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STOCHASTIC VOLATILITY AND TIME DEFORMATION : AN APPLICATION OF TRADING VOLUME AND LEVERAGE EFFECTS

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RÉSUMÉ

Dans ce papier, nous étudions un modèle de volatilité stochastique avec déformation du temps. Le processus de volatilité est supposé évoluer dans un temps opérationnel déterminé par l'arrivée de l'information sur le marché boursier. Cette arrivée étant non observable, nous utilisons le volume de transactions et les prix passés comme approximation de la déformation du temps. Nous admettons également un effet de levier par l'asymétrie de la déformation du temps par rapport aux prix passés.

La spécification économétrique est basée sur le modèle espace d'états pour les modèles de volatilité stochastique proposés notamment par Harvey, Ruiz et Shephard (1992) et combinée avec le filtre de Kalman pour processus avec déformation du temps étudié par Stock (1988). Nous utilisons des données quotidiennes de la bourse de New York, S&P 500 et volume de transactions, pour estimer le modèle et trouvons une forte évidence pour la déformation du temps. Notamment, lorsque le volume monte, la volatilité devient plus erratique (moins persistante et sujette à des chocs plus grands). Ce résultat s'obtient aussi suite à une baisse des prix. Par contre, une hausse des prix rend la volatilité plus persistante. Nous présentons également plusieurs extensions du modèle de base.

Mots clés : volatilité stochastique, filtre de Kalman, processus de diffusion avec temps local, mouvements de valeurs boursières, volume de transactions.

ABSTRACT

In this paper, we study stochastic volatility models with time deformation. In our setup, the latent process of stochastic volatility evolves in an operational time which differs from calendar time. The time deformation can be determined by past volume of trade, past price changes, possibly with an asymmetric leverage effect, and other variables setting the pace of information arrival.

The econometric specification exploits the state-space approach for stochastic volatility models proposed by Harvey, Ruiz and Shephard (1992) and uses the Kalman filter framework proposed by Stock (1988) to handle continuous time processes with time deformation. Daily data on the price changes and volume of trade of the S&P 500 over a 1950-1987 sample are investigated. Strong evidence for time deformation is found, and its impact on the behavior of price series is analyzed. We find that increases in volume accelerate operational time, resulting in volatility being less persistent and subject to shocks with a higher innovation variance. Downward price movements have similar effects, while upward price movements increase persistence in volatility and decrease the dispersion of shocks by slowing down the operational time clock. We present the basic model as well as several extensions.

Key words : stochastic volatility, Kalman filter, diffusion processes and local time, stock price movements, volume of trade.

1. INTRODUCTION

Stochastic processes used in finance are most often assumed to be generated by a first-order stochastic differential equation of the form :

$$(1.1) \qquad dX(s) = a(s, X(s), \theta)\, d\tau + b(s, X(s), \theta)\, dM(s)$$

where $X(s)$ is a n-dimensional process adapted to a filtered probability space (Ω, F, P) evolving in some *operational* time. The process is parameterized by $\theta \in \mathbb{R}^P$ with $dM(s)$ a m-dimensional semimartingale process, while $a(s, X(s), \theta)$ and $b(s, X(s), \theta)$ are both bounded predictable processes of dimensions n and $n \times m$ respectively. Equations like (1.1) have been adopted to describe security, bond and derivative prices as well as information flows, mortgage values, inventories and other state variables such as technology. Depending on the context, the operational time s is most often assumed to be calendar time, denoted $s = t$, or sometimes market time when trading dates are explicitly modeled [see, e.g., Melino (1991) or Sawyer (1993) for discussion]. Whenever the assumed operational time scale s differs from t, there is so-called time deformation. A very simple example of the separation of operational time and calendar time can be found in the early works of Clark (1973) and Tauchen and Pitts (1983) on trading volume.[1] Madan and Seneta (1990), and Madan and Milne (1991) introduced a Brownian motion evaluated at random time changes governed by independent gamma increments as an alternative martingale process for the uncertainty driving stock market returns. Carr and Jarrow (1990) show a connection between local time and the Black-Scholes option-pricing model. More recently, Detemple and Murthy (1993) have used local time scales to characterize an intertemporal asset pricing with heterogeneous beliefs in which assets are held in equilibrium.

In this paper, time deformation is used to model stochastic volatility. Several authors, including Chesney and Scott (1989), Clark (1973), Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Tauchen and Pitts (1983) and Wiggins (1987), proposed and discussed stochastic volatility (henceforth SV) models which attempted to describe the joint process of a security price, say $y(t)$,

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1 There is now an extensive literature on trading volume, including both theoretical and empirical papers. See, for instance, Anderson (1992), Easley and O'Hara (1992), Foster and Viswanathan (1993a,b), Gallant, Rossi and Tauchen (1992), Hausman and Lo (1991), Huffman (1987), Karpoff (1987), Lamoureux and Lastrapes (1990, 1993), Wang (1993), among others.

and its conditional volatility $\sigma(t)$, i.e., $X(t) \equiv (y(t), \sigma(t))'$ in calendar time t. A SV model typically takes the form :

$$(1.2a) \qquad dy(t) = \mu y(t)\, dt + \sigma(t) y(t) dw_1(t)$$

$$(1.2b) \qquad d\log\sigma(t) = a(b - \sigma(t))dt + c dw_2(t)$$

where $w_1(t)$ and $w_2(t)$ are two standard independent Wiener processes. Our paper does not assume that the volatility process moves continuously and smoothly through calendar time, as is usually assumed and described in (1.2b). It is clear that key variables affecting volatility, like the arrival of information to the market, tend not to evolve continuously and smoothly through time. Therefore, we assume an operational time scale s for the volatility processes, with $s = g(t)$, a mapping between operational and calendar time, such that :[2]

$$(1.3a) \qquad dy(t) = \mu y(t)\, dt + \sigma(g(t))\, y(t)\, d\omega_1(t)$$

$$(1.3b) \qquad d\log\sigma(s) = a(b - \sigma(s))\, ds + c d\omega_2(s)\, .$$

Hence, volatility moves continuously but only when the pace of information arrival and other state variables set the clock of time evolution. The set of equations in (1.3) can be viewed as a special case of (1.1) letting $X(s) \equiv (y(g^{-1}(s)), \sigma(s))'$ or alternatively $X(t) \equiv (y(t), \sigma(g(t)))'$. Several extensions of (1.3) fitting the general structure described in equation (1.1) will be discussed in the paper.

What can be accomplished by letting stochastic volatility evolve in an operational time scale which differs from calendar time? Obviously, the flow of information and other variables which determine the evolution of s are latent. Hence, we must specify the mapping $s = g(t)$ in terms of observable processes. We propose to use past volume of trade and other variables such as past price changes allowing possibly for an asymmetric response to create a leverage effect. Therefore, our setup provides a way of introducing data on trading volume in the specification of stochastic volatility models. Furthermore, it provides a rationale for leverage effects through the specification of asymmetric responses of s to past price changes, i.e., operational time evolves differently in bull and bear markets. It also appears from our empirical results that our specification provides an alternative to a class of processes put forward by Merton (1976a,b) for option pricing, where jumps in the underlying security returns

2 The mapping $s = g(t)$ must satisfy certain regularity conditions which will be discussed later.

are permitted. Merton suggested to include a Poisson jump process to distinguish between the arrival of *normal* information, modeled as a standard lognormal diffusion, and the arrival of *abnormal* information, modeled as a Poisson process. We find that operational time typically moves slowly, but every so often one finds dramatic increases in market speed. In Merton's setup, the information arrival spells are purely exogenous, whereas our approach has the sources of these changes modeled both in a multivariate sense, via the introduction of volume series, and in an endogenous fashion through past price changes. Using daily S&P 500 data and NYSE volume from 1950–1987, we find that increases in volume accelerate operational time, resulting in volatility being higher and less persistent and subject to shocks with a higher innovation variance. Downward price movements have similar effects, while upward price movements increase persistence in volatility and decrease the dispersion of shocks by slowing down the operational time clock.

There are other advantages to time deformation in stochastic volatility models, but before discussing them, we need to elaborate briefly on econometric issues. Estimating SV models represents some stiff challenges for econometricians. Lately, several methods were proposed involving the use of simulated methods of moments, the Kalman filter or Bayesian inference. Recent contributions include Gallant and Tauchen (1992), Gouriéroux, Monfort and Renault (1992), Harvey, Ruiz and Shephard (1992) and Jacquier, Polson and Rossi (1992). The approach suggested by Nelson (1988) and Harvey et al. (1990) relies on the Kalman filter and a state-space representation of the SV model. The resulting estimator is a quasi-maximum likelihood estimator, henceforth QMLE, and therefore has the disadvantage of being asymptotically inefficient. Yet, its main advantage is in terms of numerical computations which are easy to perform. It has been argued, however, notably by Jacquier et al. (1992) that the QMLE algorithm entails important efficiency losses in relevant parts of the parameter space. Following Nelson (1988) and Harvey, Ruiz and Shephard (1993), we also adopt a state-space representation of a stochastic volatility model but use results of Stock (1988) to incorporate the effects of time deformation. This leads to a time-varying state-space model for which we consider the QMLE approach as advocated by Harvey et al. (1993). It appears that the time-deformation time-varying state-space representation considerably improves the fit, particularly of the tail behavior of stock returns. Hence, allowing for time deformation in SV models may reduce the inefficiency of the QMLE approach.

