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**FERTILITY, CONSUMPTION AND BEQUESTS
IN A MODEL WITH NON-DYNASTIC PARENTAL ALTRUISM**

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RÉSUMÉ

Un modèle économique de la fertilité est étudié dans lequel le bien-être des parents dépend de leur consommation, du nombre d'enfants qu'ils ont et de la consommation de leurs enfants, mais pas du bien-être des enfants en tant que tel. Par conséquent, les parents sont altruistes à l'égard de leurs enfants, mais dans une moindre mesure que dans le modèle dynastique de la fertilité récemment présenté par Becker et Barro (1988). Le taux de natalité à l'état stationnaire est plus faible dans le modèle étudié dans ce papier que dans le modèle de Becker et Barro, mais il s'avère que les principales prédictions qualitatives concernant le comportement de la fertilité, de la consommation et des legs sont fort similaires dans les deux modèles.

Mots clés : fertilité, consommation, legs, altruisme, préférences non-dynastiques.

ABSTRACT

A model of fertility choice is studied in which the utility of parents depends on how much they consume, on how many children they have and on the consumption of their children, but not on the utility of descendants per se. Hence parents are altruistic towards their children, but in a more limited sense than in the dynastic fertility model recently presented by Becker and Barro (1988). While the steady state birth rate is lower in the model presented in this paper than in the Becker and Barro model, it appears that the key qualitative predictions concerning the behavior of fertility, consumption and bequests are quite similar in both models.

Key words : fertility, consumption, bequests, altruism, non-dynastic preferences.



1. Introduction

This paper develops an economic model of fertility choice in which the utility level of agents depends on how much they consume, on how many children they have and on the consumption of their children. Hence parents are altruistic towards their children, but in a more limited sense than in the dynastic model of fertility choice recently proposed by Becker and Barro (1988) in which the utility of parents depends on the utility of their children and hence (indirectly) on the consumption and fertility decisions made by all subsequent generations (henceforth, I will refer to Barro and Becker (1988) as 'BB').¹

In contrast to the dynastic model, the non-dynastic form of altruism considered in the present paper implies that the preferences of different generations diverge: parents want their children to spend as little as possible on their own offsprings. The model is solved by applying the notion of a (subgame perfect) bequest equilibrium introduced by Bernheim and Ray (1987) and Leininger (1986).²

¹Other work which uses a dynastic framework to analyze fertility issues includes Razin and Ben-Zion (1974), Becker and Murphy (1988), Barro and Becker (1989), Becker, Murphy and Tamura (1990), Benhabib and Nishimura (1989, 1990), Pestieau (1989), Lapan and Enders (1990) and Nishimura and Raut (1992). Dynastic preferences in a setting with exogenous fertility are studied by Barro (1974), Loury (1981), Ray (1987), Weil (1987), Bernheim (1989) and Hori (1992), among others.

²In contrast to the present paper, Bernheim and Ray (1987) and Leininger (1986) study growth models with non-dynastic altruism in which fertility is *exogenous*. Other research on the same class of non-dynastic models with exogenous fertility includes Arrow (1973), Kohlberg (1973), Dasgupta (1974), Lane and Mitra (1981), Lane and Leininger (1984) and Veall (1986).

Kollmann (1987), Raut (1992) and Nishimura and Zhang (1992) make fertility endogenous in a growth model with non-dynastic altruism (Eckstein and Wolpin (1985) and Raut (1987, 1990) study overlapping generations models with endogenous fertility, but without altruism). The present paper is closest to Raut (1992), as that paper also focuses on subgame perfect equilibria. In contrast to Raut, the use of a utility function inspired by BB allows to identify conditions under which a unique bequest equilibrium exists and to fully characterize the dynamics of fertility and consumption (Raut assumes weaker restrictions on utility functions and hence is merely

Steady state fertility is lower in the model with non-dynastic preferences than in the BB model. However, despite the fact that BB stress that the dynastic preference specification is a central feature of their model, the non-dynastic model developed in this paper captures well the key qualitative features of the behavior of fertility, bequests and consumption which obtains in the BB model. It appears that, in both models, the steady state birth rate does not depend on the cost of raising children; a permanent change in the cost of raising children therefore only has transitory effects on the birth rate. In both models, a temporary increase in the cost of raising children leads to a temporary drop in the birth rate which is followed by a temporary increase; a shock to the inherited wealth of a given generation does not affect the per capita wealth of subsequent generations (i.e. per capita wealth regresses to the mean in one generation).

Section 2 of the paper presents the assumptions of the non-dynastic model and it shows how to solve for equilibria in that model. Section 3 summarizes the BB model. Section 4 describes the implications of the non-dynastic model. Section 5 concludes.

2. A non-dynastic model of fertility

2.1. Preferences and budget constraints

The model is closely related to those of BB, Leininger (1986) and Bernheim and Ray (1987). The economy consists of an infinite sequence of generations which are labelled $i=0,1,2,\dots$. All members of the same generation are assumed to be identical. Each generation lives two periods. Agents consume

able to analyze steady states). I learnt about Raut (1992) after the present research was completed (I thank Christophe Faugère from bringing that paper to my attention). The present paper is thus a complementary and independent analysis.

and give birth to children during the second period of their life (no consumption takes place during the first period). The utility of a member of generation i depends on that person's consumption, on how many children the person has and on the consumption of these children.³ The utility function of a member of generation i is given by:

$$u_i = c_i^\sigma + \alpha n_i^{1-\epsilon} c_{i+1}^\sigma \quad (1)$$

Here c_i is the consumption of the member of generation i , n_i is the number of children which the person has and c_{i+1} is the consumption of each child. σ , ϵ and α are preference parameters. The parameters α and ϵ determine the degree of parental altruism towards children. I assume $0 < \sigma, \epsilon$ and $\sigma + \epsilon < 1$.⁴ Although c_{i+1} appears in the utility function of generation i , this variable is not (directly) controlled by parents: parents cannot dictate their children's consumption decisions.

In contrast to (1), BB have assumed a dynastic utility function according to which the utility of generation i depends on the utility of each child and hence on the consumption and the birth rate of all subsequent generations:

$$u_i = c_i^{\sigma + \alpha n_i^{1-\epsilon}} u_{i+1} = c_i^{\sigma + \alpha n_i^{1-\epsilon}} c_{i+1}^\sigma + \alpha^2 (n_i n_{i+1})^{1-\epsilon} c_{i+2}^\sigma + \dots \quad (2)$$

Hence the present paper maintains BB's assumption that utility is time-separable in consumption, but it only keeps the first two terms on the right-hand side of (2).

