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ON PERIODIC TIME SERIES AND TESTING THE UNIT ROOT HYPOTHESIS

Eric Ghysels[1] and Alastair Hall[2]

1 Département de sciences économiques and Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal.

2 Department of Economics, North Carolina State University

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C.P. 6128, succursale A
Montréal (Québec)
H3C 3J7

Télécopieur (FAX): (514) 343-5831
Courrier électronique (E-Mail): econo@tornade.ERE.Umontreal.CA

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RÉSUMÉ

Nous proposons plusieurs tests de racines unitaires pour des structures périodiques. Les tests sont de types Wald et multiplicateur de Lagrange ainsi que du type Dickey-Fuller. Leur distribution asymptotique ne dépend que de la fréquence périodique. Une étude de Monte Carlo est présentée et démontre les avantages d'utiliser ces nouveaux tests par rapport aux tests usuels. Des données trimestrielles d'inventaires sont utilisées pour fins d'illustration empirique.

Mots clés : stationnarité, saisonnalité, tests de racines unitaires.

ABSTRACT

Periodic ARMA models have gained considerable interest in recent years to model seasonal time series. We derive a number of tests for an I(1) versus I(0) characterization for periodic structures. A LM and Wald test for the unit root hypothesis in periodic models are proposed. The nonstandard asymptotic distribution only depends on the periodicity of the model. Critical values for quarterly and monthly models are presented. A periodic Dickey-Fuller-type test is also proposed. The tests have better power properties than standard tests for the unit root hypothesis such as Dickey-Fuller tests which ignore the possibility of periodic parametric variations. Power and size properties are studied via simulations. The paper concludes with an empirical example involving quarterly finished goods inventory data.

Key words : stationarity, seasonality, unit root test.

1. INTRODUCTION

Many time series variables are observed at regular intervals throughout the year, and their dynamic properties reflect their seasonal frequency. Consequently, considerable interest has been focused on developing models which can capture this type of behavior. There have been two main approaches taken in the literature. The first, and most common, is the use of "seasonal" time series models; e.g., see Box and Jenkins (1976), chapter 9; Granger and Newbold (1986), chapter 3. In this approach, the lag structures of the autoregressive and moving average polynomials reflect the seasonal frequency, but the parameters are assumed constant over time. The second approach is to fit "periodic" time series models. In this approach, the parameters are a deterministic periodic function whose period is the seasonal frequency. Although originally proposed over three decades ago by Gladyshev (1961), periodic models have only attracted attention relatively recently.[1]

Most of the analysis of either seasonal or periodic time series models is within the ARMA framework. However, before an appropriate model can be estimated, it is important to characterize the nonstationary properties of the series. This is most often done by testing whether time series contain a unit root at either the zero and/or seasonal frequency against a suitable alternative. Nearly all of these tests have been developed in the context of seasonal time series.[2] Whereas the issue of unit root testing has received very little attention in the context of periodic time series models. Of course, one could apply the tests cited above which ignore the periodic structure of the time series. However, intuition suggests that this would at best lead to inefficiencies and at worst could result in bias.

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1 See, for example, Jones and Brelsford (1967), Pagano (1978), Tiao and Grupe (1980), Todd (1983), Anděl (1983), Cipra (1985), Vechia (1985), Anděl (1987), Osborn (1988), Osborn and Smith (1989), Hansen and Sargent (1990), Todd (1990), Lütkepohl (1991a,b) and Sakai (1991). Empirical applications involving economic time series appear in Osborn (1988), Osborn and Smith (1989), Franses (1991), Ghysels and Hall (1992), among others. A class of nonlinear periodic Markov switching regime models and their empirical evidence are also discussed in Ghysels (1991a, b, 1992).

2 See, for instance, Hasza and Fuller (1982), Dickey, Hasza and Fuller (1984), Hylleberg, Engle, Granger and Yoo (1990), Engle, Granger, Hylleberg and Lee (1993), Ghysels, Lee and Noh (1993), among others.

In this paper, we develop a number of tests which allow a researcher to determine whether a periodic time series model possesses a unit root. The approach is based on estimating autoregressive time series models, but the specifics of the test depend on the type of trend included in the model. We examine three cases : (1) when no deterministic terms are included; (2) when a periodic intercept is included and (3) when a periodic linear time trend is included. The tests are based on the Wald and Lagrange Multiplier principles, and we derive their limiting distributions for an arbitrary seasonal frequency, say, S. We tabulate the tail percentiles of these distributions for the cases where S equals 4 and 12, which are of most interest because they correspond to quarterly and monthly data. In some cases, one may be prepared to assume that the long-run behavior of the series is aperiodic and that the periodicity is confined to the short-run dynamics. For these situations, we propose a modification of the Augmented Dickey-Fuller statistic [Dickey and Fuller (1979)] which takes account of this periodic structure. We refer to this statistic as the "Periodic ADF" test, and show that it converges to the distribution tabulated by Dickey and Fuller.

The paper is outlined as follows. In section 2, we discuss the relationship between periodic time series models and time invariant ARMA models. This provides the motivation for our analysis. In section 3, we derive the limiting distribution of the Wald and Lagrange Multiplier tests. In section 4, we introduce the periodic ADF test and demonstrate that it converges to the Dickey-Fuller distribution. The results from a simulation study are reported in section 5, which indicates that our tests have good finite sample properties. We also describe the results from an empirical application to quarterly series on finished goods inventories in two industries. These examples illustrate the value of explicitly modeling any periodic structure. Finally, section 6 contains some concluding remarks. All proofs are relegated to a mathematical appendix.

2. PERIODIC STRUCTURES AND NONSTATIONARITY

Let us first focus on the simple periodic AR(p) model without intercept, namely :

$$y_t = \alpha_t y_{t-1} + \sum_{j=1}^{p} \theta_{tj} z_{t-j} + u_t \tag{2.1}$$

where $\alpha_t = \sum_{s=1}^{S} D_{st} \alpha_s$, $\theta_{tj} = \sum_{s=1}^{S} D_{st} \theta_{sj}$, $z_t = y_t - y_{t-1}$ while $D_{st} = 1$ if $t \bmod S = s$, $D_{st} = 0$ otherwise, and u_t is i.i.d. $N(0, \sigma^2)$.

Equation (2.1) is very similar to the standard Dickey-Fuller (henceforth denoted DF) setup, except that the autoregressive parameters may take different values in each of the S periods. The hypothesis of interest is whether in fact $\alpha_t = 1$ for all t. More formally, the null and alternative hypotheses are :

$$H_0 : \alpha_s = 1 \qquad s = 1, \ldots, S \tag{2.2}$$

$$H_A : \alpha_s \neq 1 \qquad \text{for at least some s} \tag{2.3}$$

If H_0 is true, then the process is said to be integrated of order 1, denoted I(1), and the first difference of the series has a stationary periodic autoregressive representation. A first important observation to make is that under the alternative H_A, the process may still satisfy the conditions for *periodic integration*. Indeed, testing for I(1) in periodic models is quite distinct from testing for periodic integration. The latter concerns the null hypothesis whether :

$$H_0^{PI} : \prod_{s=1}^{S} \alpha_s = 1 \tag{2.4}$$

against the alternative :

$$H_A^{PI} : \prod_{s=1}^{S} \alpha_s < 1 \tag{2.5}$$

Under this null hypothesis, annual differencing yields a covariance stationary representation. Clearly, a process which is both periodic and I(1) must also be periodically integrated, but the converse is not true. Therefore, if one rejects H_0, then one can investigate whether the process is periodically integrated using, for instance, the test recently proposed by Boswijk and Franses (1992).

Given the popularity of the ADF test, it is natural to wonder about its properties when the data are generated by an integrated but periodic process. Or, put differently : do we really need another test and why? To test the hypothesis of a zero frequency unit root in seasonal time series, one can apply the DF statistics under quite general circumstances. Seasonal time series processes may, for instance, be characterized by unit roots on the unit circle at some or all of the seasonal frequencies. Despite the presence of unit roots at frequencies other than at zero, Ghysels, Lee and Noh (1993) show that DF statistics can be used provided that some precaution is taken regarding the AR lag augmentation to compute the test statistics. Moreover, DF statistics can also be used when the seasonal process is *of the periodic type* as described by equation (2.1) or (2.2). To clarify this, we rely on some fundamental results regarding the relationship between periodic processes and time-invariant processes presented in Tiao and Grupe (1980), also elegantly presented and generalized by Hansen and Sargent (1990). In particular, let H_0 hold. Then, using the standard techniques of stacking $T = S \times N$ observations into N skip-sampled annual vectors of length S, one exhausts all possible parameter variations and obtains the time-invariant representation :

$$\underline{z}_n = \psi\, \underline{z}_{n-1} + \underline{u}_n \tag{2.6}$$

where $\underline{z}_n = (\underline{z}^{P\prime}_{(n-1)S+1}, \ldots , \underline{z}^{P\prime}_{nS})'$ with $\underline{z}^P_t = (z_t, \ldots , z_{t-p+1})'$ and $\underline{u}_\tau$ have the same structure, while :

$$\psi \equiv \begin{bmatrix} I & & 0 \\ -\psi_1 & \ddots & \\ 0 & -\psi_{S-1} & I \end{bmatrix}^{-1} \begin{bmatrix} 0 & \cdots & \psi_S \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \text{ and } \psi_s = \begin{bmatrix} \theta_{s1} & \cdots & \theta_{sp} \\ 1 & 0 & 0 \\ 0 & \ddots\ 1 & 0 \end{bmatrix}. \tag{2.7}$$

