that is abstraction. Therefore, we may say abstraction is the most essential epistemic procedure in Carnap's philosophy by which we arrive at elementary experiences as well as quasi-objects. Although elementary experiences are based on psychology, they are of the same type so that we may introduce (construct) our prescientific and scientific objects based upon them.

> The elementary experiences are to be the basic elements of our constructional system. From this basis we wish to construct all other objects of prescientific and scientific knowledge, and hence also those objects which one generally calls the constituents of experiences or components of psychological events and which are found as the result of psychological analysis (for example, partial sensations in a compound perception, different simultaneous perceptions of different senses, quality and intensity components of a sensation, etc.). (Carnap, 1967, p. 109)

Consequently, elementary experiences are not analyzable (nonetheless quasi-analyzable). "The basic elements of the constructional system cannot be analyzed through construction. Thus, the elementary experiences cannot be analyzed in our system since this system takes them as basic elements" (*Ibid.*, p. 110). According to Carnap, the unanalyzability of elementary experience leads us only to synthetic levels of construction (or, as we interpreted above, levels of abstraction).

> We remember that class and relation extension are to be the only ascension forms of the constructional system. Starting from any basic elements and basic relations, we can form only objects of the following kinds in the constructional system: on the first constructional level, classes of elements and relations between elements; on the second level, only (1) classes of such classes, or classes of relations of the first level, and (2) relations between such classes, or relations between relations on the first level, or relations between classes on the first level and elements, etc. It is obvious that construction, when carried out with the aid of these ascension forms is always synthetic, never analytic. (Carnap, 1967, p. 110)

Carnap introduces the method of quasi-analysis precisely to overcome the problem of the unanalyzability of elementary experience, "which, even though synthetic, leads from any basic elements to objects which can serve as formal substituents for the constituents of the basic elements" (*Ibid.*, §69). Elementary experiences are unanalyzable since "in their immediate given-ness, do not exhibit any constituents or properties or aspects." (*Ibid.*). Although elementary experiences are given and therefore, we may only treat them synthetically, we can ascribe various properties and/or characteristics to them, or think of them as having constituents, via quasi-analysis. "Properties and constituents are here taken to be the same thing; with psychological processes, for example, one cannot use the expression 'constituent' in its original, spatial sense, but only in the sense of the equally figurative expression of 'different aspects' or 'characteristics'" (*Ibid.*). Carnap clearly describes what quasi-analysis is set to achieve in the following quote:

> Generally speaking, and without restriction to the particular problem of elementary experiences, quasi analysis is to achieve the following: unanalyzable units of any kind, a pair list[37] of which is presupposed, are to be manipulated with the constructional ascension forms of class and relation extension (i.e., with synthetic methods) in such a way that the result is a formal substitution for proper analysis (i.e., the analysis into constituents or properties), which cannot be carried out in this case. Because of the required formal analogy between the results of quasi analysis and those of proper analysis, one can *suppose* that a certain formal analogy will obtain between these two procedures themselves. (Carnap, 1967, p. 111) (emphasis mine)

Just to take a quick detour, here is where we have a better understanding of the conventionality of logic or of logical pluralism (as well as the conventionality of scientific theoretical statements) in Carnap's philosophy. It is on this ground that one may realize that manipulations of synthetic methods (along with the suppositions we make) in order to find formal substitutions for proper analysis require a great deal of conventionality. Carnap is clear in that it is up to us to imagine individual experience as manifolds or as having constituents. "An individual experience, taken by itself, is unanalyzable. Experiences, taken as a manifold, can be compared and ordered, and only through their order result the (quasi) constituents of the individual experiences" (*Ibid.*, p. 149). For Carnap, even the notion of "being identical" is a fictional notion.

> There is another assumption which is connected with the fiction of the retainability of the given, namely, that each element of the given (each elementary experience) is identically retained, so that, during the synthesis, it can be utilized more than once and can be identified each time as identically the same. In our fiction, we could express this, for example, by saying that each individual elementary experience is provided with an arbitrary, but permanent, token, for example, an (arbitrary) number. (Carnap, 1967, p. 160)

[37] The translator is right in saying this in the footnote: "Since only dyadic relations are discussed in the sequel, I have translated '*Relationsbeschreibung*' as "pair list", even though "list of n-tuples" would have been more precise" (*Ibid.*).

In conclusion of this section, we saw that unanalyzable elementary experiences are the basic elements, and abstraction is the essential procedure for performing quasi-analysis, according to Carnap. Another important point that may enhance our understanding of abstraction in the Carnapian sense is that elementary experiences are psychologically based, and that abstraction is supposed to bridge auto-psychological (subjective) statements and hetero-psychological (intersubjective) statements, which we will explain in the subsequent subsection.

2.1.4 From Auto-Psychological to Hetero-Psychological

One of the specific features of Carnap's notion of abstraction (unlike Frege's) is that abstraction does not have to be a non-psychological process. It is actually on the basis of this psychological abstraction that Carnap could manage to base his constructional system on empirical facts. Carnap's thesis is that "it is possible to have a constructional system with a psychological basis. The logical justification for this system form is independent of any metaphysical standpoint and rests solely upon the […] proof that all cultural as well as physical objects are reducible to psychological objects" (Carnap, 1967, p. 96).

As said above, Carnap has quite a relativistic view about "object", in that it seems that there is no deference between objects and concepts except for their level of construction (or, as we said, abstraction).

> In opposition to the customary theory of concepts, it seems to us that the generality of a concept is relative, so that the borderline between general and individual concepts can be shifted, depending on the point of view (cf. §158). Thus, we will say that even general concepts have their "objects". It makes no logical difference whether a given sign denotes the concept or the object, or whether a sentence holds

> for objects or concepts. There is at most a psychological difference, namely, a difference in mental imagery. Actually, we have here not two conceptions, but only two different interpretative modes of speech. Thus, in construction theory we sometimes speak of constructed objects, sometimes of constructed concepts, without differentiating. (Carnap, 1967, p. 10)

As we will see, this relativistic view on objects contrasts substantially with Frege's view on objects and concepts. Carnap also considers an epistemic order of objects that he calls "epistemic primacy", which he defines in this way: "An object (or an object type) is called *epistemically primary* relative to another one, which we call *epistemically secondary*, if the second one is recognized through the mediation of the first and thus presupposes, for its recognition, the recognition of the first" (*Ibid.*, §54, pp. 88-89). For example, an auto-psychological experience may lead one to recognizing a physical object and uses the name "apple" or construct the sentence "the apple is red"; recognition of "apple" is mediated by an elementary experience (immediately given and unanalyzable) and the recognition of "red" is mediated by the recognition of the apple. The main thesis in *Aufbau*, as Carnap says in the introduction of its latest edition, is that "it is in principle possible to reduce all concepts to the immediately given" (*Ibid.*, p. vi). We may see the picture that Carnap draws, regarding a (re)-constructing system, could be completely captured in a conceptual hierarchy that we have presented so far, with abstraction as the upward movement and interpretation (or reduction) as the downward one. Carnap is explicit in claiming that rational reconstruction "is the searching out of new definitions for old concepts" (*Ibid.*, p. v). There are many changes in his view (in the *Aufbau*) that Carnap wants us to pay attention to, one of which is the realization that "the reduction of higher level concepts to lower level ones cannot always take the form of explicit definitions; generally more liberal forms of concept introduction must be used" (*Ibid.*, p. viii). "Analogous considerations hold for the physicalist thesis of the reducibility of scientific concepts to thing concepts and the reducibility of hetero-psychological concepts to thing", which we will explain in a moment (*Ibid.*). According to Carnap, in *Aufbau*, "the reducibility of thing concepts to auto-psychological concepts remains valid, but the assertion that the former can be defined in terms of the latter must now be given up [...]" (*Ibid.*). While Carnap believes it is possible to draw conclusions concerning properties of individuals from "relation descriptions", he is well aware that "in the case of structure descriptions, this no longer holds true" because that is the limit of formalization since structure descriptions (i.e., syntax) "form the highest level of formalization and dematerialization" (*Ibid.*, p. 23). Regarding scientific statements compared to mathematical ones Carnap says the following:

> Thus, our thesis, namely that scientific statements relate only to structural properties, amounts to the assertion that scientific statements speak only of forms without stating what the elements and the relations of these forms are. Superficially, this seems to be a paradoxical assertion. Whitehead and Russell, by deriving the mathematical disciplines from logistics, have given a strict demonstration that mathematics (viz., not only arithmetic and analysis, but also geometry) is concerned with nothing but structure statements. However, the empirical sciences seem to be of an entirely different sort: in an empirical science, one ought to know whether one speaks of persons or villages. This is the decisive point: *empirical science must be in a position to distinguish these various entities*; initially, it does this mostly through definite descriptions utilizing other entities. But ultimately the definite descriptions are carried out with the aid of structure descriptions only. (Carnap, 1967, p. 23)

In other words, roughly speaking, the language we use in mathematics ought to be uninterpreted (i.e., having the possibility for alternative interpretations) while the language of science ought to be an interpreted language (more on this later in the next chapter). Carnap is clear that "in a structure description, only the structure of the relation is indicated, i.e., the totality of its formal properties" and "by formal properties of a relation, we mean those that can be formulated *without reference to the meaning of the relation and the type of objects* between which it holds"

(*Ibid.*, p. 21) (emphasis mine; more on "distancing from meaning" in the next chapter). Thus, we may see that, according to Carnap, referential meaning relation in formal systems is out of the question (while it is not the case for scientific systems). To give an example of what Carnap means by the "structure of a relation", he gives the following topological example:

> In order to understand what is meant by the structure of a relation, let us think of the following arrow diagram: Let all members of the relation be represented by points. From each point, an arrow runs to those other points which stand to the former in the relation in question. A double arrow designates a pair of members for which the relation holds in both directions. An arrow that returns to its origin designates a member which has the relation to itself. If two relations have the same arrow diagram, then they are called structurally equivalent, or isomorphic. The arrow diagram is, as it were, the symbolic representation of the structure. Of course, the arrow diagrams of two isomorphic relations do not have to be congruent. We call two such diagrams equivalent if one of them can be transformed into the other by distorting it, as long as no connections are disrupted (topological equivalence). (Carnap, 1967, p. 22)

Carnap considers advanced theoretical scientific statements to be in the same camp as the formal systems, except that the theoretical terms are not uninterpreted (unlike the formal ones), but their interpretations are always "incomplete". For this reason, and to have complete interpretations, one needs to appeal to "corresponding rules".

> The correspondence rules connect the theoretical terms with observation terms. Thus, the theoretical terms are interpreted, but this interpretation is always incomplete. Herein lies the essential difference between theoretical terms and explicitly defined terms. The concepts of theoretical physics and of other advanced branches of science are best envisaged in this way. At present I am inclined to think that the same holds for all concepts referring to hetero-psychological objects whether they occur in scientific psychology or in daily life. (Carnap, 1967, p. ix)

One should pay special attention to that, because, for Carnap, the terms "auto-psychological" and "hetero-psychological" relate to psychology in a philosophical sense (something similar to cognitive or intellectual faculty), and that they do not relate (directly) to psychology as a scientific discipline. Respectively, one may consider them as “subjective” and “intersubjective” in today’s literature. I believe that the reason Carnap avoided using terms such as “subjective” or “intersubjective” rests on the fact that he was not sympathetic to the idea of recognizing epistemology (as a theory of knowledge) as a part of philosophy since he is clear in that the main theme in *Aufbau* “is the aim of eliminating pseudo-problems from epistemology” (*Ibid.*, p. x); although, this topic is obviously out of the scope of our discussion, I will only refer you to the following quote as an example, regarding Carnap’s position on epistemology[38].

> *Philosophy is the logic of science*, i.e., the logical analysis of the concepts, propositions, proofs, theories of science, as well as of those which we select in available science as common to the possible methods of constructing concepts, proofs, hypotheses, theories. What one used to call epistemology or theory of knowledge is a mixture of applied logic and psychology (and at times even metaphysics); insofar as this theory is logic it is included in what we call logic of science; insofar, however, as it is psychology, it does not belong to philosophy, but to empirical science. (Carnap, 1984, p. 6)

Carnap is explicit, in the latest introduction to *Aufbau*, that the analytic system he is trying to present (in general, i.e., not limited to *Aufbau*) is based on the “auto-psychological domain” (Carnap, 1967, p. vii). As said above, Carnap’s goal is to present a method for analyzing reality via the theory of relations[39]. By referring to the examples of the applications of such a theory to non-logical objects, such as Whitehead’s “theory of extensive abstraction” and “theory of

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[38] Another example is:

> The designation “theory of knowledge” (or “epistemology”) is a more neutral one, but even this appears not to be quite unobjectionable, since it misleadingly suggests a resemblance between the problems of our logic of science and the problems of traditional epistemology; the latter, however, are always permeated by pseudo-concepts and pseudo-questions, and frequently in such a way that their disentanglement is impossible. (Carnap, 1937, p. 280)

[39] “The fundamental concepts of the theory of relations are found as far back as Leibniz’ ideas of a *mathesis universalis* and of an *ars combinatoria*. The application of the theory of relations to the formulation of a constructional system is closely related to Leibniz’ idea of a characteristics *universalis* and of a *scientia generalis*.” (Carnap, 1967, p. 8)

occasions", and Russell's construction of the external world, Carnap intended to use Russell's principle (i.e., wherever possible, logical constructions are to be substituted for inferred entities) in a more radical way: "We shall, however, employ this principle in an even more radical way than Russell (for example, through the choice of an auto-psychological basis, in the construction of that which is not seen from that which is seen, and in the construction of hetero-psychological objects)" (*Ibid.*, p. 8).

Carnap is clear that the first point one may draw from considering an auto-psychological domain for a constructional system is the unity of science (uniformity of objects[40]); in this way all diversities and varieties of objects basically come from the difference of methods and levels of abstraction (remember, as said above, for Carnap, even physical objects are abstractions).

> If a constructional system of concepts or objects (it can be taken in either sense) is possible in the manner indicated, then it follows that the objects do not come from several unrelated areas, but that *there is only one domain of objects and therefore only one science*. We can, of course, still differentiate various types of objects if they belong to different levels of the constructional system, or, in case they are on the same level, if their form of construction is different. (Carnap, 1967, p. 9)

By distinguishing between a "logical complex" and a "whole"[41], we find out that "the object state, for example, will have to be constructed in this constructional system out of psychological

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[40] Carnap gives the following analogy for the uniformity of objects:

> An analogy for the uniformity of objects and the multiplicity of different constructs is found in synthetic geometry. It starts from points, straight lines, and surfaces as its elements; the higher constructs are constructed as complexes of these elements. The construction takes place in several steps, and the objects on the different levels are essentially different from one another. Nevertheless, all statements about these constructs are ultimately statements about the elements. Thus we find different types of objects in this case, too, and yet a unified domain of objects from which they all arise. (Carnap, 1967, pp. 8-9)

[41] If an object is logically reducible to others, then it is a *logical complex* of these other objects, which are called its *elements*. If, on the other hand, an object, in relation to other objects, in such that they are its parts relative to an extensive medium, e.g., space or time, then we can call the first object the *extensive whole* of the other objects. The whole consists of its parts. (Carnap, 1967, §36)

processes, but it should by no means be thought of as a sum of psychological processes" (*Ibid.*, p. 8). Analyzing psychological processes would be the job of psychology as a scientific discipline. In other words, as we pointed out before, what we are facing is an impression; the final result of the interaction of our cognitive faculty as a whole with the reality (as a whole). Thus, what would be considered as an auto-psychological object is the first-hand result of such an interaction. Carnap finds the customary use of the terms "physical objects" and "psychological objects" vague and "logically impure" (*Ibid.*, §18). Using the word "object" in its widest sense, if we consider a "physical body" as a complex, we may say that this complex belongs to other objects, such as place, shape, size, and position, as well as sensory qualities such as color, weight, temperature, etc., which are all the determining characteristics of any physical body (*Ibid.*). Carnap makes no distinction between objects and events: "we make no distinction between events and objects" (*Ibid.*, p. 32). In the same manner "to the psychological objects belong, to begin with, the acts of consciousness: perceptions, representation, feelings, thoughts, acts of will, and so on" (*Ibid.*).

> The psychological objects have in common with the physical ones that they can be temporally determined. In other respects, a sharp distinction must be drawn between the two types. A psychological object does not have color or any other sensory quality and, furthermore, no spatial determination. Outside of these negative characteristics, psychological objects have the positive characteristic that each of them belongs to some individual subject. (Carnap, 1967, p. 33)

As we may see, objects of different types and ranges may appear at any level of construction (abstraction). As soon as we have object-words and means for communication (e.g., sentences, expressions, statements, etc.) we may enter the world of language and start logical analysis. Here is where we may disconnect ourselves with the actual world and start constructing logically (i.e., by preserving the "logical value" of the signs). Carnap contrasts the "logical values" of the signs, which would remain unchanged during construction, with their "epistemic values", which could change: "as regards object names, statements, and propositional functions, *it is concerned exclusively with logical, not with epistemic, value; it is purely logical, not psychological*" (*Ibid.*, p. 84). "A constructional transformation of a statement (or propositional function) always leaves the logical value, but not necessarily the epistemic value, unchanged. (In contradistinction to translations from one natural language to another, these transformations do not have to preserve the intuitive content)" (*Ibid.*). Carnap is quite clear that epistemic value is more related to the intuitive meaning (*der vorstellungsmässig sinn*) of the statements, which may frequently change.

> […] a constructional transformation leaves the logical value of a propositional function, as well as of a statement, untouched. We contrast this logical value with the "epistemic value". A constructional transformation may, for example, turn a true, epistemically valuable statement into a triviality; in such a case, we say that the "epistemic value" has been changed. But, since the trivial statement is also true, the logical value has not changed. (Carnap, 1967, p. 84)

The domain of auto-psychological objects is exactly the domain where we may start our logical analysis, during which the resulting abstractive objects would be gradually transformed into hetero-psychological ones. One may raise an objection to constructing hetero-psychological objects (i.e., the psychological occurrences in another person) on the basis of the physical indicators (namely, expressive motions and bodily reactions, including linguistic utterances) of the other person, and on the basis that, realistically, hetero-psychological occurrences are in reality something different from their reaction behavior, which plays only the role of an indicator. To this objection Carnap replies:

> Let *K* stand for the physical reaction behavior which is the indicator of a certain hetero-psychological process. The objection amounts, then, to the following: the concept of this hetero-psychological process is not itself identical with *K*, and therefore requires its own symbol, for example, *F*. To this objection, we make the following reply: all scientific (though not all metaphysical) statements about *F*, especially all statements which are made within psychology itself, can be transformed into statements about *K* that have the same logical value. Now, since *K* and *F* satisfy the same propositional functions, they are to be considered as identical (as far as logical value is concerned). No meaning for *F*, which is not identical with *K*, can be given in scientific (i.e., constructable) expressions. (This question is connected with Leibniz' thesis of the identity of indiscernible, cf. §51; and also with the problem of introjection and with the metaphysical component of the problem of reality, §175 f.). (Carnap, 1967, p. 86)

Although auto-psychological objects are available primarily to one subject, Carnap is explicit that, by the mediation of physical objects including the language used by other subjects, there is a way to recognize and be sure about hetero-psychological objects.

> It turns out that psychological processes of other subjects can be recognized only through the mediation of physical objects, namely, through the mediation of expressive motions (in the wider sense) or, if we assume a state of brain physiology which has not yet been reached, through the mediation of brain processes. On the other hand, the recognition of our own psychological processes does not need to be mediated through the recognition of physical objects, but takes place directly. (Carnap, 1967, p. 94)

Now, the question is how one could sort the physical and psychological objects according to their epistemic order. To this question Carnap provides the following answer:

> […] we have to split the domain of psychological objects into two parts: we separate the hetero-psychological objects from the auto-psychological objects. The auto-psychological objects are epistemically primary relative to the physical objects, while the hetero-psychological objects are secondary. Thus, we shall construct the physical objects from the auto-psychological and the hetero-psychological from the physical objects. (Carnap, 1967, p. 94)

Thus, we may depict the order of objects according to their epistemic primacy in the Table VII.

Epistemic Order	**1**	**2**	**3**
Objects	Auto-psychological	Physical	Hetero-psychological

Table VII Objects sorted by epistemic order

Carnap firmly believes that "it is possible to have a constructional system with a psychological basis", and that the logical justification for such a system is regardless of "any metaphysical standpoint" and rests solely upon proving that all cultural as well as physical objects are reducible to psychological objects, which he proves (Carnap, 1967, §§54-60). Carnap emphasizes that the psychological basis is independent of any subscription to "being real" or "being non-real", since those terms will appear later on in the constructional hierarchy.

> The differentiation between real and nonreal objects does not stand at the beginning of the constructional system. As far as the basis is concerned, we do not make a distinction between experiences which subsequent constructions allow us to differentiate into perceptions, hallucinations, dreams, etc. This differentiation and thus the distinction between real and nonreal objects occurs only at a relatively advanced constructional level. At the beginning of the system, the experiences must simply be taken as they occur. We shall not claim reality or nonreality in connection with these experiences; rather, these claims will be "bracketed" (i.e., we will exercise the phenomenological "withholding of judgment", "*εποχη*", in Husserl's sense). (Carnap, 1967, p. 101)

Carnap would rather consider the term "psychological", regarding "auto-psychological" basis, "as comprehending unconscious occurrences, but the basis consists only in conscious appearances (in the widest sense): all experiences belong to it, no matter whether or not we presently or afterward reflect upon them" (*Ibid.*). As said above, the term "auto-psychological" does not have any epistemological connotation as well.

> The expressions "auto-psychological basis" and "methodological solipsism" are not to be interpreted as if we wanted to separate, to begin with, the "*ipse*", or the "self", from the other subjects, or as if we wanted to single out one of the empirical subjects and declare it to be the epistemological subject. At the outset, we can speak neither of other subjects nor of the self. Both of them are constructed simultaneously on a higher level. (Carnap, 1967, p. 104)

Carnap emphasizes that the basis (on its own) should be considered completely neutral, even with regard to being "psychological" or "physical": "Before the formation of the system, *the basis is neutral* in any system form; that is, in itself, it is neither psychological nor physical" (*Ibid.*, p. 104). "With the aid of the qualitative, the spatial, and the temporal order, the world of physical objects is then to be constructed [on the basis of auto-psychological objects], and finally the further object domains, especially the hetero-psychological […]" (*Ibid.*, p. 134).

According to Carnap, the construction of hetero-psychological objects is fundamentally dependent on the "expression relation" (i.e., "the relation between expressive motions, i.e., facial expressions, gestures, bodily motions, even organic processes, on the one hand, and the simultaneous psychological events which are 'expressed' through them, on the other"[42]) (*Ibid.*, p. 212). The construction of the expression relation consists in the following: "to a class of auto-psychological events which frequently occur simultaneously with certain recognizable physical events of my body, we correlate the class of these physical events as 'expression'" (*Ibid.*). Hetero-psychological objects are to be constructed on the basis of "other persons", which are already constructed as "physical objects" via the expression relation just described. "There exists a certain correspondence between the world which we have constructed up to this point,

[42] Regarding this explanation, Carnap adds: "This explanation is not meant to be the constructional definition of the expression relation, since it would clearly be circular. It is really meant to refer to already known facts in order to provide a clearer understanding of the word" (*Ibid.*).

namely, "my world", and this "world of the other". Upon this correspondence, the construction of the *intersubjective world* is based" (*Ibid.*). Communication is the necessary means for the construction of hetero-psychological objects. "Aside from the expression relation, we shall also utilize the 'production of signs', i.e., information that the other person gives me" (*Ibid.*, p. 215). It is upon these interactions with (the) other person(s) that one produces new signs for new objects that go up the levels of construction. We should not forget that arriving at hetero-psychological objects may lead to rearranging the realm of the auto-psychological ones: "Thus, the entire *experience sequence of the other person consists of nothing but a rearrangement of my own experiences and their constituents*" (*Ibid.*). Carnap attracts our attention to two important points in this regard:

> [First] the construction of the hetero-psychological can be an assignment only to the *body* of the other, not to his mind, which, after all, cannot be constructed in any other way than through this assignment thus, constructionally, the other mind does not even exist before this assignment is carried out. Secondly, the assigned psychological events are auto-psychological events for the very same reason: the only psychological entities which have been constructed up to this point are auto-psychological entities, and no other can be constructed prior to this assignment; there is no possibility of constructing non-auto-psychological entities other than with the aid of precisely this assignment. (Carnap, 1967, p. 215)

Considering the basis of the construction (i.e., auto-psychological objects) and its products (i.e., hetero-psychological ones), and the procedure of arriving at the products (i.e., abstraction, as we explained), we may say that the construction basically consists of a move from auto-psychological objects to hetero-psychological ones by abstraction (i.e., the construction is a system of objects). Thus, you may have realized by now that in a Carnapian framework there is essentially no need for the object-concept distinction as what we are intended to do is to construct a system of objects. What this system entails will be discussed in the next section.

As a side note, I would like to add if we consider this move (from auto-psychological objects to hetero-psychological ones), as it may be represented in the discourses, inversely proportional to "disagreement" as an indicator of epistemic differences, we may be able to say that the more abstract the construction, the less disagreement in the discourses; since the move is from subjective matters to the intersubjective ones.

So far, and according to the all above explanations, we may clearly see that, in a Carnapian construction system, the following hold:

1. All objects are the results of abstraction (auto-psychological objects are abstractions and quasi-analysis is an abstractive method of construction).
2. One would arrive at each level of construction via abstraction.
3. "Objects" and "concepts" are not fundamentally different; they are relative terms which could be employed for objects of different levels of construction.
4. There is no distinction between objects and events.
5. Auto-psychological objects are the basis of the construction.
6. In the process of construction, the properties of objects (objects themselves) could be drawn from relation descriptions.
7. The whole construction could be considered as a voluntary move from the auto-psychological realm to the realm of hetero-psychological objects.

2.2. Characteristics of a Carnapian Framework

One of the key elements in Carnap's philosophy is the conception of "linguistic framework" (see chapter one), that he articulates in his famous paper (Carnap, 1950). A linguistic framework, in general, consists of three related parts, i.e., pragmatics, semantics, and syntax, and that by abstracting from pragmatics we move to semantics and from semantics to syntax. You may have noticed that Carnap uses the terms “pure semantics” and “pure syntax” but never uses “pure pragmatics”. The reason is that, as mentioned above, Carnap’s goal was to base everything on empirical data; appealing to pure pragmatics would therefore be a revocation of this goal. Here is what Charles Morris says in Schilpp’s volume:

> It is to be noted that Carnap calls these terms "descriptive" rather than "logical", and entitles the chapter which deals with them "Empirical Analysis of Confirmation and Testing". This fits in with his general tendency to regard pragmatics as an empirical discipline, and not to recognize the possibility of a pure pragmatics coordinate with pure semantics and pure syntactics. (Schilpp, 1963, p. 88)

How does Carnap consider language in terms of interpretation? According to Hintikka, Carnap is the first philosopher who considers language as an uninterpreted system[43] (see the following quote). Even in his *Logical Syntax* (Carnap, 1937), as noted by Hintikka (1975, p. LVII), Carnap conceived of languages only as uninterpreted calculi, but he rounds out this treatment of formal languages with a theory of interpretation of such languages.

> Carnap was the first philosopher to show the necessity to sharply distinguish between two levels of language in the logical analysis of language: The Object language, in which one represents the objects of inquiry, and which is construed by analysis as an uninterpreted calculus; and the metalanguage, which is an interpreted language and which is used to talk about the object language. Carnap was able to show that the confusion of these two levels of language, a failure to distinguish between object-linguistic and metalinguistic concepts, is responsible for the introduction of certain contradictions, and that this confusion led even such distinguished logicians and mathematicians as Bertrand Russell and David Hilbert into serious errors. (Hintikka, 1975, p. LV)

Thus, according to the above explanations, instead of the given-ness of the first-order concepts, the elementary experience is a given; that is, the elementary terms upon which we can construct

[43] Other Carnap commentators and scholars also looked at Carnap’s uninterpreted language system. For example, see Awodey and Carus (2007); Bueno (2016); Friedman (2009); McCarthy (1990).

a language (a language within which we try to reconstruct our knowledge of experience, and wherefrom we can perform analysis by abstraction) are given. Therefore, our existential claims would be deemed internal claims within the framework in question, assuming the given-ness of the experience. As explained before in chapter one, where we articulated the first and second method of construction in Carnap's philosophy, it is only in the first method of construction (bottom-up) that we make use of abstraction. In each method, there is a certain sense of arbitrariness but not in the absolute sense of the word. We may choose our pragmatic words and phrases at the bottom of the hierarchy, but there are constraints to be met, which may stem from, for example, the scientific phenomena under investigation, from our psychology, and from the structure of the employed language itself. Carnap is fairly clear that, in the first method, pragmatic and empirical criteria can be regarded as "practical guides" (or constraints) in setting up rules or making conventions (Carnap, 1939, p. 6). There are also systematic constrains, since "if we have chosen some rules arbitrarily, we are no longer free in the choice of others" (Carnap, 1939, p. 25). So, even though our first terms, phrases, or claims might be formed arbitrarily (in that sense) or depending on psychology (auto-psychology), through analysis they become more and more hetero-psychological and eventually logical.

Now we are able to assess a Carnapian framework. A Carnapian framework assumes or is committed to:

(1) An uninterpreted language system (i.e., abstract statements are neither true nor false).

(2) Intensional definition of object (i.e., defining objects in terms of concepts; as having state-descriptions).

(3) Existential claims are knowable assuming the given-ness of the elementary experience.

(4) Dependency of the conceptual hierarchy and its rules on pragmatics and/or pragmatic considerations.

We will see that if we are to understand abstraction as a process of producing new concepts, it would have a different sense compared to the next setting. If we want to distinguish these two senses, we may as well want to call this one *empirical abstraction*.

2.3 Frege and Fregeans

In order to compare and contrast a Carnapian system of object with the Fregean alternative (considering the inclusion of the object-concept distinction) and see the characteristics of the alternative versus the former, we may want to review a Fregean system and some of its philosophical basis in this regard. In this section, first, we are going to review some of Frege's philosophical positions. Our discussion will be followed by a very short review on the philosophical aspect of neo-Fregeans' modification without going into much detailed discussion. It seems that for neo-Fregeans the most important issue is the mathematical sense or application of abstraction as they are often silent about the general philosophical positions regarding abstraction and its role in linguistic or thought analysis. Thus, in the following, it is our assumption that the neo-Fregeans share some of their philosophical positions with that of Frege (such as Frege's famous anti-psychological position). Frege does not propose a psychological basis for logic, which leads, as we will see, to his well-known anti-psychological position against concepts deemed to be logical[44].

> The most reliable way of carrying out a proof, obviously, is to follow pure logic, […]. Accordingly, we divide all truths that require justification into two kinds, those for which the proof can be carried out purely by means of logic and those for which it must be supported by facts of experience. But that a proposition is of the first kind is surely compatible with the fact that it could nevertheless not have come to consciousness in a human mind without any activity of the senses. Hence it is not the psychological genesis but the best method of proof that is at the basis of the classification. (Frege, 1967, p. 5)

Based on this division in the above quote, Frege is explicit that by "concept" he means strictly a logical (and not psychological) entity.

> The word 'concept' is used in various ways; its sense is sometimes psychological, sometimes logical, and sometimes perhaps a confused mixture of both. Since this licence exists, it is natural to restrict it by requiring that when once a usage is adopted it shall be maintained. What I decided was to keep strictly to a purely logical use; the question whether this or that use is more appropriate is one that I should like to leave on one side, as of minor importance. (Frege, 1951, p. 168)

Frege is also clear that a concept has nothing to do with properties of an object.

> From the prevailing logic I cannot hope for approval of the distinction that I make between the mark of a concept and the property of an object, for it seems to be thoroughly infected by psychology. If people consider, instead of things themselves, only subjective representations of them, only their own mental images -then all the more delicate distinctions in the things themselves are naturally lost, and others appear instead which are logically quite worthless. (Frege & Geach, 1960, pp. 145-146)

[44] Russell seems to be in complete agreement with Frege's anti-psychological position since in a letter to Frege he says "I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [Moment] in logic and when you place a high value upon an ideography [Begriffsschrift] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished" (Van Heijenoort, 1967, p. 124).

Elsewhere Frege dismisses the role of psychology in the structure of concepts (meaning that concepts have their own rules) and puts mathematics over and above science and philosophy.

> Thought is in essentials the same everywhere: it is not true that there are different kinds of laws of thought to suit the different kinds of objects thought about. Such differences as there are consist only in this, that the thought is more pure or less pure, less dependent or more upon psychological influences and on external aids such as words or numerals, and further to some extent too in the finer or coarser structure of the concepts involved; but it is precisely in this respect that mathematics aspires to surpass all other sciences, even philosophy. (Frege, 1960a, pp. xv-xvi)

Thus, it is important to know the laws that govern the concepts irrespective of objects. Frege explicitly denies the role of psychology in any forms and shapes that may contribute to the foundation of arithmetic.

> It may, of course, serve some purpose to investigate the ideas[45] and changes of ideas which occur during the course of mathematical thinking; but psychology should not imagine that it can contribute anything whatever to the foundation of arithmetic. (Frege, 1960a, p. xviii)
>
> We suppose, it would seem, that concepts sprout in the individual mind like leaves on a tree, and we think to discover their nature by studying their birth: we seek to define them psychologically, in terms of the nature of the human mind. But this account makes everything subjective, and if we follow it through to the end, does away with truth. What is known as the history of concepts is really a history either of our knowledge of concepts or of the meanings of words. (Frege, 1960a, p. xix)

Thus, one of the Frege's fundamental principles is "always to separate sharply the psychological from the logical, the subjective from the objective" (Frege, 1960a, p. xxii). Regarding abstraction, Frege clearly denies that there is any such process that links objects and concepts when he says that: "An object, again, is not found more than once, but rather, more than one

[45] "Idea" is a psychological term for Frege. Austin explain this in the footnote of page xvii (*Ibid.*): "*vorstellungsmässig* I have translated this word consistently by 'idea', and cognate words by 'imagine', 'imagination', etc. For Frege it is a psychological term".

object falls under the same concept. […] a concept need not be acquired by abstraction from the things which fall under it […]" (Frege, 1960a, p. 69).

Now, let us take a closer look at some of Frege's philosophical positions, to clearly see why he does not want to deal with abstraction at all, aside from it being psychological. By downgrading abstraction, nevertheless, there remain some important philosophical questions that Fregeans have to deal with. Those questions, even when abstraction is brought to the fore (by neo-Fregeans), remain open. It seems the main problem is that there is a philosophical twist in Frege's method for conceptual analysis, which is exactly located at the level of first-order predication. From this level up, the rules for arriving at new concepts are clear and given via Hume's Principles (or its generic version: abstraction principle), while to arrive at objects the only possible way is via extensional reference (downward[46]). The basis of this twist could be tracked back to the collision of two of Frege's most famous distinctions: the object-concept distinction and the sense-reference distinction.

2.3.1 Frege

For Frege, "[to] abstract from something simply means: not to attend to it specially" (Frege & Geach, 1960, p. 84). Although Frege reserves room for abstraction in arriving at some concepts, as it is evident in the following quote (pay attention to the chain of generic terms), he believes appealing to abstraction epistemologically is of no value for arriving at certain kind of

[46] During our discussion, we will have the above-described conceptual hierarchy in our minds (objects at level zero and all concepts at the higher levels); hence, any reference or use of the words "upwards" or "downwards" is meant to be with regard to this hierarchy.

knowledge (i.e., mathematical knowledge) at least. Given Frege's project (Logicism), we may find Frege's general philosophical standpoint quite like that of the Cartesian rationalists (or that of the Platonists): there already exist ontologically pure distinct species (entities and relations) that are mind/experience independent; we just come to realize them through our thoughts, and ought to follow them inevitably.

> By abstraction we do indeed get certain concepts, viz. satellite of the Earth, satellite of a planet, non-self-luminous heavenly body, heavenly body, body, object. But in this series 1 is not to be met with; for it is no concept that the moon could fall under. In the case of 0, we have simply no object at all from which to start our process of abstracting. It is no good objecting that 0 and 1 are not numbers in the same sense as 2 and 3. (Frege, 1960a, p. 44)

Abstraction from actual phenomena to concepts (including the first-order concepts) makes no sense, for Frege, since the actual phenomena are transient while concepts are fixed, constant, and governed by their own rules. Thus, if we ask what the first-order concepts are he would say they are basically impressions of "a unique sort" (which means either they are already there, or they are being constructed and wired in us in a sort of intrinsic way).

> It would indeed be remarkable if a property abstracted from external things could be transferred without any change of sense to events, to ideas and to concepts. […] It does not make sense that what is by nature sensible should occur in what is non-sensible. When we see a blue surface, we have an impression of a unique sort, which corresponds to the word "blue"; this impression we recognize again, when we catch sight of another blue surface. (Frege, 1960a, p. 31)

We could say Frege does not construe abstraction as a general intellectual tool for arriving at concepts in general, especially when it comes to discovering the foundations of mathematics; that might be why he did not need/try to emphasize abstraction and give it a major role in his philosophy. According to Greimann (2007, p. 120), Frege himself never called Hume's

Principle an "abstraction principle" (as it is later called by the neo-Fregeans), despite recognizing Russell's position on "definition by abstraction".

2.3.1.1 Two Major Distinctions

As said above, the two major distinctions in Frege's philosophy are: the basic object-concept distinction and the sense-reference distinction. Whether we consider the distinctions as purely epistemological, or as having a bearing on ontology, it seems that they may have conflicting outcomes regardless; so that one ought to resolve the conflict in one way or another. In the following, we want to unfold, or highlight, some of the consequences of these distinctions that are important and/or related to our topic at hand, namely, abstraction.

Mendelsohn (2005, p. 64) sees the conflict as what leads to "the prevalent confusion" about Frege's semantics for function-expressions which, he thinks, "is not solely the fault of Frege's commentators". It may as well be due the philosophical setting of Frege's philosophy. Mendelsohn singles out two points of tension in this regard. The first point is related to the object-concept or object-function distinction:

> On the one hand, there is the concept/object distinction Frege had drawn in the course of analyzing the notion of cardinal number: Number, Frege had insisted, is a property of concepts, not of objects. On the other hand, there is the later distinction Frege had drawn that was keyed to underpinning the construction of linguistic expressions: here the crucial feature of the distinction was the "unsaturatedness" of functions and the "saturatedness" of objects. (Mendelsohn, 2005, p. 64)

Later on, Frege identifies concepts with the referents of concept-expressions; but the arguments for the concept/object distinction would lead one to suppose that concepts ought to be identified with the senses of function-expressions instead. For example, the concept of "having heart" and "having kidney" might be referentially identical (if they have the same extension), but they have different senses, of course. On the other hand, if one is to identify concepts with the sense of functional expressions (rather than with what they refer-to), since concepts are meaningful if, and only if, they have objective referential extensions, then, reference to objects is still necessary for meaningful concepts so that the concept of "pink elephant" becomes meaningless. This situation might be caused because, even though structurally, we know that first-order concepts range over objects (second-order concepts range over the first-order concepts and so on), epistemologically, access to the higher-order concepts is possible via Hume's Principle (upward). At the same time, due to the fundamental object-concept distinction, the access to the objects is only possible by reference (this is also true for a higher-order concept i.e., the access from the higher-order concept to the lower ones is possible via looking for its extension downward). As mentioned above, Frege considers the first-order concepts as constant "impressions". Frege does not want to restrict himself to a definition of object, since he regards a regular definition as "impossible", thus he defines object unrestrictedly (basically defines concepts in terms of objects) this way: "An object is anything that is not a function, so that an expression for it does not contain any empty place" (Frege, 1960b, p. 32).

