Entrepreneurship, Outside options and Constrained Efficiency

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May 2019

Abstract

The literature on search frictions has often adopted the assumption of free entry. In this paper we forgo of this restriction by proposing a more realisitic framework in which individuals are constantly making the decision whether or not to open a firm. Namely, firms are created through endogenous choices and business-owners and workers are drawn from the same pool. We show that in this framework, the Nash bargaining parameter is crucial for internal dynamics. In particular, workers and business owners share the same outside-options. As a result, the wage is no longer unambiguously positively related to the value of unemployment. The constrained efficient solution to this model takes the same form as the standard search model implying the same form for the Hosios condition. However, at this efficient solution changes in the rate of unemployment are either exacerbated or muted conditional on the value of the match elasticity parameter. (**JEL Classification** : E24 J63 J64 D61)

Keywords : Search and Matching, Entrepreneurship, Outside Options, Constrained Efficiency

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‡We would like to thank Jesse Perla, Henry Siu, Yaniv Yedid-Levi and Giovanni Gallipoli for feedback and discussions.

1 Introduction

Understanding the process and choices that drive the creation of new firms and hence spur employment is crucial to a complete understanding of employment, productivity growth, wages, vacancy creation and a host of other labor market variables. The focus of this contribution is on the entrepreneurship margin. We make a theoretical contribution to the literature on firm creation by placing the decision of individuals to create a firm inside the search and matching framework. Jobs are created by ex-ante identical individuals who face a choice between entrepreneurship and wage work. By modelling the start-up decision as an endogenous choice in this manner we forgo the inclusion of the typical free entry condition to close the model as in Mortensen and Pissarides (1994). Instead the model is closed by a condition whereby entrepreneurs are indifferent between remaining unemployed or creating a business, conditional on their productivity draw.

Modelling firm creation in this manner implies an interesting distinction between our framework and the baseline search model. For instance, in the standard search framework wages are unambiguously increasing in the value of unemployment. In contrast, the direction, and not just the magnitude of this relationship is dependent on the Nash bargaining parameter in our framework. For threshold values of the bargaining parameter the slope of the wage can become negative, or indeed flat. The intuition underlying this result is that firms are created by individuals whose outside option is to search for wage work through rejoining the pool of unemployed workers. Due to this, the outside option value for both firms and workers in this model is the value of unemployment. This contrasts with exogenous models of firm creation where the outside option value for the firm is the value of an unfilled vacancy. As a result the equilibrium wage equation in our framework includes an additional term coming from the firm side. This relationship also appears counter-intuitive: for high values of workers bargaining power wages are negatively related to the value of unemployment.

Additionally, the inclusion of endogenous firm creation here implies the existence of an additional externality in the model, in addition to the standard congestion and thick margin externalities, which we refer to as the 'job-creation margin'. This margin arises from the endogeneous choice to search for a business idea or a job. If the labor market is tight then individuals will prefer to search for wage work and hence entrepreneurs will be more selective on which business ideas they implement. However, those deciding between searching for a job or a business venture do not take into account the effect of their choice on the search process of other potential entrepreneurs or other job seekers. Their choices also affect the choices of entrepreneurs currently operating in the market. This effect again operates through changes in the entrepreneurs outside-option term, which is the value of unemployment.

Given the inclusion of this additional externality and the distinct difference in the wage function, it is not ex-ante clear what form the efficient solution to the model will take. In solving for the planners problem we find that the solution is identical to that of the Hosios condition for the standard search framework: externalities are balanced when agents bargaining power is equated to the elasticity of the matching function. However, this socially efficient solution does not pin down a clear direction for the wage, and hence a clear direction of adjustment to equilibrium. The dynamics of the model following a shock remain sensitive to the size of the elasticity parameter. In particular, wages do not necessarily exert a dampening effect in response to exogenous shocks.

This paper contributes to the theoretical literature evaluating constrained efficiency in search theoretic models of the labor market. The Hosios rule (Hosios, 1990) states that a standard search model a là Pissarides (2000) is constrained efficient when the Nash bargaining parameter is equal to the elasticity of the matching function. Literature in this area has sought to examine the set of conditions under which the Hosios rule gives the socially efficient outcome[1] or generalizes the Hosios rule to alternative environments[2].

