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ON THE PROFITABILITY OF PRODUCTION CONSTRAINTS IN A DYNAMIC NATURAL RESOURCE OLIGOPOLY*

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Résumé

Dans le contexte d'un équilibre oligopolistique statique à la Cournot, il est clair que si une seule des firmes était forcée de façon exogène à réduire marginalement sa production, ses profits diminueraient. Par contre, si la même contrainte était imposée simultanément à toutes les firmes, faisant ainsi se déplacer l'industrie vers la production de monopole, les profits de chaque firme augmenteraient. Nous démontrons que ces résultats très intuitifs peuvent ne pas tenir s'il s'agit d'un oligopole dynamique, tel qu'un oligopole de ressource naturelle non renouvelable, où la contrainte exogène prendrait la forme d'un déplacement marginal du sentier de production durant un certain intervalle de temps. Il existe en effet des situations où la firme qui subit seule une telle contrainte verra son profit augmenter et d'autres situations dans lesquelles toutes les firmes y perdront si la même contrainte leur était imposée simultanément. Les résultats tirés de l'analyse d'un oligopole statique peuvent donc être complètement inversés dans certains cas d'oligopole dynamique.

Mots clés : Contraintes de production, oligopole dynamique, jeux différentiels, ressources naturelles non renouvelables.

Abstract

Static oligopoly analysis predicts that if a single firm in Cournot equilibrium were to be constrained to contract its production marginally, its profits would fall. On the other hand, if all the firms were simultaneously constrained to reduce their production, thus moving the industry towards monopoly output, each firm's profit would rise. We show that these very intuitive results may not hold in a dynamic oligopoly, such as a nonrenewable natural resource oligopoly, where the exogenous constraint would take the form of a contraction of the firm's output path over some fixed interval of time : there are situations where a firm will gain from being the lone firm constrained in this way and cases where each firm will lose if all the firms in the industry are so constrained, thus exactly reversing the conclusions obtained from purely static analysis.

Keywords : Production constraints, dynamic oligopoly, differential games, nonrenewable resources.

1 Introduction

It is normally in the interest of a Stackelberg leader to choose a higher equilibrium quantity than if he were one of a set of identical players in a simultaneous game à la Cournot. Indeed, if the goods are substitutes in demand and strategic substitutes, a marginal increase of the output of a single firm away from its Nash-Cournot equilibrium output will result in a decrease in the output of all its competitors. As a result, a leader who can commit to such an increase will improve his profits. For this reason, an exogenously imposed marginal contraction of the production of a single firm in a Cournot equilibrium can never improve its profits[1].

We show in this paper that things can be different in the context of a differential game. We consider an oligopoly where each firm owns a stock of a nonrenewable natural resource and competes in quantity with the other firms. In that context, even in the case of a duopoly, a single firm's profits can be increased by forcing it to marginally contract its production path over a fixed interval of time. Furthermore, this can happen no matter what the number of firms in the industry and the number of firms subject to the exogenous output contraction. Conversely, if all the firms in the industry are subject to the exogenously supported output contraction over some fixed interval of time, then, paradoxically, profits may fall. The reason for these results is that the contraction affects not only the output during the period of contraction, but affects also the stocks that remain at the end of the period of contraction. This in turn affects the equilibrium output paths and values of the game that is played from that time on.

The exogenous output contraction to which the analysis applies may be one that is the direct result of some outside intervention, such as a strike, for instance. It may also be one that is supported indirectly by some policy decisions, such as taxes or quotas, which are expected only to be in effect for some predetermined length of time. The analysis applies as

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[1]As shown in Gaudet and Salant (1991), the imposition of such a contraction can be profitable only if the firm is one of a larger subset of firms being imposed the same constraint and that subset is sufficiently large relative to the number of firms in the industry.

well — albeit with the direction of the effects being reversed — to policy variables, such as subsidies, that support an output expansion rather than a contraction. The approach can also be useful to analyze, in some cases, the effects on profits of shifts in the output paths due to decisions to merge or to cartelize all or part of the industry. The usefulness of the methodology itself is not limited to nonrenewable natural resource industries, although the qualitative results that follow from it may be different in other cases. It could be usefully applied to analyze the effect of similar outside interventions in other dynamic oligopoly situations — such as learning by doing — where the current state depends on past actions.

We will consider both an open-loop and a closed-loop production game. In the open-loop game, the firms commit to a production path at the beginning of the game. Each firm chooses its extraction path taking as given the extraction paths of its competitors. Hence firms do not try to manipulate strategically their competitors' extraction paths through the choice of their own path, since they know that the rivals extraction decision is also strictly a function of time. In the closed-loop game, each firm chooses an extraction rule taking as given its competitors extraction rules. These extraction rules specify for each instant an extraction rate for each level of its own stock and those of its competitors. Thus each firm is aware of the impact of its decisions on the rate of production of the other firms.

