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ON THE RESIDUAL DYNAMICS IMPLIED BY THE RATIONAL EXPECTATIONS HYPOTHESIS

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RÉSUMÉ

Ce texte contient essentiellement deux résultats. Le premier porte sur la représentation de l'hypothèse d'anticipations rationnelles à partir de fonctions d'anticipation à la Hicks-Grandmont. Une telle réconciliation est possible et complète la théorie en ce qu'elle permet de relier la pluralité des équilibres à la pluralité des fonctions d'anticipation: l'absence de coordination au niveau des anticipations rationnelles se traduit par une multiplicité de fonctions d'anticipation. Le second résultat porte sur la caractérisation d'un système de demande temporaire dans lequel on a substitué la fonction d'anticipations rationnelles. On calcule alors la décomposition de Slutsky et relie, en une seule équation, les conditions des théorèmes d'impossibilité de Sonnenschein-Mantel-Debreu et de Polemarchakis.

Mots-clés: fonction d'anticipation, anticipations rationnelles, équilibre temporaire, décomposition de Slutsky.

ABSTRACT

There are essentially two results in this paper. The first concerns the representation of the rational expectations hypothesis with the help of expectation functions à la Hicks-Grandmont. If expectation functions are defined so as to fit uncertainty, such a reconciliation is possible and more complete than expected. Indeed the eventual plurality of equilibria is then equivalent to the plurality of expectation functions. The eventual coordination failure of the rational expectations hypothesis (agents may choose different equilibria) is then translated into a plurality of expectation functions (agents can be endowed with different expectation functions). The second result applies the first: in order to characterize a temporary demand system, a rational expectations function is substituted into it. We then compute the Slutsky decomposition and link, in a single equation, the conditions of the Sonnenschein-Mantel-Debreu and Polemarchakis impossibility theorems.

Keywords: expectation function, rational expectations, temporary equilibrium, Slutsky decomposition.

The purpose of this paper is twofold: first, to study the possible links between the expectation functions à la Hicks-Grandmont (or their generalizations) and the rational expectations hypothesis; second, to scrutinize the rational expectations hypothesis itself in order to characterize, with the help of the previous point, the excess demand function of a temporary equilibrium.

To do so, a two-period economy is considered, the second period involving S states of the world. An Arrow-Debreu equilibrium can be defined and, under the conventions of Arrow (1953) and Malinvaud (1953), can be splitted into: a) a temporary equilibrium involving n_0 commodity markets and S financial markets, b) a future contingent equilibrium involving $\sum_{s=1}^{S} n_{1s}$ markets ($n = n_0 + \sum_s n_{1s}$ is the total number of commodities). Consequently, the future of this economy can be conceived as a contingent equilibrium.

If each agent conceives the same contingent equilibrium by automatic coordination or if these expectations are "strongly rational " (Guesnerie (1992a),(1992b)), the contingent equilibrium (which is parametrized on current prices) is representable by an expectation function (perhaps a generalized one) ψ^*, namely a rational expectations function.

A temporary equilibrium under rational expectations is then defined by the substitution of ψ^* in the part a) of the previous splitting. Such a construction could be thought of as a standard static equilibrium. That this intuition is false was shown recently by Balasko (1994) in a model similar to ours. In this paper, we will explain why this intuition is false: even in the pure case of the above construction, the rational expectations hypothesis implies something new, namely a new dynamics in the variations of real incomes.

To make explicit this residual dynamics, we shall characterize the local structure of the temporary excess demand (or equivalently, the local structure of the temporary demand system in the aggregate). We shall find, essentially, that the total price effect decomposes into a) a pure substitution effect, b) a current income effect, c) a spill-over in the present period of the expected income effects. The importance of this point can be stressed as follows: if c) vanishes, a necessary condition for the arbitrariness of the price-effect given by the Sonnenschein-Mantel-Debreu impossibility theorem ((1972),(1974)); if b) vanishes the Sonnenschein-Mantel-Debreu condition and the Polemarchakis condition (1983) are both necessary. Finally, when income effects are identical across individuals, it is found that the aggregate current static demand is recovered with all its usual properties. This last point implies that the static formulas found in Barten (1967), Deaton and Muellbauer (1980), Barten and Bettendorf (1989), and Hildenbrand (1994) can indeed be used in a dynamic framework (a sequence of temporary equilibria, say).

