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ON THE ANALYSIS OF BUSINESS CYCLES THROUGH THE SPECTRUM OF CHRONOLOGIES

Eric GHYSELS[1] and F. Haldun SARLAN[2]

[1] Département de sciences économiques and Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal, and CIRANO

[2] Département de sciences économiques and Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal

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C.P. 6128, succursale Centre-ville
Montréal (Québec) H3C 3J7

Télécopieur (FAX): (514) 343-5831
Courrier électronique (E-Mail): econo@tornade.ERE.Umontreal.CA

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RÉSUMÉ

Burns et Mitchell (1947) ont proposé d'analyser les cycles d'affaires à travers la chronologie des points de retournement. Dans ce papier, au lieu d'appliquer des méthodes spectrales directement aux données, nous suggérons l'analyse spectrale de la chronologie des points de retournement. La chronologie des cycles peut être vue comme une réalisation d'un processus aléatoire sur un espace discret, usuellement à deux états, qui constitue une séquence alternante des expansions et récessions comme celle du NBER qui couvre un échantillon mensuel débutant en 1854. L'application des méthodes spectrales à de tels processus discrets constitue un outil pratique pour montrer les ressemblances et les différences entre les chronologies de référence alternatives. En effet, la comparaison formelle à travers la cohérence peut nous montrer comment deux chronologies sont reliées l'une à l'autre. En plus, en utilisant un algorithme tel que proposé par Bry et Boschan (1971), on peut déterminer une chronologie pour des séries individuelles comme le taux d'intérêt, l'offre de monnaie, etc. Ceci nous permet d'étudier les comouvements entre le processus identifié par la chronologie du NBER et les points de retournement d'une série individuelle. Une telle analyse nous permet de mettre dans une nouvelle perspective la relation entre les cycles des différentes séries mesurant l'activité réelle et les agrégats monétaires.

Mots clés : algorithme de Bry-Boschan, cycles d'affaires, analyse spectrale Walsh-Fourier.

ABSTRACT

Burns and Mitchell (1947) proposed to study business cycles via chronologies. In this paper, we suggest to apply spectral methods not to the data directly but instead to time series consisting of business cycle chronologies. A business cycle chronology can be viewed as a realization of a random variable over a discrete space, usually two states, resulting in an alternating sequence of expansions and recessions like the one produced by the NBER which covers a sample of monthly observations starting in 1854. Applying spectral methods to such discrete processes provides an easy tool to assess the similarities and differences between alternative reference chronologies. Indeed, a formal comparison via the coherence can inform us how the two chronologies are related. Moreover, using an algorithm such as proposed by Bry and Boschan (1971), we can date peaks and troughs in a set of individual time series like interest rates, money supply, etc., allowing us to study the comovements between the process identified by the NBER chronology and the turning point process associated with any individual series. Such analysis allows us to describe the association of cycles between different series measuring real activity and monetary aggregates in a very novel perspective.

Key words: Bry-Boschan dating algorithm, business cycles, Walsh-Fourier spectral analysis.

1. INTRODUCTION

The measurement of business cycle phenomena has been a very active area of research since at least the thirties, when Burns, Mitchell and Tinbergen proposed a variety of statistical methods to examine macroeconomic data. To some, the phenomenon of business cycles was one of regimes, like expansions and recessions, which led to the work by Burns and Mitchell (1947) who proposed to study business cycles via chronologies. To others, when one talked about cyclical phenomena, the picture that came to mind was a sine wave with its regular and recurrent pattern. This led to the more modern techniques of spectral analysis initiated in econometrics by Hannan (1960), Granger and Hatanaka (1964) and Nerlove (1964). See also Sargent (1987) for a fairly extensive coverage of business cycle phenomena and spectral decompositions. Business cycle chronologies and spectral analysis of time series have been largely independent developments, as they were techniques associated with two very different views about modeling business cycles. In this paper, we propose to pair the two developments. Indeed, we suggest to apply spectral methods not to the data directly but instead to time series consisting of business cycle chronologies. A business cycle chronology, such as the one produced by the NBER which covers a sample of monthly observations starting in 1854, can be viewed as a realization of a random variable over a discrete space, usually two states, resulting in an alternating sequence of expansions and recessions. Such a time series, when stationary, has a spectral representation allowing us to take advantage of tools developed over the last several decades but hitherto not exploited.[1]

Before we get into the technicalities about how to apply spectral methods to such discrete processes, let us explain what the advantages would be. First, they provide an easy tool to assess the similarities and differences between alternative reference chronologies. Indeed, two chronologies may be different not only in the dating of peaks

1 Hatanaka (1964) proposed an approach somewhat similar to ours when he estimated the spectral density of a zigzagged pattern of the U.S. business cycles with a discrete-valued triangular pattern, taking its maximum values at the business cycle peaks and its minimum values at the troughs. Moore and Zarnowitz (1986) also displayed such zigzagged patterns to show the matching time of reference chronologies for four countries.

and troughs but also in the number of recessions and expansions. A formal comparison via the coherence can inform us how the two chronologies are related. Second, using an algorithm such as the one proposed by Bry and Boschan (1971), we can date peaks and troughs in a set of individual time series like interest rates, money supply, etc., allowing us to study the comovements between the process identified by the NBER chronology and the turning point process associated with any individual series. Such is an alternative way to describe the association of cycles between different series measuring real activity and monetary aggregates. Third, while it is true that by focusing on business cycle phases instead of the actual series like GNP much information is thrown out, it should be noted that applying spectral analysis to chronologies aims at investigating nonlinear properties of the data instead of the linear ones. Namely, the spectral methods are applied to time series reflecting duration of cycles, regime switches and turning points. A related issue is the critical dependence of empirical stylized facts regarding business cycles on the detrending of the data. The methods we propose put emphasis on turning points instead of trends, which has certain advantages.

Applying spectral methods to the rectangular processes of two states requires some technical discussions. The well-known Fourier transform based on sinusoidal functions is one of at least two ways to proceed and compute a spectral decomposition of a series within a class of orthogonal functions. An alternative approach consists of a frequency-based analysis of time series via the Walsh-Fourier transform based on Walsh functions, which are similar to trigonometric functions, except that they take rectangular shapes.[2] Both Fourier and Walsh-Fourier representations have their merits in the analysis of discrete-valued time series and will be used throughout the paper. There are, however, some clear advantages to using the Walsh-Fourier analysis for decomposing chronologies. Section 2 includes a brief introduction of the Walsh-Fourier analysis with the technical material appearing in an Appendix. We discuss the univariate spectral analysis of two alternative U.S. reference cycle chronologies given by the NBER and Romer (1992).

2 References regarding Walsh functions and their use include Ahmed and Rao (1975); Kohn (1980a,b); Morettin (1981) and Stoffer (1987, 1990, 1991).

Several major individual chronologies are also considered. The reference dates of the latter were selected by the Bry and Boschan dating algorithm for cyclical turning points.

The univariate spectral analysis of the NBER and the Romer chronologies reveal a double peak in the spectrum for cycles between two and six years. Such heterogeneity suggests that not all cycles are alike and that probably different sources of impulses and propagation mechanisms may be at work. This result holds before WWII as well as in the post-WWII era. There is also a peak at the seasonal frequency before the WWII. When we compute the coherency of the NBER and the Romer chronologies, we find that it averages to about 0.95 in the post-WWII era, yet only to 0.79 before WWII. Outside the business cycle frequency band, the two chronologies do not match very closely, as the average coherencies at high and low frequencies are at most 50 to 60 percent.

Studying comovements among individual series also yields interesting insights about business cycle comovements. Using Walsh coherencies, we first compare pre- and post-WWII cycles and find striking differences in the cyclical behavior of prices, bond yields and the stock market across the two eras. The pattern of comovements between industrial production and the NBER reference cycle also shows dramatic changes at the short end of the spectrum around seasonal frequencies.

Besides comparisons of pre- and post-WWII eras, we also investigate the stylized facts for the latter period for a larger set of series. Alternative detrending methods tend to affect business cycle frequencies differently, as noted by Canova (1991) for instance, in his elaborate study of detrending and stylized facts. As a result, some important empirical evidence regarding business cycle behavior critically depends on detrending methods. The approach via spectral decompositions of chronologies has the advantage that it does not depend in any direct way on detrending.[3] We therefore study post-WWII

3 Obviously, some algorithms for selecting turning points proceed according to a certain trend specification.

business cycle facts, via coherence analysis, through the spectral representation of several chronologies associated with a set of major economic time series.

