THE DURATION OF UNEMPLOYMENT AS A SIGNAL:
IMPLICATIONS FOR LABOR MARKET EQUILIBRIUM

Robert Kollmann

Département de sciences économiques and Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal.

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RÉSUMÉ

Un modèle est étudié dans lequel les firmes sont imparfaitement informées sur les aptitudes des travailleurs qui font des demandes d’emploi. Dans le modèle, la durée du chômage des demandeurs d’emploi est un critère dans les décisions de recrutement des firmes. À l’équilibre, le salaire offert aux demandeurs d’emploi est une fonction décroissante de la durée du chômage de ces travailleurs. Une fraction des travailleurs dont la durée du chômage excède une durée critique quitte le marché du travail. Cette durée critique est fonction croissante de la productivité du travail.

Mots clés : durée du chômage, recrutement, taux de salaire, marché du travail.

ABSTRACT

A model is studied in which firms lack accurate information about the skills of job applicants. The length of the unemployment spells of job applicants is used as a criterion in firms’ hiring decisions, as workers with long unemployment spells are, on average, less employable than workers with short spells. In equilibrium, the wage rate offered to job applicants is decreasing in the length of their unemployment spell and a fraction of the workers whose spell exceeds a critical duration drops out of the labor market. This critical duration is positively related to labor productivity.

Key words: unemployment duration, recruitment, wage rate, labor market.
1. Introduction

Frequently, firms are unable to accurately determine whether job applicants meet their skill requirements without incurring considerable costs. Firms thus have an incentive to use any signal which can help them in their recruitment decisions. In this respect, information on the length of the job applicants' unemployment spells is useful, because this information is easily available, and because workers with long unemployment spells are typically less employable than workers with shorter spells, as workers who meet the job requirements of a high proportion of firms are likely to leave unemployment relatively quickly.¹

The model presented in this paper focuses on this signaling role of the unemployment duration. Although, in the model, the productivity of a given worker is unaffected by unemployment, the signaling role of the unemployment duration implies that, in equilibrium, the wage offered to jobless workers is decreasing in the length of their unemployment spells. If the wage falls sufficiently low for long unemployment spells, then a fraction of the jobless (and possibly all jobless) stops searching for jobs once they reach a critical unemployment duration. This critical duration is positively related to market sector productivity. The model is consistent with empirical evidence that the reservation wage of the jobless, the intensity of job search and the probability of leaving unemployment are decreasing in the unemployment duration.²

¹Anecdotal evidence suggests that firms frequently use the duration of unemployment to screen job applicants.
²See Kasper (1967), Kiefer and Neumann (1979), Layard et al. (1991) and Nickell (1979). This paper is related to stigma theories of unemployment due to Lockwood (1991) and Berkovitch (1988). The present model was developed independently of their work (see Kollmann (1986)). The structure and some of the key predictions of their models differ from those of the present model (e.g., while in Lockwood's paper firms do not hire workers who exceed a cut-off unemployment duration, simulations suggest that--in contrast to the present model--the cut-off duration is decreasing
2. The Model

Preferences, Technology and the Information Structure

The model assumes a continuum of workers and free entry of firms. There are two types of workers, denoted A and B. The probability that a random type A worker meets the skill requirements of a random firm (this probability will be denoted by \( p^A \)) is greater than the corresponding probability for a type B worker (\( p^B \)), i.e., \( p^A > p^B \). This is the only difference between the two types. Regardless of his type, the output of a worker in a given firm is \( y > 0 \) per period if the worker meets that firm's skill requirements and output is zero otherwise. A firm only hires a worker if that worker meets its skill requirements. A jobless worker has a non-market productivity which equals \( w \)\(^3\). Time is continuous. For an exogenous reason (death) workers withdraw from the labor force at the rate \( z > 0 \).\(^4\) There is a continual inflow of new workers into the economy and the size and the composition (between type A and type B workers) of the total labor force is constant. All workers are born jobless.

A firm can costlessly determine the length of the unemployment spell of a given worker. In addition, it can accurately determine the skills of an unemployed worker, but only by interviewing (testing) the worker. A firm bears a cost \( c > 0 \) for each interview which it conducts.

Firms which are willing to employ duration \( \tau \) jobless announce the wage at which they accept to hire these workers, conditional on the outcome of an interview. All employment contracts are signed for the life-time of the worker and the wage rate is constant over the entire employment period.