The second source of confusion, according to Mendelsohn (2005, p. 64) is due to the fact that Frege holds the object-function distinction both at the level of sense and reference. Mendelsohn thinks that, although there might be nothing wrong with this, there is a point of confusion: while the interpretation of the distinction, referentially, has a *function-argument* structure, it has a *part-whole* structure at the level of sense. Mendelsohn believes that this part of Frege's philosophy is "hopelessly obscure".

> […] at the level of reference, it is interpreted as a function/argument structure, while at the level of sense it is interpreted as a part/whole structure. It is not clear how these two very different instantiations of the function/object structure are supposed to be integrated. Furthermore, because Frege himself confused levels and sometimes spoke of part/whole at the level of reference, and sometimes spoke of function/argument at the level of sense, the lessons supposed to be learned by his reader are hopelessly obscured. (Mendelsohn, 2005, pp. 64-65)

Regarding the sense-reference distinction, there is yet another question that we may want to pay attention to, and that is whether language ought to be considered as an interpreted system or not. In a Fregean analytic system, the core of the famous discussion on the ontological commitments of a theory could rest upon the fact that general terms are meaningful as long as they are associated with objective referential extensions. In other words (in the case of monadic predicates), if we know F, "$(\exists x)Fx$" is true if and only if F is satisfied by some object a; or (in the case of substitutional interpretation of quantifiers) if there exist some substitution instances of "$(\exists x)Fx$" by proper names or definite descriptions. With this description in mind let us talk about highly abstract statements such as axioms (or general laws in physics) and see how the truth of such statements is considered in a Fregean system.

From yet another angle, we may look at abstract statements instead of abstract terms. It is customary nowadays, in the literature, to speak of axiomatic systems (and/or general laws of physics) and interpretations[47] as particular examples of abstract systems specially since "[i]n his

[47] See for example Wilder (2013, p. 24), where he is clear that "If $\sum$ is an axiom system, then an *interpretation* of $\sum$ is an assignment of meaning to the undefined technical terms of $\sum$ such that the axioms become simultaneously true statements for all values of the variables".

mature work Frege is very explicit that axioms are abstract thoughts" (Beaney & Reck, 2005, p. 324). We may consider axioms (and/or general laws, in science) as abstract statements in the sense that they do have the possibility of having alternative interpretations. To give an example in the modern days, we may say the axioms of measure theory[48] in mathematics are abstract in the sense that they may have interpretations for measuring probabilities, as well as other examples of continuous measurable spaces, e.g., length. Or, the laws of simple harmonic oscillators may be interpreted equally to explain local laws of spring systems, strings, and even some electromagnetic phenomena. This could mean that the statement in question itself (alone and regardless of its interpretations) is neither true nor false, unless it is interpretable (in my view, it could be nonetheless meaningful simply by the virtue of interpretability).

As we will see in the following, since Frege considers language as a fully *interpreted* system, as noted by Antonelli and May (2000), meaningful axioms cannot therefore be anything other than true. This position is hard to digest, especially regarding axiomatic non-Euclidian geometries (in which the axioms are not only interpretable differently but also independent from each other).

According to Antonelli and May (2000, p. 224), Frege's endorsement of the claim that "Axioms are true and could not be otherwise", during his famous debate with Hilbert on the foundation of geometry, not only means that no false proposition could be an axiom but also means that if a proposition is genuinely an axiom, then it is not even sensible to consider it to be other than true. This position is, of course, a consequence of Frege's doctrine of sense and reference, so

[48] See for example Bogachev (2007); Halmos (2013); Tao (2011).

that if it were possible to consider axioms (propositions themselves) to be other than true, they would be excluded from the domain of meaningful propositions (i.e., those that express thoughts that determine a truth value; note: they cannot be false), which is an "absurd result to Frege's mind" (*Ibid.*).

> All we need observe is Frege's presumption that propositions too have references, to the True or the False, and because this reference is determined by the thoughts they express, each proposition comes immutably equipped with one and only one truth value. Now we just transpose the argument just outlined—we can make no greater sense of considering a true thought (that is, a thought whose reference is the True) to be false, for to do so would not be to consider that thought, but another. (Antonelli & May, 2000, p. 246)

As Antonelli and May (2000, p. 245) point out, given that, for Frege, a basic linguistic notion is made up of a sign, pairings of a symbol (or a formal mark), and sense, which is expressed by the symbol. Language is a system of signs, where any alteration in the pairing of symbols and senses constitutes a change in the signs, and thereby creates a different system of signs, that is, a different language.

> What is important to observe here is that for Frege, since a language is a system of signs, it is an interpreted system. This simply follows from the doctrine that sense determines reference. Moreover, it is a uniquely interpreted system since sense uniquely determines reference; thus, a change in the system of signs would not just be a different interpretation for the language but a different language altogether. (Antonelli & May, 2000, pp. 245-246)

It is important to notice that once we consider, as Frege does, an interpretation(s) as a necessarily inherent assignment(s), then we are already presupposing a rationalist philosophical position about the relation between an axiom (as an abstract statement) and its interpretation(s) as a requirement for meaning, under which changing the interpretation necessarily requires the change of language. Axioms are necessarily true/meaningful due to them having a true interpretation from which the axioms are abstracted; thus, an alternative interpretation that disturbs the fixed meaning-relation between the axiom and its interpretation, which is an absolute requirement in recognizing a language, therefore implies that one should abandon the language in question altogether.

Antonelli and May (2000, p. 284) are certain that "there is no room in Frege's conception for the notion of an alternative interpretation for a given language; indeed 'the word 'interpretation', Frege says 'is objectionable, for when properly expressed, a thought leaves no room for different interpretations'"

> As if it were permissible to have different propositions with the same wording! This contradicts the rule of unambiguousness, the most important rule that logic must impose on written or spoken language. If propositions having the same wording differ, they can do so only in their thought-content. Just how could there be a single proof of different thoughts? This looks as though what is proved is the wording alone, without the thought-content; and as though afterwards different thoughts were then supposed to be correlated with this wording in the different disciplines. Rubbish! A mere wording without a thought-content can never be proved. (Frege, 1971, pp. 79-80, as cited by Antonelli & May, 2000, p. 2284)

As shown above, the fixed axiom-interpretation relation stems from the sense-reference distinction[49]. One could easily see that the latter is reflected on yet another aspect of Frege's philosophy, which is the object-concept distinction (especially at the first level, which could be held between an actual object and a concept). It is famously known that, for Frege, objects are strictly the values of first-order concepts. In other words, if $F(x)$ is a sentence and F is a first-order concept, then x ranges over the objects and only objects. In the same way that an axiom

[49] Carnap also believes that Frege's sense and reference distinction has "serious disadvantages" (Carnap, 1947, p. 2).

stays in a fixed relation with certain interpretation, the term "object" is limited to be referred to by certain concepts; therefore, it might be obvious that holding on to a referential conception of "object" needs (makes it mandatory) an appeal to a meta-ontology of concepts; this conception could be deemed as independent of the observer and experience in some sense.

Furthermore, Frege's insistence on language as being an interpreted system, and his conception of "object", puts him in a peculiar position against logic, if one considers logic as an object of its own. I agree with Antonelli and May (2000, p. 251) in saying that "Frege's conception of logic sharply varies from the modern conception in ways that do not easily translate over to the contemporary viewpoint". As Antonelli and May noted (*Ibid.*), two different perspectives toward logic are distinguishable: in one view, logic is exhausted by the practice of deriving consequences; while, in another view, logic, itself, is an object of a formal investigation. Based on the first view, "one can use logic to prove theorems without any conception that the ways by which logic does this can itself be the locus of inquiry"; whereas, per the second view, the main question is what gives rise to logic and how it works; or, what properties logic has in order to make it suitable for a derivation tool, and why is it as such. To answer the latter types of questions, one, of course, needs an appeal to a meta-theory (i.e., a theory within which one could study properties such as soundness, completeness, independence, and so on). According to the above explanation, it seems that Frege's adherence to his philosophical position and assumptions about language preclude him from being meta-theoretical about logic. Antonelli and May (2000, p. 258) show us the similarity between Klein's famous Erlangen Program (i.e., to identify geometric notions as those that are invariant under a certain class of transformations) and Tarski's treatment of logical notions (i.e., to characterize the logical notions as those that are invariant under all permutations of the domain). Based on this similarity and by showing that Frege's method for showing independence is quite like the method of permutation in the Tarskian characterization of the logical terms, although they argue about that, in principle, Frege's access to a meta-theory is not completely blocked, but, they are also clear that for Frege himself, at the time, it was.

> There is a property that all logical truths have, but it is not a property that distinguishes them from non-logical truths. It is that they are all propositions in Frege's sense, and hence inherently contentful, that is, express thoughts. […] this is sufficient, given Frege's conception of language as an interpreted system with its roots in the doctrine of sense and reference to rule out a conception of metatheory that requires us to be able to consider alternative interpretations for a given language. If we are seeking a locus of the difference between Frege's view of logic and the modern views, we find it here in this incompatibility with a model-theoretic conception which formally requires an uninterpreted language. (Antonelli & May, 2000, p. 255)

According to the above explanations, we may conclude that language, for Frege, is an interpreted system and the conception of "object" is unrestricted and is fixed via reference. We will elaborate on the latter in the following section.

2.3.1.2 Individuation, Sameness and Identity

The question in this section is to see how one should look at the objectual identity relation from a Fregean stand point. We will find out that identity (of an object) could be only achieved from the intersection of the concepts. The philosophical issue in this regard is that, in this way, the understanding of concepts becomes prior to the understanding of objects. Given the independent ontological status of concept, one may be forced to believe that it is the already-established internal concepts that eventually identify factual objects; especially since, as said above about

Frege, our relationship with the objects primarily rests upon recognizing what are our "impressions" and whether they are the same.

Frege is quite clear that "objective ideas can be divided into objects and concept"(Frege, 1960b, p. 37). Frege's conceptual analysis may consider objects (physical ones and/or otherwise) on which he builds up a hierarchy of concepts. For example, the concept "… is red" is a first-order concept because it ranges over the objects (physical ones). It also is the case for the concept of "… is prime", which also ranges over objects (abstract ones). Dummett (1973, p. xx) is clear in that "Frege holds that there are mathematical objects which are not created by us (either in the mind or on paper), have not come into being nor will cease to be, but exist independently of us". In order to resolve some of the issues with respect to identity and sameness (mentioned below), some modern scholars suggest that one should distinguish between the application of the abstraction principle whence it is applied for abstracting concepts from objects in distinction of when it is applied for abstracting concepts from concepts. According to Fine (2002, p. 11), "a principle of abstraction is said to be *conceptual* when the items upon which it abstracts are concepts and it is said to be *objectual* when the items upon which it abstracts are objects".

In his attempt to define numbers explicitly, Frege (1960a) defines the extension of a concept as the objects (or concepts of a lower order) falling under the concept in question, e.g., the extension of "red" is the collection of red objects or is all the objects that satisfy the concept of "being red". Thus, by appealing to Hume's Principle (*Ibid.*, §63), which is considered implicit or contextual definition of numbers, and states that: the number of *F*s is equal to the number of *G*s if and only if *F* and *G* are equinumerous (i.e., there is a bijection or a one-to-one relationship between what falls under F and what falls under of G). Via this contextual definition, Frege then gives an explicit definition of numbers, i.e., the number of Fs is the extension of the concept "being equinumerous to F". Frege unsuccessfully[50] defines the number of Fs as the extension of the concept "being equinumerous to *F"*.

Let us go back a little bit to the way in which we identify objects, and to ease the issue, let us talk about material objects (and natural language). In a Fregean way, as mentioned above, there is a distinction between objects and concepts (reminder: a firs-order predicate expresses a sense, and, by virtue of expressing this sense, it stands for a concept and its extension); thus, the first-order *abstraction* of concepts from objects is out of the question since, at the first level, we identify concepts in terms of objects and not vice versa. For example, in the sentence "this is an apple", the concept "… is an apple" (or "being apple") is referring to a bundle of objects among which the particular apple (the object in question) exists[51]. An obvious problem associated with this method, once employed for defining objects, is that "an object" is a concept itself by definition (but the concept of "object" is not a concept), and if we consider it as such, identifying its extension is not like other first-order concepts such as "apple" since it has to be stipulated (without stipulation its extension could be finite or un-countably infinite). Frege dismisses this problem (the first clause), blaming it on the "awkwardness of language".

[50] It is famously known that due to the inconsistency in Frege's Basic Law V (which says: the extension of the concept F is equal to the extension of the concept G if and only if F and G are equinumerous; or simply: given any property, there is a set whose members are just those entities possessing that property. Basic Law V was eventually refuted due to Russell's Paradox) and the problem which is commonly referred to as the "Julius Caesar Problem" (the problem is that there is no axiom to exclude statements such as "Julius Caesar is number nine", for example), his project for giving an explicit definition of numbers failed. See Greimann (2003); Heck (1997, 2005); Sullivan and Potter (1997).

[51] The alternative way which allows abstraction, as we will see, is to define objects in terms of concept, hence considering the object in question as a bundle of concepts. Thus, in principal, an object would be identified via a list of properties that exactly identify that particular object (including the knowns and the ones yet to be known).

> It must indeed be recognized that here we are confronted by an awkwardness of language, which I admit cannot be avoided, if we say that the concept horse is not a concept, whereas, e.g., the city of Berlin is a city and the volcano Vesuvius is a volcano. Language is here in a predicament that justifies the departure from custom. (Frege, 1951, p. 172)

Apart from this problem, the main question (of value for philosophers of science) is how one can identify the exact same particular, using this analytic method. This brings us to the question of sameness and the identity relation, on one hand, and the question of making sense of what exactly, in this case, "being an apple" means, on the other hand. It seems that the only way of making sense of, for example, "being an apple" is to repeatedly point out its instances[52]; since we understand (and individuate) objects by the rules of concepts, i.e., we understands object by understanding concepts and what falls under them. Wiggins (2001), in his attempt to launch a theory of individuation, discusses this issue in more detail.

> To understand a predicate and know what concept it stands for is to grasp a rule that associates things that answer to it with the True and things that don't answer to it with the False. (The extension of the concept is therefore the inverse image of the True under the function determined by this rule). To grasp the rule is to grasp how or what a thing must be (or what a thing must do) in order to satisfy the predicate. To grasp this last is itself to grasp the Fregean concept. (Wiggins, 2001, pp. 9-10)

This means that the question of "if *a* is the same as *b*?" should be, in fact, broken down to if "*a* is the same *F* as *b*" or "the same *G* as *b*" or "the same… as *b*"? And the only way for looking for all the mentioned properties in one place is to look for the intersection of *F* & *G* & …. Wiggins formalize this idea in the following way:

$$a = b \leftrightarrow (\exists f)\big(a =_f b\big)$$

[52] We may observe that this is exactly the method that some computer softwares (that are built upon Boolean/Fregean logic) employ today for concept formation, which are also considered as being not very efficient; see Lake, Salakhutdinov, and Tenenbaum (2013) for example.

In other words, in Frege's system, the identity relation has to be delivered by a second-order identity relation (since it quantifies over concepts[53]). Despite its rationality, the idea behind this relationship seems to be contrary to the actual situations in which one normally encounters a first-order concept (like red); otherwise an assembly of concepts (like apple) is enough to identify the next instance. Frege, of course, would say that the ability of identifying "red" totally depends on our impressions from the actualities which are in certain ways constant; thus, arriving at "red" or any other "color", for that matter, is not a question that we should be concerned about; and, as mentioned above, this rests upon the assumption that we do not abstract first-order concepts from the object, *per se* (for example abstracting red from observing a red apple); rather we are reciting our constant impressions on the observed entities. Thus, the identity of the object in question (via the identity relation among objects) ought to be delivered by a second-order identity relation among concepts (just explained). It should be clear now that, according to a Fregean system, the identity of a zero-level object is not viewed as what makes an object an object, rather it is only accessible via a second-order identity among the already-known concepts.

In fact, it has been proven by some neo-Fregeans that appealing to the second-order logic in some cases, such as arithmetic, can give an account for some first-order relations, such as Peano Axioms. Whether this project is extendable to all cases (including the analysis of natural language) is still an open question, but it remains a goal for neo-Fregeans.

[53] The detailed and more technical discussion is intentionally avoided; for that see Wiggins (2001).

2.3.2 Neo-Fregeans

The important development, made by neo-Fregeans, was the realization that by taking Hume's Principle in a restricted form, one could avoid Russell's paradox. Thus, Neo-Fregeans abandoned the explicit definition of number, yet preserved a generic version of Hume's Principle and called it "abstraction principles" (among other names). In what follows, we will give an overview of their project and why some of the problems mentioned above persist even in this new form of logicism. We want to show that there is a cognitive element associated with abstraction (namely, recognizing objects as assemblies of concepts) that is hard for many forms of logicism. As it stands the principle is meant to be purely mathematical or perhaps logical; whether it could have applications in linguistic analysis is another question.

After the failure of the above-mentioned attempt in explicitly defining numbers, the so-called "Frege's theorem" (i.e., the Peano axioms can be derived from Hume's principle), formulated in the second-order logic, was formally proven by Wright (1983)[54]. Since then, the principle was taken to be the bedrock of a new school in the philosophy of mathematics, known as neo-Logicism, neo-Fregeanism, or simply Abstractionism (Cook, 2007, p. xvi). As said above, Hume's Principle (i.e., the number of *F*s is equal to the number of *G*s if and only if *F* and *G* are equinumerous) is considered to be an implicit definition of numbers, while an explicit definition is that the number of *F*s is the extension of the concept of "being equinumerous with *F*". Frege abandons the implicit definition in favor of the explicit one, and it seems that the neo-Fregeans are taking just the opposite strategy. To wit:

[54] According to Cook (2007), the Peano axioms from Hume's Principle was "extrapolated" from Frege's comments, and also by Boolos (1990); Boolos and Heck (1998); Heck (1993).

> While Frege, according to Hale and Wright, "abandons" the contextual definition, which he initially considers, and adopts the explicit one, the neo-Fregeans do precisely the opposite: they retain the contextual definition, or Hume principle, and abandon the explicit one. Their reason is that the latter brings in sets (extensions), and with sets, the Russell contradiction. (Angelelli, 2004, p. 88)

The generalized version of Hume's Principle is now known, in today's literature, as the "principle of abstraction", and can be formulated in the following way:

$$\forall\alpha\forall\beta[\Phi(\alpha) = \Phi(\beta) \leftrightarrow E(\alpha, \beta)]$$

in which Φ denotes the new concept (mathematically, it is a one-place function that is mapping entities of the type ranged over by α to some objects), and *E* is an equivalence relation, which is a two-place relation denoting any sort of resemblance, a similarity (in a strict sense) or equivalency that is held by α and β[55]. "Thus, an abstraction principle is meant to act as an implicit definition of sorts, providing [...] an account of the meaning of novel terms of the form [$\Phi(\alpha)$]" (Cook, 2007, p. xvii). An equivalence relation *E* defines "clusters" of elements over the domain *D*, which determines equivalence classes, that could partition the domain. More formally, we may say for each $a \in D$ there is a set of,

$$[a]_E = \{b \in D: aEb\}$$

that is called the equivalence class of α (with respect to *E*). From a practical point of view, what the principle is saying is that if, for instance, one can spot any similarity between two objects (or instances of a similarity that is necessarily transitive), one may simultaneously have access to a new concept by which one can define a term and vice versa. Linguistically speaking, we may say an instance φ_1(e.g., being red) of Φ is just the name accessible via the instance e_1(e.g., sameness of color) of *E*; that is to say e_1 means φ_1 implicitly (and vice versa), in the same way

[55] As a side note, I would like mention that there also exists modifications of the abstraction principle (Jaakko & Hintikka, 1956, 1957), that were critically assessed (Lake, 1973) and are not popular.

that “being parallel” for straight lines means the “sameness of direction” (Frege, 1960a, p. 64), where the concept of “direction” is more generic than the specific case of being “parallel”; that is to say, according to Frege, “we carve up the content in a way different from the original way, and this yields us a new concept” (*Ibid.*); thus, here, he points out a process by which one obtains a new concept. (Put this in a linguistic perspective, we may say to call two different instances “red” is achievable by appealing to the “sameness of color”, while “color” is more generic than “red”.) Or, as Frege says, “from geometrical similarity is derived the concept of shape, so that instead of ‘the two triangles are similar’ we say ‘the two triangles are of identical shape’ or ‘the shape of the one is identical with that of the other’” (*Ibid.*).

As we said above, the mentioned philosophical twist regarding the upward accessibility by Hume’s/Abstraction Principle and the downward accessibility by reference persists in the neo-Fregean project as well. Angelelli (2004, pp. 87-96) argues that the neo-Fregean program is flawed by a basic inconsistency, that is inherited from Frege’s method (i.e., combining the explicit definition with Hume’s principle, as indicated), and which causes philosophical problems. In the following, Angelelli points to a situation in which only the upward move is allowed, and the downward move is not consistent philosophically.

> If the philosopher of arithmetic pledges to be totally contextual with regard to the semantics of the singular terms of the form "the number of the concept F", and does not plan to assign any denotation to them (equivalently, if the philosopher wants to use only the Hume principle as the source of arithmetical knowledge), then there is no problem[…]If, however, the philosopher of arithmetic pledges to answer the question "What is number?", and assumes that the answer is to assign a denotation to "5", then he will find himself in the situation in which Frege found himself […], and will have to either transcend pure contextuality, adopting some explicit definition (as Frege did) or rethink the very significance of the biconditional called "Hume's principle" and restart the project in an entirely different way. (Angelelli, 2004, p. 89)

According to Angelelli (*Ibid.*, p. 94), "a philosopher or any ordinary person interested in learning what is, for example, 5, or the number of the concept fingers of my left hand, will be disappointed both by Frege himself as much as she was disappointed by the neo-Fregeans". By extension, one could assume that this defect persists in speaking of any objects (other than just numbers), for that matter. Neo-Fregeans, in their pure contextuality plan of basing everything upon Hume's principle, say nothing about what the number of the concept of "fingers of my left hand" is (Frege gives an answer, but it is unjustified).

It might be fair to say it is because of the mentioned-philosophical twist that neo-Fregeans eventually ought to take a rationalistic position and submit to assuming *a priori* knowable existential claims. Cook (2007, p. xxiv) acknowledges that the lack of some general criteria for distinguishing between acceptable and unacceptable abstraction principles would leave us with "too many abstraction principles". By referring to Boolos' objection[56], Cook describes how abstractionists hopelessly have to take a rationalistic position.

> One of the main (supposed) advantages of abstractionism is that abstraction principles imply the existence of more objects than we would expect from logic and definitions alone. Some (including, of course, Boolos) have objected to this, on the grounds that logic (or analytic statements, or a priori knowledge more generally) should not imply the existence of all (or most) of the objects studied by working mathematicians.[…] Nevertheless, abstractionism is hopeless without the assumption that at least some existential claims are analytic, or a priori knowable, or something similar – the position in question is (on one reading) nothing more than a detailed philosophical account of how such [a position] is possible. (Cook, 2007, pp. xxvii-xxviii)

[56]As cited by Cook (2007, p. xxviii),

> It was a central tenet of logical positivism that the truths of mathematics were analytic. Positivism was dead by 1960 and the more traditional view, that analytic truths cannot entail the existence either of particular objects or of too many objects, has held sway ever since. (Boolos & Heck, 1998, p. 305)

As we may see, assuming *a priori* knowable existential claims are the textbook definition of a rationalist position. Thus, according to the above quotation, we may say abstractionists, in a neo-Fregean sense, ought to take a rationalist position in this regard eventually.

2.3.2.1 Aprioricity and Objects

It is evident that individuation is intimately tied with how we consider an "object" to be. Neo-Fregeans tends to consider the abstraction principle as "analytic" in the sense that it is universally applicable, it stands on its own, and is a priori knowable. After Boolos (2007, pp. 3-6) raises serious concerns about the aprioricity of the principle itself, he raises some epistemological concerns regarding the ontological status of objects that, again, might be due to the mentioned philosophical twist and downward reference to the objects that are supposed to be fundamentally separated from the concepts. Boolos' concerns[57] leads him to ask: "just how do we know, what kind of guarantee do we have, why should we believe, that there is a function that maps concepts onto objects in the way that the denotation of octothorpe, #, does if HP [Hume's Principle] is true?" (*Ibid.*, p. 8)[58] (consider HP as: $\#(f) = \#(g) \leftrightarrow fEg$, where E stands for "is equinumerous to"). He continues:

> If there is such a function then it is quite reasonable to think that whichever function octothorpe denotes, it maps non-equinumerous concepts to different objects and equinumerous ones to the same object, and this moreover because of the meaning of octothorpe, the number-of-sign or the phrase "the number of." But do we have any analytic guarantee that there is a function that works in the appropriate manner? (Boolos, 2007, p. 8)

[57] As cited by Boolos; see also Hodes (1984).

[58] Wright replies to Boolos' criticism in Wright (2001).

Boolos here is basically questioning the analyticity of Hume's principle, since one cannot think of any analytic reason or way that would guarantee it works appropriately. There is no analytic guarantee if the function that is denoted by octothorpe (for example what is denoted by #(chairs) in which chair is a concept) maps non-equinumerous concepts to different objects and equinumerous ones to the same object[59]. Looking at these concerns from the perspective of philosophy of language, seems even more complicated.

Elsewhere, MacBride (2003) thinks that the best understanding of Hale's and Wright's neo-Logicism is on the basis of "three related though independent theses" which are: (1) a general conception of the relation between language and reality; (2) the method of abstraction as a particular method for introducing concepts into language; (3) the scope of logic (*Ibid.*, p. 103). By explaining and assessing the criticisms of Boolos, Dummett, Field, Quine and others of these theses, he then concludes:

> It is incontestable that any thoroughgoing defence of neo-logicism must deal with an encompassing range of some of the most fundamental questions in epistemology, metaphysics, philosophical logic and the philosophy of language. […] An understanding of arithmetic appears intimately bound up with the ordinary apparatus of individuation, an arguably constitutive feature of cognition. It remains to be established what shape a completed epistemology must take to capture Frege's insight. (MacBride, 2003, p. 151)

Perhaps, one of the ways to avoid some of these concerns is appealing to the fundamental division of objects into physical and abstract categories.

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[59] For example, suppose we have one table for every five people sitting on five chairs. if the function # maps the chairs to the equinumerous people, there is no analytic guarantee if it maps the non-equinumerous tables to the chairs or to the people.

2.4 Characteristic of a Fregean Framework

Now, if we apply the criteria mentioned at the beginning of the chapter to a typical Fregean setting, we will get the following results. Based upon what we have discussed so far, a Fregean setting assumes or is committed to:

(1) An interpreted language system (i.e., abstract statements are intrinsically true).

(2) Unrestricted definition of object (i.e., defining concepts in terms of objects).

(3) Existential claims are knowable assuming the given-ness of the first-order concepts.

(4) Independency of the conceptual hierarchy and its rules for abstraction.

Accordingly, I believe that, in a Fregean setting, it is better to understand abstraction from a rationalistic perspective, according to which, and in consideration of the whole setting, one might split the conception of object into two: physical objects, and abstract objects. If we want to distinguish this conception of abstraction from other senses of abstraction, we may want to call it *rational abstraction*.

2.5 Conclusion

As we saw, by eliminating the object-concept distinction Carnap introduces a flexible and relative notion of object for the purpose of constructing an analytic system in form of a linguistic framework. Consequently, the general system develops some properties and acquires some characteristics that are different than a Fregean system, which includes the object-concept distinction. One of the major differences is the way in which one understands abstraction in each system. Abstraction in a Carnapian system, which is still a vague conception and it will be clearer in the next chapter, as explained, is the main constructional procedure and it is more in accord with the traditional constructive move from the old objects to concepts (or the new objects, i.e., quasi-objects). On the other hand, in a Fregean system, abstraction is more in accord with the opposite move (i.e., from concepts to objects) since objects are nothing but the extension of concepts.

As we mentioned, some of the Carnap's main philosophical tendencies, regarding his constructional system and the method of quasi-analysis, remains unchanged from *Aufbau* to his later works (some of the more radical changes regarding logical analysis of language has been discussed in the previous chapter). Among the unchanged ideas is the role of abstraction as the main process of construction. Hence, we may realize that, as abstraction is the major player of the Carnap's proposed constructional system in *Aufbau* (1928), it remains the major player of constructing linguistic frameworks in the *Foundations of logic and mathematics* (1939). The sense in which Carnap regards abstraction is linked to but substantially different from his predecessors. As we explained in this chapter, for Carnap, for example, there is no fundamental distinction between "objects" and "concepts", or between "objects" and "events", or the like; whereas, there exists a fundamental object-concept distinction for Frege. For Carnap, it is all about spheres of objects and the relations among them.

Another one of our main concern was determining how to understand abstraction as a process of producing new concepts in the two different philosophical settings of assumptions, conceptions, and methodology on the basis of which abstraction is supposedly understood. Thus, we end up with two different senses of abstraction; one in a Fregean setting and another in a

Carnapian setting. According to the former, abstraction has no dependency on psychology and works on an interpreted language, while in the latter, abstraction as a linguistic (voluntary) activity originally depends on psychology and could work on an uninterpreted language. In one, abstraction could result in existential claims with ontological bearing, in the other, abstraction reveals structural information within which existential claims are possible to be made. To distinguish these different conceptions of abstraction, we proposed to call the first one *rational abstraction* and the second one *empirical abstraction*.

Chapter 3. Abstraction in Carnap's Philosophy

Based on the previous chapter, we now are closer to the notion of abstraction in Carnap's philosophy. Perhaps one of the shortest ways of explaining the difference between Fregean and Carnapian philosophical notions of abstraction is to put it in the following way: if we agree on the classical definition of abstraction, i.e., separating the forms from the matter, then, for Frege, the assertion of a sentence like "the apple is red" is deemed as a report of this separation; in which we already separated "redness" from "the apple-ness". Whereas, for Carnap the whole sentence "the apple is red" itself is the matter in question from which we want to (or should) separate its forms. The latter point of view exactly corresponds to the notions of "object language" and "metalanguage" in Carnap's philosophy, according to which language itself is viewed as an object. One could also see that, by adopting the latter point of view, one simultaneously appeals to an extended (or enhanced) notion of "object", compared to the prior view. After a preliminary discussion on "object", in what follows we intend to show how a Carnapian framework would change after eliminating Fregean sense-reference distinction, and, consequently, how abstraction actually works in a Carnapian framework under this condition. We will see the meaning of many terms (such as "extension") changed under this new condition (i.e., without the sense-reference distinction) compared to the corresponding terms in a Fregean system, which would again entail that the two systems are substantially different even though they sometimes use the same terms. To see this clearly, we focus on some key concepts such as "designation", "interpretation", and "range", in Carnap's framework, and on the different interpretations of some other concepts such as "extension" and "intension", compared to the corresponding Fregean ones. Since the Carnapian abstraction (and system), in distinction from the Fregean counterpart, has never been coherently formalized, at the end we propose some intuitive suggestion as an introduction to finding a way for articulating the notion of abstraction in a Carnapian framework.

One of the important features of Carnap's philosophy is that there is no essential difference between the methodology (analysis) of mathematical statements and the ones of empirical sciences (especially at the theoretical part) in terms of an abstractive analysis (the objects of both fields linguistically/epistemologically function in the same way). Carnap is clear that abstraction to semantics (from pragmatics; i.e., the empirical part) is not just an "accidental help" to pure logic but that it will also supply the very basis for logic (Carnap, 1942, p. viii); meaning that there is no non-empirical (rational) basis for constructing logic. In other words, we may say pure logic could have an empirical basis after all. Thus, for Carnap, the methodology of analyzing mathematical (logical) assertions is on a par with that of empirical sciences and it is "plausible" to regard pure logic and the methodology of science as the results of the same method of abstraction.

> […] it seems plausible to assume that both pure logic and the methodology of science will continue to require a method which—like that of semantics and syntax at present—sacrifices through abstraction some of the features which a full, pragmatical investigation of language would take into account, and thereby gains an exactness not attainable by the empirical concepts of pragmatics. (Carnap, 1942, p. viii)

As it is clear from this passage, for Carnap, exactness (and perhaps the universality), in general (whether in pure logic or pure science), is regarded as the result of sacrificing some pragmatic features through abstraction[60]. As we represented Carnap in the previous chapter as an advocator

[60] With regard to the sameness of analytic methods in mathematics and empirical science, he also says: "I believe, semantics will be of great importance for the so-called theory of knowledge and the methodology of mathematics and of empirical science" (Carnap, 1942, p. viii).

of empirical abstraction, we explained that he is open to the possibility of psychological contributions (psychology in its philosophical sense; i.e., intellectual and cognitive faculties) into the whole system. In his later works, although, Carnap refers to "psychology" strictly as a scientific line of investigation (with no philosophical connotation, and therefore a part of pragmatics and not logic), but he still acknowledges that the result of such an investigation could give us a better understanding about the formation of our object language and its semantics; worthy to investigate but not a job for philosophers. The following are two examples:

> Examples of pragmatical investigations are: a physiological analysis of the processes in the speaking organs and in the nervous system connected with speaking activities, a psychological analysis of the relations between speaking behavior and other behavior, a psychological study of the different connotations of one and the same word for different individuals, ethnological and sociological studies of the speaking habits and their differences in different tribes, different age groups, social strata, a study of the procedures applied by scientists in recording the results of experiments, etc. *Semantics* contains the theory of what is usually called the meaning of expressions, and hence the studies leading to the construction of a dictionary translating the object language in to the metalanguage. (Carnap, 1942, p. 10)

> The equivalence [between '*p*' and '*p* is true'] holds certainly if 'true' is understood in the sense of the semantical concept of truth. I believe with Tarski that this is also the sense in which the word 'true' is mostly used both in everyday life and in science. However, this is a psychological or historical question, which we need not here examine further. (Carnap, 1949, p. 121)

Given that semantics is an abstraction from pragmatics, it is clear that the door to the psychological contributions in the resulting abstract construction is open. Having said that, it should be clear that it is also possible to proceed studying the semantics of our object language disregarding the psychologically involved factors; since, in abstracting, we are distancing ourselves from meaning. Carnap is completely clear about (and committed to) the "empirical character of the language of science", since by holding this view "we see more easily whether and how each term proposed for introduction [to semantics] is connected with possible observations" (Carnap, 1938, p. 34); thus he privileges a bottom-up construction method because it is directly under the influence of observation. As we said earlier, what Frege calls "first-order concepts" Carnap calls "primitive elementary terms", for which he considers the following main possibilities (*Ibid.*):

1. physical elementary terms (e.g., "blue", "hot", as attributed to a physical body)
2. psychological elementary terms (e.g., "having a perception of blue", "having a tooth-ache", as attributed to an organism).

Carnap considers the two as "compatible".

> The thesis that a sufficient basis for the whole of the scientific language can be found among terms of the kind (1), in other words that all scientific terms are reducible to these terms, is the main thesis of Physicalism. The analogous thesis about kind (2) may be taken as one of the main theses of positivism (in a certain sense of this word). Construed in this way, the two assertions are compatible. (Carnap, 1938, p. 34)

Nevertheless, he prefers to take the first one into consideration, because, although both could be inter-subjectively confirmed by the language community, only the first one is inter-subjectively "observable" (accessible), while the second is only subjectively observable (*Ibid.*, p. 35). Although Carnap is clear that "the words and expressions of a language have a close relation to actions and perceptions, and in that connection they are the objects of psychological study" (Carnap, 1937, p. 5), he wants to separate the formal study of language from its empirical component by distinguishing the empirical part as "sematology" (*Ibid.*, p. 8) as opposed to "semantics" for the formal component.

With this introduction, we now return to *Aufbau* to take a closer look at the concept of "object spheres". Carnap is explicit that his concept of "object spheres" is basically the application of Russell's type theory on the concrete objects and concepts, except that "[…] Russell has applied this theory only to formal-logical structures, not to a system of concrete concepts (more precisely: only to variables and logical constants, not to nonlogical constants). Our object spheres are Russell's "types" applied to extralogical concepts" (Carnap, 1967, p. 53).

3.1 Object Spheres

As we have shown, one of the major differences between Frege and Carnap is that for Carnap, unlike Frege, there is no fundamental distinction between objects and concepts. A Carnapian theory of construction is all about object-object relations. Obviously, in such a "system of objects", one of the major tasks would be identifying the objects and their type.

> The problem of object types and their mutual relations is of great importance for construction theory since its aim is *a system of objects*. The various differences and relations which can be indicated, and especially the differences between the various "object spheres", must somehow be reflected in the system that we are about to develop This is an especially important test for our form of construction theory, since we subscribe to the thesis that the concepts of all objects can be derived from a *single common basis*. (Carnap, 1967, p. 31) (emphasis mine)

According to Carnap, there are three distinct types of objects: "psychological", "physical", and "cultural"[61], that are all based on a single auto-psychological basis (as we saw in the previous

[61] We did not talk about the "cultural objects" since it was not quite necessary for our discussion. In short, cultural objects, for Carnap, are the most important types of objects, after the physical and the psychological ones, which belong to the domain of cultural and sociological sciences. "Among the cultural objects, we count individual incidents and large-scale occurrences, sociological groups, institutions, movements in all areas of culture, and also properties and relations of such processes and entities" (Carnap, 1967, p. 23).

chapter). These object types are "autonomous" and belong to different object spheres (*Ibid.*, §25). Carnap's general plan, in *Aufbau*, is to show that:

> […] in what way the assertion of the unity of the entire domain of objects of knowledge refers to the derivation ('construction') of all objects starting from one and the same basis, and that the assertion that the various spheres of objects are different means that there are different constructional levels and forms" (*Ibid.*, pp. 39-40).

As we may see in this quote, the concept of "spheres of objects" is closely tied to the levels and forms of construction. Thus, Carnap is clear that there is no substantial difference between objects and concepts, and that the main goal of his theory of construction is to formulate a constructional system, i.e., "a stepwise ordered system of objects". "The stepwise ordering is a result of the fact that the objects on each level are 'constructed' from the objects of the lower levels" (*Ibid.*, p. 47) in a sense to be made precise later in the subsequent sections. Therefore, according to Carnap, to provide such a theory four things have to be determined: (1) *basis*: the lowest level upon which all others are founded, (2) *ascension form*: "the recurrent forms through which we ascend from one level to the next", (3) *object form*: "how the objects of various types can be constructed through repeated applications of the ascension forms", and (4) *system form*: "the over-all form of the system as it results from the stratified arrangement of the object types" (*Ibid.*). As we explained, (1) could be any one of the three above mentioned categories of objects (i.e., "psychological", "physical", and "cultural"), (2) could be achieved by abstraction (which we will call *vertical abstraction*, later), (3) is the ways in which we could recognize, organize and (re)produce object types and spheres (which relates to what we will later call *horizontal expansion*), and finally (4), which is the form of the entire linguistic framework in which our analysis is embedded.

Carnap defines the sphere of an object as "the class of all objects which are isogenous with the given object" (*Ibid.*, §29). "Two objects (and this always includes quasi objects) are said to be *isogenous* if there is an argument position in any propositional function for which the two object names are permissible arguments" (otherwise they are *allogeneous*) (*Ibid.*). Note that isogeny is transitive, and hence the object spheres are mutually exclusive. The way in which one could observe whether or not two objects are isogenous (linguistically) is, of course, via their corresponding object-names, so that if the statements about these objects are expressed in a word language, then we would have ultimately to ascertain whether or not a string of words forms a meaningful assertion (sentence). Formally speaking, for example, let us take a two-place propositional function $P(\varphi, o) = \pi$, which consists of an independent variable for predicates "φ" (ranges over F, G, etc.), taking the first place, an independent variable for objects "o" (ranges over a, b, etc.), taking the second place, and a dependent variable as a complete sentence π (ranges over p, q, r etc.). Then, for a fixed predicate F, if $P(F, a) \wedge P(F, b)$, i.e., if $p \wedge q$, is meaningful, then a and b are isogenous. In the same way, we may say, in the case of a fixed object a, if $P(F, a) \wedge P(G, a)$ is meaningful, then F and G are isogenous. But since substitutes for φ occupy different places in the propositional function P than the substitutes for o, then they are allogeneous. Object types may have two properties; they might be "pure" or "impure": "we call an object type *pure*, if all its objects are isogenous with one another, i.e., if the type is a subclass of an object sphere. All other types we call *impure*" (*Ibid.*, p. 52). Carnap is explicit that only the pure types are logically unobjectionable concepts, and that only they have classes as extensions (*Ibid.*, §32). "However, in the practical pursuit of science the impure types play an important role. Thus, the main object types, namely, the physical, the psychological, and the cultural are impure types […]" (*Ibid.*, p. 52).