This paper also relates to the literature on the individual choice between working and opening a business. The empirical literature is vast as exemplified by the seminal papers

[1] see Albrecht et al. (2010), Gavrel (2011)

[2] see Acemoglu and Shimer (1999), Julien et al. (2016)

of Hamilton (2000) and Quadrini (2000) as well as more recent research such as that of Humphries (2016) and Poschke (2013). Finally entrepreneurship is important to the extent that the extensive margin of firm creation is important for the macroeconomy. In that respect there is evidence on the importance of the firm creation process for persistence in firm outcomes (Sedlácek and Sterk (2014), Moreira (2015)), wealth inequality (Quadrini (2000) and Cagetti et al. (2006)) and the importance of young firms for job creation (Haltiwanger et al. (2013)). This paper contributes with a richer theoretical framework to investigate these decisions of individuals to open a business.

The remainder of this paper is structured as follows: In section 2 we present our theoretical model, the dynamics of which are discussed in section 3. In section 4 we discuss the Hosios condition for efficiency and we present concluding remarks in section 5.

2 Model

At a given point in time an individual can be one of four types; a worker, an entrepreneur, a searcher for paid work, or a searcher for a business idea. Individuals search for a 'business idea' while unemployed only. Each business idea represents the productivity level of the firm and is modelled as an exogenous productivity draw, ϵ. To maintain tractability we assume there is no direct entry into entrepreneurship from wage work[3], and that there is no recall of productivity draws. Labor market tightness, defined in the standard manner, includes as the unemployed only those seeking wage work. It is worth noting however that this does not mean that entrepreneurship only affects market tightness on the vacancy side. The size of the unemployed pool is in part the result of the endogenous choice between searching for a job or an idea.

A choice is made by all unemployed workers to either accept a job, or implement a business idea. If the unemployed chooses to search for business idea, he or she receives one

[3]This would require a separate threshold rule for each realized wage. The comparative statics of the model in that case becomes more complex. For every shock that increases the selection on business projects among unemployed, there would be a corresponding decrease in selection among the workers.

at rate ψ from distribution $F(\epsilon)$, which he or she then chooses whether to implement or not. Individuals finding sufficiently productive business ideas post a vacancy next period, in which case they receive the value of a unfilled vacancy of productivity ϵ, $V(\epsilon)$ and lose the value of being a searcher U. If the unemployed decide to search for a job, they receive one at rate $p(\theta)$, in which case they draw a job from the endogenous firm productivity distribution $\mu(\epsilon)$.[4] Upon finding an employer of productivity ϵ, the worker receives the value of being a worker in a firm of productivity ϵ, $W(\epsilon)$. From the assumptions above it follows that the value function of a searcher U is given by

$$rU = b + \max(p(\theta) \int (W(\epsilon^*) - U)\mu(\epsilon^*)d\epsilon^*, \psi \int_{\underline{\epsilon}} (V(\epsilon^*) - U)dF(\epsilon^*)) \tag{1}$$

As all individuals are ex-ante identical, in equilibrium individuals will be indifferent between searching for a job or for a business idea.

The productivity threshold, below which no entrepreneurs will implement their business idea is characterized by:

$$V(\underline{\epsilon}) = U \tag{2}$$

The cost of posting a vacancy is given by c, $q(\theta)$ defines the firm match probability. Let $J(\epsilon)$ denote the value of a filled vacancy of productivity ϵ. The value functions for a vacant job ($V(\epsilon)$) is standard and given by

$$rV(\epsilon) = -c + q(\theta)(J(\epsilon) - V(\epsilon)). \tag{3}$$

When working for an employer of productivity ϵ the worker receives wage $w(\epsilon)$. Firms experience exogenous destruction shocks at rate λ. The value of being a worker, $W(\epsilon)$, is

[4] This distribution is a equilibrium object that depends on which business opportunities individuals choose to implement.

given by

$$rW(\epsilon) = w(\epsilon) + \lambda(U - W(\epsilon)). \tag{4}$$

The value of a filled job ($J(\epsilon)$) takes into account that at exogenous rate λ the firm is destroyed and the entrepreneur transitions back to unemployment

$$rJ(\epsilon) = \epsilon - w(\epsilon) + \lambda(U - J(\epsilon)). \tag{5}$$

Wages are formed through Nash bargaining. The threat point in the bargaining process for both workers and entrepreneurs is the value of unemployment, U. If either party chooses to walk away from the match then the firm will shut down. Equilibrium in this model is characterized by the set of equations

1. $V(\underline{\epsilon}) = U$
2. $(1-\beta)(W(\epsilon) - U) = \beta(J(\epsilon) - U)$
3. $W(\epsilon) > U \quad \forall \epsilon > \underline{\epsilon}$
4. $p(\theta) \int (W(\epsilon^*) - U)\mu(\epsilon^*)d\epsilon^* = \psi \int_{\underline{\epsilon}} (V(\epsilon^*) - U)dF(\epsilon^*)$

where the last equation ensures individuals are indifferent between searching for a job or a business idea.

