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# USING EX ANTE PAYMENTS IN SELF-ENFORCING RISK-SHARING CONTRACTS

Céline GAUTHIER and Michel POITEVIN

Département de sciences économiques, Université de Montréal and Centre de recherche et développement en économique (C.R.D.E.).

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C.P. 6128, succursale A
Montréal (Québec)
H3C 3J7

Télécopieur (FAX): (514) 343-5831
Courrier électronique (E-Mail): econo@tornade.ERE.Umontreal.CA

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## RÉSUMÉ

Dans ce papier, nous analysons les propriétés des contrats de partage de risque auto-exécutoires entre deux agents riscophobes. L'espace contractuel est élargi pour permettre un transfert au début de chaque période avant la réalisation de l'état de la nature. Nous analysons l'arbitrage que doit faire un contrat optimal entre les contraintes auto-exécutoires des deux agents par une spécification appropriée des transferts de début et de fin de période. Nos principaux résultats sont à l'effet que les transferts de début de période sont utilisés et ne sont pas stationnaires. Ces transferts dépendent du surplus généré par le contrat et anticipé par chaque agent. Nous démontrons que le transfert de début de période est inversement proportionnel au surplus qu'un agent anticipe. Le modèle génère également des propriétés dynamiques intéressantes. Par exemple, même si les chocs aléatoires sont distribués de façon identique et indépendante, la dynamique du contrat réplique celle d'un modèle avec tarification selon l'expérience, et ceci, en dépit du fait que le modèle fasse abstraction de tout apprentissage ou information privée.

Mots clés : contrats auto-exécutoires, relations de long terme.

## ABSTRACT

In this paper, we analyze a risk-sharing contract between two risk-averse agents facing self-enforcing constraints. We enlarge the contracting space to allow for an ex ante transfer (at the beginning of the period) before the state of nature is realized. We analyze the trade-off between the self-enforcing constraints of the two agents by characterizing the optimal ex ante and ex post transfer payments. Our main results are that ex ante payments are optimally used and that these payments are non-stationary. They optimally depend on the surplus from the relationship each agent expects. We show that the size of the ex ante payment an agent makes is inversely related to the surplus she expects to get from the relationship. We can also show that interesting dynamics properties emerge from our model even though shocks are independently and identically distributed across periods. For instance, in a two-state example, we show that the dynamics of the optimal contract exhibits experience-rating, even though there is no private information or learning taking place.

Key words : self-enforcing contracts, long-term relationships.

# 1 Introduction

Long-term contracts are useful for the governance of long-term relationships. Such contracts can help improve incentives as well as risk-sharing between two agents. An optimal contract will trade-off between incentives and risk-sharing to attain an efficient allocation; however this efficient allocation is often time-inconsistent. For example, an ex ante efficient allocation may not be ex post efficient (once certain actions have been undertaken or some information has been revealed). This time-consistency problem has led to the recent literature on renegotiation. Or, an ex ante profitable contract may not be ex post profitable following a given history. In this case, if enforcement costs are high (or mobility costs are low), agents may be tempted to renege on the contract to seek more profitable opportunities elsewhere. The literature on self-enforcing contracts studies this problem.

Consider two agents that enter into a long-run relationship to share risk and for which enforcement costs are high. Their relationship is governed by a contract that prescribes in every period transfer payments from one agent to the other contingent on the realization of the state of nature (and possibly the complete history of the relationship). If the two agents can commit not to default on any prescribed transfer payment then the optimal contract achieves an efficient risk-sharing allocation; however, if an agent cannot commit not to default, efficient risk sharing may be impeded as the optimal contract is constrained by the possibility of ex post default. The contract should then prescribe payments that are slef-enforcing, that is, an agent will make a transfer payment if and only if it is in her interest to do so. In any period, the surplus one expects from the relationship conditions the transfer that she can make in this period. If an agent expects a high surplus in the future, she has low incentives to break the relationship and she is therefore willing to make a high payment to continue the relationship. On the other hand, if her expected surplus is low, she has high incentives to break the relationship and she must therefore be induced not to do so by requiring a low (possibly negative) payment. Self-enforcing constraints generally limit transfer payments and therefore reduce the opportunity for efficient risk sharing.

The risk-sharing problem analysed in the literature usually has the following structure.[1] There are two agents. In every period, a risk-averse agent receives a stochastic endowment. This endowment is independently and identically distributed across periods. Risk-sharing between the two agents is implemented by a contract specifying transfer payments between the two agents. The contracting space is such that all transfers take place at the end of the period once the state of nature has been observed. In an environment in which the two agents cannot commit to making all prescribed payments, ex post self-enforcing constraints must be satisfied, that is, for any realization of the state of nature, the contractually specified payment must satisfy a participation constraint for each agent. It is possible that these constraints be quite stringent for one agent, say agent 1. This effectively limits the payments agent 1 can make to agent 2. In this case, agent 1 would like to make a transfer to agent 2 *before* the state of nature is realized. At this point, agent 1's self-enforcing constraints only have to hold in expectation over all states of nature. Such ex ante transfer would effectively relax agent 1's ex post self-enforcing constraints. When the two agents face self-enforcing constraints, if one agent makes such payment to relax her own self-enforcing constraints it usually makes the other agent's self-enforcing constraints more stringent by leaving the ex post burden to that agent to make the necessary transfers for optimal risk-sharing. Consequently, the ex ante payment must trade-off between the self-enforcing constraints of the two agents.

In this paper we analyse a risk-sharing contract between two risk-averse agents facing self-enforcing constraints. We enlarge the contracting space to allow for an ex ante transfer (at the beginning of the period) before the state of nature is realized. We analyse the trade-off between the self-enforcing constraints of the two agents by characterizing the optimal ex ante and ex post transfer payments.

Our main results are that ex ante payments are optimally used and that these payments are non-stationary. They optimally depend on the surplus from the relationship each agent expects. This expected surplus evolves with the history of past realizations of states of nature. When an agent expects a low share of the

[1] For example, see Thomas and Worrall (1988).

surplus of the relationship, her ex post self-enforcing constraints are relatively stringent and she cannot be required to make a high payment. In this case, she is optimally asked to pay up front before the realization of the state of nature. This effectively relaxes her ex post self-enforcing constraints. In general, however, these constraints cannot be completely eliminated because a high ex ante payment by one agent increases the incentives of the other agent to break the relationship and run away with this payment. We show that the size of the ex ante payment an agent makes is inversely related to the surplus she expects to get from the relationship. We can also show that interesting dynamics properties emerge from our model even though shocks are independently and identically distributed across periods. For example, in a two-state example, we show that the dynamics of the optimal contract exhibits experience rating even though there is no private information or learning taking place.

Ex ante payments are observed in many different contractual relationships. For example, in a financing relationship the firm commits ex ante to a certain amount of collateral in case it defaults on the contract. Alternatively, the financier can commit to a given amount of financing by providing the firm with a credit line. Such payments can be interpreted as ex ante transfers from one agent to the other. In other contexts, the ex ante payment can be reinterpreted as a breach-of-contract penalty which is in fact a payment agents are committed to if they default on the contract. In an insurance-contract example, insurance premia are paid before the state of nature is realized as an ex ante payment from the insuree to the insuror. We discuss the economic implications of our model more at length in Section 5.

Section 2 presents the basic model. In Section 3 we analyze the role of ex ante payments when only one agent faces self-enforcing constraints. Section 4 presents the main results of the paper when the two agents face self-enforcing constraints. Section 5 provides different economic interpretations of the contractual form with an ex ante payment. A conclusion follows.

## 2 The model

The environment we consider can be described by an infinite sequence of periods, $t = 1, 2, ..., \infty$, and for each period, a finite set of states of nature, $s \in \{1, 2, ..., S\}$, with $S \geq 2$. We assume that the states are distributed independently and identically across all periods, and therefore, in each period, the state of nature $s$ occurs with probability $p^s$ where $\sum_{s=1}^{S} p^s = 1$. It is assumed that each period $t$ is divided into three dates, $t_0, t_1$, and $t_2$, where $t_1$ is the date at which the state of nature is realized; the dates $t_0$ and $t_2$ denote respectively the dates preceding and following the realization of the state of nature.

Two infinitely-lived agents evolve in this environment. Both agents are risk averse. In each period, agent 1's preferences over consumption $c$ are represented by a state-independent strictly concave quadratic utility function $u(c)$ for $c \in [a, b]$. In each period, agent 1 obtains a state-contingent endowment $y^s$. We adopt the convention that $y^s > y^{s-1}$ for all states $s$. We assume that $a < y^1 < y^S < b$. In each period, agent 2's preferences over consumption $c$ are given by $v(c)$ which is also a state-independent concave quadratic function. In each period, agent 2 obtains a state-independent endowment $b$.[2] Both agents discount the future by a common factor $\beta \in (0, 1)$.

In such an environment there exists gains from trade to be exploited: both agents are risk averse, and agent 1 obtains a risky endowment, while agent 2 obtains a constant endowment. We assume that there are no contingent markets that would allow the agents to diversify their risk and therefore the two agents enter into a risk-sharing relationship. For example, agent 1 may represent a firm and agent 2, a financier; or alternatively, agent 1 may be a sovereign agent and agent 2, a consortium of financiers. A relationship is characterized by transfer payments between the two agents that take place at various time periods and dates. We call the governance of such relationship a contract where the term contract is interpreted in a broad sense, namely it can encompass implicit as well as explicit agreements. A contract then specifies various transfers between the

[2]The analysis can be easily generalized to the case in which the endowment of agent 2 is stochastic.

two agents for all periods of the relationship. In each period $t$, a contract can specify the following structure of transfer payments.

1. A (positive or negative) ex ante transfer $B_t$ from agent 2 to agent 1 at date $t_0$ (before the state of nature is realized).

2. Ex post (positive or negative) transfers $a_t^s$ from agent 1 to agent 2 at date $t_2$ (after the state of nature $s$ is realized).

Consumption takes place at the end of the period. Agent 1's consumption in period $t$ if state $s$ is realized is $c_t^s = y_t^s + B_t - a_t^s$; agent 2's consumption is $b - B_t + a_t^s = b + y_t^s - c_t^s$.

In a typical relationship the prescribed transfers can potentially be contingent on the complete past history of the relationship. The history up to period $t$ is the vector of all previous realizations of the state of nature. Let $s_t$ denote the realized state of nature in period $t$. The history at the end of period $t-1$ (date $(t-1)_2$) or at the beginning of period $t$ (date $t_0$) is denoted by $h_{t-1} = (s_1, s_2, ..., s_{t-1})$. We assume that $h_0 = \emptyset$. Assume that the two agents enter into a long-term (infinite) contractual relationship. We can then define formally a contract between the two agents.

**Definition 1** *A contract, $\delta$, is a sequence of two functions: $\{B(h_{t-1}), a(h_t)\}_{t=1}^{\infty}$ where $B_t = B(h_{t-1})$ represents the transfer from agent 2 to agent 1 at the beginning of period t (date $t_0$) when history is $h_{t-1}$, and where $a_t^s = a(h_{t-1}, s)$ represents the transfer from agent 1 to agent 2 at the end of period t (date $t_2$) when history is $h_{t-1}$ up to period t and $s$ is the realized state of nature in period t.*

For any contract, $\delta$, and any history, $h_{t-1}$, agent 1's expected surplus of the relationship from the beginning of period $t$ onwards is

$$U(\delta; h_{t-1}) \equiv E \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{u(y_\tau^s + B_\tau - a_\tau^s) - u(y_\tau^s)\}$$

where $E$ is the expectation operator taken over all states in all future periods and $y_\tau^s$ denotes that the endowment $y^s$ is realized in period $\tau$. Similarly, the expected surplus of agent 2 from the beginning of period $t$ onwards is

$$V(\delta; h_{t-1}) \equiv E \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ v(b - B_\tau + a^s_\tau) - v(b) \right\}.$$

The surplus of the two agents are measured with respect to autarky, that is, it gives the surplus one agent can get from the relationship over autarky where it would consume its endowment.

