



Université de Montréal  
Faculté des arts et des sciences  
Département de sciences économiques

CAHIER 9504

INDEXATION LAGS AND HETERODOX STABILIZATION PROGRAMS

Emanuela CARDIA<sup>1</sup> and Steve AMBLER<sup>2</sup>

- <sup>1</sup> Département de sciences économiques and Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal.
- <sup>2</sup> Centre de recherche sur l'emploi et les fluctuations économiques, Université du Québec à Montréal.

January 1995

We gratefully acknowledge financial support from the SSHRC, the Fonds FCAR, and the PARADI program of CIDA. We thank Stéphane Vachon for able research assistance and Vittorio Corbo and Camilo Dagum for comments. The usual caveat applies.

C.P. 6128, succursale Centre-ville  
Montréal (Québec) H3C 3J7

Télécopieur (FAX): (514) 343-5831  
Courrier électronique (E-Mail): econo@tornade.ERE.Umontreal.CA

Ce cahier a également été publié au Centre de recherche et développement en économie  
(C.R.D.E.) (publication no 0495).

Dépôt légal - 1995  
Bibliothèque nationale du Québec  
Bibliothèque nationale du Canada

ISSN 0709-9231

## RÉSUMÉ

Nous examinons le rôle de l'indexation et des délais d'indexation pour la stabilisation économique. Nous analysons si l'inertie accrue due aux délais peut rendre plus importants les aspects hétérodoxes des programmes anti-inflation, aspects qui comprennent l'utilisation de contrôles directs sur les prix et les salaires. À cette fin, nous utilisons un modèle d'une petite économie semi-industrialisée avec une rigidité nominale qui provient de contrats de salaire chevauchés. Le modèle permet un degré d'indexation ainsi qu'un délai d'indexation variables.

Mots-clés : inflation, programmes de stabilisation hétérodoxe, indexation des salaires.

## ABSTRACT

This paper examines the role of wage indexation and indexation lags in stabilization programs. We analyze whether the added inflation inertia caused by such lags increases the importance of the heterodox aspects of anti-inflation stabilization programs, which involve the imposition of direct wage and price controls. For this purpose, we build a model of a small, open semi-industrialized economy which incorporates nominal rigidities via staggered wage contracts. The model allows for a variable degree of indexation to the cost of living which is implemented with a variable lag.

Key words: inflation, heterodox stabilization programs, wage indexation.



## 1. Introduction

Following the failure of many stabilization programs in semi-industrialized countries that suffered from bouts of high inflation in the 70's and 80's, some authors (for example Cukierman, 1988, and Dornbusch, 1986) have suggested that explicit wage-price controls may be a crucial element for the success of stabilization policies. These policies may be more important in the context of widespread indexing of nominal contracts with indexation lags causing a large degree of inflation inertia. For example, Simonsen (1983, p.131), writing of the Brazilian experience, suggests that indexation lags "may produce highly unfavorable output-inflation trade-offs in the short run when market flexibility is deterred by rigid indexation schemes, such as those imposed by law. As a result of accommodating policies and of staggered indexation rigidities, inflation may follow a random walk, as it apparently did in Brazil since 1968." In this context, wage-price controls could help decrease inflation immediately where indexation lags would inhibit deflationary policies (like fiscal spending cuts) that would otherwise decrease wages quickly. It has become current practice to refer to stabilization programs with wage-price controls as "heterodox".

In a previous paper (Ambler and Cardia, 1992), we characterized the qualitative features of an optimal, time-consistent anti-inflation stabilization program in a continuous-time model of a small open economy with staggered wage setting and partial indexation. We showed that such a program involves a rapidly decelerating rate of downward crawl of the exchange rate and a decrease in the budget deficit via cuts in fiscal spending. We found that the general shape of optimal stabilization programs remains the same over a wide range of parameter values. The downward crawl of the exchange rate is quickly halted, and government spending is rapidly reduced. The optimal program includes a modest degree of wage and price controls. One strong conclusion of the paper is that the ability to impose wage and price controls does not

make a significant difference to the costs of fighting inflation, or to the time paths of inflation and output.

To understand the role that lagged indexation may play in the choice of heterodox versus orthodox stabilization programs we build a discrete-time simulation model with staggered wages and indexation, which allows for wage indexation to be subject to a lag. We simulate the model with and without indexation lags to analyze whether the added inflation inertia caused by such lags increases the importance of the heterodox aspects of anti-inflation stabilization programs.

The paper is organized as follows. The model is presented in the next section. In section 3, the policy game played between the government and the private sector is specified and its solution is described. In section 4, the simulation results are presented. We analyze optimal policy and the time paths of various endogenous variables with indexation lags of zero, one and two periods. Section 5 contains concluding remarks. An appendix gives the values of the coefficients of the state space form of the model and discusses in detail the algorithm used to derive the optimal anti-inflation program and to simulate the model.

## 2. The Model

The model is a fairly standard New Keynesian model of a small open economy, with some changes which are designed to capture stylized facts peculiar to semi-industrialized economies. Domestic credit markets are assumed to be thin so that money creation is the primary means of financing government budget deficits; this leads to a particularly simple formulation of the government budget constraint. It is assumed that capital controls restrict capital flows to respond slowly to uncovered interest rate differentials. Workers and firms sign contracts which fix the base wage in nominal terms, but the model allows for the possibility of varying degrees of wage indexation to

compensate for price level changes that occur after contracts are signed. The model is similar to the one in Ambler and Cardia (1992), but is cast in discrete time in order to allow wage indexation to be subject to discrete indexation lags.

The specification of wage determination is based on Calvo (1983), extended to allow for partial wage indexation. Calvo's model in turn is a continuous-time formulation of the staggered contracts model of Taylor (1979, 1980). Contract lengths are assumed to be stochastic,<sup>1</sup> with a given probability of expiry from one period to the next equal to  $(1-d)$  for a constant parameter  $d$ . The mean contract length is then given by  $1/(1-d)$ .<sup>2</sup> The (logarithm of) the base nominal wage of a contract signed at time  $s$  is denoted by  $v_s$ . The current wage  $x_t$  (measured in logs) for a contract signed at time  $s$  is given by:

$$x_t = v_s + \theta(p_{t-k} - p_{s-k}) + (z_t - z_s), \quad (1)$$

where  $\theta$  is a parameter which gives the degree of indexation of contracts,  $p_t$  denotes (the logarithm of) the aggregate price level, and  $(z_t - z_s)$  is a wage-control parameter. Workers are compensated for increases in the price level since their contract was signed; this compensation is partial, depending on the parameter  $\theta$ , and is also subject to a  $k$ -period lag. The lag captures delays in information and implementation. The policy authorities are assumed to be able to affect the current wage directly via a wage control parameter  $(z_t - z_s)$ , which can vary over different contracts as indexed by the subscript  $s$ . This parameter can be thought of as affecting the ex post realized degree of indexation. The average wage in the economy, denoted by  $w_t$ , is given by summing over

<sup>1</sup> This assumption allows for a particularly simple solution to the problem of aggregating over contracts, as will become clear below.

