Positively responsive collective choice rules and majority rule: a generalization of May's theorem to many alternatives

Sean Horan, Martin J. Osborne, and M. Remzi Sanver*

April 2, 2018

Abstract

A collective choice rule selects a set of alternatives for each collective choice problem. Suppose that the alternative x is in the set selected by a collective choice rule for some collective choice problem. Now suppose that x rises above another selected alternative y in some individual's preferences. If the collective choice rule is "positively responsive", x remains selected but y is no longer selected. If the set of alternatives contains two members, an anonymous and neutral collective choice rule is positively responsive if and only if it is majority rule (May 1952). If the set of alternatives contains three or more members, a large set of collective choice rules satisfy these three conditions. We show, however, that in this case only the rule that assigns to every problem its strict Condorcet winner satisfies the three conditions plus Nash's version of "independence of irrelevant alternatives" for the domain of problems that have strict Condorcet winners. Further, no rule satisfies the four conditions for the domain of all preference relations.

Horan: Department of Economics, Université de Montréal, and CIREQ. Osborne: Department of Economics, University of Toronto. Sanver: Université Paris-Dauphine, PSL Research University, CNRS, UMR [7243], LAMSADE, 75016 Paris, France. We thank Salvador Barberà, Jean-Pierre Benoît, Felix Brandt, Markus Brill, Donald E. Campbell, Christopher Dobronyi, John Duggan, Justin Kruger, Hervé Moulin, Matías Núñez, and Maurice Salles for discussions and comments. Horan's work was partly supported by FRQSC. Sanver's work was partly supported by the projects ANR-14-CE24-0007-01 "CoCoRICo-CoDec" and IDEX ANR-10-IDEX-0001-02 PSL "MIFID".

of alternatives. But what if we restrict to a limited set of preference profiles? Is a natural generalization of majority rule characterized by May's conditions plus an additional reasonable condition?

We show that the answer is affirmative. The condition we add is a set-valued version of the independence condition used by Nash (1950, condition 7 on p. 159) in the context of his bargaining model, which we call Nash independence.[6] This condition says that removing unchosen alternatives does not affect the set of alternatives selected. We show that for the domain of collective choice problems that have a strict Condorcet winner,[7] May's conditions plus Nash independence characterize the collective choice rule that selects the strict Condorcet winner (Theorem 2). We show also that when preferences are unrestricted, no collective choice rule satisfies these conditions if there are at least three individuals and three alternatives (Theorem 3), and if the only restriction on preferences is that no individual is indifferent between any two alternatives, no collective choice rule satisfies these conditions if there are either at least three individuals and four alternatives or at least four individuals and at least three alternatives (Theorem 4).

A strict Condorcet winner is an appealing outcome if it exists, and the conditions of anonymity, neutrality, positive responsiveness, and Nash independence also are appealing. We interpret Theorem 2 to increase the appeal of both the collective choice rule that selects the strict Condorcet winner and the four conditions. It shows that the combination of the conditions is "just right" for collective choice problems with a strict Condorcet winner: this combination implies the collective choice rule that selects the strict Condorcet winner. If no collective choice rule were to satisfy the conditions on this domain, the combination of conditions would be too strong, and our subsequent results that no collective choice rule satisfies it on a larger domain would be less significant. If collective choice rules other than the one that selects the strict Condorcet winner were to satisfy the combination of conditions on the domain of collective choice problems with a strict Condorcet winner, then the combination of conditions would be too weak.

[6] The condition is known by several other names, including "strong superset property". See Brandt and Harrenstein (2011) (who call the condition "$\widehat{\alpha}$") for an analysis of the condition and account of its previous use. It neither implies nor is implied by the related "Chernoff condition"; see the discussion following Definition 10.

[7] An alternative a such that for every other alternative b a strict majority of individuals prefer a to b.

The key condition in our results is an adaptation of May's "positive responsiveness" to an environment with many alternatives. Suppose that the alternatives a and b are both selected, in a "tie", for some problem, and some individual ranks b above a. Now suppose that the individual's preferences change to rank a above b. Our condition requires that a remains one of the selected alternatives, b is no longer selected, and no alternative that was not selected originally is now selected. This condition captures the spirit of May's condition: a change in the relative ranking of two alternatives by a single individual breaks a tie between the alternatives. More loosely, the condition ensures that every individual's preferences matter. A stronger condition is sometimes given the same name: if a is among the alternatives selected, and then a's ranking relative to some other alternative improves in some individual's preferences, a becomes the unique alternative selected.[8] That is, a's rising in some individual's preferences relative to any other alternative disqualifies all other alternatives. Our condition requires that a's rising in some individual's preferences disqualifies only alternatives that it surpasses as it rises.

2. Model

Throughout we fix a finite set N of individuals and a finite set A of all possible alternatives, and assume that both N and A contain at least two elements. In any given instance, the set of individuals has to choose an alternative from some subset of A, and each individual has preferences over this subset (the set of "available" alternatives). For reasons we discuss after the following definition, it is convenient, however, to allow the preferences specified by a "collective choice problem" to rank alternatives *outside* the set of available alternatives. As usual, we define a *preference relation on a set X* to be a complete and transitive binary relation on X.

Definition 1 (Collective choice problem)**.** A *collective choice problem* is a pair $(X, \succsim)$, where X is a subset of A with at least two members and $\succsim$ is a profile $(\succsim_i)_{i \in N}$ of preference relations over some set Y with $X \subseteq Y \subseteq A$.

We use $\succ_i$ to denote strict preference: $x \succ_i y$ if and only if $x \succsim_i y$ and not $y \succsim_i x$.

[8] See, for example, Barberà (1977, Definition 7, p. 1579) or Blair et al. (1976, p. 370).

The reason we allow the preference profile in a collective choice problem $(X, \succsim)$ to rank alternatives outside X is that we frequently consider a collective choice problem that is derived from another problem by shrinking the set of alternatives. Our definition allows us to write the problem derived from $(X, \succsim)$ by shrinking the set of available alternatives to $Z \subset X$ as $(Z, \succsim)$ rather than $(Z, \succsim|_Z)$. (Throughout we use $\subset$ to denote strict inclusion and $\subseteq$ to denote weak inclusion.)

For every collective choice problem, we would like to identify an alternative that the individuals collectively "like best". However, for some problems we cannot select a single such alternative without discriminating among individuals or alternatives. For example, for the collective choice problem with two individuals and two available alternatives, x and y, in which one individual prefers x to y and the other prefers y to x, we need to declare x and y "tied" if we want to refrain from discriminating between the individuals and between the alternatives.[9] Thus we define a collective choice rule to be a function that associates with every collective choice problem $(X, \succsim)$ a *subset* of X.

