Université de Montréal
Faculté des arts et des sciences
Département de sciences économiques

CAHIER 9326

EXACT NONPARAMETRIC ORTHOGONALITY AND RANDOM WALK TESTS

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August 1993

This work was supported by the Social Sciences and Humanities Research Council of Canada, the Natural Sciences and Engineering Research Council of Canada, and the Government of Québec (Fonds FCAR). The authors are grateful to Peter Nagan for providing the data used in Section 4 and thank Marc Hallin for helpful comments. All correspondence should be addressed to the authors at the Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal, C.P. 6128, succursale A, Montréal (Québec) Canada H3C 3J7.

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Ce cahier a également été publié au Centre de recherche et développement en économique (C.R.D.E.) (publication no 1893).

Dépôt légal - 1993
Bibliothèque nationale du Québec
Bibliothèque nationale du Canada

ISSN 0709-9231

RÉSUMÉ

L'hypothèse qu'une variable est indépendante de l'information disponible à une date donnée, par exemple, le propre passé de la variable et les valeurs réalisées d'autres variables observables, est une implication fréquente de la théorie économique. Toutefois les tests paramétriques usuels d'orthogonalité basés sur des régressions peuvent facilement ne pas avoir le niveau affiché s'il y a rétroaction des innovations sur les valeurs futures des régresseurs. Dans ce texte, nous développons des tests non paramétriques d'orthogonalité basés sur les signes et les rangs signés, dont on peut montrer qu'ils sont valides pour une grande classe de modèles avec rétroaction. Ces tests sont aussi robustes par rapport à divers problèmes de non-normalité et d'hétéroscédasticité. De plus, en étudiant par simulation deux modèles de régression avec rétroaction (un modèle d'attentes rationnelles, précédemment considéré par Mankiw et Shapiro, et un modèle de promenade aléatoire), nous trouvons que les tests non paramétriques ont une excellente puissance. Nous concluons l'article en appliquant nos résultats à des données d'attentes sur les taux d'intérêt précédemment étudiées par B. Friedman.

Mots clés : tests non paramétriques; test d'orthogonalité; test de promenade aléatoire; test de signes; test de rangs signés; attentes rationnelles; rétroaction; non-normalité; hétéroscédasticité; taux d'intérêt.

ABSTRACT

The hypothesis that a variable is independent of past information, such as its own past and past realizations of other observable variables, is a frequent implication of economic theory. Yet, standard regression-based tests of orthogonality may not have the correct size if there is feedback from innovations to future values of the regressors. In this paper, we develop nonparamteric tests of orthogonality based on signs and signed ranks which are proved to reject at their nominal levels over a wide class of models admitting feedback. The tests are robust to problems of non-normality and heteroskedasticity. Further, in simulation studies of two specifications of feedback --a rational expectations model considered by Mankiw and Shapiro, and the random walk model-- we find that the nonparametric tests display remarkable power. The paper concludes with an application which assesses the efficiency of survey data on interest rate expectations previously studied by B. Friedman.

Key words : nonparametric tests; orthogonality test; random walk test; sign test; signed rank test; rational expectations; feedback; non-normality; heteroskedasticity; interest rates.

I. Introduction

The hypothesis that markets are efficient implies the testable proposition that forecast errors made by the market are independent of any information available to the market when the forecast was made. This orthogonality property of efficient markets or, more generally, of rational expectations is an instance of the wider statistical issue of determining whether two time series are stochastically independent given they are independent of other past values of the same variables. Yet standard regression testing procedures which attempt to evaluate conditional independence may reject too often, even with fairly large samples.

We have two apparently dissimilar examples in mind. The first is a simple linear regression with predetermined variables considered by Mankiw and Shapiro (1986, referred to as MS in what follows) who found by Monte Carlo techniques that the true level of the t-test may be considerably larger than its nominal level even for fairly large samples. Even though asymptotic inference based on a normal distribution for the t-statistic is correct in their specification, the finite-sample distribution of the t-statistic differs considerably from its asymptotic distribution. The issue has been treated further by Banerjee and Dolado (1987, 1988), Galbraith, Dolado and Banerjee (1987), Banerjee, Dolado and Galbraith (1990), as well as by Mankiw and Shapiro (1985). The second example is the random walk model without drift which has necessitated even more radical readjustment, since the t-statistic associated with the OLS estimate does not have the usual asymptotic normal distribution. These two examples are illustrative of a class of models which involve feedback: future values of the regressors are affected by disturbances which are contemporaneously uncorrelated with the regressors.

In this paper, we introduce nonparametric analogues of the t-test, based on sign and signed rank statistics, that are applicable to a specific class of feedback models including both the MS model and the random walk without drift. The sign tests are provably exact for this class of models, irrespective of the nature of feedback, even if the disturbances are non-normal or heteroskedastic; similar results are obtained for a class of linear signed rank statistics (e.g. Wilcoxon-type statistics). Modifications of these results are obtained as well for cases involving discrete random variables, possibly with a mass at zero. Most importantly, simulations indicate that the nonparametric tests considered have good power relative to the t-test, using either the asymptotic or size-corrected critical values for the MS model or the Dickey-Fuller critical values as can be found in Fuller (1976) for the random walk model. The results of this paper involve a considerable generalization of those in Campbell and Dufour (1991), where various nonparametric statistics are introduced to deal with a variant of the MS model. In particular, the nature of the allowed feedback is considerably more general and exact distributional results are established for a class of Wilcoxon-type statistics.

The paper is organized as follows. In section 2, we introduce the relevant test statistics in the general feedback context and derive distributional results for various sign and signed-rank statistics. In section 3, two specific cases illustrating such feedback are introduced, and we present Monte Carlo results on the level and power of the proposed tests applied to these two cases. A relevant application is presented in Section 4: the orthogonality of forecast errors are tested using the same survey data considered by Friedman (1980). Section 5 offers some concluding remarks.

2. Nonparametric statistics in the feedback context

In many tests of orthogonality between two random variables, the null hypothesis asserts that a variable Y_t is independent of its own past as well as past realizations of a second variate X_t. Our goal is to introduce tests of this assertion which are exact under very weak assumptions concerning the distribution of Y_t and the relationship between Y_t and X_t. For one group of tests, we simply assume that Y_t has median zero; for the other, we suppose that the distribution of Y_t is symmetric about zero. No additional assumption other than the independence of Y_t with respect to the past (denoted in what follows by I_{t-1}) governs the relationship between Y_t and X_t. In more precise language, we work within the framework of the following general specification involving the random variables $Y_1, \dots, Y_n, X_0, \dots, X_{n-1}$, and the corresponding information vectors $I_t = (X_0, X_1, \dots, X_t, Y_1, \dots, Y_t)'$, where $t = 0, \dots, n\text{-}1$, with the convention $I_0 = (X_0)$:

$$Y_t \text{ is independent of } I_{t-1}\ , \text{ for each } t = 1, \dots, n\ ; \tag{1}$$

$$P[Y_t > 0] = P[Y_t < 0]\ , \quad \text{for } t = 1, \dots, n\ . \tag{2}$$

Assumption (1) states that Y_t is independent of the past values of Y_t and X_t, while Assumption (2) means that $Y_1, \dots, Y_n$ have median zero. These assumptions leave open the possibility of feedback from Y_t to current and future values of the X-variable without specifying the form of feedback. The variables Y_t and X_t may have discrete distributions (which includes the possibility of non-zero probability mass at zero); as well, the variables Y_t need not be normal nor identically distributed. In what follows, we shall also consider the additional assumption that $Y_1, \ldots, Y_n$ have distributions symmetric about zero:

$$Y_1, \ldots, Y_n \text{ have continuous distributions symmetric about zero.} \tag{3}$$

Clearly, the latter assumption implies (2), but the converse is not true.