<sup>2</sup> We recognize that the average rate of inflation may be an important determinant of contract length. Endogenizing  $d$  would introduce an important element of nonlinearity into the model. It is implicitly assumed that contract lengths would start to adjust only with a considerable lag after the implementation of a successful anti-inflation program, so that we can ignore this consideration.

all contracts which are still in force at time  $t$ :

$$w_t = \sum_{i=0}^{\infty} (1-d)d^i \left[ v_{t+i} + \theta(p_{t+k} - p_{t,k-i}) + (z_t - z_{t-i}) \right]. \quad (2)$$

The base wage of a contract signed at time  $t$  is determined by the following equation:

$$v_t = \sum_{i=0}^{\infty} (1-d)d^i E_t \left[ w_{t+i} + \gamma(p_{t,k+i} - p_{t,k}) + \beta y_{t+i} \right]. \quad (3)$$

Here,  $E_t$  denotes the mathematical expectations operator conditional on information available at time  $t$ . Workers are concerned about their nominal wage relative to the expected future average wage. Wage negotiations are influenced by current and future expected output, which is a proxy variable for expected labor market tightness. The base wage is also influenced by expected increases in the price level. The parameter  $\gamma$  is chosen so that output is equal to its natural level (normalized to equal zero) in the long run.

Define the lag operator  $L$  such that  $LE_t z_{t+i} = E_t z_{t+i-1}$ . Then, applying the filters  $(1-dL)$  and  $(1-dL^{-1})$  respectively to the average wage equation and the contract wage equation gives:

$$w_t = d w_{t-1} + (1-d) v_t + d\theta (p_{t,k} - p_{t,k-1}) + d (z_t - z_{t-1}), \quad (4)$$

$$v_t = d \hat{v}_{t+1} + (1-d) w_t + d\gamma (p_{t,k+1} - p_{t,k}) + (1-d)\beta y_t. \quad (5)$$

We use a caret as a shorthand notation for the  $E_t$  operator. The contract-specific wage control variable collapses to a single wage control variable which directly affects the average wage in the economy.



The main effect of indexation in this model is to reduce the degree of "front end loading" of wage contracts. In the absence of indexation, workers are compensated immediately for expected future inflation. The real wage is at a maximum when the contract is first signed, and declines gradually throughout the life of the contract. Indexation obviates the need to incorporate future expected price increases into the current base wage, and thus moderates wage settlements at a point in time in an inflationary environment. However, the presence of indexation makes current wage settlements less sensitive to a reduction in the future expected rate of inflation. For a given initial average wage in the economy, this can in principle increase the costs of an anti-inflation program.

The wage-price sector of the model is completed by the specification of the aggregate price level. The economy produces a good which is an imperfect substitute for importables. The price index is given by:

$$p_t = \alpha w_t + (1-\alpha) (e_t + p_t^*) \quad (6)$$

Here,  $e_t$  is the logarithm of the nominal exchange rate, measured in units of domestic currency per unit of foreign currency. The log of the foreign currency price of foreign output is given by  $p_t^*$ . The price of domestic output is determined by a fixed markup over wage costs.

Aggregate demand is assumed to depend on the real exchange rate, the real interest rate, government purchases of goods and services, and foreign aggregate demand:

$$y_t = \tau_1 (e_t + p_t^* - w_t) - \tau_2 (i_t - p_{t+1} + p_t) + \tau_3 g_t + \tau_4 y_t^* \quad (7)$$

where  $i_t$  is the domestic nominal interest rate,  $g_t$  is the log of government spending, and  $y_t^*$  is foreign aggregate demand (output is demand determined in the short run). The log-linearized domestic demand for nominal money balances is given by:

$$m_t - p_t = y_t - \lambda i_t \quad (8)$$

For simplicity, the income elasticity of money demand is set equal to one: none of the results of the paper depend on this simplification. The base money supply can be decomposed into its domestic and foreign reserve components:

$$M_t = D_t + F_t \quad (9)$$

where

$$F_t - F_{t-1} \equiv E_t (R_t^* - R_{t-1}^*) \quad (10)$$

$E_t$  is the nominal exchange rate and  $R_t^*$  denotes reserves in terms of the foreign currency. Capital letters are used to denote the levels rather than the logs of variables. Equation (10) specifies that the foreign component of the monetary base remains unaffected by exchange rate changes that affect the domestic currency value of the central bank's stock of foreign reserves. The central bank absorbs these capital gains and losses via changes in its net worth. Log linearizing (9) gives:

$$m_t = \omega d_t + (1-\omega) f_t \quad (11)$$

Capital mobility is imperfect due to the presence of capital controls which are costly for private agents to avoid. Following Frenkel and Rodriguez (1982), capital flows are taken to depend on the uncovered interest differential. The simplest possible specification is adopted for the current account, which depends on the real exchange rate and national income. Reserves earn interest. This gives the following equation for the balance of payments:

$$R_t^* - R_{t-1}^* = \pi_1 (i_t - i_t^* - \hat{e}_{t+1} + e_t) + \pi_2 (e_t + p_t^* - w_t) + \pi_3 y_t + i^* R_t^* \quad (12)$$

The model is closed with a specification of the government budget constraint. Government spending is financed by printing money and by the interest earnings on the central bank's foreign reserves.