After describing the model in section 2, we present results for the open-loop game in section 3. The reason for considering the open-loop game before going on to the closed-loop game is twofold. First it makes clear that our conclusions do not rest on the possibility for the individual firm to strategically manipulate its rivals' production paths through the choice of its own path. Second, it will make it easier to isolate the effects of the exogenous contraction which can be attributed specifically to the closed-loop strategies. In the section 4 we show how our qualitative results carry over to the closed-loop game. Section 5 offers a brief conclusion.

2   The model

Consider a nonrenewable natural resource industry composed of N identical firms, each one owning at date $t = 0$ identical resource stocks[2]. Let $u_i(t)$ denote the rate of extraction of firm i at time t, $x_i(t)$ its remaining stock at time t and T_i its terminal date (date after which $u_i(t) \equiv 0$). We will denote respectively by $x(t) = (x_1(t), x_2(t), \ldots x_N(t))$, $u(t) = (u_1(t), u_2(t), \ldots, u_N(t))$ and $T = (T_1, T_2, \ldots T_N)$ the vectors of stocks, of extraction rates and of terminal dates, and by $\Pi_i(u(t))$ firm i's profits at time t. $J_i(u)$ will denote the present value of those profits over the interval $[0, T_i]$. Thus,

$$J_i(u) = \int_0^{T_i} \Pi_i(u(t)) e^{-rt} dt$$

We assume $\Pi_i(u(t))$ to be twice continuously differentiable and concave with respect to $u_i(t)$. We also assume that resources i and j are, at each date t, substitutes in demand $(\partial \Pi_i / \partial u_j < 0)$ and strategic substitutes[3], which means that firm i's best response to an aggressive action from firm j (an increase of $u_j(t)$) at time t is to reduce its own production at that same date. Given the assumptions just made on $\Pi_i(u(t))$, this is equivalent to assuming $\partial^2 \Pi_i / \partial u_i \partial u_j < 0$.

In the open-loop game, firm i's problem is to determine, at time $t = 0$, the rates $u_i(t)$ at which it will be extracting the resource at each instant t, taking as given its competitors extraction paths. It can commit at time $t = 0$ to this extraction path. In the absence of any exogenous constraints on its extraction path, firm i chooses its best response to solve the following problem:

$$\max_{\{u_i(t)\}, T_i} J_i(u)$$

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[2]We make the assumptions of identical firms and identical initial stocks because this significantly simplifies the derivations by allowing us to restrict attention to symmetrical equilibria. There is no loss in insight in doing so. Similar results can be derived with non identical firms and initial stocks.

[3]See Bulow *et al.* (1985).

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subject to

$$\dot{x}_i(t) = -u_i(t) \tag{1}$$

and

$$x_i(0) = x^0, \quad x_i(T_i) \geq 0. \tag{2}$$

An open-loop equilibrium is a vector of extraction paths that solves the above problem simultaneously for all $i = 1, 2, \ldots, N$. Denote this equilibrium vector $u^*(x(0))$, where $x(0)$ is the vector of initial endowments. Note that the vector $x(0)$ has scalar elements $x_i(0) = x^0$, which represent the *identical* individual initial stocks. The equilibrium present value of firm i is therefore also a function of x^0 and will be written $V_i(x(0)) \equiv J_i(u^*(x(0)))$.

In the closed-loop game, each firm's problem is to determine at the initial date a strategy which takes the form of a decision rule based on time and the vector of remaining stocks at the moment the decision is to be made. We thus limit our attention to Markov strategies, in the sense that the decision rule depends only on the current stocks and not on their entire history. Firm i chooses its strategy taking as given the strategies of the $N - 1$ other firms.

Denote firm i's decision rule by $u_i(t, x(t))$. Then its problem can be stated as that of choosing $u_i(t, x(t))$ in order to solve

$$\max_{\{u_i(t,x)\}, T_i} \int_0^{T_i} \Pi_i\left(u\left(t, x\right)\right) e^{-rt} dt$$

subject to

$$\dot{x}_i = -u_i\left(t, x\right)$$

and

$$x_i(0) = x^0, \quad x_i(T_i) \geq 0$$

taking as given the decision rules of the $N - 1$ other firms. The strategy chosen by firm i therefore constitutes a best response to the set of strategies of the $N - 1$ other firms. A closed-loop, or Markov perfect, equilibrium is a set of N such strategies, one for each firm,

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each of which is a best response to the other $N-1$ strategies.

Let us suppose now that each firm in a subset composed of S firms ($S \leq N$) is exogenously constrained, during a given interval of time, to marginally reduce its production, in a neighborhood of the prevailing unconstrained equilibrium. Without loss of generality, we may assume that this subset is composed of the first S firms. Thus, for $i = 1, \ldots, S$, firm i is constrained to produce $u_i^*(t) + \varepsilon h(t) \geq 0$ for all $t \in [0, \tau]$, where ε is an arbitrarily small positive parameter and $h(t)$ is a negative piecewise continuous function and $\tau \in (0, T_i)$. Notice that we could just as well consider an exogenously imposed marginal expansion of the production path by choosing $h(t)$ to be positive.