The approach we take is to assume that the two agents enter into a relationship at the beginning of a period called period 1. This relationship is governed by a contract. The characterization of the implemented contract depends on the available technology to legally enforce the prescribed payments. The objective of the paper is to study the effects of limited enforceability of payments on optimal contracts. The literature on self-enforcing contracts[3] has tackled such analysis and we will discuss later how our paper relates to some of these papers.

We first establish a benchmark case in which the two agents sign a contract at the beginning of the first period and all prescribed transfers are legally enforceable. We refer to this case as the full-commitment case. In this case, the optimal contract, $\delta^{fc}$, is the solution to the following maximization problem where, for simplicity, it is assumed that agent 1 has the bargaining power and $V_0$ is agent 2's reservation utility.

$$\delta^{fc} = \arg\max_{\delta} \left\{ U(\delta; h_0) \text{ s/t } V(\delta; h_0) \geq V_0 \right\} \tag{1}$$

This maximization problem simply states that the optimal contract maximizes the discounted expected utility of agent 1 subject to agent 2's participation constraint. This constraint states that the contract must provide agent 2 with a discounted expected surplus of at least $V_0$. A solution to this maximization problem exists and is characterized in the following proposition.[4]

**Proposition 1** *When both agents can commit to the terms of the contract, the optimal contract, $\delta^{fc}$, is characterized by the equalization of marginal rates of sub-*

[3] For example, see Harris and Holmström (1982), Bull (1987), MacLeod and Malcomson (1988), Thomas and Worrall (1988), and Kletzer and Wright (1990).

[4] The proof of this proposition is trivial and is therefore omitted.

*stitution of consumption of the two agents across all states and periods. Formally, for all periods $t, \tau$, all states $s, q$, and all histories $h_{t-1}$, $\frac{u'(y_t^s + B_t - a_t^s)}{u'(y_\tau^q + B_\tau - a_\tau^q)} = \frac{v'(b - B_t + a_t^s)}{v'(b - B_\tau + a_\tau^q)}$.*

The optimal full-commitment contract specifies perfect risk-sharing with a stationary consumption rule. This consumption rule can be written as $c_t^s = c^*(c_{t-1}, y_{t-1}, s)$ where

$$\frac{u'(c^*(c_{t-1}, y_{t-1}, s))}{u'(c_{t-1})} = \frac{v'(b + y_t^s - c^*(c_{t-1}, y_{t-1}, s))}{v'(b + y_{t-1} - c_{t-1})}.$$

Two aspects of this characterization deserve mention. First, in problem (1), the functions $U(\delta; h_0)$ and $V(\delta; h_0)$ depend only on the net transfers $B_t - a_t^s$ and therefore, in each state, only optimal net transfers are determined. This implies that the value of $B_t$ is arbitrary. With full commitment, there is no role for the ex ante transfer $B_t$ in the optimal contract.

Second, in some states of nature, net transfers from agent 1 to agent 2 are positive, and in other states, the reverse is true. Complete legal enforcement of the contract is a sufficient condition to make these transfers feasible. In the next sections we relax the assumption of complete legal enforcement to study the characterization of optimal contracts under incomplete legal enforcement.

# 3 Contracting under one-sided commitment

In this section we consider an environment in which legal enforcement of all prescribed payments is limited. We first examine the situation in which only agent 1 cannot commit to making all transfers prescribed by the contract.[5] We say that agent 1 faces self-enforcing constraints. These constraints impose that, at any point in time, agent 1 should always do as well obeying the contract as reneging on it. When the self-enforcing constraints are satisfied we say that the contract is self-enforcing.

When legal enforcement cannot provide a sufficient incentive for agent 1 to obey the contract, she must be incite to do so differently. In a long term relationship

[5] The analysis of the opposite case in which agent 2 can renege on the contract is symmetric.

such incentive arises endogenously from the interaction of the two agents over time. One approach to study this incentive would be to model the relationship as a strategic game where each agent's strategy would be a sequence of payments for the complete history and following any history. In this case, the incentive for agent 1 to obey her equilibrium strategy would come from the anticipation of agent 2's response to a deviation. Any payment by agent 1 would therefore be enforced by the strategy of player 2. The more severe would be player 2's punishment, the higher would be cooperation between the two agents. In this case, the Folk theorem states that given a high enough discount factor any individually rational feasible allocation can be sustained in equilibrium. For our purposes, such an approach is unsatisfactory for two reasons. First, as is well known in the theory of supergames, the multiplicity of equilibria creates significant coordination problems between the two agents. Second, we are interested here in characterizing allocations for any value of the discount factor and not just allocations for high values of the discount factor.

We therefore adopt the following approach. We assume that if agent 1 reneges on the contract she suffers maximal punishment in that she must stay in autarky forever after. This punishment strategy by agent 2 allows us to characterize the best possible contract satisfying self-enforcing constraints.[6] The optimal contract is then the solution to a well-defined maximization problem. This approach resolves the coordination problem in effectively coordinating the two agents on a Pareto optimal allocation. Furthermore it allows us to characterize optimal allocations for any value of the discount factor.

When agent 1 can renege on the contract at any point in time, she will make a transfer to agent 2 if and only if it is in her interest to do so. Agent 1 will compare the benefit of making the transfer and obeying the contract with the payoff of reneging on the contract and staying in autarky thereafter. For example, suppose the two agents have signed a contract $\delta$ prescribing transfers $\{B(h_{t-1}), a(h_t)\}$ for

[6]MacLeod and Malcomson (1989) model a situation similar to ours as an explicit game and show that the maximal punishment is indeed subgame perfect. Any deviating agent is punished in the future by not being able to enter a successful relationship, all parties expecting the deviating agent to deviate again in the future.

all histories $h_t$. In period $t$, agent 1 may decide to renege on the contract at date $t_0$ before receiving the (possibly negative) transfer $B_t$. Her surplus from staying in the contract is then $U(\delta; h_{t-1})$. Agent 1 may also decide to renege on the contract after the state of nature has been realized at date $t_2$. In this case her surplus from staying in the contract is $u(y_t^s + B_t - a_t^s) - u(y_t^s + B_t) + \beta U(\delta; h_{t-1}, s)$, where the first two terms represent her current surplus from the relationship and the last term, her discounted expected future surplus. We can now define a self-enforcing contract for agent 1.

**Definition 2** *A contract $\delta$ is **self-enforcing for agent 1** if and only if, for all histories $h_{t-1}$, periods $t$, and states $s$, the following constraints hold.*

$$(i) \quad U(\delta; h_{t-1}) \geq 0$$

$$(ii) \quad u(y_t^s + B_t - a_t^s) - u(y_t^s + B_t) + \beta U(\delta; h_{t-1}, s) \geq 0$$

This definition states that a contract is self-enforcing for agent 1 if at any time during the relationship agent 1 prefers making the contractual transfer rather than reneging on the contract and be reduced to autarky from then on. Constraint (i) is an ex ante self-enforcing constraint in that it holds at date $t_0$; constraint (ii) is an ex post self-enforcing constraint in that it holds at date $t_2$ after the state of nature has been realized. Note that all ex post self-enforcing constraints being satisfied does not necessarily imply that the ex ante self-enforcing constraint is also. For example, if $B_t$ is negative, the ex ante constraint may bind while ex post constraints may not once the ex ante payment $B_t$ has been paid. It is therefore necessary to consider these two sets of constraints.

When designing the optimal contract the two agents will anticipate agent 1's incentive to renege, and the terms of the contract will take into account such incentive. To solve for the optimal contract we must therefore add self-enforcing constraints to the maximization problem (1). The optimal contract with non-commitment by agent 1, $\delta^1$, is then the solution to the following maximization problem where, for simplicity, we assume that agent 2's reservation utility is equal to zero.

$$\delta^1 = \arg\max_{\delta} U(\delta; h_0)$$

$$\text{s/t} \quad V(\delta; h_0) \geq 0 \tag{2}$$
$$U(\delta; h_{t-1}) \geq 0 \quad \forall t, h_{t-1}$$
$$u(y_t^s + B_t - a_t^s) - u(y_t^s + B_t) + \beta U(\delta; h_{t-1}, s) \geq 0 \quad \forall s, t, h_{t-1}$$

The next proposition characterizes the optimal contract $\delta^1$.[7]

**Proposition 2** *Suppose that the maximum ex ante payment agent 1 can make is* $\underline{B}$.

*(i) For all values of* $\beta \in (0,1)$ *the optimal contract with non-commitment by agent 1 is the optimal full-commitment contract, that is,* $\delta^1 = \delta^{fc}$, *if and only if* $\underline{B} \geq y^S - c^{Sfc}$, *where* $c^{sfc}$ *is the optimal consumption in state s under the full-commitment contract.*

*Suppose that* $\underline{B} < y^S - c^{Sfc}$.

*(ii) There exists a* $\beta_1$ *which depends on* $\underline{B}$ *such that for* $\beta \in [\beta_1, 1)$, *the optimal contract with non-commitment by agent 1 is the optimal full-commitment contract, that is,* $\delta^1 = \delta^{fc}$.

*(iii) For all* $\beta \in (0, \beta_1)$, *the following characterization forms part of an optimal contract* $\delta^1$:

1. *Agent 1 makes the highest ex ante payment in every period, that is,* $B_t = -\underline{B}$ *for all periods t;*

2. *Agent 2's expected profit is non-increasing in time, that is,* $V(\delta^1; h_{t-1}, s) \leq V(\delta^1; h_{t-1})$ *for all histories* $h_{t-1}$, *time periods t, and states s;*

3. *Agent 1's implicit discount factor is no greater than agent 2's, that is,* $\beta \frac{Eu'(c_t^s)}{u'(c_{t-1})} \leq \beta \frac{Ev'(b+y_t^s-c_t^s)}{v'(b+y_{t-1}-c_{t-1})}$ *for all histories* $h_{t-1}$ *and time periods t where* $c_t^s = c(h_{t-1}, s)$ *and* $c_{t-1} = c(h_{t-1})$ *and* $c(h)$ *defines the optimal consumption following history h.*

This proposition states that if agent 1 can make a high enough ex ante payment ($\underline{B} \geq y^S - c^{Sfc}$), then the optimal full-commitment contract can satisfy agent 1's self-enforcing constraints. A large enough ex ante payment effectively allows all

[7]All proofs are relegated to the Appendix.

ex post transfers $a_t^s$ to be negative which in turn implies that all ex post self-enforcing constraints are satisfied.[8]

When the maximum ex ante payment agent 1 can make is not high enough, the optimal contract with full-commitment cannot be supported for all values of the discount factor. If the discount factor is high enough, that is, no lower than $\beta_1$, then agent 1's ex post self-enforcing constraints are not binding.[9] In this case, the future benefits of perfect risk-sharing exceed the short-run cost of making the prescribed transfer in any state $s$. Contrary to the full-commitment case however, the transfer $B_t$ is not a matter of indifference. It will optimally be set to the maximum level agent 1 can pay. A maximal ex ante payment reduces ex post transfers $a_t^s$ from agent 1 to agent 2 and hence the incentive for the former to renege ex post on the contract. It therefore allows the optimal full-commitment contract $\delta^{fc}$ to be supported for the largest interval of discount factors.