If we let Ω_u denote the covariance matrix of $\underline{u}_n$, then one can derive a time-invariant representation for the vector process $\{\underline{z}^P_t\}$ from (2.6) via :

$$S^P_z(\omega) = S^{-1}[I\, e^{-i\omega} I \ldots e^{-i\omega(S-1)}I]\,[I - \psi e^{-i\omega S}]\,\Omega_u[I - \psi e^{-i\omega S}]'\,[I\, e^{-i\omega} I \ldots e^{-i\omega(S-1)}I]' \tag{2.8}$$

by applying the so-called Tiao-Grupe formula [see Tiao and Grupe (1980) and Hansen and Sargent (1990)]. Consequently, the implied time-invariant representation for $\{z_t\}$, obtained by "unstacking" the p-dimensional vector $\underline{z}^P_t$, will be

$$\theta(L)z_t = \delta(L)\varepsilon_t \tag{2.9}$$

with $\theta(L) = \sum_{j=1}^{q} \theta_j L^j$ and $\delta(L) = \sum_{j=1}^{r} \delta_j L^j$ where $r \geq 1$ and $q \geq S$. However, in contrast to (2.1) under H_0, the time-invariant representation in (2.9) involves MA terms and possibly longer AR expansions as well. Similar arguments apply if the alternative holds, provided H_A^{IP} is satisfied; see Tiao and Grupe (1980). Therefore, one can test for a unit root in z_t using a DF statistic by allowing the order of the AR approximation to increase with the sample size [see Said and Dickey (1984)]. However, it is clear that we may experience a considerable loss in the finite sample power of the tests due to the long AR augmentations needed to whiten the innovation series. Intuition suggests that we can considerably reduce the complexity of a time-invariant parameterization of seasonal processes by explicitly modelling the periodic variation. This, in turn, is anticipated to yield a significant increase in the power of unit root tests.

Finally, it is important to note, at this point, that the tests introduced in the next section are *two-sided*, unlike the DF statistic for I(1) which is one-sided. Here again, we exploit the particular properties of periodic processes under H_A. Indeed, for any given s, the parameter variation in α_s may easily take on values greater than one and, in fact, most empirical evidence yields such values in combination, of course, with seasons having α_s less than one.

3. TESTING FOR A UNIT ROOT IN PERIODIC TIME SERIES

Consider again the process $\{y_t\}$ generated by (2.1) under H_0 in (2.2), i.e.,

$$y_t = y_{t-1} + \sum_{j=1}^{p} \theta_{tj} z_{t-j} + u_t \qquad t = 1, 2, \ldots, T \tag{3.1}$$

$$z_t = y_t - y_{t-1}$$

and assume the following :

Assumption 3.1 :

(i) $t = n(S - 1) + s$, $n = 1, 2, \ldots, N$; $s = 1, 2, \ldots, S$;

(ii) $\theta_{tj} = \sum_{s=1}^{S} D_{st}\, \theta_{sj}$, where D_{st} is the dummy variable which equals 1 if modS(t) = s and zero otherwise;

(iii) $\{u_t\}$ is a sequence of independent normal random variables with mean zero and variance ω^2.

Under these conditions, z_t follows a periodic autoregressive model of order p. For expositional brevity, we assume that the autoregressive order is the same for each period and our analysis is easily modified to allow this order to be period-dependent. It will be required to impose certain conditions on the autoregressive parameters, but these will be presented later. It is worth noting at the outset that the normality assumption is made for convenience and our analysis can be conducted under much weaker conditions [see Phillips and Durlauf (1986)].

We start by considering the case where inference about a unit root is based on the following regression model :

$$y_t = \alpha_t\, y_{t-1} + \sum_{j=1}^{p} \theta_{tj}\, z_{t-j} + u_t, \tag{3.2}$$

where $\alpha_t = \sum_{s=1}^{S} D_{st}\, \alpha_s$. Let

$$\Psi = (\alpha_1, \dots, \alpha_S, \underline{\theta}_1', \dots, \underline{\theta}_S', \omega_1^2, \dots, \omega_S^2),$$

where $\underline{\theta}_s' = (\theta_{s1}, \dots, \theta_{sp})$, s = 1, ... , S and $\hat{\Psi}$, Ψ denote the unrestricted and restricted maximum-likelihood estimators respectively implied by the normality assumption. The Wald test statistic for our null hypothesis is

$$W = [\hat{\alpha} - \iota]' \left\{ \left[\frac{\partial^2 LLF_T(\hat{\Psi})}{\partial\Psi\partial\Psi'} \right]^{-1} \right\}_{1:S,1:S}^{-1} [\hat{\alpha} - \iota], \tag{3.3}$$

where $\hat{\alpha}$ is the unrestricted MLE of $\alpha = [\alpha_1, \ldots, \alpha_S]'$, ι is a $(S \times 1)$ vector of ones, $LLF_T(\cdot)$ is the sample log-likelihood function and $\{\cdot\}_{1:S,1:S}$ denotes the submatrix consisting of the elements in both the first S rows and first S columns of the matrix in the braces. The Lagrange Multiplier statistic is

$$LM = \left[\frac{\partial LLF_T(\Psi)}{\partial \alpha}\right]' \left\{\left[\frac{\partial^2 LLF_T(\Psi)}{\partial \Psi \partial \Psi'}\right]^{-1}\right\}_{1:S,1:S} \left[\frac{\partial LLF_T(\Psi)}{\partial \alpha}\right]. \tag{3.4}$$

Let us assume that we have a sample of N years, each containing S observations. Hence, the sample size is $T = S \times N$ and the log likelihood function is consequently

$$LLF_T(\Psi) = c - \frac{1}{2}\sum_{t=1}^{T}\left\{\ln \omega^2 + \sum_{s=1}^{S} D_{st}\,(y_t - \alpha_s y_{t-1} - \sum_{j=1}^{p} \theta_{sj} z_{t-j})^2/\omega^2]\right\}, \tag{3.5}$$

where c is a constant. Therefore, the unrestricted MLE's are

$$[\hat{\alpha}_s, \hat{\theta}_s']' = \left[\sum_{t=1}^{T} D_{st}\, x_t x_t'\right]^{-1} \sum_{t=1}^{T} D_{st}\, x_t y_t; \tag{3.6}$$

$$\hat{\omega}^2 = \sum_{t=1}^{T} (y_t - \hat{\alpha}_s y_{t-1} - \sum_{j=1}^{p} \hat{\theta}_{sj} z_{t-j})^2/T \tag{3.7}$$

and the restricted MLE's are $\tilde{\alpha}_s = 1$,

$$\tilde{\theta}_s = \left[\sum_{t=1}^{T} D_{st}\, x_{1t} x_{1t}'\right]^{-1} \sum_{t=1}^{T} D_{st}\, x_{1t} z_t, \tag{3.8}$$

$$\tilde{\omega}^2 = \sum_{t=1}^{T} (z_t - \sum_{j=1}^{p} \tilde{\theta}_{sj} z_{t-j})^2/T \tag{3.9}$$

for $s = 1, \ldots, S$ where $x_t' = (y_{t-1}, x_{1t}')$ and $x_{1t}' = (z_{t-1}, \ldots, z_{t-p})$.

If we let Ψ_0 be the true value of Ψ, then, under both H_0 and H_A, it is easily shown that appropriately normalized Hessian $\partial^2 LLF_T(\Psi_0)/\partial\Psi\partial\Psi'$ converges in probability to a block diagonal matrix with the first block referring to the second derivative with respect to $(\alpha_1, \dots, \alpha_S, \theta_1', \dots, \theta_S')$ and the second block referring to the second derivative with respect to ω^2. Taking advantage of this fact, we consider the following versions of the Wald and Lagrange Multiplier statistics

$$W_{1S} = \sum_{s=1}^{S} (\hat{\alpha}_s - 1)^2/\hat{\omega}^2 \left\{ \left[\sum_{t=1}^{T} D_{st} x_t x_t' \right]^{-1} \right\}_{11} \tag{3.10}$$

$$LM_{1S} = \sum_{s=1}^{S} \left[\left[\sum_{t=1}^{T} D_{st} y_{t-1} \tilde{u}_t \right]^2 / \tilde{\omega}^2 \left\{ \left[\sum_{t=1}^{T} D_{st} x_t x_t' \right] \right\}_{11} \right] \tag{3.11}$$

where $\tilde{u}_t = z_t - \sum_{j=1}^{p} \tilde{\theta}_{tj} z_{t-j}$ and $\{\cdot\}_{11}$ is the 1-1 element of the matrix in curly brackets.

