THE EXCHANGE RATE IN A DYNAMIC-OPTIMIZING CURRENT ACCOUNT MODEL WITH NOMINAL RIGIDITIES: A QUANTITATIVE INVESTIGATION

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Résumé

Cet article étudie un modèle dynamique optimisant d'une économie ouverte semi-petite. Les prix et salaires nominaux sont rigides dans cette économie. Le modèle prédit un phénomène de "overshooting" du taux de change suite à des changements de l'offre de monnaie. La volatilité du taux de change nominal et du taux de change réel générée par ce modèle est compatible avec celle des taux de change effectifs des pays du G7 pendant la période de l'après-Bretton Woods. Le modèle prédit que des augmentations de l'offre de monnaie domestique, de la productivité domestique du travail et du taux d'intérêt mondial conduisent à une dépréciation nominale et réelle de la monnaie du pays.

Mots clés : taux de change nominaux et réels, rigidités nominales, cycles des affaires

Abstract

This paper studies a dynamic-optimizing model of a semi-small open economy with sticky nominal prices and wages. The model exhibits exchange rate overshooting in response to money supply shocks. The predicted variability of nominal and real exchange rates is roughly consistent with that of G7 effective exchange rates during the post-Bretton Woods era. The model predicts that increases in the domestic money supply, in domestic labor productivity and in the world interest rate induce a nominal and real depreciation of the country's currency.

Key words : nominal and real exchange rates, nominal rigidities, business cycles
1. Introduction

During the last decade, much effort has been devoted to the development of dynamic open economy business cycle models with explicit microfoundations. This work is often referred to as the dynamic optimizing approach to the current account or as the international Real Business Cycle approach (see, e.g., Razin (1995) and Backus, Kehoe and Kydland (1995) for detailed surveys of that work). That research studies models with forward-looking rational agents who trade in international goods and asset markets. With rare exceptions (see discussion below) that literature has either considered models without money or models in which money is neutral (or almost neutral) as prices and wages are assumed fully flexible.¹ In these models, non-monetary shocks (shocks to technologies, preferences, fiscal policy or the terms of trade) are the main source of economic fluctuations.

One of the most striking limitations of models of this type is their inability to capture important aspects of actual exchange rate behavior. In particular, these models tend to underpredict sharply the high variability of nominal and real exchange rates observed during periods of floating


exchange rates.  

It has repeatedly been suggested that models with nominal rigidities might be needed for a proper understanding of exchange rate behavior (see, e.g., Mussa (1990)), and recently several authors have begun to study dynamic-optimizing open economy models that depart from the assumption that nominal prices are fully flexible. The present paper contributes to this recent research effort.

Specifically, the work here builds on papers by Obstfeld and Rogoff (1995) and by Beaudry and Devereux (1995) who develop dynamic-optimizing monetary open economy models in which nominal goods prices are fixed in the short run, as firms set their prices one period in advance. However, these recent models too seem unable to generate sufficient nominal and real exchange rate volatility.

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2 For example, the recent flexible-price monetary model studied by Schlagenhauf and Wrase (1995) generates standard deviations of nominal and real exchange rates that are roughly five to ten times smaller than the actual standard deviations observed for industrialized countries since the end of the Bretton Woods system; non-monetary models generate standard deviations of (real) exchange rates that are smaller still; see, e.g., Backus, Kehoe and Kydland (1995).

3 Sticky prices are a key ingredient of Keynesian exchange rate models developed during the 1970s and 1980s (e.g., Dornbusch (1976)). However, those models lack the rigorous micro-foundations regarding the private sector's consumption and investment decisions that characterize the dynamic-optimizing approach.

The work by Obstfeld and Rogoff and by Beaudry and Devereux is also closely related to recent research that has introduced money and nominal rigidities into closed economy Real Business Cycle models (see, i.a. Cho and Cooley (1990), Cho (1993), Cho and Phaneuf (1993), Hairault and Portier (1993), Yun (1994), Benassy (1995) and Bordo, Erceg and Evans (1995)).

4 The Beaudry and Devereux (1995) model predicts that nominal and real exchange rates are less volatile than output, whereas the reverse is observed historically. Obstfeld and Rogoff (1995) show that (at least in the baseline version of their model) the assumption of preset prices reduces exchange rate volatility due to money supply shocks.

While working on the present project, papers by Hau (1995), Betts and
The present paper studies a dynamic optimizing open economy model in which, in contrast to the work that was just discussed, nominal prices and nominal wages are set two or four periods in advance (the model is calibrated to quarterly data, i.e. one period represents one quarter in calendar time). In addition, a price and wage adjustment process inspired by Calvo (1983 a,b; 1987) is considered that assumes that nominal prices and wages are changed after time intervals of random length.

The paper assumes a semi-small open economy with four types of exogenous shocks: shocks to the domestic money supply, to domestic labor productivity, to the price level in the rest of the world and to the world interest rate.

It appears that the predicted variability of nominal and real exchange rates generated by the model is roughly consistent with that of Hodrick-Prescott filtered quarterly G7 effective exchange rates during the post-Bretton Woods era. The nominal rigidities assumed in this paper allow also to improve model predictions for other business cycle statistics. For example, the version of the model in which prices and wages are set four periods in advance captures better the observed variability of output, consumption and nominal interest rates than a version of the model without nominal rigidities.

Devereux (1996) and Sutherland (1996) came to my attention that also explore the effect of nominal rigidities in open economies, using models closely inspired by Obstfeld and Rogoff (1995). Unfortunately, these authors do not present stochastic model simulations, and hence it remains to be seen whether their models are consistent with exchange rate data. However, only the model proposed by Hau seems to have the potential for generating highly volatile exchange rates, as in his model (in contrast to those of Betts and Devereux and of Sutherland) money supply shocks can generate strong short-run responses of the exchange rate; however, this is only the case when the share of non-tradables in the households' consumption basket is very high.
Among the four types of shocks assumed in this paper, money supply changes have the strongest impact on nominal and real exchange rates. In response to money supply shocks, the model with nominal rigidities considered here exhibits exchange rate overshooting, similar to that generated by Keynesian open economy models with sticky prices (Dornbusch (1976)).

The model predicts that an expansionary money supply shock lowers the domestic nominal interest rate, that it raises output and that it leads to a nominal and real depreciation of the country's currency. Likewise, an increase in the foreign interest rate is predicted to induce a nominal and real depreciation of the country's currency. These predictions are consistent with recent empirical evidence on the effects of monetary policy shocks on the exchange rate reported by Eichenbaum and Evans (1995), among others. The model here predicts furthermore that an increase in domestic total labor productivity triggers a nominal and real depreciation of the country's currency, while an increase in the price level in the rest of the world induces a nominal appreciation (foreign price shocks have little impact on the real exchange rate).

The structure of the remainder of the paper is as follows: the model is outlined in Section 2. Section 3 discusses empirical regularities that characterize international business cycles. Section 4 presents simulation results. Section 5 concludes.
2. The model

The paper assumes a semi-small open economy with a representative household, with firms and a government.  

2.1. Preferences

Household preferences are described by:

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, M_t/P_t, L_t). \]  

(1)

\( E_0 \) denotes the mathematical expectation conditional on information available in period t=0. \( 0<\beta<1 \) is a subjective discount factor and \( U(\cdot) \) is an instantaneous utility function. \( C_t \) is an index of period t consumption. \( M_t/P_t \) represents real balances, where \( M_t \) is nominal nominal balances held at the beginning of period t. while \( P_t \) is a consumption price index for period t. \( L_t \) represents labor effort in period t. The utility function \( U \) is of the following form:

\[ U(C, M/P, L) = \left( \frac{1}{1-\psi} \right) \left[ C^{\sigma} + \kappa (M/P)^\Gamma \right]^{1-\psi} \]

- L.

where \( \psi, \sigma, \Gamma \) and \( \kappa \) are parameters.  

The consumption index \( C_t \) is defined as  

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5 In contrast, existing dynamic optimizing open economy models with nominal rigidities have assumed a two-country world. The work here builds on Real Business Cycle models of (semi-)small open economies (e.g., Cardia (1991), Mendoza (1991), Schmitt-Grohé (1993) and Akitoby (1995)). The economy considered here is semi-small in the sense that (as discussed below) it faces a downward-sloping aggregate export demand function, while import prices and the international interest rates are exogenous (this distinguishes the model here from models of small economies that face exogenous prices in all international markets).

6 Note that labor effort enters linearly in the period utility function. Such a specification is widely used in the Real Business Cycle literature, as it seems best suited for capturing the observed volatility of hours worked (e.g., Hansen (1985)).
\[ C_t = D_t^{1-\alpha} F_t^\alpha, \]

where \( D_t \) is an index of consumption goods produced in the country, while \( F_t \) is an index of imported consumption goods (\( \alpha \) is a parameter; \( 0 < \alpha < 1 \)). There exists a continuum of home produced goods indexed by \( s \in [0, 1] \) and a continuum of imported goods indexed by \( \tau \in [0, 1] \). All consumption goods are perishable. \( D_t \) and \( F_t \) are defined as follows:

\[
D_t = \left\{ \int_0^1 d_t(s)^{1/(1+\nu)} \, ds \right\}^{1+\nu} \quad \text{and} \quad F_t = \left\{ \int_0^1 f_t(\tau)^{1/(1+\nu)} \, d\tau \right\}^{1+\nu},
\]

where \( \nu > 0 \) is a parameter. \( d_t(s) \) and \( f_t(\tau) \) denote the date \( t \) consumption of home produced and of imported goods of types \( s \) and \( \tau \), respectively. Let \( p_t^D(s) \) and \( p_t^F(\tau) \) be the prices of these goods (in domestic currency) and let \( p_t^D \) and \( p_t^F \) be price indexes defined as:

\[
p_t^D = \left\{ \int_0^1 p_t^D(s)^{-1/\nu} \, ds \right\}^{-\nu} \quad \text{and} \quad p_t^F = \left\{ \int_0^1 p_t^F(\tau)^{-1/\nu} \, d\tau \right\}^{-\nu}.
\]

The consumption price index \( P_t \) is defined as:

\[
P_t = (1-\alpha)^{\alpha - 1} \alpha^{-\alpha} (P_t^D)^{1-\alpha} (P_t^F)^{\alpha}. \tag{7}
\]

Optimal consumption behavior implies:

\[
D_t = (1-\alpha) P_t C_t / P_t^D \quad \text{and} \quad F_t = \alpha P_t C_t / P_t^F \quad \text{as well as}
\]

\[
d_t(s) = D_t \left( p_t^D(s)/P_t^D \right)^{(1+\nu)/\nu} \quad \text{and} \quad f_t(\tau) = F_t \left( p_t^F(\tau)/P_t^F \right)^{(1+\nu)/\nu}. \tag{2}
\]

The household can provide labor services of different types. There exists a continuum of labor types, indexed by \( h \in [0, 1] \). Let \( l_t(h) \) denote the number of hours of type \( h \) labor. The variable \( L_t \) that appears in the

\[ \text{The price indices } p_t^D, p_t^F \text{ and } P_t \text{ represent the minimal expenditure (in domestic currency) needed to buy one unit of the composite D, F and C goods in period } t, \text{ respectively.} \]
utility function is defined as: \( L_t = \int_0^1 l_t(h) \, dh \).