Let $J_i(u^* + \varepsilon h, \tau)$ denote the payoff of constrained firm i over the interval $[0, \tau]$. Firm i considers these payoffs as given. Its problem now reduces to determining its extraction rate for $t \geq \tau$, with stock $x_i(\tau)$ given by

$$x_i(\tau) = x^0 - \int_0^\tau \left(u_i^*(t) + \varepsilon h(t) \right) dt. \tag{3}$$

In the open-loop case, it does this by choosing at date $t = 0$ its extraction path $\{u(t)\}$ for $t \geq \tau$, taking as given the full extraction paths of its rivals. In the closed-loop case, it chooses at $t = 0$ its extraction rule $u(t, x(t))$ for $t \geq \tau$, taking as given the extraction rules of its rivals. As for the unconstrained firm j, $j = S + 1, \ldots, N$, it continues to choose at $t = 0$ its extraction path (rule) for $t \geq 0$ in the open-loop (closed-loop) game taking as given that of its $N-1$ rivals.

We will let $v(t) = (v_1(t), v_2(t), \ldots, v_N(t))$ denote the vector of equilibrium extraction rates when the S firms are constrained. Thus $v_i(t) = u_i^*(t) + \varepsilon h(t)$ for $i = 1, 2, \ldots, S$ and $t \in [0, \tau]$ and $v(t) = u^*(t)$ for $\varepsilon = 0$. Notice that the value function of firm i's program, which we will denote V_i, depends on the parameters ε and τ as well as on the initial stocks. In particular, when evaluated at $\varepsilon = 0$, it yields the value of its program in the initial unconstrained equilibrium being considered.

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3 The case of open-loop equilibria

Consider first the case of open-loop equilibria. In that case, an unconstrained equilibrium must satisfy, in addition to (1) and (2), the following set of conditions:

$$\frac{\partial \Pi_i}{\partial u_i} - \lambda_i = 0 \qquad (4)$$

$$\dot{\lambda}_i = r\lambda_i \qquad (5)$$

$$\lambda_i(T_i) \geq 0, \quad \lambda_i(T_i)x_i(T_i) = 0 \qquad (6)$$

$$\Pi_i(u(T_i)) - \lambda_i(T_i)u_i(T_i) = 0 \qquad (7)$$

where λ_i is the shadow value of firm i's resource stock. Notice that the transversality condition (6) implies $x(T_i) = 0$, since $\lambda_i(T_i) = \lambda_i(0)e^{-rT_i}$ by (5) and $\lambda_i(0)$ is positive, x^0 being finite.

Suppose each of the first S firms is imposed a marginal reduction of its rate of extraction over the interval $[0, \tau]$, as defined in the previous section. Given τ, this exogenously imposed contraction of its production path will, at least in a neighborhood of the unconstrained equilibrium, be beneficial to the firm if $dV_i(\varepsilon = 0)/d\varepsilon$ is positive for $i = 1, 2, \ldots, S$. Taking into account the fact that conditions (4) to (7) must hold when $\varepsilon = 0$ and the fact that the initial unconstrained equilibrium is symmetrical, we find that

$$\frac{dV_i(\varepsilon = 0)}{d\varepsilon} = \int_0^\tau \left[\frac{\partial \Pi_i(u^*)}{\partial v_k}e^{-rt} - \frac{\partial V_i(x^*(\tau))}{\partial x_k}e^{-r\tau} \right] \left[(N - S)\frac{dv_j}{d\varepsilon} + (S - 1)h(t) \right] dt \qquad (8)$$

where k denotes any firm other than firm i and j denotes any firm in the unconstrained subset $\{S + 1, S + 2, \ldots, N\}$. The details of the derivation are provided in Appendix A.

We notice immediately that a marginal contraction of firm i's production has no *direct* effect on its equilibrium profits. This is due to the fact that a reduction of its production at any date $t \in [0, \tau]$ is necessarily compensated by an equivalent increase in the stock it has

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remaining at τ. But along the equilibrium path of an open-loop equilibrium, the present value of a marginal unit of the resource left unexploited at date $t \in [0, \tau]$ must be equal to the present value of a marginal unit left unexploited at date τ, by (5), and the latter measures the present value of the contribution of a marginal unit of resource stock to the value of firm i's optimal program from date τ on. A marginal contraction of firm i's equilibrium production path therefore has only an *indirect* effect on its profits, via the change it induces in the extraction path of its rivals.

To analyze this effect, let

$$E_1^o(t) = \frac{\partial \Pi_i(u^*)}{\partial v_k} e^{-rt} - \frac{\partial V_i(x^*(\tau))}{\partial x_k} e^{-r\tau} \tag{9}$$

and

$$E_2^o(t) = (N - S) \frac{dv_j}{d\varepsilon} + (S - 1) h(t). \tag{10}$$

The superscript o denotes the fact that each of those expressions is evaluated along an open-loop equilibrium.