To derive the limiting distributions of these statistics under H_0, it is necessary to obtain the limiting distributions of such functions of the data as $N^{-1} \sum_{t=1}^{T} D_{st} y_{t-1} u_t$. This is most easily achieved by rewriting the periodic univariate equation in (3.1) as a constant parameter $S \times 1$ system. Define $Y_n = [y_{Sn}, y_{Sn-1}, \dots, y_{S(n-1)+1}]'$, $Z_n = [z_{Sn}, z_{Sn-1}, \dots, z_{S(n-1)+1}]'$, $U_n = [u_{Sn}, u_{Sn-1}, \dots, u_{S(n-1)+1}]'$ and P equal to the smallest integer greater than or equal to p/S. It is shown in the Appendix that (3.1) implies that Y_n is generated by a cointegrated system of rank S - 1. This representation is central to our analysis, and we assume it satisfies certain conditions which place restrictions on $\{\theta_{sj}\}$. These are detailed in Assumption A.2 which is also stated in the Appendix. Using Theorem 2.1 from Phillips and Durlauf (1986), it follows that

$$N^{-1/2} \sum_{i=1}^{[Nr]} U_i \Rightarrow \Omega B_S(r) \tag{3.12}$$

where $B_S(r)$ is a $S \times 1$ standard Brownian notion, $\Omega = \text{diag}(\omega, \omega, \dots, \omega)$ and "$\Rightarrow$" denotes weak convergence of the underlying probability measures. It is convenient to introduce the following notation :

$$G(r) = \sum_{i=1}^{S} B_{Si}(r) \qquad (3.13)$$

where $B_{Si}(r)$ is the i^{th} element of $B_S(r)$. Equations (A.3)-(A.4) in the Appendix, (3.12) and Lemma 3.1 from Phillips and Durlauf (1986) can be used to characterize the limiting distribution of W_{1S} and LM_{1S}. These distributions are given in the following theorem.

Theorem 3.1 : If the data are generated by (3.1), and Assumptions 3.1 and A.1 hold. Then : (i) $W_{1S} \Rightarrow \Psi_{1S}$; (ii) $LM_{1S} \Rightarrow \Psi_{1S}$; where $\Psi_{1S} = \sum_{j=1}^{S} [\int_0^1 G(r)dB_{Sj}(r)]^2 / \int_0^1 G(r)^2 dr$.

Proof : See Appendix.

The distribution in Theorem 3.1 depends only on S, and its percentiles are tabulated in Table 5.1 for the quarterly case and in Table 5.2 for the monthly one. Both tables appear in section 5.

We now consider the extension of our analysis to cover the case where an intercept or an intercept and a linear time trend are included in the regression model. Consider the case where either one of the following models is estimated

$$y_t = \alpha_t y_{t-1} + \mu_t + \sum_{j=1}^{p} \theta_{tj} z_{t-j} + u_t \qquad (3.14)$$

$$y_t = \alpha_t y_{t-1} + \mu_t + \beta_t(n - N/2) + \sum_{j=1}^{p} \theta_{tj} \tilde{z}_{t-j} + u_t, \qquad (3.15)$$

where $\mu_t = \sum_{s=1}^{S} D_{st} \mu_s$, $\beta_t = \sum_{s=1}^{S} D_{st} \beta_s$ and the remaining parameters and variables are the same as above. Before proceeding, it is worth commenting on the nature of the linear trend term. It is more convenient for our asymptotic analysis if we include the linear trend (n - N/2) rather than the more usual (t - T/2) trend. Both trends combined with the periodic intercepts are capable of capturing the same behavior; the only difference is in the interpretation of the parameters. To see this, note that if the trend in period s is $\nu_s + \gamma_s(t - T/2)$, then from Assumption 3.1(i), this can be written as

$$\nu_s + \gamma_s(t - T/2) = \nu_s + \gamma_s(Sn - j - SN/2) = \mu_s + \beta_s(n - N/2)$$

for $\mu_s = \nu_s - j\gamma_s$, $\beta_s = S\gamma_s$ and $j = S - s + 1$, $s = 1, 2, \ldots, S$.

Using identical arguments as before, it is easily shown that the unrestricted and restricted MLEs of the regression parameters are just the unrestricted and restricted least squares estimators from the appropriate model. Similarly, the unrestricted and restricted MLEs of ω^2 are the unrestricted and restricted residual sum of squares divided by T. For notational convenience, we again use $\hat{\phi}$ and $\tilde{\phi}$ to denote the unrestricted and restricted MLE, respectively, of any parameter ϕ, and use the context to specify the model from which the estimators are calculated. We also define $x_{2t} = (y_{t-1}, 1, x_{1t})'$ and $x_{3t} = (y_{t-1}, 1, n - N/2, x_{1t})'$.

Armed with these definitions, we can now present Wald and Lagrange Multiplier statistics for testing $H_0 : \alpha_s = 1$ for all s based on the regressions in (3.14) and (3.15). For the intercept model, equation (3.14), the Wald test statistic is

$$W_{2S} = \sum_{s=1}^{S} (\hat{\alpha}_s - 1)^2 / \hat{\omega}^2 \left\{ \left[\sum_{t=1}^{T} D_{st}(x_{2t}\, x_{2t}') \right]^{-1} \right\}_{1,1} \tag{3.16}$$

and the LM statistic is

$$LM_{2S} = \sum_{s=1}^{S} \left[\sum_{t=1}^{T} D_{st}\, (y_{t-1}\, \tilde{u}_t) \right]^2 / \tilde{\omega}^2 \left\{ \left[\sum_{t=1}^{T} D_{st}\, (x_{2t}\, x_{2t}') \right]^{-1} \right\}_{1,1} \tag{3.17}$$

where $\tilde{u}_t = z_t - \tilde{\mu}_s - \sum_{j=1}^{p} \tilde{\theta}_{sj}\, z_{t-j}$.

For the time trend model, equation (3.15), the Wald and Lagrange Multiplier tests are then :

$$W_{3S} = \sum_{s=1}^{S} (\hat{\alpha}_s - 1)^2 / \hat{\omega}^2 \left\{ \left[\sum_{t=1}^{T} D_{st}\, x_{3t}\, x_{3t}' \right]^{-1} \right\}_{1,1} \tag{3.18}$$

$$LM_{3S} = \sum_{s=1}^{S} \left[\sum_{t=1}^{T} D_{st}\, y_{t-1}\, \tilde{u}_t \right] / \tilde{\omega}^2 \left\{ \left[\sum_{t=1}^{T} D_{st}\, x_{3t}\, x_{3t}' \right]^{-1} \right\}_{1,1} \tag{3.19}$$

where $\tilde{u}_t = z_t - \tilde{\mu}_s - \tilde{\beta}_s(n - N/2) - \sum_{j=1}^{p} \tilde{\theta}_{tj}\, z_{t-j}$. The limiting distributions of these statistics are given in the next theorem.

Theorem 3.2 : If the data are generated by (3.1) and Assumptions 3.1 and A.1 hold, then (i) $W_{2S} \Rightarrow \Psi_{2S}$; (ii) $LM_{2S} \Rightarrow \Psi_{2S}$; (iii) $W_{3S} \Rightarrow \Psi_{3S}$; (iv) $LM_{3S} \Rightarrow \Psi_{3S}$; where

$$\Psi_{2S} = \sum_{j=1}^{S} \left[\int_0^1 G(r)\, d\, B_{Sj}(r) - \int_0^1 G(r)\, dr\, B_{Sj}(1) \right]^2 \left[\int_0^1 G(r)^2\, dr - \left[\int_0^1 G(r)\, dr \right]^2 \right]^{-1},$$

while $\Psi_{3S} = \sum_{j=1}^{S} A_j^2/D$ with

$$A = 6B_{Sj}(1) \int_0^1 r\, G(r)dr - 4B_{Sj}(1) \int_0^1 G(r)dr - 12 \left[B_{Sj}(1) - \int_0^1 B_{Sj}(r)dr \right] \times$$

$$\left[\int_0^1 r\, G(r)dr - (1/2) \int_0^1 G(r)dr \right] + \int_0^1 G(r)dB_{Sj}(r);$$

$$D = (1/12) \left[\int_0^1 G(r)^2 dr - 12 \left[\int_0^1 rG(r)dr \right]^2 + 12 \int_0^1 G(r)dr \int_0^1 rG(r)dr - 4 \left[\int_0^1 G(r)dr \right]^2 \right].$$

Proof : See Appendix.

These distributions again only depend on S, and their percentiles are presented in Tables 5.1 and 5.2 for the quarterly and monthly case respectively.

Our analysis has focused on the case where y_t contains no drift, but can easily be extended to allow for the presence of a drift term. Dickey and Fuller (1979) derived and tabulated the distribution of two tests for a unit root in the aperiodic analogue to (4.11) (i.e., with $\mu_s = \mu$, $\underline{\theta}_s = \underline{\theta}$, $\beta_s = \beta$) when y_t is generated by (3.1) with $\underline{\theta}_s' = \underline{\theta}'$. They demonstrate that their tests converge to the same distributions if y_t is generated by an ARIMA (p, 1, 0) process plus drift. One of these tests is the regression t-statistic for $\alpha = 1$. Now, LM_{3S} and W_{3S} are functions of regression t-statistics [see equation (A.26) in the Appendix]. Therefore, similar arguments to Dickey and Fuller's imply that LM_{3S} and W_{3S} converge to Ψ_{3S} if μ_t is added to the right-hand side of (3.1).