2.2. Firms and the structure of goods markets

There are two types of firms in the country: (1) producers of consumption goods (home produced goods can be sold in the domestic market or exported); (2) firms that import foreign consumption goods in order to sell them in the domestic market. All firms are owned by the domestic household.

Following Obstfeld and Rogoff (1995) and Beaudry and Devereux (1995), monopolistic competition in goods markets is assumed: each good is produced (or imported) and sold by a single firm (consumers purchase all goods from the country's firms—they cannot buy goods directly in foreign markets).

Domestic producers have identical technologies that use domestic labor as the only input (labor is immobile internationally). The period \( t \) output of the firm producing domestic good \( s \) is:

\[
y_t(s) = \theta_t \, L_t(s),
\]

where \( y_t(s) \) is the firm's output, while \( \theta_t \) is period \( t \) labor productivity (N.B. productivity is identical for all domestic producers). \( \theta_t \) is an exogenous random variable. \( L_t(s) \) is an index of the different types of labor used by the firm in period \( t \):

\[
L_t(s) = \left( \int_0^1 l_t(h;s) \phi \, dh \right)^{1/\phi},
\]

where \( l_t(h;s) \) represents the quantity of type \( h \) labor used by firm \( s \) at date \( t \); \( \phi < 1 \) is a parameter. Cost minimization implies that the demand for type \( h \) labor by firm \( s \) satisfies:

\[
l_t(h;s) = (y_t(s)/\theta_t) \left( w_t(h)/w_t \right)^{1/(\phi-1)},
\]

where \( w_t(h) \) is the wage rate for type \( h \) labor, while
\[ \Pi_t = \left( \int_0^1 w_t(h)^{\phi-1} dh \right)^{(\phi-1)/\phi} \]

is an aggregate wage index.  

The date \( t \) profit of the firm that produces good \( s \) is:

\[ \Pi^D_t(s) = p^D_t(s) d_t(s) + e_t p^X_t(s) x_t(s) - \int_0^1 w_t(h) l_t(h; s) dh, \]

where \( e_t \) is the country's exchange rate in period \( t \), quoted as the local currency price of one unit of foreign currency. \( p^X_t(s) \) is the price (in foreign currency) of good \( s \) in the export market, while \( x_t(s) \) represents exports of the good (the determinants of export demand are discussed below).

The period \( t \) profit of the firm that sells the imported good of type \( \tau \) is:

\[ \Pi^F_t(\tau) = (p^F_t(\tau) - e_t P^*_t) f_t(\tau), \]

where \( P^*_t \) is the foreign currency price of the imported good in period \( t \) (the foreign currency prices of all imported goods are identical). It is assumed that \( P^*_t \) equals the price level in the rest of the world. \( P^*_t \) is treated as an exogenous variable in the following analysis.

The producer of domestic good \( s \) maximizes

\[ \Pi^D_t(s) = \sum_{i=0}^{1} E_t \rho_{t,t+1} \Pi^D_{t+1}(s)/P_{t+1} \]

while the importer of foreign good \( \tau \) maximizes

\[ \Pi^F_t(\tau) = \sum_{i=0}^{1} E_t \rho_{t,t+1} \Pi^F_{t+1}(\tau)/P_{t+1} \]

Here, \( \rho_{t,t+1} \) is the pricing kernel used to value random date \( t+1 \) pay-offs (denominated in units of the composite consumption good), in terms of units composite consumption at date \( t \). As firms are owned by the representative

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\( W_t \) represents the minimal expenditure (in domestic currency) needed to purchase one unit of the composite labor in period \( t \).
household, it is assumed that firms value future payoffs according to the consumer's intertemporal marginal rate of substitution in consumption. Hence, $\rho_{t,t+1} = \beta^1 U_{C,t+1} / U_{C,t}$ is assumed, where $U_{C,t+1}$ is the household's marginal utility of consumption in period $t+1$.\footnote{See, e.g., Sargent (1987), Blanchard and Fischer (1989) and Romer (1996) for discussions of this pricing kernel.}

2.3. Foreign demand

Let $P^X_t$ and $X_t$ be an index of date $t$ export prices (in foreign currency) and a quantity index of date $t$ exports, respectively. $P^X_t$ and $X_t$ are defined analogously to the indices $P^D_t$ and $D_t$:

$$P^X_t = \left( \int_0^1 P^X_t(s)^{-1/\nu} ds \right)^{-\nu}, \quad X_t = \left( \int_0^1 x_t(s)^{1/(1+\nu)} ds \right)^{1+\nu}.$$ 

It is assumed that aggregate exports are determined by

$$X_t = (P^X_t/P^*_t)^{-\eta}, \quad \eta > 0.$$ 

Hence, $X_t$ is negatively related to the ratio of export prices to the price level in the rest of the world.

It is assumed that the export demand function for good $s$ resembles the domestic demand function for that good (see (2)):

$$x_t(s) = X_t \left( P^X_t(s)/P_t \right)^{-(1+\nu)/\nu}. \quad (5)$$

2.4. Government

The country's government prints the local currency. Increases in the money stock are paid out to the representative household in the form of lump-sum transfers. The money stock is exogenous. The government makes no attempt to
influence the exchange rate, i.e. the exchange rate floats freely.

2.5. Household budget constraint

The household can hold three financial assets: local money, nominal bonds denominated in foreign currency and domestic currency bonds. The bonds are risk-free and have a maturity of one period. As all firms are owned by the domestic household, the household's budget constraint in period $t$ is:

$$ M_{t+1} + P_{t}C_t + e_tB_{t+1} + A_{t+1} = \int_0^1 \pi^D_t(s) \, ds + \int_0^1 \pi^F_t(r) \, dr + \int_0^1 \pi^L_t(h) w_t(h) \, dh + e_tB_t(1+i^*_t) + A_t(1+i^*_{t-1}) - M_t + T_t. \tag{6} $$

Here, $T_t$ is the government cash transfer in period $t$. $B_t$ and $A_t$ are, respectively, the household's (net) stock of foreign currency bonds and its (net) stock of local currency bonds that become due in period $t$. $i^*_t$ and $i^*_{t-1}$ are the nominal interest rates on these two types of bonds. The interest rate on foreign currency bonds ($i^*_t$) is exogenous.

2.6. Price and wage determination

Most of the discussions below assume that nominal prices and wages are set a fixed number of periods in advance. In addition, a price and wage adjustment mechanisms inspired by Calvo (1983a,b; 1987) is considered that postulates overlapping price and wage contracts of random duration. Throughout the analysis, it is assumed that export prices are set in foreign currency.\(^{10}\)

\(^{10}\)In contrast, Obstfeld and Rogoff (1995) and Beaudry and Devereux (1995) abstract from nominal wage rigidity and they assume that export prices are predetermined in terms of the exporter's currency—in their models, the export price in foreign currency adjusts instantaneously to changes in the nominal exchange rate, in a manner that ensures that the law
2.6.1. Predetermined prices and wages

The first framework assumes that the period $t$ prices and nominal wages are set at date $t-k$ (the simulations below consider $k=2$ and $k=4$).\textsuperscript{11}

Maximizing the period $t-k$ objective function of the domestic producer of good $s$ ($\pi_t^D(s)$) with respect to $p_t^D(s)$ and $p_t^X(s)$, subject to the firm's production function (equation (3)) and to the demand functions for the domestic good of type $s$ (see (2) and (5)), and aggregating over all $s \in [0,1]$ yields the following aggregate price equations:

\begin{equation}
 p_t^D = (1+\nu) W_t E_t \cdot k(\rho_{t-k,t} D_t / \theta_t) / E_t \cdot k(\rho_{t-k,t} D_t) \tag{7} 
\end{equation}

and

\begin{equation}
 p_t^X = (1+\nu) W_t E_t \cdot k(\rho_{t-k,t} X_t / \theta_t) / E_t \cdot k(\rho_{t-k,t} X_t) \tag{8} 
\end{equation}

Similarly, maximization of $\pi_t^F(\tau)$ with respect to $p_t^F(\tau)$, subject to the demand function for imported goods of type $\tau$ (see (2)) and aggregation over all $\tau \in [0, 1]$ yields:

\begin{equation}
 p_t^F = (1+\nu) E_t \cdot k(\rho_{t-k,t} F_t / \theta_t) / E_t \cdot k(\rho_{t-k,t} F_t) \tag{9} 
\end{equation}

These price equations are based on the assumption that, although prices are fixed in advance, firms always satisfy the demand that they face.\textsuperscript{12}

\textsuperscript{11}A similar framework is considered by Bordo, Erceg and Evans (1995) who develop a dynamic general equilibrium model of a closed economy in which the wage is set $k=4$ periods in advance (prices, however, are fully flexible in that model).