When the discount factor is smaller than $\beta_1$, the optimal full-commitment contract cannot obtain if $\underline{B} < y^S - c^{Sfc}$. In this case, the transfer that agent 1 must make to agent 2 in state $S$ is large compared to the discounted future benefits of perfect risk-sharing. Agent 1 then has the incentive to renege on the contract. The optimal contract must therefore account for this possibility. In this case, agent 1 can make the maximum ex ante payment $B_t$ in all periods, namely, $B_t = -\underline{B}$.[10] The intuition for this result can be found by looking at the maximization problem (2). Suppose the optimal contract specifies $B_t > -\underline{B}$ and transfer payments $a_t^s$ for some $t$. It is then possible to construct a contract $\hat{\delta}$ with, in period $t$, $\hat{B}_t = -\underline{B}$ and $\hat{a}_t^s = a_t^s - B_t - \underline{B} < a_t^s$ for all states $s$, and leave all other periods unchanged. These modifications leave the two agents' consumption unchanged and therefore do not change the value of agent 2's participation constraint, nor the value of ex ante self-enforcing constraints; however they do relax the ex post self-enforcing

[8]Note that the transfer $y^S - c^{Sfc}$ is the largest transfer agent 1 makes to agent 2 in the full-commitment contract $\delta^{fc}$.

[9]This result is akin to results in the theory of supergames where any efficient outcome of a static game can be supported as an equilibrium of its associated supergame provided that the discount factor is high enough.

[10]Although making the largest ex ante payment is not necessary for an optimal contract for all values of the discount factor, it is clearly sufficient.

constraints of agent 1. When one of these constraints is binding, this new contract (weakly) increases the utility of agent 1.

The optimal contract also specifies that agent 2's expected profit be non-increasing in time. The optimal contract seeks two objectives: (1) to insure agent 1 against shocks to her endowment and (2) to smooth her consumption across periods. When agent 1 cannot commit to make any transfer at the end of a period, risk-sharing and intertemporal smoothing are partially impeded. These objectives can be improved upon by having agent 1 save in the early periods and good states of the world and withdraw her savings in later periods in bad states of the world. This is possible given that agent 2 can commit not to "steal" agent 1's early savings. The optimal contract therefore uses agent 2 as a savings account. The objective of this savings account is precisely to insure future consumption against bad states of the world. This is optimally achieved by having agent 2's expected profits be non-increasing in time. This can also be seen in the fact that agent 1's implicit discount factor is not larger (and sometimes strictly smaller) than agent 2's which reflects agent 1's relative preference for the future. Self-enforcing constraints force agent 1 to save more than she would in a full-commitment environment. It is therefore as if she could earn a high interest rate on her savings (a low discount factor).

The following corollary gives a more precise characterization of the optimal consumption path.

**Corollary 1** *Assume that* $\underline{B} < y^S - c^{Sfc}$ *and that* $\beta < \beta_1$.
*(i) For all states s, there exists a time-invariant consumption level* $\underline{c}^s$ *such that* $c_t^s \geq \underline{c}^s$ *for all time periods t.*
*(ii) The lower bounds of consumption,* $\underline{c}^s$*, are increasing in the state of the world, that is,* $k > q \Rightarrow \underline{c}^k > \underline{c}^q$*, and are decreasing in the maximal payment* $\underline{B}$ *that agent 1 can make.*
*(iii) For any history* $(h_{t-2}, q, s)$*, optimal consumption at time t is such that:*

$$c(h_{t-2}, q, s) = \begin{cases} \underline{c}^s & \text{if } c^*(c_{t-1}, y_{t-1}^q, s) \leq \underline{c}^s \\ c^*(c_{t-1}, y_{t-1}^q, s) & \text{otherwise} \end{cases}$$

*where* $c^*(c_{t-1}, y_{t-1}^q, s)$ *is implicitly defined by* $\frac{u'(c^*(c_{t-1}, y_{t-1}^q, s))}{u'(c_{t-1})} = \frac{v'(b + y_t^s - c^*(c_{t-1}, y_{t-1}^q, s))}{v'(b + y_{t-1}^q - c_{t-1})}$.

This corollary shows that, in each state, there exist time-invariant lower bounds on agent 1's consumption. These bounds are increasing with the state of the world and are decreasing in the maximum ex ante payment that agent 1 can make. Thus the higher the payment $\underline{B}$, the larger are the intervals of consumption that can be supported in each state.

If, given consumption in period $t-1$, consumption smoothing satisfies agent 1's ex post self-enforcing constraint, then consumption in period $t$ is equal to $c^*(c_{t-1}, y^q_{t-1}, s)$. If it does not satisfy agent 1's ex post self-enforcing constraint, then consumption in period $t$ is equal to $\underline{c}^s$. Consumption follows a stationary first-order Markov process where period $t$ consumption depends on period $t-1$ consumption and the realized states in periods $t-1$ and $t$. The dynamics of consumption also imply that there is convergence to perfect risk-sharing and consumption smoothing. In the steady state, consumption only depends on the current state. Moreover, since optimal risk-sharing at actuarially fair prices is impossible when $\beta < \beta_1$, the steady-state consumption in every state must be higher than optimal consumption in the full-commitment case. This implies that agent 1 gets, in the steady state, optimal risk-sharing at prices lower than actuarially fair prices. This is acceptable to agent 2 because he gets a compensating surplus at the beginning of the relationship in order to make zero-profit overall. In terms of the implicit discount factor the two agents face, agent 1's is lower than agent 2's until the steady state is reached, after which they become equal (as is implied by perfect risk sharing and consumption smoothing).

The results of Proposition 2 and Corollary 1 are similar to results obtained by Harris and Holmström (1982) in a model of labor contracts. They showed that under the assumption of non-commitment by the employee, wages are downward rigid as the risk-neutral employer fully insures the worker against bad states of the world. Our characterization is, first, a generalization to the case of two risk-averse agents. It shows in fact that in this case, consumption can decrease in some states. Secondly, it shows that the non-committed party (agent 1) would like to make, in each period, ex ante transfers to relax the ex post self-enforcing constraints, that is, an optimal characterization sets $B_t = -\underline{B}$.

A similar analysis could be performed for the case in which agent 2 cannot commit not to renege on the contract and similar results would obtain. Having the non-committed agent making the maximal ex ante payment relaxes its ex post self-enforcing constraints. The non-committed agent effectively uses the committed agent as a savings account. This improves risk-sharing and consumption smoothing as consumption eventually achieves perfect risk-sharing. The possibility that the committed agent has of making an ex ante payment allows to shift (some or all) the burden of ex post transfers to the committed agent. However if both agents face self-enforcing constraints, the above characterization may not be feasible. One agent may run away with the ex ante payment of the other agent as its ex post self-enforcing constraints would become too stringent. The next section studies the optimal contract when the two agents face self-enforcing constraints.

## 4 Contracting under no commitment

In this section, we relax the assumption of commitment by either agent and study the properties of the optimal contract. We have seen that when agent 1 cannot commit she makes the maximal ex ante payment at date $t_0$. Agent 2 does similarly when he cannot commit. This payment reduces the non-committed agent's ex post transfers at date $t_2$ and thus relaxes its ex post self-enforcing constraints. For constant consumption, the substitution from ex post to ex ante payments does not affect ex ante constraints since they only have to hold in expectation over the possible states of the world. However when both agents simultaneously face self-enforcing constraints, an ex ante payment that relaxes one agent's ex post self-enforcing constraints may strengthen the other agent's constraints. The optimal ex ante payment should therefore trade off between the two sets of self-enforcing constraints. This section studies the details of that trade-off.

We first define the concept of a self-enforcing contract under the non-commitment assumption.

**Definition 3** *A contract $\delta$ is* ***self-enforcing*** *if and only if, for all histories $h_{t-1}$,* *periods t, and states s, the following constraints hold.*

$$
\begin{aligned}
&(i) \quad U(\delta; h_{t-1}) \geq 0 \\
&(ii) \quad u(y_t^s + B_t - a_t^s) - u(y_t^s + B_t) + \beta U(\delta; h_{t-1}, s) \geq 0 \\
&(iii) \quad V(\delta; h_{t-1}) \geq 0 \\
&(iv) \quad v(b - B_t + a_t^s) - v(b - B_t) + \beta V(\delta; h_{t-1}, s) \geq 0
\end{aligned}
$$

This definition simply states that a contract is self-enforcing if it is self-enforcing for agent 1 (constraints i and ii) as well as for agent 2 (constraints iii and iv).

Before proceeding with the analysis, we assume that there are no exogenous bounds on the ex ante payment $B_t$. This assumption is motivated by the fact that we want to study how self-enforcing constraints rather than some exogenous bound limit the use of the ex ante payment.

The optimal contract without commitment, $\delta^{nc}$, is the solution to the following maximization problem.

$$
\begin{aligned}
\delta^{nc} = \quad & \arg\max_{\delta} \; U(\delta; h_0) \\
\text{s/t} \quad & U(\delta; h_t) \geq 0 \quad \forall t, h_t \\
& u(y_t^s + B_t - a_t^s) - u(y_t^s + B_t) + \beta U(\delta; h_{t-1}, s) \geq 0 \quad \forall s, t, h_t \qquad (3) \\
& V(\delta; h_t) \geq 0 \quad \forall t, h_t \\
& v(b - B_t + a_t^s) - v(b - B_t) + \beta V(\delta; h_{t-1}, s) \geq 0 \quad \forall s, t, h_{t-1}
\end{aligned}
$$

Following any time period and any history, the optimal contract $\delta^{nc}$ will necessarily be efficient, since if it was not it would be possible to replace the non-efficient path by an efficient path thus (weakly) increasing the utility each agent derives from the contract and hence relaxing all previous self-enforcing constraints. This new contract would necessarily be self-enforcing and would dominate the old contract at the beginning of the relationship. This argument implies that the optimal contract from the start of period $t$ onwards is the solution to the following maximization problem.

$$
\begin{aligned}
f(V_t) = \quad & \max_{B_t, (a_t^s)_s, (V_{t+1}^s)_s} E\left\{u(y_t^s + B_t - a_t^s) - u(y_t^s) + \beta f(V_{t+1}^s)\right\} \\
\text{s/t} \quad & f(V_{t+1}^s) \geq 0 \quad \forall s
\end{aligned}
$$

$$
\begin{aligned}
&u(y_t^s + B_t - a_t^s) - u(y_t^s + B_t) + \beta f(V_{t+1}^s) \geq 0 \quad \forall s \\
&V_{t+1}^s \geq 0 \quad \forall s \\
&v(b - B_t + a_t^s) - v(b - B_t) + \beta V_{t+1}^s \geq 0 \quad \forall s \\
&V_t \leq E\left\{v(b - B_t + a_t^s) - v(b) + \beta V_{t+1}^s\right\}
\end{aligned} \tag{4}
$$

where $f(V_t)$ represents the Pareto frontier that can be attained through a self-enforcing contract after an arbitrary history $h_{t-1}$. Denote by $\Lambda(h_{t-1})$ the set of contracts satisfying the self-enforcing constraints following the history $h_{t-1}$. The Pareto frontier is then given by a time-independent function

$$
f(V_t) = \max_{\delta \in \Lambda(h_{t-1})} \left\{U(\delta; h_{t-1}) \text{ s/t } V(\delta; h_{t-1}) \geq V_t\right\}.
$$

In problem (4), the variable $V_{t+1}^s$ is to be interpreted as $V(\delta; h_{t-1}, s)$, that is, agent 2's expected surplus from period $t+1$ onwards when contract $\delta$ is signed and $s$ is the realized state of nature in period $t$. The first two constraints represent agent 1's ex ante and ex post self-enforcing constraints respectively. The next two constraints represent agent 2's self-enforcing constraints. The last constraint of the problem ensures that the contract is dynamically consistent. Before characterizing the properties of the optimal contract we need some technical results.

**Lemma 1** *(i) The set of self-enforcing contracts following history $h_{t-1}$, $\Lambda(h_{t-1})$, is convex.*

*(ii) The set of values of $V_t$ for which a self-enforcing contract exists is a compact interval $[0, \bar{V}]$.*

*(iii) The Pareto frontier $f(V_t)$ is decreasing, strictly concave, and continuously differentiable on $(0, \bar{V})$.*

*(iv) For each value of $V_t \in [0, \bar{V}]$ there exists a unique continuation of the contract $\delta$ at time $t$ in which $V(\delta; h_{t-1}) = V_t$ and $U(\delta; h_{t-1}) = f(V_t)$.*

To get a better understanding of the role of the ex ante payment in this contracting problem, we will first state the solution to this problem assuming that no ex ante payments are allowed.[11]

[11] This corresponds to the generalization to the case of bilateral risk aversion of Thomas and Worrall's (1988).