$$G_t W_t = D_t - D_{t-1} + i^* R_t^* E_t + T, \quad (13)$$

where  $G_t$  is the level of government spending in real terms, and  $T$  is a constant, politically or institutionally determined level of lump sum taxation. For simplicity, we assume that fiscal spending falls only on domestic output so that  $W_t$  is the relevant price used to calculate government spending in nominal terms. The rate of change of fiscal spending is a control variable for the government:

$$\dot{g}_t - g_{t-1} = \eta_t. \quad (14)$$

#### *Steady State*

The model is simulated using a linear approximation in the neighborhood of its steady state. Some of the parameters of the linearized version of the model depend on the steady-state values of certain endogenous variables. In addition, calculating the steady state of the model is useful in order to establish a coherent set of initial values for the predetermined state variables of the model. Dropping time subscripts, and using the convention that upper case letters denote the levels of variables and that lower case letters denote variables measured in logs, we have:

$$m - p = y - \lambda i, \quad (15)$$

$$y = \gamma_1 (e - w) - \gamma_2 i + \gamma_2 \Delta e + \gamma_3 g, \quad (16)$$

$$G = \Delta d D/W + i^* R^* E/W, \quad (17)$$

$$0 = \pi_1 (i - i^* - \Delta e) + \pi_2 (e - w) + \pi_3 y + i^* R^*, \quad (18)$$

$$(v - w) = (d(1-\theta)/(1-d)) \Delta e - (d/(1-d)) \Delta Z, \quad (19)$$

$$\exp(m - p) = \left[ \exp(f - w) + \exp(d - w) \right] \left[ \exp(-(1-\alpha)(e - w)) \right], \quad (20)$$

$$\Delta f F/W = 0, \quad (21)$$

where  $\Delta k$  denotes the rate of change of a nominal variable  $k$  in the steady state, and where we have set foreign variables other than the foreign interest rate equal to zero. To derive these equations, we suppose that all real variables are constant in the steady state and that all nominal variables grow at the same rate. Equation (20) follows from the average wage equation together with the contract wage equation. We impose  $y = 0$ , so that output is equal to its natural rate in the long run, which implies that  $r = -\theta$ . Equation (21) implies that if inflation is positive in the steady state, the real value of the foreign reserve component of the monetary base equals zero. This follows from the assumption that foreign reserves are kept on the central bank's balance sheet at their initial book value, with capital gains and losses affecting the central bank's net worth. In a non-inflationary steady state, with  $\Delta f = 0$ , the long run level of  $F/W$  is determined by the model's transitional dynamics, so that there is hysteresis.

### *State Space Form*

The model, after linearizing around its steady state, can be reduced analytically to a set of equations which can be written in matrix notation as:

$$\hat{A} \hat{x}_{t+1} = \hat{B} x_t + \hat{C} y_t + \hat{D} u_t + \hat{E} \hat{u}_{t+1}, \quad (22)$$

$$F y_t = \hat{G} x_{t+1} + \hat{H} x_t + \hat{J} u_t + \hat{K} \hat{u}_{t+1}. \quad (23)$$

The  $x_t$  variables are endogenous state variables, and the  $u_t$  variables are government policy variables. In the case of no indexation lag, the variables of vectors are

defined as follows:

$$\hat{x}'_{t+1} = \left[ r'_t, (d_t-w_t), (f_t-w_t), g_t, (e_t-w_t), (\hat{v}_{t+1}-\hat{w}_{t+1}) \right], \quad (24)$$

$$u'_t = \left[ (z_t-z_{t-1}), \eta_t, (e_t-e_{t-1}) \right], \quad (25)$$

$$y'_t = \left[ y_t, i_t, (\hat{e}_{t+1}-\hat{w}_{t+1}), (\hat{p}_{t+1}-p_t) \right]. \quad (26)$$

The government's control variables are the wage control variable, which is appears as a rate of change due to the transformation in equation (4), the rate of change of fiscal spending, and the rate of change of the exchange rate. We assume a crawling peg regime in which the overall money supply is endogenously determined in the long run. Kiguel and Liviatan (1992) develop a similar model in which the rate of crawl of the exchange rate is exogenous rather than optimally determined to minimize a loss function as in our model. In the case of a one-period indexation lag, the vector of endogenous state variables is redefined as follows:

$$\hat{x}'_{t+1} = \left[ (z_t-z_{t-1}), \eta_t, (e_t-e_{t-1}), r'_t, (d_t-w_t), (f_t-w_t), g_t, (e_t-w_t), (e_{t-1}-w_{t-1}), (\hat{v}_{t+1}-\hat{w}_{t+1}) \right]. \quad (24a)$$

The addition of the current levels of the policy variables to the state vector results from the dependence of the system on the first lag of the policy variables. With a two-period indexation lag, the system depends on the second lag of the policy variables. The state vector is augmented with the introduction of the first lag of the policy variables and the second lag of the real exchange rate,  $(e_{t-2}-w_{t-2})$ . The coefficients of the various matrices are given in Appendix A at the end of the text. This system is reduced numerically to its "quasi-state-space" form:

$$\hat{x}_{t+1} = A x_t + B u_t + C \hat{u}_{t+1}, \quad (27)$$

$$y_t = D x_t + E u_t + F \hat{u}_{t+1}. \quad (28)$$

We refer to this system of equations as a quasi-state-space form because of the presence of future expected control variables in the structural equations. The system always has one jump state variable, the contract wage deflated by the average wage, independent of the number of lags in wage indexation.

This is the form of the model which is used to derive optimal government policies. The optimization problem is more complicated than for a standard state space form because of the presence of future expected control variables in the equations of motion of the system. The appendix describes our solution methodology in detail.

### 3. The Optimal Policy Game

Given a loss function, initial conditions for the predetermined state variables, and a specification of the policy game, optimal time paths of the policy variables can be calculated. Following Miller and Salmon (1986), Cohen and Michel (1988), and others, we specify a quadratic loss function. The policy maker is assumed to care about the squared deviations of output, of the rate of inflation, of government expenditures, of the balance of trade and of the level of foreign exchange reserves. Adding foreign exchange reserves to the loss function is designed to capture the concern that the policy makers may have for avoiding balance of payments crises, the possibility of which is not explicitly modeled in the paper.

The loss function is given by:

$$\begin{aligned} \mathcal{L} = .5 \sum_{s=0}^{\infty} \delta^s \{ & q_1 y_{t+s}^2 + q_2 \Delta p_{t+s}^2 + q_3 i_{t+s} + q_4 (e_{t+s} - w_{t+s}) + q_5 r_{t+s}^2 \\ & + q_6 (d_{t+s} - w_{t+s}) + q_7 (f_{t+s} - w_{t+s}) + q_8 g_{t+s}^2 + r_1 \Delta z_{t+s}^2 + r_2 \eta_{t+s}^2 + r_3 \Delta e_{t+s}^2 \}. \end{aligned} \quad (29)$$

where the  $q_i$  and the  $r_i$  are positive constants. The loss function also penalizes deviations of the policy instruments themselves. The direct presence of the instruments in the loss function serves a technical purpose: it helps avoid the instrument instability problem in which the controls fluctuate wildly along the optimal path to the new steady state. In addition, micro-theoretic justifications (which are not explicitly part of the model) can be invoked to justify their presence. High rates of downward crawl increase the distortionary effects of the inflation tax. Wage and price controls can have negative effects on wealth and income redistribution. Their redistributive effects can also be a source of political instability.