\textsuperscript{12}This assumption is standard in business cycle models with price rigidities (e.g., Mankiw (1994), Romer (1996)). Note that, as all firms
\( w_t(h) \), the nominal hourly wage of type \( h \) labor in period \( t \) is also determined at date \( t-k \). It is assumed that the household makes a commitment at date \( t-k \) to provide \( \xi_{t-k}(h;s) \cdot l_t(h;s) \) hours of type \( h \) labor to the firm that produces good \( s \) in period \( t \) (at the predetermined hourly wage rate \( w_t(h) \)). \( \xi_{t-k}(h;s) \) is a decision variable that the household sets at date \( t-k \) (clearly, \( \xi_{t-k}(h;s) = 1 \) has to hold in equilibrium--see below). In contrast, \( l_t(h;s) \) (the input of type \( h \) labor used by firm \( s \) at date \( t \)) is not predetermined, but is chosen by firm \( s \) in period \( t \) (according to equation (4)).

As shown in the Appendix, optimizing household behavior implies that the following first-order condition has to hold:

\[
\frac{w_t(h)}{P_t} = \frac{\{E_{t-k} \cdot l_t(h;s)\}}{\{E_{t-k} \cdot U_{C,t} \cdot l_t(h;s)\}}
\]  
(10)

(note that, in the absence of uncertainty, equation (11) implies \( w_t(h)/P_t = 1/U_{C,t} \); this condition corresponds to the familiar equalization of the real wage rate to the marginal rate of substitution between consumption and leisure\(^{13}\)). As (10) has to hold for all \( h, s \in [0, 1] \), the aggregate wage index, \( \bar{W}_t \), satisfies the following condition (see Appendix):

\[
\frac{\bar{W}_t}{P_t} = \frac{\{E_{t-k} \cdot L_t\}}{\{E_{t-k} \cdot L \cdot U_{C,t}\}}
\]  
(11)

have identical technologies and face identical demand functions, \( p_t^D(s) = p_t^D \), \( p_t^X(s) = P_t^X \) and \( p_t^F(\tau) = P_t^F \) holds for all \( s, \tau \). Up to a certainty equivalent approximation, equations (7)-(9) show thus that each firm's price equals expected unit costs multiplied by a constant mark-up factor, \( 1+\nu > 1 \). Unless unanticipated shocks raise the actual unit costs in period \( t \) above the predetermined prices, it is thus not in the interest of firms to ration their customers in period \( t \).

\(^{13}\)N.B. For the utility function used here, the marginal utility equals unity; hence \( 1/U_{C,t} \) is the marginal rate of substitution between consumption and leisure.
2.6.2. Calvo-type price and wage determination

In addition, a model of price determination inspired by Calvo (1983 a,b; 1987) is considered that assumes that firms are not allowed to change their prices, unless they receive a random "price-change signal". The probability that a given price can be changed in any particular period is \(1-\delta\), a constant (as there is a continuum of goods, \(1-\delta\) represents also the fraction of all prices that are changed in each period; furthermore, the average time between price changes is \(1/(1-\delta))\).\(^{14}\)

Consider a domestic producer that is "allowed" at date \(t\) to set a new sales price in the domestic market. Let \(\rho_t^D\) be the price selected by that firm. If this price is still in effect at date \(t+1\), then the firm's sales in the domestic market at that date are given by
\[
d_{t+1} = D_{t+1} \left(\frac{\rho_t^D}{P_{t+1}}\right)^{-\frac{(1+\nu)}{\nu}},
\]
as can be seen from (2) (here, it is again assumed that firms always satisfy the demand that they face). The probability that the price \(\rho_t^D\) is still in effect at date \(t+1\) is given by \(\delta^t\). Thus, the firm selects the price \(\rho_t^D\) that maximizes the following expression (N.B. \(W_{t+1}/\theta_{t+1}\) is the firm's unit cost in period \(t+1\)):
\[
\sum_{i=0}^{i=\infty} \delta^t E_t \left(\rho_{t+1} - \frac{\rho_t^D}{P_{t+1}}\right)^{-\frac{(1+\nu)}{\nu}} \left(\frac{\rho_t^D}{P_{t+1}} - \frac{W_{t+1}/\theta_{t+1}}{P_{t+1}}\right).
\]
The solution of this maximization problem is:
\[
\rho_t^D = \frac{(1+\nu)}{\nu} \left(\sum_{i=0}^{i=\infty} \delta^t E_t \frac{W_{t+1}/\theta_{t+1}}{P_{t+1}}\right) \left(\frac{\sum_{i=0}^{i=\infty} \delta^t E_t W_{t+1}^D}{\sum_{i=0}^{i=\infty} \delta^t E_t P_{t+1}^D}\right),
\]
where \(E_{t+1}^D = \rho_{t+1}^D (1+\nu) / P_{t+1}^D\). In period \(t\), a fraction \((1-\delta)\delta^t\) of domestic producers are posting prices in the domestic market.

\(^{14}\)Calvo (1983 a,b; 1987) considers a continuous time model. Here, a discrete time version is used that builds on Rotemberg (1987), Chadha (1987) and Yun (1994). The original Calvo model does not assume sluggish nominal wages; however, the wage adjustment equation derived below follows closely the spirit of Calvo's work.
that were set \( j \geq 0 \) periods ago. Hence, the price index for home produced consumption goods is:

\[
P^D_t = \left(1 - \delta\right) \sum_{j=0}^{\infty} \delta^j (\rho^D_{t-j})^{-1/\nu}.
\]

Analogously, it can be shown that a firm that is allowed in period \( t \) to set a new export price (in foreign currency) selects the following price:

\[
\rho^X_t = (1+\nu) \left\{ \sum_{i=0}^{\infty} \delta^i E_t \{X^X_{t,t+1} w^X_{t+1}/\theta_{t+1} \} \right\} \left( \sum_{i=0}^{\infty} \delta^i E_t \{X^X_{t,t+1} e^X_{t+1} \} \right)^{-1/\nu}.
\]

where \( \sum_{t,t+1}^{X} \sum_{t+1}^{P^X} \frac{X^X_{t,t+1}}{(1+\nu)} X^X_{t+1}/P^X_{t+1} \). The index of export prices is:

\[
P^X_t = \left(1 - \delta\right) \sum_{j=0}^{\infty} \delta^j \left(\rho^X_{t-j}\right)^{-1/\nu}.
\]

An importer of foreign goods that is allowed at date \( t \) to set a new price of its good in the domestic market selects the following price:

\[
\rho^F_t = (1+\nu) \left\{ \sum_{i=0}^{\infty} \delta^i E_t \{X^F_{t,t+1} e^F_{t+1} P^F_{t+1} \} \right\} \left( \sum_{i=0}^{\infty} \delta^i E_t \{X^F_{t,t+1} e^F_{t+1} \} \right)^{-1/\nu}.
\]

where \( \sum_{t,t+1}^{F} \sum_{t+1}^{P^F} \frac{F^F_{t,t+1}}{(1+\nu)} F^F_{t+1}/P^F_{t+1} \). The price index of imported goods is thus:

\[
P^F_t = \left(1 - \delta\right) \sum_{j=0}^{\infty} \delta^j \left(\rho^F_{t-j}\right)^{-1/\nu}.
\]

Wages too are changed after time intervals of random length. With an exogenously given probability \( 1 - \Delta \), the wage rate of a given labor type is changed in any particular period (hence, in each period, the hourly wage of a constant fraction \( 1 - \Delta \) of labor types changes). Assume that the wage for type \( h \) labor is changed in period \( t \) and let \( w^i_t(h) \) denote the new wage. With probability \( \Delta^1 \), \( w^i_t(h) \) is still in effect at date \( t+1 \) (\( 1 \geq 0 \)). It is assumed that the household makes a commitment at date \( t \) to provide \( \xi^i(t;h) i^i_{t+1}(h;\cdot)s \) hours of type \( h \) labor to firm \( s \) at date \( t+1 \), provided that the wage rate \( w^i_t(h) \) is still in effect at that date. \( \xi^i_t(h;\cdot)s \) is a decision variable that the household sets at date \( t \). In contrast, \( i^i_{t+1}(h;\cdot)s \)
(the type h labor input used by firm s at date t+1) is not determined in period t, but is chosen by firm s at date t+1 (as a function of output demand at that date). As shown in the Appendix, optimizing household behavior implies that the following first-order condition has to hold:

\[ w_t = w_t(h) = \sum_{i=0}^{1} (\beta \Delta)^{i} E_t \chi_{t+1} / \sum_{i=0}^{1} (\beta \Delta)^{i} E_t (U_{c,t+1} (1/P_{t+1}) \chi_{t+1}), \]  

(12)

where \[ \chi_{t+1} = (W_{t+1})^{1/(1-\phi)} Y_{t+1}/\theta_{t+1}; \]

here, \[ Y_{t+1} = \int_{0}^{1} y_{t+1}(s)ds \] is total physical output of domestic producers in period t+1.\(^{15}\)

For a fraction \((1-\Delta)\Delta^j\) of labor types, the hourly wage in effect at date t was set in period t-j \((j > 0)\). Hence, the aggregate wage index is given by:

\[ w_t = \left( (1-\Delta) \sum_{j=0}^{\infty} \Delta^j (w_{t-j})^{\phi/(\phi-1)} \right)^{(\phi-1)/\phi}. \]

2.7. The household's intertemporal decisions

The representative household's intertemporal consumption decisions and her demand for money can be determined by maximizing the expected life-time utility function specified in (1) subject to the restriction that the budget constraint (6) holds in all periods and for all states of the world. Ruling out Ponzi schemes, that decision problem has the following first-order conditions:

\[ 1 = \beta (1+\tau_t) E_t \left\{ \frac{c_{t+1}}{\Omega_{t+1} P_{t}} \frac{\sigma^{-1} c_{t}}{\Omega_t P_{t+1}} \right\}. \]  

(13a)

\(^{15}\)Note that \(w_t(h)\) does not depend on the labor type h, i.e. the same wage rate is set for all labor types for which a wage change occurs in period t.
and 
\[ 1 = \beta (1+i_t^*) E_t \left( \frac{c_{t+1}^{\sigma-1} \Omega_{t+1} P_t e_{t+1}}{c_t^{\sigma-1} \Omega_t P_{t+1} e_t} \right), \]  
\text{(13b)} \]

where \( \Omega_{t+1} = \left( c_{t+1}^{\sigma-1} \kappa (M_{t+1}/P_{t+1})^\Gamma \right)^{-1+(1-\psi)/\sigma} \). Furthermore,
\[ \kappa \left( \Gamma/\sigma \right) E_t \left\{ \Omega_{t+1} \left( M_{t+1}/P_{t+1} \right)^{\Gamma-1}/P_{t+1} \right\} = l_t E_t \left\{ \Omega_{t+1} c_{t+1}^{\sigma-1} (1/P_{t+1}) \right\}. \text{(14)} \]

Equations (13 a) and (13 b) are Euler conditions, while equation (14) can be interpreted as a money demand condition.

