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Abstract. This paper reviews the welfarist approach to population ethics. We provide an overview of the critical-level utilitarian population principles and their generalized counterparts, examine important properties of these principles and discuss their relationships to other variable-population social-evaluation rules. We illustrate the difficulties arising in population ethics by means of an impossibility result and present characterizations of the critical-level generalized-utilitarian principles and of three of their sub-classes. *Journal of Economic Literature* Classification Number: D63.

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1. Introduction

The traditional framework of social-choice theory as initiated by Arrow [1951, 1963] addresses the issue of aggregating profiles of individual preference relations into a social preference relation. One way of escape from the negative conclusion of his impossibility theorem consists of expanding the informational base of collective choice by assuming that individual preferences are represented by utility functions and allowing for interpersonal comparisons of utility, thereby moving away from the narrow confines of Arrow's assumption of ordinal measurability and interpersonal non-comparability; see, for instance, d'Aspremont and Gevers [1977], Hammond [1979] and Sen [1977] for possibility results and characterizations under various informational assumptions. An extensive survey of the literature on social choice with interpersonal utility comparisons is provided by Bossert and Weymark [2004].

Most of the literature on social-choice theory treats the population as fixed, and the notion of variable-population social evaluation has its origins in applied ethics. In particular, Parfit [1976, 1982, 1984] is usually credited with introducing the axiomatic approach to populations ethics, and his contribution continues to be one of the most influential in the area; see, for instance, Ryberg and Tännsjö [2004]. The approach we follow is welfarist: in order to compare any two alternatives (whose populations may differ), the only information required consists of the sets those alive in the respective alternatives and their lifetime utilities. The extension of fixed-population social-evaluation methods to a variable-population context is important because so many public-policy decisions involve endogenous population. For instance, when determining public spending on pre-natal care, on foreign-aid packages with population consequences or on intergenerational resource allocation, the assumption that the population is fixed is difficult to justify. Therefore, more comprehensive criteria are called for.

Following the usual convention in population ethics, we assume that utilities represent individual lifetime well-being and are normalized so that a lifetime utility of zero represents *neutrality.* Above neutrality, a life, as a whole, is worth living; below neutrality, it is not. From the viewpoint of an individual, a neutral life is a life which is as good as one in which the person has no experiences; see, for instance, Broome [1993, 2004, ch. 8], Heyd [1992, ch. 1], McMahan [1996] and Parfit [1984, Appendix G] for discussions. People who do not exist do not have interests or preferences and, therefore, we take the view that it is not possible to say that an individual can gain (or lose) by being brought into existence with a utility level above (or below) neutrality. Someone who is alive might have an attitude, such as a desire or preference, toward a world in which the person does not exist but that attitude could hardly be construed as individual betterness or worseness. Similarly, a person who is alive and expresses satisfaction with her or his existence (that is, with having been born) cannot be claiming that existence is better (for him or her) than nonexistence. Note that this does not prevent an individual from gaining or losing by continuing to live—the continuation or termination of life is a matter of length of life, not existence itself.

A commonly-used principle is *classical utilitarianism,* also referred to as *total utilitarianism.* It ranks any two alternatives by comparing the total utilities of the individuals alive in them. Parfit [1976, 1982, 1984, ch. 19] observed that classical utilitarianism leads to the *repugnant conclusion.* A population principle implies the repugnant conclusion if every alternative in which everyone alive experiences a utility level above neutrality is ranked as worse than an alternative in which each member of a larger population has a utility level that is above neutrality but may be arbitrarily close to it. This means that population size can always be used to substitute for quality of life as long as lives are (possibly barely) worth living. As Parfit's analysis demonstrates, the repugnant conclusion is implied by any population principle that: (i) declares the ceteris-paribus addition of an individual above neutrality to a given population to be a social improvement; (ii) ranks any alternative with an equal utility distribution as at least as good as any alternative involving the same population, the same total utility but an unequal distribution of well-being; and (iii) ranks same-population equal-utility alternatives by declaring that with a higher common utility level to be better.

For any alternative, the *critical level* of utility is that level which, if experienced by an added person without changing the utilities of the existing population, leads to an alternative which is as good as the original. Clearly, the choice of critical levels has important consequences for the properties of a population principle and is closely linked to the possibility of avoiding the repugnant conclusion.