In order to motivate the nonparametric statistics introduced in this paper, it is useful to consider the following linear model:

$$Y_t = \beta X_{t-1} + e_t\,, \quad t = 1, \ldots, n \tag{4}$$

where e_t has the same properties as Y_t in (1) and (2). Suppose we wish to test the null hypothesis that $\beta = 0$. It seems reasonable to focus on nonparametric analogues of Student's t-statistic, which in this environment are derived from

$$T = \frac{\hat\beta}{\hat\sigma\left(\sum_{t=1}^{n} X_{t-1}^2\right)^{-1/2}} = \frac{\sum_{t=1}^{n} Y_t X_{t-1}}{\hat\sigma\left(\sum_{t=1}^{n} X_{t-1}^2\right)^{1/2}} = \sum_{t=1}^{n} V_t\,, \tag{5}$$

where $\hat\beta = \sum_{t=1}^{n} Y_t X_{t-1} \Big/ \sum_{t=1}^{n} X_{t-1}^2$, $\hat\sigma^2 = \sum_{t=1}^{n} \left(Y_t - \hat\beta X_{t-1}\right)^2 \Big/ (n-1)$ and $V_t = Y_t X_{t-1} \Big/ \hat\sigma\left(\sum_{\tau=1}^{n} X_{\tau-1}^2\right)^{1/2}$. Nonparametric procedures abstract from the specific values of V_t to consider simply its sign and possibly the rank of its absolute value among $|V_1|, \ldots, |V_n|$. In such a context the denominator $\hat\sigma\left(\sum_{t=1}^{n} X_{t-1}^2\right)^{1/2}$ plays no role and we are led to consider the simple products $Z_t = Y_t X_{t-1}$ as the basic building block in the definition of various nonparametric statistics. More generally, to test $\beta = \beta_0$ in the context of (4), we would start with $Z_t = (Y_t - \beta_0 X_{t-1})X_{t-1}$ as the basic product. In particular, if X_{t-1} is identified with Y_{t-1} in (4), we can develop in this way tests of the random walk hypothesis without drift ($\beta = 1$).

A natural nonparametric analogue of the statistic T is thus the sign statistic given by

$$S_0 = \sum_{t=1}^{n} u(Y_t X_{t-1})\,, \tag{6}$$

where $u(z) = 1$, if $z > 0$, and $u(z) = 0$ for $z \le 0$. In this paper, we shall in fact study a more general sign statistic of the form

$$S_g = \sum_{t=1}^{n} u(Y_t g_{t-1}) , \tag{7}$$

where $g_t = g_t(I_t)$, $t = 0, \ldots, n-1$, is a sequence of measurable functions of the information vector I_t. Clearly S_0 is a special case of S_g obtained by taking $g_t = X_t$. The functions $g_t(\cdot)$ allow one to consider various (possibly nonlinear) transformations of the data, provided g_t depends only on past and current values of X_τ and Y_τ ($\tau \le t$). The role of such transformations will be elaborated further below.

The statistic S_g is an integer-valued random variable assuming values between 0 and n. Under the quite general conditions described by (1) and (2), the following proposition establishes the exact distribution of S_g when Y_t and g_t have no probability mass at zero. This result represents a considerable generalization of the main theorem of Campbell and Dufour (1991). The proofs of all the propositions given in this section can be found in the Appendix. We denote by Bi(n, p) the binomial distribution with number of trials n and probability of success p.

Proposition 1: Let $Y = (Y_1, \ldots, Y_n)'$ and $X = (X_0, \ldots, X_{n-1})'$ be two $n \times 1$ random vectors which satisfy assumptions (1) and (2). Suppose further that $P[Y_t = 0] = 0$, for $t = 1, \ldots, n$, and let $g_t = g_t(I_t)$, $t = 0, \ldots, n-1$, be a sequence of measurable functions of I_t such that $P[g_t = 0] = 0$ for $t = 0, \ldots, n-1$. Then the statistic S_g defined by (7) follows a Bi(n, 0.5) distribution, i.e. $P[S_g = x] = C_n^x (1/2)^n$ for $x = 0, 1, \ldots, n$, where $C_n^x = n!/[x!(n-x)!]$.

This distributional result obviously also holds for S_0. It must be stressed that S_0 and, more generally, S_g have well-known distributions under very general conditions. The assumption $P[Y_t = 0] = P[g_t = 0] = 0$ simply means that the variables Y_t and g_t have no mass at zero, which of course will hold when they have continuous distributions. Otherwise, the nature of the distribution of each Y_t is left open; there are no assumptions concerning the existence of moments; heteroskedasticity of unknown form is permitted; the nature of the feedback mechanism between Y_t and current and future values of X_{t+s} ($s \ge 0$) is not specified. As long as Y_t has median 0 and is independent of the past, the sign statistics S_0 and S_g are all binomial with mean n/2 and variance n/4.

Under the further assumption that each Y_t has a continuous distribution symmetric around zero, i.e. under (3), it is natural to introduce ranks as well. In this paper, we consider two basic types of signed rank statistics:

$$W_g = \sum_{t=1}^{n} u(Y_t g_{t-1}) R_{1t}^{+} , \qquad (8)$$

$$SR_g = \sum_{t=1}^{n} u(Y_t g_{t-1}) R_{2t}^{+} , \qquad (9)$$

where R_{1t}^{+} in W_g is the rank of $|Z_{1t}| = |Y_t g_{t-1}|$, i.e. $R_{1t}^{+} = \sum_{j=1}^{n} u(|Z_{1t}| - |Z_{1j}|)$ the rank of $|Z_{1t}|$ when $|Z_{11}|$, ... , $|Z_{1n}|$ are put in ascending order, while R_{2t}^{+} in SR_g denotes the rank of $|Y_t|$ among $|Y_1|$, ... , $|Y_n|$. We also call W_0 and SR_0 the statistics obtained by taking $g_t = X_t$ in (8) and (9):

$$W_0 = \sum_{t=1}^{n} u(Y_t X_{t-1}) R_{1t}^{+}, \qquad (10)$$

$$SR_0 = \sum_{t=1}^{n} u(Y_t X_{t-1}) R_{2t}^{+}. \qquad (11)$$

The statistics W_0 and W_g defined above are standard signed rank analogues of the statistics S_0 and S_g: the statistics are computed by weighting the sign of each positive product $Y_t X_{t-1}$ (or $Y_t g_{t-1}$) by the rank of its absolute value. The possibility of feedback makes it impossible to establish in general that W_0 and W_g are distributed as a Wilcoxon signed rank variate, i.e. as $W = \sum_{t=1}^{n} tB_t$ where B_1, ... , B_n are independent random variables such that $P[B_t = 0] = P[B_t = 1] = 0.5$ for t = 1, ... , n (independent uniform Bernoulli variables on {0, 1}). A counter-example can be found in Campbell (1990). However, simulation studies indicate that W_0 and W_g reject at their nominal levels for the two specifications of (4) considered in this paper and, consequently, these statistics are included in the empirical studies of power in the next section. Without feedback, it is easy to establish the following proposition, which slightly extends a standard result of the theory of linear signed rank tests.

Proposition 2: Let $Y = (Y_1, \dots , Y_n)'$ and $X = (X_0, \dots , X_{n-1})'$ be independent $n\times 1$ random vectors such that (1) and (3) hold. Let $g_t = g_t(X)$, t = 0 ,... , n-1, be a sequence of measurable functions of the vector X such that $P[g_t = 0] = 0$. Then the statistic W_g defined in (8) is distributed like $W = \sum_{t=1}^{n} t B_t$, where $B_1, \dots, B_n$ are independent uniform Bernoulli variables on $\{0, 1\}$.