The resources of a member of generation i consist of an inheritance

³Following BB, the model assumes that people have children without marriage.

⁴ $0 < \sigma, \epsilon < 1$ implies that u_i is increasing and concave in c_i and in n_i . $\sigma + \epsilon < 1$ is assumed, in order to ensure that the decision problem of generation i is well defined (see the discussion below).

received from the previous generation; this endowment can be used for consumption, in order to raise children and as a bequest. Parents cannot leave negative bequests to their children. I assume that the budget constraint of a member of generation i is given by:

$$k_i(1+r) = c_i + n_i(k_{i+1} + b_i). \quad (3)$$

Here $k_i(1+r)$ is the inheritance received by the person. $b_i > 0$ is the cost of raising one child. $r > 0$ is the return on the capital k_i .⁵

Although parents cannot dictate their children's consumption, they can influence it through the bequests which they make. Assume the relation between the consumption of generation $i+1$ and its wealth is given by the consumption schedule g_{i+1} :

$$c_{i+1} = g_{i+1}(k_{i+1}(1+r)). \quad (4)$$

Given g_{i+1} , the decision problem of a member of generation i consists in maximizing the following objective function subject to the budget constraint (3) and subject to $k_{i+1} \geq 0$:

$$u_i = c_i^\sigma + \alpha n_i^{1-\epsilon} [g_{i+1}(k_{i+1}(1+r))]^\sigma. \quad (5)$$

2.2. Equilibrium

The paper applies the concept of a (subgame perfect) bequest equilibrium developed by Leininger (1986) and by Bernheim and Ray (1987).

⁵The budget constraint (3) follows the one used by BB, except that they assume that $w_i + (1+r)k_i = c_i + n_i(k_{i+1} + b_i)$, where w_i is the wage income of generation i (BB also allow for a variable interest rate). Letting $k'_i = w_i/(1+r) + k_i$ and $b'_i = b_i - w_{i+1}/(1+r)$, this constraint can be expressed in a way which is formally identical to (3): $(1+r)k'_i = c_i + n_i(k'_{i+1} + b'_i)$. The k_i variable in (3) should thus be interpreted as including both human capital and other assets (physical and financial capital).

Given an initial capital stock k_0 and a sequence of costs of raising children $\{b_i\}_{i \geq 0}$, an equilibrium is a sequence of consumption schedules, consumptions, bequests and birth rates $\{g_i^*, c_i^*, k_i^*, n_i^*\}_{i \geq 0}$ with the following properties:

(i) for all $i \geq 0$ and for all $y \geq 0$,

$$g_i^*(y) \in \text{Arg Max}_{0 \leq c \leq y} \left\{ \text{Max}_{n, k \in \Gamma_i(c, y)} c^\sigma + \alpha n^{1-\varepsilon} [g_{i+1}^*(k(1+r))]^\sigma \right\},$$

where $\Gamma_i(c, y) = \{n, k | c + n(k + b_i) \leq y; n, k \geq 0\}$.

(ii) $k_0^* = k_0$. For all $i \geq 0$ we have $c_i^* = g_i^*(k_i^*(1+r))$ and

$$n_i^*, k_{i+1}^* \in \text{Arg Max}_{n, k \in \Gamma_i(c_i^*, k_i^*(1+r))} \{[(c_i^*)^\sigma + \alpha n_i^{1-\varepsilon} [g_{i+1}^*(k(1+r))]^\sigma\}.$$

Part (i) of the definition of an equilibrium says that the equilibrium consumption schedule g_i^* reflects optimal consumption decisions by generation i given that the consumption schedule of generation $i+1$ is g_{i+1}^* . In equilibrium, generation i 's wealth is $k_i^*(1+r)$ and hence its equilibrium consumption is $c_i^* = g_i^*(k_i^*(1+r))$. The equilibrium birth rate and bequests of generation i (n_i^*, k_{i+1}^*) are determined by maximizing i 's utility function given the schedule g_{i+1}^* and subject to i 's inheritance $k_i^*(1+r)$.

Appendix A shows that, irrespective of the consumption schedule of generation $i+1$,⁶ the consumption schedule of generation i has the following properties: $0 \leq g_i^*(y) \leq y$ for all $y \geq 0$; g_i^* is differentiable and strictly increasing; the elasticity of g_i^* with respect to i 's wealth does not exceed

⁶ Provided that the ensuing optimization problem of generation i is well defined ($0 \leq g_{i+1}^*(y) \leq y$ for all $y \geq 0$ and continuity of g_{i+1}^* are sufficient conditions which ensure that this is the case).

unity; this elasticity is continuous and decreasing in wealth.

This implies that, in equilibrium, the consumption schedules of all generations have the properties which were just listed. This is very useful, because when the consumption schedule of generation $i+1$ has these properties, then the first-order conditions of the decision problem of generation i can be used to solve for the optimal optimal consumption, fertility and bequest decisions of generation i (see appendix A).

These first-order conditions are given by the following equations:⁷

$$n_i = c_i^{(1-\sigma)/\epsilon} (k_{i+1} + b_i)^{-1/\epsilon} g_{i+1}^{\sigma/\epsilon} [\alpha(1-\epsilon)/\sigma]^{1/\epsilon}, \quad (6)$$

$$(\sigma/(1-\epsilon)) \eta_{i+1} = k_{i+1}/(k_{i+1} + b_i), \quad (7)$$

$$k_i(1+r) = c_i + c_i^{(1-\sigma)/\epsilon} \psi_i, \text{ with } \psi_i = g_{i+1}^{\sigma/\epsilon} (k_{i+1} + b_i)^{1-1/\epsilon} (\alpha(1-\epsilon)/\sigma)^{1/\epsilon} > 0. \quad (8)$$

In (7), η_{i+1} denote the elasticity of generation $i+1$'s consumption schedule: $\eta_{i+1} = [\partial g_{i+1}(y)/\partial y][y/g_{i+1}(y)]$. (7) determines the optimal value of k_{i+1} . We see that (7) has a unique solution, provided $\eta_{i+1} \leq 1$ and provided η_{i+1} is continuous and decreasing in k_{i+1} (as is the case in equilibrium), and given the assumption that $\sigma + \epsilon < 1$.⁹

Equation (8) implies that the elasticity of i 's consumption schedule with respect to the inheritance ($k_i(1+r)$) can be expressed as:¹⁰

⁷The analysis disregards integer restrictions on the number of children.