The paper is divided in three sections. In the first, a model allowing us to define generalized expectation functions is considered. The second section is devoted to characterizing the rational expectations function. In the third section the temporary demand system is developed and characterized.

1 The model

The preferences of a given consumer are representable by a utility function u which is

a) defined on an open, convex and bounded from below subset X of $\Re^n$ ($\Re^n$ is the commodity space and $x \in X$ is a physically possible consumption plan).

b) of class C^k, $k \geq 2$

c) strongly monotonic ($\frac{\partial u}{\partial x} > 0$)

d)strongly quasi-concave ($\zeta' \frac{\partial^2 u}{\partial x \partial x'} \zeta < 0$ for any $\zeta \in \Re^n \setminus \{0\}$ such that $\zeta' \frac{\partial u}{\partial x} = 0$).

(These assumptions are standard (see, for instance, Mas Colell (1985) and Balasko (1988))).

Let p be a strictly positive vector of prices, R a level of wealth (or income). The budget constraint is written $px = R$. The survival condition $\{x \mid px = R\} \cap X \neq \Phi$ is assumed. Then, an optimum exists and is unique. The Marshallian demand function ξ such that $x = \xi(p, R)$ exists, is continuously differentiable, and characterized by a Slutsky local structure.

In order to study the economic content of the previous model, let us consider an a priori really more complex one.

A consumer has to optimize over two periods. In period 0, he can purchase a quantity vector x_0 and a financial asset A_1 to be deliverable at the beginning of period 1. His current budget constraint is

$$p_0 x_0 + \beta A_1 = R_0 \tag{1}$$

where p_0 is the vector of current prices, β is a discount factor (consequently βA_1 can be seen as a bank deposit) (β can also be seen as the price of a portfolio to be delivered in period 1 and fitting every state of the world) and finally R_0 is his current income plus the balance of his previous financial operations (so that a real balance effect can be introduced).

Of course, the consumer cannot operate without forecasting his future budget constraint. He knows that, at the beginning of period 1, markets for risky financial assets open first (Arrow convention I) and that there will be S possible states of nature. Assuming that risky financial assets are of the Arrow type, that is, contingent elementary assets, his future budget constraints are

$$\sum_s \overline{q}_s y_s = A_1 \tag{2}$$

$$\hat{p}_{1s} x_{1s} - y_s = \hat{R}_{1s} \tag{3}$$

where $\overline{q}_s$ is a state price, y_s is the amount of money the consumer will receive if state s obtains, $\hat{p}_{1s}$ is the conditional price vector at date 1 (prices to be paid only if state s obtains), x_{1s} is the vector of consumption in the sth state and $\hat{R}_{1s}$ is the consumer's future income if state s obtains.

Of course, in this problem, $\overline{q}_s$, $\hat{p}_{1s}$ and $\hat{R}_{1s}$ have to be known or forecasted for every state.

Such is the more complex problem. Now if the risky assets are unconstrained, equations (2) and (3) are clearly equivalent to

$$\overline{p}_1 x_1 - A_1 = \overline{R}_1 \tag{4}$$

where $\overline{p}_1 = (\overline{q}_1 \hat{p}_{11}, ..., \overline{q}_s \hat{p}_{1s})$ (Arrow convention II) is a contingent price system, $x_1 = (x_{11}, ..., x_{1S})$ a strategy against uncertainty, and $\overline{R}_1 = \sum_s \overline{q}_s \hat{R}_{1s}$ a future income.

Finally, if bank operations are unrestricted, (1) and (4) can be written as

$$px = R \tag{5}$$

where $p = (p_0, p_1) = (p_0, \beta \overline{p}_1)$ (Malinvaud convention) is an intertemporal price system, $R = R_0 + \beta \overline{R}_1$ a level of wealth, and $x = (x_0, x_1)$ a consumption plan.

In other words, we have reverted to our initial problem so far as static optimization is implied. Consequently, a solution exists and is unique (x being determined, the risky assets follow from (3) and A_1 follows from (2)).

The main point here is that if markets for risky assets are complete, (1),(2) and (3) can be reduced to (1) and (4). We assume this completeness in order to focus on expectations.