The policy makers are assumed to inherit both a situation of high inflation and a credibility problem. It is well known that policy making in a rational expectations environment gives rise to problems of time inconsistency. Unless policy makers can commit to their policy, they will not be believed by the public. Optimal time-consistent policies can be derived using dynamic programming techniques. Such policies are time-consistent by construction.

The value of the loss function under the optimal policy rule will depend on the initial deviations of the predetermined state variables from their optimal levels. Finding a consistent set of initial conditions for an inflationary steady state involves solving the non-linearized version of the model numerically. We do this for a given level of government expenditures. Equation (21) shows that the foreign component of the money base tends towards zero in an inflationary steady state. We therefore set this foreign component equal to a small value, and the domestic component is set equal to the difference between the monetary base and its foreign component. We assume that when the stabilization program is implemented, the economy is in an inflationary steady state with a rate of crawl of 100%.

#### 4. Simulation Results

The parameter values used to simulate the model are given in Table 1. Table 1a gives the values of the model's structural parameters, and Table 2 gives the values of the loss function parameters for the base case scenario. It is difficult to choose these values on an a priori basis. Instead, we simulated the dynamic response of the model for a range of values and report the results of three representative policy scenarios. Aside from the base case, we consider a case where wage and price controls are particularly costly to apply, with  $r_1$  set equal to 10,000, and a case where it is particularly costly to bring fiscal spending under control, with  $r_2$  set equal to 10,000.

Figure 1 illustrates the time paths of various endogenous variables during the stabilization program for the base-case parameter values. The general shape of the anti-inflation program is the same as in the continuous-time model of Ambler and Cardia (1992).<sup>3</sup> Fiscal spending is rapidly brought under control. The rate of crawl of the exchange rate initially exceeds the rate of crawl in the pre-stabilization period before being gradually reduced towards zero. The high initial rate of depreciation leads to a rapid real exchange rate depreciation: the real exchange rate is initially well below its final steady state level. Direct wage controls are used at the beginning of the stabilization program to reduce the rate of domestic wage growth and domestic inflation and to help achieve the real exchange rate depreciation. After an initial recession, output approaches its final steady state level from above.

Figure 2 illustrates the time paths of the same endogenous variables when we increase the penalty against the use of direct wage controls, with  $r_1 = 10,000$ . The time paths of the variables are qualitatively similar to those in Figure 1. As expected, there is a sharp reduction in the use of direct wage controls. Inflation

---

<sup>3</sup> There, we report results for partial wage indexation ( $\theta = 0.5$ ). In that case, and as we note below, wage-price controls are used to a lesser extent.



becomes more persistent, and is higher initially. In addition, indexation lags now lead systematically to an increase in the persistence of inflation, although the effect is quite small. After the first three periods, the time path of inflation with a two-period indexation lag is above that of inflation with a one-period indexation lag, which is in turn above that of inflation with no indexation lag. Wage controls are used less initially when there are indexation lags, but are more persistent. Output now approaches its final steady state level from below. With no indexation lags, output is below the steady state level for the whole of the stabilization program. With indexation lags, there is an initial boom caused by the depreciation of the real exchange rate, followed by a recession that is deeper and more persistent than with no indexation lags. The recession induced by the stabilization program is most persistent with a two-period indexation lag.

Table 2 compares the previous two scenarios by looking at the discounted sum of squared deviations of three key variables: output, inflation and the wage control variable itself. The table indicates clearly that inflation is more persistent when wage controls are costly to implement. It also clearly indicates that, with costly wage controls, indexation lags increase the persistence of inflation, but the increase in inflation persistence is not very large. The sum of squared output deviations is actually larger when wage controls are less costly to use. This reflects the fact that the loss function is symmetric around zero. The initial boom induced by the stabilization program is greater when wage controls are less costly (as noted in the previous paragraph, there is no boom at all with costly wage controls and  $k = 0$ ), and the loss function penalizes this as much as the recession which occurs at a later stage in the optimal stabilization program when wage controls are costly. With costly wage controls, the sum of squared deviations of output changes very little as the length of indexation lag is varied.

Finally, Figure 3 illustrates the response of the economy in a scenario where it is particularly costly to bring fiscal spending. The time paths of inflation are almost identical to those in the base case scenario. Output deviations are more persistent, but they are more positive. This results from the fact that output is aggregate demand determined and because fiscal spending is higher during the stabilization program when it is more costly to bring it down.

## 5. Conclusions

We built a discrete-time simulation model of a small open economy with staggered contracts and indexation which is subject to discrete lags. We analyzed the optimal anti-inflation policy when the economy begins in a high-inflation steady state. Our results show that, for the simulated model, indexation lags make little difference to the basic shape of the optimal anti-inflation policy. The paths of the control variables are very similar with an indexation lag of zero, one or two periods. Indexation lags can make a difference to the shape of the response of output during an optimal stabilization program, but the discounted sum of squared deviations of output is not greatly affected. However, wage controls are useful for bringing down inflation faster. When direct wage controls are costly to use, inflation does have slightly more inertia, and indexation lags make inflation still more persistent. When wage controls are costly, the anti-inflation program is not more costly in terms of the squared deviations of output, but there is a recession during the later stages of the optimal stabilization program. This recession is deeper and more persistent with indexation lags than without them.

The conclusions we reach may be model specific. Domestic prices in our model are determined by a markup over wage costs, and labor demand and production are not modeled explicitly. There is room to extend the model to incorporate a richer analysis of the

labor market, the joint determination of wage indexation and contract length (along the lines of Gray, 1978), optimal price setting by firms (along the lines of Bonomo and Garcia, 1994), and other features that may or may not affect the importance of the heterodox aspects of stabilization policies. This is left for future research.