In section 2, we review some of the basic tools of the Fourier and Walsh-Fourier analysis used in the remainder of the paper. Section 3 is devoted to the spectral decomposition of some of the basic reference chronologies. Section 4 covers comovements between individual series, including a study of pre- and post-WWII as well as a review of stylized facts since WWII. Conclusions appear in section 5.

2. MOTIVATION

Spectral analysis is well covered in many textbooks of time series analysis. Its conventional use involves Fourier transforms of weakly stationary processes. For a univariate time series x_t which has time invariant mean and autocovariances $\gamma_\tau = \mathrm{cov}(x_t, x_{t-\tau})$, one has two fundamental relationships : namely, the Cramer representation

$$x_t = \int_{-\pi}^{\pi} e^{it\omega} dz(\omega) \tag{2.1}$$

where $E[dz(\omega)\overline{dz(\lambda)}]=f(\omega)d\omega$ when $\omega=\lambda$ and zero otherwise, and the spectral representation of the autocovariances

$$\gamma_\tau = \int_{-\pi}^{\pi} e^{i\tau\omega} f(\omega)d\omega \; . \tag{2.2}$$

Equations (2.1) and (2.2) both involve Fourier transforms. The first equation states that a stationary process can be thought of as a noncountably infinite sum of uncorrelated components, and the second equation provides the spectral representation $f(\omega)$ of the covariance structure of the process. When a sample of size N of the process x_t is available, one usually draws on the periodogram, denoted $I_x^F(\omega)$ where F refers to Fourier, to estimate the spectrum. For any frequency $\omega \in [-\pi, \pi]$, one defines the periodogram as :

$$I_x^F(\omega) = C_x^2(\omega) + S_x^2(\omega) \tag{2.3}$$

where

$$C_x(\omega) = \left(\frac{2}{N}\right)^{1/2} \sum_{t=1}^{N} x_t \cos(\omega t) \tag{2.4}$$

$$S_x(\omega) = \left(\frac{2}{N}\right)^{1/2} \sum_{t=1}^{N} x_t \sin(\omega t) . \tag{2.5}$$

The $C_x(\omega)$ and $S_x(\omega)$ functions are the cosine and sine transformations of the observed series $\{x_t\}_{t=1}^{N}$. The computed frequencies are $\omega_j = 2\pi j/N$ for $j = 0, 1, ..., [N/2]$. For most economic time series, spectral representations have been documented in the literature. Granger (1966) reported, for instance, the "typical" spectral shape of series.

It is not uncommon to apply spectral analysis to transformed series instead of the raw data x_t. Perhaps the best example is that of seasonal adjustment, where the spectral properties of x_t^{SA} instead of x_t are studied. Such filters are, at least in principle, designed to extract from the raw series those features of the data that are of interest to the researcher. We essentially apply a similar principle here in a different context. Namely, let us construct a binary time series b_t, where :

$$b_t \equiv CH(X_{t-k}, ..., X_t, ... X_{t+l}, t) \tag{2.6}$$

$$b_t \in \{-1, 1\} \ \forall t \tag{2.7}$$

$$X_t = (x_t, y_t, z_t, ...) . \tag{2.8}$$

For convention, the rectangular patterns are scaled according to whether the economy is in expansion, $b_t = +1$ or in recession, $b_t = -1$.[4] Note that the function CH, generating a chronology, may be a function of several series when X_t is multivariate. Such is the case, for instance, with the NBER chronology, based on dating committees gathering evidence from a multitude of series. The function mapping X_t into b_t may also vary through time, hence CH(., t), since the dating committees may change the *modus operandi* of defining the phases of the business cycle. Moreover, producing a chronology may involve future as well as past observations, hence the leads and lags appearing in (2.6). The algorithm proposed by Bry and Boschan (1971) is another example where a specific rule applies to a single series, i.e., $b_t \equiv CH(x_{t-k}, \ldots, x_t, \ldots x_{t+l}, t)$. Yet, another example is the algorithm proposed by Hamilton (1989) based on a Markov switching regime framework.

Figure 2.1 displays the business cycle rectangular patterns for a variety of series ranging from the NBER and the Romer chronologies to several major individual patterns generated by the Bry and Boschan dating algorithm for turning points. The switching points with zero crossings from +1 to -1 correspond to downturns and from -1 to +1 reflect upturns. Hence, the length between sign changes reflects the durations of cycles. Table A.1 in the Appendix reports some summary statistics for the duration data appearing in Figure 2.1. A distinction is made between the pre-WWII and post-WWII data series for two reasons : (1) in some cases, the series involved are not exactly the same over those two eras, and (2) there has been much discussion about the distinct business cycle patterns.[5]

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4 The values +1 and -1 are arbitrary and the procedures we use are invariant to scaling.

5 Some of the most recent papers on this subject include Diebold and Rudebusch (1992), Romer (1992), Watson (1992), among many others.

Figure 2.1
Historical Plots of U.S. Business Cycles

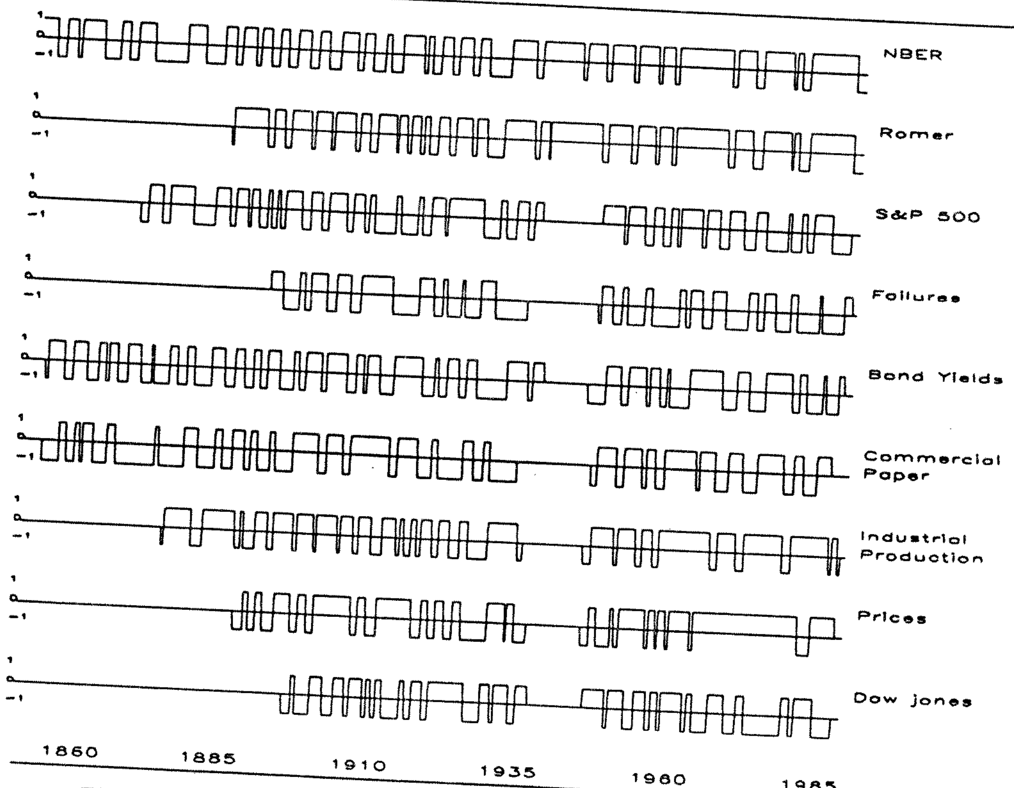

Figure 2.1 : Historical Plots of U.S. Business Cycles

We would now like to investigate several features of the series b_t. First, in analogy with standard spectral analysis, we would like to decompose the square wave pattern of series b_t in orthogonal harmonic components. We expect, of course, that the business cycle frequencies will be dominant in the spectral shape, yet other cycles may be revealed as well. Second, we would also like to study comovements across chronologies using a frequency-domain representation. Such analysis serves two purposes : namely, (1) to examine competing chronologies, like the NBER and the alternative proposed by Romer (1992) for instance, and (2) to study relationships between different series of economic activity through their chronology transforms. For instance, one may investigate the stock market and cyclical comovements with b_t series obtained from the Dow Jones and the NBER. It is clear that spectral analysis enables us to formally asses the similarities and differences between two chronologies of the business cycle. Such comparisons are generally not straightforward, since chronologies may not only differ with respect to the location of a turning point, but may also involve a different number of recessions and expansions. Furthermore, when the b_t series of say the NBER and the Dow Jones are examined, it is clear that we apply spectral analysis in the context of a regime switching framework. Namely, we by-pass the linear properties of the series through the CH(., .) filter and study the association of business cycle phase patterns across series.