**Proposition 3** *Assume that $B_t = 0$ for all time periods $t$.*
*(i) For all states $s$, there exists optimal time-invariant consumption levels $\underline{c}^s$ and $\bar{c}^s$ such that $\underline{c}^s \leq c_t^s \leq \bar{c}^s$ for all time periods $t$.*
*(ii) The optimal lower bounds $\underline{c}^s$ and upper bounds $\bar{c}^s$ are increasing with the states of the world, that is, $k > q \Rightarrow \underline{c}^k > \underline{c}^q$ and $\bar{c}^k > \bar{c}^q$.*
*(iii) For any history $(h_{t-2}, q, s)$, optimal consumption at time $t$ is such that:*

$$c(h_{t-2}, q, s) = \begin{cases} \underline{c}^s & \text{if } c^*(c_{t-1}, y_{t-1}^q, s) \leq \underline{c}^s \\ c^*(c_{t-1}, y_{t-1}^q, s) & \text{if } \underline{c}^s < c^*(c_{t-1}, y_{t-1}^q, s) < \bar{c}^s \\ \bar{c}^s & \text{otherwise} \end{cases}$$

*(iv) There are no values of $\beta$ such that the contract with non-commitment, $\delta^{nc}$, is the optimal contract with full commitment, $\delta^{fc}$.*

When no ex ante payments are allowed, there are upper and lower bounds on optimal consumption. These bounds are determined by the two agents' ex post self-enforcing constraints. Agent 1's consumption follows a simple stationary first-order Markov process. In period $t$, consumption depends on period $t-1$ consumption and the states of the world realized in periods $t-1$ and $t$. This implies that the consumption of the two agents is smoothed as much as possible subject to ex post self-enforcing constraints.

We now characterize the optimal solution when the ex ante payment is chosen optimally. An implication of Lemma 1 is that problem (4) is a concave program and therefore first-order conditions are both necessary and sufficient for a solution. Denote respectively by $\beta p^s \alpha^s$, $p^s \theta^s$, $\beta p^s \phi^s$, $p^s \lambda^s$, and $\psi$ the multipliers of the five constraints in problem (4). The first-order conditions are

$$\begin{aligned} B_t: \quad & \sum_s p^s u'(y_t^s + B_t - a_t^s) + \sum_s p^s \theta^s \left(u'(y_t^s + B_t - a_t^s) - u'(y_t^s + B_t)\right) \\ & - \sum_s p^s(\lambda^s + \psi) v'(b - B_t + a_t^s) + \sum_s p^s \lambda^s v'(b - B_t) = 0 & (5) \\ a_t^s: \quad & -p^s(1 + \theta^s) u'(y_t^s + B_t - a_t^s) + p^s(\lambda^s + \psi) v'(b - B_t + a_t^s) = 0 \quad \forall s & (6) \\ V_{t+1}^s: \quad & (1 + \alpha^s + \theta^s) f'(V_{t+1}^s) + \lambda^s + \phi^s + \psi = 0 \quad \forall s & (7) \end{aligned}$$

and the envelope condition is $f'(V_t) = -\psi$. Lemma 2 provides some basic properties of the solution.

**Lemma 2** *(i) There exists a $\beta_{nc}$ such that the optimal non-commitment contract, $\delta^{nc}$, yields the same consumption as the optimal full-commitment contract, $\delta^{fc}$, if and only if $\beta \in [\beta_{nc}, 1)$.*
*Suppose that $\beta < \beta_{nc}$.*
*(ii) For $i = 1, 2$, there exists a state $s_i$ in which agent i's ex post self-enforcing constraint is binding.*

This lemma shows that if the discount factor is high enough, the optimal full-commitment contract is feasible with non-commitment and is therefore optimal. Agent 2 pays up front a high enough payment ($B_t = c^{1fc} - y^1$) such that the resulting ex post payments, $a_t^s$, are all positive. These payments yield zero expected utility to agent 2 in every period and therefore his self-enforcing constraints are all satisfied. If the discount factor is high enough, agent 1 prefers to make the ex post payments in all states of nature and be optimally insured in the future rather than keep the up front payment, renege on the contract, and revert to autarky thereafter. The critical discount factor $\beta_{nc}$ is defined as the lowest discount factor such that agent 1 does not renege on the contract in all states of nature. This contrasts with the case $B_t = 0$ where optimal risk-sharing is not feasible with non-commitment for any value of the discount factor. The intuition for the difference in these two results is the following. Optimal risk-sharing yields zero expected utility to agent 2 in every period. Ex post self-enforcing constraints then hold if and only if $a_t^s \geq 0$ for all states $s$. But this is incompatible with optimal risk-sharing and hence there is no value of the discount factor such that optimal risk-sharing is feasible. Lemma 2 provides a first indication that the use of ex ante payments can strictly improve the utility of the two agents (at least for some values of the discount factor).

When the discount factor is such that optimal risk-sharing is not feasible, at least one agent is constrained by its ex post self-enforcing constraints. The second result of Lemma 2 states that each agent always has at least one ex post self-enforcing constraint binding. The intuition for this result is the following. Suppose only one agent is constrained. This implies that the constrained agent could increase marginally its up front payment, adjust its ex post payments to maintain its level of consumption, and hence relax its ex post self-enforcing constraints. At the margin, this would not violate the other agent's ex post self-enforcing constraints which were not binding before the increase in the ex ante payment. This would therefore increase the utility of a least one agent. Such increase in the ex ante payment by one agent is possible until one of the other agent's self-enforcing constraint becomes binding, in which case further increases may not be self-enforcing anymore. Therefore, in the optimal contract each agent always has at least one ex post self-enforcing constraint binding.

The next proposition provides a characterization of the optimal ex ante payment when the discount factor does not allow optimal risk-sharing.

**Proposition 4** *Assume that* $\beta < \beta_{nc}$.
*(i) The optimal value of the ex ante payment in period t is strictly decreasing in the expected surplus that agent 1 has to concede to agent 2 in period t, that is,* $V_t' > V_t'' \Rightarrow B_t' < B_t''$ *where* $B_t'$ $(B_t'')$ *is optimal if the expected surplus in period t is* $V_t'$ $(V_t'')$.
*(ii) The optimal ex ante payment is strictly positive when agent 2 has a zero expected surplus and negative when agent 2 has maximal expected surplus, that is,* $B_t > 0$ *if* $V_t = 0$ *and* $B_t < 0$ *if* $V_t = \bar{V}$.

This result states that the ex ante payment is used optimally to relax ex post self-enforcing constraints. Furthermore, the optimal ex ante payment is decreasing in the expected surplus of agent 2. Suppose that, following a given history, the contract promises a low expected surplus to agent 2. This makes the contract not much more profitable than autarky and thus agent 2's ex post self-enforcing constraints are likely to be more constraining than agent 1's ex post self-enforcing constraints. In this case, agent 2 optimally pays out a relatively large ex ante payment to relax his ex post self-enforcing constraints. The size of the optimal ex ante payment is inversely related to the expected surplus of agent 2. This logic can easily be extended to show that the optimal ex ante payment is negative when agent 2 expects a high surplus from the relationship, that is, agent 1 pays out to agent 2 a high ex ante transfer.

It is difficult to provide a more complete characterization of the solution in the general case given the number of inequality constraints; however, we can do so in a special case in which there are only two states. This simple example is sufficient to illustrate the role of the ex ante payment. We then compare our results with the case in which no ex ante payments are allowed. Suppose that $S = 2$ and assume that the discount factor is such that optimal risk-sharing is not self-enforcing.

**Proposition 5** *Suppose that $S = 2$ and $\beta < \beta^{nc}$.*
*(i) The expected profit of agent 2 for period $t+1$ is larger (smaller) than that of period $t$ if state 1 (2) occurs in period $t$, that is, $V(\delta^{nc}; h_{t-1}, 2) \leq V(\delta^{nc}; h_{t-1}) \leq V(\delta^{nc}; h_{t-1}, 1)$, with strict inequality if $V(\delta^{nc}; h_{t-1}) \notin \{0, \bar{V}\}$.*
*Suppose that $0 < V_t < \bar{V}$ and define $c_t^s = c(h_{t-1}, s)$ and $c_{t-1} = c(h_{t-1})$.*
*(ii) Agent 1's intertemporal marginal rate of substitution is larger (smaller) than agent 2's if state 1 (2) occurs, that is, $\frac{u'(c_t^1)}{u'(c_{t-1})} > \frac{v'(b+y_t^1-c_t^1)}{v'(b+y_{t-1}-c_{t-1})}$ and $\frac{u'(c_t^2)}{u'(c_{t-1})} < \frac{v'(b+y_t^2-c_t^2)}{v'(b+y_{t-1}-c_{t-1})}$.*
*(iii) Agent 1's consumption in period $t$ is smaller (larger) than $c^*(c_{t-1}, y_{t-1}^q, s)$ if $s = 1$ (if $s = 2$) in period $t$ where $q$ is the realized state in period $t-1$, that is, $c_t^1 < c^*(c_{t-1}, y_{t-1}^q, 1)$ and $c_t^2 > c^*(c_{t-1}, y_{t-1}^q, 2)$.*

This proposition states that in a two-state example if the bad state occurs (state 1) agent 1 borrows from agent 2 to smooth her consumption, that is, $V_{t+1}^1 \geq V_t$. Alternatively, agent 1 lends (or reimburses) to agent 2 if state 2 occurs, that is $V_{t+1}^2 \leq V_t$. This allows the best consumption smoothing possible for agent 1. This same result can be translated in terms of each agent's intertemporal marginal rate of substitution. In state 1, self-enforcing constraints are such that agent 1 has stronger preferences than agent 2 for the present and she therefore borrows from agent 2. The opposite is true in state 2 and agent 2 lends to agent 1. The last result of the proposition states that in general the contract cannot achieve optimal risk-sharing because of self-enforcing constraints. Consequently the two agents bear more risk than they do in the full-commitment case.

These results may seem quite similar to those one would obtain when no ex ante payments are allowed. The dynamics of consumption is however quite different in the two cases. Consider first the case where no ex ante payments are allowed. The results of Proposition 3 imply that optimal consumption takes place at $\bar{c}^1$ ($\underline{c}^2$) if state 1 (2) occurs. These consumption levels are time-invariant and therefore consumption can only take one of these two values depending on the realized state. At any given period, expected consumption for next period is the same regardless of the history. Now consider the case where ex ante payments are allowed. Proposition 5 states that $V_{t+1}^2 \leq V_t \leq V_{t+1}^1$. Proposition 4 states that the ex ante payment from agent 1 to agent 2 will decrease (increase) in period $t+1$ compared to that of period $t$ if state 1 (2) occurs. Since the optimal consumption bounds $\bar{c}^1$ and $\underline{c}^2$ are increasing with the ex ante payment,[12] then if state 1 occurs in period $t$, agent 2 is promised a higher expected surplus for period $t+1$ and this implies that he will make a lower ex ante payment in period $t+1$ thus reducing the two consumption bounds for that period. Expected consumption will then be lower in period $t+1$ than in period $t$. The opposite holds if state 2 occurs in period $t$. Expected consumption rises in period $t+1$ compared to period $t$. The consumption pattern with an ex ante payment looks like "experience rating", that is, average consumption in one period is inversely related to the state variable and thus is positively related to the previous realization of the state of nature. This implies that the complete history up to period $t$ may be potentially relevant in explaining period $t$ consumption. Our model can therefore generate higher order correlation in consumption even though endowments are independently distributed. In fact, the introduction of the ex ante payment allows better risk-sharing across states within a period at the expense of worse consumption smoothing across periods. This shows that neither asymmetric information nor uncertainty and learning are necessary to explain experience rating in insurance contracts. This simple example shows that allowing for an ex ante payment yields predictions that are significantly different from those without ex ante payments.

