Stress Testing
A Complement of Value-at-Risk

Rapport de Stage pour l'obtention du Grade de Maîtrise en Sciences (M.Sc.)
en Finance Mathématique et Computationnelle.

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Special thank.

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Executive Summary

In the past, risks were measured using a variety of ad hoc tools, none of which was satisfactory. These included notional amounts, sensitivity measures, and scenarios. While these measures provide some intuition of risk, they do not measure what matters, i.e., the downside risk for the total portfolio. They fail to take into account the probability of adverse moves in the risk factors as well as correlation.

A sensitivity measure such as duration is somewhat helpful. This duration measure reveals the extreme sensitivity of the bond but does not answer the question of whether such a disastrous movement in interest rates is likely. It also ignores the non-linearity between the note price and yields.

Scenario analysis is the best measure so far, as it allows the investor to investigate non-linear, extreme effects in price. But again, the method does not associate the loss with a probability.

Another general problem is that these sensitivity or scenario measures do not allow the investor to aggregate risk across different markets.

The great beauty of value at risk (VAR) is that it provides a neat answer to all these questions. One number aggregates the risks across the whole portfolio, taking into account leverage and diversification, and providing a risk measure with an associated probability.

However VAR is based on a model designed to forecast short term volatility in portfolio values accurately. It is successful in doing that, but the model is not designed to forecast extreme events. Therefore it should be complemented by stress testing, which aims at identifying potential losses under extreme market conditions.

As other financial Institutions National Bank of Canada is also using Stress Testing for its; (1) internal management and decisions and (2)capital allocation challenges for its credit risk. This document starts with the an introduction of the evolution in risk management , followed by the evolution in Regulation and compliance. Value at Risk, Credit Risk, Measuring Credit Risk and Stress Testing and its characteristic are then introduced. The last part of this document is concentrated on Stress Testing at National Bank of Canada on its newly development model which is used by BNC to forecast its average probability of default for its corporate portfolio.

Introduction

As recent events have illustrated, the task of managing financial risk is growing more complex as firms increasingly use more advanced quantitative tools to manage risk. This is certainly not a surprise to anyone in the financial world. Today's risk managers are challenged by the ever-increasing integration of financial products and services, pace of capital flows and the task of applying risk management practices across diverse business operations and geographic locations. The growth of the financial services industry and its increasingly diverse scope means that institutions must be able to manage risks across a far wider range of counterparties, products and business units than in prior times. The securities lending markets have been an excellent example of this trend, with many institutions structuring securities lending transactions off-balance sheet through total return swaps and other derivative structures. Moreover, the increasing pace of capital flows has led global markets to place an increased premium on strong risk management practices and impose greater punishment upon market participants that fail to recognise this new reality.

Indeed, risk management has undergone a dramatic transformation in recent years through advances in both theory and technology. The entire risk management process is now more quantitative, reflecting not only a greater capability and lower cost of collecting and processing data, but also improved techniques for measuring and managing risk. The market for risk management systems illustrates this trend: vendors and consultants have saturated the market with new and advanced applications that incorporate new methodologies and integrated capabilities to capture a wider product base.

These new approaches to risk management have contributed to what economists herald as "more efficient resource allocation." Under this theory, more stringent evaluation has lead to more effective pricing, which, in turn, has lead to a more robust distribution among market participants. As a result, the overall risk-bearing capacity of the market is improved. In simpler terms, improvements in risk modelling have led to the development of innovative markets for the transfer of credit risk, such as credit derivatives and collateralised debt obligations. These relatively new markets have broadened the methods by which all market participants share credit risk and, in turn, have contributed to more efficient pricing of that risk. Despite today's challenging market environment, we are currently witnessing the benefits of stronger risk management with very few firms experiencing capitalisation problems.

From a market risk perspective, most market participants have adopted Value-at-Risk (VAR) analysis and related system implementations in the mid-to-late 1990s. Many firms, however, are continuing to allocate resources to improve existing market risk capabilities and integrate various forms of analytical techniques. Scenario-based stress testing has become the most widely-used form of market risk analysis. Scenario-based stress testing provides a basis for making risk profile determinations, reporting to senior management and setting market risk limits. As most firms' market VAR models have progressed to the point of handling complex structured products such as Collateralized Mortgage Obligation, Collateralized Bond Obligation, Collateralized Loan Obligation, Mortgage Backed

Security, risk practitioners now confront the challenge of incorporating stress testing, event risk, and liquidity risk into VAR frameworks.

From a credit perspective, it is interesting that despite the difficult economic environment and poor performance of the credit markets, lending and other credit standards have not tightened as much as might have been expected. Possible explanations for this trend may be that firms adjusted their lending standards earlier in the economic cycle, the wide-spread use of more robust risk management tools, or simply, that firms are better capitalised and have been taking advantage of favourable credit spreads (i.e., credit premiums for riskier asset classes have remained high relative to historical data). It is also possible that this trend may reflect a growing acceptance of a risk/return-based pricing philosophy, a better understanding of sophisticated risk analytics (e.g., scoring and pricing) and the proliferation of hedging tools. Key developments in credit risk analysis include efforts by many firms to conduct regular testing of their internal risk ratings, using external ratings as benchmarks, in conjunction with internal historical loss experience (in place of the traditional loss avoidance approach). These firms have an advantage in understanding the performance or effectiveness of internal risk-rating systems in contrast to those firms that do not regularly benchmark and, therefore, remain unaware of their own performance.

However, despite advancements in risk analytics, significant challenges remain. Firms recognise the limited ability of statistical models such as VAR to accurately capture what happens in exceptional circumstances. In part , this is due to modelling assumptions that make it easier to compute VAR. However , there is a more fundamental problem with using statistical models like VAR for assessing risks in exceptional circumstances. By definition, exceptional circumstances occurs rarely, and statistically inference is imprecise without a sufficient number of observations. Stress tests partially fill this gap, and thus complement VAR, but offering a quantitative picture of the exposure associated with a possible extreme event. In the absence of a reliable statistical measure of the probability of such an event, stress testing calls on the informed judgement of risk managers and senior executives to assess whether, and to what degree, the firm should move to limit or modify such an exposure. Even if a statistical model could be built that accurately captured risk in extreme circumstances, risk managers and senior management appear likely to prefer to continue using stress testing, because the assumptions underlying such a statistical model would not be transparent.

Since the 1997 Asian crisis, Russian crisis and collapse of LTCM in 1998, the importance of stress testing as a risk management tool has been widely recognised by financial institutions. The committee on the Global Financial System of the Bank for international Settlement has recognised the significance providing for market-wide stress environments, and has conducted various studies. Individual financial institutions also utilise stress tests as an opportunity to consider possible reactions to plausible future events on a firm-wide basis.

The goal is to ensure that the institution can ride out the turmoil. In other words, stress testing can help to guarantee the very survival of the institution.

Regulation and Compliance

One could ask at the outset why regulation are necessary. After all, the owners of a financial institution should be free to set their own economic risk capital. Economic risk capital is the amount of capital that institution would devote to support their financial activities in the absence of regulatory constraints, after careful consideration of the risk-return trade off it's involved.

Indeed shareholders are putting their own capital at risk and suffer the direct consequences of failure to control the respective risk. Essentially, this is what happened to Barings, where complacent shareholders failed to monitor the firm's management. Poor control over traders led to increasingly risky activities and bankruptcy.

The bank of England is reported to have agonised over the decision of whether it should bail out Barings. In the end, it let the bank fail. Many observers said this was the correct decision. In freely functioning capital markets, badly managed institutions should be allowed to fail. This failure also serves as a powerful object lesson in risk management.

Nevertheless, regulation generally is viewed as necessary when free market appear to be unable to allocate resources efficiently. For financial institutions, this is the case for two situations, *externalities* and deposit *insurance.*

Externalities arise when an institution's failure affects other firms. Here , the fear is that of systemic risk. Systemic risk arises when default by one institution has a cascading effect on other firms, thus posing a threat to the stability of the entire financial system. Systemic risk is rather difficult to evaluate because it involves situations of extreme instability, thus happening infrequently. In recent years, however, two medium-sized institutions, Drexel and Barings, failed without creating other defaults.

Deposit insurance also provide a rationale for regulation. By nature, bank deposits are destabilising. Depositors are promised to be repaid the full face value of their investment on demand. They may then rationally cause a "run on the bank" if they fear that a bank's assets have fallen behind its liabilities. Given that bank assets can be invested in illiquid securities or in real estate, the run will force liquidation at great costs.

One solution to this problem is government guarantees for bank deposits, which eliminate the rationale for bank runs. These guarantees are also viewed as necessary to protect small depositors who cannot efficiently monitor their bank. Such monitoring is complex, expensive and time consuming for the thousands of small depositors who entrust their funds to a bank.

This government guarantee is no panacea, for it creates a host of other problems, generally described under the rubric of moral hazard. Given government guarantees, there is even less incentive for depositors to monitor their banks, but rather to flock to institutions offering high deposit rates. Banks owners are now offered what is the equivalent of a "put" option. If they take risks and prosper, they partake in the benefits. If they lose, the

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government steps in and pays back the depositors. As long as the cost of deposit insurance is not related to the riskiness of activities, there will be perverse incentives to take on additional risk. These incentives no doubt played a part in the great savings and loans debacle, where total losses are estimated above $150 billion in US, most of which was paid for by taxpayers. The national commission set up to consider the lessons of this fiasco called deposit insurance the "necessary condition" without which this debacle would not have occurred.

The moral hazard problem due to deposit insurance explain why regulators attempt to control risk taking activities. This is achieved by forcing banks to carry minimum levels of capital, thus providing a cushion to protect the insurance fund. Capital adequacy requirements also can serve as a deterrent to unusual risk taking if the amount of capital to set aside is tied to the amount of risk undertaken.

Still, a remaining issue is the appropriate level of capital required to ensure a "safe and sound" financial system. Historically, regulators have been tempted to set high capital-adequacy levels, just to be safe. Perhaps the bast warning against imposing capital standards that are too high was articulated by Alan Greenspan, chairman of the Federal Reserve, in May 1994

Bank shareholders must earn a competitive rate of return on capital at risk, and returns are adversely affected by high capital requirements.

In times of stress, banks can take steps to reduce their exposure to market risks

"When market forces….break loose of economic fundamentals, sound policy actions, and not just bank capital, are necessary to preserve financial stability"

In Greenspan's views, the management of systemic risk is "properly the job of the central banks", which offer a form of catastrophe insurance against such events.

A more radical approach to the deposit insurance-moral hazard dilemma is to rely on market discipline only. The central bank of New Zealand, for instance, recently has abolished deposit insurance. Thus the Reserve Bank will not bail out failing banks, although it is still responsible for protecting the overall banking system. As a result, depositors must now rely on information provided by commercial banks and ratings agencies to decide whether their funds will be safe. This system puts an increased responsibility on bank directors to ensure that their institution is sound, since failure may lead to creditor lawsuits.

The New Zealand experiment surely will be watched intensely by bank regulators all over the world. In the meantime, the mainstream regulatory path is evolving toward a system where capital requirements are explicitly linked to the risk of activities undertaken by commercial banks.

The 1988 Basel Accord

The Basel Accord represents a landmark financial agreement for the regulation of commercial banks. It was concluded on July 15, 1988, by the central bankers from the Group of Ten(G-10) countries.

The main purposes of the accord were to strengthen the soundness and stability of the international banking system by providing a minimum standard for capital requirements and to create a level playing field among international banks by harmonising global regulations.

The 1988 agreement defined a common measure of solvency, the cookie ratio, that only covers credit risks. Although not statutory, the new ratios were fully implemented in the G-10 countries by December 1992. By now, over 100 countries have adopted the accord, making for more consistent prudential regulations world-wide.

The Cookie Ratio

The Basel Accord required capital to be equal to at least 8 percent of the total risk-weighted assets of the bank. Capital, however, is interpreted more broadly than the usual definition of equity book value, since its goal is to protect deposits. It consists of two components.

Tier 1 capital or "core" capital. Tier 1 capital includes paid up stock issues and disclosed reserve, most notably from the after-tax retained earnings. Such capital is permanent and is regarded as a buffer of the highest quality. This definition is common to all countries' banking systems, and is most visible basis of capital strength. Of the 8 percent capital charge, at least 50 percent must be covered by tier 1 capital.

Tier 2 capital or "supplementary" capital. Tier 2 capital includes perpetual securities, undisclosed reserves, subordinated debt with maturity greater than 5 years, and shares redeemable at the option of the issuer. Since long-term debt has a junior status relative to deposits, it acts as a buffer to protect depositors (and the deposit insurer)

Risk capital weights were classified into four categories, depending on the nature of the asset. These ratios are described in Table 1 on page 10. For instance, U.S Treasuries, being obligations of an Organisation for Economic Co-operation and Development(OECD) government, are assigned a weight of zero. So is cash and gold held by banks. As the perceived credit risk increases, so does the risk weight. At the other end of the scale, claims on corporations, including loans, bonds, and equities, receive a 100 percent weight, which means that effectively they must be covered by 8 percent capital.

The credit risk charge (CRC) is defined as

$$CRC = 8\% * (\text{risk- weighted assets}) = 8\% * (\sum_i w_i \ asset_i)$$

Where w_i is the risk weight attached to asset i. In addition, these guidelines include capital requirements for the credit exposure of derivatives contracts. Signatories to the Basel Accord are free to impose higher capital requirements in their own countries. Accordingly, shortly after the Basel Accord, U.S. legislators passed the Federal Deposit Insurance Corporation soundness of American financial institutions. Among the newly established bank capital requirements, U.S regulators have added the restriction that tier 1 capital must be no less than 3 percent of total assets; this ratio can be set higher for banks deemed to be weaker. The European Union also has issued its own capital requirement rules, known as the Capital Adequacy Directive(CAD), which are in line with the Basel guidelines.

Table 1

Risk Capital Weights by Asset Class

Weights (%)	Asset Type
0	cash held
	claims on OECD central governments
	claims on central governments in national currency
20	cash to be received
	claims on OECD banks and regulated securities firms
	claims on non-OECD banks below 1 year
	claims on multilateral development banks
50	residential mortgage loans
100	claims on the private sector (corporate debt, equity,....)
	Real estate
	Plant and equipement
0-50	(at national discretion)
	claims on domestic OECD public-sector entities, e.g., claims on U.S agencies:20

Activity Restrictions

In addition to capital adequacy requirements, the Basel Accord sets limits on "excessive risk takings". These are restrictions on large risks, defined as positions that exceed 10 percent of a bank's capital. Large risks must be reported to regulatory authorities. Positions that exceed 25 percent of a firm's capital are not allowed, and the total of large risks must not exceed 800 percent of capital. In practice, however, the rules behind these ratios have not always been defined formally and sometimes need clarification from regulatory authorities.