Note that g_t in Proposition 2 can be a function of all the variables $X_0, \dots, X_{n-1}$, but does not depend on Y. When $g_t = X_t$, the result applies to S_0. By contrast, exact distributional results can be established for SR_0 and SR_g without the additional assumption that the vectors Y and X are independent. In the definitions of these Wilcoxon-type statistics, the absolute ranks are defined with respect to $Y_1, \dots, Y_n$ which are mutually independent according to (1). It is this feature which is crucial in establishing the following proposition.

Proposition 3: Let $Y = (Y_1, \dots, Y_n)'$ and $X = (X_0, \dots, X_{n-1})'$ be two $n \times 1$ random vectors such that (1) and (3) hold. Let also $g_t = g_t(I_t)$, $t = 0, \dots, n-1$, be a sequence of measurable functions of $I_t = (X_0, \dots, X_t, Y_1, \dots, Y_t)'$ such that $P[g_t = 0] = 0$ for $t = 0, \dots, n-1$, let $|Y| = (|Y_1|, \dots, |Y_n|)'$, and define the sign variables $s_t = u(Y_t g_{t-1})$ for $t = 1, \dots, n$. Then the following two properties hold:

(a) the signs $s_1, \dots, s_n$ are mutually independent and, provided $|Y_t| \neq 0$ for $t = 1, \dots, n$,

$$P[s_t = 0 \mid |Y|] = P[s_t = 1 \mid |Y|] = 0.5, \text{ for } t = 1, \dots, n;$$

(b) the statistic SR_g defined by (9) is distributed like $W = \sum_{t=1}^{n} t B_t$, where $B_1, \dots, B_n$ are independent uniform Bernoulli variables on $\{0, 1\}$.

Again it is clear that the result of Proposition 3 also holds for SR_0 by taking $g_t = X_t$. For a general discussion of the variable W, see Lehmann (1975). The distribution of W has been extensively tabulated [see, for example, Wilcoxon, Katti and Wilcox (1970)] and the normal approximation with $E(W) = n(n+1)/4$ and $Var(W) = n(n+1)(2n+1)/24$ works well even for small values of n. Proposition 3(a) also provides the basic property for establishing the distributions of more general linear signed rank statistics analogous to SR_g, i.e. statistics of the form $\sum_{t=1}^{n} u(Y_t g_{t-1})\, a_n(R_{2t}^{+})$ where $a_n(\cdot)$ is a "score" function. The distribution of such statistics, however, are not well tabulated and studying the choice of the score function is beyond the scope of the present paper. For further discussion of linear signed rank statistics, see Hájek and Šidák (1967), Dufour (1981), and Dufour and Hallin (1992a, 1992b).

Up to this point we have assumed that Y_t and X_t (or more generally g_t) had no probability mass at zero. In the following proposition, we relax totally or partially these assumptions.

Proposition 4: Let $Y = (Y_1, \dots, Y_n)'$ and $X = (X_0, \dots, X_{n-1})'$ be two $n\times 1$ random vectors such that (1) and (2) hold, let $g_t = g_t(I_t)$, $t = 0, \dots, n-1$, be a sequence of measurable functions of $I_t = (X_0, \dots, X_t, Y_1, \dots, Y_t)'$, and set $\bar{g}_t = g_t + \delta(g_t)$, where $\delta(x) = 1$ if $x = 0$, and $\delta(x) = 0$ if $x \neq 0$. Let also S_g and SR_g be defined as in (7) and (9), set

$$\bar{S}_g = \sum_{t=1}^{n} u(Y_t \bar{g}_{t-1}) , \quad \bar{SR}_g = \sum_{t=1}^{n} u(Y_t \bar{g}_{t-1}) R_{2t}^{*} ,$$

$\delta(Y) = [\delta(Y_1), \dots, \delta(Y_n)]'$, and let $n^* = n - \sum_{t=1}^{n} \delta(Y_t)$, the number of non-zero Y_t's. Then the following properties hold:

(a) $0 \le S_g \le \bar{S}_g$, $0 \le SR_g \le \bar{SR}_g$, and the conditional distribution of $\bar{S}_g$ given $\delta(Y)$ is $Bi(n^*, 0.5)$;

(b) if $P[g_t = 0] = 0$ for $t = 0, \dots, n-1$, we have $S_g = \bar{S}_g$ and $SR_g = \bar{SR}_g$ with probability 1, and the conditional distribution of $\bar{S}_g$ given $\delta(Y)$ is $Bi(n^*, 0.5)$;

(c) if assumption (3) holds, $\bar{SR}_g$ is distributed like $W = \sum_{t=1}^{n} t\, B_t$, where $B_1, \dots, B_n$ are independent uniform Bernoulli variables on $\{0, 1\}$.

Part (b) of Proposition 4 shows that, provided $g_0, \dots, g_{n-1}$ have no probability mass at zero, tests based on S_g can be performed conditionally on the non-zero Y_t's, i. e. after dropping the zero $Y_t g_{t-1}$ products. For the more general case where $g_0, \dots, g_{n-1}$ may have a mass at zero, the distribution of S_g appears difficult to determine. Proposition 4(a), however, shows that a simple alternative consists in replacing S_g by the closely related statistic $\bar{S}_g$, to which the result of part (b) applies. When $P[g_t = 0] = 0$ for $t = 0, \dots, n-1$, the two statistics coincide with probability 1. Similarly under assumption (3), we can use the statistic $\bar{SR}_g$ instead of SR_g; by Proposition 4(c), $\bar{SR}_g$ follows the usual Wilcoxon distribution. We do not study here the distribution of SR_g when $Y_1, \dots, Y_n$ have masses at zero, because in such a situation it is a more complicated linear signed rank statistic. For a further discussion of such statistics, see again Dufour and Hallin (1992a, 1992b).

Given that the exact distribution of both a sign statistic and a class of signed rank statistics is known under quite general conditions, the issue of power becomes crucial in determining the usefulness of these nonparametric tests. For example, in model (4) we have:

$$Y_t X_{t-1} = \beta X_{t-1}^2 + e_t X_{t-1} .$$

The sign and Wilcoxon tests based on $Z_t = Y_t X_{t-1}$ test in effect whether the random variable Z_t is centered at zero, more precisely whether Z_t has median zero. Under the null hypothesis ($\beta = 0$), the median is determined by the behavior of $e_t X_{t-1}$ which is zero under assumption (2). When $\beta \neq 0$, the median of Z_t is displaced from zero by the expression βX_{t-1}^2 and, as β gets larger, it is expected that the displacement is more severe and the test more powerful. This intuition is confirmed by the empirical studies in the following section.

There remain two points which are relevant to the application of these nonparametric tests. First, the assumption in (1) that the disturbances are mutually independent cannot be relaxed without compromising the distributional results established in this section. But the approach can be modified to deal with certain patterns of dependence such as MA(q) disturbances. For example, if Y_t represented a two-period forecast error, it is entirely consistent with the efficiency hypothesis associated with rational forecasting that Y_t behave as an MA(1) process. In this instance, the independence required to use the nonparametric procedures can be recaptured by splitting the sample into two with alternate points assigned to different subsamples. At least two simple testing strategies are then available. In the first, a nonparametric test with level $\alpha/2$ is applied to each subsample and the null hypothesis is rejected if one of the tests is significant; by Bonferroni inequality, this yields a test whose level does not exceed α. In the second strategy, a single test with level α is applied to a randomly chosen subsample; this procedure is not conservative but involves dropping half the sample. Both procedures can be adapted to deal with MA(q) disturbances or, more generally, to situations where Y_t is q-dependent.