When the b_t series is weakly stationary, then the fundamental spectral representation theorem tells us we can apply Fourier transforms of the series and compute :

$$C_b(\omega) = \left(\frac{2}{N}\right)^{1/2} \sum_{t=1}^{N} b_t \cos(\omega t) \tag{2.9}$$

$$S_b(\omega) = \left(\frac{2}{N}\right)^{1/2} \sum_{t=1}^{N} b_t \sin(\omega t) \tag{2.10}$$

and proceed as usual to compute a spectral decomposition. Yet, as the b_t series is a square-waved series, an alternative spectral decomposition may be considered as well. To approximate square waves, one can replace the sine and cosine functions by so-called Walsh functions. Such functions, which will be discussed shortly, are displayed in Figure 2.2 on top of the standard Fourier harmonic functions. The spectral analysis based on such functions is called the Walsh-Fourier analysis and is, to our knowledge, new in terms of econometric applications. Obviously, one may expect Walsh functions, appearing in Figure 2.2, to be a better approximation to decompose the patterns displayed in Figure 2.1 for the various macroeconomic chronologies. There are, however, some technical issues which need to be addressed. In the Appendix to the paper, we provide a summary of some of the issues. Here, we shall limit ourselves to a presentation of some basic results which will aid the reader to follow the analysis without being distracted by the details.

Walsh functions are square wave functions that take only two values, +1 and -1 (up and down).[6] This is in contrast to the sinusoids $\cos(2\pi n\omega)$ and $\sin(2\pi n\omega)$ ($n = 1, 2, \ldots$) which are characterized by their frequency of oscillation n in terms of the complete cycles they make on the interval $0 \leq \omega < 1$. Figure 2.2 displays the first eight continuous Walsh functions $W(n, \omega)$, ($n = 0, 1, \ldots, 7, 0 \leq \omega < 1$) superimposed on Fourier harmonics. The Walsh functions $W(n, \omega)$ are characterized by the number of times n they switch signs in the unit interval $0 \leq \omega < 1$. For example, $W(4, \omega)$ switches four times on the interval $0 \leq \omega < 1$ from +1 to -1 at 1/8, then from -1 to +1 at 3/8, from +1 to -1 at 5/8 and, finally, from -1 to +1 at 7/8.[7]

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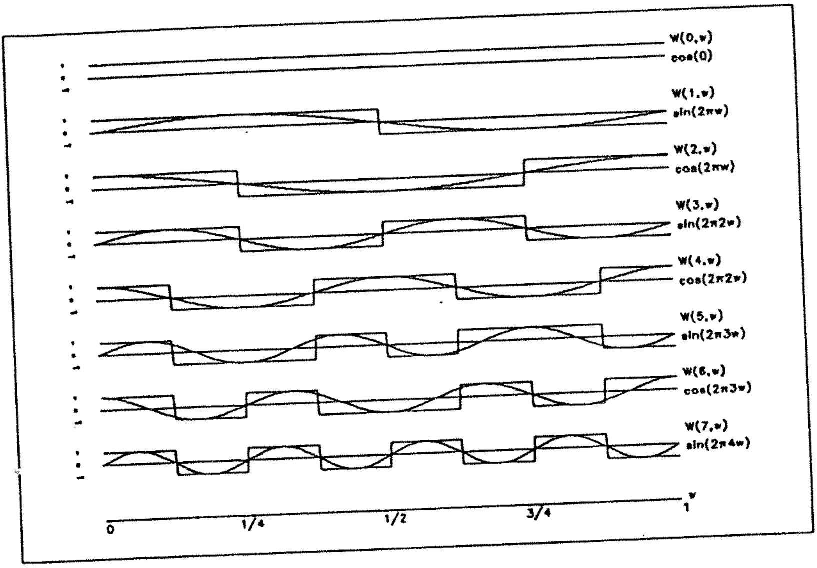
6 In our setup, the series b_t only assumes two values, yet Walsh functions can be used for the spectral decomposition of series taking a finite number of states. See Stoffer (1991) for examples and applications.

7 As discussed in detail in the Appendix, it should be noted that Walsh functions, unlike trigonometric functions, are aperiodic. Therefore, the notion of frequency was generalized to handle Walsh functions. This generalized frequency, called sequency or Harmuth sequency, measures half cycle lengths. Twice the sequency corresponds to the Fourier-based frequency. We conducted both sequency-based and frequency-based Walsh-Fourier analyses. As the results with both approaches were similar, we have chosen to report only the frequency-based Walsh-Fourier analysis, which is directly comparable with the Fourier spectrum.

Figure 2.2
Walsh and Fourier Harmonics



We now turn our attention to a short discussion of computational issues. The discrete-valued Walsh functions W(n, m/N) n = m = 0, 1, ..., N - 1, corresponding to sample of length $N = 2^p$ where p is a positive integer, are generated via a (N x N) Walsh-ordered Hadamard matrix $H_W(p)$ described in the Appendix.

The finite-order Walsh-Fourier transform of a series b_t is expressed as :

$$d_b(\omega_j) = H_W(p)\, \underline{b}_N' / \sqrt{N} \tag{2.11}$$

where $\omega_j = m/N$, n = 0, ..., N - 1 and $\underline{b}_N$ is the vector $(b_1, \ldots, b_N)$. By analogy to the Fourier analysis, the Walsh periodogram of a series b_t is obtained by squaring each element of (2.11).

$$\tilde{I}_b^W(\omega_j) = d_b^2(\omega_j) = [N^{-1/2} \sum_{n=0}^{N-1} b_n \; W(n,\omega_j)]^2 \qquad (2.12)$$

In order to make the Fourier and Walsh-Fourier spectra comparable, one computes

$$I_b^W(\omega_j) = (\tilde{I}_b^W(\omega_{2j-1}) + \tilde{I}_b^W(\omega_{2j}))/2 \qquad j = 1,...(n-2)/2 \qquad (2.13)$$

where $I_b^W(\omega_j)$ represents the periodogram ordinate corresponding to frequency ω_j.[8]

A useful measure of the degree of association between two time series, i.e., two chronologies, b_t^1 and b_t^2, is the coherency,

$$K_{12}(\omega) = \frac{f_{12}(\omega)}{[f_{11}(\omega)f_{22}(\omega)]^{1/2}} \quad 0 < \omega < 1 \qquad (2.14)$$

where $f_{11}(\omega)$ and $f_{22}(\omega)$ are the Walsh-Fourier spectra of two series, while $f_{12}(\omega)$ is the cross-spectrum of two series. The cross-spectrum is a measure of covariance between the series in much the same way as the Fourier spectrum is computed.

One advantage of the Walsh-Fourier coherency analysis is that the coherency is real and takes on negative as well as positive values, i.e., it satisfies the usual correlation inequality $-1 \leq K_{12}(\omega) \leq +1$. In the trigonometric (Fourier) case, cross-spectra are complex-valued, and hence squared coherency rather than coherency is used. A consequence of this is that Walsh-Fourier coherencies do not only measure a strength of association, but also its sign.

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8 Various asymptotic results relating the convergence of $\tilde{I}_b^W(.)$ to Walsh-Fourier spectral density function $f(\omega)$ exist. Many of those that are applicable to categorical time series are discussed in Stoffer (1987). Other references include Kohn (1980a,b) and Morettin (1981). More details appear in the Appendix.

Beauchamp (1984, chapter 3) provides numerous comparisons between Walsh-based and Fourier-based spectral analyses. Not surprisingly, he finds that the Fourier analysis is most relevant for smooth-varying time series, while the Walsh-Fourier analysis is more suitable for series with sharp discontinuities and a limited number of discrete-valued realizations. In the remainder of this paper, we will use both types of spectral methods as complementary rather than substitutes. One must keep in mind though the relative advantage that Walsh-Fourier methods may have for series such as those appearing in Figure 2.1.

3. REFERENCE CHRONOLOGIES AND THEIR SPECTRAL DECOMPOSITION

The plots in Figure 2.1 represent a subset of the chronologies we investigated. The most well known is the business cycle chronology produced by the National Bureau of Economic Research, starting in 1854. From the turning points we reproduced a time series :

$$b_t^N = \begin{cases} +1 \text{ if month } t \text{ is in expansion era according to NBER chronology} \\ -1 \text{ if month } t \text{ is in recession era according to NBER chronology} \end{cases}$$

where the superscript N refers to the NBER. This reproduces the time series appearing at the top of Figure 2.1. Second from the top is a time series corresponding to an alternative chronology proposed by Romer (1992). The analysis of Romer starts from the observation that there appear to be inconsistencies between the determination of NBER dates before and after World War II. Romer proposed an algorithm that chooses postwar turning points which match the NBER dates, but produces a chronology quite different both in terms of length of cycles and number of recessions and expansions. The binary series obtained from the Romer chronology will be denoted b_t^R.