Criticisms of the 1988 Approach.

The 1988 Basel regulations have been criticised on several fronts. As in usually the case with binding regulatory requirements, institutions may find way to get around the restrictions or, even worse, may engage in distorted lending patterns. For instance, banks that are subject to binding regulatory capital may move into areas where expected returns from lending exceed regulatory costs in an attempt to equalise regulatory capital with economic capital.

This has led to regulatory arbitrage, which generally can be defined as behaviour that defeats the regulatory requirements. An example is securitization, which transforms loans into tradable securities, some of which can be sold off or moved into the trading books, which lowers the capital requirements without necessarily decreasing the remaining economic credit risk. Indeed, about a quarter of U.S. banks' balance sheets have been securitised in recent years and the credit quality of loan books has deteriorated.

Another example is credit derivatives, which are akin to credit insurance and can be used to shuffle credit exposures into areas with lower risk weights. This problem, however, arises with any regulatory requirements. As William McDonough, chairman of the Basel Committee, has said: "There isn't a system in the world that can't be gamed." The real issue is whether the 1988 guidelines were so grossly out of step with economic charges for credit risk so as to actually induce dangerous behaviour.

The criticisms of the 1988 Basel Accord can be classified as follows:

- ## Inadequate differentiation of credit risks.

The four risk-weight categories are widely viewed as too crude. The same 100 percent ratio, for instance, is applied to low-risk and high-risk borrowers. Thus a loan to General Electric, which is among the biggest corporations in terms of market capitalisation with an Aaa credit rating, would require the same regulatory capital as a loan to a near-bankrupt company. Also, the original low capital charge for OECD banks become inadequate as the OECD started to include "emerging" economics with shaky banking institutions.

For example, Peregrine's Downfall

Peregrine Investments Holdings, one of Hong Kong's leading investment banks, was one of the victims of the Asian crisis of 1997. The bank suffered from losses such as a loan to PT Steady Sage, an Indonesian taxicab operator, that amounted to $235 million, a quarter of the bank's equity capital. After Asian currencies crashed in the summer of 1997, many Asian borrowers were unable to repay their foreign-currency loans. Peregrine's exposure to Asia led to its collapse on January 13, 1998.

While Peregrine was not subject to the capital adequacy requirements of commercial banks, it did not control credit risk adequately. The head of credit-risk management , John Lee, said that he had not even been informed of the Steady Safe loans, which was the

single biggest item in the firm's bond portfolio! The firm's ruin can be traced to insufficient diversification and lack of proper risk-management controls.

The Asian crisis also revealed patterns in bank lending that had become distorted due to the 1988 capital requirements. In 1997, 60 percent of $380 billion bank lending to Asia had a maturity of less than 1 year, which only carries a 20 percent risk weight for non-OECD banks. Asian banks and borrowers buckled under the combined effects of local currency devaluation and liquidity problems due to these short maturities.

- **Non recognition of term structure effects**. Even when controlling for the credit rating, the term of the loan is an important factor in measuring credit risk. A 2 year loan to an AA-rated company , for instance, has very little risk of default. In contrast, a 30 year loan to the same company is much riskier.

- **Non recognition of risk-mitigation techniques**. These techniques, such as netting or the use of collateral, decrease the economic credit risk but are not recognised under the 1988 rules. Netting refers to a legal agreement whereby payment obligations between two parties are amalgamated into one single, net obligation. As a result of netting, counterparty failure will lead to a smaller loss if the amount lent is matched by the amount borrowed. Similarly , credit losses will be lessened if the bank holds collateral. The fact that these prudent risk –mitigation techniques are not recognised under the 1988 Basel Accord is a significant problem because it discourages and even penalises banks for attempting to control credit risk better.

- **Non recognition of diversification effects**. The rules do not recognise that credit risk can, and should , be mitigated through spreading risks across issuers, industries, and geographic locations. As long as correlations between components of the portfolio are below one, simple summing the capital charges will overstate the true risk. Again, this is significant problem because the 1988 Basel Accord discourages prudent diversification.

- **Non recognition of market risk**. Finally, the 1988 Basel Accord did not account for the market risk assumed by banks. This omission was particularly glaring with the growth in proprietary trading activities(i.e., trading for their own account) and derivatives. In recognition this drawback, the Basel Committee has added a capital charge for market risk.

The bottom line is that besides falling to encourage prudent diversification, the 1988 Basel Accord leads to capital charges that have been estimated to be twice the economic charges estimated by major U.S banks.

The 1996 Amendment on Market Risks

In 1996, the Basel Committee amended the Basel Capital Accord to incorporate market risks. This amendment, which came into force at the end of 1997, added a capital charge for market risk based on either of two approaches, the standardised method or the internal models method.

The amendment separated the bank's assets into two categories:

- **Trading book**. This is the bank portfolio containing financial instruments that are intentionally held for short-term resale and typically are marked-to-market

- **Banking book.** This consists of other instruments, mainly loans.

The amendment adds a capital charge for the market risk of trading books, as well as for the currency and commodity risk of the banking book. The credit risk charge now excludes debt and equity securities in the trading book and positions in commodities but still includes all OTC derivatives, whether in the trading or banking books.

To obtain total capital-adequacy requirements, banks should add their credit risk charge to their market risk charge(MRC)

TRC= CRC + MRC

In exchange for having to allocate additional capital, banks were allowed to use a new class of capital, *tier 3 capital,* which consists of short-term subordinated debt. The amount of tier 3 capital(tier2 capital or both) is limited to 250 percent of tier 1 capital allocated to support market risks.

The standardised method

The first approach, originally proposed in April 1993, is based on a pre-specified "building-block approach". The bank's market risk is first computed for portfolios exposed to interest rate risk, exchange rate risk, equity risk, and commodity risk using specific guidelines. The bank's total risk is then obtained from the summation of risks across the four categories. Because the construction of the risk charge follows a highly structured and standardised process, this approach is sometimes called the standardised method.

For interest rate risk, the rules define a set of maturity banks, within which net positions are identified across all on-and off-balance sheet items. A duration weight is then assigned to each of the 13 bands, varying from 0,20 percent for positions under 3 months to 12,50 percent for positions over 20 years. The sum of all weighted net positions then yields on overall interest rate risk indicator. Note that the netting of positions within a band and aggregation across bands assume perfect correlation across debt instruments. For currency and equity risk, the market risk capital charge is essentially 8 percent of the net position,

whereas for commodities, the charge is 15 percent. In total, the market risk charge is the arithmetic sum of market risk charges for individual positions.

$$MRC^{STD} = \sum_i MRC_i$$

Although this approach aims at identifying banks with unusual exposure, it is still beset by problems. The duration of some instruments cannot be identified easily. Mortgages, for instances, contain prepayments options that allow the homeowner to refinance the loan if interest rates fall. This risk is known as contraction risk. Conversely homeowners will make payments over a longer period if interest rates increase. This risk is called extension risk. The effective duration of mortgages changes with the level of interest rates and the past history of prepayment for a mortgage pool. Assigning a duration band to one of these instruments becomes highly questionable. More generally, the risk classification is arbitrary. The capital charges of 8 percent are applied uniformly to equities and currencies(and gold) without regard for their actual return volatilities.

Another issue is that the standardised method does not account for diversification across risks. Loss correlation imply that the risk of a portfolio can be much less than the sum of individual component risks. This diversification effect applies across market risks or across different types of financial risks.

Diversification across market risk is the easiest to measure. Historical data reveal that correlation across markets are not perfect. Investing across global fixed-income markets, for instance, is less risky than investing in a single market. Similarly, exchange rate movement are not perfectly correlated, no are movements between interest rates and exchange rates. Assuming perfect correlation across various types of risks overestimates portfolio risk and leads to capital adequacy requirements that are too high.

Correlation across different types of risks are more difficult to deal with. Most notably, credit risk may be related to interest rate risk. This is true for most floating-rate instruments, where borrowers may default should interest rates increase to insufferable amounts. By simple adding up the credit and market risk charges, the Basel Committee has implicitly assumed perfect correlation between these risks, which is the worst-case scenario.

The Internal Models Approach.

In response to industry criticisms to the standardised method, the Basel Committee came forth with a major alternative in April 1995. For the first time, it would allow banks the option of using their own risk-measurement models to determine their capital charge. This decision stemmed from a recognition that many banks had developed sophisticated risk-management systems, in many cases far more complex that could be dictated by regulators. As for institutions lagging behind the times, this alternative provided a further impetus to create sound risk-management systems.

To use this approach, banks first have to satisfy various qualitative requirements. The bank must demonstrate that it has a sound risk management system, which must be integrated into management decisions. It must conduct regular stress tests. The bank also must have an independent risk-control unit as well as external audits. When these requirements are satisfied, the market risk charge is based on the following steps.

- **Quantitative parameters.** The computation of VAR shall be based on a set of uniform quantitative inputs

  1. A horizon of 10 trading days or 2 calendar weeks

  2. A 99 percent confidence interval

  3. An observation period based on at least a year of historical data and updated at least once a quarter.

- **Treatment of correlation.** Correlation can be recognised in broad categories(e.g., fixed income) as well as across categories(e.g., between fixed income and currencies)

- **Market risk charge.** The general market capital charge shall be set at the higher of the previous day's VAR or the average VAR over the last 60 business days times a "multiplicative" factor k. The exact value of this multiplicative factor is to be determined by local regulators, subject to an absolute floor of 3. Without this risk factor, a bank would be expected to have losses that exceed its capital in one 10-day period out of a hundred, or about once in 4 years. Also, this factor(sometimes called a hysteria factor) is intended to provide additional protection against environments that are less stable than historical data would lead one to believe.

- **Plus factor.** A penalty component , or plus factor, shall be added to the multiplicative factor k if backtesting reveals that the bank's internal model incorrectly forecasts risks. The purpose of this factor is to give incentives to banks to improve the predictive accuracy of their models and to avoid overly optimistic projection of profits and losses due to model fitting. Since the penalty factor may depend on the quality of internal controls at the bank, this system is designed to reward truthful internal monitoring, as well as developing sound risk management systems.

To summarise , the internal models approach(IMA) market risk charge on any day t is

$$MRC_t^{IMA} = \text{Max} \left(k \frac{1}{60} \sum_i^{60} VAR_{t-i} , VAR_{t-1} \right) + SRC_t)$$

where SRC is the specific risk charge. In practice, banks are allowed to base their 10-day VAR from scaling up their 1-day VAR by the square root of 10. Also note that, due to the multiplier, the charge generally will be driven by the 60-day average instead of the latest VAR. The bank would have to experience a sharp increase in its risk positions or in the market volatility for the previous day's VAR to become the dominant factor.

Value at Risk(VAR)

Definition- Value at Risk

Value at risk, VAR is a summary measure of the downside risk, expressed in dollars.

A general definition is:

VAR is the maximum loss over a target horizon such that there is a low, pre-specified probability that the actual loss will be larger.

Consider for instance a position of $4 billion dollars short the yen, long the dollar. This position corresponds to a well-known hedge fund that look a bet that the yen would fall in value against the dollar. How much could this position lose over a day?

To answer this question, we could use 10 years of historical daily data on the yen/dollar rate and simulate a daily return. The simulated daily return in dollars is then

$$\Re_t(\$) = \phi_0 [S_t - S_{t-1}] / S_{t-1}]$$

Where ϕ_0 is the current dollar value of the position and S is the spot rate in yen per dollar measured over two consecutive days.

For instance , for two hypothetical days $S_1 = 122.0$ and $S_2 = 111.8$. We then have a hypothetical return of

$$\Re_2(\$) = \$ 4000m *[111.8-112.0]/112.0 = \$7.2 \text{ m}$$

So, the simulated return over the first day is –7.2m. Repeating this operation over the whole sample, or 2527 trading days, creates a time-series of fictitious returns.

We can now construct a frequency distribution of daily returns and order the losses from worst to best return

Figure 1
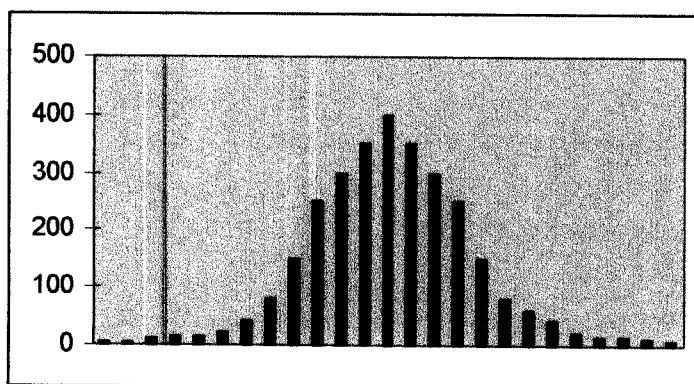
Perhaps the greatest advantage of value at risk (VAR) is that it summarises in a single, easy to understand number the downside risk of an institution due to financial market variables. No doubt this explains why VAR is fast becoming an essential tool for conveying trading risks to senior management, directors and shareholders.

VAR assumes that the portfolio is "frozen" over the horizon or more generally that the risk profile of the institution remains constant. In addition , VAR assumes that the current portfolio will be marked-to-market on the target horizon.

Steps in Constructing VAR

The following steps are required to compute VAR

- Mark-to-Market of the current portfolio(e.g., $100 millions)
- Measure the variability of the risk factors(e.g., 15 percent per annum)
- Set the time horizon, or the holding period(e.g., adjust to 10 business days)
- Set the confidence level(e.g., 99 percent, which yields a 2.33 factor assuming a normal distribution)
- Report the worst loss by processing all the preceding information(e.g., a $7 million VAR)

These steps are illustrated below.
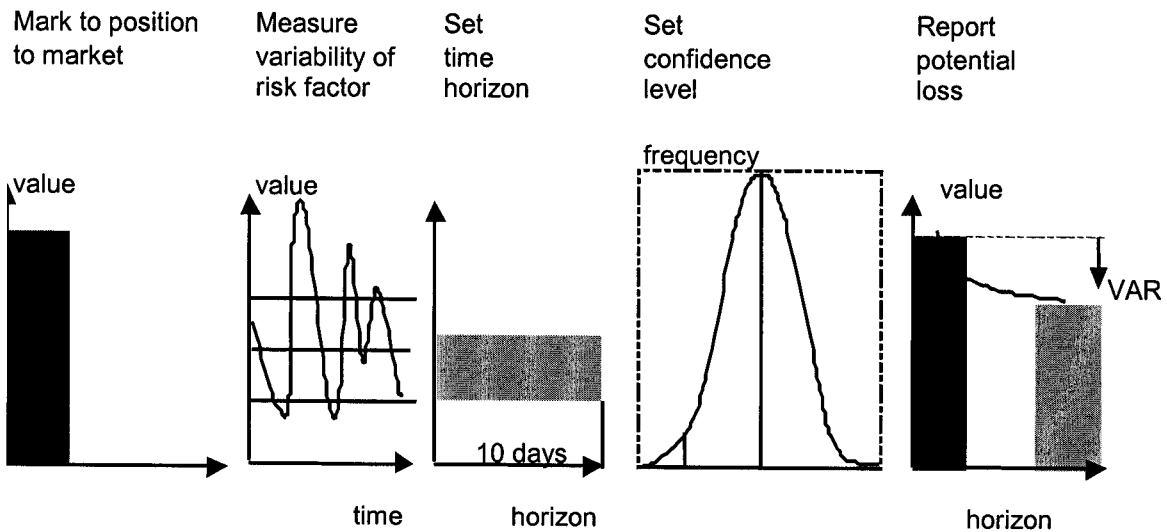
We now wish to summarise the distribution by one number. We could describe the quantile, i.e., the level of loss that will not be exceeded at some high confidence level. Select for instance this confidence level as c = 95 percent. This corresponds to a right-tail cumulative probability. We could as well define VAR in terms of a left-tail probability, which we write as p=1-c

We want to find the cut-off value R^* such that the probability of a loss worse than R^* is p=1-c = 5 percent. With a total of T= 2527 observations, this corresponds to a total of pT = 0,05*2527= 126 observations in the left tail, for example $R^* = \$47$ million. We pick from the ordered distribution the cut-off value. We can now make a statement such as

The maximum loss over one day is about 47$ million at the 95 percent confidence level

Desirable Properties of Risk Measures.