Second, in many applications, it is more appropriate to consider the following variant of model (4):

$$Y_t = \beta(X_{t-1} - \mu_{t-1}) + e_t , \qquad (4)'$$

where μ_t is a centering parameter for X_t, such as the mean, the median or the trend of X_t; for example, if X_t is stationary, $\mu_t = \mu$ may represent the mean of X_t. If Y_t represent forecast errors which are centered at zero and X_t is a macroeconomic variable which assumes only positive values, it is clear that the nonparametric statistics S_0, W_0 and SR_0 introduced above will have no power whatever the value of β. In this context, the rejection of the null is associated with comovements of Y_t around zero and X_{t-1} around its mean. However, as the proofs of Propositions 1, 2 and 4 reveal, μ_t should be estimated using only information available at time t if the exact distribution of the nonparametric statistic is to be preserved. The functions $g_t(I_t)$ then represent any such estimation attempt based on partial information; various ways of centering the X_t variable are considered in the application presented in Section 4. For given functions $g_t(I_t)$, the sign statistic is defined in (7) and the signed rank statistics W_g and SR_g are given in equations (8) and (9).

3. A simulation study of two examples

Two specifications of model (4) are now introduced to contrast the behavior of nonparametric statistics with standard regression procedures. In Mankiw and Shapiro (1986), X_t is assumed to follow a stationary autoregressive process given by

$$X_t = \theta_0 + \theta_1 X_{t-1} + \varepsilon_t\,, \quad t = 1, \ldots, n, \tag{12}$$

where the ε_t are assumed to be mutually independent and each ε_t is independent of X_{t-j}, $j \geq 1$; the disturbances e_t and ε_t are also assumed to follow a bivariate normal distribution with correlation coefficient ρ. It follows that e_t in (4) is related to X_t through ε_t and hence to future X_{t+j} $(j > 1)$ by the autoregressive process. Since the disturbance vector $(e_1, \ldots, e_n)'$ is not independent of the explanatory variable vector $(X_0, \ldots, X_{n-1})'$, the t-test associated with the least squares estimate of β in model (4) can only be justified in large samples. Mankiw and Shapiro (1986) investigated the finite-sample properties of the usual t-test in a Monte Carlo study and found that it over-rejects the null hypothesis when ρ and θ_1 are close to one and asymptotic critical points are used. By contrast, since the exact finite-sample distribution of S_0 and SR_0 are given by Propositions 1 and 3, the reliability of the associated sign and Wilcoxon tests under the null is not an issue.

To investigate empirically the relative behavior of the nonparametric versus the asymptotic regression-based procedures in the MS specification, data were generated from model (4), with the X process specified as (12), by setting $\varepsilon_t = \rho e_t + w_t\sqrt{1-\rho^2}$, $\theta_0 = 0$ and $X_0 = w_0\big/\sqrt{1-\theta_1^2}$, where e_t and w_t are independent with the same distribution either N(0, 1), t(3), Cauchy or lognormal;

the asymmetric lognormal disturbances are centered at their median. Experiments illustrating the impact of heteroskedasticity involve modifications of standard normal disturbances as described in Table 3. Five tests are usually considered: the non-centered T-test, the centered t-test and the three nonparametric tests based on S_0, W_0 and SR_0, as defined in equations (6), (10) and (11). Asymptotic 5% critical values are used in applying the parametric tests; in situations where the t-test over-rejects, it is also applied using size-corrected critical points which are determined empirically. Because the sign and Wilcoxon statistics have discrete distributions, it is not possible (without randomization) to obtain tests whose level is precisely 5%; here, the levels of the sign test are 4.33%, 6.49%, 3.52%, 4.00% for sample sizes n = 25, 50, 100, 200 respectively; for n = 25, 50, the levels of the Wilcoxon tests are 4.82% and 4.94%, while the normal approximation is used for the larger sample sizes. Each experiment comprises 2000 replications.

Table 1 presents simulation results based on normal disturbances for the specification given by $\rho = 0.8$ and $\theta_1 = 0.99$ for various sample sizes. The results confirm the MS finding that asymptotic regression-based tests are unreliable when ρ and θ_1 are close to one: the t-test rejects at over twice its nominal level for sample sizes as large as 200. It is curious that the T-test appears to reject at its nominal level except perhaps when the sample size is small, and it should also be noted that the test based on W_0 rejects at its nominal level. But the striking message of Table 1 is that not much power relative to the T-test is lost in applying the sign test with small sample sizes or either of the Wilcoxon tests for any sample size. Overall, the t-test applied using corrected critical points is a poor last.

Two general types of heteroskedasticity, again restricted to $(\rho, \theta_1) = (0.8, 0.99)$, with sample size n = 100, are considered in Table 2. In the first, the variance of the underlying normal disturbances jumps from 1 to 16; the break occurs at one of three possible points (t = 25, 50, 75). In the second, the variability of the disturbances grows exponentially through the sample [i.e. e_t is a N(0, 1) variable multiplied by exp(t)]. Along with the four statistics considered throughout the study, we consider in this context an attempt due to MacKinnon and White (1985) to correct in a general manner for heteroskedasticity through the preliminary estimation of a heteroskedastic-consistent covariance matrix which is then used in a GLS estimation of the model coefficients. Consistent quasi-T and quasi-t statistics (WM and wm, respectively) can be computed and their performance is compared here with the other statistics.

The results of Table 2 are interesting indeed. Both types of heteroscedasticity compromise the reliability of the four parametric tests, including the MacKinnon-White procedures; again the W_0 test rejects at its nominal level in all the specifications considered. Accordingly, the power performance of the parametric tests should be assessed using the (empirically) correct critical points. It is apparent that in the context of break heteroskedasticity these corrected tests are outperformed by all the nonparametric tests. Under the extreme form of exponential heteroscedasticity, the parametric tests over-reject considerably and the corresponding size-corrected tests show no power whatsoever. By contrast the nonparametric tests behave quite well even under this extreme specification.

Table 3 presents results for homoskedastic non-normal disturbances which again show the nonparametric tests in a favorable light. The power of these tests improves when the disturbances are fat-tailed. With Cauchy disturbances, the sign and the Wilcoxon tests both outperform the parametric tests by a wide margin. Under lognormal disturbances, the sign test performs best; notice here that the signed rank tests appear to over-reject (asymmetric disturbances).

