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GOVERNMENT EXPENDITURE AND
THE DYNAMICS OF HIGH INFLATION

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RÉSUMÉ

Nous développons un modèle dynamique de l'inflation où l'offre de monnaie est déterminée par le recours du gouvernement à la "planche à billets" pour financer son déficit budgétaire. Ce déficit est influencé par le taux d'inflation passée qui réduit la valeur réelle des taxes perçues par le gouvernement. L'offre de monnaie et le déficit budgétaire sont déterminés de façon endogène dans le modèle, tandis que les dépenses du gouvernement sont supposées exogènes, fixées par le décideur politique. Les changements de fiscalité sont alors intégrés par le biais d'une modélisation des dépenses gouvernementales sous la forme d'un processus autorégressif avec changements discrets de régime. Par ailleurs, il est admis que les agents ont accès à un ensemble d'informations plus vaste que celui dont le chercheur dispose. Cette information additionnelle est incorporée par le taux d'inflation au travers de la demande de monnaie des agents. L'économètre construit des estimateurs de la probabilité du régime des dépenses à chaque point du temps et procède au raffinement de son inférence en exploitant la relation structurelle entre l'inflation et les dépenses du gouvernement.

Mots-clés: inflation élevée, changement de régime, information asymétrique, offre endogène de monnaie.

ABSTRACT

This paper develops a dynamic model of inflation in which the money supply is determined by the government's use of newly created money to finance its budget deficit. In turn, the government's deficit is influenced by past inflation rates that reduce the real value of tax receipts. While the money supply and the budget deficit are modeled as endogenous, government expenditure is assumed to be exogenously determined by the policy maker. Changes in fiscal policy are allowed by modeling expenditure as an autoregressive process subject to discrete changes in regime. The agents are conjectured to have access to a larger set of information than the researcher. This additional information is incorporated in the rate of inflation through the agents' money demand decision. The econometrician constructs probability assessments concerning the regime of the spending process at every point in time and refines his/her inferences by exploiting the structural relationship between inflation and government expenditure.

Key words: high inflation, changes in regime, asymmetric information, endogenous money supply.
I. Introduction

This paper develops a dynamic model of inflation in which the money supply is endogenously determined by the government's use of newly created money to partially finance its budget deficit. In turn, the government's deficit is influenced by past inflation rates as a result of the Olivera-Tanzi effect [Olivera (1967) and Tanzi (1977)]. This effect designates the reduction in the real value of tax receipts between the moment the liability toward the state is created and the moment in which the government effectively receives the payment. In the presence of high rates of inflation, the Olivera-Tanzi effect can lead to substantial losses in revenue by the government and further increase its budget deficit. While the money supply and the budget deficit are modeled as endogenous, the level of government spending is assumed to be exogenously determined by the policy maker. Thus, government expenditure is the variable that drives the dynamics of the model. Empirically, expenditure is postulated to follow an autoregressive process subject to discrete changes in regime [Hamilton (1989)]. The two states are fully characterized by their level and variance of spending. The regime switches are conjectured to arise from changes in the government's fiscal policy and are modeled as the outcome of a discrete stochastic process that follows a Markov chain.

The demand function for money on the part of the economic agents is a function of the expected inflation rate [Cagan (1956)]. Agents form their expectations rationally on the basis of an information set that is assumed to be strictly larger than the small number of time series used by the econometrician to estimate and test the model. This asymmetry of information is embodied in the assumption that the agents can unambiguously infer the current spending regime. On the other hand, with a limited information set, the econometrician can only construct probability assessments concerning the regime using data on government expenditure and the inflation rate.

Note that as part their formation of inflation expectations, agents still need to construct a forecast about the state of the expenditure process in the incoming period. Since (i) the agents use their
knowledge of the current spending regime when constructing the forecast about the future regime, and
(ii) the price level is determined (in the aggregate) by the agents as they adjust their portfolios, it follows
that the time series of inflation contains additional information about the state of the expenditure process
which could be exploited by the econometrician. Earlier applications of Hamilton’s regime-switching
procedure implicitly assume that the agents in the model are subject to same limitation as the researcher,
namely the unobservability of the state variable [see, e.g., Hamilton (1988) and Kaminsky (1993)]. On
the other hand, this paper explicitly models and employs this plausible source of information asymmetry
between the agents and the econometrician in order to refine the researchers’ inferences about the
expenditure regime and to increase the precision of the parameter estimates.

The determinant role of monetized government budget deficits on the inflation process has been
pointed out by some of the previous literature on hyperinflation.1 Keynes (1923), Stolper (1940), and
Sargent (1986) explicitly refer to the government’s use of seigniorage as a source of revenue in their
analysis of the German hyperinflation. Cagan (1956), Bailey (1956), Friedman (1971), and Phelps (1973)
examine the optimal use of the inflation tax in public finance. Sargent and Wallace (1981, 1987), Kigel
(1989), Marcet and Sargent (1989), Dornbusch and Fischer (1986), and Bruno and Fischer (1990) develop
theoretical models in which the government’s deficit is completely financed with newly created money.
In contrast with the above literature, the empirical research has typically employed the convenient
assumption that the money supply follows an exogenous autoregressive process [see, among others, Cagan
(1956), Flood and Garber (1980), Burmeister and Wall (1982), and LaHaye (1985)].2 Thus an important

1 Alternatively, the "balance of payments" view stresses the effect of exchange rate depreciation on
the inflation rate through Purchasing Power Parity, accommodating monetary policies, and wage indexation
[see Liviatan (1986) and Montiel (1989)].

2 Two important exceptions are Burmeister and Wall (1987) and İmrohoroğlu (1993). Burmeister and
Wall assume that the government considers current inflationary expectations when determining the rate
of money growth; İmrohoroğlu estimates the stochastic specification proposed by Sargent and Wallace
contribution of this paper is that it econometrically estimates and tests a fully-specified model where the money supply is endogenously determined by the government’s spending decision and by the effect of past rates of inflation on the value of tax receipts. In this setting, the dynamics of the inflation rate is described in terms of the exogenous expenditure process and lagged rates of inflation. The postulated non-linear specification for government expenditure permits more general dynamics than the one obtained using standard ARIMA models. In particular, the agents in the model are allowed to perform more-than-proportional adjustments in their portfolios as a result of observed changes in government policy, and to consider the possibility of such changes when constructing their expectations.

The model is estimated using monthly data for Brazil between 1980 and 1989. Specification tests provide overall support for the overidentifying restrictions of the model. The estimates indicate that a change in regime from the low to the high spending regime is associated in the long run with (i) an increase of 10.7 points in the proportion of GDP devoted to government outlays, and (ii) a monthly rate of inflation that is 15.9 percentage points higher than in the low expenditure state. The probability inferences indicate that such a regime switch occurred in Brazil in 1985. It is possible that this change in fiscal policy is associated with the change in government that took place in March of that year. The probabilities also suggest there was not a significant modification in the spending process during the inflation stabilization programs applied in the late 1980s. If, despite the government’s pledge to reduce spending to a level consistent with the inflation target, the economic agents still concluded that expenditure continued in the high regime, then the plan could not have been credible. Therefore, up to the extent that (i) the money supply is determined by the expenditure level and (ii) the money demand is affected by the agent’s beliefs about government policy, the empirical results would support the hypothesis that the eventual failure of these stabilization plans is primarily attributable to the government’s inability to substantially reform its fiscal policy.

The paper is organized as follows: Section 2 presents the analytical model, Section 3 outlines the
estimation technique, Section 4 presents the empirical results and specification testing, and finally, Section 5 contains the main conclusions.

