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On the Optimal Order of Natural Resource Use When the Capacity of the Inexhaustible Substitute is Limited*

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Résumé

Nous étudions, dans un contexte d'équilibre général, l'ordre d'exploitation de plusieurs gisements d'une ressource naturelle épuisable qui ne diffèrent que par leur coût marginal d'exploitation. Ce coût marginal est supposé constant lorsqu'exprimé en terme de quantité d'une ressource inépuisable, laquelle constitue un parfait substitut pour la ressource épuisable. Quoique ce substitut inépuisable puisse être exploité indéfiniment, son taux instantané d'exploitation est sujet à une contrainte de capacité. Nous démontrons que non seulement peut-il être optimal d'entamer un gisement d'une ressource à coût élevé avant que ne soit épuisé un gisement à coût faible, comme l'ont démontré Kemp et Long (1980), mais il peut être optimal de l'entamer avant même que ne soit entamé le gisement moins coûteux. Il s'ensuit que le principe que suggère l'analyse d'équilibre partiel dans ce genre de problème, à l'effet que si le taux d'actualisation est positif, les ressources naturelles doivent nécessairement être exploitées en ordre strictement croissant de coût, non seulement ne tient pas, mais peut être complètement inversé.

Mots clés: Ressources épuisables, ressources inépuisables, contrainte de capacité, ordre d'exploitation.

Abstract

Consider a general equilibrium framework where the marginal cost of extraction from several deposits of an exhaustible resource is constant in terms of an inexhaustible perfect substitute and differs between deposits. The instantaneous rate of production from the inexhaustible resource is subject to a capacity constraint. We show, under standard assumptions, that not only may it be optimal to begin using a high cost resource before a lower cost one is depleted, as shown in Kemp and Long (1980), but it may be optimal to begin using it strictly before the lower cost one is even put into use. Thus the intuitive principle, derived from partial equilibrium analysis, that when the rate of discount is positive natural resources should always be exploited in strictly increasing order of costs, not only does not hold in a general equilibrium context, but may be totally reversed.

Keywords: Exhaustible resource, inexhaustible resource, capacity constraint, order of exploitation.

1 Introduction

The intuitive principle that, with a positive rate of discount, natural resource deposits should always be exploited in strictly increasing order of marginal costs has been shown by Kemp and Long (1980) not to hold in a general equilibrium framework[1]. They show that if the marginal cost of extraction is constant in terms of a inexhaustible perfect substitute, then it may be optimal to exploit both high and low cost exhaustible resource deposits simultaneously. Furthermore, they show that it may be optimal to begin using the inexhaustible substitute before having completely drawn down the exhaustible resource, although its cost is higher than that of even the highest cost exhaustible resource deposit. Their results therefore show that a strict sequencing in order of cost is not a necessary condition for optimality. This seems to leave intact a weaker formulation of the principle, which would be that a high cost resource must never be put into use *before* the lower cost ones have also been put into use. We show that this weaker version of the principle also fails. In other words, it may be optimal, in some entirely plausible situations, to begin using a high cost resource strictly before some lower cost one, thus totally reversing the intuition obtained from partial equilibrium analysis.

We show this by constructing an example of an optimal program where the inexhaustible substitute is put into use strictly before some lower cost deposit of the exhaustible resource. We make one fundamental change to the model of Kemp and Long, which consists in assuming that the productive capacity of the inexhaustible resource is finite. This seems like a natural assumption: an inexhaustible resource may be capable of indefinitely generating some output, but the rate of output will itself generally be limited by some capacity constraint. To fix ideas, one may think of the exhaustible resource as oil, from which energy can be produced. The inexhaustible resource may be thought of as a river that flows indefinitely and can be damned to also

[1]The first explicit statement of this principle in a partial equilibrium framework is due to Herfindahl (1967). It was also developed more formally by Solow and Wan (1976). More recently, Chakravorty and Krulce (1994) have shown that, in a partial equilibrium context, if the resources drawn from two deposits of exhaustible resources are differentiated on the demand side as well as on the cost side, then it may be optimal to exploit the two deposits simultaneously over some interval of time. On the other hand, Lewis (1982) has shown that in the general equilibrium model of Kemp and Long, if the resource can be converted into capital which may either be consumed or stored for future consumption, and this capital grows while stored, then the optimality of a strict sequencing of extraction is reestablished.

produce energy. Our assumption is that the production capacity of the river is fixed and finite. When this constraint is binding, there arises an incentive to make intertemporal tranfers which may only be made by conserving some of the exhaustible resource. We show that this incentive may lead to delaying the opening of a deposit of the exhaustible resource to the point of beginning the production from the more costly inexhaustible resource before that from the deposit.

We present the details of the model in the next section. We will restrict attention to the case of two deposits of an exhaustible resource and one inexhaustible resource, since this is sufficient to prove our point. It will be clear to the reader that our result is easily extended to the case of more than two exhaustible resource deposits. In section 3, we develop the method of construction of the solution in the case where the economy has access to only one deposit of the exhaustible resource, along with the higher cost inexhaustible substitute. We then use the results of that section to show, in section 4, that when the economy has access to (at least) two deposits of the exhaustible resource, each of different extraction cost but each less costly than the inexhaustible substitute, then there exists some nondegenerate configuration of initial stocks for which it is optimal to keep the more costly of the two deposits intact until after the inexhaustible substitute has been brought into use. We conclude in section 5 with a discussion of the intuition behind this result.

2 The model

We pose the problem in a general equilibrium framework similar to that of Kemp and Long. The economy has access to two deposits of an exhaustible resource and a flow of some inexhaustible substitute. Labor is used either to draw from the exhaustible resource deposits or from the flow of the inexhaustible resource. Like Kemp and Long, we assume that the inexhaustible substitute can indefinitely yield a continuous flow of output. Unlike them we assume that this flow is bounded from above, which imposes a capacity constraint to its instantaneous rate of production. The production capacity of the inexhaustible resource is not storable. This means that whatever part of it is not used at some instant of time is lost forever.