⁸Conditions (6) and (7) are obtained by equating the marginal rate of substitution between c_i and n_i and between k_{i+1} and n_i to the corresponding marginal rates of transformation. (8) is obtained from (6), (7) and the budget constraint (3).

⁹We see from this condition that the optimal bequests per child made by generation i do not depend on the wealth of generation i --they merely depend on the shape of the consumption schedule of generation $i+1$ and on the cost of raising children (b_i) (ceteris paribus, an increase in b_i increases k_{i+1}).

¹⁰Note that the term ψ_i in (8) does not depend on i 's wealth.

$$\eta_i = \left(c_i + c_i^{(1-\sigma)/\epsilon} \psi_i \right) / \left(c_i + ((1-\sigma)/\epsilon) c_i^{(1-\sigma)/\epsilon} \psi_i \right) \quad (9).$$

In equilibrium, equations (6)-(9) hold for all $i \geq 0$.

In order to solve for an equilibrium, it is convenient to write equations (6)-(9) in terms of ratios of consumption to wealth. Denote generation i 's consumption to wealth ratio by γ_i : $\gamma_i = c_i / (k_i(1+r))$. Equations (7), (8) and (9) imply:

$$k_{i+1} = \left(\sigma \epsilon / ((1-\sigma-\epsilon)(1-\epsilon)) \right) b_i / \left(1/(1-\epsilon) - \gamma_{i+1} \right) \text{ for } i \geq 0. \quad (10)$$

Substituting this expression into (8) for $i=0$ gives:

$$1 = \gamma_0 + \gamma_0^{(1-\sigma)/\epsilon} (k_0(1+r))^{(1-\sigma-\epsilon)/\epsilon} \gamma_1^{\sigma/\epsilon} \left(1/(1-\epsilon) - \gamma_1 \right)^{(1-\sigma-\epsilon)/\epsilon} \cdot \left((1-\sigma)/(1-\sigma-\epsilon) - \gamma_1 \right)^{1-1/\epsilon} (1+r)^{\sigma/\epsilon} b_0^{(\sigma+\epsilon-1)/\epsilon} \cdot \Gamma_0, \quad (11)$$

where $\Gamma_0 = [\sigma \epsilon / ((1-\sigma-\epsilon)(1-\epsilon))]^{\sigma/\epsilon} [\alpha(1-\epsilon)/\sigma]^{1/\epsilon}$.

Making the same substitution for $i \geq 1$ we obtain:

$$1 = \gamma_i + \gamma_i^{(1-\sigma)/\epsilon} \left(1/(1-\epsilon) - \gamma_i \right)^{-(1-\sigma-\epsilon)/\epsilon} \gamma_{i+1}^{\sigma/\epsilon} \cdot \left(1/(1-\epsilon) - \gamma_{i+1} \right)^{(1-\sigma-\epsilon)/\epsilon} \left((1-\sigma)/(1-\sigma-\epsilon) - \gamma_{i+1} \right)^{1-1/\epsilon} \cdot (1+r)^{(1-\epsilon)/\epsilon} (b_{i-1}/b_i)^{(1-\sigma-\epsilon)/\epsilon} \Gamma_i \text{ for } i \geq 1, \quad (12)$$

where $\Gamma_i = \alpha^{1/\epsilon} ((1-\epsilon)/\sigma) (\epsilon/(1-\sigma-\epsilon))^{(1-\epsilon)/\epsilon}$ 11

¹¹The expression relating γ_0 to γ_1 (i.e. equation (11)) differs from that which relates γ_i to γ_{i+1} for $i \geq 1$ (see (12)) because k_0 is historically given, i.e. we cannot use (10) to eliminate k_0 from the budget constraint of generation $i=0$.

2.3. Existence and Uniqueness of the Equilibrium

To solve for an equilibrium, we have to find a sequence of consumption ratios $\{\gamma_i\}_{i=0}^{\infty}$ satisfying (11) and (12). Once such a sequence has been found, the equilibrium consumptions, birth rates and bequests can easily be determined.¹²

(11) and (12) define reaction functions which relate the consumption ratio selected by generation i (γ_i) to γ_{i+1} :

$$\gamma_i = f^i(\gamma_{i+1}) \text{ for } i \geq 0. \quad (13)$$

It can be verified that f^i has the following properties:

$$f^i(0) = 1, f^i(1) > 0, f^i \text{ is continuous and strictly decreasing.} \quad (14)$$

For $j \geq 1$, denote by $F^j(\gamma)$ the consumption ratio selected by generation 0 if generation j selects the ratio γ and if generations $0, 1, \dots, j-1$ behave according to (13): $F^j(\gamma) = f^0(f^1(\dots(f^{j-1}(\gamma))\dots))$. Let $S^j = \{F^j(\gamma) \mid 0 \leq \gamma \leq 1\}$. S^j is a closed interval. Furthermore $S^{j+1} \subseteq S^j$.¹⁴ This implies that $S = \bigcap_{j=0} S^j$ is non-empty. A sequence of equilibrium consumption ratios $\{\gamma_i^*\}_{i \geq 0}$ which satisfy (13) for all $i \geq 0$ can be constructed by choosing $\gamma_0^* \in S$ and by

¹²(10) can be used to solve for equilibrium bequests. See appendix C for formulae which give the birth rate as a function of consumption ratios.

¹³Generation i derives utility from the consumption expenditures of its children, but not from the resources which these children spend on their own children. Hence it is not surprising that when generation $i+1$ consumes a larger share of its inheritance, then generation i devotes a larger share of its resources to its descendants and hence a smaller share to its own consumption (which explains why f^i is decreasing). When generation $i+1$ selects a zero consumption ratio, then generation i does not derive any utility from having children. Hence it consumes all its resources (thus $1 = f^i(0)$). As the marginal utility of consumption is infinite when $c_{i+1} = 0$, generation i always consume a positive fraction of its resources and thus $f^i(1) > 0$.