3.1.1 Confusing Cases

Carnap is well aware of the linguistic ambiguities that may lead to confusing object spheres and, hence, may impose some philosophical difficulties in analysis. Nevertheless, Carnap explains that there are linguistic ambiguities with which we should not be concerned as philosophers.

> We are here not concerned with straightforward ambiguity (homonymy) as it occurs, e.g., in such words as "cock", "spring", etc., nor with somewhat more subtle ambiguities as they occur in many expressions of ordinary life, of science and of philosophy, as, for example, in the words "representation", "value", "objective", "idea", etc. In our daily lives, we are well aware of the first type of ambiguity, while in philosophy we concern ourselves with the second, and we can thus avoid at least the more obvious mistakes. (Carnap, 1967, p. 53)

The ambiguities with which one should be concerned (as Carnap explains in an example[62]) are the ones that are due to mixing up the spheres of objects or "confusion of spheres" and may lead to contradictions. To show how tests for isogeny (i.e., test for admissibility into the same position in a proposition) may create confusion, based on the criteria of meaningfulness, consider the following four sentences about a specific piece of stone:

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[62] Carnap gives the following example of the ambiguities which we should be concerned with (*Ibid.*, p. 53).

> Let me explain, by way of example, the third type of ambiguity, the one which concerns us here. The expression "thankful" seems unambiguous when it is taken in its root sense (i.e., setting aside any use of the term in a metaphorical sense; this would fall under the second type of ambiguity considered above, e.g., when "thankful" is used relative to a task or work). However, we not only say of a person that he is thankful, but also of his character, of a look, of a letter, of a people. Now each of these five objects belongs to a different sphere. It follows from the theory of types that the properties of objects which belong to different spheres themselves belong to different spheres. Thus, there are five concepts, "thankful", which belong to different spheres, the confusion of which would lead to contradictions. However, generally speaking, there is no danger that we might draw an invalid conclusion since precisely the fact that these objects are of different spheres keeps us from misunderstanding which of the five concepts is meant. In general, using only one word for these different objects is innocuous, and therefore useful and justifiable. This ambiguity must be noted only if finer distinctions between concepts are to be made, distinctions which are important for epistemological and metaphysical problems. Neglect of the difference between concepts of different spheres, we call *confusion of spheres*.

1. The stone is red.
2. The stone is hard.
3. The stone weighs 5kg.
4. The stone lies in Switzerland.

Regardless whether these sentences are true or false they are all unobjectionably meaningful. The object-name “the stone” can be substituted with any other specific object-name, such as “the apple”, “the chicken”, etc.; hence, all these objects are isogenous within the sphere of physical bodies. Now in the list of the sentences, if we replace “the stone” by “aluminum”, the first two sentences are still meaningful (regardless if they are true or false) while the last two are not. If we consider just the first two, we may claim that “aluminum” belongs to the same sphere of objects as “the stone” does; yet considering the whole list one realizes that “aluminum” belongs to a different sphere of objects. As Carnap points out, “it is frequently necessary to consider several different sentences in testing isogeny; otherwise, one may be misled by the fact that words are frequently impure as far as spheres are concerned” (*Ibid.*, p. 55). We may have different representations of objects (impure objects) such as:

a) *Physical* Objects: a particular stone, aluminum
b) *Psychological* Objects: a (certain, particular) worry, the vivacity of Mr. N.
c) *Cultural* Objects: the constitution of the Reich, expressionism
d) *Biological* Objects: the Mongolian race, heredity of acquired traits;
e) *Mathematical and Logical* Objects: the Pythagorean theorem, the number 3
f) *Phenomenal* Objects: the color green, a certain melody
g) *Objects of Physics*: the electrical elementary quantum, the melting point of ice;
h) *Ethical* Object: the categorical imperative
i) *Temporal* Object: the present day

This list is there simply because, in the ordinary language (or even scientific language), there are meaningful things you can say about "the stone" which you simply cannot say about "Mongolian race", for example. Given that the list can be easily extended and with the objects on the list belonging to different object spheres (yet of the same level given their position in the propositional function), the question is how large this list of objects can be. Another important question is how long the list of the sentences about them should be, for the test of isogeny. To the first question Carnap answers:

> At the moment, there is no way of telling whether this number is finite. In other words, not only is the number of object types which are coordinated with one another (e.g., as the species in a classification) very large, but also the number of object types which are *toto coelo* different from one another. (They are *toto coelo* different from one another in that each of them has its own *coelum*, its own object sphere.) (Carnap, 1967, p. 55)

Although the number of object types (and object spheres) of the whole system might be very large, if one limits his/her concentration to the actual (impure) terms of the object language for the sake of constructing a system of objects we may have a finite basis to start with.

3.1.2 Object Spheres of Different Levels

As said in the previous chapter, we treat quasi-objects (such as classes, extensions, etc.) the same as objects. It seems that another differentiating factor among the spheres of objects is their relative constructional level (level of abstraction). This should be obvious from the following quote:

> We must emphasize the fact that classes are quasi objects *in relation to* their elements, and that they belong to *different spheres*. This is important because a class is frequently confounded with the whole that consists of the elements of that class.

> These wholes, however, are not quasi objects relative to their parts, but are isogenous with them. We shall discuss the difference between classes and wholes, and the fact that elements belong to different spheres from their classes, more thoroughly in the sequel. (Carnap, 1967, p. 58) (emphasis mine).

This means that the object (in fact a quasi-object; see chapter two, §1.1) which corresponds to the incomplete sentence (open sentence, see below) "… is red" is of a different *allogeneous* sphere with respect to the sphere of the objects that comes in the blank space. If we consider the concept of "red" as of a higher-order than the red objects, then it is clear that the level of abstraction is another differentiating factor among the object spheres[63], i.e., objects of different levels are allogeneous with respect to one another. In the same way, we may say relation extensions are quasi-objects. Thus, according to Carnap, since classes and extensions are not isogenous with their elements, as the wholes are (with respect to their parts), then classes and extensions are allogeneous to the wholes (as complexes) that correspond to them.

> Thus, not only is it not the case that a class is identical with the whole corresponding to it; it even belongs to a different sphere. As we have seen, extensions are quasi objects relative to their elements. Thus we see that it is part of the logical doctrine that an extension cannot be a permissible argument for the same argument position of the propositional function for which its elements are permissible arguments. *Nothing can be asserted of a class that can be asserted of its elements, and nothing can be asserted of a relation extension that can be asserted of its members.* (The well-known theorem of logic, that one cannot say of a class either that it does, or that it does not, belong to itself, is only a special case of this.) (Carnap, 1967, p. 64)

[63] Carnap explains how his theory inspired by Russell's theory and corresponds to it.

> Frege has already shown that extension symbols, and thus the class symbols, are incomplete symbols. According to Russell, it is irrelevant for logic whether or not there are actual objects which are designated by class symbols, since classes are not defined by themselves, but only in the context of total sentences ("no class theory"). More recently, Russell has expressed himself even more strongly and has called classes logical fictions or symbolic fictions. This corresponds to our notion of classes as quasi objects. Furthermore, according to Russell, classes are sharply distinguished from their elements in that no statement can be meaningful for a class (i.e., either true or false), if it is meaningful for one of its elements (theory of types). This corresponds to our notion that classes and their elements belong to different spheres. (Carnap, 1967, p. 58)

Carnap is quite clear that, in the course of building up a system (bottom-up), “construction takes place through definition”; i.e., showing how the statements about the object in question can be transformed into the statements about the basic objects of the system or the objects which have been constructed prior (at the lower level) to the object in question (*Ibid.*, §38). This can be done in two different ways, according to Carnap (*Ibid.*, §§38-39): (1) by *explicit definitions*, i.e., “the new symbol is declared to have the same meaning as the compound one” (in this case, the new object is not a quasi-object relative to the older objects[64]); and, (2) by *definition in use*, i.e., introducing a quasi-object. For example, in the first case, one may define “2” as “1+1” or “bachelor” as “unmarried”; while in the second case one may define “2” as “a number” or “bachelor” as “a marital status”. Although the objects that have been defined by the first method remain in the same sphere (and at the same level), the quasi-objects, introduced by the second method, are of different spheres (and of different level). In order to transform the statements into statements about the basic objects, simple substitutions would suffice; while, in the second case, one needs to appeal to some rules of translation (or *interpretation*; since there is no symbol for the new object which is composed of the symbols of the already constructed objects; see §4.2 below for a discussion on interpretation)[65]. Carnap thinks that in the case of introducing new quasi-objects,

> […] we must have a translation rule which generally determines the transformation operation for the statement form in which the new object name is to occur. In contrast to an explicit definition, such an introduction of a new symbol is called a *definition in use*, since it does not explain the new symbol itself which, after all, does not have any meaning by itself—but only its use in complete sentences. (Carnap, 1967, pp. 65-66)

[64] “Thus, it remains within one of the already formed object spheres, even if we should consider it as a representative of a new object type” (*Ibid.*, p. 65).

[65] For example, “… is red” can be replaced by its French version (or any English synonym) “… est rouge” while, considering the higher-order concept “color”, “x is red” would have to be translated into “x has a color”. The latter would be the same sentence (structurally) but containing less empirical information (empirical content).

As it is explained, quasi-objects are meaningless by themselves; their meaning is totally dependent on their presence in the system's hierarchy and on the lower-level objects. Going back to the external-internal questions for a moment, here is another angle in which we can clearly see why some existential claims (in the absolute sense), for example, about numbers, do not make sense in a Carnapian system.

On the other hand, we may see that quasi-objects are of the spheres which belong to the higher-levels of construction (which is exactly the case for Carnap), hence it shows that there is fundamental relativity involved in recognizing these objects (and a fundamental dependency to the system, i.e., the linguistic framework).

> If, in a constructional system of any kind, we carry out a step-by-step construction of more and more object domains by proceeding from any set of basic objects by applying in any order the class and relation construction, then these domains, which are all in different spheres and of which each forms a domain of quasi objects relative to the preceding domain, are called *constructional levels*. Hence, constructional levels are object spheres which are brought into a stratified order within the constructional system by constructing some of these objects on the basis of others. Here, the relativity of the concept "quasi object", which holds for any object on any constructional level relative to the object on the preceding level, is especially obvious. (Carnap, 1967, pp. 69-70)

As Carnap explains, here, we may clearly see how the unity of object domain and the multiplicity of the independent objects are to be reconciled (*Ibid.*). Thus, on one hand, assuming all (basic) objects are transformable into the statements about them, we may say science is concerned with only one domain of objects ("as far as the logical *meaning* of its statements is concerned") (*Ibid.*). On the other hand, in science, we usually do not transform the statements to their basic forms and most of the time we form statements out of the constructed objects. "And these constructed entities belong to different constructional levels which are all allogeneous to one another" (*Ibid.*). Thus, it is fair to say that "the compatibility of these two theses rests on the fact that it is possible to construct different allogeneous levels from the same basic objects" (*Ibid.*).

Philosophically, Carnap explains that the presence of objects in different allogeneous spheres (stemming from one object domain) correspond to the different "modes of being" (*Ibid.*, §42). According to Carnap, "fundamentally, the difference between being and holding, […] goes back to the difference between object spheres, more precisely, to the difference between proper objects and quasi objects" (*Ibid.*). That is to say "being" applies to proper objects (with respect to a higher-level concept, e.g., "the table is …") while constructed objects "hold" for the ones from which they were ascended (e.g., "object holds for …", or "red holds for …"). Even though we are more apt to say a relation "holds" between the members of its extension, we may say a class "holds" for its elements. Consequently, "being" and "holding" are relative concepts (just like "object and concept", or "synthetic and analytic", etc.) with respect to their levels of construction (abstraction).

> what holds for objects of the first level has a second mode of being, and can in turn become the object of something that holds of it (on a third level) etc. So far as construction theory is concerned, this is the logically strict form of the dialectic of the conceptual process. Hence the concepts being and holding are relative and express the relation between each constructional level and the succeeding one.

Now we can clearly see how Carnap distributes objects in different levels, so that each of which expresses a different mode of being. For example, as said before, consider the case of "classes", "classes are constructed from things. These classes do not consist of the things. They do not have being in the same sense as the things; rather, they hold for the things" (*Ibid.*, p. 71). This "being" and "holding" can recursively be repeated if we proceed from the classes of the second level to the cardinals of the third level and so on.

> These classes, even though they hold of things, can now be envisaged as having a second mode of being. From them we can proceed, for example, to the cardinal numbers, which hold for these classes. (For the construction of cardinal numbers as classes of classes) Cardinal numbers belong to a third mode of being and allow us to construct the fractions as relation extensions which hold for certain cardinal numbers. (Carnap, 1967, p. 71)

As Carnap points out, by employing this method of construction "eventually we shall arrive at objects which do not disclose, at first sight, nay for which it seems impossible, that they are constructed from the basic objects" (*Ibid.*). Accordingly, Carnap sides with Kronecker in saying that

> Hence, the appearance of paradox in Kronecker's saying that all of mathematics treats of nothing but natural numbers, and even more in the thesis of construction theory that the objects of all sciences are constructed from the same basic objects through nothing but the application of the ascension forms of class and relation extension. (Carnap, 1967, p. 71)

In short, we have seen that Carnap's construction theory is basically a system of objects distributed at different levels in which the isogenous objects of the same level belong to the same sphere while the spheres of different levels are allogeneous. Also, abstraction certainly is the basic cognitive operation that would allow us to construct quasi-objects and arrive at the ascension forms of higher-levels. Thus, Carnap's construction theory can be equally considered as a theory of abstraction in which the multiplicity of object types and spheres could be the result of the stepwise (and recursive) abstraction based upon a single and finite domain of proper objects.

In the following discussion, you will see that abstraction, in a general linguistic analysis (in which the objects in question are linguistic expressions themselves), is considered as a process of distancing ourselves from the "meaning" of the expressions. In a sense, this process can be characterized as *formalization*, although given the explained-background "formalization" in this sense is slightly different than our normal understanding of the word. In a Carnapian sense, "formalization of logic" means the same as "abstraction to logic". Carnap has a definition of *being formal* and, in that, he considers all forms that a proposition (or a class of propositions) may take.

> A theory, a rule, a definition, or the like is to be called *formal* when no reference is made in it either to the meaning of the symbols (for example, the words) or to the sense of the expressions (e.g. the sentences), but simply and solely to the kinds and order of the symbols from which the expressions are constructed. (Carnap, 1937, p. 1)

As you may see, this definition of being formal is strictly structural. Thus, if we consider logic to be a formal study of language, then the basis of the study should not be put on judgements (i.e., thoughts or content of thoughts, etc.) "but rather on linguistic expressions, of which sentences are the most important, because only for them is it possible to lay down sharply defined rules" (*Ibid.*). It seems obvious through Carnap's work that Carnap maintains a reductionist's view regarding propositional structure (propositional logic, in particular) in language analysis. Carnap distinguishes between being closed with respect to an operator (universal, existential, lambda, and the like) and being closed with respect to a certain relation. "An expression is called open, if it contains a free variable; otherwise closed (a class of sentences is called closed if all its sentences are closed, this concept must be distinguished from that of a class closed with respect to a certain relation.)" (*Ibid.*, p. 17); thus, propositional logic makes a strong case for analysis, given its proven consistency and completeness. Starting with complete sentences of an object language and introducing variables in various sentential places, in which the place itself could be taken as an indicator of the type, is what enables us to go to the next level of abstraction (ready for undergoing different kinds of expansions; see the last section). Carnap is explicitly clear about two things: first, *being formal*, for him, strictly means being syntactical, and secondly abstraction is not about generalization and/or validity of structures; it only means disregarding meaning (in a stepwise manner).

> It is to be noted that we use the term 'formal' here always in the strict sense of "in abstraction from the meaning" [III], hence as synonymous with 'syntactical'[66] […], in contradistinction to the weaker meanings "general" (meaning I), and "logically valid" (meaning II). The difference between II and III might be described in this way: in using the term 'formal' in meaning II, abstraction is made from the meaning of the descriptive signs but not from that of the logical signs [Thus, for instance, the sentence '$P(a) \vee \sim P(a)$' is called formally true (II) because its truth is logically necessary on the basis of the meaning of '$\vee$' and '$\sim$' (as given by the truth tables), independent of the meaning of 'P' and 'a']. On the other hand, in the method which we call formal (in meaning III) or syntactical, abstraction is made from the meaning of all signs, including the logical ones [For instance, in a suitable calculus, the sentence '$P(a) \vee \sim P(a)$' is shown to be C-true (provable) on the basis of rules which are formal in the strict sense III inasmuch as they do not refer to the meaning of any signs, not even of the connectives]. (Carnap, 1943, p. 6)

Thus, as we said previously (chapter 1, §2.3), syntactically pure statements (abstracting from meaning of the connectives) will only give us structural information, or "the geometry of finite, discrete, serial structures of a particular kind" (Carnap, 1937, p. 7), such as $Var(x)Con(x')Var(x'')$, in the case of the given example, in which Var stands for variable and Con for constant. As with Carnap famous appeal to a metalinguistic level for analyzing any objects languages, one should pay special attention to the point that the meaning requirements

[66] Note, as said earlier, that within the syntax there is "descriptive syntax" which is at a lower level of abstraction, and "pure syntax" which at a higher level according to Carnap.

are conceptually prior to the requirements for truth, which come later at a higher level of abstraction. The burden of meaning is on two things: the structure of the sentence and the appropriate assignment of object spheres with respect to different places in the structure in question. Despite the criticism raised by Church (1943) that the designata of sentences, whether the language is extensional or intensional[67], are truth-values, Carnap did not change his position in this regard, that the designata of sentences are always propositions and not the truth-values in general. It could not always be truth values since it depends on the type of the designator as well as the pertinent equivalency (and *L*-equivalency) relation (see §3.4) of the constituents (in the case of composed or compound sentences). Taking a closer look at this criticism, and Carnap's reply to it, will serve us two purposes; we will become more familiar with how abstraction works in a Carnapian framework, as well as seeing the differences between a Carnapian and a Fregean analysis (which is the dominate method in today's literature).

3.2 Church's Criticism

As said above, Alonzo Church (1943) shows that, whether the language is extensional or intensional, the designata of sentences ought to be the truth values, if it contains the abstraction operator λ. And, thus, he criticizes Carnap for saying that the designata of sentences are propositions.

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[67] It might be useful to explain that some languages (like the language of first-order logic) are taken to be extensional in the sense that if one changes a proper name (like "Scott") with a definite description of the same name (like "the author of Waverley") in a closed sentence, the truth value of the sentence would not change. Some other languages do not have this property and are called intensional such as the language of modal logic in which there are sentential operators like "it is necessary that …", and "it is possible that …". In these languages, the change of expressions would change the truth value of the complete sentence. For example, while the sentence "it is necessary that Scott is Scott" is necessarily true, "it is necessary that Scott is the author of Waverley" is not necessarily true.

> Carnap takes it as an assumption that the designata of sentences are propositions and makes this his primary usage (although he does also mention the possibility of truth-values as designata of sentences). However, if a language, in addition to certain other common properties, contains an abstraction operator '(λx)' such that '$(\lambda x)(\ldots)$' means 'the class of all *x* such that…', then- independently of the question whether the language is intensional or extensional-it is possible to prove that the designata of sentences of the language must be truth-values rather than propositions. (Church, 1943, p. 299)

Church's proof goes as follows. Let the following hold:

(1) '…' is a true sentence in the language *S* (*F*-true but not *L*-true; i.e., not analytic)

(2) $\mathfrak{A}$ is "$(\lambda x)(x = x \land \sim \ldots)$"

(3) $\mathfrak{S}_1$ is "$(\lambda x)(x = x \land \sim \ldots) = \Lambda$" in *S* (in which, Λ is the "null class")

(4) $\mathfrak{S}_2$ is "$\Lambda = \Lambda$" in *S*

(5) S' is a metalanguage of *S* that must contain expressions, at least, synonymous with those in *S*, (and "they may as well be taken to be the same expressions"), and in addition to contain semantical terms appropriate to *S*, in particular the predicate '*Des*' ('designates').

Then, the following are true sentences in S':

I. $Des(\mathfrak{S}_1, (\lambda x)(x = x \land \sim \ldots) = \Lambda)$

II. $Des(\mathfrak{S}_2, \Lambda = \Lambda)$

According to Carnap's definition of "synonymous"[68], Church concludes "$\mathfrak{A}$ and Λ are synonymous, whether in *S* or S'" (Church, 1943, p. 300) since they have the same designatum, namely the null class. Then, Church, by referring to Carnap's definition regarding the interchangeability of synonymous expressions (see below), says: "Also, synonymous

[68] D12-2. $\mathfrak{A}_i$ in S_m is synonymous to $\mathfrak{A}_j$ in $S_n \overset{\text{def}}{=} \mathfrak{A}_i$ in S_m designates the same entity as $\mathfrak{A}_j$ in S_n. (Carnap, 1942, p. 55)

expressions are interchangeable. Hence, using the interchangeability of $\mathfrak{U}$ and Λ in S', we obtain a third true sentence of S'" (*Ibid.*):

$$Des(\mathfrak{S}_1, \Lambda = \Lambda)$$

"Hence, again using the definition of synonymy, but this time within S' and in the sense of synonymy in *S*, we obtain, as a true sentence of S'" (*Ibid.*):

$$Syn(\mathfrak{S}_1, \mathfrak{S}_2)$$

where "*Syn*" is the predicate "are synonymous". Church believes that up to this point it is already enough to show that "that the designata of $\mathfrak{S}_1$, and of $\mathfrak{S}_2$cannot be propositions, since the corresponding propositions are certainly not the same for any ordinary meaning of the word 'proposition' (one sentence is *L*-true and the other not!)" (*Ibid.*).

Moreover, according to Church's view, Carnap's defined relation between semantic and syntax (we may summarize this relation of syntax as being abstracted from semantic) amounts to no good since, in effect, "various syntactical definitions are now replaced by semantical definitions which are expected better to serve the intended purpose. E.g., 'analytic' is abandoned in favor of '*L*-true'" (*Ibid.*, p. 304). And if the designatum of a sentence is always truth-value, then Carnap's definition of extensionality fails.

> The thesis of extensionality is said to be still held as a supposition, but on the basis of a semantical concept of extensionality—However, if the designatum of a sentence is always a truth-value, then Carnap's definition of 'extensional' fails in that under it every language (every semantical system) is extensional, even those which contain names of propositions and modal operators, or which contain names of properties as opposed to class names. Apparently a more satisfactory definition of extensionality of a language, or of a semantical system, must be found before the thesis of extensionality can be considered. (Church, 1943, p. 304)

In §32 of Carnap (1947), Carnap presents and compares Frege's, Church's, Quine's and Russell's methods for dealing with the problems regarding name-relations. In that, Carnap finds Church's as a modified version of the Fregean method. With regard to Church's criticism, Carnap says:

> Church's statement that the designatum of a sentence is not a proposition but a truth-value is—on the basis of Frege's method of the name-relation—correct for Church's use of 'designatum' in the sense of 'nominatum'; not, however, for my use of 'designatum' […] in the sense of 'intension'.

As it is evident in this quote, Carnap takes an intensional stance regarding "designatum". What exactly the difference is, related to Carnap's understanding of designators, interchangeability, and some other concepts, we will have to recall in the following section before getting to Carnap's reply. According to Church (1943, p. 300), one should believe "synonymous expressions are interchangeable" for Carnap, because of what Carnap says here:

> In many systems, 'interchangeable' and 'synonymous' coincide, and also 'L-interchangeable' and 'L-synonymous'. But, in general, the first concept in each pair is weaker than the second. If $\mathfrak{A}_i$ and $\mathfrak{A}_j$are interchangeable in S, then their designata have all properties in common which can be expressed (by closed sentences) in S but are not necessarily identical. If they are L-interchangeable, then this is the case for logical reasons, i.e., on the basis of the semantical rules, but the designata may still be different. If, however, $\mathfrak{A}_i$ and $\mathfrak{A}_j$ are synonymous, then their designata are Identical; therefore, they have all properties in common whether expressible in S or not. And if $\mathfrak{A}_i$ and $\mathfrak{A}_j$ are, moreover, L-synonymous, then the semantical rules show us that the designata are identical, hence the expressions have, so to speak, the same meaning. (Carnap, 1942, p. 75)

Now, in the following section, we are going to see some of Carnap's definitions, results, and also what he calls the "serious disadvantages" (Carnap, 1947, p. 2) of a Fregean name-relation method of analysis. Carnap believes Church's method also suffers from these disadvantages as it is a "variant of Frege's method" (*Ibid.*, p. 96).

3.3 Carnap's Proposal for Analysis of Meaning

In the following discussions, we will go through Carnap's method of extension-intension (the two major ingredients of meaning) which he claims to be an improvement over Frege's distinction of sense and reference. Interchangeability of names and definite descriptions are generally considered as, extensionally, truth preservative (i.e., the truth value of the statements would not be changed considering the extensions only; i.e., references only). Unlike Frege, Carnap prefers to start his formalization from *neutral* concepts; neutral with respect to both extension and intension. In what follows, the key concept is "designation", and more precisely the concept of "*designator*" (see chapter two, Table III), which at any level belongs to a higher level of abstraction (i.e., to a metalanguage which is "neutral with regard to extension and intension"; Carnap, 1947, p. 2). We may want to recall the following from what we have said so far about a Carnapian system:

- there is no fundamental difference between objects and concepts (they may differ with respect to their relative level of abstraction; relative to a specific designator). In fact, what we are looking for is just the relations between different objects.
- *L*-truths (*L*-terms in general) are abstracted from *F*-truths (*F*-terms) and *C*-truths (*C*-terms in general) are abstracted from *L*-truths (*L*-terms), in which abstraction (in this context) strictly means becoming more and more independent from "meaning" (of the lower level; see the last quote of section two, meaning [III]).
- the assignment (or admissibility) of objects to different places (gaps in the incomplete form of expressions) is conditioned to the meaningfulness of the expression in question; not just syntactically but also semantically[69] (this assignment primarily has nothing to do with truth as much as it has something to do with meaning and the place of the assignment).

[69] To explain this further, for example, the expression "the apple is or" is meaningless syntactically, while the expression "John is a prime number" is meaningless semantically due to the confusion of object spheres as explained earlier and will be explained later in consideration of the term "designator".

In short, Carnap is explicit that his consideration of extension and intension has advantages over Frege's sense and reference in the following sense:

> The chief disadvantage of the method applying the latter pair [sense and reference] is that, in order to speak about, say, a property and the corresponding class, two different expressions are used. The method of extension and intension needs only one expression to speak about both the property and the class and, generally, one expression only to speak about an intension and the corresponding extension. (Carnap, 1947, p. 2)

Carnap believes that Frege's method for analyzing meaning, using the customary name-relation, has an intrinsic ambiguity which will lead to "an unnecessary multiplication of the entities leading to a complicated language structure, or unnecessary restrictions in the construction of languages" (*Ibid.*, p. 97). According to Carnap, "the name-relation is customarily conceived as holding between an expression in a language and a concrete or abstract entity (object), of which that expression is a name". Thus, this relation is essentially semantical, in Carnap's terminology. To express the name-relation Carnap uses these expressions: "*x* is a *name* for *y*" or "the *nominatum* of *x* is *y*". Carnap emphasizes that the expression "*x* denotes *y*" is often used in a quite different sense, namely, "in the case where *x* is a predicator for a certain property (e.g., the word 'human') and *y* is an entity having that property (e.g., the man Walter Scott)" (*Ibid.*, p. 97), which Carnap considers as a semantical relation of "special kind" (*Ibid.*). The mentioned ambiguity in the Fregean method is there since the name-relation "is applicable not to designators in general but only to predicators and, moreover, only to predicators of degree one, unless one is willing to regard a sequence of entities as the entity denoted" (*Ibid.*). We saw (chapter two, Table III) that the "predicator" is of a special kind of designators (along with "individuator", "sentence", etc.) for Carnap. Carnap specifies that "the method of the name-relation", regardless of how one would express it, rests on three principles (*Ibid.*, p. 98).

1. *The principle of univocality*: Every expression used as a name (in a certain context) is a name of exactly one entity; we call it the nominatum of the expression.
2. *The principle of subject matter*: A sentence is about (deals with, includes in its subject matter) the nominata of the names occurring in it.
3. *The principle of interchangeability* (or substitutivity); This principle occurs in either of two forms:
   a. If two expressions name the same entity, then a true sentence remains true when the one is replaced in it by the other; in our terminology: the two expressions are interchangeable (everywhere).
   b. If an identity sentence "… = _ _ _" (or "… is identical to _ _ _" or "… is the same as _ _ _") is true, then the two argument expressions "…" and "_ _ _" are interchangeable (everywhere).

According to Carnap (*Ibid.*, p. 96), if the third principle is applied without restriction, in certain cases, it will lead to contradiction, which he calls "the antinomy of the name-relation". It might be obvious that by accepting the first two principles one does not need the third one. According to Carnap, if one accepts the first two principles, one will hardly reject the third one, because "if $\mathfrak{A}_j$ and $\mathfrak{A}_k$ have the same nominatum and if the sentence "$\ldots\mathfrak{A}_j\ldots$" says something true about this nominatum, then the sentence "$\ldots\mathfrak{A}_k\ldots$", saying the same about the same nominatum, must also be true" (Carnap, 1947, pp. 98-99). It may seem that the third principle has nothing to do with the name-relation while, in fact, the name-relation is implicit in this principle. To see this clearly, Carnap gives the following definitions (of *identity expression* and *identity sentence*), which he believes are presupposed in the form *b* of the third principle:

A. a predicator $\mathfrak{A}_l$, is an *identity expression* (for a certain type) $\stackrel{\text{def}}{=}$ for any closed expressions (names) $\mathfrak{A}_j$ and $\mathfrak{A}_k$ of the type in question, the full sentence of $\mathfrak{A}_l$ with $\mathfrak{A}_j$, and $\mathfrak{U}_k$ as argument expressions (i.e., $\mathfrak{A}_l(\mathfrak{A}_j, \mathfrak{A}_k)$ or $(\mathfrak{A}_j)\mathfrak{A}_l(\mathfrak{A}_k)$ is true if and only if $\mathfrak{U}_j$and $\mathfrak{U}_k$ name the same entity.

B. $\mathfrak{S}_i$is an *identity sentence* $\stackrel{\text{def}}{=}$ $\mathfrak{S}_i$ is a full sentence of an identity expression.

Carnap is clear that, regarding the third principle, "on the basis of these definitions, form b of the principle of interchangeability follows immediately from form a. Thus, granted the adequacy of these definitions, form b is just as plausible as form a" (Carnap, 1947, p. 99). According to Carnap, Frege formulates the principle of interchangeability in the form a, after distinguishing between nominatum (reference) and sense, in this way: "The truth-value of a sentence remains unchanged if we replace an expression in it by one which names the same [entity]" (*Ibid.*). Note that the concept of the extension of an expression is similar to the concept of nominatum here. Now let us see to what extent the analogous of the above-mentioned principle holds in terms of extension, according to Carnap. The analogous of the first principle would be: Every designator has exactly one extension, which holds. "The analogue of the principle of subject matter holds, too, but with restrictions" (*Ibid.*, p. 100). Carnap then clarifies the source of the ambiguity between the name and relation in the following (which we may take as the source of the difference between his method of extension-intension and Frege's method).

> In general, a sentence containing a designator $\mathfrak{A}_j$ may be interpreted as speaking about the extension of $\mathfrak{A}_j$. However, it may be interpreted alternatively as speaking about the intension of $\mathfrak{U}_j$; and, […], the latter interpretation is sometimes more appropriate. The decisive difference emerges with respect to the principle of interchangeability. (Carnap, 1947, p. 100)

According to Carnap (*Ibid.*), for extensions, instead of the analogous of the form a of the third principle, only a restricted principle holds, of which he says: "if two expressions have the same extension, in other words, if they are equivalent, then they are interchangeable in extensional contexts". The most important part is the form b of the third principle which speaks about identity, which, in the Carnapian method, we cannot simply speak of identity without distinguishing between identity of extensions and the identity of intensions; or in other words, between "equivalence and L-equivalence" (*Ibid.*). Therefore, instead of the one principle for identity, we have two principles in the following forms, one for equivalence and the other for L-equivalence. Before we continue our discussion about the principles, paying attention to the following notice is important.

> **Important notice:** in what follows S stands for either a semantical system of object language (i.e., part of English), including a conventional (constructed) and symbolic semantical system of metalanguage (i.e., L-semantic) which could be further specified as S_1(*extensional*), S_2(*intensional*), or S_3(*extensional*). In the following statements, for brevity and simplicity, we avoid the specifications in each case especially since we are not going to discuss Carnap's system in more detail in that direction.
>
> You may find the distinction between different semantical systems as follows (note that the purpose of designing S_3 is to abandon the difference between proper names and definite/individual descriptions due to the inadequacy of the difference; see, Carnap, 1947, §18):

S: contains descriptive predicates, and hence factual sentences, along with individual descriptions with those predicates, and also contains variables for the non-individual types of designators.

S_1: contains the customary connectives of negation "∼" ('not'), disjunction "∨" ('or'), conjunction "∧" ('and'), conditional (or material implication) "⊃" ('if … then …'), and bi-conditional (or material equivalence) "≡" ('if and only if'). The only variables occurring are individual variables $x, y, z, ...$, and for these variables the customary universal and existential quantifiers are used.

S_2: is an extension of S_1 which also contains modal signs of necessity "□" ('it is necessary that …') and possibility "◊" ('it is possible that …').

S_3: is a coordinate language in which the individuals are positions in discrete linear order O,O', O'', O''' (which is suited for arithmetical expressions, according to Carnap, *Ibid.*).

If S is an object language we have the following principles (*Ibid.*, p. 51-52):

The First Principle of Interchangeability: Let "…$\mathfrak{A}_j$…" be a sentence (in S) which is extensional with respect to a certain occurrence of the designator $\mathfrak{A}_j$, and "…$\mathfrak{A}_k$…" the corresponding sentence with an occurrence of $\mathfrak{A}_k$ instead of that of $\mathfrak{A}_j$; analogously for "… *u* …" and "… *v*…" in S.

a. . If $\mathfrak{A}_j$ and $\mathfrak{A}_k$ are equivalent (in S), then the occurrence in question of $\mathfrak{A}_j$ within "…$\mathfrak{A}_j$…" is interchangeable with $\mathfrak{A}_k$ (in S).
b. $(\mathfrak{A}_j \equiv \mathfrak{U}_k) \supset (\text{"}\ldots \mathfrak{A}_j\ldots\text{"} \equiv \text{"}\ldots \mathfrak{A}_k\ldots\text{"})$ is true in S.
c. Suppose that S contains variables for which $\mathfrak{A}_j$ and $\mathfrak{A}_k$ are substitutable, say *u* and *v*; then $\forall u \forall v\ [(u \equiv v) \supset (\text{"}\ldots u\ldots\text{"} \equiv \text{"}\ldots v\ldots\text{"})]$ is true in S.

The Second Principle of Interchangeability: Let "…$\mathfrak{A}_j$…" be a sentence (in *S*) which is either extensional or intensional with respect to a certain occurrence of the designator $\mathfrak{A}_j$, and "…$\mathfrak{A}_k$…" the corresponding sentence with $\mathfrak{A}_k$.

a. If $\mathfrak{A}_j$ and $\mathfrak{A}_k$ are *L*-equivalent (in *S*), then the occurrence in question of $\mathfrak{A}_j$ within "…$\mathfrak{A}_j$…" is *L*-interchangeable and hence interchangeable with $\mathfrak{A}_k$ (in *S*).
b. (the same as above)
c. (the same as above)

As you may see, these principles could be translated to our earlier discussion on the admissibility of objects into certain structures (e.g., a propositional structure/function) and object spheres to which $\mathfrak{A}_j$ and $\mathfrak{A}_k$ belong. Thus, the following two theorems are directly derivable from the theses two principles (*Ibid.*, p. 52).

T3.1: ***First theorem***: If *S* is an *extensional system*, then:

a. Equivalent expressions are interchangeable in *S*.
b. *L*-equivalent expressions are *L*-interchangeable in *S*.

T3.2: ***Second theorem***: If *S* is an *intensional system* (e.g., modal logic or S_2), then:

a. Equivalent expressions are interchangeable in *S* except where they occur in an intensional context[70] (e.g., in a context of the form "it is necessary that…")
b. *L*-equivalent expressions are *L*-interchangeable in *S*.

[70] Note that in Carnap's system "context" (in the Fregean sense; i.e., extensionality or intensionality of an expression depends on the context within which it occurs) is abandoned; instead, Carnap (1947, p. 5) introduces "*sentential matrix*" (in short "matrix"), that is used for expressions which are either sentences or formed from sentences by replacing individual constants with variables; the extensionality or intensionality of the expressions does not depend on the matrices; it rather depends on the type of the designators in question and whether or not equivalency (or *L*-equivalency) between designators holds such that the extension (or intension) of the resulting sentence is a function of the extension (or intension) of the designators.

These theorems are based on the following definitions (and theorems) where $\mathfrak{S}$ is a closed sentence in *S*:

D 3.1: $\mathfrak{S}_i$is ***equivalent*** to $\mathfrak{S}_j$(in *S*) $\stackrel{\text{def}}{=} \mathfrak{S}_i \equiv \mathfrak{S}_j$ is true (in *S*)[71]

D 3.2: A sentence $\mathfrak{S}_i \equiv \mathfrak{S}_j$ is true if and only if either both components are true, or both are false

D 3.3: $\mathfrak{S}_i$is ***L-equivalent*** to $\mathfrak{S}_j$(in *S*) $\stackrel{\text{def}}{=} \mathfrak{S}_i \equiv \mathfrak{S}_j$ is L-true (in *S*)

D 3.4: $\mathfrak{S}_i$is ***L-equivalent*** to $\mathfrak{S}_j$ if and only if $\mathfrak{S}_i$and $\mathfrak{S}_j$hold in the same state-description

D 3.5: $\mathfrak{A}_i$is ***equivalent*** to $\mathfrak{A}_j$(in *S*) $\stackrel{\text{def}}{=}$ the sentence $\mathfrak{A}_i \equiv \mathfrak{A}_j$ is true (in *S*)

D 3.6: $\mathfrak{A}_i$is ***L-equivalent*** to $\mathfrak{A}_j$(in *S*) $\stackrel{\text{def}}{=}$ the sentence $\mathfrak{A}_i \equiv \mathfrak{A}_j$ is *L*-true (in *S*)

D 3.7: Tow closed sentence $\mathfrak{S}_i$ and $\mathfrak{S}_j$ are ***X-equivalent*** (*X*: *F*-, *L*-, *C*-) if and only if $\mathfrak{S}_i \equiv \mathfrak{S}_j$ is ***X-true***.

D 3.8: Two predicators ($\mathfrak{A}_j$ and $\mathfrak{A}_k$) *have the **same extension*** if and only if they are equivalent.