The paper is organized as follows. In section 2, we present the basic model, starting with a brief review of time-deformation models. Econometric issues are dealt with in section 3. Empirical results appear in section 4. Several extensions of the basic model are presented in section 5. Conclusions follow.

2. A TIME DEFORMATION APPROACH TO STOCHASTIC VOLATILITY

In this section, we present the basic model. It will be useful to first provide a short discussion on time deformation, following closely the work of Stock (1988). Then we will move to stochastic volatility models with deformation of time.

Let us consider a continuous time process $\xi(s)$ evolving in an operational time which we shall denote by s. The time deformation model relates the latent process $\xi(s)$ to the observable process Y_t, appearing in calendar time t as follows :

$$(2.1) \qquad Y_t = \xi(g(t)) \qquad\qquad t = 1, \dots, T$$

where Y_t is a discrete n-dimensional variable. The mapping $s = g(t)$ determines the relation between operational time s and calendar time t. Obviously, the mapping g(t) will have to satisfy certain regularity conditions, which will be discussed shortly. Stock considered a stable linear r^{th} order stochastic differential equation :

$$(2.2) \qquad d[D^{r-1}\,\xi(s)] = [A_1 D^{r-1}\xi(s) + \dots + A_{r-1} D\,\xi(s) + A_r\,\xi(s) + X(s;\beta)]ds + d\eta(s)$$

where D is the mean-square differential operator, A_i $n \times n$ real matrices for $i = 1, \dots, r$ and $X(s; \beta)$ is an exogenous variable process, depending on a parameter vector β. The innovation process $\eta(s)$ is Gaussian with zero-mean increments and covariance matrix $E[d\eta(s)\, d\eta(s')'] = \Sigma ds$ for $s = s'$ and 0 otherwise. For the purpose of presentation, we will adopt Stock's linear process specification, while the discussion will be extended to a larger class of processes later in the section. To describe an investor's information, let us consider the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and the nondecreasing family $F = \{\mathcal{F}_t\}_{t=0}^{+\infty}$ of sub-σ-algebras in calendar time. Furthermore, let Z_t be a m-dimensional vector process adapted to the filtration F, i.e., Z_t is $\mathcal{F}_t$-measurable. The increments of the time deformation mapping g will be assumed to be $\mathcal{F}_{t-1}$ measurable via the logistic transformation :

$$\text{(2.3)} \qquad \frac{dg(\tau;Z_{t-1})}{d\tau} \equiv \dot{g}(\tau;\, Z_{t-1}) \equiv \exp(c'Z_{t-1}) \,/\, \left\{\frac{1}{T}\,\Sigma_{t=1}^{T}\exp(c'Z_{t-1})\right\}$$

for $t - 1 \leq \tau < t$. Equation (2.3), setting the speed of change of operational time as a measurable function of calendar time process Z_{t-1}, is complemented with additional identification assumptions :

$$\text{(2.4a)} \qquad 0 < \dot{g}(\tau;\, Z_{t-1}) < \infty$$

$$\text{(2.4b)} \qquad g(0) = 0$$

$$\text{(2.4c)} \qquad \frac{1}{T}\,\Sigma_{t=1}^{T}\, \Delta g(t) = 1\ .$$

These three technical conditions, which will not be discussed at length here as they are covered in detail in Stock (1988), guarantee that the operational time clock progresses in the same direction as calendar time without stops or jumps.[3] Given that $\dot{g}$ is constant between successive calendar time observations via (2.3), its discrete time analogue $\Delta g(t) \equiv g(t) - g(t-1)$ takes the same logistic form appearing in (2.3).

The first-order autoregressive case, i.e., setting $r = 1$ in (2.2) will be of particular value to us and therefore further developed here. The solution in operational time of a first-order linear process can be expressed as :[4]

$$\text{(2.5)} \qquad \xi(s) = e^{A(s-s')}\,\xi(s') + \int_{s'}^{s} e^{A(s-e)}\, d\eta(e)$$

where $s' < s$. To recover the solution in calendar time, we let $s = g(t)$ and $s' = g(t-1)$ and take advantage of equation (2.1), yielding :

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3 Excluding jumps for the time deformation process must not be confused with the presence of jumps in the stock return process, as proposed by Merton (1976a, b). The time deformation will govern the (stochastic) volatility of the return process. Arbitrarily large (yet finite) changes in operational time will make the stock return process extremely volatile through the conditional variance.

4 For notational convenience, we shall drop the index to the matrix A_1, since we will exclusively treat the first-order AR case in the remainder of the section.

$$\text{(2.6a)} \qquad Y_t = e^{A\Delta g(t)}\, Y_{t-1} + v_t \qquad\qquad t = 1, \dots, T$$

$$\text{(2.6b)} \qquad v_t \sim N(0, -\Sigma(1 - e^{2A\Delta g(t)})\, A^{-1}/2)$$

$$\text{(2.6c)} \qquad \Delta g(t) = \exp(c'Z_{t-1}) \,/\, \tfrac{1}{T}\, \{\Sigma_{t=1}^{T} \exp(c'Z_{t-1})\}\ .$$

Hence, the process $\xi(s)$ while linear in operational time becomes a doubly stochastic process in calendar time also featuring conditional heteroskedasticity governed by $\Delta g(t)$.[5]

The brief digression on time deformation facilitates the presentation of the process of main interest, which is a SV model with time deformation. Let $y(t)$ be any price process which, as most often assumed, follows a geometric Brownian motion :

$$\text{(2.7)} \qquad dy(t) = \mu y(t)dt + \sigma(g(t))\, y(t)\, dw(t)$$

where $w(t)$ is a Wiener process. As noted in the introduction, we assume that instantaneous volatility of the process, i.e., $\sigma(\cdot)$, does not evolve in calendar time, but instead has its evolution determined by $g(t)$. The technical restrictions on the mapping $g(\cdot)$ make this a legitimate process.[6] For the moment, we will assume that the instantaneous volatility process obeys :

$$\text{(2.8)} \qquad d\log\sigma(s)^2 = A[\log\sigma(s)^2]\, ds + d\eta(s)\ ,$$

i.e., follows a first-order autoregression in operational time s. Equation (2.8) will be replaced later on by an Ornstein-Uhlenbeck process. The logarithmic transformation allows us to deal with the nonnegativity of the volatility process. Equations (2.7) and (2.8) together with the time deformation mapping $g(t) = s$ form the basic

5 Doubly stochastic processes have been discussed in detail by Tjøstheim (1986). Stability conditions and existence of moments have been studied for cases where $\Delta g(t)$ is Markovian. It may be worth noting at this point that the Z_{t-1} process need not be exogenous. Indeed, Stock (1988) showed that by setting $Z_{t-1} = Y_{t-1}^2$, one obtains an ARCH-like process having the additional feature of a random coefficient model. We will not be concerned with ARCH processes, but instead apply the deformation to the volatility, as noted in the introduction.

6 A formal discussion of the stochastic process theory with time deformation or local time appears, for instance, in Jacod and Shiryayev (1987).

SV model. Suppose now that $\{y_t\}$ represents a discrete time sample of the process in (2.7).[7] A standard Euler approximation to (2.7) yields :

$$\text{(2.9)} \qquad \log y_t = \lambda + \log y_{t-1} + \sigma_t \varepsilon_t \qquad\qquad \varepsilon_t \sim \text{i.i.d. } N(0, 1)\ .$$

Following Harvey, Ruiz, and Shephard (1992), we can rewrite (2.9) as :

$$\text{(2.10)} \qquad \log[\log y_t - \log y_{t-1} - \lambda]^2 = h_t + \log \varepsilon_t^2$$

where $h_t = \log \sigma_t^2$, $E \log \varepsilon_t^2 = -1.27$ and $\text{Var} \log \varepsilon_t^2 = \pi^2/2$. Since $\log \sigma_t^2 \equiv \log \sigma(g(t))^2$, we use the discrete time approximation to (2.8) appearing in (2.6), yielding the complete representation of our model :

$$\text{(2.11a)} \qquad \log[\log y_t - \log y_{t-1} - \lambda]^2 = -1.27 + h_t + \zeta_t$$

$$\text{(2.11b)} \qquad h_t = e^{\Lambda \Delta g(t)}\, h_{t-1} + v_t$$

where $\zeta_t \equiv \log \varepsilon_t^2 + 1.27$ and v_t satisfies (2.6.2). Apart from the parameter λ, whose treatment is discussed, for instance, by Gouriéroux, Monfort and Renault (1992), we obtain a state-space model with time-varying coefficients similar to that obtained by Stock (1988), except for the properties of the ζ_t process which is no longer Gaussian.[8] Consequently, the estimation procedure based on the Kalman filter will result here in a quasi-maximum likelihood estimator, similar to Harvey, Ruiz and Shephard (1992). For convenience of the exposition, we took an AR(1) model in continuous time, i.e., r = 1, in (2.2). The Kalman filter procedure can handle higher order cases by stacking the equation as a r-dimensional VAR(1).