¹⁴N.B. $S^{j+1} = \{F^j(f^j(\gamma)) \mid 0 \leq \gamma \leq 1\} \subseteq S^j$ because $0 < f^j \leq 1$.

selecting $\gamma_1^*, \gamma_2^*, \gamma_3^*, \dots$ using $\gamma_0^* = f^0(\gamma_1^*), \gamma_1^* = f^1(\gamma_2^*), \gamma_2^* = f^2(\gamma_3^*), \dots$

The fact that the set S is nonempty therefore guarantees the existence of an equilibrium. Under relatively weak assumptions, we can guarantee that the equilibrium is unique.¹⁵

3. The Becker and Barro (1988) fertility model

Before discussing the predictions of the non-dynastic model, it seems useful to summarize the key features of the BB fertility model. Maximizing the dynastic utility function (2) subject to the restriction that (3) is satisfied for all $i \geq 0$, gives the following equilibrium conditions:

$$c_i = [\sigma / (1 - \sigma - \epsilon)] b_{i-1} (1+r) \text{ for } i \geq 1 \text{ and } n_i = [\alpha(1+r)(c_{i+1}/c_i)^{\sigma-1}]^{1/\epsilon} \text{ for } i \geq 0.$$

When b is constant, there exists a unique steady state. The steady state birth rate is given by:

$$n = (\alpha(1+r))^{1/\epsilon} \tag{15}$$

A prediction of this model is thus that the steady state birth rate is independent of the cost of raising children and that it is positively related to the interest rate and to the altruism parameter α . Other important predictions are: (i) A permanent reduction in the cost of raising children leads to a temporary rise in the birth rate, but it does not affect the birth rate in the long run. (ii) A fall in b during one generation induces a rise in the birth rate of that generation which is followed by a reduction of the birth rate in the following generation; the birth rates of subsequent generations are unaffected. (iii) A change in the wealth of the initial generation (k_0) does not affect the birth rate, the per capita wealth and the per capita consumption of subsequent generations, i.e. per capita wealth regresses to the mean in one generation (an increase in k_0 merely increases consumption and the birth rate in the initial

¹⁵ Appendix B shows that if there exists a number q such that for all generations $i \geq q$ the cost b_i grows at a constant rate, then $\sigma \leq .5$ guarantees uniqueness of the equilibrium.

generation). (iv) Per capita consumption only exhibits long term growth if the cost of raising children grows over time. More rapid growth of b reduces the birth rate.

4. Implications of the non-dynastic model

4.1. Steady states

When the cost of raising children is constant, then the non-dynastic model has a unique steady state in which consumption, bequests and the birth rate are constant.¹⁶ Equations (3), (6) and (7) imply that the steady state birth rate is determined by the following equation:

$$n = [\alpha(1+r) - n \alpha(1-\epsilon)(1-\sigma)/(\epsilon\sigma)]^{1/\epsilon}. \quad (16)$$

The steady state consumption to wealth ratio is given by:

$$1 = \gamma + \gamma^{1/\epsilon} (1-\sigma)/(1-\sigma-\epsilon) - \gamma^{1-1/\epsilon} (1+r)^{(1-\epsilon)/\epsilon} \alpha^{1/\epsilon} \Gamma, \quad (17)$$

where $\Gamma = ((1-\epsilon)/\sigma) (\epsilon/(1-\sigma-\epsilon))^{(1-\epsilon)/\epsilon}$.¹⁷

We see that, as in the BB model, the steady state birth rate is independent of the cost of raising children. In both models, increases in the interest rate and in the altruism parameter α increase the birth rate.¹⁸ A comparison between equations (15) and (16) shows that the steady state birth rate is lower in the non-dynastic model. This is not surprising, as parents are less altruistic in the non-dynastic model than in the BB model.

The model can also generate steady states in which per capita

¹⁶In other words, in a setting in which b is constant, there exists a unique equilibrium in which all endogenous variables are constant across generations. In that setting, other equilibria may exist, but by setting $c \leq 5$ we can guarantee that the steady state equilibrium is the only equilibrium (see section 2.3).

¹⁷Steady state bequests per child and the steady state consumption are given by: $k = (b/(1/(1-\epsilon) - \gamma)) (\sigma\epsilon/((1-\sigma-\epsilon)(1-\epsilon)))$; $c = \gamma k (1+r)$.

¹⁸Note however that, in the BB model, increases in α and r raise per capita consumption and per child bequests, while the opposite can be shown to be the case in the non-dynastic model.

consumption and bequests grow at constant rates if one assumes that the cost of raising children grows at a constant rate.¹⁹ Growth of b at a constant rate implies that the last term on the right-hand side of (17) has to be multiplied by $\varphi_b^{-(1-\sigma-\epsilon)/\epsilon}$, where $\varphi_b = b_{i+1}/b_i$. Thus an increase in φ_b increases the steady state consumption ratio γ and--as in the BB model--it reduces the birth rate.²⁰ As in the BB model, an increase in the interest rate does not affect the growth rate of per capita consumption, although it increases the growth rate of aggregate consumption, because it raises the population growth rate.

4.2. Dynamic effects of changes in exogenous variables

The following changes in exogenous variables will be considered: a permanent and a temporary change in the cost of raising children²¹ and a change in k_0 (i.e. a wealth shock). In all three cases, the predictions of the model are similar to those of the BB model.

The exogenous changes discussed in this section have in common that they do not alter the reaction function f^i of generations $i \geq q$ for $q=1$ or $q=2$. If the equilibrium is unique, these exogenous changes therefore do not affect the equilibrium values of $\gamma_q, \gamma_{q+1}, \dots$. The following discussion assumes that the equilibrium is unique.²²

¹⁹ Steady state growth in the cost b can be due to technical progress which induces growth in the marginal product of labor in the production of physical goods and hence in the time-cost to parents of raising children.

²⁰ From (10) and (3), we see that the steady state birth rate can be expressed as $n = [(1-\gamma)/((1-\sigma)/(1-\sigma-\epsilon)-\gamma)] \cdot [1/\varphi_b] \cdot [\sigma\epsilon/((1-\sigma-\epsilon)(1-\epsilon))] \cdot (1+r)$. This expression is decreasing in γ and in φ_b .

²¹ As discussed by BB, a reduction in infant mortality can be interpreted as a fall in the cost b (such a reduction lowers the cost of raising one child which survives to adulthood).

²² Note that, even if the equilibrium is not unique, there exists an equilibrium after the change in the exogenous variables in which the consumption ratios for generations $i \geq q$ are the same as before the change.