D 3.9: Two predicators ($\mathfrak{A}_j$ and $\mathfrak{A}_k$) *have the **same intension*** if and only if they are *L*-equivalent

D 3.10: The ***extension*** of a predicator (of degree one) is the corresponding class (e.g., the class of humans).

D 3.11: The ***intension*** of a predicator (of degree one) is the corresponding property (e.g., the property of being human).

[71] Regarding this definition Carnap brings out the conventional element (agreement) with respect to determining meaning:

> It is to be noticed that the term 'equivalent' is here defined in such a manner that it means merely agreement with respect to truth-value (truth or falsity), a relation which is sometimes called 'material equivalence'. The term is here not used, as in ordinary language, in the sense of agreement in meaning, sometimes called 'logical equivalence'; for the latter concept we shall later introduce the term 'L-equivalent'. (Carnap, 1947, p. 6)

Carnap uses "*L*-true" in order to explicate Leibniz's "necessary truth" and Kant's "analytic truth"; therefore, he makes this convention: "A sentence $\mathfrak{S}_i$ is *L*-true in a semantical system *S* if and only if $\mathfrak{S}_i$ is true in *S* in such a way that its truth can be established on the basis of the semantical rules of the system *S* alone, without any reference to (extra-linguistic) facts" (Carnap, 1947, p. 10). This, without a doubt, is one of the pivotal conventions in the Carnapian approach. Accordingly, Carnap gives the following definition (*Ibid.*).

D 3.12: An atomic sentence $\mathfrak{S}_i$ (in *S*) consisting of a predicate followed by an individual constant is *true* if and only if the individual to which the individual constant refers possesses the property to which the predicate refers.

D 3.13: A sentence $\mathfrak{S}_i$ is ***L-true*** (in *S*) $\stackrel{\text{def}}{=}$ $\mathfrak{S}_i$ holds in every *state-description* (in *S*).

D 3.14: A sentence $\mathfrak{S}_i$ is ***F-true*** (in *S*) $\stackrel{\text{def}}{=}$ $\mathfrak{S}_i$ is true but not *L*-true (in *S*).

D 3.15: ***State-description*** $\stackrel{\text{def}}{=}$ a class of sentences (in *S*) which contains for every atomic sentence either this sentence or its negation, but not both, and no other sentences.

D 3.16: The class of all those state-descriptions in which a given sentence $\mathfrak{S}_i$ holds is called the ***range***[72] of $\mathfrak{S}_i$.

As said above, the key concept here is the concept of *designator* which includes "(declarative) sentences, individual expressions (i.e., individual constants or individual descriptions) and predicators (i.e., predicate constants or compound predicate expressions, including abstraction expressions)" (*Ibid.*, p. 1). Although "designator" is considered to be applied to the mentioned cases its meaning is not limited to them; its application is quite flexible and may vary also with respect to the levels of abstraction. In general, "designator" is meant to be used for all those expressions to which a semantical meaning analysis is applied.

[72] Carnap is explicit that it is according to the "rules of range" that one could find an interpretations of $\mathfrak{S}_i$ since "by determining the ranges, they give, together with the rules of designation for the predicates and the individual constants, an interpretation for all sentences in *S*, since to know the meaning of a sentence is to know in which of the possible cases it would be true and in which not" (1947, p. 9-10).

> I propose to use the term 'designator' for all those expressions to which a semantical analysis of meaning is applied, the class of designators thus being narrower or wider according to the method of analysis used. [The word 'meaning' is here always understood in the sense of 'designative meaning', sometimes also called 'cognitive', 'theoretical', 'referential', or 'informative', as distinguished from other meaning components, e.g., emotive or motivative meaning. Thus, here we have to do only with declarative sentences and their parts.] Our method takes as designators at least sentences, predicators (i.e., predicate expressions, in a wide sense, including class expressions), functors (i.e., expressions for functions in the narrower sense, excluding propositioned functions), and individual expressions; other types may be included if desired (e.g., connectives, both extensional and modal ones). (Carnap, 1947, pp. 6-7)

Carnap defines "designator" in a way in which alternative interpretations are possible (see below). Keep in mind that "designator" (whatever it is) always belongs to a higher level of abstraction (i.e., to an allogeneous sphere with respect to what it designates) and its construction rests upon our purpose of analysis whether it is logical-linguistic analysis or scientific-theoretical one.

D 3.17: Two designators *have the same extension* (in S) $\stackrel{\text{def}}{=}$ they are equivalent (in S).

D 3.18: Two designators *have the same intension* (in S) $\stackrel{\text{def}}{=}$ they are L-equivalent (in S).

Note that, as Carnap notifies (*Ibid.*, p. 23), in these definitions the terms "extension" and "intension" have not been defined, but only the phrases "have the same extension" and "have the same intension". And, the use of the latter expressions in these definitions are "entirely free of the problematic nature of the terms 'extension' and 'intension'", since we already defined them via "equivalent" (D3.5), "L-equivalent" (D3.6), "true" (D3.12), and "L-true" (D3.13).

According to Carnap (*Ibid.*, p. 7), the designators are not meant to be *names* of some entities (i.e., not existential claims) but they are merely meant to show the relative independence of their expressions from meaning (i.e., showing relative level of abstraction), which to a great extent is conventional. "Only (declarative) sentences have a (designative) meaning in the strictest sense, a meaning of the highest degree of independence [(abstraction)]. All other expressions derive what meaning they have from *the way* in which they contribute to the meaning of the sentences in which they occur" (*Ibid.*, emphasis mine); hence, they could be evaluated by means of their relational and structural properties. From this just-mentioned quote (and from the following quote) one may easily derive that what we normally call "second-order logic", is in fact of a lower level of abstraction compared to what we call "propositional logic". Note that the higher the abstraction level, the higher the degree of independence from meaning. Carnap continues on giving an example, showing the conventionality of designation, in the following way:

> Thus, for instance, I should attribute a very low degree to '(', somewhat more independence to '∨', still more to '+' (in an arithmetical language), still more to 'H' ('human') and 's'('Scott'); I should not know which of the last two to rank higher. This order of rank is, of course, highly subjective. And where to make the cut between expressions with no or little independence of meaning ('syncategorematic' in traditional terminology) and those with a high degree of independence, to be taken as designators, seems more or less a matter of convention. (Carnap, 1947, p. 7)

It is, perhaps, for this reason (the "subjectivity" mentioned in the quote) that Carnap advises us that "we shall not try to give an exact definition for 'degree of abstractness'", although he is clear that "between quite elementary concepts and those of high abstraction there are many intermediate levels" (Carnap, 1939, p. 61).

Considering the above definitions, you may see that the sign "≡" for equivalence could be used between different kinds of designators (sentences, predicators, individuals) and thus its usage is slightly different than its customary use only with respect to sentences. It could be used between predicators as well as individual expressions (see the examples below). Different possibilities of interpretation are implied in the following definitions (*Ibid.*, p. 16):

D 3.19: If two designator signs are ***equivalent***, then any two sentences of simplest form (in *S*: atomic form) which are alike except for the occurrence of the two designator signs are likewise equivalent.

D 3.20: If two designators (which may be compound expressions) are ***L-equivalent***, then any two sentences (of any form whatever) which are alike except for the occurrence of the two designators are likewise *L*-equivalent.

Thus, on the basis of equivalence and *L*-equivalence, where $\mathfrak{A}_i$ is a designator in *S* we have the following pair of definitions regarding its class:

D 3.21: The equivalence class of $\mathfrak{A}_i \overset{\text{def}}{\equiv}$ the class of those expressions (in *S*) which are equivalent to $\mathfrak{A}_i$.

D 3.22: The *L*-equivalence class of $\mathfrak{A}_i \overset{\text{def}}{\equiv}$ the class of those expressions (in *S*) which are *L*-equivalent to $\mathfrak{A}_i$.

As said above, the equivalency relation "≡" between two designators[73] could be interpreted differently depending on the kind of the designator in question (one of which is an identity relation). Also, *extensionality* and *intentionality* are always considered relative to the designator in question such that,

> A sentence is said to be *extensional* with respect to a designator occurring in it if the extension of the sentence is a function of the extension of the designator, that is to say, if the replacement of the designator by an equivalent one transforms the whole sentence into an equivalent one. A sentence is said to be *intensional* with respect to a designator occurring in it if it is not extensional and if its intension is a function of the intension of the designator, that is to say, if the replacement of this designator by an L-equivalent one transforms the whole sentence into an L-equivalent one. (Carnap, 1947, p. 1)

[73] As Carnap explains (in the footnote of 1947, p. 13), the terms ending with "-or" such as "predicator", "individuator", "functor", and the like (could be "descriptor", "connector", "abstractor", etc., as well) are intentionally chosen to give a uniform feature to the metalanguage terminology.

So far, we have realized that the general interchangeability principle based on $\equiv$ does not hold for all systems (e.g., S_2). Accordingly, the equivalency of two designators, $\mathfrak{A}_i \equiv \mathfrak{A}_j$, may have a different meaning (designative meaning). Thus,

D 3.23: $\mathfrak{A}_i \equiv \mathfrak{A}_j$ for:

a. ***predicators*** of the same degree n in S is $\forall x_1 \forall x_2 \dots \forall x_n [\mathfrak{A}_i x_1 x_2 \dots x_n \equiv \mathfrak{A}_j x_1 x_2 \dots x_n]$(for the first degree: $\forall x[\mathfrak{A}_i x \equiv \mathfrak{A}_j x]$)

i. For the two predicators of the first degree in S, $\mathfrak{A}_i \wedge \mathfrak{A}_j$ is $(\lambda x)[\mathfrak{A}_i x \wedge \mathfrak{A}_j x]$[74]

b. ***individuators***, where f is a predictor variable, is $\forall f[f(\mathfrak{A}_i) \equiv f(\mathfrak{A}_j)]$

c. ***functors*** in S is $\forall x[\mathfrak{A}_i x \equiv \mathfrak{A}_j x]$

According to the above explanations, an immediate philosophical implication would be that the sign "≡" is not to be understood as universal. While the identity relation is an example of equivalence relation, neither of which, by no means, are supposed to be understood as a universal concept, upon which one could pronounce a general principle for constructing an abstraction hierarchy (as it is the case in a Fregean method). The type of the equivalence relation, as we saw, depends on the type of the designators in question, which, in turn, is a voluntary choice and a pragmatic issue[75].

As we may see, in Carnap's method (of extension-intension), as opposed to Frege's referential method, there is no need to speak about, say a property, and its corresponding class in two different expressions. (Frege's method also requires a clarification of the "context"; since the

[74] This line shows how predicate logic can essentially be obtained from propositional logic by introducing variables. It is also obvious that this introduction can basically only be justified for pragmatic reasons depending on the goal of our analysis.

[75] Rejecting universality as an inevitable consequence of abstraction has a history among medieval philosophers; see the appendix for more information.

same name may have different nominata in different contexts, as opposed to Carnap's method in which "context" is irrelevant and a name simply either has a nominatum or not in a matrix).

For more clarity, let us first go through some examples to see how various types of designators behave with respect to their own equivalency relation.

(Example 1) Let "*P*" and "*Q*" be two isogenous predicators of degree one (in *S*). According to D3.1, D3.11, and D3.5, $P \equiv Q$ if and only if "$P \equiv Q$" is true; hence, according to 3.21.a.i, if and only if $\forall x[P(x) \equiv Q(x)]$ (i.e., *P* and *Q* hold for the same individuals).

(Example 2) Assuming that: H stands for "… is a human", F for "… is featherless", B for "… is a biped", and "all humans are featherless bipeds" and vice versa, the sentence "$\forall x[Hx \equiv (Fx \wedge Bx)]$" is true (in *S*) but not *L*-true; therefore, it is *F*-true, and, according to D3.21.a.i, we may say $H \equiv F \wedge B$. Thus, H and $F \wedge B$ are equivalent (in *S*), but not *L*-equivalent, hence they are *F*-equivalent.

(Example 3) Assuming that according to the standard English dictionary, "being human" (*H*), and "being a rational animal" (*RA*) is the same. Thus, the truth of the sentence "$\forall x[Hx \equiv RAx]$" can be established only based on the semantical rules of *S* with no reference to the facts. Therefore, "$\forall x[Hx \equiv RAx]$" is *L*-true, and *H* is *L*-equivalent to *RA*.

(Example 4) Given *s* stands for the proper name "Scott", then the direct translation of *Hs* is "Scott is human". There are also two other translations possible using the terms "property", or "class" (both of which have the same logical content as the just-given translation; see Carnap, 1947, p. 17); they are respectively "Scott has the property Human", and "Scott belongs to (is an element of) the class Human". Similarly, in the case of "$\forall x[Hx \supset Bx]$" the direct translation is "for every x, if x is human then x is biped", and two other translations are "the property Human implies (materially) the property Biped" and "the class Human is a subclass of the class Biped"[76].

In the last example, as Carnap points out (*Ibid.*, p. 17), since it is possible to have a translation of the mentioned symbolic sentences without using the terms "property" (or "concept" in a Fregean sense) or "class" (the "extension of a concept" in a Fregean sense), these terms seem unnecessary. Now the important question is whether it is necessary to admit both kinds of entities, classes and properties, or whether those of the one kind are definable with the help of those of the other. Carnap believes it is possible to avoid using both terms. In §33 of Carnap (1947), Carnap describes four methods (including Russell's) for taking properties as primitive and defining classes in terms of properties, and shows the problems associated with each one of them. In short, all methods end up with having two different expressions such as:

(1) "$H \equiv F \wedge B$" meaning "$\forall x[Hx \equiv Fx \wedge Bx]$"
(2) "$H\hat{z} \neq F\hat{z} \wedge B\hat{z}$"

where (1) is showing that the property Featherless Biped and the property Human are equivalent, and (2) is showing that they are not identical. Carnap also does not want to consider the inverse of such methods, i.e., taking classes as primitive and defining properties in terms of classes, for

[76] Capitalization in the quotation marks are intentional: "I prefer now the method of capitalizing; I shall use it not only in connection with 'property' and 'class' but likewise with other words designating kinds of entities, e.g., 'relation', 'function', 'concept', 'individual', 'individual concept', and the like" (*Ibid.*, p. 17-)

a property might be even more obscure than a class. Then Carnap proposes (*Ibid.*, §34) that the best way is to construct a "*neutral metalanguage*" (M') by eliminating the terms "class", "property", etc., "in favor of neutral formulations" such as the following, in which "Human" is a neutral expression (M is a non-neutral object language):

D 3.24: The extension of "Human" (in M') is the class Human (in M).

D 3.25: The intension of "Human" (in M') is the property Human (in M).

One may see, here, that classes and properties have different identity conditions, and since, generally speaking, identity is different for extensions and intensions, a neutral formulation cannot speak about identity. "Hence, identity phrases like 'is identical with' or 'is the same as' are not admissible in M'"(*Ibid.*, p. 154). Thus, given the following general rules of translations (D3.26 & 27), and what we said above regarding equivalency and *L*-equivalency, we can derive these statements:

C 3.1: The class Human is the same as the class Featherless Biped.

C 3.2: The property Human is not the same as the property Featherless Biped.

C 3.3: The property Human is the same as the property Rational Animal.

The following rules are for translating identity sentences from a non-neutral object language *M* into neutral formulations in M' (in which identity phrases like 'is identical with' or 'is the same as' are not admissible).

D 3.26: A sentence stating identity of extensions is translated into M' as a sentence stating equivalence of neutral entities.

D 3.27: A sentence stating identity of intensions is translated into M' as a sentence stating L-equivalence of neutral entities.

Therefore:

D 3.28: The extension of “Human” in M' is the class Human in *M*.

D 3.29: The intension of “Human” in M' is the property Human in *M*.

D 3.30: The class Human in *M* is the same as the class Featherless Biped in *M*.

D 3.31: The property Human in *M* is not the same as the property Featherless Biped in *M*.

D 3.32: The property Human in *M* is the same as the property Rational Animal in *M*.

Thus, considering the following sentences in *M*:

D 3.33: Human is equivalent to Featherless Biped.

D 3.34: Human is not *L*-equivalent to Featherless Biped.

D 3.35: Human is *L*-equivalent to Rational Animal.

We will have the following in M':

D 3.36: “Human” is equivalent to “Featherless Biped”.

D 3.37: “Human” is not *L*-equivalent to “Featherless Biped”.

D 3.38: “Human” is *L*-equivalent to “Rational Animal”.

According to Carnap (*Ibid.*, p. 155), we have to distinguish D3.33-35 from D.3.36-38, which “look similar but are fundamentally different in their nature”. Carnap also shows that M' is not weaker than *M*.

Hence, since the translation to a neutral formulation (neutral to classes and properties, which stems from Frege’s sense-reference distinction), as we saw, is possible, we can eliminate both terms, “class” and “property”, and take neither of them as primitive. In the alternative way we speak about *equivalency* or *L-equivalency* of expressions (with respect to designators) and their extensions and intensions via the rules that govern them; the rules are the following four:

(i) Rules of *formation*, on the basis of a classification of the signs; these rules constitute a definition of "sentence" (based on a constructed matrix).
(ii) Rules of *designation* for the primitive descriptive constants, namely, individual constants and predicates (types of designators).
(iii) Rules of *truth*.
(iv) Rules of *ranges*.

Since we have identity conditions that are depending on different designators (their extensions, or intensions), and given that in a Carnapian system there is no fundamental difference between objects and concepts, we expect that the just-explained relative identity situation could be applied to any chosen levels of abstraction (including the so-called "abstract" objects or quasi-objects). Thus, as the grand conclusion of this part, one may say, according to Carnap, there is no unique and universal identity relation at any rate, even among the so-called abstract objects. Interestingly enough, there are some recent studies on the foundations of mathematics that point to this direction and that support this conclusion. While showing that "there are important parts of mathematics in which the notion of identity at work is not based on the purely extensional point of view" (Marquis, 2013b), Marquis comes to the conclusion that "there is no unique, global, and universal relation of identity for abstract objects" (2013a).

Given that the majority of the different areas of mathematics have been developed under the dominant influence of the Fregean notion of extension and intension (i.e., extension and intension are two sides of the first-order concepts), Marquis' results not only show the inadequacy of Fregean methods (i.e., identity relation cannot be deemed as purely extensional) but also support the just-described Carnapian method of extension and intension.

3.3.1 Carnap's Criticism of Frege's Method and Reply to Church

According to Carnap, Frege regards logical structure of natural language as being defective in the sense that some natural language expressions, in some cases, may appear as a name of one specific object whereas in other cases not; thus, Frege suggests constructing the rules of a language system in such a way that "every description has a descriptum" (in Carnap's terminology, 1947, p. 35). "This requires certain conventions which are more or less arbitrary; but this disadvantage seems small in comparison with the gain in simplicity for the rules of the system" (*Ibid.*). Carnap does not claim that his method (of distinguishing between extension and intension) is theoretically incompatible with that of Frege's (of distinguishing between nominatum and sense); he believes that the difference is "merely a practical difference of method" (*Ibid.*, p. 124): Which distinction ought to be counted to result in a less complicated construction. As we said above, one of the advantages is that, in Carnap's method, the concepts are independent from the context. "A decisive difference between our method and Frege's consists in the fact that our concepts, in distinction to Frege's, are independent of the context" (*Ibid.*, p. 125). Carnap emphasizes that the difference between his method and that of Frege's is not a difference of opinion, since both methods serve the same goal.

> Thus, it becomes clear—and I wish to emphasize this point—that the difference between Frege's method and that here proposed is not a difference of opinion. In other words, it is not the case that there is one question to which different and incompatible answers have been given. There are two questions, and, more precisely, these are not even theoretical questions but merely practical aims. While the general aim is the same, namely, the construction of a pair of concepts suitable as instruments for semantical analysis, the specific aims are different. Frege tries to achieve the general aim by an explication of one pair of concepts [(sense and nominatum)], I by the explication of another pair [(extension and intension]. (Carnap, 1947, pp. 127-128)

Let us first see what the problem (which is called the problem of name-relation by Carnap) is, to which both methods (Carnap's and Frege's) offer their own solution. The problem is that, in semantic analysis, regarding the occurrences of the names (or description) of concrete or abstract entities in a sentence, the name (or description) should satisfy three conditions: (1) Every name should stand for one nominatum (i.e., the entity named by it). (2) Every sentence should speak about the nominata of the names occurring in it. (3) If a name occurring in a true sentence is replaced by another name with the same nominatum, the sentence should remain true. We know that, in natural language, a name (or description) sometimes has a unique nominatum (or descriptum), sometimes multiple, and sometimes none; also, a nominatum (or descriptum) sometimes has multiple names (or descriptions), and sometimes a unique one. In order to satisfy the mentioned conditions, in addition to making a fundamental distinction between objects and concepts, Frege proposes a distinction between the sense of a name (or description) and its reference, or its nominatum (or descriptum), upon which he could construct his language system.

According to Carnap, Frege suggests that "the rules of a language system should be constructed in such a way that every description has a descriptum" (1947, p. 35). Carnap explains (*Ibid.*, §8) that there are two methods by which one could construct a language system upon Frege's proposal (including Carnap's own method). These are:

(I). To construct a system with no type difference between individuals and classes (i.e., both classes and their elements as objects which could be considered as values of the individual variables). And, to assign a descriptum (a class of objects) to all descriptions which do not satisfy the condition of uniqueness.

(II). To select, once for all, a certain entity from the range of the values of the variable in question and assign it (as descriptum) to all descriptions which do not satisfy the condition of uniqueness.

According to Carnap (*Ibid.*), who chooses the second procedure, (II), for construction, this procedure could be executed in three ways:

i. If the individuals of the system are numbers, the number 0 seems to be the most natural choice. (as noted by Carnap, this way has been applied by Gödel for his epsilon-operator, and by Carnap himself for the K-operator)
ii. For variables to whose values the null class Λ belongs, this class seems to be the most convenient choice.
iii. If we construct the system with space-time points as individuals in such a way that the spatiotemporal part-whole relation is one of its concepts. Thus, every individual in such a system corresponds to a class of space-time points. Therefore, it is possible, although not customary in the ordinary language, to count among the things also the null thing, which corresponds to the null class of space-time points. In the language system of things (thing language) it could be characterized as that thing which is part of every thing; let us call it a_0 in the thing language and call it a^* in the corresponding semantical system.

Carnap clearly wants to merge option i and iii of the second method[77] into one; but the question is how? One of the ways in which Carnap can accomplish this goal, as he actually does, is to construct a neutral matrix (I would say ontologically-neutral) with an expressive ability with which one can express things about things. In this way one can speak about, say numbers, with the same tool (matrix) one may speak about, say physical objects. Regarding descriptions with variables of other types than the individual type, especially predicator variables, functor variables, and sentential variables, Carnap make the following remarks:

> Here it is easy to make a natural choice of a value of the variable as a descriptum for those descriptions which do not satisfy the condition of uniqueness. If an individual has been chosen as a^* (it may be [spatiotemporal] a_0 or 0 or anything else), then we might call one entity in every type the null entity of that type, in the

[77] One may realize that (II)iii relates the linguistic analysis of what we have previously presented from *Aufbau* and our discussion on the notion of object so far.

> following way: In the type of individuals it would be a^*; in any predicator type, the null class or null relation of that type, e.g., for level one and degree one the null class Λ; in the type of propositions, the L-false proposition; in any type of functions, that function which has as value for all arguments the null entity of the type in question. Then we may take as descriptum in the case of nonuniqueness the null entity of the type of the description variable. (Carnap, 1947, p. 38)

It is obvious from this quote that the nature of the neutral zeroing assumption, a^*, depends on the type of the chosen designators[78].

Carnap believes (*Ibid.*, p. 126) that perhaps Frege's goal in making a distinction between sense and nominatum (reference), was to explicate some pairs of concepts in traditional logic such as "the distinction between 'extension' and 'comprehension' in the Port-Royal Logic" or "the distinction between 'denotation' and 'connotation' made by John Stuart Mill". To show the shortcomings of Frege's method, Carnap considers two situations in which a sentence could be expressed: *Ordinary* and *oblique*. In the ordinary situation Frege has the following principles:

- The (ordinary) sense of a sentence is the proposition expressed by it.
- The (ordinary) nominatum of a sentence is its truth value.

According to Carnap (1947, p. 124) these principles "compel him [Frege] to regard certain cases as exceptions to these results and thereby to make his whole scheme rather complicated". Those exceptions include examples like embedding the false sentence "the planetary orbits are circles" within the oblique context "Copernicus asserts that the planetary orbits are circles" which is

[78] Carnap is explicit that "we leave it open which individual is meant by a^*" (*Ibid.*, p. 37). Accordingly, Carnap analyses the structure of the sentences containing individual descriptions such as "Scott is the author of Waverley" as: Either there is an individual y such that y is the only individual for which "… y …" holds, and "--- y ---"; or there is no such individual, and "--- a^*---". In this example: either there is an individual y such that y is the only author of Waverley, and y is human; or there is no such individual y (that is to say, there is either no author or several authors of Waverley), and $a^*$is human.

true. “Now Frege says that the sentence within the oblique context has not its ordinary nominatum but a different one, which he calls its oblique nominatum, and not its ordinary sense but a different one, which he calls its oblique sense” (*Ibid.*, p. 123). According to Carnap (*Ibid.*), Frege, hence, makes the following statements in which the second is a special case of the first:

- ❖ The oblique nominatum of a name is the same as its ordinary sense.
- ❖ The oblique nominatum of a sentence is not its truth-value but the proposition which is its ordinary sense.

Consequently, we may say: the oblique nominatum of the sentence “the planetary orbits are circles”, (that is, the entity named by it) in an oblique context like “Copernicus asserts that the planetary orbits are circles”, is the proposition that the planetary orbits are circles and not its truth value. Thus, for Frege, “an expression in a well-constructed language system always has the same extension and the same intension; but in some contexts, it has its ordinary nominatum [(hence extension)] and its ordinary sense [(hence intension)], in other contexts its oblique nominatum and its oblique sense” (*Ibid.*, p. 125). According to Carnap it is difficult to see clearly what constitutes the ordinary sense (in Frege’s method) due to “the lack of precise explanation and especially of a statement as to the condition of identity of sense” (*Ibid.*, p. 126); on the other hand, this condition is quite clear, in Carnap’s method, and that is the condition of *L*-equivalency, as said above. Also, according to Carnap again, “in one respect, Frege’s concept of proposition (‘*Gedanke*’) is not quite clear; he does not state an identity condition for propositions” (*Ibid.*, p. 124). Comparing the methods of Frege and Carnap, as noted by Carnap (*Ibid.*, pp. 126-127), one reaches to the following results:

- ❖ For any expression, its ordinary nominatum (in Frege’s method) is the same as its extension (in Carnap’s method).

- For any expression, its ordinary sense (in Frege's method) is the same as its intension (in Carnap's method).

Thus, for ordinary occurrences of expressions, Carnap's two concepts (extension and intension) coincide with those of Frege. The differences arise only with respect to expressions in an oblique context in which following Carnap's method "lead to the same entities as for the ordinary occurrences of the same expressions, while Frege's concepts lead to different entities" (*Ibid.*, p. 126). According to Carnap, "this complication is not introduced by Frege arbitrarily but is an inevitable consequence of his general principles" (*Ibid.*). Nevertheless, Carnap does not say Frege's system is invalid; on the contrary, given Frege's choice of concept (nominatum and sense) Frege's system, although more complicated, it is still valid.

> The results found by Frege, including the complication in the case of oblique contexts, are consequences of his principles and hence share their analytic validity (assuming that Frege made no mistake in reasoning from the principles to the results). Therefore, I am in complete agreement with Frege's results in this sense: they are valid for his concepts. The same holds for Church's results on the same (or a somewhat modified) basis. (Carnap, 1947, p. 128)

But, in the oblique situations, Carnap is clear that "Frege's analysis of sentences with terms like 'asserts', 'believes', etc., is not quite correct; because a sentence of this kind may change its truth-value and hence, a fortiori, its sense if the subsentence is replaced by an *L*-equivalent one" (*Ibid.*, p. 124). In Carnap's terminology, on the other hand, one may say in the oblique cases like those just mentioned neither the extension of the whole sentence is the function of the extension of the sub-sentence nor the intension (i.e., neither the replacement of the subsentence with an equivalent one transforms the whole sentence into an equivalent version, nor the replacement with a *L*-equivalent sub-sentence turns the whole sentence into a *L*-equivalent one). Once again, you see that what is important (and decisive) is the question of equivalency and interchangeability.

Furthermore, Carnap identifies a bigger problem with Frege's appeal to the nominatum-sense distinction, which is the unnecessary multiplication of names (or expansion) within the same type which leads to a complicated system. "Further, […] the method of the name-relation may lead to a complicated duplication or multiplicity of names within the same type. If Frege's form of the method is adopted, the situation becomes complicated" (*Ibid.*, p. 130). Carnap illustrates this problem with two examples. Let n_i be the names of the entities e_i. Now, in the first example, let n_1 be the name of the sentence *Hs*, "Scott is a human"; and, in the second example n_1 be the name of the predicator *H*, "… is a human". According to nominatum-sense distinction, every name relates to an entity we call its "nominatum" (let us call it relationship *Nu*), on the one hand; and, on the other hand, to another entity we call its "sense" (let us call this relationship *Se*). The following depiction illustrates how this situation leads to multiplication of objects.

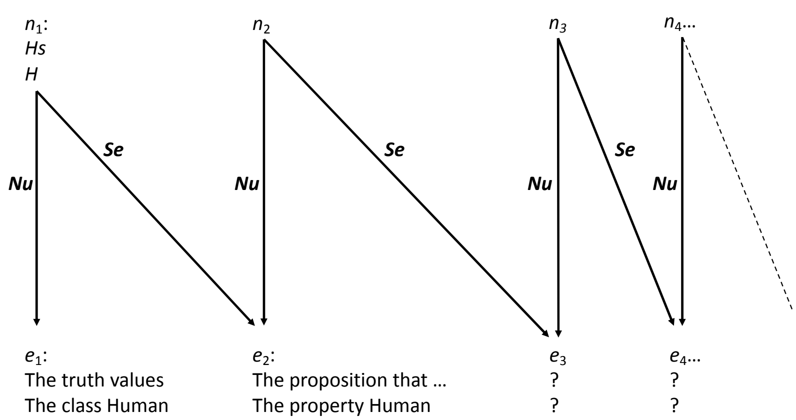

Figure 4 Sense and nominatum relations

Figure 4 shows that for a name n_1, say the sentence "Hs", according to Frege's method, there is an entity $e_1$which is the nominatum named by n_1; in this case, the truth value of "Hs". There is also another entity, e_2, that is the sense of "Hs", which is the proposition that Scott is human. Since e_2 is different than e_1, it can have its own name, say n_2; now, n_2 has its own sense that is another entity e_3 which can have its own name, n_3, and so on *ad infinitum*.

Here we can see how the sense-nominatum distinction may lead to an infinite and unnecessary multiplication. If we are to compare Frege's and Carnap's method, we may say the following: Frege utilizes an absolute (i.e., type-independent) pair of concepts (sense and nominatum), whereas Carnap makes use of a relative (type-dependent) pair (extension and intension). In Frege's method every name (and description) is viewed as a coincidence of two entities called sense and nominatum, regardless of the type of the expression. In Carnap's method every designator has an extension and an intension depending on its type, subjected to the equivalency or L-equivalency conditions of that specific type. Furthermore, as we saw, equivalency relation is not universal for Carnap (as it is for Frege), and it may have a different interpretation depending on the involved designators in the formation of the equivalency sentences which are relied on the definitions of truth and L-truth in turn.

As a side note, which is closely related to this discussion, I would like to add that, regarding the definition of cardinal numbers, Carnap believes that if Frege, instead of defining "the extension of the property Equinumerous to f" (in which f is a function/property), would have chosen "the property Equinumerous to f", the result would have been much simpler (for a complete discussion see Carnap, 1947, §27)[79]. Therefore, for defining cardinal numbers, Carnap follows

[79] According to Carnap, Frege actually does consider the use of "property" instead "the extension of property" but "does not pursue it any further" (*Ibid.*, p. 116).

Frege's procedure half way through (i.e., that the property f is equinumerous to the property g, $Equ(f, g)$, if there is a one-to-one correlation between those individuals which have the property f and those which have the property g), but then, at the second level, he defines the cardinal number (Nc) of the property f , $Nc'f'$, as the property (of second level) Equinumerous To f: $(\lambda g)[Equ(g, f)]$. At the final step, Carnap defines "n is a cardinal number" ($NC(n)$) in this way: "there is property such that n is the cardinal number of it": $(\lambda n)[\exists f(n \equiv Nc'f')]$. According to Carnap, "in this way the whole system of mathematics constructed on the basis of logic by Frege and Russell can be reconstructed in a simpler form without the use of class expressions distinct from property expressions and of class variables distinct from property variables" (*Ibid.*, p. 117). In short, in a Carnapian system, classes are a translation of names (or expressions), similar to other names of the same kind.

Thus, if *S* (different from S_1) contains "not only individual variables but also those of other types" (*Ibid.*, p. 44-45) then it would contain, for example, variables 'f', 'g', etc., which are of the type of predicators of level one and degree one. With respect to a predicator, say "*H*" in *S*, we have distinguished between its extension, the class Human, and its intension, the property Human. A sentence "… H …" containing "H" can be translated into the metalanguage *M* in different ways; "we may use either the word "human" alone or the phrase 'the class Human' or 'the property Human'"; you may see that the difference between these expressions is just the formulation. Hence, according to existential generalization, from "… H …", we can derive the existential sentence "$\exists f(\ldots f \ldots)$" which can be translated into *M* in the three following forms:

(i) "There is a f such that ... f ...",
(ii) "There is a class f such that ... f ...",
(iii) "There is a property f such that ... f ..."

As you see, f is a variable for both classes (the extensions) and properties (intensions). In the case of the "*H*", Carnap says:

> Since we regarded the class Human as the extension of 'H', we shall now regard it as one of the ***value extensions*** of 'f'; and, analogously, we take the property Human as one of the ***value intensions*** of 'f'. Let us call the closed expressions substitutable for a certain variable of any kind the ***value expressions*** of that variable.

Carnap then defines the extension and intension of "value expression" of a variable as follows (*Ibid.*, p. 45):

D 3.39: The *extension* of a ***value expression*** of a variable is one of the value extensions of that variable.

D 3.40: The *intension* of a ***value expression*** of a variable is one of the value intensions of that variable.

Thus, in a Carnapian method, if (and only if) we take "sentence" as a designator then the following holds (see also: D3.17-28; D3.6-7; D3.12-13) (note that the sentence in question might as well be an identity or equivalence sentence):

D 3.41: The *extension of a sentence* is its truth-value.

D 3.42: The *intension of a sentence* is the proposition expressed by it.

Hence:

- The extension of the sentence "*Hs*" (in S_1) is the truth-value that Scott is human, which happens to be the truth.
- The intension of the sentence "*Hs*" is the proposition that Scott is human.

Carnap makes the following remarks regarding what he means by using the term "proposition":

> Some remarks may help to clarify the sense in which we intend to use the term 'proposition'. Like the term 'property', it is used neither for a linguistic expression nor for a subjective, mental occurrence, but rather for something objective that may

> or may not be exemplified in nature. We apply the term 'proposition' to any entities of a certain logical type, namely, those that may be expressed by (declarative) sentences in a language. By the property Black we mean something that a thing may or may not have and that this table actually has. Analogously, by the proposition that this table is black we mean something that actually is the case with this table, something that is exemplified by the fact of the table's being as it is. (Carnap, 1947, p. 27)

As one may see, in the absence of the sense-nominatum distinction and starting with a neutral formulation, if the designator in question is "sentence", truth values only constitute its extension, but its intension remains the proposition it expresses. In a Carnapian system, what the designatum of "sentence" is depends on whether the sentence itself is *extensional* (i.e., whether its extension, with respect to a designator occurring in it, is a function of the extension of the designator) or *intensional* (a sentence is intentional if it is not extensional). If the sentence is extensional (or taken to be as one), then it should be analyzed in an extensional semantical system such as S_1; otherwise one should choose an intensional system such as S_2 for an appropriate analysis[80] (see T1 and T2 above, and below from *Ibid.*, §11). In general:

D 3.43: The *expression* $\mathfrak{A}_i$ is ***extensional*** *with respect to a certain occurrence of* $\mathfrak{A}_j$ *within* $\mathfrak{A}_i$ (in the system *S*) $\stackrel{\text{def}}{=}$ $\mathfrak{A}_i$ and $\mathfrak{A}_j$, are designators; the occurrence in question of $\mathfrak{A}_j$ within $\mathfrak{A}_i$ is interchangeable with any expression equivalent to $\mathfrak{A}_j$ (in *S*).

D 3.44: The *expression* $\mathfrak{A}_i$ is ***extensional*** (in *S*) $\stackrel{\text{def}}{=}$ $\mathfrak{A}_i$ is a designator (in *S*); $\mathfrak{A}_i$ is extensional with respect to any occurrence of a designator within $\mathfrak{A}_i$ (in *S*).

D 3.45: The *semantical system S* is ***extensional*** $\stackrel{\text{def}}{=}$ every sentence in *S* is extensional.

[80] Note that there are forms of sentences that are neither intensional nor extensional with respect to an internal designator. For example, what Carnap calls "*belief-sentences*" in the form that "x believes that P" (in which x is the name of a person and P stands for a proposition) are neither extensional nor intensional with respect to P (1947, §13); one should find a way to transform them for analysis.

According to Carnap (*Ibid.*, p. 48), if these conditions are not satisfied, we use the term "***non-extensional***" which is not necessarily synonymous to *intentional* (as it is in some cases).

D 3.46: The *expression* $\mathfrak{A}_i$ is ***intensional*** *with respect to a certain occurrence of* $\mathfrak{A}_j$ *within* $\mathfrak{A}_i$ (in the system *S*) $\stackrel{\text{def}}{=}$ $\mathfrak{A}_i$ and $\mathfrak{A}_j$, are designators; the occurrence in question of $\mathfrak{A}_j$ within $\mathfrak{A}_i$ is *L*-interchangeable with any expression *L*-equivalent to $\mathfrak{A}_j$ (in *S*).

D 3.47: The *expression* $\mathfrak{A}_i$ is ***intensional*** (in *S*) $\stackrel{\text{def}}{=}$ $\mathfrak{A}_i$ is a designator (in *S*); $\mathfrak{A}_i$ is either extensional or intensional and is intensional with respect to at least one occurrence of a designator $\mathfrak{A}_i$ (in *S*).

D 3.48: The *semantical system S* is ***intensional*** $\stackrel{\text{def}}{=}$ every sentence in *S* is either extensional or intensional, and at least one is intensional.

As we have said, according to Carnap, Church's method is a modified version of that of Frege's and a lot of what we have said about Frege's method applies to Church as well (especially since both methods share the sense-nominatum distinction). Thus, it should be clear by now that Church's criticism is only valid in the case of taking "proposition" as the nominatum of "sentence" and not in the way in which Carnap constructs his system. If you remember, Church claims that whether the language is extensional or intensional, the designata of sentences ought to be the truth values; Carnap shows that this claim is wrong for the following reasons (*Ibid.*, §11). Consider the following definitions:

D 3.49: An occurrence of the *expression* $\mathfrak{A}_j$, *within the expression* $\mathfrak{A}_i$ is (1) ***interchangeable***, (2) ***L-interchangeable*** with $\mathfrak{A}'_j$ (in *S*) $\stackrel{\text{def}}{=}$ $\mathfrak{A}_i$, is a designator and is (1) equivalent, (2) *L*-equivalent to the expression $\mathfrak{A}'_i$ constructed out of $\mathfrak{A}_i$. by replacing the occurrence of $\mathfrak{A}_i$ in question by $\mathfrak{A}'_i$.