Average utilitarianism uses average rather than total utility to rank alternatives. It does not imply the repugnant conclusion but has other defects, such as declaring the ceteris-paribus addition of an individual with a lifetime utility well below neutrality desirable as long as the existing population's average utility is even lower. Thus, other population principles are called for, and avoidance of the repugnant conclusion has become an important criterion that acceptable principles should satisfy. We believe that the *critical-level utilitarian* principles with positive critical levels and their generalized counterparts are the most satisfactory; see Blackorby, Bossert and Donaldson [2005a] and Blackorby and Donaldson [1984]. Critical-level utilitarianism is a one-parameter family of principles. The parameter is a *fixed* critical level of utility that applies to all alternatives, and the criterion used to rank the alternatives is the sum of the differences between individual utilities and the critical level. Critical-level generalized utilitarianism uses transformed utilities, thereby allowing for inequality aversion in individual well-being: if the transformation is (strictly) concave, the resulting principle is (strictly) inequality-averse.

Due to space limitations, we cannot go beyond a brief introduction to the subject and refer the reader to Blackorby, Bossert and Donaldson [2005a] for an extensive treatment. We focus on critical-level generalized-utilitarian principles because we consider those with positive critical levels to be the most suitable for social evaluation. In addition to characterizing these and other critical-level principles, we discuss an impossibility result as an example for the dilemmas and conflicts that arise in population ethics.

Section 2 introduces a welfarist and anonymous approach to population ethics, along with the population principles that are of interest in this survey. Section 3 illustrates the dilemmas that arise in populations ethics by means of an impossibility result. In Section 4, we provide a characterization of critical-level generalized utilitarianism and three of its sub-classes. Some issues that are not addressed in the previous sections are discussed briefly in Section 5. Section 6 concludes.

2. Variable-population anonymous welfarism

We use $\mathcal{R}$ to denote the set of all real numbers, and $\mathcal{Z}_{++}$ is the set of positive integers. $\mathbf{1}_n$ is the vector consisting of $n \in \mathcal{Z}_{++}$ ones. Suppose there is a set of alternatives X. Each element $x \in X$ is a full description of all relevant aspects of the world, including the identities of everyone alive in x and everything that may affect a person's lifetime well-being. We assume that, for each possible (finite but arbitrarily large) population, there are at least three alternatives with that population. Potential individuals are indexed by positive integers and, for an individual $i \in \mathcal{Z}_{++}$, X_i is the subset of X consisting of all alternatives x such that i is alive in x. An individual utility function for i is a mapping $U_i\colon X_i \to \mathcal{R}$, interpreted as an indicator of lifetime well-being. Thus, for $x \in X_i$, $U_i(x)$ is i's lifetime utility in alternatives x. Note that the domain of U_i is X_i and, therefore, i's well-being is only defined for alternatives in which the person exists. A profile of utility functions is an infinite-dimensional vector $U = (U_1, U_2, \ldots, U_i, \ldots)$ containing one utility function for each potential person.

A social-evaluation functional assigns a social ordering of the alternatives to each possible profile. An ordering is a reflexive, complete and transitive binary relation, and the social ordering is interpreted as a goodness relation. Because we employ a welfarist approach, a single social-evaluation ordering R defined on the set $\Omega = \cup_{n \in \mathcal{Z}_{++}} \mathcal{R}^n$ of all utility vectors is sufficient to perform all comparisons for any profile under consideration. The asymmetric factor of R is denoted by P and I is the symmetric factor of R. The relation R is interpreted as an at-least-as-good-as-relation and P and I are the corresponding better-than and as-good-as relations. According to welfarism, an alternative $x \in X$ is at least as good as an alternative $y \in X$ given the profile U if and only if the utility vector $U(x)$ is at least as good as the utility vector $U(y)$ according to R. Thus, knowledge of those alive in two alternatives and their lifetime utilities is sufficient to rank any two alternatives for any profile. We refer the reader to Blackorby, Bossert and Donaldson [2005a, ch. 3] for the details of obtaining R in a variable-population setting. Fixed-population versions of the welfarism theorem are discussed in Blackorby, Bossert and Donaldson [2005b], Blau [1976], Bossert and Weymark [2004], d'Aspremont and Gevers [1977], Guha [1972], Hammond [1979] and Sen [1977], for instance. In addition, we assume social evaluation to be anonymous in the sense that the identities of the individuals are irrelevant in ranking alternatives—only the lifetime utilities achieved in two alternatives can influence the social ranking of the two. Thus, R is assumed to be such that uIv for all $n \in \mathcal{Z}_{++}$ and for all $u, v \in \mathcal{R}^n$ such that u is a permutation of v. Because of this anonymity property, we can without loss of generality assume that if there are $n \in \mathcal{Z}_{++}$ individuals alive, they are the individuals labelled from 1 to n.