4. A PERIODIC ADF TEST

In section 2, it was observed that a periodic AR model possesses a constant parameter ARMA representation. Therefore, one could use the ADF statistic to test for a unit root in this class of series by using a constant AR approximation whose order increases at a certain rate with the sample size; see Said and Dickey (1984). Intuitively, one would anticipate that this test would exhibit low finite sample power because it ignores the periodic structure, and this is corroborated by the simulation evidence reported in the next section. One potential method of improving the ADF test's performance is to modify the regression equation from which it is based to allow the coefficients on the lagged first differences to be periodic. We refer to this new version as the "Periodic ADF test". In this section, we demonstrate that the Periodic ADF test converges to the appropriate Dickey-Fuller distribution. Our simulation results indicate that this modification to the ADF test can considerably improve its power. However, it is important to note that both the original and Periodic ADF tests are calculated under the assumption that coefficients on lagged y, the intercept and time trend are constant across periods. This restriction may be inappropriate and cause a loss in power relative to the Wald and LM statistics described in the previous section.

For simplicity, we focus our attention on the case where we estimate the model

$$y_t = \alpha y_{t-1} + \sum_{j=1}^{p} \theta_{tj} z_{t-j} + u_t \tag{4.1}$$

and discuss the extension of our analysis to the intercept and time trend models at the end of the section. Let $(\bar{\alpha}, \underline{\bar{\theta}}_1', \dots, \underline{\bar{\theta}}_S', \bar{\omega}_1^2, \dots, \bar{\omega}_S^2)$ be the maximum likelihood estimators of $(\alpha, \underline{\theta}_1', \dots, \underline{\theta}_S', \omega_1^2, \dots, \omega_S^2)$ and define $X_t' = [y_{t-1}, D_t' \otimes x_{t-1}']$ for $D_t = \text{diag}(D_{1t}, \dots, D_{St})$. It is easily shown that

$$[\bar{\alpha}, \bar{\theta}_1', \bar{\theta}_2', \ldots, \bar{\theta}_S']' = \left[\sum_{t=1}^{T} X_t X_t'\right]^{-1} \sum_{t=1}^{T} X_t y_t \tag{4.2}$$

Now, consider the statistic

$$\bar{\tau} = (\bar{\alpha} - 1) / \bar{\omega} \left\{ \left[\sum_{t=1}^{T} X_t X_t' \right]^{-1} \right\}_{11} \tag{4.3}$$

where $\bar{\omega}^2 = S^{-1} \sum_{s=1}^{S} \bar{\omega}_s^2 = T^{-1} \sum_{t=1}^{T} \bar{u}_t^2$ and $\bar{u}_t = y_t - \bar{\alpha}\, y_{t-1} - \sum_{t=1}^{p} \bar{\theta}_{tj}\, z_{t-j}$. Notice that $\bar{\tau}$ is the regression t-statistic for $\alpha = 1$ from the OLS estimation of (3.17) ignoring the periodicity in the variances. In the following theorem, proven in the Appendix, we present the limiting distribution of $\bar{\tau}$ under $H_0 : \alpha = 1$.

Theorem 4.1 : Under the conditions of Theorem 3.1, $\bar{\tau} \Rightarrow \hat{\tau}$ where $\hat{\tau}$ is the random variable tabulated in Fuller (1976), Table 8.52, p. 373.

Proof : See Appendix.

As noted above, our analysis is easily extended to the case where an intercept or an intercept and a time trend are included in the model. Suppose it is desired to base inference about a unit root on the maximum likelihood estimators of α in the following models :

$$y_t = \alpha\, y_{t-1} + \mu_t + \sum_{j=1}^{p} \theta_{tj}\, z_{t-j} + u_t, \tag{4.4}$$

$$y_t = \alpha\, y_{t-1} + \mu_t + \beta(t - T/2) + \sum_{j=1}^{p} \theta_{tj}\, z_{t-j} + u_t. \tag{4.5}$$

Define the Periodic ADF test (henceforth denoted P-ADF) to be the regression t-statistic for $\alpha = 1$. Using similar arguments to Theorem 4.1, it can be shown that the Periodic ADF tests based on (4.4) and (4.5) converge in distribution to $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$, respectively. The percentiles of these distributions are given in Fuller (1976), Table 8.5.2, p. 373.

5. SIMULATIONS OF SIZE AND POWER PROPERTIES

The critical values of the W_{iS} and LM_{iS} statistics are reported in Table 5.1 (for S = 4) and Table 5.2 (for S = 12), the former covering S = 4 and the latter S = 12. The percentiles were computed for six sample sizes, namely, 10, 20, 30 and 40 years of data, either quarterly or monthly. In all cases, 10,000 replications were used. Computations were done with MATLAB Version 3.5j using the normal random generator.

In the remainder of the section, we report simulation evidence on the finite sample behavior of the tests for the case where S = 4.

Data were generated from the following model :

$$y_t = \sum_{s=1}^{S} D_{st}\, \alpha_s\, y_{t-1} + u_t \tag{5.1}$$

where u_t is i.i.d. N(0, 1) and D_{st} is the seasonal indicator function defined in equation (2.1). The choice of $\{\alpha_s\}$ was determined as follows : $\alpha_1 = \alpha$, $\alpha_2 = \theta/\alpha$, $\alpha_3 = \alpha$, $\alpha_4 = \theta/\alpha$. Hence, two parameters, α and θ, governed the periodic (and nonperiodic) structure of the DGP. We examined four cases of particular interest, namely :

Case I : $\alpha = 1, \theta = 1 \Rightarrow$ $\alpha_s = 1\ \forall s$; (5.2.I)

Case II : $\alpha = 0.8, \theta = 1 \Rightarrow$ Not integrated but periodically integrated; (5.2.II)

Case III : $\alpha = 1.0, \theta = 0.8 \Rightarrow$ Neither integrated nor periodically integrated; (5.2.III)

Case IV : $\alpha = 0.8, \theta = 0.95 \Rightarrow$ Similar to Case III but combination of II and IV. (5.2.IV)

Notice that Case I corresponds to out null hypothesis that the process is I(1). Case II implies that the process is periodically integrated but not I(1). Cases III and IV represent genuine periodic stationary alternatives, although, the former has unit root behavior in two out of four periods since $\alpha = 1.0$.

Table 5.1 : Critical Values Asymptotic Distribution
W_{iS} and LM_{iS} Statistics i = 1, 2, 3 - Quarterly Case S = 4

	Percentiles												
Number of years	1	5	10	20	30	40	50	60	70	80	90	95	99
						W_{14} and LM_{14}							
10	0.0866	0.2021	0.3021	0.4747	0.6327	0.7987	0.9720	1.1749	1.4215	1.7598	2.3541	2.9167	4.3382
20	0.0793	0.1936	0.2964	0.4610	0.6049	0.7601	0.9234	1.1161	1.3477	1.6692	2.1653	2.6231	3.7735
30	0.0780	0.1910	0.2937	0.4480	0.6011	0.7474	0.9045	1.0916	1.3190	1.6164	2.0731	2.5140	3.5508
40	0.0818	0.1929	0.2846	0.4425	0.5829	0.7277	0.8893	1.0749	1.2878	1.5716	2.0616	2.4986	3.5990
						W_{24} and LM_{24}							
10	0.1210	0.2895	0.4348	0.6655	0.8739	1.1001	1.3256	1.5939	1.9242	2.3561	3.1387	3.8698	5.6350
20	0.1115	0.2613	0.3943	0.6016	0.7925	0.9888	1.1902	1.4161	1.6832	2.0716	2.6861	3.2630	4.5428
30	0.1132	0.2666	0.3852	0.5806	0.7584	0.9415	1.1507	1.3919	1.6521	1.9902	2.5529	3.0955	4.2115
40	0.1019	0.2524	0.3718	0.5608	0.7488	0.9249	1.1149	1.3311	1.5962	1.9501	2.5107	3.0579	4.2347
						W_{34} and LM_{34}							
10	0.2310	0.5293	0.7458	1.0976	1.4007	1.7062	2.0266	2.3878	2.8627	3.4437	4.4418	5.4137	7.9101
20	0.2199	0.4873	0.6981	1.0[illegible]59	1.3049	1.5628	1.8226	2.1297	2.4850	2.9734	3.6928	4.4107	6.0667
30	0.2400	0.4925	0.7011	1.0028	1.2583	1.5420	1.8068	2.0833	2.4314	2.8601	3.5198	4.1926	5.5084
40	0.2109	0.4841	0.6987	0.9934	1.2690	1.5272	1.7881	2.0629	2.3757	2.8162	3.4691	4.0978	5.4337

All critical values were computed with 10,000 replications using MATLAB Version 3.5j.