The remainder of our analysis considers the economy in steady state. From steady state equations we derive a tight, negative relationship between the threshold productivity and market tightness which we refer to as the Job Creation Curve,

$$p(\theta) = \psi(1 - F(\underline{\epsilon})) \tag{6}$$

Furthermore, using the entrepreneur's value functions combined with the derived endogenous productivity distribution we derive a second curve showing a positive relationship between tightness and threshold productivity, which we refer to as the Entrepreneurship curve.

Theorem 1. *The indifference condition determining* $\underline{\epsilon}$ *implies the following positive relationship between* $\underline{\epsilon}$ *and* θ*.*

$$\frac{b(r+\lambda)+\beta\psi\int_{\underline{\epsilon}}(\epsilon^*)dF(\epsilon^*)}{r(r+\lambda+2\psi\beta(1-F(\underline{\epsilon})))}=\frac{-c(r+\lambda)+q(\theta)[(\underline{\epsilon})(1-\beta)]}{r(r+\lambda+2(1-\beta)q(\theta))} \tag{7}$$

The intuition underlying this curve is that a tighter labor market implies a greater benefit to seeking wage work. To remain indifferent, the benefits to entrepreneurship must be high for potential entrants, which translates into a higher threshold productivity, $\underline{\epsilon}$. The interaction of the Entrepreneurship curve and the Job Creation curve pins down an equilibrium pair of $(\theta, \underline{\epsilon})$.

Finally, wages in equilibrium are given by

$$w(\epsilon)=\beta\epsilon+(1-2\beta)rU, \tag{8}$$

which differs from the traditional search and matching model due to the inclusion of an additional $-\beta rU$ term coming from the outside option term of the entrepreneur. The direction of fluctuations in the wage due to movements in the value of unemployment are determined by the size of the bargaining parameter. Wages are increasing in the value of unemployment, U, for $\beta < \frac{1}{2}$ and decreasing for $\beta > \frac{1}{2}$.

3 Model Exploration

In this section we present some comparative statics outlining the underlying mechanisms present in the model. From equation (8) it is quite clear that the direction of wage movements is dependent upon the value of the bargaining parameter. Intuitively two effects are operating here. Firstly, an increase in U pushes wages upward as the threat point of job seekers has increased. Secondly, failure becomes less costly to the entrepreneur as the outside option to firm closure is higher. As a result, wages are negatively related to U when workers have a greater share of bargaining power. The intuition behind this seemingly counter-intuitive result is that when β is high a greater weight is placed upon the decline in the cost of failure to the entrepreneur than on the cost of job loss to the worker. The reverse is true if $\beta < \frac{1}{2}$. However, even when the direction of wage movements are unknown the following result holds true:

Theorem 2. *An increase in the value of unemployment income, b leads to a higher value of threshold productivity, $\underline{\epsilon}$, and a lower value of market tightness, θ, in equilibrium, implying an increase in average firm productivity.*

This means that even if wages fall in response to an increase in the flow value of unemployment, net job creation will still fall. The intuition for this result is simply that a higher b provides greater insurance to potential entrepreneurs and as a result those searching for ideas become more selective on the business ideas they implement. This raises average productivity and decreases net job creation as captured by an increase in the threshold productivity $\underline{\epsilon}$. Furthermore regardless of how wages are affected there will be a decline in market tightness. This results from lower net vacancy creation. The implication of our model therefore, is that there is a trade-off between entrepreneur quality and quantity. If a greater social safety net is provided for entrepreneurs then there will be a fall in the rate of entrepreneurship but an increase in the average quality of entrepreneurs. Furthermore it is interesting to note that this relationship holds regardless of how wages adjust. In particular it is possible to have an increase in aggregate productivity with flat or even decreasing wages. This contrasts with Mortensen and Pissarides (1994) where wages are the key channel through which the flow value of unemployment affects vacancy creation.

Similarly, without knowing the direction of wage movements it is possible to pinpoint a relationship between the other endogenous variables and the flow cost of posting a vacancy.

