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BALANCED EGALITARIAN REDISTRIBUTION OF INCOME

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RÉSUMÉ

Cet article réexamine la question de la redistribution du revenu entre des individus caractérisés par des niveaux de talent et d'effort différents. Deux principes de redistribution, *l'équivalence égalitaire équilibrée* et *l'égalitarisme conditionnel équilibré*, sont définis et axiomatisés.

Mots-clés: redistribution, égalitarisme.

ABSTRACT

This paper reconsiders the issue of how income should be redistributed when people endowed with different levels of talent exert different levels of effort. Two new schemes, called *balanced egalitarian equivalence* and *balanced conditional egalitarianism*, are proposed and characterized.

Key words: redistribution, egalitarianism.

1. INTRODUCTION

How should income be redistributed among people endowed with different skills, handicaps or needs? The ideal of equal opportunity, advocated in a broader context by Arneson (1989) and Cohen (1989), is based on the postulate that some determinants of an individual's income are within his control while others are not. Throughout the paper I will use the term *effort* to refer to the former category and *talent* to refer to the latter. I assume that both are real variables; this tremendous simplification does not destroy the interest of the analysis. Equalizing opportunities, it may be argued, requires that income be redistributed so as to offset inequalities that are due to differences in talent while preserving those that arise from differences in effort.

As it turns out, there is a tension between these two desiderata. In fact, it is generally impossible to guarantee an equal income to individuals exerting a same effort while performing identical income transfers to those who are equally talented. This remarkable impossibility was first pointed out (in a more general framework) by Fleurbaey (1994a, b) and later reinforced by Bossert (1995).

Weakening the defining requirements of equal opportunity opens the way to a variety of interesting redistribution schemes. Egalitarian equivalence and conditional egalitarianism are among the most appealing ideas. *Egalitarian equivalence* insists that the principle of equal income for equal effort be preserved. The simplest egalitarian-equivalent schemes work as follows. First, a reference level of talent is fixed. Then, each individual receives a post-transfer income equal to the pre-transfer income he would earn were he endowed with the reference talent level, plus a uniform amount dictated by the budget constraint. These schemes were introduced in a slightly different setting and given an axiomatic characterization by Fleurbaey (1994c). An alternative axiomatization was proposed by Bossert and Fleurbaey (1994). The main difficulty lies in the arbitrary nature of the reference level of talent. In an effort to overcome this difficulty, Moulin (1994) suggested the *average* egalitarian-equivalent scheme: successively choose each individual's level of talent as the reference level, compute the corresponding egalitarian-equivalent post-transfer income distributions, and take the average. That method was recently axiomatized by Bossert and Fleurbaey (1994). Yet another solution, mentioned in Moulin (1994), is to choose the median individual talent as the reference.

Conditional egalitarianism is essentially dual to egalitarian equivalence. Here the principle of equal transfer for equal talent is given priority. The simplest conditionally egalitarian schemes are based on a fixed reference level of effort. The first component of every individual's post-transfer income is the average income that would be generated if all individuals exerted the reference level of effort. To that component is added the income increment created by the individual's deviation from the reference level. *Median* conditional egalitarianism selects the median value of the individual efforts as the reference level. *Average* conditional egalitarianism averages the conditionally egalitarian post-transfer income distributions obtained by successively choosing each individual's effort as the reference level. Conditionally egalitarian schemes and their average version were studied by Fleurbaey (1994c) and Bossert and Fleurbaey (1994). Roemer's (1993) approach, though somewhat different, also makes use of reference levels of effort.

This paper proposes and defends two new income redistribution schemes to which I will refer by the phrase *balanced egalitarianism.* They are motivated by weaknesses of the average and median egalitarian-equivalent and conditionally egalitarian approaches that I shall now describe.

Consider first the median and average egalitarian-equivalent schemes. By the aforementioned impossibility result of Fleurbaey, the transfers received by two equally talented individuals under either of these two schemes will generally differ. What is more disturbing, however, is that one individual may be taxed while the other is subsidized. As it turns out, the gap between the tax and the subsidy is unbounded. This is a rather extreme violation of the principle of equal transfer for equal talent. I will suggest a simple scheme, called *balanced egalitarian-equivalent*, that guarantees that equally talented individuals never receive transfers of opposite signs, and delivers equal income for equal effort. I will show that these properties are characteristic of the balanced egalitarian-equivalent scheme in the limit case where all configurations of effort and talent are present in society.