Table 5.2 : Critical Values Asymptotic Distribution
W_{iS} and LM_{iS} Statistics i = 1, 2, 3 - Monthly Case S = 12

Number of years	Percentiles 1	5	10	20	30	40	50	60	70	80	90	95	99
						W_{112} and LM_{112}							
10	0.3235	0.4770	0.5749	0.7172	0.8386	0.9484	1.0629	1.1917	1.3321	1.5143	1.7919	2.0504	2.6310
20	0.3055	0.4557	0.5474	0.6892	0.7969	0.9007	1.0061	1.1178	1.2499	1.4172	1.6740	1.9020	2.4166
30	0.3138	0.4582	0.5505	0.6777	0.7875	0.8901	0.9936	1.1012	1.2258	1.3856	1.6343	1.8522	2.3391
40	0.3049	0.4527	0.5465	0.6770	0.7827	0.8844	0.9814	1.0868	1.2137	1.3720	1.6230	1.8454	2.2634
						W_{212} and LM_{212}							
10	0.3907	0.5767	0.6944	0.8701	1.0095	1.1454	1.2783	1.4188	1.5872	1.7974	2.1273	2.4684	3.1250
20	0.3489	0.5262	0.6270	0.7789	0.9012	1.0234	1.1377	1.2598	1.4070	1.5834	1.8659	2.1210	2.6613
30	0.3454	0.5049	0.6054	0.7549	0.8771	0.9848	1.0972	1.2173	1.3480	1.5283	1.8054	2.0520	2.5625
40	0.3406	0.5033	0.6070	0.7463	0.8648	0.9671	1.0802	1.1927	1.3276	1.4992	1.7492	1.9744	2.4655
						W_{312} and LM_{312}							
10	0.4359	0.6545	0.8037	1.0039	1.1702	1.3190	1.4807	1.6414	1.8367	2.0943	2.5022	2.8778	3.6802
20	0.3754	0.5647	0.6818	0.8446	0.9788	1.1033	1.2318	1.3681	1.5356	1.7367	2.0465	2.3382	2.9643
30	0.3850	0.5497	0.6553	0.8084	0.9309	1.0393	1.1625	1.2946	1.4494	1.6323	1.9075	2.1740	2.7214
40	0.3733	0.5389	0.6463	0.7931	0.9116	1.0298	1.1468	1.2716	1.4214	1.6057	1.8600	2.0842	2.5617

All critical values were computed with 10,000 replications using MATLAB Version 3.5j.

Three classes of test statistics are included in our study. The first class consists of statistics designed to test for I(1) in periodic time series. For brevity, we only report results for W_{24}, LM_{24} and the periodic ADF test based on (4.4), denoted $P - ADF_2$. The first two tests are calculated with $p = 0$ and 1; $P - ADF_2$ is calculated with $p = 1$ and 2. Recall that the P - ADF test restricts $\alpha_S = \alpha$ and so (4.4) does not correspond to (5.1) under Cases II-IV. Therefore, the regression model in (4.4) is only an approximation to the data-generation process in these cases and so the tests' behavior may be sensitive to the choice of p. The second class consists of statistics designed to test for periodic integration. For brevity, we focus on just one statistic, namely, the likelihood ratio test proposed by Boswijk and Franses (1992). This statistic is calculated as :

$$LR_{PI} = T\left[\text{sign}\left[\left[\prod_{s=1}^{4} \hat{\alpha}_s\right] - 1\right]\right] \ln(RSS_0/RSS_1)$$

where RSS_0 is the residual sum of squares from estimating (3.14) subject to the nonlinear restriction that $\sum_{s=1}^{4} \alpha_s = 1$; RSS_1 is the unrestricted residual sum of squares and $\{\hat{\alpha}_s\}$ are the unrestricted parameter estimators. Boswijk and Frances show that LR_{PI} converges to the Dickey-Fuller distribution denoted $\hat{\tau}_\mu$ in Table 8.5.1 of Fuller (1976). The last class consists of the Augmented Dickey-Fuller statistic, which is designed to test for I(1) in aperiodic series. The discussion in section 2 indicates that this test is still valid when the data are generated by a periodic process, but it is anticipated to be inefficient. This test is based on the regression of y_t on an intercept y_{t-1} and $\{y_{t-i} - y_{t-i-1}; i = 1, 2, ..., p\}$. We report results for the cases where p is set equal to 4 and 8.

All experiments were based on 2,000 replications and involved sample sizes covering 10, 20, 30 and 40 years of data. A combination of RATS Version 4.01 and MATLAB Version 3.5j were used to conduct the simulations. The results are reported in Table 5.3.

Table 5.3 : Size and Power of Tests for Unit Roots in Periodic Time Series

	$t_\alpha(p=4)$	$t_\alpha(p=8)$	$W_{24}(p=0)$	$W_{24}(p=1)$	P-ADF$_2$(p = 1)	P-ADF$_2$(p = 2)	LR_{PI}
Case I : $\alpha = 1.0,\ \theta = 1.0$							
T = 40	0.067	0.049	0.055	0.048	0.039	0.041	0.065
T = 80	0.065	0.051	0.057	0.047	0.041	0.044	0.061
T = 120	0.059	0.052	0.056	0.049	0.047	0.047	0.059
T = 160	0.053	0.054	0.051	0.048	0.049	0.049	0.055
Case II : $\alpha = 0.8,\ \theta = 1.0$							
T = 40	0.071	0.053	0.063	0.078	0.046	0.128	0.185
T = 80	0.094	0.078	0.121	0.111	0.067	0.292	0.179
T = 120	0.156	0.117	0.154	0.179	0.111	0.498	0.149
T = 160	0.201	0.165	0.143	0.228	0.168	0.651	0.049
Case III : $\alpha = 1.0,\ \theta = 0.8$							
T = 40	0.146	0.126	0.993	0.991	0.003	0.065	0.001
T = 80	0.624	0.210	1.000	1.000	0.000	0.823	0.000
T = 120	0.727	0.510	1.000	1.000	0.000	0.897	0.000
T = 160	0.809	0.715	1.000	1.000	0.000	0.928	0.000
Case IV : $\alpha = 0.8,\ \theta = 0.95$							
T = 40	0.098	0.063	0.151	0.111	0.068	0.118	0.107
T = 80	0.190	0.116	0.413	0.291	0.180	0.332	0.272
T = 120	0.382	0.253	0.687	0.483	0.362	0.586	0.498
T = 160	0.582	0.404	0.877	0.678	0.550	0.802	0.742

It is most convenient to first compare the ADF test with the Wald, LM and P-ADF tests. None of these tests exhibit any significant size distortions[3] and, in most cases, there are clearly gains in power from explicitly modeling the periodic structure. The one exception is the P-ADF_2 test with $p = 1$, which does not perform well. This can be explained as follows. Recall that the P-ADF test is calculated under the assumption that $\alpha_S = \alpha$. Therefore, in Cases II-IV, for which this restriction is invalid, the AR polynomial in (4.4) is approximating $a_t = \Sigma\, D_{st}(\alpha_S - \alpha)\, y_{t-1}$. Our results suggest that $p = 1$ in (4.4) does not yield a good approximation to the omitted term a_t.

Now, consider the LR_{PI} test for periodic integration. This test exhibits no size distortions in Case I, but substantial size distortions in Case II. In other words, the test for periodic integration is close to its nominal size when the process is periodic and I(1), but not when the process is only periodically integrated.[4] Moreover, the DGP of Case III reveals that with $\alpha = 1$ and $\theta = 0.8$, i.e., unit root like behavior in two out of four quarters but no periodic integration since $\alpha_1\alpha_2\alpha_3\alpha_4 = (0.8)^2$, the LR_{PI} test does not appear to have power. The fourth and last case shows a better performance, however.

The simulation results therefore indicate that acknowledging the presence of periodicity may greatly improve the power of tests in comparison to the more standard tests one can apply for investigating unit root behavior.

6. AN EMPIRICAL EXAMPLE AND CONCLUDING REMARKS

In this section, we illustrate the use of the test statistics for a set of series exhibiting strong seasonal patterns and at the same time having no clearly established

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3 One should expect, though, that designing a DGP with MA components entailing a near-cancellation effect on the autoregressive unit root would produce serious size distortions. The issue of near-cancellations and size distortions has been widely discussed; see, for instance, Schwert (1989) for the zero frequency case and Ghysels, Lee and Noh (1993) for the seasonal case.

4 Testing for periodic integration can be viewed as testing for common trends in a skip-sampled vector system [see, e.g., Franses (1991)]. It has been noted that a combination of relatively large systems and small samples, when testing for common trends, may result in severe size distortions, see, e.g., Reimers (1991). As periodic systems are typically large, particularly with $S = 12$ but even with $S = 4$, it seems that our findings agree with the Monte Carlo evidence regarding cointegration tests.

unit root behavior. Finished goods inventories of two-digit SIC industries were selected on that basis. The unadjusted series are seasonal, yet not so easy to fit with standard fixed parameter time series models, and standard unit root tests yield ambivalent results regarding their long-run behavior.[5] We concentrated on two industries : SIC 20 (Food), SIC 30 (Rubber), using quarterly data covering the 1967:2-1989:2 period.

We first examine whether the data series exhibit periodic features in the univariate autoregressive representation. The point estimates and standard errors of $\hat{\alpha}_s$ are reported in Table 6.1. In addition, we report the results from testing whether $\theta_{ij} = \theta_j$ in (3.14) using the Wald statistic proposed by Ghysels and Hall (1992). It can be shown that this statistic is asymptotically distributed χ^2_{3p} if $\theta_{ij} = \theta_j$ for all j under both H_0 and H_A in (2.2)-(2.3).[6] The p-values of this Wald test are reported in Table 6.1 and provide rather strong evidence of periodic autoregressive behavior for both series. Moreover, the parameter estimates and their standard errors also suggest significantly different coefficients across the four quarters.

Let us now turn our attention to the question whether we can entertain the unit root hypothesis for both series. For that, we rely on the tests described in section 5, namely, the standard unit root tests as well as those exploiting periodic features. It may be worth noting that the DGP of Cases III and IV are most relevant to gauge our empirical study judging on the basis of the point estimates appearing in Table 6.1. Indeed, both series do not seem to be periodically integrated, since the product of the four coefficients $\hat{\alpha}_s$ is quite below one. The unit root test results are reported in Table 6.2. In both cases, we cannot reject the null hypothesis of a unit root via standard ADF tests. Whereas both versions of the Wald test reject the unit root in each of the series and the P-ADF test rejects the unit root for the Rubber industry (SIC 30). This pattern of rejections and nonrejections is broadly consistent with the simulation results for Cases III and IV reported in the previous section.