Essentially , a risk measure "maps" the whole distribution of dollar returns X into one summary measure $\rho(X)$. Artzner et al.(1999) list four desirable properties for risk measure for capital adequacy purposes.

- **Monotonicity**: If $X_1 < X_2$, $\rho(X_1) \geq \rho(X_2)$.

In other words, if a portfolio has systematically lower returns than another(in each state of the world), it must have greater risk. Risk is a measure of the extent of downside realisations.

- **Translation Invariance**: $\rho(X_1 + k) = \rho(X_1) + k$

In other words, adding cash k to a portfolio should reduce its risk by k, As with X, k is measured in dollars.

- **Homogeneity**: $\rho(b X_1) = b \rho(X_1)$

In other words, increasing the size of a portfolio by a factor b should scale its risk measure by the same factor b. This property applies to the standard deviation.

- **Subadditivity**: $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$.

In other words, merging portfolio cannot increase risk.

The usefulness of these criteria is that they force us to think about ideal properties and more importantly, potential problems with simplified risk measures.

Indeed, Artzner et al. Show that the quantile-based VAR measure fails to satisfy the latter property. They give some "pathological" examples of positions that combine to create

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portfolios with larger VAR. They also show that the shortfall measure E[-X/X≤-VAR], which is the expected loss conditional on exceeding VAR, satisfies all these desirable coherence properties.

- Note that , with a normal distribution, the standard deviation-based VAR satisfies the subadditivity property. This is because the volatility of a portfolio is less than the sum of volatilities $\sigma(X_1 + X_2) \leq \sigma(X_1) + \sigma(X_2)$. We only have a strict equality when the correlation is perfect(positive for long positions). More generally, this property holds for elliptical distributions, for which contours of equal density are ellipsoids.

Example: Why VAR is not necessarily subadditive.

Consider a trader with an investment in a corporate bond with face value of $100,000 and default probability of 0.5%. Over the next period, we can either have no defaults, with payoff say of $500(consisting of a coupon payment), or default with a loss of $100,000.The payoff are thus –$100,000 with probability of 0.5% and + $500 with probability 99.5%. Since the probability of + $500 is greater that 99%, the VAR at the 99 percent confidence level is $500(abstracting from the mean).

Now, if we consider three identical positions, the VAR numbers add up to

$\sum_i VaR_i$ =$1500. But this is not the same as the portfolio VAR. If the defaults are independent, the payoffs and probabilities are:

State	Probability	Payoff
No default	0,995 * 0,995 * 0,995 = 0,985075	1 500,00 $
1 default	3* 0,005 * 0,995 * 0,995 = 0,014850	-99 000,00 $
2 default	3 * 0,005 * 0,005 * 0,995 = 0,000075	-199 500,00 $
3 default	0,005 * 0,005 * 0,005 = 0,0000001	-300 000,00 $

Here, there is a higher than 1 percent probability of some default. In fact interpolating between the probabilities of 1 and 2 defaults, we find a true VAR of -131,991$. Thus we have that $\rho(X_1 + X_2 + X_3) = 131,991$ is greater than $\rho(X_1) + \rho(X_2) + \rho(X_3)$=1500 $. In this situation , VAR is not subadditive. Adding up the 3 VAR figures totally underestimates the portfolio VAR. This is an undesirable property as it creates disincentives to aggregate the portfolio since it would appear to have higher risk.

Admittedly, this example is a bit contrived, but is still shows the danger in conventional VAR quantiles. The portfolio may be structured so that it appears to have low risk. When a loss occurs, however, this may be a huge loss.

VAR's Caveats

VAR is a useful summary measure of risk. Its interpretation, however, should bear in mind the following points:

- **VAR does not describe the worst loss**

This is not what VAR is designed to measure. Indeed we would expect the VAR number to be exceeded with a frequency of p, i.e., 5 days out of a hundred for a 95 percent confidence level. This is perfectly normal. In fact, backtesting procedures are designed to check whether the frequency of exceedences is in line with p.

- **VAR does not describe the losses in the left tail**

VAR does not say anything about the distribution of losses in its left tail. It just indicates the probability of a such value occurring. For the same VAR number, however, we can have very different distribution shapes. In the case of Figure1, on page16 , the average value of the losses worse than $47 m is around $75m, which is 60 percent worse than the VAR. So, it would be unusual to sustain many losses beyond $200m.

Instead, figure 2 shows a distribution with the same VAR, but with 125 occurrences of larger losses, beyond $160 million. This graph shows that, while the VAR number is still $47, there is a high probability of sustaining very large losses.

Figure 2
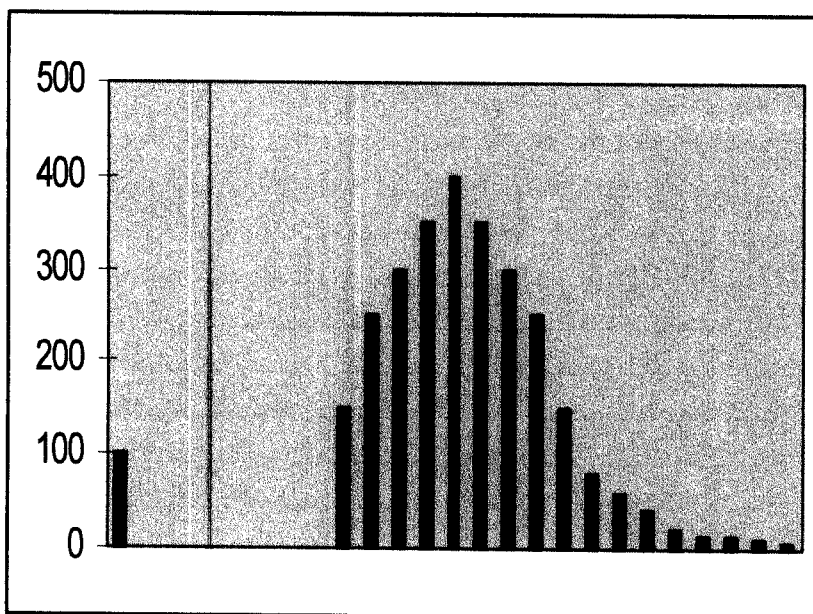
- **VAR is measured with some error**

The VAR number itself is subject to normal sampling variation. In our example, we used ten years of daily data. Another sample period, or a period of different length will lead to a different VAR number. Different methodologies or mapping procedures can also lead to different VAR numbers. One can experiment with sample periods and methodologies to get a sense of the precision in VAR. Hence it is useful to remember that there is limited precision in VAR numbers. What matters is the first-order magnitude.

VAR Parameters

To measure VAR, we need to define first two quantitative parameters, the confidence level and the horizon.

Confidence level

The higher the confidence level c, the greater the VAR measure. Varying the confidence level provides useful information about the return distribution and potential extreme losses.

The problem is that as c increases, the number of occurrences becomes rarefied, leading to poor measures of large but unlikely losses. Assuming return distributions can be modelled by parametric distributions such as the normal distribution, we have to realise that these distributions have infinite tails. Thus it is not clear whether one should stop at 99%,99.9%, 99.99% and so on. Each of these values will create an increasingly larger loss.

The choice of the confidence level depends on the use of VAR. For most applications, VAR is simply a benchmark measure of downside risk. If so, what really matters is consistency of the VAR confidence level across trading desks or time.

In contrast, if the VAR number is being used to decide how much capital to set aside to avoid bankruptcy, then a high confidence level is advisable. Obviously, institutions would prefer to go bankrupt very infrequently. The capital adequacy use, however applies to the overall institution and not to trading desks.

Another important point is that VAR models are only useful insofar as they can be verified. This is the purpose of backtesting, which systematically checks whether the number losses exceeding VAR is inline with p. For this purpose, the risk manager should not choose too high a value of c. Picking, for instance, c=99,99% should lead, on average, to one exceedence out of 10,000 trading days, or 40 years. In other words, it is going to be impossible to verify if the true probability associated with VAR is indeed 99,99 percent

For all these reasons, the usual recommendation is to pick a confidence level that is not too high, i.e., 95 to 99 percent.

Horizon

The longer the horizon(T) , the greater the VAR measure. This extrapolation depends on two factors: the behaviour of the risk factors, and the portfolio positions.

To extrapolate form a one-day horizon to a longer horizon, we need to assume that returns are independently and identically distributed. This allows us to transform a daily volatility to a multiple-day volatility by multiplication by the square root of time. We also need to assume that the distribution of daily returns is the same as that at longer horizons, which restricts the class of distribution to the so-called "stable" class, of which the normal is a member. If so, we have

$$VAR(T \text{ days}) = VAR(1 \text{ day}) * \sqrt{T}$$

This requires (1) the distribution to be invariant to the horizon(i.e., the same α , as for the normal), (2) the distribution to be the same for various horizons(i.e., no time decay in variance), and (3) innovations to be independent across days.

The choice of the horizon also depends on the characteristics of the portfolio. If the positions change quickly , or if exposures(e.g. option deltas) change as underlying prices changes, increasing the horizon will create "slippage" in the VAR measure.

Again , the choice of the horizon depends on the use of VAR. If the purpose is to provide an accurate benchmark measure of downside risk, the horizon should be relatively short, ideally less than the average period for major portfolio rebalancing.

In contrast, if the VAR number is being used to decide how much capital to set aside to avoid bankruptcy, then a long horizon is advisable. Institutions will want to have enough time for corrective action as problems start to develop.

We should also recognise that the horizon cannot be less than the frequency of reporting of profits and losses. Typically, banks measure P&L on a daily basis, and corporate on a longer interval(ranging from daily to monthly) .This interval is the minimum horizon for VAR.

Another criteria relates to the backtesting issue. Shorter time intervals creates more data points matching the forecast VAR with the actual , subsequent P&L. As the power of the statistical tests increases with the number of observations, it is advisable to have a horizon as short as possible.

For all these reasons, the usual recommendation is to pick a horizon that is as short as feasible, i.e., 1-days for trading desks. The horizon needs to be appropriate to the asset classes and the purpose of risk management. For institutions such as pension funds, for instance, a 1-month horizon may be more appropriate.

It should also be recognised that for capital adequacy purposes, institutions should select a high confidence level and a long horizon. There is also a fundamental indeterminacy in this choice: we have one objective function, the amount of capital to hold, and two variables, the confidence level and horizon. Increasing one or the other will increase VAR.

Credit Risk

Credit risk can be broadly defined as the risk of financial loss due to counterparty failure to perform their obligations. Time and again lack of diversification of credit risk has been the primary culprit for bank failures. The dilemma is that banks have a comparative advantage in making loans to entities with whom they have an ongoing relationship, thereby creating excessive concentrations in geographic or industrial sectors.

It is only recently that the banking industry has learned to measure credit risk in the context of a portfolio. These newer models truly started to blossom as a result of the risk-management revolution started by value-at-risk. After all, VAR aggregates risks across an institution, taking into account portfolio effects. Once measured, credit risk can be managed and better diversified, like any financial risk. This is why the banking sector is busily developing sophisticated internal models for credit risk.

These recent developments can be traced to the risk analysis of swaps, by now the largest class of derivatives. Initially, swaps were arranged between top-rated credit risks, and margins were fat enough to absorb the very few defaults in these markets. Later, as the market matured, greater volume exposed participants to deteriorating credit risks. This led to a need for more precise measurement of credit risk.

Credit risk, unfortunately , is much more difficult to quantify than market risk. There are many more factors driving credit risk, some of which are extremely difficult to measure due to their infrequency . This includes default probabilities, their correlation, and recovery rates. In addition, credit risk models suffer from a verification problem. Unlike market risk, for which backtesting can be performed on a daily basis, the longer horizon of credit risk models makes it difficult to compare risk forecasts with their realisation.

The Nature of Credit Risk

Credit can be ascribed to two factors

1. **Default risk**, which is the objective assessment of the likelihood that a counterparty will default, or default probability combined with the loss given default

2. **Market risk**, which drives the market value of the obligation, as known as credit exposure.

Consider, for instance, the credit risk of a forward contract on a foreign currency. The credit exposure is the positive value of the contract, whose value depends on movements in exchange rates. Thus credit risk involves both default and market risk.

As a result, the risk-management function for credit risk focuses on issues that are quite different from those facing market risk managers, as shown below on page 25. First, credit risk deals with the combined effect of market risk and default risk. Second, risk limits

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apply to different units. For market risk, limits apply to levels of the trading organisation; for credit risk, limits apply to the total exposure to each counterparty, a legally defined entity. Third, the time horizon is generally quite different, usually very short (days) in the case of market risk but much longer (years) for credit risk. This longer horizon makes it important to consider changes in the portfolio, as well as any mean reversion in the risk factors. Fourth legal issues are very important for evaluating credit risk, whereas they are not applicable for market risk. Recovery from credit losses depends on national laws and on the application of bankruptcy rules.

Overall, credit risk is much less amenable to precise measurement than market risk, for all the reasons listed above. In addition due to their infrequent nature, default probabilities and their correlation are much more difficult to measure than dispersion in market movements.