We conclude this section by introducing the population principles that are of particular interest in this paper. For the definition of these principles, it is important to keep in mind that neutrality is normalized to a lifetime-utility level of zero. For other normalizations, the formulation of the principles has to be amended accordingly; see, for instance, Dasgupta [1993] who, somewhat unconventionally, uses a negative utility level to represent neutrality.

The first principle we define is classical utilitarianism, which ranks utility vectors (and, thus, alternatives) on the basis of the total utilities obtained in them. According to classical utilitarianism,

$$uRv \;\Leftrightarrow\; \sum_{i=1}^{n} u_i \geq \sum_{i=1}^{m} v_i$$

for all $n, m \in \mathcal{Z}_{++}$, for all $u \in \mathcal{R}^n$ and for all $v \in \mathcal{R}^m$.

Average utilitarianism employs average utilities instead of total utilities for social evaluation. Thus, the average-utilitarian principle is defined by

$$uRv \;\Leftrightarrow\; \frac{1}{n}\sum_{i=1}^{n} u_i \geq \frac{1}{m}\sum_{i=1}^{m} v_i$$

for all $n, m \in \mathcal{Z}_{++}$, for all $u \in \mathcal{R}^n$ and for all $v \in \mathcal{R}^m$.

Critical-level utilitarianism with a parameter value of $\alpha \in \mathcal{R}$ generalizes classical utilitarianism by replacing utilities with the differences between utilities and the critical-level parameter. This leads to the principle defined by

$$uRv \;\Leftrightarrow\; \sum_{i=1}^{n} [u_i - \alpha] \geq \sum_{i=1}^{m} [v_i - \alpha]$$

for all $n, m \in \mathcal{Z}_{++}$, for all $u \in \mathcal{R}^n$ and for all $v \in \mathcal{R}^m$. Clearly, classical utilitarianism is obtained for $\alpha = 0$.

All of the above principles produce identical fixed-population comparisons, namely, those corresponding to utilitarianism.

Generalizations are obtained by replacing utility levels (including critical levels) with transformed utilities. Letting $g\colon \mathcal{R} \to \mathcal{R}$ be a continuous and increasing function such that $g(0) = 0$, the classical generalized-utilitarian principle corresponding to g is defined by

$$uRv \;\Leftrightarrow\; \sum_{i=1}^{n} g(u_i) \geq \sum_{i=1}^{m} g(v_i)$$

for all $n, m \in \mathcal{Z}_{++}$, for all $u \in \mathcal{R}^n$ and for all $v \in \mathcal{R}^m$, average generalized utilitarianism is given by

$$uRv \;\Leftrightarrow\; \frac{1}{n}\sum_{i=1}^{n} g(u_i) \geq \frac{1}{m}\sum_{i=1}^{m} g(v_i)$$

for all $n, m \in \mathcal{Z}_{++}$, for all $u \in \mathcal{R}^n$ and for all $v \in \mathcal{R}^m$, and the critical-level generalized-utilitarian principle for g is

$$uRv \;\Leftrightarrow\; \sum_{i=1}^{n} [g(u_i) - g(\alpha)] \geq \sum_{i=1}^{m} [g(v_i) - g(\alpha)]$$

for all $n, m \in \mathcal{Z}_{++}$, for all $u \in \mathcal{R}^n$ and for all $v \in \mathcal{R}^m$.

Again, fixed-population comparisons are the same for all of the generalized principles—they reduce to those according to generalized utilitarianism. If g is (strictly) concave, the resulting principle is (strictly) inequality-averse with respect to lifetime well-being.

3. An impossibility result

There are numerous impossibility results in the population-ethics literature that establish the incompatibility of seemingly plausible axioms. The purpose of this section is to illustrate this observation by means of an impossibility theorem (Blackorby, Bossert and Donaldson [2006a]). The axioms that follow are employed.

A weakening of the well-known weak-Pareto principle is obtained if social betterness is required whenever one equal utility distribution strictly dominates another equal distribution. We call this axiom minimal increasingness; it is satisfied by all population principles introduced in Section 2.

Minimal increasingness: For all $n \in \mathcal{Z}_{++}$ and for all $a, b \in \mathcal{R}$, if $a > b$, then $a\mathbf{1}_n P b\mathbf{1}_n$.