Let us first investigate whether there are noticable differences between the Fourier spectrum and the Walsh-Fourier spectrum. Figure 3.1 displays both types of spectral densities for the NBER chronology over three samples. Recall from section 2 that there are restrictions in choosing a sample length ($N = 2^p$ where p is an integer) for computing Walsh-Fourier spectra. We selected the following samples : (1) 1905:3-1990:7, (2) 1896:1-1938:6, (3) 1948:1-1990:8. The first sample is approximately the "entire sample," though the earlier part of the chronology was deleted.[9] The second sample covers the pre-WWII era and, finally, the last sample covers the post-WWII era. The Fourier and Walsh-Fourier spectra for each of the three samples appear in Figure 3.1. For the chronology proposed by Romer, the spectra are reported in Figure 3.2.

The two curves in each of the plots match fairly closely, yet there are some significant differences worth noting. Namely, the Walsh-Fourier spectrum of the NBER and Romer series gives rise to extra spectral peaks.[10] The economic interpretation of several peaks in business cycle bands will be discussed below. The differences appear to be in the number of spectral peaks and their location, which means that the average period of oscillation in rectangular b_t patterns differ remarkably if one approximates them via Fourier or Walsh functions. The frequency band of cycles of two to six years long are identified via two vertical lines appearing in each plot. For the entire sample of the NBER series in Figure 3.1, we notice three distinct peaks in the Walsh-Fourier spectrum for cycles of two years and more. The Fourier spectrum, on the other hand, has only a single peak located almost exactly at a dip in the Walsh-Fourier spectrum. A similar exercise applied to the Romer chronology, which appears in Figure 3.2, confirms this finding. In fact, with the Romer chronology, the three peaks in the Walsh-Fourier

9 We chose to delete the 19th century part of the chronology to have sample size matching N = 1024 data points (p = 10). The earlier part was deleted, as there is greater uncertainty regarding the locaton of turning points [see, e.g., Diebold and Rudebusch (1989) for discussion]. This choice of entire sample also allowed for a direct comparison of the NBER and Romer chronologies.

10 Stoffer (1991) also reports and discusses peaks uncovered by Walsh-Fourier which do not appear in the Fourier spectra.

spectrum are much stronger. This seems to indicate that there is a certain degree of heterogeneity in business cycle patterns uncovered by the Walsh-Fourier analysis which remains concealed with the Fourier spectrum. The heterogeneity, suggested by the Walsh-Fourier spectrum, can be attributed to at least two sources. As the sample includes both pre- and post-WWII observations one may expect heterogeneity in business cycle lengths to emerge because of the distinct character of business cycles before and after World War II. Another source of heterogeneity can be explained in the context of the impulse propagation framework introduced by Frisch (1933) and Slutsky (1937). There are different views regarding the nature of shocks and their propagation mechanism. This leads to the question, as noted, for instance, by Blanchard and Watson (1986), whether all business cycles are alike.

Figure 3.1
Spectral Decomposition of NBER Chronology

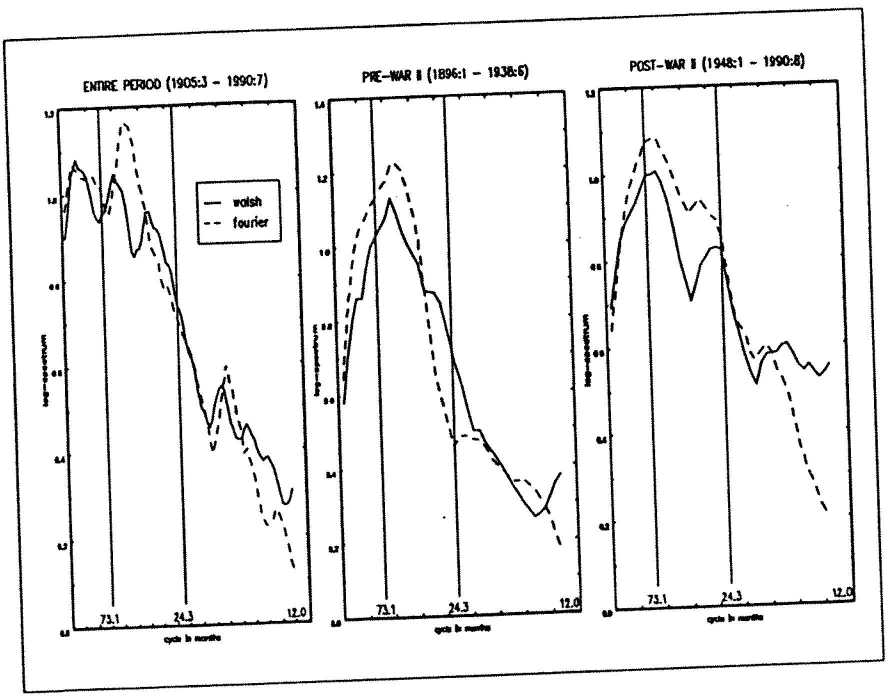
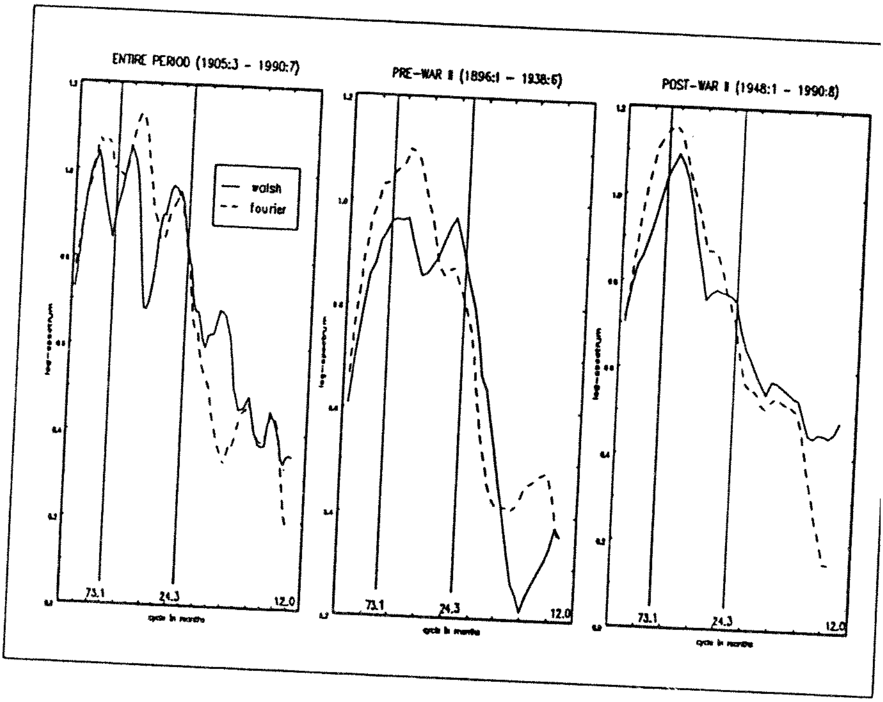


Figure 3.2
Spectral Decomposition of Romer Chronology



The first possible source of heterogeneity, namely the pre- versus the post-WWII eras having different characteristics can be investigated by simply studying the subsamples. Let us therefore focus on the separate pre- and post-WWII samples. The Fourier and Walsh-Fourier spectra once again do not entirely agree on some critical issues. In particular, for the Romer pre-WWII spectra, we notice again a double-dip pattern with the Walsh-Fourier spectrum, not revealed by the Fourier spectrum. This is also the case with post-WWII NBER chronology. Several other observations emerge from the pre- and post-WWII comparison. We notice a very different spectral shape for the two eras, particularly with the Walsh-Fourier analysis, but also with the standard spectral representation. Moreover, one also observes significant differences between the NBER and Romer chronologies. Indeed, before WWII, we found a double-dip pattern with Walsh-Fourier applied to the Romer chronology, but no such pattern emerged with the NBER chronology. Both post-WWII chronologies suggest two distinct peaks in the business cycle frequency bands, though this is more evident for the NBER chronology. Hence, the only chronology not exhibiting a double-peak is the NBER one before WWII. The double peak spectrum emerging from our analysis suggests that the heterogeneity does not only appear to be related to the so-called stabilization hypothesis after WWII. There is indeed evidence of a mixture of business cycle patterns, relatively long cycles of over five years and cycles that are much shorter, that is, less than three years. The advantage of spectral decompositions is to uncover such peaks. We cannot, of course, from this univariate decomposition derive the sources of shocks and propagation mechanism which generate the heterogeneity.