Definition 2 (Collective choice rule)**.** A *collective choice rule* is a function F that associates with every collective choice problem $(X, \succsim)$ a nonempty subset $F(X, \succsim)$ of X with the property that $F(X, \succsim) = F(X, \succsim')$ whenever $\succsim$ and $\succsim'$ agree on X.

Our definition of a collective choice problem allows the individuals' preferences to rank alternatives that are not available, but the property of F in Definition 2 requires that the set of alternatives assigned by a collective choice rule depends only on the individuals' preferences over *available* alternatives. This restriction is important for our results on the nonexistence of a collective rule satisfying our conditions.

An alternative that plays a prominent role in the collective choice rules we discuss is the Condorcet winner, defined as follows.

Definition 3 (Condorcet winner)**.** For the collective choice problem $(X, \succsim)$, an alternative $x \in X$ is

- a *Condorcet winner* if for each alternative $y \in X \setminus \{x\}$, the number of individuals $i \in N$ for whom $x \succ_i y$ is at least the number for whom $y \succ_i x$

[9] More generally, declaring "ties" is necessary whenever the number of alternatives is the sum of the divisors of the number of individuals different from 1 (Moulin 1988, Exercise 9.9(b), p. 253).

- a *strict Condorcet winner* if for each alternative $y \in X \setminus \{x\}$ the number of individuals $i \in N$ for whom $x \succ_i y$ exceeds the number for whom $y \succ_i x$.

A collective choice problem may have more than one Condorcet winner. For example, if every individual is indifferent between all alternatives, then every alternative is a Condorcet winner. But every problem has at most one strict Condorcet winner. Some problems have no Condorcet winner: suppose that $N = \{1, 2, 3\}$ and consider the problem $(X, \succcurlyeq)$ for which $X = \{x, y, z\}$ and $x \succ_1 y \succ_1 z$, $y \succ_2 z \succ_2 x$, and $z \succ_3 x \succ_3 y$. However, every problem $(X, \succcurlyeq)$ for which X contains two alternatives has a Condorcet winner (although not necessarily a strict one).

We take the standard axiomatic approach to finding collective choice rules that select alternatives that reflect well the individuals' preferences. That is, we look for rules that satisfy a list of apparently desirable properties for all members of certain sets of collective choice problems. We refer to such sets as "domains". One domain is the set of all possible collective choice problems. The other domains we consider restrict the individuals' preferences. For example, one domain we consider is the set of all collective choice problems in which no individual is indifferent between any two alternatives.[10]

Definition 4 (Domain)**.** A *domain* is a (set-valued) function $\mathscr{D}$ that associates with every set $X \subseteq A$ of available alternatives with two or more members a set $\mathscr{D}(X)$ of preference profiles over X.

The first two properties that we impose on a collective choice rule are that it treats the individuals equally and the alternatives equally. For any profile $\succcurlyeq$ of preference relations we say that the profile $\succcurlyeq'$ of preference relations is a *permutation* of $\succcurlyeq$ if there is a one-to-one function $\pi : N \to N$ such that $\succcurlyeq'_i = \succcurlyeq_{\pi(i)}$ for all $i \in N$.

Definition 5 (Anonymous collective choice rule)**.** For any domain $\mathscr{D}$, a collective choice rule F is *anonymous on* $\mathscr{D}$ if for every collective choice problem $(X, \succcurlyeq)$ with $\succcurlyeq \in \mathscr{D}(X)$ and every permutation $\succcurlyeq'$ of $\succcurlyeq$ with $\succcurlyeq' \in \mathscr{D}(X)$,

$$F(X, \succcurlyeq) = F(X, \succcurlyeq').$$

[10]That is, each individual's preference relation is a "linear order".

For any profile $\succsim$ of preference relations and any one-to-one function $\sigma : X \to X$, we say that the profile $\succsim^\sigma$ of preference relations is the σ-*transformation* of $\succsim$ if for each $i \in N$ we have $x \succsim_i y$ if and only if $\sigma(x) \succsim_i^\sigma \sigma(y)$.

Definition 6 (Neutral collective choice rule)**.** For any domain $\mathscr{D}$, a collective choice rule F is *neutral on* $\mathscr{D}$ if for every collective choice problem $(X, \succsim)$ with $\succsim \in \mathscr{D}(X)$ and every one-to-one function $\sigma : X \to X$,

$$F(X, \succsim^\sigma) = \sigma(F(X, \succsim))$$

whenever $\succsim^\sigma \in \mathscr{D}(X)$, where $\succsim^\sigma$ is the σ-transformation of $\succsim$.

May's (1952) condition of "positive responsiveness", a variant of Arrow's "positive association of social and individual values" (1963, p. 25), is a key component of his characterization of majority rule. A generalization of this condition to an environment with many alternatives plays a key role also in our analysis. Many generalizations of the condition are possible. Suppose that the alternatives x and y (among possibly others) are both selected by a collective choice rule, in a "tie". The spirit of May's condition is that an improvement in one individual's ranking of one of these alternatives relative to the other breaks the tie. Specifically, if one individual's preferences change from ranking x below y to ranking x above y, then the collective choice rule should still select x but no longer select y.

Precisely, our condition requires that if x is one of the alternatives selected by a collective choice rule for some collective choice problem and, for some other alternative y, one individual's preferences change either (*i*) from ranking x below y to ranking x at least as high as y, or (*ii*) from ranking x at most as high as y to ranking x above y then (*a*) x remains a selected alternative, (*b*) y is not selected for the new problem, and (*c*) no alternative that was not selected originally is selected for the new problem.[11]

Definition 7 (Positively responsive collective choice rule)**.** For any domain $\mathscr{D}$, the collective choice rule F is *positively responsive on* $\mathscr{D}$ if for every collective choice problem

[11]In a different context, Núñez and Valletta (2015, p. 284) formulate a similar condition.

$(X,\succsim)$ with $\succsim\in\mathscr{D}(X)$, if $x\in F(X,\succsim)$ and a preference profile $\succsim'$ for which, for some individual $j\in N$,

- $\succsim'_i=\succsim_i$ for every $i\in N\setminus\{j\}$
- $w\succsim'_j z$ if and only if $w\succsim_j z$ for every $w\in X\setminus\{x\}$ and $z\in X\setminus\{x\}$
- either $y\succsim_j x$ and $x\succ'_j y$ for some $y\in X$, or $y\succ_j x$ and $x\succsim'_j y$ for some $y\in X$ (or both)

is in $\mathscr{D}(X)$, then

- $x\in F(X,\succsim')$
- $y\notin F(X,\succsim')$
- $F(X,\succsim')\subseteq F(X,\succsim)$.