A permanent change in the cost of raising children

Assume that for all generations $i \geq 0$ the cost of raising children falls by the same proportion. From equation (12) we see that this change does not affect the reaction function f^i for generations $i \geq 1$. Under the assumption that the equilibrium is unique, the consumption ratios of generations $i \geq 1$ do not change and hence the per capita wealth of generations $i \geq 1$ is unaffected (see (10)), and the same thing is true for the per capita consumptions of these generations (and for the bequests per child made by these generations). The budget constraints of generations $i \geq 1$ thus imply that the birth rates of these generations are unaffected by a permanent equiproportional drop in b . A permanent fall in b does, however, increase the birth rate of generation $i=0$.²³ Hence we predict that a permanent equiproportional fall in the cost of raising children increases the birth rate during generation $i=0$, but that it does not affect the birth rates of subsequent generations.

A temporary change in the cost of raising children

When the cost of raising children in the initial generation (b_0) falls (while b_1 stays constant for $i \geq 1$), then the birth rate of that generation

To see this, note that the set of sequences of consumption ratios for generations $i \geq q$ which are consistent with an equilibrium is given by $\mathcal{S}^q = \{ \{ \gamma_i \}_{i \geq q} \mid \gamma_i = f^i(\gamma_{i+1}) \text{ for all } i \geq q \}$. Clearly, \mathcal{S}^q does not change when f^i for $i \geq q$ does not change.

²³As the right-hand side of (11) is increasing in γ_0 and decreasing in b_0 , a permanent fall in the cost of raising children reduces γ_0 and hence the consumption of the initial generation. (10) implies that generation $i=0$ reduces the bequest which it gives to each child. The budget constraint of generation $i=0$ implies therefore that the birth rate of that generation increases.

rises, while the birth rate of generation 1 falls.²⁴ n_1 falls because a reduction in b_0 lowers the bequest which generation 0 makes to each of its children. Bequests per child made by generation 1 however do not change and hence the per capita wealth and the birth rates of subsequent generations are unaffected.²⁵

A wealth change

We see from equation (12) that a change in the wealth of the initial generations (k_0) does not alter the reaction functions f^i of generations $i \geq 1$. Hence we see from equation (10) that generation $i=0$ does not change the bequest which it gives to each child; generation $i=0$ merely increases its consumption and the number of children when its wealth rises (see (3) and (6)). As the per capita wealth of generations $i \geq 1$ does not change when k_0 varies, the birth rates (and other decision variables) of generations $i \geq 1$ are unaffected by a change in k_0 .

5. Conclusion

A model is studied in which the utility of parents depends on how much they consume, on how many children they have and on the consumption of their children, but not on the utility of their children per se. This distinguishes the model from the dynastic fertility model recently studied

²⁴Note that the reaction functions f^i for $i \geq 2$ are unaffected by a change in b_0 and, assuming uniqueness of the equilibrium, $\gamma_2, \gamma_3, \dots$ do not change.

²⁵See appendix C for a derivation of this response pattern. It can be shown that a fall in b_j for $j > 0$ raises n_j and lowers n_{j+1} , under the assumption that, in the absence of the change in b_j , generations j and $j+1$ would select identical consumption ratios (such an assumption is not needed in order to obtain the response to a change in b_0 which is described in the text).

by Becker and Barro (1988). While steady state fertility is lower in the model studied in this paper than in the Becker-Barro model, it appears that the key qualitative predictions concerning the behavior of consumption, fertility and bequests are similar in both models.

Appendix A

Irrespective of the shape of the consumption schedule of generation $i+1$ (provided that the ensuing optimization problem of generation i is well-behaved), the consumption schedule of generation i has the following properties: $0 \leq g_i(y) \leq y$; g_i is differentiable and strictly increasing; the wealth elasticity of g_i does not exceed unity and this elasticity is a continuous and decreasing function of wealth.

Consider first the problem which consists in finding the values of c_i and n_i which maximize the objective function u_i (see (5)) subject to the budget constraint (3), holding constant bequests k_{i+1} at an arbitrary value. The objective function is strictly concave in c_i and n_i and the budget constraint is linear in these variables. Hence the first order conditions of this decision problem (equations (3) and (6)) are necessary and sufficient for an optimum. Substituting the expression for n_i given by (6) into the objective function u_i yields:

$$u_i = c_i^\sigma + \alpha(\alpha(1-\epsilon)/\sigma)^{(1-\epsilon)/\epsilon} c_i^{(1-\sigma)(1-\epsilon)/\epsilon} g_{i+1}^{\sigma/\epsilon} (k_{i+1} + b_i)^{1-1/\epsilon}. \quad (A1)$$

Substituting the expression for n_i given by (6) into (3) gives equation (8). (8) determines the optimal value of c_i conditional on k_{i+1} . In what follows, I denote this optimal value by $\bar{c}_i(k_{i+1})$. Using (8) to eliminate the term involving g_{i+1} and k_{i+1} from (A1) gives the following expression for the maximal utility level which generation i can reach if its bequests per child are set to k_{i+1} :

$$u_i(k_{i+1}) = ((1-\sigma-\epsilon)/(1-\epsilon)) [\bar{c}_i(k_{i+1})]^\sigma + (\sigma/(1-\epsilon)) [\bar{c}_i(k_{i+1})]^{-(1-\sigma)} k_{i+1}(1+r).$$

$\sigma + \epsilon < 1$ and $\bar{c}_i \leq k_i(1+r)$ imply that this expression is decreasing in $\bar{c}_i(k_{i+1})$. Hence the optimal bequest (\bar{k}_{i+1}) can be found by minimizing the value of $\bar{c}_i(k_{i+1})$: $\bar{k}_{i+1} \in \text{Arg Min}_{k_{i+1} \geq 0} \bar{c}_i(k_{i+1})$. We see from (8) that \bar{k}_{i+1} can

equivalently be found as follows:

$$\bar{k}_{i+1} \in \text{Arg Max}_{k_{i+1} \geq 0} \chi_i(k_{i+1}), \quad (A2)$$

where $\chi_i(k_{i+1}) = [g_{i+1}(k_{i+1}(1+r))]^{\sigma/\epsilon} (k_{i+1} + b_i)^{1-1/\epsilon}$.

Generation i 's consumption schedule is thus given by

$$g_i(y) = \{c_i \mid y = c_i + c_i^{(1-\sigma)/\epsilon} g_{i+1}(\bar{k}_{i+1}(1+r))^{\sigma/\epsilon} (\bar{k}_{i+1} + b_i)^{1-1/\epsilon} (\alpha(1-\epsilon)/\sigma)^{1/\epsilon}; \\ \bar{k}_{i+1} \text{ given by (A2)}\}.$$

Clearly $0 \leq g_i(y) \leq y$ holds and g_i is differentiable and strictly increasing. η_i , the elasticity of g_i with respect to $k_i(1+r)$, is given by (9). η_i is continuous in c_i and hence in i 's wealth. $\sigma + \epsilon < 1$ implies that $\eta_i \leq 1$ and that η_i is decreasing in i 's wealth.