It is also worth noting that at the end of the frequency domain plotted in Figures 3.1 and 3.2, we observe a peak at yearly cycles for the entire sample as well as the pre-WWII sample, particularly for the Romer chronology. The appearance of such a peak at the seasonal frequency is related to the observations made in Ghysels (1991, 1992) regarding the nonuniform distribution of turning points throughout the calendar year. Namely, it suggests that the propensity of the economy to emerge from a recession or end an expansion is calendar-dependent. Obviously, the peak which emerges is not as dominant as those in the business cycle frequency band, yet it is clearly present in almost all the plots. This finding, which is essentially obtained via nonparametric methods, i.e., spectral methods, complements the nonparametric duration analysis discussed in Ghysels (1991).

The spectral plots in Figure 3.1 suggest differences between the NBER and Romer chronologies. We can measure the association of the two chronologies via the multivariate spectral analysis discussed in section 2. In particular, we can compute the coherence between two chronologies. In Table 3.1, we report the average coherencies over different frequency bands.

Table 3.1
Average Coherences Between NBER and Romer Chronologies

	Over 73.1 Months	24.4 to 64.0 Months	23.3 to 12.1 Months
		Entire Sample, 1905:3 - 1990:7	
Walsh-Fourier	0.82	0.85	0.77
Fourier	0.81	0.87	0.67
		Pre-WWII Sample, 1896:1 - 1938:6	
Walsh-Fourier	0.63	0.80	0.45
Fourier	0.68	0.79	0.47
		Post-WWII Sample, 1948:1 - 1990:8	
Walsh-Fourier	0.95	0.95	0.89
Fourier	0.97	0.96	0.78

Clearly, after WWII, the two spectra are much in agreement, as the coherencies run on average at 0.95 and higher in the business cycle frequency band. However, before WWII, the two spectra are substantially more in disagreement with a coherency of 80 % or less. Over the entire sample, the coherency is below 90 %. Outside the business cycle band, there is far less agreement. In fact, at the low and high frequencies, there appears to be less than 60 % to 50 % coherence. These measures quantify much of the discussion regarding the differences between the two alternative chronologies. It may also be worth noting that this time the results obtained from the standard spectral methods appear to be in agreement with the Walsh-Fourier coherencies.

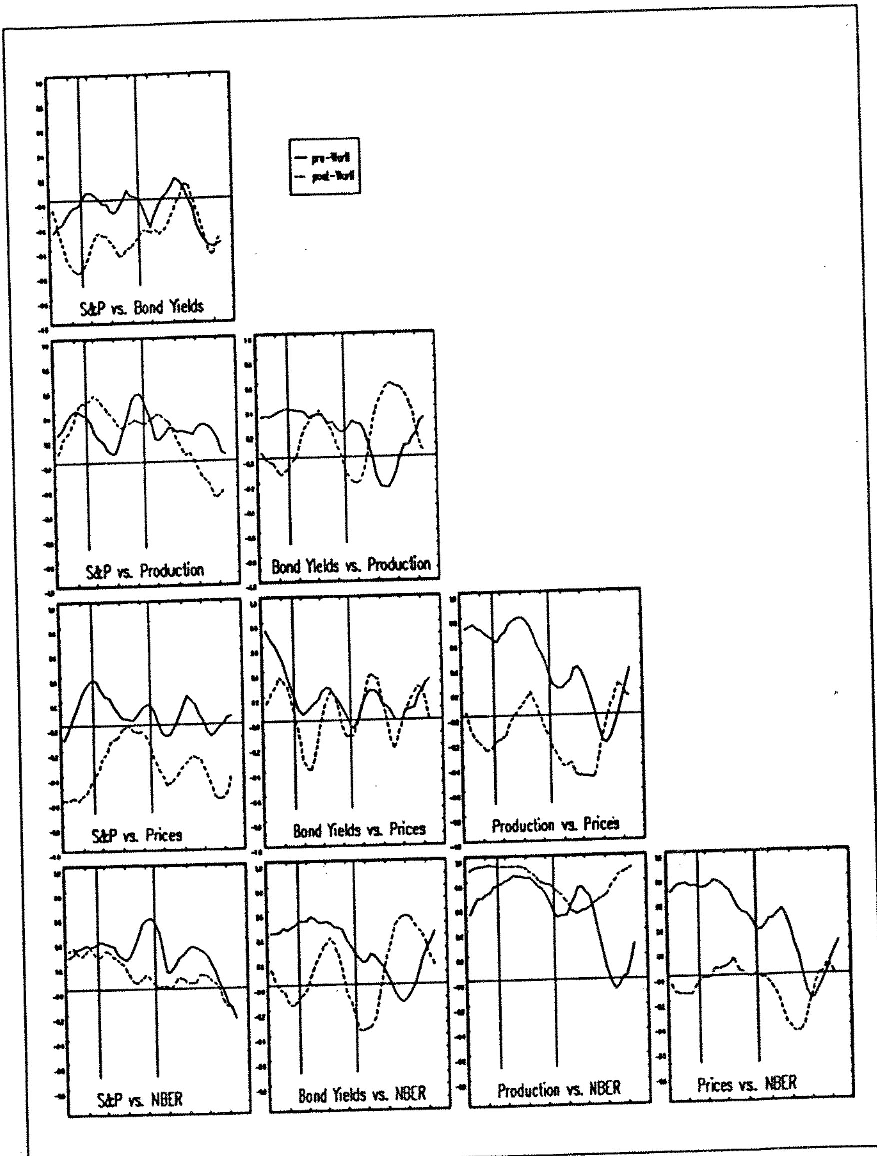
4. COMOVEMENTS BETWEEN INDIVIDUAL SERIES

A key characteristic of the business cycle is that fluctuations are common across sectors of the economy. We turn our attention here to a set of individual series covering real activity, prices and financial indicators. We will be interested in a pre- and post-WWII comparison of comovements as well as a more detailed study of post-WWII series, since a wider range of data series are available for this era. Unlike reference chronologies, which are the output of some CH(., t) procedure defined in (2.6), there is no direct turning point chronology available for individual series. Hence, for each series we need to construct a binary business cycle phase series. Like Watson (1992), we opted to use the Bry and Boschan (1971) algorithm to date business cycle phases. The merit of this method is that it reproduces the NBER chronology quite accurately. All chronologies appearing in the remainder of this section will be based on the Bry-Boschan algorithm. A first subsection will be devoted to pre- and post-WWII comparisons, while a second section covers the post-WWII era.

4.1 Business cycle comovements before and after WWII

Comparing business cycle features before and after WWII has been the subject of many research papers. A very incomplete list of the most recent papers includes Moore and Zarnowitz (1986), Romer (1992), Diebold and Rudebusch (1992) and Watson (1992). The question whether there has been a fundamental change in the nature of business cycles has been vigorously debated among economists for several reasons, particularly with respect to the success of postwar stabilization policies. A comparison of both eras is limited to a relatively small set of series, as there are not many matching pairs of uniformly defined or approximately similar series. A total of eight series were considered similar to those studied by Watson (1992). Sources of all the data series are described in the Appendix, while Figure 2.1 displays the binary processes extracted via the Bry-Boschan algorithm for a subset of those series. We focus our attention on four series of broad measures of economic activity, namely the industrial production (IP) turning point series, denoted b_t^{IP}, the S&P common stock price index b_t^{SP}, the producer price index b_t^{PP} and bond yields b_t^{BY}. Figure 4.1 shows the coherency among these four individual series as well as their coherency with the NBER reference cycle. The coherency was computed before and after WWII so that each plot in Figure 4.1 has two curves. The frequency band of cycles of two to six years are again marked on each plot. It is worth recalling from section 2 that the Walsh-Fourier cross-spectrum is real, unlike the Fourier cross-spectrum. Consequently, the Walsh-Fourier coherency can assume negative values, a clear advantage over its Fourier counterpart, as it reveals the magnitude as well as the sign of comovements. The last row of plots in Figure 4.1 shows the coherency of b_t^{IP}, b_t^{PP}, b_t^{BY} and b_t^{SP} respectively with the NBER reference cycle. Among the four individual series, b_t^{PP} shows a most dramatic change in cyclical pattern. After WWII, there was virtually no cyclical pattern in prices, while before the war, prices moved strongly pro-cyclical. Kydland and Prescott (1990) also noted the change in price level business cycle patterns, yet they claimed that prices moved countercyclical after WWII. Our results do not support such a view of post-WWII price behavior. The bond yield chronology also displays very different patterns across the two samples with two strong and distinct peaks after WWII, including one of a short-cycle comovement with the NBER series (under two years). At the short end of the spectrum, we also notice a sharp change of b_t^{IP} and b_t^{N} comovements with a strong seasonal coherency before WWII, which virtually disappeared in the last forty years. It is also interesting and not surprising to note that b_t^{IP} and b_t^{PP} show the same dramatic change in coherency as b_t^{N} and b_t^{PP} do. The stock market was strongly negatively correlated across the frequency domain with prices, but this is no longer the case since WWII. Bond yields and the stock market also appear negatively related across all frequencies before the war but little remains since.