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5 See, for instance, Ghysels (1987) and Hall (1993) for a discussion of unit root and seasonal properties of inventory data.

6 Clearly, we cannot test $\alpha_s = \alpha$ because the distribution theory depends on whether H_0 in (2.2) or H_A in (2.3) is correct.

Table 6.1 : Evidence of Periodicity in Autoregressive Models of Finished Goods Inventories

	SIC 20		SIC 30	
	p = 1	p = 2	p = 1	p = 2
Regression Model : $y_t = \alpha_t y_{t-1} + \mu_t + \sum_{j=1}^{p} \theta_{tj} z_{t-j} + u_t$ [eq. (3.14)]				
$\hat{\alpha}_1$	0.798	0.800	0.887	0.873
	(0.142)	(0.143)	(0.054)	(0.060)
$\hat{\alpha}_2$	1.402	1.391	0.870	0.809
	(0.154)	(0.161)	(0.047)	(0.059)
$\hat{\alpha}_3$	0.672	0.814	0.786	0.773
	(0.130)	(0.168)	(0.040)	(0.056)
$\hat{\alpha}_4$	0.803	0.792	1.071	1.067
	(0.127)	(0.152)	(0.059)	(0.057)
$\prod_{s=1}^{4} \hat{\alpha}_s$	0.603	0.717	0.650	0.583
Test $\theta_{tj}=\theta_j$	0.000	0.000	0.031	0.014

Samples : Quarterly data 1967:2–1988:4. Standard errors are reported between parenthesis.

Table 6.2 : Empirical Evidence on Unit Roots in Finished Goods Inventories, SIC 20 and SIC 30

	$t_{\hat{\alpha}}(p = 6)$	$t_{\hat{\alpha}}(p = 12)$	P-ADF$_3$(p = 1)	P-ADF$_3$(p = 2)	$W_{312}(p = 0)$	$W_{312}(p = 1)$
SIC 20*	-2.406	-1.642	-1.564	-0.750	4.887	3.998
SIC 30	-2.898	-2.632	-4.701	-3.761	5.011	4.881

Note : Critical value for $t_{\hat{\alpha}}$, $t^*_{\hat{\alpha}}$ and P-ADF tests : -3.41 (5 %); for the W_{312} statistic (T = 20), 2.96 (5 %).

There are several directions in which one can extend the theoretical results of our paper. For instance, the theoretical developments can be used to formulate tests for unit roots at seasonal frequencies against alternatives which exhibit seasonal parameter variation, as discussed in Ghysels, Hall and Lee (1993).

REFERENCES

Anděl, J. (1983), "Statistical Analysis of Periodic Autoregression", *Aplikace Mathematiky* 28, 364-385.

Anděl, J. (1987), "On Multiple Periodic Autoregression", *Aplikace Mathematiky* 32, 63-80.

Andrews, D.W.K. (1992), "Heteroskedasticity and Autocorrelation Consistent Covariance Estimation", *Econometrica* 59, 817-858.

Boswijk, H.P. and P.H. Franses (1992), "Testing for Periodic Integration", Discussion Paper, Econometric Institute, Erasmus University.

Box, G. and G. Jenkins (1976), *Time Series Analysis : Forecasting and Control*, Prentice Hall, Englewood Cliffs, NJ.

Burridge, P. and K.F. Wallis (1990), "Seasonal Adjustment and Kalman Filtering : Extension to Periodic Variances", *Journal of Forecasting* 9, 109-118.

Cipra, T. (1985), "Periodic Moving Average Process", *Aplikace Mathematiky* 30, 218-229.

Dickey, D.A., W. Bell and R. Miller (1986), "Unit Roots in Time Series Models : Tests and Implications", *The American Statistician* 40, 12-26.

Dickey, D.A. and W. Fuller (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association* 74, 427-431.

Dickey, D.A., D.P. Hasza and W.A. Fuller (1984), "Testing for Unit Roots in Seasonal Time Series", *Journal of the American Statistical Association* 79, 355-367.

Engle, R.F., C.W.J. Granger, S. Hylleberg and H.S. Lee (1993), "Seasonal Cointegration : The Japanese Consumption Function", *Journal of Econometrics* 55, 275-298.

Franses, P.H. (1991), "A Multivariate Approach to Modeling Univariate Seasonal Time Series", *Journal of Econometrics* (forthcoming).

Fuller, W. (1976), *Introduction to Statistical Time Series*, Wiley, New York.

Ghysels, E. (1987), "Cycles and Seasonals in Inventories : Another Look at Nonstationarity and Induced Seasonality", Discussion Paper No. 1487, C.R.D.E., Université de Montréal.

Ghysels, E. (1991a), "On the Periodic Structure of the Business Cycle", *Journal of Business and Economic Statistics* (forthcoming).

Ghysels, E. (1991b), "On Scoring Asymmetric Periodic Probability Models of Turning-Point Forecasts", *Journal of Forecasting* (forthcoming).

Ghysels, E. (1992), "A Time Series Model with Periodic Stochastic Regime Switching", Discussion Paper, C.R.D.E., Université de Montréal.

Ghysels, E. and A. Hall (1992), "Testing for Periodicity in Some Linear Macroeconomic Models", Discussion Paper, C.R.D.E., Université de Montréal.

Ghysels, E., A. Hall and H.S. Lee (1993), "Testing for Unit Roots in Periodic Time Series", mimeo.

Ghysels, E., H.S. Lee and J. Noh (1993), "Testing Unit Roots in Seasonal Time Series : Some Theoretical Extensions and a Monte Carlo Investigation", *Journal of Econometrics* (forthcoming).

Gladyshev, E.G. (1961), "Periodically Correlated Random Sequences", *Soviet Mathematics* 2, 385-388.

Granger, C. and P. Newbold (1986), *Forecasting Economic Time Series*, 2nd Edition, Academic Press, New York.

Hall, A.R. (1993), "Testing for a Unit Root in Time Series with Pretest Data-Based Model Selection", Discussion Paper, North Carolina State University.

Hansen, L.P. and T.J. Sargent (1990), "Recursive Linear Models of Dynamic Economics", manuscript, Hoover Institution, Stanford University.

Hasza, D.P. and W.A. Fuller (1982), "Testing for Nonstationarity Parameter Specifications in Seasonal Time Series Models", *Annals of Statistics* 10, 209-216.

Hylleberg, S, R.F. Engle, C.W.J. Granger and B.S. Yoo (1990), "Seasonal Integration and Cointegration", *Journal of Econometrics* 44, 215-238.

Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models", *Econometrica* 59, 1551-1580.

Jones, R.H. and W.M. Brelsford (1967), "Time Series with Periodic Structure", *Biometrika* 54, 408-408.

Lee, H.S. (1992), "Maximum Likelihood Inference on Cointegration and Seasonal Cointegration", *Journal of Econometrics* 54, 1-49.

Lütkepohl, H. (1991a), *Introduction to Multiple Time Series Analysis*, Berlin : Springer Verlag, New York.

Lütkepohl, H. (1991b), "Testing for Time Varying Parameters in Vector Autoregressive Models", in W.E. Griffiths, H. Lütkepohl and M.E. Bock (eds.), *Readings in Econometric Theory and Practice*, Amsterdam : North Holland.

Osborn, D.R. (1988), "Seasonality and Habit Persistence in a Life-Cycle Model of Consumption", *Journal of Applied Econometrics* 3, 255-266.

Osborn, D.R. and J.P. Smith (1989), "The Performance of Periodic Autoregressive Models in Forecasting Seasonal U.K. Consumption, *Journal of Business and Economic Statistics* 7, 117-127.

Pagano, M. (1978), "On Periodic and Multiple Autoregressions", *Annals of Statistics* 6, 1310-1317.

Pantula, S. and A.R. Hall (1991), "Testing for Unit Roots in Autoregressive Moving Average Models : An Instrumental Variable Approach", *Journal of Econometrics* 48, 325-353.

Park, J.Y. and P.C.B. Phillips (1988), "Statistical Inference in Regressions with Integrated Processes : Part 1", *Econometric Theory* 4, 468-498.

Phillips, P.C.B. and S. Durlauf (1986), "Multiple Time Series Regression with Integrated Processes", *Review of Economic Studies* 53, 473-495.

Phillips, P.C.B. and P. Perron (1988), "Testing for a Unit Root in Time Series Regression", *Biometrika* 75, 335-346.

Reimers, H.E. (1991), "Comparisons of Tests for Multivariate Cointegration", Statistical Papers (forthcoming).

Said, S.E. and D.A. Dickey (1984), "Testing for Unit Roots in Autoregressive Moving Average Models of Unknown Order", *Biometrika* 71, 599-608.

Said, S.E. and D.A. Dickey (1985), "Hypothesis Testing in ARIMA(p,1,q) Models", *Journal of the American Statistical Association* 80, 369-374.

Sakai, H. (1991), "On the Spectral Density Matrix of a Periodic ARMA Process", *Journal of Time Series Analysis* 18, 73-82.