Moreover, in order to study expectations, the decomposition of (5) into (1) and (4) is "optimal". On the one hand, equation (4) contains a contingent price system and since financial markets are complete at period one, the consumer can forecast as if contingent markets exist at period one; moreover, in order to determine his present purchases of commodities and assets, the expectations of $\overline{p}_1$ and $\overline{R}_1$ are sufficient. On the other hand, what is known (namely p_0, β, R_0) appears in (1) while what has to be forecasted (namely $\overline{p}_1$ and $\overline{R}_1$) appears in (4).

Under this decomposition, the demand system can be written

$$x_0 = \xi_0(p_0, \beta\overline{p}_1, R_0 + \beta\overline{R}_1)$$

$$A_1 = \overline{p}_1\xi_1(p_0, \beta\overline{p}_1, R_0 + \beta\overline{R}_1) - \overline{R}_1$$

$$x_1 = \xi_1(p_0, \beta\overline{p}_1, R_0 + \beta\overline{R}_1).$$

It contains current commodity demand, asset demand and planned commodity demand. Of course, its comparative statics will depend on the role played by expectations.

2 The Rational Expectations Function

Let us consider a private ownership economy with m consumers each one being indicated by an index i=1,2,...,m. Each consumer is characterized by the axioms and conventions presented in the beginning of the preceding section and the following convention on incomes:

$$R_{0i} = p_0\omega_{0i} + A_{0i}$$

$$\overline{R}_{1i} = \overline{p}_{1i}\omega_{1i}$$

where A_{0i} is the consumer's initial credit position[1] and ω_{0i} and ω_{1i} are individual initial endowments. Consequently the total demands are defined

[1]The presence of the consumer's initial credit position amounts to assuming that the economy has a history.

by

$$\sum_i x_{0i} = \sum_i \xi_{0i}(p_0, \beta\overline{p}_{1i}, p_0\omega_{0i} + \beta\overline{p}_{1i}\omega_{1i} + A_{0i}) \tag{6}$$

$$\sum_i A_{1i} = \sum_i \overline{p}_{1i}\xi_{1i}(p_0, \beta\overline{p}_{1i}, p_0\omega_{0i} + \beta\overline{p}_{1i}\omega_{1i} + A_{0i}) - \overline{p}_{1i}\omega_{1i} \tag{7}$$

$$\sum_i x_{1i} = \sum_i \xi_{1i}(p_0, \beta\overline{p}_{1i}, p_0\omega_{0i} + \beta\overline{p}_{1i}\omega_{1i} + A_{0i}). \tag{8}$$

In these relations, $\overline{p}_{1i}$ have to be specified in some way if at date 0 exist only spot markets for goods and services (x_{0i}) and a futures market to trade the A_{1i}'s. A first device amounts to endow each consumer i with an expectation function (ψ_i) such that

$$\overline{p}_{1i} = \psi_i(p_0, \beta, (A_{0j})) \qquad i, j = 1, 2, ...m \tag{9}$$

where $(A_{0j}) = (A_{01}, ..., A_{0j}, ...A_{0m})$ is the distribution over consumers of initial credit positions.

This first device can be attributed to Hicks (1946) and Grandmont (1983). As a matter of fact, both authors used "point-expectations" in the above-mentioned treatises (the future involved a single state of nature). For Grandmont, this was clearly a pedagogical affair: in his previous works (for instance Grandmont (1977)) probabilistic interpretations were considered. Accordingly, in this paper, an expectation function, or a Hicks-Grandmont expectation function, is a collection of vector-valued functions ψ_{si} such that $\psi_i = (\psi_{1i}, ...\psi_{si}, ...\psi_{Si})$. A consumer has to anticipate the vector of prices within each state of the world.

A second device amounts to assume that agents have uniform expectations ($\overline{p}_{1i} = \overline{p}_1, i = 1, 2, ...m$) and know or can reconstitute the conditions of a future equilibrium

$$\sum_i \xi_{1i}(.) = \omega_1 \tag{10}$$

where $\omega_1 = (\omega_{11}, ...\omega_{1s}, ...\omega_{1S})$ is a vector of future initial endowments (of common knowledge).

In our context this second device cannot be confused with the perfect foresight hypothesis (since we have in principle several states of the world) and, on the contrary, can be used to rationalize it (if states of nature are reduced to one). This is the rational expectations hypothesis as specified by

Arrow (1953) and Radner (1972) which can be seen as a special case of a more general concept (Green (1973), Lucas (1972), Radner (1979), Laffont (1991)) and whose logical foundations were explicited in Guesnerie ((1992a),(1992b)).