Average, or median, conditional egalitarianism suffers from a weakness that is technically dual to that of average or median egalitarian equivalence, though maybe conceptually less serious: individuals exerting the same effort may enjoy post-transfer incomes that are on opposite sides of society's average income. The *balanced conditionally egalitarian* scheme defined in this paper avoids that difficulty.

Since the two schemes that I propose are dual, and because I find the property of equal income for equal effort more fundamental than equal transfer for equal talent, I shall analyze balanced egalitarian equivalence in some detail but content myself with a brief description of balanced conditional egalitarianism. The beginning of Section 2, which is a bit informal, serves as a motivation for introducing balanced egalitarian equivalence. I recall the definition of average egalitarian equivalence in (the one-dimensional version of) Bossert's (1995) model and offer an example where two equally talented individuals receive transfers of opposite signs. A similar example could be constructed for the median egalitarian-equivalent scheme. Next, I set up the basic model used in the paper, which is a continuous version of Bossert's. I define and characterize balanced egalitarian equivalence. Resorting to a continuum of individuals is necessary for the characterization result but balanced egalitarian equivalence is, of course, well defined in the finite case as well. A shortcoming of Bossert's model is that an individual's income does not depend on the talent and effort of other members of society. This assumption is relaxed in Section 3. The definition of balanced egalitarian equivalence is generalized to a richer setting and conditions under which its characterization remains valid are discussed. Section 4 defines and characterizes balanced conditional egalitarianism and Section 5 gathers some concluding remarks.

2. A BASIC FORMULATION OF BALANCED EGALITARIAN EQUIVALENCE

Consider a society composed of n agents. Agent i's effort is e_i, his talent is t_i, and his income before redistribution is $f(e_i, t_i)$. The function f is increasing in both arguments. Let us fix a reference talent t_0. Under the *t_0- egalitarian-equivalent* redistribution scheme, agent i's post-transfer income is

$$x_i^{t_0} = f(e_i, t_0) - \frac{1}{n}\sum_{k=1}^{n} [f(e_k, t_0) - f(e_k, t_k)] .$$

Under *average* egalitarian equivalence, agent i receives the post-transfer income

$$x_i = \frac{1}{n}\sum_{j=1}^{n} x_i^{t_j} .$$

Simple computations show that the corresponding income transfer, or compensation, takes the form

$$c_i := x_i - f(e_i, t_i) = \left[\frac{1}{n}\sum_{j=1}^{n} f(e_j, t_j) - \frac{1}{n^2}\sum_{j,k=1}^{n} f(e_k, t_j)\right] + \left[\frac{1}{n}\sum_{j=1}^{n} f(e_i, t_j) - f(e_i, t_i)\right].$$

Suppose now that $n = 3$, $e_1 > e_2 > e_3$, and $t_1 = t_2 =: t < t_3$. Thus, agents 1 and 2 are equally talented. Assume that

$$f(e_1, t) = 5 \ , \quad f(e_1, t_3) = 10 \ ,$$

$$f(e_2, t) = 2 \ , \quad f(e_2, t_3) = 3 \ ,$$

$$f(e_3, t) = 1 \ , \quad f(e_3, t_3) = 2 \ .$$

Then $c_1 = 11/9 > 0 > -1/9 = c_2$: agent 1 is subsidized whereas agent 2 is taxed. In fact, if $f(e_1, t_3) = 5 + a$, where $a > 2$, then $c_1 = (1 + 2a)/9$ and $c_2 = (4 - a)/9$: agent 1's subsidy and agent 2's tax are unbounded.

There is a natural solution to the above problem. Choose the talent t^* such that $\Sigma_{k=1}^{n} f(e_k, t^*) = \Sigma_{k=1}^{n} f(e_k, t_k)$. Agent i's post-transfer income under the t^* - egalitarian-equivalent scheme is merely $f(e_i, t^*)$. No uniform amount is needed to balance the budget: each agent simply gets the pre-transfer income he would earn if he were endowed with the talent t^*. The transfer to agent i being $f(e_i, t^*) - f(e_i, t)$, it is obvious that equally talented agents cannot receive transfers of opposite signs. Let us call the scheme just described *balanced egalitarian equivalent.* It turns out that when all combinations of effort and talent are present in society, balanced egalitarian equivalence is the only way to guarantee transfers of equal sign to equally talented agents and equal income for equal effort. The rest of this section is devoted to a formal statement and proof of this assertion.