2.8. Equilibrium and solution method

Demand equals supply in all goods markets because, by assumption, firms always satisfy the demand that they face at prevailing prices. Likewise, hours worked at the prevailing wage rate are determined by firms' demand for labor. In equilibrium, the amount of type h labor purchased by each firm has to equal the supply of type h hours to that firm by the representative household:

\[ \xi_t(h;s) = 1 \text{ for all } t \text{ and all } h, s \in [0, 1], \]

and, hence, \( l_t(h) = \int_0^1 \xi_t(h;s)ds \) for all \( t \) and all \( h \in [0, 1] \).

Equilibrium in the market for domestic money requires that the demand for money equals the supply. It is assumed that only residents of the country hold the local currency. Equilibrium in the money market requires thus:

\[ M_{t+1}^* = M_{t+1} \text{ for all } t, \]

where \( M_{t+1}^* \) represents the household's desired money balances, as determined by equation (14). The law of motion of the money supply is:

\[ M_{t+1} = M_t + T_t. \]
where $T_t$ is the government transfer to the household in period $t$ (see (6)).

It is assumed that the government does not issue bonds and that foreign investors do not hold bonds denominated in domestic currency. Hence, the household’s (net) stock of domestic currency bonds has to be zero in equilibrium: 

$$A_t = 0 \text{ for all } t.$$

Given a stochastic process for the exogenous variables of the model, an equilibrium can be defined as a stochastic process for the endogenous variables that satisfies the equilibrium conditions that were just discussed and the equations of the model discussed earlier. No analytical model solution exists. In this paper, a numerical solution is obtained by taking a linear approximation of the equations of the model around a deterministic steady state (i.e. around an equilibrium in which all exogenous and endogenous variables are constant). This approximation yields a system of expectational difference equations that can easily be solved (for example, using the method described in Blanchard and Kahn (1980)).

2.9. Parameterization

2.9.1. Preferences and foreign demand

The simulations assume a coefficient of relative risk aversion of $\psi=2$. This value lies in the range of risk aversion coefficients usually assumed in the business cycle literature (Friend and Blume (1975) present evidence consistent with this value of the risk aversion coefficient).

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16This solution method is widely used in business cycle research; see, e.g., King, Plosser and Rebelo (1988), Rotemberg and Woodford (1992), Cooley and Hansen (1995), Uhlig (1995). In the simulations discussed below, the model is linearized around a steady state in which the country’s (net) stock of foreign bonds is zero.
The preference parameter $\alpha$ determines the share of consumption expenditures that is devoted to imported consumption goods. The simulations assume $\alpha=0.33$ (this value corresponds to the arithmetic average of the ratios of imports to private consumption in the G7 countries during the period 1973-91).

As mentioned above, equation (14) can be interpreted as a money demand equation. The elasticities of money demand with respect to consumption and with respect to the domestic nominal interest rate are (approximately) given by $e_{m,c}=(1)/(1)$ and $e_{m,i}=1/(1)$, respectively. The simulations assume $e_{m,c}=0.20$ and $e_{m,i}=-0.04$ (the values of $\sigma$ and $\Gamma$ that correspond to these choices for $e_{m,c}$ and $e_{m,i}$ are: $\sigma=-4$, $\Gamma=-24$). These values of $e_{m,c}$ and $e_{m,i}$ are in the range of estimates of the (quarterly) transactions elasticity and interest rate elasticity of money demand that can be found in econometric work on U.S. money demand (e.g., McCallum (1989) and Goldfeld and Sichel (1990)) as well in Fair's (1987) study of money demand in 27 industrialized countries.

The preference parameter $\kappa$ is set in such a way that the steady state consumption velocity (ratio of nominal consumption expenditure to the money

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17 To understand these expressions, note that, up to a certainty equivalent approximation, the money demand condition (14) can be written as: $k (1/\sigma) (M_{t+1}/P_{t+1})^{-1} = e_{t+1}^{\sigma-1} t+1 t+1$, where $e_{t+1}$ is a forecast error ($E_t e_{t+1}=0$).

18 Note, however, that (as is common in the literature) the money demand functions estimated by these authors use GNP as a scale variable, and not consumption per se. Because, the focus of the present paper is on high frequency exchange rate fluctuations, estimates of quarterly money demand elasticities are used to calibrate the model (long-run elasticities of money demand with respect to the transactions proxy are generally higher than short-run elasticities---e.g., estimation results presented by McCallum (1989) suggest that the long run elasticity is approximately 0.50).
stock) equals unity. ¹⁹

Business cycle models that are calibrated to quarterly data commonly assume a steady state real interest rate in the range of 1% per quarter, and that is also the value of the steady state interest rate used here. ²⁰

The price elasticity of export demand is set to n=1.2, a value consistent with estimated export demand elasticities for industrialized countries (e.g., Goldstein and Khan (1978)). ²¹

2.9.2. Price and wage adjustment

The simulations of the version of the model with predetermined prices and wages consider the following values of k: k=2, k=4 (most of the discussions below focus on the case k=4).

Rotemberg (1987) points out that the aggregate price equations of the Calvo model are observationally equivalent to those implied by a model of price determination developed in Rotemberg (1982 a,b) that assumes that firms can freely alter their prices at any time, but that they face

¹⁹The key model predictions discussed below are not sensitive to the assumed steady state velocity (a unit velocity is roughly consistent with data on the M1 consumption velocity in the G7 countries; e.g., in the U.S. that velocity was 0.93 in 1994).

²⁰Thus β=1/1.01 is assumed (the existence of a deterministic steady in the present model requires that β (1+r)=1 holds, where r is the steady state interest rate).

²¹In order to solve the model for the aggregate price and quantity variables on which the discussions below focus, no specific values need to be assigned to the parameters γ and φ that determine the elasticity of substitution between different types of consumption goods and different types of labor (the linearization of the model yields a system of equations in the aggregate variables that does not depend on γ or φ). It can be verified that for the values of the preference parameters assumed in the simulations, the utility function is strictly increasing and concave in consumption and real balances, in the neighborhood of the steady state around which the model is linearized.
quadratic costs of changing their prices. Econometric results (based on aggregate U.S. price data) presented in Rotemberg (1982 a), yield the following estimate of the price adjustment parameter $\delta$: $\delta=0.92$. This is the value of $\delta$ used in the simulations discussed below. That value implies that the average time between price changes at the firm level is 12.5 quarters. It is assumed that the average time between wage changes too equals 12.5 quarters, i.e. $\Delta=0.92$ is used (support for this value is provided by Backus (1984) who finds that Canadian wage contracts have a mean length of 12.7 quarters).

2.9.3. Exogenous variables

The exogenous variables follow autoregressive processes. In the following equations, $\rho^M_{}$, $\rho^\theta_{}$, $\rho^*_{}$ and $\rho^R_{}$ are parameters, while $\epsilon^M_t$, $\epsilon^\theta_t$, $\epsilon^*_t$ and $\epsilon^R_t$ are white noise random errors whose standard deviation are denoted by $\sigma^M_{}$, $\sigma^\theta_{}$, $\sigma^*_{}$ and $\sigma^R_{}$, respectively. These error terms are assumed to be mutually independent.

The money supply process assumed in the simulations is identical to that used in a recent monetary business cycle model developed by Cooley and Hansen (1995):

$$\ln(M_{t+1}/M_t) = \rho^M_{} \ln(M_t/M_{t-1}) + \epsilon^M_t,$$

where $M_t$ is the money supply at the beginning of period $t$. Following Cooley and Hansen, $\rho^M_{}=0.491$ is assumed and the standard deviation of the money supply innovation ($\epsilon^M_t$) is set to $\sigma^M_{}=0.0089$ (Cooley and Hansen obtain these parameter values by fitting the above money supply equation to U.S. money stock data).

The process for productivity is:

$$\ln(\theta_t) = \rho^\theta_{} \ln(\theta_{t-1}) + \epsilon^\theta_t.$$
As is common in the Real Business Cycle literature, productivity is assumed to be highly serially correlated. Prescott (1986) presents evidence that the autocorrelation of quarterly productivity is in the range of 0.95, and hence $\rho^\theta=0.95$ is assumed here. The standard deviation of the productivity innovation ($\sigma^\theta_t$) is set to $\sigma^\theta=0.007$ (this is approximately the value of $\sigma^\theta$ suggested by Prescott (1986)).

The behavior of the foreign price level is described by:

$$\ln(P_t^*/P_{t-1}^*) = \rho^* \ln(P_{t-1}^*/P_{t-2}^*) + \epsilon_t^*.$$  

The simulations assume the following values of $\rho^*$ and of the standard deviation of $\epsilon_t^*$: $\rho^*=0.80$ and $\sigma^*=0.005$.

Finally, a stochastic process for the foreign interest rate has to be specified. Let $R_t = (1+i_t^*) \ E_t(P_t^*/P_{t+1}^*)^{-1}$ denote the expected foreign real interest rate. The simulations assume that $R_t$ follows an AR(1) process:

$$R_t = (1-\rho^R) r + \rho^R R_{t-1} + \epsilon_t^R,$$

where $r$ is the steady state real interest rate. In the simulations, $\rho^R=0.79$ is assumed and the standard deviation of the interest rate innovation $\epsilon_t^R$ is: $\sigma^R=0.0043$.

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22 Many business cycle studies have used these (or very similar) values for $\rho^\theta$ and $\sigma^\theta$ (e.g., Hansen (1985), Gomme (1993), Ambler and Paquet (1994)).