To extract one unit from deposit i requires μ_i units of labor. To produce one unit from the flow of inexhaustible resource requires η units of labor. We assume $\eta > \mu_2 > \mu_1$. We will denote by $Y_i(t)$ the stock remaining in deposit

i at time t. The amount extracted from deposit i at instant t will be denoted by $y_i(t)$ and that produced from the inexhaustible substitute by $x(t) \leq \bar{x}$. The amount $\bar{x}$ is the production capacity of the inexhaustible substitute. If $c(t)$ denotes the total consumption and $w(t)$ the amount of labor required to produce it, then $c(t) = x(t) + \sum_{i=1}^{2} y_i(t)$ and $w(t) = \eta x(t) + \sum_{i=1}^{2} \mu_i y_i(t)$. We will assume that population is constant and normalized to one, so that $c(t)$ and $w(t)$ also denote per capita quantities. Hence, $w(t) \in [0, 1]$.

Consumers are identical and their utility function is given by $u(c(t), l(t))$, where $l(t) = 1 - w(t)$ is leisure per capita[2]. The utility function is twice continuously differentiable, strictly increasing in c and l, strictly concave and bounded from above. It also satisfies $\lim_{l \downarrow 0} u_l(c, l) = +\infty$ and $\lim_{c \downarrow 0} u_c(c, l) = +\infty$. Note that these assumptions imply that the isocline curves, in the (c, l) plane are increasing: the locus $I^\sigma = \{(c, l) | u_c(c, l)/u_l(c, l) = \sigma, \sigma > 0\}$ may be seen as the graph of some function $l^\sigma : (0, +\infty) \rightarrow (0, 1)$ such that $dl^\sigma/dc > 0$.

We will assume that the inexhaustible resource is scarce, in the sense that

$$u_c(\bar{x}, \bar{l}) - \eta u_l(\bar{x}, \bar{l}) > 0,$$

where $\bar{l} = 1 - \eta\bar{x}$. The stationary state which will prevail after exhaustion of the exhaustible resource is constrained by the flow of the inexhaustible resource. Let T be the date of exhaustion of the exhaustible resource and let $\bar{U}(T)$ denote the present value of the stationary program $(\bar{x}, \bar{l})$ beginning at T. Thus

$$\bar{U}(T) = e^{\rho T} \frac{u(\bar{x}, \bar{l})}{\rho}$$

where $\rho > 0$ is the rate of discount.

The problem of the planner who seeks to maximize total welfare can therefore be stated as

$$\max_{T, \{x(t), y_1(t), y_2(t)\}_{t=0}^{T}} \int_0^T e^{-\rho t} u(c(t), l(t)) dt + \bar{U}(T)$$

subject to

$$c(t) = x(t) + \sum_{i=1}^{2} y_i(t) \text{ and } l(t) = 1 - \eta x(t) + \sum_{i=1}^{2} \mu_i y_i(t) \qquad (1)$$

[2] Contrary to Kemp and Long, we do not restrict attention to the case of a perfectly inelastic supply of labor. Hence it is clear that our result (as, for that matter, that of Kemp and Long), does not depend on such a restrictive assumption.

$$\dot{Y}_i(t) = -y_i(t), \quad i = 1,2 \tag{2}$$

$$y_i(t) \geq 0 \quad i = 1,2 \tag{3}$$

$$Y_i(0) = Y_i^0 > 0 \quad \text{and} \quad Y_i(t) \geq 0, \quad i = 1,2 \tag{4}$$

$$\bar{x} - x(t) \geq 0 \quad \text{and} \quad x(t) \geq 0 \tag{5}$$

If we denote by $\lambda_i(t)$ the costate variable associated with the stock of the exhaustible resource i, then the Lagrangian associated with the maximization of the Hamiltonian at each instant t may be written

$$\begin{aligned}\mathcal{L} &= e^{-\rho t}u(x(t) + \sum_{i=1}^{2} y_i(t), 1 - \eta x(t) + \sum_{i=1}^{2} \mu_i y_i(t)) - \sum_{i=1}^{2} \lambda_i(t) y_i(t) \\ &\quad + \underline{\alpha}(t)x(t) + \overline{\alpha}(t)(\bar{x} - x(t)) + \sum_{i=1}^{2} \nu_i(t) y_i(t) - \sum_{i=1}^{2} \delta_i(t) y_i(t)\end{aligned}$$

where the multipliers $\nu_i(t)$ and $\delta_i(t)$ relate respectively to the nonnegativity constraints on $y_i(t)$ and $Y_i(t)$ and the interpretation of the multipliers $\underline{\alpha}(t)$ and $\overline{\alpha}(t)$ is self-evident.

An optimal program $\{x^*(t), y_i^*(t), i = 1,2\}_0^T$ must satisfy, for all $t \in [0, T^*]$, the following first-order conditions[3]:

$$e^{-\rho t}[u_c(c^*(t), l^*(t))) - \eta u_l(c^*(t), l^*(t))] + \underline{\alpha}(t) - \overline{\alpha}(t) = 0 \tag{6}$$

$$e^{-\rho t}[u_c(c^*(t), l^*(t))) - \mu_i u_l(c^*(t), l^*(t))] - \lambda_i(t) + \nu_i(t) - \delta_i(t) = 0 \tag{7}$$

together with the complementary slackness conditions

$$\underline{\alpha}(t)x^*(t) = 0 \quad \text{and} \quad \underline{\alpha}(t) \geq 0 \tag{8}$$

$$\overline{\alpha}(t)[\bar{x} - x^*(t)] = 0 \quad \text{and} \quad \overline{\alpha}(t) \geq 0 \tag{9}$$

$$\nu_i(t) y_i^*(t) = 0 \quad \text{and} \quad \nu_i(t) \geq 0 \tag{10}$$

$$\delta_i(t) Y_i^*(t) = 0, \ -\delta_i(t) y_i^*(t) = 0 \quad \text{and} \quad \delta_i(t) \geq 0 \tag{11}$$

as well as

$$\dot{\lambda}_i(t) = 0 \tag{12}$$

and, at T^*, the transversality condition

$$e^{-\rho T} u(c^*(T^*), l^*(T^*)) - \sum_{i=1}^{2} \lambda_i(T^*) y_i^*(T^*) = d\bar{U}(T^*)/dT. \tag{13}$$

[3] Along with (2), these conditions are also sufficient in the present context.