Theorem 3. *An increase in the cost of posting a vacancy, c, leads to a higher value of threshold productivity, $\underline{\epsilon}$, and a lower value of market tightness θ in equilibrium, implying an increase in average firm productivity.*

An increase to the flow cost c leads to a shift in the entrepreneurship curve, while the job creation curve remains unchanged. In a model with exogenous firms and a free entry condition, the value of unemployment is affected only indirectly through a change in the firm productivity distribution. In our framework there is an additional effect whereby the value of a vacancy enters the value of unemployment through the agents choice to become an entrepreneur. In both models the value of unemployment falls as a result. The wage effect serves to dampen the employment response in the case of exogenous firms, but in our model the employment response is exacerbated if wages are negatively related to the value of unemployment.

4 The Constrained Efficient Solution: Deriving the Hosios Condition

We solve the planners problem for the model to derive the constrained efficient solution which is summarised by the following theorem:

Theorem 4. *The competitive equilibrium allocation is constrained efficient when* $\beta = 1 - \alpha$.

The condition under which the model is at a social optimum, is the same as the original Hosios condition (Hosios, 1990), whereby externalities are fully balanced when the bargaining parameter is equal to the elasticity of the matching function. Given the unique relationship the bargaining parameter plays in our model in determining the direction of the wage response to changes in the value of unemployment, it is worth exploring this result somewhat.

In particular, the constrained efficient wage equation now takes the form:

$$w(\epsilon) = (1 - \alpha)\epsilon + (2\alpha - 1)rU \tag{9}$$

Wages are declining in the value of unemployment for values of α lower than $\frac{1}{2}$ and strictly non-negative otherwise. A high value of α implies that workers match probability is sensitive to market tightness, relative to that of firms.

Consider the case in which $\alpha > \frac{1}{2}$. Holding wages constant, an increase in market tightness (θ) has three effects. First, the value of job search increases due to a rise in $p(\theta)$ holding constant the value of unemployment (U) and the threshold productivity ($\underline{\epsilon}$). We refer to this as the *job search effect.* Secondly, for entrepreneurs, the likelihood of matching with a worker falls ($q(\theta)$) but by a lesser amount than the increase in $p(\theta)$, again fixing U and $\underline{\epsilon}$. This we call the *worker finding effect.* Thirdly, the *outside option effect* comes from fluctuations in the value of unemployment (U) which affects entrepreneurs via the outside option channel. Note that via the outside option channel (changes in U), entrepreneurs are affected by changes in $p(\theta)$, holding changes in $q(\theta)$ constant. This contrasts with more standard search models where changes in U and $p(\theta)$ affect firms only indirectly via wages once we hold $q(\theta)$ constant.

This channel, where the decision to enter entrepreneurship is dependent on the value of unemployment and the value of wage work introduces novel externalities. The first being that when agents choose to search for a business idea they do not take into account the impact of this decision on the choice between job creation and wage work for other individuals. Secondly, they do not take into account how their choice affects the outside-option value of entrepreneurs currently operating in the market. These externalities arise from the presence of the *job search effect* and the *outside option effect*, respectively, both not present in a model with exogenous firms and free entry.

In the absence of wage effects, the value of unemployment will rise, and due to the indifference condition on entry (equation 2), the value of $\underline{\epsilon}$ increases. A rise in $\underline{\epsilon}$ decreases firm creation and increases average firm productivity. Under the constrained efficient allocation ($\beta = 1 - \alpha$), as from equation (8), a larger share of productivity accrues to the firm, and hence a smaller increase in $\underline{\epsilon}$ maintains the equality. However, assuming that $\alpha > \frac{1}{2}$, wages are increasing in the value of unemployment, which further increases the gains to job search, exacerbating the rise in $\underline{\epsilon}$ required to make the individual indifferent. As a result, the constrained efficient allocation generates less firm creation (higher $\underline{\epsilon}$) relative to the allocation where wages do not adjust. In other words, compared to the socially efficient allocation, there is excessive firm entry coming from an increase in the value of unemployment, despite the rise in $q(\theta)$.

The intuition is that with $\alpha > \frac{1}{2}$, $p(\theta)$ is more responsive to changes in θ relative to $q(\theta)$. As a result, for a rise in θ, the *job search* and the *outside option effects* are quite large while the *worker finding effect* is small. Since the *job search* and the *outside option effect* pull $\underline{\epsilon}$ in opposite directions the overall result is a small response of $\underline{\epsilon}$ to a rise in θ. Therefore, to attain the constrained efficient allocation prices have to move so as to generate more selection than would happen otherwise. This is achieved through wage adjustment where the bargaining parameter is such that wages are increasing in the value of unemployment.