II. The Model

A. Money Demand

It is assumed that the agents' demand for real balances is suitably described by a Cagan money demand function with unitary income elasticity [Cagan (1956)],

\[ m_t^d \cdot p_t = y_t \cdot \alpha E(\pi_{t+1} | I_t) + v_t, \quad \alpha > 0, \tag{1} \]

where \( m_t^d \), \( p_t \), and \( y_t \) denote the natural logarithms of nominal money holdings, price level, and real income respectively; \( I_t \) is the set of information available to the agents at time \( t \) and includes current and past observations of inflation and government expenditure; \( E(\pi_{t+1} | I_t) \) is the conditional expectation of inflation at time \( t+1 \); and the coefficient \( \alpha \) represents the semi-elasticity of money demand with respect to the rate of inflation. I follow the literature in conveniently assuming that the disturbance term \( v_t \) follows a random walk [see, among others, Sargent and Wallace (1973), Salerni and Sargent (1979), Flood and Garber (1980), Burmeister and Wall (1982, 1987), LaHaye (1985), and Casella (1989)]. Thus, \( v_t = v_{t-1} + \epsilon_t \), where \( E(\epsilon_t) = E(\epsilon_i \epsilon_j) = 0 \) for all \( i \neq j \), and \( E(\epsilon_t) = \sigma^2 \). First-differencing (1),

\[ \mu_t^d - \pi_t = y_t - y_{t-1} + \alpha [E(\pi_t | I_{t-1}) - E(\pi_{t+1} | I_t)] + \epsilon_t, \tag{2} \]

where \( \mu_t^d \) is the rate of growth of money demand at time \( t \). Finally, I postulate that real income grows at a constant rate. This premise reflects the relative stability of income growth when compared with the large fluctuations in the rate of inflation and real balances during the period considered. Consequently, equation (2) can be rewritten as,

\[ \mu_t^d - \pi_t = n + \alpha [E(\pi_t | I_{t-1}) - E(\pi_{t+1} | I_t)] + \epsilon_t, \tag{3} \]

where \( n \) denotes the (constant) growth rate of real income.
B. Money Supply

In an economy in which the government actively employs seigniorage as a source of revenue, the rate of growth of the money supply would be fundamentally determined by the proportion of the budget deficit that is financed with newly created money.\footnote{The effect of foreign exchange operations by the central bank on the money supply would be reasonably small if the exchange rate is approximately floating. In the case of Brazil, the monetary authority financing of the central government deficit was (on average) 81.6\% for the period 1982-1986. Author’s calculations based on data from \textit{Government Finance Statistics Yearbook}, International Monetary Fund, 1990 (p. 175).} Under this postulate, Olivera (1967), Sargent and Wallace (1981, 1987), Kiguel (1989), Marcet and Sargent (1989), Dornbusch and Fischer (1986), and Bruno and Fischer (1990) develop theoretical models in which the government’s budget deficit is wholly financed with seigniorage in every period. In the spirit of this papers, consider the linear projection [see Ruge-Murcia (forthcoming)],

\[ \mu_t^s = a + \gamma D_t + \xi_t, \]

\[ \gamma > 0, \]  \hspace{1cm} (4)

where $\mu_t^s$ is the rate of growth of the money supply, $D_t$ is the government’s budget deficit as percentage of GDP, $a$ is an intercept term, $\gamma$ is a coefficient that measures the response of money growth to a change in the government deficit amounting to one percentage of GDP, and $\xi_t$ is a serially uncorrelated error term with zero mean and variance $\sigma^2_\xi$. In turn, the budget deficit is defined as the difference between the level of government expenditure (represented by $G_t$) and the value of tax receipts (denoted by $T_t$). Formally,

\[ D_t = G_t - T_t, \]

\hspace{1cm} (5)

where $G_t$ includes the interest payments on public debt and all variables are expressed as percentage of GDP.

The value of tax revenue raised by the government is assumed to be endogenously influenced by
past inflation rates as a result of the Olivera-Tanzi effect. This effect designates the reduction in the real value of tax receipts that arises in an institutional setting in which tax liability is assessed in nominal terms but subject to a collection delay. Hence, a general process for the value of taxes receipts would take the form,

\[ T_t = \tau + \sum_{i=1}^{m} \psi_i T_{t-i} + \sum_{j=1}^{q} \alpha_j \pi_{t-j} + \eta_t, \]  

(6)

where \( \tau \) is a constant term and \( \eta_t \) denotes an error term. In particular, consider the restricted version of (6),

\[ T_t = \tau + \sum_{j=1}^{2} \alpha_j \pi_{t-i} + \eta_t, \]  

(7)

where \( \eta_t \) is assumed serially uncorrelated with mean zero and variance \( \sigma_{\eta_t}^2 \), and could be contemporaneously correlated with \( \xi_t \). Use (5) and (7) into (4) to obtain the money supply process,

\[ \mu_t = c + \gamma G_t + \sum_{j=1}^{2} \theta_j \pi_{t-i} + \zeta_t, \]  

(8)

where \( c = a - \gamma \tau \) and \( \theta_i = \gamma a_i \) for \( i = 1, 2 \). The composite error term, \( \zeta_t = \xi_t + \gamma \eta_t \), satisfies \( E(\zeta_t) = 0, \)

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4 Keynes (1923, p. 52) and Stolper (1940, p. 76) mention this effect in their studies of the German hyperinflation. Olivera (1967) examines the dynamics of inflation under the general postulate that government income follows the increase in the level of prices more slowly than expenditure does. Tanzi (1977) focuses on the impact of lags in tax collection.

5 In countries like Brazil where inflation has been traditionally high and indexation is widespread, the Olivera-Tanzi effect can result from less-than-perfect indexation in tax collection and/or backward-looking indexation in the presence of rapidly increasing inflation rates [see Cukierman (1988)].

6 Rather than finding the solution of the model for a general tax process [say, equation (6)], I pursue the simpler strategy of focusing only on a representation that parsimoniously describes the tax process for Brazil. Using monthly data on fiscal revenue (taxes plus dividends and other income) for the period January 1980 to December 1989, I tested (and rejected at the 1% level) the joint significance of six lags of \( T_t \) in (6). I also tested (and rejected) the significance of seasonal dummies.
\[ E(\zeta_i \zeta_j) = 0 \text{ for all } i \neq j, \text{ and has variance } \sigma^2 e = \sigma^2 \xi + \gamma^2 \sigma^2 \eta + 2 \gamma \sigma \xi \eta, \text{ where } \sigma \xi \eta \text{ denotes the (possibly non-zero) covariance of the disturbance terms } \xi_i \text{ and } \eta_i. \]

The specification in (8) characterizes the effect on the money supply of (i) the government's policy rule of partially financing its budget deficit with newly created money, and (ii) the inflation-induced erosion in the value of real tax receipts that further increases the government's deficit. While the money supply and the budget deficit are treated as endogenous, the level of government expenditure is assumed to be exogenously determined by the policy maker. Hence, government spending is the variable that drives the inflation dynamics. This modeling decision rests on the premise that, although not strictly exogenous, it is considerably more plausible to model government expenditure as exogenous than to model either the money supply or the budget deficit as exogenous. The possibility that the government could (endogenously) change its level of spending in light of increasing rates of inflation will be discussed in Section D.

Finally, it could be argued that in modeling the budget deficit (and the money supply), this paper attributes a somewhat more prominent role to government expenditure than to tax revenue. Consider figure (1) that presents annual data on government spending, tax receipts, and the budget deficit as percent of GDP between 1980 and 1988.\(^8\) It is clear from this figure that the main variable driving the budget deficit is government expenditure. In addition, notice that tax revenues are substantially lower in the second part of the decade, when the rate of inflation was higher (as the Olivera-Tanzi effect would suggest). This reduction in tax revenue also contributes to the observed increase of the budget deficit in

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\(^7\) The reader might wonder whether, from the purely statistical perspective, an exogenous autoregressive process could fit the money supply data better than the process postulated in (8). In order to address this query, I ran ordinary least squares on equation (8) and on an AR(6) representation for the growth rate of the money supply. The adjusted \(R^2\)s were respectively 0.38 and 0.17, suggesting that the endogenous specification (8) might provide a more accurate description of the money supply process than the AR representation.

\(^8\) The data for figure (1) was obtained from *Government Finance Statistics Yearbook*, Volume 14, International Monetary Fund 1990 (p. 79 and 91).
the late 1980s. Thus, it would appear to be the case that the simple model postulated in this section captures the most important features of the budget deficit process during the period considered.

C. RE Solution of the Model

Having specified the demand and supply of money in the economy, I proceed to establish the rational expectations (RE) solution of the model.\(^9\) The first step is finding the appropriate expression for \(E(\pi_t | I_{t-1})\). Equating money supply (8) with money demand (3), and taking conditional expectations as of time \(t-1\), yield a third-order difference equation in \(E(\pi_t | I_{t-1})\) with roots denoted by \(\lambda, \lambda_1\) and \(\lambda_2\). Assume that the system is saddle-path stable, that is, \(|\lambda| > 1\) and \(|\lambda_i| < 1\) for \(i = 1, 2\).\(^10\) Then, the solution of this difference equation (ruling out speculative bubbles) is,

\[
E(\pi_t | I_{t-1}) = \kappa + (\lambda_1 + \lambda_2) \pi_{t-1} - (\lambda_1 \lambda_2) \pi_{t-2} + (\gamma/\alpha) \sum_{i=0}^{\infty} \lambda^{-i+1} E(G_{t+i} | I_{t-1}) ,
\]

where \(\kappa = (n-c)/\alpha(1-\lambda)\) is a constant term. Notice that since \(\gamma/\alpha > 0\), expected inflation increases with the weighted sum of current and expected future government expenditure. Using (9) and (9) forwarded one period, the RE solution for the rate of inflation can be written as,

\[
\pi_t = \kappa + (\lambda_1 + \lambda_2) \pi_{t-1} - (\lambda_1 \lambda_2) \pi_{t-2} + (\gamma/\alpha) \sum_{i=0}^{\infty} \lambda^{-i+1} E(G_{t+i} | I_{t-1})
+ \varphi_1 \lambda^i \sum_{i=0}^{\infty} \lambda^{-i+1} [E(G_{t+i} | I_{t}) - E(G_{t+i} | I_{t-1})] + u_t ,
\]

where \(\varphi = \gamma[1-\alpha(\lambda_1 + \lambda_2)]\) is a positive coefficient, and \(u_t = \varepsilon_t \xi_t\) is a serially uncorrelated disturbance term.