For ease of exposition, we will denote in what follows by $v^r(x(t), y(t))$ and by $v^i(x(t), y(t))$ the marginal net benefits of respectively the inexhaustible resource and the exhaustible resource i, where $y(t) = (y_1(t), y_2(t))$. That is

$$v^r(x(t), y(t)) = u_c(c(t), l(t))) - \eta u_l(c(t), l(t))$$

$$v^i(x(t), y(t)) = u_c(c(t), l(t))) - \mu_i u_l(c(t), l(t)).$$

To investigate the properties of the optimal program and illustrate how the solution can be constructed, we will consider first the case where only one deposit (deposit i) of the exhaustible resource is available. Consequently, in that case we write, by a slight abuse of notation, $v^h(x(t), y_i(t))$, $h \in \{1, 2, r\}$, instead of $v^h(x(t), (y_i(t), 0))$.

To illustrate the properties of the optimal program, we will refer frequently either to Figure 1 (in the case of one deposit of the exhaustible resource), or to Figure 2 (in the case of two deposits). On these figures, the straight line F_i is the instantaneous production frontier in the (c, l) plane if only deposit i is being exploited. Similarly, the line F_r is the production frontier when only the inexhaustible resource is being used. Assuming that the inexhaustible resource is scarce means first that only the upper part of the frontier F_r (the segment $[(0, 1), (\bar{x}, \bar{l})]$) is relevant and, second, that at $(\bar{x}, \bar{l})$ the slope of the indifference curve is greater in absolute value than the slope of F_r. For any isocline I^σ, we denote by $A_h^\sigma = (c_h^\sigma, l_h^\sigma)$, $h \in \{1, 2, r\}$, the point at which the isocline intersects the frontier F_h. Finally, $\bar{F}_i$ is the frontier when the inexhaustible resource and deposit i of the exhaustible resource are being exploited simultaneously, with the inexhaustible resource being exploited at full capacity. We denote by $\bar{A}_h^\sigma = (\bar{c}_h^\sigma, \bar{l}_h^\sigma)$ the point at which the isocline I^σ intersects $\bar{F}_i$.

3 The case of one exhaustible resource deposit and one inexhaustible resource

Consider the situation where only one deposit of the exhaustible resource is available, along with the inexhaustible substitute, and begin by assuming that only the exhaustible resource is exploited. From conditions (11) and (12), we know that $\lambda_i(t)$ is a constant. Denote this constant λ_i. Then, from (7), the first-order condition that determines $y_i(t)$ is

$$v^i(0, y_i(t)) = e^{\rho t}\lambda_i. \tag{14}$$

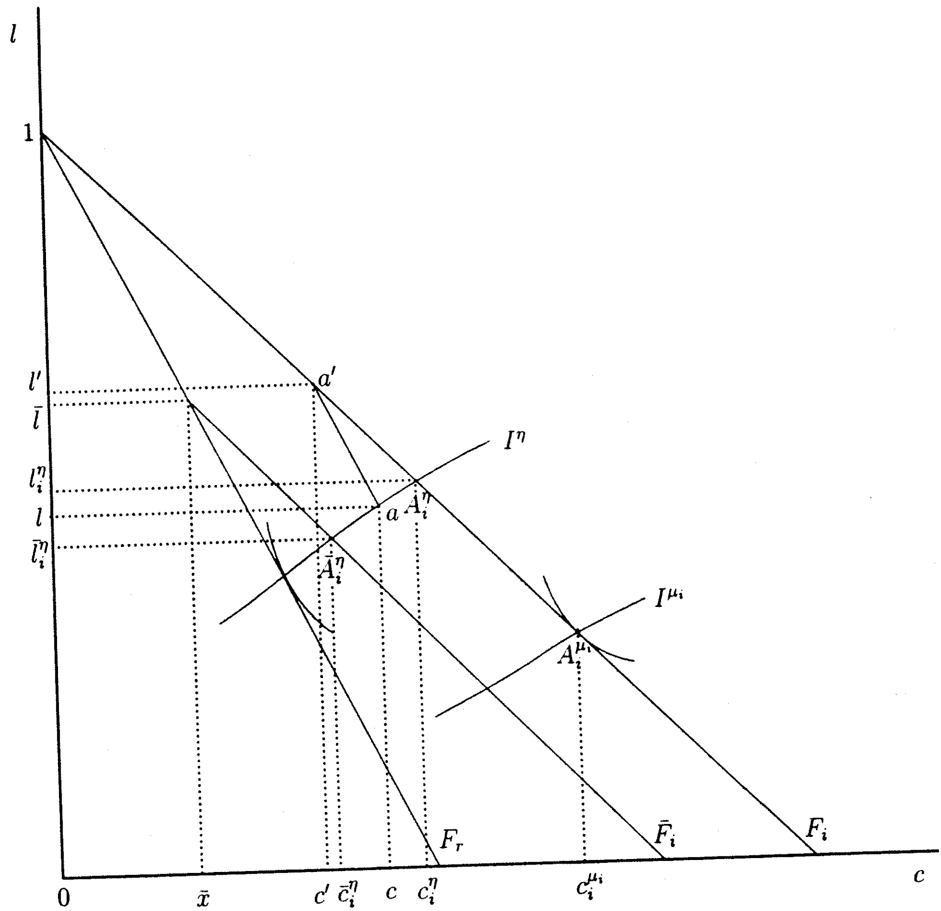

Figure 1: One exhaustible resource deposit

For any $\lambda_i \in (0, (\eta - \mu_i)u_l(c_i^\eta, l_i^\eta))$, define the interval $[0, t^a(\lambda_i))$, where $t^a(\lambda_i)$ is the solution of $e^{\rho t}\lambda_i = (\eta - \mu_i)u_l(c_i^\eta, l_i^\eta)$ and let $y_i^a(t, \lambda_i)$ be the unique solution of (14) on this interval. Hence at $t = t^a(\lambda_i)$, we have $u_c(c(t), l(t))/u_l(c(t)$ η. A glance at Figure 1 could be useful. Note that along the F_i frontier, from $A_i^{\mu_i}$ to $(0, 1)$, the marginal rate of substitution between leisure and consumption (u_c/u_l) increases from μ_i to $+\infty$, going through η at A_i^η, where the isocline I^η crosses the straight line F_i. Should λ_i have been set to 0, we would have had $y_i^a(t, \lambda_i) = c_i^{\mu_i}$ for all $t \geq 0$. For any $\lambda_i \in (0, (\eta - \mu_i)u_l(c_i^\eta, l_i^\eta))$, the per capita leisure-consumption path, $\{1 - \mu_i y_i^a(t, \lambda_i), y_i^a(t, \lambda_i)\}_0^{t^a(\lambda_i)}$ moves up over time along the frontier F_i, starting from some point between $A_i^{\mu_i}$ and A_i^η, and attaining A_i^η at time $t = t^a(\lambda_i)$.