Consider first the expression for $E_2^o(t)$. This measures the effect of the exogenous contraction on the combined production of firm i's $N - 1$ rivals: $E_2^o(t) < 0(> 0)$ means that the combined reaction of the $(N - 1)$ other firms is to reduce (increase) production. Of the $N - 1$ rivals of firm i, $S - 1$ are also constrained to reducing their production, while the other $N - S$ can freely adjust their rate of extraction in reaction to the reduction of firm i's own rate of extraction. Because, by assumption, we are dealing with strategic substitutes, each of them will react by increasing its own production and hence $dv_j/d\varepsilon$ is positive. Since $h(t) < 0$, the sign of $E_2^o(t)$ is therefore ambiguous.

In a purely static Cournot oligopoly framework, the sign of $E_2^o(t)$ is all that matters in order to determine the effect on the profit of the constrained firm i: if and only if $E_2^o(t) < 0$, so that the combined reaction of the $N - 1$ other firms is to reduce production, the exogenously

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imposed contraction will be beneficial to firm i[4]. This will be the case if the number (S) of firms being constrained is sufficiently large relative to the number $(N - S)$ of firms left unconstrained[5].

The expression $E_1^o(t)$ is the sum of two terms. The first term is the present value of the effect on the profit of firm i of a change in the extraction rate of firm k, $k \neq i$, at date $t \in [0, \tau]$. This term is negative, by assumption: resources i and k, $i \neq k$, are substitutes in demand. The second term is the present value of the effect of a change in firm k's stock at τ on the value of the optimal program of firm i over the interval $[\tau, T_i]$. This term is also negative. The expression $E_1^o(t)$ can therefore be of either sign. Furthermore, it can change sign over the interval $[0, \tau]$.

It is therefore clear that knowing the sign of $E_2^o(t)$ is no longer sufficient, as it was in the static case, to establish whether the exogenous constraint is profitable or not for each firm in the constrained subset. For we may have $dV_i\,(\varepsilon = 0)\,/d\varepsilon > 0$ with $E_2^o(t) > 0$ as well as with $E_2^o(t) < 0$. We may also have $dV_i\,(\varepsilon = 0)\,/d\varepsilon < 0$ with $E_2^o(t)$ of either sign.

Now consider the case of a duopoly where one of the firms is constrained to reduce its production over the interval $[0, \tau]$. We therefore have $N = 2$ and $S = 1$ and hence $E_2^o(t) > 0$. For the sake of argument, assume $E_1^o(t) > 0$ for all $t \in [0, \tau]$[6]. The fact that $E_1^o(t) > 0$ for all $t \in [0, \tau]$ means that the competition which results from an extra unit of stock to the other firm at τ is more harmful to the constrained firm than is the competition from an extra unit of production by the other firm at some date $t \in [0, \tau]$. If this were a static oligopoly, we would therefore get the not surprising result that forcing one of the firms to marginally reduce its production cannot increase its equilibrium profits. In the present context however, since $E_1^o(t) > 0$, $dV_i\,(\varepsilon = 0)\,/d\varepsilon > 0$ and the constrained firm gains. Thus

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[4]Notice that the static Cournot oligopoly outcome is obtained if one assumes that the stocks of the resource are infinite. In that case, the marginal effect on the optimal value of the remaining program of firm i at any date t of resources left in the ground by any firm, including itself, is zero. Hence $E_1^o(t)$ reduces to $(\partial \Pi_i(u^*)/\partial v_k)e^{-rt}$, which is negative, since, by assumption, resources i and k are substitutes in demand.

[5]For a necessary and sufficient condition on the number of constrained firms relative to the number of unconstrained firms in the static oligopoly case, see Gaudet and Salant (1991). The condition generally depends also on the curvature of the demand and cost functions.

[6]This is clearly possible. An example where it occurs is presented in Appendix B.

we have the paradoxical situation where even a duopolist would gain from an exogenously imposed marginal contraction of its production.

With more than two firms in the industry, we may have $E_2^o(t) < 0$ if the number of firms in the constrained subset is sufficiently large. As pointed out above, in the static oligopoly, this is sufficient for each firm in the subset to benefit from the output contraction they are being imposed. The reason is that although each unconstrained firm reacts by increasing its output, the combined reaction of a constrained firm's $N-1$ rivals is to reduce production, hence resulting in a fall in the total industry output and an increase in price[7]. In the dynamic resource oligopoly however, the change in output by each of the $N-1$ rivals at any date $t \in [0, \tau]$ — an increase for each of the $N - S$ unconstrained firm and a decrease for the other $S - 1$ constrained firm — results in an equivalent change in the opposite direction in its stock at date τ. It may be that the net outcome is $E_1^o(t) > 0$ for some subinterval of $[0, \tau]$ — possibly the whole interval — and that the overall net effect over the interval $[0, \tau]$ is detrimental to each constrained firm, resulting in a reduction of the present value of its profits even though $E_2^o(t) < 0$.