Figure 4.1
Walsh Coherencies : Pre- and Post-WWII Comparision



4.2 Coherency since WWII

Stylized facts of business cycle comovements over the post-WWII era have been documented by a large variety of authors, sometimes using quite diverse statistical methods and data transformations for detrending, seasonal adjustment, etc. In general, one analyzes the timing relation between various series and some reference series, usually real GNP, by means of cross-correlation coefficients. There exist more complicated procedures, however, such as VAR impulse response analysis, common factor and index models. Documenting stylized facts is quite sensitive to prefiltering data. Such prefiltering occurs either when detrending or seasonally adjusting series. For instance, Canova (1991) shows in detail that a multitude of key stylized facts in business cycle analyses are inconclusive because of prefiltering effects.[11] There probably is less disagreement regarding the location of turning points, particularly for the post-WWII era, than there is regarding the specification of the secular component of macroeconomic time series. Therefore, we suggest to use the Walsh-Fourier coherency methods here as an alternative tool of studying post-WWII business cycle features.

Over the typical business cycle, it is claimed that employment varies substantially, while the determinants of labor supply, like real wages and real interest rates, vary only slightly [see, e.g., Mankiw (1989)]. The Walsh-Fourier coherencies between the NBER and individual series plotted in Figure 4.2 confirm this finding to a large degree, except for the comovements between real interest rates and the NBER chronology. They appear indeed important, compared to the de facto zero coherence between NBER and real wages at all frequencies of the spectrum.[12] In contrast to the real wage, we observe strong procyclicality of labor productivity. It was already noted that the price level, measured

---

11 Several other papers have raised this question, including Singleton (1988), Cogley (1990) and Ghysels, Lee and Siklos (1993).

12 It is often claimed that real wages are procyclical. While they are for certain business cycle frequencies, they also appear negatively correlated with the reference cycle over other business cycle frequencies.

via the PPI, is neither procyclical nor countercyclical. Also, in Figure 4.2, we notice that the unemployment rate is strongly countercyclical, yet there appears to be a sharp (positive) peak at the seasonal frequency. Hours worked and real wages are typically found to have low correlation. Figure 4.3 shows a zigzag coherence pattern which decomposes the low correlation in a sharp positive peak around the seasonal frequency as well as a large dip in the business cycle frequency band. Unemployment and real wages also show mostly a positive coherency, as would be expected, but again labor productivity and the real wage are basically uncorrelated across frequencies. Finally, inflation against the nominal interest rate as well as against the real interest rate also yields some peculiar patterns. Inflation and real interest rates show a strong negative correlation in the business cycle frequency band. For the nominal rate, there are two sharp positive peaks decomposing the comovements.

5. CONCLUSIONS

In this paper, we have introduced spectral methods as a tool for analyzing business cycle chronologies. It is a fairly convenient way to examine the nature of comovements across the chronologies of different series, and it is also an ideal tool to compare competing chronologies of reference or other cycles. We uncovered interesting features regarding (1) the relation between the NBER and Romer chronology, (2) the nature of pre- and post-WWII business cycle fluctuations and (3) some stylized facts with respect to the post-WWII era.

Of course, as with any application of spectral analysis, one can only rely on it as a method for decomposing observed series in orthogonal cycles. It does not readily yield economic interpretations of the decomposition. But if one is only paying attention to stylized facts, there are some clear advantages to pairing spectral methods with chronologies.

Figure 4.2
Some Stylized Facts with Walsh-Fourier Coherency - Post-WWII

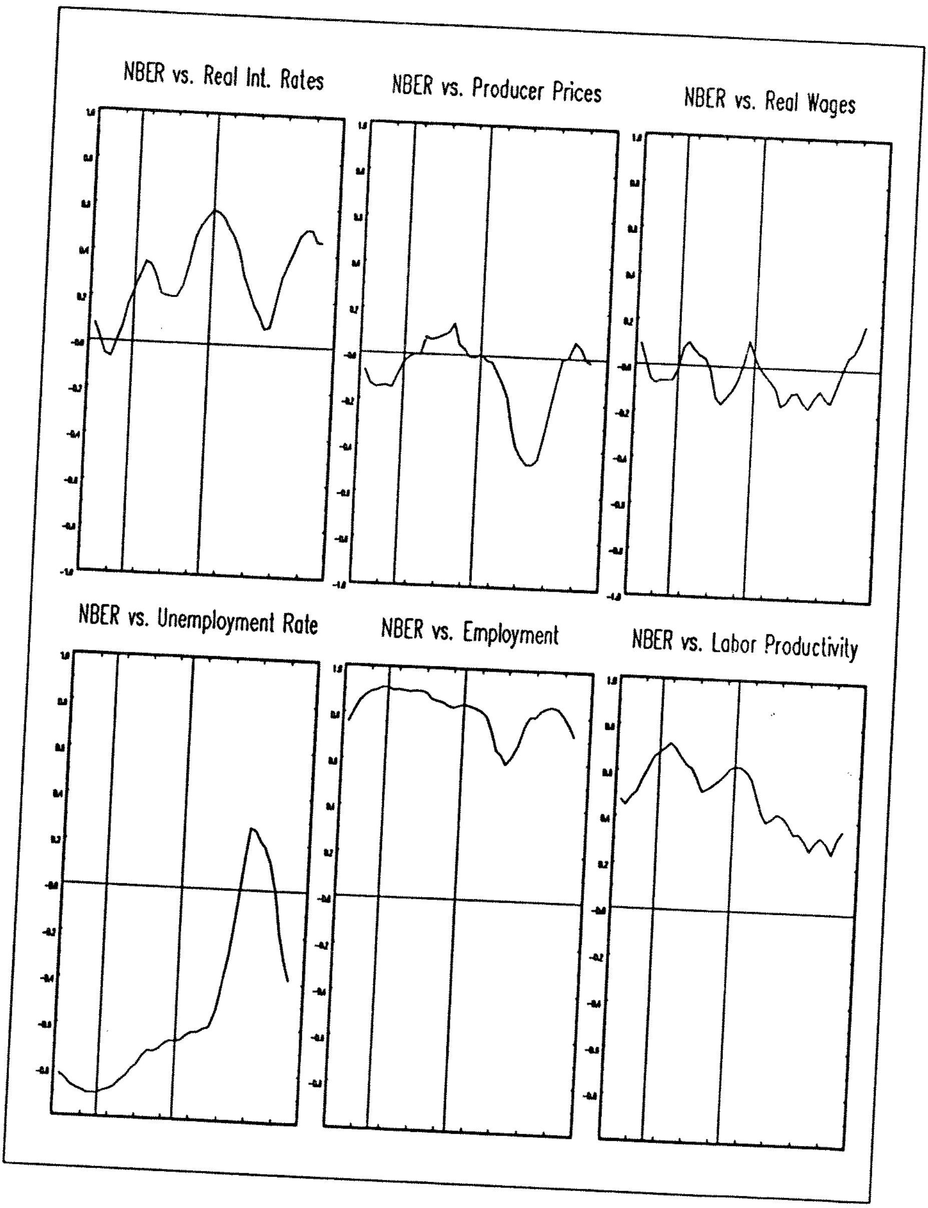
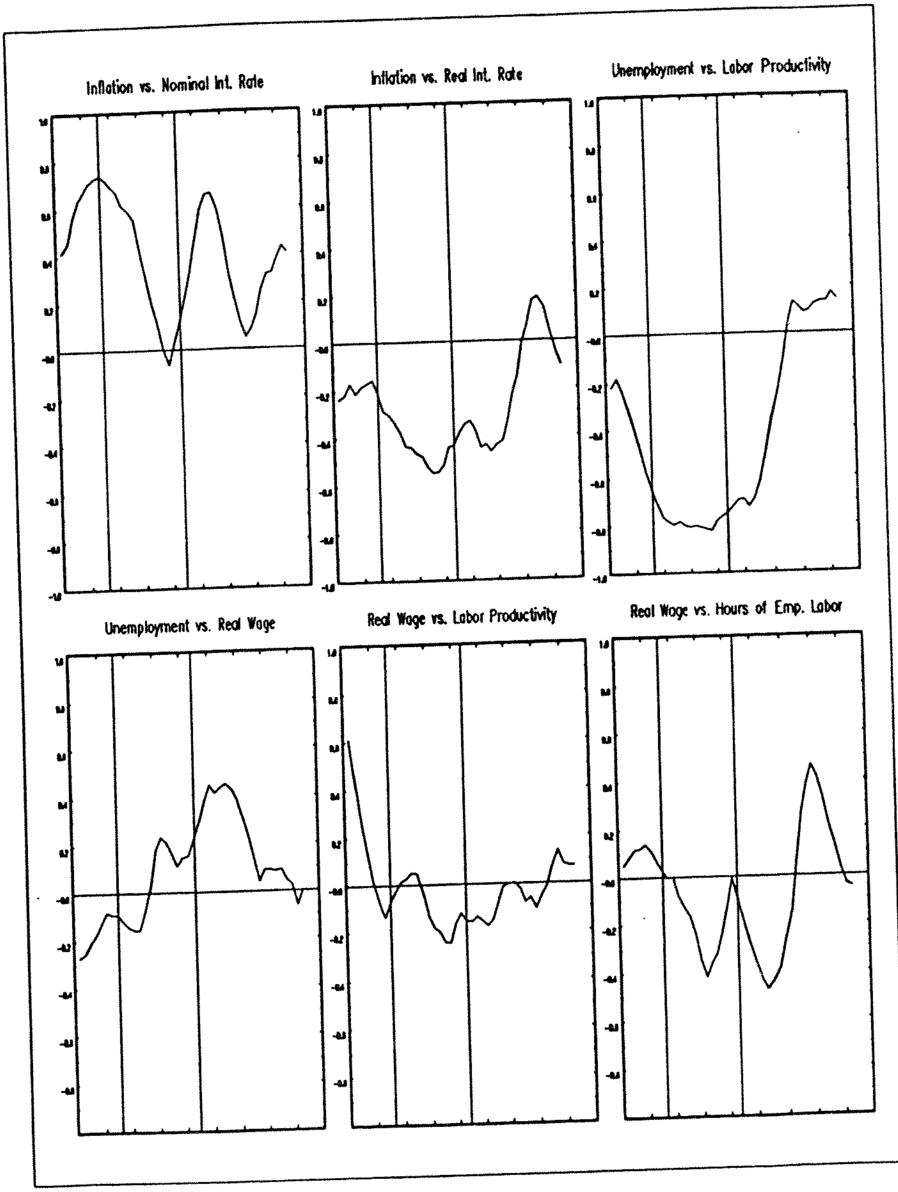