\(^9\) The complete solution of the model is presented in Appendix A.

\(^10\) I am implicitly assuming that the roots of the difference equation are real. For the econometric estimation of the model no constraints will be imposed on \(\lambda, \lambda_1\) and \(\lambda_2\). Thus, the data will be allowed to either support or reject the assumption that \(|\lambda_i| < 1\) for \(i = 1, 2\). Finally, notice that the roots satisfy \(\lambda + \lambda_1 + \lambda_2 = (\alpha+1)/\alpha > 1, \lambda(\lambda_1 + \lambda_2) + \lambda_1 \lambda_2 = \theta_1/\alpha,\) and \(\lambda_1 \lambda_2 = -\theta_2/\alpha.\)
with mean zero and variance \( \sigma_u^2 = \sigma_c^2 + \sigma_t^2 \). Equation (10) postulates that the rate of inflation is a function of (i) past rates of inflation, (ii) the discounted sum of current and expected future government expenditures, and (iii) the discounted sum of the agent's revision of the expected path of government spending as a result of the information become available between period \( t-1 \) and period \( t \). Regarding the latter component of the solution, note that in the case when expenditure follows a standard ARIMA process, the agent's revision about the future path of spending is history-independent and exactly proportional to the size of the current innovation. These properties follow solely from the linearity of the process of the forcing variable [see Koop, Pesaran, and Potter (1993)]. In contrast, the non-linear specification that I shall postulate for government spending allows this revision to depend on the particular fiscal policy in effect when the innovation takes place, and permits more-than-proportional movements in the inflation rate as a result of observed shifts in the expenditure process.

**D. The Government Expenditure Process**

In order to obtain a closed form solution for the inflation rate in (10), it is necessary to specify a process for the level of government spending (as a percentage of GDP). In particular, I assume that it follows a stationary, autoregressive process subject to discrete regime changes [Hamilton (1989)] of the form,

\[
G_t = \beta_{s(t)} + \phi_1 G_{t-1} + \phi_2 G_{t-2} + \ldots + \phi_r G_{t-r} + \psi d_t + \sigma_{s(t)} u_t. 
\]  

where \( s(t) \) is a discrete variable that denotes the state government expenditure is in at time \( t \); \( \sigma_{s(t)} \) and \( \beta_{s(t)} \) are variables whose value depends on the spending regime; and \( \psi \) and \( \phi_i \) for \( i = 1, 2, \ldots, r \) are constant coefficients. Without loss of generality, I postulate \( \beta_2 > \beta_1 \). Thus, \( s(t) = 2 \) indexes the high expenditure

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11 I also considered more general specifications in which (in addition to \( \beta \) and \( \sigma \)) the autoregressive coefficients and/or the coefficient of the seasonal dummy were modeled as state-dependent. However, after performing the appropriate likelihood ratio tests, I was unable to reject the null hypothesis of constant (i.e., non-state-contingent) coefficients at the 10% significance level.
regime and \( s(t) = 1 \) designates the low spending state. The term \( e_t \) denotes a serially uncorrelated disturbance with zero mean, variance equal to 1, and possibly (contemporaneously) correlated with \( u_t \).\(^{12}\) Finally, \( d_t \) is a dummy that takes value one when the observation \( t \) corresponds to December and zero otherwise. This specification intends to account for the important seasonal element in government expenditure during the month of December.\(^{13}\)

Figure (2) contains the monthly average of the observations on government expenditure, money growth, and inflation in Brazil between 1980 and 1989.\(^{14}\) Notice that the seasonality in government expenditure in December is mirrored in the money growth process. This observation is consistent with the hypothesis that the government employs newly created money to cover its current spending. In addition, it suggests that the government does not (or is unable to) actively use debt to smooth out the strong seasonality in expenditure.\(^{15}\) Finally, note the seasonal pattern in the rate of inflation in the lower panel of figure (2). Inflation increases steadily throughout the year to reach its peak in December after which it falls. My model will explain this empirical observation as the result of the direct effect of seasonal spending on the inflation rate in December, and the agents' expectations of high levels of government expenditure and money growth during the month of December that affect the rate of inflation in all months: forward-looking agents smooth out the inflationary effect of seasonal expenditure throughout

\(^{12}\) This correlation might arise because shocks to the money supply process (8) (e.g., an unexpected reduction in tax revenues), could be partially smoothed out by the government's adjusting the level of spending.

\(^{13}\) This seasonal component is due primarily to the annual bonus paid to all government employees in December, in addition to the regular stipend. This bonus is equivalent to a monthly salary so that effectively the government's wage bill is twice as large in December than in any other month of the year.

\(^{14}\) The data for this figure was obtained by taking the arithmetic average of the observations for a given month during the years considered.

\(^{15}\) This could be the result of the government's facing exogenous borrowing constraints. After the beginning of the debt crisis in August 1982, most Latin American countries were unable to borrow in the international markets. In addition, the governments of these countries face underdeveloped financial markets at home.
the year.\textsuperscript{16}

For the state variable $s$, denoting the regime of the government spending process, define the $1 \times 2$
vector $L_{a(0)}$ with 1 in the $i^{th}$ element if $s(t) = i$ and zero everywhere else, and the $2 \times 1$ vector $\beta = [\beta_1 \ \beta_2]'$ with the possible values the state-dependent constant $\beta_{a(0)}$ can take. I assume the current realization of $s(t)$ depends only on $s(t-1)$ through a Markov chain with matrix of transition probabilities

$$
P = \begin{pmatrix}
    p_{11} & p_{12} \\
    p_{21} & p_{22}
\end{pmatrix},$

where $p_{jk} = Pr(s(t)=k | s(t-1)=j)$ and $p_{11} + p_{12} = 1$ for $j = 1, 2$.

In theory, it would be possible to parameterize the transition probabilities as a non-linear function of predetermined variables (see Diebold et al. (1992) and Filardo (1994)). In the case examined here, parameterizing the transition probabilities as a function of lagged inflation would (endogenously) alter the likelihood of the government's modifying its fiscal policy in light of increasing rates of inflation. Alternatively, the probabilities could be modeled as a function of government spending. However, notice that these specifications would substantially complicate the solution and estimation of the model. Specifically, it would be necessary to (numerically) compute the conditional expectation of a non-linear function of endogenous and/or exogenous variables for every observation in the sample [see equation (10)]. Thus, in order to keep the scope of this project manageable, I shall assume that the elements of

\textsuperscript{16} Sims (1990) and Hansen and Sargent (1993) assert that econometricians who misspecify the model of seasonality may do much worse than those who discard the additional information contained in the seasonal fluctuations. On the other hand, Sargent (1978) and Singleton (1988) maintain that since the economic agents being modeled are assumed to solve signal extraction and forecasting problems using seasonally unadjusted data, the cross equation restriction implied by their behavior applies only to unadjusted data. Furthermore, Ghysels (1991) has pointed out to the efficiency gains that may be possible if seasonally unadjusted data is employed. I am indebted to Toni Braun for helpful discussions on this topic.
\( P \) are time-invariant.

An important feature of the model is that the agents are postulated to have access to a larger set of information than the econometrician. This asymmetry of information is embodied in the assumption that agents observe the current expenditure state while the researcher can only construct probability assessments concerning the regime using data on inflation and spending. Nonetheless, the agents still have to generate inferences about the future regime of the expenditure process as part of their formation of inflation expectations. Since the agents’ expectations are constructed on the basis of their available information set (that includes the current expenditure regime), the time series of inflation is conjectured to contain additional information about the expenditure process. This information is exploited by the econometrician to refine the inferences regarding the spending regime in the framework of an analytical model that captures the relationship between government expenditure and inflation.

In order to write the government spending process in vector form, define the \( r \times 1 \) column vector 
\[ G_t = [G_t, G_{t-1}, \ldots, G_{t-r+1}] \]; the \( 1 \times r \) selection vector \( h \) with 1 in its (1,1) element and zero everywhere else; the \( r \times 1 \) column vectors \( X_t \) and \( B_t \) with (1,1) elements given respectively by \( \sigma_{k,t} \) and \( \beta_{k,t} + \psi_{k,t} \), and zero everywhere else; and the \( r \times r \) matrix \( \Phi \),

\[
\Phi = \begin{pmatrix}
\phi_1 & \phi_2 & \cdots & \phi_{r-1} & \phi_r \\
1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{pmatrix}
\]

Then, write \( G_t = hG_{t-1} \), where \( G_t = \Phi G_{t-1} + B_t + X_t \).