<u>Comparison of Value at Risk to Credit Risk</u>

Item	Value at risk	Credit Risk
Source of risk	Market risk	Market risk and default
Units to which risk limits to apply	Some level of trading organisation	Legal entity of counterparty
Time horizon	short term(days) Static portfolio mean reversion not important	long term(years) Dynamic portfolio mean reversion essential
Legal issues	Not applicable	very important

Settlement Risk

Pre-Settlement vs. Settlement Risk

Counterparty credit risk consists of both presettlement and settlement risk. Presettlement risk is the risk of loss due to the counterparty's failure to perform on an obligation during the life of the transaction. This includes default on a loan or bond, or failure to make the required payment on a derivative transaction. Presettlement risk can exit over long periods, often years, starting from the time it is contracted until settlement.

In contrast, settlement risk is due to the exchange of cash flows and is of a much shorter-term nature. This risk arises as soon as an institution makes the required payment until the offsetting payment is received. The risk is greatest when payments occurs in different time zones, especially for foreign exchange transactions where notionals are exchanged in different currencies. Failure to perform on settlement can be caused by counterparty default, liquidity constraints or operational problems.

Most of the time, settlement failure due to operational problems leads to minor economic losses, such as additional interest payments. In some cases, however, the loss can be quite large, extending to the full amount of the transferred payment. An example of major settlement risk is the 1974 failure of Herstatt Bank. The day it went bankrupt, it had received payments from a number of counter parties but defaulted before payments were made on the other legs of the transactions.

Handling Settlement Risk

In March 1996, the BIS issued a report warning that the private sector should find ways to reduce settlement risk in the $1,2 trillion-a-day global foreign exchange market. The report noted that central banks had "significant concerns regarding the risk stemming from the current arrangements for settling FX(Foreign Exchange) trades" . It explained that "the amount at risk to even a single counterparty could exceed a bank's capital ," which creates systemic risk. The threat of regulatory action led to a re-examination of settlement risk.

The status of a trade can be classified into five categories:

1. **Revocable:** when the institution can still cancel the transfer without the consent of the counterparty.

2. **Irrevocable:** after the payment has been sent and before payment from the other party is due

3. **Uncertain**: after the payment from the other party is due but before it is actually received

4. **Settled:** after the counterparty payment has been received

5. **Failed:** after it has been established that the counterparty has not made the payment.

Settlement risk occurs during the periods of irrevocable and uncertain status, which can take from one to three days.

While this type of credit risk can lead to substantial economic losses, the short nature of settlement risk makes it fundamentally different from presettlement risk. Managing settlement risk requires unique tools, such as real-time gross settlement systems. These systems aim at reducing time interval between the time an institution can no longer stop a payment and the receipt of the funds from the counterparty.

Settlement risk can be further managed with netting agreements. One such form is bilateral netting, which involves two banks. Instead of making payments of gross amounts to each other, the banks would tot up the balance and settle only the net balance outstanding in each currency. At the level of instruments, netting also occurs with contracts for differences. Instead of exchanging principals in different currencies, the contracts are settled in dollars at the end of the contract term.

Drivers of Credit Risk

Credit risk measurement systems attempts to quantify the risk of losses due to counterparty default. The distribution of credit risk can be viewed as a compound process driven by the following variables:

- **Default**, which is a discrete state for the counterparty; either the counterparty is in default or not
- **Credit exposure**, also known as exposure at default(EAD), which is the economic value of the claim an the counterparty at the time of default
- **Loss given default(LGD),** which represents the fractional loss due to default. As an example, take a situation where default results in a fractional recovery rate of 30% only. LGD is then 70% of the exposure.

Traditionally, credit risk has been measured in the context of loans or bonds for which the exposure, or economic value of the asset is close to its notional, or face value. This is an acceptable approximation for bonds but certainly not for derivatives, which can have positive or negative value.

Credit Risk as a Short Option

For default risk to create losses, two conditions must be satisfied. First there must be net claim against the counterparty(or credit exposure) and second that counterparty must default.

Traditionally , credit risk only applied to bonds and loans, for which the exposure is simply the face value of the investment. Derivatives, in contrast, can have either positive value(a net asset to the solvent party) or negative value(a liability of the solvent party). There is credit exposure when the contract has positive values, or is in the money.

In effect, the loss due to default is much like that of an option. Define V_t as the current, or replacement value of the asset to the solvent party. Assuming no recovery in case of default, the loss is the current exposure V_t if positive:

$$Loss_t = \max(V_t, 0)$$

This asymmetric treatment stems from the fact that if the counterparty defaults while the contracts has negative value, the solvent party is typically not free to "walk away" from the contract. In contrast, a loss may occur if the defaulting party goes bankrupt, in which case payment will be only a fraction of the funds owned. Therefore, the current exposure from default has an asymmetrical pattern, like a short position in an option.

Time and Portfolio Effects

Credit risk, however, should include not only the current replacement value but also the potential, or future, loss from default. Indeed, the G-30 report recommends that users "measure credit risk in two ways:(1) current exposure and (2) potential exposure, which is an estimate of the future replacement cost of derivative transactions." The peak credit exposure is often measured as

Peak credit $exposure_t = \max(V_t + \Delta V_r, 0)$

Where ΔV_r represents the maximum increase in value over the horizon, r, at a specified confidence level c.

This approach has the merit of simplicity. Unfortunately it ignores the time variation of credit exposure as well as that of the default probability. Credit exposure can evolve in complicated ways over time. Also, a counterparty with a high credit rating has low default risk initially but higher risk later.

More sophisticated approaches rely on the potential exposure profile, which describes, which describes the worst potential loss, measured at some confidence level, at a set futures dates(e.g., monthly intervals) . The pattern of dynamic credit exposure can be combined with future default probabilities to create a credit risk profile across time.

Even with these adjustments, the traditional approach is on a transaction-by-transaction basis, which essentially ignores portfolio effects. Consider, for instance, a portfolio consisting of a long yen forward position and a short yen forward with two different counterparties. The portfolio is hedged as to market risk. The transaction-by-transaction approach would consider the effect of default on each position separately . A loss on the long position occurs if the yen appreciates and the first counterparty defaults; a loss on the short position occurs if the yen depreciates and the second counterparty defaults . In this approach, the potential credit losses from the two positions are added up.

Since appreciation and depreciation of the yen are two mutually exclusive events, however, this method overstates the true potential loss from credit risk. Instead, a portfolio approach would take into account interactions between market movements and then determine the potential loss. In this case, assuming equal probability of appreciation/depreciation and of default by the two counterparties, the potential loss is only half the previous measure.

Accounting for time and portfolio effects, however, is no simple matter. This requires Monte Carlo simulation methods that combine market risk with credit risk. The benefits of an integrated portfolio approach can be substantial though. In one case, a large U.S. financial institution calculated a $27 billion peak credit exposure using traditional methods. Using simulations, the firm found a peak portfolio exposure over all time horizons of $5,5 billion only. This substantially lowered the amount of economic capital required to support the transactions.

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Thus it is essential to measure credit risk within the context of a portfolio, which is the purpose of internal portfolio credit risk models.

Measurement of Credit Risk

The evolution of credit risk management tools has gone through the following steps:

- Notional amounts
- Risk-weighted amounts
- External/internal credit ratings
- Internal portfolio credit models

Initially, risk was measured by the total notional amount. A multiplier, say 8 percent, can be applied to this amount to establish the amount of required capital to hold as reserve against credit risk.

The problem with this approach is that it ignores variations in the probability of default. In 1988, the Basel Committee instituted a very rough categorisation of credit risk by "risk class" providing risk weights to scale each notional amount. This represented a first attempt to force banks to carry enough capital in relation to the risks they were taking.

These risk weights proved to be too simplistic, however, creating incentives for banks to alter their portfolio in order to maximise their shareholder returns subject to the Basel capital requirements. This had the perverse effect of creating more risk into the balance sheets of commercial banks, which was certainly not the intended purpose of the 1988 rules. As an example, there was no differentiation between AAA-rated and C-rated corporate credits. Since loans to C-credits are much more profitable than those to AAA-credits, given the same amount of regulatory capital, the banking sector responded by shifting its loan mix toward lower-rated credits.

This has led to the 2001 proposal by the Basel Committee to allow banks to use their own internal , or external credit ratings. These credit ratings provide a better representation of credit risk, where "better" can be defined as more in line with economic measures.

Credit Risk Diversification

Modern banking was built on the sensible notion that a portfolio of loans is less risky than single loans. As with market risk, the most important feature of credit risk is the ability to diversify across defaults.

To illustrate this point, Figure 1(below) presents the distribution of losses for a $100 million loan portfolio. The probability of default is fixed at 1 percent. If default occurs, recovery is zero.

In the first panel, we have one loan only. We can either have no default, with probability 99%, or a loss of $100m with probability 1%. The expected loss is

EL= 0.01 * $100m + 0.99*0 = $ 1m.

The problem of course is that, if default occurs, it will be a big hit to the bottom line, possibly bankrupting the lending bank.

Basically, this is what happened to Peregrine Investments Holdings, one of Hong Kong's leading investments banks, that failed due to the Asian crisis of 1997. The bank failed in large part from a failed loan to PT Steady Safe, an Indonesian taxi-cab operator, that amounted to $235 million, a quarter of the bank's equity capital.

In the case of our single loan, the spread of the distribution is quite large, with a variance of 99, which implies a standard deviation (SD) of about $10m. Simply focusing on the standard deviation, of course, is misleading given the severe skewness in the distribution.

In the second panel, (Figure 2), we consider ten loans, each for $10m. The total notional is the same as before. We assume that defaults are independent. The expected loss is still 1$m, or 10*0.01*$10m. The SD, however, is now $ 3m, much less than before.

Next, the third panel considers a hundred loans of $1m each (Figure 3). The expected loss is still $1m, but the SD is now $1m, even lower. Finally, the fourth panel considers a thousand loans of $100,000, which create a SD of $0.3m(Figure 4)

For comparability, all these graphs use the same vertical and horizontal scale. This, however , does not reveal the distribution fully. This is why the fifth panel expands the distribution with 1000 counterparties, which looks remarkably similar to a normal distribution(Figure 5). This reflects the Central Limit Theorem, which states that the distribution of the sum of independent variables tends to a normal distribution.

Remarkably, even when we started with a highly skewed distribution , we end up with a normal distribution due to diversification. This ability to diversity provides effective protection against credit risk. This explains why portfolios of consumer loans, which are spread over a large number of credits, are less risky than typical portfolio of corporate loans.

With N events that occur with the same probability p, define the variable $X= \sum_{i}^{N} b_i$ as the number of defaults(where $b_i = 1$ when default occurs). The expected credit loss on our portfolio is then

E[CL] = E[X]* $100/N = pN*$100/N= p * $ 100; which does not depend on N but rather on the average probability of default and total exposure, $100 million. When the events are independent, the variance of this variable is, using the results from a binomial distribution

$$V[CL] = V[X] * (\$100/N)^2 = p(1-p)N* (\$100/N)^2,$$

which gives a standard deviation of

$$SD[CL] = \sqrt{p(1-p)} * \$100/\sqrt{N}$$

For a constant total notional, this shrinks to zero as N increases.

We should note the crucial assumption that the credits are independent. When this is not the case, the distribution will lose its asymmetry more slowly. Even with a very large number of consumer loans, the dispersion may not tend to zero due to credits. Indeed , many more defaults occur in a recession than in an expansion.

Institutions loosely attempt to achieve diversification by concentration limits. In other words, they limit the extent of exposure, say loans, to a particular industrial correlated among sectors than across sectors. Conversely, concentration risk is the risk that too many defaults could occur at the same time.

Figure 1; Distribution of Credit Losses

Figure 1

Loss PDF: 1 credit of $100M
N=1, E(loss) = $ 1m, V(Loss)= $ 99M
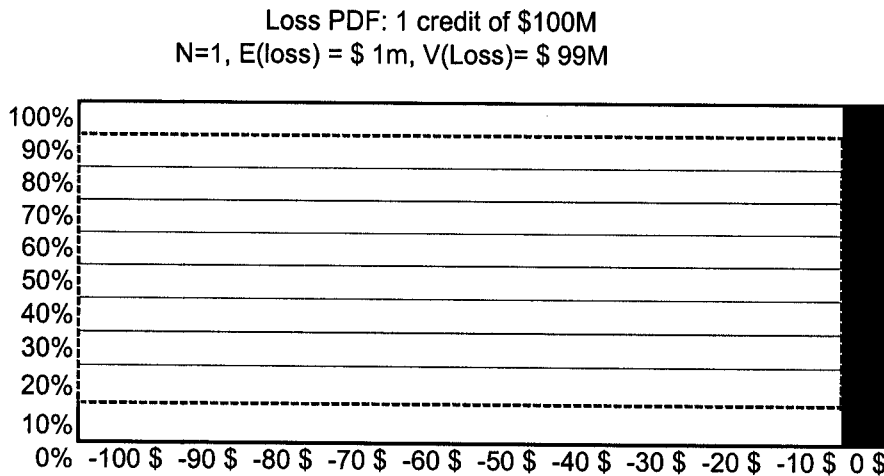
Figure 2

Loss PDF: 10 credit of $10M
N=10, E(loss) = $ 1m, V(Loss)= $ 9,9M
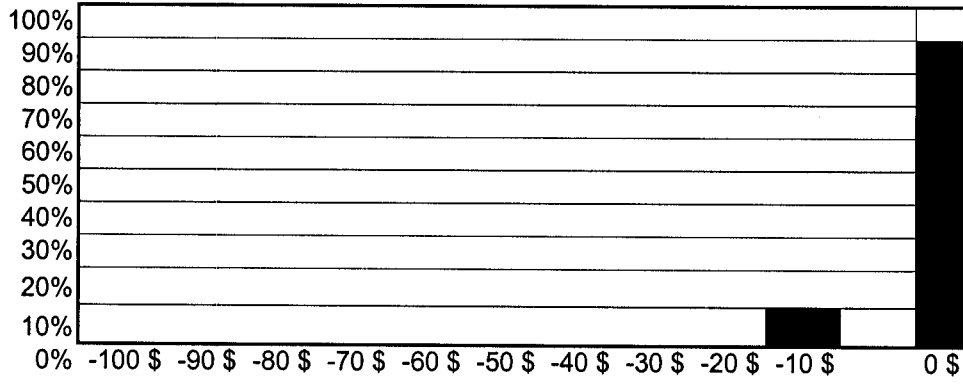
Figure 3