Suppose now that both resources are exploited but that $x(t) < \bar{x}$. Then the first-order conditions are

$$v^r(x(t), y_i(t)) = 0 \tag{15}$$

and

$$v^i(x(t), y_i(t)) = e^{\rho t}\lambda_i. \tag{16}$$

Condition (15) implies that

$$MRS(t) = \frac{u_c(c(t), l(t))}{u_l(c(t), l(t))} = \eta.$$

In Figure 1, consider the curve segment $[\bar{A}_i^\eta, A_i^\eta]$. This segment corresponds to that part of the isocline I^η between the frontiers $\bar{F}_i$ and F_i. A point (c, l) interior to this segment corresponds to a situation where the two resources are being utilized in some uniquely defined proportion, with the inexhaustible resource being utilized at less than full capacity. For example, point a in Figure 1 is obtained by exploiting the quantity $c - c'$ of the inexhaustible resource and the quantity c' of the exhaustible resource, the slope of the line segment $[a, a']$ being equal to $-\eta$.

For a point $(c, l) = (c, l^\eta(c))$ on the segment $[\bar{A}_i^\eta, A_i^\eta]$, where, we recall, $l^\eta(c)$ denotes per capita leisure as a function of per capita consumption along the isocline I^η, let us define $x^{i\eta}(c)$ and $y_i^\eta(c)$ as the quantities consumed of the inexhaustible and the exhaustible resource that are the unique solution to

$$x^{i\eta}(c) + y_i^\eta(c) = c$$

$$1 - \eta x^{i\eta}(c) + \mu_i y_i^\eta(c) = l^\eta(c).$$

The function $x^{i\eta}(c)$ takes the value $\bar{x}$ when $c = \bar{c}_i^\eta$ and 0 when $c = c_i^\eta$ and decreases monotically on that interval. The function $y_i^\eta(c)$ increases monotically from $\bar{c}_i^\eta$ to c_i^η on the same interval[4].

For any $\lambda_i \in (0, v^i(\bar{x}, \bar{c}_i^\eta - \bar{x}))$, consider the time interval $[\bar{t}^a(\lambda_i), t^b(\lambda_i))$, where

$$\bar{t}^a(\lambda_i) = \begin{cases} t^a(\lambda_i) & \text{if } \lambda_i < (\eta - \mu_i)u_l(c_i^\eta, l_i^\eta) \\ 0 & \text{otherwise,} \end{cases}$$

and $t^b(\lambda_i)$ is the solution of $v^i(\bar{x}, \bar{c}_i^\eta - \bar{x}) = e^{\rho t}\lambda_i$. On this interval, let $x^{ib}(t, \lambda_i) = x^{i\eta}(c(t, \lambda_i))$ and $y_i^b(t, \lambda_i) = y_i^\eta(c(t, \lambda_i))$, where $c(t, \lambda_i)$ is the unique solution to $v^i(x^{i\eta}(c), y_i^\eta(c)) = e^{\rho t}\lambda_i$. Therefore, over the interval $[\bar{t}^a(\lambda_i), t^b(\lambda_i))$, the per capita leisure-consumption path moves down the isocline I^η towards $\bar{A}_i^\eta$, from either A_i^η, if $\lambda_i \leq (\eta - \mu_i)u_l(c_i^\eta, l_i^\eta)$, or some point between A_i^η and $\bar{A}_i^\eta$, if $\lambda_i > (\eta - \mu_i)u_l(c_i^\eta, l_i^\eta)$.

Suppose finally that both resources are being exploited, but that the inexhaustible resource is exploited at full capacity, so that $x(t) = \bar{x}$. The first-order condition determining the extraction rate of the exhaustible resource then becomes

$$v^i(\bar{x}, y_i(t)) = e^{\rho t}\lambda_i. \tag{17}$$

For any $\lambda_i \in (0, v^i(\bar{x}, 0))$, consider the time interval $[\bar{t}^b(\lambda_i), t^c(\lambda_i))$, where

$$\bar{t}^b(\lambda_i) = \begin{cases} t^b(\lambda_i) & \text{if } \lambda_i < v^i(\bar{x}, \bar{c}_i^\eta - \bar{x}) \\ 0 & \text{otherwise,} \end{cases}$$

and $t^c(\lambda_i)$ is the solution to $v^i(\bar{x}, 0) = e^{\rho t}\lambda_i$. For any t in this time interval, define $y_i^c(t, \lambda_i)$ as the solution to $v^i(\bar{x}, y_i(t)) = e^{\rho t}\lambda_i$. Therefore the per capita leisure-consumption path moves over time along $\bar{F}_i$ from A_i^η if $\lambda_i \leq v^i(\bar{x}, \bar{c}_i^\eta - \bar{x})$, or some point between $\bar{A}_i^\eta$ and $(\bar{x}, \bar{l})$ otherwise, attaining $(\bar{x}, \bar{l})$ at $t = t^c(\lambda_i)$.

Now for any $\lambda_i \in (0, v^i(\bar{x}, 0))$, let

$$y_i(t, \lambda_i) = \begin{cases} y_i^a(t, \lambda_i) & \text{if } t \in [0, \bar{t}^a(\lambda_i)) \text{ and } \bar{t}^a(\lambda_i) > 0 \\ y_i^b(t, \lambda_i) & \text{if } t \in [\bar{t}^a(\lambda_i), \bar{t}^b(\lambda_i)) \text{ and } \bar{t}^b(\lambda_i) > 0 \\ y_i^c(t, \lambda_i) & \text{if } t \in [\bar{t}^b(\lambda_i), t^c(\lambda_i)) \end{cases}$$

and

$$x(t, \lambda_i) = \begin{cases} 0 & \text{if } t \in [0, \bar{t}^a(\lambda_i)) \text{ and } \bar{t}^a(\lambda_i) > 0 \\ x^{ib}(t, \lambda_i) & \text{if } t \in [\bar{t}^a(\lambda_i), \bar{t}^b(\lambda_i)) \text{ and } \bar{t}^b(\lambda_i) > 0 \\ \bar{x} & \text{if } t \in [\bar{t}^b(\lambda_i), t^c(\lambda_i)). \end{cases}$$

[4] $dy_i^\eta/dc = -dx_i^\eta/dc = [\eta + dl^\eta/dc]/(\eta - \mu_i) > 0$.

and let $Y_i(\lambda_i) = \int_0^{t^c(\lambda_i)} y_i(t, \lambda_i)dt$. Thus $Y_i(\lambda_i)$ denotes the initial size of the exhaustible resource deposit which sustains the extraction path $y_i(t, \lambda_i)$. The value of λ_i at the optimum, λ_i^*, must be the solution to $Y_i(\lambda_i) = Y_i^0$. There remains to verify that all the first-order conditions together with the complementary slackness conditions are satisfied, and not only those conditions already used to construct the different phases of the path.