With the notation introduced above, the appropriate expression for the weighted sum of expected future and current expenditures can easily be found. The interested reader is encouraged to see Appendix A for details. Then, the reduced form of the rational expectations solution (10) can be written as,
\[ \pi_t = \kappa + (\lambda_1 + \lambda_2) \pi_{t-1} - (\lambda_1 \lambda_2) \pi_{t-2} + \varphi(1+\delta) \beta \pi_{t-1} + \varphi(\lambda-1) \delta \Phi (\lambda I - \Phi)^{-1} G_{s.t} + \varphi(1+\delta) L_{s(0)-L_{s(t-1)}} (\Phi(1-P)^{-1} \beta + \Gamma(\text{month } t) + \varphi(1+\delta) \sigma_{s(t)} e_t) + u_t. \] (12)

where \( q = h\lambda(\lambda I - \Phi)^{-1} h \) and \( \delta = h\Phi(\lambda I - \Phi)^{-1} h \) denote constant scalars, and \( I \) is the 2x2 identity matrix.

Notice that \( \pi_t \) depends on past inflation rates and on government expenditure through (i) lagged values of \( G_t \), that partially determine the agent's forecast of future government spending, (ii) the current regime of government expenditure (since \( \varphi(1+\delta) > 0 \) and \( \beta_2 > \beta_1 \), the inflation rate is higher for the high-spending process than for the lower one), and (iii) the change in regime. The latter component captures the effect of the agents' reassessment about the future path of government expenditure in light of an observed switch in its underlying process. Consider a change from the low to the high spending regime at time \( t \). Since \( \varphi(1+\delta) > 0 \) and \( \beta_2 > \beta_1 \), the change-in-regime component will be positive, meaning there is a once-and-for-all increase in inflation rate at time \( t \) as agents expect this new, high-spending regime to remain in place for some time in the future.\(^{17}\) Conversely, should the agents infer that the expenditure process has switched from the high to the low state, this component will have the same magnitude but a negative sign. Finally, should the agents understand that no change in the process of \( G_t \) has taken place, then this element would be zero because \( L_{s(t)} = L_{s(t-1)} \). In addition, note that the variance of the inflation process depends on the variance associated with the government spending regime. For example, should the data indicate that \( \sigma_2 > \sigma_1 \), then the researcher would conclude that the rate of inflation is more volatile when expenditure is in the high state than when it is in the low regime.

The seasonal function \( \Gamma(\text{month } t) \) takes the value \( \varphi\psi(1+\delta) + \varphi\psi\psi(1-\lambda^{11})/(\lambda^{12} - 1) \) if period \( t \) corresponds to December and \( \varphi\psi\psi\lambda^{j}(\lambda-1)/(\lambda^{12} - 1) \), otherwise, where \( j_t = 0 \) if \( t \) corresponds to January, 1 if \( t \) corresponds to February, 2 if \( t \) corresponds to March, \ldots, and 10 if \( t \) corresponds to November. \( \Gamma(\ast) \)

\(^{17}\) Under the Markov process driving the state variable \( s_t \), the expected duration of regime \( j \) is \( 1/(1 - p_{jj}) \) periods.
embody the direct effect of seasonal spending on the inflation rate in December, and the expectations of high levels of government expenditure and seigniorage during the month of December that affect the inflation rate in all months. The latter component increases geometrically throughout the year at a rate given by the parameter \( \lambda \). The intuition of this result is as follows: under the expectations of high levels of money growth and inflation in December, agents will choose to hold a smaller level of real balances in November. Other things equal, as agents adjust their portfolios, they will bid the price level up, causing inflation in November to be higher than it would have been otherwise. Although the seasonal component in November’s inflation will be smaller than the one in December, it will affect the demand for real balances in October likewise. By induction, it becomes apparent that seasonal expenditure in December, and its associated money growth, affect the inflation rate in all months of the year. As noted before, the seasonal function \( \Gamma(\cdot) \) implied by the analytical model is consistent with the observed seasonal pattern of the inflation rate in Brazil [see figure (2)].

III. Estimation of the Model

For the estimation of the joint system of government expenditure and the RE solution of the inflation rate, I employ the procedure proposed by Hamilton (1989). This non-linear technique provides a filter for the maximum likelihood estimation of state-space models in which the state variable \( s_t \) is discrete. In this case, the disturbance term in the stochastic process driving the state variable is not normally distributed since it can only take a finite number of discrete values. Thus, the Kalman filter does not generate optimal forecasts or evaluation of the likelihood function.\(^{18}\) In addition, Hamilton’s procedure yields probability assessments about the state or regime of the system for every observation in the sample.

\(^{18}\) However the Kalman filter would still provide the "best" predictor and updated estimator among the class of linear procedures. See Harvey (1988), p. 102.
Define the vector of current observations \( Z_t = [G_t, x_t] \)^T, the set of parameters \( \Theta = \{ \beta_1, \beta_2, \sigma_1, \sigma_2, \sigma_u, \rho_{11}, \rho_{22}, \lambda_1, \lambda_2, \rho, \kappa, \phi_1, \ldots, \phi_r, \omega \} \), and the set \( S_t = \{ s_t, s_{t-1} \} \), with the state of the government expenditure process in the current and previous period. Note that by assumption \( \{ Z_{t-1}, Z_{t-2}, Z_{t-3}, \ldots \} \in I_{t-1} \). In the case when the error terms \( \epsilon_t \) and \( \epsilon_{t-1} \) are normally distributed, the conditional density for a given observation of the joint system is,

\[
\Pr(Z_t \mid S_t, I_{t-1}; \Theta) = (2\pi)^{-1/2} \left| \Omega_{\Theta(0)} \right|^{-1/2} \exp \left\{ -\frac{1}{2} \left[ \frac{\{ Z_t - E(Z_t \mid S_t, I_{t-1}; \Theta) \}^T \Omega_{\Theta(0)}^{-1} \{ Z_t - E(Z_t \mid S_t, I_{t-1}; \Theta) \}} \right] \right\}, \tag{13}
\]

where the elements of \( E(Z_t \mid S_t, I_{t-1}; \Theta) \) are obtained by taking the expectation of (11) and (12) conditional on \( S_t, I_{t-1}, \) and \( \Theta \). The variance-covariance matrix of the residuals is given by,

\[
\Omega_{\Theta(0)} = \begin{bmatrix}
\varphi(1+\delta)\sigma_{\epsilon(0)}^2 + \sigma_{u(0)} \rho \sigma_u & \sigma_{u(0)} \rho \sigma_u \\
\sigma_{u(0)} \rho \sigma_u & \sigma_{u(0)} \rho \sigma_u + \sigma_u^2
\end{bmatrix},
\]

where \( \rho \) denotes the correlation coefficient of \( \epsilon_t \) and \( u_t \).

The estimation filter works recursively taking as input \( \Pr(s_{t+1}, s_{t+2} \mid t_1, \Theta) \), obtained from the previous iteration,\(^{19}\) and has as output, \( \Pr(s_t, s_{t+1} \mid t_1, \Theta) \), along with (as a byproduct) the conditional likelihood of \( Z_t \), namely \( \Pr(Z_t \mid s_t, t_1, \Theta) \). The value of the likelihood function for any given set of values taken by \( \Theta \) can be calculated from

\[
\Pr(Z_T^N \mid \Theta) = \prod_{t=t+1}^N \Pr(Z_t \mid I_{t-1}; \Theta), \tag{14}
\]

where \( N \) is the number of observations. Estimates of the parameters values that are most likely to have generated the data series under consideration are obtained by numerically maximizing the function (14)

\(^{19}\) The filter is started with the unconditional probabilities \( \Pr(s_1=1) = (1-\rho_{22})/(2-\rho_{11} \cdot \rho_{22}) \) and \( \Pr(s_1=2) = (1-\rho_{11})/(2-\rho_{11} \cdot \rho_{22}) \) [see Hamilton (1989)].
with respect to $\Theta$. It is also possible to calculate,

$$
Pr(s_t|I_t; \Theta) = \sum_{s_{t-1}=1}^2 Pr(s_t, s_{t-1}|I_t; \Theta),
$$

(15)

where $Pr(s_t|I_t; \Theta)$ is termed the filter probability of $s_t$ and denotes the probability that the system is in state $s_t$ given the parameters of the model and all current and previous observation of spending and inflation.