If we consider the opposing case where α is less than one half a similar logic applies. An increase in tightness, θ, generates a large response of $q(\theta)$ and a small response of $p(\theta)$. As a result, the *job search* and *outside option effects* are small while the *worker finding effect* is large. The overall response of $\underline{\epsilon}$ to the change in θ is larger than would be induced in a constrained efficient allocation. Therefore, to attain efficiency, prices must adjust to reduce selection on productivity. This is achieved when wages are decreasing in the value of unemployment. The same is true if we consider a fall in θ. The wage response will mitigate the increased entry to entrepreneurship when $\alpha < \frac{1}{2}$ and exacerbate entry when $\alpha > \frac{1}{2}$. If the slope of the wage equation with respect to U was α rather than $1 - 2\alpha$ then this mitigating effect on $\underline{\epsilon}$ would not operate when $\alpha < \frac{1}{2}$.

Therefore, this model includes a range of values over which the unemployment response to a shock is more severe, and a region where it is lessened. This is driven by the ambiguity in the direction of the wage response arising from entrepreneurs and workers sharing a threat point in the bargaining process. In a search model with exogenous firm creation and a free entry condition a shock that increases tightness, θ, always feeds into a larger wage, w, via a increase in the value of unemployment, U, which partially offsets the increase in θ by decreasing the incentives for firms to hire. The wage response here has a mitigation effect like in standard models without entrepreneurs and with a free entry condition when $\alpha > \frac{1}{2}$ but has a amplification effect when $\alpha < \frac{1}{2}$. The constrained efficient solution therefore balances traditional search externalities with additional externalities arising from the dependency of the gains to firm creation on the value of unemployment. In particular the social planner weighs the additional effects on currently operating entrepreneurs and those deciding between entrepreneurship or wage work, of the endogenous choice of agents to search for a job or an entrepreneurial venture.

5 Conclusion

We forgo of the traditional free entry condition by proposing a more realistic framework in which individuals are constantly making the decision whether or not to open a firm. We endogenize firm entry as a dynamic entry process where both workers and business owners are drawn from the same population. We do so by allowing ex-ante individuals to search for either a job or an idea. In the stylized framework considered here the threat point of the entrepreneur and the worker become one and the same. This generates a novel interplay between the bargaining parameter and the direction of wage changes to any feasible shock.

In deriving the planners solution to the model we find that unemployment effects are either muted or intensified in response to a shock at the social optimum. This mechanism operates through wages and contrasts with the standard search framework where wages serve to only dampen unemployment fluctuations. This result is particularly interesting given that the Hosios condition takes the same form.

A Appendix

Proof of Theorem 1. To get this expression first evaluate $V(\epsilon)$ at $\epsilon = \underline{\epsilon}$ and using $U = V(\underline{\epsilon})$ to obtain the following expression for U

$$U = \frac{-c(r+\lambda) + q(\theta)(1-\beta)(\underline{\epsilon})}{r(r+2q(\theta)(1-\beta)+\lambda)}$$

Now using the condition that individuals must be indifferent between searching for a job or for an idea we know that

$$rU = b + p(\theta)\int (W(\epsilon^*) - U)d\mu(\epsilon^*)$$

Replacing $W(\epsilon^*) - U = \frac{w-rU}{r+\lambda} = \frac{\beta(\epsilon)-2\beta rU}{r+\lambda}$, $\mu(\epsilon) = \frac{f(\epsilon)}{1-F(\epsilon)}$ and $p(\theta) = \psi(1 - F(\underline{\epsilon}))$ gives

$$rU = b + \psi \int_{\underline{\epsilon}} \frac{\beta(\epsilon) - 2\beta rU}{r+\lambda} f(\epsilon)d\epsilon$$

$$U = \frac{b(r+\lambda) + \beta\psi \int_{\underline{\epsilon}}(\epsilon^*)f(\epsilon^*)d\epsilon^*}{r(r+\lambda+2\beta\psi(1-F(\underline{\epsilon})))}$$