As we noted in the introduction, this condition is weaker than a condition sometimes given the same name that requires that if an alternative rises in some individual's preferences, then it becomes the unique selected alternative, whether or not it has risen above other alternatives that were was selected originally. An example of a collective choice rule that satisfies our condition but not the stronger condition is a variant of "Black's rule" that selects the set of Condorcet winners if this set is nonempty and the set of alternatives with the lowest Borda count[12] otherwise.[13] (This rule does not satisfy the stronger version of positive responsiveness because the set of Condorcet winners for a problem with two individuals, one who prefers x to z to y and the other of whom prefers y to x to z, is $\{x,y\}$, and this set remains the same if the first individual's preferences change to prefer x to y to z.)

Note that this definition is equivalent to one in which the modified profile $\succsim'$ raises some alternative x in the preferences of several individuals, not just one. (That condition can be obtained by repeatedly applying ours.)

[12] The Borda count of an alternative is the sum of its ranks in the individuals' preferences.

[13] The rule Black (1958, p. 66) suggests differs in that it selects the strict Condorcet winner if one exists, rather than the set of all Condorcet winners, and the set of alternatives with the lowest Borda count otherwise.

May (1952) shows that if the set A of all possible alternatives has two members, the only anonymous, neutral, and positively responsive collective choice rule is majority rule, which selects the alternative favored by a majority of individuals, or both alternatives in the case of a tie.

Definition 8 (Majority rule)**.** If the set A of all possible alternatives has two members, *majority rule* is the collective choice rule that associates with every collective choice problem its set of Condorcet winners.

May establishes his result for the domain consisting of all preference profiles, but it holds also for the domain consisting of all preference profiles without indifference.[14] Before stating his result, we give names to these domains and define them precisely.

Definition 9 (Domains $\mathscr{U}$ and $\mathscr{S}$)**.** For the domain $\mathscr{U}$, for every set $X \subseteq A$ the set $\mathscr{U}(X)$ consists of all preference profiles over X. For the domain $\mathscr{S}$, for every set $X \subseteq A$ the set $\mathscr{S}(X)$ consists of all preference profiles over X for which no individual is indifferent between any two members of X.

Theorem 1 (May)**.** *Let the domain $\mathscr{D}$ be either $\mathscr{U}$ or $\mathscr{S}$. If the set A of all possible alternatives has two members then the only collective choice rule that is anonymous, neutral, and positively responsive on $\mathscr{D}$ is majority rule.*

Note that an implication of this result is that for any collective choice rule F that is anonymous, neutral, and positively responsive on either $\mathscr{U}$ or $\mathscr{S}$, for any set $X \subseteq A$ with two members $F(X, \succsim)$ is the set of Condorcet winners of $(X, \succsim)$.

For problems with three or more alternatives, many collective choice rules are anonymous, neutral, and positively responsive. For example, consider scoring rules, defined as follows. For each $j = 1, \ldots, k$, where k is the number of members of X, let $\alpha(j)$ be a number, with $\alpha(1) \geq \alpha(2) \geq \cdots \geq \alpha(k)$. For each individual $i \in N$ and each $x \in X$, denote by $\rho_i(x)$ the rank of x according to $\succsim_i$. The *scoring rule with weights* $\alpha(1), \ldots, \alpha(k)$ selects the set of alternatives x that maximize $\sum_{i \in N} \alpha(\rho_i(x))$. Any scoring rule with $\alpha(1) > \alpha(2) > \cdots > \alpha(k)$ is anonymous, neutral, and positively responsive.

[14] See, for example, Moulin (1988, Exercise 11.2, p. 313).

Every scoring rule, however, has an undesirable feature: for some set of individuals and some preference profile $\succsim$ over $\{x, y, z\}$ for these individuals, the rule selects x for the collective choice problem $(\{x, y, z\}, \succsim)$ and y for the problem $(\{x, y\}, \succsim)$.[15] That is, the alternative selected depends on the presence of an alternative that is not selected.

The condition that we add to May's list rules out such behavior. It says that removing unchosen alternatives does not affect the set of alternatives selected. This condition is an adaptation to collective choice rules of a condition for point-valued solutions used by Nash (1950, condition 7 on p. 159) that is called "independence of irrelevant alternatives" in the literature on axiomatic models of bargaining.[16] We call it Nash independence.[17]

Definition 10 (Nash independence)**.** For any domain $\mathscr{D}$, the collective choice rule F is *Nash independent on* $\mathscr{D}$ if for every collective choice problem $(X, \succsim)$ with $\succsim \in \mathscr{D}(X)$ and every set $X' \subset X$ for which $\succsim|_{X'} \in \mathscr{D}(X')$,

$$F(X, \succsim) \subseteq X' \quad \text{implies} \quad F(X', \succsim) = F(X, \succsim).$$

The "Chernoff condition" (Chernoff 1954, Postulate 4, p. 429)[18] is another possible adaptation of Nash's condition to collective choice rules. This condition says that if the set of alternatives selected from X includes y (among possibly other alternatives), then y is among the alternatives selected from any subset of X that contains y (but not necessarily other alternatives that were selected from X). The condition neither implies nor is implied by Nash independence.[19] We argue that Nash independence, which treats the

[15] For example, there are seven individuals and $z \succ x \succ y$ for three of them, $x \succ y \succ z$ for two of them, $x \succ z \succ y$ for one of them, and $y \succ z \succ x$ for the remaining individual. (Fishburn 1984 gives this example.)

[16] Nash himself did not call the condition "independence of irrelevant alternatives". Note that it differs from the axiom with the same name used by Arrow (1963, p. 27).

[17] Chernoff (1954, p. 430) calls it Postulate 5*; some authors refer to it as "Aizerman", after its appearance in Aizerman and Malishevski (1981, p. 1033) (who call it "Independence of rejecting the outcast variants").

[18] Called "property α" by Sen (1969, p. 384).

[19] The Chernoff condition, unlike Nash independence, requires that if the alternative y is selected from X then it is selected also from subsets of X that contain y but not other alternatives that the rule selects from X. Nash independence requires that if Y is the set selected from X, then Y is exactly the set selected from any subset of X that contains Y, whereas the Chernoff condition allows the set selected from such a subset to include additional alternatives.

selected set as a unit, is appropriate in our model, where the collective choice rule is set-valued. If the *set* of alternatives selected from X is contained in $Y \subset X$ then it should be selected from Y; the fact that a *member* of the set of alternatives selected from X is a member of Y is not convincing evidence that this alternative should be a member of the set selected from Y.

If there are two individuals and any number of alternatives, the collective choice rule that selects the set of Condorcet winners (which in this case coincides with the set of Pareto efficient alternatives) satisfies all four of our conditions (anonymity, neutrality, and positive responsiveness, and Nash independence).