If $\lambda_i^* < (\eta - \mu_i)u_l(c_i^\eta, l_i^\eta)$ and $t < t^a(\lambda_i^*)$, then the per capita leisure-consumption path follows some segment $[A_0, A_i^\eta)$ of F_i, with $A_0 \in (A_i^{\mu_i}, A_i^\eta)$. For any point (c, l) on this segment, $u_c/u_l < \eta$ and hence $u_c - \eta u_l < 0$. So let $\underline{\alpha}(t) = -(u_c^*(t) - \eta u_l^*(t))$, where $u_k^*(t) = u_k(y_i^a(t, \lambda_i^*), 1 - \mu_i y_i^a(t, \lambda_i^*))$, $k \in \{c, l\}$, and $\overline{\alpha}(t) = \nu_i(t) = \delta_i(t) = 0$. Then conditions (6) to (12) are all satisfied. If $\lambda_i^* < v^i(\bar{x}, \bar{c}_i^\eta - \bar{x})$ and $t \in [\bar{t}^a(\lambda_i^*), t^b(\lambda_i^*))$, then the first-order conditions (6) and (7) are satisfied by construction for $\underline{\alpha}(t) = \overline{\alpha}(t) = \nu_i(t) = \delta_i(t) = 0$. Finally, if $\lambda_i^* < v^i(\bar{x}, 0)$ and $t \in [\bar{t}^b(\lambda_i^*), t^c(\lambda_i^*))$, the per capita leisure-consumption path is moving up over time along the frontier $\bar{F}_i$ between $\bar{A}_i^\eta$ and $(\bar{x}, \bar{l})$, so that $u_c/u_l > \eta$ and hence $u_c - \eta u_l > 0$. So let $\underline{\alpha}(t) = 0$, $\overline{\alpha}(t) = u_c^*(t) - \eta u_l^*(t)$, where $u_k^*(t) = u_k(\bar{x} + y_i^c(t, \lambda_i), 1 - \eta\bar{x} + \nu_i y_i^c(t, \lambda_i^*))$, $k \in \{c, l\}$, $\nu_i(t) = \delta_i(t) = 0$. Then conditions (6) to (12) are all satisfied. Also, since at $t = t^c(\lambda_i^*)$, $y_i^c(t, \lambda_i^*) = 0$, then $c(t^c(\lambda_i^*)) = \bar{x}$ and $l(t^c(\lambda_i^*)) = \bar{l}$ and the transversality condition (13) is satisfied.

If the initial stock of the exhaustible resource is sufficiently high, that is if $Y_i^0 > Y_i((\eta - \mu_i)u_l(c_i^\eta, l_i^\eta))$, then the optimal per capita leisure-consumption path is constituted of four phases: the three phases just described, followed by a stationary phase $[t^c(\lambda_i^*), +\infty)$, that begins when the exhaustible resource is depleted and during which $c(t) = \bar{x}$ and $l(t) = 1 - \eta\bar{x}$. If $Y_i(v^i(\bar{x}, \bar{c}_i^\eta - \bar{x})) < Y_i^0 \leq Y_i((\eta - \mu_i)u_l(c_i^\eta, l_i^\eta))$, then the first phase, during which only the exhaustible resource is exploited is dropped. Finally, if $Y_i^0 < Y_i(v^i(\bar{x}, \bar{c}_i^\eta - \bar{x}))$, then the initial stock of the exhaustible resource is sufficiently low that the inexhaustible resource is exploited at full capacity from the start.

It is worth noting two characteristics of the optimal program. The first is that although the unitary labor cost of the inexhaustible resource is higher than that of the exhaustible resource, the exploitation phases always overlap. Such an overlap also occurs in Kemp and Long (1980). In fact, the case they consider is one where the inexhaustible substitute is produced at all points along the optimal path. Their case therefore corresponds to one where circumstances dictate that $\bar{t}^a(\lambda_i) = 0$. This is clearly not required for an overlap to occur. In fact an overlap will always occur if the inexhaustible resource is exploited at all. This is not due to the assumption that the productive capacity of the inexhaustible resource is limited, but to the necessity of smoothing consumption along the optimal path.

The second characteristic is that whereas consumption is always decreasing before attaining its stationary level, leisure may fluctuate along its optimal path. If the initial stock Y_i^0 is sufficiently large, leisure first increases, then decreases and finally increases again up to its stationary level. It is only if Y_i^0 is relatively small that leisure will increase monotically along the optimal path.

4 The case of two exhaustible resource deposits and one inexhaustible resource

Suppose now that the two deposits of the exhaustible resource are available, along with the flow of the inexhaustible resource. We show in this section that for a nondegenerate vector $Y^0 = (Y_1^0, Y_2^0)$ of initial stocks in the two deposits, we may have the following sequence of six phases along the optimal paths (see Figure 2), a sequence which involves the inexhaustible resource being put into use before the less costly deposit 2. Recall that $\eta > \mu_2 > \mu_1$.

During a first phase, only the least costly deposit 1 of the exhaustible resource is exploited, at a decreasing rate. Starting from a point A_0 between $A_1^{\mu_1}$ and A_1^{η} on the frontier F_1, the per capita leisure-consumption path moves over time along F_1 to attain A_1^{η}.

During the second phase, the leisure-consumption path moves along the isocline I^{η} from A_1^{η} to $\bar{A}_1^{\eta}$. Deposit 1 is still being exploited, deposit 2 is not, and the inexhaustible resource flow is brought into use. The rate of extraction from deposit 1 is decreasing while usage of the inexhaustible resource is increasing. At the end of the phase, the rate of exploitation of the inexhaustible resource is maximal ($x(t) = \bar{x}$) and the rate of extraction from deposit 1 is strictly positive.