From now on, I postulate a continuum of agents. Formally, the *agent space* is the probability space $(I, \mathcal{B}, \ell)$, where $I = [0, 1]$, $\mathcal{B}$ is the Borel σ-algebra of I, and ℓ is Lebesgue's measure on $\mathcal{B}$. *Effort* ε and *talent* τ are random variables on the agent space taking values in $E = T = (0, 1)$. *Society* is a joint probability density function s of effort and talent. The corresponding distribution function is denoted by σ. The marginal density functions of effort and talent are s_ε and s_τ, with associated distribution functions σ_ε and σ_τ. Mean effort and mean talent in society s are $\bar{e}(s) = \int_E e\, s_\varepsilon(e)\, de$ and $\bar{t}(s) = \int_T t\, s_\tau(t)\, dt$. Society s is called *regular* if for every measurable subset A of ExT, $\sigma(A) > 0$ if and only if $\lambda(a) > 0$, where λ is Lebesgue's measure on the Borel σ-algebra of ExT.

The (pre-transfer) *income* of an agent who exerts the effort level e and is endowed with the talent level t in society s is denoted f(e, t). Note that this quantity does not vary with s: in this model, an agent's income is entirely determined by his own characteristics. Think of a society of farmers who all cultivate corn on their own identical parcel of land. This assumption that all agents are completely independent of each other will be relaxed in the next section. I suppose that the *income function* $f : ExT \to \mathbb{R}_+$ satisfies the following conditions:

(A.1) *for each* $t \in T$, *f(e, t) is continuous and increasing in e, and* $\lim_{e \to 0} f(e, t) = 0$;

(A.2) *for each* $e \in E$, *f(e, t) is continuous and increasing in t, and* $\lim_{t \to 0} f(e, t) = 0$.

The mean income in society s is $\bar{y}(s) = \int_E \int_T f(e, t)\, s(e, t)\, de\, dt$.

In what follows, every uncountable subset of an Euclidean space is endowed with the induced Euclidean topology and every finite set is endowed with the discrete topology. The measurability of a mapping between two topological spaces is always understood with respect to the Borel σ-algebras of these spaces. A *redistribution (of income in society s)* is a measurable mapping $F : ExT \to \mathbb{R}_+$ satisfying the feasibility condition $\int_E \int_T F(e, t)\, s(e, t)\, de\, dt = \bar{y}(s)$. The number F(e, t) is the post-transfer income of any agent exerting effort e and endowed with talent t in society s.[1] The *compensation mapping* C associated with F is defined by $C(e, t) = F(e, t) - f(e, t)$ for all $(e, t) \in ExT$.

In order to define balanced egalitarian equivalence, consider a society where the distribution of effort is the same as in society s but where all agents are endowed with a same level of talent. The level ensuring that the mean income in this society is the same as in s is called the *balanced talent level* (in society s) and denoted by t^*. Formally, t^* satisfies the condition

$$\int_E f(e, t^*)\, s_\varepsilon(e)\, de = \bar{y}(s) .$$

Under assumptions (A.1) and (A.2), t^* exists, is unique, and belongs to T. A redistribution F in society s is *balanced egalitarian-equivalent* if

$$\text{for } \sigma\text{-a.e. } (e, t) \in ExT, \;\; F(e, t) = f(e, t^*) .$$

Balanced egalitarian-equivalent redistributions are unique up to differences on sets of zero σ-measure. As already mentioned, they guarantee transfers of equal sign to equally talented agents and equal income for equal effort. Formally, a redistribution F satisfies the principle of *Equal Income for Equal Effort* (EIEE) if there exists a measurable mapping $\alpha : E \to \mathbb{R}_+$ such that for σ-a.e. (e, t), $F(e, t) = \alpha(e)$. It satisfies the principle of *Fairly Signed Transfers* (FST) if there exists a measurable mapping $\beta : T \to \{-1, +1\}$ such that for σ-a.e. (e, t), $\beta(t)\, C(e, t) \geq 0$.

Proposition 0. Suppose that the income function $f : ExT \to \mathbb{R}_+$ satisfies assumptions (A.1) and (A.2) and let s be a regular society. Then a redistribution of income in society s satisfies the principles of Equal Income for Equal Effort and Fairly Signed Transfers if and only if it is balanced egalitarian-equivalent.