Obviously, the SV model with time deformation can be viewed simply as a model with a doubly stochastic process for h_t, replacing the usual linear or Ornstein-Uhlenbeck processes. Yet, the stochastic variation in the autoregressive coefficient has a very specific interpretation through the specification of the mapping g(t). Let us therefore turn our attention now to a description of the functional form that

7 We use lower case y_t instead of upper case in order to distinguish scalars from vectors.

8 The innovations v_t and ζ_t are assumed i.i.d. Correlation between the two processes would create asymmetries in the conditional variance [see Harvey and Shephard (1993)]. We do not need to assume such a correlation, since the asymmetry will come through the time deformation (as will be discussed later in the text).

will be adopted here. The work by Tauchen and Pitts (1983) and Gallant, Rossi and Tauchen (1992) suggests that Z_{t-1} should include both past volume and price movements. With respect to price movements, we will adopt a functional form which can allow for asymmetries in the time deformation when prices move upward or downward. Such asymmetry allows us to investigate so-called leverage effects in the conditional variance [cfr. Black (1976) and Christie (1982)]. Several recent empirical studies, including Gallant, Rossi and Tauchen (1992), Nelson (1989, 1991), and Pagan and Schwert (1990) indeed suggest asymmetries in the conditional variance function. Finally, we also include a set of predetermined processes to account for nontrading day effects and possibly other periodic patterns discussed, for instance, by Baillie, Bollerslev and Ghysels (1993) (see section 5 for further discussion). This results in the following specification of the logistic function :[9]

$$(2.12) \qquad \exp(c' Z_{t-1}) \equiv \exp(c_d' d_t + c_v v_{t-1} + c_p \Delta \log y_{t-1} + c_\ell \mid \Delta \log y_{t-1} \mid) .$$

The specification of the time deformation function is chosen in light of certain existing stylized facts we would like the model to fit. Other specifications can be chosen, however. The general model we develop holds for any process Z_{t-1}, which is assumed to capture the flow of information. The specification in (2.12) is just one of possibly many, yet is directly related to the existing literature on conditional variance models. Further research may find other series appropriate as well.

It might be useful to describe the stochastic behavior of the process obtained so far. Referring to some of the empirical results, discussed later, we must first observe that coefficient A in (2.11b) is found to be negative. Therefore, when $c_v > 0$, the model predicts that increases in volume make $\Delta g(t)$ increase. This acceleration in operational time results in a decline in $a_t \equiv \exp A\Delta g(t)$ and an increase in σ^2_{vt} defined in (2.6b). These two effects imply that the h_t process becomes more erratic since its persistence declines and it is subject to larger shocks. Thus, trading volume increases are paired with volatility increases, an empirical fact documented via SNP fitting by

9 Note first that the timing of d_t differs from the other processes. Since the variables entering d_t are predetermined, they are measurable with respect to $\mathscr{I}_{t-1}$, and therefore legitimate for setting the pace of operational time changes $\Delta g(t)$. Moreover, it should be observed that c_d is a vector of parameters, since d_t may be multivariate, as will be discussed in further detail in section 5. The volume series v_t has not yet been explicitly specified. Gallant, Rossi and Tauchen (1992) report a strong upward trend in volume. To accommodate this, we will adopt the approach of Gallant, Rossi and Tauchen who detrend the original data with a quadratic trend.

Gallant, Rossi and Tauchen. If we find $c_p < 0$ combined with $c_\ell > 0$, while $|c_\ell| > |c_p|$ to ensure $\Delta g(t) > 0$, then a change in price of the same magnitude but of the opposite sign will result in $\Delta g(t)$ to be smaller with upward price movements and larger with falling prices. Consequently, declining stock prices have an effect of making the volatility process more erratic (i.e., a_t declines and σ^2_{ν} increases), while a positive price move of the same size has an opposite effect : namely, a_t increases and σ^2_{ν} decreases.

So far, we studied a continuous time first-order AR(1) model. Since time deformation makes this process nonlinear in calendar time, there is a certain degree of arbitrariness in choosing the structure process $\xi(s)$. Yet, when no time deformation is present, we may want to mimic standard processes considered in the literature. We therefore extend our analysis in this section to the standard Ornstein-Uhlenbeck (henceforth O-U) process often considered in stochastic volatility models. Nelson (1990) shows that the EGARCH model approximates in discrete time a diffusion model of the type :

(2.13a) $$d[\log y(t)] = \theta\, h(g(t))\, dt + \sqrt{h(g(t))}\, dw_1(t)$$

(2.13b) $$d\, h(s) = \alpha[\beta - h(s)]\, ds$$

where w_1 and w_2 are two standard independent Wiener processes. Obviously, when equation (2.13b) evolves in calendar time instead of operational time, we recover the standard formulation considered by Nelson and others. We now let the stochastic volatility evolve in the operational time clock $s = g(t)$ and solve equation (2.13b) for $s > s'$, yielding :

(2.14) $$h(s) = [1 - e^{-\alpha(s-s')}]\,\beta + h(s')e^{-\alpha(s-s')} + \int_{s'}^{s} e^{-\alpha(s-r)}\, dw_2(r)\,.$$

Let $s = g(t)$ and $s' = g(t-1)$. We then obtain the following discrete time calendar time representation :

(2.15) $$h_t = [1 - e^{A\Delta g(t)}]\,\beta + e^{A\Delta g(t)}\, h_{t-1} + \nu_t$$

where $(-\alpha)$ has been replaced by A to facilitate comparison, while :

(2.16a) $$\nu_t = \int_{g(t-1)}^{g(t)} e^{A(g(t)-s)}\, dw_2(s)\,.$$

From (2.16a), it follows that :

$$(2.16b) \quad \nu_t \sim N(0, -\Sigma(1 - e^{2A\Delta g(t)}) / 2A) .$$

Note that equation (2.15) differs from (2.6a), since we introduced an intercept term which has a time-varying pattern. With A < 0, as noted before, we find from (2.15) that $\Delta g(t) > 0$ results in a *larger* intercept. Hence, increases in volume, for instance, will result in an overall increase in volatility through this additional level effect.

We can modify the specification in (2.15) so that it better fits the Kalman filter that will be used for estimation. Namely, let $\bar{h}_t \equiv h_t - \beta$. Then from (2.15) we have that

$$(2.17) \quad \bar{h}_t = e^{-A\Delta g(t)} \bar{h}_{t-1} + \nu_t$$

so that we recover the AR(1) case, except that h_t is replaced by $\bar{h}_t$. Accordingly, equation (2.11a) is modified and thus becomes

$$(2.18) \quad \log[\log y_t - \log y_{t-1} - \lambda]^2 = -1.27 + \beta + \bar{h}_t + \xi_t .$$

Hence, the difference between the AR(1) and O-U processes is the presence of an intercept in the measurement equation of the time-varying state-space model.

3. ECONOMETRIC ANALYSIS

Estimating SV models is a challenging task for econometricians since the direct computation of the likelihood function is quite complex. Several approaches have been suggested recently to circumvent the difficult task of using standard maximum likelihood theory. We shall adopt one of the recently developed methods, namely a quasi-maximum likelihood procedure based on the state-space representation of the SV model proposed by Harvey, Ruiz and Shephard (1992). We combine their method with the Kalman filter algorithm proposed by Stock (1988) to estimate continuous time processes with time deformation. As our approach is based on a quasi-maximum likelihood procedure, there is room for improvement regarding asymptotic efficiency. Other estimation methods for SV models, such as those proposed by Gallant and Tauchen (1992) and Gouriéroux, Monfort and Renault (1992), which are based on a simulated method-of-moment approach and the Bayesian inference procedure developed by Jacquier, Polson and Rossi (1993), are all computationally more involved. While one can obviously get efficiency gains from using any such methods, we have decided to leave this for further research and have settled for the computationally fairly straightforward Kalman filtering algorithm which we will discuss in the remainder of this section. We devote a first subsection to the transition equation of the state-space model. A second subsection deals with the measurement equation.