D 3.50: $\mathfrak{A}_j$ is (1) ***interchangeable***, (2) ***L-interchangeable*** with $\mathfrak{A}'_j$ *in the system S* $\stackrel{\text{def}}{=}$ any occurrence of $\mathfrak{A}_j$ within any sentence of *S* is (1) interchangeable, (2) *L*-interchangeable with $\mathfrak{A}'_j$.

Suppose that C stands for a F-true sentence (i.e., true but not L-true; such as Hs), and N stands for logical necessity such that if "…" is a L-true sentence $N(\ldots)$ is true, and if not (e.g., it is F-true) $N(\ldots)$ is false. We know that, given these assumptions, "$C \vee \sim C$" is a L-true sentence, then by the definitions given above "C" and "$C \vee \sim C$" are equivalent but not L-equivalent. And, according to the given definition for $N(\ldots)$:

- $N(C \vee \sim C)$ is true (and also L-true)
- $N(C)$ is false

Hence, according to D3.1:

- "$N(C \vee \sim C)$" and "$N(C)$" are not equivalent

which means (according to D3.48-49):

- The occurrences of "C" in "$N(C)$" is not interchangeable with "$C \vee \sim C$"

Thus, according to D3.42:

- "$N(C)$" is non-extensional with respect to the sub-sentence "C"
- "$N(C \vee \sim C)$" is non-extensional with respect to the sub-sentence "$C \vee \sim C$"

Given that the non-extensionality of a statement (or language) implies that the statement (or language) in question is not truth-functional, Church's argument does not apply to intensional statements (or languages) if our system also contains factual statements. As Carnap notifies, under the given definitions and formulations, "it is certainly not the case that […] all sentences and all semantical systems fulfill the defining condition for extensionality" (*Ibid.*, p. 50-) regardless what the designata of sentences are.

> Church qualifies his statement [‘Carnap’s definition of ‘extensional’ fails in that under it every language (every semantical system) is extensional, even those which contain names of propositions and modal operators’] by the following condition: ‘if the designatum of a sentence is always a truth-value.’ However, this qualification does not change the situation. Any assumption as to what are the designata (nominata) of sentences is irrelevant to the question of whether the examples stated [...] are extensional or not on the basis of my definition, because in this definition the concept of the designatum (nominatum) of a sentence is not used. (Carnap, 1947, p. 50)

Thus, it is obvious that Church’s criticism does not apply to Carnap’s system, given Carnap’s own definitions. On the other hand, even though “Church does not simply adopt Frege’s method in its original form” Church’s own extensional approach still suffers from admitting the sense-nominatum distinction because “he agrees with Frege’s conclusion that the nominatum of an oblique (non-extensional) occurrence of a name must be different from its ordinary nominatum and must be the same as its ordinary sense” (*Ibid.*, p. 138). Also, according to Carnap, “Church is in accord with Frege’s intentions when he regards a class as the (ordinary) nominatum of a predicator (of degree one)—for instance, a common noun—and a property as its (ordinary) sense” (*Ibid.*, p. 125); and also defends and develops Frege’s distinction in his publications (*Ibid.*, p. 127). According to Carnap, since Carnap’s use of the term “designatum” with respect to individual expressions (definite description) is always the same as “nominatum” in Frege’s method, and his use of this term was not used uniformly in his earlier work (1942), it is likely that Church misunderstood his use of the term.

> It is probably due to this fact that Church understood my term ‘designatum’ in all cases in the sense of ‘nominatum’; and presumably Quine likewise believes himself to be in accord with my use when he applies designatum in this sense. I regret that the lack of a clear explanation in [(1942)] has caused these misunderstandings. This lack was not accidental but was caused by an obscurity of long standing in some of the fundamental semantical concepts. If I see it correctly, this obscurity has been overcome only by the analysis made in this book. Church’s statement that the designatum of a sentence is not a proposition but a truth-value is—on the basis of

> Frege's method of the name-relation—correct for Church's use of 'designatum' in the sense of 'nominatum'; not, however, for my use of 'designatum' in [(1942)] in the sense of 'intension'. (Carnap, 1947, p. 166)

In concluding this section, as we went through Carnap's criticism of Frege's method and his reply to Church, we saw clearly the advantage of Carnap's method of extension and intention compared to any Fregean method based on the distinction between sense and nominatum (including that of Church). We also saw, in the absence of a universal notion of equivalency, in a Carnapian system, equivalency relation may have different interpretations depending on the type and admissibility of designators into the equivalency sentence that is based on the semantical concept of truth (and *L*-truth). Although, implicitly and not quite directly, according to all of the given explanations and definition, we might have a sense of how abstraction from *F*-truth to *L*-truth (or, in general from semantic to *L*-semantic) is taking place on the basis on equivalency and *L*-equivalency. In a constructed semantical system based on a set of rules (e.g., rules of formation, translation, etc.) for that particular system, as we may have noticed, even the abstract concept of a general semantical system S might be interpreted to S_1, S_2, S_3, etc. The final step of abstraction is to abstract from semantic to syntax (from *L*-true to *C*-true), and this is what we are going to discuss in the next section.

3.4 Abstraction to Syntax: formalization of logic

If we consider language as an expressive medium, i.e., a medium by which one is able to express and share one's thoughts, ideas, feelings, opinions, etc., within one's community (*using signs*), then it is a genuine line of study to see the ways in which linguistic expressions make sense *to the people* of that community (mathematicians, physicists, chemists, philosophers, etc.); and that is what we mean by *linguistic analysis*. In terms of linguistic analysis, Carnap gives a rather clear understanding of what abstraction means; as we saw, it only means *becoming independent from meaning*. In this sense semantic expressions are more abstract than that of pragmatics, and syntactical expressions are more abstract than that of semantic. Note that it might not be necessary to have an exact definition of what "meaning" is (to start our analysis); all we need is to take a unit which we can be sure contains a complete meaning. If we take complete sentences as units of meaning, then from the ways in which other constitutive expressions (or signs) participating in the formation of sentences, we may realize their participating share in the meaning as a whole. Thus, the template we use for studying declarative sentence is what we call "*sentential matrix*" or "*matrix*", i.e., "expressions which are either sentences or formed from sentences by replacing individual constants with variables"[81] (Carnap, 1947, p. 5). With respect to meaning, Carnap describes the two tendencies or points of view among modern logicians: "The one tendency emphasizes form, the logical structure of sentences and deductions, relations between signs in abstraction from their meaning [(i.e., syntax)]. The other [(i.e., semantic)] emphasizes just the factors excluded by the first [(meaning, designation, interpretation, relations of entailment, compatibility, incompatibility, etc.]" (Carnap, 1943, p. ix). Carnap finds the two not only theoretically compatible but also "complementary" (*Ibid.*).

So far, we have seen the importance, role, and relativity of "designation" in the first step of the abstraction ladder, that is abstracting from factual sentences of pragmatics to logical sentences of semantics (or L-semantic containing only linguistic terms of the metalanguage). We also saw

[81] According to Carnap (*Ibid.*), "If a matrix contains any number of free occurrences of n different variables, it is said to be of degree n" (e.g., "$Axy \wedge Hx$" is a degree-two matrix); thus, complete sentences are matrices of degree zero.

that "meaning" (as it applies to the constructed matrices) could be considered as the result of two factors: (1) the structural validity and applicability of matrices (given the purpose of the construction), and (2) the admissibility of objects with respect to the occurrences of their designators within the matrices in question. The next step of becoming independent from meaning is to set aside "designation" altogether and built up a calculus (constituting C-concepts/terms) upon which the rules of designation could be employed. To reach this goal, in what follows, our main source would be from Carnap (1943). (Note that in what follows, PC stands for propositional calculus, which is the same as propositional logic; and, FC stands for functional calculus which is the same as predicate logic)

What we have seen, in the previous sections, is that, in a Carnapian system, equivalency relation is not universal, and that it could be interpreted once it is used in an equivalency sentence, which could have different interpretations under the rules of designation. We have also seen that the abstract term "S" may have different interpretations ($S_1, S_2, S_3, \ldots$), and we will see that the same holds for the term "K" as it stands for calculus. Carnap calls attention to an important fact regarding the customary formulation of modern logic, as follows: "this customary language contains both syntactical and semantical terms" (*Ibid.*, p. xiii). For example, according to Carnap (*Ibid.*, p. xi), for the first-order predicate logic, Gödel's completeness theorem is formulated in two ways: (1) "Every formula (i.e., sentential function, or matrix in our terminology, of the calculus in question) which is universally valid is provable", and (2) "Every formula is either refutable or satisfiable". In these formulations, the terms "provable" and "refutable" are obviously syntactical, because "they are exactly defined on the basis of the rules of the calculus in question; and those rules are explicitly stated in the form of primitive sentences (axioms) and rules of inference" (*Ibid.*). Nonetheless, this is not the case for the terms "universally valid" (*allgemeingultig*) and "satisfiable" (*erfullbar*); which, according to Carnap, are semantical terms because:

> They are explained in the following way a formula (a sentential function of the calculus in question) is called universally valid if it is true for all values of the free variables, it is called satisfiable if there are values of the free variables for which it is true. Clearly these two terms are not of a syntactical but of a semantical nature. In a theory of semantics, they could be exactly denned on the basis of the concept of entities satisfying a sentential function (this is the basic concept in Tarski's semantics […]). Gödel's theorem is accordingly of a peculiar nature, which is usually not recognized, it combines syntactical and semantical concepts, in a more exact formulation it would state a relation between a syntactical and a corresponding semantical system. (Carnap, 1943, pp. xi-xii)

As it is obvious from this quote, what is missing (to make the exact definitions) and hence make the whole construction well-defined is exactly the abstractive link between semantic and syntax, that is "a relation between a syntactical and a corresponding semantical system". And that is what exactly we are going to see in some of the examples in this section. Carnap also considers some other terms (other than "true" and "false"), such as, "truth-value", "values of a variable", etc., as essentially of semantical nature. Carnap is explicit that the missing *corresponding rules* are a defect in the construction of modern logical systems and that it requires urgent attention from modern logicians; otherwise, with the present form, the whole construction is *not* well-constructed.

> The decisive point is this while the syntactical terms used by logicians are exactly denned and belong to a well-constructed and recognized theory (namely syntax), the same is not true for the semantical terms. These are merely explained in an informal manner, without a theory as framework for them. No rules constituting semantical systems corresponding to the calculi in question are given, although such rules would serve as a basis for the semantical terms used. Thus the understanding and the use of these terms is left to common-sense and instinct. It is assumed that the reader knows how to interpret and use them on the basis of his

knowledge of everyday language. This assumption is perhaps correct to some extent. Similarly, however, most people know how to use the terms “all” and “some” before a logician expounds Aristotle’s rules to them. Once we concede that it is essential for the development of logic to give explicit rules for all terms which play a central role, then we see that the demand for such rules in the case of the semantical terms is at least as urgent as in the case of “all” and “some”. It should be noted that the semantical terms used in recent investigations do not merely serve for incidental explanations or illustrations outside of the theory dealt with, but are essential to that theory; this is shown by the fact that they occur in the very formulations of the problems and theorems. (Carnap, 1943, p. xii)

In semantic, we managed to abstract/formalize (i.e., becoming independent from meaning) in such a way that certain features of expressions (such as the question of whether a sentence $\mathfrak{S}$ is true or L-true in S, for example) become a matter of semantic. Now, the point is if we can further formalize in the same abstractive manner by mirroring those semantical concepts in a syntactic way within a constructed calculus, i.e., if we can construct a calculus K in such a way that one could say $\mathfrak{S}$ is C-true (provable) in K, for example. Thus, Carnap defines “formalized” in the following way (*Ibid.*, p. 3). (see also D3.90 and D3.92 below for true and L-true interpretations)

D 3.51: A radical semantical property F of an expression $\mathfrak{A}_i$, is ***formalized*** in $K \overset{\text{def}}{=} \mathfrak{A}_i$ has the property F in every semantical system which is a ***true interpretation*** for K.

D 3.52: An L-semantical property F of $\mathfrak{A}_i$, is ***formalized*** in $K \overset{\text{def}}{=} \mathfrak{A}_i$, has F in every ***L-true interpretation*** for K. Analogously for semantical relations.

Given these definitions, one should be also aware that abstracting from semantic to syntax (as a process of becoming independent from meaning) means that some semantical properties necessarily ought to be set aside in this regard; that is to say some properties of a semantical nature cannot be formalized further. For example, “having certain designatum” is one of those semantical properties which cannot be formalized (and hence abstracted) further; because:

> it is not possible to formalize the property of "*a*" designating Chicago and the property of "*P*" designating the property of being large; in other words, it is not possible to construct a calculus K in such a way that in every true interpretation for K "*a*" and "*P*" have the designata mentioned. If a true interpretation for K with these designata is given, another true interpretation for K with different designata can always be constructed. (Carnap, 1943, p. 4)

Let us take a quick detour to see Carnap's view on the possibility of a full formalization. The fact that not all semantical properties can be formalized relates to a foundational question in logic, namely whether or not logic can be fully formalized. According to Carnap (*Ibid.*, p. 4), this question may be asked in two senses. If the question is referring to the formalization of logical deduction (in other words, to a formalization of the relation of L-implication) then the answer is affirmative; "L-implication can in general be formally represented by C-implication" (*Ibid.*). On the other hand, we may ask the question in the sense of whether we can construct a calculus K as such that the calculus would be a *full formalization* of propositional or functional logic, according to the following definitions:

- If a calculus K (containing the ordinary connectives of propositional logic) could be constructed (on the basis of *S*) in such a way that it could formalize all essential properties of these connectives so that it would exclude the possibility of interpreting the connectives in any other than the ordinary way, then we should say that K was a *full formalization of propositional logic* (*Ibid.*).
- If K should, in addition to the above, impose the ordinary interpretation on the universal and existential operators, we should speak of a *full formalization of functional logic* (*Ibid.*).

According to Carnap (1939, p. 23-), for the latter sense of the question, "the answer depends upon the degree of complexity of *S*". We may formulate the question in terms of the possibility of constructing an *L-exhaustive* calculus (K is an *L-exhaustive* calculus with respect to *S* if K is not only in accordance with S, but also that the extensions of "C-implicate", "C-true", and possibly "C-false" coincide with those of "L-implicate", "L-true", and possibly "L-false", respectively), while making it clear whether we would like to construct a "*finite*" or "*transfinite*" calculus (i.e., to what degree of complexity we wish our calculus to have). The "transformation rules" (i.e., the set of rules stating explicitly the primitive sentences and the rules of inference) could be either "*finite rules*" (i.e., primitive sentences and rules of inference, each of which refer to a finite number of premises[82]) which may be employed for constructing a "*finite calculus*"[83], or they could be transfinite (in the case of an infinite number of premises) for constructing an "*transfinite calculus*" (Carnap, 1939, §10). If a finite set of the transformation rules is considered, constructing a finite L-exhaustive calculus K is possible. On the other hand, if, in S, there is a sentence S_2 and an infinite class of sentences C_1 such that S_2 is an L-implicate of C_1 but not an L-implicate of any finite subclass of C_1[84], then an L-exhaustive calculus K can be constructed if and only if transfinite rules are admitted (because, since C_1 is infinite, S_2 cannot be derivable from C_1). "If we decide in a given case to admit transfinite rules, we have to accept the complications and methodological difficulties connected with them" (Carnap, 1939, p. 23); it is well-known (as shown by Gödel) that in this case we cannot construct a finite calculus.

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82 The terms "*primitive sentence*" and "*premise*" must not be confused:

> A primitive sentence is a feature of the calculus, when the calculus is interpreted, the primitive sentences are asserted as true. On the other hand, a premise is a feature of a particular derivation in the calculus. Any sentence of the calculus occurs as a premise in some derivation. A premise is not asserted, it is only investigated with respect to its consequences. (Carnap, 1942, p. 167).

83 According to Carnap (Carnap, 1939, p. 23), it was first shown by Gödel that a finite calculus cannot be constructed for the whole of arithmetic.

84 As an example, (Carnap, 1939, p. 23), consider the case where S contains a name for every object of an infinite domain: $'a_1', 'a_2', 'a_3',\ etc$. And $'P'$ is a descriptive predicate. And C_1 is the (infinite) class of all sentences of the form "… is a P" where "…" is one of the object names. And S_2 is the sentence "$\forall x P(x)$"; i.e., "for every x, x is a P".

Going back to our discussion and only considering constructing a finite calculus, for example, what is important in this section, as we said above, is to show what is the relation between semantic and syntax (where formalization, in the sense just described, could be on par with the Carnapian notion of abstraction). Before that, one has to familiarize oneself with Carnap's terminology (*C*-terminology) for syntax, which is different than the customary terminology. Carnap uses this terminology, as mentioned above, for two reasons; first to be able to distinguish semantical and syntactical terms more clearly, and secondly, to illustrate the relation between semantic and syntax more explicitly in a manner which fits into his conception of abstraction. In the following tables we see the customary terms and their corresponding *C*-terms, and metalanguage signs.

Customary Terms	***C*-Terms**
derivable	*C*-implicate ("$\underset{C}{\rightarrow}$")
directly derivable	direct *C*-implicate ("$\underset{dc}{\rightarrow}$")
provable	*C*-true
primitive sentence (directly provable)	(directly) *C*-true
directly derivable from **Λ**	direct *C*-implicate of Λ
refutable	*C*-false
directly refutable	(directly) *C*-false
equipollent	*C*-equivalent
decidable	*C*-determinate
undecidable	*C*-indeterminate
incompatible	*C*-exclusive
compatible	non-*C*-exclusive
---	*C*-disjunct

Table VIII *C*-terminology

Metalanguage Signs	
$\boldsymbol{K}, \boldsymbol{C}$	calculus
$\mathfrak{T}, \mathfrak{S}, \mathfrak{K}, \boldsymbol{S}$	semantical systems
$\mathfrak{S}$	sentences (including propositional signs)
$\mathfrak{A}$	expressions
$\mathfrak{T}$	sentences and classes of sentences
$\mathfrak{K}$	sentential classes
$\mathfrak{R}$	proof (sequence of sentences)
$\mathfrak{f}$	functor variables
$\mathfrak{fu}$	functors (including variables)
$\mathfrak{p}$	predicate variables
$\mathfrak{pr}$	predicates (including variables)
$\mathfrak{i}$	(the class of) individual variables
$\mathfrak{in}$	individual signs (including variables)
$\mathfrak{s}$	propositional variables
$\mathfrak{se}$	propositional signs (including variables)
$\mathfrak{a}$	singes
$\mathfrak{c}$	constants
$\mathfrak{v}$	variables (of any kind)

Table IX Metalanguage vocabulary for syntax

The followings are selected definitions or theorems regarding the basic concepts of K that have been introduced by or derived from Carnap (1942, §§28-32; 1943, §23). From this selection, it is intended to show a sense of the relation between semantic and syntax in the case of propositional logic as described by Carnap.

D 3.53: $\mathfrak{T}_i \underset{C}{\rightarrow} \mathfrak{T}_j$ in K ($\mathfrak{T}_j$ is a ***C*-implicate** of $\mathfrak{T}_i$ in K) $\stackrel{\text{def}}{=}$ $\mathfrak{T}_j$ is derivable from $\mathfrak{T}_i$, or $\mathfrak{T}_i$ is *C*-false

D 3.54: $\mathfrak{T}_i$is ***C*-true** in $K \stackrel{\text{def}}{=} \Lambda \underset{C}{\rightarrow} \mathfrak{T}_i$

D 3.55: $\mathfrak{T}_i$ is ***C*-equivalent** to $\mathfrak{T}_j$ in $K \stackrel{\text{def}}{=} \mathfrak{T}_i \underset{C}{\rightarrow} \mathfrak{T}_j$ and $\mathfrak{T}_j \underset{C}{\rightarrow} \mathfrak{T}_i$

D 3.56: $\mathfrak{S}_i$is a ***primitive sentence*** in $K \stackrel{\text{def}}{=} \mathfrak{S}_i$ is directly derivable from Λ.

D 3.57: $\mathfrak{T}_i$ is ***provable*** in $K \stackrel{\text{def}}{=} \Lambda \underset{C}{\rightarrow} \mathfrak{T}_i$

T3.3: $\mathfrak{T}_i$is ***C*-true** if and only if $\mathfrak{T}_i$ is *C*-equivalent to Λ.

T3.4: Λ is a ***C*-implicate** of every $\mathfrak{T}_i$.

T3.5: $\mathfrak{T}_i$ is ***C*-true** in K if and only if $\mathfrak{T}_i$ is true in every true interpretation of K.

T3.6: $\mathfrak{T}_i$ is ***C*-false** in K if and only if $\mathfrak{T}_i$ is false in every true interpretation of K.

T3.7: $\mathfrak{T}_i$ is ***C*-equivalent** to $\mathfrak{T}_j$ in K if and only if $\mathfrak{T}_i$ is equivalent to $\mathfrak{T}_j$ in every true interpretation of K.

T3.8: $\mathfrak{T}_i \underset{c}{\rightarrow} \mathfrak{T}_j$ in K ($\mathfrak{T}_j$ is a ***C*-implicate** of $\mathfrak{T}_i$ in K) if and only if $\mathfrak{T}_i \rightarrow \mathfrak{T}_j$ in S ($\mathfrak{T}_j$ is an implicate of $\mathfrak{T}_i$ in S) for every true interpretation S for K.

Of course, one of the important relations between an abstract term and the terms of the lower levels is “interpretation”; in general, an abstract term could have multiple interpretations. “We call *S* an *interpretation* of *C* if the rules of *S* determine truth criteria for all sentences of *C*; in other words, if to every formula of *C* there is a corresponding proposition of *S*; the converse is not required” (Carnap, 1939, p. 21). Regarding the possible relations between *S* and *C*, Carnap also gives the following definitions (*Ibid.*, some modifications is made for the consistency of signs):

D 3.58: S is a ***true interpretation*** of $C \stackrel{\text{def}}{=}$ for any$\mathfrak{T}_1$, $\mathfrak{T}_2$, $\mathfrak{T}_3$, and $\mathfrak{T}_4$, if $\mathfrak{T}_2$ is a C-implicate of $\mathfrak{T}_1$ in C, then $\mathfrak{T}_2$ is an implicate of $\mathfrak{T}_1$ in S; and, if $\mathfrak{T}_3$ is C-true in C, it is true in S; and if $\mathfrak{T}_4$ is C-false in C, it is false in S. (otherwise S is a *false interpretation* of C)

D 3.59: If the semantical rules suffice to show that S is a true interpretation of C, then S an ***L-true interpretation*** of C. (in this case C-implication becomes L-implication; every C-true sentence becomes L-true, and every C-false sentence becomes L-false)

D 3.60: If semantical rules suffice to show that S is a false interpretation of C, then S an *L-false interpretation* of C.

D 3.61: If S is an interpretation but neither an L-true nor an L-false interpretation of C, then S a ***factual interpretation*** of C. (in this case, in order to find out whether the interpretation is true, we have to find out whether some factual sentences are true; for this task we have to carry out empirical investigations about facts)

After classification of the signs of K (i.e., specifying as many classes of signs as are necessary for the formulation of the syntactical rules) the first step is to setup the *rules of formation* (i.e., giving the definition of "sentence" in K). "There is a difference between these rules and the rules of formation of a semantical system. In the latter rules we may refer to the designata of the signs, although it is not often done. But in the syntactical rules of formation this is not permitted" (Carnap, 1942, p. 157). As said above, the essential set of rules in constructing a calculus are the *rules of transformation* (or rules of deduction), in which "first, *primitive sentences* are laid down, either by an enumeration, or by the stipulation that all sentences of certain forms are admitted as primitive sentences" (*Ibid.*, p. 157) (note that the number of primitive sentences could be transfinite); secondly, rules of inference are laid down such that the rules exactly specify under what condition (and only under those conditions) one would be able to formulate $\mathfrak{S}_j$ is a direct C-implicate of (i.e., directly derivable from) $\mathfrak{K}_i$.

To see the relation between semantic and syntax more clearly, in a Carnapian system, and innovative features in constructing a syntactical system (introduced by Carnap), let us take propositional logic (PC) as an example. Carnap, of course was well aware of the two methods of constructing a system for logical connectives. "There are two customary ways of constructing a system for the propositional connectives, one by the use of primitive sentences and rules of inference, the other by the use of truth tables.[85]" (1943, p. 10). Since the second method is a method for interpreting sentences, "hence, it does not belong to syntax but to semantics" (*Ibid.*). Carnap distinguishes between different forms of PC (e.g., PC_1, PC_1^D, PC_2, PC_3, etc.). The difference between forms of PC is based on which and how many of the logical connectives are being considered (as primitive) in the form in question (under the condition of being isomorphic to a calculus which contains PC_1^D[86]). Carnap identifies four different definitions for singulary (unary) connectives (negation) and sixteen ones for binary ones. PC_1 is the main form for Carnap (he calls it the "*Hilbert-Bernays form*"[87]). PC_1 only takes the connectives "negation" and "disjunction" as its primitives, in accordance with the following definition of connective.

D 3.62: $\mathfrak{a}_i$, is a *general connective* of degree n in a calculus K (or in a semantical system S) $\overset{\text{def}}{=}$ K (or S) contains closed sentences, and for every n-term sequence of closed sentences in K (or S respectively) there is a full sentence of $\mathfrak{a}_i$, in K (or S) with that sequence of components.

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85 In the modern notation we usually use the sign "⊢" for derivations by the first method, and it is called "*the semantic consequence relation of PC*"; and we use this sign "⊨" for the derivations by the second method, and it is called "*syntactic consequence relation*" (Epstein, 2001, pp. 51 & 65).

86 PC_1^D is PC_1 plus all other connectives.

87 According to Carnap (1943, p. 18),

> A number of other forms of PC besides PC_1 are known. Thus, e.g., each of the following sets of primitive signs is a sufficient basis for expressing all connections$_C$: signs for negation$_C$ and conjunction$_C$ (PC_2); negation$_C$ and implication$_C$ (PC_3), exclusion$_C$ (PC_4, shown by Sheffer), bi-negation$_C$ (PC_5, Sheffer). Suitable rules of deduction for these forms have been constructed for $PC_3$by Frege, for PC_4 by Nicod and Quine, for PC_5 by Quine.

For example, if a connective $\mathfrak{a}_j$ of degree one ("degree one" means that it is a singulary connective like "negation") is applied to a complete sentence $\mathfrak{S}_i$, then the resulting sentence would be "$\mathfrak{a}_k(\mathfrak{S}_i)$". Similarly, if a degree two connective $\mathfrak{a}_l$ (or a binary connective like "disjunction") is applied to two complete sentences $\mathfrak{S}_i$ and $\mathfrak{S}_j$, the resulting sentence would be "$\mathfrak{a}_l(\mathfrak{S}_i, \mathfrak{S}_j)$". Accordingly, one may start constructing PC_1 in the following way (note that the domain to which the connective belongs is specified by the subscript of its sign):

D 3.63: K ***contains*** PC_1 (with *neg*$_c$ as sign of *negation*$_c$ and *dis*$_c$ as sign of *disjunction*$_c$)$\overset{\text{def}}{=}$ the calculus K fulfills the following conditions:

a. $neg_C$is a singulary and dis_C a binary general connective in K.

b. The relation of **direct *C*-implication** ($\underset{dc}{\rightarrow}$) (*direct derivability*) holds in the following cases for any $\mathfrak{S}_i$, $\mathfrak{S}_j$, and $\mathfrak{S}_k$ (but not necessarily only in these cases):

i. $\Lambda \underset{dc}{\rightarrow} dis_C(neg_C(dis_C(\mathfrak{S}_i, \mathfrak{S}_i)), \mathfrak{S}_i)$

ii. $\Lambda \underset{dc}{\rightarrow} dis_C(neg_C(\mathfrak{S}_i), dis_C(\mathfrak{S}_i, \mathfrak{S}_j))$

iii. $\Lambda \underset{dc}{\rightarrow} dis_C(neg_C(dis_C(\mathfrak{S}_i, \mathfrak{S}_j)), dis_C(\mathfrak{S}_j, \mathfrak{S}_i))$

iv. $\Lambda \underset{dc}{\rightarrow} dis_C(neg_C\left(dis_C\left(neg_C(\mathfrak{S}_i), \mathfrak{S}_j\right)\right), dis_C\left(neg_C\left(dis_C\,(\mathfrak{S}_k, \mathfrak{S}_i)\right), dis_C\left(\mathfrak{S}_k, \mathfrak{S}_j\right)\right))$

v. $\{\mathfrak{S}_i, dis_C(neg_C(\mathfrak{S}_i), \mathfrak{S}_i)\} \underset{dc}{\rightarrow} \mathfrak{S}_j$

D 3.64: $\mathfrak{T}_j$ is a ***C*-implicate** of $\mathfrak{T}_i$ in K **by PC_1** $\overset{\text{def}}{=}$ K contains $PC_1$and $\mathfrak{T}_i \underset{C}{\rightarrow} \mathfrak{T}_j$ in virtue of the rules of deduction as given in D3.58 (b), analogously for any other *C*-term defined on the basis of "*C*-implication".

For PC_1, *rules of inference* are:

I. *Rule of substitution*: $\mathfrak{S}_i\binom{\mathfrak{s}_k}{\mathfrak{S}_l}$ is directly derivable from $\mathfrak{S}_i$.

II. *Rule of implication* (in disjunctive form): $\mathfrak{S}_j$ is directly derivable from $\mathfrak{S}_i$ and $\mathfrak{S}_j \vee {\sim}\mathfrak{S}_i$.

As it may have been noticed, in constructing a calculus for PC there is no designation relation (as there is in semantic). Formalizing (abstracting) further into pure syntax (from descriptive syntax, i.e., the syntax that contains descriptive signs for different connectives) is still possible, and this is the innovation by Carnap, which is different from customary syntactical formalization. To give an example let us take the formalization of a singulary connective (negation) that has been illustrated in Table X:

Connection$_C$		**Connective**			
(1) Abbreviation	**(2) Ordinary name**	**(3) Customary symbol**	**(4) Syntactic symbol**	**(5) Expression in PC$_1$**	**(6) Characteristic**
${}_c\boldsymbol{Conn}_1^1$	$Tautology_C$		${}_c\mathfrak{b}^1$	$\mathfrak{S}_i \vee \sim\mathfrak{S}_i$	TT
${}_c\boldsymbol{Conn}_2^1$	$(Identity_C)$		${}_c\mathfrak{b}^2$	$\mathfrak{S}_i$	TF
${}_c\boldsymbol{Conn}_3^1$	$Negation_C$	~	${}_c\mathfrak{b}^3(neg_C)$	$\sim\mathfrak{S}_i$	FT
${}_c\boldsymbol{Conn}_4^1$	$Contardiction_C$		${}_c\mathfrak{b}^4$	$\sim(\mathfrak{S}_i \vee \sim\mathfrak{S}_i)$	FF

Table X The four singulary $connection_C$

D 3.65: $\mathfrak{S}_k$ is a *sentence of* ${}_cConn_1^1$ (or a $tautology_C$*sentence*) with $\mathfrak{S}_i$ (as component) in PC_1 in $K \overset{\text{def}}{=} K$ contains PC_1 and $\mathfrak{S}_k$ C-equivalent in K to $dis_C(\mathfrak{S}_i, neg_C(\mathfrak{S}_i))$.

D 3.66: (3) $\mathfrak{S}_i$a *sentence of* ${}_cConn_3^1$ (or a $negation_C$ *sentence*) with $\mathfrak{S}_i$, (as component) in PC_1 in $K \overset{\text{def}}{=} K$ contains PC_1 and $\mathfrak{S}_k$ is C-equivalent in K to $neg_C(\mathfrak{S}_i)$

D 3.67: (2) and (4) are analogous.

What is interesting with these definitions is that by them we manage to define the negation sign fully syntactically. We also see that, syntactically, there are four possible definitions for the negation sign (given only disjunction). Column (6) includes all possible assignments to the closed sentence $\mathfrak{S}_i$ (i.e., the first value/sign from the left) and to the resulting sentence (expressed in column 5) "$\mathfrak{a}_k(\mathfrak{S}_i)$" (i.e., the second value/sign from the left). "The expressions in column (5) of the table show how all singulary [...] $connections_C$ can be expressed in PC_1. Therefore, these expressions may be taken as definientia in definitions of signs for these $connections_C$, on the basis of the signs for $negation_C$ [...] as primitives" (Carnap, 1943, p. 15); this is one of the features that we do not normally see in the customary logic books. The advantage of having this feature is that it enables us to separate semantical rules from syntactic ones; as we said, in a Fregean formalization we ought to accept a mix of both. The schema that would allow the above four definitions for "~" is the following:

D 3.68: $\mathfrak{a}_q$is a sign (or connective) for ${}_cConn_1^1$ (q = 1 to 4) in PC_1in $K \stackrel{\text{def}}{=} K$ contains PC_1, $\mathfrak{a}_q$ is a general connective in K, and, for any closed sentence[88] $\mathfrak{S}_i$ in K, the full sentence $\mathfrak{a}_q(\mathfrak{S}_i)$ is a sentence for ${}_cConn_1^1$ in PC_1 in K.

Carnap gives similar definitions for "disjunction", e.g., "${}_cConn_1^2$", (and other connectives which we do not discuss here) by which he is able to give a formalization of PC (up to pure syntax)[89].

Meanwhile, Carnap notices some issues with the customary formulation of propositional logic that is very much related to the construction of our abstraction-interpretation hierarchy (one of which is specifically related to the exclusion of factual sentences from interpretation as you will see below); hence, we will explain it here briefly. In the customary formulation in which propositional variables (such as p, q, r, etc.) are being used, primitive sentences of PC_1 are the following:

---

[88] In general:
$\mathfrak{A}_i$ is an open expression $\stackrel{\text{def}}{=}$ $\mathfrak{A}_i$ contains a free variable.
$\mathfrak{A}_i$ is a closed expression $\stackrel{\text{def}}{=}$ $\mathfrak{A}_i$ contains no free variable.

[89] For a simpler version of formalization of PC (without the use of German Gothic scripts) see Carnap (1972, pp. ii-vii); you may also see a copy of that in the appendix.

a. "$\sim(p \vee p) \vee p$"
b. "$\sim p \vee (p \vee q)$"
c. "$\sim(p \vee q) \vee (q \vee p)$"
d. "$\sim(\sim p \vee q) \vee (\sim (r \vee p) \vee (r \vee q))$"

According to Carnap, normally (i.e. in the customary way of propositional logic) PC is formalized "in isolation" and not as a part which could be contained in calculi; "this difference seems slight, but it is essential for the later discussion of interpretation" (1943, p. 18). Thus, in the customary method of construction, PC is represented in its so called "*pure form*" (i.e., as a calculus containing propositional variables as the only ultimate components, as presented right above; *Ibid.*). "But in a calculus of this kind, every sentence is *open* and is either *C*-true or *C*-comprehensive" (*Ibid.*). Thus, the interpretation (inverse of abstraction) will be stopped at the sentences expressing logical truths (L-determinate sentences) and never developed to factual examples (L-indeterminate expression). "This is a disadvantage for a discussion of interpretations. The customary interpretation is L-true, and hence all sentences in a pure form of PC become here L-determinate; there are no factual sentences" (*Ibid.*). Carnap also points out another reason that could be considered as a disadvantage for formalizing PC *in isolation* and hence giving us reasons to formalize it in ways in which it could be contained in a calculus.

> Moreover, the most convenient and customary formulation of semantical rules for the normal interpretation, namely the truth-tables, cannot be directly used for such a form of PC, because the truth tables apply only to closed sentences. Therefore, for the discussion of interpretations we shall have to take into consideration not pure forms but calculi containing PC with or without propositional variables, but in any case containing closed sentences and hence other constants in addition to the connectives (in the simplest case propositional constants). (Carnap, 1943, pp. 18-19)

If PC could be contained in a calculus K, the following basic theorem could be derived on the basis of the above definitions (*Ibid.*, p. 20):

T3.9: If K contains PC_1, then any sentence of one of the following forms is *C-true* in K by PC_1:

a. $dis_C(\mathfrak{S}_i, neg_C(\mathfrak{S}_i))$
b. $dis_C(neg_C(\mathfrak{S}_i), \mathfrak{S}_i)$

Thus, the following holds if (1) K contains PC_1, and all rules of inference in K are extensible[90], and (2) K contains no rule of refutation (or $(2')$, i.e., if it does contain a rule of refutation, every directly C-false $\mathfrak{T}_i$ in K is such that every sentence in K is derivable from it).

T3.10: If $\mathfrak{S}_i$ is closed and $\mathfrak{S}_i \underset{c}{\rightarrow} \mathfrak{S}_j$ in K, then any $implication_C$ sentence with $\mathfrak{S}_i$ and $\mathfrak{S}_j$, (e.g., $dis_C(neg_C(\mathfrak{S}_i), \mathfrak{S}_i)$), is C-true in K.

T3.11: If $\mathfrak{S}_i$ is a closed sentence in K then:

c. Every sentence in K which is a C-implicate both of $\mathfrak{S}_i$, and of $neg_C(\mathfrak{S}_i)$ is C-true.
d. $neg_C(\mathfrak{S}_i)$ is the strongest sentence which has the relation to $\mathfrak{S}_i$, stated in (a); that is to say, if $\mathfrak{S}_l$ is such that every sentence which is a C-implicate both of $\mathfrak{S}_i$ and of $\mathfrak{S}_l$is C-true, then $neg_C(\mathfrak{S}_i) \underset{c}{\rightarrow} \mathfrak{S}_j$.

Accordingly, the rules for making truth tables (for negation), would be as follows (in consideration of column 6 of Table X). N1: if $\mathfrak{S}_i$ is true, $neg_C(\mathfrak{S}_i)$ is false; and N2: if $\mathfrak{S}_i$is false, $neg_C(\mathfrak{S}_i)$ is true.

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[90] According to Carnap (1943, p. 23),

> A rule of inference R in a calculus K is (extensible with respect to a disjunctive component, or briefly) ***extensible*** $\overset{\text{def}}{=}$ K contains PC, and for any $\mathfrak{S}_j$, $\mathfrak{K}_i$, and $\mathfrak{S}_k$ in K, if $\mathfrak{S}_j$ is a direct C-implicate of $\mathfrak{S}_j$ in virtue of R, and $\mathfrak{S}_k$ is either closed (i.e. does not contain a free variable) or at least does not contain a free variable also occurring freely in $\mathfrak{S}_j$, or any sentence of $\mathfrak{K}_i$, then $dis'_C(\mathfrak{S}_k, \mathfrak{K}_i) \underset{c}{\rightarrow} dis_C(\mathfrak{S}_k, \mathfrak{S}_j)$ in K.

Name of the rule	$\mathfrak{S}_i$	$neg_C(\mathfrak{S}_i)$
N1.	T	F
N2.	F	T

Table XI Rules of truth table for negation

As Carnap has noted, "it is necessary to restrict the application of the *truth tables* to *closed sentences*" (1943, p. 40); otherwise, "p" and "$\sim p$" are both false regardless of N2; because they are variables and not constants thus they are syntactically false (in our formulation $\mathfrak{S}_i$ is a propositional variable that ranges over p, q, etc., as propositional constants). Carnap believes (*Ibid.*), "this is often overlooked because of a confusion between propositional variables and propositional constants", in the customary formalization[91].