Workers face an exogenous constraint on the number of job applications which they can make. At any given point in time, an exogenous random mechanism determines which workers can make a job application at that instant. It is assumed that, with probability \( \Delta \), a worker gets the opportunity to make a job application during a short time interval of length \( \Delta \). Hence a jobless worker has the opportunity to make one job application, on average, per period of unit length.

Workers do not know which firms require the specific skills which they possess. Hence they randomly apply for jobs at firms offering to pay the highest wage rate to workers with their duration. In what follows, \( w(\tau) \) denotes the highest wage rate offered to the duration \( \tau \) jobless.

Workers and firms are risk neutral and they use a constant discount factor \( \delta \). For a firm, the expected present value of the profits from employing a worker at a salary \( w \) is \( (y-w)/(z+1) \) (recall that \( z \) is the death rate of workers). Hence the expected gain for a firm from interviewing a worker with unemployment duration \( \tau \) is \( w(\tau)(y-w(\tau))/(z+1) - c \), where \( w(\tau) \) is the probability that the worker meets the skill requirements of the firm.

A jobless worker with duration \( \tau \) does not want to get a job unless \( w(\tau)w \) holds. Hence, no duration \( \tau \) jobless is hired unless

\[
\frac{w(\tau)c(z+1)/(y-w)}{w} \leq \min_{\tau}\quad (1)
\]

Let \( \Pi(\tau) \) denote the value of \( w(\tau) \) which obtains in an economy in which all jobless workers with durations \( \tau \)'s use every opportunity to make job applications. We have

\[
\text{in the market sector productivity; also, the wage rate does not depend on the unemployment duration). Other related work includes Flinn and Heckman (1982), Totsch (1988), Viswanath (1989), Graafland and Huizinga (1990), and Blanchard and Diamond (1990).}
\]

\(^3\)Job search is costless in the sense that a jobless worker has the same non-market productivity \( w \), irrespective of whether the worker searches for a job or whether she does not search. Costly job search would not affect the main conclusions of the paper.

\(^4\)The probability that a given worker dies during a short time interval of length \( \Delta \) is \( z\Delta \).
\[ \Pi(t) = \left\{ a \exp(-p^A t) + (1-a) \exp(-p^B t) \right\} \left\{ \frac{a \exp(-p^A t) + (1-a) \exp(-p^B t)}{a \exp(-p^A t) + (1-a) \exp(-p^B t)} \right\}. \] (2)

where 'a' is the proportion of type A workers among the newborn workers.  
\[ p^A \leq p^B \] implies that \( \Pi(t) \) is decreasing in \( t \) (note that \( \lim_{t \to \infty} \Pi(t) = p^B \)).

It will be assumed that \( p^B < \min \leq \Pi(0) \).  
(3)

This condition guarantees that there exists an unemployment duration \( \tau^* \) such that \( \Pi(\tau^*) = \min \).

(4)

**Equilibrium**

When (3) holds, then all jobless workers whose unemployment spells are shorter than \( \tau^* \) use every opportunity to make job applications. Hence, for a firm, the probability that a random job applicant with unemployment duration \( \tau \leq \tau^* \) meets its skill requirements equals \( w(t) = \Pi(t) \). Free entry of firms ensures that the expected gain to firms from interviewing workers with durations \( \tau \leq \tau^* \) is zero, i.e. \( \Pi(t)(y-w(t))/(r+1)-\sigma = 0 \) holds for \( \tau \leq \tau^* \). As \( \Pi(t) \) is decreasing in \( t \), this implies that \( w(t) \) (and hence also the reservation wage of jobless workers) is decreasing in \( t \) (for \( \tau \leq \tau^* \)). The intuition for this is simple: type A workers tend to find jobs more rapidly than type B workers; hence a firm which interviews workers with short unemployment spells is more likely to find workers who meets its skill requirements than a firm which interviews workers with longer unemployment spells. Therefore, the wage rate consistent with zero expected gains from interviewing workers is lower for workers with longer unemployment spells.

\( w(t) \) approaches the non-market sector productivity \( \omega \) and \( \pi(t) \) approaches \( \pi^{\min} \) as the unemployment duration approaches \( \tau^* \). Figure 1 plots the wage schedule \( v(t) \), and it also illustrates the determination of the critical duration \( \tau^* \).