Figure 4.3
Comovements - Individual Series



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APPENDIX

A) DATA DEFINITIONS

The reference chronologies were taken from Romer (1992), while the pre-WWII series were obtained from Watson (1992). All the post-WWII series were extracted from CITIBASE. The CITIBASE mnemonics are given in parantheses.

Pre-WWII data

- Pig Iron production, 1877:1 - 1941:12 (NBER, BCD, ID number n01585).
- S&P common stock price index, 1871:1 - 1940:12 [SPPRWARR in Watson (1992)].
- Wholesale price index, 1890:1 - 1940:12 (NBER, BCD, ID number n04010).
- RR Bond yields, 1857:1 - 1940:12 (NBER, BCD, ID number n13024).

Post-WWII data

- Industrial production index, total, 1947:1 - 1993:8 (IP).
- Real S&Ps common stock price index, 1947:1 - 1993:8 (FSPCOM/PUNEW).
- Producer price index, all commodities, 1946:1 - 1993:8, NSA (PW).
- Bond yield, Moody's BAA corporate, percentage per annum, 1947:1 - 1993:8 (FYBAAC).
- Price inflation, 1948:1 - 1993:8, INFLP = $[100 * \log(PUNEW_t / PUNEW_{t-12})]$.
- Wage inflation, 1947:1 - 1993:8, INFLW = $[100 * \log(LEHM_t / LEHM_{t-12})]$.
- Interest rate, U.S. treasury bills, 1947:1 - 1993:8 (FYGM3).
- Real short-term interest rate, 1948:1 - 1993:8, FYGM3R = (FYGM3 - INFLP).
- Unemployment rate, men, 20 years and over, percentage, sa, 1948:1 - 1993:8 (LHMUR).
- Man-hours of employed labor force, 1947:1 - 1993:7 (LHOURS).
- Labor productivity, 1947:1 - 1993:7, LPROD = (IP / LHOURS).

Table A.1
Average Phase Durations* in Months

Series	Sample Period	P-P	T-T	P-T	T-P
NBER (entire)	1905:3 - 1990:7	55.4	51.2	14.6	40.4
NBER (prewar)	1896:1 - 1938:6	45.0	44.7	19.0	25.7
NBER (postwar)	1948:1 - 1990:8	63.4	56.7	11.0	51.5
Romer (entire)	1905:3 - 1990:7	52.4	53.8	12.7	40.0
Romer (prewar)	1896:1 - 1938:6	40.7	41.4	12.4	29.0
Romer (postwar)	1948:1 - 1990:8	63.3	57.4	12.4	50.2
M01585	1878:4 - 1938:6	44.5	44.8	15.0	29.3
SPPRWARR	1872:8 - 1939:7	42.3	42.8	18.5	23.8
M0401X	1890:10 - 1939:8	46.8	47.3	19.8	27.5
M13024	1857:12 - 1940:11	43.3	44.0	20.1	23.2
IP	1948:7 - 1991:3	63.4	57.0	37.1	19.8
FSPCOMR	1949:7 - 1990:10	41.5	45.1	19.2	25.9
PW	1948:9 - 1990:10	63.2	62.8	13.2	50.0
FYBAAC	1948:4 - 1990:10	51.1	51.7	24.5	26.6
INFLP	1949:9 - 1993:2	45.6	46.3	23.6	21.8
INFLW	1949:12 - 1992:11	36.6	36.8	20.0	16.8
FYGM3	1949:2 - 1989:3	53.5	55.8	16.9	36.7
FYGM3R	1949:11 - 1993:4	42.0	42.2	21.6	20.1
FMBASE6	1952:12 - 1992:1	41.7	40.7	16.6	24.6
IPXMCA	1949:11 - 1991:3	50.4	49.7	23.1	26.6
LHMUR	1949:11 - 1992:6	56.9	57.0	37.1	19.8
LPNAG	1948:10 - 1992:2	71.6	72.6	12.6	60.0
LHOURS	1948:5 - 1991:8	62.6	62.7	16.4	46.4
LPROD	1948:6 - 1991:4	43.9	45.7	16.3	29.0

* P-P : Peak to peak; T-T : trough to trough; P-T : peak to trough; T-P : trough to peak.

B) TECHNICAL NOTES ON THE WALSH-FOURIER SPECTRAL ANALYSIS

In the remainder of this Appendix, we present a brief technical review of the Walsh-Fourier theory. The proof of the results have been omitted, but the references will be provided. The presentation is divided into several subsections.

A fundamental difference between sinusoids and Walsh functions is that the latter are aperiodic. Consequently, the value of n in W(n, ω) does not have the same straightforward interpretation as in sinusoidals. The notion of *sequency* is introduced to describe a generalized concept of frequency appropriate for functions such as the Walsh functions that are not necessarily periodic. The frequency parameter n in sinusoidals may also be interpreted as one half the number of zero crossings or sign changes per unit time. So the term sequency will simply denote half of the frequency. Roughly speaking, while the frequency is inversely related to the length of a full cycle, the sequency is inversely related to the length of half a cycle. Henceforth, for the sake of comparability between Fourier and Walsh-Fourier spectral representations, we will adopt the generalized concept of frequency in reference to the Walsh functions. The reader may easily convert back and forth between the two measures, since generalized frequency is one half of sequency.

Sequency and frequency analyses

- *Walsh functions*

The square-wave Walsh functions form a complete orthonormal sequence on [0, 1) and take on only two values, +1 and -1. Suppose that a sample of length $N = 2^p$, where $p > 0$ integer, is available.