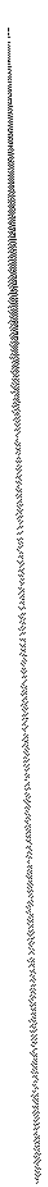
Schwert, G.W. (1989), "Tests for Unit Roots : A Monte Carlo Investigation", *Journal of Business and Economic Statistics* 7, 147-160.

Tiao, G. and M.R. Grupe (1980), "Hidden Periodic Autoregressive-Moving Average Models in Time Series Data", *Biometrika* 67(2), 365-373.

Todd, R. (1983), "A Dynamic Equilibrium Model of Seasonal and Cyclical Fluctuations in the Corn-Soybean-Hog Sector", Ph.D. dissertation, University of Minnesota.

Todd, R. (1990), "Periodic Linear Quadratic Models of Seasonality", *Journal of Economic Dynamics and Control* 14, 763-796.

Vecchia, A.V. (1985), "Maximum Likelihood Estimation for Periodic Autoregressive Moving Average Models", *Technometrics* 27, 375-384.



APPENDIX

A.1 We first present a representation for Y_n which is central to our analysis.

From Lütkepohl (1991a), p. 397-398, it follows that there exist $S \times S$ matrices $\{\Theta_i\}$ such that :

$$Z_n = HCY_{n-1} + \sum_{i=0}^{P} \Theta_i Z_{n-i} + A U_n \qquad n = 1, \dots, N \tag{A.1}$$

where H, C' are $S \times (S - 1)$ matrices whose nonzero elements are given by $H_{ij} = -1$ for $i = 2, \dots, S$; $j = i - 1$; $C_{ij} = -1$ for $i = 2, 3, \dots, S$; $j = 1$, $C_{ij} = 1$ for $i = 1, 2, \dots, S - 1$; $j = i + 1$, and A is the $S \times S$ matrix whose nonzero elements are given by $A_{ij} = 1$ for $i \leq j$.

Observe that Z_n appears on both the left- and right-hand sides of (A.1). Therefore, we rearrange (A.1) to give

$$Z_n = H^* C Y_{n-1} + \sum_{i=1}^{P} \Theta_i^* Z_{n-i} + A^* U_n, \tag{A.2}$$

where $H^* = (I - \Theta_0)^{-1} H$, $\Theta_i^* = (I - \Theta_0)^{-1} \Theta_i$ and $A^* = (I - \Theta_0)^{-1} A$. Moreover from (3.1), it follows that only the nonzero elements in Θ_0 lie above the leading diagonal and so $(I - \Theta_0)$ is an upper triangular matrix with ones along the leading diagonal. Therefore, $(I - \Theta_0)^{-1}$ always exists and is also an upper-triangular matrix. Equation (A.2) is a cointegrated system of rank $S - 1$. This representation is central to our analysis, and we assume that it satisfies the following assumption.

Assumption A.1 : It is assumed that (A.2) implies $\Pi(L)Z_n = A U_n$ where $\Pi(L)$ is a P^{th} order polynomial in the lag operator L which satisfies $|\Pi(m)| \neq 0$ for $m \leq 1$.

This assumption ensures that Z_n has a well-defined infinite order moving average representation in $A^* U_n$, denoted $Z_n = \Phi(L) A^* U_n$ [see Lütkepohl (1991a), p. 17]. Then from Johansen's (1991a) Theorem 4.1, it follows that Y_n has the following representation :

$$Y_n = Y_0 + D A^* \sum_{i=1}^{n} U_i + F(L) A^*(U_n - U_0), \tag{A.3}$$

where $D = C_\perp'(H_\perp^{*\prime} R(1) C_\perp')^{-1} H_\perp^{*\prime}$; with $C_\perp'$, $H_\perp^*$ (S × 1) vectors which are orthogonal to C', H^* respectively; $R(1) = I_S + H^*C - \Theta^*(1)$; $\Theta^*(L) = \sum_{i=1}^{P} \Theta_i^* L^i$; and $F(L) = [\Phi(L) - \Phi(1)] / (1 - L)$. Now, by construction, we have $H_\perp^* = (I - \Theta_0)' H_\perp$, $H_\perp = (1, 0, \dots, 0)$, $C_\perp = \iota'$.

Therefore,

$$D A^* = g^{-1} \iota H_\perp' A = g^{-1} u' \tag{A.4}$$

where g is the scalar $H_\perp^{*\prime} R(1) C_\perp'$.

A.2 PROOF OF THEOREM 3.1

We first present the formula for the score function and the hessian. From (3.5), it follows that the score function consists of the following elements of the form

$$\frac{\partial LLF_T(\Psi)}{\partial \alpha_s} = \sum_{t=1}^{T} D_{st}\, y_{t-1}\, u_t(\Psi) / \omega^2 \tag{A.5}$$

$$\frac{\partial LLF_T(\Psi)}{\partial \theta_{sj}} = \sum_{t=1}^{T} D_{st}\, z_{t-j}\, u_t(\Psi) / \omega^2 \tag{A.6}$$

$$\frac{\partial LLF_T(\Psi)}{\partial \omega^2} = \sum_{t=1}^{T} [u_t^2(\Psi) - \omega^2] / 2\omega^4 \tag{A.7}$$

where $u_t(\Psi) = \alpha_t y_{t-1} - \sum_{j=1}^{p} \theta_{tj} z_{t-j}$.

From (A.5)-(A.7), it follows that the only nonzero terms in the hessian are of the following form :

$$\frac{\partial^2 LLF_T(\Psi)}{\partial \alpha_s^2} = -\sum_{t=1}^{T} D_{st} y_{t-1}^2 / \omega^2 \tag{A.8}$$

$$\frac{\partial^2 LLF_T(\Psi)}{\partial \theta_{sj} \partial \theta_{sk}} = -\sum_{t=1}^{T} D_{st} z_{t-j} z_{t-k} / \omega^2 \tag{A.9}$$

$$\frac{\partial^2 LLF_T(\Psi)}{\partial \alpha_s \partial \theta_{sj}} = -\sum_{t=1}^{T} D_{st} y_{t-1} z_{t-j} / \omega^2 \tag{A.10}$$

$$\frac{\partial^2 LLF_T(\Psi)}{\partial \alpha_s \partial \omega^2} = -\sum_{t=1}^{T} D_{st} y_{t-1} u_t(\Psi) / \omega^4 \tag{A.11}$$

$$\frac{\partial^2 LLF_T(\Psi)}{\partial \theta_{sj} \partial \omega^2} = -\sum_{t=1}^{T} D_{st} z_{t-j} u_t(\Psi) / \omega^4 \tag{A.12}$$

$$\frac{\partial^2 LLF_T(\Psi)}{(\partial \omega^2)^2} = -\sum_{t=1}^{T} D_{st} [u_t^2(\Psi) \omega^{-6} - 0.5 \omega^{-4}]. \tag{A.13}$$

We first prove (i). Define $H_N = \text{diag}(h_{1N}, h_{2N} T^{1/2})$ where h_{1N} is the $(S \times 1)$ vector with all its elements equal to N, and h_{2N} is the $Sp \times 1$ vector with all its elements equal to $N^{1/2}$.

Using similar arguments to Fuller (1976), Theorem 8.5.1, p. 374 [or see Pantula and Hall (1990), Lemma 1], it can be shown that

$$H_N^{-1} \frac{\partial^2 LLF_T(\hat{\Psi})}{\partial\Psi\, \partial\Psi'} H_N^{-1} = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} + o_p(1) \tag{A.14}$$

where $M_1 = N^{-2} \sum_{t=1}^{T} (F_t \otimes y_{t-1})$, $M_2 = N^{-1} \sum_{t=1}^{T} (F_t \otimes x_{1t} \ x'_{1t})$, $M_3 = 0.5\omega^{-4}$ and $F_t = \text{diag}(\omega^{-2} D_{1t}, \dots, \omega^{-2} D_{St})$.

Similarly, if we let $H_{1N} = \text{diag}(h_{1N})$, it can be shown that

$$H_{1N}[\hat{\alpha} - \iota] = M_1 H_{1N} v_1 + o_p(1) \tag{A.15}$$

where $v'_1 = \left[\sum_{t=1}^{T} D_{1t}\, y_{t-1}\, u_t / \omega^2, \dots, \sum_{t=1}^{T} D_{St}\, y_{t-1}\, u_t / \omega^2\right]$.

Combining (3.3), (A.14) and (A.15), we have

$$W_{1S} = \sum_{s=1}^{S} \left[\left[\sum_{t=1}^{T} D_{st}\, y_{t-1}\, u_t / \omega^2\right]^2 / \sum_{t=1}^{T} D_{st}\, y_{t-1}^2 / \omega^2\right] + o_p(1). \tag{A.16}$$

We now derive the limiting distribution of the numerator and denominator of W_{1S}. From (A.3)-(A.4), (3.12) and Phillips and Durlauf (1986), Lemma 3.1, it follows that

$$N^{-1} \sum_{n=1}^{N} Y_{n-1} U_n' \Rightarrow g^{-1} \iota' \Omega^{1/2} \int_0^1 B_S(r)\, d\, B_S(r)' \Omega^{1/2} = g^{-1} \omega^2 \iota \int_0^1 G(r)\, d\, B_S(r)' \quad (A.17)$$

and

$$N^{-2} \sum_{n=1}^{N} Y_{n-1} Y_{n-1}' \Rightarrow g^{-2} \iota' \Omega^{1/2} \int_0^1 B_S(r) B_S(r)' dr\, \Omega^{1/2} \iota' = g^{-2} \omega^2 \iota' \int_0^1 G(r)^2 dr. \quad (A.18)$$

Now,

$$N^{-1} \sum_{t=1}^{T} D_{st}\, y_{t-1} u_t / \omega^2 = \left\{ \Omega^{1/2} N^{-1} \sum_{n=1}^{N} Y_{n-1} U_n' \Omega^{1/2} \right\}_{jj} \quad (A.19)$$

$$N^{-2} \sum_{t=1}^{T} D_{st}\, y_{t-1}^2 / \omega^2 = \left\{ \Omega^{1/2} N^{-2} \sum_{n=1}^{N} Y_{n-1} Y_{n-1}' \Omega^{1/2} \right\}_{jj} \quad (A.20)$$

for $j = S - s + 1$.