The econometrician's best inference about the state of the system at any date is given by the smooth probability of $s_t$, that is, the probability that the joint process of inflation and expenditure is in state $s_t$ at time $t$, given all the observations in the sample. The smooth probability of $s_t$ is denoted by $Pr(s_t|I_N; \Theta)$ and can be calculated using the procedure suggested by Kim (1991). Recall the previously defined set $S_t$, that condenses all possible values $s_t$ can take in periods $t$ and $t-1$, and notice that $S_t$ indexes 4 possible states. It is straightforward to verify that [see Hamilton (1993)],

$$
Pr(S_t=j|I_N) = \sum_{i=1}^2 \left( \frac{Pr(S_t=j|I_t) Pr(S_{t+1}=i|S_t=j)}{Pr(S_{t+1}=i|I_t)} \right).
$$

(16)

The smooth probability for the full-state $S_t$ can be obtained by iterating on (16) backwards for $t = N, N-1, N-2, \ldots, 1$. Finally, the smooth probability for $s_t$ is calculated from

$$
Pr(s_t|I_N) = \sum_{s_{t-1}=1}^2 Pr(s_t|I_N).
$$

(17)

Using the smooth probabilities, the econometrician can (i) assess the government's claim to have reformed its spending policy during the stabilization programs, and (ii) evaluate whether any temporary reduction in the rate of inflation was the result of a change, albeit transitory, in the government's fiscal policy or solely the effect of the incomes policies applied as part of these programs. Formally, if $Pr(s_t=2|I_N) > 0.5$, then the researcher would conclude that the joint observation of inflation and expenditure is generated by the high-spending state. Conversely, if $Pr(s_t=2|I_N) \leq 0.5$, then the econometrician would
infer that the variables are produced by the low-spending regime.

IV. Empirical Results and Specification Testing

The model was estimated for Brazil using monthly observations of government expenditure and inflation between January 1980 and December 1989. The rate of inflation was calculated employing the Broad Consumer Price Index, and government spending was measured by total (actual rather than programmed) outlays of the National Treasury. The National Treasury budget covers the obligations to government contractors and suppliers, transactions related with public debt to commercial banks and non-financial institutions, debts of states and municipalities, and all (quasi-fiscal) expenditures previously in the Monetary budget administered by the Banco Central and the Banco do Brasil (e.g., subsidies to energy programs, subsidies linked to the formation of buffer stocks, agricultural loans, etc).20 The General Industrial Production Index of the Brazilian Institute of Geography and Statistics (IBGE), was used as a proxy for real GDP.21 The time series of inflation and expenditure (as percent of GDP) are respectively presented in Figure (3) and in the lower panel of Figure (6).

Initially, the appropriate lag length for the government spending process (11) was determined. Processes with lag length \( r = 1, r = 2, r = 3, r = 4, r = 5, \) and \( r = 6 \) were considered. After applying the pertinent Likelihood Ratio tests, the AR(2) specification was selected as the most suitable to describe the expenditure process. Then, the parameters of the joint process of inflation and government expenditure were estimated by the numerical maximization of the likelihood function. In the maximization procedure,

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20 For more details on the Brazilian budgetary system, see Country Profile: Brazil, The Economist Intelligence Unit 1989-1990, p. 36-38.

21 The time series on government expenditure as percentage of GDP, was obtained by dividing every observation of nominal government expenditure by its corresponding nominal Gross Domestic Product. A measure of the nominal Gross Domestic Product was constructed by multiplying the General Index of Industrial Production by the Consumer Price Index of the current month. The base for nominal Gross Domestic Product was set as March 1986 =100.
the transition probabilities $p_{11}$ and $p_{22}$ were constrained to lie between zero and one, and the correlation coefficient $\rho$ was kept between one and minus one. Similarly, $\lambda$ was reparameterized so that $\lambda > 1$ under the assumption of saddle-path stability. The variance-covariance matrix was calculated by the inverse of the hessian estimated at the maximum. Several randomly chosen starting values were tried, and four local maxima were found. The values of the parameters corresponding to the global maximum are reported below.

The estimated process for government expenditure (as percent of GDP) is,\textsuperscript{22}

$$G_t = \beta_4 + 0.04G_{t-1} + 0.22G_{t-2} + 23.87d_t + \sigma_4e_{t-1}$$

where the state dependent parameters are $\beta_1 = 14.81 (1.88)$, $\beta_2 = 22.73 (3.03)$, $\sigma_1 = 3.72 (0.33)$, and $\sigma_2 = 9.8 (1.0)$. Consider a change in regime from the low to the high expenditure state. The empirical estimates above indicate that such a change is associated with an \textit{immediate} increase in the share of GDP devoted to government spending of $\beta_2 - \beta_1 = 7.92 (1.72)$ percent, and a long-run increase of 10.70 percent. In other words, $G_t$ eventually rises from 22.73 percent of GDP in the low-spending regime, to 33.43 percent in the high-expenditure state.\textsuperscript{23} The converse result occurs when there is a regime switch in the opposite direction. Figure (4) presents the impulse response function for government expenditure following a change from the low to the high spending regime. Below, I will examine the effect of this switch in the state of the government spending process on the endogenous variables, namely the rates of inflation and money growth.

In addition to the mean, the standard deviation of the government expenditure process is also sharply different across regimes. Specifically, the standard deviation of spending in the high-expenditure

\textsuperscript{22} The standard errors of the estimates are presented in parentheses.

\textsuperscript{23} The (conditional) mean of government expenditure in state $j$ is calculated as $(\beta_j + \psi/12)/(1 - \Phi_1 - \Phi_2)$ for $j = 1, 2$. 
state is almost three times larger than in low-spending regime. A Likelihood Ratio test was employed to examine whether the difference between $\sigma_1$ and $\sigma_2$ is statistically significant. Since the restriction $\sigma_1 = \sigma_2$ is rejected by the data at the 1 percent level (the p-value is well below 0.1 percent), it is reasonable to conclude that government expenditure is substantially more volatile in the high-spending regime than in the lower one. As we shall see below, this observation partially accounts for the increased variability of the rate of inflation after 1985.

The transition probabilities are estimated by $p_{11} = 0.991 (0.011)$ and $p_{22} = 0.9898 (0.013)$, where $p_{ii}$ denotes the probability that $s_t = i$ given that in the previous period $s_{t-1} = i$ for $i = 1, 2$. These results indicate a very strong persistence in the expenditure regimes in Brazil during the sample period studied. All previous estimates and their low standard errors would appear to support the hypothesis that two different processes generated the observations of government expenditure during the 1980s with the high-spending process exhibiting larger and more volatile levels of expenditure than the lower state.\textsuperscript{24}

The parameter $\varphi$, that measures the impact of the discounted sum of current and expected future government expenditure on the inflation rate, is estimated as $\varphi = 0.148 (0.048)$. Notice it is positive, as expected, and significantly different from zero. The estimated process for the monthly inflation rate is

\[
\pi_t = -2.64 + 1.18 \pi_{t-1} - 0.28 \pi_{t-2} + 0.17 \beta_{s(t)} + 0.01 G_{t-1} + 0.01 G_{t-2} \\
(1.21) \quad (0.09) \quad (0.09) \\
+ 2.82 \quad \text{[If change from low to high state at time t]} \\
- 2.82 \quad \text{[If change from high to low state at time t]} \\
+ \text{Seasonal component} + 0.17 \sigma_{s(t)} \varepsilon_t + u_t,
\]

\textsuperscript{24} Nevertheless, these results (specially, the apparent significance of $\beta_2 - \beta_1$) should not be construed as a formal statistical test of the model. It is well known that under the null hypothesis of linearity (i.e., $\beta_1 = \beta_2$), the transition probabilities are unidentified and standard inference theory does not hold. Some specification tests will be presented in the next subsection.
where $\beta_1 = 14.81 (1.88)$, $\beta_2 = 22.73 (3.03)$, $\sigma_1 = 3.72 (0.33)$, $\sigma_2 = 9.8 (1.0)$, $\sigma_e = 3.27 (0.22)$, and $\sigma_c = 1$ (by definition). The coefficient on the state-dependent variables $\beta_{s(t)}$ and $\sigma_{s(t)}$ is positive as expected. A Lagrange Multiplier test was performed to assess whether the restriction imposed by the model on this coefficient is in agreement with the data. The estimated $\chi^2$ statistic was $\chi^2(1) = 0.997$, indicating that indeed the constraint cannot be rejected at the 5 percent level (the p-value is approximately 21.5 percent). Hence, since $\beta_2 > \beta_1$, and $\sigma_2 > \sigma_1$, it follows that the rate of inflation is significantly higher and more volatile in the high-expenditure state than in the low-spending one. In particular, the model implies that there is a steady-state inflation rate associated with each of the two possible government spending regimes. The low regime would be characterized in equilibrium by a rate of inflation of 6.42% per month and a share of GDP devoted to government outlays of 22.73%; the high spending regime would be associated with an expenditure level amounting to 33.43% of GDP and a monthly rate of inflation of 22.32%.

Regarding the volatility of inflation, earlier literature has empirically found a positive correlation between the level and the variability of inflation [see, for example, Okun (1971), Taylor (1981), and Garcia and Perron (forthcoming)]. For the case of Brazil, the results above provide an intuitive explanation for this observation, as arising directly from the positive relationship between the level and variability of the fundamentals driving the inflation process.