Weak inequality aversion requires that, for any given population and any given total utility, the equal distribution is at least as good as any unequal distribution with the same total utility. The axiom is satisfied by all of the generalized principles (critical-level utilitarian as well as average) associated with a concave transformation g.

Weak inequality aversion: For all $n \in \mathcal{Z}_{++}$ and for all $u \in \mathcal{R}^n$, $\big((1/n)\sum_{i=1}^n u_i\big)\mathbf{1}_n R u$.

Sikora [1978] suggests extending the standard Pareto principle to non-existing individuals, an axiom he refers to as the Pareto-plus principle. It is usually defined as the conjunction of strong Pareto (defined formally in the following section) and the requirement that the addition of an individual above neutrality to a utility-unaffected population is a social improvement. Because the full force of strong Pareto is not required (our impossibility theorem stated below merely assumes minimal increasingness), we retain strong Pareto as a separate axiom and define Pareto plus as follows.

Pareto plus: For all $n \in \mathcal{Z}_{++}$, for all $u \in \mathcal{R}^n$ and for all $a > 0$, $(u, a)Pu$.

In the axiom statement, the common population in u and (u, a) is unaffected. To defend the axiom, therefore, it must be argued that a life above neutrality is better than non-existence. While it is possible to compare alternatives with different populations from a social point of view (which is, after all, the fundamental issue addressed in population ethics), such comparisons are implausible if made from the viewpoint of an individual if the person is not alive in all alternatives to be compared. It is therefore difficult to interpret this axiom as a Pareto condition because it appears to be based on the idea that people who do not exist have interests that should be respected. For that reason, we do not consider Pareto plus to be very compelling. Pareto plus is satisfied by all critical-level generalized-utilitarian principles with non-positive critical levels. Average generalized utilitarianism and critical-level generalized utilitarianism with positive critical levels do not possess this property.

We follow Parfit [1984] in considering the repugnant conclusion an unacceptable property of a population principle. Thus, our final axiom requires that this conclusion be avoided. A population principle implies the repugnant conclusion if and only if, for any population size $n \in \mathcal{Z}_{++}$, any positive utility level ξ and any utility level $\varepsilon \in (0, \xi)$, there exists a population size $m > n$ such that an m-person alternative in which every individual experiences utility level ε is ranked as better than an n-person society in which every individual's utility level is ξ. The axiom that requires the repugnant conclusion to be avoided is defined as follows.

Avoidance of the repugnant conclusion: There exist $n \in \mathcal{Z}_{++}$, $\xi \in \mathcal{R}_{++}$ and $\varepsilon \in (0, \xi)$ such that, for all $m > n$, $\xi\mathbf{1}_n R \varepsilon\mathbf{1}_m$.

As is straightforward to verify, critical-level generalized utilitarianism satisfies Pareto plus if and only if the critical level α is non-positive and satisfies avoidance of the repugnant conclusion if and only if α is positive. Thus, no critical-level generalized-utilitarian principle can satisfy Pareto plus and, at the same time, avoid the repugnant conclusion.

However, this incompatibility extends well beyond these principles. As an illustration, we reproduce an impossibility theorem due to Blackorby, Bossert and Donaldson [2006a]. In particular, we show that all minimally increasing and weakly inequality-averse population principles that satisfy Pareto plus lead to the repugnant conclusion. Similar theorems can be found in Arrhenius [2000], Blackorby, Bossert and Donaldson [2005a], Blackorby, Bossert, Donaldson and Fleurbaey [1998], Blackorby and Donaldson [1991], Carlson [1998] and Ng [1989], for instance.

Theorem 1: *There exists no anonymous social-evaluation ordering R that satisfies minimal increasingness, weak inequality aversion, Pareto plus and avoidance of the repugnant conclusion.*