3.1 Transition equation

In operational time s, the r^{th}-order linear differential equation representing a n-dimentional O-U process can be written in a stacked form as :

(3.1.1) $d\psi^*(s) = A[R\beta - \psi^*(s)]\, ds + Rd\eta(s)$,

where

$$\psi^*(s) = \begin{bmatrix} \xi(s) \\ D\xi(s) \\ \vdots \\ D^{r-1}\xi(s) \end{bmatrix}, R = \begin{bmatrix} 0 \\ \vdots \\ I \end{bmatrix}, A = \begin{bmatrix} 0 & & & \\ \vdots & & I & \\ 0 & & & \\ A_r & A_{r-1} & \cdots & A_1 \end{bmatrix}.$$

The vector $\psi^*(s)$ is of dimension $nr \times 1$ and the matrix R is $nr \times n$. The matrix of coefficients A is of dimention $nr \times nr$, it's elements being $n \times n$, while the mean vector β is $n \times 1$. We denote the mean-square differential operator by D. The innovation process $\eta(s)$ is Gaussian with zero-mean increments and covariance matrix $E[d\eta(s)\, d\eta(s')'] = \Sigma\, ds$ for $s = s'$ and 0 otherwise. The real parts of the roots of matrix A are required to be negative for stability. We will also assume that they are distinct in order to adopt a useful eigenvalue decomposition $A = G\Lambda G^{-1}$, where Λ is a diagonal matrix of eigenvalues of A, which are, in general, complex numbers, while G is a matrix of eigenvectors of A. Following Stock (1988), we set $\psi(s) = G^{-1}\psi^*(s)$ and observe that in operational time the transformed variable satisfies the following equation :

$$\text{(3.1.2)} \quad \psi(s) = [I - e^{\Lambda(s-s')}]\, G^{-1}R\beta + e^{\Lambda(s-s')}\, \psi(s') + \int_{r=s'}^{s} e^{\Lambda(s-r)}\, G^{-1}R\,d\eta(r)\,,$$

where $s > s'$. Let the calendar time state vector be $h(\tau) = \psi(g(\tau))$. Evaluating the previous equation at $s = g(\tau)$ and $s' = g(t-1)$, we find that $h(\tau)$ satisfies

$$\text{(3.1.3)} \quad h(\tau) = [I - e^{\Lambda(g(\tau)-g(t-1))}]\, G^{-1}R\beta + e^{\Lambda(g(\tau)-g(t-1))}\, h(t-1) + \int_{r=g(t-1)}^{g(\tau)} e^{\Lambda(g(\tau)-r)}\, G^{-1}R\,d\eta(r)\,.$$

Developing the first term on the r.h.s of (3.1.3), we obtain

$$\text{(3.1.4)} \quad h(\tau) = G^{-1}R\beta - e^{\Lambda(g(\tau)-g(t-1))}\, G^{-1}R\beta + e^{\Lambda(g(\tau)-g(t-1))}\, h(t-1) + \int_{r=g(t-1)}^{g(\tau)} e^{\Lambda(g(\tau)-r)}\, G^{-1}R\,d\eta(r)\,,$$

and hence,

$$\text{(3.1.5)} \quad h(\tau) - G^{-1}R\beta = e^{\Lambda(g(\tau)-g(t-1))}\, [h(t-1) - G^{-1}R\beta] + \int_{r=g(t-1)}^{g(\tau)} e^{\Lambda(g(\tau)-r)}\, G^{-1}R\,d\eta(r)\,.$$

Now, set $\bar{h}(\tau) = h(\tau) - G^{-1}R\beta$. It is easy to note that equation (3.1.5) can be written as

$$\text{(3.1.6)} \quad \bar{h}(\tau) = e^{\Lambda(g(\tau)-g(t-1))}\, \bar{h}(t-1) + \int_{r=g(t-1)}^{g(\tau)} e^{\Lambda(g(\tau)-r)}\, G^{-1}R\,d\eta(r)\,.$$

Equation (3.1.6) evaluated at $\tau = t$ yields the final representation of the transition equation :

(3.1.7) $\tilde{h}_t = T_t \tilde{h}_{t-1} + \nu_t$,

where $T_t = \exp(\Lambda\ \Delta g(t))$ and $\nu_t = \int_{r=g(t-1)}^{g(\tau)} \exp[\Lambda(g(t) - r)]\ G^{-1}Rd\eta(r)$.

3.2 Measurement equation

The multivariate analogue of equation (2.11b) can be written as :

(3.2.1) $Y_t = -1.27\iota + Gh_t + \zeta_t$,

or in terms of the state vector $\tilde{h}_t$ as :

(3.2.2) $Y_t = -1.27\iota + G\tilde{h}_t + R\beta + \zeta_t$,

where Y_t and ζ_t are $n \times 1$ vectors with elements $y_{it} = \log[\Delta \log y_{it} - \lambda]^2$, $\zeta_{it} = \log \varepsilon_{it}^2 + 1.27$, $i = 1, \ldots, n$, and ι is a $n \times 1$ vector of ones.

Treating the system equations consisting of (3.1.7) and (3.2.1) as a Gaussian state space with uncorrelated disturbance terms allows us to obtain QML estimators by means of the Kalman filter.

According to Stock (1988), the initial conditions for a Kalman filter algorithm can be obtained, taking unconditional expectations, assuming that prior to the sample $\Delta g(t) = 1$. Adopting the usual notation, we find the one-step ahead forecast of the state, $a_{1|0} = 0$ and its covariance matrix, $P_{1|0} = \Sigma_{i=0}^{\infty} T^j\ Q\ \overline{T}^{j\prime}$, where T and Q denote T_t and $Q_t = E(\nu_t\ \overline{\nu}_t')$ evaluated at $\Delta g(t) = 1$. The matrix Q_t will be computed in the following way : the (i - j) element of Q_t is known to be equal to $q_{ij} \int_{r=0}^{\Delta g(t)} \exp[(\lambda_i + \overline{\lambda}_j)(\Delta g(t) - r)]dr = -q_{ij}(1 - T_{it}\overline{T}_{jt}) \ / \ (\lambda_i + \overline{\lambda}_j)$, where q_{ij} is the (i - j) element of the matrix $G^{-1}R\ \Sigma\ R'\overline{G}^{-1\prime}$

Maximizing the quasi-log-likelihood L_T yields a vector-of-parameters estimate, which is consistent and asymptotically normal. The asymptotic covariance matrix obviously differs from that of the maximum likelihood and equals $J_0^{-1} I_0 J_0^{-1}$ [Gouriéroux and Monfort (1989)], where $J_0 \equiv \lim_{T\to\infty} -\frac{1}{T}\frac{\partial^2 L_T(\theta_0)}{\partial\theta\,\partial\theta'}$ and $I_0 = \lim_{T\to\infty} \frac{1}{T} V_0(\frac{\partial L_T(\theta_0)}{\partial\theta})$ with θ_0 being the true value of the parameter vector. All computations were performed with GAUSS version 3.1 on a PC 486-25.

4. EMPIRICAL RESULTS

In this section, we turn our attention to an empirical study of SV models subject to time deformation. The data used consist of the daily closing value of the S&P composite stock index and the daily volume of shares traded on the NYSE. The data set is identical to that used by Gallant, Rossi and Tauchen (1992), who describe its sources in detail. The data are plotted in Figure 4.1 which consists of two parts : namely, 4.1a displays the price series, while 4.1b contains volume. Both series were adjusted for seasonal and trend factors, as described in Gallant, Rossi and Tauchen. The empirical section is based exclusively on the adjusted series appearing in Figure 4.1. In the next section, where extension of SV models with time deformation will be discussed, we will propose models suitable for unadjusted data, i.e., models explicitly taking into account nontrading day effects, trends, etc.

[Insert Figure 4.1 here]

Two types of continuous time models were introduced in section 2, namely the AR(1) and the Ornstein-Uhlenbeck process. The parameter estimates of the former appear in Table 4.1, while the second volatility process specification is covered in Table 4.2. A total of six models were estimated in each case, with the sixth being a SV model without time deformation, i.e., imposing $c_\ell = c_V = c_p = 0$. The other five specifications involve time deformation, yet with different functional forms. The most general specification is the unconstrained model with $\Delta g(t)$ as a function of past volume and prices with a leverage effect. The second model only involves volume, the third only prices with leverage effect, the fourth prices and volume without leverage and, finally, the fifth model has $\Delta g(t)$ determined by past price changes. A total of seven coefficients in the AR(1) and eight in the O-U case were estimated. The parameter λ was estimated as a sample average of $\Delta \log y_t$ following the suggestion of Gouriéroux, Monfort and Renault (1992). Moreover, as there appears to be some minor autocorrelation left in $\Delta \log y_t$, we first fitted first-order autoregressive models to $\Delta \log y_t$

and replaced $\Delta \log y_t$ by the residuals to estimate the SV models. The autoregressive coefficients appear as a_1 in both tables.

The empirical results in Tables 4.1 and 4.2 support the view that at least some form of time deformation is present. Indeed, if we rely on simple Wald tests, we observe that in most instances, but not all, as will be discussed shortly, the coefficients of the $\Delta g(t)$ function appear to be significant. It is important to recall at this stage that the standard errors appearing in both tables are in fact based on an inefficient estimation procedure. Hence, the already strong evidence for the presence of time deformation would only be enforced if a more efficient estimator were used. There are some exceptions though, i.e., cases where there appears not to be any statistically significant time deformation. The most striking case is the unconstrained model appearing in both tables involving all three coefficients c_v, c_ℓ and c_p. Somehow, there appears to be some sort of overfit yielding imprecise estimates in the sense that the constrained models often yield parameter estimates quite similar to unconstrained models, but in most cases, the constrained model yields a significant coefficient while the full model doesn't. In terms of the actual path of $\Delta g(t)$, it turns out that the results are quite often similar, regardless of the functional specification for time deformation. This will be discussed in more details later.