# 5 Economic applications

This section illustrates some economic applications of contracting models with self-enforcing constraints of the type modeled here. The possibility to make ex

[12]This is a straight extension of Proposition 3 and Corollary 1.

ante payments allows agents to increase somewhat their commitment possibilities with the result that they generally can do better with this payment than without. The economic interpretation that one can give to the ex ante payment can vary with the economic environment and we will try to motivate the relevance of the contractual form with ex ante payments for different environments.

### 5.1 Insurance contracts

The model presented here can represent the contracting relationship between a risk-averse insuree and a risk-neutral insurer. Although the model is a very abstract representation of the insurance market, it still includes some important features of it. At first, we would not think of insurance companies as facing self-enforcing constraints because every insuree is small compared to the complete portfolio of insurees the insurer deals with. However, at a more aggregate level, insurors appear to face self-enforcing constraints. For example, regulation aiming at forcing insurors to keep a certain reserves ratio can be seen as a measure to weaken self-enforcing (or limited liability) constraints. Such measures necessarily increase the cost to the insurer of defaulting on its insurance compensations. Furthermore, insurees face self-enforcing constraints and this is reflected in the fact that insurance companies always require insurees to pay their premium up front before the insurance policy takes effect. The premium is such that all ex post payments are made to the insuree.

In this context, the ex ante payment $B$ could be interpreted as the liquidity reserves that the insurer sets aside for the insuree in case of defaulting net of the premium paid by the insuree. The net ex ante payment can then be positive or negative depending on the expected surplus (solvability) of the insurer. With this interpretation, the difference $B - a^s$ is just the net payment made to the insuree in state $s$. The model then offers some interesting predictions about insurance premia. Consider the two-state example of the previous section. Suppose liquidity reserves remain relatively fixed compared to insurance premia. Most changes in the net payment $B$ can therefore be attributed to changes in insurance premia. We have seen that the expected surplus of the insurer (given by the state variable) increases if state 1 occurs and decreases if state 2 occurs. This can be interpreted as the insurance premium increasing when an accident occurs and decreasing when no accident occurs. Furthermore the dynamics of our model implies that the complete history is relevant for explaining contemporeanous insurance premium, that is, a sequence of accidents results in successive increases in insurance premia and therefore a drop in expected consumption. This feature is absent without ex ante payments as expected consumption remains fixed. The dynamic properties of our model mimics experience rating. Experience rating arises from the desire of the insuree to share risk and smooth consumption over time. This is optimally achieved by having a premium increase when an accident occurs. In this case, the current marginal utility of the insuree is high and it promises higher premia in the future in exchange for a high current compensation. Our model therefore predicts that experience rating can take place in an insurance market even though information is symmetric and shocks are identically and independently distributed. The optimal characterization seems to be consistent with stylized facts. However, a thorough analysis of the insurance market would require a more complete formalization that would take into account more institutional details. Such details could include the possibility for the insurer to hold a whole portfolio of insurees, to have many insurors competing in every period, etc.

### 5.2 Long-term supply relationships

Suppose two agents enter a long-term relationship in which a supplier offers some service to a buyer. Suppose the net value of this service to the buyer is stochastic and given by $y^s$. The two parties sign a contract specifying for each period net payments $B - a^s$ contingent on the buyer's value of the service. Assume that the buyer's value for the service $y^s$ is observable to both agents but non-verifiable by third parties. Furthermore, the contractual relationship itself is assumed to be verifiable and it is therefore feasible to include in the contract a penalty if the contract is broken. Our model represents the case in which courts cannot observe who breached the contract and thus the penalty is not contingent on the identity of the agent who defaulted. If the contractual relationship is ever broken, a breach-of-contract penalty $B$ has to be paid to the buyer. In this framework, ex post self-enforcing constraints must be satisfied to ensure that all prescribed payments $B - a^s$ are effectively made. Ex ante self-enforcing constraints simply represent participation constraints for both agents at the beginning of each period. In this framework, the explicit part of the contract specifies that a service must be provided by the supplier in every period. If, for some unspecified reason, the service is not provided then a (possibly negative) penalty $B$ is imposed on the supplier. The implicit part of the contract specifies the contingent prices at which this service is to be provided. These prices cannot be included in the explicit contract as they are not legally enforceable.

In this situation, our model would predict that perfect risk-sharing is not achieved in general and that the buyer's profits are serially correlated as its consumption depends on the complete history of past values of the service. Furthermore, the breach-of-contract penalty paid to the buyer is history-dependent and inversely related to the surplus the supplier expects from the relationship. This seems consistent with legal rules that impose that, upon a breach of contract, penalties should compensate the parties involved proportionally to the value of the contract to them. For example, if the supplier expects a low surplus from the relationship (the state variable is low), a breach of contract will result in a high penalty paid by the supplier to the buyer since most surplus from the contract was expected to accrue to the buyer. If, however, the supplier expects a high surplus from the relationship, a breach of contract will result in a low penalty (possibly negative) paid by the supplier to the buyer. The model therefore shows that having breach-of-contract penalties proportional to the value of the contract is optimal even in an environment in which courts cannot verify which agent broke the relationship.

### 5.3 Sovereign debt

Self-enforcing constraints have been emphasized in the analysis of sovereign debt.[13] The current model can contribute to this literature to help explain the dynamics of transfer payments between a sovereign agent, say agent 1 and financiers, say agent 2. In this case, the reimbursement or borrowing of new debt can be represented by the net ex post contingent payments $B - a^*$; the ex ante payment $B$ can represent a credit line that has been opened up by financiers for the benefit of the sovereign agent. Such a credit line is opened up at the beginning of the period and is committed to until the end of the period at which time the country can draw upon it. A credit line is an actual commitment by financiers to pay a certain amount to the sovereign agent regardless of the realization of the state of nature. This interpretation makes sense for positive values of the ex ante payment $B$ but is not as convincing for negative values as sovereign agents may not be able to commit to such payments. The model can easily be adjusted to take this institutional detail into account by adding the constraint that $B \geq 0$. The optimal solution would have a similar characterization with the difference that the ex ante payment would be zero (instead of negative) for certain values of the state variable. The use of a credit line can therefore be seen as a technology allowing some commitment to be introduced in the financing relationship.

Credit lines therefore have a value in risk-sharing contracts with self-enforcing constraints as they increase the gains from trade. They allow financiers to commit to a given level of financing even though they expect a low surplus in the future. In fact, such credit lines have been used, namely by the IMF. Furthermore our model predicts that financial flows will be state-contingent. This seems to be consistent with empirical observation which suggests that sovereign financial contracts do not always specify fixed reimbursement payments. For example, Mexico, an oil exporting country, has some financial contracts which specify reimbursement to be contingent on the price of oil.

[13] See for example Bulow and Rogoff (1989), Fernandez and Rosenthal (1990), and Kletzer and Wright (1990).

## 5.4 A dynamic theory of the firm

The last interpretation of our model we would like to offer bears similarities with the previous interpretation of the sovereign agent. Suppose that agent 1 is some entrepreneur or residual claimant that produces every period a stochastic output valued at $y^s$. This entrepreneur can diversify her risk by issuing financial claims to financiers. The ex ante payment $B$ can be interpreted either as a credit line to which financiers commit to at the beginning of a period (when $B$ is positive) or as collateral put up by the entrepreneur (when $B$ is negative). A credit line allows the entrepreneur to draw upon it regardless of the state of nature, while collateral compensates financiers in case of default. The contingent payments $B - a^s$ represent dividend payments or debt reimbursements to financiers. The two agents face self-enforcing constraints that specify that no agent can be forced to continue the relationship if its expected surplus is negative. In this context self-enforcing constraints are naturally associated with limited-liability (or bankruptcy) constraints.

The model may explain the endogenous presence of financial intermediary that have the necessary technology to commit to a given level of financing through the use of credit lines. Such technology is not generally available for firms financed exclusively through small investors. An intermediary using a credit line can commit to better risk-sharing opportunities for the entrepreneur by providing a minimum level of financing even though the expected profitability for the financier is low. The model can also explain the use of collateralized loans as a way of committing to reimburse due claims. Our explanation contrasts with other explanations of collateral based on asymmetric information.[14] The model can also provide a basic framework to analyze flows of funds between a firm and its financiers. The model would predict some correlations between a firm's profit ($y^s$), its market value ($V^s$), and the net flows of funds to the financial market ($B - a^s$); however we do believe at this stage that the model is too abstract to tackle such issues. For example, it would be better to include investment, correlated shocks, etc. Nevertheless a model with self-enforcing constraints provides a simple

[14] For example, see Bester (1985), Besanko and Thakor (1987), and Stiglitz and Weiss (1986).

framework to develop more realistic models of firm behaviour.

## 6 Conclusion

We develop a model of contracting for risk-sharing purposes. Complete insurance is impeded by ex post opportunism in that agents can break the relationship at any time if it is in their own interest to do so. However, agents can commit partially by making payments at the beginning of a period before the state of nature is realized. We show that these payments can increase the potential gains from trade but cannot generally restore perfect risk-sharing.

The ex ante payment can be interpreted as a bond. It is well known that posting a bond is one way of avoiding self-enforcing constraints, namely, the agent that cannot commit or that must be disciplined simply posts a bond that it may lose if it does not perform satisfactorily. For example, Williamson (1983) illustrates how the use of a bond can promote efficient trade. In this context, our model can be reinterpreted as modeling a situation in which the two agents must be disciplined and each agent can run away with the bond that has been posted by the other agent. This puts endogenous limits to the size of the bond that each agent can post and therefore bonding becomes a non-trivial solution to the incentive problem. Our model would therefore predict that the net bond posted by one agent is inversely proportional to its expected surplus from the relationship.

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## APPENDIX

**Proof of Proposition 2** (i) Consider the optimal full-commitment contract $\delta^{fc}$ characterized in Proposition 1. The per-period surplus to agent 1 is $Eu(c^{qfc}) - Eu(y^q)$ which is positive. Hence $U(\delta^{fc}; h_{t-1}) > 0$ for all histories $h_{t-1}$ and periods $t$. This implies that all ex ante self-enforcing constrains are satisfied. Suppose that $\underline{B} \geq y^S - c^{Sfc}$ and that agent 1 makes the maximum ex ante payment, namely, $B_t = -\underline{B}$. From Proposition 1 we know that $a_t^s = y_t^s + B_t - c^{sfc}$. If $B_t = -\underline{B}$, then $a_t^s \leq y_t^s - y^S + c^{Sfc} - c^{sfc}$. Since the transfer from agent 1 to agent 2 is largest when $s = S$, we have that $a_t^s \leq 0$ for all $s$, and hence no ex post self-enforcing constraints are binding. The contract $\delta^{fc}$ can then be supported as the optimal contract $\delta^1$.

(ii) Assume that $\underline{B} < y^S - c^{Sfc}$. Consider the optimal full-commitment contract $\delta^{fc}$. As argued above, all ex ante self-enforcing constraints are satisfied. Ex post self-enforcing constraints are satisfied if and only if

$$u(c^{sfc}) - u(y_t^s + B_t) + \frac{\beta}{1-\beta}\left(Eu(c^{qfc}) - Eu(y^q)\right) \geq 0 \quad \forall s.$$

These constraints become less binding if $B_t$ is set at its lowest level, namely, $B_t = -\underline{B}$ and net transfers are adjusted such that agent 1's consumption be $c^{sfc}$. Setting $B_t = -\underline{B}$ and solving for $\beta$ in the ex post self-enforcing constraint yields

$$\beta \geq \frac{u(y_t^s - \underline{B}) - u(c^{sfc})}{u(y_t^s - \underline{B}) - Eu(y^q) + Eu(c^{qfc}) - u(c^{sfc})} \text{ for all } s.$$

The critical value $\beta_1$ above which all ex post self-enforcing constraints are satisfied is given by the maximum over $s$ of the right-hand-side of the inequality.[15] Hence all (ex ante and ex post) self-enforcing constraints for agent 1 are satisfied for $\beta \in [\beta_1, 1)$. For values of $\beta$ outside this interval, one of the ex post self-enforcing constraints for agent 1 is violated and thus the optimal full-commitment contract is not feasible when agent 1 can renege on the contract.