The second specification of model (4) identifies the X and Y processes to obtain the autoregressive model:

$$Y_t = \theta Y_{t-1} + e_t , \qquad t = 1, \dots , n, \tag{13}$$

where the vector $(e_1, \dots , e_n)'$ is independent of Y_0. We wish to test H_0: $\theta = 1$ (random walk without drift) against the one-sided alternative that the process is stationary ($\theta < 1$). Here, under H_0, the disturbances have permanent effect, and the t-statistic associated with the usual regression estimate of θ does not have the usual asymptotic normal distribution. To test $\theta = 1$, it will be convenient to consider the following equivalent form of (13):

$$Y_t - Y_{t-1} = \beta Y_{t-1} + e_t , \qquad t = 1, \dots , n, \tag{13$'$}$$

where $\beta = \theta - 1$. The null hypothesis is then equivalent to $\beta = 0$, with $\beta < 0$ under the alternative. Clearly, under the null hypothesis and provided Y_0 and $(e_1, \dots , e_n)'$ have continuous distributions, the assumptions of Propositions 1 and 3 are satisfied when Y_t is replaced by $Y_t - Y_{t-1}$ and X_t by Y_t. This suggests considering the following statistics for testing the random walk hypothesis:

$$S_{RW} = \sum_{t=1}^{n} u[(Y_t - Y_{t-1}) Y_{t-1}] , \qquad SR_{RW} = \sum_{t=1}^{n} u[(Y_t - Y_{t-1}) Y_{t-1}] R_{1t}^{+} , \tag{14}$$

where R_{t}^{*} is the rank of $|Y_t - Y_{t-1}|$ among $|Y_\tau - Y_{\tau-1}|$, $\tau = 1, \ldots, n$. The critical regions against the one-sided alternative of stationarity have the form $S_{RW} < c_1(\alpha)$ and $SR_{RW} < c_2(\alpha)$, where the critical values are determined by the distributions given in Propositions 1 and 3 respectively. As in the MS specification, we also consider a second Wilcoxon statistic based on the ranks R_{t}^{*} associated with $|(Y_t - Y_{t-1})Y_{t-1}|$:

$$W_{RW} = \sum_{t=1}^{n} u[(Y_t - Y_{t-1})Y_{t-1}] R_{t}^{*} \,. \tag{15}$$

On the assumption that the e_t are i.i.d. normal and that $Y_0 = 0$, the appropriate parametric tests to consider in this context are based on $n(\bar{\theta} - 1)$ [the P-test in Tables 4, 5 and 6] and the T-statistic both defined using the OLS estimate of θ in (13). Since these statistics are sensitive to the value of the point of departure, it is usual practice to consider tests based on $n(\hat{\theta} - 1)$ [the p-test] and the t-statistic both defined using $\hat{\theta}$, the OLS estimate of θ in the presence of an intercept term. The correct critical points for the various parametric tests have been determined by simulation; see Fuller (1976, pp. 371, 373) for the relevant tables. It should be noted that the theoretical results of the previous section establish the robustness of S_{RW} and SR_{RW} to the point of departure Y_0.

To assess the relative merits of the seven parametric and nonparametric tests of the random walk null, we follow the same pattern of Monte Carlo simulation used in the analysis of the MS specification. The results are presented in Tables 4, 5 and 6. For the experiments with normal and heteroskedastic errors, Y_0 was assumed to be standard normal under the null and, under the alternative, to be drawn from a normal distribution with the appropriate variance determined by the alternative. For the analysis with non-normal errors, Y_0 is taken to be zero under both the null and the alternative; for such cases, $n+1$ observations were generated using model (13), and the summations in (14) and (15) run from $t = 2$ to $t = n+1$ (because the first sign variable is always zero).

The power of the nonparametric statistics as revealed in Table 4 is striking. S_{RW} uniformly outperforms both the centered parametric tests which are usually applied in the context of independent homoskedastic normal disturbances. For larger samples the signed rank statistic with known distribution under the null displays more power than the sign test; there is some indication that the other Wilcoxon statistic rejects somewhat too much than its nominal level. In the presence of heteroskedastic disturbances (Table 5), the parametric tests perform irregularly. For example, when the variance suddenly jumps at a point in the sample, the t-test may either be conservative if the break point is early or somewhat liberal if the point occurs later in the sample. By contrast, both S_{RW} and SR_{RW} are both reliable and show similar powers, both superior to that of the centered parametric tests.

As in the MS specification, the nonparametric statistics display considerable power in the context of fat-tailed distributions, outperforming by a considerable margin the parametric alternatives (Table 6). With asymmetric lognormal disturbances, both signed rank statistics appear highly conservative; again the sign test exhibits remarkable power.

4. An application

B. Friedman (1980) studied interest rate expectations based on survey data published by The Goldsmith-Nagan Bond and Money Market Letter, a publication with a wide circulation among money market professionals. Late in the concluding month of each quarter, a selected group of its subscribers were asked to forecast the values of ten interest rates on the last business day of the two following quarters. The means of the different forecasts were subsequently published along with the names of the participants in the survey. In his study based on data from 1969 to 1977, Friedman focused on six rates of the most highly traded assets; here we consider three: U.S. Treasury Bills (3-month), Utility Bonds and Municipal Bonds. We follow Friedman in considering one aspect of the rationality hypothesis: whether the forecasters made efficient use of readily available information concerning, to use his examples, the unemployment rate, the growth rates of the consumer price index (CPI), industrial production and M1, and the federal deficit (in levels). Our goal in this section is to illustrate the details of the nonparametric approach and to compare the results with those of the more standard regression-based approach used by Friedman. A more thorough analysis of a nonparametric methodology to assess the adequacy or rationality of federal budget forecasts can be found in Campbell and Ghysels (1992).

The nonparametric statistics considered in this section have the form:

$$S_j^c = \sum_{t=1}^{n} u[(r_t - r_t^e)X_{t-j}^c] , \quad SR_j^c = \sum_{t=1}^{n} u[(r_t - r_t^e)X_{t-j}^c]\, R_{1t}^{*} , \quad W_j^c = \sum_{t=1}^{n} u[(r_t - r_t^e)X_{t-j}^c]\, R_{2t}^{*} , \tag{16}$$

for $j = 1, \dots, 4$. Here r_t^e is the forecast of r_t determined in the previous period. X_t^c denotes a centered value of X_t where the centering (or detrending) is based only on information available at time t; see the end of Section 2 for a discussion of the motivation for using such variables. In this application, we use one of two general centering methods: (1) the distance relative to a cumulative moving average of the mean, and (2) the distance from a recursively estimated linear time trend. Specifically to obtain X_t^c, the unemployment rate and the growth rates of the CPI, industrial production and M1 were centered using the first method. The fifth macroeconomic variable considered by Friedman was the Federal Deficit (in levels) and, given its evident non-stationarity, it is more appropriate to center this variable using the second method. All the variables are taken from the Main Economic Indicators of the OECD data base for a sample beginning in 1962. Each of the nonparametric statistics is then computed for $j = 1, \dots, 4$; p-values (for two-sided tests) are reported in Table 7. The results of a joint nonparametric test are also reported: here the null of efficiency is rejected if the smallest p-value (among $j = 1, \dots, 4$) is less than 0.0125. This procedure yields a test whose level does not exceed 0.05. Table 7 presents analogous regression-based results: we consider the t-statistic calculated from a one-variable regression (with constant) corresponding to the nonparametric tests introduced in (16) for each $j = 1, \dots, 4$ [i. e., the equation $(r_t - r_t^e) = \alpha + \beta X_{t-j}^c + w_t$], as well as the results of the appropriate F-test in a linear regression with four lags [i. e., the F-test of the hypothesis $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ in the regression $(r_t - r_t^e) = \alpha + \sum_{j=1}^{4} \beta_j X_{t-j}^c + e_t$].

Table 7 indicates there is core agreement between the parametric and nonparametric approaches. In sixteen of the twenty single-variable cases considered, neither the nonparametric nor parametric tests of the efficiency of the Treasury Bills forecasts is significant and, in the other four cases, the evidence is mixed. The joint efficiency tests are not significant for all five macroeconomic variables under both approaches. In sum, there is little evidence that the information contained in the five variables is not efficiently used in the Treasury Bills forecasts. By contrast, the interest rate forecasts for Utility Bonds do not appear to be as efficient: both the nonparametric and parametric results reject the efficiency hypothesis regarding specified lags of the Federal Deficit, as well as the joint efficiency hypothesis for this variable. However, there is interesting divergence between the two approaches applied to the Utility Bonds forecasts, which are found to be inefficient with regard to the information contained in the Unemployment Rate only by the parametric tests and inefficient with regard to M1 only by the nonparametric tests. Such divergence is somewhat less dramatic in the analysis of the efficiency of the Municipal Bonds forecasts which are found to be inefficient according to the parametric and nonparametric approaches with regard to information contained in both Industrial Production and the Federal Deficit. It is noteworthy that in contrast to the nonparametric results the t-test is significant for two of the lags of the Unemployment Rate in the single-equation efficiency tests and in one lag of the CPI. These may represent examples of spurious rejection as underscored by Mankiw and Shapiro (1986). Whatever the ultimate interpretation of these results, the important point is that the nonparametric results are more credible than the regression-based alternatives.