Note that if a regime change occurs in period $t$, the regime change has an effect in addition to the direct effect of the new regime in the inflation rate through the state dependent constant $\beta_{s(t)}$. This once-and-for-all effect is the result of the agents' revising their expectations about the future path of government expenditure, in the light of the observed switch in the state driving its process. It can take any of three values as follows: (i) if $s_{t-1} = 1$ and $s_t = 2$, that is, government spending has switched from the low to the high regime, then its value is 2.82, (ii) if $s_{t-1} = 2$ and $s_t = 1$, that is, government spending has switched

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25 For this calculations, I have employed the arithmetic average of the seasonal components reported below.
from the high to the low regime, then its value is -2.82, or \((iii)\) if \(s_{t-1} = s_t\), that is, there has been no change in the expenditure process the its value is exactly zero. The seasonal components for each month (in percentage points) increase throughout the year as expected and take values,

<table>
<thead>
<tr>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.021</td>
<td>0.031</td>
<td>0.045</td>
<td>0.065</td>
<td>0.099</td>
<td>0.137</td>
<td>0.199</td>
<td>0.289</td>
<td>0.419</td>
<td>0.608</td>
<td>0.881</td>
<td>1.277</td>
</tr>
</tbody>
</table>

Finally, the correlation coefficient between the error terms \(e_t\) and \(u_t\) was found to be -0.065 (0.14) and insignificantly different from zero.

Figure (5) presents the impulse-response function of a change in the regime of the government spending on the inflation rate. A change from the low to the high expenditure state has an immediate effect on the inflation rate of \(0.17(\beta_2 - \beta_1) + 2.82 = 4.17\) percentage points. The first component \(0.17(\beta_2 - \beta_1) = 1.35\) is the result of the direct effect of higher levels of expenditure on the rate of inflation. The second element corresponds to the change-in-regime component that captures agents' expectations that the new, higher level of spending will remain in place for some time in the future. As noted before the latter is a once-and-for-all effect while the former will be in place for as long as the economy remains in the high-expenditure state. Thereafter, past rates of inflation and government spending affect current inflation rate as well, yielding a long-run monthly inflation rate that is 15.9 percentage points higher that in the lower spending regime.

The roots of the third-order difference equation in expected inflation are \(\lambda = 1.45 (0.29)\), \(\lambda_1 = 0.85 (0.08)\), and \(\lambda_2 = 0.32 (0.13)\). The roots \(\lambda_1\) and \(\lambda_2\) were assumed to be less than one in absolute value as a requirement for saddle-path stability. Their (unconstrained) empirical estimates are consistent with the assumption above. Recall that the root \(\lambda\) was constrained to be greater than 1 during the maximization procedure.
From the estimates of the $\lambda$'s it is possible to recover some of the structural parameters. In particular, one can calculate the semi-elasticity of money demand with respect to the inflation rate ($\alpha$) and the coefficients of the money supply process (7). The estimate for the semi-elasticity of money demand is $\alpha = 0.619$ (0.156). This value has the expected positive sign though a somewhat smaller magnitude than previous empirical estimates. LaHaye (1985) finds values ranging from 0.58 (0.09) to 2.22 (1.23) for the Greek and Polish hyperinflations respectively, Cagan (1956, p. 43) obtains estimates between 2.30 (for Poland, April 1923 to November 1923) and 8.70 (for Hungary, July 1922 to February 1924), and Barro (1970) finds values between 2.56 (Poland) and 5.53 (Hungary). Finally, Frenkel (1977) estimates $\alpha = 2.144$ (0.325) for the German hyperinflation.

Let us examine the implication of the model on the money supply. The process for the growth rate of the money supply is:

$$\mu_t^q = c + 0.04 \gamma_t + 0.88 \pi_{t-1} - 0.25 \pi_{t-2} + \epsilon_t$$

(0.02) (0.21) (0.09)

The coefficient of current government expenditure is positive and significantly different from zero. This is the result one would expect to find in an institutional setting in which a substantial proportion of the budget deficit is financed with newly created money. The coefficients of $\pi_{t-1}$ and $\pi_{t-2}$ support the hypothesis that past inflation rates affect the rate of money growth and provide substantial empirical evidence for the endogeneity of the money supply process.

Figure (6) presents the impulse-response function of a change in the regime of government spending on the rate of money growth. A change in the state of the expenditure process from the low to the high spending regime requires an immediate increase in money growth of $0.04(\beta_2 - \beta_1) = 0.32$ percentage points. However, in the long-run, money growth must be $10.37$ percentage points higher than in the low spending regime in order to finance the additional level of spending and to make up for the

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26 The constant term $c$ cannot be recover from the estimated structural parameters.
loss in the value real tax collection. This implication of the model is consistent with empirical evidence for Brazil. The average rate of money growth (per month) rose from 6.03 percent in the period January 1980-December 1984, to 16.89 percent between January 1986 and December 1989.

From figures (5) and (6), it is apparent that immediately after a change from the low to the high spending regime, inflation rises by more than the rate of money growth. This observation is explained by the different nature of these two endogenous variables. While the rate of money growth depends on (actual) current government spending, the inflation rate depends on agent’s expectations of future government expenditure. After observing the change in regime, households understand that the new state will continue to drive the expenditure process for some time in the future. Consequently agents adjust their portfolios (i.e., reduce their money holdings) by more than the increase in money growth would appear to require.

A. Specification Testing

In this sub-section, I report the results of tests of the overidentifying restrictions of the model. First, a Likelihood Ratio test was performed employing an alternative model designed to encompass the null hypothesis as a nested case. It consisted of the joint system of (i) the exogenous process of government expenditure subject to regime changes, and (ii) a specification for the inflation rate equivalent to (12) in which the coefficients of the explanatory variables were freely estimated. A seasonal component was estimated for each of eleven months. The estimated test statistic was $\chi^2_{(14)} = 25.2$, indicating that the model restrictions cannot be rejected at the 1 percent significance level (the p-value is approximately 3.4 percent). If one were to follow Sims (1980, p. 17) suggestion of adjusting the chi-square statistic of the LR test to correct for its small sample properties,\(^{27}\) then the null hypothesis would not be rejected at the

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\(^{27}\) It is well known that for small samples the likelihood ratio test is biased against the null hypothesis.
5 percent level (the p-value would be 8.3 percent).  

The constraints imposed by the model on the seasonality of inflation were separately tested. In this case, all overidentifying restrictions except the ones concerning the seasonal pattern of the inflation rate were imposed on the alternative model. The computed LR statistic was $\chi^2_{(11)} = 18.47$, indicating that the seasonality restrictions would not be rejected at the 5 percent level (the p-value is approximately 7.6 percent).  

Recall that in the model, the inflation rate is assumed to be determined (in the aggregate) by agents behaving under a larger information set than the econometrician has access to. Thus, this data series was conjectured to contain additional information about the government expenditure process. A test of the restrictions of the off diagonal elements $\Omega_{x(1)}$ would provide evidence on this hypothesis. A LR test was performed and the estimated statistic was $\chi^2_{(2)} = 4.83$. Hence, the restrictions of the model on the off-diagonal elements of the variance-covariance matrix are not rejected by the data at the 5 percent level (the p-value is approximately 9.2 percent). The unconstrained estimates of these components are 4.18 (1.97) and 8.37 (3.37), and significantly different from zero.  

In light of the results in this section, one can conclude that the data provides some reasonable support for the restrictions implied by the model.  

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28 For the Sims-adjusted likelihood ratio test, I multiplied twice the difference in the value of the likelihood function by $(T - k)/T$, where $T = 117$ is the sample size and $k = 15$ is the number of coefficients estimated in the unrestricted system divided by the number of equations.  

29 Note, however, that the econometrician's inability to reject this null hypothesis might be the result of the seasonal dummies being imprecisely estimated. The sample period 1980-1989 only provides ten observations to estimate each dummy.  

30 This test was suggested to me by an anonymous referee.
B. Probability Inferences

The time series of government spending and the probability estimates that the expenditure process is in the high-spending regime for every observation between June 1980 and December 1989 are presented in figure (7). The upper panel presents the smooth probabilities that are obtained using all observations in the sample; the middle panel displays the filter probabilities that are computed employing only current and past observations of expenditure and inflation; and the lower panel contains the level of government spending (as percent of GDP). The periods in which the stabilization programs were in effect are indicated between lines in the lower panel.

The smooth probability constitutes the econometrician’s best inference about the spending regime because they make use of (i) the additional information about the regime that is contained in the inflation rate and (ii) all the observations in the sample. These probabilities unambiguously indicate a change in the government spending regime from the low to the high state in 1985. Specifically, the probability that the expenditure process is in the high-spending state continuously increases from zero to (almost) one between June to November 1985. The estimated probabilities for the transition period are 0.9 percent (June), 2.1 percent (July), 4.1 percent (August), 7.6 percent (September), 23.5 percent (October) and 94.8 percent (November). These observations are contemporaneous with important political developments in Brazil. In March 1985, the first democratically elected government took office after 21 years of military rule. It seems plausible that the shift in fiscal policy could be directly associated with this change in government.\(^{31}\) Clearly, from figure (7), the fiscal process in Brazil follows a different rule prior and after 1985 [see also figure (1)].