Proof. Suppose R satisfies minimal increasingness, weak inequality aversion and Pareto plus. We complete the proof by showing that R must imply the repugnant conclusion. For any population size $n \in \mathcal{Z}_{++}$, let ξ, ε and δ be utility levels such that $0 < \delta < \varepsilon < \xi$. Choose the positive integer r such that

$$r > n\frac{(\xi - \varepsilon)}{(\varepsilon - \delta)}. \tag{1}$$

Because the numerator and the denominator of the right side of this inequality are both positive, r is positive. By Pareto plus, $(\xi\mathbf{1}_n, \delta\mathbf{1}_r)P\xi\mathbf{1}_n$. Average utility in $(\xi\mathbf{1}_n, \delta\mathbf{1}_r)$ is $(n\xi + r\delta)/(n + r)$ so, by minimal inequality aversion, $[(n\xi + r\delta)/(n + r)]\mathbf{1}_{n+r}R(\xi\mathbf{1}_n, \delta\mathbf{1}_r)$. By (1),

$$\varepsilon > \frac{n\xi + r\delta}{n + r}$$

and, by minimal increasingness, $\varepsilon\mathbf{1}_{n+r}P[(n\xi + r\delta)/(n + r)]\mathbf{1}_{n+r}$. Using transitivity, it follows that $\varepsilon\mathbf{1}_{n+r}P\xi\mathbf{1}_n$ and, letting $m = n+r > n$, avoidance of the repugnant conclusion is violated. ■

If weak inequality aversion is dropped from the list of axioms in the theorem, the remaining axioms are compatible. For example, *geometrism,* a principle proposed by Sider [1991], satisfies all axioms other than weak inequality aversion. The principle uses a constant $k \in (0, 1)$ between zero and one and ranks alternatives with a weighted sum of utilities, where the weights are such that the j^{th}-highest non-negative utility level receives a weight of k^{j-1} and the ℓ^{th}-lowest negative utility receives a weight of $k^{\ell-1}$. Critical levels are all zero and the repugnant conclusion is avoided but, because weights on higher positive utilities exceed weights on lower ones, the principle prefers inequality of positive utilities over equality (see Arrhenius and Bykvist [1995]).

4. A characterization of critical-level generalized utilitarianism

Critical-level generalized utilitarianism can be characterized by means of a set of plausible and intuitively appealing axioms. The first of these applies to fixed-population comparisons only. It is the well-known strong-Pareto requirement which demands that unanimity be respected.

Strong Pareto: For all $n \in \mathcal{Z}_{++}$ and for all $u, v \in \mathcal{R}^n$, if $u_i \geq v_i$ for all $i \in \{1, \ldots, n\}$ with at least one strict inequality, then uPv.

The standard definition of strong Pareto encompasses Pareto indifference, requiring that if everyone in a fixed population has the same level of well-being in two alternatives, the two should be ranked as equally good. In our welfarist framework, this property is automatically satisfied because the relation R is reflexive.

Our second axiom is another fixed-population requirement. Continuity requires that small changes in utilities should not lead to large changes in the social ranking.

Continuity: For all $n \in \mathcal{Z}_{++}$ and for all $u \in \mathcal{R}^n$, the sets $\{v \in \mathcal{R}^n \mid vRu\}$ and $\{v \in \mathcal{R}^n \mid uRv\}$ are closed in $\mathcal{R}^n$.

Existence of a critical level is an axiom regarding the comparison of alternatives with different population sizes, ensuring that at least some non-trivial trade-offs between population size and well-being are possible. It requires the existence of a critical level for at least one utility vector. Critical levels need not exist for other utility vectors and if they do, they need not be constant. Thus, the axiom is very weak.

Existence of a critical level: There exists $\bar{u} \in \Omega$ and $c \in \mathcal{R}$ such that $(\bar{u}, c)I\bar{u}$.

Strong Pareto, continuity and existence of a critical level are satisfied by all of the principles introduced in Section 2. In contrast, the final axiom used in our characterization is satisfied by all critical-level generalized-utilitarian principles but violated by average utilitarianism and its generalized counterpart. Existence independence is a separability axiom that applies not only to fixed-population comparisons but also to those involving different populations. It requires the social ranking to be independent of the existence of the unconcerned—individuals who are not affected by the ranking of two alternatives. See d'Aspremont and Gevers [1977], for example, for a fixed-population version of this independence property.

Existence independence: For all $u, v, w \in \Omega$, $(u, w)R(v, w) \Leftrightarrow uRv$.

These axioms characterize critical-level generalized utilitarianism, as established in the following theorem.