### References

- Ambler, S. and E. Cardia, 1992, Optimal Anti-Inflation Programs in Semi-Industrialized Economies: Orthodox versus Heterodox Policies, *Journal of Development Economics* 38, pp.41-61
- Ambler, S. and A. Paquet, 1994a, Fiscal Spending Shocks, Endogenous Government Spending and Real Business Cycles, *Journal of Economic Dynamics and Control* forthcoming
- Ambler, S. and A. Paquet, 1994b, Recursive Methods for Computing Equilibria of General Equilibrium Dynamic Stackelberg Games, cahier de recherche 21, Centre de recherche sur l'emploi et les fluctuations économiques, UQAM
- Blanchard, O. and C. Kahn, 1980, The solution of linear difference models under rational expectations, *Econometrica* 48, pp.1305-1311
- Bonomo, Marco and René Garcia (1994), "Indexation, Staggering and Disinflation", *Journal of Development Economics* 43, 39-58
- Calvo, G., 1983, Staggered Contracts and Exchange Rate Policy, in: J. Frenkel, ed., *Exchange Rates and International Economics* (University of Chicago Press, Chicago)
- Cohen, D. and P. Michel, 1988, How Should Control Theory be Used to Calculate a Time-Consistent Government Policy?, *Review of Economic Studies* 60, pp.265-274
- Cukierman, A., 1988, The End of the High Israeli Inflation: An Experiment in Heterodox Stabilization, in: M. Bruno et al, eds., *Inflation Stabilization* (MIT Press, Cambridge, MA)
- Dornbusch R., 1986, Inflation, Exchange Rates, and Stabilization, *Essays in*

- International Finance no. 165* (Princeton University Press, Princeton)
- Frenkel, J. and C. Rodriguez, 1982, Exchange Rate Dynamics and the Overshooting Hypothesis, *International Monetary Fund Staff Papers* 29, pp.674-688
- Gray, Jo Anna (1978), "On Indexation and Contract Length", *Journal of Political Economy* 86, 1-18
- Hansen, G. and E. Prescott, 1994, Recursive Methods for Computing Equilibria of Business Cycle Models, forthcoming in: T.F. Cooley, ed., *Frontiers of Business Cycle Research* (Princeton University Press, Princeton, NJ)
- Kiguel, Miguel A. and Nissan Liviatan (1992), "The Business Cycle Associated with Exchange Rate-Based Stabilizations", *The World Bank Economic Review* 6, 279-305
- Miller, M. and M. Salmon, 1985, Dynamic Games and the Time Inconsistency of Optimal Policy in Open Economies, *Economic Journal* 95, pp.124-137
- Simonsen, M.H. 1983, Indexation: Current Theory and the Brazilian Experience, in: R. Dornbusch and M.H. Simonsen, eds., *Inflation, Debt and Indexation* (MIT Press, Cambridge, MA)
- Taylor, J., 1979, Staggered Wage Setting in a Macroeconomic Model, *American Economic Review* 69, pp.108-113
- Taylor, J., 1980, Aggregate Dynamics and Staggered Contracts, *Journal of Political Economy* 88, pp.1-23

#### Appendix A: State Space Form

With  $k = 0$ :

$$\bar{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & -\pi_2 & 0 \\ 0 & -\phi_2 & 0 & \phi_1 & \phi_2 - \phi_1 & 0 \\ -\psi_2 & 0 & \psi_1 & 0 & -\psi_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d(1-\alpha\theta) & 0 \\ 0 & 0 & 0 & 0 & -d(1+\alpha\gamma) & 0 \end{bmatrix} \quad (A1)$$

$$\hat{B} = \begin{bmatrix} 1+i & 0 & 0 & 0 & 0 & 0 \\ \phi_3 & -\phi_2 & 0 & 0 & \phi_2 & 0 \\ -\psi_2 & 0 & \psi_1 & 0 & -\psi_1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -d(1-\alpha\theta) & 1-d \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (\text{A2})$$

$$\hat{C} = \begin{bmatrix} -\pi_3 & \pi_1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ (1-d)\beta & 0 & -d(1+\alpha\gamma) \end{bmatrix} \quad (\text{A3})$$

$$\hat{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \phi_2 \\ 0 & 0 & -\psi_1 \\ 0 & 1 & 0 \\ d & 0 & -d(1-\theta) \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{A4})$$

$$\hat{E} = \begin{bmatrix} 0 & 0 & -\pi_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d(1+\gamma) \end{bmatrix} \quad (\text{A5})$$

$$\hat{F} = \begin{bmatrix} 1 & \gamma_2 & \alpha\gamma_2 \\ 1 & -\lambda & 0 \\ 0 & 0 & -d(1-\theta\alpha) \end{bmatrix} \quad (\text{A6})$$

$$\hat{G} = \begin{bmatrix} 0 & 0 & 0 & \gamma_3 & \gamma_1 + \alpha\gamma_2 & 0 \\ 0 & \omega & 1-\omega & 0 & -(1-\alpha) & 0 \\ 0 & 0 & 0 & 0 & -d(1-\alpha\theta) & 1-d \end{bmatrix} \quad (\text{A7})$$

$$\hat{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A8})$$

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{A9})$$

$$K = \begin{bmatrix} 0 & 0 & r_2 \\ 0 & 0 & 0 \\ d & 0 & -d(1-\theta) \end{bmatrix} \quad (\text{A10})$$

The parameters that are not explicitly defined in the text arise from the linearization of the model around its steady state. They depend on the steady-state levels of the endogenous variables of the model. We have:

$$\phi_1 \equiv G(E/W), \phi_2 \equiv (D/W)/(E/W), \phi_3 \equiv i^*/R^*, \quad (\text{A11})$$

$$\psi_1 \equiv (F/W)/(E/W), \psi_2 \equiv R^*. \quad (\text{A12})$$

### Appendix B: Model Solution

In this appendix, we discuss in detail the problem of calculating the optimal time-consistent government policy in a discrete-time linear rational expectations model in which the expected future levels of policy variables enter the structural model explicitly. Our method can be considered an extension of the techniques described in Cohen and Michel (1988) and Miller and Salmon (1985). Ambler and Paquet (1994a, 1994b) apply a similar methodology to a model with optimizing private agents. We assume that the model can be reduced analytically or numerically to the following "quasi state space form":

$$\hat{x}_{t+1} = A x_t + B u_t + C \hat{u}_{t+1}, \quad (\text{B1})$$

$$y_t = D x_t + E u_t + F \hat{u}_{t+1}, \quad (\text{B2})$$

where  $x_t$  is a vector of state variables which can be divided into a subvector  $x_1$  of  $n_1$

predetermined variables and a subvector  $x_2$  of  $n_2$  jump variables,  $y_t$  is a vector of non-dynamic endogenous variables, and  $u_t$  is a vector of control variables. We assume that the policy maker's objective is to minimize a quadratic loss function which can be written in terms of endogenous variables, states, future states, and policy variables, in the following manner:

$$Q_t = .5 E_t \sum_{i=0}^{\infty} \delta^i \{ [x_{t+i}, x_{t+i}, y_{t+i}, u_{t+i}]' Q [x_{t+i}, x_{t+i}, y_{t+i}, u_{t+i}] \}. \quad (B3)$$

where  $Q$  is a positive definite matrix of constants, and where the vectors inside square brackets are understood to be stacked vertically.