Note that when $0 \leq g_{i+1}(y) \leq y$ holds for all $y \geq 0$, then $\chi_i(k_{i+1}) \leq k_{i+1}(k_{i+1} + b_i)^{1-1/\epsilon} (1+r)^{\sigma/\epsilon}$, which implies $\chi_i(0) = 0$ and $\lim_{k_{i+1} \rightarrow \infty} \chi_i(k_{i+1}) = 0$ (assuming $\sigma + \epsilon < 1$). Hence a sufficient condition which ensures that χ_i has a global maximum (and hence that the decision problem of generation i is well defined) is that $0 \leq g_{i+1}(y) \leq y$ holds all $y \geq 0$ and that g_{i+1} is continuous.

Assume: $0 \leq g_{i+1}(y) \leq y$ for all $y \geq 0$; g_{i+1} is differentiable and strictly increasing; the wealth elasticity of g_{i+1} does not exceed unity and this elasticity is continuous and decreasing in wealth. Then the first-order conditions (6), (7) and (8) determine generation i 's optimal decisions concerning c_i , n_i and k_{i+1} .

Equations (6) and (8) are necessary and sufficient for optimal c_i and n_i choices conditional on a given value of k_{i+1} . The assumptions about g_{i+1} imply that χ_i has a maximum at which $\chi_i'(k_{i+1}) = 0$ holds (χ_i is defined earlier in this appendix). Note that the condition $\chi_i'(k_{i+1}) = 0$ is equivalent to the first-order condition (7). The assumptions about g_{i+1} guarantee that (7) has a unique solution, which implies that (7) determines a global maximum of the function χ_i .

Appendix B.

Assume that there exists a number q such that (b_i/b_{i-1}) is constant for all $i \geq q$. Then $\sigma \leq .5$ guarantees that the equilibrium is unique.

When b_i/b_{i-1} is constant for generations $i \geq q$, then the reaction function (13) is the same for these generations. Denote this common reaction function by f : $\gamma_i = f(\gamma_{i+1})$ for $i \geq q$. Define $G^1(\gamma) = f(\gamma)$ and let $G^j(\gamma) = f(G^{j-1}(\gamma))$ for $j > 1$, i.e. $G^j(\gamma)$ is the consumption ratio chosen by generation q if generation $q+j$ selects γ and generations $q, q+1, \dots, q+j-1$

act according to (13). Let $\Sigma^j = \{G^j(\gamma) \mid 0 \leq \gamma \leq 1\}$ and $\Sigma = \bigcap_{j \geq 1} \Sigma^j$. There exists a unique equilibrium if and only if Σ is a singleton.²⁶ It is easy to see that $\Sigma^{j+1} \subseteq \Sigma^j$. Therefore $\Sigma = \bigcap_{j \geq 1} \Sigma^{2j}$. Note that $\Sigma^{2j} = [\text{Min}(G^{2j}(0), G^{2j}(1)), \text{Max}(G^{2j}(0), G^{2j}(1))]$ for $j \geq 1$ as G^{2j} is strictly increasing in the interval $[0, 1]$. Let $\bar{\gamma}$ and $\hat{\gamma}$ respectively denote the smallest and the largest value of $0 \leq \gamma \leq 1$ for which $\gamma = G^2(\gamma)$. We have $\gamma \leq G^2(\gamma) \leq \bar{\gamma}$ for $0 \leq \gamma \leq \bar{\gamma}$ and $\hat{\gamma} \leq G^2(\gamma) \leq \gamma$ for $\hat{\gamma} \leq \gamma \leq 1$. As illustrated in figure 1, the sequence $\{G^{2j}(0)\}_{j \geq 1}$ is increasing and it converges to $\bar{\gamma}$, while the sequence $\{G^{2j}(1)\}_{j \geq 1}$ is decreasing and converges to $\hat{\gamma}$. Hence $G^{2j}(0) < G^{2j}(1)$ holds for all $j \geq 1$, which implies that $\Sigma^{2j} = [G^{2j}(0), G^{2j}(1)]$, and hence $\Sigma = [\bar{\gamma}, \hat{\gamma}]$. Thus there exists a unique equilibrium if and only if $\bar{\gamma} = \hat{\gamma}$ holds.

Insert figure 1 here.

Note that $G^{2j}(1) = f(G^{2j}(0))$. We thus have $\hat{\gamma} = f(\bar{\gamma})$ and similarly $\bar{\gamma} = f(\hat{\gamma})$. When there exists a unique equilibrium, then $\Sigma = \gamma^* = f(\gamma^*)$ and the equilibrium consumption ratio for generations $i \geq q$ is constant: $\gamma_i = \gamma^*$ for all $i \geq q$.

To derive a sufficient condition for uniqueness of the equilibrium, consider the function $\phi(x) = (1-x)x^{(2\sigma-1)/\varepsilon} \cdot (1/(1-\varepsilon)-x)^{2(1-\sigma-\varepsilon)/\varepsilon}$.

$$((1-\sigma)/(1-\sigma-\varepsilon)-x)^{1-1/\varepsilon} \cdot ((1-\sigma)/(1-\sigma-\varepsilon)-x)^{1-1/\varepsilon}.$$

Using (12), we can see that $\bar{\gamma} = f(\hat{\gamma})$ and $\hat{\gamma} = f(\bar{\gamma})$ imply that $\phi(\bar{\gamma}) = \phi(\hat{\gamma})$ holds. Hence $\sigma \leq .5$ is a necessary and sufficient condition under which the function ϕ is strictly monotonic in the interval $[0, 1]$. Hence $\phi(\bar{\gamma}) = \phi(\hat{\gamma})$ cannot hold for $\bar{\gamma} \neq \hat{\gamma}$ when $\sigma \leq .5$ and therefore $\sigma \leq .5$ is a sufficient condition for the existence of a unique equilibrium in an economy in which the growth rate of the cost of raising children is constant for all generations $i \geq q$.

²⁶ $S = \{F^q(\gamma) \mid \gamma \in \Sigma\}$ and hence S is a singleton if and only if Σ is a singleton (N.B. S and the function F^q are defined following equation (14) in the text).