If the numbers of individuals and alternatives are both at least three, collective choice rules that satisfy Nash independence and any two of our other conditions (anonymity, neutrality, and positive responsiveness) are also easy to find.

Anonymity, neutrality, and Nash independence The collective choice rule that assigns the set X of all alternatives to the problem $(X, \succsim)$ is anonymous, neutral, and Nash independent on any domain. Two other rules that satisfy these properties are the rule that assigns to every problem the set of Pareto efficient alternatives and the one that assigns the top cycle set.[20]

Anonymity, positive responsiveness, and Nash independence For any given complete, irreflexive, and transitive binary relation on A, the rule that selects the set of maximal elements according to the relation is anonymous, positively responsive, and Nash independent on any domain. (We refer to such rules as *constant rules*.)

Neutrality, positive responsiveness, and Nash independence The rule that selects the favorite alternatives of individual 1, breaking ties according to the preferences of individual 2, and further breaking any ties according to the preferences of individual 3, and so on ("serial dictatorship"), is neutral, positively responsive, and Nash independent on any domain.

[20] The smallest (nonempty) set with respect to set inclusion such that every alternative in the set is preferred by a majority of individuals to every alternative outside the set.

We now show that, for an arbitrary number of individuals, only the collective choice rule that selects the strict Condorcet winner satisfies all four conditions on the domain of collective choice problems that have strict Condorcet winners. We show also that on the domain of all preference profiles, no rule satisfies all four conditions.

3. Results

3.1 Condorcet winners

For a collective choice problem that has a strict Condorcet winner, selecting that alternative is attractive. We now establish that our four conditions imply the selection of the strict Condorcet winner on the domain for which such an alternative exists and no individual is indifferent between any two alternatives.

Definition 11 (Domain $\mathscr{C}$)**.** For the domain $\mathscr{C}$, for every set $X \subseteq A$ the set $\mathscr{C}(X)$ consists of the set of preference profiles $\succcurlyeq$ over X for which no individual is indifferent between any two alternatives and $(X, \succcurlyeq)$ has a strict Condorcet winner.

Theorem 2. *A collective choice rule F is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{C}$ if and only if for every $X \subseteq A$ and preference profile $\succcurlyeq \in \mathscr{C}(X)$, $F(X, \succcurlyeq)$ contains a single alternative, the strict Condorcet winner of $(X, \succcurlyeq)$.*

Proof. A collective choice rule F that selects the strict Condorcet winner of each collective choice problem $(X, \succcurlyeq)$ with $\succcurlyeq \in \mathscr{C}(X)$ satisfies the four properties on $\mathscr{C}$.

Now let $X \subseteq A$ and $\succcurlyeq \in \mathscr{C}(X)$, and let $c \in X$ be the strict Condorcet winner for the collective choice problem $(X, \succcurlyeq)$. Let F be a collective choice rule that is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{C}$ and suppose, contrary to our claim, that $F(X, \succcurlyeq)$ contains an alternative different from c.

Step 1. *$F(X, \succcurlyeq)$ contains at least two alternatives different from c.*

Proof. If $F(X, \succcurlyeq) = \{x, c\}$ or $F(X, \succcurlyeq) = \{x\}$ for some $x \in X \setminus \{c\}$ then by Nash independence we have $F(\{x, c\}, \succcurlyeq) = F(X, \succcurlyeq)$. Given that c is the strict Condorcet winner of $(X, \succcurlyeq)$, it is the strict Condorcet winner of $(\{x, c\}, \succsim)$, so that by May's theorem $F(\{x, c\}, \succsim) = \{c\}$, a contradiction. $\lhd$

Step 2. *Let $x_1, \ldots, x_k$ be the distinct alternatives in $F(X, \succsim)$ in addition, possibly, to c. Then for each individual i there exist x_j and x_l in $\{x_1, \ldots, x_k\}$ such that $x_j \succ_i c \succ_i x_l$.*

Proof. By Step 1, $k \geq 2$. Suppose, contrary to the claim, that for some individual i either $c \succ_i x_l$ for all $l \in \{1, \ldots, k\}$ or $x_l \succ_i c$ for all $l \in \{1, \ldots, k\}$. Assume, without loss of generality, that $x_l \succ_i x_k$ for all $l \in \{1, \ldots, k-1\}$. Derive the preference profile $\succsim'$ from $\succsim$ by raising x_k in i's preferences as in the definition of positive responsiveness, with $x_k \succ'_i x_j$ for $j = 1, \ldots, k-1$ and, if $c \succ_i x_l$ for all $l \in \{1, \ldots, k\}$, $c \succ'_i x_k$. Then by positive responsiveness we have $x_j \notin F(X, \succsim')$ for $j = 1, \ldots, k-1$, so that either $F(X, \succsim') = \{x_k\}$ or $F(X, \succsim') = \{x_k, c\}$. But the ranking of c relative to every other alternative is the same in $\succsim$ as it is in $\succsim'$, so c is the strict Condorcet winner of $(X, \succsim')$, contradicting Step 1 applied to $\succsim'$. $\lhd$

Step 3. *$F(X, \succsim)$ contains at least three alternatives different from c.*

Proof. By Step 1, $F(X, \succsim)$ contains at least two alternatives different from c. If it contains exactly two such alternatives, say x and y, then by Step 2 we have either $x \succ_i c \succ_i y$ or $y \succ_i c \succ_i x$ for every individual i, in which case c is not the strict Condorcet winner of $(X, \succsim)$. $\lhd$

Now let $x_1, \ldots, x_k$ be the alternatives in $F(X, \succsim)$ in addition, possibly, to c, and denote by $x \in \{x_1, \ldots, x_k\}$ the alternative that individual 1 ranks lowest among those in $\{x_1, \ldots, x_k\}$ that she prefers to c. (Such an alternative exists by Step 2.) Derive $\succsim'$ from $\succsim$ by raising x to the top of 1's ranking (without changing the ordering of the other alternatives or changing any other individual's preferences). Then c is a Condorcet winner for $(X, \succsim')$ (the ranking of c relative to every other alternative is the same in $\succsim$ and $\succsim'$), so that $\succsim' \in \mathscr{C}(X)$. Thus by positive responsiveness, $x \in F(X, \succsim')$ and the only other alternatives that can belong to $F(X, \succsim')$ are c and the alternatives in $\{x_1, \ldots, x_k\}$ worse for individual 1 than c. Denote the alternatives in $\{x_1, \ldots, x_k\}$ that belong to $F(X, \succsim')$ by $\{y_1, \ldots, y_r\}$. By Step 3, $r \geq 2$. Denote the alternative in $F(X, \succsim')$ that individual 1 least prefers by y.