In order to use dynamic programming techniques, the loss function must be expressed in terms of predetermined states and policy variables only. The direct dependence of both the states and the endogenous variables on future expected policy variables makes it difficult to eliminate these variables from the loss function in order to arrive at a loss function only in terms of states and instruments. In order to get around this problem, we use our knowledge of the general form of the solution to the problem. The control variables will depend linearly on the predetermined state variables. The feedback rule is of the form:

$$u_t = \theta x_t, \quad (B4)$$

where the matrix  $\theta$  is a function of the matrix of eigenvectors of the state space matrix of the optimally controlled dynamic system. Leading this expression by one period and taking expectations dated at time  $t$  allows us to for the true state space form of the model for a given arbitrary set of values of the elements of the  $\theta$  matrix:

$$\hat{u}_{t+1} = \theta x_{t+1} = [\theta|0] \hat{x}_{t+1}$$

$$\hat{x}_{t+1} = (I - C [\theta | 0])^{-1} (A x_t + B u_t) = A1 x_t + B1 u_t. \quad (B5)$$

In order to apply dynamic programming methods, the model has to be written with recursive dynamics, that is, by eliminating the forward-looking or jump variables from the state vector. This can be done by first using the feedback rule (B4) to eliminate the control variables from (B5):

$$\hat{x}_{t+1} = A1 x_t + B1 \theta x1_t = A2 x_t. \quad (B6)$$

This is a system of homogeneous first order difference equations. If it satisfies the standard properties of saddlepoint stability (as many stable roots as predetermined variables), it can be solved as a special case of the Blanchard and Kahn (1980) forward-backward algorithm. This solution gives a linear relationship between the predetermined state variables and the jump state variables of the form:

$$x2_t = \phi x1_t. \quad (B7)$$

Using this linear relationship, we can substitute back into equation (B5), which yields, using the first  $n_2$  equations of (B5), a system of the form:

$$x1_{t+1} = A3 x1_t + B3 u_t. \quad (B8)$$

This transition equation is of the form used by Hansen and Prescott (1994) to derive an optimal feedback rule for  $u_t$  in terms of predetermined state variables. In order to use this algorithm, the loss function must also be expressed in terms of predetermined state variables and controls only. This can be done using (B2), (B4) and (B7) to eliminate the non-state endogenous variables, the jump state variables, and the future expected policy variables from the objective function, giving a loss function of the form:



$$\Omega_i' = .5 E_i \sum_{i=0}^{\infty} \delta^i \left\{ [x_{1+i}, u_{1+i}]' Q^* [x_{1+i}, u_{1+i}] \right\}, \quad (B9)$$

with a suitably redefined matrix  $Q^*$ . Since the objective function is quadratic and the state transition equation (B8) is linear, the Hansen-Prescott algorithm can be applied without modification to derive the optimal feedback rule.

The following pseudo-algorithm was used to derive the feedback rule used to simulate the model in the paper.

- Initialize the feedback rule  $\theta_0$ .
- Set  $i = 0$ .
- Do until convergence.
  - Use  $\theta_i$  to derive A1 and B1 from equation (B5).
  - Derive A2 from equation (B6).
  - Derive A2 from equation (B6).
  - Use the Blanchard-Kahn algorithm to derive  $\phi$  in (B7).
  - Use these results to derive the transition equation (B8) and the loss function (B9).
  - Apply the Hansen-Prescott algorithm to calculate an updated feedback rule  $\theta_{i+1}$ .
  - Check to see whether or not  $\theta_i$  is sufficiently close to  $\theta_{i+1}$ .
  - Increment  $i$  by one.
- End do.

.....

TABLE I  
Base Case

Table 1A

Model Parameter Values

Parameter	Value
$\alpha$	0.50
$\beta$	0.10
$\theta$	1.00
$\gamma$	-1.00
$\tau_1$	0.20
$\tau_2$	1.00
$\tau_3$	0.23
$\lambda$	1.50
$\omega$	0.50
$\pi_1$	20.00
$\pi_2$	0.10
$\pi_3$	4.00
$i^*$	0.02
$\delta$	0.96

Table 1B

Loss Function Parameter Values

Parameter	Value
$q_1$	10.0
$q_2$	10.0
$q_3$	10.0
$q_4$	3.0
$q_5$	3.0
$q_6$	3.0
$q_7$	3.0
$q_8$	3.0
$r_1$	100.0
$r_2$	100.0
$r_3$	100.0