5. Concluding remarks

It is a testable implication of expectations models which imply that some observed variable is a rational forecast of another unobserved variable that the forecast error is independent of information available to the forecaster. Generally, this information is not strictly exogenous, and the issue of finite-sample bias associated with the usual regression procedures arises. The sign and signed rank procedures introduced in this paper are exact in such situations and are robust to problems of heteroskedasticity and non-normality. Moreover, as revealed in two simulation studies, the power of the nonparametric tests can be considerably superior to that of the parametric t-tests, particularly in the presence of heteroskedasticity or non-normal disturbances.

The general feedback model considered in this paper does not contain an intercept term. In the presence of such a term α, the methods used in this paper could readily be modified if α were known. Various methods of dealing with such situations are discussed in Campbell (1990). In particular, it is possible to adapt the procedures of Dufour (1990) to obtain exact nonparametric tests in the presence of the unknown nuisance parameter α. These results will be presented in a forthcoming paper.

Appendix

Proof of Proposition 1: Let $s_t = u(Y_t g_{t-1})$ and consider the characteristic function of S_g :

$$\phi_g(\tau) = E[\exp(i\tau S_g)] = E\left[\prod_{t=1}^{n} \exp(i\tau s_t)\right],$$

where $\tau \in \mathbf{R}$ and $i = \sqrt{-1}$. Conditional on the vector $I_{n-1} = (X_0, X_1, \ldots, X_{n-1}, Y_1, \ldots, Y_{n-1})'$, the variables $s_1, \ldots, s_{n-1}, g_{n-1}$ are fixed. We can thus write

$$\phi_g(\tau) = E\left\{\prod_{t=1}^{n-1} \exp(i\tau s_t) E\left[\exp(i\tau s_n) \mid I_{n-1}\right]\right\}.$$

When computing $E\left[\exp(i\tau s_n) \mid I_{n-1}\right]$, we can assume without loss of generality that $g_{n-1} \neq 0$ (an event with probability 1). Then, from (1), (2) and the assumption that Y_n has no probability mass at 0, we have $P[s_n = 0 \mid I_{n-1}] = P[s_n = 1 \mid I_{n-1}] = 0.5$ almost everywhere. It follows that

$$\phi_g(\tau) = (0.5)[1 + \exp(i\tau)]E\left[\prod_{t=1}^{n-1} \exp(i\tau s_t)\right].$$

Applying the same argument to $E\left[\prod_{t=1}^{n-j} \exp(i\tau s_t)\right]$ for $j = 1, \ldots, n-1$, we find

$$\phi_g(\tau) = \{(0.5)[1 + \exp(i\tau)]\}^n,$$

which is the charateristic function of the binomial distribution with number of trials n and probability of success 0.5. Thus S_g follows a Bi(n, 0.5) distribution. □

Proof of Proposition 2: Let $Z_{1t} = Y_t g_{t-1}$, $t = 1, \ldots, n$. Without loss of generality, we can only consider the case where $g_t \neq 0$ for $t = 1, \ldots, n$ (an event with probability 1). Conditional on $|g| = (|g_0|, \ldots, |g_{n-1}|)'$, the variables Z_{1t}, $t = 1, \ldots, n$, are mutually independent with $P[Z_{1t} > 0 \mid |g|] = P[Z_{1t} < 0 \mid |g|] = 0.5$; further, the rank vector $R_1^+ = (R_{11}^+, \ldots, R_{1n}^+)'$, which is a function of $|g|$, is a fixed permutation of the integers 1, 2, ... , n. Conditional on $|g|$, W_g is thus distributed like $W = \sum_{t=1}^{n} tB_t$. Since the distribution does not depend on $|g|$, the result also holds unconditionally. □

Proof of Proposition 3: (a) Let $z = (z_1, \dots, z_n)' \in \mathbf{R}^n$ and $\bar{S}_t = \sum_{\tau=1}^{t} z_\tau s_\tau$ for $t = 1, \dots, n$. The conditional charateristic function of the random vector $s = (s_1, \dots, s_n)'$ given $|Y|$ can be written:

$$\phi_s(z) = E\{\exp(iz's) \mid |Y|\} = E\{\exp(i\bar{S}_n) \mid |Y|\} = E\{\exp[i(\bar{S}_{n-1} + z_n s_n)] \mid |Y|\}$$
$$= E\{\exp(i\bar{S}_{n-1}) E[\exp(iz_n s_n) \mid I_{n-1}, |Y_n|] \mid |Y|\} .$$

When computing $E[\exp(iz_n s_n) \mid I_{n-1}, |Y|]$, we can assume that $g_{n-1} \neq 0$ (an event with probability 1). Further, by (1), (3) and the assumption that Y_n has no mass at zero,

$$E[\exp(iz_n s_n) \mid I_{n-1}, |Y_n|] = E[\exp(iz_n s_n) \mid I_{n-1}] = (0.5)[1 + \exp(iz_n)] ,$$

so that

$$\phi_s(z) = (0.5)[1 + \exp(iz_n)]\, E\{\exp(i\bar{S}_{n-1}) \mid |Y|\} .$$

Applying the same argument to $E\{\exp(i\bar{S}_{n-j}) \mid |Y|\}$ for $j = 1, \dots, n-1$, we find

$$\phi_s(z) = (0.5)^n \prod_{t=1}^{n} [1 + \exp(iz_t)] ,$$

which is the characteristic function we obtain when $s_1, \dots, s_n$ are mutually independent with uniform Bernoulli distributions over $\{0, 1\}$. Thus $s_1, \dots, s_n$ are mutually independent conditional on $|Y|$, with $P[s_t = 0 \mid |Y|] = P[s_t = 1 \mid |Y|] = 0.5$, for $t = 1, \dots, n$.

(b) With probability 1, we have $|Y_t| \neq 0$ for $t = 1, \dots, n$. Conditional on $|Y|$ such that $Y_t \neq 0$ for $t = 1, \dots, n$, the rank vector $(R_{21}^{+}, \dots, R_{2n}^{+})'$ is a fixed permutation of the integers $(1, \dots, n)'$. Hence, using part (a) of the proposition, SR_g is distributed like $W = \sum_{t=1}^{n} tB_t$ conditional on $|Y|$. Since this distribution does not depend on $|Y|$, this result also holds unconditionally. □

Proof of Proposition 4: It will be convenient to prove (b) first.