The parameter values all appear to agree with the stochastic process behavior described in Section 2. In particular, we find that A is negative and always significantly different from zero. Parameters λ and a_1 are also significant. Past volume has a positive impact on $\Delta g(t)$ since $c_v > 0$. This implies, as noted in section 2, that the marginal effect of increases in trading volume is a volatility process that is less persistent (in calendar time) as the random AR(1) coefficient decreases, while the innovation variance increases. The leverage coefficient is also positive, while past price change always enter with a negative coefficient in the $\Delta g(t)$ specification. However, since $|c_\ell| > |c_p|$ with $c_p > 0$ and $c_p < 0$ it follows that whenever $\Delta \log p_{t-1}$ is negative, we find a greater positive effect of past prices on $\Delta g(t)$ than when $\Delta \log p_{t-1}$ is positive. Hence, bull markets tend to make volatility more persistent, while bear markets are associated with more erratic behavior of volatility. Of course, both effects are marginal, as one must also take into account the influence of trading volume when it appears in $\Delta g(t)$. Parameter β, appearing only in Table 4.2, since it is specific to the O-U model, yields mixed results because it is significant in four out of six specifications. This would mean that we should have a preference for the O-U specification if one were to choose between the two models. It is interesting to note though that the time deformation process is not very different across the two tables.

We complement the Wald tests in Tables 4.1 and 4.2 with LR-type tests and also investigate joint hypotheses. Tests regarding the time-deformation hypothesis appear in Table 4.3. The results indicate that when $\Delta g(t)$ is determined by either one of the individual series, volume or prices the Wald and LR tests are not in agreement and there is also a difference depending on the process specification. However, prices combined with either a leverage effect or trading volume yield robust and strong results supporting significant time deformation. Finally, the three series combined again yield mixed results with the joint LR test favoring time deformation, though none of the coefficients are individually significant for the AR(1) model. In Table 4.4, we turn our attention to a number of LR tests regarding the functional specifications of time deformation. We test whether $\Delta g(t)$ is determined by (1) volume only against the alternative of volume and prices with leverage, (2) prices with leverage only against the same alternative and (3) prices only without leverage and volume once again against all three series. In each case, the restricted model is rejected. We also test whether leverage should be introduced once prices and volume determine time deformation and found mixed results. For the AR(1) model, there is a significant leverage effect, while the O-U process appears to have a very flat likelihood surface, making the marginal contribution of leverage to $\Delta g(t)$ negligible.

We turn our attention now to the sample path of the time deformation process $\Delta g(t)$ for a number of specifications. As we could not plot all possible combinations, since it would be quite repetitive, we selected a few representative cases. We first examine the path of time deformation for the AR(1) model with two alternative specifications of $\Delta g(t)$: one involving prices and volume, the other adding leverage effects. Four plots appear in Figure 4.2. Each $\Delta g(t)$ specification yields a pair of plots, one for $\Delta g(t)$, the other for the innovation variance which also depends on $\Delta g(t)$. Figures 4.2a and 4.2b display the patterns of time deformation, both involving prices and volume with leverage effects included in the latter. They appear to be quite similar, though $\Delta g(t)$ with leverage seems to be slightly less erratic. One key feature emerging from both figures, as well as the adjacent plots containing the innovation variance to the volatility process, is the infrequent appearance of sharp peaks in operational time acceleration. Since A is found to be negative, this means that the conditional variance function becomes locally extremely erratic, unattached to the previous period and subject to a large variance innovation shock. As noted in the introduction, this finding complements a competing specification of laws of motion via diffusion processes involving jumps. Such processes, proposed by Merton (1967a,b) were built on the premise that one would occasionally observe abnormal information leading to the incidence of a jump in asset prices. Through the time deformation specification, one can view such information arrival as extremely rapid acceleration of market time through the increased trading and price movement per unit of calendar time. The advantage of SV models with time deformation over jump-diffusion processes is that the former might be relatively easier to estimate, at least if one is satisfied with the asymptotically inefficient QMLE algorithm. Indeed, the ML estimation of jump-diffusion processes can be quite involved [see, for instance, Lo (1988) for details].

[Insert Figure 2 here]

We turn our attention now to the volatility process itself, i.e, the h_t process as extracted via the Kalman filter procedure. A first caveat to note is that the filtering algorithm we use, like the estimation procedure, is only an approximation of the true latent volatility process. Indeed, the Kalman filtering algorithm ignores all non-Gaussian features of the DGP, as noted in section 3. Jacquier, Polson and Rossi (1992) proposed a procedure that yields an exact extraction algorithm for the volatility process as a by-product of their Bayesian inference procedure for SV models. Their algorithm is numerically quite more involved in comparison to that described in section 3 and is probably not so easy to modify so that a time deformation SV model can be handled [see Ghysels and Jasiak (1993) for further discussion]. Figure 4.3a displays the approximate filter extraction of the volatility process h_t. The figure consists of two parts, namely, 4.3a displays stochastic volatility as extracted under the assumption of no time deformation. Hence, Figure 4.3a corresponds to a volatility process that one would obtain from the approach proposed by Nelson (1988) and Harvey, Ruiz and Shephard (1992). Figure 4.3b plots h_t extracted from a model with time deformation. In sharp contrast to the standard SV specification, we uncover a very smooth volatility process. This may not be as surprising, given the plots in Figure 4.2 where $\Delta g(t)$ and the innovation variance appeared. Indeed, most of the erratic behavior of h_t obtained through a specification without time deformation is absorbed through the doubly stochastic random coefficient stochastic volatility specification. Once time deformation is taken into account, it appears that the underlying volatility process evolves smoothly in operational time. This yields an alternative interpretation. Indeed, the smooth evolution of h_t in operational time implies that the process is easier to predict over long horizons. This smooth and predictable component appears to be separated from the more erratic behavior of market time through $\Delta g(t)$. This separation into two components is interesting as it decomposes a volatility process that is itself latent.

[Insert Figure 4.3 here]

5. EXTENSIONS OF THE BASIC MODEL

Having introduced the basic model, let us now propose several extensions. The first one will explore the possibility of periodic structures in stochastic volatility and volume. This extension is closely related to the periodic GARCH structures studied by Baillie, Bollerslev and Ghysels (1993) and also sheds light on the seasonal adjustments often performed on conditional variance models. The second one will entail a joint model for volatility and volume. Such joint modeling of both series is directly related to the research by Gallant, Rossi and Tauchen (1992). They proposed a seminonparametric, i.e., SNP as dubbed by Gallant and Tauchen (1989), modeling strategy for the joint price-volume process. Our approach will consist of a multivariate state space model with time deformation. Each of these two extensions will be discussed in separate subsections.

5.1 Periodicity and time deformation

Price movements and volume of trade exhibit strong seasonal patterns. Some of these movements, like the so-called January effect, have been widely documented and belong to the large body of literature on stock market anomalies [see, e.g., Dimson (1988)]. Most often, one estimates seasonal dummy coefficients for variation in the mean and in the variance. Such seasonal mean shifts are then removed and the "seasonally adjusted" series are used for further analysis involving fitting a conditional variance function. This strategy, adopted, for instance, by Gallant, Rossi and Tauchen (1992) and many others, has many of the pitfalls encountered in dealing with seasonality in econometric models [see, for instance, Ghysels (1990) for a survey]. An alternative sometimes considered consists of fitting a conditional variance model with seasonal lags. A recent example of such a model specification strategy can be found in Bollerslev and Hodrick (1992) in their study of the behavior of monthly real dividend series on the NYSE. They fitted a seasonal lag in the mean as well as a seasonal lag in the ARCH specification to model the series. We propose a different strategy which, though it will take advantage of the time deformation specification, has links with a body of literature dealing with periodic models.

The arrival of information on the stock market is reasonably well structured throughout the year. Obviously, there is the closure of the market producing the nontrading day effect already alluded to, but there is more structure beyond this effect. At the aggregate level, many government statistics are released with a specific calendar. Quarterly GNP and monthly inflation, unemployment, industrial production, capacity utilization, inventories, housing starts and car sales releases are among the most widely publicized data. Also, the weekly money supply announcements and the Wednesday T-Bill auctions are closely watched. Most firms also have a fixed calendar of publishing their annual report and deciding on dividends. These are some examples explaining the seasonality found in both volume and volatility [see, for instance, Tables 1 and 2 in Gallant, Rossi, and Tauchen (1992) for empirical evidence].[10]

The presence of a periodic structure, even after the adjustments are made yielding the series plotted in Figure 4.1, can be investigated quite easily in the context of the results produced in the previous section. Tables 5.1 and 5.2 summarize the average $\Delta g(t)$ per month and per day of the week over the entire sample, as identified with the five different functional forms for the time-deformation process in Table 4.2. The two tables differ with respect to the sample size involved. Indeed, the second table excludes the year 1987 of the October crash.[11] It affects significantly the Monday and October $\Delta g(t)$ average for most specifications, except for the second column which corresponds to a $\Delta g(t)$ involving trading volume only.