Loss PDF: 100 credit of $1M
N=100, E(loss) = $ 1m, V(Loss)= $ 0,99M

Figure 4

Loss PDF: 1000 credit of $ 0,1M
N=1000, E(loss) = $ 1m, V(Loss)= $ 0,1M

Figure 5

Loss PDF: 100,000 independent
of $0.3m
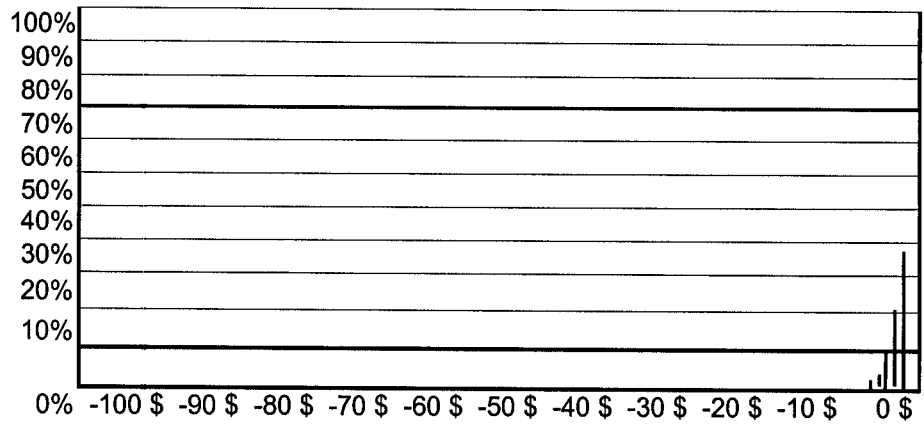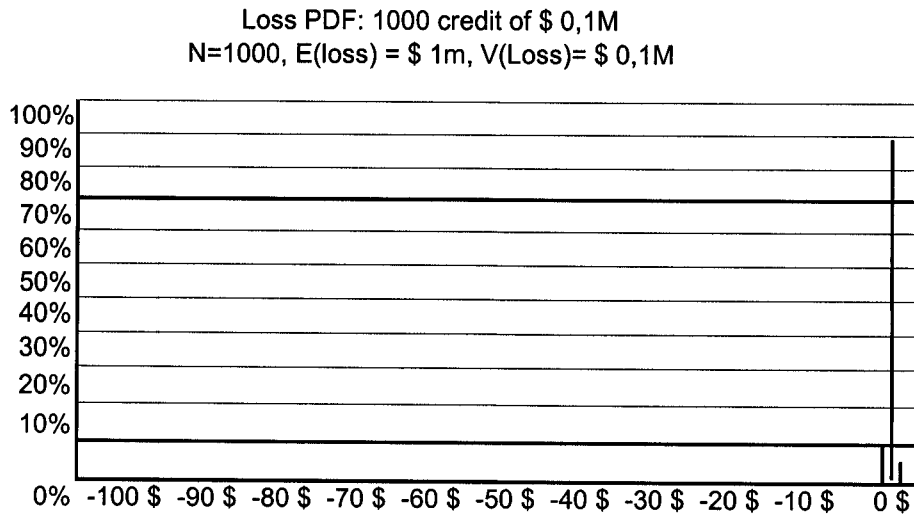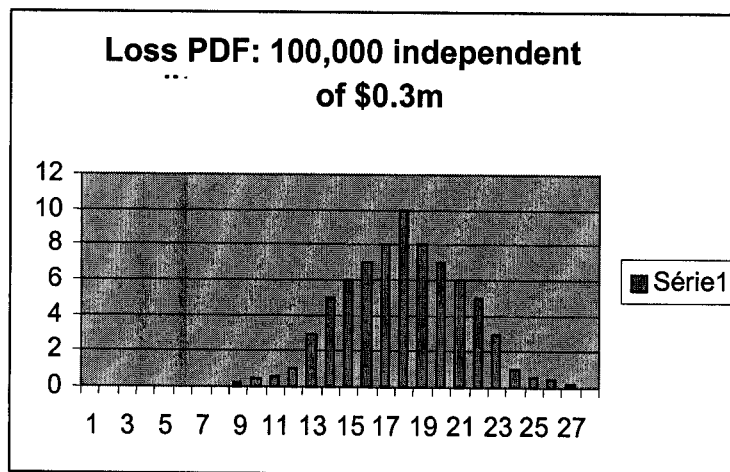
The y-axes values are in percentage, we noticed that most of the value are below 10% and the loss are less than 2 millions. This is reason why the standard deviation is at its lowest.

Measuring Credit Risk

As mentioned before credit risk measurement systems attempts to quantify the risk of losses due to counterparty default. The distribution of credit risk can be viewed as a compound process driven by the following variables:

- **Default**, which is a discrete state for the counterparty; either the counterparty is in default or not
- **Credit exposure**, also known as exposure at default(EAD), which is the economic value of the claim an the counterparty at the time of default
- **Loss given default(LGD),** which represents the fractional loss due to default. As an example, take a situation where default results in a fractional recovery rate of 30% only. LGD is then 70% of the exposure.

Credit losses

To simplify, consider only credit risk due to the effect of defaults. This is what is called default mode. The distribution of losses due to credit risk from a portfolio of N instruments can be described as

$$\text{Credit loss} = \sum_{i=1}^{N} b_i * CE_i * (1 - f_i) \quad ., \text{ where}$$

- b_i is a (Bernoulli) random variable that takes the value of 1 if default occurs and 0 otherwise, with probability p_i
- CE_i is the credit exposure at the time of default
- f_i is the recovery rate, or (1-f) the loss given default

In theory , all of these could be random variables. We will assume that the only random variable is default.

Measuring Actuarial Default Risk

Default risk is the primary component of credit risk. It represents the probability of default, as well as the loss given default(LGD). When default occurs, the actual loss is the combination of exposure at default and loss given default.

Default risk can be measured using two approaches:

Actuarial methods, which provide "objective", as opposed to risk-neutral, measures of default rates, usually based on historical default data
Market-price methods, which infer from traded prices the market's assessment of default risk, along with a possible risk premium. The market price of debt, equity or credit derivatives can be used to derive "risk-neutral" measures of default risk.

Risk-neutral measures provide a useful short-cut to price assets, such as options. For risk management purposes, however, they are contaminated by the effect of risk premia and therefore do not exactly measure default probabilities. In contrast, objective measures describe the "actual" or "natural" probability of default. On the other hand, since risk-neutral measures are derived directly from market data, they should incorporate all the news about a creditor's prospects

Credit Event

A credit event is a discrete state, either it happens or not. The issue is the definition of the event, which must be framed in legal terms.

One could say, for instance, that the definition of default for a bond obligation is quite narrow. Default on the bond occurs when payment on that same bond is missed.

Default on a bond, however, reflects the creditor's financial distress and is almost always accompanied by default on other obligations. This is why rating agencies give a credit rating for the issuer. Likewise, the state of default is defined by Standard & Poor (S&P), a credit rating agency, as

" the first occurrence of a payment default on any financial obligation, rated or unrated, other than a financial obligation subject to a bona fide commercial dispute; an exception occurs when an interest payment missed on the due date is made within the grace period."

However the definition of credit events have been recently formalised by the ISDA Master Netting Agreement for credit derivatives, which lists the following events:

- **Bankruptcy**, which is a situation involving:

1. The dissolution of the obligor(other than merger)
2. The insolvency, or inability to pay its debt
3. The assignment of claims
4. The institution of bankruptcy proceeding
5. The appointment of receivership
6. The attachment of substantially all assets by a third party

- **Failure to pay**, which means failure of the creditor to make due payment; this is usually triggered after an agreed-upon grace period and above a certain amount.
- **Obligation/cross default**, which means the occurrence of a default (other than failure to make a payment) on any other similar obligation

- **Obligation/cross acceleration**, which means the occurrence of a default (other than failure to make a payment) on any other similar obligation that results in that obligation becoming due immediately.

- **Repudiation/moratorium,** which means that he counterparty is rejecting, or challenges the validity of the obligation.

- **Restructuring,** which means a waiver, deferral, rescheduling of the obligation with he effect that the terms are less favourable than before.

In addition, other events sometimes included are:

- **Downgrade,** which means the credit rating is lower than previously, or withdrawn

- **Currency inconvertibility,** which means the imposition of exchange controls or other currency restrictions imposed by a governmental or associated authority

- **Government action,** which means either (1) declarations or actions by a government or regulatory that impair the validity of the obligation, or (2) the occurrence of war or other armed conflict that impairs the functioning of the government or banking activities.

The ISDA definitions are designed to minimise legal risks, by precisely wording the definition of credit event. When Russia defaulted on its domestic debt, for instance, it maintained payments on its external debt The question was whether this default constituted a credit event for foreign debt. Even now, it is sometimes not clear whether a restructuring creates impairment in the value of a creditor's obligations, leading to a disagreement as to whether a credit event has occurred.

Credit Ratings

A credit rating is an "evaluation of creditworthiness" issued by a rating agency. More technically, it has been defined by Moody's, a rating agency, as an "opinion of the future ability, legal obligation, and willingness of a bond issuer or other obligor to make full and timely payments on principal and interest due to investors."

The table below presents the interpretation of various credit ratings issued by the two major ratings agencies. Moody's and Standard and Poor's. These ratings correspond to long-term debt: other ratings apply to short-term debt. Generally, the two agencies provide similar ratings for the same issuer.

Classification by Credit Ratings

Explanation	Standard & Poor's	Moody's Services
Investment grade		
Highest grade	AAA	Aaa
high grade	AA	Aa
Upper medium grade	A	A
Medium grade	BBB	Baa
Speculative grade		
lower medium grade	BB	Ba
Speculative	BB	B
Poor standing	CCC	Caa
Highly speculative	CC	Ca
lowest quality, no interest	C	C
In default	D	
Modifiers: Example	A+, A, A-	A1,A2, A3

Ratings are broadly divided into:

- **Investment grade**, i.e., at and above BBB for S&P and Baa for Moody's and

- **Speculative grade**, or below investment grade, for the rest.

These ratings represents objective (or actuarial) probabilities of default. Indeed the agencies have published studies that track the frequency of bond default in the US, classified by initial ratings and for different horizons. These frequencies can be used to convert ratings to default probabilities.

The tables below display historical default rates as reported by Moody's and Standard and Poor's respectively. These describe the proportion of firms that default, $\bar{X}$, which is a statistical estimate of the true default probability:

$$E[\bar{X}] = p$$

For example, borrowers with an initial Moody's rating of Baa experienced an average 0.30% default rate over the next year, and 7.92% over the following ten years. Similar rates are obtained for S&P's BBB-rated credits, who experienced an average 0.22% default rate over the next year, and 5.23% over the following ten years.

Thus higher ratings are associated with lower default rates. As a result, this information could be used as estimates of default probability for an initial rating class. In addition, the tables show that the default rate increases with the horizon, for a given initial credit rating. Credit risk increases with the horizon.

Moody's Cumulative Default Rates, 1920-1999(Percent)

Rating	year									
	1	2	3	4	5	6	7	8	9	10
Aaa	0	0	0,02	0,09	0,2	0,31	0,43	0,62	0,83	1,09
Aa	0,08	0,25	0,41	0,61	0,97	137	1,81	2,26	2,67	3,1
A	0,08	0,27	0,6	0,97	1,37	1,78	2,23	2,63	3,1	3,61
Baa	0,3	0,94	1,73	2,62	3,51	4,45	5,34	6,21	7,12	7,92
Ba	1,43	3,45	5,57	7,8	10,04	12,09	13,9	15,73	17,31	19,05
B	4,48	9,16	13,73	17,56	20,89	23,68	26,19	28,32	30,22	31,9
Invest. Grade	0,16	0,49	0,93	1,43	1,97	2,54	3,12	3,68	4,27	4,85
Specul. Grade	3,35	6,76	9,98	12,89	15,57	17,91	19,96	21,89	23,59	25,31
All corporates	1,33	2,76	4,14	5,44	6,65	7,76	8,77	9,71	10,61	11,49

S&P's Cumulative Default Rates, 1920-1999(Percent)

Rating	year														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0	0	0,04	0,09	0,14	0,26	0,4	0,63	0,73	0,84	0,84	0,84	0,84	0,84	0,84
AA	0,01	0,04	0,12	0,23	0,38	0,56	0,75	0,91	1,02	1,11	1,16	1,22	1,22	1,22	1,22
A	0,04	0,12	0,21	0,36	0,56	0,77	1,03	1,32	1,66	2,02	2,35	2,52	2,62	2,68	2,76
BBB	0,22	0,54	0,88	1,55	2,28	3,06	3,67	4,26	4,76	5,23	5,59	5,85	6,17	6,43	6,59
BB	1,01	3,33	6,28	9,31	12,25	15,61	17,63	19,8	21,8	23,41	25,15	26	26,71	26,71	26,7
B	5,82	13,2	19,8	25,2	29,5	32,87	35,9	38,7	40,9	42,88	43,87	44,44	44,63	44,63	44,6
C	24,27	33,7	41,5	47	53,41	55,46	56,93	57,3	58,8	59,91	59,91	59,91	59,91	59,91	59,9
Invest. G	0,08	0,19	0,34	0,59	0,87	1,19	1,49	1,8	2,08	2,36	2,58	2,72	2,82	2,9	2,97
Spec. G	4,5	9	14,5	18,7	22,51	25,78	28,23	30,6	32,6	34,36	35,62	36,28	36,7	36,7	36,7

One problem with such historical information, however is the relative paucity of data. There are simply not many instances of highly rated borrowers that default over long horizon. For instance, S&P reports default rates up to 15 years using data form 1981 to 1999. The one-year default rates represent 20 years of data, i.e., 1981,1982 and so on to 1999. There are, however, only five years of data for the 15-year default rates, i.e., from 1981 to 1995, 1982 –96, 1983-97, 1984-98,1985-99. Thus the sample size is much shorter. If so, omitting or adding a few borrowers can drastically alter the reported default rates.

This can leads to some inconsistencies in the tables. For instance, the default rates for CCC-borrowers is the same, at 59.91 percent , form year 10 to 15. This would imply that there is no further risk of default after ten years, which is unrealistic.

The default rates reported in the above two tables are cumulative default rates for an initial credit rating, i.e., measure the total frequency of default at any time between the starting date and year T.

Transition Probabilities

As we have seen the measurement of long-term default rates can be problematic with small sample sizes. The computation of these default rates can be simplified by assuming a Markov process for the rating migration, described by a transition matrix. Migration is a discrete process that consists of credit ratings changing from one period to the next.

The transition matrix gives the probability of moving to one rating conditional on the rating at the beginning of the period. It is then usual to assume that these moves follow a Markov process., or that migrations across states are independent from one period to the next. This type of process exhibits no carry-over effect. More formally, a Markov chain describes a stochastic process in discrete time where the conditional distribution, given today's value, is constant over time. Only present values are relevant.

The table below gives the example of a simplified transition matrix for 4 states, A, B, C, D, where the latter represents default.