The Walsh-ordered Hadamard matrix $H_W(p)$ is obtained by counting the number of sign changes in each row (or by symmetry in each column) of the Hadamard matrix H(p) and then by reordering the rows (columns) in ascending order according to the number of switches. So the first column of $H_W(p)$ makes no sign changes, the second changes one time, the third two times, etc., and $H_W(3)$ summarizes a discrete-valued version ($\omega = m/N$) $n = m = 0, 1, ..., 7$, instead of a continuous-valued version $0 \leq \omega < 1$ exhibited in Figure 2.2.

$$H_W(3) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{pmatrix}.$$

In order to discuss the fast Walsh-ordered Hadamard transform, consider the recursive generation of the Hadamard matrix by setting initially $H(0) = 1$ and then processing by

$$H(k+1) = \begin{vmatrix} H(k) & H(k) \\ H(k) & -H(k) \end{vmatrix} \quad k = 0, 1, 2, ..., (p-1).$$

The Hadamard matrix gives the discrete Walsh functions as rows (or columns). To obtain the Walsh functions in sequency order $W(j, w_j)$, one will reorder the rows of H(p) according to the number of sign changes. The Walsh-ordered Hadamard matrix $H_W(p)$ can be computed as :

$$H_W(p) = \prod_{i=1}^{p} H_i(p) \cdot B \qquad \text{(A.1)}$$

where

$$H_i(p) = \begin{vmatrix} F_s & & & & 0 \\ & G_s & & & \\ & & \cdot \; \cdot \; \cdot & & \\ & & & F_s & \\ 0 & & & & G_s \end{vmatrix}, \qquad s = 2^{i-1} \, , \tag{A.2}$$

with $F_s = \begin{vmatrix} I_s & I_s \\ I_s & -I_s \end{vmatrix}$, $G_s = \begin{vmatrix} I_s & -I_s \\ I_s & I_s \end{vmatrix}$ and I_s being the (s x s) identity matrix. Matrix B in (A.1) is a matrix which bit reverses the order of the matrix H(p). Namely, matrix B counts essentially the number of sign changes in each row (column) of the H(p) and then reorders the rows (columns) to obtain $H_W(p)$.

The basic properties of Walsh functions are given in Kohn (1980a, Lemma 1).

- *Finite Walsh transform and the logical covariances*

Assume that {X(n)} is a discrete-time, zero-mean, second-order stationary time series. Let γ_j be the autocovariance function of $\{b_t\}$. For $0 \le \omega < 1$, the finite order Walsh transform is defined as [Kohn (1980a), Stoffer (1987, 1990)] :

$$d_N(\omega) = N^{-1/2} \sum_{j=0}^{N-1} b(j)\, W(j, \omega) \, . \tag{A.3}$$

Then

$$\mathrm{var}\{d_N(\omega)\} = N^{-1} \sum_{j=0}^{N-1} \tau(j)\, W(j, \omega) \tag{A.4}$$

where $\tau(j)$ is the logical covariance function of X(t), given [Kohn (1980a)] as :

$$\tau(j) = 2^{-q} \sum_{k=0}^{2^q-1} \gamma(j \oplus k - k) \quad 2^q \le j < 2^{q+1}$$

where $m \oplus r$ is the dyadic addition of m and r, and equal to $\sum_{j=0}^{q} \mid m_j - n_j \mid 2^j$, where m_j and n_j are the coefficients of binary expansion of n and r respectively. The logical covariance functions $\tau(j)$ play the same role in the Walsh analysis as $\gamma(j)$ do when working with trigonometric functions.

- *The second-order properties of transform and the Walsh spectrum*

The limiting behavior of (A.4) gives the Walsh-Fourier spectrum. That is, $\text{var}\{d_N(\omega)\} \rightarrow f(\omega)$, as $N \rightarrow \infty$ where :

$$f(\omega) = \sum_{j=0}^{\infty} \tau(j)\ W(j, \omega), \quad 0 \leq \omega < 1 \tag{A.5}$$

is the Walsh-Fourier spectrum. We note that $f(\omega)$ exists, since the absolute summability of $\gamma(j)$ implies the absolute summability of $\tau(j)$. Specifically, Kohn (1980a, Lemma 3) shows that if $\lim_{n\rightarrow\infty} \sum_{|j|<2^n} \left(1 - \frac{|j|}{2^n}\right) |\gamma(j)| < \infty$, then $\sum_{j=0}^{\infty} |\tau(j)| < \infty$ and $f(\omega)$ are well defined.

As with the usual Fourier analysis, if the mean of the series is unknown, the only sequency for which the transform in (A.3) cannot be evaluated is at the zero (m = 0) sequency. Let $\mu = E\{b(n)\}$, all n, and note that for m = 0, 1, ..., N - 1,

$$N^{-1} \sum_{n=0}^{N-1} W(n, m / N) = \delta_0^m \tag{A.6}$$

where δ is the Kronocker delta. From (A.6), the mean-centred transform will be the uncentred transform, except at m = 0 and, particularly,

$$E\{d_N(m / N)\} = N^{-1} \sum_{n=0}^{N-1} \mu\ W(n, m / N) = \sqrt{N}\ \mu\ \delta_0^m,\ m = 0, ..., N - 1.$$

Let ω_N be dyadically rational. If $\omega_N \oplus \omega \to 0$ as $N = 2^p \to \infty$, then

$$E\{d_N^2(\omega_n)\} \to f(\omega).$$

The asymptotic covariance of the Walsh-Fourier transform at two distinct sequencies $\omega_{1,N}$ and $\omega_{2,N}$ is not generally zero. This is in contrast to the trigonometric case, where the Fourier transform of the data at two distinct frequencies are, under suitable conditions, asymptotically independent. If $\omega_{1,N}$ and $\omega_{2,N}$ are dyadically rational and $|\omega_{1,N} - \omega_{2,N}| N^{-1}$ with $\omega_{i,N} \oplus \omega \to 0$, $i = 1, 2$ as $N = 2^p$, then [Kohn (1980a,b)]

$$E\{d_N(\omega_{1,N})d_N(\omega_{2,N})\} \to 0 .$$

The basic result is that (Stoffer 1990) under appropriate conditions, $d_N(\omega_N)$ converge in distribution with mean zero and variance $f(\omega)$. Under these same conditions and using the results above, if $\{\omega_{1,N}, \ldots, \omega_{M,N}\}$ is a collection of M sequencies close to a sequency of interest, ω, then

$$\sum_{j=1}^{M} d_N^2(\omega_{j,N}) \to f(\omega)\chi_M^2$$

where χ_M^2 denotes a chi-squared distribution with M degrees of freedom. From this, one can deduce that $M^{-1} \sum_{j=1}^{M} d_N^2(\omega_{j,N})$ is an estimate of $f(\omega)$, having variance $wf^2(\omega) / M$. If we let $M \to \infty$ as $N \to \infty$, then this estimate is a mean-squared consistent estimate of $f(\omega)$ $(0 < \omega < 1)$.

C) NOTES ON THE ESTIMATION OF SPECTRA AND COHERENCIES

As illustrated in Figure 2.1, the data have different lengths, and hence to utilize the Fourier and Walsh-Fourier transformations, the length of the series were truncated to the nearest power. For the sake of comparability between sinusoidal and asinusoidal waves, the spectral density estimates were computed using an asinusoidal window generator, namely a tent-type kernel with eleven equally weighted periodogram ordinates in the frequency domain.

The sample periods in Figures 3.1 and 3.2 consist of either 512 monthly observations or, in the case of the entire sample of reference chronologies, 1,024. Thus, any arbitrary padding schemes were avoided by the truncation of the series, as both sample sizes are integer powers of 2. The spectral density ordinates $f(\omega_j)$ were decomposed over the following three nonoverlapping business cycle bands : for the number of observations $N = 512$ ($p = 9$), the frequency domain was decomposed as $j = 1, ..., 7$, $j = 8, ..., 21$, $j = 22, ..., 42$. Hence, these bands are centered at periodicities of 189.6, 38.5 and 16.6 months respectively. The first band ranges between periodicities of 512 to 73.1 months and is thus the band in which long oscillations occur. The remaining bands are considered as business cycle oscillations and higher frequencies. When $N = 1,024$ ($p = 10$), then the bands considered were quite similar to the previous ones by setting $j = 1, ..., 14$, $j = 15, ..., 42$, $j = 43, ..., 85$ with averaged periodicities of 237, 39.3 and 16.6 months.

Let $d_A(\omega)$ and $d_B(\omega)$ denote the Walsh-Fourier transforms of the rectangular cyclical pattern of b_1 and b_2. The sample coherency between b_1 and b_2 was computed according to (2.15), where the cross-spectral estimation $\hat{f}_{12}(\omega)$ was obtained by averaging the values of the product $d_1(m / N)\, d_2(m / N)$ over values of m in the neighborhood of ω. Similarly, $\hat{f}_{11}(\omega)$ and $\hat{f}_{22}(\omega)$ were obtained by averaging the Walsh periodograms $d_1^2(m / N)$ and $d_2^2(m / N)$ over values of m in the neighborhood of ω.

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Département de sciences économiques
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