Proof. The proof of sufficiency is obvious. Conversely, fix a regular society s and a redistribution F satisfying EIEE and FST. By EIEE, there exists a measurable mapping $\alpha : E \to \mathbb{R}_+$ such that $F(e, t) = \alpha(e)$ for σ-a.e. (e, t). Extend the income function f to $\mathbb{R}^2_+$ in such a way that $f(e, \cdot)$ is continuous, increasing and unbounded for each $e \in E$ and $f(\cdot, t)$ is continuous for each $t \in \mathbb{R}_+$. For each $e \in E$ there is a unique number $t_1(e)$ such that $f(e, t_1(e)) = \alpha(e)$. The mapping $t_1 : E \to \mathbb{R}_+$ is such that

$$\text{for } \sigma\text{-a.e. } (e, t), \;\; F(e, t) = f(e, t_1(e)) . \tag{1}$$

This mapping is measurable. (To check this assertion, it suffices to verify that the set $E(t^o) := \{e \in E \mid t_1(e) < t^o\}$ is measurable for each $t_0 \in \mathbb{R}_+$. But for every $e \in E$, $t_1(e) < t^o \Leftrightarrow f(e, t_1(e)) < f(e, t^o) \Leftrightarrow \alpha(e) < f(e, t^o)$. Therefore $E(t^o) = \{e \in E \mid \alpha(e) - f(e, t^o) < 0\}$, which is clearly measurable for each t^o since α is measurable and $f(\cdot, t^o)$ is continuous, hence measurable as well). Define

$$A^- = \{(e, t) \in ExT \mid F(e, t) < f(e, t^*)\} ,$$

$$E^- = \{e \in E \mid t_1(e) < t^*\} ,$$

and define A^+ and E^+ by reversing the inequality signs in the above definitions. All these sets are measurable because F, f, and t_1 are measurable mappings. By (1) and because f is increasing in t, A^- and A^+ are almost equal to E^-xT and E^+xT respectively. Formally, if Δ denotes the symmetric difference operator, $\sigma(A^- \Delta (E^-xT)) = \sigma(A^+ \Delta (E^+xT)) = 0$. We must prove that $\sigma(A^-) = \sigma(A^+) = 0$. Since σ is regular, this is equivalent to

$$\lambda_\varepsilon(E^-) = \lambda_\varepsilon(E^+) = 0 , \tag{2}$$

where λ_ε is Lebesgue's measure on the Borel σ-algebra of E.

Suppose, by way of contradiction, that

$$\lambda_\varepsilon(E^-) + \lambda_\varepsilon(E^+) > 0 . \tag{3}$$

1 Of course, there need not exist such agents. This is irrelevant, however, since the behavior of F on sets of zero σ-measure is unconstrained by the feasibility condition.

By the condition defining t^* and the feasibility constraint on F,

$$\int_E \int_T F(e, t)\, s(e, t)\, de\, dt = \int_E f(e, t^*)\, s_\varepsilon(e)\, de$$

$$\Rightarrow \quad \int_E f(e, t_1(e))\, s_\varepsilon(e)\, de = \int_E f(e, t^*)\, s_\varepsilon(e)\, de$$

$$\Rightarrow \quad \int_{E^-} [f(e, t^*) - f(e, t_1(e))]\, s_\varepsilon(e)\, de = \int_{E^+} [f(e, t_1(e)) - f(e, t^*)]\, s_\varepsilon(e)\, de. \qquad (4)$$

From (3) and (4) follows that $\lambda_\varepsilon(E^-) > 0$ and $\lambda_\varepsilon(E^+) > 0$. This conclusion can be slightly strengthened. For n = 1, 2, ... , define $E_n^- = \{e \in E \mid t_1(e) < t^* - \frac{1}{n}\}$ and define E_n^+ similarly. There exists a positive integer N^- such that $\lambda_\varepsilon(E_{N^-}^-) > 0$ since otherwise $\lambda_\varepsilon(E^-) = \lambda_\varepsilon(\cup_{n=1}^\infty E_n^-) \le \Sigma_{n=1}^\infty \lambda_\varepsilon(E_n^-) \le 0$. Likewise, there exists a positive integer N^+ such that $\lambda_\varepsilon(E_{N^+}^+) > 0$. Letting $N = \max\{N^-, N^+\}$,

$$\lambda_\varepsilon(E_N^-) > 0 \quad \text{and} \quad \lambda_\varepsilon(E_N^+) > 0 . \qquad (5)$$