If we judge these statistics via simple pairwise comparisons with a two-standard error rule, we certainly find significant Monday and Friday effects. Moreover, the months of January and May-June seem above while November-December often below what most other months yield.

A SV model with deformation can accommodate this structure through the d_t variable in the specification of $\Delta g(t)$ appearing in (2.12). Since each month and each day of the week may have their own rhythm in information arrival, we propose to specify a mean-shift function setting the pace of $\Delta g(t)$. In particular, let $c_d' d_t$ be equal to :

$$(5.1.1) \qquad c_d' d_t = c_d^n n_t + \Sigma_{s=1}^{12} c_{ds}^m \alpha_{st} + \Sigma_{j=1}^{5} c_{dj}^w \alpha_{jt}$$

---

10 Volume of transactions also has a particular pattern due to the regular expirations of stock derivative contracts. Some of the most active trading days on the NYSE are related to the so-called triple-witching expirations of stock-index options, and futures and options on individual stocks, which typically lead to increased trading as money managers execute computer-driven arbitrage transactions.

11 The $\Delta g(t)$ process was not reestimated, however, only the sample size was modified. Hence, the $\Delta g(t)$ in Table 5.2 no longer averages to 1 over the shorter sample.

where $c_d' \equiv (c_d^n, c_{d1}^m, \ldots, c_{d12}^m, c_{d1}^w, \ldots, c_{d5}^w)$, $n_t = 1$ if the previous day was a nontrading day and zero otherwise, $\alpha_{st} = 1$ if $t = s \bmod 12$ and zero otherwise and, finally, $\alpha_{jt} = 1$, the j-th day of the week and zero otherwise.

It is interesting to observe that the mean-shift function in (5.1.1) establishes a relationship with the periodic GARCH processes studied by Baillie, Bollerslev, and Ghysels (1993). Indeed, suppose for the moment that time deformation only depends on the function specified in (5.1.1), i.e., we set $c_v = c_p = c_\ell = 0$ in (2.12). Then the random coefficient model for h_t appearing in (2.11.2) becomes a periodic autoregressive model of order one, i.e.,

$$(5.1.2) \quad h_t = \alpha_p h_{t-1} + \nu_t$$

where $\alpha_p = \exp A(c_d' d_t)$ is specified in (5.1.1) and hence purely deterministic. Since GARCH processes can be viewed as discrete time approximations to diffusion processes, it is not surprising to find an implicit link with the specification of time deformation having periodic features and periodic GARCH processes. Obviously, the specification in (5.1.2) is not exactly a periodic GARCH, as it is still a SV model with an innovation process ζ_t. In addition, it should also be noted that the process ν_t exhibits periodic features as a result of (2.6.2). Gallant, Rossi and Tauchen (1992) removed seasonal effects in volatility and volume through regression-based methods prior to their SNP model fitting. It is interesting to note that Baillie, Bollerslev and Ghysels (1993) also found that evidence of periodicity could still be uncovered in the adjusted volatility and volume data used by Gallant, Rossi and Tauchen (1992), like Tables 5.1 and 5.2 seem to indicate.

Modeling seasonal effects directly and explicitly into the model specification is generally preferred, though there are many unresolved questions regarding the econometric practice of seasonal adjustment versus the use of unadjusted series [see, e.g., Ghysels (1990) for a survey]. The fact that we use periodic structures has an advantage over, say, seasonal lag specifications. Tiao and Grupe (1980), who established the relationship between time-invariant linear models involving seasonal lags and periodic ARMA models, showed the loss of information and prediction accuracy involved in the transition from the latter to the former. Indeed, the periodic models involve sigma algebra where events are conditioned on the season they occur since autocorrelations are season-dependent, while linear seasonal processes involve seasonal conditioning of events [see also Hansen and Sargent (1990, Chapter 10) for further details]. This analysis has been extended by Baillie, Bollerslev, and Ghysels (1993) to periodicity in conditional heteroskedasticity, showing the link between periodic ARCH and seasonal ARCH models and the loss of information and misspecification involved in going from the former to the latter. By analogy, we should find the period specification appearing in the time deformation model to outperform the other nonperiodic but seasonal or nonseasonal specifications involving adjusted data.

Of course, the arrival of information on the stock market is determined by factors other than the purely periodic patterns. Therefore, the d_t is not the only component appearing in $\Delta g(t)$. The fact that volume has a (linear) periodic structure is also a contributing factor, yet it also adds to the dimension of information arrival due to informed traders and noise traders in the market. Further research along these lines is certainly necessary.

5.2 A multivariate model of price volatility and volume with time deformation

As Harvey, Ruiz, and Shephard (1992), and Harvey and Shephard (1993) observed, it is relatively easy to extend the state space representation of stochastic volatility models to a multivariate context. This straightforwardly applies to the price process y_t governed by equation (2.11), since the Kalman filter procedure developed by Stock also applies to a vector process.[12] Such extensions, while certainly not without interest, will not be further explored here. Instead, we will exploit the possibility of multivariate modeling to propose a structure accommodating both price volatility and volume subject to time deformation. Volume series show a strong upward trend, and a first-differenced series or alternatively a detrended series as in Gallant, Rossi and Tauchen (1992) has been proposed to accommodate the nonstationarity. Here, we will use a generic process denoted v_t which is assumed stationary. For the purpose of presentation, we will again focus on the AR(1) case. Higher order stochastic differential equations, either for $\sigma(g(t))$ or $v(g(t))$ can be accommodated for by the usual stacking argument. Following equations (2.5) and (2.6), we obtain :

$$(5.2.1) \qquad \begin{bmatrix} \log[\log y_t - \log y_{t-1} - \lambda]^2 \\ v_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_t \\ v_t \end{bmatrix} + \begin{bmatrix} -1.27 \\ 0 \end{bmatrix} + \begin{bmatrix} \zeta_t \\ 0 \end{bmatrix}$$

12 It should be noted that all asset prices are subject to the same time deformation. This obviously constraints the volume effect somewhat, since the trading volume of each asset separately cannot be taken easily into account.

$$(5.2.2) \qquad \begin{bmatrix} h_t \\ v_t \end{bmatrix} = \exp(A\ \Delta g(t)) \begin{bmatrix} h_{t-1} \\ v_{t-1} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}$$

where A in (5.2.2) now represents a 2×2 matrix of coefficients. The system of equations in (5.2.1) and (5.2.2) is intrinsically nonlinear, through the time deformation specification, and can therefore be put to use for the same purpose as the SNP approach to the joint process proposed by Gallant, Rossi, and Tauchen (1992). Several expansions can be made, notably by adding more lags, as noted earlier, and adopting a more elaborate specification for $\Delta g(t)$ than the one appearing in (2.12).

The bivariate system (5.2.1 and 5.1.2) is of interest for at least two reasons. First, a SV model with time deformation involving volume series requires predictions regarding trading volume if, for instance, an option-pricing model with SV of the type Hull and White (1987) is considered. The bivariate system clearly provides a formal representation of the joint process which can be used to predict volume (and prices) in an option-pricing model. The second use of the bivariate system is for the purpose of impulse response function analysis in the context of SV models. Since with time deformation such models are nonlinear, one faces issues similar to those studied by, for instance, Gallant, Rossi and Tauchen (1993) and Potter (1991).

6. CONCLUSIONS

In the paper, we introduced SV models with volatility evolving in an operational time, determined by the arrival of information on the market. As proxy for the information flow, we used trading volume and past prices, though other series could be used as well. Significant time deformation was found yielding a volatility process evolving smoothly in operational time, yet the changes in operational time, per unit of calendar time, were found to be very erratic with sporadic large increases associated with extreme market time accelerations.

The notion of time deformation to model volatility still needs further research. In particular, in section 5, links between time deformation and periodicity in market activity were discussed. Moreover, pricing of derivative assets like options with SV subject to time deformation requires more research as well, particularly via multivariate modeling of volume and prices, as discussed in section 5.