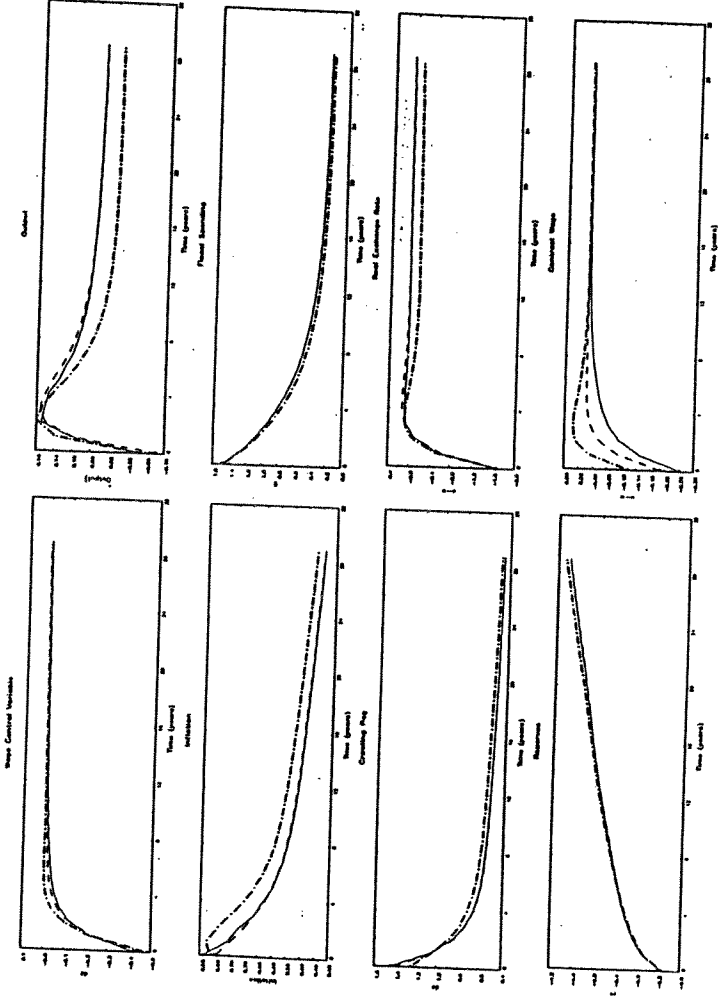
TABLE 2

Discounted Sum of Squared Deviations

Variable:	y	$\Delta p$	$\Delta z$
Scenario:			
Base case (Moderate use of wage controls):			
0-period lag	0.2137	7.5549	0.1049
1-period lag	0.2079	6.5109	0.1665
2-period lag	0.1831	6.5405	0.1519
Heavy penalty against use of wage controls:			
0-period lag	0.0874	7.7269	0.0010
1-period lag	0.0628	8.2010	0.0009
2-period lag	0.0956	8.3672	0.0008

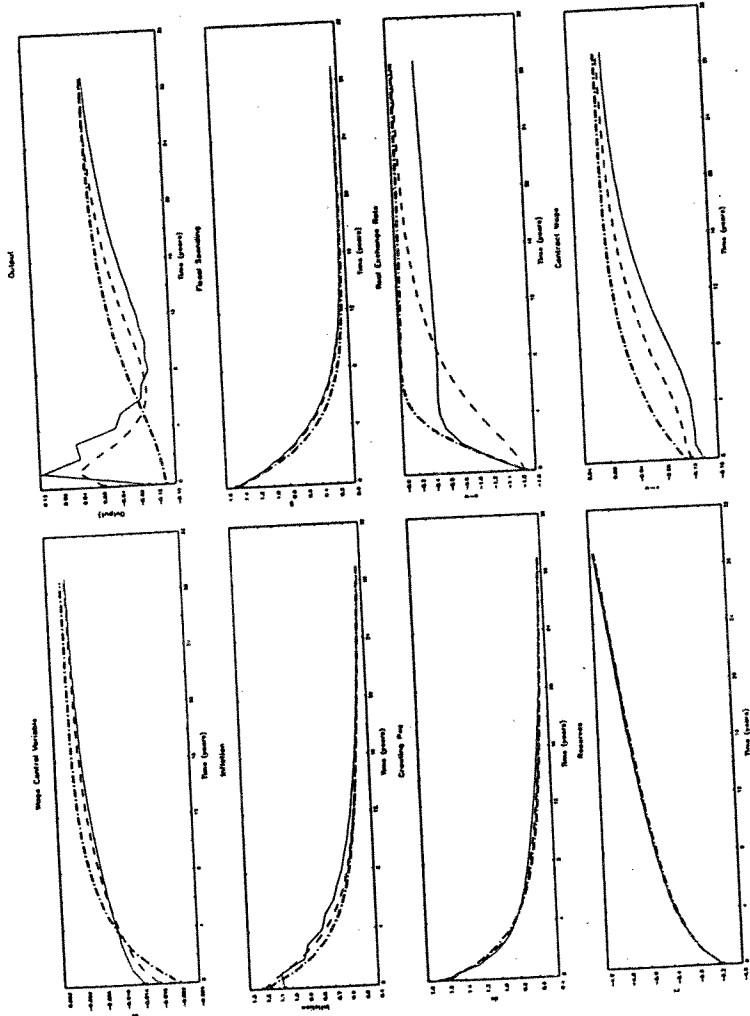
The model is simulated over 30 periods, and a discount rate of  $\delta = 0.96$  is used to discount the squared deviations of the variables.

Figure 1  
Base Case



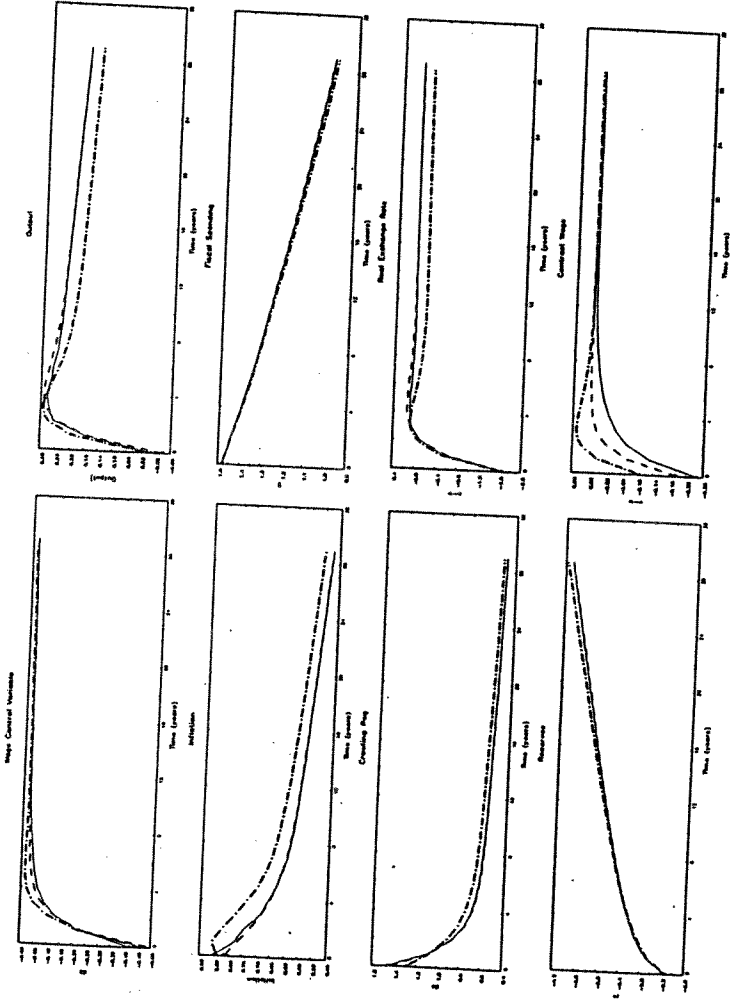
Legend:  $k = 0$  (dots and dashes);  $k = 1$  (dashes);  $k = 2$  (solid)

Figure 2  
Costly Wage Controls



Legend:  $k = 0$  (dots and dashes);  $k = 1$  (dashes);  $k = 2$  (solid)  
 $r_1 = 10,000$