(iii) The proof of this part of the proposition is more involved and we need to introduce some notation. Define by $\Omega(h_{t-1})$ the set of contracts satisfying the self-enforcing constraints for agent 1 following history $h_{t-1}$. The Pareto frontier that is attainable by a self-enforcing contract is given by

$$g(V_t) = \max_{\delta \in \Omega(h_{t-1})} \{U(\delta; h_{t-1}) \text{ s/t } V(\delta; h_{t-1}) \geq V_t\}.$$

The Pareto frontier is time-independent as all constraints defining $\Omega(h_{t-1})$ and the functions $U(\delta; h_{t-1})$ and $V(\delta; h_{t-1})$ are forward looking. This Pareto frontier can be used to characterize the optimal contract. Following any history, the optimal contract $\delta^1$ will necessarily be efficient, since if it was not it would be possible to replace the non-efficient path by an efficient path thus relaxing all previous self-enforcing constraints. This new contract would necessarily be self-enforcing and would dominate the old contract at the beginning of the relationship. This argument implies that the optimal contract from the start of period $t$ onwards is the solution to the following maximization problem.

$$\begin{aligned} g(V_t) = \max_{B_t, (a_t^s)_s, (V_{t+1}^s)_s} & E\left\{u(y_t^s + B_t - a_t^s) - u(y_t^s) + \beta g(V_{t+1}^s)\right\} \\ & g(V_{t+1}^s) \geq 0 \quad \forall s \\ & u(y_t^s + B_t - a_t^s) - u(y_t^s + B_t) + \beta g(V_{t+1}^s) \geq 0 \quad \forall s \\ & V_t \leq E\left\{v(b - B_t + a_t^s) - v(b) + \beta V_{t+1}^s\right\} \end{aligned} \qquad (8)$$

[15]Since the number of states is finite, this maximum clearly exists.

In this problem, the variable $V^s_{t+1}$ is to be interpreted as $V(\delta;h_{t-1},s)$, that is, agent 2's expected utility from period $t+1$ onwards when contract $\delta$ is signed, $h_{t-1}$ is the history up to period $t$, and $s$ is the realized state of nature in period $t$. The first two constraints represent agent 1's self-enforcing constraints. The last constraint of the problem ensures that the contract is dynamically consistent. We will first show that problem (8) is a concave program and then use its first-order conditions to characterize the optimal contract $\delta^1$.

To show that problem (8) is a concave program, we show that the Pareto frontier is strictly concave and continously differentiable.[16] We first have to show that the set $\Omega(h_{t-1})$ is convex. Consider two self-enforcing contracts for agent 1, $\delta'$ and $\delta''$ with associated payments $\{a'(h_t), B'(h_t)\}$ and $\{a''(h_t), B''(h_t)\}$ respectively. It is easily shown that because the utility function $u(\cdot)$ is concave and quadratic, any linear combination of these two contracts $\delta^\lambda$ where $a^\lambda(h_t) = \lambda a'(h_t)+(1-\lambda)a''(h_t)$ and $B^\lambda(h_t) = \lambda B'(h_t) + (1-\lambda)B''(h_t)$ is also self-enforcing for agent 1. Secondly, following history $h_{t-1}$, the set of $V_t$ such that a self-enforcing contract for agent 1 exists is a compact interval $[-K_1, \bar{V}]$ where $-K_1$ is the discounted utility of agent 2 when it pays out to agent 1 transfers yielding her a consumption of $b$ in every state and period.[17] Such transfers are obviously self-enforcing for agent 1. There exists an upper bound on the surplus agent 1 can concede to agent 2 in a self-enforcing contract. Denote this upper bound by $\bar{V}$. If $\bar{V}$ is attainable by a self-enforcing contract, then any $V_t \in [-K_1, \bar{V}]$ is also. The closedness of this interval can be shown by constructing a sequence of self-enforcing contracts yielding some utility level to agent 2 converging to $\bar{V}$. Since $u(\cdot)$ is continuous and $\beta \in (0,1)$, the Dominated Convergence Theorem implies that the limiting contract is also self-enforcing and hence $\bar{V}$ is included in the interval. Finally, we show that the Pareto frontier is decreasing, strictly concave and continuously differentiable. It is obvious that the function $g(\cdot)$ is decreasing. The strict concavity property follows from the strict concavity of $u(\cdot)$, the concavity of $v(\cdot)$, and the convexity of $\Omega(\cdot)$. The differentiability property follows from the continuity and differentiability of $u(\cdot)$. Consider an efficient self-enforcing contract $\delta$ such that $V(\delta;h_{t-1}) = V_t \in (-K_1, \bar{V})$. Construct a contract $\delta^\gamma$ which differs from the contract $\delta$ in that $a^\gamma(h_{t-1},s) = a(h_{t-1},s)+\gamma$. The state $s$ is chosen such that agent 1's ex post self-enforcing constraint is not strictly binding. The contract $\delta^\gamma$ is therefore self-enforcing for $\gamma$ small enough. Define the function $\hat{g}$ such that $U(\delta^\gamma;h_{t-1}) = \hat{g}(V(\delta^\gamma;h_{t-1})) \le g(V(\delta;h_{t-1}))$ with equality if $\gamma = 0$. As $\gamma$ is varied, it is easy to show that the function $\hat{g}$ is concave and differentiable at $V_t$. Therefore it satisfies Lemma 1 reported in Benveniste and Scheinkman (1979). The function $g(\cdot)$ is then differentiable. Since it is monotonic, it is also continously differentiable. This implies that for any value $V_t \in [-K_1, \bar{V}]$, there exists a unique efficient continuation of the contract $\delta$ at time $t$ in which $V(\delta;h_{t-1}) = V_t$ and $U(\delta;h_{t-1}) = g(V_t)$. Existence is guaranteed by the compactness of the interval $[-K_1, \bar{V}]$; uniqueness is guaranteed by the convexity of $\Omega(\cdot)$ and the strict concavity of $u(\cdot)$. These results effectively imply that problem (8) is a concave program, and therefore its first-order conditions are both necessary and sufficient for a solution. Let $\beta p^s\alpha^s$, $p^s\theta^s$ and $\psi$ be the respective multipliers of the constraints in problem (8), and $\mu_1$ and $\mu_2$ the multipliers for the constraints on the lower and upper bounds of $B_t$. The first-order conditions are then

$$B_t: \ \sum_s p^s u'(y^s_t + B_t - a^s_t) + \sum_s p^s\theta^s\left(u'(y^s_t + B_t - a^s_t) - u'(y^s_t + B_t)\right) - \sum_s \psi p^s v'(b - B_t + a^s_t) + \mu_1 - \mu_2 = 0 \tag{9}$$

$$a^s_t: \ -p^s(1+\theta^s)u'(y^s_t + B_t - a^s_t) + p^s\psi v'(b - B_t + a^s_t) = 0 \quad \forall s \tag{10}$$

$$V^s_{t+1}: \ (1+\alpha^s+\theta^s)g'(V^s_{t+1}) + \psi = 0 \quad \forall s \tag{11}$$

[16]Most of the arguments used here follow those of Lemma 1 of Thomas and Worrall (1988).
[17]Remember that agent 1's utility function is defined over the interval $[a, b]$.

and the envelope condition is $g'(V_t) = -\psi$.

1. Summing over $s$ all conditions (10) and substituting in condition (9) yield $-\sum_s p^s\theta^s u'(y_t^s + B_t) + \mu_1 - \mu_2 = 0$. If there is at least one self-enforcing constraint that binds, the multiplier $\mu_1 > 0$ and therefore $B_t = -\underline{B}$. If no self-enforcing constraint binds, only net payments matter and hence $B_t$ can be set arbitrarily at its highest level $\underline{B}$, namely the level for which ex post self-enforcing constraints are the least binding.

2. Condition (11) and the envelope condition jointly imply that $(1 + \alpha^s + \theta^s)g'(V_{t+1}^s) = g'(V_t)$. Because $\alpha^s + \theta^s \geq 0$ and the Pareto frontier $g(\cdot)$ is decreasing and concave, it implies that $V_{t+1}^s \leq V_t$ with strict inequality when $\alpha^s + \theta^s > 0$.

3. We now show that agent 1's implicit discount factor is not larger than agent 2's. Suppose first that $V_t < \bar{V}$. The above result implies that $\alpha^s = 0$ for all states $s$. Conditions (10) and (11) then imply that $\frac{u'(y_t^s - \underline{B} - a_t^s)}{v'(b + \underline{B} + a_t^s)} = -g'(V_{t+1}^s)$. Furthermore, efficiency of the optimal contract in period $t-1$ implies that $\frac{u'(y_{t-1} - \underline{B} - a_{t-1})}{v'(b + \underline{B} + a_{t-1})} = -g'(V_t)$. Using the result that $V_{t+1}^s \leq V_t$, we then have that $\frac{u'(y_{t-1} - \underline{B} - a_{t-1})}{v'(b + \underline{B} + a_{t-1})} \geq \frac{u'(y_t^s - \underline{B} - a_t^s)}{v'(b + \underline{B} + a_t^s)}$ for all states $s$. Rearragning terms and taking expectation over the states $s$ then prove the result. Suppose now that $V_t = \bar{V}$. When $V_{t+1}^s = \bar{V}$, the condition $(1 + \alpha^s + \theta^s)g'(V_{t+1}^s) = g'(V_t)$ implies that $\alpha^s + \theta^s = 0$. We then have that $\frac{u'(y_t^s - \underline{B} - a_t^s)}{v'(b + \underline{B} + a_t^s)} = -g'(V_{t+1}^s) = -g'(V_t)$. A similar argument for period $t-1$ implies that $\frac{u'(y_{t-1} - \underline{B} - a_{t-1})}{v'(b + \underline{B} + a_{t-1})} = -g'(V_t)$. When $V_{t+1}^s < \bar{V}$, the multiplier $\alpha^s = 0$. We then have that $\frac{u'(y_t^s - \underline{B} - a_t^s)}{v'(b + \underline{B} + a_t^s)} = -g'(V_{t+1}^s) < g'(V_t)$. This implies that $\frac{u'(y_{t-1} - \underline{B} - a_{t-1})}{v'(b + \underline{B} + a_{t-1})} \geq \frac{u'(y_t^s - \underline{B} - a_t^s)}{v'(b + \underline{B} + a_t^s)}$ for all states $s$ and the result follows. Finally note for future reference that these arguments imply that $\alpha^s = 0$ for all states $s$ and time periods $t$.

*Q.E.D.*

**Proof of Corollary 1** (i) Denote by $\bar{V}^s$ and $\underline{c}^s$ the optimal maximum and minimum values for $V_{t+1}^s$ and $c_t^s$ respectively such that there exists a self-enforcing contract for agent 1. These values are implicitly defined by

$$\begin{aligned} \frac{u'(\underline{c}^s)}{v'(b + y^s - \underline{c}^s)} &= -g'(\bar{V}^s) \\ u(\underline{c}^s) - u(y^s - \underline{B}) + \beta g(\bar{V}^s) &= 0. \end{aligned}$$

The first equation follows from first-order conditions to problem (8) and the fact that $\alpha^s = 0$ for all $s$ while the second represents agent 1's ex post self-enforcing constraint in state $s$. Note that these equations are time-independent. After substituting for $\bar{V}^s$ in agent 1's ex post self-enforcing constraint, the optimal bound on consumption, $\underline{c}^s$, is then implicitly defined by

$$u(\underline{c}^s) - u(y^s - \underline{B}) + \beta g\left(g'^{-1}\left(-\frac{u'(\underline{c}^s)}{v'(b + y^s - \underline{c}^s)}\right)\right) = 0. \tag{12}$$

The left-hand side of the ex post self-enforcing constraint is increasing in $\underline{c}^s$ which implies that it is satisfied for $c_t^s \geq \underline{c}^s$ for all $t$ and $s$.