Appendix C. Effects of a temporary change in the cost of raising children
 This appendix considers the effects of a change in b_0 on the birth rate (for $i>0$, b_i is held fixed in this experiment). Using equations (3) and (10), we obtain the following expressions for the birth rate:

$$n_0 = (1-\gamma_0)k_0(1+r)/(k_1+b_1) = (1-\gamma_0)k_0(1+r)(1/b_0)[1/(1-\varepsilon)-\gamma_1]/[(1-\sigma)/(1-\sigma-\varepsilon)-\gamma_1]$$

$$\text{and } n_i = \Lambda_i(1-\gamma_i)[(b_{i-1}/b_i)/(1/(1-\varepsilon)-\gamma_i)][1/(1-\varepsilon)-\gamma_{i+1}]/[(1-\sigma)/(1-\sigma-\varepsilon)-\gamma_{i+1}]$$

for $i \geq 1$, where $\Lambda_i = (1+r)\sigma\varepsilon/((1-\sigma-\varepsilon)(1-\varepsilon))$.

We see from (12) that a change in b_0 does not affect the reaction function $\gamma_i = f^i(\gamma_{i+1})$ for $i \geq 2$, which implies (assuming that the equilibrium is unique) that the equilibrium values of $\gamma_2, \gamma_3, \dots$ are not altered by a change in b_0 ; hence the birth rates of generations $i \geq 2$ also are unaffected by such a change.

Log-linearizing equation (11) and (12) (for $i=1$) and the above expressions for n_0 and n_1 around the values of γ_0, γ_1 and b_0 which would hold in the absence of the change in b_0 we obtain the following system of equations: $\Delta\gamma_0 = \eta_{11}\Delta\gamma_1 + \eta_{12}\Delta b_0$, $\Delta\gamma_1 = \eta_{22}\Delta b_0$, $\Delta n_0 = -Z_0^0\Delta\gamma_0 - \Delta b_0 + (Z_2^1 - Z_1^1)\Delta\gamma_1$, $\Delta n_1 = -Z_0^1\Delta\gamma_1 + \Delta b_0 + Z_1^1\Delta\gamma_1$. Here Δx denotes the percentage deviation of variable x from its value in the absence of the change in b_0 ; furthermore, $Z_0^S = \gamma_S/(1-\gamma_S)$, $Z_1^S = \gamma_S/((1/(1-\varepsilon))-\gamma_S)$ and $Z_2^S = \gamma_S/((1-\sigma)/(1-\sigma-\varepsilon)-\gamma_S)$, where γ_S is the value of the consumption ratio of generation s around which the

log-linear approximation is taken (note that $0 \leq Z_2^S \leq Z_1^S \leq Z_0^S$) and $\eta_{11} = -[\sigma/\varepsilon - Z_1^1(1-\sigma-\varepsilon)/\varepsilon - Z_2^1(1-1/\varepsilon)]/[Z_0^0 + (1-\sigma)/\varepsilon]$, $\eta_{12} = [(1-\sigma-\varepsilon)/\varepsilon]/[Z_0^0 + (1-\sigma)/\varepsilon]$ and $\eta_{22} = -[(1-\sigma-\varepsilon)/\varepsilon]/[Z_0^1 + (1-\sigma)/\varepsilon + Z_1^1(1-\sigma-\varepsilon)/\varepsilon]$, with $\eta_{11} \leq 0$, $\eta_{12} \geq 0$ and $\eta_{22} < 0$.

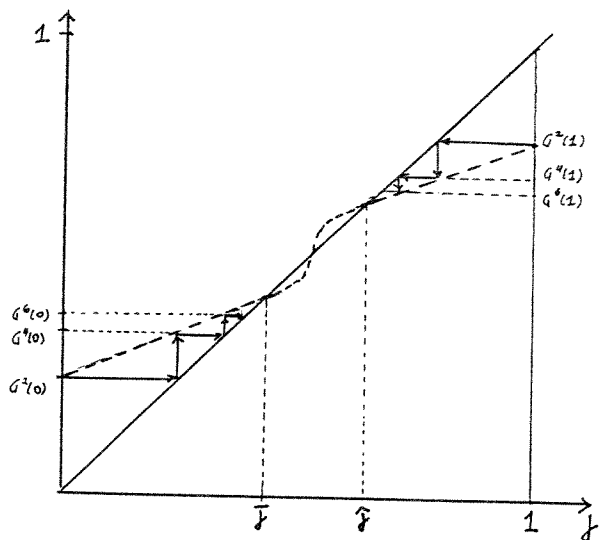
The above equations imply that $\Delta n_0/\Delta b_0 = -Z_0^0(\eta_{11}\eta_{22} + \eta_{12}) + (Z_2^1 - Z_1^1)\eta_{22}^{-1} = [-Z_0^0 \left(\frac{\eta_{12}}{-\eta_{22}} - \eta_{11} \right) - Z_2^1 + Z_1^1] (-\eta_{22}) - 1$. $\Delta n_0/\Delta b_0 < 0$ holds because $(Z_1^1 - Z_2^1)(-\eta_{22}) < 1$. Furthermore, $\Delta n_1/\Delta b_0 = (Z_1^1 - Z_0^1)\eta_{22} + 1$ holds. Using the above formula for η_{22} , it is straightforward to check that $\Delta n_1/\Delta b_0 > 0$.

The analysis of the effects of a change in b_j with $j > 0$ proceeds similarly. The key difference compared to a change in b_0 is that an (anticipated) change in b_j affects the wealth of generation j . For example, a fall in b_j with $j > 0$ lowers the consumption ratio of generation j , which

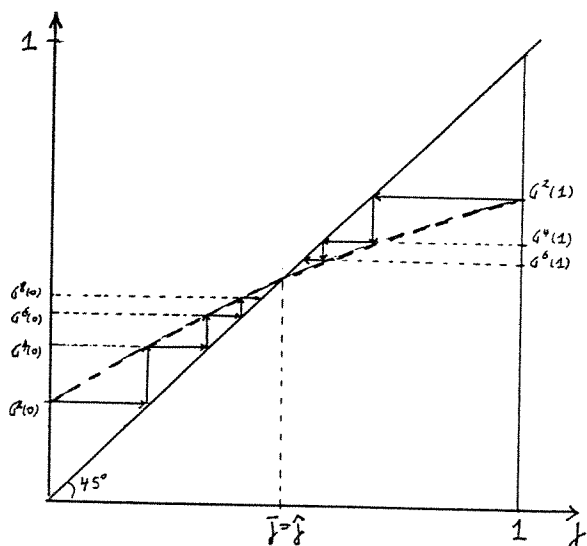
induces generation $j-1$ to reduce its bequests to each member of generation j . If the fall in the per capita wealth of generation j is sufficiently great, then a fall in b_j can actually lower the birth rate of generation j . It can, however, be shown that if in the absence of the change in b_j generation j and $j+1$ would select the same consumption ratios, then $\Delta n_j / \Delta b_j < 0$ and $\Delta n_{j+1} / \Delta b_j > 0$ obtain.