In equilibrium, a positive fraction of the jobless with durations \( \tau \leq \tau^* \) abstains from applying for jobs. To see why this is so, assume that none of these jobless withdraw from the job market; \( w(t) \) would then be driven below \( \pi^{\min} \), and hence \( w(t) \) would fall below \( \omega \) (and thus it would not be optimal for the long term jobless to continue searching for jobs).

In this sense, the model captures the empirical finding that the intensity of job search and the probability of switching from unemployment to employment are smaller for the long-term unemployed than for workers with short unemployment spells.

However the model does not uniquely pin down the behavior of the jobless workers with durations \( \tau \leq \tau^* \). One possibility is that \( w(t) = \omega \) holds for \( \tau \leq \tau^* \), which implies that jobless workers with these durations are indifferent between applying for jobs and not applying. Hence one possible equilibrium is that all jobless with durations \( \tau \leq \tau^* \) withdraw from the job market. However, there exist other equilibria in which fractions of both

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5Let \( A(t) \) (\( B(t) \)) be the measure of type A (B) jobless with duration \( t \). We have \( \Pi(t) = (p^A A(t) + p^B B(t)) / (A(t) + B(t)) \), where \( A(t) = A(0) \exp(-z p^A t) \) and \( B(t) = B(0) \exp(-z p^B t) \) (N.B. \( a = A(0)/(A(0)+B(0)) \)). With probability \( A \), a jobless worker gets the opportunity to make a job application during a short time span of length \( A \); when type j workers use every opportunity for job applications, then the proportion of type j workers who find a job or die during a time span of length \( A \) is \( (z p^j) A \). Hence \( da(t)/dt = -(z p^A) A(t) \) and \( dB(t)/dt = -z p^B B(t) \), which explains the above formulae for \( A(t) \) and \( B(t) \).

6\( p^B < \min \) implies that if there only existed type B workers, then no worker would ever be interviewed.

7The appendix presents proofs of this and the following statements.

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8The figure assumes \( w(t) = \omega \) for \( \tau \leq \tau^* \) (see the discussion below).

9It can be shown that, in equilibrium, \( w(t) = \omega \) has to hold for all \( \tau \).
types of jobless workers with durations $\tau^*$ stay in the job market.  

We see from (1) and (4) that an increase in the market sector productivity $y$ raises the critical duration $\tau^*$. Hence a productivity increase is likely to increase employment (Kollmann (1986) provides a detailed analysis of the dynamic effects of productivity shocks in the model).  

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10 The fractions have to satisfy the restriction that $\pi(\tau) \leq \pi^* \text{ min}$ holds for $\tau^*$, because otherwise firms would not be willing to interview job applicants with these durations.

If we, realistically, assumed that the market sector productivity of a jobless worker is negatively related to $\tau$, then all jobless exceeding a critical duration would withdraw from the job market (and that even if the negative relation between $y$ and $\tau$ were very weak). Hence the symmetric equilibrium in which all workers with durations $\tau^*$ withdraw from the market seems to be the most appealing equilibrium.

11 Note that, because the model does not uniquely pin down the behavior of the jobless for $\tau^*$, it does not yield a definite relation between $y$ and total employment; however it is easy to see that if all workers with durations $\tau^*$ withdraw from the job market, then unambiguously employment rises when $y$ increases.
APPENDIX: Detailed Analysis of Equilibria

This appendix presents a formal discussion of equilibria in the economy considered in the paper. It will be shown that, if (3) holds, then equilibria have the following features:

(i) For \( \tau \) (with \( \tau \) determined by \( \Pi(\tau) = \Pi^{m} \)), we have \( w(\tau) = \Pi(\tau) \) and \( \Pi(\tau)(y - w(\tau)) / (z+1) = 0 \); furthermore, all jobless workers with durations \( \tau \) want to get jobs, and hence they use every opportunity to make job applications.

(ii) \( w(\tau) = 0 \) holds for \( \tau \geq \tau^{*} \).

To start the analysis of equilibria, the optimal behavior of the jobless workers will be characterized.