Credit Ratings Transition Probabilities

State	Ending				Total
Starting	A	B	C	D	Prob.
A	0,97	0,03	0,00	0,00	1,00
B	0,02	0,93	0,02	0,03	1,00
C	0,01	0,12	0,64	0,23	1,00
D	0,00	0,00	0,00	1,00	1,00

Consider a company in year 0 in the B category . This company could default:

- in year 1, with probability $D[t_1 / B(t_0)] = P(D_1/B_0) = 3\%$
- in year 2, with a total probability of

$$P(D_2/A_1)P(A_1) + P(D_2/B_1)P(B_1) + P(D_2/C_1)P(C_1)$$
$$= 0.00 * 0.02 + 0.03*0.93 + 0.23*0.02 = 3.25\%$$

The cumulative probability of default over the two years is then 3% + 3.25 % = 6.25%

The following figure illustrates the various paths to default in years 1, 2, and 3.

Paths to Default
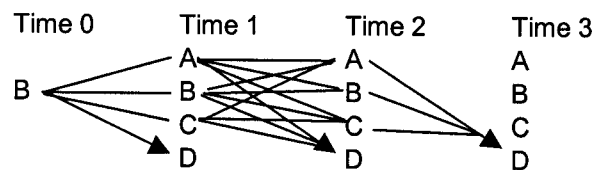
Measuring Default risk from Market Prices.

Credit risk can also be assessed from market prices of securities whose value is affected by default. This includes corporate bonds, equities, and credit derivatives. In principle, these should provide more up-to-date and accurate measures of credit risk since financial markets have access to a large amount of information. We will concentrate on default risk models based on stock prices, as equity prices are available for a larger number of companies and are now more actively traded than corporate bonds.

The Merton Model views equity as akin to a call option on the assets of the firm , with an exercise price given by the face value of debt.

The Merton Model

To simplify to the extreme, consider a firm with total value V that has one bond due in one period with face value K. If the value of the firm exceeds the promised payment, the bond is repaid in full and stockholders receive the remainder. On the other hand if V is less than K, the firm is in default and the bondholders receive V only. The value of equity goes to zero. Throughout, we assume that there are no transaction costs. Hence, the value of the stock at expiration is

$$S_T = \text{Max}(V_T - K, 0)$$

Since the bond and equity add up to the firm value, the value of the bond must be

$$B_T = V_T - S_T = V_T - \text{Max}(V_T - K, 0) = \text{Min}(V_T, K)$$

The current stock price, therefore, embodies a forecast of default probability , in the same way that an option embodies a forecast of being exercised. The following figures describe how the value of the firm can be split up into the bond and stock values.

Note that the bond value can also be described as

$$B_T = K - \text{Max}(K - V_T, 0).$$

In other words, a long position in a risky bond is equivalent to a long position in a risk-free bond plus a short put option, which is really a credit derivatives.

Equity as an Option on the Value of the firm
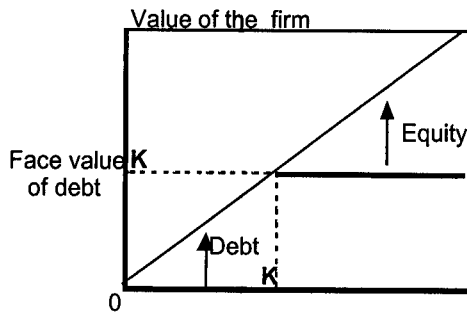
Components of the Value of the firm

Value of the firm

Equity

Face value of debt K

Equity

Debt

0 K

Equity

0 K

Debt

K

Value of the firm

0
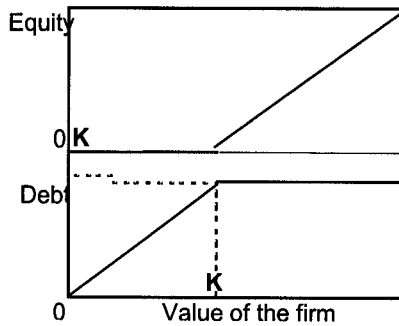
This approach is particularly illuminating as it demonstrates that corporate debt has a payoff akin to a short position in an option, explaining the left skewness that is so characteristic of credit losses. In contrast, equity is equivalent to a long position in an option due to its limited liability feature, i.e., investor can lose no more than their equity investment.

To illustrate, we proceed along with lines of the usual Blank-Scholes framework, assuming the firm value follows the usual "Geometric Brownian Motion" process

$$dV = \mu V dt + \sigma V dz$$

If we assume that markets are frictionless and that there are no bankruptcy costs, the value of the firms is simply the sum of the firm's equity and debt: $V = B + S$.

Any claim on the value of the firm then follows the usual Partial Differential Equation with the appropriate boundary conditions. The corporate bond value is obtained from

$$B = F(V, t), \qquad F(V, T) = Min [V, B_F]$$

Where $B_F = K$ is the face value of the bond to be repaid at expiration, or strike price. Similarly, the equity value is

$$S = f(V, t), \quad f(V, T) = Max[V - B_F, 0]$$

The value of the stock is given by the Black -Scholes formula

$S = Call = VN(d_1) - K e^{-rt} N(d_2)$., where N(d) is the cumulative distribution function for the standard normal distribution.

Note that in practice, this application is different of the Black Scholes model where we plug in the value of V, of its volatility $\sigma = \sigma_V$, and solve for the value of the call. Here, we observe the market value of the firm S and the equity volatility σ_S and must infer the values of V and its volatility such that the above equation (S= Call = VN(d_1)-$Ke^{-rt}N(d_2)$) is satisfied. This can only be done iteratively. Given the hedge ratio Δ,

$$dS = \frac{\partial S}{\partial V} dV = \Delta dV.,$$

Defining σ_S as the volatility of dS/S, we have a relationship between the stock volatility and the asset volatility:

$$\sigma_S S = \sigma_V V \Delta., \text{ from which we can infer } \sigma_V$$

Bond Valuation

The value of the bond is given by B= V- S, or

$$B = Ke^{-rt} N(d_2) + V[1- VN(d_1)]$$

$$B/Ke^{-rt} = [N(d_2) + (V/Ke^{-rt})/N(-d_1)]$$

Risk-Neutral Dynamics of Default

In Black-Scholes model, N(d_2) is also the probability of exercising the call, or that the bond will not default. Conversely, $1- N(d_2) = N(-d_2)$ is the probability of default. It is important to note that these are "risk-neutral" instead of objective probabilities.

In practise, default is much more complex. We would have to collect information about all the nominal, fixed liabilities of the company, as well as their maturities. Default can also happen at any intermediate point. So, instead of default on the target date, we could measure default probabilities as a function of the distance relative to a moving floor that represents liabilities.

The Merton approach has many advantages. First, it relies on equity prices rather than bond prices. There are many more firms with an actively traded stock price than bonds. Second, correlations between equity prices can generate correlations between defaults, which would be otherwise exceedingly difficult to measure. Perhaps the most important advantage of this model is that it generates movements in EDFs that seem to lead changes in credit ratings.

On the other hand, these models have disadvantages. The first limitation of the model is that it cannot be used to price sovereign credit risk, as countries obviously do not have a stock price. This is a problems for credit derivatives, where a large share of the market consists of sovereign risks. A more fundamental drawback is that it relies on a "static" model of the firm's capital and risk structure. The debt level is assumed to be constant over the horizon. Also, the model needs to be expanded to a more realistic setting where debt matures, at various points in time, which is not an obvious extension.

Another problem is that management could also undertake new projects that increase the value of equity but also its volatility, thereby increasing the credit spread. This runs counter to the fundamental intuition of the Merton model, which is that, all else equal, a higher stock price reflects a lower probability of default and hence should be associated with a smaller credit spread.

KMV- Application of Merton Method

KMV is a private firm based in San Franciso that provides forecasts of Estimated Default Frequencies(EDFs) for approximately 29,000 companies in 40 countries. Much of its technology is considered proprietary and unpublished and the BNC's uses KMV to forecast its EDF for its Corporate and Commercial Portfolio.

The basic idea, however, is an application of the Merton approach to credit risk. The value of equity is viewed as a call option on the value of the firm's assets.

$$S = f(A, K, r, \sigma_A, \tau),$$

,where K is the value of liabilities, taken as the value of all short-term liabilities(one year and under) plus half the book value of all long-term debt. This has to be iteratively estimated from observable variables, in particular the stock market value S and its volatility σ_S. This model generates an estimated default frequency based on the distance between the current value of assets and the boundary point.

The strength of this approach is that it relies on what is perhaps the best market data for a company-namely, its stock price. KMV claims that his model predicts defaults much better than credit ratings.

The problem with this approach is that it relies on accounting data to measure liabilities, and assumes that managers will keep the level of debt constant over the horizon. In practice, this approach provides only one component of the portfolio credit risk construction process. We also need correlation of defaults as well as distributions of exposures and loss given default

Measuring the Distribution of Credit Losses.

We can pool together the information in credit exposures, default rates and recovery rates to measure the distribution of losses due to credit risk. For simplicity, we only consider losses in default mode, i.e., due to defaults.

For one instruments, the current or potential credit loss is

Credit Loss = b * Credit Exposure *LGD   (1)

The distribution of credit loss is quite complex. Without going into details, a typical distribution is as follows:

Frequency Distribution

Credit Loss

Unexpected credit Loss
at 99%

Expected Credit Loss
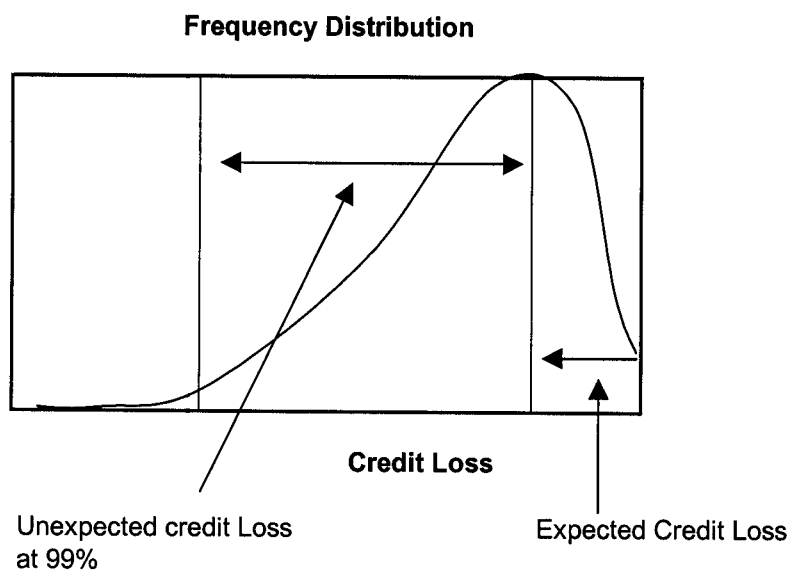
The distribution of credit losses is highly skewed to the left, in contrast to that of market risk factors, which is in general roughly symmetrical. This distribution is actually similar to a short position in an option. This analogy is formalised in the Merton model, which equates a risky bond to a risk-free plus a short position in an option.

Expected Credit Loss(ECL)

The Expected Credit Loss represents the average credit loss. The pricing of the portfolio should be such that it covers the expected loss. In others words, the price should be advantageous enough to offset average credit losses. In the case of a bond, the price should be low enough, or yield high enough, to compensate against expected losses. In a case of a derivative, the bank that takes on the credit risk should factor this expected loss into the pricing of its product. Loan loss reserves should also be accumulated as a provision against expected losses.

Worst Credit Loss(WCL)

The Worst Credit Loss represents the loss that will not be exceeded at some level of confidence. Like a VAR figure, the unexpected credit loss(UCL) is the deviation from the expected loss. The institution should have enough capital to cover the unexpected loss. Unexpected Loss depends on the distribution of joint default rates, among other factors. Notably , the dispersion in the distribution narrows as the number of credits increases and when correlations among default decreases.

Remuneration of Capital.

The measure of worst credit loss is also important for the pricing of credit-sensitive instrument. Say that the distribution has an Expected Credit Loss of $1 billion and Unexpected Credit Loss of $5 billion. The bank then needs to set aside $5 billion just to cover deviations from expected credit losses. This equity capital, however, will require remuneration. So, the pricing of loans should not only cover expected losses, but also the remuneration of his economic capital. This is what we call a risk premium and explains why observed credit spread are larger than simply to cover actuarial losses.

Measuring Expected Credit Loss

For pricing purpose, we need to measure the expected credit loss, which is

$$E[CL] = \int f(b, CE, LGD)(b*CE*LGD)db \ dCE \ dLGD.$$

If the random variables are independent, the joint density reduces to the product of densities and we have

$$E[CL] = [\int f(b) (b) \ db \ ] \ [\int f(CE)(CE)dCE \ ] \ [\int f(LGD)(LGD) \ dLGD .$$

Which is the product of the expected values. In other words, the expected loss is

Expected Credit Loss = Prob[default] * E[Credit Exposure] * E[LGD]

However if we assume Credit Exposure and LGD to be constant then,

Expected Credit Loss = Prob[default] * [Credit Exposure] * [LGD]

Measuring Credit VAR

The other component of the credit loss distribution is the Credit VAR, defined as the unexpected credit loss at some confidence level. Using the measure of credit loss, in equation 1(above on page 44) , we construct a distribution of the credit loss $f(CL)$ over a target horizon. At a given confidence c, the worst loss (WCL) is defined such that

$$1-c = \int_{WCL}^{\infty} f(x)dx$$

The Credit VAR is then measured as the deviation away from the expected credit loss:

CVAR= WCL-ECL

This CVAR number should be viewed as the economic capital to be held as a buffer against unexpected losses. Its application is fundamentally different from the expected credit loss, which aggregates expected losses over time and takes their present values.

Instead, the CVAR is measured over a target horizon, say one year, which is deemed sufficient for the bank to take corrective actions should credit problems start to develop. Corrective action can take the form of exposure reduction or adjustment of economic capital, all of which take considerably longer than the typical horizon for market risk.

Stress Testing

Stress Testing is a risk management sound practice aimed at protecting market participants from the effects of financial crises and stress.

The practice includes procedures for

- Identifying potential future stress conditions
- Measuring the effects on the market participant's portfolio
- Reporting these results timely to policy makers
- Acting on the findings timely and appropriately
- Examine how large losses could be if prices move sharply against the portfolio

The main purpose of Value-at-risk(VAR)- type risk measure is to quantify potential losses under "normal" market conditions. In principle, increasing the confidence level could progressively uncover large but unlikely losses. The problem is that VAR measures based on recent historical data can fail to identify extreme unusual situations that could cause severe losses.

The reason why we need to stress test is mainly due to the fact the VAR is based on a model which is not designed to forecast extreme events. Most model is based on Random walk model with the assumption that returns are jointly normally distributed, but yet, returns are generally not exactly normally distributed. Some returns might be quite far from normally distributed. Therefore risk measurement techniques for extreme events supplementing Classic VAR is essential.