Now further modify individual 1's preferences: derive $\succsim''$ from $\succsim'$ by raising y so that it is the best alternative for individual 1 among those worse than c. Then as before, $\succsim'' \in \mathscr{C}(X)$. So by positive responsiveness we have $y \in F(X, \succsim'')$ and the only other alternatives that can belong to $F(X, \succsim'')$ are x and c, contradicting Step 3. □

3.2 Nonexistence for large domains

Theorem 2 shows that for the domain $\mathscr{C}$ of problems that have a strict Condorcet winner, the only collective choice rule that is anonymous, neutral, positively responsive, and Nash independent is the one that selects the strict Condorcet winner. We now argue that while collective choice rules satisfying the four conditions exist for some larger domains, no rule satisfies them for the domain of all preference profiles (Section 3.2.1) or for the domain of all profiles in which no individual is indifferent between any two alternatives (Section 3.2.2).

Let $\mathscr{D}$ be a domain for which for each set $X \subseteq A$, $\mathscr{D}(X)$ consists of $\mathscr{C}(X)$ plus exactly one profile $\succsim$ for which $(X, \succsim)$ does not have a strict Condorcet winner. Consider the collective choice rule that assigns the strict Condorcet winner to each problem that has such a winner, and the set of all alternatives to any problem without a strict Condorcet winner. This rule is clearly anonymous, neutral, and Nash independent.[21] It is also positively responsive, by the following argument. The rule specifies a tie only for problems without a strict Condorcet winner. If the preference profile for such a problem is changed by raising alternative x in some individual's preferences in such a way that a profile within the domain is generated, then the collective choice problem associated with this new profile has a strict Condorcet winner. Raising x in one individual's preferences cannot make some *other* alternative a strict Condorcet winner, so it must make x a strict Condorcet winner and hence the unique alternative chosen by the collective

[21] Note that the rule does not satisfy the Chernoff condition. For a problem without a strict Condorcet winner, let x and y be alternatives such that a majority of individuals prefer x to y. Then the Chernoff condition requires that for the problem in which the set of alternatives is $\{x, y\}$ both x and y are chosen, violating May's theorem. We view this argument as an illustration of the inappropriateness of the Chernoff condition when the collective choice rule is set-valued. For a problem $(X, \succsim)$ without a strict Condorcet winner, the rule we define selects X *as a unit* and does not have implications for the set of alternatives chosen from subsets of X.

choice rule. (We do not know the characteristics of the largest domain for which a collective choice rule satisfying the four conditions exists.)

We now show the nonexistence of a collective choice rule that is anonymous, neutral, positively responsive, and Nash independent. We first establish a key component of our arguments: if a collective choice problem has no Condorcet winner then any anonymous, neutral, positively responsive, and Nash independent collective choice rule assigns to the problem a set consisting of at least two alternatives.

Lemma 1. *Let $\mathscr{D}$ be either $\mathscr{U}$ or $\mathscr{S}$ and suppose that the collective choice problem $(X, \succsim)$, with $\succsim \in \mathscr{D}(X)$, has no strict Condorcet winner. If the collective choice rule F is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{D}$ then $F(X, \succsim)$ is not a singleton.*

Proof. Suppose $F(X, \succsim) = \{x\}$ for some $x \in X$. The fact that x is not a strict Condorcet winner means that for some alternative $y \in X \setminus \{x\}$ the number of individuals i for whom $y \succ_i x$ is at least the number for whom $x \succ_i y$, so that y is a Condorcet winner of $(\{x, y\}, \succsim)$. Thus May's theorem implies that $y \in F(\{x, y\}, \succsim)$. But $F(X, \succsim) = \{x\} \subset \{x, y\} \subseteq X$, so by Nash independence, $F(\{x, y\}, \succsim) = F(X, \succsim) = \{x\}$, a contradiction. □

3.2.1 Domain of all preference profiles

If there are at least three individuals and three alternatives, no collective choice rule is anonymous, neutral, positively responsive, and Nash independent on the domain $\mathscr{U}$ of all preference profiles.

Theorem 3. *If both the set A of all possible alternatives and the set N of individuals have at least three members, no collective choice rule is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{U}$.*

Proof. Let F be a collective choice rule that is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{U}$.

Denote the number of members of N by n. Choose a subset X^* of A with three members, x, y, and z. Let $\succsim^*$ be the preference profile in which the preference orderings of individuals 1, 2, and 3 are

$$x \succ_1^* y \succ_1^* z$$
$$z \succ_2^* x \succ_2^* y$$
$$y \succ_3^* z \sim_3^* x$$

(notice that individual 3 is indifferent between x and z) and every other individual i is indifferent between all three alternatives:

$$x \sim_i^* y \sim_i^* z \text{ for } i = 4, \ldots, n.$$

The problem $(X^*, \succsim^*)$ has no strict Condorcet winner: only individual 1 prefers x to z, only individual 2 prefers z to x, and only individual 3 prefers y to x. Thus by Lemma 1, $F(X^*, \succsim^*)$ contains at least two alternatives, so that if $z \notin F(X^*, \succsim^*)$ then $F(X^*, \succsim^*) = \{x, y\}$. Let $\succsim^{**}$ be the preference profile that differs from $\succsim^*$ only in that y is raised to be indifferent with x, so that $y \sim_1^{**} x \succ_1^{**} z$. Then $F(X^*, \succsim^{**}) = \{y\}$ by positive responsiveness, so that $F(\{x, y\}, \succsim^{**}) = \{y\}$ by Nash independence, contradicting May's theorem because x and y are both Condorcet winners of $(\{x, y\}, \succsim^{**})$. Thus $z \in F(X^*, \succsim^*)$.

Now let $\succsim'$ be the preference profile that differs from $\succsim^*$ only in individual 3's preference relation, in which y is indifferent to z rather than being preferred to it:

$$y \sim_3' z \succ_3' x.$$

Then by positive responsiveness, $F(X^*, \succsim') = \{z\}$. Thus $F(\{y, z\}, \succsim') = \{z\}$ by Nash independence, contradicting May's theorem because y and z are both Condorcet winners of $(\{y, z\}, \succsim')$. □

3.2.2 Domain of strict preference profiles

For the domain $\mathscr{S}$ of preference profiles in which no individual is indifferent between any two alternatives, no collective choice rule is anonymous, neutral, positively responsive, and Nash independent if both the number of alternatives and the number of individuals are at least three and one of them is at least four. (This result implies Theorem 3 except when there are exactly three alternatives and three individuals.)

Theorem 4. *If both the set A of all possible alternatives and the set N of individuals have at least three members and at least one of these sets has at least four members, no collective choice rule is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{S}$.*

Proof. Let F be a collective choice rule that is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{S}$.