Optimal behavior of the jobless

Let \( U^{j}(\tau) \) denote the expected life-time utility of a rational jobless type \( j \) worker with unemployment duration \( \tau \). We assume that workers do not have access to financial markets, so that their consumption equals their wage (or their non-market productivity). Assuming that the workers have an instantaneous utility function \( u(x) = x \), we have:

\[
(z+1)U^{j}(\tau) = \left( \max\left(0, w(\tau) / (z+1) - U^{j}(\tau) \right) \right) + \frac{dU^{j}(\tau)}{d\tau}. \tag{A1}
\]

In equilibrium, the wage schedule \( w(\tau) \) is differentiable for \( \tau < \tau^{*} \) and there exists a unique solution to (A1) which satisfies:

\[
w(z+1)U^{j}(\tau) = 0 \] for all \( \tau < \tau^{*}. \tag{A2}
\]

Note that, as type A workers are more likely to be hired than type B workers \( (p^{A} > p^{B}) \), the expected life-time utility of a type A worker cannot fall below that of a type B worker: \( U^{B}(\tau) \leq U^{A}(\tau) \).

A type \( j \) jobless with unemployment duration \( \tau \) does not want to be hired at the wage \( w(\tau) \) when \( (z+1)U^{j}(\tau) < w(\tau) \) holds, i.e. \( (z+1)U^{j}(\tau) \) is the reservation wage of that worker.

As described in the text, we assume that, at any point in time, an exogenous random mechanism picks the jobless workers who can make a job application at that instant. Not all workers picked by this mechanism will necessarily use the opportunity of making a job application. Let \( \phi^{j}(\tau) \) denote the share of type \( j \) jobless with unemployment duration \( \tau \) who choose to use this opportunity. We have:

\[
\begin{align*}
\phi^{j}(\tau) &= 1 \quad \text{when} \quad w(\tau) > (z+1)U^{j}(\tau), \\
\phi^{j}(\tau) &= 0 \quad \text{when} \quad w(\tau) < (z+1)U^{j}(\tau).
\end{align*}
\]  \tag{A3}

Note that when \( w(\tau) = (z+1)U^{j}(\tau) \), then the type \( j \) jobless with unemployment duration \( \tau \) are indifferent between getting a job and remaining jobless and hence \( \phi^{j}(\tau) \) can take any value between 0 and 1.

The probability of a successful interview (from a firm's point of view)

Free entry implies also that, subject to the exogenous restriction on the number of job applications described in the text, duration \( \tau \) jobless who wish to get a job at the prevailing wage \( w(\tau) \) can be matched to firms which are willing to interview them. This requirement means that:

\[
w(\tau)(y - w(\tau)) / (z+1) = 0 \] has to hold, whenever \( \phi^{A}(\tau) + \phi^{B}(\tau) = 0 \). \tag{A4}
\]

where \( w(\tau) \) denotes the probability for a firm of conducting a successful interview (i.e. the probability that a job applicant meets the skill requirements of the firm).

When \( \phi^{A}(\tau) + \phi^{B}(\tau) > 0 \), then \( w(\tau) \) is given by \( w(\tau) = p^{A}a(\tau) + p^{B}(1-a(\tau)) \), \tag{A5}

where \( a(\tau) = \phi^{A}(\tau)A(\tau) / (\phi^{A}(\tau)A(\tau) + \phi^{B}(\tau)B(\tau)) \) is the proportion of type A workers among the duration \( \tau \) jobless who apply for jobs. Here \( A(\tau) \) \( (B(\tau)) \) is the measure of type A \( (B) \) workers with unemployment duration \( \tau \).

When \( \phi^{A}(\tau) + \phi^{B}(\tau) = 0 \), I set \( w(\tau) = 0 \). \tag{A6}

\( A(\tau) \) and \( B(\tau) \) are determined by:

\[
dA(\tau)/d\tau = -\left( z + \phi^{A}(\tau)p^{A}(\tau) \right)A(\tau), \quad A(0) = \tilde{A}; \quad dB(\tau)/d\tau = -\left( z + \phi^{B}(\tau)p^{B}(\tau) \right)B(\tau), \quad B(0) = \tilde{B}, \tag{A7}
\]

where \( \tilde{A} \) and \( \tilde{B} \) denote the measure of new-born type A and type B workers.
respectively.\footnote{A(\tau+\Delta)=(1-\Delta z)(1-\Delta \phi^A(\tau)p^A)A(\tau) holds for small values of \Delta. Subtracting \phi^A from both sides of the equation, dividing by \Delta and taking the limit \Delta \to 0 gives (A7). Note that during a short time span of length \Delta, a fraction \Delta of the type A workers gets the opportunity to make a job application; a fraction \phi^A of these workers uses this opportunity and a fraction p^A of the type A workers who make job applications is hired. As the death rate is z, (1-\Delta z)(1-\Delta \phi^A(\tau)p^A) is the fraction of the type A workers with unemployment duration \tau at a given point in time who are still jobless a time span \Delta later.}