These two devices are often seen as antagonist. A first element of reconciliation between both representations is immediate. Suppose that for any $(p_0, \beta, (A_{0j}))$, equation (10) admits one and only one solution $\overline{p}_1$ (assuming either an automatic coordination of agents or that the future equilibrium is, in fact, a strongly rational equilibrium, see Guesnerie (1992a), (1992b)).

Then, by definition, there exists a function ψ^* such that, for any i,

$$\overline{p}_{1i} = \overline{p}_1 = \psi^*(p_0, \beta, (A_{0j})).$$

Consequently if $\psi_i = \psi^*$ in equation (9), both representations give the same result. This first element is refined in the proposition 1.

Proposition 1 *If the conditions of a future equilibrium*

$$\sum_i \xi_{1i}(p_0, \beta\overline{p}_1, p_0\omega_{0i} + \beta\overline{p}_1\omega_{1i} + A_{0i}) = \omega_1 \tag{11}$$

a) are defined on an open set $V_0 \times \Omega \times V_1$ *where* $(p_0, \beta, (A_{0j})) \in V_0$, $((\omega_{0i}), (\omega_{1i})) \in \Omega$, $(\overline{p}_1) \in V_1$

b) admit one and only one solution for any $(p_0, \beta, (A_{0j}), (\omega_{0i}), (\omega_{1i})) \in V_0 \times \Omega$

c) have their functions ξ_{1i} *sufficiently differentiable*

then the rational expectations hypothesis is representable by a Hicks-Grandmont expectation function (a rational expectations function ψ^**) such that*

a) ψ^* *is defined on an open set* $V_0 \times \Omega$

b) $\overline{p}_1 = \psi^*(.)$ *is an equilibrium price vector*

c) $\psi^* \in C^{k-1}$ *almost everywhere on* $V_0 \times \Omega$.

Proof:

Because the sum of the intertemporal Slutsky matrices has a full rank south-east block (the substitution effects associated to $\frac{\partial x_1}{\partial p_1}$), the Jacobian matrix of equation (11) has full rank in $\overline{p}_1$. By transversality this implies that the matrix of gross substitution effects

$$\tilde{K}_{11} = \sum_i \frac{\partial \xi_{1i}}{\partial p_1} + \sum_i \frac{\partial \xi_{1i}}{\partial R_i}\omega'_{1i} \tag{12}$$

(where $R_i = p_0\omega_{0i} + \beta\overline{p}_1\omega_{1i} + A_{0i}$) has full rank almost everywhere if $\sum_i \xi_{1i}(.)$ is sufficiently differentiable (see Mas Colell (1985) p.320). Consequently the implicit function theorem can be applied: ψ^* exists and is of class C^{k-1} almost everywhere.

Q.E.D.

Now, what happens if the rational expectations representation admits many equilibria. (Because $\sum_i \xi_{1i}$ is regular in $\overline{p}_1$, each one is locally unique.) For each given $\sigma = (p_0, \beta, (A_{0j}))$ there exists at least one equilibrium but it can also happen that many equilibria exist. Because the total future demand $\sum_i \xi_{1i}$ is still a regular mapping, we can still express locally $\overline{p}_1$ as a function of σ but now we have many expectation functions, let us say, $\overline{p}_{1a} = \psi_a^*(.)$, $\overline{p}_{1b} = \psi_b^*(.)$, and $\overline{p}_{1c} = \psi_c^*(.)$ each of them being defined on an open neighborhood of σ. This looks like the end of any possible reconciliation but it is its achievement. Indeed, the plurality of equilibria is equivalent to the plurality of expectation functions: the coordination failure of the rational expectations hypothesis (agents may choose different equilibria) is translated into a plurality of expectation functions (agents can be endowed with different expectation functions). Conversely, the harmonization problem can be attacked from both fronts.

We shall now characterize the local structure of the rational expectations function ψ^* in order to scrutinize the rational expectations hypothesis itself in its strong form.