Define $T_N = [t^* - \frac{1}{N}, t^* + \frac{1}{N}]$. Since σ is regular, (5) implies

$$\sigma(E_N^- x T_N) > 0 \quad \text{and} \quad \sigma(E_N^+ x T_N) > 0 . \qquad (6)$$

Now, for σ-a.e. $(e^-, t) \in E_N^- x T_N$ and σ-a.e. $(e^+, t) \in E_N^+ x T_N$,

$$F(e^-, t) \;=\; f(e^-, t_1(e^-)) \;<\; f(e^-, t^* - \tfrac{1}{N}) \;\le\; f(e^-, t) ,$$

$$F(e^+, t) \;=\; f(e^+, t_1(e^+)) \;>\; f(e^+, t^* + \tfrac{1}{N}) \;\ge\; f(e^+, t) .$$

This means that there exists full σ_ε-measure subsets of E_N^- and E_N^+, say E_N^- and E_N^+ themselves, and a full σ_τ-measure subset of T_N, say T_N itself, such that

$$C(e^-, t) < 0 < C(e^+, t)$$

for all $e^- \in E_N^-$, $e^+ \in E_N^+$, and $t \in T_N$. It is easily seen that this contradicts the FST principle. Indeed, let β be any measurable function from T into $\{-1, +1\}$. For all $t \in T_N$, one of the following statements holds:

$$\beta(t)\, C(e^-, t) < 0 < \beta(t)\, C(e^+, t) \quad \text{for all } e^- \in E_N^- \text{ and } e^+ \in E_N^+ ,$$

$$\beta(t)\, C(e^+, t) < 0 < \beta(t)\, C(e^-, t) \quad \text{for all } e^- \in E_N^- \text{ and } e^+ \in N .$$

Let $B^- = \{(e^-, t) \in E_N^- x T_N \mid \beta(t)\, C(e^-, t) < 0\}$ and define B^+ similarly. The measurability of F and f ensures that these sets are measurable. I claim that $\sigma(B^- \cup B^+) = \sigma(B^-) + \sigma(B^+) > 0$. Suppose $\sigma(B^-) = 0$. Then $\beta(t)\, C(e^-, t) > 0$ for σ-a.e. $(e^-, t) \in E_N^- x T_N$, hence $\beta(t)\, C(e^+, t) < 0$ for σ-a.e. $(e^+, t) \in E_N^+ x T_N$. Therefore $\sigma(B^+) = \sigma(E_N^+ x T_N)$, which is positive by (6). Similarly, $\sigma(B^+) = 0$ implies $\sigma(B^-) = 0$. This shows that $\sigma(B^- \cup B^+) > 0$, in contradiction to FST. ■

3. BALANCED EGALITARIAN EQUIVALENCE IN MORE COMPLEX ENVIRONMENTS

This section allows each agent's pre-transfer income to depend not only on his own characteristics, but also on the characteristics of other members of society.

The agent space, effort, talent and society are defined as in the previous section. The (pre-transfer) income of an agent who exerts effort e and is endowed with talent t in society s is now written f(e, t, s). Letting S stand for the set of all societies, a *(generalized) income function* f is defined on ExTxS. Assumptions (A.1) and (A.2) are reexpressed as follows:

(A.3) *for each $t \in T$ and $s \in S$, $f(e, t, s)$ is continuous and increasing in e, and $\lim_{e \to 0} f(e, t, s) = 0$;*

(A.4) *for each $e \in E$ and $s \in S$, $f(e, t, s)$ is continuous and increasing in t, and $\lim_{t \to 0} f(e, t, s) = 0$.*

These are restrictions on the pre-transfer income function within any society. In addition, I shall impose two restrictions on how income varies across societies. Consider a society in which the distribution of talent is concentrated at a single level. Provided that the distribution of effort remains unchanged, a small increase of the uniform talent level is likely to induce a small increase in income at every level of effort. Formally, let s^t denote the society where talent is concentrated at level t and effort is distributed as in s:

$$s^t(e, t') = \begin{cases} 0 & \text{for all } e \in E \text{ and } t' \in T \setminus \{t\}, \\ s_E(e) & \text{for all } e \in E \text{ and } t' = t. \end{cases}$$

The assumption is:

(A.5) *for each* $e \in E$ *and* $s \in S$, $f(e, t, s^t)$ *is continuous and increasing in* t.