23 These values were obtained by taking the quarterly U.S. CPI series as a measure of the foreign price $P_t^*$ and fitting the above equation to that series (a constant was also included in the regression; sample period 1973-94).

24 N.B. Note that, approximately, $R_t = i_t^* - E_t \ln(P_{t+1}^*/P_t^*)$ and, hence $R_t = i_t^* - \rho^* \ln(P_t^*/P_{t-1}^*)$ if the above equation for the foreign price level $P_t^*$ holds. Using the U.S. interest rate on three-month Certificates of Deposit as a measure of $i_t^*$, the U.S. CPI as a measure of $P_t^*$ as well as the value of $\rho^*$ reported above to construct a quarterly time-series for $R_t$ and fitting
3. Stylized facts about economic fluctuations (Post-Bretton Woods era)

Table 1 presents empirical information on the behavior of output, private consumption, hours worked, net exports, the price level, the money supply and short term nominal interest rates in the G7 countries since 1973. The table also provides information on the effective exchange rates of the G7 countries and on bilateral exchange rates between the U.S. dollar and the currencies of the remaining G7 countries. Standard deviations and autocorrelations of the variables are reported, as well as correlations with domestic output. All series used in Table 1 are sampled at a quarterly frequency. Detailed information on the data is provided in the Appendix.

The empirical series have all been detrended using the Hodrick-Prescott filter; before applying this filter, all series (with the exception of net exports and nominal interest rates) were logged. The empirical regularities discussed below do not depend on this particular filter—other detrending methods, e.g. linear detrending, lead to similar stylized facts.

In most G7 countries, the standard deviation of output is about 2%. Generally, consumption and hours worked are less volatile than output. The standard deviation of money typically exceeds that of output, while the price level is generally less volatile than output. Consumption and hours worked are procyclical (i.e., positively correlated with output), while net exports are countercyclical. Money is procyclical, while the price level is countercyclical. The nominal interest rate is procyclical in four of the G7 countries. All variables considered in Table 1 are highly serially

an AR(1) process to that series yields $\rho^R=0.79$ and $\sigma^R=0.0043$ (sample period 1973-91).
correlated (to save space, Table 1 only shows autocorrelations of output and effective exchange rates).

Nominal and real exchange rates are more volatile than any of the other variables considered in Table 1. The standard deviations of real exchange rates are very similar to those of nominal exchange rates. For all G7 countries, the correlation between nominal and real effective exchange rates is high (correlations of 0.95 or above). The autocorrelations of effective exchange rates mostly exceed 0.70. The U.S. effective exchange rate (nominal and real) is procyclical, while the effective exchange rates of the remaining G7 countries are generally countercyclical (here, exchange rates are measured as the national currency price of foreign currency; thus the external value of a country's currency is typically positively correlated with domestic output).

Among the G7 countries, Germany, France, Italy and Canada have the least volatile effective exchange rates, while Japan has the most volatile effective exchange rate. The arithmetic average of the standard deviations of the nominal effective exchange rates of the G7 countries is 4.80% while the average standard deviation of the real effective exchange rate series is 4.75%. Bilateral U.S. dollar exchange rates are typically more volatile than the effective exchange rates of the G7 countries (the standard deviations of these bilateral exchange rates range mostly between 8% and 9%).

The comparatively low volatility of the effective exchange rates of Germany, France and Italy reflects attempts by these countries to reduce

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25 Likewise, the nominal bilateral U.S. dollar exchange rates are highly correlated with the corresponding real rates. To save space, these additional statistics are not reported in Table 1.
fluctuations in the bilateral exchange rates among the member states of the European Community (EMS). Likewise, the comparatively low volatility of the Canadian effective exchange rate reflects attempts by the Canadian authorities to reduce fluctuations in the U.S. dollar-Canadian dollar exchange rate. As the model considered in this paper assumes that the exchange rate floats freely, particular attention in the discussion below will be devoted to the ability of the model to capture the historical volatility of U.S., Japanese and U.K. effective exchange rates and of the bilateral U.S. dollar exchange rates.

4. Simulation results

Simulation results are presented in Tables 2-3. The statistics reported in these tables are averages of moments calculated for 1000 model simulations with a sample length of 89 periods each (this number of periods corresponds to the length of the empirical nominal effective exchange rate series used for Table 1).

In Tables 2 and 3, the output variable corresponds to \( Y_{t}^{26} \), consumption is \( C_{t} \), hours worked is \( L_{t} \), the price level is \( P_{t} \) and the real exchange rate is defined as \( e_{t}^{*}P_{t}^{e}/P_{t} \).

All simulated series were logged (with the exception of net exports and the nominal interest rate) and passed through the Hodrick and Prescott (1980) filter. To facilitate the comparison between model predictions and the data, the column labelled "Data" in Tables 2-3 reports arithmetic averages, across the G7 countries, of the empirical statistics presented in

\[ y_{t+1}^{26} = \int_{0}^{1} y_{t}(s)ds \] (total output of domestic producers).
Table 1 (the "Data" column is identical to the last column of Table 1; the exchange rate statistics in the "Data" column pertain to effective exchange rates).

The methodology developed by Gregory and Smith (1991) was used to formally evaluate how close the model predictions are to the data. Following these authors, the frequency distribution of the simulated statistics was used to construct confidence intervals for each of the statistics considered in Tables 2 and 3. In these Tables, a § (†) next to a given theoretical statistic indicates that the 95% (99%) confidence interval for that statistic includes the historical statistic that is reported in the "Data" column.27 (when a given historical statistic is not included in the relevant confidence interval, this suggests a rejection of the hypothesis that the statistic generated by the model is compatible with the data).

4.1. Predetermined prices and wages

Table 2 presents results for the version of the model with predetermined prices and wages. Results are reported for simulations in which the model is subjected to each of the four types of exogenous shocks separately, as well as for simulations in which the four types of shocks are used simultaneously. For each configuration of shocks, versions of the model with k=0 and with k=4 are compared (the case k=0, i.e. absence of nominal rigidities, is considered here as most earlier dynamic-optimizing open

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27 The 95% confidence intervals run from the 0.025 to the 0.975 quantiles of the frequency distributions of the simulated statistics obtained by simulating the model 1000 times. The 99% confidence intervals run from the 0.005 to the 0.995 quantiles.
4.1.1. Money supply shocks

Columns 1-3 of Table 2 report results for the case in which just money supply shocks are assumed. When \( k=0 \) (see column 1), then money supply shocks have almost no effect on output, consumption, net exports and the real exchange rate (the predicted standard deviations of these variables are all smaller than 0.03%). In contrast, the predicted standard deviation of the domestic price level is roughly consistent with the data. As the real exchange rate shows little response to money supply shocks, the predicted standard deviation of the nominal exchange rate (1.97%) is (basically) identical to that of the price level, and it is thus much too small, compared to the data. These results are consistent with the failure of earlier monetary open economy models without nominal rigidities to explain the historical variability of nominal and real exchange rates.

Money supply shocks have a much stronger impact on real variables when there are nominal rigidities (\( k=2, 4 \); see columns 2 and 3 of Table 2): the standard deviations of output and the real exchange rate rise from close to zero (when \( k=0 \)) to 0.72% and 2.17%, respectively, when \( k=2 \) and to 1.20% and 3.64% when \( k=4 \). Nominal rigidities increase also the standard deviation of the nominal exchange rate (from 1.97% when \( k=0 \) to 2.83% when \( k=4 \)).

For the case \( k=4 \), Figure 1 shows the impact of a one standard deviation (i.e. 0.89%) innovation to the money supply process. In these, as

28 The simulations that just assume money supply shocks also consider a version of the model in which \( k=2 \).
well as in all following Figures, the responses of all variables (with the exception of the interest rate) are expressed as relative deviations from the steady state around which the model is linearized.\footnote{29}

As prices are predetermined, an increase in the nominal money supply induces a short-run rise in the real money supply. This lowers the domestic nominal interest rate, as can be seen in Panel (b) of Figure 1 (a fall in the interest rate is required to induce an increase in the household's demand for real money balances).

The drop in the interest rate triggers a rise in the household's consumption and thus it increases output. But note that the increase in consumption and output is short-lived: it only lasts four periods, i.e. until the price level starts to adjust to the rise in the money supply.

Figure 1 shows that, on impact, a 0.89% money supply innovation induces a depreciation of the nominal exchange rate by about 2.5%, when \( k = 4 \) is assumed. In the periods that follow the shock, the exchange rate appreciates and converges to its new long-run level.\footnote{30} The long-run effect of the money supply shock is a depreciation of the nominal exchange rate by approximately 1.8%. As in Dornbusch's (1976) exchange rate model, the

\footnote{29}The response of a given variable \( z_t \) is expressed as \((z_t - z)/z\), where \( z \) is the steady state value of that variable. In contrast, interest rate responses are shown as differences from steady state: \( i_t - i \), where \( i \) is the steady state interest rate.

\footnote{30}Conditions (13 a) and (13 b) imply that, up to a certainty equivalent approximation, uncovered interest parity holds in equilibrium: \((1 + i_t)(1 + i_t')/e_t = E_t e_{t+1}/e_t\)--the drop in the domestic interest rate triggered by a positive money supply shock requires thus an appreciation of the country's currency in the periods after the money supply shock.
initial response of the exchange rate to a money supply shock exceeds thus the long-run response, i.e. exchange rate "overshooting" occurs (it appears, in contrast, that no exchange rate overshooting takes place when there are no nominal rigidities,\textsuperscript{31} which explains why, as discussed above, the nominal exchange rate is more volatile when k=4 than when k=0).

During the first three periods after the money supply shock, the domestic price level does not respond to that shock, but thereafter the price level converges rapidly to its new long-run value. In the long-run, the price level rises by approximately 1.8\% (note that a 0.89\% innovation to the money supply raises the money stock by about 1.8\%, in the long-run). In the long-run, the money supply shock has, hence, little impact on the real exchange rate. However, in the short-run, the nominal depreciation of the exchange rate is accompanied by a real depreciation. This helps understand why nominal and real exchange rates are highly positively correlated when k=4 is assumed (see Table 2, column 3).