In what follows, \hat{x}(\tau) denotes the probability of a successful interview which obtains if all 'eligible' type A and type B jobless of duration \tau apply for jobs (i.e. when \hat{\phi}^A(\tau)=\phi^A(\tau)=1):

\[
\hat{x}(\tau)=A(\tau)/(A(\tau)+B(\tau)),
\]

where \(A(\tau)=A(\tau)/A(\tau)+B(\tau)\).

The probability \(I(\tau)\) (see equation (2) in the text) corresponds to the value of \(x(\tau)\) which obtains when \(\hat{\phi}^A(\tau)=\phi^A(\tau)=1\) for all durations \(\tau \geq \tau^*\), i.e. when all workers with durations \(\tau \geq \tau^*\) use every opportunity to make a job application.

Free entry of firms and the wage rate

Unless a firm offers the duration \(\tau\) jobless a wage which equals or exceeds the highest wage rate which other firms offer these jobless, it is unable to attract job applicants. Recalling that \(w(\tau)\) denotes the highest wage rate offered to the duration \(\tau\) jobless, free entry of firms implies that, in equilibrium, there does not exist a wage \(w(\tau)\) for which the expected gain from interviewing the duration \(\tau\) jobless would exceed zero. This implies that, in equilibrium, one of the following two conditions has to be satisfied for each duration \(\tau \geq \tau^*\):

\[
\hat{x}(\tau)(y-U^A(\tau)(z+1))/(z+1)-c>0 \quad \text{and} \quad \hat{x}(\tau)(y-w(\tau))/(z+1)-c=0; \quad (A9^a)
\]

\[
\hat{x}(\tau)(y-U^A(\tau)(z+1))/(z+1)-c<0. \quad (A9^b)
\]

To understand these conditions, note that when \((A9^a)\) holds, then \(w(\tau)>U^A(\tau)(z+1)\); hence, for a firm offering a wage \(w(\tau)\), the probability of a successful interview is \(\hat{x}(\tau)\), which implies that the expected present value of the profits (net of the interview cost \(c\)) made by such a firm are negative.

To understand \((A9^b)\), note that when this condition holds, no firm offers a wage greater than \(U^A(\tau)(z+1)\), as this would entail negative expected profits (net of \(c\)); hence, in equilibrium, \(w(\tau)>U^A(\tau)(z+1)\) has to hold when \((A9^b)\) is satisfied.

When \(w(\tau)=U^A(\tau)(z+1)\), then \((A9^b)\) implies that the expected present value of the profits (net of the interview cost \(c\)) of a firm offering \(w(\tau)\) are negative.

Consider next the case where \(w(\tau)>U^A(\tau)(z+1)\). In that case, a firm offering a wage \(w(\tau)<U^A(\tau)(z+1)\) cannot attract type A workers and hence (by \(A9^b\)) its profits (net of \(c\)) are negative in expected present value.

For a firm offering \(wU^A(\tau)(z+1)\) too, the expected present value of profits (net of \(c\)) does not exceed zero. To show this, we shall separately consider the case when \(U^B(\tau)>U^A(\tau)\) and when \(U^B(\tau)=U^A(\tau)\) (as mentioned above, we have \(U^B(\tau)>U^A(\tau)\)).

(1) If \(U^B(\tau)>U^A(\tau)\), then a firm which offers a wage \(w=U^A(\tau)(z+1)\) at a given point in time gets job applications from all type B workers who are able to make a job application at that instant. Hence the probability of a successful interview cannot exceed \(\hat{x}(\tau)\) for such a firm (see \((A5)\), \((A9)\)) and thus \((A9^b)\) implies that the expected present value of the profits of the firm (net of \(c\)) do not exceed zero.