Theorem 2: *An anonymous social-evaluation ordering R satisfies strong Pareto, continuity, existence of a critical level and existence independence if and only if R is critical-level generalized-utilitarian.*

Proof. That critical-level generalized utilitarianism satisfies the required axioms is straightforward to verify. To prove the reverse implication, consider first the case of a fixed population size $n \geq 3$. Applying Debreu's [1959, pp. 56–59] representation theorem, continuity implies the existence of a continuous function $f^n: \mathcal{R}^n \to \mathcal{R}$ such that, for all $u, v \in \mathcal{R}^n$,

$$uRv \;\Leftrightarrow\; f^n(u) \geq f^n(v).$$

By strong Pareto, f^n is increasing in all arguments, and the anonymity of R implies that f^n is symmetric. Existence independence implies that $\{1, \ldots, n\} \setminus M$ is separable in f^n from its complement M for any choice of M such that $\emptyset \neq M \subset \{1, \ldots, n\}$. Gorman's [1968] theorem on overlapping separable sets of variables (see also Aczél [1966, p. 312] and Blackorby, Primont and Russell [1978, p. 127]) implies that f^n is additively separable. Therefore, there exist continuous and increasing functions $H^n: \mathcal{R} \to \mathcal{R}$ and $g_i^n: \mathcal{R} \to \mathcal{R}$ for all $i \in \{1, \ldots, n\}$ such that

$$f^n(u) = H^n\left(\sum_{i=1}^{n} g_i^n(u_i)\right)$$

for all $u \in \mathcal{R}^n$. Because f^n is symmetric, each g_i^n can be chosen to be independent of i, and we define $g^n = g_i^n$ for all $i \in \{1, \ldots, n\}$. Therefore, because f^n is a representation of the restriction of R to $\mathcal{R}^n$ and H^n is increasing,

$$\begin{aligned} uRv \Leftrightarrow\;& H^n\left(\sum_{i=1}^{n} g^n(u_i)\right) \geq H^n\left(\sum_{i=1}^{n} g^n(v_i)\right) \\ \Leftrightarrow\;& \sum_{i=1}^{n} g^n(u_i) \geq \sum_{i=1}^{n} g^n(v_i) \end{aligned} \tag{2}$$

for all $u, v \in \mathcal{R}^n$. Without loss of generality, g^n can be chosen so that $g^n(0) = 0$.

Next, we prove that there exists a utility level $\alpha \in \mathcal{R}$ which is a critical level for all utility vectors in Ω. Let $u \in \Omega$ be arbitrary. By existence of a critical level, there exist $\bar{u} \in \Omega$ and $c \in \mathcal{R}$ such that $(\bar{u}, c)I\bar{u}$. Applying existence independence twice, we obtain $(u, \bar{u}, c)I(u, \bar{u})$ and $(u, c)Iu$. Thus, c is a critical level not only for $\bar{u}$ but also for any $u \in \Omega$. Letting $\alpha = c$ establishes the claim.

Now we show that, for all $n \geq 3$, the functions g^n and g^{n+1} can be chosen to be the same. Let $u, v \in \mathcal{R}^n$. Because $\alpha \in \mathcal{R}$ is a critical level for all utility vectors in Ω, we have

$$uRv \;\Leftrightarrow\; (u, \alpha)R(v, \alpha). \tag{3}$$

By (2),

$$uRv \;\Leftrightarrow\; \sum_{i=1}^{n} g^n(u_i) \geq \sum_{i=1}^{n} g^n(v_i) \tag{4}$$

and

$$\begin{aligned}(u,\alpha)R(v,\alpha) &\Leftrightarrow \; \sum_{i=1}^{n} g^{n+1}(u_i) + g^{n+1}(\alpha) \geq \sum_{i=1}^{n} g^{n+1}(v_i) + g^{n+1}(\alpha) \\ &\Leftrightarrow \; \sum_{i=1}^{n} g^{n+1}(u_i) \geq \sum_{i=1}^{n} g^{n+1}(v_i).\end{aligned} \tag{5}$$

Therefore, using (3), (4) and (5),

$$\sum_{i=1}^{n} g^n(u_i) \geq \sum_{i=1}^{n} g^n(v_i) \;\Leftrightarrow\; \sum_{i=1}^{n} g^{n+1}(u_i) \geq \sum_{i=1}^{n} g^{n+1}(v_i),$$

which means that the same function can be used for g^n and for g^{n+1}. Because this is true for all $n \geq 3$, it follows that the functions g^n can be chosen independently of n, and we write $g = g^n$ for all $n \geq 3$. Together with (2), it follows that, for all $n \geq 3$ and for all $u, v \in \mathcal{R}^n$,

$$uRv \;\Leftrightarrow\; \sum_{i=1}^{n} g(u_i) \geq \sum_{i=1}^{n} g(v_i). \tag{6}$$