Stress testing in indeed required by the Basel Committee as one of seven conditions to be satisfied to use internal models. It is also endorsed by the Derivatives Policy Group and by the G-30.

Stress Testing can be described as a process to identify and manage situations that could cause extraordinary losses. This can be made with a set of tools, including (1) scenario analysis, (2) stressing models, votalilities, and correlations, and (3) policy responses.

Scenario analysis consists of evaluating the portfolio under various states of world. Typically , these involves large movements in key variables, which require the application of full-valuation methods. The first application of stress tests consisted of sequentially moving key variables by a large amount. This, however, ignores correlations. More generally , scenarios provide a description of the joint movements in financial variables and can be either historical prospective, i.e., drawn from historical events or from plausible economic and political developments.

More recently, the industry has realised that the identification of scenarios should be driven by the particular portfolio at hand. For example, a highly leverage portfolio with a long

position in corporate bonds offset by a short position in Treasuries could suffer sharp losses if correlation broke down. The scenario should then be structured to create unusual decreases in correlations.

Whenever the stress tests reveal some weakness, management must take steps to handle the identified risks. One solution could be set aside enough capital to absorb potential large losses. Too often, however, this amount will be too large, reducing the return on capital.

The goal is to ensure that the institution can ride out the turmoil. In other words, stress testing can help to guarantee the very survival of the institution.

Stress Testing and Value-at-Risk

There are normally two ways that VAR can be supplemented by Stress Testing.

Case 1.

Stress test and then calculate the Value at Risk.

We examine scenarios in which market prices behave in a way that is highly unlikely in the Classical Model. We first identifies the risk factors $R_{t,1}$, ..., $R_{t,K}$ with horizon (e.g., daily, weekly) returns $r_{t,1}$, ...,$r_{t,K}$. With a pricing model, we estimate; $V(r_{t,1}, ...,r_{t,K})$; the change in portfolio values given the returns. Here we are focusing on the scenarios, not on the pricing models. It is important to note that this is not a sensitivity analysis and is not oriented towards model risk.

Case 2.

Value at Risk is first calculated which is then followed by Stress Testing.

We first generates scenarios ($\bar{r}_{t,1},........,\bar{r}_{t,K}$) that are in some sense "true" to the probability distribution of return $f(r_{t,1}, ...,r_{t,K})$. We then estimate distribution $V(r_{t,1}, ...,r_{t,K})$ the change in portfolio values given the returns and then find the 95-th or 99-th percentile of the distribution of $V(r_{t,1}, ...,r_{t,K})$. Stress Testing is then performed. We first used posit scenarios $\bar{r}_{t,1},........,\bar{r}_{t,K}$ using judgement as well as statistical reasoning and then consider the scenario losses $V(\bar{r}_{t,1},........,\bar{r}_{t,K})$

Types of Stress Tests

There are mainly two types of Stress Tests scenarios. Scenarios which are either based on actual historical events or on anticipated possible future events.

Black Monday, Gulf war, Mexican peso devaluation are all examples of historical stress testing scenarios. Historical scenarios ensure realism of stress tests, particularly with respect to joint movements of risk factors. The disadvantage of historical scenarios is that history rarely repeat itself.

For anticipating possible future events, stress tests can be further divided into User-defined stress tests and Predictive stress tests.

Examples of User-defined stress tests are:

- Selective scenarios based on portfolio vulnerabilities. VAR, expected shortfall and related marginal measures can be used to identify vulnerabilities.

- Senior Management and Poll traders can also put forward their suggestions on what events they are most concerned about. (for example 100 % increase in interest rate)

- Mechanical approach such as maximum moves in selected risk factors in recent experiences.

Examples of Predictive stress tests are:

- Shock subset of risk factors and let other risk factors vary in accordance with their correlations.

- Move in each non-specified factor in accordance with multiple regression on specified factors

- Take into account correlations conditional on the occurrence of market stress or shock of given size.

Description of the groupings of similar scenarios.

Equity

"Black Monday 1987"; Scenarios capturing market events on and after 19 October 1987. The main focus in on shocks to equity indices. Some of the scenarios include related events in other markets.

"Hypothetical stock market crash"; Similar to Black Monday in terms of risk factors addressed, but the shocks used are not directly based on historical experience.

"New Economy scenarios": The stress tests in this group almost exclusively concentrate on equity prices. The main focus is on market correction in New Economy equities and on the relative performance of New And Old Economy stocks.

Interest rates

"Other historical interest rate increases/declines"; These groups combine a number of historical episodes focusing on specific interest rate changes. The focus is almost exclusively on interest rate changes as such, typically in only one or two countries(often the home country of the reporting bank)

"Bond market crash 1994"; Scenarios capturing events in international bond market in the first quarter of 1994. However , the focus of all of the stress tests in this group is not exclusively on bond market but rather on all relating market events including shocks to equity prices. FX, an swap rates as well as other risk factors.

"Global tightening"; Focus on increasing short and long-term interest rates and , in some cases, interest rate volatilities across several developed markets. The underlying scenario is similar to the one of the 1994 bond market crash, however other risk factors(FX rates, equities) are only of minor importance.

"US tightening ". Hypothetical events that could follow a US Interest rate increases across several developed markets. The risk factors shocked are similar to those in the "global tightening " group including interest rate increases in several developed markets. However, the number of countries and currencies involved is smaller.

Emerging market

"Asia" Eight historical scenarios simulating the events of the Asian financial crises in 1997-98, and seven hypothetical scenarios inspired by the Asian financial crisis. The focus is usually on equities, FX and interest rates in Asian countries as well as in G7 countries.

"Latin America" for example historical scenarios based on the devaluation of the Mexican Peso on 14 December 1994.

"Country risk" stress tests focus on particular(sets of) countries and tend to stress interest and FX rates as well as equities . In many cases the different countries are stress-tested one after another, with the results moves are applied in more than one Latin American country, but not on risk factors in other regions.

"Russia" Scenarios are based on the Russian devaluation on 21 August 1998 and the associated market events. The focus of both the hypothetical and historical scenarios is on the region as such as in some cases on rates in G7 countries, but not on risk factors in other regions.

"Eastern Europe" This group combines a number of hypothetical scenarios focusing on Eastern European FZ and interest rates. Some of these set-ups also shock interest rates in developed countries.

"Global emerging market crises"; Scenarios which are hypothetical and simulate a crisis across all emerging market, including spillovers into the developed economies, without a clear focus on a particular region or set of risk factors.

Credit

"Spread widening"; A widening of various credit and/or swap spreads over a wide range of countries, less frequently including shocks to equity prices and interest rates in the major currencies.

"Fall 1998" Scenarios focusing on the historical crisis episode following the Russian devaluation and default in August 1998. The main risk factors are similar to the spread widening described above.

Europe

"European stress 1992"; These scenarios refer to the crisis of the European Exchange. Rate Mechanism(ERM) in 1992. The focus is thus on the interest and FX rates of European countries, but also in a few cases on the US dollar and the yen.

"European stress/weak Euro"; Hypothetical scenarios dealing with the impact of a stressful shock in European stock, bond and /or currency markets on market in Europe, Japan, an the US.

"European divergence" These scenarios concentrate on divergent interest and FX rate developments between the Euro-zone and other European countries. Contrary to the weak Euro stress test, non-European currencies and typically not among those being shocked.

"European boom/strong Euro"; The focus is on the impact of FX and interest rate movements driven by rising Euro interest and/or exchanges rates.

Japan

"Interest rate increase scenarios"; These scenarios stress banks' portfolio with historical and hypothetical increase of Japanese interest rates. Some of the scenarios also include shocks to Japanese equity prices and interest rates in the major currencies. Some explicitly consider an end to the zero-interest-rate policy.

"Japan market-wide stress"; Historical and hypothetical events with a stressful impact on all Japanese markets. In terms of risk factors the focus is thus on Japanese equity prices, interest rates, and the yen exchange rate, while repercussions in the other main markets are also considered.

"December 1998"; Scenarios focusing on the termination of the Japanese MOF-Bond Purchase Operation on 20 December 1998 and its subsequent impact on the JGB bond market. The main risk factors shocked are thus Japanese short and long-term interest rates.

"Strong yen"; scenarios apply various historical appreciation of the Japanese yen. Shocks are limited to FX rates and volatilities.

Commodities

"Middle East Crisis"; Scenarios focusing on financial spillovers to equities, FX and interest rates in industrialised countries from an oil shock. Four of the scenarios are historical , based on events of August 1990. Only three of the scenarios actually contain a shock to oil prices.

"Commodity stress"; These scenarios focus on shocks to commodity prices and volatilities.

North America

"Weak dollar" Scenarios focusing on historical and hypothetical depreciation of the US dollar. The main risk factors are thus FX rates and volatilities, but in some cases also interest and swap rates. Three historical scenarios based on the 1985 Plaza Accord are included.

"Strong dollar"; Like the weak dollar scenarios, these scenarios focus on movements on the US dollar. However this time the dollar appreciates. These scenarios almost exclusively concentrate on FX rate and volatility movements.

"US market-wide stress" scenarios feature declines in the US dollar of 5-15%, declines in the S&P 500 index of 15-30%, increases in the US long bond yield of 75-125 basis points, and knock-on effects on other G7 markets.

Other
"Volatility disruptions" stress portfolio against shocks to interest rate, equity and FX volatilities as well as, in some cases, a number of additional risk factors.

What is difficult about Stress Testing?

The main problem about Stress Testing is the probability weighting and scenario design. VAR measures have probabilities attached and scenarios are generated in accordance with posited probability law. Stress scenarios are arrived at judgementally , hence less statistical underpinning. Scenario must be realistic yet rare. Historical experience and judgement lead to very wide range of scenarios, too wide to practically implement.

A stress Testing program

Stress Testing should be carried out regularly, as part of risk reporting cycle. Permanent scenarios or Varying scenarios could be used.

Examples of Permanent scenarios are; Standard historical scenarios, large moves in a limited set of risk factors, discount curve shifts or twists , currency, equity index, and commodity shocks. These scenarios chosen with regard to durable characteristics of institution's exposures could be used in a Stress Testing Program.

Scenario based on warnings signals or on changes in portfolio concentrations are examples of Varying scenarios, which are chosen in accordance with changes in exposure or market conditions.

Stress Testing at National Bank of Canada.

National Bank of Canada(BNC) is using Stress Testing for its internal management and decisions and capital allocation challenges for its credit risk. BNC wanted to estimate the likely impact of economic changes on it's Corporate portfolio.

As mentioned earlier, the drivers of credit-risk are:

- Default
- Credit exposure
- Loss given default

Normally Stress Testing should be carried out on all of these three components, however due to lack of historical data on credit exposure and loss given default. Stress Testing is almost carried out only on the probability of default. We assume that the credit exposure and loss given default are constant over time.

Empirically driven econometric method is being used to stress-test BNC's Corporate portfolio performance under alternative economic conditions and environment. There have been two phases of this engagement:

- Development of econometric models to analyse the linkage between economic conditions and expected default frequencies (EDF) using KMV data.
- Using forecast expected default frequencies and other parameters to estimate the Expected Loss and the Economic Capital.

The BNC econometric models identify the economic attributes that impact portfolio performance, and quantify that relationship. These models have been used to conduct "what-if" simulation exercises under different macro-economic environments.

In a first attempt, BNC has developed three models based on loans grade. Each model is applied to a particular range of grade within BNC's credit portfolio. In the second phase, only one model is applied to the whole Corporate portfolio. The first models are briefly described below, whereas, the construction of the second phase model are explained much more in details. The same methodology, as mention for the second phase model, has been applied to all the three models in the first phase, however, I concentrated on the second phase method for comparison as the second phase model is more recent and it is more appropriate to reflect BNC's portfolio performance.

Phase 1: the first three models

BNC's corporate portfolio was divided into three different portfolios.

- Corporate loans– percentage outstanding on the watch list among loans rated 3 or higher

- Corporate loans– average expected default frequency for medium rated loans (graded 3 to 6,5)
- Corporate loans– average expected default frequency for highest rated loans (graded 1 and 2,5)

Loans are placed into different grades and each grade corresponds to a range of probability of default. We have 19 grade for our credit portfolio. Grade 1 is considered to have the lowest probability of default, for grade 1.5, the probability of default is higher than that of grade 1, and that of grade 3 is higher than of grade 2 and so on. The worst grade is grade 10 which has the highest probability of default.

These models predictions are then extrapolated to BNC's commercial and small business portfolio by modelling the structure between corporate loan performance and the commercial loan performance or small business loan performance.

Corporate loans- Percentage on the Watch List

BNC provided monthly loan-level data for corporate loans. These data were used to create aggregate monthly data for the corporate portfolio. The variable analysed for business loans was the portion of BNC's corporate loan volume rated 3 or higher that is on BNC's watch list. Loans with rating of 7 or higher are placed on BNC's watch list.

$$\% \text{ loans rated 3 or higher on watch list} = \frac{\text{Total number of loans rated 7 or higher}}{\text{Total number of loans rated 3 or higher}}$$

For example, in November 2001, approximately 10 percent of the loans outstanding rated 3 or higher were on the watch list(i.e., they were rated 7 or higher). Thus the value of the analysis variable for November 2001 would be (approximately) 10 percent.

On the watch list, we have loans that might default in the future. Firms are placed on the watch list for higher degree of monitoring or for restructuring to prevent default.

By modelling the portion of BNC's riskier corporate loan volume on the watch list, the relationship between the economy and the portion of the BNC's loans in serious risk of default can be captured.

The proposed modelled for the business model is:

% loans rated 3 or higher on watch list = f(economic variables, internal portfolio changes) + ϵ, where ϵ is the error term.

The economics variables for this model are:

1. Shipment to inventory ratio for finished goods,
2. Percentage change in the Canadian bank rate between one and two months ago,
3. Percentage change in Canadian durable goods sales between 2 and three months ago,
4. Percentage change in Canadian GDP
5. Percentage change in housing starts in Quebec between one and two months ago,
6. A variable controlling for the change in policy after October 2000.

The descriptive statistics for this model is on page 59 under model 1.

For commercial and small business loan portfolio, the impact of economic factors must be extrapolated. Within a period, say the last 12 month, the percent of the amount of commercial and small business loans rated three or higher on BNC's watch list is compared to the percent of corporate loans rated three of higher on BNC's watch list. Predictions from the corporate model should be inflated by the factor when applied to commercial and small business loans. We make an assumption that the commercial and small business loans behave in the same manner as corporate loans under economics changes. The main reasons why the extrapolation is done, is mainly due to the lack of quality historical data for the commercial portfolio.