Notice that the smooth probability that the system is in the high-expenditure state remains close to one during the periods when inflation stabilization programs were in effect in the late 1980s. These

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\(^{31}\) Note, however, that since there is only one observation of a change in government during the sample period, it is not possible to rule out (statistically) the hypothesis that the joint change in fiscal policy and government is solely the outcome of chance.
inferences suggest that there was not a significant modification in the spending process associated with the stabilization plans launched by the government. Thus, the temporary reduction in inflation observed during this period might be solely attributable to the effect of the incomes policies applied as part of these programs.

Recall that the filter probabilities are constructed using only the subset of the agents' information set that is available to the researcher, namely current and past observations of the economic variables. These probabilities are in general agreement with the conclusions obtained using all the observations in the sample. During the periods corresponding to the Cruzado and Bresser programs, the probabilities that government expenditure is in the high regime decrease substantially, but they still remain above 50 percent. Although the agents' beliefs about the spending state are unobserved, one can conjecture that an individual, using only current and past observations of government expenditure and inflation, would have concluded that the joint observation of these variables was more likely to be generated by the high-than by the low-expenditure regime during the inflation stabilization programs. In the situation when the economic agents correctly concluded that no change in government expenditure was associated with the stabilization programs, the plan could not have been credible to the agents. Hence, up to the extent that the agents' beliefs affect their money demand decision, and the level of spending determines the money supply, the empirical results support the hypothesis that the eventual failure of the stabilization plans is attributable to the government's inability to substantially reform its expenditure policy.

V. Conclusions

This paper has developed a dynamic, rational expectations model of inflation where the money supply is endogenously determined by the government's use of newly created money to finance its current spending and by the effect of past rates of inflation on the real value of taxes. Changes in the government's fiscal policy are allowed by modeling expenditure as an autoregressive process subject to
discrete changes in regime (Hamilton (1989)). Agents are presumed to have access to a larger set of information that the econometrician. This information is encapsulated, though not fully-revealed, in the rate of inflation through the agents' money demand decision. The additional information is exploited by the econometrician to refine his/her inferences about the regime and to increase the precision of the parameter estimates.

In an empirical application to Brazil, results provide overall support for the overidentifying restrictions of the model. Empirical estimates indicate that there is a steady-state inflation rate associated with each of the two possible government spending regimes. The low regime would be characterized in equilibrium by a rate of inflation of 6.42% per month and a share of GDP devoted to government outlays of 22.73%; the high spending regime would be associated with an expenditure level amounting to 33.43% of GDP and a monthly rate of inflation of 22.32%. The probability inferences about the regime conclusively indicate a change in government spending from the low to the high-expenditure regime during 1985. These probabilities also suggest that no substantial change in the expenditure regime occurred as part of the inflation stabilization programs introduced by the government in the late 1980s. Up to the extent that these plans were not credible to the agents, results support the hypothesis that the eventual failure of the stabilization plans is attributable to the government's inability to substantially reform its expenditure policy.
Appendix A: RE Solution of the Model (Not intended for final publication)

The analytical model consists of the structural process of money supply (7), and the money demand function (3),

\[ \mu_t = n + \alpha E(\pi_{t+1}) + \alpha E(\pi_{t+1}^t) + \nu_t \quad (3) \]
\[ \mu_t^d = c + \gamma G_t + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \xi_t. \quad (8) \]

Equating (3) and (7) and rearranging,

\[ \alpha E(\pi_{t+1}) - \alpha E(\pi_{t+1}^t) - \pi_t + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} = (n-c) - \gamma G_t + u_t \quad (a1) \]

where \( u_t \) is a disturbance term resulting from the linear combination of \( \xi_t \) and \( \nu_t \). Taking \( E(\pi_{t+1}) \) in both sides of (a1), and dividing through by \( \alpha \) obtain,

\[ E(\pi_{t+1}^t) - \frac{(\alpha+1)}{\alpha} E(\pi_{t+1}^t) + \frac{\theta_1}{\alpha} E(\pi_{t-1}^t) + \frac{\theta_2}{\alpha} E(\pi_{t-2}^t) = \frac{n-c}{\alpha} - \frac{\gamma}{\alpha} E(G_t) \quad (a2) \]

For the solution to this third-order difference equation in \( E(\pi_{t+1}^t) \) define the lag operator, \( LE(\pi_{t+1}^t) = E(\pi_{t+1}^t) \), and the forward operator \( FE(\pi_{t+1}^t) = E(\pi_{t+1}^t) \). Then write,

\[ \left( F^3 - \frac{(\alpha+1)}{\alpha} F^2 + \frac{\theta_1}{\alpha} F + \frac{\theta_2}{\alpha} \right) L^2 E(\pi_{t+1}^t) = \frac{n-c}{\alpha} - \frac{\gamma}{\alpha} E(G_t) \quad (a3) \]

or equivalently,

\[ (F-\lambda)(F-\lambda_1)(F-\lambda_2)L^2 E(\pi_{t+1}^t) = \frac{n-c}{\alpha} - \frac{\gamma}{\alpha} E(G_t) \quad (a4) \]

where \( \lambda, \lambda_1 \) and \( \lambda_2 \) are the roots of the second-order polynomial in the left-hand side of (a3). Note \( \lambda + \lambda_1 + \lambda_2 = (\alpha+1)/\alpha > 1, \lambda \lambda_1 + \lambda_2 + \lambda_1 \lambda_2 = \theta_1/\alpha \) and \( \lambda \lambda_1 \lambda_2 = -\theta_2/\alpha \). Assume the system is saddle-path stable, that is, \( |\lambda| > 1 \), and \( |\lambda_i| < 1 \) for \( i = 1, 2 \). Then rewrite (a4) as,

\[ \lambda(1-F/\lambda)(1-\lambda_1 L)(1-\lambda_2 L)E(\pi_{t+1}^t) = \frac{-\gamma}{\alpha} E(G_t) \]

\[ + \frac{n-c}{\alpha} \quad \text{(as desired)}. \]
Dividing through by $\lambda(1-F/\lambda),$ solving for $E(\pi_t|I_{t-1})$, and with $E(\pi_t|I_{t-1}) = \pi_t$, for $i = 1, 2$, we have

$$E(\pi_t|I_{t-1}) = \kappa + (\lambda_1 + \lambda_2)\pi_{t-1} - (\lambda_1, \lambda_2)\pi_{t-2} + \sum_{j=0}^{\infty} \lambda^{-j+1}E(G_{t+j}|I_{t-1})$$

(a5)

where $\kappa = (n-c)/\alpha(1-\lambda)$. A necessary condition for the summation in (a5) to converge is that $(1/\lambda) < 1$, satisfied under the assumption of saddle-path stability. A sufficient condition for $E(\pi_t|I_{t-1})$ to be finite is that $E(G_{t+1}|I_{t-1})$ is of mean exponential order less than $\lambda$. This is a requirement that government spending does not grow too fast. With (a5), and (a5) forward one period, into (a1), obtain the RE solution for the inflation rate,

$$\pi_t = \kappa + (\lambda_1 + \lambda_2)\pi_{t-1} - (\lambda_1, \lambda_2)\pi_{t-2} + \sum_{j=0}^{\infty} \lambda^{-j+1}E(G_{t+j}|I_{t-1})$$

$$+ \psi \sum_{j=1}^{\infty} \lambda^{-j+1}[E(G_{t+j}|I_{t}) - E(G_{t+j}|I_{t-1})] + u_t$$

(a6)

where $\psi = \gamma(1-c\alpha) > 0$.

Assume the real government spending process follows the autoregressive process

$$G_t = \beta_{3t(0)} + \phi_1 G_{t-1} + \phi_2 G_{t-2} + \ldots + \phi_l G_{t-l} + \psi d_t + \sigma_{3t(0)} e_t$$

(a7)

32 Sargent (1987, p. 395) points out that given the particular definition of the lag operator, which lags the variable but not the information set the expectation is conditioned upon, it is only legitimate to operate in both sides of an equation with its inverse, the forward operator.