(1) If \(U^B(\tau)=U^A(\tau)\), then type A and type B jobless with unemployment duration \(\tau\) are indistinguishable between applying and not applying for a job paying \(wU^A(\tau)(z+1)\). Under the plausible assumption that a firm which considers offering \(wU^A(\tau)(z+1)\) believes that equal fractions of the jobless of both types will apply for a job, then—for that firm—the probability of a successful interview is \(\hat{x}(\tau)\), and hence \((A9^b)\) again implies that the expected present value of the firm's profits (net of the interview cost \(c\)) does not exceed zero.
Definition of an Equilibrium

An equilibrium in an economy in which (3) holds is given by \( R \equiv R \), functions \( \phi^A(t), \phi^B(t), \pi(t), w(t), A(t), B(t), U^A(t) \) and \( U^B(t) \) which satisfy equations (A1)-(A9).

Existence of an Equilibrium

Establishing the existence of an equilibrium is straightforward. For example, it is easy to verify that the functions \( \phi^A(t), \phi^B(t), \pi(t), w(t), A(t), B(t), U^A(t) \) and \( U^B(t) \) defined as follows constitute an equilibrium:

(1) \( \phi^A(t) = 1 \) for \( t < t^* \) and \( \phi^A = 0 \) for \( t > t^* \), \( j=A,B \) (where \( t^* \) is defined in (4)).

(2) \( \pi(t) = p(t) \) for \( t < t^* \) and \( \pi(t) = \min \) for \( t > t^* \).

(3) \( w(t) = y - c(z+1) / \Pi(t) \) for \( t < t^* \) and \( w(t) = \omega \) for \( t > t^* \).

(4) \( A(t) = A_0 \exp(-c^Bt), B(t) = B_0 \exp(-c^Bt) \) for \( t < t^* \) and \( A(t) = A_0 \exp(-c^A t^* - c^B t), B(t) = B_0 \exp(-c^B t^* - c^B t) \) for \( t > t^* \).

(5) \( U^A(t) = w^A((s+t)(z+1)) \) for \( t < t^* \)

\[
\int_t^{t^*} \left[ w^A \left( \frac{w^B(y)}{(z+1)} \right) \exp(-c^B (s+t)(z+1)) \right] ds 
\]

and \( U^B(t) = w^B(z+1) \) for \( t > t^* \), \( j=A,B \).

This equilibrium corresponds to the symmetric equilibrium discussed at the end of the text, in which all jobless workers with durations \( t > t^* \) withdraw from the labor market.

It is less straightforward to prove that, when (3) is assumed, then any equilibrium has the features listed at the beginning of this appendix. In what follows, this fact will be proved.

Proposition 1. Assume (3). Then, in equilibrium, \( \phi^B(t) = \phi^A(t) \) holds for all \( t < 0 \), and hence \( \pi(t) \) is decreasing.
In case (i), (A3) implies that $\phi_B^*(\tau) = 1$ (given our assumption that $u_B^*(\tau) = c^*(\tau)$) and hence (A4) requires that $x(\tau)(y-u_B^*(\tau)(z+1))/(z+1) = c^B$; therefore $x(\tau)x(\tau)$ has to hold, which requires $u_B^*(\tau) = u_B^*(\tau)$ (see (A5), (A8)) and hence $\phi_B^*(\tau) = 1$.

Consider next case (ii). If $\phi_B^*(\tau) > 0$, then $x(\tau) = p_B^*$, and (A6) thus requires $p_B^*(y-w(\tau))/(z+1) = c^B$, but this is inconsistent with (3). Therefore, $\phi_B^*(\tau) = 0$ has to hold when $w(\tau) = u_B^*(\tau)(z+1)$.

Finally, in case (iii), $\phi_B^*(\tau) = 0$ has to hold (see (A3)).

(c) Assume next that $x(\tau)(y-u_A^*(\tau)(z+1))/(z+1) = c^A$. In that case, $w(\tau)u_A^*(\tau)(z+1)$ cannot occur in equilibrium. Hence $w(\tau)u_A^*(\tau)(z+1)$ has to hold, and therefore $\phi_A^*(\tau) = 0$. When $w(\tau)u_A^*(\tau)(z+1)$, then $\phi_A^*(\tau) = 0$.