Proposition 2 *Under the assumptions of Proposition 1, the effects of a variation in current prices and initial credit positions on expected future prices are the image (by $\tilde{K}_{11}^{-1}$) of their intertemporal effects:*

a)

$$\frac{\partial \psi^*}{\partial p_0} = -\frac{1}{\beta}\tilde{K}_{11}^{-1}\tilde{K}_{10} \tag{13}$$

b)

$$\frac{\partial \psi^*}{\partial \beta} = -\frac{1}{\beta}\tilde{K}_{11}^{-1}[\tilde{K}_{11}\overline{p}_1] = -\frac{1}{\beta}\overline{p}_1 \tag{14}$$

c)

$$\frac{\partial \psi^*}{\partial A_{0j}} = -\frac{1}{\beta}\tilde{K}_{11}^{-1}\frac{\partial \xi_{1j}}{\partial R_j} \qquad j = 1, 2, ..., m. \tag{15}$$

Consequently the rational expectations function ψ^* is homogeneous of degree one in p_0 and (A_{0j}) and homogeneous of degree zero in $(p_0, \beta, (A_{0j}))$.

In order to translate the previous characterization in terms of pure substitution effects a lemma is needed to characterize the indirect utility function.

Lemma 1 *Let V_i be an intertemporal indirect utility function and v_i its counterpart when rational expectations functions are substituted into it. One has*

$$v_i(p_0, \beta, (A_{0j})) = V_i(p_0, \beta\psi_1^*, p_0\omega_{0i} + \beta\psi_1^*\omega_{1i} + A_{0i})$$

and then,

$$\frac{\partial v_i}{\partial p_0} = -\lambda_i[z'_{0i} - z'_{1i}\tilde{K}_{11}^{-1}\tilde{K}_{10}]$$

$$\frac{\partial v_i}{\partial \beta} = 0$$

$$\frac{\partial v_i}{\partial A_{0i}} = \lambda_i[1 + z'_{1i}\tilde{K}_{11}^{-1}\frac{\partial \xi_{1i}}{\partial R_i}]$$

$$\frac{\partial v_i}{\partial A_{0j}} = \lambda_i z'_{1i}\tilde{K}_{11}^{-1}\frac{\partial \xi_{1j}}{\partial R_j}$$

where $\lambda_i = \frac{\partial V_i}{\partial R_i}$, $z_{0i} = x_{0i} - \omega_{0i}$, $z_{1i} = x_{1i} - \omega_{1i}$.

Proof: One has

$$\frac{\partial v_i}{\partial p_0} = \frac{\partial V_i}{\partial p_0} + \frac{\partial V_i}{\partial p_1}\beta\frac{\partial \psi^*}{\partial p_0} + \frac{\partial V_i}{\partial R_i}\omega_0 + \frac{\partial V_i}{\partial \beta}\beta\frac{\partial \psi^*}{\partial p_0}\omega_1.$$

By Roy identities, this can be written

$$\frac{\partial v_i}{\partial p_0} = -\lambda_i x'_{0i} - \lambda_i x'_{1i}\beta\frac{\partial \psi^*}{\partial p_0} + \lambda_i\omega'_{0i} + \lambda_i\beta\frac{\partial \psi^*}{\partial p_0}\omega'_{1i}$$

and, by equation (13),

$$\frac{\partial v_i}{\partial p_0} = -\lambda_i[z'_{0i} - z'_{1i}\tilde{K}_{11}^{-1}\tilde{K}_{10}].$$

Q.E.D.

The other relations are derived accordingly.

Proposition 3 *Under the assumptions of Proposition 1, the effects of a variation in current prices and initial credit position on expected future prices can be written in terms of their pure substitution effects*

a)

$$\frac{\partial\psi^*}{\partial p_0} = -\frac{1}{\beta}K_{11}^{-1}[K_{10} + \sum_i \frac{\partial\xi_1}{\partial R_i}\frac{1}{\lambda_i}\frac{\partial v_i}{\partial p_0}] \tag{16}$$

b)

$$\frac{\partial\psi^*}{\partial\beta} = -\frac{1}{\beta}\overline{p}_1 \tag{17}$$

c)

$$\frac{\partial\psi^*}{\partial A_{0j}} = -\frac{1}{\beta}K_{11}^{-1}\sum_i \frac{\partial\xi_1}{\partial R_i}\frac{1}{\lambda_i}\frac{\partial v_i}{\partial A_{0j}} \tag{18}$$

where

$$K_{11} = \sum_i[\frac{\partial\xi_{1i}}{\partial p_1} + \frac{\partial\xi_{1i}}{\partial R_i}x'_{1i}] = \tilde{K}_{11} + \sum_i \frac{\partial\xi_{1i}}{\partial R_i}z'_{1i}$$

$$K_{10} = \sum_i[\frac{\partial\xi_{1i}}{\partial p_0} + \frac{\partial\xi_{1i}}{\partial R_i}x'_{0i}] = \tilde{K}_{10} + \sum_i \frac{\partial\xi_{1i}}{\partial R_i}z'_{0i}$$

represent pure substitution effects.