However, there are other ways in which truth tables can be formulated for open sentences (using propositional variables such as p, q, r, etc., as they are being used in the customary formulation; and, as opposed to them being used as constants in Carnap's formulation), and those are the cases in which the truth of the propositions is considered as an *absolute* concept (outside the language system; and hence outside of linguistic analysis). In those cases, we have to make the following conventions (1942, p. 89):

[91] The same consideration may be regarded in the first-order predicate logic (FC), where, in the customary interpretation, open sentences are interpreted as universal (unlike Carnap's definition of "universal operator" below, 1943, p. 140). Also, such a case, "if there is an individual which is not P, and another one which is P and not Q, then "$P(x)$" and "$\sim P(x)$" (which is L-equivalent to "$\forall x(\sim P(x))$") are both false, regardless of N2" (*Ibid.*, p. 40).
The ***universal operator*** in FC_1 has a ***normal interpretation*** in $S \stackrel{\text{def}}{=} S$ is a true interpretation for FC_1, and any closed sentence of the form $\mathfrak{i}_k(\mathfrak{S}_p)$ is true in S if and only if every individual in S has the property determined by $\mathfrak{S}_p$.

Convention (I): A term used for a radical semantical property of expressions will be applied in an absolute way (i.e. without reference to a language system) to an entity u if and only if every expression $\mathfrak{A}_i$ which designates u in any semantical system S has that semantical property in S (analogously with a semantical relation between two or more expressions).

On the basis of this convention we may say: (1) $\mathfrak{S}_i$, is *true* in $S \overset{\text{def}}{=}$ there is a (proposition) p such that $\mathfrak{S}_i$, designates p in S, and p, and (2) $\mathfrak{S}_i$is *false* in $S \overset{\text{def}}{=}$ there is a p such that $\mathfrak{S}_i$, designates p in S, and $\sim p$. (If we remember our above discussion regarding the designata of the sentences, which Carnap believes are "propositions" and not the truth values, here we may have another look at the reason why this is the case.) As one may realize by now, in the semantical application of the radical term "truth" (in a Carnapian system), we may not only use this term in the expression "the sentence '$P(a)$' is true in S", but we may also apply it to the designatum of this expression, i.e., "the proposition $P(a)$ is true". "In the second case, no reference to a language system is made" (1942, p. 88), i.e., the truth of the proposition (truth as an "absolute concept") is outside of both semantical and syntactical analysis and belongs to pragmatics. Thus, Carnap believes (1943, p. 40), while the formulation of the following sentence, "If 'p' is false and 'q' is false, then '$p \equiv q$' is true" is not correct, the formulation of this following sentence (using propositional variables in an open sentence), "If p is false and q is false, then $p \equiv q$ is true" is correct; because "in the latter sentence, 'p', 'q', and '$\equiv$' are regarded as belonging to the English language [(not to the metalanguage)]. The sentence refers to the absolute concept of truth for propositions, not to the semantical concept of truth for sentences. Therefore, it cannot serve as a rule for a language system" (*Ibid.*).

A similar procedure (in which L-true is deemed as an absolute concept) could be carried out for L-terms (such as L-true), but, then, this would require the metalanguage to be non-extensional, "*a non-extensional metalanguage is needed for this purpose*" (*Ibid.*, p. 88). This is an important requirement which we will discuss in the subsequent section.

3.4.1 The Requirement of Non-extensionality for Metalanguage

In this section we want to see what is required from a metalanguage if we are to apply L-terms to open sentences as absolute concepts. Before we get into the main discussion of this section let us summarize how Carnap uses "semantical metalanguage" in distinction of syntactical metalanguage (Carnap, 1972, p. 17) in linguistic analysis. We may think of metalanguage as a language with different vocabulary-packages, each of which could be used depending on the subject matter of the investigation (whether it is semantic or syntax, for example). The general vocabulary of a semantical metalanguage consists of the following parts and we may see their relations to the object language in the following diagram.

The vocabulary of the semantical metalanguage M for an object language L consists of the following four parts:

(1) *The logical vocabulary*: logical constants ('not', 'or', 'every', etc.) and general variables 'x', 'F', 'p', etc.).
(2) *The syntactical vocabulary*: names of the signs in L, and a notation for concatenation. Thus, a spelling description for any expression in L can be formulated. Further, syntactical Variables (e.g., $\mathfrak{pr}_i$, $\mathfrak{in}_i$, etc.).

(3) *The non-semiotical vocabulary* (*translation vocabulary*): descriptive constants referring to non-linguistic entities (e.g., things in the world). This vocabulary, together with (1), must be sufficient for a translation of all sentences in L.

(4) *The semantical vocabulary*: The semantical terms are defined on the basis of the terms of the three other parts.

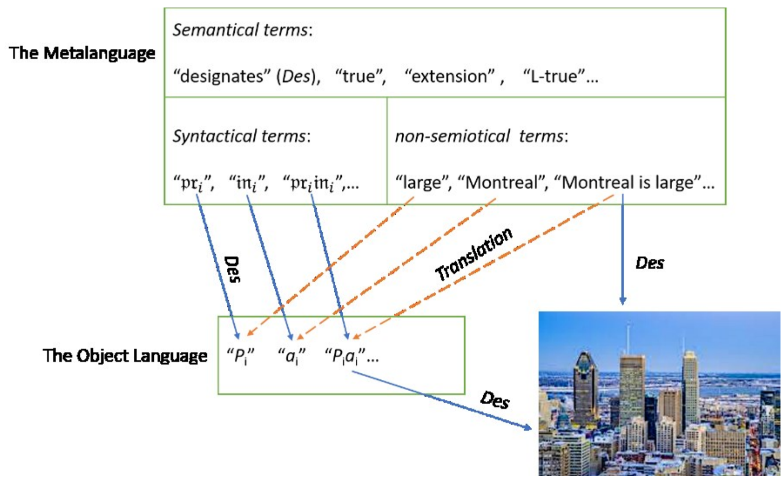

Figure 5 The main relations between the object language and the metalanguage

Carnap (*Ibid.*) introduces three metalanguages: M^e(material), M^i(logical), and M^s (synonymous); and hence three different corresponding designation relations (Dse^e, Des^i, Des^s) for speaking of semantic. Carnap is explicit that M^i and M^s need to be non-extensional: "Des^i and Des^s require non-extensional metalanguages" (1972, p. 14).

As said above, in the construction of a calculus some of the sematic relations and/or properties (including "designation") are not to be formalized (or abstracted) further; since the goal of abstraction is to disregard meaning (i.e., focusing on structure) as much as possible. We may also want to remember that the Fregean concept of "extension" is different than that of Carnap's; in a constructed formal system, while the sameness of Fregean extensions is always verifiable (via interchangeability of expressions), the sameness of Fregean intensions is not; on the other hand, in a Carnapian system, we saw that the sameness of both extensions and intensions are always verifiable since they rely respectively on the concepts of equivalency and L-equivalency defined within the system (relative to the designators).

With this introduction, we now go back to our discussion about the absolute use of the L-terms under the above convention (I). "The convention is not itself a definition for the absolute terms in question; it merely states under what conditions we will accept such definitions" (1942, p. 90). If we want to apply the convention for truth, first we have to prove that, (a proposition) p is true if and only if for every S and every $\mathfrak{S}_i$, if $\mathfrak{S}_i$ designates p in S, then $\mathfrak{S}_i$ is true in S (*Ibid.*). Then, fulfilling this requirement, this definition might be given as, p is true $\stackrel{\text{def}}{=}$ for every S and every $\mathfrak{S}_i$, if $\mathfrak{S}_i$ designates p in *S*, then $\mathfrak{S}_i$ is true in *S*. And, it is only then that we can propose the following definition:

- (a proposition) p is **true** $\stackrel{\text{def}}{=}$ p.
- p is **false** $\stackrel{\text{def}}{=}$ $\sim p$.
- q is an **implicate** of P $\stackrel{\text{def}}{=}$ $p \supset q$.
- p is **equivalent** to q $\stackrel{\text{def}}{=}$ $p \equiv q$.

Accordingly, we may say that a proposition p will be called L-true if and only if every sentence designating p in some system S is L-true in *S*. "However, these absolute L-concepts are non-extensional (i.e., not truth-functions)" (1942, p. 91). Otherwise, the metalanguage may lead to wrong statements. For example, by applying the absolute truth concept, we know that not only may we say: "the sentence '$P(a) \vee \sim P(a)$' is L-true in *S*", but also "the proposition $P(a) \vee \sim P(a)$ is L-true in *S*" (*Ibid.*). Now, from the above D3.1 and D3.2 (with some modification) we obviously may derive that:

D 3.69: $\mathfrak{T}_i$ is ***equivalent to*** $\mathfrak{T}_j$, (in S) $\overset{\text{def}}{=}$ $\mathfrak{T}_i$ and $\mathfrak{T}_j$, belong to S, and either both are true or neither of them is true.

Based on this definition '$P(a) \vee \sim P(a)$' and '$P(a)$' are equivalent; but, if we replace the equivalent sentence in the statement "the proposition $P(a) \vee \sim P(a)$ is L-true in S" the whole statement becomes false, because we cannot say "the proposition $P(a)$ is L-true in S". Hence, for the absolute use of the L-terms, we need a non-extensional language (*Ibid.*, p. 92). Once again, we may see here why Carnap insists that the designata of the sentences should be "propositions" and not the truth-values (Carnap has an analogous discussion regarding the first order predicate logic and reaches the same conclusion for the metalanguage, i.e., metalanguages ought to be non-extensional; see 1942, §17).

> In general […] we have tried to frame definitions and theorems in a neutral way, so as not to require the language used—especially the metalanguage used for semantics and syntax—either to be non-extensional or to be extensional. Absolute L-concepts apply to *propositions*, not merely to truth-values. We construe propositions in such a way that L-equivalent sentences designate the same proposition. Hence, the absolute concept of L-equivalence is the same as identity among propositions. (Carnap, 1942, p. 92)

Disregarding the non-extensionality requirement for metalanguage in modern formulation of syntax and semantics is at least a questionable issue. In modern formulation, the sign "$\vdash$" is normally introduced for "*syntactic consequence relation*" (for derivations according to the rules of inference) as a binary relation that can be held between a metavariable for a class of sentences, Γ (on the left), and a metavariable for a proposition (atomic or compound, on the right), A. Similarly, we have "$\vDash$" for "*semantic consequence relation*", which could also be held between

Γ, and A. Thus, a meta-theorem, which is called "*strong completeness theorem*"[92] (Epstein, 2001, p. 68), could supposedly be formulated in metalanguage using an equivalency (bi-conditional) relation in order to show a relation between semantical and syntactical consequences, given the same language.

Strong Completeness Theorem (SCT): $\Gamma \vDash A \equiv \Gamma \vdash A$

This theorem has been proven for some languages such as the language of PC (see *Ibid.*, pp. 71-79, for a sample of the proof). If we take metalanguage to be extensional regardless (as in Fregean systems), we are forced to say "$\vdash$" and "$\vDash$" are *one and the same* relations (in PC); thus, there is no difference between syntactic and sematic consequences (in the process of deduction) in PC (although we know there is). In the modern formulation, it seems that we use truth as an absolute concept. Thus, under the above convention (I), we see this issue will not appear in a Carnapian system because the metalanguage ought to be non-extensional. In a Carnapian system, as we saw, although it is incorrect to say "the sentence '$\Gamma \vdash A \equiv \Gamma \vDash A$' is true in PC", it is not incorrect to say: "the proposition $\Gamma \vdash A \equiv \Gamma \vDash A$ is true in PC", given the adoption of the convention and non-extensionality of the metalanguage. If we do not adopt the convention, and we agree with Carnap that "PC is not a full formalization of propositional logic" (1943, p. 94) (i.e., full formalization requires a calculus K which may or may not contain PC, in which we can formulate its syntax at a higher level of abstraction, or make sentences like "$\Gamma \vdash A$"), then the equivalency relation, stated in the strong completeness theorem, is not an equivalency relation as much as it is an abstraction-interpretation relation; we may abstract "$\vDash$"

[92] As examples, see Epstein (2001, pp. 51 & 66); Gabbay and Guenthner (2004, p. 26); Halmos and Givant (1998, pp. 91-93); Lightstone (2012, p. 212); Makinson, Malinowski, and Wansing (2008, p. 187). Sometimes only the forward relation is referred to the same name.

to "⊢" and/or interpret "⊢" to "⊨" in a given semantic. That is to say, the abstracted syntactical statement "$\Gamma \vdash A$" (in K) could be interpreted as "$\Gamma \vDash A$" in some semantical system S (like PC) and may not be interpreted in others (more on that later).

Carnap speaks about the question of extensionality and interpretation of the connectives extensively (in 1943, §§11-24; 1942, §§18-20) via focusing on the defined concepts of their "characteristic", "range", and "states", which, of course, serves his main intention that focuses more on the structure (relations) for the sake of abstracting from meaning rather than on the other involved entities. To understand an intuitive sense of the concept of "range" consider the following example (in that you may include the sentence "my pencil is not blue" as well, and compare the range of negation):

> A semantical system will, in general, contain not only true but also false sentences. If a false sentence is not L-false, hence not self-contradictory, it describes a situation which is possible though not real Let us compare the following sentences. "My pencil is blue" ($\mathfrak{S}_1$), "My pencil is blue or red" ($\mathfrak{S}_2$), "My pencil is blue or green" ($\mathfrak{S}_3$). None of them specifies precisely the color of my pencil; each admits a plurality of colors as possible. Even $\mathfrak{S}_1$ still admits all the various shades of blue. But the range of possible colors admitted by $\mathfrak{S}_1$ is narrower than those admitted by $\mathfrak{S}_2$and by $\mathfrak{S}_3$, $\mathfrak{S}_1$ is therefore more precise. Between $\mathfrak{S}_2$ and $\mathfrak{S}_3$, there is no simple way of comparing preciseness. Their ranges overlap, but none of them is contained in the other. (Carnap, 1942, pp. 95-96)

In order to give a sense of Carnap's extensive discussion on extensionality and interpretation, let us again give an example for the case of the singulary connection "negation". Carnap admits that the general concept of range (given in the above example) is vague and one of the ways to make it precise is to consider "L-range" instead. Thus, for the concept of range:

> We shall use for it the term 'L-range' because it turns out to be an L-concept. Whenever we understand a sentence we know what possibilities it admits. The semantical rules determine under what conditions the sentence is true, and that is just the same as determining what possible cases are admitted by it. Therefore, the L-range of a sentence is known if we understand it—in other words, if the semantical rules are given, factual knowledge is not required. Thus, in the above example, we found certain relations between the L-ranges without knowing which color the pencil really had.

Therefore, just like the other semantical concepts, the concept L-range will be applied to sentential classes as well as to sentences; thus, we may abbreviate "the L-range of $\mathfrak{T}_i$ in S" to "$Lr_S\mathfrak{T}_i$" (in short "$Lr\mathfrak{T}_i$") or "R_i". We may also use "V_S" for "the universal (comprehensive) range in S", and "Λ_S" for "the null range in S". In the following general definitions, we also make use of set-theoretic symbols, e.g., "$\in$", "$\subset$", "$\times$", "$+$", "$-$" (there are also some theorems included in the form of definitions that are not separately identified for brevity).

Theorem of state-description: Let S contain negation. Then every L-state with respect to S is designated by exactly one maximum state-description in S.

D 3.70: ***L-states*** with respect to $S \stackrel{\text{def}}{=}$ either completely specified possible states of affairs of the objects dealt with in S, or other entities corresponding to them (e.g. state-descriptions).

D 3.71: a ***state-description*** $\stackrel{\text{def}}{=}$ the linguistic expression for an L-state in the form of a sentence or a sentential class. (thus, L-states are propositions of certain kind).

D 3.72: $\mathfrak{T}_i$ is (an L-state-description or briefly) a ***state-description*** in $S \stackrel{\text{def}}{=} \mathfrak{T}_i$ is L-equivalent to an *atomic sentential selection* in S.

D 3.73: $\mathfrak{K}_i$ is an ***atomic sentential selection*** in $S \stackrel{\text{def}}{=}$ for every atomic sentence $\mathfrak{S}_i$ in S, $\mathfrak{K}_i$ either contains $\mathfrak{S}_i$, or $\sim\mathfrak{S}_i$ but not both, and no other sentences.

D 3.74: $\boldsymbol{rs}$ (the ***real L-state*** with respect to S) $\stackrel{\text{def}}{=}$ the p such that p is an L-state and true $\stackrel{\text{def}}{=}$ The conjunction of the class which contains, for every p in $Des(S)$, either p, if p is true, or $\sim p$, if p is false.

D 3.75: V_S (the ***universal L-range*** in S) $\stackrel{\text{def}}{=}$ the class of all L-states (i.e., the universal class of the second type of individuals).

D 3.76: Λ_S (the ***null L-range*** in S) $\stackrel{\text{def}}{=}$ the null class (of L-states).

D 3.77: $\mathfrak{T}_i$ is ***true*** in $S \stackrel{\text{def}}{=} rs \in Lr\mathfrak{T}_i$

D 3.78: $\mathfrak{T}_i$ is ***L-true*** (in S) $\stackrel{\text{def}}{=} Lr\mathfrak{T}_i = \mathrm{V}_S$

D 3.79: $\mathfrak{T}_i$ is ***L-false*** (in S) $\stackrel{\text{def}}{=} Lr\mathfrak{T}_i = \Lambda_S$

D 3.80: $\mathfrak{T}_j$ is an ***L-implicate*** of $\mathfrak{T}_i \stackrel{\text{def}}{=} Lr\mathfrak{T}_i \subset Lr\mathfrak{T}_j$

D 3.81: $\mathfrak{T}_i$ is ***L-equivalent*** to $\mathfrak{T}_j \stackrel{\text{def}}{=} Lr\mathfrak{T}_i = Lr\mathfrak{T}_j$

D 3.82: $\mathfrak{T}_i$ is ***L-disjunct*** with $\mathfrak{T}_j \stackrel{\text{def}}{=} Lr\mathfrak{T}_i + Lr\mathfrak{T}_j = \mathrm{V}_S$

D 3.83: $\mathfrak{T}_i$ is ***L-exclusive*** of $\mathfrak{T}_j \stackrel{\text{def}}{=} Lr\mathfrak{T}_i \times Lr\mathfrak{T}_j = \Lambda_S$ (Note: $Lr\mathfrak{T}_i \times Lr\mathfrak{T}_j = Lr(\mathfrak{T}_i + \mathfrak{T}_i)$)

D 3.84: $\mathfrak{T}_i$ is ***L-non-equivalent*** to $\mathfrak{T}_j \stackrel{\text{def}}{=} Lr\mathfrak{T}_i = -Lr\mathfrak{T}_j$ ($\mathfrak{T}_i$ is *L-non-equivalent* to $\mathfrak{T}_j$ if and only if $\mathfrak{T}_i$ is both L-disjunct and L-exclusive of $\mathfrak{T}_j$)

Based on the given definitions, and Tables X and XI above, Carnap shows us how one can construct a truth table for a defined-connection. As said above, we only give an example for a singulary connection (negation) but the method is analogous for the binary connections as well. If $\mathfrak{a}_k$ is a sign for a singulary connective applied to a closed sentence $\mathfrak{S}_i$, such that $\mathfrak{S}_k$ is $\mathfrak{a}_k(\mathfrak{S}_i)$, given that t is the numeric name of the truth value distribution of the connection in constructing a normal truth table (NTT), we have the following definitions (Table XII):

D 3.85: S ***contains*** NTT $\stackrel{\text{def}}{=}$ all sentences of S are closed, S contains a sign for each singulary (or binary) connection (in accordance to the following table and definitions).

t	Degree One Connective (singulary)	Degree Two Connective (binary)
1	T	TT
2	F	TF
3		FT
4		FF

Table XII t-th Distribution

D 3.86: $\mathfrak{a}_k$ ***satisfies the t-th rule*** ($t = 1$ to 2) in NTT for $Conn_q^1$ ($q = 1$ to 4) with respect to $\mathfrak{S}_i$ in $S \stackrel{\text{def}}{=} \mathfrak{a}_k$ is a singulary general connective in S, $\mathfrak{S}_i$ is closed and have the t-th distribution of truth-values; $\mathfrak{a}_k(\mathfrak{S}_i)$ has the t-th truth-value in the characteristic of $Conn_r^1$; as given in column (6) of the Table XII.

D 3.87: $\mathfrak{a}_k$ has the [***L-***]***characteristic*** value X (T or F) for the t-th distribution ($t = 1$ to 2) of degree one in $S \stackrel{\text{def}}{=} \mathfrak{a}_k$ is a singulary general connective in S, and if $\mathfrak{S}_i$ is any closed sentences in S and $\mathfrak{S}_k$ is the full sentence $\mathfrak{a}_k(\mathfrak{S}_i)$, then the class specified below contains rs [is V_S].

t	VALUE X	CLASS
1	T	$-Lr_i + Lr_k$
1	F	$-Lr_i + (-Lr_k)$
2	T	$Lr_i + Lr_k$
2	F	$Lr_i + (-Lr_k)$

Table XIII t-th distribution for a singulary connective

As you may see, defining truth tables (and truth values) in terms of L-range (with the inclusion of rs) guarantees a relation with corresponding F- and C-terms. The appeal to L-range is another one of Carnap's novel techniques, which he developed under the influence of Wittgenstein[93], showing not only the objects (individual) but also the relations (and connection) could be abstracted to higher levels. The relation with the factual state of affairs (F-terms) is particularly obvious from the following theorems (see also D3.68-74)

T3.12: Every L-state with respect to S is the conjunction of a class of propositions, every one of which is either itself designated in S or is the negation of a proposition designated in S.

T3.13: For any atomic sentential selections $\mathfrak{K}_i$ and $\mathfrak{K}_j$ (in S) the following holds:

a. $\mathfrak{K}_i$ is not L-true.

b. $\mathfrak{K}_i$ is not L-false.

c. $\mathfrak{K}_i$ is L-complete.

d. If $\mathfrak{K}_i$ and $\mathfrak{K}_j$ are different, they are L-exclusive.

T3.14: For any state-descriptions $\mathfrak{T}_i$and $\mathfrak{T}_j$, (in S) the following holds:

a. $\mathfrak{T}_i$ is not L-true.

b. $\mathfrak{T}_i$ is not L-false.

c. $\mathfrak{T}_i$ is L-complete.

d. $\mathfrak{T}_i$ and $\mathfrak{T}_j$ are either L-equivalent or L-exclusive.

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93 Carnap attributes the main idea of L-range to Wittgenstein:

> Wittgenstein uses the concept of the range of a proposition for informal, intuitive explanations, he shows that L-truth (tautology), L-falsity (contradiction), and L-implication are determined by the ranges. "The truth-conditions determine the range which is left to the facts by the proposition. Tautology leaves to reality the whole infinite logical space, contradiction fills the whole logical space and leaves no point to reality neither of them, therefore, can in any way determine reality" ([Tractatus] 4463). Wittgenstein explains the concept of range for molecular sentences only. (Carnap, 1942, p. 107)

T3.15: For any L-states p and q (with respect to S) the following holds:

a. p is not L-true.

b. p is not L-false.

c. p is L-complete (with respect to S).

d. If p and q are different, they are L-exclusive.

The application of this method also shows that the construction of truth tables follows the general rules of semantic and abstracting to C-terms is essentially possible (e.g., C-range, C-content, etc.). Carnap shows (1942, §§18-19) that, by adopting the concepts of L-range and L-state, the theorems should be proven in a non-extensional metalanguage. Following Russell, Carnap agrees that, in the cases in which we can construct a truth table for a connective, if the truth-value of a full sentence depends solely upon the truth-values of the components, then we may call the connection *extensional* (or truth-functional). Now, let us take a singulary connective (negation) as an example.

As we saw, for a singulary connective $\mathfrak{a}_k$ we formulated the condition of extensionality in this way: if $\mathfrak{S}_i$, and $\mathfrak{S}'_i$ are any closed sentences, which have the same truth-value and hence are equivalent, then $\mathfrak{a}_k(\mathfrak{S}_i)$, and $\mathfrak{a}_k(\mathfrak{S}'_i)$, which we will call $\mathfrak{S}_k$ and $\mathfrak{S}'_k$ respectively, also have the same truth-value and hence are equivalent. We also know that there is a distinction between L- and F-concepts in a Carnapian system. This distinction, with respect to the general "extensionality", would be translated into the distinction in the condition for extensionality, in the sense that whether the condition is fulfilled by the contingency of the facts, or the case in which it is fulfilled necessarily. In the latter case, it means that whether we can find out that the condition is fulfilled on the basis of the semantical rules of the system in question alone, without using factual knowledge. Thus, we may translate the condition in this way: "either $\mathfrak{S}_i$, and $\mathfrak{S}'_i$ are not equivalent, or $\mathfrak{S}_k$ and $\mathfrak{S}'_k$ are equivalent" (1943, p. 53). In terms of L-range the condition would be "either $[rs \in \left((Lr_i \times -Lr'_i) + (-Lr_i \times Lr'_i)\right)]$ or $[rs \in \left((Lr_k \times Lr'_k) + (-Lr_k \times -Lr'_k)\right)]$", which means that if "$rs \in \left((Lr_i \times -Lr'_i) + (-Lr_i \times Lr'_i) + (Lr_k \times Lr'_k) + (-Lr_k \times -Lr'_k)\right)$", then the condition of extensionality is fulfilled. It is obvious that, in the general case (for radical concepts such as "true"), $\mathfrak{T}_i$ does require factual knowledge in order to see if the condition for extensionality is fulfilled or not (i.e., to find out whether rs is included in the class in question or not; see D3.79). On the other hand, according to D3.81, for "L-true" we do not require factual knowledge and we know that the condition for extensionality is fulfilled; since the class in question is V_S in which case we certainly know that rs is included, without knowing which L-state is rs (i.e., without factual knowledge). Therefore, in those cases where the condition for extensionality is fulfilled, i.e., in the cases which we do know rs is included without knowing factual knowledge, then we call $\mathfrak{a}_k$ ***L-extensional***. If, on the other hand, the condition for extensionality is not fulfilled we call $\mathfrak{a}_k$ ***non-extensional***[94].

One of the other cases that we know certainly (without requiring factual knowledge) that rs is *not* included (hence the condition of extensionality is *not* fulfilled) is the case in which the class in question is the null class Λ_S. This is the case for some of the basic *C*-concepts (see D3.54-57 and T3.3-4, for example) which means some of the basic concepts of K , in their pure syntactical form and regardless of the containment (or the possibility of the containment) of any semantical

[94] Carnap gives this explanation regarding the choice of the term "non-extensional" instead of "intensional": "The term 'intensional' is often used in this case. Since, however, this term is used in traditional logic in another sense, it might be advisable not to use it here" (1943, p. 53).

system (in K), are essentially non-extensional. Let us take a closer look on the extensional (and non-extensional) interpretations of K, again in the case of "negation" as an example, in the subsequent section after we explain what we mean by "interpretation".

3.4.2 Interpretation: A relation between syntax and semantics

As we said above, semantic is an abstraction from observational statements in pragmatics. The way in which we distance ourselves from meaning (in this part) is by constructing a system of rules for making assertions; including the rules for designation, specifying the types of designators, and the rules of how to assign the objects in question into their appropriate places in the matrices. In turn, syntax is an abstraction of semantics. The way in which we abstract from semantic is by setting aside the rules of designation (and the concept of designation) altogether and rely only on the rules of formation, of truth, of range, and of translation for constructing a calculus upon which the calculus in question may or may not contain (see D3.63) various semantical systems in order to be interpreted. More clearly, sentences of a calculus K may (or may not) be interpreted by the truth-conditions stated in the semantical rules of a system S, provided that S contains all sentences of K. In other words, "if the direct C-concepts of K (and, hence, also the other C-concepts of K) are in agreement with the corresponding radical concepts in an interpretation S for K, then S is called a *true* interpretation for K, otherwise, a *false* interpretation" (1942, p. 202). Of course, to complete the ladder of abstraction there should be another interpretation that relates semantic to pragmatic. "Other kinds of interpretations [are]: L-false, L-determinate, factual, F-true, F-false, logical, and descriptive interpretations" (*Ibid.*). In what follows it is quite important to notice that while "Λ in K" and "Λ in S" mean the same thing (thus, we do not label Λ), "V in K" and "V in S" might be different; thus, we label them respectively as V_K and V_S . Basic definitions regarding interpretations are as follows:

D 3.88: S is an ***interpretation*** for $K \overset{\text{def}}{\equiv} K$ is a calculus and S is a semantical system and every sentence of K is a sentence of S. (therefore, if S is an interpretation for K, then $\mathrm{V}_K \subset \mathrm{V}_S$.

D 3.89: S is a ***true interpretation*** for $K \overset{\text{def}}{\equiv} S$ is an interpretation for K such that the following two conditions are fulfilled:

a. If $\mathfrak{T}_i \underset{dC}{\rightarrow} \mathfrak{T}_j$ K, then $\mathfrak{T}_i \rightarrow \mathfrak{T}_j$ in S.

b. If $\mathfrak{T}_i$ is directly *C*-false in K, $\mathfrak{T}_i$ is false in S.

(thus, if $\mathfrak{S}_j$ is derivable from $\mathfrak{K}_i$ in K, then $\mathfrak{K}_i \rightarrow \mathfrak{S}_j$ in S)

D 3.90: S is a ***[L-] false interpretation*** for $K \overset{\text{def}}{\equiv} S$ is an interpretation for K but not a [L-]true interpretation for K. (an L-false interpretation is a false interpretation)

D 3.91: S is an ***L-true interpretation*** for $K \overset{\text{def}}{\equiv} S$ is an interpretation for K such that the following two conditions are fulfilled:

a. If $\mathfrak{T}_i \underset{dC}{\rightarrow} \mathfrak{T}_j$ in K, then $\mathfrak{T}_i \underset{L}{\rightarrow} \mathfrak{T}_j$ in S.

b. If $\mathfrak{T}_i$ is directly *C*-false in K, $\mathfrak{T}_i$ is L-false in S.

(thus, if $\mathfrak{S}_j$ is derivable from $\mathfrak{K}_i$ in K, then $\mathfrak{K}_i \underset{L}{\rightarrow} \mathfrak{S}_j$ in S)

D 3.92: S is an ***L-determinate interpretation*** for $K \overset{\text{def}}{\equiv} S$ is an L-true or an L-false interpretation for K.

D 3.93: S is an (L-indeterminate or) ***factual interpretation*** for $K \overset{\text{def}}{\equiv} S$ is an interpretation for K but not an L-determinate interpretation for K.

D 3.94: S is an ***F-true interpretation*** for $K \overset{\text{def}}{\equiv} S$ is a true interpretation for K but not an L-true interpretation for K.

D 3.95: S is an ***F-false interpretation*** for $K \overset{\text{def}}{\equiv} S$ is a false interpretation for K but not an L-false interpretation for K.

D 3.96: S is a ***logical interpretation*** for $K \overset{\text{def}}{\equiv} S$ is an interpretation for K and every sign of K is logical in S.

D 3.97: S is a ***descriptive interpretation*** for $K \overset{\text{def}}{\equiv} S$ is an interpretation for K and at least one sign of K is descriptive in S.

Accordingly, in the Table XIV, for any instance (i.e., for any $\mathfrak{T}_i$ or pair $\mathfrak{T}_i$, $\mathfrak{T}_j$) for which the concept **(1)** in one of the following rows holds in K, concept **(2)** holds in S, if S is a true or L-true interpretation of K, and concept **(3)** holds, if S is a false interpretation of K.

(1) in *K*	**(2) [*L*-]true interpretation in *S***	**(3) false interpretation in *S***
primitive sentence	[*L*-]true	false
$\boldsymbol{\mathfrak{T}_i}$ ***is derivable from*** $\boldsymbol{\mathfrak{T}_j}$	[*L*-]implicate	$\mathfrak{T}_i$ true, $\mathfrak{T}_j$ false
provable	[*L*-]true	false
C-false	[*L*-]false	true
$\boldsymbol{\mathfrak{T}_i \underset{C}{\rightarrow} \mathfrak{T}_j}$	[*L*-]implicate	$\mathfrak{T}_i$ true, $\mathfrak{T}_j$ false
C-true	[*L*-]true	false
C-equivalent	[*L*-]equivalent	non-equivalent

Table XIV Corresponding *C*-terms of *K* in the true, *L*-true, and false interpretations in *S*

As explained, K could be a constructed calculus for PC that may (or may not) contain multiple sub-calculi such as PC_1, which only has two connectives (negation and disjunction; or, more precisely, $negation_C$ and $disjunction_C$). In that case, NTT (normal truth table) could be considered as its *L*-true interpretation (but not its only interpretation). To see this (Table XI) let $\mathfrak{S}_q^1$ be a closed sentence in K that includes $\mathfrak{S}_i$ and $Conn_q^1$ ($q = 1$ to 4), $\mathfrak{S}_r^2$ be a closed sentence in K that includes $\mathfrak{S}_i$, $\mathfrak{S}_j$ and $Conn_r^2$ ($r = 1$ to 16) (we did not show the disjunction in Table XI; see 1943, §3 for the full table), then $\mathfrak{S}_q^1$ and $\mathfrak{b}_q(\mathfrak{S}_i)$, also $\mathfrak{S}_r^2$ and $\mathfrak{c}_r(\mathfrak{S}_i, \mathfrak{S}_j)$, are *L*-equivalent by NTT in S. "It can easily be verified by showing with the help of truth-tables that in each case the two sentences have the same *L*-characteristic" (*Ibid.*, p. 70). It is shown that the sign for negation fulfills the condition of extensionality under NTT in the following way:

T3.16: $\mathfrak{a}_k$ [*L*-]satisfies generally the rule N1 and N2 for negation in NTT in S if and only if $\mathfrak{a}_k$ is a singulary general connective in S, and for any closed $\mathfrak{S}_i$ with the full sentence $\mathfrak{S}_k$ the following condition (stated in three forms for each rule) is fulfilled:

a. $-R_i + (-R_k)$ contains rs [is V_S].

b. $R_i \times R_k$ does not contain rs [is Λ].

c. $\mathfrak{S}_i$ and $\mathfrak{S}_k$ are [*L*-]exclusive.

i. $R_i + R_k$ contains rs [is V_S].

ii. $-R_i \times -R_k$ does not contain rs [is Λ].

iii. $\mathfrak{S}_i$ and $\mathfrak{S}_k$ are [*L*-]disjunct.

With the theorems such as the above (to see the complete proof see 1943, §13) Carnap shows under which conditions the signs of PC_1(and hence the entire system) becomes extensional.

T3.17: Let S contain a singulary general connective $\mathfrak{a}_k$ and signs of equivalence$_{[\mathrm{L}]}$ and implication$_{[\mathrm{L}]}$. Then each of the following conditions is a necessary and sufficient condition for $\mathfrak{a}_k$ to be **[*L*-]extensional** for any closed $\mathfrak{S}_i$ and $\mathfrak{S}'_i$ with the full sentences $\mathfrak{S}_k$ and $\mathfrak{S}'_k$,

a. $equ_{[L]}(\mathfrak{S}_i, \mathfrak{S}'_i) \underset{[L]}{\rightarrow} equ_{[L]}(\mathfrak{S}_k, \mathfrak{S}'_k)$.

b. $imp_{[L]}(equ_{[L]}(\mathfrak{S}_i, \mathfrak{S}'_i), equ_{[L]}(\mathfrak{S}_k, \mathfrak{S}'_k))$ is [*L*-]true.

T3.18: Let S contain a singulary general connective $\mathfrak{a}_k$ and signs of equivalence$_{[\mathrm{L}]}$. Then $\mathfrak{a}_k$ is **[*L*-]non-extensional** for any closed $\mathfrak{S}_i$ and $\mathfrak{S}'_i$ with the full sentences $\mathfrak{S}_k$ and $\mathfrak{S}'_k$,if and only if $equ_{[L]}(\mathfrak{S}_i, \mathfrak{S}'_i)$ is [L]-true and $equ_{[L]}(\mathfrak{S}_k, \mathfrak{S}'_k)$ is [*L*-]false.

T3.19: Let S contain a singulary general connective $\mathfrak{a}_k$ and signs of negation$_{[\mathrm{L}]}$. Then, for any closed $\mathfrak{S}_i$ and $\mathfrak{S}'_i$ with the full sentences $\mathfrak{S}_k$ and $\mathfrak{S}'_k$, the following holds:

a. $\mathfrak{S}_k \underset{[L]}{\rightarrow} \mathfrak{S}'_k$ if and only if $\mathfrak{S}_i \underset{[L]}{\rightarrow} \mathfrak{S}'_i$.

b. $\mathfrak{S}_k$ and $\mathfrak{S}'_k$ are [*L*-]equivalent if and only if $\mathfrak{S}_i$ and $\mathfrak{S}'_i$ are [*L*-]equivalent.

As you may see, the above condition specifies how and under which conditions NTT could (or could not) be a contained-extensional system for a form of PC (i.e., PC_1) and hence for K. Now the question is whether it is possible for PC to contain a non-extensional semantic system under NTT. The answer to this question is affirmative, according to Carnap.

> The concepts of normal and L-normal interpretations for the connectives in a calculus are defined with the help of NTT. It 1s shown that, under certain conditions, if a calculus contains two signs for the same connection_C and the first has a normal or L-normal interpretation, then the second has, too (This result might mislead us into the erroneous assumption that non-normal interpretations are impossible). A non-normal interpretation of a connective would involve the violation of a truth-table. Therefore, the consequences of supposed violations of the single rules in NTT for disjunction and negation (N1 and N2) are examined. Some of the results [regarding disjunction] are generally satisfied, if N1is once violated, then it is always violated, and all sentences are true, if N1 is once violated, then the sign of negation_C is non-extensional, [...]. (Carnap, 1943, p. 73)

Note that the violation of the rules of NTT does not necessarily means we are to adopt another truth table; this just shows that if there is such a violation, then the semantical system ought to be non-extensional; which means it is possible, for a calculus (a syntax) like PC_1, to contain either extensional or non-extensional semantics given the same truth table rules. To see this more clearly, let us suppose that K and S fulfill the following conditions (regardless of whether or not this is possible):

A. K contains PC_1
B. S is a true interpretation for K.
C. neg_C in K violates the rule N2 of NTT at least once is S, say with respect to $\mathfrak{S}_1$, let $\mathfrak{S}_3$ be $dis_C(\mathfrak{S}_1, neg_C(\mathfrak{S}_1))$.

then the following holds, according to Carnap (1943, p. 77):

a. Both $\mathfrak{S}_1$ and $neg_C(\mathfrak{S}_1)$ are false.
b. neg_C in K generally satisfies N1 in S.
c. $\mathfrak{S}_3$ is true.
d. dis_C in K violates a rule with respect to $\mathfrak{S}_1$, $neg_C(\mathfrak{S}_1)$.
e. $neg_C(\mathfrak{S}_3)$ is false
f. $neg_C(neg_C(\mathfrak{S}_3))$ is true.
g. neg_C satisfies N2 with respect to $neg_C(\mathfrak{S}_3)$.
h. neg_C in K is non-extensional in S.

With the help of similar theorems and others (for the disjunction), Carnap clearly shows that it is always possible to have both extensional and non-extensional *true* interpretations of the same connectives for PC_1, given the same rules for constructing a truth table. If this is the case (i.e., if it is possible to have extensional and non-extensional semantical connections, or relations in general), it is obvious that the semantical metalanguage ought to be non-extensional in order to be able to speak about both cases. Carnap distinguishes between two kinds of non-normal interpretations of NTT; in the first kind, every sentence in K is true in S (1st kind), in the second kind, at least one is false (2nd kind). The results only for the case of negation is shown in the Table XV (similar results have been obtained for disjunction; see 1943, §16). In the Table XV, these questions have been answered, Q1: normal (n) or non-normal (nn)?; Q2: name of the violated rule (N1 or N2; see Table XI)?; Q3: extensional (e) or non-extensional (ne)?.