When $w(\tau)u_A^*(\tau)(z+1)$, then we know from (A3) that $\phi_A^*(\tau) = (0, 1)$. Note that $\phi_A^*(\tau) = 0$ would imply $x(\tau) = p_A^*$, and (A4) thus would require $p_A^*(y-w(\tau))/(z+1) = c^A$. This however would be inconsistent with (3) and hence $\phi_A^*(\tau) = 0$ has to hold.

Cases (a)-(c) show that when (3) is assumed, then in equilibrium, $\phi_B^*(\tau) = u_B^*(\tau)$ has to hold for any duration $\tau$ for which $u_B^*(\tau) = u_B^*(\tau)$. As mentioned above, it can also be shown that when, in equilibrium, $u_B^*(\tau) = u_B^*(\tau)$ holds for some duration $\tau$, then $\phi_B^*(\tau) = u_B^*(\tau)$.

Because we assume $p_B^* < p_A^*$, the result that $\phi_B^*(\tau) = u_B^*(\tau)$ holds in equilibrium implies that type B workers leave unemployment at a rate which does not exceed that of the type A workers. Hence, in equilibrium, the ratio $A(\tau)/B(\tau)$ is decreasing (not necessarily strictly) in $\tau$, which implies that $x(\tau)$ too is decreasing (not necessarily strictly) in $\tau$.

Proposition 2
To prove this proposition, assume that, in some equilibrium, $w(u_A^*(\tau)(z+1)$ and $w(u_B^*(\tau)(z+1)$ holds for duration $\tau$. The first inequality implies that there exists a duration $\tau_0 > 0$ such that: (1) $u_A^*(\tau_0)(z+1) = w(\tau_0)$ and (11) $u_B^*(\tau_0)(z+1) = w(\tau_0)$.

To see why this is so, let $W(\tau_0) = \sup_{\tau_0}(\tau_0) w(\tau)$. From (A1) we see that if a jobless type A worker faces a constant wage $w(\tau) = w^*$ for all durations $\tau \geq \tau_0$, then that worker's expected lifetime utility is $U_A^*(w^*/(z+1)) = w^*/(z+1)$, hence $U_B^*(w^*/(z+1)) = w^*/(z+1) = w(\tau_0)$. Hence $U_A^*(w^*) = (w^*/(z+1)) / (z+1)$ and $U_B^*(w^*) = w(\tau_0) / (z+1)$. For all $\tau_0 \geq \tau_0$, where $\phi_B^*(\tau) = w(\tau_0)/(z+1) = w(\tau_0)$ holds for all $\tau_0$. By picking a duration $\tau_0 > 0$ such that $W(\tau_0) = w(\tau_0)/(z+1)$, we can ensure that $w(\tau_0) = u_A^*(\tau_0)(z+1)$ and $w(\tau_0) = u_B^*(\tau_0)(z+1)$ (see the illustration in figure A).

$U_A^*(\tau_0)(z+1) = w(\tau_0)$ implies that $\phi_A^*(\tau_0) = \phi_B^*(\tau_0) = 1$, and hence $x(\tau_0) = \phi_A^*(\tau_0)$ and $x(\tau_0) = \phi_B^*(\tau_0)(z+1) = c^A$ (see (A4)), and therefore $x(\tau_0) = \phi_B^*(\tau_0)(z+1) = c^B$.

We know from the fact that either (A8) or (A9) has to be satisfied in equilibrium, that if $w(\tau_0) = u_A^*(\tau_0)(z+1)$ holds in equilibrium, then (A9) has to hold for period $\tau_0$, i.e. $x(\tau_0) = \phi_A^*(\tau_0)(z+1)$ does not hold. Because $\tau_0 < \tau_0$, we have $x(\tau_0) = \phi_A^*(\tau_0)$. For this reason, and since $w(\tau_0) = u_A^*(\tau_0)(z+1)$, conditions (A10) and $x(\tau_0) = \phi_B^*(\tau_0)(z+1)$ cannot both be satisfied. We therefore conclude that $w(\tau_0) = u_A^*(\tau_0)(z+1)$ and $w(\tau_0) = u_B^*(\tau_0)(z+1)$ cannot hold in equilibrium: if $w(\tau_0) = u_A^*(\tau_0)(z+1)$ holds for duration $\tau_0$, then $U_A^*(\tau_0)(z+1) = w(\tau_0)$. 