Somewhat surprisingly, this may happen even in the case where each of the N firms is simultaneously being forced to a marginal contraction of its output path over the interval $[0, \tau]$. In that case, $S = N$ and $E_2^o(t) = (N-1)h(t) < 0$. Since the extraction paths of each firms over the interval $[0, \tau]$ is exogenously given, this is equivalent to a game starting at τ with $x(\tau)$ as the vector of initial stocks. Since each firm's resource stock at τ is larger than it would have been along the unconstrained equilibrium path, each firm's discounted equilibrium rent from τ on will be smaller[8]. If the gains from jointly producing closer to the monopoly output over the interval $[0, \tau]$ are insufficient to compensate for this, then $E_1^o(t) > 0$ and each firm loses. Paradoxically, this means that if the N firms were to form a cartel to last *some exogenously set interval of time* $[0, \tau]$, $\tau < T_i^*$, they might choose to

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[7]See Gaudet and Salant (1991) for further details on the static oligopoly case.
[8]Even though the previous production paths for $t \geq \tau$ are still feasible, they cannot constitute an equilibrium anymore, for this would mean leaving some of the stock unexploited.

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expand their production path over that interval.

Although these results provide some important insights into the possible effect on the present value of natural resource oligopolists of some exogenous constraint on their production path, they suffer from the drawback that it is has been assumed the firms can commit at time zero to entire production paths. It is well known that unless the only information accessible to each firm as the game evolves is that which it has available at the initial date, such open-loop equilibria will not be subgame perfect. We now turn to consideration of the effect of the same type of constraints in Markov perfect equilibria, where firms commit to an extraction rule as a function of time and the current resource stocks rather to an extraction path.

4 The case of closed-loop equilibria

We will now assume that the firms know, at each instant, the vector of remaining resource stocks and that they are able to adjust their extraction rates to this information. A strategy specifies for each date t a rate of extraction which depends on the vector of stocks remaining at that date[9].

One important difference with the open-loop game considered in the previous section is that each firm now imputes a shadow value to the resource stocks of its competitors, because its own decision rule depends on them. Thus the current value Hamiltonian corresponding to firm i's optimization problem in the closed-loop game is

$$H_i(u, x, \lambda_i, \gamma_{ik}) = \Pi_i(u(t, x)) - \lambda_i(t)u_i(t, x) - \sum_{k \neq i} \gamma_{ik}(t)u_k(t, x)$$

where, as before, $\lambda_i(t)$ is the shadow value associated to firm i's own stock, and now, in addition, $\gamma_{ik}(t)$ represents the shadow value associated by firm i to firm k's stock.

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[9]We assume the strategies to be continuous with respect to t and continuously differentiable with respect to x.

Any equilibrium must satisfy[10] :

$$\frac{\partial \Pi_i}{\partial u_i} - \lambda_i = 0 \tag{11}$$

$$\dot{\lambda}_i = r\lambda_i - \sum_{k \neq i} \left(\frac{\partial \Pi_i}{\partial u_k} - \gamma_{ik} \right) \frac{\partial u_k}{\partial x_i} \tag{12}$$

$$\dot{\gamma}_{ik} = r\gamma_{ik} - \sum_{k \neq i} \left(\frac{\partial \Pi_i}{\partial u_k} - \gamma_{ik} \right) \frac{\partial u_k}{\partial x_k} \tag{13}$$

$$\lambda_i(T_i) \geq 0, \quad \lambda_i(T_i)x_i(T_i) = 0 \tag{14}$$

$$\gamma_{ik}(T_i)x_k(T_i) = 0 \tag{15}$$

$$\Pi_i\left(u\left(T_i, x\left(T_i\right)\right)\right) - \lambda_i\left(T_i\right)u_i\left(T_i\right) - \gamma_{ik}\left(T_i\right)u_k\left(T_i\right) = 0 \tag{16}$$

Other than the fact that each firm now imputes a shadow value to the resource stocks held by its competitors, there is another important difference that characterizes a closed-loop equilibrium. It comes from the fact that each firm now considers as given the other firms' extraction strategies — which are functions of the vector of resource stocks — and not the other firms' extraction paths, as in an open-loop equilibrium. Thus each firm is aware of the influence of its own resource stock on the production decisions of the other firms and knows it can manipulate the other firms production decisions. For this reason, there is now a strategic element to the value of the firm's own stock, which explains the second term of condition (12).

As is clear from condition (12), an immediate consequence of this strategic consideration is that the discounted shadow value to the firm of a unit of resource stock $(\lambda_i(t)e^{-rt})$ is not constant, as it was along an open-loop equilibrium. Therefore, along the equilibrium path of a closed-loop equilibrium, the present value of a marginal unit of the resource left unexploited at date $t \in [0, \tau)$ is not equal to the present value of a marginal unit left unexploited at date

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[10]See Basar et Olsder (1982), Starr et Ho (1969) or Fudenberg and Tirole (1991, chapter 13).

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τ and, hence, not equal to the present value of the contribution of a marginal unit of resource stock to the value of the firm's optimal program from date τ on. As a consequence, and unlike in the open-loop equilibrium, the firm is not indifferent, along the equilibrium production path of a closed-loop equilibrium, between a marginal decrease of its *rate of extraction* at some date t and a compensating marginal increase of its available *stock* at $\tau > t$. This is because it is aware of the fact that its' rivals base their production decisions directly on the level of its remaining resource stock.