During phase three, deposit 1 continues to be exploited at a decreasing rate, while the inexhaustible resource continues to be exploited at full capacity. Deposit 2 is still not being exploited, even though it is less costly than the inexhaustible resource. The per capita leisure-consumption path is moving up along $\bar{F}_1$, from $\bar{A}_1^{\eta}$ to $\bar{A}_1^{\sigma}$, where $\bar{A}_1^{\eta}$ is the point where the frontier $\bar{F}_1$ is crossed by some isocline I^{σ} such that $\eta < \sigma < \bar{\sigma} = u_c(\bar{x}, \bar{l})/u_l(\bar{x}, \bar{l})$.

During the fourth phase, the rate of extraction from deposit 1 is still positive but decreasing to attain zero at the end of the phase, at which point

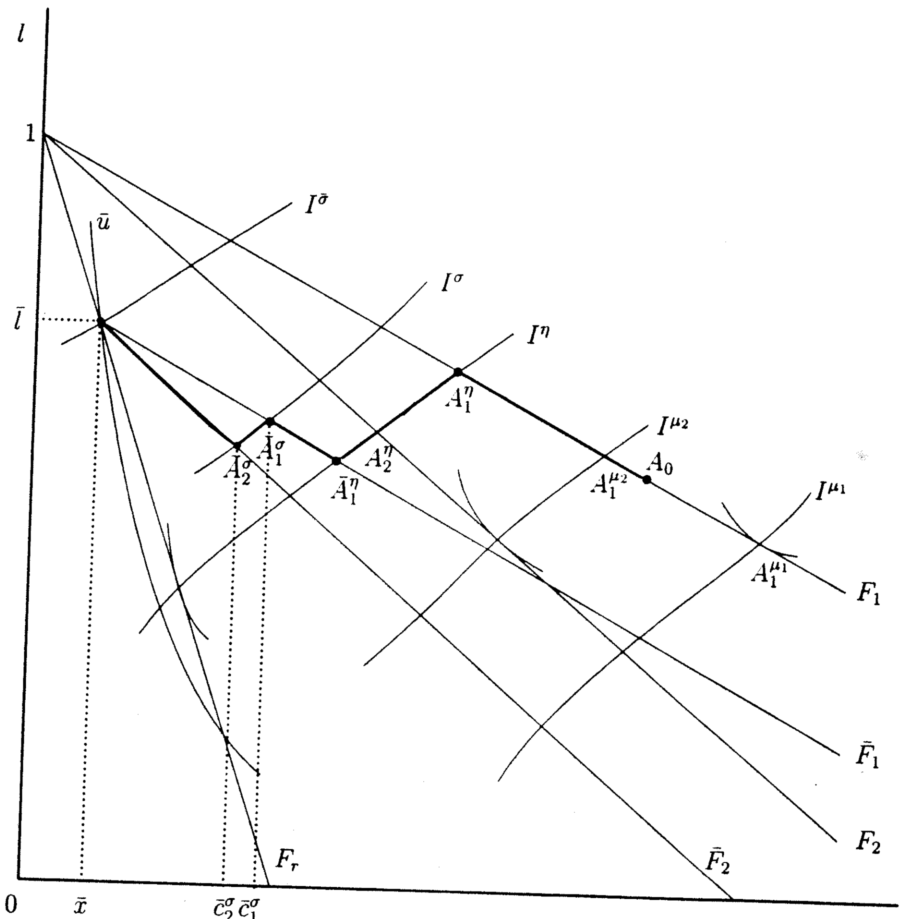

Figure 2: Two exhaustible resource deposits

deposit 1 is exhausted. Extraction from deposit 2 starts at the beginning of the phase, at a rate which is increasing throughout the phase. The inexhaustible resource is still being exploited at full capacity. The leisure-consumption path moves over time along the isocline I^σ, from $\bar{A}_1^\sigma$ to $\bar{A}_2^\sigma$, where $\bar{A}_2^\sigma$ is the crossing point of isocline I^σ and the frontier $\bar{F}_2$.

During the fifth phase, the rate of extraction from deposit 2 gradually decreases to zero and the deposit is exhausted at the end of the phase. The inexhaustible resource is still being exploited at full capacity. The leisure-consumption path is moving up along the frontier $\bar{F}_2$, from $\bar{A}_2^\sigma$ to $(\bar{x}, \bar{l})$.

The sixth and final phase is the stationary state $(\bar{x}, \bar{l})$.

This sequence of exploitation is, as we shall see, the optimal sequence provided that the initial stock in deposit 1 is sufficiently large and that in deposit 2 sufficiently small. In order to show this, we proceed to show that there exists values of λ_1 and λ_2 such that a path of the type just described satisfies all the first-order conditions, along with all the complementary slackness conditions and the transversality condition. These values imply definite extraction paths for both deposits. Therefore, if the vector of initial stocks Y^0 sustains these extraction paths, the values of these λ_i's are the optimal values and the paths are the optimal paths.

In order to generate an optimal path of the type just described, we first choose $\lambda_1 \in (0, (\eta - \mu_1)u_l(c_1^\eta, l_1^\eta))$. Then, set $t_a(\lambda_1, \lambda_2)$ equal to $t^a(\lambda_i)$ for $i = 1$, as defined in section 3. The value of λ_2 will be chosen later (see (18) and (19) below). Since $\lambda_1 < (\eta - \mu_1)u_l(c_1^\eta, l_1^\eta)$, then $t_a(\lambda_1, \lambda_2) > 0$. For any $t \in [0, t_a(\lambda_1, \lambda_2))$, set $y_1(t, \lambda_1, \lambda_2)$ equal to $y_i^a(t, \lambda_i)$ for $i = 1$, again as defined in section 3, and set $y_2(t, \lambda_1, \lambda_2)$ and $x(t, \lambda_1, \lambda_2)$ both equal to 0.

Similarly, let $t_b(\lambda_1, \lambda_2) = t^b(\lambda_1)$ and for any $t \in [t_a(\lambda_1, \lambda_2), t_b(\lambda_1, \lambda_2))$ let $y_1(t, \lambda_1, \lambda_2) = y_1^b(t, \lambda_1)$, $y_2(t, \lambda_1, \lambda_2) = 0$ and $x(t, \lambda_1, \lambda_2) = x^{1b}(t, \lambda_1)$, where $t^b(\lambda_1)$, $y_1^b(t, \lambda_1)$ and $x^{1b}(t, \lambda_1)$ are the functions $t^b(\lambda_i)$, $y_i^b(t, \lambda_i)$ and $x^{ib}(t, \lambda_i)$ defined in the previous section, with $i = 1$.