From (A.17)-(A.20), it follows that

$$\sum_{t=1}^{T} D_{st}\, y_{t-1} u_t / \omega^2 / \sum_{t=1}^{T} D_{st}\, y_{t-1}^2 / \omega^2 \Rightarrow \left[\int_0^1 G(r)\, d\, B_{Sj}(r) \right]^2 / \int_0^1 G(r)^2\, dr \quad (A.21)$$

for $j = S - s + 1$.

The result then follows from (A.16) and (A.21). To prove (ii), it is only necessary to observe from (3.11) and (A.16) that $LM_{1S} = W_{1S} + o_p(1)$.

A.3 PROOF OF THEOREM 3.2

We prove (i) first. Part (ii) follows directly from noting $W_{2S} = LM_{2S} + o_p(1)$.

Using standard arguments, it can be shown that

$$W_{2S} = \sum_{s=1}^{S} \left\{ \left[\left[N^{-1} \sum_{t=1}^{T} D_{st}\, y_{t-1}\, u_t / \omega^2 - \left[N^{-3/2} \sum_{t=1}^{T} D_{st}\, y_{t-1} / \omega \right] \left[N^{-1/2} \sum_{t=1}^{T} D_{st}\, u_t / \omega \right] \right]^2 \right.$$

$$\left. \times \left[N^{-2} \sum_{t=1}^{T} D_{st} (y_{t-1} - \bar{y}_s)^2 / \omega^2 \right] \right\} + o_p(1), \qquad \text{(A.22)}$$

where $\bar{y}_s = N^{-1} \sum_{t=1}^{T} D_{st}\, y_{t-1}$. From (A.3) and Phillips and Durlauf (1986), Lemma 3.1, it follows that

$$N^{-3/2} \sum_{n=1}^{N} Y_n \Rightarrow g^{-1}\, u'\, \Omega^{1/2} \int_0^1 B_S(r)\, dr, \qquad \text{(A.23)}$$

$$N^{-2} \sum_{n=1}^{N} (Y_{n-1} - \bar{Y})(Y_{n-1} - \bar{Y}) \Rightarrow g^{-2}\, u'\, \Omega^{1/2} \left[\int_0^1 B_S(r)\, B_S(r)'\, dr \right.$$

$$\left. - \int_0^1 B_S(r)\, dr \int_0^1 B_S(r)'\, dr \right] \Omega^{1/2}\, u', \qquad \text{(A.24)}$$

$$N^{-1/2} \sum_{n=1}^{N} U_n \Rightarrow \Omega^{1/2} B_S(1). \qquad \text{(A.25)}$$

The result then follows immediately.

We prove that (iii) and (v) follow directly from $W_{3S} = LM_{3S} + o_p(1)$: First, note that

$$W_{3S} = \sum_{s=1}^{S} \hat{\tau}^2_{\tau s} \qquad \text{(A.26)}$$

where $\hat{\tau}^2_{\tau s}$ is the t-statistic for $H_0 : \alpha_s = 1$ in the model

$$y_t = \alpha_s y_{t-1} + \mu_s + \beta_s(n - N/2) + \sum_{j=1}^{p} \theta_{sj} z_{t-j} + u_t$$

for $t = n(S - 1) + s$. Using similar arguments to Dickey and Fuller (1979) and Phillips and Perron (1988), it can be shown that

$$\hat{\tau}_{\tau s} = M_s^{1/2} (\hat{\alpha}_s - 1) / \hat{\omega} [N^2(N^2 - 1) / 12]^{1/2} + o_p(1), \quad \text{(A.27)}$$

where

$$\hat{\alpha}_s - 1 = M_s^{-1}[(N(N + 1) / 2)] \sum_{t=1}^{T} D_{st}\, n\, y_{t-1} \sum_{t=1}^{T} D_{st} u_t$$

$$- (N(N + 1) (2N + 1) / 6) \sum_{t=1}^{T} D_{st} y_{t-1}$$

$$\times \sum_{t=1}^{T} D_{st} u_t - N \sum_{t=1}^{T} D_{st}\, n\, y_{t-1} \sum_{t=1}^{T} D_{st}\, n\, u_t + (N(N + 1) / 2) \sum_{t=1}^{T} D_{st} y_{t-1}$$

$$\times \sum_{t=1}^{T} D_{st}\, n\, u_t + (N^2(N^2 - 1) / 12) \sum_{t=1}^{T} D_{st} u_t y_{t-1} + o_p(1) \quad \text{(A.28)}$$

$$M_s = (N^2(N^2 - 1) / 12) \sum_{t=1}^{T} D_{st} y^2_{t-1} - N\left[\sum_{t=1}^{T} D_{st}\, n\, y_{t-1}\right]^2$$

$$+ N(N + 1) \sum_{t=1}^{T} D_{st}\, n\, y_{t-1} \sum_{t=1}^{T} D_{st} y_{t-1}$$

$$- (N(N + 1) (2N + 1) / 6) \left[\sum_{t=1}^{T} D_{st} y_{t-1}\right]^2. \quad \text{(A.29)}$$

From (A.3), (A.4) and Park and Phillips (1988), Lemma 2.1, we have

$$N^{-3/2} \sum_{n=1}^{N} n\, u_n \Rightarrow \Omega^{1/2} \int_0^1 r\, d\, B_S(r) \qquad \text{(A.30)}$$

$$N^{-5/2} \sum_{n=1}^{N} n\, y_{n-1} \Rightarrow \Omega^{1/2} \int_0^1 r\, B_S(r)\, dr \qquad \text{(A.31)}$$

and so

$$N^{-3/2} \sum_{n=1}^{N} D_{st}\, n\, u_n \Rightarrow \omega \int_0^1 r\, d\, B_{Sj}(r), \qquad j = S - s + 1, \qquad \text{(A.32)}$$

$$N^{-5/2} \sum_{n=1}^{N} D_{st}\, n\, y_{n-1} \Rightarrow \omega \int_0^1 r\, G(r)\, dr. \qquad \text{(A.33)}$$

Therefore, combining (A.17)-(A.20), (A.23)-(A.25), (A.26)-(A.33), we obtain the desired result.

A.4 PROOF OF THEOREM 4.1

Define H_{3N} to be the $(pS + 1) \times (pS + 1)$ matrix $H_{3N} = \text{diag}(T, N^{1/2}, \ldots, N^{1/2})$ and rewrite $\bar{\tau}$ as

$$\bar{\tau} = (\bar{\alpha} - 1) / \omega \left\{ H_{3N}^{-1} \sum_{t=1}^{T} X_t X_t' H_{3N}^{-1} \right\}_{11} .$$

Using similar arguments to the proof of Theorem 3.1, it can be shown that

$$\bar{\tau} = \left[T^{-1} \sum_{t=1}^{T} y_{t-1} u_t \right] / \omega \left[T^{-2} \sum_{t=1}^{T} y_{t-1}^2 \right]^{1/2} + O_p(1). \qquad \text{(A.34)}$$

Now,

$$T^{-1} \sum_{t=1}^{T} y_{t-1} u_t = S^{-1} \sum_{s=1}^{S} N^{-1} \sum_{n=1}^{N} D_{st} y_{t-1} u_t \tag{A.35}$$

$$T^{-2} \sum_{t=1}^{T} y_{t-1}^2 = S^{-2} \sum_{s=1}^{S} N^{-2} \sum_{n=1}^{N} D_{st} y_{t-1}^2, \tag{A.36}$$

therefore, combining (A.35)-(A.36) with (A.17)-(A.20), we obtain

$$T^{-1} \sum_{t=1}^{T} y_{t-1} u_t \Rightarrow g^{-1} S^{-1} \sum_{j=1}^{S} \omega^2 \int_0^1 G(r)\, d\, B_{Sj}(r) = g^{-1} S^{-1} \omega^2 \int_0^1 G(r)\, d\, G(r), \tag{A.37}$$

$$T^{-2} \sum_{t=1}^{T} y_{t-1}^2 \Rightarrow g^{-2} S^{-2} \omega^2 \sum_{j=1}^{S} \int_0^1 G(r)^2\, dr = g^{-2} S^{-1} \omega^2 \int_0^1 G(r)^2\, dr. \tag{A.38}$$

Now, $G(r) = S\, \omega^2\, B_1(r)$, so from (A.34)-(A.38),

$$\bar{\tau} \Rightarrow \int_0^1 B_1(r)\, d\, B_1(r) \Big/ \left[\int_0^1 B_1(r)^2\, dr\right]^{1/2} = \hat{\tau}.$$

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