Proof: One has

$$K_{11} = [I + \sum_j \frac{\partial\xi_{1j}}{\partial R_j}z'_{1j}\tilde{K}_{11}^{-1}]\tilde{K}_{11}$$

and

$$\tilde{K}_{11}^{-1} = K_{11}^{-1}[I + \sum_j \frac{\partial\xi_{1j}}{\partial R_j}z'_{1j}\tilde{K}_{11}^{-1}].$$

Therefore (13) can be written

$$\frac{\partial\psi^*}{\partial p_0} = -\frac{1}{\beta}K_{11}^{-1}[I + \sum_j \frac{\partial\xi_{1j}}{\partial R_j}z'_{1j}\tilde{K}_{11}^{-1}]\tilde{K}_{10}$$

$$\frac{\partial\psi^*}{\partial p_0} = -\frac{1}{\beta}K_{11}^{-1}\tilde{K}_{10} - \frac{1}{\beta}K_{11}^{-1}\sum_j \frac{\partial\xi_{1j}}{\partial R_j}z'_{1j}\tilde{K}_{11}^{-1}\tilde{K}_{10}$$

$$\frac{\partial\psi^*}{\partial p_0} = -\frac{1}{\beta}K_{11}^{-1}K_{10} + \frac{1}{\beta}K_{11}^{-1}\sum_j \frac{\partial\xi_{1j}}{\partial R_j}z'_{0j} - \frac{1}{\beta}K_{11}^{-1}\sum_j \frac{\partial\xi_{1j}}{\partial R_j}z'_{1j}\tilde{K}_{11}^{-1}\tilde{K}_{10}$$

and, by Lemma 1,

$$\frac{\partial \psi^*}{\partial p_0} = -\frac{1}{\beta} K_{11}^{-1} K_{10} - \frac{1}{\beta} K_{11}^{-1} \sum_j \frac{\partial \xi_{1j}}{\partial R_j} \frac{1}{\lambda_j} \frac{\partial v_i}{\partial p_0}.$$

This is (16). The other relations are derived accordingly. Q.E.D.

3 THE TEMPORARY DEMAND SYSTEM

The results of the two previous sections can now be utilized in order to characterize the complete system of current demands under rational expectations.

Let us consider equations (6) and (7). After substituting the rational expectations function ψ^*, we differentiate with respect to p_0. One has

$$\frac{\partial x_0}{\partial p_0} = \tilde{K}_{00} + \beta \tilde{K}_{01} \frac{\partial \psi^*}{\partial p_0} \tag{19}$$

where the price effects on the current total quantities ($x_0 = \sum_i x_{0i}$) involve a spill-over effect from the future.

Using Proposition 2, this can be written in terms of gross substitution effects:

$$\frac{\partial x_0}{\partial p_0} = \tilde{K}_{00} - \tilde{K}_{01} \tilde{K}_{11}^{-1} \tilde{K}_{10}. \tag{20}$$

Using Proposition 3, it can also be written in terms of pure substitution effects:

$$\frac{\partial x_0}{\partial p_0} = K_{00}^* + \sum_j \frac{1}{\lambda_j} k_j^* \frac{\partial v_j}{\partial p_0}$$

where $K_{00}^* = K_{00} - K_{01} K_{11}^{-1} K_{10}$ and $k_j^* = \frac{\partial \xi_{0j}}{\partial R_j} - K_{01} K_{11}^{-1} \frac{\partial \xi_{1j}}{\partial R_j}$.