(ii) Differentiating along equation (12) yields

$$\frac{d\underline{c}^s}{dy^s} = -\frac{\beta g' g'^{-1\prime} * \left(\frac{u'(\underline{c}^s)v''(b+y^s-\underline{c}^s)}{v'(b+y^s-\underline{c}^s)^2}\right) - u'(y^s - \underline{B})}{u'(\underline{c}^s) + \beta g' g'^{-1\prime} * \left(-\frac{u''(\underline{c}^s)v'(b+y^s-\underline{c}^s) + u'(\underline{c}^s)v''(b+y^s-\underline{c}^s)}{v'(b+y^s-\underline{c}^s)^2}\right)}$$

which is positive since $g' g'^{-1\prime} \geq 0$.[18] Hence, $\underline{c}^s$ is increasing in the states of the world, that is, $\underline{c}^k > \underline{c}^q$ if and only if $y^k > y^q$.

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[18] Since the function $g$ is continuously differentiable and concave we know that $g'^{-1\prime}$ exists almost everywhere. Where it does not exists, we know that the right-hand and left-hand derivatives are negative.

Finally, differentiating along equation (12) yields

$$\frac{d\underline{c}^s}{d\underline{B}} = -\frac{u'(y^s - \underline{B})}{u'(\underline{c}^s) + \beta g' g'^{-1'} * \left(-\frac{u''(\underline{c}^s)v'(b+y^s-\underline{c}^s)+u'(\underline{c}^s)v''(b+y^s-\underline{c}^s)}{v'(b+y^s-\underline{c}^s)^2}\right)} < 0.$$

(iii) From the first-order conditions we know that

$$(1+\theta^s)\left(u'(c_t^s)/v'(b+y_t^s-c_t^s)\right) = u'(c_{t-1})/v'(b+y_{t-1}-c_{t-1}).$$

Suppose first that $c^*(c_{t-1}, q, s) \geq \underline{c}^s$. This condition is satisfied when $c_t^s = c^*(c_{t-1}, q, s) \geq \underline{c}^s$ and $\theta^s = 0$. Now suppose that $c^*(c_{t-1}, q, s) < \underline{c}^s$. This condition is satisfied when $c_t^s = \underline{c}^s$ and $\theta^s > 0$. *Q.E.D.*

**Proof of Lemma 1** (i) From the proof of Proposition 2 we know that $\Omega(h_{t-1})$ is convex. By symmetry, the set of self-enforcing contracts for agent 2 is also convex. The set $\Lambda(h_{t-1})$ is the intersection of these two convex sets and is therefore convex.
(ii), (iii), (iv) The rest of the proof follows that of Proposition 2 with minor modifications. *Q.E.D.*

**Proof of Proposition 3** Denote respectively by $\beta p^s \alpha^s$, $p^s\theta^s$, $\beta p^s \phi^s$, $p^s\lambda^s$, and $\psi$ the multipliers of the five constraints in problem (4). The first-order conditions when $B_t = 0$ for all time periods are

$$a_t^s: \quad -p^s(1+\theta^s)u'(y_t^s + B_t - a_t^s) + p^s(\lambda^s + \psi)v'(b - B_t + a_t^s) = 0 \quad \forall s \tag{13}$$
$$V_{t+1}^s: \quad (1+\alpha^s+\theta^s)f'(V_{t+1}^s) + \lambda^s + \phi^s + \psi = 0 \quad \forall s \tag{14}$$

and the envelope condition is $f'(V_t) = -\psi$.
(i) First-order conditions imply that

$$(1+\theta^s)\frac{u'(c_t^s)}{v'(b+y_t^s-c_t^s)} = -(1+\alpha^s+\theta^s)f'(V_{t+1}^s) - \phi^s. \tag{15}$$

We first define the optimal lower bounds on consumption. Suppose first that $\phi^s = \alpha^s = 0$. Condition (15) then implies $u'(c_t^s)/v'(b+y_t^s-c_t^s) = -f'(V_{t+1}^s)$. This can be rewritten as $V_{t+1}^s = f'^{-1}\left(-u'(c_t^s)/v'(b+y_t^s-c_t^s)\right)$. It is easily shown that the right-hand-side of this expression is decreasing in $c_t^s$. A lower bound on consumption is therefore associated with an upper bound on $V_{t+1}^s$. If $\alpha^s > 0$, then $V_{t+1}^s = \bar{V}$. The lower bound on consumption, $\underline{c}^s$, is defined by the intersection of this expression and agent 1's ex post self-enforcing constraint. More formally,

$$u(\underline{c}^s) - u(y^s) + \beta f\left(\min\left\{f'^{-1}\left(-u'(\underline{c}^s)/v'(b+y^s-\underline{c}^s)\right), \bar{V}\right\}\right) = 0 \tag{16}$$

where $f(\bar{V}) = 0$. This expression states that optimal lower bounds on consumption are defined by the intersection of first-order conditions and agent 1's ex post self-enforcing constraint to the extent that they respect the ex ante self-enforcing constraints; otherwise the expression reduces to $\underline{c}^s = y^s$. It is clear from expression (16) that $\underline{c}^s \leq y^s$. Note that these optimal lower bounds are time-independent.
We now define the optimal upper bounds on consumption. The expression

$$V_{t+1}^s = f'^{-1}\left(-u'(c_t^s)/v'(b+y_t^s-c_t^s)\right)$$

is substituted in agent 2's ex post self-enforcing constraint. More formally,

$$v(b+y^s-\bar{c}^s) - v(b) + \beta\left(\max\left\{f'^{-1}\left(-u'(\bar{c}^s)/v'(b+y^s-\bar{c}^s)\right), 0\right\}\right) = 0. \tag{17}$$

This expression states that optimal upper bounds on consumption are defined by the intersection of first-order conditions and agent 2's ex post self-enforcing constraint to the extent that they respect the ex ante self-enforcing constraints; otherwise the expression reduces to $\bar{c}^s = y^s$. It is clear from expression (17) that $\bar{c}^s \geq y^s$. Again, note that these optimal upper bounds are time-independent. The preceding arguments show that in any time period $t$ and state $s$, consumption $c_t^s$ must be included in the interval $[\underline{c}^s, \bar{c}^s]$; otherwise one of the self-enforcing constraints or first-order conditions would be violated.

(ii) We now show that the optimal lower bounds are increasing in the states of the world. The optimal lower bounds are implicitly defined as a function of $y^s$ in expression (16). This expression is continuous in $\underline{c}^s$ and $y^s$ but is not differentiable at one point (where the minimum switches from $f'^{-1}\left(-u'(\underline{c}^s)/v'(b + y^s - \underline{c}^s)\right)$ to $\bar{V}$). When the minimum equals $\bar{V}$, $\underline{c}^s = y^s$ and clearly the optimal bound is increasing in the state of the world. When the minimum equals the first expression, total differentiation of the implicit function yields

$$\frac{d\underline{c}^s}{dy^s} = -\frac{\beta f' f'^{-1\prime} * \left(\frac{u'(\underline{c}^s)v''(b+y^s-\underline{c}^s)}{v'(b+y^s-\underline{c}^s)^2}\right) - u'(y^s)}{u'(\underline{c}^s) + \beta f' f'^{-1\prime} * \left(-\frac{u''(\underline{c}^s)v'(b+y^s-\underline{c}^s)+u'(\underline{c}^s)v''(b+y^s-\underline{c}^s)}{v'(b+y^s-\underline{c}^s)^2}\right)}$$

which is positive since $f' f'^{-1\prime} > 0$.[19] Hence, because $\underline{c}^s$ is a continous implicit function of $y^s$, these results imply that $\underline{c}^s$ is increasing in the states of the world, that is, $\underline{c}^k > \underline{c}^q$ if and only if $y^k > y^q$. We now show that the optimal upper bounds are increasing in the states of the world. The optimal upper bounds are implicitly defined as a function of $y^s$ in expression (17). This expression is continuous in $\bar{c}^s$ and $y^s$ but is not differentiable at one point (where the maximum switches from $f'^{-1}\left(-u'(\bar{c}^s)/v'(b + y^s - \bar{c}^s)\right)$ to 0). When the maximum equals 0, $\bar{c}^s = y^s$ and clearly the optimal bound is increasing in the state of the world. When the maximum equals the first expression, total differentiation of the implicit function yields

$$\frac{d\bar{c}^s}{dy^s} = -\frac{v'(b + y^s - \bar{c}^s) + \beta f'^{-1\prime} * \left(\frac{u'(\bar{c}^s)v''(b+y^s-\bar{c}^s)}{v'(b+y^s-\bar{c}^s)^2}\right)}{-v'(b + y^s - \bar{c}^s) + \beta f'^{-1\prime} * \left(-\frac{u''(\bar{c}^s)v'(b+y^s-\bar{c}^s)+u'(\bar{c}^s)v''(b+y^s-\bar{c}^s)}{v'(b+y^s-\bar{c}^s)^2}\right)}$$

which is positive since $f'^{-1\prime} \leq 0$. Hence, because $\bar{c}^s$ is a continous implicit function of $y^s$, these results imply that $\bar{c}^s$ is increasing in the states of the world, that is, $\bar{c}^k > \bar{c}^q$ if and only if $y^k > y^q$.

(iii) The proof of this part of the proposition requires proving the following preliminary result.

**Lemma 3** *The multipliers $\alpha^s = \phi^s = 0$ for all $s$ and $t$.*

**Proof of Lemma 3** This proof consists of two parts. First we show that $\alpha^s = 0$ for all $s$. Consider the optimal lower bounds $\underline{c}^s$. From agent 1's self-enforcing constraints, we know that $y^s \geq \underline{c}^s$. Furthermore it is easy to show that $0 < d\underline{c}^s/dy^s < 1$ when $V_{t+1}^s < \bar{V}$. Since $\underline{c}^s$ is continuous in $y^s$ it must be the case that if $\underline{c}^s = y^s$ then $s = 1$. In this case if $c_t^s = \underline{c}^1$, then $V_{t+1}^1 = \bar{V}$. Given that the optimal lower bounds are increasing in the states of nature and that first-order conditions imply an inverse relationship between $c_t^s$ and $V_{t+1}^s$, it must be the case that $\alpha^s = 0$ for all $s \geq 2$. Now suppose that in period $t-1$ we had $c_{t-1} = \underline{c}^1$ and $V_t = \bar{V}$. First-order conditions in periods $t-1$ and $t$ imply that

$$\begin{aligned} \frac{u'(c_{t-1})}{v'(b + y^1 - c_{t-1})} &= \frac{(1 + \alpha_{t-1} + \theta_{t-1})}{1 + \theta_{t-1}} \left\{ (1 + \theta^s) \frac{u'(c_t^s)}{v'(b + y_t^s - c_t^s)} - \lambda^s \right\} - \frac{\phi_{t-1}}{1 + \theta_{t-1}} \\ f'(V_t) &= (1 + \alpha^s + \theta^s) f'(V_{t+1}^s) + \phi^s + \lambda^s \end{aligned}$$

[19] Since the function $f$ is continuously diffrentiable and concave we know that $f'^{-1\prime}$ exists almost everywhere. Where it does not exists, we know that the right-hand and left-hand derivatives are negative.