(b) Since g_t and g_t' differ only when $g_t = 0$ (an event with probablity zero by assumption), it is clear that $S_g = \bar{S}_g$ and $SR_g = \bar{SR}_g$ with probablity 1. By assumption (2), we have

$$P[Y_t > 0 \mid \delta(Y_t)] = P[Y_t < 0 \mid \delta(Y_t)] = 0 \text{ , if } \delta(Y_t) = 1$$
$$= 0.5\text{, otherwise,}$$

for $t = 1, \dots, n$. Set $p_t = P[Y_t < 0 \mid \delta(Y_t)]$, $t = 1, \dots, n$. Then, by assumption (1), we have when $g_{t-1} \neq 0$ (an event with probability 1):

$$p_t = P[g_{t-1}Y_t > 0 \mid I_{t-1}, \delta(Y)] = P[g_{t-1}Y_t < 0 \mid I_{t-1}, \delta(Y)]$$

for $t = 1, \ldots, n$. Consider now the charateristic function of S_g conditional on $\delta(Y)$: for $z \in \mathbb{R}$,

$$\begin{aligned}\phi_g[z \mid \delta(Y)] &= E[\exp(izs_g) \mid \delta(Y)] \\ &= E\left\{\prod_{t=1}^{n-1} \exp(izs_t) E[\exp(izs_n) \mid I_{n-1}, \delta(Y)] \,\middle|\, \delta(Y)\right\} \\ &= [(1 - p_n) + p_n \exp(iz)] E\left\{\prod_{t=1}^{n-1} \exp(izs_t) \,\middle|\, \delta(Y)\right\},\end{aligned}$$

where $i = \sqrt{-1}$, hence

$$\begin{aligned}\phi_g[z \mid \delta(Y)] &= \prod_{t=1}^{n} [(1 - p_t) + p_t \exp(iz)] \\ &= \{(0.5)[1 + \exp(iz)]\}^{n^*},\end{aligned}$$

where n^* is the number of non-zero Y_t's. Since this is the characteristic function of a $Bi(n^*, 0.5)$ variable, we can conclude that the conditional distribution of S_g given $\delta(Y)$ is $Bi(n^*, 0.5)$.

(a) Since $\bar{g}_t = g_t$ when $g_t \neq 0$ and $\bar{g}_t = 1$ when $g_t = 0$, we have $u(Y_t g_{t-1}) \leq u(Y_t \bar{g}_{t-1})$ for all t, hence

$$0 \leq S_g = \sum_{t=1}^{n} u(Y_t g_{t-1}) \leq \sum_{t=1}^{n} u(Y_t \bar{g}_{t-1}) = \bar{S}_g,$$

$$0 \leq SR_g = \sum_{t=1}^{n} u(Y_t g_{t-1}) R_{2t}^{+} \leq \sum_{t=1}^{n} u(Y_t \bar{g}_{t-1}) R_{2t}^{+} = \bar{SR}_g .$$

Further $\bar{S}_g$ satisfies all the assumptions of part (a), so that its conditional distribution given $\delta(Y)$ is $Bi(n^*, 0.5)$.

(c) Since $g_t' \neq 0$ for all t, the distribution of $\bar{SR}_g$ follows from Proposition 3(b). □

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Table 1
Mankiw-Shapiro Model: Normal Disturbances[a]
$\rho = 0.8$, $\theta_1 = 0.99$
Various Sample Sizes

β_1	t-test		T-test	S_0	SR_0	W_0
	Asymptotic	Size-Corrected[b]				
n = 25						
0.00	19.6	5.0	7.1	3.8	5.3	4.6
0.04	11.8	3.0	25.3	19.3	26.1	24.6
0.07	9.5	2.2	43.3	36.0	43.4	41.5
n = 50						
0.00	18.7	5.0	6.2	3.9	4.4	4.9
0.02	12.3	3.1	16.7	11.9	18.8	17.2
0.07	11.4	4.1	57.8	48.1	57.3	57.1
n = 100						
0.00	14.7	5.0	5.0	4.2	4.4	4.7
0.02	7.8	2.7	25.4	18.1	26.2	25.5
0.04	12.4	5.6	52.1	40.9	51.2	50.9
n = 200						
0.00	11.9	5.0	5.5	4.3	4.9	4.7
0.01	7.1	3.2	16.2	12.1	17.9	16.6
0.02	11.7	6.1	39.1	27.1	37.1	37.0

[a] Entries represent percentage rejections. The statistics S_0, W_0 and SR_0 are defined in equations (6), (10) and (11).
[b] Empirical critical points are used in power calculations. For $\beta = 0$, the rejection frequency for the size-corrected t-test is 5.0% by construction.

Table 2
Mankiw-Shapiro Model: Heteroskedastic Disturbances[a]
$\rho = 0.8$, $\theta_1 = 0.99$, $n = 100$

β_1	t-test	T-test	wm-test	WM-test	S_0	SR_0	W_0
Break at t = 25[b]							
0.00	12.5	7.8	11.2	6.0	3.7	5.4	5.3
0.04	13.4 6.5	37.0 32.7	12.5 6.1	32.4 30.7	28.2	36.3	35.5
0.07	36.2 25.0	63.0 58.9	35.1 24.7	59.7 58.3	54.6	60.5	62.0
Break at t = 50							
0.00	15.2	10.4	8.8	5.7	4.0	4.8	4.5
0.04	20.1 8.1	35.2 28.1	13.7 9.3	26.5 24.8	31.7	37.1	36.3
0.07	38.3 24.6	59.3 51.4	30.6 25.6	52.1 49.7	55.9	61.7	63.4
Break at t = 75							
0.00	24.7	13.9	8.9	5.8	4.3	4.9	4.8
0.04	25.8 7.8	35.7 21.7	9.3 6.2	21.2 19.8	34.5	40.9	41.9
0.07	36.1 18.2	59.3 44.0	20.2 15.0	46.3 44.4	59.4	66.4	67.5
Exponential							
0.00	89.2	89.4	12.3	12.2	3.2	4.4	4.5
0.50	89.1 6.0	89.2 6.1	12.9 5.9	12.9 5.6	19.6	18.7	18.5
0.70	89.2 6.3	89.5 6.4	13.6 5.8	13.5 5.7	35.7	32.4	32.1

[a] Entries represent percentage rejections; empirical critical points are used in the power calculations for the second entry in a cell. The statistics wm and WM are defined in the text.
[b] In the Break model, the variance of the disturbances jumps from 1 to 16 at the indicated point; in the exponential model, the variance grows exponentially with time [i. e., e_t is a N(0, 1) variable multiplied by exp(t)].

Table 3
Mankiw-Shapiro Model: Non-normal Disturbances*
$\rho = 0.8$, $\theta_1 = 0.99$, $n = 100$

β_1	t-test Asymptotic	t-test Size-Corrected	T-test	S_0	SR_0	W_0
t(3) Distribution						
0.00	15.7	5.0	5.5	3.4	4.4	4.9
0.02	8.7	2.9	24.5	27.9	33.4	35.7
0.07	35.8	24.6	76.6	75.3	80.4	83.6
Cauchy Distribution						
0.00	14.2	5.0	5.4	3.7	5.7	5.5
0.005	15.8	7.0	11.3	51.5	49.7	56.1
0.02	20.8	12.9	31.8	88.5	87.9	91.4
Lognormal Distribution						
0.00	13.9	5.0	70.1	3.3	27.4	22.6
0.02	10.1	4.3	44.0	47.9	42.6	44.6
0.04	17.8	10.7	46.6	76.7	71.5	76.1

* Entries represent percentage rejections.

Table 4
Random Walk Without Drift: Normal Disturbances[a]

θ		P-test	T-test	p-test	t-test	S_{RW}	SR_{RW}	W_{RW}
	n = 50							
1.0		5.5	5.7	4.1	5.1	6.7	6.1	7.3
.97		7.0	12.1	7.0	6.6	11.3	10.6	13.9
.95		12.4	18.0	9.5	7.9	15.8	14.6	20.0
	n = 100							
1.0		5.2	5.3	5.0	5.3	4.7	6.1	6.9
.97		15.4	21.3	10.2	8.2	14.5	16.9	23.5
.95		30.6	36.9	16.7	12.7	21.0	28.1	37.5
	n = 250							
1.0		5.3	5.4	4.6	4.7	4.6	5.1	6.4
.99		12.0	18.0	9.1	7.1	11.5	16.1	20.5
.98		30.6	38.0	18.1	12.7	20.3	27.8	37.0

[a] Entries represent percentage rejections. The statistics S_{RW}, SR_{RW} and W_{RW} are defined in equations (14) and (15). The critical values for the four parametric tests can be found in Tables 8.5.1 and 8.5.2 in Fuller (1976).