This last relation is interesting for two reasons. First, K_{00}^* and k_j^* can be seen respectively as constrained (or conditional) substitution effects and the income effects.[2] Both effects are additive as $p_0' K_{00}^* = 0$ and $p_0' k_j^* = 1$ and the

[2]That these substitution effects are constrained (or conditional) is not really surprising as the perception of a given ω_1 and of a future equilibrium amounts to internalize some kind of rationing. This internalization appears in K_{00}^* where intertemporal substitution effects K_{10} are transformed into intratemporal substitution effects via the spill-over effects $K_{01} K_{11}^{-1}$.

matrix K^*_{00} is symmetric and negative semidefinite (this last property is not asserted at the individual level). Second, it looks like the aggregate standard Slutsky decomposition but is not. Indeed, using Lemma 1, the income effect itself can be decomposed into a current part and an expected one:

$$\frac{\partial x_0}{\partial p_0} = K^*_{00} - \sum_j k^*_j z'_{0j} + \sum_j k^*_j z'_{1j} \tilde{K}^{-1}_{11} \tilde{K}_{10}.$$

(The first income effect being associated to the Sonnenschein-Mantel-Debreu arbitrariness, the second one defines a new kind of arbitrariness.)

These considerations are formally represented in

Proposition 4 *The current total price effects admit the following decompositions*

$$\frac{\partial x_0}{\partial p_0} = K^*_{00} + \sum_j \frac{1}{\lambda_j} k^*_j \frac{\partial v_j}{\partial p_0} \tag{21}$$

$$\frac{\partial x_0}{\partial p_0} = K^*_{00} - \sum_j k^*_j [z'_{0j} - z'_{1j} \tilde{K}^{-1}_{11} \tilde{K}_{10}] \tag{22}$$

$$\frac{\partial x_0}{\partial p_0} = K^*_{00} - \sum_j k^*_j z'_{0j} + \sum_j k^*_j z'_{1j} \tilde{K}^{-1}_{11} \tilde{K}_{10}. \tag{23}$$

In these relations, the matrix K^*_{00} is symmetric, additive and negative semidefinite:

$$K^*_{00} = [K^*_{00}]', K^*_{00} p_0 = 0, \zeta'_0 K^*_{00} \zeta_0 < 0 for \zeta_0 \neq \theta p_0, \theta \in \Re. \tag{24}$$

The vectors k^*_j are additive:

$$p'_0 k^*_j = 1, \qquad j = 1, 2, ...m. \tag{25}$$

In equation (23) a current price effect decomposes into a pure substitution effect, a current wealth (or real balance) effect and an expected real wealth effect. The expected real wealth effect ($\sum_j k^*_j z'_{1i} \tilde{K}^{-1}_{11} \tilde{K}_{10}$) is adding[3] *a new kind of arbitrariness to the Sonnenschein-Mantel-Debreu arbitrariness associated with the current wealth effect ($\sum_j k^*_j z'_{0j}$).*

[3]Of course this addition of a new term does not necessarily imply that arbitrariness is increasing. On the contrary, in a compensated equilibrium (see Bronsard and Salvas-Bronsard (1988)), each one of these effects is just the additive inverse of the other one.

The links with the standard neoclassical results can be studied with the help of the three following corollaries.

Corollary 1 *If the expected future contingent equilibrium is a no-trade equilibrium (that is, if $z_{1i} = 0$ for any i) then, in the aggregate, the price decomposition behaves as usual:*

$$\frac{\partial x_0}{\partial p_0} = K^*_{00} - \sum_j k^*_j z'_{0j}. \tag{26}$$

Consequently, the price effect is arbitrary only if the Sonnenschein-Mantel-Debreu condition, $m > n_0$, is fulfilled (m being the number of consumers and n_0 the number of current commodities).

Corollary 2 *If the current temporary equilibrium is a no-trade equilibrium (that is, if $z_{0i} = 0$ for any i), then, in the aggregate, the price decomposition is written*

$$\frac{\partial x_0}{\partial p_0} = K^*_{00} + \sum_j k^*_j z'_{1j} \tilde{K}^{-1}_{11} \tilde{K}_{10} \tag{27}$$

and, consequently, the price effect is arbitrary only if a) the Sonnenschein-Mantel-Debreu condition is fulfilled and b) the Polemarchakis condition, $n_1 > n_0$, is fulfilled.

Corollary 3 *Suppose that the constrained income effects are identical across individuals (that is, there exists k^* such that $k^*_j = k^*$ for any j) then, in the aggregate, the price decomposition behaves as the usual individual price decomposition:*

$$\frac{\partial x_0}{\partial p_0} = K^*_{00} - k^* \sum_j z'_{0j} = K^*_{00} - k^* z'_0 \tag{28}$$

and consequently the econometric structural forms based on standard static results can be rationalized in a temporary context.(Such structural forms can be found in Barten (1967), Barten and Bettendorf (1989), Deaton and Muellbauer (1980) and, more recently, in Hildenbrand (1994))

The general meaning of Proposition 4 and of its three Corollaries implies that even if the future can be conceived as a contingent equilibrium, the corresponding temporary equilibrium is not generally equivalent to a static equilibrium. This temporary equilibrium contains some dynamic elements, namely the expected effect of variations in real income which are not wiped off by the rational expectations hypothesis. They are exhibited in Proposition 4 and cannot easily be ignored as seen in Corollaries 1, 2 and 3.