Setting both expressions for U equal we get the desired expression. This expression can also be written as

$$\begin{aligned}(r+\lambda+2\psi\beta(1-F(\underline{\epsilon})))(-c(r+\lambda)+q(\theta)(z+\underline{\epsilon})(1-\beta))& \\ = (r+\lambda+2(1-\beta)q(\theta))(b(r+\lambda)+\beta\psi\int_{\underline{\epsilon}}(\epsilon^*)dF(\epsilon^*))&\end{aligned}$$

To see the expression implies a positive relationship between θ and $\underline{\epsilon}$ totally differentiate with respect to both getting

$$[-\beta\psi(r+\lambda)f(\underline{\epsilon})\underline{\epsilon} - \beta\psi 2(1-\beta)q(\theta)\underline{\epsilon}f(\underline{\epsilon}) - f(\underline{\epsilon})c2\beta(r+\lambda)$$
$$- q(\theta)(1-\beta)(r+\lambda) - 2\beta(1-F(\underline{\epsilon}))q(\theta)(1-\beta) + \beta\psi 2(1-\beta)q(\theta)\underline{\epsilon}f(\underline{\epsilon})]d\underline{\epsilon}$$
$$= [q'(\theta)(1-\beta)(r+\lambda)[\underline{\epsilon} - 2b] + 2\beta(1-\beta)q'(\theta)\psi\int_{\underline{\epsilon}}(\epsilon^* - \underline{\epsilon})dF(\epsilon^*)]d\theta$$

Note that $\underline{\epsilon} - 2b > 0$ in equilibrium otherwise for the marginal entrepreneur both parties would be better if the match separated. This would contradict with the individuals initial decision to become an entrepreneur.

It follows that both sides of the expression above are negative implying a positive relationship between θ and $\underline{\epsilon}$. □

Proof of Theorem 2. To show the result, totally differentiate the Entrepreneurship equation with respect to $\underline{\epsilon}$ and b holding θ constant to obtain

$$[-\beta\psi(r+\lambda)f(\underline{\epsilon})\underline{\epsilon} - f(\underline{\epsilon})c2\beta(r+\lambda) - q(\theta)(1-\beta)(r+\lambda) - 2\beta(1-F(\underline{\epsilon}))q(\theta)(1-\beta)]d\underline{\epsilon}$$
$$= -(r+\lambda+2(1-\beta)q(\theta))(r+\lambda)db$$

From there we see that $\underline{\epsilon}$ will increase for all θ levels. Since the Job Creation curve does not shift, it follows that θ will decrease and $\underline{\epsilon}$ will increase. To see that aggregate productivity increases remember that

$$\mu(\epsilon) = \frac{f(\epsilon)}{1-F(\underline{\epsilon})}$$

It follows aggregate productivity can be written as

$$\int_{\underline{\epsilon}} \frac{\epsilon f(\epsilon)}{1-F(\underline{\epsilon})}dz$$

$\square$

Proof of Theorem 3. To show the result, totally differentiate the Entrepreneurship equation with respect to $\underline{\epsilon}$ and c holding θ constant to obtain

$$[-\beta\psi(r+\lambda)f(\underline{\epsilon})\underline{\epsilon} - f(\underline{\epsilon})c2\beta(r+\lambda) - q(\theta)(1-\beta)(r+\lambda) - 2\beta(1-F(\underline{\epsilon}))q(\theta)(1-\beta)]d\underline{\epsilon}$$

$$= -(r+\lambda+2\beta\psi(1-F(\underline{\epsilon})))(r+\lambda)dc$$

From there we see that $\underline{\epsilon}$ will increase for all θ levels. Since the Job Creation curve does not shift, it follows that θ will decrease and $\underline{\epsilon}$ will increase. To see that aggregate productivity increases remember that

$$\mu(\epsilon) = \frac{f(\epsilon)}{1-F(\underline{\epsilon})}$$

It follows aggregate productivity can be written as

$$\int_{\underline{\epsilon}} \frac{\epsilon f(\epsilon)}{1-F(\underline{\epsilon})} dz$$

Since $\underline{\epsilon}$ increases with c, average firm productivity will increase following the shock.

$\square$

Proof of Theorem 4. The law of motion for match surplus of average productivity

$$(10)\ \ rS(\bar{\epsilon}) = \frac{\xi\underline{\epsilon}}{\xi-1} - b - ((\delta\int_{\underline{\epsilon}} p(\theta)\beta S(\epsilon)\frac{v(\epsilon)}{v}d\epsilon) + (1-\delta)(\psi\int_{\underline{\epsilon}}(V(\epsilon)-U)dF(\epsilon))) - \lambda S(\bar{\epsilon}) + \dot{S}(\bar{\epsilon})$$

where $\frac{\xi\underline{\epsilon}}{\xi-1} = \int_{\underline{\epsilon}} \frac{\epsilon f(\epsilon)}{1-F(\underline{\epsilon})} d\epsilon$.