Table 5
Random Walk Without Drift: Heteroskedastic Disturbances[a]
n = 100

θ	P-test	T-test	p-test	t-test	S_{RW}	SR_{RW}	W_{RW}
Break at t = 25							
1.0	8.6	8.1	4.5	2.3	4.3	5.2	6.7
.97	21.0	21.4	10.7	6.0	13.8	14.1	18.0
	14.0	14.0	11.7	12.1			
.95	35.3	35.9	18.0	10.4	19.3	23.2	29.6
	24.2	24.8	19.1	20.1			
Break at t = 50							
1.0	11.4	10.5	7.1	4.6	5.5	5.8	7.7
.97	26.0	25.6	14.8	9.4	13.4	15.1	18.2
	11.0	12.0	11.2	10.2			
.95	40.7	39.7	22.5	14.1	19.6	22.9	28.3
	21.7	21.0	17.1	16.1			
Break at t = 75							
1.0	13.6	12.4	10.5	6.6	4.5	4.8	6.5
.97	30.1	28.6	20.9	13.3	13.6	15.5	19.5
	13.7	14.6	9.8	10.0			
.95	46.1	42.8	29.5	19.8	19.4	23.5	28.6
	23.6	24.1	15.9	14.7			
Exponential							
1.0	54.9	51.2	53.9	48.4	4.9	4.8	5.0
.8	54.3	50.9	53.5	48.2	11.1	10.8	11.0
	5.7	5.6	5.7	5.6			
.7	55.7	52.0	54.9	49.8	16.4	15.0	15.2
	5.7	6.0	5.6	6.1			

[a] Entries represent percentage rejections; empirical critical points are used in the power calculations for the second entry in a cell. In the Break model, the variance of the disturbances jumps from 1 to 16 at the indicated point; the variance grows exponentially with time in the exponential model [e, is a N(0, 1) variable multiplied by exp(t)].

Table 6
Random Walk Without Drift: Non-normal Disturbances[a]
n = 100

θ	P-test	T-test	p-test	t-test	S_{RW}	SR_{RW}	W_{RW}
t(3) Disturbances							
1.0	4.2	4.1	4.8	5.7	4.1	5.2	5.8
.98	10.3	10.5	7.8	5.7	17.4	17.7	24.6
.97	15.3	15.9	9.5	6.5	24.3	24.5	36.2
Cauchy Disturbances							
1.0	3.2	3.1	5.7	7.6	4.8	5.6	5.9
.99	4.7	5.0	5.6	6.4	69.3	66.2	74.7
.98	6.7	7.2	5.7	4.8	85.0	84.7	92.2
Lognormal Disturbances							
1.0	0.0	0.0	0.0	0.1	4.8	0.0	0.0
.99	0.0	0.0	0.0	5.9	47.7	4.8	13.5
.98	0.0	0.0	2.0	9.0	78.2	24.4	50.2

[a] Entries represent percentage rejections. In the Break model, the variance of the disturbances jumps from 1 to 16 at the indicated point; the variance grows exponentially with time in the Exponential model.

Table 7

Goldsmith-Nagan Interest Rate Forecasts
Nonparametric Orthogonality Results: 1-Period Forecast Errors

	Lag	Treasury Bills				Utility Bonds				Municipal Bonds			
	joint	S_j	SR_j	W_j	t_j F-test	S_j	SR_j	W_j	t_j F-test	S_j	SR_j	W_j	t_j F-test
Unemployment Rate													
Mean[a]	j = 1	.099	.210	.123	.218	.585	.918	.681	.165	.362	.465	.285	.340
	j = 2	.200	.245	.175	.189	.856	.999	.607	.058	.585	.491	.149	.111
	j = 3	.362	.465	.267	.189	.999	.622	.666	.015*	.856	.629	.084	.032*
	j = 4	.585	.821	.459	.375	.856	.579	.869	.028*	.999	.727	.096	.023*
	Joint	.099	.210	.123	.400	.585	.579	.607	.031*	.362	.465	.084	.134
CPI Growth Rate													
Mean	j = 1	.362	.213	.258	.385	.585	.999	.636	.369	.362	.943	.758	.211
	j = 2	.999	.902	.934	.315	.099	.051	.044*	.030*	.585	.079	.074	.001*
	j = 3	.999	.766	.902	.829	.099	.031*	.053	.071	.585	.094	.139	.068
	j = 4	.585	.329	.365	.322	.999	.334	.537	.751	.999	.614	.696	.244
	Joint	.362	.213	.258	.129	.099	.031	.044	.108	.362	.079	.074	.031*
Industrial Production (Growth Rate)													
Mean	j = 1	.855	.781	.523	.986	.200	.061	.092	.149	.099	.006*	.015*	.104
	j = 2	.999	.829	.711	.298	.362	.067	.025*	.004*	.043*	.009*	.002*	.003*
	j = 3	.585	.355	.323	.133	.362	.853	.918	.818	.585	.267	.303	.736
	j = 4	.362	.304	.144	.024*	.043*	.232	.144	.207	.856	.837	.934	.619
	Joint	.362	.304	.144	.068	.043	.061	.025	.025*	.043	.006*	.002*	.048*
M1 Growth Rate													
Mean	j = 1	.855	.992	.967	.339	.585	.789	.696	.699	.099	.399	.113	.501
	j = 2	.099	.092	.038*	.141	.043*	.056	.005*	.074	.099	.136	.025*	.329
	j = 3	.999	.593	.773	.562	.362	.484	.355	.981	.999	.975	.934	.816
	j = 4	.200	.053	.113	.042*	.016*	.003*	.010*	.074	.200	.159	.241	.661
	Joint	.099	.053	.038	.107	.016	.003*	.010*	.224	.099	.136	.025	.843
Federal Deficit													
Regression	j = 1	.200	.241	.181	.369	.362	.294	.294	.215	.200	.217	.144	.430
	j = 2	.099	.249	.175	.098	.099	.036*	.056	.017*	.016*	.014*	.013*	.080
	j = 3	.043*	.060	.074	.129	.016*	.007*	.014*	.007*	.005*	.002*	.002*	.018*
	j = 4	.361	.478	.472	.674	.200	.031*	.048*	.076	.016*	.002*	.001*	.056
	Joint	.043	.060	.074	.120	.016	.007*	.048	.026*	.005*	.002*	.001*	.104

Notes: The sample covers 1969:4 to 1977:1 (n = 30). The statistics S_j, SR_j and W_j are given in equation (16) for j = 1 to 4. Exact p-values (two-sided tests) are calculated for the sign statistics; the normal approximation is used for SR_j and W_j; p-values less than or equal to 0.05 are starred. A joint nonparametric test is significant if the smallest p-value (among j = 1, ... , 4) is less than or equal to 0.0125. We also report the corresponding p-values of the t-statistics (t_j) for the explanatory variable in a regression (with intercept) of the forecast error on the indicated lag of the centered macroeconomic variable considered, as well as the p-value of the standard F-test for the joint significance of the explanatory variables associated with the appropriate regression (with intercept).

[a] The macroeconomic variables are centered recursively according to the indicated procedure: Mean corresponds to method (1) in the text; Regression, to method (2).

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