The prediction that an expansionary money supply shock induces a nominal and real depreciation of a country's currency, that it reduces the domestic interest rate, and that it raises domestic output is consistent with recent empirical evidence on the effects of monetary policy shocks (e.g., Eichenbaum (1992), Eichenbaum and Evans (1995), Schlagenauf and Wrase (1995), Grilli and Roubini (1995)).\textsuperscript{32}

\textsuperscript{31}Impulse response functions for the version of the model with k=0 not shown in Figure 1 (available from the author, upon request).

\textsuperscript{32}Note that the model predicts that the maximal effect of a money supply shock on the exchange rate occurs in the same period as the shock. In contrast, empirical research suggests that the maximal effect occurs after the shock (following an expansionary monetary policy shock, the exchange rate appears to depreciate for some time, before it starts to appreciate). In fact, Eichenbaum and Evans (1995) argue that the maximal effect is reached after a period of two to three years. However, Grilli and
4.1.2. Productivity shocks

Columns 4 and 5 of Table 2 report simulation results for the case where just productivity shocks are assumed. It appears that technology shocks have a relatively weak impact on the price level and on nominal and real exchange rates, and that irrespectively of whether prices and wages are fully flexible \((k=0)\) or not \((k=4)\).

For the case with nominal rigidities \((k=4)\), Figure 2 shows the effect of a one standard deviation (i.e. 0.7%) innovation to productivity. This productivity shock causes an immediate nominal and real depreciation of the country’s currency.\(^{33}\)

4.1.3. Shocks to foreign price level

Next, simulations are considered in which just shocks to the foreign price level are assumed (see columns 7 and 8 of Table 2).\(^{34}\) These shocks have a non-negligible effect on the nominal nominal exchange rate (the predicted standard deviation of the nominal exchange rate is 1.6%), but the standard deviation of the real exchange rate is close to zero, and that irrespective

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\(^{33}\) Roubini (1995) present empirical results according to which the maximal effect is reached fairly rapidly (within a few months), which is much more consistent with the predictions of the model here.

\(^{34}\) As can be seen in Panel (b) of Figure 2, a positive productivity shock increases output; interestingly, the rise in output is (initially) much smaller (in percentage terms) than the rise in productivity. This is so because, on impact, hours worked fall sharply when a productivity increases (this is due to the fact that, on impact, exports cannot expand in response to a positive productivity shock, when export prices are predetermined).

The foreign expected real interest rate is held constant in these simulations; as foreign price shocks affect the foreign inflation rate, they thus have an impact on the foreign nominal interest rate.
of whether nominal rigidities are assumed or not.

Figure 3 helps understand why this is so. For the case $k=4$, the Figure shows the effect of a one standard deviation (i.e. 0.5%) innovation to the foreign price level. The shock triggers an appreciation of the nominal exchange rate that matches almost exactly (in percentage terms) the increase in the foreign price level. Thus, the shock has little effect on the real exchange rate (note that, because of the nominal exchange rate appreciation, the domestic currency price of imported consumption goods and, thus, the domestic price level are basically unaffected by the shock). 35

4.1.4. Shocks to expected foreign real interest rate

Shocks to the expected foreign real interest rate have a sizable effect on nominal and real exchange rates: when $k=4$, the predicted standard deviations of these variables exceed 2% when changes in the foreign interest rate are the only exogenous shock (see Column 10 in Table 2).

For the case $k=4$, Figure 4 shows the response of nominal and real exchange rates and of the domestic price level to a one standard deviation (i.e. 0.43%) innovation to the expected foreign real interest rate. Note that, as in this experiment the foreign price level is held fixed, this shock to the foreign expected real interest rate raises the foreign nominal

35 When prices are predetermined, then an increase in the foreign price level raises the demand for the country's exports (as export prices are set in foreign currency), and it thus increases domestic output, as can be seen in Panel (b) of Figure 3. In contrast, when there are no nominal rigidities ($k=0$), then the price of exports, in foreign currency, adjusts immediately to foreign price shocks in a manner that keeps the relative price between exported goods and foreign goods constant. Thus foreign price shocks have no effect on exports, and hence no effect on domestic output, when $k=0$ (as can be seen in column 7 of Table 2).
interest rate. On impact, the shock induces a depreciation of the country's nominal exchange rate. In the periods after the interest rate shock, the exchange rate appreciates.\textsuperscript{36} The response of the real exchange rate mimics that of the nominal exchange rate, as the interest rate shock only has a comparatively weak effect on the domestic price level.

The prediction that positive shocks to the foreign interest rate induce a depreciation of the country's currency is consistent with recent empirical research on the macroeconomic effects of monetary policy shocks (e.g., Elchenbaum and Evans (1995), Roubini and Grilli (1995)).

4.1.5. Combined effect of four types of shocks

Finally, columns (11) and (12) of Table 2 consider the case where the model is subjected to all four types of shocks simultaneously. In that case, the predicted standard deviations of nominal and real exchange rates are significantly closer to the data when nominal rigidities are assumed (k=4) than when k=0 (the predicted standard deviations of the nominal and real exchange rates are 3.91% and 4.25%, respectively, when k=4, compared to standard deviations of 3.24% and 1.61% when k=0).

The version of the model with k=4 explains 80% [90%] of the average historical standard deviations of G7 nominal [real] effective exchange rates—the average historical standard deviations are included in the 95% confidence intervals generated by the model with k=4. In contrast, the

\textsuperscript{36} The appreciation of the country's currency in the periods after the shock occurs because, as discussed above, interest parity holds in the present model, up to a certainty equivalent approximation (N.B. the foreign interest rate shock raises the foreign nominal interest rate above the domestic nominal interest rate, as can be seen in Panel (b) of Figure 4).
average standard deviations are not included in the 95% (or even in the 99%) confidence intervals generated by the model with k=0. The model with k=4 captures a somewhat smaller fraction of the historical standard deviations of U.S. and U.K. effective exchange rates (between 65% and 80%)—however, the historical standard deviations of the U.S. real effective rate and of the U.K. nominal and real effective exchange rates are included in the 99%, or even 95%, confidence intervals for these statistics. In contrast, the model captures only about 50% of the standard deviations of Japanese effective exchange rates and of bilateral U.S. dollar exchange rates.

Note that when k=4 is assumed, the predicted standard deviations of nominal and real exchange rates that are generated when the four types of shocks are used exceed by no more than roughly 1 percentage point the standard deviations reported for the case where there are only money supply shocks. Among the four types of exogenous shocks considered here, money supply changes have thus the strongest impact on nominal and real exchange rates, when nominal rigidities are assumed.

Compared to the case k=0, model performance improves also in several other dimensions (besides predicted exchange rate volatility) when nominal rigidities (k=4) are assumed. Note, for example, that the model with k=4 captures much better the historical standard deviations of the nominal interest rate, output and consumption and that it is consistent with the

37 When k=4 is assumed, the 95% confidence interval for the standard deviations of the nominal and real exchange rates is [2.97%, 5.04%], while that of the real exchange rate is [3.19%, 5.38%]; the corresponding 99% confidence intervals are [2.77%, 5.64%] and [2.89%, 5.90%], respectively (for k=0, the corresponding 99% confidence intervals are [2.11%, 4.55%] and [1.16%, 2.14%]).
stylized fact that consumption is procyclical, and that net exports and the price level are countercyclical (in contrast, the version with k=0 predicts that consumption is countercyclical and that net exports and the price level are procyclical).

However, neither of the two versions of the model (k=0 and k=4) captures the empirical fact that the exchange rate tends to be countercyclical in the G7 countries. The predicted correlation between nominal and real exchange rates is higher when k=4 than when there are no nominal rigidities; however, even when k=4 is assumed, the predicted correlation (0.77) is too small compared to that observed in the data (0.97). Note also that, although the versions of the model with k=0 and k=4 both predict that nominal and real exchange rates are highly serially correlated, the predicted autocorrelations of exchange rates are smaller than those observed in the data.

4.2. Calvo-type price and wage adjustment

Table 3 presents simulation results for the version of the model with Calvo-type nominal rigidities (to save space, results are only shown for the case in which the model is just subjected to money supply shocks and for the case in which the model is subjected to the four types of shocks simultaneously).

When just money supply shocks are assumed, the predicted standard deviations of nominal and real exchange rates are 5.45% and 5.33%, respectively; when all four exogenous shocks are assumed, the corresponding standard deviations are 6.15% and 5.78%. As in the version of the model with predetermined prices and wages, money supply changes have thus the strongest impact on nominal and real exchange rates, among the four types
of exogenous shocks considered here. The predicted standard deviations of exchange rates are larger than those generated by the version of the model with predetermined prices and wages discussed previously. For the case where the four types of exogenous shocks are assumed, the average standard deviations of G7 effective exchange rates (nominal and real) as well the historical standard deviations of U.S., Japanese and U.K. nominal effective exchange rates and of U.S. and U.K. real effective exchange rates are included in 95% confidence interval for these statistics.  

Note also from Table 3 that, in contrast to the version of the model with predetermined prices and wages, the version with Calvo-type nominal rigidities matches quite well the observed high serial correlation of nominal and real exchange rates, as well as the observed high correlation between nominal and real exchange rates. However, other historical statistics, particularly the cross-correlations of the variables considered in the Table with output, are less well captured when Calvo-type nominal rigidities are assumed.  

Figure 5 shows the effect of a one standard deviation (i.e. 0.89%) innovation to the money supply. This shock generates substantial exchange rate overshooting. With Calvo price adjustment, the domestic price level begins to rise as soon as the money supply shock occurs. Also, the adjustment of the price level to its new long-run level is much slower than in the setting with predetermined prices and wages.  

The 95% confidence intervals of the standard deviations of the nominal exchange rates and of the real exchange rate are [4.60%, 7.96%] and [4.39%, 7.42%], respectively.  