Next, choose some $\sigma \in (\eta, \bar{\sigma})$, where $\bar{\sigma}$ is the absolute value of the slope of the indifference curve through $(\bar{x}, \bar{l})$, and, for the interval $[t_b(\lambda_1, \lambda_2), t_c(\lambda_1, \lambda_2))$, let $y_1(t, \lambda_1, \lambda_2) = y_1^c(t, \lambda_1)$, $y_2(t, \lambda_1, \lambda_2) = 0$ and $x(t, \lambda_1, \lambda_2) = \bar{x}$. The upper bound of the time interval, $t_c(\lambda_1, \lambda_2)$, is the time at which $y_1^c(t, \lambda_1)$ satisfies

$$\bar{x} + y_1^c(t, \lambda_1) = \bar{c}_1^\sigma$$

where $\bar{c}_1^\sigma$ is the consumption coordinate of $\bar{A}_1^\sigma = (\bar{c}_1^\sigma, \bar{l}_1^\sigma)$, the point at which the isocline I^σ crosses the frontier $\bar{F}_1$.

Whenever both deposits of the exhaustible resource and the inexhaustible resource are all being exploited simultaneously, with the inexhaustible resource being exploited at full capacity, the first-order conditions determining $y_1(t)$ and $y_2(t)$ are

$$v^i(\bar{x}, y_1(t), y_2(t)) = e^{-\rho t}\lambda_i, \quad i = 1, 2. \tag{18}$$

From conditions (18) it follows that

$$\lambda_2 = \frac{\sigma - \mu_2}{\sigma - \mu_1}\lambda_1. \tag{19}$$

Given the value initially chosen for λ_1, we choose λ_2 to satisfy (19).

Note that any point a between $\bar{A}_1^\sigma = (\bar{c}_1^\sigma, \bar{l}_1^\sigma)$ and $\bar{A}_2^\sigma = (\bar{c}_2^\sigma, \bar{l}_2^\sigma)$ on the isocline I^σ determines a unique combination of the per capita exploitation rates of deposits 1 and 2, given that the inexhaustible resource is being exploited at full capacity. Hence, for any $c \in (\bar{c}_2^\sigma, \bar{c}_1^\sigma)$ we may define two functions $y_i^\sigma(c)$, $i = 1, 2$, such that

$$\bar{x} + \sum_{i=1}^{2} y_i^\sigma(c) = c$$

$$1 - \eta\bar{x} + \sum_{i=1}^{2} \mu_i y_i^\sigma(c) = l^\sigma(c).$$

Then function $y_1^\sigma(c)$ is increasing from 0, at $c = \bar{c}_2^\sigma$, to $\bar{c}_1^\sigma - \bar{x}$, at $c = \bar{c}_1^\sigma$, whereas the function $y_2^\sigma(c)$ is decreasing from $\bar{c}_2^\sigma - \bar{x}$ to 0 over the same interval[5].

Consider the time interval $[t_c(\lambda_1, \lambda_2), t_d(\lambda_1, \lambda_2))$ where $t_d(\lambda_1, \lambda_2)$ is the solution to $v^1(\bar{x}, 0, y_2^\sigma(\bar{c}_2^\sigma)) = v^1(\bar{x}, 0, \bar{c}_2^\sigma - \bar{x}) = e^{\rho t}\lambda_1$. For all t in this interval, define $y_i(t, \lambda_1, \lambda_2)$, for $i = 1, 2$, as $y_i^\sigma(c(t, \lambda_1, \lambda_2))$, where $c(t, \lambda_1, \lambda_2)$ is given indifferently, because of the choice of λ_2, either by the solution of $v^1(\bar{x}, y_1^\sigma(c), y_2^\sigma(c)) = e^{\rho t}\lambda_1$ or the solution of $v^2(\bar{x}, y_1^\sigma(c), y_2^\sigma(c)) = e^{\rho t}\lambda_2$.

Finally, consider the time interval $[t_d(\lambda_1, \lambda_2), t_e(\lambda_1, \lambda_2))$ where $t_e(\lambda_1, \lambda_2)$ is the solution to $v^2(\bar{x}, 0, 0) = e^{\rho t}\lambda_2$. For all t in this interval, set $x(t, \lambda_1, \lambda_2) = \bar{x}$, $y_1(t, \lambda_1, \lambda_2) = 0$ and $y_2(t, \lambda_1, \lambda_2) = y_2^c(t, \lambda_2)$, where $y_2^c(t, \lambda_2)$ is the function $y_i^b(t, \lambda_i)$ defined in the previous section, with $i = 2$.

[5] $dy_1^\sigma/dc = (\mu_2 + dl^\sigma/dc)/(\mu_2 - \mu_1) > 0$ and $dy_2^\sigma/dc = -(\mu_1 + dl^\sigma/dc)/(\mu_2 - \mu_1) < 0$, where $l^\sigma(c)$ is per capita leisure as a function of per capita consumption along the isocline I^σ.

Having described a path that has the desired characteristics, there remains to show that it satisfies all the first-order conditions, the complementary slackness conditions and the transversality condition. Concerning conditions (6), (8) and (9), that relate to $x(t, \lambda_1, \lambda_2)$, the proof runs exactly as in the case examined in the previous section. This is because, as in the case where there is only one deposit of the exhaustible resource, on the first time interval, $[0, t_a(\lambda_1, \lambda_2))$, we have $MRS(t) < \eta$ and $x(t, \lambda_1, \lambda_2) = 0$, on the second time interval, $[t_a(\lambda_1, \lambda_2), t_b(\lambda_1, \lambda_2))$, we have $MRS(t) = \eta$ and $0 < x(t, \lambda_1, \lambda_2) < \bar{x}$ and from $t_b(\lambda_1, \lambda_2)$ onwards, we have $MRS(t) > \eta$ and $x(t, \lambda_1, \lambda_2) = \bar{x}$. Thus we are left with the conditions relating to $y_i(t, \lambda_1, \lambda_2)$, $i = 1, 2$, for the time intervals on which one of these extraction rates is equal to 0.