Next, we prove that (6) must be true for $n \in \{1, 2\}$ as well. Let $u, v \in \mathcal{R}^1$. By strong Pareto and the increasingness of g,

$$uRv \;\Leftrightarrow\; u_1 \geq v_1 \Leftrightarrow g(u_1) \geq g(v_1). \tag{7}$$

If $u, v \in \mathcal{R}^2$, existence independence and (6) together imply

$$\begin{aligned}uRv \;\Leftrightarrow\; (u,\alpha)R(v,\alpha) &\Leftrightarrow \; \sum_{i=1}^{2} g(u_i) + g(\alpha) \geq \sum_{i=1}^{2} g(v_i) + g(\alpha) \\ &\Leftrightarrow \; \sum_{i=1}^{2} g(u_i) \geq \sum_{i=1}^{2} g(v_i).\end{aligned} \tag{8}$$

(6), (7) and (8) imply that all fixed-population comparisons are carried out according to generalized utilitarianism with the same transformation for all population sizes.

To complete the proof, let $n, m \in \mathcal{Z}_{++}$ with $n \neq m$, $u \in \mathcal{R}^n$ and $v \in \mathcal{R}^m$. Suppose $n > m$. By definition of the critical level α,

$$\begin{aligned} uRv &\Leftrightarrow uR(v, \alpha \mathbf{1}_{n-m}) \\ &\Leftrightarrow \sum_{i=1}^{n} g(u_i) \geq \sum_{i=1}^{m} g(v_i) + (n-m)g(\alpha) \\ &\Leftrightarrow \sum_{i=1}^{n} [g(u_i) - g(\alpha)] \geq \sum_{i=1}^{m} [g(v_i) - g(\alpha)] \end{aligned}.$$

An analogous argument applies to the case $n < m$ and it follows that R is critical-level generalized-utilitarian. ■

As mentioned earlier, adding Pareto plus (respectively avoidance of the repugnant conclusion) to the axioms of Theorem 2 leads to a characterization of the sub-class of critical-level generalized-utilitarian principles the members of which have a non-positive (respectively positive) critical level. These observations are summarized in the following two theorems.

Theorem 3: *An anonymous social-evaluation ordering R satisfies strong Pareto, continuity, existence of a critical level, existence independence and Pareto plus if and only if R is critical-level generalized-utilitarian with a non-positive critical-level parameter.*

Theorem 4: *An anonymous social-evaluation ordering R satisfies strong Pareto, continuity, existence of a critical level, existence independence and avoidance of the repugnant conclusion if and only if R is critical-level generalized-utilitarian with a positive critical-level parameter.*

Because we consider the repugnant conclusion unacceptable and see minimal increasingness and weak inequality aversion as obviously appealing, Theorems 3 and 4 suggest that Pareto plus should be abandoned. Furthermore, we advocate weakly inequality-averse principles satisfying the axioms of Theorem 2 and, as a consequence, we recommend the critical-level generalized-utilitarian principles with a positive critical level α and a concave utility transformation g characterized in our final theorem.

Theorem 5: *An anonymous social-evaluation ordering R satisfies weak inequality aversion, strong Pareto, continuity, existence of a critical level, existence independence and avoidance of the repugnant conclusion if and only if R is critical-level generalized-utilitarian with a positive critical-level parameter and a concave utility transformation.*

5. Some issues and extensions

In this section, we address several additional issues that are not discussed above. Each is examined in Blackorby, Bossert and Donaldson [2005a]. Some are present in both fixed-population and variable-population environments; others appear in variable-population environments only.

5.1. Utility measurement and interpersonal comparisons

If utilities are ordinally measurable and interpersonally non-comparable, Arrow's [1951, 1963] theorem, appropriately modified, leads to an impossibility. Utilities can be assumed to be numerically measurable and interpersonally comparable in order to allow for the largest class of principles. Although this assumption is strong, if utilities are cardinally measurable (unique up to increasing affine transformations) and interpersonally comparable at two utility levels, full numerical comparability results from choosing utility numbers for the two levels.

5.2. The neutrality normalization

We follow the standard practice in the literature and assign a utility level of zero to neutrality. The idea of neutrality is not necessary for many theorems, including Theorems 1 and 2. Indeed, Dasgupta [1988, 1993] and Hammond [1988] do without neutrality and normalize the critical level in critical-level generalized utilitarianism to zero. Such a normalization is not without its difficulties, however. If best alternatives are compared when critical levels differ, individual utilities must change.