This requirement is fairly weak because it bears only on very comparable societies. By contrast, the last assumption involves societies that may be quite different from each other. Recall that the balanced talent level t* in society s was defined in Section 2 by the condition that shifting everyone's talent to that level does not affect mean income. Since society itself is now an argument of the income function, the proper formulation of that condition becomes

$$\int_E f(e, t^*, s^{t^*})\, s_E(e)\, de = \bar{y}(s),$$

where $\bar{y}(s) := \int_E \int_T f(e, t, s)\, s(e, t)\, de\, dt$ is the mean income in society s. Assumptions (A.3) to (A.5) ensure that t* exists, is unique and belongs to T. I now assume:

(A.6) *for each* $e \in E$, $t \in T$ *and* $s \in S$, $f(e, t^*, s) = f(e, t^*, s^{t^*})$.

This is a limited neutrality property. It says that bringing all talents to the balanced level does not affect the agents who are endowed with precisely the balanced level of talent.

Of course, assumptions (A.3) to (A.6) are satisfied if f is constant in s and $f(\cdot, \cdot, s)$ satisfies (A.1) and (A.2) for any s: the present model encompasses the no-interaction model of Section 2. Assumptions (A.3) to (A.6), however, allow for interesting patterns of influence among the agents, as the following example illustrates.

Example. Suppose that each agent combines effort and talent to supply some standardized input through the mapping x, say, $x(e, t) = et$. Suppose that the mean input in society s, $\bar{x}(s)$, determines the mean income through some aggregate production function $\varphi : (0, 1) \to \mathbb{R}_+$ which is continuous, increasing, and such that $\lim_{x \to 0} \varphi(x) = 0$. Finally, suppose that each agent's income is proportional to his input contribution:

$$f(e, t, s) = (x(e, t) / \bar{x}(s))\, \varphi(\bar{x}(s)) \quad \text{for all } e, t, s. \tag{7}$$

This technological environment is considered in Mirrlees (1974), Moulin (1990), and Maniquet (1994). The traditional story is that of a group of fishers catching fish in a lake. The proportionality constraint (7) simply means that each fisher eats his own catch. In that context, it is natural to assume that $\varphi(x) / x$ is decreasing in x: individual income then decreases with society's mean input. I shall not make that assumption here, thereby allowing for a greater variety of possible interactions among agents. It is obvious that assumptions (A.3) and (A.4) are satisfied. Next, since

$$f(e, t, s^t) = (e / \bar{e}(s))\, \varphi(\bar{e}(s)\, t) \quad \text{for all } e, t, s,$$

assumption (A.5) is met as well. Finally, since the balanced level of talent in society s is

$$t^* = \bar{x}(s) / \bar{e}(s),$$

it follows that $f(e, t^*, s) = (x(e, t^*) / \bar{x}(s))\, \varphi(\bar{x}(s)) = (e / \bar{e}(s))\, \varphi(\bar{e}(s)\, t^*) = f(e, t^*, s^{t^*})$, in agreement with assumption (A.6).

Returning to the general model, define a redistribution F in society s as in the previous section. The compensation mapping associated with F is $C = F - f(\cdot, \cdot, s)$ and the EIEE and FST principles are unchanged. A redistribution F in s is balanced egalitarian-equivalent if

$$\text{for } \sigma\text{-a.e. } (e, t), \quad F(e, t) = f(e, t^*, s^{t^*}). \tag{8}$$

(Of course, under assumption (A.6), this amounts to imposing $F(e, t) = f(e, t^*, s)$ for σ-a.e. (e, t). Yet, the latter condition may contradict feasibility when (A.6) is violated whereas (8) never does).

Obviously, a balanced egalitarian-equivalent redistribution F satisfies the EIEE principle. To check the FST property, observe that for σ-a.e. (e, t),

$$C(e, t) \geq 0 \iff f(e, t^*, s) \geq f(e, t, s) \iff t^* \geq t .$$

If β is defined on T by $\beta(t) = 1$ if $t \leq t^*$ and $\beta(t) = -1$ otherwise, then $\beta(t)\, C(e, t) \geq 0$ for σ-a.e. (e, t), as required. Conversely, it is a simple matter to modify the proof of Proposition 0 to obtain:

Proposition 1. Suppose that the generalized income function $f : E \times T \times S \to \mathbb{R}_+$ satisfies assumptions (A.3) to (A.6), and let s be a regular society. Then a redistribution of income in s satisfies the principles of Equal Income for Equal Effort and Fairly Signed Transfers if and only if it is balanced egalitarian-equivalent.