$\int_{\underline{\epsilon}} p(\theta)\beta S(\epsilon)\frac{v(\epsilon)}{v}d\epsilon$ is the private opportunity cost of it unemployed always transition back to wage work from unemployment. Note that $\beta S(\epsilon) = W(\epsilon) - U$. Similarly, $\int_{\underline{\epsilon}}(V(\epsilon)-U)dF(\epsilon))$ is the private opportunity cost if unemployed always transition to business ownership from unemployment. δ is the fraction of unemployed that search for a job. In equilibrium, individuals are indifferent whether to search for a job or a business opportunity and so we can rewrite the condition above as

$$(11) \qquad rS(\bar{\epsilon}) = \frac{\xi \underline{\epsilon}}{\xi - 1} - b - \left(\int_{\underline{\epsilon}} p(\theta)\beta S(\epsilon)\frac{v(\epsilon)}{v}d\epsilon\right) - \lambda S(\bar{\epsilon}) + \dot{S}(\bar{\epsilon})$$

Finally using the fact that $S(\epsilon)$ is linear in ϵ, we can rewrite it as

$$(12) \qquad rS(\bar{\epsilon}) = \frac{\xi \underline{\epsilon}}{\xi - 1} - b - [\lambda + p(\theta)\beta]S(\bar{\epsilon})) + \dot{S}(\bar{\epsilon})$$

From the Job Creation Curve,using the expression for $rV(\underline{\epsilon})$

$$(13) \qquad J(\underline{\epsilon}) = \frac{rU + c}{q(\theta)} + U$$

which gives

$$(14) \qquad c = q(\theta)(1 - \beta)S(\underline{\epsilon}) - rU$$

The social planner maximizes total welfare

$$(15) \qquad \max_{v} \int_0^\infty e^{-rt}\left[\frac{\xi \underline{\epsilon}(\theta)}{\xi - 1}(1 - u - v) + ub - cv\right]dt$$

$$(16) \qquad s.t. : \dot{u} = \lambda(1 - u - v) - \left(\frac{v}{\frac{u}{2}}\right)^{\alpha} u$$

where $\theta \equiv \frac{v}{\frac{u}{2}}$

Setting up the hamiltonean and taking FOCs (using $\underline{\epsilon} = \psi^{\frac{1}{\xi}}\theta^{-\frac{\alpha}{\xi}}$) gives

$$(17) \qquad c = \frac{\xi}{\xi - 1}\left(-\frac{\alpha}{\xi}\right)\psi^{\frac{1}{\xi}}\theta^{-\frac{\alpha}{\xi}-1}\frac{1}{\frac{u}{2}}(1 - u - v) - \frac{\xi \underline{\epsilon}}{\xi - 1} - \mu_1[\lambda + p'(\theta)] \quad \text{(optimality for v)}$$

and

$$r\mu_1 - \dot{\mu}_1 = \frac{-\xi}{\xi - 1}\underline{\epsilon} + b - \lambda\mu_1 - (1-\alpha)(\frac{v}{\frac{u}{2}})^{\alpha} \quad \text{(optimality for u)} \tag{18}$$

Now define $\pi = -\mu_1$, and rewrite both conditions as

$$r\pi = \frac{\xi}{\xi - 1}\underline{\epsilon} - b - \pi[\lambda + (1-\alpha)p(\theta)] + \dot{\pi} \tag{19}$$

and

$$c = \frac{\xi}{\xi - 1}(\frac{-\underline{\epsilon}(1-u)}{v}) + \pi[\lambda + p'(\theta)] \tag{20}$$

For a given value of π the two equations together with the expressions for $S(\bar{\epsilon})$ and $\underline{\epsilon} = \psi^{\frac{1}{\xi}}\theta^{-\frac{\alpha}{\xi}}$ allow us to solve for θ and β. π is the additional benefit of increasing marginally the measure of non-unemployed in the economy. Note that in equilibrium $\pi = S(\bar{\epsilon})$.