The smoother response of the price level to shocks explains why the standard deviation of the price level is much smaller with Calvo-type nominal rigidities than with predetermined prices and wages, as can be seen in Table 3.
adjustment of the nominal exchange rate to its new long-run level too is
much slower and why the effect of a money supply shock on the real exchange
rate (as well as on output and consumption) is much less short-lived than
when prices and wages are predetermined.

5. Conclusion
This paper has studied a dynamic-optimizing model of a semi-small open
economy with nominal rigidities. Money is incorporated into the model by
using a 'money-in-the utility function' framework. As in the Dornbusch
(1976) model, money supply shocks induce exchange rate overshooting.
The predicted variability of nominal and real exchange rates is roughly
consistent with that of G7 effective exchange rates during the post-Bretton
Woods era. Increases in the domestic money supply, in domestic labor
productivity and in the world interest rate induce a nominal and real
depreciation of the country's currency. An increase in the price level in
the rest of the world induces a nominal appreciation (foreign price shocks
have little impact on the real exchange rate).
APPENDIX

DERIVATION OF WAGE EQUATIONS

Wage equation in version of model with predetermined prices and wages ((10), (11))
Suppose that the household changes $\xi_{t-k}(h;s)$ by an infinitesimal amount $\varepsilon$. This implies that her supply of type $h$ labor in period $t$ changes by $l_t(h;s)\varepsilon$ and, that her real wage income in period $t$ changes by $(w_t(h)/P_t)l_t(h;s)\varepsilon$. Hence, the following first-order condition has to hold when the household behaves optimally:

$$-E_{t-k} U_{L_t} l_t(h;s) = E_{t-k} \lambda_t (w_t(h)/P_t) l_t(h;s),$$

where $U_{L_t}$ is the marginal disutility of labor effort in period $t$, while $\lambda_t$ is the shadow value of household wealth in period $t$, i.e. $\lambda_t=U_{C_t,t}$. Using the fact that $U_{L_t}=1$ and that $w_t(h)$ and $P_t$ belong to the information set of period $t-k$ (in the version of the model with predetermined prices and wages), it can be seen that equation (10) is equivalent to the above first-order condition.

As all producers set identical prices in the version of the model with predetermined prices (see discussion in text) and as domestic producers have identical technologies that are symmetric in the different type of labor, $l_t(h;s)=l_t=L_t$ and $w_t(h)=W_t$ has to hold for all $h,s \in [0,1]$. Thus (11) follows immediately from (10).

Calvo-type wage adjustment (equation (12))
The derivation of the wage equation (12) resembles that of (10). Suppose that the household changes $\xi_t(h;s)$ by an infinitesimal amount $\varepsilon$. If the wage $w_t(h)$ is still in effect in period $t+1$, then her hours worked in that period changes by $l_{t+1}(h;s)\varepsilon$, and hence her real wage income in that period changes by $(w_{t+1}(h)/P_{t+1})l_{t+1}(h;s)\varepsilon$ (N.B. the probability that $w_t(h)$ is still in effect in $t+1$ is $\Delta^1$). A reasoning similar to that used to derive (10) then yields the following first-order condition:

$$\sum_{i=0}^{1} (\beta\Delta)^i E_t U_{C_t,t+1} (w_t(h)/P_{t+1}) = \sum_{i=0}^{1} (\beta\Delta)^i E_t l_{t+1}(h;s).$$

Note that $l_{t+1}(h;s) = (w_t(h)/W_{t+1})^{1/(\psi-1)} y_{t+1}(s)/\theta_{t+1}$ holds if the wage $w_t(h)$ is in effect in period $t+1$ (see equation (4)). Substituting this expression into the above condition, integrating over all $s \in [0,1]$ and solving the resulting expression for $w_t(h)$ yields equation (12) in the
Note that (12) depends on $Y_{t+1}$, total physical output of domestic producers in period $t+1$. Using the demand functions (2) and (5) and the fact that $y_{t+1}(s) = d_{t+1}(s) \times x_{t+1}(s)$, it can be shown that

$$Y_{t+1} = (1-\alpha) P_{t+1} C_{t+1} \left( P^D_{t+1} \right)^{1/\nu} P^D_{t+1} X_{t+1} P^X_{t+1} \left( P^D_{t+1} \right)^{(1+\nu)/\nu} P^X_{t+1},$$

where

$$F^D_{t+1} = \sum_{j=0}^{\infty} \delta^j (P^D_{t+1-j})^{-(1+\nu)/\nu}, \quad P^X_{t+1} = \sum_{j=0}^{\infty} \delta^j (P^X_{t+1-j})^{-(1+\nu)/\nu}.$$

**DESCRIPTION OF DATA USED TO COMPUTE HISTORICAL STATISTICS (TABLE 1)**

Unless otherwise indicated, all data are taken from International Financial Statistics (published by the International Monetary Fund).


Hours/employment series for the U.S., the U.K. and Canada are provided in seasonally adjusted form by the data sources. ILO series seem to be presented in seasonally unadjusted form, but inspection of the ILO series for Japan and France suggests that these series do not exhibit seasonality. The ILO employment series for Italy, however, exhibits seasonality, and it was seasonally adjusted using the Census X-11 procedure (using the EZ-X11 program available from Doan Associates, Evanston, IL.). Sample period of hours worked series: 73:Q1-91:Q3.
Net exports—defined as exp-imp/(exp+imp), where exp and imp denote the value of exports and imports, in domestic currency, of goods and services, respectively. Sample period: 73:Q1-91:Q4.


Nominal interest rate—short term rates from Citibase. U.S.: CD rate (Citibase series FYUSCD); Japan, Germany, France: call money rate (FYJPCM, FYGECM, FYFRCM); U.K.: interest rate on prime bank bills (FYGBBB); Italy: bond yields, credit institutions (FYITBY); Canada: prime corporate paper, 60 days (FYCACP). These interest rates are provided at a monthly frequency by Citibase. Observations for the second month of each quarter are used to construct quarterly series. Sample period: 73:Q1-91:Q4.

Nominal effective exchange rate—effective exchange rate (HERM) computed by IMF. Sample period: 73:Q1-95:Q1.

Real effective exchange rate—Sample period: 75:Q1-95:Q1. For the period 75:Q1-78:Q4, the real effective exchange rate is based on relative value added deflators, while the real effective exchange rate for 79:Q1-95:Q1 is based on relative consumer price indexes; series for two sub-periods were multiplicatively spliced together.


Nominal exchange rate series (bilateral or effective) are measured as domestic currency prices of foreign currency; hence, an increase in the nominal exchange rate of a country represents a depreciation of the domestic currency. Likewise, an increase in a real exchange rate (bilateral or effective) represents a real depreciation.
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Table 1. Economic fluctuations in the Post-Bretton Woods era

<table>
<thead>
<tr>
<th>Statistic</th>
<th>U.S.</th>
<th>Japan</th>
<th>Germany</th>
<th>France</th>
<th>U.K.</th>
<th>Italy</th>
<th>Canada</th>
<th>Average</th>
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<td>Standard deviation (in %):</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>2.37</td>
<td>1.49</td>
<td>1.78</td>
<td>1.29</td>
<td>2.21</td>
<td>2.57</td>
<td>2.17</td>
<td>1.85</td>
</tr>
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<td>1.38</td>
<td>1.29</td>
<td>1.05</td>
<td>2.38</td>
<td>1.46</td>
<td>1.57</td>
<td>1.59</td>
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<tr>
<td>Hours</td>
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<td>1.23</td>
<td>1.81</td>
<td>0.80</td>
<td>1.65</td>
<td>0.67</td>
<td>2.32</td>
<td>1.46</td>
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<td>Net exports</td>
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<td>2.07</td>
<td>2.19</td>
<td>2.78</td>
<td>1.63</td>
<td>2.42</td>
</tr>
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<td>1.05</td>
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<td>2.31</td>
<td>1.75</td>
<td>1.40</td>
<td>1.62</td>
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<td>2.79</td>
<td>2.18</td>
<td>1.38</td>
<td>2.85</td>
<td>4.20</td>
<td>2.50</td>
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<td>0.42</td>
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<td>0.38</td>
<td>0.52</td>
<td>0.43</td>
<td>0.45</td>
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<td>Nominal effective exchange rate</td>
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<td>3.35</td>
<td>3.81</td>
<td>5.15</td>
<td>4.03</td>
<td>4.02</td>
<td>4.80</td>
</tr>
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<td>9.05</td>
<td>2.81</td>
<td>2.85</td>
<td>5.65</td>
<td>3.34</td>
<td>4.30</td>
<td>4.75</td>
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<td>8.94</td>
<td>8.81</td>
<td>9.07</td>
<td>8.93</td>
<td>9.04</td>
<td>2.93</td>
<td>7.95</td>
</tr>
<tr>
<td>Real U.S. dollar exchange rate</td>
<td>-</td>
<td>8.93</td>
<td>8.43</td>
<td>8.40</td>
<td>8.54</td>
<td>8.28</td>
<td>3.18</td>
<td>7.63</td>
</tr>
<tr>
<td>Correlation with domestic output:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.92</td>
<td>0.78</td>
<td>0.78</td>
<td>0.28</td>
<td>0.88</td>
<td>0.78</td>
<td>0.84</td>
<td>0.75</td>
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<tr>
<td>Hours</td>
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<td>0.75</td>
<td>0.35</td>
<td>0.66</td>
<td>0.29</td>
<td>0.63</td>
<td>0.58</td>
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<tr>
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<td>-0.14</td>
<td>0.12</td>
<td>-0.19</td>
<td>-0.56</td>
<td>-0.20</td>
<td>-0.24</td>
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<td>-0.78</td>
<td>-0.76</td>
<td>-0.75</td>
<td>-0.54</td>
<td>-0.35</td>
<td>-0.48</td>
<td>-0.63</td>
</tr>
<tr>
<td>Money supply</td>
<td>0.28</td>
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<td>0.31</td>
<td>0.00</td>
<td>0.39</td>
<td>0.62</td>
<td>0.09</td>
<td>0.27</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.08</td>
<td>-0.28</td>
<td>-0.19</td>
<td>0.11</td>
<td>-0.11</td>
<td>0.51</td>
<td>0.11</td>
<td>0.03</td>
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<tr>
<td>Nominal effective exchange rate</td>
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<td>-0.09</td>
<td>-0.09</td>
<td>-0.06</td>
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<td>Real effective exchange rate</td>
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<td>-0.41</td>
<td>-0.04</td>
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<td>0.09</td>
<td>0.06</td>
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<tr>
<td>Output</td>
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<td>0.61</td>
<td>0.76</td>
<td>0.80</td>
<td>0.67</td>
<td>0.82</td>
<td>0.86</td>
<td>0.77</td>
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<tr>
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<td>0.76</td>
<td>0.83</td>
<td>0.85</td>
<td>0.78</td>
<td>0.77</td>
<td>0.90</td>
<td>0.82</td>
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<tr>
<td>Real effective exchange rate</td>
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<td>0.85</td>
<td>0.79</td>
<td>0.72</td>
<td>0.77</td>
<td>0.68</td>
<td>0.89</td>
<td>0.79</td>
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<td>0.97</td>
<td>0.98</td>
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<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.97</td>
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</table>