4. BALANCED CONDITIONAL EGALITARIANISM

The symmetric structure of the model laid down in Section 3 suggests a dual approach to balanced egalitarian equivalence. Denote by s^e the society in which effort is concentrated at level e and talent is distributed as in society s. Maintain assumptions (A.3) and (A.4) and replace (A.5) by the following assumption:

(A.7) *for each $t \in T$ and $s \in S$, $f(e, t, s^e)$ is continuous and increasing in e.*

Define the balanced effort level e^* in society s by the condition

$$\int_T f(e^*, t, s^{e^*})\, s_\tau(t)\, dt = \bar{y}(s) ,$$

and replace (A.6) by

(A.8) *for each $e \in E$, $t \in T$ and $s \in S$, $f(e^*, t, s) = f(e^*, t, s^{e^*})$.*

Call a redistribution F in society s *balanced conditionally egalitarian* if its associated compensation mapping satisfies the condition:

$$\text{for } \sigma\text{-a.e. } (e, t),\; C(e, t) = \bar{y}(s) - f(e^*, t, s^{e^*}) .$$

Such a redistribution satisfies the principle of *Equal Transfer for Equal Talent*: there exists a measurable mapping $\gamma : T \to \mathbb{R}$ such that for σ-a.e. (e, t), $C(e, t) = \gamma(t)$. Moreover, agents who exert the same effort receive post-transfer incomes that are almost never on opposite sides of the mean income: either almost all of them are richer than average, on almost all are poorer. This I call the principle of *Fairly Ranked Incomes*: there is a measurable mapping $\delta : E \to \{-1, +1\}$ such that for σ-a.e. (e, t), $\delta(e)\, (F(e, t) - \bar{y}(s)) \geq 0$. These two principles are characteristic of balanced conditional egalitarianism:

Proposition 2. Suppose that the generalized income function $f : E \times T \times S \to \mathbb{R}_+$ satisfies assumptions (A.3), (A.4), (A.7) and (A.8), and let s be a regular society. Then a redistribution of income in s satisfies the principles of Equal Transfer for Equal Talent and Fairly Ranked Incomes if and only if it is balanced conditionally egalitarian.

The proof of Proposition 2 mimics those of Propositions 0 and 1 and is therefore omitted.

5. CONCLUDING REMARKS

Perhaps the main weakness of the approach to income redistribution outlined in this paper is the assumption that talent and effort are summarized by real numbers. Balanced egalitarian equivalence does not extend in any obvious way to the case when talent is multi-dimensional and balanced conditional egalitarianism is restricted to those environments where effort is one-dimensional. Note that this shortcoming is (essentially) shared by the median versions of egalitarian equivalence and conditional egalitarianism discussed in the introduction, which require a preorder structure on the set of talents or efforts. The average approach, by contrast, is well defined regardless of the structure of those sets.

As in Bossert (1995) and Bossert and Fleurbaey (1994), preferences are not a part of the model I used. Suppose that an agent is characterized by a preference u over effort-income pairs, and a talent level. To fix ideas, suppose that u satisfies the usual assumptions and is also separable and linear in income. Call a density function r on preference-talent pairs an economy. How can balanced egalitarianism be generalized to this richer setting?

The answer depends on the technological environment. Consider the example discussed in Section 3. If the aggregate production function φ is linear, decentralized preference maximization is unambiguous. It yields for every economy r a society, i.e., a density s over effort-talent pairs. Moreover, since effort is efficiently distributed in that society, it is natural to operate balanced egalitarianism on s to redistribute the corresponding income.

If φ is not linear, decentralized preference maximization may be interpreted in different ways. If we assume Nash behavior, the equilibrium density function of effort will generally be inefficient. In that case, *any* income redistribution would perpetuate that inefficiency. It seems to me that this does not destroy the relevance of balanced egalitarianism: even if effort is inefficiently distributed, it is of interest to fairly redistribute income.

But balanced egalitarianism is compatible with efficiency. To each preference-talent pair corresponds an efficient effort level. Using this correspondence, it is straightforward to transform an economy r into a society s* whose marginal s^*_ε is efficient. Balanced egalitarianism may now be applied to s*.

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