Finally, note that an anticipated increase in b_j also affects the birth rates of generations preceding generation j . For example, it can be shown that an anticipated fall in b_j increases n_{j-1} .²⁷

²⁷ $\Delta n_{j-1} / \Delta b_j < 0$ obtains when, in the absence of a change in b_j , $\gamma_{j-1} = \gamma_j = \gamma_{j+1}$ would hold.



1.a: Multiple equilibria ($\bar{y} < \hat{y}$).



1.b: Unique equilibrium ($\bar{y} = \hat{y}$).

Figure 1 The function $G^2(\bar{y})$ (----), see appendix B.

The figure illustrates the convergence of the series $G^2(0), G^4(0), G^6(0), \dots$ and $G^2(1), G^4(1), G^6(1), \dots$ to \bar{y} and \hat{y} respectively.



Bibliography

- Arrow, K. (1973). Rawls's Principle of Just Saving. *Swedish Journal of Economics*, 75: 323-335.
- Barro, R. (1974). Are Government Bonds Net Wealth? *Journal of Political Economy*, 82: 1095-1117.
- Becker, G. & Barro, R. (1988). A Reformulation of the Economic Theory of Fertility. *Quarterly Journal of Economics*, 103: 1-25.
- Barro, R. & Becker, G. (1989). Fertility Choice in a Model of Economic Growth. *Econometrica*, 57: 481-501.
- Becker, G. & Murphy, K. (1989). Economic Growth, Human Capital and Population Growth. Manuscript, University of Chicago.
- Becker, G., Murphy, K. and Tamura, R. (1990). Human Capital, Fertility and Economic Growth. *Journal of Political Economy*, 98: S12-S37.
- Benhabib, J. and Nishimura, K. (1989). Endogenous Fluctuations in the Barro-Becker Theory of Fertility. In: *Demographic Change and Economic Development*, A. Wenig and K. Zimmermann (eds.), Springer Verlag, 29-41.
- Benhabib, J. and Nishimura, K. (1990). Endogenous Fertility and Growth. Research Report 90-20, C.V. Starr Center for Applied Economics, New York University.
- Bernheim, D. & Ray, D. (1987). Economic Growth with Intergenerational Altruism. *Review of Economic Studies*, 54: 227-243.
- Bernheim, D. (1989). Intergenerational Altruism, Dynastic Equilibria and Social Welfare. *Review of Economic Studies*, 56: 119-128.
- Dasgupta, P. (1974). On Some Alternative Criteria for Justice Between Generations. *Journal of Public Economics*, 3: 405-423.
- Eckstein, Z. and Wolpin, K. (1985). Endogenous Fertility and Optimal Population Size. *Journal of Public Economics*, 27: 93-106.
- Hori, H. (1992). Utility Functionals with Nonpaternalistic Intergenerational Altruism: The Case Where Altruism Extends to Many Generations. *Journal of Economic Theory*, 56: 451-467.
- Kohlberg, E. (1976). A Model of Economic Growth with Altruism Between Generations. *Journal of Economic Theory*, 13: 1-13.
- Kollmann, R. (1987). Fertility, Consumption and Bequests in a Model with Non-Dynastic Parental Preferences. Working Paper, Economics Department, University of Chicago.
- Lane, J. & Leininger, W. (1984). Differentiable Nash Equilibria in Altruistic Economies. *Zeitschrift für Nationalökonomie (Journal of Economics)*, 44: 329-347.
- Lane, J. & Mitra, T. (1981). On Nash Equilibrium Programs of Capital Accumulation Under Altruistic Preferences. *International Economic Review*, 22: 309-331.
- Lapan, H. and Enders, W. (1990). Endogenous Fertility, Ricardian Equivalence, and Debt Management Policy. *Journal of Public Economics*, 41: 227-248.
- Leininger, W. (1986). The Existence of Perfect Equilibria in a Model of Growth with Altruism Between Generations. *Review of Economic Studies*, 53: 349-367.
- Loury, G. (1981). Intergenerational Transfers and the Distribution of Earnings. *Econometrica*, 49: 843-867.
- Nishimura, K. & Raut, L. (1992). Family Expansion and Capital Expansion of a Dynasty. Discussion Paper 92-40, Department of Economics, University of California San Diego.
- Nishimura, K. and Zhang, J. (1992). Pay-as-you-go Public Pensions With Endogenous Fertility. *Journal of Public Economics*, 48: 239-258.

- Pestieau, P. (1989). Endogenous Fertility and Fixed Input in a Growth Model With Altruism. In: *Economic Theory of Optimal Population*, K. Zimmermann (ed.), Springer Verlag, 91-103.
- Raut, L. (1987). Effects of Social Security on Fertility and Saving: an Overlapping Generations Model. Discussion Paper 87-13, Economics Research Center, National Opinion Research Center, University of Chicago.
- Raut, L. (1990). Capital Accumulation, Income Distribution and Endogenous Fertility in an Overlapping Generations General Equilibrium Model. *Journal of Development Economics*, 34: 123-150.
- Raut, L. (1992). Subgame Perfect Manipulation of Children by Overlapping Generations of Agents With Two-Sided Altruism and Endogenous Fertility. Working Paper, Economics Department, University of California San Diego.
- Ray, D. (1987). Nonpaternalistic Intergenerational Altruism. *Journal of Economic Theory*, 41: 112-132.
- Razin, A. & Ben-Zion (1974). An Intergenerational Model of Population Growth. *American Economic Review*, 65: 923-33.
- Veall, M. (1986). Public Pensions as Optimal Social Contracts. *Journal of Public Economics*, 31:237-251.
- Weil, P. (1987). Love Thy Children, Reflections on the Barro Debt Neutrality Theorem. *Journal of Monetary Economics*, 19: 377-91.

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