An exogenous contraction of firm i's extraction path over the interval $[0, \tau]$ will therefore now have a *direct* effect on its present value, unlike in an open-loop equilibrium. Indeed, if we constrain a subset of S firms in the same way as we did in the open-loop case, we now find that along a symmetric closed-loop equilibrium, if it exists,

$$
\begin{aligned}
\frac{dV_i\,(\varepsilon = 0)}{d\varepsilon} &= \int_0^\tau \left[\frac{\partial \Pi_i(u^*)}{\partial v_k} e^{-rt} - \frac{\partial V_i\,(x^*\,(\tau))}{\partial x_k} e^{-r\tau} \right] \left[(N - S)\frac{dv_j}{d\varepsilon} + (S - 1)\,h\,(t) \right] dt \\
&+ \int_0^\tau \left[\frac{\partial \Pi_i(u^*)}{\partial v_i} e^{-rt} - \frac{\partial V_i\,(x^*\,(\tau))}{\partial x_i} e^{-r\tau} \right] h(t)dt,
\end{aligned}
\tag{17}
$$

where, as in (8), k denotes any firm other than firm i and j denotes any firm in the unconstrained subset $\{S + 1, S + 2, \ldots, N\}$. Note the additional term when compared to the open-loop case. This additional term captures the *direct* effect of the exogenous contraction of firm i's production path on its present value.

Since at any date t, we will have

$$
\gamma_{ik}(t) = \frac{\partial V_i(x(t))}{\partial x_k} \quad \text{and} \quad \lambda_i(t) = \frac{\partial V_i(x(t))}{\partial x_i},
$$

if we define

$$
E_1^c(t) = \frac{\partial \Pi_i(u^*)}{\partial v_k} e^{-rt} - \gamma_{ik}(t) e^{-r\tau},
\tag{18}
$$

$$
E_2^c(t) = (N - S)\frac{dv_j}{d\varepsilon} + (S - 1)\,h\,(t).
\tag{19}
$$

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and

$$E_3^c(t) = \lambda_i(t)e^{-rt} - \lambda_i(\tau)e^{-r\tau}, \tag{20}$$

we may rewrite (17) as

$$\frac{dV_i\,(\varepsilon = 0)}{d\varepsilon} = \int_0^\tau E_1^c(t)E_2^c(t)dt + \int_0^\tau E_3^c(t)h(t)dt. \tag{21}$$

In defining $E_3^c(t)$ we have also made use of condition (11), which must hold along any unconstrained equilibrium path.

$E_1^c(t)$ and $E_2^c(t)$ have the same interpretation as $E_1^o(t)$ and $E_2^o(t)$ in the previous section. They differ however by the fact that, being evaluated along a closed-loop equilibrium, not only are the equilibrium paths not the same, but the derivatives of V_i with respect to x_k, $k \neq i$ and the derivative of v_j with respect to ε must take into account the fact that the extraction strategy of each firm is now a function of the vector of remaining resource stocks and that these remaining stocks will be affected by a contraction of the production path of the S firms in the constrained subset.

The shadow value to firm i of a unit of firm k's stock, $\gamma_{ik}(t)$, is negative if a marginal increase of firm k's stock at date t along the equilibrium path decreases the present value, at t, of the flow of profits of firm i. Assume this is the case along the equilibrium path being considered. This is certainly plausible, given the structure of the problem and especially the fact that the goods are substitutes in demand. In that case, the sign of $E_1^c(t)$ is indeterminate. As for the sign of $E_2^c(t)$, it depends, as in the open-loop case, on the relative size of the constrained and unconstrained subset. The sign of $E_3^c(t)$ will be negative, and hence $E_3^c(t)h(t)$ positive, if the discounted shadow value of firm i's own stock, $\lambda_i(t)e^{-rt}$, is growing over time along the equilibrium path. Hence, as in the dynamic open-loop equilibrium and contrary to the static equilibrium, knowing the sign of $E_2^c(t)$ is not sufficient to determine the sign of $dV_i\,(\varepsilon = 0)/d\varepsilon$. In particular, it cannot be ruled out, once again, that forcing one firm in a nonrenewable resource duopoly to contract its production path over a given interval of time

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may improve its profits.

5 Conclusion

Static — or steady-state, if it exists — oligopoly analysis predicts that if a single firm in the oligopoly were forced to contract its production marginally, its profits would fall. On the other hand, if all the firms were simultaneously forced to reduce their production, thus moving the industry towards monopoly output, each firm's profit would rise. We have shown that these results may not hold in a dynamic oligopoly, such as a nonrenewable resource oligopoly, where the exogenous constraint would take the form of a contraction of the firm's output path over some fixed interval of time: there are situations where a firm will gain from being the lone firm constrained in this way and cases where each firm will lose if all the firms in the industry are so constrained. As in the static oligopoly, such constraints have the effect of inducing all the unconstrained firms, if any, to adjust their output at each date at which the constraint holds. But in a dynamic oligopoly they also modify the initial conditions of the remaining game that follows the lifting of the constraints. These effects may have opposite consequences for profits and the net result will depend on the actual parameters. Clearly, the very intuitive results obtained from a static analysis of the effects of exogenous phenomena, such as strikes, taxes or quotas, which cause firms to contract their production path, can be misleading when the context is one of dynamic oligopoly. Their effects on profits may be exactly reversed.