Denote by n the number of individuals (members of N) and first suppose that $n \geq 4$. Choose a subset X^* of A with three members and let $X^* = \{x, y, z\}$. Define the preference relations $\succsim^1$, $\succsim^2$, and $\succsim^3$ by

$$x \succ^1 y \succ^1 z$$
$$y \succ^2 z \succ^2 x$$
$$z \succ^3 x \succ^3 y$$

and denote by $\succsim^*$ the preference profile that consists of the first n members of the infinite sequence of preference relations consisting of repetitions of the triple $(\succsim^1, \succsim^2, \succsim^3)$. (Expressing n as $3s + t$ for integers s and t with $t \in \{0, 1, 2\}$, $\succsim^*$ consists of $s + 1$ copies of $\succsim^j$ for $j \leq t$ and s copies of $\succsim^j$ for $j \geq t + 1$.)

We first argue that $z \notin F(X^*, \succsim^*)$. Suppose to the contrary that $z \in F(X^*, \succsim^*)$. Let $\succsim'$ be the preference profile that differs from $\succsim^*$ only in individual 1's preference relation, in which z becomes the top-ranked alternative:

$$z \succ'_1 x \succ'_1 y.$$

Then $F(X^*, \succsim') = \{z\}$ by positive responsiveness and hence $F(\{y, z\}, \succsim') = \{z\}$ by Nash independence. Now, for the preference relations $\succsim^1$ and $\succsim^2$ the alternative y is preferred to z, so y is a Condorcet winner of the problem $(\{y, z\}, \succsim')$. (For $n = 4$, individuals 1 and 3 prefer z to y and individuals 2 and 4 prefer y to z; for $n \geq 5$, more individuals prefer y to z than z to y.) Thus $y \in F(\{y, z\}, \succsim')$ by May's theorem, contradicting $F(\{y, z\}, \succsim') = \{z\}$.

Now, the problem $(X^*, \succsim^*)$ has no strict Condorcet winner. (More individuals prefer x to y than y to x, more prefer y to z than z to y, and at least as many prefer z to x as x to z.) So given $z \notin F(X^*, \succsim^*)$, Lemma 1 implies that $F(X^*, \succsim^*) = \{x, y\}$, and hence $F(\{x, y\}, \succsim^*) = \{x, y\}$ by Nash independence. But x is the strict Condorcet winner for $(\{x, y\}, \succsim^*)$, so $F(\{x, y\}, \succsim^*) = \{x\}$ by May's theorem, a contradiction.

Now suppose that $n = 3$ and the number of alternatives is at least four. Choose a subset X^* of A with four members and let $X^* = \{w, x, y, z\}$. Define the preference profile $\succsim$ by

$$z \succ_1 w \succ_1 x \succ_1 y$$
$$x \succ_2 y \succ_2 z \succ_2 w$$
$$y \succ_3 w \succ_3 x \succ_3 z.$$

Suppose that $z \in F(X^*, \succsim)$. Let $\succsim'$ be the preference profile that differs from $\succsim$ only in individual 3's preference relation, in which z becomes the second-ranked alternative:

$$y \succ_3' z \succ_3' w \succ_3' x.$$

Then $z \in F(X^*, \succsim')$ by positive responsiveness and hence $z \in F(\{y, z\}, \succsim')$ by Nash independence, contradicting May's theorem because y is the unique Condorcet winner for $(\{y, z\}, \succsim')$. Thus $F(X^*, \succsim) \subseteq \{w, x, y\}$.

Suppose that $y \notin F(X^*, \succsim)$. Then Lemma 1 implies that $F(X^*, \succsim) = \{w, x\}$, and hence $F(\{w, x\}, \succsim) = \{w, x\}$ by Nash independence, contradicting May's theorem because w is the strict Condorcet winner for $(\{w, x\}, \succsim)$. The same argument applies to the alternatives w and x, so we conclude that $F(X^*, \succsim) = \{w, x, y\}$.

Now let $\succsim''$ be the preference profile that differs from $\succsim$ only in individual 3's preference relation, in which w becomes the top-ranked alternative:

$$w \succ_3'' y \succ_3'' x \succ_3'' z.$$

Then $F(X^*, \succsim'') \subseteq \{w, x\}$ by positive responsiveness, so $F(\{w, x\}, \succsim'') = \{w, x\}$ by Nash independence and Lemma 1, contradicting May's theorem because w is the unique Condorcet winner for $(\{w,x\},\succsim'')$. □

Remark For a society with three individuals and three alternatives, a collective choice rule that is anonymous, neutral, positively responsive, and Nash independent on $\mathscr{S}$ *does* exist: the rule that assigns the top cycle set (see footnote 20) to each problem. As we remarked on p. 10, this rule is anonymous, neutral, and Nash independent on every domain. It is positively responsive by the following argument. If the top cycle set contains a single alternative, this alternative is a strict Condorcet winner, and remains a strict Condorcet winner if it improves in some individual's preferences. Otherwise, the top cycle set contains all three alternatives and no alternative is a strict Condorcet winner; if an alternative improves in some individual's preferences then it becomes the strict Condorcet winner.

Remark The proof of Theorem 3 shows that the result holds for a weaker combination of conditions. Specifically, replace anonymity and neutrality with the condition that the rule coincides with majority rule for sets of alternatives with two members; require positive responsiveness only for changes in preferences that raise x to the top of a ranking (a condition we refer to as *weak positive responsiveness*); and weaken Nash independence to require only $F(X',\succsim)\subseteq F(X,\succsim)$) (a condition we refer to as *weak Nash independence*[22]). Theorem 4 holds also for this set of conditions except in the case of five individuals and three alternatives; details are included in the Appendix.

4. Related work

Two lines of work are related to our characterization of the collective choice rule that selects the strict Condorcet winner (Theorem 2).

Like us, Alemante et al. (2016) extend May's result to three or more alternatives. A

[22]The condition is called ε^+ by Bordes (1983, p. 125), Aizerman by Moulin (1985, p. 154), and δ^* by Deb (2011, p. 340).

key difference is that they restrict the choice rule to be single-valued.[23] In our view, this restriction is not appealing; it leads to difficulty even in the simplest collective choice problems, with two individuals and two alternatives where one individual prefers a to b and the other prefers b to a. Another difference is that their version of positive responsiveness (called "monotonicity", p. 768) requires the raised alternative to remain chosen *and* the resulting preference profile to remain in the domain. By contrast, our condition imposes the weaker requirement that the raised alternative remains chosen *if* the resulting profile remains in the domain.