Name of the Connection$_C$ in *K*	**Definiens for the sign in *K***	**NTT characteristic in *S***	**Non-normal interpretations**					
			1st kind			2nd kind		
			Q1	Q2	Q3	Q1	Q2	Q3
Tautology$_C$	$\mathfrak{S}_i \vee \sim\mathfrak{S}_i$	TT	n		e	n		e
(Identity$_C$)	$\mathfrak{S}_i$	TF	n		e	n		e
Negation$_C$	$\sim\mathfrak{S}_i$	FT	nn	N1	e	nn	N2	ne
Contradiction$_C$	$\sim(\mathfrak{S}_i \vee \sim\mathfrak{S}_i)$	FF	nn	N1	e	n		e

Table XV Non-normal interpretations of propositional negation connective of PC

As you may see in this table (the third row), given the same rules for identifying NTT characteristic, we have at least two kinds of the non-normal interpretations (extensional and non-extensional) with different truth-value assignments. Carnap mentions that "in general [...] we have tried to frame definitions and theorems in a neutral way, so as not to require the language used—especially the metalanguage used for semantics and syntax—either to be non-extensional or to be extensional" (1942, p. 92). Carnap is also completely clear that "the metalanguage M in which we speak about S and propositions and L-ranges with respect to S, must be non-extensional" (*Ibid.*, p. 98).

As we said before, given the above explanations, one may observe some anomalies in the modern metalinguistic formulation, due to Fregean extensional analysis, specifically with regard to the expression of the *strong completeness theorem*[95] (henceforth SCT) in metalanguage. For

[95] Much research has been done on this theorem, in modern logic, all of which presupposes Fregean concepts of extension and intension (as well as a sharp distinction between objects and concepts). Here are some examples: Amor (2003, 2009); Makkai (1988); Minari (1983); Silver (1980).

example, we know that the sign for the sematic consequence relation "$\vDash$" belongs to a semantical metalanguage that can be interpreted for a given syntax (propositional logic as well as predicate logic and others). Let us consider one of the definitions of the sign in the current literature, for example (Amor, 2009, pp. 173-174):

> Let us assume that the notions of first order language with equality and the interpretation for it—as they are presented in most mathematical logic books—are well known. In this section we will go over some basic notions such as "logical consequence", "formal derivation in an axiomatic system" and the related theorems of strong soundness–completeness and compactness, in order to clarify concepts or simply refresh the reader's memory. [...] In what follows we will refer only to first order formal languages with equality and to first order classical logic. [...] The basic semantic relational concept of being forced to be true by other truths is known as "logical consequence".
>
> DEFINITION 1.1 φ *is a logical consequence of* Σ or Σ *logically implies* φ ($\Sigma \vDash \varphi$) if and only if in every interpretation A every variable–assignment s that satisfies α for every $\alpha \in \Sigma$, also satisfies φ.

Here it is apparent that s works on the normal true interpretations of NTT, and it is obvious that the given definition does not apply to the non-normal interpretations, since under the same rules, they assign different values to the closed sentences, even though those interpretations, as Carnap shows, are still true interpretations of NTT. On the other hand, it is not clear that what is the relationship; for which does the sign "$\equiv$" stand in SCT? Is this relationship also supposed to be interpreted under the same extensional semantic? If the answer is affirmative, then, given PC, it should be interpreted extensionally, which would require us to say a semantic relation "$\vDash$" is practically *one and the same* with the corresponding syntactic one, i.e., "$\vdash$". Could this mean sematic and syntax are the same in the cases that equivalency holds? Obviously, we cannot say that by virtue of those cases that the equivalence relation does not hold; but, we may definitely claim some sort of duality between semantic and syntax, given the Fregean philosophical assumptions (i.e., object-concept distinction and the Fregean notion of extension and intention based on the sense-reference distinction). In fact, this is exactly the claim that has been made and proven by Makkai (1987) in terms of model theory (upon Fregean assumptions).

> The most interesting phenomena in model theory are conclusions concerning the syntactical structure of a first order theory drawn from the examination of the models of the theory. With these phenomena in mind, it is natural to ask if it is possible to endow the collection of models of the theory with a natural abstract structure so that from the resulting entity one can fully recover the theory as a syntactical structure. We report here on results intended to constitute a positive answer to this question. (Makkai, 1987, p. 97)

In a Carnapian system, on the other hand, one expresses the forward relation of the equivalency (right to left) in the SCT by simply saying this sign "⊨" is abstractable to "⊢" (i.e., it is possible to ignore the rules of designation and still make a well-formed formula); and the backward relation (right to left) is simply saying that this sign "⊢" is interpretable to this sign "⊨" and can be interpreted further, given the semantical system, but the language, in which SCT is formulated, should be non-extensional, and in each case we have to give a different sense to "≡" (and to the other relational signs). Taking all the differences between basic assumptions of a Carnapian system and that of a Fregean one, we may realize that the Carnapian expression of SCT, using a metalanguage (in the case of PC a non-extensional one), is clearer than the Fregean expression.

3.4.3 Support for a Carnapian System

Among all the documents that I have studied in the current literature, which are or could be considered as support for a Carnapian system of analysis, one, in particular, caught my attention, and it is worth discussing for several reasons because, although the author is constructing models for language (in category theory) on the basis of Fregean assumptions (such as sense-reference distinction and extensionality) of language, and in a Fregean framework, she gets to similar results, as we saw when constructing a Carnapian system. Accordingly, one may easily realize that a Carnapian system has a clear advantage over a Fregean one for reasons of simplicity and the incorporation these results in the construction of the system.

In her paper, Wybraniec-Skardowska (2009) takes a categorical approach to provide an answer to a classical philosophical question: when is our language knowledge in agreement with our cognition of reality? To achieve this goal, she focusses on addressing a problem she calls “the problem of logical adequacy of language knowledge” on the basis of the following concerns (*Ibid.*, p. 320):

1) an adequacy of syntax and two kinds of semantics,
2) concord between syntactic forms of language expressions and their two correlates: meanings and denotations, and,
3) an agreement of three notions of truth: one syntactic and two semantic ones.

The basis of her approach, “following Frege”, is a triangular of three relations between three entities: “cognition”, “language”, and “reality”, so that our knowledge about this triangle constitutes our “meta-knowledge” (*Ibid.*, p. 321). Accordingly, our meta-knowledge could be manifested by our knowledge about the three following spaces (*Ibid.*):

1) *language reality* **S** (the set of all *well-formed expressions*, of L), in which results of cognitive activities such as concepts[96] and propositions are expressed,
2) *conceptual reality* **C**, in which products of cognition of ordinary reality such as logical concepts and logical propositions (*meanings* of language expressions) are considered, and
3) *ontological reality* **O** which contains objects of cognition (among others, *denotations* of language expressions).

Further, she specifies *indexation reality* **I** as "certain metalinguistic space of objects (indices) serving the purpose of indication of categories of expressions of **S**, categories of conceptual objects of **C** and ontological categories of objects of **O**" (*Ibid.*). Accordingly, **syntax** is being considered as the ways in which **S** (with respect to L) relates to two kinds of semantics: **intensional** (conceptual) **semantics**; comprising the relationship between **S** and cognition in describing conceptual **C**; and, **extensional** (denotational) **semantics**, describing the relationships between L and ordinary reality, i.e., ontological reality **O** to which the language refers (*Ibid.*). Thus, she constructs three models for L (one syntactic and two semantic) given the following conditions:

> Every compound expression of L has a **functor-argument structure** and both it and its constituents (the main part—the main **functor** and its complementary parts—**arguments of that functor**) have determined:
>
> - the *syntactic*, the *conceptual* and the *ontological* categories defined by the functions ιL, ιC, ιO of the indications of categorial indices assigned to them, respectively,
> - *meanings* (*intensions*), defined by the *meaning operation* μ,
> - *denotations* (*extensions*), defined by the *denotation operation* δ. (Wybraniec-Skardowska, 2009, p. 323)

[96] Note that she uses "concept" (as well as words such as "conceptual" and the like) in the Fregean sense; i.e., an entity sharply distinct from "object".

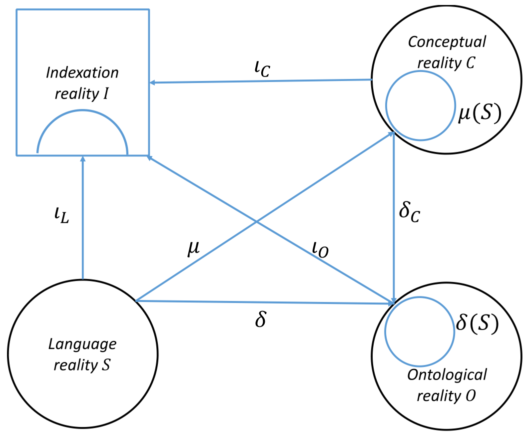

Figure 6 Categorical representation of the syntax and semantics of the language S (the set of well-formed expressions of L)

In this construction "meaning" is different than extensional "denotation" since "the *denotation operation* δ is defined as the composition of the *operation* μ and the operation δC of conceptual denotation" (*Ibid.*, p. 326), i.e., for any well-formed expression $e \in \mathbf{S}$,

$$(\delta C) \quad \delta(e) = \delta_C(\mu(e))$$

Hence, if both e and e' have the same meaning then they have the same denotation, i.e.,

$$\mu(e) = \mu(e') \Rightarrow \delta(e) = \delta(e'), \quad for\ any\ e, e' \in \mathbf{S}$$

And "it is well-known that the converse implication does not hold" (*Ibid.*). What is interesting about this research is that, given the just mentioned requirement for "meaning" is satisfied, she defines three notions of "truth" with regard to the three models of the language: $L\iota_L(\mathbf{L})$, $\mu(\mathbf{L})$ and $\delta(\mathbf{L})$; corresponding to different images of $\mathbf{S}$, i.e., "$\iota_L(\mathbf{S})$: a fragment of the indexation reality $\mathbf{I}$; $\mu(\mathbf{S})$: a fragment of the conceptual reality $\mathbf{C}$; and $\delta(\mathbf{S})$: a fragment of the ontological reality $\mathbf{O}$ as some algebraic structures, as some partial algebras" (*Ibid.*, p. 334). Accordingly, all of the three definitions of a true sentence in one of the models ($L\iota_L(\mathbf{L})$, $\mu(\mathbf{L})$ and $\delta(\mathbf{L})$) could follow the following scheme of definitions where $h = \iota, \mu, \delta$; and $T\iota_L$, $T\mu$, and $T\delta$ are nonempty subsets of **I**, **C** and **O**, respectively:

$The\ sentence\ e\ is\ true\ in\ the\ model\ h(L)\ iff\ h(e)\ \in\ Th$ (*Ibid.*, p. 335)

Thus, we may say (*Ibid.*):

- e is *syntactically true* iff $\iota_L(e)\ \in\ T\iota_L$,
- e is *intensionally true* iff $\mu(e)\ \in\ T\mu$,
- e is $extensionally$ *true* iff $\delta(e)\ \in\ T\delta$.

It should be obvious that these notions of truths, although defined for three different models of language L, are quite similar to Carnap's *C*-true, *L*-true, and *F*-true concepts within one language, respectively. Furthermore, we also saw that the *L*-interchangeability and *L*-equivalency of designators and predicators (see D3.1-11 and D3.49-50) are based on the sameness of intensions (of the expressions). It is quite interesting, and perhaps philosophically significant, that a category-theoretic analysis based on Fregean assumptions could give similar results (with regard to the explication of the general concept of truth) as Carnap's analysis.

In this section, we saw the difference between Fregean and Carnapian notions of "extension" and "intension". We also saw how abstraction, as a process of distancing from meaning, works at different levels. We saw that abstraction might be considered, formally, as a process of introducing variable(s) to closed expressions. As a simple example, we know that "$F(a)$" designates a statement (with full meaning), in which the constant a designates a specific object (as an autonomous complex); the extension of a is the object in question and its intension is the individual concept of that object. If we replace a by a variable x to form "$F(x)$", while we distance ourselves from the full meaning, the incomplete "$F\_$" now corresponds to another object (as an autonomous complex), which is the extension of F (i.e., the class of objects bearing the property F if the designator of F is a predicator) and its intension is the property expressed by F. On the other hand, the extension of the complete closed sentence "$F(a)$" (which can be replaced by a propositional constant "p") is its F-truth (i.e., the factual conditions under which one could announce the statement is "true" or "false") and its intension is the report about a expressed by the statement in question. As we may realize, the diversity of the method by which one could form propositional functions and introduce variables (for abstraction) could relate to the variety of possible methods of abstraction; the preferences, then, depend on the subject matter of the investigation and pragmatic criteria such as simplicity. As we saw, in a Carnapian system, the bottom-up abstracting (or formalizing) logic first requires constructing a semantical system, in which the concept of "designation" and "range" are well-defined. Abstracting from semantic via constructing L-semantics for logical constants (logical individuals) explicates the concept of truth to L-truth. We may then set aside the rules of designation and abstract only the structures, by which a C-true statement could be made, and the rules of designation could be applied upon them.

3.5 Conclusion: Suggestions for Describing Abstraction in a Carnapian Framework

Given all of the above explanations, based on recursive method of construction (recursive use of "*being*" and "*holding*" relation), we may say that a Carnapian construction system should satisfy the following general conditions. The basic philosophical assumption is that, in principle, if there exists a thing, one can always make a statement(s) about it (i.e., it has a state-description). And thus, the system:

1. Should provide an un-interpreted language system (i.e., always leaves room for an alternative interpretation).
2. Should have a flexible (or relative) notion of "object" (i.e., should consider object-concept distinction as a relative distinction between the objects of two immediate levels, but not as an absolute one).
3. Should distribute allogeneous objects on different levels, and isogenous objects on the same levels, so that "*being*" and "*holding*" could alternate with respect to the objects of different levels.
4. Should introduce variables so that they range over isogenous objects.

As we may have realized by now, Carnap proposes a theory of construction (in *Aufbau*) which could be employed for various purposes of investigation (Figure 7). Later, based on philosophical considerations, he takes a linguistic approach against philosophical problems, where he employs this construction system for linguistic analysis, and hence, for constructing a linguistic framework. If we consider abstraction as a general method for separating forms from the matters, then, right away, we can distinguish the philosophical difference between Frege's and Carnap's method in linguistic analysis. In Frege's method, declarative statements, such as “the apple is red”, are considered as reports of form-matter separation, in which separating form (e.g., “redness”) from the matter (e.g., “apple”) is already done; in Carnap’s method, on the other hand, the whole sentence “the apple is red” is the matter in question that we are supposed to separate its forms.

For linguistic analysis, then, the objects (the matter) of the investigation are linguistic expressions of all kinds; language is considered, in its widest sense, as any communicative medium that uses signs. The atoms are complete closed sentences which we assume have complete meaning, regardless the exact definition of meaning. The goal of the investigation is to search for the structures (forms) that are independent from the meaning. The method of the investigation is to look for describable relations among the atoms (and molecules), on one hand, and to identify the constituents of the atoms and the relations among them, on the other hand, via abstraction. With this method, the independency from meaning often coincides with the ability to introduce variables and clearly distinguishing variables from constants in each case and at each stage. In the just-described setting, abstraction is the main process for achieving distance from meaning (hence separating forms from the matter, given the object and goal of the investigation). Abstraction, in general, is considered as an intellectual activity that could be purposefully employed to construct frameworks in which the constructor is willing to separate some forms from some matters. The choice of the forms and matters (and hence the method of abstraction) is based upon pragmatic considerations relative to the goal of the investigation. Abstraction may start subjectively, as we explained in the previous chapter, by making assertions at lower-levels which may be accompanied with a higher degree of disagreement (among the speakers). But as the statements gradually become abstracted and distanced more from the meaning, they become more "objective", so to speak, and thus gain more agreement. As we saw, the independency from meaning (using abstraction), in the described setting, could be achieved in a stepwise manner.

Given the object and the goal of the investigation, and the basic flexibility of the notion of object, abstraction could be translated into a recursive process for introducing new objects at each level, in a Carnapian framework. The objects of higher-levels could be called concepts with respect to the objects of lower-levels. Thus, we may consider abstraction as a process that could be employed between two immediate levels of allogeneous objects that can be used recursively for constructing the entire framework. Given the above explanation we may summarize the whole Carnapian construction theory in the following diagram, in which "in", in any script, stands for an individual; "R" for relation and "S" for structure.

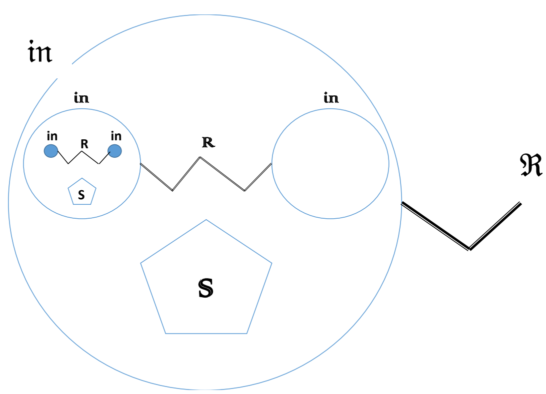

Figure 7 Visual schematic of Carnap's method of construction

I should explain that, for Carnap, "structure" is a specific notion; it is considered as a specific type of relation description. Once we describe a relation, it means that the relation in question could become independent from the elements that are involved in the relation (thus, we could introduce a variable in their place, disregarding the properties of the elements). Describable structures are in the same position with respect to their involved relations as describable relations are with respect to their elements.

> There is a certain type of relation description which we shall call *structure description*. Unlike relation descriptions, these not only leave the properties of the individual elements of the range unmentioned, they do not even specify the relations themselves which hold between these elements. In a structure description, only the *structure* of the relation is indicated, i.e., the totality of its formal properties. (Carnap, 1967, p. 21)

Carnap considers arrow diagrams as representations of the structures in which structures may or may not be equivalent (here is another interpretation of equivalency). "If two relations have the same arrow diagram, then they are called *structurally equivalent*, or *isomorphic*" (*Ibid.*). Carnap is clear that the equivalency of structure does not mean congruency (*Ibid.*). "We call two such diagrams equivalent if one of them can be transformed into the other by distorting it, as long as no connections are disrupted (topological equivalence)" (*Ibid.*). Carnap finds the structure descriptions in a domain as "the highest level of formalization and dematerialization" in that domain relative to that relation (*Ibid.*, p. 27). "The structure description forms the highest level of formalization in the representation of a domain" (*Ibid.*, p. 43). Regarding scientific theories, Carnap believes that scientific statements also tend to speak of structural properties, disregarding the involved elements.

> […] the representation of the world in science is fundamentally a structure description. […] Hence it is in principle possible to transform all statements of science into structure statements; indeed, this transformation is necessary if science is to advance from the subjective to the objective: all genuine science is structural science. (Carnap, 1967, p. 43)

Now, here is an important point where the above statement about scientific assertions (being about structures), seems to be "paradoxical", although if we say all mathematics (arithmetic, analysis, geometry, etc.) is only about structures, it does not seem that way.

> Superficially, this seems to be a paradoxical assertion. Whitehead and Russell, by deriving the mathematical disciplines from logistics, have given a strict demonstration that mathematics (viz., not only arithmetic and analysis, but also geometry) is concerned with nothing but structure statements. However, the empirical sciences seem to be of an entirely different sort: in an empirical science, one ought to know whether one speaks of persons or villages. This is the decisive point: *empirical science must be in a position to distinguish these various entities*; initially, it does this mostly through definite descriptions utilizing other entities. But ultimately the definite descriptions are carried out with the aid of structure descriptions only. (Carnap, 1967, p. 23)

It is obvious that the recursive method described above pictorially, in Figure 7, could be employed indefinitely. Now, let us go back to the basics; on the one hand, we said that the goal of linguistic analysis is to give an account of linguistic expressions of any sort (regardless the subject matter), and to construct a theory to analyze them. According to what we have explained, this investigation seemingly ends at the level of syntax, since there is nothing more to be formalized beyond pure syntax, where the only relation is the simple concatenation of pure signs (interpretable but not designated). But we also said that within each major step (i.e., from pragmatic to semantic and from semantic to syntax) toward abstraction may take several intermediate steps[97]. This feature is confusing since the investigation (i.e., linguistic analysis)

[97] Carnap gives an example of a six-level construction:

> EXAMPLE. Stepwise progress of construction, in which the relationship between being and holding recurs several times: Classes are constructed from things. These classes do not consist of the things. They do not have being in the same sense as the things; rather, they hold for the things. These classes, even though they hold of things, can now be envisaged as having a second mode of being. From them we can proceed, for example, to the cardinal numbers, which hold for these classes. (For the construction of cardinal numbers as classes of classes.) Cardinal numbers belong to a third mode of being and allow us to construct the fractions as relation extensions which hold for certain cardinal numbers. These fractions can also be reified, that is, they can be envisaged as belonging to a fourth

starts from a finite domain of objects and it takes finite steps to reach to highest level of abstraction (at syntax), yet, in some fields of study (that we are to give analysis for their expressions), there could be infinitely many objects. One of the prime examples is in the field of number theory, in which we are able to produce infinitely many objects and even to distinguish two different types of infinity; i.e., countable and uncountable infinity. In geometry, the same thing could happen; manipulation of points, lines, surfaces, etc. may lead to producing infinite objects as well. The results of these productions, such as the distinction between ordinal and cardinal numbers (or the invention of vector spaces), certainly enhances the versatility of mathematical objects and mathematical vocabulary. But the production of these objects does not seem to be in the same direction of disengaging from meaning, such as the way in which the concept of "operation" in group theory is disengaged from the meaning of "addition", "subtraction", and the like, for example. As such, at these points, abstraction could take another direction and expand in another dimension. Given that the objects that are produced in this way are isogenous, and that there is always a defined rule for how to produce or replicate them, we certainly could consider and use these features as the requirements for constructing another dimension in our overall abstractive construction, at these points that would allow expansions at these points. Looking at abstractive constructions in this way, i.e. assuming another dimension in abstractive constructions that would allow abstract objects to expand but not necessarily become disengaged from the meaning they preserve up to that level, it would certainly be less confusing than looking at the abstraction only in one dimension. This is the reason that we intend

mode of being, and can be made elements of certain classes which hold for them, namely the real numbers. The latter belong to a fifth mode of being, while the complex numbers, being relation extensions that hold for certain real numbers, belong to a sixth mode of being, etc. (Carnap, 1967, p. 71) to propose an intuitive two-dimensional sketch for Carnap's construction theory incorporating the Carnapian notion of abstraction for constructing a linguistic framework. We hope this sketch could also simplify what we have explained so far.

Based on the given analysis of abstraction in Carnap's philosophy, it seems that it would be less confusing if one could dissociate the inter-level relations of object spheres (based on the isogeny of spheres) in which the expansion of the given objects would be possible (we will call it *horizontal expansion*) from the intra-level relations among allogeneous spheres, in which "being" and "holding" could alternate (we will call this *vertical abstraction*) recursively.

As we said, abstraction is a recursive process of disengaging from meaning; thus, it would be enough to establish the relation between two immediate levels. In order to construct a semantic, as we saw when the old objects (or terms) are viewed with respect to their designators of a higher level, from which we could have different interpretations of equivalency. Let us take another look at the designators we have seen (Carnap, 1972, pp. 9 & 30):

	Closed Expression	**Constants**	**Variables**
(1) Designator Formulas; including (2), (3), and (4)	Designators	Designator Constants	Designator Variables
(2) Sentential Formulas	Sentences	Sentential/Propositional Constants	Sentential/Propositional Variables
(3) Individual Formulas	Individuators	Individual Constants	Individual Variables
(4) Predicator Formulas	Predicators	Predicates	Predicates Variables

Table XVI Terminology of Designators

Class	Designator Formula		Extension
	Open or Closed	Closed	
$\mathfrak{S}$	Sentential Formulas	Sentences	Truth-values
Type 0	Individual Formulas	Individuators	Individual
Type 1 (one-place) Type 2 (two-place) Type 3 (three-place) Etc.	Predicator Formulas	Predicators	Classes

Table XVII Classification of Designator Formulas

We saw how *L*-terms were defined based upon the designators in question. We also saw how the signs of syntax (*C*-terms) correspond to (imitates) *L*-terms, and that the reason that they belong to a higher-level of abstraction is their disengagement from "designation" (hence meaning). We saw that the definition of all *C*-terms imitates corresponding *L*-terms (except the ones regarding "designation"), being conditioned to find true interpretations. Thus, the core formalization (abstraction) process is the construction of *L*-semantic in which *L*-state is the most important description (state-description), since based on that one could determine *L*-range and eventually the extensionality of the language for which one is looking to find a true interpretation. For example, it can be shown that if a semantic system contains negation, every *L*-state is designated by a state-description (Carnap, 1942, §18). State-description is basically a conjunction of some atomic sentences (see D3.70-76):

$$Z_j = \bigwedge_{i=1}^{n} p_i$$

If, for example, we want to describe a lower level object a by its higher-level designators A_i, and we consider a conjunction of atomic sentences about that object such that the atomic sentences are in the form of $p_i = A_i(a)$, say A as a type 1 predicator, then the description of that object is "a is $A_1$and $A_2$and A_3…" or,

$$\text{(I)} \ \ a = \bigwedge_{i=1}^{n} A_i$$

On the other hand, if an object A of some higher-level holds for lower-level objects $a_1, a_2, a_3, ...$ (could be a pair, triplet, … of objects), we may say "A holds for a_1 and a_2 and a_3 and …"

$$\text{(II)} \ A \approx \bigwedge_{i=1}^{n} a_i$$

As we may see, (I) and (II) could be recursively employed. Of course (I) and (II) may be held in a different scope of abstraction; *F*-scope (pragmatics), *L*-scope (semantic) or C-scope (syntax). In the case of (II), we could also say "A is interpretable to a_1 or a_2 or a_3 or …", depending on the scope of abstraction. If the conditions of (I) and (II) could be held between objects of allogeneous spheres, then it means that "A is ***abstracted*** (vertically) from a_1 and a_2 and a_3 and …" or,

$$(a_i) \uparrow A$$

For example, as Carnap explains (1972, §40, pp. 68-69), if (a_i) are classes of five objects, then their common property could be abstracted to A, which could be the phrase "number 5" or just the numeral sign "5". Note that, here, we do not consider the classes as wholes consisting of their elements, but as *autonomous complexes*, as previously explained in the previous chapter. For this, Carnap gives an example (*Ibid.*) that the class of one's right hand's fingers are not ones' whole hand.

If we could abstract A from (a_i), in some cases, we could also say "A is interpretable to a_i" which means "A could be a_1 or a_2 or a_3 or …", depending on the scope of abstraction (i.e., the range of the involved entities), or,

$$\text{(III)}\ \ A \downarrow (a_i) \equiv A = (\bigvee_{i=1}^{n} a_i)$$

Given the non-extensionality of the metalanguage, about which we talked in the previous section, one could see that there are cases in which (II) holds, but where (III) does not, and that there are cases in which both hold. Thus, for example, the relation between "⊢" and "⊨" in SCT (see previous section) could be reformulated in the following way, where "a_1" could be "$\vDash_{PC}$" (i.e., the semantic consequence relation in PC) and "A" could be "⊢" (i.e., the syntactic consequence relation); given syntax is abstracted from semantic we will have:

$$a_1 \uparrow\downarrow A$$

What we just described could be considered as intuitive suggestions for formalizing the notion of abstraction in Carnap's philosophy. But it could also be the description of what I call ***vertical abstraction***, in distinction with ***horizontal expansion***. As said above, in terms of a Carnapian framework, we could observe the mass-producing of isogenous objects (of the same spheres) that at some points of abstraction may occur, that would allow the objects of the same level to expand dramatically, but not in the same abstractive direction (the direction of distancing from meaning) we just described. In what follows I would like to give further suggestions for describing the characteristic of these expansions, which I believe could constitute another dimension in the whole of abstractive construction. Normally, in these cases of expansions, there exists a rule according to which one is able to produce more objects. It seems that the semantic of these expansions strictly follows the rules of S_3. There is another feature that is associated with these expansions, that is the imposition of what I call ***zeroing assumption***. Zeroing assumption is sometimes necessary for determining the rules of production (such as assuming an empty set for constructing so-called abstract sets), and sometimes serves as a means for increasing accuracy, and, hence, being able to focus more on the structure of some scientific statements (such as assuming frictionless planes, perfectly rigid rods, totally elastic impacts, and the like).

The philosophical idea behind *horizontal expansion* is that, by the first encounter (observation, realization, etc.) with an object (an event, a quasi-object, a relation, etc.), two states are immediately imaginable: First, the state in which the content of the object in question is empty, while its designator still holds regardless, and, secondly, the state in which multiple versions of the same object exists (with respect to the same designator)[98]. One should pay attention to the fact that these states are the result of our faculty of imagination, and, in principle, require no further external experience. Considering all characteristics of the expansions, we may say ***horizontal expansion*** is distinguishable from *vertical abstraction*; firstly, by the fact that their objects do not get distanced from the meaning of their main designator (be it “number”, “set”,

[98] To give an example, by only one encounter with an instance of “red”, provided the designator is “color”, one could imagine the “colorless” situation, in which the content of “color” is empty, and the situations in which there exist other colors. Note that these situations could be arrived at totally based on imagination, regardless of whether or not such objects actually exist.

"oscillator", etc.), although they could enhance the meaning of the designator to its ideal state, in the sense that they get distanced from the original meaning of the designator in question, but in another direction. Secondly, the semantic of the expansions follows the rule of S_3 for producing more objects in which either or both of the following could be identified (at the same level, and with respect to the same designator):

(1) The zeroing-assumption: Introducing a new isogenous object a_0 (at the lower-level) that has no specification(s), for which a given higher-level designator A stands for but still could be considered in the same object sphere.

(2) The rule of replication/reproduction: $a_i \xrightarrow{re_A} a_j$. This rule specifies the method for (re)producing a_j from a_i (an isogenous object with respect to a given higher-level designator A). This rule, "$\xrightarrow{re_A}$", in general, could be articulated in the form of a well-defined operation (such as addition), simple iteration, and the like, or could stem from pure imagination, etc. (Note that horizontal expansion, potentially, could produce concepts without extensions).

We can clearly see that this analysis of abstraction for constructing a language system satisfies all the above-mentioned conditions in a Carnapian construction. We could always have an uninterpreted language, either by dismissing the lower level or by expanding the objects. Furthermore, we could always have a very flexible notion of "object" which is totally relative to the notion of "levels of abstraction". As we may see, the choice of which forms to ascend entirely depends on the abstractor who can justifiably make his/her decision with respect to the purpose of the analysis. In this way, one could avoid much confusion and reach interesting conclusions, such as: It is possible to rest everything on the experience even though one could open horizons of thought and rationality which primarily were not evident. Or, there is no fundamental difference between the methodology of science and mathematics; the difference is only in their respective starting level of their investigation about the objects in question.

In the way we presented horizontal expansion, cognitively speaking, we may suggest some interesting hypothesis. With respect to a concept, it seems that in practice a concept could be considered as a combination of a content and what is holding that content, in a way in which one is able to dissociate the two from one another (for example, for the purpose of zeroing the content, as in constructing "colorless" from "color"). This dissociation is only possible if there is a higher-level designator. This might be clearer if we present it graphically. In the following picture, we present the content of a concept by a circle in the middle of a square (which represents what the concept holds for).

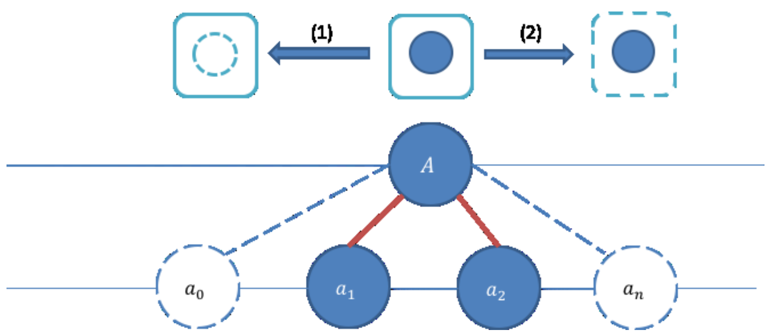

Figure 8 Cognitive schema of the horizontal zeroing assumption (1) and reproducing rule (2)

We must differentiate between what we mean by *content* here and what we meant by content in speaking of the empirical *content* earlier (although the two are related). As we said above, *empirical content* corresponds to the *statements* (that one might make in a framework), which could be emptied in a stepwise manner as we elevate toward higher levels of abstraction (this has something to do with the purity or impurity of the involved objects); whereas, *content*, here, corresponds to what a concept stands for at each level, which could be emptied at once, and at the same level in order to construct a zero-version of that specific concept.

Conclusion

As discussed, rejecting the object-concept distinction leads to a system of objects instead of a system of concepts, which is the result of keeping the distinction. Abstraction in a system of objects, could be considered as a move from the old objects to the new ones (which we could label the new ones as concepts relative to the old ones); whereas, in a system of concepts, abstraction could be considered as a move from the old concepts to the new ones and it is only via the concepts one could identify (or single out) the objects. Hence, we labeled the former notion of abstraction *empirical abstraction* and the latter *rational abstraction*; respecting the essentiality of experience (and inclusion of psychology) in the former, and the essentiality of independent existence of concepts (and exclusion of psychology) in the latter.

Likewise, the elimination of the other distinction, namely the sense-reference distinction entails some other changes in the analytic system, among which the meaning of “extension” and “intension”. As mentioned, in a Carnapian system, the “extension” of a term is considered as an autonomous logical complex, which is defined in terms of the equivalency of the designators, and the “intension” of it is defined in terms of the logical equivalency of the designators. Accordingly, metalanguage could no longer be considered as extensional.

Hence, due to the elimination of both the object-concept and the sense-reference distinctions, consequently, the Carnapian analytic system will acquire the following general characteristics:

- Ontological status: *language-dependent* (*linguistic*).
- Analytic system: *system of objects*.

- Equivalency relation: *non-universal*.
- Meta-language: *non-extensional*.
- Abstraction: *mind-dependent*.

In contrast, the Fregean system has the following general characteristics:

- Ontological status: *independent*.
- Analytic system: *system of concepts*.
- Equivalency relation: *universal*.
- Meta-language: *extensional*.
- Abstraction: *mind-independent*.

Given all of our discussions, perhaps another angle of viewing the philosophical differences between a Fregean and a Carnapian framework (for linguistic analysis) is by looking at the two frameworks from the perspective of *meaning theories*. One of the legitimate questions regarding language analysis, in general, is what is the relation of language (linguistic expressions) with respect to the real world? And what do we mean by "meaning" of the expressions? Of course, one of the important aspects of what we may call "meaning" relates to the transmission of information from observable phenomena or subjective experiences into the forms of communicable linguistic expressions. There is a family of theories of meaning which share this principle; "meaning is a relation between the symbols of a language and certain entities which are independent of that language. These theories may collectively be designated as *correspondence theories of meaning*" (Gamut, 1991, p. 1). From this point of departure, theories may take different lines of thought.

> There are, for example, theories which say that the meaning of a symbol resides in the use which is made of that symbol. The 'meaning is use' theory defended by the later Wittgenstein is an example of one such theory. And then there are theories which identify the meaning of a symbol with the set of all stimuli which elicit the use of that symbol as a response. There, meaning is defined in terms of the disposition of language users to display certain kinds of behavior. As examples we have the behavioristic theories of meaning of Bloomfield, Morris, and Skinner. And finally, there are theories which accept the correspondence theory as a partial account of meaning, in the sense that correspondence to entities is thought to account for just one aspect of the total meaning of symbols. Grice's theory of implicatures is an example of such a theory. (Gamut, 1991, p. 2)

As presented by Gamut (1991, §1.4), there are various theories in the family of *correspondence theories of meaning*.

1. *Conceptualism*: meaning is a relation between symbols and the contents of consciousness. Concepts, expressed by means of predicates, and propositions, expressed by means of sentences, are mental entities, with language functioning as a system of observable symbols which mediates between individuals, thus making communication possible.
2. *Platonism*: concepts and propositions are not mental entities but real things. Only they do not belong to the world of observable phenomena but to the world of ideas. Linguistic symbols refer to things in the observable world only in an indirect manner, via the reflections of the world of ideas in the observable world.
3. *Realism*: the entities to which linguistic symbols bear the relation of meaning all belong to the concrete, observable reality around us: they are individuals, properties, relations, and states of affairs. A typical example of this position is the 'picture theory of meaning', which was presented by Wittgenstein in the *Tractatus Logico-Philosophicus*.

Accordingly, a Fregean theory of meaning, extracted from the third view, could be called a "*referential theory of meaning*" (*Ibid.*), and is essentially compatible to all three views since "it only states that the meaning of a symbol is that to which it refers. So that a theory of meaning is referential in itself says nothing about the nature of the entities to which symbols refer" (*Ibid.*).

As we saw, in a Fregean system, there is a strict parallelism between the syntactic constructions and their semantic interpretations so that "the truth definition mirrors the syntactic definition of the formulas of the language in question" (*Ibid.*, p. 5). The fundamental idea behind Frege's methodology is that "every sentence, no matter how complex, is the result of a systematic syntactic construction process which builds it up step by step, and in which every step can receive a semantic interpretation. This is the well-known *principle of semantic compositionality*" (*Ibid.*). With this view, one consequence is that standard propositional and predicate logic are *extensional*. "A logical system is said to be extensional if expressions with the same reference (or extension) may be freely substituted for each other" (*Ibid.*).

This methodology, as explained, stems from the Frege's famous distinction between sense and reference. Thus, as Frege proceeds to dissect a complete closed sentence into two parts in accordance to his object-concept distinction (as the major contributors to meaning, so that each have their own reference), the world would be spilt in two; the world of objects and world of concepts, each of which is governed by their own rules. Upon reflection, it turns out that the world of concepts is ruled by perfect, precise, and exact rules, which could be constructed rationally; and, as we explained, one could construct a system of concept only by appealing to a universal notion of identity relation (following the principle of abstraction). The world of objects, on the other hand, would be imprecise, imperfect and not exact. Therefore, given the above background of meaning theories, Frege's conceptual analysis may appear to be a discovery with ontological bearing, if we are convinced that the object-concept distinction is *the* dissection of the sentence which corresponds to mind-independent realities. Please note that the dissection of sentences into two parts is one of the many possible dissections, and perhaps the most effective one; dissecting a sentence into its constituents in order to recognize the contributors to the meaning, in principle, could be done in different ways; for example, one may dissect the sentence into three (e.g., the old Aristotelian way with *copula*, in categorical propositions), or to the level of characters so that the empty spaces between the words could also be counted as the participants in the meaning.

In a Carnapian system, on the other hand, as we explained, "being meaningful" is considered prior to "being true". For Carnap, language itself is an object of the real world which, just like other objects, maintains certain relations with other objects of the world (including human mental faculty). And, just like any other object, it is essentially subjected to change and evolution, structurally and/or otherwise, in reaction to the changes in its environment; there also could be changes in human metal capacity (caused by facing new observations, and/evolutionary changes or both). Nonetheless, we could take a closed sentence like "$\mathfrak{S}$" as the unit of meaning. which, in the case of being meaningful, it would designate a proposition. The relation of the sentence to the real world, thus, is "designation", being a semantic top-down relation, which makes the entire sentence an abstraction from the world. The position of meaningfulness, for Carnap, is characterized (one may say topologically-positioned) in the sense that, no matter how we dissect the sentence in question, the meaning of the sentence is always the result of placing the right object in the right position. In other words, no matter how we build our matrices, we also have to consider the question of admissibility of objects in the appropriate places in the matrix in question. The sequentiality (or concatenation), or the order of the spaces (syntax), is one factor, but there are also properties associated with the particular spaces, only by virtue of being a space in the matrix, that have to be met by the admissible objects (semantic). Therefore, in principle, there is no dissection as *the* dissection for a typical sentence, since it is a question of how one dissects the sentence in question, and ultimately, it is a matter of choice, convenience, simplicity or, in general, a matter of pragmatic considerations. In a sense, we follow a version of Ockham's razor here, in linguistic analysis, i.e., the best analysis usually is the simplest one (even if we go on with the Fregean dissection), but, unlike the Fregean system (extensional), the method we choose has no ontological bearing outside of the analytic framework. Consequently, according to Carnapian analysis, we would be able to analyze scientific and mathematical statements based on the same methodology. Another consequence, of course, is that a scientific theory, at best, would be only one possible reading of the real world among many in constructing theories, it all boils down to the methods of abstraction.