5.3. Welfarism and the account of well-being

Sen [1987, p. 11] criticizes welfarism on the grounds that "the battered slave, the broken unemployed, the hopeless destitute, the tamed housewife, may have the courage to desire little." Because we use accounts of well-being that include all aspects of well-being whether they accord with preferences or not, such as those of Griffin [1986] and Sumner [1996], this difficulty does not arise.

5.4. One or many profiles

It can be argued that, when comparing complete histories, multiple profiles are inappropriate. Although the single-profile approach is less well developed than the multi-profile approach, we have argued that a richness condition on the set of alternatives together with adapted versions of axioms such as anonymity, are sufficient to make the results of the multi-profile case apply in the single-profile environment (Blackorby, Bossert and Donaldson [2006b]).

5.5. *Dynamics*

The model presented in this article can be modified to accommodate multiple time periods. If this is done, Pareto indifference rules out discounting of future lifetime utilities. That axiom can be modified to allow discounting of lifetime utilities, however, if the axiom is conditional on birth dates.

Sometimes, population principles are applied to single periods using per-period utilities. If this is done and critical levels are not zero, difficulties arise. Suppose, for example, that a person lives one period longer in alternative x than in y with a utility level of zero in the additional period, all else the same. If a per-period utility level of zero represents neutrality in the period, every person is equally well off in the two alternatives from the timeless perspective, Pareto indifference requires x and y to be ranked as equally good, and consistency between per-period rankings and the timeless ranking requires the critical level to be zero for the per-period ranking.

5.6. *Uncertainty*

The critical-level generalized-utilitarian principles can be used to rank actions or combinations of institutional arrangements (including legal and educational ones), customs, and moral rules, taking account of the constraints of history and human nature. If each of these leads with certainty to a particular social alternative, they can be ranked with any welfarist principle. But consequences may be uncertain and, in that case, probabilities may be assigned to outcomes and the resulting uncertain alternatives ranked with extended population principles. One class of such principles, which can be justified axiomatically, consists of the ex-ante critical-level generalized-utilitarian principles; see Blackorby, Bossert and Donaldson [2005a, ch. 7, 2006c]. These principles employ value functions that are equivalent to the critical-level generalized-utilitarian value functions applied to expected utilities.

5.7. *Incomplete rankings*

There are population principles which declare alternatives to be unranked in some circumstances. One such class of principles is the critical-band generalized-utilitarian class, which uses an interval (the band). Two alternatives are ranked if and only if one is declared better than the other by *all* critical-level generalized-utilitarian principles with critical levels in the band.

5.8. Choice functions

Because many policy decisions have population consequences, it is natural to use population principles to guide them. These decision problems are, in most cases, choice problems: one or more options must be selected from a set of feasible alternatives. The maximizing approach to solving choice problems requires the selection of a best feasible alternative, according to a social ranking. Although this is a reasonable way to proceed, it excludes consideration of choice procedures that are not based on social orderings from the outset.

A natural way to proceed, therefore, is to focus on choice functions and ask whether the choices can be rationalized by a social ordering. Axioms must therefore be employed that apply to choices rather than rankings of alternatives. This is a complex problem, but it is possible to find a set of such axioms that characterizes a choice-theoretic version of critical-level generalized utilitarianism (Blackorby, Bossert and Donaldson [2005a, ch. 10].

6. Concluding remarks

This survey provides but a brief introduction to the many issues that arise in population ethics. There are numerous other principles that have been suggested and analyzed in the literature. For example, number-dampened principles, their restricted counterparts, and restricted versions of critical-level principles (see Blackorby, Bossert and Donaldson [2005a, ch. 5], Hurka [1983, 2000] and Ng [1986]) fail to satisfy existence independence, whereas variable-population versions of leximin become possible if continuity is dropped as a requirement. And, if social relations are not required to be complete, principles such as critical-band utilitarianism, based on an interval of critical levels, can be characterized.

As mentioned above, population issues can be analyzed in an intertemporal framework and in choice-theoretic settings. Although we restrict attention to a model with certain outcomes in this brief survey, it is possible to include uncertainty in population ethics. These extensions as well as related issues and applications are discussed in detail in Blackorby, Bossert and Donaldson [2005a].

There are, however, open questions in population ethics. Some principles for fixed-population social evaluation, such as that corresponding to the the Gini social-evaluation function (see Blackorby and Donaldson [1984]) are not additively separable. It is not known whether they can be extended to population problems in a reasonable way.

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