Note.—All series were logged (with exception of net exports and nominal interest rates) and passed through the Hodrick and Prescott (1980) filter. Series are quarterly. Nominal interest rates are expressed at a gross quarterly rate prior to filtering. The net exports variable is defined as \((\text{exp}-\text{imp})/(\text{exp}+\text{imp})\), where exp and imp denote, respectively, the value of exports and imports of goods and services (in domestic currency). See Appendix for detailed information on data. The last column reports arithmetic average of statistics for G7 countries.
### TABLE 2. Model predictions with predetermined prices and wages

<table>
<thead>
<tr>
<th>Statistics</th>
<th>k=0</th>
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<th>k=4</th>
<th>Productivity shocks</th>
<th>k=0</th>
<th>k=4</th>
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<td></td>
<td>(1)</td>
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<td>(3)</td>
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<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
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<td>Standard deviation (in %):</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
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<td>0.72</td>
<td>1.20</td>
<td>0.70</td>
<td>0.48</td>
<td>1.85</td>
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<td>1.09</td>
<td>1.82$\dagger$</td>
<td>0.31</td>
<td>0.22</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>Hours worked</td>
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<td>0.72</td>
<td>1.20$\dagger$</td>
<td>0.18</td>
<td>0.84</td>
<td>1.46</td>
<td></td>
</tr>
<tr>
<td>Net exports</td>
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<td>1.13</td>
<td>1.90$\dagger$</td>
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<td>0.52</td>
<td>2.42</td>
<td></td>
</tr>
<tr>
<td>Price level</td>
<td>1.97$\dagger$</td>
<td>2.12$\dagger$</td>
<td>2.14$\dagger$</td>
<td>0.15</td>
<td>0.12</td>
<td>1.62</td>
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<tr>
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<td>1.77$\dagger$</td>
<td>1.77$\dagger$</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Nominal interest rate</td>
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<td>0.02</td>
<td>0.03</td>
<td>0.45</td>
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<tr>
<td>Nominal exchange rate</td>
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<td>4.80</td>
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<td>3.64$\dagger$</td>
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<td>0.99</td>
<td>0.99</td>
<td>0.65$\dagger$</td>
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<td>-0.99</td>
<td>0.99</td>
<td>0.78</td>
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<td>-0.51$\dagger$</td>
<td>-0.63$\dagger$</td>
<td>-0.99</td>
<td>-0.96</td>
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<td>$\mu$</td>
<td>$\mu$</td>
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<td>0.99</td>
<td>-0.18</td>
<td>0.56</td>
<td>0.40</td>
<td>-0.16</td>
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<td>Real exchange rate</td>
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<td>0.99</td>
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<td>0.97</td>
<td>0.63</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Output</td>
<td>u</td>
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<td>0.58</td>
<td>0.56</td>
<td>0.60</td>
<td>0.63</td>
<td>0.77</td>
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<td>0.91</td>
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<td>0.72</td>
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<td>0.62</td>
<td>0.79</td>
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<td>0.77</td>
<td>0.97</td>
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</table>

Note. --The Table assumes that prices are set 'k' periods in advance.

All series are logged (with exception of net exports and interest rate) and passed through the Hodrick and Prescott (1980) filter. In accordance with Table 1, the net exports variable is defined as \( \frac{\text{exp-imp}}{(\text{exp}+\text{imp})} \), where \( \exp P_t X_t \) and \( \text{imp} P_t F_t \) denote, respectively, the value of exports and of imports, in domestic currency. The nominal interest rate is expressed at a gross quarterly rate prior to filtering.

"Data" column shows average (across G7 countries) of historical statistics reported in Table 1 (the exchange rate statistics in the "Data" column pertain to effective exchange rates).

u: correlation is not defined (series with zero variance).

\( \dagger \) indicates that a 95% (99%) confidence interval includes the historical statistic reported in the "Data" column.
### TABLE 3. Model predictions with Calvo-type nominal rigidities

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Money shocks</th>
<th>M,θ,P,R</th>
<th>Data</th>
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<td>Standard deviations (in %):</td>
<td></td>
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<td></td>
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<tr>
<td>Output</td>
<td>2.00§</td>
<td>2.07§</td>
<td>1.85</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.67</td>
<td>2.68</td>
<td>1.59</td>
</tr>
<tr>
<td>Hours worked</td>
<td>2.00§</td>
<td>2.17†</td>
<td>1.46</td>
</tr>
<tr>
<td>Net exports</td>
<td>2.56§</td>
<td>2.61§</td>
<td>2.42</td>
</tr>
<tr>
<td>Price level</td>
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<td>0.62</td>
<td>1.62</td>
</tr>
<tr>
<td>Money supply</td>
<td>1.77†</td>
<td>1.77†</td>
<td>2.50</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.33§</td>
<td>0.38§</td>
<td>0.45</td>
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<tr>
<td>Nominal exchange rate</td>
<td>5.45§</td>
<td>6.15§</td>
<td>4.80</td>
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<tr>
<td>Real exchange rate</td>
<td>5.33§</td>
<td>5.78§</td>
<td>4.75</td>
</tr>
</tbody>
</table>

| Correlation with domestic output: |            |         |      |
| Consumption                      | 0.98        | 0.95    | 0.75 |
| Hours worked                     | 1.00        | 0.91    | 0.58 |
| Net exports                      | -0.92       | -0.86   | -0.24|
| Price level                      | 0.29        | 0.28    | -0.63|
| Money supply                     | 0.57        | 0.56    | 0.27 |
| Nominal interest rate            | -0.59       | -0.45   | 0.03 |
| Nominal exchange rate            | 0.99        | 0.80    | -0.16|
| Real exchange rate               | 0.98        | 0.87    | -0.06|

| Autocorrelation:                 |            |         |      |
| Output                           | 0.71§       | 0.73§   | 0.77 |
| Nominal exchange rate            | 0.68†       | 0.69†   | 0.82 |
| Real exchange rate               | 0.68§       | 0.66§   | 0.79 |

| Correlation between nominal and real exchange rate: |         |         |      |
|                                                    | 0.99     | 0.95§   | 0.97 |

Note.—Average time between price and wage changes is 12.5 quarters (Δ=Δ=0.92).

All series are logged (with exception of net exports and interest rate) and passed through the Hodrick and Prescott (1980) filter. In accordance with Table 1, the net exports variable is defined as (exp-imp)/(exp+imp), where exp_{t,t}^{P,t} and imp_{t,t}^{P,t} denote, respectively, the value of exports and of imports, in domestic currency. The nominal interest rate is expressed at a gross quarterly rate prior to filtering.

"Data" column shows average (across G7 countries) of historical statistics reported in Table 1 (the exchange rate statistics in the "Data" column pertain to effective exchange rates).

§ (†) indicates that a 95% (99%) confidence interval includes the historical statistic reported in the "Data" column.
(a) Response of money supply (M), nominal exchange rate (e), real exchange rate (rer) and domestic price level (P).

(b) Response of output (Y), consumption (C) and domestic nominal interest rate (I).

FIGURE 1--Prices and wages set 4 periods in advance. Responses to a 1 standard deviation (i.e. 0.89%) innovation to money supply process. Response of interest rate expressed as difference from steady state; responses of other variables shown as relative deviations from steady state. Money supply response (Panel (a)) pertains to end of period money stocks. Abscissa: quarters after shock.
(a) Response of nominal exchange rate (e), real exchange rate (rer) and domestic price level (P).

(b) Response of productivity (θ), output (Y), consumption (C), exports (X) and hours worked (L).

FIGURE 2—Prices and wages set 4 periods in advance. Responses to a 1 standard deviation (i.e. 0.7%) innovation to labor productivity. Responses expressed as relative deviations from steady state. Abscissa: quarters after shock.
(a) Response of foreign price level ($P^*$), nominal exchange rate ($e$), real exchange rate ($rer$) and domestic price level ($P$).

(b) Response of output ($Y$), consumption ($C$) and exports ($X$).

FIGURE 3—Prices and wages set 4 periods in advance. Responses to a 1 standard deviation (i.e. 0.5%) innovation to foreign price level. Responses expressed as relative deviations from steady state. Abscissa: quarters after shock.
(a) Response of nominal exchange rate (e), real exchange rate (rer) and domestic price level (P).

(b) Response of foreign nominal interest rate (i*) and of domestic nominal interest rate (i).

FIGURE 4--Prices and wages set k=4 periods in advance. Responses to innovation that raises expected real foreign interest rate by 1 standard deviation (i.e. 0.43 percentage points). Response of interest rate expressed as difference from steady state; responses of other variables shown as relative deviations from steady state. Abscissa: quarters after shock.
(a) Response of money supply (M), nominal exchange rate (e), real exchange rate (rer) and domestic price level (P).

(b) Response of output (Y), consumption (C) and domestic nominal interest rate (i).

FIGURE 5--Model with Calvo-type price and wage adjustment. Responses to a 1 standard deviation (i.e. 0.89%) innovation to money supply process. Response of interest rate expressed as difference from steady state; responses of other variables shown as relative deviations from steady state. Money supply response (Panel (a)) pertains to end of period money stocks. Abscissa: quarters after shock.
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