Then comparing equation (19) and equation (12) gives

$$[\lambda + p(\theta)\beta] = [\lambda + (1-\alpha)p(\theta)] \tag{21}$$

which implies

$$\beta = (1-\alpha) \tag{22}$$

□

B Deriving the Endogenous Productivity Distribution

The law of motion for the measure of vacancies of a particular productivity ϵ is

$$\dot{v}(\epsilon) = \psi(1-\gamma)uf(\epsilon) - q(\theta)v(\epsilon) \quad \forall\epsilon \geq \underline{\epsilon}$$

$$v(\epsilon) = 0 \quad \forall\epsilon < \underline{\epsilon}$$

Setting the expression above to zero implies

$$q(\theta)v(\epsilon) = \psi(1-\gamma)uf(\epsilon)$$

Integrating with respect to ϵ and dividing the integrated expression in the equation above yields

$$\mu(\epsilon) = \frac{v(\epsilon)}{v} = \frac{f(\epsilon)}{1 - F(\underline{\epsilon})}$$

This equation states that the distribution of enacted ideas, or firm productivity in the economy, is fully characterized by the underlying distribution of feasible ideas and the threshold rule for entering entrepreneurship.

C Deriving the Job Creation Curve

For the remainder of our analysis we will consider the economy to be in steady state. The law of motion of the measure of jobs in the economy is given by

$$\dot{n} = p(\theta)\gamma u - \lambda n$$

The law of motion of the measure of total entrepreneurs in this economy is given by

$$\dot{e} = \psi(1 - F(\underline{\epsilon}))(1-\gamma)u - \lambda n$$

Setting these two to zero implies that

$$p(\theta) = \frac{\psi(1 - F(\underline{\epsilon}))(1 - \gamma)}{\gamma}$$

The law of motion of the total unemployed looking for a job in this economy is

$$\dot{u}^w = \gamma 2\lambda n - p(\theta)\gamma u$$

and that for those looking for an idea is

$$\dot{u}^v = (1 - \gamma)2\lambda n - (1 - \gamma)u\psi(1 - F(\underline{\epsilon}))$$

Setting these two to zero implies that

$$p(\theta) = \psi(1 - F(\underline{\epsilon}))$$

References

ACEMOGLU, D. AND R. SHIMER, "Holdups and Efficiency with Search Frictions," *International Economic Review* 40 (1999), 827–849.

ALBRECHT, J., L. NAVARRO AND S. VROMAN, "Efficiency in a search and matching model with endogenous participation," *Economics Letters* 106 (2010), 48–50.

CAGETTI, M., M. DE NARDI ET AL., "Entrepreneurship, frictions, and wealth," *Journal of political Economy* 114 (2006), 835–870.

GAVREL, F., "On the efficiency of participation with vertically differentiated workers," *Economics Letters* 112 (2011), 100–102.

HALTIWANGER, J., R. S. JARMIN AND J. MIRANDA, "Who creates jobs? Small versus large versus young," *Review of Economics and Statistics* 95 (2013), 347–361.

HAMILTON, B. H., "Does entrepreneurship pay? An empirical analysis of the returns to self-employment," *Journal of Political economy* 108 (2000), 604–631.

HOSIOS, A. J., "On the efficiency of matching and related models of search and unemployment," *The Review of Economic Studies* 57 (1990), 279–298.

HUMPHRIES, J. E., "The Causes and Consequences of Self-Employment over the Life Cycle," Technical Report, Mimeo, University of Chicago, 2016.

JULIEN, B., S. MANGIN ET AL., "Efficiency in search and matching models: a generalized Hosios condition," *Unpublished manuscript. Retrieved from¡ http://www. sephorahmangin. info/current_research/working_papers/GeneralizedHosiosCondition. pdf* (2016).

MOREIRA, S., "Firm Dynamics, Persistent Effects of Entry Conditions, and Business Cycles," (2015).

Mortensen, D. T. and C. A. Pissarides, "Job creation and job destruction in the theory of unemployment," *The review of economic studies* 61 (1994), 397–415.

Pissarides, C. A., "Short-run equilibrium dynamics of unemployment, vacancies, and real wages," *The American Economic Review* (1985), 676–690.

———, *Equilibrium unemployment theory* (MIT press, 2000).

Poschke, M., "Who becomes an entrepreneur? Labor market prospects and occupational choice," *Journal of Economic Dynamics and Control* 37 (2013), 693–710.

Quadrini, V., "Entrepreneurship, saving, and social mobility," *Review of Economic Dynamics* 3 (2000), 1–40.

Sedlácek, P. and V. Sterk, "The growth potential of startups over the business cycle," (2014).