Now, we have a clear understanding with respect to the philosophical differences between a Fregean and a Carnapian system. As we saw, in a Carnapian system, there is no fundamental difference between objects and concepts, and that the metalanguage for talking about propositions ought to be non-extensional, and that we will have a better understanding of radical concepts such as "truth", "equivalency", "being", and the like, once we could explicate them in a linguistic framework by stepwise abstraction. Considering all of our discussion, we may say that the main thesis of this dissertation is a call for attention to the fact that there is something fundamentally and primitively, at the very least, inappropriate about our presently dominant Fregean method of analysis by a purely extensional language. From the stand point of a field mathematician or logician, the Carnapian system (especially regarding some problems, e.g., object-concept, and/or sense-reference distinctions) may seem trivial, that eventually just leads to another way of formulation. But, in general, the choice of the method is philosophically significant, since each choice is committed to a fundamentally different world-view.

Once one is firmly convinced, in a Fregean framework, which means there exist well-defined, exact, and mind-independent realities in the world of concepts, with universally valid rules and relations, such as equivalency, one also may come to a conclusion that actual world phenomena (including natural languages) are rough, inaccurate, and approximate versions of those perfect (mind-independent) realities. In this framework, it is not hard to make a distinction between superior and inferior knowledge. A Fregean set of beliefs may give us the illusion of superiority, given the advancements of mathematically-expressed scientific and technological statements. While, in a Carnapian framework, one would remain humbler and more cautious with respect to one's research results and interpretations, even in the hypothetical event of acquiring perfect explanations and predictions. A simple and updated version of formulations on the basis of Carnapian notions of "object", "extension", and "intension", is what is definitely missing in our present literature on logic. Given the contemporary advancements in some fields of mathematics, such as category theory, one could be optimistic that future work on the new formulations would not only be possible but also greatly beneficial and productive, both logically and philosophically.

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Appendix A. A Historic Anecdote

> Abstraction is, as it were, a mixture of perceptual induction and intellectual deduction based upon the difference of what is *per se* and what is *per accident*. (Avicenna, Madkur, & Afifi, 1956)

We have characterized the notion of the Carnapian abstraction as a mind-dependent process of acquiring non-universal knowledge; in a sense a move from the auto-psychological entities to the hetero-psychological ones in a constructed linguistic framework. Does this notion of abstraction have a history in the history of philosophy? It seems that Avicenna (980-1037), if not the only one, is one of the medieval philosophers whose view on abstraction is quite similar to that of Carnap's. In what follows, a brief presentation of Avicenna's view on abstraction, as an anecdote to the whole discussion, will be presented, in which the following points are worthy of attention:

- Abstraction is one of the activities of our intellectual faculty and its final goal is dissociating forms from the matters.
- The basic assumption of Avicenna's perceptual abstraction differs from that of Frege.
- The activity of abstraction (dissociation of form from the matter) is a gradual activity, i.e., it is a matter of degree.
- At some point forms become free of the matters and free they would remain.
- There is no absolute sense of universality with regard to the gained knowledge, i.e., universality is an internal feature.

1. The Philosophical Roots of Abstraction

The word "abstraction" (Greek: αφαρεσις [aph΄iresis]; Latin: abstractio— detachment, division, retention), according to the Universal Encyclopedia of Philosophy (Maryniarczyk, 2010), philosophically designates "a specific operation of the intellect consisting in detaching and retaining some property from a thing." One can trace the discussion on abstraction in philosophy to Aristotle's theory of substantial forms (Coniglione, 2004, p. 60). Although Aristotle has no treaties on abstraction (Bäck, 2014, p. 2), it is generally accepted that the theory of abstraction was formulated and developed by Aristotle following a discussion with Plato on the nature and genesis of mathematical objects (Maryniarczyk, 2010). Aristotle held the idea that "mathematical objects are created by the intellect by detaching (abstracting) and retaining the form that characterizes the relation of quantitative order in what is individual and material" (*Ibid.*). Thus, they cannot exist on their own. Aristotle founds the process of abstraction on the faculty of sense perception which he construes as identical to the imagination but different in "being" (Bäck, 2014, p. 138). There are no doubts among scholars of Aristotle that he proposes some kind of empiricism, at least in the sense that all of our knowledge ultimately derives from sense data:

> Since according to common agreement there is nothing outside and separate in existence from sensible spatial magnitudes, the objects of thought are in the sensible forms, viz. both the abstract objects and all the states and affections of sensible things. Hence (1) no one can learn or understand anything in the absence of sense, and (2) when the mind is actively aware of any thing it is necessarily aware of it along with an image; for images are like sensuous contents except in that they contain no matter. (Aristotle, as cited by Cleary, 1985)

However, it is not clear whether Aristotle himself considers abstraction as the main process by which we grasp universal thoughts through sense experience (Cleary, 1985). Given the premise that, according to Aristotle, the main source of knowledge is sense data received from observable phenomena, the question is how the process of abstraction fits into the Aristotelian picture of knowledge.

When Aristotle describes the structure of the soul's rational faculty, where abstraction is construed as the main operation of the intellect, he distinguishes what he calls the “passive intellect” from the “active intellect” (Maryniarczyk, 2010). One may describe the passive intellect as simply the reservoir of forms (where the first form to be reserved is the form of the soul), while the active intellect (the light) enables us to see the relationship between the forms and make judgments accordingly. Based on the adopted criteria, one may distinguish various kinds of abstraction:

> If we consider the intellect's function of detaching the universal from the matter of the thing known, we may distinguish universal and formal abstraction. […] In the case where the form is detached from the matter of the individual thing, we are dealing with so-called physical (or natural) abstraction. […] If we take as our criterion [sic.] for division the degree of reflection in the operation of abstraction, we may distinguish between pre-scientific and scientific abstraction. […] In scholastic logic we encounter the division between negative and positive abstraction. This division is based on the way judgments are formed. In positive abstraction we may create judgments by the composition or joining of concepts, and in negative abstraction we create judgments by dividing or disconnecting concepts. (Maryniarczyk, 2010)

Bäck (2014) emphasizes that the meaning of abstraction for Aristotle could be summarized as the process of “selective attention”, i.e. of “focusing on an aspect, typically a general one, and then looking at features belonging to that aspect, while ignoring the remaining ones” (p. 2). Aristotle’s use of the word “*qua*” is an indication he considered such an abstraction (*Ibid.*). Bäck presents a sustained, convincing argument for why we should locate abstraction in the category of relations, since Aristotle explicitly places perception and knowledge in that category. Bäck concludes, "Aristotle takes his universals, like other abstracta, to have the structure of *relata*" (*Ibid.*, p. 6). Since *relata* exist in *re*, abstraction should have an ontic connotation for Aristotle. Bäck also distinguishes what he calls concrete *relata* from abstract *relata*:

> For *relata* Aristotle complicates the situation as he discusses examples like 'master' and 'slave', 'wing' and 'bird', where the nouns being used to signify *relata* are concrete, as well as examples like 'perception' and 'knowledge', and 'standing' and 'sitting', where the nouns being used to signify *relata* are themselves abstract. So then some of the *relata*, although described by the concrete term '*relata*', will be items that we might well call "relations", as they themselves are named by abstract terms like 'perception'. (Bäck, 2014, p. 29)

Holding the ontic connection of abstraction in *re* is what makes us prone to hold the fundamentalist attitude with respect to abstraction. Here one might easily see why abstraction, construed as a psychological process, is dismissed by, e.g. Frege, and, instead, its ontic character becomes predominant. The most fundamental distinction in Frege's philosophy is the ontological object-concept distinction. Indeed, Frege is quite clear that "objective ideas can be divided into objects and concept." (Frege, 1960b, p. 37). On this basis, Frege builds up his own hierarchical system of concepts, in which any assertion about objects, using a concept, stays at the first level. Moreover, any assertion about first level concepts must use a second-order concept and so on. Bäck (2014, p. 6) summarizes Frege's conception of abstraction as follows:

- The ordinary: from an individual (object) to its features. This gets us from individuals, '*a*', '*b*', '*c*', to their predicate functions or features (concepts), '*Fx*', '*Rxy*'... This amounts to selective attention.
- The contextual: this concerns introducing a new "abstract" term through its use in various definitions.
- The "magical": do ordinary abstraction and then claim that what is abstracted is an individual in its own right (Aristotle allows for this in the mode of "as if").

If abstraction is a relation, its epistemic value could be secured via its ontic connection to the world. Therefore, if what is abstracted is considered to be an individual in its own right (like numbers, for example), then we would privilege its ontological status at the expense of whatever psychological process might generate our knowledge of it. That is, the latter would be less important.

If we consider Aristotle's conception of abstraction as a relation, confusion might arise concerning the *relata* that are involved in the relation in question:

> [...] On the one hand, Aristotle says, there is the perception or knowledge, while on the other there is the object perceived or thought. "Knowledge is the knowledge of something." What is confusing is that he uses the same term, 'perception' or 'knowledge', for both for the relation and for one of its *relata*. Yet the two need to be distinguished. [...]So too being related, the relation, is one thing; the *relatum*, what has the relation, is another. Aristotle has made this very distinction in general already in introducing paronymy as holding between two objects (not: expressions!), like bravery and the thing that is brave, where one is "said from" another. It is one thing to be bravery; it is another to have bravery, to be a brave person. (Bäck, 2014, p. 48)

Thus, when we speak of "quality" there is normally only one object involved. On the other hand, when we speak of a "relation" there are two objects involved: the *relata*. In terms of abstraction the difference between relations and qualities is that "the latter have only one object said from the original paronym signified by the abstract term, while the former have two" (*Ibid.*, p. 49). Bäck is clear that for Aristotle the first *relatum* of knowledge would be a mental state, which is similar to the case of perception[99]. Therefore, 'knowledge' has two paronyms: 'what knowledge

[99] The example Bäck discusses is the perception of a bird. An image of a bird in a mirror just is the image of the bird: "[i]f the bird goes away, there is no longer present an image or perception of that bird. In this way we

we have' and 'the thing about which we have knowledge' (*Ibid.*, p. 49, ff. 8). Both are, in a sense, pointed to by the term 'knowledge': the latter seems to point to something that is known as well as to a knower. In this sense, what knowledge we have and the thing about which we have knowledge are referred to or signified by the expression 'what is known'. Bäck puts this in a more cryptic way: the expressions 'what knowledge we have' and 'the thing about which we have knowledge' are both *said from* the term 'knowledge' (*Ibid.*, p. 47, ff. 8). So, for Avicenna, 'the knower', or 'what has the knowledge', also seems to be *said from* 'knowledge' (*Ibid.*, p. 49, ff. 8). Bäck continues:

> Thus, despite using an abstract noun, 'perception' or 'knowledge', what is being signified is its paronym, a concrete thing. This concrete thing is not itself the substance itself but that state in its substance. Likewise what is being perceived strictly is not the bird, but the bird qua being perceived, i.e., the individual substance qua being in that relation. (*Ibid.*, p. 48)

Despite Aristotle's insistence that there is a natural basis for abstract objects, he argued that universals are present only indistinctly, secondarily, and potentially. Indeed, sense perceptions of universals are far less reliable than sense perceptions of particulars (*Ibid.*, p. 109). Nevertheless, Bäck argues that Aristotle essentially agreed with Avicenna's construal of abstraction as a process that "strip[s] away all the accidental attributes" (*Ibid.*). Furthermore, Aristotle could somehow reconstruct the universals in *re* from our sense perceptions of particular accidents using the *noûs*, which is an active participant in the process (*Ibid.*). The importance of having a memory in this reconstruction (synthesis) process in the *noûs* is

should understand 'perception' in 'the perception is the perception of a bird' to concern a mental state" (Bäck, 2014, p. 48). The same is true in the case of knowledge via abstraction. A question immediately arises: what will happen to that image if we continuously employ abstraction? We deal with this question later in the paper. The relationship between numbers, for example, remain even when the individuals "go away", so to speak. Rather, the relationships seem to "go away" only when the *knower* is removed (more on this later).

undeniable: “[t]o see something as moving, like Avicenna’s example of a drop of rain falling, requires that a sequence of images from past sense perception be represented, in memory and imagination, and superimposed, somehow, so as to generate an experience of ‘seeing’ the drop move” (*Ibid.*, p. 145). Moreover, the act of thinking is more involved in this reconstructive process than is the act of perceiving. In contrast to perceiving, thinking grasps the essences of perceived objects (e.g. we do not merely grasp *flesh* but *being flesh*; not *water* but *being water*; not *magnitude* but *being a magnitude*). When thinking of objects, just as when we perceive objects, the thinker focuses selectively on certain aspects of the perceived objects (such as their color, their magnitude, their shape, etc.). According to Bäck, “even those essences may serve as output for a final stage, so as to produce those that are in abstraction” (*Ibid.*, p. 149). Aristotle does not tell us how we are able to recognize and focus on these necessary attributes while excluding the universal, albeit contingent, features of objects (*Ibid.*, p. 149). Avicenna, on the other hand, has much to say on this issue (*Ibid.*, p. 149).

While this kind of selective perception is important to the Aristotelian conception of abstraction, the process of abstraction, in general, is not always a process of exclusion. Instead, there is a parallel additive process (i.e. synthesis) that is involved in abstraction. In this sense, the process of abstraction has a dual character:

> Aristotle gives an instance of how both abstraction and synthesis arise in mathematical thinking. He holds that thinking always requires a phantasm. The imagination constructs the phantasm from sense perceptions but does not mirror them. Thus in thinking of a triangle, though we do not make any use of the fact that the quantity in the triangle is determinate, we nevertheless draw it determinate in quantity. (Bäck, 2014, pp. 141-142)

Avicenna makes similar remarks:

> So we imagine a triangle with a definite shape but then abstract away from that. Again we can imagine something without definite quantitative features but then add on those features as if it had them. Likewise imaginations can have features of both abstraction and synthesis. (as cited by *Ibid.*)

When discussing the Aristotelian conception of abstraction, Bäck refers extensively to Avicenna's reading of Aristotle. For Avicenna (as cited by Bäck, 2014, p. 165), "abstraction is, as it were, a mixture of perceptual induction and intellectual deduction based upon the difference of what is *per se* and what is *per accidents*".

In Avicenna's works, we find a sustained, focused discussion on abstraction and the integral role that perceptions play in the process of abstraction. It is also one of the earliest works in which we find a constructive, gradable[100] conception of abstraction. According to Avicenna, all our knowledge of things in the world has its origin in the abstraction of forms by the soul (Knuuttila, 2008, p. 9). Sense perception, in particular, is the lowest mode of abstraction (Knuuttila, 2008, p. 9). Indeed, Hasse (2007) regards Avicenna as a champion of abstraction theory relative to his Arabic predecessors.

2. The Basis of Avicenna's Theory of Abstraction

In his theory of abstraction, Avicenna discusses abstraction in terms of the intellect's capacity to derive universal knowledge from sense data (Hasse, 2007). McGinnis (2006) also evaluates Avicenna's theory of abstraction. He argues that the debate whether Avicenna's conception of

[100] That is, a conception of abstraction that admits of *degrees* of abstraction.

abstraction is a metaphor for emanation or whether it should be taken literally is superficial[101]. According to Hasse (2007, p. 41), Avicenna, following Al-Farabi, assumes that the "active intellect" is a distinct substance of the incorporeal intelligences of the universe. Due to the influence of the active intellect, potentially intelligible things become *actually* intelligible (*Ibid.*, p. 41). Hasse argues the use of the term 'intellect' by Al-Farabi (and hence by Avicenna) is different from Aristotle's use of the term in *De anima*. According to Al-Farabi, one of the definitions of the potential intellect is that its essence is disposed or able *to extract*[102] the quiddities of all objects and their forms from their matter:

> When we say that something is known for the first time, we mean that the forms which are in matter are extracted from their matter and that they receive an existence different from their previous existence. If there are things that are forms to which does not belong any matter, then this essence [i.e., the intellect] does not need to extract them from matter at all but finds them as something abstract. (Al-Farabi, as cited by Hasse, 2007, p. 42)

Hasse argues that we should not construe Al-Farabi's position on the intellect's "extraction" to be one where the form of a thing is separated from its matter and whereby the form acquires a new mode of existence (*Ibid.*, p. 42). Instead, Al-Farabi holds an alternative position where "the forms in matter are imitated in the intellect (as Avicenna once mentions) rather than put into a new mode of existence, or that intelligibles arise from sense data […], or that the active intellect is involved in the process" (*Ibid.*, p. 42).

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[101] McGinnis thinks this debate "stems from the deeper philosophical question of whether humans acquire intelligibles externally from an emanation by the Active Intellect, which is a separate substance, or internally from an inherently human cognitive process, which prepares us for an emanation from the Active Intellect". (*Ibid.*, p. 169)

[102] The original Arabic word (*entezaā*) also means to abstract, but it seems that the author chose this particular translation to emphasize the transformational aspect of intelligence.

Hasse also enumerates many of Avicenna's early doctrines that are related to his theory of abstraction (*Ibid.*, p. 43):

- The cooperation between the intellect and the internal senses (and the limits of this cooperation).
- The distinction between common and special, accidental and essential forms.
- The involvement of a separate universal intellect in the intellective process.
- The thesis that all perception, sensual as well as intellectual, is the abstraction of forms from matter.
- The comparison of the different modes of abstraction in the senses and in the intellect.

The form-matter dichotomy is a crucial dichotomy in Avicenna's philosophy. Indeed, Avicenna considers our knowledge of the world to be a coincidental association of the two. Moreover, we are somehow innately equipped with the faculties that enable us to dissociate forms from matter. It is in this way that the transformation of sense data into intelligibles can occur:

> The faculty which grasps such concepts (i.e. intelligibles that are not self-evident) acquires intelligible forms from sense-perception by force of an inborn disposition, so that forms, which are in the form-bearing faculty (*scil. common sense*) and the memorizing faculty, are made present to [the rational soul] with the assistance of the imaginative and estimative [faculties] Then, looking at [the forms], it finds that they sometimes share forms and sometimes do not, and it finds that some of the forms among them are essential and some are accidental. (Avicenna, as cited by Hasse, 2007, p. 43)

In general, as we will see below, Avicenna's construction of the hierarchy of abstractions can be characterized as a move from diversity to unicity in which derivation plays a pivotal role. The process of abstraction is considered to be dependent on the activity of human intellect and the only access to the abstracted forms is through sense data via intellectual derivation. For Avicenna, unlike Aristotle, ontology is less colorful than epistemology. That is, if we believe

Aristotle has a fundamentalist attitude, we can certainly ascribe a constructive attitude to Avicenna. According to Hasse, Avicenna gives a complete account of the cooperation between the intellect and the internal senses, as well as the distinction between common, special, accidental and essential forms (*Ibid.*, p. 44). Moreover, Avicenna also explains how a separate universal intellect is involved in the intellective process and how abstraction has different modes with regard to the senses and the intellect (*Ibid.*, p. 44):

> When it [the rational faculty] has found them being forms in this way [i.e., essential, accidental, common, etc.], each of these essential, accidental, common, or special forms becomes a single, intellectual, universal form by itself. Hence it discovers by force of this natural disposition intellectual kinds, species, differences, properties and accidents. It then composes these single concepts by way of first particular and later syllogistic composition; from there it concludes derivations from conclusions. (Avicenna, as cited by Hasse, 2007, p. 44)

What is interesting about Avicenna's approach to the knowledge of forms is the objectivity he ascribed to them. Forms are objective not because they are mind independent (as someone like Frege would say) or inter-subjective, but because there are limited ways of deriving (abstracting) them from sense data and from one another. It is not that objectivity explains why it is that there may be a limited number of ways that forms could be derived or abstracted from sense data. Rather, that there are limited ways of doing so is taken to be constitutive of objectivity. Consider the following passage

> Even though this faculty [the rational faculty] receives help from the faculty of sense-perception in DERIVING intellectual, single forms from sense-perceived forms, it does not need such assistance in forming these concepts in themselves and in composing syllogisms out of them, neither when granting assent to, nor when conceiving the two propositions, as we will explain below. Whenever the necessary corollaries have been DERIVED from sense-perception through the afore-mentioned natural disposition, it dispenses with the assistance of the faculties of sense-perception; instead it has enough power by itself for every action dealt with by it. (Avicenna, as cited by *Ibid.*, p. 44)

Avicenna continues building his abstraction hierarchy on the basis of sense perception by treating the abstracted forms at each level as entities similar to the objects of the first level undergoing similar processes but with less restrictions and, thus, more degrees of freedom. For example, should I abstract a sphere from, e.g., an orange, the sphere would share certain properties with the orange, but it would not have the same physical limitations of that particular orange. The sphere could undergo various size transformations, rotations, reflections, etc., and in these kinds of processes we might reveal properties of the sphere which are not restricted by the physical properties of the orange from which we abstracted. Here is where the intellect finds the freedom to perform the previously mentioned additive process and shows its activity. We move to a higher level of abstraction by dissociating forms from the matter, and we do this with the help of sense perception. However, in hindsight, we may regard the perceptual faculty as more passive than the intellectual faculty. We may now have a clear sense of what having an "active intellect" means:

> Just as the faculties of sense perception perceive only through imitation of the object of sense perception, likewise the intellectual faculties perceive only through imitation of the object of intellection. This imitation is the ABSTRACTION of the form from matter and the union with [the form]. The sensible form, however, does not come about when the faculty of sensation wishes to move or act, but when the essence of the object of sensation reaches the faculty either by accident or through the mediation of the moving faculty; the ABSTRACTION of the form [occurs] to the faculty because of the assistance of the media which make the forms reach the faculty. The case is different with the intellectual faculty, because its essence performs the ABSTRACTION of forms from matter by itself whenever it wishes, and then it unites with [the form]. For this reason one says that the faculty of sense-perception has a somehow passive role in conceiving [forms], whereas the intellectual faculty is active, or rather one says that the faculty of sense-perception cannot dispense with the organs and does not reach actualization through itself, while it would be wrong to apply this statement to the intellectual faculty. (Avicenna, as cited by Hasse, 2007, p. 44)

Obviously, for Avicenna, level zero is where abstraction begins constructively. A person's intellectual faculty works like a template for transforming sense data into intelligibles due to the faculty's own specific properties. Indeed, Hasse notes that "Avicenna plainly states that in contrast to sense-perception, the rational faculty is an active faculty which can perform the abstraction of a format will. The power to form concepts is innate" (Hasse, 2007, p. 45).

As is well known, the dualism of light and darkness, as well as analogical references to this dualism, are common features of many eastern schools of thought. Normally, the existential and essential division between intellect(s) and matter is drawn by associating matter with darkness, and intellect with light. In this way, pure intellect stands independently and has nothing to do with sensation and matter. For Avicenna, on the other hand, the intellect needs the help of the senses, of the universal intellect, and of naturally inborn axioms. The natural inborn axioms and the universal intellect are needed for syllogistic forms of reasoning. Avicenna invokes the traditional analogy of light:

> Light is similar to this intellect in that it enables the faculty of sight to perceive without, however, providing it with the perceived forms: This substance (i.e. the universal intellect), in turn, supplies by the sole force of its essence the power of perception unto the rational soul, and makes the perceived form arise in it as well, as we have said above. (Avicenna, as cited by *Ibid.*)

Therefore, one could argue that the light-darkness analogy also applies to the matter-form dichotomy in Avicenna's works. That is, the intellect's primary access (or introduction) to the realm of forms is via the forms that have been illuminated by the light of matter. From the above passage, it appears (ontologically speaking) that there exist two very different powers in the process of abstraction rather than two necessary accompanying conditions: (1) the human intellect; (2) the separate universal intellect. In the following passage, Hasse argues against this idea:

> For the relation between the human and the universal intellect is clearly described as an act of "assistance": "All this [the conceptualizing faculty is able to do] with the service of the animal faculties and the assistance of the universal intellect". Without doubt, in this early version of Avicenna's theory of abstraction, it is the powerful abstracting force of the human intellect which is the focus of the theory. The senses are indispensable, for they provide the necessary sense-data. The universal intellect is indispensable as well; its function is hardly described at all but seems to consist in somehow providing the necessary intellectual surrounding for the activity of the rational soul, in a manner similar to light with respect to the human ability to see. Hence both the senses and the universal intellect are necessary accompanying conditions rather than powers active in the process. (Hasse, 2007, p. 46)

In short, and in light of what has been said thus far, it is evident that Avicenna's approach to abstraction is a constructive one. The basis of his theory of abstraction is the sense data received by perception. The sense data is then treated by the active intellect according to the intellect's innate properties. These properties make the active intellect susceptible to receive help from (or connect with) the universal intellect[103]. The universal intellect then shines a new light on the whole process, and it leads us to new or improved sense data from which we proceed to another cycle of abstraction. One of the important features of Avicenna's theory of abstraction is the mediatory and gradual characteristic of the abstraction. I will discuss these further in subsequent sections.

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[103] There are reasons to believe that the abetting role (of the universal intellect) would be better understood if we consider it in connection with similar ideas that existed in pre-Islamic doctrines of popular Persian schools of thought. The latter include Mithraism, Zoroastrianism, etc. In Zoroastrianism, for example, Immortal Bounteous (*Aməša Spənta*) refers to six helpers for realization of the ultimate wisdom, *Ahura-Mazda* (*əˌhʊrəˌmæzdə*), (Frye, 1984, p. 58), (Boyce, 1979, p. 17). The first one is called *Vohu Manah*, which literally means "Good Mind",that could correspond to "active intellect". There are other strong similarities between the other helpers and some other features of Avicenna's philosophy that should be discussed elsewhere.

3. Degrees of Abstraction

In his mature theory of abstraction, Avicenna speaks of different kinds as well as of four different degrees of abstraction: (1) sense perception; (2) imagination; (3) estimation; (4) intellect. Here, once again, Avicenna gives central attention to the form-matter relationship. Forms, as long as they are associated with matter, are subjected to certain conditions and states of affairs. We may call them *material forms* at this stage. However, as soon as they are understood by an active intellect they are liberated from those conditions and can be treated (and studied) as objects in their own right:

> It seems that all perception is but the grasping of the form of the perceived object in some manner. If, then, it is a perception of some material object, it consists in an apprehension of its form by ABSTRACTING it from matter in some way. But the kinds of ABSTRACTION are different and their degrees various. This is because, owing to matter, the material form is subject to certain states and conditions, which do not belong to [the form] by itself insofar as it is this form. So sometimes the ABSTRACTION from matter is effected with all or some of these attachments, and sometimes it is complete in that the concept is ABSTRACTED from matter and from the accidents it possesses on account of the matter. […] (Avicenna, as cited by Hasse, 2007, pp. 47-48)

Thus, the word ‘object’ may refer to the accidental association of form and matter (physical objects) at the lowest level of abstraction (the level of perception) as well as to the liberated forms at the higher levels of abstraction. Avicenna is clear that since the fixed forms are either (1) the forms of non-material objects (and do not occur accidentally in matter[104]), (2) the forms of objects which are accidentally non-material, (3) the forms of material objects purified in every respect from material attachments, then such a faculty obviously perceives the forms by grasping them as abstracted from matter in every respect:

[104] For example, one might think of different forms of geometry.

> […] This is evident in the case of objects which are in themselves FREE from matter. As to those objects which are present in matter, either because their existence is material or because they are by accident material, this faculty completely ABSTRACTS them both from matter and from their material attachments and grasps them in the way of ABSTRACTION; hence in the case of 'man' which is predicated of many, this faculty takes the unitary nature of the many, DIVESTS it of all material quantity, quality, place, and position. If [the faculty] did not ABSTRACT it from all these, it could not be truly predicated of all. (Avicenna, as cited by *Ibid.*)

In this scheme, going through higher levels of abstraction is coincidental with the gradual loss of material attachments[105]. It is clear from the previous passage that spatiotemporality is one of the material characteristics that could be removed when abstracting. We may also see an emphasis on the abstractive nature of our intellectual faculty to grasp the forms (should we want to). This becomes crucial to the very meaning of "active intellect". Hasse notices the following important changes in Avicenna's mature theory of abstraction (2007, pp. 48-49):

1. The main difference between sense perception and intellection is no longer described in terms of passivity and activity but the difference lies in the faculties' widely diverging powers to divest forms of their material attachments.
2. There is no explicit link to the theory of the separate active intellect.
3. There is no mention of "imitation" or "assimilation".
4. There is now an explicit connection between the fully abstracted status of a form with the fact that the form has many instances, i.e., it can be predicated of many things.
5. The terminology of "form" and "matter" had not yet served to develop a theory about the ontological status of concepts. The latter was one of Avicenna's major concerns during his middle period.

[105] We will see, in the subsequent chapters, that this loss of material properties is quite similar, in Carnap's philosophy, to the loss of "factual content" in a linguistic framework.

Therefore, it is the activity of the soul that leads to the distinction between what is accidental to the form and what belongs to the form "insofar as it is this form" (Hasse, 2007, p. 49). Avicenna adds the following important component to his abstraction theory in order to explain the exact nature of this form (which is the object of abstraction):

> To give an example: the form or essence of man is a nature in which all the individuals of the species share equally, while in its definition it is a single unit: although it is merely by accident that it happens to exist in this or that individual and is thus multiplied. (Avicenna, as cited by *Ibid.*, p. 49)

Accordingly, the form in question (the object of abstraction) is a single unit (an object of its own) and multiplicity is only accidental to the form. That is to say that the form gets multiplied only once it comes into contact with matter[106]. What is interesting about Avicenna's theory of abstraction is that after perceiving sense data (at the first degree of abstraction, i.e., perception) and abstracting a form in our imagination (the second degree of abstraction), we are still not to consider this form as *the* form (the intelligible one). We may only do so once the form goes through estimation (the third degree) and intellectual deduction (the forth degree)[107]. For Avicenna both universality and particularity are accidents to the intelligible form (Hasse, 2007, p. 49). According to Hasse, it is plausible that, for Avicenna, the theory of abstraction belongs to psychology whereas the theory of forms belongs to metaphysics. This is because Avicenna never treated both together. Another interesting feature of Avicenna's theory is that from the

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[106] This discussion is largely in line with Frege's discussion on the definite and indefinite articles. The former identifies an object whereas the later identifies a concept (a class of objects). On this view, we are faced with an awkwardness of language where "the concept horse" is not a concept (Frege, 1951, p. 172). Since the expression "the concept horse" is prefaced by the definite article, it must refer to an object.

[107] I will not explain the third and fourth degrees in more detail since this would make for a much too lengthy discussion and detract from the issues we are most concerned with.

level of imagination onward, all data that our faculty receives are internal and it is exactly for this reason that the attribution of universality or particularity to the forms are nonsensical[108]:

> Embedded in this theory [the theory that neither multiplicity nor particularity belong to the form as such] is a lucid description of the process of abstraction: the intellect works on data presented to it by the senses and stored in imagination; these data themselves are not imported from outside but are creations of the senses. The form that is in the intellect is a single concept only with respect to the intellect; it is universal with respect to the objects outside, and in itself it is neither universal nor particular, since - and here the distinction between essence and existence is involved again - it is independent of its existence outside and inside the intellect: as Avicenna says in his psychological works, it is the "unitary nature of the many" or "a nature in which all the individuals of the species share equally". (Hasse, 2007, p. 50)

Avicenna construes thoughts as “movements of the human intellect produced before the reception of abstract forms” (*Ibid.*, p. 57). It is important to notice that the intelligible forms, for Avicenna, bear a dual nature. That is, they ultimately derive from the particulars in imagination and still resemble them: “they are partly of their kind and partly not” (*Ibid.*, p. 58). This is an important point to note. The “partly not” is due to the above-mentioned additive (or creative) process. Now we can clearly see that Avicenna characterizes abstraction, in general, as a move from multiplicity to unicity and regards this move as an intrinsic property of human intellect[109]. Thus, the abstracted unit may seem universal with regard to the objects that it comes from, but

[108] This discussion is closely related to Carnap’s external-internal discussion regarding ontological questions. Carnap argues that ontological questions are meaningless if they are questions external to what he calls a “linguistic framework”. However, they are meaningful if they are questions internal to this framework (Carnap, 1950).

[109] If, as Avicenna suggests, this move is a basic one in human psychology, we should be able to see manifestations of it in different aspects of human behavior. Throughout human history we may identify many instances of moving from multiplicity to unicity. One example would be the move from polytheism to monotheism. In fact, without this characterization of abstraction, we may have a hard time explaining why there is such propensity among scientists to find a unified theory of science or to have a general theory from which we could derive local laws. Why is this phenomenon admired and considered to be an advantage for scientific theorists and regarded as providing us with a “better understanding” of the physical world, if it is not that our thoughts are movements that intrinsically move in this direction. In social and political phenomena, it may even explain why social unity and collective action is common despite the fact that diversity is generally valued. I will discuss the intrinsicness of this move in greater detail below.

it is not universal in the sense of being an entity outside of the abstraction system (or the intellect). For example, if we look at the universality of mathematical laws from this perspective, we need not say they are universal because they are mind-independent. Rather, they are universal because human minds are essentially the same in terms of abstraction. That is, the process of abstraction in any particular mind is essentially the same as in any other mind.

In conclusion, Hasse ends his argument for the originality of Avicenna's theory of abstraction by saying: "It seems impossible to deny that Avicenna was convinced of the human power of abstraction, that he meant what he said and that he was fully capable of developing a theory of impressive quality […]" (*Ibid.*).

The retrospective effect of the active and universal intellect in the process of abstraction is obvious in Avicenna's abstraction theory. D'Ancona (2008), who acknowledges that Avicenna's understanding of human knowledge is quite different from Aristotle's, wants to present Avicenna as a philosopher who manages to combine both the Aristotelian and the Neo-Platonic models into "a unique and consistent description" (*Ibid.*, p. 52). D'Ancona presents the four degrees of abstraction (perception, imagination, estimation, and intellection) as a purifying process:

> All this gives rise to a doctrine of knowledge which may be represented as a double arrow, one coming from sense perception and the other coming from above, i.e. from the intelligible forms as they are in themselves, with the two arrows having their meeting point in abstraction. Abstraction is a process of disentangling forms from matter, and it culminates in the intellect, which "completely abstracts them both from matter and from their material attachments in every respect and perceives them in pure abstraction." In the meeting point, human knowledge grasps the forms as they are in themselves, at the end of a process of "purifying" the forms: first they are grasped by sense-perception in association with matter, then imagined still in association with matter, then again judged or, if you want, named, and finally theoretically known. (D'Ancona, 2008, p. 58)

Here we see, once again, that there are intermediate forms gradually disentangling from their material attachments; hence, they bear a dualistic perceptual-intellectual nature. Later on, we will see how this perceptual-intellectual dichotomy might correspond to the modern synthetic-analytic dichotomy.

Regardless of where abstraction exactly fits into Avicenna's philosophy, McGinnis (2006), in line with Hasse, reminds us that abstraction refers to an in internal operation in the human soul and not to the activity of a separate intellect. Nevertheless, McGinnis believes construing abstraction in this way will raise the following philosophical puzzle:

> […] if abstraction were an operation internal and proper to each human intellect, then the product of such an operation, namely, the intelligible concepts, would be as numerous and diverse as the human intellects that produce them; however, when there is scientific knowledge, what is known is not unique to the various intellects, but is universal and is at least potentially common to all intellects. Thus, if abstraction is an internal human operation, whose product is something particular to the individual performing the act of abstraction, what is it that explains the universal nature of scientific knowledge as it exists in different intellects? (McGinnis, 2006, p. 170)

Thus, in order to explain the inter-subjective unity of scientific discourse, and why intelligible concepts are not as diverse as human intellects, McGinnis suggests we adopt the view that the above-mentioned abstraction process is an essential and intrinsic property of human intellectual activity. In this way, abstraction can be an internal operation to the human mind without endangering the inter-subjectivity of scientific discourse.

McGinnis points out that if we consider Avicenna's doctrine of essences as the starting point, we will see how Avicenna's theory of abstraction fits into his overall philosophy consistently.

McGinnis notes the three following important features about Avicenna's account of essences (2006, pp. 170-171):

1. There is a close relation between essences and conceptualization, where conceptualization for Avicenna is one aspect of scientific knowledge and is closely linked with concept acquisition and mastery. Indeed, for Avicenna all knowledge begins with conceptualization, and what is conceptualized are essences.
2. For Avicenna essences exist *only* either in concrete particulars or in conceptualization[110], he also believed that they could be considered in three, not just two, respects: they can be considered as they exist in concrete particulars and in conceptualization as well as in themselves[111].
3. Finally, there is lingering concern about how we should understand the "particularized existence of essences" concretely and conceptually and the relation between the particularized existence of an essence and the essence in itself (*Ibid.*, p. 171). This relation is such that the accidents that "follow upon being in concrete particulars are owing to the matter in which the essences occur" (*Ibid.*, p. 171). These accidents are the features that make something sensible or perceptible. As for the accidents that "follow upon conceptualization", Avicenna construed them as "being a logical subject and predicate, universality and particularity as well as essentiality and accidentally in predication" (*Ibid.*, p. 171). McGinning continues: "[i]n effect these are the features that give our thinking a logical character and allow us to employ logic in scientific inquiry" (*Ibid.*, p. 171).

Accordingly, McGinnis concludes that, for Avicenna, "essences-in-themselves are a common element in both concrete particulars and objects of conceptualization" (*Ibid.*, p. 171):

[110] Conceptualization is the immediate image we receive in our imagination from our perceptual faculty (see degrees of abstraction above).

[111] In order to see what it means for the essences to exist in themselves rather than the other two respects, McGinnis gives us the following example: "Consider natural or counting numbers. Any instance of such a number can only ever exist as either odd or even, but certainly natural numbers can be considered just as natural numbers independent of any features that follow upon being odd or even".

> There is a difference between the thingness and the existence in concrete particulars; for the account [of what something is] has an existence in concrete particulars and in the soul [i.e., as conceptualized] and is something common [to both]. That common thing, then, is the thingness. (Avicenna, as cited by McGinnis, 2006, p. 171)

Since abstraction, as described above, is a process of dissociating forms (essences) from the matter (which makes the forms particularly and accidentally concrete), one can clearly acknowledge that there is no need to consider a special ontological status for products of abstraction. This is an important conclusion one might draw from Avicenna's treatment of abstraction.

To conclude this chapter, we may construe abstraction as an innate and internal process that tends to move from multiplicity to unicity by disentangling forms from the matter (or their material attachments) in a gradual way. This process is initiated by a voluntary and creative focus of attention. Although material attachments of the sense data as they are received by our imagination could be stripped away differently (creatively), the resulting forms, which still preserve some characteristics of their material attachments, eventually converge to more essential forms. Thus, the forms would have a dual perceptual-intellectual nature. In virtue of this dual nature they should not be considered particular or universal. The essential freedom of pure forms is something that we discussed briefly. We did, however, note that it could, for example, manifest itself as different formulations of geometry. If space happens to be this or that way, it is purely accidental. We are not construing the *concept* of space in this or that way. In other words, the abstract concept of space is free of total particularity, but it is not free from partial particularity. The latter is true because of it preserves some material attachment.

Therefore, the resulting image that we may draw of abstraction from ancient philosophy (though mostly from Avicenna) is that abstraction is an intellectually active construction in which the activity of the intellect manifests itself, primarily, by voluntarily noticing (which is different from involuntary pure observation). Later on, it manifests itself by imposing constraints and conditions in order to separate forms from matter (in a stepwise manner) and to reach the (relatively) pure and free forms.