Appendices

A Derivation of equations (8) and (17)

We show in this appendix that $dV_i\,(\varepsilon = 0)\,/d\varepsilon$ is given by (8) in the case of open-loop equilibria and by (17) in the case of closed-loop equilibria.

Recall that $V_i(x(t))$ denotes the equilibrium value of the program of firm i beginning at date t with resource stock $x(t)$, as measured by the flow of future profits, discounted to date t, along the equilibrium path beginning at t. In particular, the value of constrained firm i's equilibrium program at time $t = 0$ in a constrained equilibrium is a function of ε and τ, in addition to $x(0)$. To denote this fact, we might write $V_i(x(0); \varepsilon, \tau)(= J(v(x(0))))$. For any constrained firm $i \in \{1, 2, \ldots, S\}$, we have

$$V_i(x(0); \varepsilon, \tau) = \int_0^\tau \Pi_i(v) dt + e^{-r\tau} V_i\left(x\left(\tau\right)\right)$$

Since $v(x(0)) = u^*(x(0))$ when $\varepsilon = 0$, we will, by a slight abuse of notation, write $V_i(\varepsilon = 0)$ to denote the equilibrium present value of firm i evaluated at $\varepsilon = 0$, that is at the initial unconstrained equilibrium.

Differentiating with respect to ε, we get

$$
\begin{aligned}
\frac{dV_i}{d\varepsilon} &= \int_0^\tau \left[\sum_{j=S+1}^N e^{-rt} \frac{\partial \Pi_i(v)}{\partial v_j} \frac{dv_j}{d\varepsilon} + \sum_{l \neq i, l=1}^S e^{-rt} \frac{\partial \Pi_i(v)}{\partial v_l} h\left(t\right) \right] dt \\
&+ e^{-r\tau} \sum_{j=S+1}^N \frac{\partial V_i\left(x\left(\tau\right)\right)}{\partial x_j} \frac{dx_j}{d\varepsilon} + e^{-r\tau} \sum_{l \neq i, l=1}^S \frac{\partial V_i\left(x\left(\tau\right)\right)}{\partial x_l} \frac{dx_l}{d\varepsilon} \\
&+ \int_0^\tau e^{-rt} \frac{\partial \Pi_i(v)}{\partial v_i} h\left(t\right) dt + e^{-r\tau} \frac{\partial V_i\left(x\left(\tau\right)\right)}{\partial x_i} \frac{dx_i}{d\varepsilon}.
\end{aligned}
$$

Since, for all $h \in \{1, 2, \ldots, N\}$,

$$x_h(\tau) = x^0 - \int_0^\tau v_h(t) dt,$$

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we have

$$\frac{dx_h}{d\varepsilon} = -\int_0^\tau \frac{dv_h}{d\varepsilon} dt.$$

Therefore,

$$\frac{dx_j}{d\varepsilon} = -\int_0^\tau \frac{dv_j}{d\varepsilon} dt,$$

$$\frac{dx_l}{d\varepsilon} = -\int_0^\tau h(t) dt$$

and

$$\frac{dx_i}{d\varepsilon} = -\int_0^\tau h(t) dt$$

and hence

$$\begin{aligned}
\frac{dV_i}{d\varepsilon} &= \int_0^\tau \left[\sum_{j=S+1}^{N} e^{-rt} \frac{\partial \Pi_i(v)}{\partial v_j} \frac{dv_j}{d\varepsilon} + \sum_{l \neq i, l=1}^{S} e^{-rt} \frac{\partial \Pi_i(v)}{\partial v_l} h(t) \right] dt \\
&\quad -e^{-r\tau} \int_0^\tau \sum_{j=S+1}^{N} \frac{\partial V_i(x(\tau))}{\partial x_j} \frac{dv_j}{d\varepsilon} dt - e^{-r\tau} \int_0^\tau \sum_{l \neq i, l=1}^{S} \frac{\partial V_i(x(\tau))}{\partial x_l} h(t) dt \\
&\quad + \int_0^\tau \left[e^{-rt} \frac{\partial \Pi_i(v)}{\partial v_i} - e^{-r\tau} \frac{\partial V_i(x(\tau))}{\partial x_i} \right] h(t) dt.
\end{aligned}$$