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Fig 4.1a: S&P 500 (Adjusted)

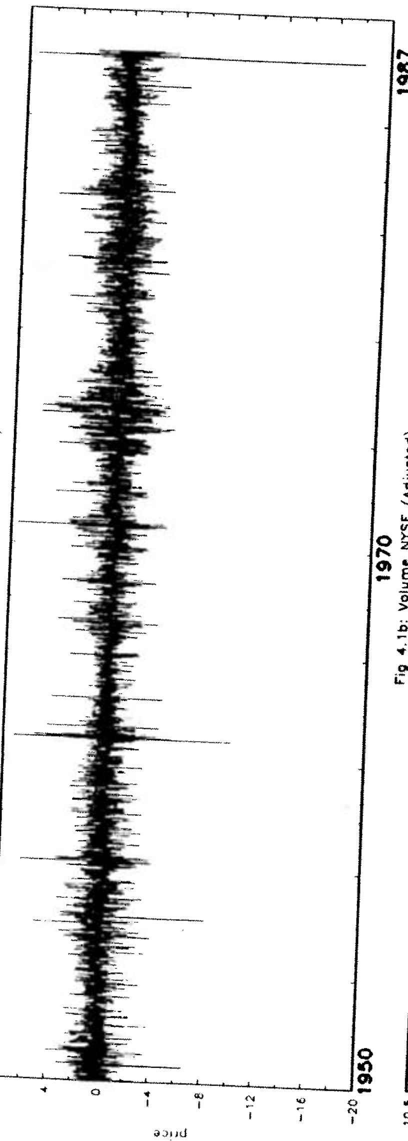

Fig 4.1b: Volume NYSE (Adjusted)

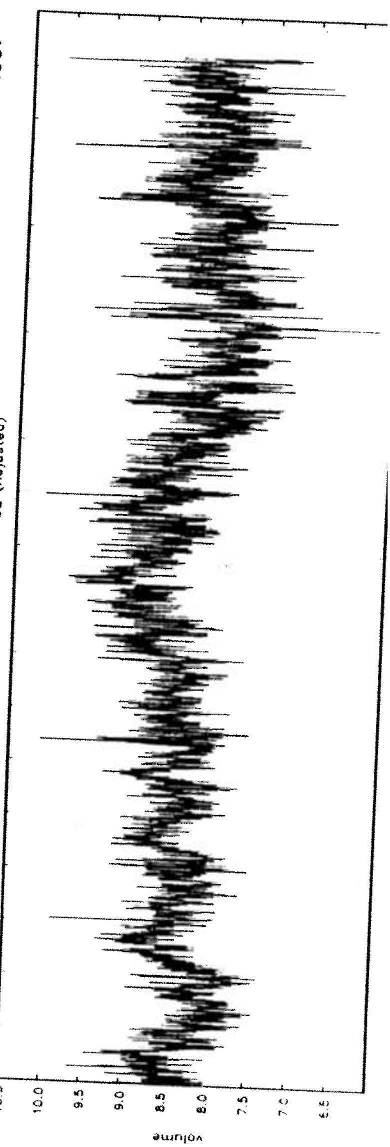

Figure 4.2 : AR(1) SV Model with Time Deformation

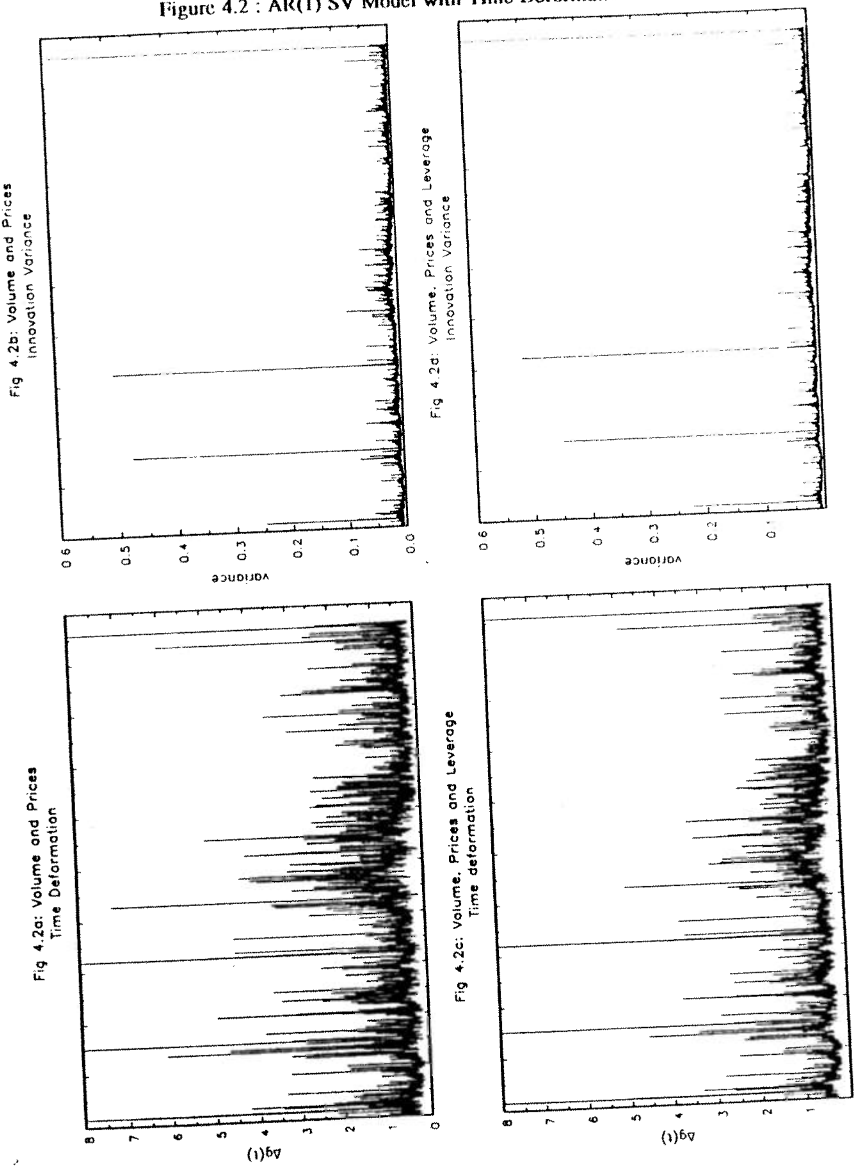

Fig 4.3a: Volatility without Time Deformation

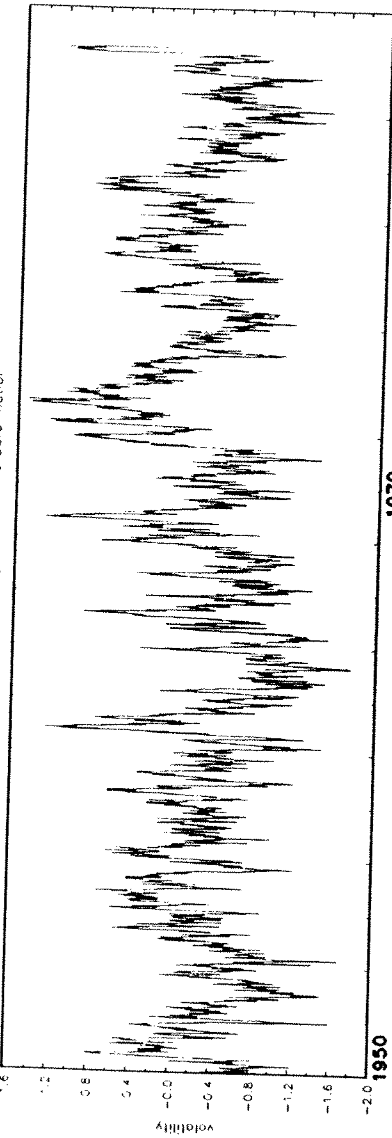

Fig 4.3b: Volatility with Time Deformation

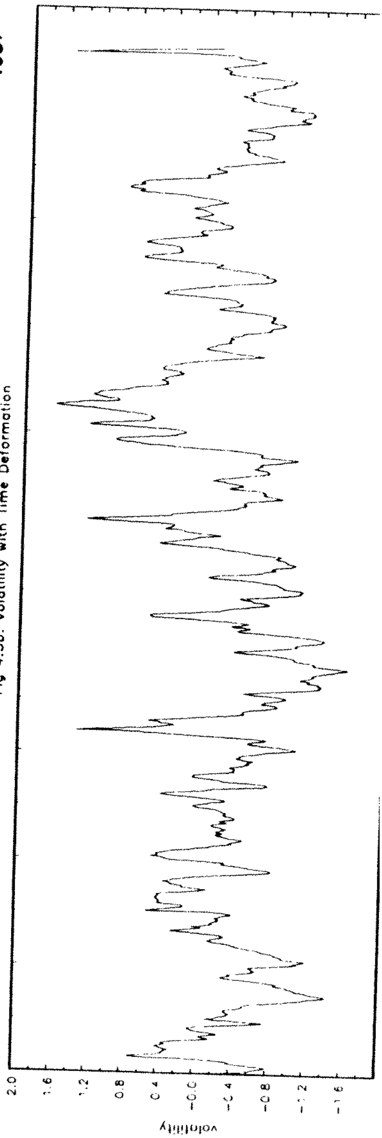

Table 4.1
Stochastic Volatility with Time Deformation Determined by Past Trading Volume and Prices with Leverage Effects

Sample : 1950-1987 : Continuous Time AR(1)

	(1)			(2)			(3)		
	Est	SE	P	Est	SE	P	Est	SE	P
c_v	1.010	0.623	0.10	0.979	0.701	0.16	-		
c_p	-0.167	0.096	0.08	-			-0.156	0.098	0.10
c_ℓ	0.256	0.136	0.06	-			0.354	0.115	0.00
Σ	0.014	0.005	0.00	0.012	0.004	0.00	0.012	0.003	0.00
A	-0.012	0.004	0.00	-0.012	0.003	0.00	-0.011	0.003	0.00
λ	0.028	0.011	0.01	0.028	0.011	0.01	0.028	0.011	0.01
a_1	0.177	0.010	0.00	0.177	0.010	0.00	0.177	0.010	0.00