14 To see why $\phi_B^*(\tau_0) > 0$ holds, note that if $w(\tau_0) = w(\tau)$ were to hold, then $w(\tau) = w(\tau)$ would be true for all $\tau \geq \tau_0$, which would imply $u_A^*(\tau_0)(z+1) = w(\tau)$. By assumption $\phi_B^*(\tau_0)(z+1) = w(\tau)$ holds and thus $\phi_B^*(\tau_0) > 0$. 

15
Proposition 3

$U^A(t_0)(z+1)^w$ can only hold if $w(t')^w$ almost everywhere for $t' \neq t_0$. This implies that $U^A(t')(z+1)^w$ holds for all durations $t' \neq t_0$. A consequence of this is that, in equilibrium, $w(t')^w$ must hold for all $t' \neq t_0$.

To see why, assume that $w(t')^w$ holds for some $t' \neq t_0$. Therefore $U^A(t'(z+1)^w < w(t')$ would obtain and hence $\phi^B(t') = \phi^A(t')$ would hold. Hence $w(t')^w = \phi^A(t'(z+1)^w$ would hold (see (A4), (A5)), and therefore $w(t')^w > w = U^A(t'(z+1)^w$ would violate (A9').

Because of (A9'), we thus see that $w(t')^w = \phi^A(t'(z+1)^w = c > 0$ would obtain. Hence $w(t')^w = \phi^A(t '(z+1)^w > 0$ would hold (as $w(t')^w = \phi^A(t')$ and $w(t')^w > w = U^A(t'(z+1)^w$). Hence $w(t')^w$ would violate (A9') at the expected present value of the profits (net of the interview cost $c$) of a firm offering a wage $\omega$ to the duration $t_0$ jobless would exceed zero.

Theorem

Propositions 2 and 3 imply immediately that any equilibrium has the following feature: there exists a critical duration $\tau$ such that:

$w(\tau)^w < w(t)$ for $\tau < t$: $\omega < U^A(t)(z+1)^w$ for all $t > t$. This implies that $\phi^A(t) = \phi^B(t) = 1$ for $t \leq \tau$. Hence $\omega = \Pi(t) = 0$ for $t \leq \tau$, and $\omega = \Pi(t)$ (see the discussion following (A5)).

Furthermore, the wage rate $w(t)$ for $t < \tau$ is determined by the equation $\Pi(t)(y-w(t))/(z+1) = c > 0$. This is so because $\phi^A(t) = \phi^B(t) = 1$ for $t < \tau$ implies $\Pi(t)(y-w(t))/(z+1) = c > 0$ (see (A4)). Hence $\Pi(t)(y-w(t))/(z+1) = c > 0$ for $t < \tau$ and thus (see (A9')) $\Pi(t)(y-w(t))/(z+1) = c > 0$ for $t < \tau$.

What remains to be shown is that $\tau = \tau^*$ has to hold in equilibrium, where $\tau^*$ is the unemployment duration defined in (4).

To see why $\tau = \tau^*$ holds, assume that $\tau < \tau^*$. Then $\omega = \Pi(\tau) \geq \omega_{\min}$. $U^A(\tau)(z+1)^w$ and $\omega(\tau)^w$ would hold, which would imply $\Pi(\tau)(y-U^A(\tau)(z+1))/z+1 = c > \omega_{\min}(y-w)(z+1) - c > 0$. $\omega(\tau)(y-U^A(\tau)(z+1))/z+1 = \alpha > 0$ and $\omega(\tau) > w = U^A(\tau)(z+1)$ imply that the expected present value of the profits (net of $c$) of a firm offering a wage slightly greater than $\omega$ to the duration $\tau$ jobless would be strictly positive. Hence $\tau < \tau^*$ cannot be an equilibrium.

$\tau < \tau^*$ cannot occur in equilibrium, because then $\omega = \omega_{\min}$ and $\omega(\tau)^w$ would hold for $t(\tau^*)$, and hence for firms interviewing workers with these durations, the expected present value of profits (net of the interview cost $c$) would be negative.

15To see why, assume that $w(t')^w$ held for a set of durations $t' = t_0$ with measure strictly greater than zero. Then a jobless worker with duration $t_0$ would, with positive probability, obtain a wage offer exceeding $\omega$ at some future date. Therefore the worker’s expected lifetime utility would exceed $\omega(z+1)$.