Figure 3  
Government Spending Costly to Change



Legend:  $k = 0$  (dots and dashed);  $k = 1$  (dashed);  $k = 2$  (solid)  
 $F_2 = 10,000$





Université de Montréal  
Département de sciences économiques  
Centre de documentation  
C.P. 6128, succursale Centre-ville  
Montréal (Québec)  
H3C 3J7

Cahiers de recherche (Discussion papers)  
1993 à aujourd'hui (1993 to date)

*Si vous désirez obtenir un exemplaire, vous n'avez qu'à faire parvenir votre demande et votre paiement (5 \$ l'unité) à l'adresse ci-haut mentionnée. / To obtain a copy (\$ 5 each), please send your request and prepayment to the above-mentioned address.*

- 9401 : Mercenier, Jean et Bernardin Akitoby, "On Intertemporal General-Equilibrium Reallocation Effects of Europe's Move to a Single Market", janvier 1994, 41 pages.
- 9402 : Gauthier, Céline et Michel Poitevin, "Using Ex Ante Payments in Self-Enforcing Risk-Sharing Contracts", février 1994, 38 pages.
- 9403 : Ghysels, Eric et Joanna Jasiak, "Stochastic Volatility and Time Deformation : an Application of Trading Volume and Leverage Effects", février 1994, 37 pages.
- 9404 : Dagenais, Marcel G. et Denyse L. Dagenais, "GMM Estimators for Linear Regression Models with Errors in the Variables", avril 1994, 33 pages.
- 9405 : Bronsard, C., Fabienne Rosenwald et Lise Salvas-Bronsard, "Evidence on Corporate Private Debt Finance and the Term Structure of Interest Rates", avril 1994, 42 pages.
- 9406 : Dinardo, John, Nicole M. Fortin et Thomas Lemieux, "Labor Market Institutions and the Distribution of Wages, 1973-1992 : A Semiparametric Approach", avril 1994, 73 pages.
- 9407 : Campbell, Bryan et Jean-Marie Dufour, "Exact Nonparametric Tests of Orthogonality and Random Walk in the Presence of a Drift Parameter", avril 1994, 32 pages.
- 9408 : Bollerslev, Tim et Eric Ghysels, "Periodic Autoregressive Conditional Heteroskedasticity", mai 1994, 29 pages.
- 9409 : Cardia, Emanuela, "The Effects of Government Financial Policies : Can We Assume Ricardian Equivalence?", mai 1994, 42 pages.

- 9410 : Kollmann, Robert, "Hidden Unemployment : A Search Theoretic Interpretation", mai 1994, 9 pages.
- 9411 : Kollmann, Robert, "The Correlation of Productivity Growth Across Regions and Industries in the US", juin 1994, 14 pages.
- 9412 : Gaudry, Marc, Benedikt Mandel et Werner Rothengatter, "Introducing Spatial Competition through an Autoregressive Contiguous Distributed (AR-C-D) Process in Intercity Generation-Distribution Models within a Quasi-Direct Format (QDF)", juin 1994, 64 pages.
- 9413 : Gaudry, Marc et Alexandre Le Leyzour, "Improving a Fragile Linear Logit Model Specified for High Speed Rail Demand Analysis in the Quebec-Windsor Corridor of Canada", août 1994, 39 pages.
- 9414 : Lewis, Tracy et Michel Poitevin, "Disclosure of Information in Regulatory Proceedings", juillet 1994, 38 pages.
- 9415 : Ambler, Steve, Emanuela Cardia et Jeannine Farazli, "Export Promotion and Growth", août 1994, 41 pages.
- 9416 : Ghysels, Eric et Haldun Sarlan, "On the Analysis of Business Cycles Through the Spectrum of Chronologies", août 1994, 37 pages.
- 9417 : Martel, Jocelyn et Timothy C.G. Fisher, "The Creditors' Financial Reorganization Decision : New Evidence from Canadian Data", août 1994, 21 pages.
- 9418 : Cannings, Kathy, Claude Montmarquette et Sophie Mahseredjian, "Entrance Quotas and Admission to Medical Schools : A Sequential Probit Model", septembre 1994, 26 pages.
- 9419 : Cannings, Kathy, Claude Montmarquette et Sophie Mahseredjian, "Major Choices : Undergraduate Concentrations and the Probability of Graduation", septembre 1994, 26 pages.
- 9420 : Nabeya, Seiji et Pierre Perron, "Approximations to Some Exact Distributions in the First Order Autoregressive Model with Dependent Errors", septembre 1994, 40 pages.
- 9421 : Perron, Pierre, "Further Evidence on Breaking Trend Functions in Macroeconomic Variables", octobre 1994, 50 pages.

- 9422 : Vogelsang, Timothy J. et Pierre Perron, "Additional Tests for a Unit Root Allowing for a Break in the Trend Function at an Unknown Time", novembre 1994, 57 pages.
- 9423 : Ng, Serena et Pierre Perron, "Unit Root Tests in ARMA Models with Data Dependent Methods for the Selection of the Truncation Lag", décembre 1994, 41 pages.
- 9424 : Perron, Pierre, "The Adequacy of Asymptotic Approximations in the Near-Integrated Autoregressive Model with Dependent Errors", décembre 1994, 37 pages.
- 9425 : Ghysels, Eric et Pierre Perron, "The Effect of Linear Filters on Dynamic Time Series with Structural Change", décembre 1994, 35 pages.
- 9426 : Boyer, Marcel, Jean-Jacques Laffont, Philippe Mahenc et Michel Moreaux, "Sequential Location Equilibria Under Incomplete Information", décembre 1994, 38 pages.
- 9427 : Perron, Pierre et Serena NG, "Useful Modifications to Some Unit Root Tests with Dependent Errors and their Local Asymptotic Properties", décembre 1994, 41 pages.
- 9428 : Garcia, René et Pierre Perron, "An Analysis of the Real Interest Rate Under Regime Shifts", décembre 1994, 42 pages.
- 9501 : Boyer, Marcel et Jean-Jacques Laffont, "Environmental Risks and Bank Liability", janvier 1995, 46 pages.
- 9502 : Margolis, David. N., "Firm Heterogeneity and Worker Self-Selection Bias Estimated Returns to Seniority", décembre 1994, 29 pages.
- 9503 : Abowd, John M., Francis Kramarz et David N. Margolis, "High-Wage Workers and High-Wage Firms", janvier 1995, 73 pages.
- 9504 : Cardia, Emanuela et Steve Ambler, "Indexation Lags and Heterodox Stabilization Programs", janvier 1995, 29 pages.