for all states $s$. Take $s = 1$ and suppose that $\alpha_{t-1} > 0$. This implies that $\phi_{t-1} = 0$ and $V_t = \bar{V}$. The solution to these two equations must then include consumption $c_t^1 = \underline{c}^1$ which implies that $\lambda^1 = 0$. But then the first equation cannot be satisfied. It must then be the case that $\alpha_{t-1} = 0$ and hence $\alpha^1 = 0$ in all periods.
The second part shows that $\phi^s = 0$ for all $s$. The argument is similar as above. We know that $y^s \leq \bar{c}^s$. It is easy to show that $0 < d\bar{c}^s/dy^s < 1$ when $V_{t+1}^s > 0$. Since $\bar{c}^s$ is continuous in $y^s$ it must be the case that if $\bar{c}^s = y^s$ then $s = S$. In this case if $c_t^s = \bar{c}^S$, then $V_{t+1}^S = 0$. Given that the optimal upper bounds are increasing in the states of nature and that first-order conditions imply an inverse relationship between $c_t^s$ and $V_{t+1}^s$, it must be the case that $\alpha^s = 0$ for all $s \leq S-1$. Suppose that in period $t-1$ we had $c_{t-1} = \bar{c}^S$ and $V_t = 0$. Use the above relationships between $c_{t-1}$ and $c_t^s$, and $V_t$ and $V_{t+1}^s$, take $s = S$, and suppose that $\phi_{t-1} > 0$. This implies that $\alpha_{t-1} = 0$ and $V_t = 0$. The solution must then include consumption $c_t^S = \bar{c}^S$ which implies that $\theta^S = 0$. But then the above equation cannot be satisfied. It must then be the case that $\phi_{t-1} = 0$ and hence $\phi^S = 0$ in all periods. *Q.E.D.*

The results of this lemma are now used to show that the optimal consumption path follows that stated in part (iii) of the proposition. First-order conditions and the envelope condition imply that

$$(1+\theta^s)\frac{u'(c_t^s)}{v'(b+y_t^s-c_t^s)} - \lambda^s = -f'(V_t)$$
$$\frac{u'(c_t^s)}{v'(b+y_t^s-c_t^s)} = -f'(V_{t+1}^s)$$

First-order conditions in period $t-1$ then imply that

$$(1+\theta^s)\frac{u'(c_t^s)}{v'(b+y_t^s-c_t^s)} - \lambda^s = \frac{u'(c_{t-1})}{v'(b+y_{t-1}-c_{t-1})}.$$

Suppose that $\underline{c}^s \leq c^*(c_{t-1}, y_{t-1}, s) \leq \bar{c}^s$. The solution must then be $c_t^s = c^*(c_{t-1}, y_{t-1}, s)$ with $\theta^s = \lambda^s = 0$. If $c^*(c_{t-1}, y_{t-1}, s) > \bar{c}^s$, then the solution must be $\lambda^s > 0$ and $c_t^s = \bar{c}^s$. If $c^*(c_{t-1}, y_{t-1}, s) < \underline{c}^s$, then the solution must be $\theta^s > 0$ and $c_t^s = \underline{c}^s$.

(iv) The contract $\delta^{fc}$ is such that $V_{t+1}^{sfc} = 0$ for all states and periods. This implies that agent 2's ex post self-enforcing constraints can be satisfied if and only if $a_t^s \geq 0$ in all states and periods. But this is inconsistent with the payments prescribed by the contract $\delta^{fc}$, hence it cannot be self-enforcing when $B_t = 0$. *Q.E.D.*

**Proof of Lemma 2** (i) We know that $V(\delta^{fc}; h_t) = 0$ for all histories $h_t$. The contract $\delta^{fc}$ is self-enforcing if and only if $a_t^s \geq 0$ for all states $s$ and periods $t$. Consider the following contract $\hat{\delta}$: the ex ante payment is set at $\hat{B}_t = c^{1fc} - y^1$ and contingent payments at $\hat{a}_t^s = y_t^s - y^1 + c^{1fc} - c^{sfc} \geq 0$ in all states and periods. This contract yields for both agents the same consumption as under the contract $\delta^{fc}$. The contract $\hat{\delta}$ is self-enforcing for agent 2. The ex ante self-enforcing constraints are trivially satisfied, that is, $\hat{V}_{t+1}^s = 0$ for all states and periods; the ex post self-enforcing constraints are also satisfied by construction since $\hat{a}_t^s \geq 0$ for all states and periods. It is self-enforcing for agent 1 if and only if all her ex post self-enforcing constraints are satisfied, that is,

$$u(c^{sfc}) - u(y^s + c^{1fc} - y^1) + \frac{\beta}{1-\beta}\left\{Eu(c^{qfc}) - Eu(y^q)\right\} \geq 0 \text{ for all } s.$$

Note that agent 1's ex ante self-enforcing constraints are satisfied by $\hat{\delta}$. Define $\beta_{nc}$ as the smallest discount factor that satisfies the above equation in all states. This shows that $\beta \geq \beta_{nc}$ is a sufficient condition for the contract $\hat{\delta}$ to be self-enforcing. It is also necessary since a contract with a smaller ex ante payment would not be self-enforcing for agent 2 as it would require at least one ex post payment to be negative; a contract with a larger ex ante payment would be self-enforcing for larger values of the discount factor than $\beta_{nc}$.

(ii) When $\beta < \beta_{nc}$, there is at least one ex post self-enforcing constraint that binds. Adding up all conditions (6) to condition (5) yields

$$\sum_s p^s \{\lambda^s v'(b - B_t) - \theta^s u'(y_t^s + B_t)\} = 0. \tag{18}$$

It therefore follows that there must be a $s_1$ for which $\theta^{s_1} > 0$ and a $s_2$ for which $\lambda^{s_2} > 0$. *Q.E.D.*

**Proof of Proposition 4** Consider the optimal solution to maximization (4) as a function of the state variable $V_t$. By the theorem of the maximum we know that the solution is continous in the state variable over the interval $[0, \bar{V}]$. Consider a marginal increase in the value of the state variable. We want to show that the optimal ex ante payment is strictly decreasing in the state variable. The proof goes by contradiction. Suppose that the optimal value of the ex ante payment is left unchanged following a marginal increase in the state variable. The envelope condition implies that $f''(V_t)dV_t = -d\psi < 0$. Consider all ex post constraints that are satisfied at equality before the increase in the state variable. Of these constraints, we choose all those that become strictly binding following the increase in the state variable. These are the only constraints that bind following the increase in $V_t$. Consider the first-order conditions (6) for all states $s$ for which one self-enforcing constraint becomes binding. In these states, consumption is left unchanged following the small increase in $V_t$.[20] For all those ex post self-enforcing constraints, for first-order conditions to continue to hold we have that $d\lambda^s = -d\psi$ if the binding constraint is that of agent 2, and $(u'(c_t^s)/v'(b + y_t^s - c_t^s))\, d\theta^s = d\psi$ if it is that of agent 1. If we substitute these changes in condition (18), we have

$$\sum_s p^s \left\{ -d\psi u'(y_t^s + B_t) * \frac{v'(b + y_t^s - c_t^s)}{u'(c_t^s)} - d\psi v'(b - B_t) \right\} = 0$$

for this condition to continue to hold. Since $d\psi > 0$, this expression cannot be equal to 0 if $B_t$ remains constant. This implies that for any marginal change in the state variable $V_t$, the ex ante payment $B_t$ must change and therefore $B_t$ is monotonic in the state variable $V_t$ over the range $[0, \bar{V}]$.

Suppose that $V_t = \bar{V}$ and fix $B_t = 0$. In this case, only agent 1 has some ex post self-enforcing constraints that bind. We know from Proposition 2 that agent 1 pays the maximum ex ante payment. Since our problem is a concave problem, this implies that at $V_t = \bar{V}$, the optimal value of $B_t$ is negative. A similar argument shows that the optimal $B_t$ is positive at $V_t = 0$. Since $B_t$ is monotonic in $V_t$ the relationship between $B_t$ and $V_t$ must be decreasing. *Q.E.D.*

**Proof of Proposition 5** We know from Lemma 2 that there exist states $s_1$ and $s_2$ such that $\theta^{s_1} > 0$ and $\lambda^{s_2} > 0$. Suppose that $\lambda^2 > 0$ and $\theta^1 > 0$. This implies that $\lambda^1 = 0$ and $\theta^2 = 0$. First-order conditions then imply

$$\begin{aligned} -(1 + \theta^1)\frac{u'(c_t^1)}{v'(b + y_t^1 - c_t^1)} &= -f'(V_t) \\ \frac{u'(c_t^2)}{v'(b + y_t^2 - c_t^2)} - \lambda^2 &= -f'(V_t) \end{aligned}$$

which yields

$$(1 + \theta^1)\frac{u'(c_t^1)}{v'(b + y_t^1 - c_t^1)} = \frac{u'(c_t^2)}{v'(b + y_t^2 - c_t^2)} - \lambda^2.$$

[20] This follows from the results of Proposition 3 which show that for $B_t = 0$, the optimal lower (and upper) optimal consumption bounds that satisfy the ex post self-enforcing constraints are time-invariant. This result can easily be generalized to any fixed value of $B_t$.

But this implies that $c_t^1 = \underline{c}^1 > c_t^2 = \bar{c}^2$ which is inconsistent with the results of Proposition 3.[21] Consequently it must be the case that $\lambda^1 > 0$ and $\theta^2 > 0$.

(i) First-order conditions imply that, for state 1,

$$(1 + \alpha^1)f'(V_{t+1}^1) + \lambda^1 + \phi^1 = f'(V_t).$$

If $\alpha^1 > 0$, then $V_{t+1}^1 = \bar{V} \geq V_t$. If $\alpha^1 = 0$, the above expression reduces to

$$f'(V_{t+1}^1) + \lambda^1 + \phi^1 = f'(V_t)$$

which implies that $V_{t+1}^1 > V_t$ by the concavity of the Pareto frontier $f(\cdot)$. In state 2, we have that

$$(1 + \alpha^2 + \theta^2)f'(V_{t+1}^2) + \phi^2 = f'(V_t).$$

If $\phi^2 > 0$, then $V_{t+1}^2 = 0 \leq V_t$. If $\phi^2 = 0$, the above expression reduces to

$$(1 + \alpha^2 + \theta^2)f'(V_{t+1}^2) = f'(V_t)$$

which implies that $V_{t+1}^2 < V_t$ by the concavity of the Pareto frontier $f(\cdot)$. We then have $V_{t+1}^2 \leq V_t \leq V_{t+1}^1$ which proves the result. For future refrence, note that these equations imply that $\phi^1 = \alpha^2 = 0$.

(ii) If $0 < V_t < \bar{V}$, first-order conditions in period $t-1$ imply that $u'(c_{t-1})/v'(b + y_{t-1} - c_{t-1}) = -f'(V_t)$. We then have

$$(1 + \theta^2)\frac{u'(c_t^2)}{v'(b + y_t^2 - c_t^2)} = \frac{u'(c_t^1)}{v'(b + y_t^1 - c_t^1)} - \lambda^1 = \frac{u'(c_{t-1})}{v'(b + y_{t-1} - c_{t-1})}.$$

This yields the following inequalities.

$$\frac{u'(c_t^2)}{v'(b + y_t^2 - c_t^2)} < \frac{u'(c_{t-1})}{v'(b + y_{t-1} - c_{t-1})} < \frac{u'(c_t^1)}{v'(b + y_t^1 - c_t^1)}$$

Rearranging terms gives the result.

$$\frac{u'(c_t^2)}{u'(c_{t-1})} < \frac{v'(b + y_t^2 - c_t^2)}{v'(b + y_{t-1} - c_{t-1})} \text{ and } \frac{u'(c_t^1)}{u'(c_{t-1})} > \frac{v'(b + y_t^1 - c_t^1)}{v'(b + y_{t-1} - c_{t-1})}$$

(iii) This result is an immediate consequence of the above inequalitites. *Q.E.D.*

[21] The results of Proposition 3 to the effect that the optimal bounds on consumption are increasing in the states of the world hold for $B_t = 0$. In any period this can be easily generalized to any value of the ex ante payment, namely the optimal value.

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