In order to complete the above characterization of the temporary demand system, (6) is differentiated relative to A_{0j} leading to

$$\frac{\partial x_0}{\partial A_{0j}} = \frac{\partial \xi_0}{\partial R_j} + \beta \tilde{K}_{01} \frac{\partial \psi^*}{\partial A_{0j}} = \frac{\partial \xi_0}{\partial R_j} - \tilde{K}_{01} \tilde{K}_{11}^{-1} \frac{\partial \xi_1}{\partial R_j}$$

and, using Proposition 3,

$$\frac{\partial x_0}{\partial A_{0j}} = \sum_i \frac{1}{\lambda_i} k_i^* \frac{\partial v_i}{\partial A_{0j}}. \tag{29}$$

From this equation, the initial wealth effect appears as a generalization of the traditional income effect.

In the same way, it is easy to prove that the discount factor has no effect on current demand[4] :

$$\frac{\partial x_0}{\partial \beta} = 0. \tag{30}$$

Finally, and in summary, considering (21), (29) and (30), we have

$$dx_0 = K_{00}^* dp_0 + \sum_j \frac{k_j^*}{\lambda_j} dv_j \tag{31}$$

which in turn, implies Proposition 4. Consequently, equations (31), (24) and (25) characterize the local structure of the temporary complete demand system. In this system, we do not have to consider A_1 because, at the aggregate level, $\sum_i A_{1i} = 0$, by equation (7), as a consequence of the rational expectations hypothesis and the private ownership convention. This is a restatement of the dichotomy between financial and real elements.

[4]The macroeconomic intertemporal substitution effects of the discount factor vanish. They are "killed" by their perfect integration into the present through K_{00}^* which includes the spill-over effects $K_{01}K_{11}^{-1}$.Grandmont (1983) can be interpreted as the negation of such a perfect integration.

4 Conclusion

As a conclusion, let us remark that Proposition 4 and its three Corollaries define a new gap between the microeconomic representation and the macroeconomic one. This gap can be summarized as follows.

At the individual level, as seen in Lemma 1, the Roy identities are generalized so as to incorporate the relevant spill-over effects,

$$\frac{1}{\lambda_i}\frac{\partial v_i}{\partial p_0} = -[z'_{0i} - z'_{1i}\tilde{K}_{11}^{-1}\tilde{K}_{10}]$$

$$\frac{1}{\lambda_i}\frac{\partial v_i}{\partial A_{0i}} = [1 + z'_{1i}\tilde{K}_{11}^{-1}\frac{\partial \xi_{1i}}{\partial R_i}]$$

$$\frac{1}{\lambda_i}\frac{\partial v_i}{\partial A_{0j}} = z'_{1i}\tilde{K}_{11}^{-1}\frac{\partial \xi_{1j}}{\partial R_j}.$$

These spill-over effects vanish at the aggregate level and "Roy-like identities" do appear

$$\sum_i \frac{1}{\lambda_i}\frac{\partial v_i}{\partial p_0} = -z'_0$$

$$\sum_i \frac{1}{\lambda_i}\frac{\partial v_i}{\partial A_{0j}} = 1.$$

In words: at the aggregate level, things behave as usual but at the individual level the sign patterns of the indirect utility function is not necessarily the opposite of the individual excess demand function. This gap has to be further examined but such a project is beyond the scope of this paper. For the moment we shall only illustrate how it completes the theory.

In a recent paper, Balasko (1994) has studied the expectational stability of Walrasian equilibria and found (by way of numerical examples) that expectational stability is different from tatonnement and Hicksian stability. This is in conformity with the result presented here. Studying expectational stability amounts to study equation (20). Because equation (20) cannot in general be reduced to equation (26), it is clear that expectational stability involves new elements not appearing in the tatonnement and Hicksian stability. Expressed in the light of the gap just mentioned above, this also means that a Hahn process cannot be defined as usual.

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