33 For homogeneous solution of (a2), the characteristic equation is,

$$X^3 - \frac{(\alpha+1)}{\alpha} X^2 + \frac{\theta_1}{\alpha} X + \frac{\theta_2}{\alpha} = 0$$

with roots given by $\lambda, \lambda_1$ and $\lambda_2$. Therefore, the complete solution for the third-order difference equation (a2) includes the component $c_1 \lambda_1^t + c_2 \lambda_2^t + c_3 \lambda^t$. For the solution of $E(\pi_t|I_{t-1})$ to be bounded for the government expenditure processes that are bounded, I require $c_3 = 0$. This terminal condition rules out inordinate expectations of inflation in the absence of runaway government spending. In addition, since $|\beta_{3t(0)}| < 1$ and $|\beta_{3t(0)}| < 1$, I assume that the time index $t$ is such that $c_1 \lambda_1^t \to 0$ and $c_2 \lambda_2^t \to 0$ for the period under consideration. See Sargent (1987), p. 197.
where $\epsilon_i$ is an i.i.d. $N(0,1)$ error term, $s_i$ denotes the regime real government spending is in, $\sigma_{s(0)}$ and $\beta_{s(0)}$ are variables whose value depends on the state the economy is in, and $d_i$ is a dummy variable equal to one when the observation $i$ corresponds to December and zero otherwise. Define the 1x2 row vector $L_{s(0)}$ with 1 in the $i$th column, if $s_i = i$, and zero everywhere else, and the 2x1 column vector $\beta = [\beta_1 \beta_2]'$.

Assume that the current realization of $s_i$ depends only on $s_{i-1}$ through a Markov chain with transition probabilities given by

$$
P = \begin{pmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{pmatrix}
$$

where $p_{ij} = \Pr(s_i=j|s_{i-1}=i)$, and $p_{11} + p_{12} = 1$ for $i = 1, 2$. Define the $r \times 1$ vector $G_i = [G_i, G_{i-1}, \ldots, G_{i-r+1}]'$; the $1 \times r$ selection vector $h$ with 1 in its (1,1) element and zero everywhere else; the $r \times 1$ vectors $X_i$ and $B_i$ with (1,1) elements given respectively by $\sigma_{s(0)}e_i$ and $\beta_{s(0)} + \psi d_i$, and zero everywhere else; and the $r \times r$ matrix $\Phi$,

$$
\Phi = \begin{pmatrix}
\phi_1 & \phi_2 & \cdots & \phi_{r-1} & \phi_r \\
1 & 0 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
\ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & 1 & 0
\end{pmatrix}
$$

Hence, we can write $G_i = hG_{i-1}$, where $G_i = \Phi G_{i-1} + B_i + X_i$, and in general,

$$
G_{i+1} = HD^{i+1}G_{i-1} + \sum_{j=0}^{i} HD^j B_{i+1-j} + \sum_{j=0}^{i} HD^j X_{i+1-j}.
$$

Taking $E(s_{i|t-1})$ in both sides obtain,

Then,
\[ E(G_{t+1} | \mathbf{H}_{t-1}) = \text{HD}^{i+1} \mathbf{G}_{t-1} + \sum_{j=0}^{i} \text{HD}^j E(B_{t+j} | \mathbf{H}_{t-1}) . \]

\[ \sum_{i=0}^{\infty} \lambda^{-i(1)} E(G_{t+1} | \mathbf{H}_{t-1}) = \sum_{i=0}^{\infty} \lambda^{-i(1)} \text{HD}^{i+1} \mathbf{G}_{t-1} + \sum_{i=0}^{\infty} \sum_{j=0}^{i} \lambda^{-i(1)} \text{HD}^j E(B_{t+j} | \mathbf{H}_{t-1}) . \quad (a8) \]

The first element in the right hand side of (a8) is,

\[ \sum_{i=0}^{\infty} \lambda^{-i(1)} \text{HD}^{i+1} \mathbf{G}_{t-1} = \text{HD}(\lambda \mathbf{I} - \mathbf{D})^{-1} \mathbf{G}_{t-1} \]

and the second element can be written as,

\[ \sum_{i=0}^{\infty} \sum_{j=0}^{i} \lambda^{-i(1)} \text{HD}^j E(B_{t+j} | \mathbf{H}_{t-1}) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \lambda^{-i(1)} \text{HD}^j E(B_{t+j} | \mathbf{H}_{t-1}) \]

\[ = \text{H} \lambda(\lambda \mathbf{I} - \mathbf{D})^{-1} \sum_{i=0}^{\infty} \lambda^{-i(1)} E(B_{t+i} | \mathbf{H}_{t-1}) . \]

Hence, the weighted sum of current and expected future government expenditures in (a8) can be written as,

\[ \sum_{i=0}^{\infty} \lambda^{-i(1)} E(G_{t+1} | \mathbf{H}_{t-1}) = \text{HD}(\lambda \mathbf{I} - \mathbf{D})^{-1} \mathbf{G}_{t-1} + \mathbf{Q} \sum_{i=0}^{\infty} \lambda^{-i(1)} E( \beta_{\mathbf{H}_{t+i}} + \psi_{\mathbf{H}_{t+i}} | \mathbf{H}_{t-1}) \quad (a9) \]

where \( \mathbf{Q} = \text{H} \lambda(\lambda \mathbf{I} - \mathbf{D})^{-1} \mathbf{h}' \) and I have used the definition \( \mathbf{B}_t = h' \beta_{\mathbf{H}_{t+i}} + \psi_{\mathbf{H}_{t+i}} \).

Under the assumptions regarding the Markovian process of \( s_{t'} \), \( E(\beta_{s(t+i)} | \mathbf{H}_{t-1}) = I_{s(t-1)} \mathbf{P}^{i+1} \beta \). Then,

\[ \sum_{i=0}^{\infty} \lambda^{-i(1)} E( \beta_{\mathbf{H}_{t+i}} | \mathbf{H}_{t-1}) = I_{\mathbf{H}_{t-1}} \mathbf{P}(\lambda \mathbf{I} - \mathbf{P})^{-1} \beta . \quad (a10) \]

Finally, note the seasonal component in (a9) depends on the month in which the expected value is taken.

For example, when \( t-1 \) corresponds to December it is equal to \( \psi(\lambda^{12} - 1) \). When period \( t-1 \) corresponds
to January, it is equal to $\psi \lambda / (\lambda^{12.1})$. When period $t-1$ corresponds to February, it is equal to $\psi \lambda^2 / (\lambda^{12.1})$, and so forth. In general,

$$\sum_{j=0}^{\infty} \lambda^{-i+1} E \left( \psi d_{t+i} | I_{t-1} \right) = \frac{\psi \lambda^j}{(\lambda^{12.1})}$$  \hspace{1cm} (a11)

where $j_t$ takes value 0 if period $t-1$ corresponds to December, 1 if period $t-1$ corresponds to January, 2 if period $t-1$ corresponds to February, and so on.

With (a10) and (a11) into (a9), obtain,

$$\sum_{i=0}^{\infty} \lambda^{-i+1} E \left( G_{t+i} | I_{t-1} \right) = HD (\lambda I - D)^{-1} G_{t-1} + q L_{t-1} P (\lambda I - P)^{-1} \beta + \frac{q \psi \lambda^j}{(\lambda^{12.1})}$$  \hspace{1cm} (a12)

and with (a12), and (a12) forwarded one period, into (a6), using $G_t = \Phi G_{t-1} + B_t + X_t$, and $G_t = \beta_{\pi(0)} + h \Phi G_{t-1} + \psi d_t + \sigma_{\pi(0)} \epsilon_t$, defining $\delta = h \Phi (\lambda I - \Phi)^{-1} h'$, and simplifying, we have the rational expectations solution for the inflation rate,

$$\pi_t = \kappa + (\lambda_1 + \lambda_2) \pi_{t-1} - (\lambda_1 \lambda_2) \pi_{t-2} + \psi (1+\delta) \beta_{\pi(0)} + \psi (\lambda-1) h \Phi (\lambda I - \Phi)^{-1} G_{t-1}$$

$$+ \psi Q (L_{\pi(0)} - L_{\pi(t-1)}) P (\lambda I - P)^{-1} \beta + \Gamma (\text{month } t) + \psi (1+\delta) \sigma_{\pi(0)} \epsilon_t + u_t$$  \hspace{1cm} (a13)

where the seasonal function $\Gamma (\text{month } t)$ takes the value $\psi \psi (1+\delta) + \psi \psi (1-\lambda^{11}) / (\lambda^{12.1})$ if period $t$ corresponds to December and $\psi \psi \lambda^j (\lambda-1) / (\lambda^{12.1})$, otherwise, where $j = 0$ if $t$ corresponds to January, 1 if $t$ corresponds to February, 2 if $t$ corresponds to March, ..., and 10 if $t$ corresponds to November.
References


Figure 1. Fiscal Variables in Brazil, 1980-1988

Government Expenditure

Tax Revenues

Budget Deficit
Figure 2. Seasonality

**Government Expenditure**

**Rate of Money Growth**

**Rate of Inflation**
Figure 5. Impulse-Response Function
Rate of Inflation

Percent per Month

Period
Figure 6. Impulse-Response Function
Rate of Money Growth
Figure 7. Probability Inferences about Expenditure Regime

Smooth Probability

Filter Probability

Government Expenditure

Percent of GDP
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