Dasgupta and Maskin (2008) (who extend work by Maskin 1995) take a different approach to highlight the appeal of "simple majority" rule. They show that among collective choice rules, this rule satisfies a set of appealing conditions on the largest domain. Two significant differences between our conditions and theirs are that they require a collective choice rule to be "almost always" single-valued, and they impose the Chernoff condition.[24] As we have discussed, we have concerns about the independent appeal of single-valuedness; also we believe that Nash independence is more compelling for collective choice rules than the Chernoff condition.

Less closely related to our work is Goodin and List (2006). Departing from most work in the field, they restrict attention to collective choice rules that depend only on the individuals' favorite alternatives. With this restriction, they show that only plurality rule[25] satisfies (natural adaptations of) May's conditions. In our view, a significant limitation of their approach is that information about favorite alternatives is not always rich enough to capture the intuitive notion of "collective preference" for a preference profile.[26]

Our results on the nonexistence of collective choice rules satisfying our conditions on large domains, Theorems 3 and 4, are part of the huge literature initiated by Arrow

[23] Campbell and Kelly (2003, 2015, 2016) also consider single-valued rules, but the axioms they use to characterize the rule that selects the strict Condorcet winner are unrelated to May's.

[24] Dasgupta and Maskin call this condition "independence of irrelevant alternatives" (though it is formally distinct from Arrow's condition of the same name). We discuss the differences between the Chernoff condition and Nash independence in the paragraph following Definition 10.

[25] Plurality rule selects the alternatives that are most preferred by the largest number of individuals.

[26] Suppose, for example, that a is the favorite alternative of slightly less than a third of the individuals and second-best for every other individual. If the favorite of these individuals is equally split between b and c, then a (which is the strict Condorcet winner) appears to better reflect the collective preference than b or c.

(1951). One of our key conditions, positive responsiveness, is a variant of the condition "positive association of social and individual values" discussed by Arrow (1963, pp. 25–26) but not used in his classic result about aggregating individual preferences into a transitive collective preference (Theorem 2, p. 97).

The first authors to impose such a requirement were Mas-Colell and Sonnenschein (1972). Their Theorem 2 shows that Arrow's result continues to hold when the requirement on the collective preference is weakened to quasi-transitivity (that is, the strict preference relation is transitive) as long as the preference aggregation rule satisfies a version of positive responsiveness (p. 186).

A quasi-transitive preference relation induces a Nash independent choice rule, so Mas-Colell and Sonnenschein's result has implications for collective choice rules. However, a comparison with our results is not straightforward. On the one hand, Arrow's conditions are more permissive than May's for sets with two alternatives. On the other hand, quasi-transitive preferences impose rationality requirements beyond Nash independence, including the Chernoff condition and Sen's property γ.[27]

Blair et al. (1976) shift the focus from preference aggregation to collective choice. Broadly, their results establish incompatibility between "normative" binary conditions (for choice from sets of two alternatives) and "rationality" conditions (linking choices from larger sets to binary sets). For binary sets, their Theorem 6 imposes Arrow's conditions[28] and the strong version of positive responsiveness described at the end of the introduction. It shows that these conditions are incompatible even with the weak rationality requirement in the Chernoff condition.

The subsequent literature extends Blair et al. in various ways, but generally maintains some version of the Chernoff condition (see, e.g., the results presented in Deb's survey, 2011). A notable exception is Duggan (2017), who instead imposes weak Nash independence and Sen's property γ. His Theorem 12 shows that these rationality conditions are incompatible with normative conditions that are even weaker than those

[27]Property γ requires that an alternative chosen from the collective choice problems $(X, \succcurlyeq)$ and $(Y, \succcurlyeq)$ must be chosen from $(X \cup Y, \succcurlyeq)$ (Sen 1971, p. 314).

[28]They impose a stronger version of Arrow's non-dictatorship condition that also rules out "weak" dictatorship.

considered by Blair et al. (with the main difference being a less demanding responsiveness requirement). Despite the obvious relationship to our work, the same difficulties (discussed in connection with Mas-Colell and Sonnenschein's work) arise in comparing this result with ours: Duggan simultaneously imposes weaker restrictions on choices from binary sets and stronger rationality restrictions on choice from larger sets.[29]

In this sense, the closest comparison with our work is Bordes (1983), who restricts his analysis to collective choice rules that coincide with majority rule on binary sets. His Theorem 2 shows that, within this restricted class of rules, the "uncovered set"[30] rules satisfy some particularly appealing properties—including the rationality properties of weak Nash independence and Sen's γ as well as weak versions of the Pareto condition and our positive responsiveness condition (which imposes only the first condition of Definition 7, namely that $x \in F(X', \succcurlyeq)$ when x is raised).

As we discuss after the proof of Theorem 3, our argument shows that no collective choice rule that coincides with majority rule on binary sets satisfies weak Nash independence and a weakening of positive responsiveness in which the second condition requires $y \notin F(X', \succcurlyeq)$ only if x is raised to the top of an individual's preference ordering. In light of Bordes' result, this raises questions about the implications of imposing a positive responsiveness condition between this version and Bordes' weaker requirement.

Intuitively, the collective choice rules that satisfy each of our two main conditions, positive responsiveness and Nash independence, differ in character. The only overlap seems to be the constant rules or dictatorial rules. This raises the question of whether our Theorems 3 and 4 can be generalized by weakening neutrality to Pareto efficiency and anonymity to non-dictatorship. In recent work, Brandt, Brill, and Harrenstein establish such a result for the domain $\mathscr{S}$ of strict preferences when the collective choice rule satisfies the strong version of positive responsiveness described at the end of the introduction.[31]

[29]As discussed, Nash independence can be weakened to weak Nash independence in our results.

[30]To define one such rule, say that x *beats* y if a strict majority of individuals prefer x to y, and x *thoroughly beats* y if x beats y, any alternative beaten by y is beaten by x, and any alternative that beats x also beats y. The rule that selects, for each choice problem, the alternatives that are not thoroughly beaten defines a restrictive version of the uncovered set (usually attributed to McKelvey—see Duggan 2013).

[31]We are grateful to the authors for making available to us a draft of their paper.

Appendix

The proof of Lemma 1 uses anonymity, neutrality, and positive responsiveness only through May's theorem, so that Lemma 1 remains true if anonymity and neutrality are replaced by the condition that the rule coincides with majority rule for sets of alternatives with two members and positive responsiveness is replaced by weak positive responsiveness. The proof also uses only weak Nash independence.

The following result shows that Theorem 4 holds under weaker conditions than the ones we state.