For $t \in [0, t_c(\lambda_1, \lambda_2))$, we must have

$$v^1(x(t, \lambda_1, \lambda_2), y_1(t, \lambda_1, \lambda_2), 0) = e^{-\rho t}\lambda_1 \tag{20}$$

$$v^2(x(t, \lambda_1, \lambda_2), y_1(t, \lambda_1, \lambda_2), 0) \leq e^{-\rho t}\lambda_2 \tag{21}$$

Conditions (20) and (21), and the choice of λ_2 (equation (19)), imply

$$\frac{MRS(t) - \mu_2}{MRS(t) - \mu_1} \leq \frac{\lambda_2}{\lambda_1} = \frac{\sigma - \mu_2}{\sigma - \mu_1}. \tag{22}$$

Since the left-hand side of (22) increases with $MRS(t)$, and since $MRS(t) < \sigma$ on that time interval, we can conclude that (22) is satisfied on the interval. Therefore, if we set $\nu_2(t) = e^{-\rho t}\lambda_2 - v^2(x(t, \lambda_1, \lambda_2), y_1(t, \lambda_1, \lambda_2), 0)$, $\nu_1(t) = 0$, $\delta_1(t) = \delta_2(t) = 0$, all the necessary conditions are met on the time interval $[0, t_c(\lambda_1, \lambda_2))$.

For $t \in [t_d(\lambda_1, \lambda_2), t_e(\lambda_1, \lambda_2))$, we must have

$$v^1(\bar{x}, 0, y_2(t, \lambda_1, \lambda_2)) \leq e^{-\rho t}\lambda_1 \tag{23}$$

$$v^2(\bar{x}, 0, y_2(t, \lambda_1, \lambda_2)) = e^{-\rho t}\lambda_2 \tag{24}$$

In this case, conditions (23) and (24), and the choice of λ_2 (equation (19)), imply

$$\frac{MRS(t) - \mu_2}{MRS(t) - \mu_1} \geq \frac{\lambda_2}{\lambda_1} = \frac{\sigma - \mu_2}{\sigma - \mu_1}. \tag{25}$$

Since the left-hand side of (25) increases with $MRS(t)$, and since $MRS(t) > \sigma$ on that time interval, it is clear that (25) is satisfied on the interval. Therefore, if we set $\nu_1(t) = e^{-\rho t}\lambda_1 - v^1(\bar{x}, 0, y_2(t, \lambda_1, \lambda_2))$, $\nu_2(t) = 0$, $\delta_1(t) = \delta_2(t) = 0$, all the necessary conditions are met on the time interval $[t_d(\lambda_1, \lambda_2), t_e(\lambda_1, \lambda_2))$.

Finally, let $Y_i(\lambda_1, \lambda_2) = \int_0^{t_e(\lambda_1,\lambda_2)} y_i(t, \lambda_1, \lambda_2)dt$, $i = 1, 2$. $Y_i(\lambda_1, \lambda_2)$ therefore denotes the initial size of deposit i of the exhaustible resource necessary to sustain the extraction paths $y_i(t, \lambda_1, \lambda_2)$ on the interval $[0, t_e(\lambda_1, \lambda_2))$. At the optimum, (λ_1, λ_2) must satisfy $Y_i(\lambda_1, \lambda_2) = Y_i^0$, $i = 1, 2$. Call these values $(\lambda_1^*, \lambda_2^*)$. Then since $l(t_e(\lambda_1^*, \lambda_2^*)), c(t_e(\lambda_1^*, \lambda_2^*)) = (\bar{l}, \bar{x})$, it is clear that the transversality condition (13) is satisfied at $T^* = t_e(\lambda_1^*, \lambda_2^*)$.

Note that the sequencing of the phases along the optimal paths is left unchanged by slight perturbations of the parameters. Hence this order of exploitation is not a mere curiosum, in the sense that the subset of initial stocks for which such paths occur at the optimum is not of measure zero.

5 Concluding remarks

The fact that the inexhaustible resource is scarce, in the sense that the capacity constraint on the instantaneous rate of production from that resource becomes binding in the stationary state prevailing once the exhaustible resource is completely depleted, is essential to the result that the resources may be brought into use in the reverse order of cost. For suppose it is not, which would obviously be the case if there were no capacity constraint at all. Then the exploitation program described in section 4 can always be improved upon in the following way. At the end of phase two, when only deposit 1 and the inexhaustible resource are being used, reduce the production from the inexhaustible resource by one unit over some subinterval of time and replace it by the extraction of one unit from deposit 2 over the same subinterval. The immediate effect is a reduction in the labor cost during phase two, since the inexhaustible resource is more costly to exploit than deposit 2 of the exhaustible resource. But since the stock in deposit 2 is finite, this requires a compensating reduction in the extraction from this deposit over some subinterval of either phase four or phase five. If there is no capacity constraint to the production from the inexhaustible resource, then this reduction can be completetly replaced by increasing the use of the inexhaustible resource. This increases the labor cost during phase four or five, but, the discount rate being positive, the net effect is a reduction in the discounted labor cost for an unchanged per capita consumption path. In fact the net increase in welfare from this transfer will exceed that due simply to the discount rate effect at a constant marginal labor cost. This is because per capita leisure is lower, and hence the instantaneous marginal labor cost in terms of utility higher, at the end of phase two than during all of phases four and five (see Figure 2). However, when there is a binding capacity constraint on the rate of exploitation from the inexhaustible resource, as is the case beginning with phase three of the program described in section 4, the increase in the production from this resource required to carry out such a welfare increasing intertemporal arbitrage is not possible.

To delay the exploitation of a higher cost exhaustible resource deposit can in fact be an optimal way of carrying out a welfare improving intertemporal transfer in the presence of a scarce inexhaustible resource. For whenever the capacity constraint on the inexhaustible resource is going to bind at some point along the optimal program, thus making it scarce, there is an underlying incentive to store some of the unused flow of the resource during a phase when it is not exploited at full capacity, in order to alleviate the effect of the constraint at some future date. Storage of the inexhaustible resource is however not possible. The next best thing, as we have shown, may be to store some of the more costly exhaustible resource by delaying the start of its exploitation until after the inexhaustible resource has already been put into use. As we have already pointed out. this completely reverses the ordering which would appear optimal from partial equilibrium analysis.

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