In the case of open-loop equilibria, we know by (4) that at $\varepsilon = 0$,

$$\frac{\partial \Pi_i(v(t))}{\partial v_i} = \lambda_i(t)$$

and by (5) that

$$\lambda_i(\tau) = e^{r(\tau-t)} \lambda_i(t).$$

Therefore

$$e^{-rt} \frac{\partial \Pi_i(v)}{\partial v_i} - e^{-r\tau} \frac{\partial V_i(x(\tau))}{\partial x_i} = 0,$$

since at any date t along the optimal path, we have

$$\lambda_i(\tau) = \frac{\partial V_i(x(\tau))}{\partial x_i}.$$

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Furthermore, the equilibrium at $\varepsilon = 0$ being symmetric, we have

$$\frac{\partial \Pi_i(u^*)}{\partial v_k} = \frac{\partial \Pi_i(u^*)}{\partial v_{k'}} \text{ for all } k, k' \neq i$$

and

$$\frac{\partial V_i\left(x\left(\tau\right)\right)}{\partial x_k} = \frac{\partial V_i\left(x\left(\tau\right)\right)}{\partial x_{k'}} \text{ for all } k, k' \neq i$$

and hence, as stated in (8),

$$\frac{dV_i\left(\varepsilon = 0\right)}{d\varepsilon} = \int_0^\tau \left[\frac{\partial \Pi_i(u^*)}{\partial v_k} e^{-rt} - \frac{\partial V_i\left(x\left(\tau\right)\right)}{\partial x_k} e^{-r\tau} \right] \left[(N-S)\frac{dv_j}{d\varepsilon} + (S-1)h\left(t\right) \right] dt$$

where k denotes any firm other than firm i and j denotes any firm in the unconstrained subset $\{S + 1, S + 2, \ldots, N\}$.

In the case of closed-loop equilibria, at $\varepsilon = 0$, we know by (12) that

$$\lambda_i(\tau) \neq e^{r(\tau - t)}\lambda_i(t).$$

and therefore

$$e^{-rt}\frac{\partial \Pi_i(v)}{\partial v_i} - e^{-r\tau}\frac{\partial V_i\left(x\left(\tau\right)\right)}{\partial x_i} \neq 0.$$

Hence, as stated in (17),

$$\begin{aligned}
\frac{dV_i\left(\varepsilon = 0\right)}{d\varepsilon} &= \int_0^\tau \left[\frac{\partial \Pi_i(u^*)}{\partial v_k} e^{-rt} - \frac{\partial V_i\left(x^*\left(\tau\right)\right)}{\partial x_k} e^{-r\tau} \right] \left[(N-S)\frac{dv_j}{d\varepsilon} + (S-1)h\left(t\right) \right] dt \\
&\quad + \int_0^\tau \left[e^{-rt}\frac{\partial \Pi_i(u^*)}{\partial v_i} - e^{-r\tau}\frac{\partial V_i\left(x^*\left(\tau\right)\right)}{\partial x_i} \right] h(t)dt.
\end{aligned}$$

where again k denotes any firm other than firm i and j denotes any firm in the unconstrained subset $\{S + 1, S + 2, \ldots, N\}$.

B   A numerical example

We have established in section 3 that if $E_2^o(t) > 0$, a marginal contraction, in a neighborhood of an unconstrained equilibrium, of the production path of a subset of the firms in an oligopoly will improve those firms' profit if, for all $t \in [0, \tau]$, we have $E_1^o(t) > 0$, where $E_2^o(t)$ and $E_1^o(t)$ are given by (9) and (10) respectively. We provide here a numerical example to show that $E_1^o(t)$ can indeed be positive.

Consider an open-loop differential game with two symmetric firms each owning an initial resource stock $x^0 = 1$. We know that if the marginal contraction is imposed to one firm (say firm 1) in a duopoly, then $E_2^o(t) > 0$. Assume that the cost of extraction is zero and that the inverse market demand is given by:

$$P(u_1, u_2) = a - b(u_1 + u_2).$$

Each firm's unconstrained equilibrium production path is then given by:

$$u^*(t) \equiv u_1^*(t) = u_2^*(t) = \frac{a}{3b}(1 - e^{r(t-T^*)}),$$

where T^* is the solution to

$$T - \left(\frac{1 - e^{-rT}}{r}\right) = \frac{3b}{a}x^0.$$

For $r = 0.05$, $a = 20$ and $b = 4$, the equilibrium date of exhaustion of the stocks is :

$$T^* = 5.10741$$

For the above demand function, we have :

$$\frac{\partial \Pi_1(u^*(t)}{\partial v_2} e^{-rt} = -bu_1^*(t) e^{-rt}$$

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and hence, for $t > 0$,

$$\frac{\partial \Pi_1(u^*(t))}{\partial v_2} e^{-rt} > -bu_1^*(0) = -1.50247$$

If we compute $\partial V_1 (x^* (\tau))/\partial x_2$ in a neighborhood of $\tau = 0$, we find that, for the above values of the parameters,

$$\frac{\partial V_1}{\partial x_2} = -52.3294 \ll -bu_1^*(0) < \frac{\partial \Pi_1}{\partial v_2} e^{-rt}.$$

We can therefore conclude that for a marginal contraction of firm 1's extraction path, $E_1^o(t)$ can be positive over a sufficiently small interval of contraction.

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