Theorem 5. *Denote the number of alternatives by k and the number of individuals by n. If $k \geq 3$, $n \geq 4$, and $(k,n) \neq (3,5)$, then no collective choice rule coincides with majority rule for sets of alternatives with two members and satisfies weak positive responsiveness and weak Nash independence.*

Proof. The argument in the proof of Theorem 4 goes through until the third sentence of the fourth paragraph (beginning "So given"). The rest of this paragraph should be replaced with the following argument. So given $z \notin F(X^*, \succsim)$, Lemma 1 implies that $F(X^*, \succsim^*) = \{x, y\}$. Let $\succsim^{**}$ denote the preference profile that differs from $\succsim^*$ only in that y is raised to the top of $\succsim_1$. By weak positive responsiveness, $F(X^*, \succsim^{**}) = \{y\}$, so that $F(\{x,y\}, \succsim^{**}) = \{y\}$ by weak Nash independence. But if $n \neq 5$, x is the strict Condorcet winner for $(\{x,y\}, \succsim^{**})$, so $F(\{x,y\}, \succsim^{**}) = \{x\}$ by the condition that the rule coincides with majority rule for sets of alternatives with two members. This contradiction establishes the result for $k \geq 3$ and $n \geq 4$ with $n \neq 5$.

For $k \geq 4$ and $n = 5$, consider the following profile $\succsim^*$:

$$x \succ_1 y \succ_1 z \succ_1 w$$

$$y \succ_2 z \succ_2 w \succ_2 x$$

$$z \succ_3 w \succ_3 x \succ_3 y$$

$$w \succ_4 x \succ_4 y \succ_4 z$$

$$x \succ_5 y \succ_5 z \succ_5 w.$$

Suppose that $y \in F(\{x,y,z,w\},\succsim^*)$. Let $\succsim^{**}$ denote the preference profile that differs from $\succsim^*$ only in that y is raised to the top of $\succsim_3$. By weak positive responsiveness and weak Nash independence, $F(\{x,y\},\succsim^{**}) = \{y\}$, violating the condition that the rule coincides with majority rule in this case since x is the strict Condorcet winner for $(\{x,y\},\succsim^{**})$. Similar arguments rule out $z \in F(\{x,y,z,w\},\succsim^*)$ and $w \in F(\{x,y,z,w\},\succsim^*)$. Thus, $F(\{x,y,z,w\},\succsim^*) = \{x\}$, which violates Lemma 1 since $(\{x,y,z,w\},\succsim^*)$ has no strict Condorcet winner. □

References

Aizerman, Mark A. and Andrew V. Malishevski (1981), "General theory of best variants choice: some aspects." *IEEE Transactions on Automatic Control*, AC-26, 1031–1041. [9]

Alemante, Fasil, Donald E. Campbell, and Jerry S. Kelly (2016), "Characterizing the resolute part of monotonic social choice correspondences." *Economic Theory*, 62, 765–783. [18]

Arrow, Kenneth J. (1951), *Social choice and individual values*, first edition. Wiley, New York. [19]

Arrow, Kenneth J. (1963), *Social choice and individual values*, second edition. Yale University Press, New Haven, Connecticut. [1, 6, 9, 20]

Barberà, Salvador (1977), "The manipulation of social choice mechanisms that do not leave 'too much' to chance." *Econometrica*, 45, 1573–1588. [3]

Black, Duncan (1958), *The theory of committees and elections*. Cambridge University Press, Cambridge. [7]

Blair, Douglas H., Georges Bordes, Jerry S. Kelly, and Kotaro Suzumura (1976), "Impossibility theorems without collective rationality." *Journal of Economic Theory*, 13, 361–379. [3, 20, 21]

Bordes, Georges (1983), "On the possibility of reasonable consistent majoritarian choice: some positive results." *Journal of Economic Theory*, 31, 122–132. [18, 21]

Brandt, Felix and Paul Harrenstein (2011), "Set-rationalizable choice and self-stability." *Journal of Economic Theory*, 146, 1721–1731. [2]

Campbell, Donald E. and Jerry S. Kelly (2003), "A strategy-proofness characterization of majority rule." *Economic Theory*, 22, 557–568. [19]

Campbell, Donald E. and Jerry S. Kelly (2015), "Anonymous, neutral, and strategy-proof rules on the Condorcet domain." *Economics Letters*, 128, 79–82. [19]

Campbell, Donald E. and Jerry S. Kelly (2016), "Correction to 'A strategy-proofness characterization of majority rule'." *Economic Theory Bulletin*, 4, 121–124. [19]

Chernoff, Herman (1954), "Rational selection of decision functions." *Econometrica*, 22, 422–443. [9]

Dasgupta, Partha and Eric S. Maskin (2008), "On the robustness of majority rule." *Journal of the European Economic Association*, 6, 949–973. [19]

Deb, Rajat (2011), "Nonbinary social choice." In *Handbook of social choice and welfare* (Kenneth J. Arrow, Amartya K. Sen, and Kotaro Suzumura, eds.), volume 2, chapter 19, 335–366, Elsevier, Amsterdam. [18, 20]

Duggan, John (2013), "Uncovered sets." *Social Choice and Welfare*, 41, 489–535. [21]

Duggan, John (2017), "Weak rationalizability and Arrovian impossibility theorems for responsive social choice." Unpublished paper (to appear in *Public Choice*). [20]

Fishburn, Peter C. (1984), "Discrete mathematics in voting and group choice." *SIAM Journal on Algebraic and Discrete Methods*, 5, 263–275. [9]

Goodin, Robert E. and Christian List (2006), "A conditional defense of plurality rule: generalizing May's theorem in a restricted informational environment." *American Journal of Political Science*, 50, 940–949. [19]

Mas-Colell, Andreu and Hugo Sonnenschein (1972), "General possibility theorems for group decisions." *Review of Economic Studies*, 39, 185–192. [20]

Maskin, Eric S. (1995), "Majority rule, social welfare functions, and game forms." In *Choice, welfare, and development* (Kaushik Basu, Prasanta K. Pattanaik, and Kotaro Suzumura, eds.), chapter 6, 100–109, Clarendon Press, Oxford. [19]

May, Kenneth O. (1952), "A set of independent necessary and sufficient conditions for simple majority decision." *Econometrica*, 20, 680–684. [0, 1, 6, 7]

Moulin, Hervé (1985), "Choice functions over a finite set: a summary." *Social Choice and Welfare*, 2, 147–160. [18]

Moulin, Hervé (1988), *Axioms of cooperative decision making*. Cambridge University Press, Cambridge. [4, 8]

Nash, John F. (1950), "The bargaining problem." *Econometrica*, 18, 155–162. [2, 9]

Núñez, Matías and Giacomo Valletta (2015), "The informational basis of scoring rules." *Review of Economic Design*, 19, 279–297. [6]

Sen, Amartya K. (1969), "Quasi-transitivity, rational choice and collective decisions." *Review of Economic Studies*, 36, 381–393. [9]

Sen, Amartya K. (1971), "Choice functions and revealed preference." *Review of Economic Studies*, 38, 307–317. [20]