	(4)			(5)			(6)		
	Est	SE	P	Est	SE	P	Est	SE	P
c_v	1.256	0.576	0.03	-			-		
c_p	-0.374	0.077	0.00	-0.510	0.047	0.00	-		
c_ℓ	-			-			-		
Σ	0.014	0.004	0.00	0.015	0.006	0.00	0.012	0.003	0.00
A	-0.012	0.003	0.00	-0.013	0.005	0.00	-0.011	0.003	0.00
λ	0.028	0.011	0.01	0.028	0.011	0.01	0.028	0.011	0.01
a_1	0.177	0.010	0.00	0.177	0.010	0.00	0.177	0.010	0.00

Table 4.2
Stochastic Volatility with Time Deformation Determined by Past Trading Volume and Prices with Leverage Effects

Sample : 1950–1987 : Ornstein–Uhlenbeck

	(1)			(2)			(3)		
	Est	SE	P	Est	SE	P	Est	SE	P
c_v	0.516	0.915	0.57	1.248	0.430	0.00	-		
c_p	-0.204	0.173	0.23	-			-0.187	0.091	0.04
c_ℓ	0.246	0.301	0.41	-			0.333	0.112	0.00
Σ	0.011	0.003	0.01	0.015	0.003	0.00	0.013	0.004	0.00
A	-0.010	0.004	0.01	-0.015	0.004	0.00	-0.011	0.003	0.00
β	-0.216	0.113	0.05	-0.274	0.085	0.00	-0.101	0.075	0.18
λ	0.028	0.011	0.01	0.028	0.011	0.01	0.028	0.011	0.01
a_1	0.177	0.010	0.00	0.177	0.010	0.00	0.177	0.010	0.00

	(4)			(5)			(6)		
	Est	SE	P	Est	SE	P	Est	SE	P
c_v	1.321	0.689	0.06	-			-		
c_p	-0.372	0.082	0.00	-0.225	0.083	0.00	-		
c_ℓ	-			-			-		
Σ	0.015	0.005	0.01	0.011	0.004	0.00	0.013	0.003	0.00
A	-0.013	0.005	0.01	-0.012	0.004	0.00	-0.013	0.003	0.00
β	-0.159	0.117	0.17	-0.247	0.098	0.01	-0.237	0.093	0.00
λ	0.028	0.011	0.01	0.028	0.011	0.01	0.028	0.011	0.01
a_1	0.177	0.010	0.00	0.177	0.010	0.00	0.177	0.010	0.00

Table 4.3
Time Deformation Hypothesis Tests (LR)

Series in $\Delta g(t)$	AR(1)	Ornstein-Uhlenbeck
Volume only[a]	2.794	5.492
Prices only[a]	10.599	1.445
Prices with leverage[b]	6.455	8.961
Prices and volume[b]	9.635	11.947
Prices with leverage and volume[c]	15.994	5.299

Note : The likelihood ratio statistic is asymptotically distributed as χ^2 with respectively a = 1, b = 2 and c = 3 degrees of freedom.

Table 4.4
Hypotheses Tests of the Time Deformation Function (LR) -
The Continuous Time AR(1) Model

Hypotheses	AR(1)	Ornstein-Uhlenbeck
$H_0 : C_v \neq 0 \; C_p = 0 \; C_l = 0$ $H_A : C_v \neq 0 \; C_p \neq 0 \; C_l \neq 0$	13.200	12.911
$H_0 : C_v = 0 \; C_p \neq 0 \; C_l \neq 0$ $H_A : C_v \neq 0 \; C_p \neq 0 \; C_l \neq 0$	9.538	10.502
$H_0 : C_v = 0 \; C_p \neq 0 \; C_l = 0$ $H_A : C_v \neq 0 \; C_p \neq 0 \; C_l \neq 0$	5.396	5.877
$H_0 : C_v \neq 0 \; C_p \neq 0 \; C_l = 0$ $H_A : C_v \neq 0 \; C_p \neq 0 \; C_l \neq 0$	6.359	0.001

Table 5.1
Average Time Deformation Obtained from O-U Processes

Sample 1950-1987

	(1)	(2)	(3)	(4)	(5)
January	0.619 (0.011)	1.039 (0.019)	0.527 (0.009)	0.630 (0.017)	0.474 (0.011)
February	0.592 (0.008)	0.999 (0.019)	0.509 (0.007)	0.585 (0.014)	0.459 (0.010)
March	0.588 (0.009)	0.997 (0.018)	0.502 (0.007)	0.583 (0.015)	0.448 (0.010)
April	0.590 (0.008)	1.021 (0.022)	0.509 (0.007)	0.586 (0.015)	0.456 (0.009)
May	0.690 (0.081)	1.016 (0.020)	0.592 (0.072)	0.709 (0.103)	0.545 (0.079)
June	0.654 (0.026)	1.030 (0.020)	0.551 (0.020)	0.684 (0.037)	0.511 (0.023)
July	0.610 (0.011)	1.033 (0.022)	0.520 (0.008)	0.628 (0.019)	0.475 (0.011)
August	0.639 (0.011)	1.102 (0.025)	0.534 (0.009)	0.665 (0.019)	0.490 (0.011)
September	0.613 (0.049)	0.947 (0.020)	0.533 (0.032)	0.627 (0.095)	0.481 (0.035)
October	5.042 (4.423)	0.958 (0.022)	5.930 (5.379)	5.008 (4.414)	6.453 (5.959)
November	0.569 (0.011)	0.915 (0.018)	0.509 (0.009)	0.531 (0.015)	0.451 (0.011)
December	0.572 (0.009)	0.932 (0.019)	0.516 (0.008)	0.540 (0.014)	0.463 (0.011)
Monday	2.669 (2.007)	0.980 (0.014)	3.014 (2.440)	2.676 (2.003)	3.223 (2.703)
Tuesday	0.609 (0.007)	1.027 (0.015)	0.523 (0.005)	0.617 (0.011)	0.474 (0.007)
Wednesday	0.601 (0.006)	1.001 (0.013)	0.522 (0.006)	0.595 (0.011)	0.472 (0.007)
Thursday	0.610 (0.008)	1.001 (0.013)	0.525 (0.007)	0.613 (0.013)	0.475 (0.008)
Friday	0.587 (0.007)	0.990 (0.013)	0.509 (0.006)	0.577 (0.011)	0.455 (0.007)

Note : Columns (1) through (5) correspond to the time-deformation specifications (2) through (5) in Table 4.2.

Table 5.2
Average Time Deformation Obtained From O-U Processes

Sample 1950-1986

	(1)	(2)	(3)	(4)	(5)
January	0.621 (0.011)	1.042 (0.019)	0.529 (0.009)	0.633 (0.018)	0.477 (0.012)
February	0.594 (0.009)	1.004 (0.020)	0.511 (0.007)	0.589 (0.014)	0.461 (0.010)
March	0.589 (0.009)	1.002 (0.018)	0.503 (0.007)	0.586 (0.015)	0.448 (0.010)
April	0.588 (0.008)	1.027 (0.023)	0.507 (0.007)	0.587 (0.015)	0.453 (0.009)
May	0.693 (0.083)	1.023 (0.021)	0.593 (0.073)	0.716 (0.106)	0.547 (0.081)
June	0.659 (0.026)	1.038 (0.021)	0.554 (0.021)	0.693 (0.038)	0.515 (0.023)
July	0.613 (0.011)	1.037 (0.023)	0.522 (0.009)	0.632 (0.020)	0.477 (0.011)
August	0.639 (0.011)	1.100 (0.025)	0.535 (0.009)	0.665 (0.020)	0.491 (0.011)
September	0.617 (0.050)	0.954 (0.021)	0.534 (0.033)	0.634 (0.098)	0.482 (0.036)
October	0.590 (0.012)	0.936 (0.019)	0.526 (0.010)	0.562 (0.017)	0.469 (0.012)
November	0.568 (0.011)	0.924 (0.018)	0.505 (0.009)	0.533 (0.016)	0.447 (0.011)
December	0.573 (0.009)	0.944 (0.020)	0.511 (0.008)	0.546 (0.014)	0.460 (0.010)
Monday	0.659 (0.043)	0.983 (0.014)	0.568 (0.036)	0.673 (0.064)	0.515 (0.040)
Tuesday	0.609 (0.007)	1.029 (0.014)	0.521 (0.005)	0.621 (0.011)	0.475 (0.007)
Wednesday	0.603 (0.006)	1.004 (0.013)	0.522 (0.006)	0.599 (0.011)	0.473 (0.007)
Thursday	0.609 (0.008)	1.012 (0.013)	0.522 (0.007)	0.614 (0.012)	0.474 (0.008)
Friday	0.584 (0.006)	0.993 (0.013)	0.506 (0.005)	0.575 (0.010)	0.452 (0.007)

Note : Columns (1) through (5) correspond to the time deformation specifications (1) through (5) in Table 4.2.

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