Medium and Higher Grade Loans- Models for EDF

Data for Canadian Companies were aggregated to form one series reflecting average EDF for medium rated companies and another series reflecting average EDF for higher rated companies. Medium graded companies are defined as those with values of EDF between 0,18 percent and 4,85 percent, (loans rated between 3 and 6,5). Higher graded companies are defined as those with EDF less than 0,18 percent(loans rated between 1 and 2,5). The average EDF for a given month is obtained by using data for the companies classified in the corresponding group that month. As EDF values vary from month to month , a given company could be in the medium rated group some months, and in the low or high rated groups other months.

Average EDF = f(economic variables) + u, where u = f(past values of u) + ϵ

The average *EDF* is calculated using *KMV* data. *KMV* is an application of the *Merton Model* which is described on page 53.

The final model for *Corporate loans- Medium Rates* includes the following variables:

1. Percentage change in Canadian durable goods sales between the current month and the previous month
2. Percentage change in seasonally adjusted Canadian GDP, between three and four month ago,
3. Percentage change in housing starts in Canada between one and two months ago,

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4. Percentage change in exchange rate between the current month and the previous months,
5. Percentage change in number of persons employed in Canada, all industries, and
6. Auto-regressive term of order 1(i.e., uses values of average EDF from the previous month as another explanatory variable)

The estimation results for this model is on page 59 under model 2.

Average EDF for the higher rated companies exhibits a very smooth evolution over time changing little from month to month. Thus most of the variability explained by the model is accounted for by its own past behaviour.

The final model for *Corporate Loans-High Rated* includes the following variables

1. Percentage change in housing starts in Canada between one and two months ago,
2. Exchange rate two months prior, and
3. Two auto-regressive terms, one of order 1, and one of order 4, i.e., values of average EDF the previous month and four months prior are considered as explanatory variables

The descriptive statistics for this model is on page 58 under model 3.

The following graphs show the time series for each independent variables

UNEMP

BANK_RATE

STOCK__INDEX_TSE_PC_TSE3

SHIPM_INV
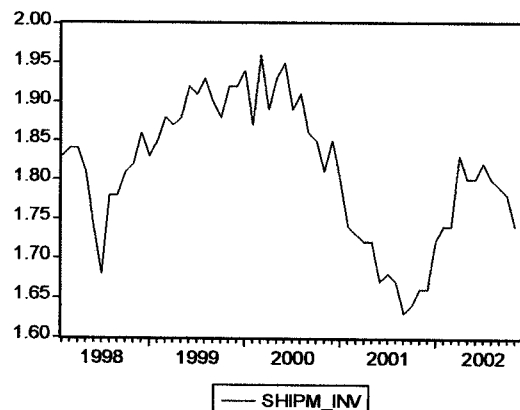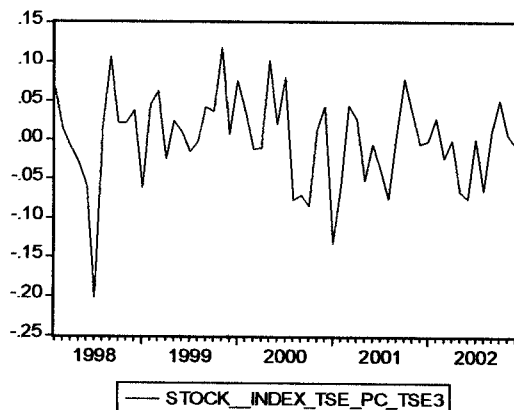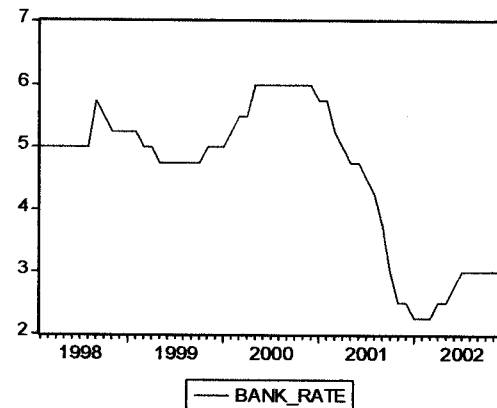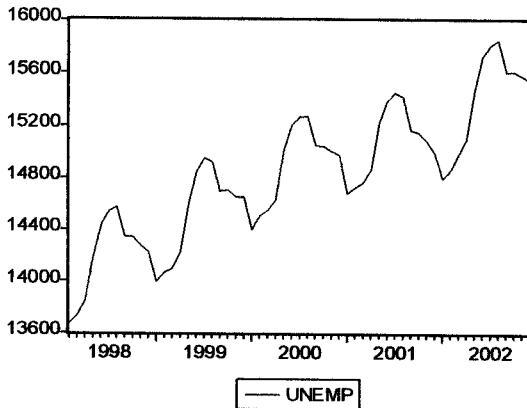
Descriptive Statistics for the first three models

The following tables show the results for the three models.

Model 1
Estimated Impact on Percent of Loans Rated 3 or Higher on Watch List

Intercept	42,019	9,96	> 99,9 %
Shipment to inventory ratio	-20,879	-9,28	> 99,9 %
Change in bank rate, one month previous	-0,0515	-1,78	91,40%
Change in durable goods production , two months previous	-0,0843	-1,81	91,90%
Change in GDP	-0,0407	-1,8	91,80%
Change in housing starts in Quebec, one month previous	-0,0205	-2,13	95,80%
October 2000 and later	1,1462	2,54	98,30%
R-square	91,70%		

Model 2 Corporate loans and Commercial
Medium Rated
Estimated Impact on Average EDF for companies rated 3 to 6,5

Intercept	1,4575	25,69	> 99,9 %
change in duratble goods production	-0,00443	-3,06	> 99,7 %
Change in seasonally adjusted GDP, three months previous	-0,02294	-2,40	98,10%
Change in housing starts in Canada, one month previous	-0,00061	-1,69	90,60%
Change in exchange rate	0,00928	2,54	98,70%
Change in employment	0,00979	-2,33	97,70%
Autoregressive term, lag 1	0,90305	19,93	99,90%
R-squared	82,70%		

Model 3; Corporate and Commercial loans
high Rated

Estimated Impact on Average EDF for companies rated 1 to 2,5

Intercept	0,0177	0,68	50,40%
Change in housing starts in Canada, one month previous	-0,000083	-3,38	99,90%
Exchange rate, two months prior	0,05	2,89	99,50%
Autoreqressive term, lag 1	0,6677	7,76	> 99,9%
Autoregre term, lag 4	0,2341	2,84	99,40%
R-square	86%		

The respectively models fit the data reasonably well with an adjusted R^2 of 91.7% for model 1, R^2 of 82.7% for model 2 and R^2 of 86.0% for the third model. The models variables are statistically significant and have signs and magnitudes consonant with

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economic theory and from the Durbin-Watson statistic, we can conclude that there is no evidence of problems with serial correlation in the estimated models.

The first phase models was a good starting point for the bank in its quest for more quantitative methodologies to measure and manage credit risk and evaluate its interaction with the economic environment.

However the effectiveness of any statistical model is subject to change and potential deterioration over time, and therefore it is essential to develop an adequate system to continuously monitor and validate the model performance.

After discussion with management and reviewing the technical analysis, we came to the conclusion that some modifications should be brought to the existing models.

Model modification: suggestion

Three models v/s one

In the first phase, BNC modelled its corporate portfolio into three independent models: one for the watch list, one for medium grade loans and one for higher grade loans. By considering each loans grade separately, indirectly they have assumed that there is no default correlation between loans grades and there is no migration between these groups.

History has revealed strong correlation in time of market stress. We believe that a single model will capture the default correlation between firms of higher grade and of lower grade and it will better reflect the performance of BNC's corporate portfolio. We propose to develop a single model instead of those three models.

Real value v/s percentage change.

In all the three models, instead of the market value, percentage change in the independent variables was used to explain the dependent variables. We believe that for certain variables market value could be more informative than percentage change.

For example, it is difficult to differential a change in the bank rate between 6 to 6,6 and 15 to 16,5 in a percentage change context. Even if in both cases, the percentage change is equal, the impact on the EDF would not be same, a 10% increase at 15 % should have a greater impact than at 6%.

A firm might default at a bank rate 16,5% while still operating at 6,6%. We propose to use market value instead of percentage change for certain variables.

Our portfolio instead of all Canadian Companies in KMV

Instead of using all the Canadian companies in KMV, we have selected only those companies that are in our Corporate Portfolio. These companies are listed on the stock exchange. The calculated EDF will be more appropriate and a better indicator for the probability of default for our Corporate Portfolio. To justify our recommendation, we calculated the correlation between the two EDF' series, between the EDF based on all Canadian companies in KMV and the EDF based on BNC's corporate portfolio. We found a very low correlation of 10% only.

We propose to use the companies from BNC's Corporate portfolio instead of all Canadian companies to calculate the average EDF.

The second phase model

Average EDF = f(economic variables) + u, where u = f(past values of u) + ϵ,

where average EDF represents the average EDF for the whole portfolio, calculated on data from KMV.

Methodology

Potential variables

We fist selected potential variables. The table below list all initial macro-economic variables that were used in constructing the final model.

STOCK__INDEX_TSE_PC_TSE3	Business leading indicators for Canada; Stock price index, TSE300
US_LEADING	Business leading indicators for Canada; United States composite leadig index
DURABLE_GOODS_SALES	Business leading indicators for Canada; Durable goods sales excluding furniture and appliances
SHIPM_INV	Business leading indicators for Canada; Shipment to inventory ratio, finished products
LEADING	Business leading indicators for Canada; Composite index of 10 indicators
EXCHG_RATE	Foreign exchange rates in Canadian dollars; United states dollars
BANK_RATE	financial market statistics, last Wednesday unless otherwise stated, Bank rate(percent)
UNEMP	labor force survey, by age group and sex; Canada; Unemployment rate; both sexes 15 years and over
HOUSING_STARTS	Housing starts, all areas; Canada(Units-thousands)
HOUSING_STARTS_QBC	Housing starts, all areas; Quebec(Units-thousands)
VAL_MFGED_GOODS_QBC	Manufacturing shipments, by North American Industry Classification System; Quebec; shipments , estimated values of goods of own manufacturing
GDP	Gross Domestic Product at basic prices in 1997 constant dollars, by NAICS;Canada
GDP_SEAS_ADJ	Gross Domestic Product at basic prices in 1997 constant dollars,
MORTGAGE_CREDIT	Chartered banks, assets and liabilities , monthly average; Canada; Residential mortgages, total(Dollars-Millions)
OTHER_CREDIT	Consumer credit, outstanding balances of selected holders; Canada; Average at month-end; Unadjusted; Total outstanding balances(Dollars-Millions)

One at a time, we regress the EDF on only one variable and recorded the corresponding R-square. This give us a table with the independent variables and corresponding R-square.

Variables	R-square	Variables	R-square
GDP_PCT	0.006519	BANK_RATE	0.147942
GDP_SEAS_ADJ_PCT	0.019453	EXCHG_RATE	0.539347
HOUSING_STARTS_PCT01	0.002677	HOUSING_STARTS_QBC_PCT01	0.000059
LEADING	0.000087	STOCK__INDEX_TSE_PC_TSE3	0.010659
MORTGAGE_CREDIT_GDP	0.289474	STOCK__INDEX_TSE_PC_TSE3(-3)	0.069374
SHIPM_INV	0.168611	UNEMP	0.497058
OTHER_CREDIT_GDP	0.449324	DURABLE_GOODS_SALES_PCT(-1)	0.000894
US_LEADING	0.359238	DURABLE_GOODS_SALES_PCT(-8)	0.025952
VAL_MFGED_GOODS_QBC_PCT	0.012647		

Correlation matrix

The following table shows the correlation between the variables: (2 digits)

	BANK	DURABLE	EMPLOY	EXCHG	GDP	GDP_SEAS	HOUSING_S	HOUSING_S_Q	LEAD	MORTGAGE	OTHER_CR	SHIPM	TS	UNE	US_LEA	VAL_MD_G
BANK_RAT	1,0															
DURABLE_G_S_P	0,0	1,0														
EMPLOYME	-0,1	-0,1	1,0													
EXCHG_RAT	-0,6	0,0	0,4	1,0												
GDP_PC	0,0	0,0	0,0	0,0	1,0											
GDP_SEAS_ADJ_P	-0,1	0,5	-0,2	0,0	0,0	1,0										
HOUSING_S_	-0,1	0,1	0,0	0,1	0,0	0,2	1,0									
HOUSING_S_Q_	-0,1	0,2	0,0	0,0	0,2	0,3	0,3	1,0								
LEADIN	-0,2	0,2	-0,1	0,0	-0,2	0,2	0,4	0,1	1,0							
MORTGAGE_CT_	-0,6	0,1	0,4	0,6	0,4	-0,1	0,1	0,1	0,0	1,0						
OTHER_CREDIT_G	-0,4	0,0	0,7	0,6	0,3	-0,2	0,1	0,1	0,0	0,9	1,0					
SHIPM_IN	0,5	-0,1	-0,1	-0,7	0,1	0,1	0,0	0,0	0,1	-0,5	-0,4	1,0				
TSE3	0,0	0,2	-0,1	0,0	-0,2	0,1	0,0	0,0	0,4	-0,2	-0,2	0,3	1,0			
UNEM	-0,2	0,0	0,8	0,5	0,0	-0,1	0,1	0,1	-0,1	0,4	0,6	-0,3	-0,1	1,0		
US_LEADIN	-0,5	-0,1	0,8	0,4	0,0	0,0	0,1	0,0	0,2	0,5	0,6	0,1	0,1	0,6	1,0	
VAL_MD_GS_Q_P	0,0	0,0	-0,1	-0,1	0,3	0,3	0,1	0,0	-0,1	0,1	0,0	0,0	-0,1	-0,1	0,1	1,0

One of the most important assumption about linear regression is that the independents variables should not have perfect correlation.

We notice that there is a strong correlation between *Bank Rate* and *Exchange Rate, Mortgage Credit, Other Credit and US_Leading*. Therefore if we include Bank Rate in our model, implicitly we are incorporating the other four variables.

Furthermore the employment and unemployment are highly correlated, $\rho = 77$ percent.

We should eliminate all highly correlated variables

The significant potential variables with a correlation < 50 %

The following tables show the final variables used in constructing the final model. We noticed that all the variables have a correlation of less than 50 percent.

	SHIPM_INV	STOCK_INDEX_TSE_PC_TSE3	UNEMP	VAL_MFGED_GOODS_QBC_PCT	HOUSING_STARTS_QBC_PCT01	HOUSING_STARTS_PCT01	BANK_RATE	DURABLE_GOODS_SALES_PCT	GDP_PCT	GDP_SEAS_ADJ_PCT
SHIPM_INV	1,000									
STOCK__IN_TE_PC_TSE3	0,256	1,000								
UNEMP	-0,266	-0,195	1,000							
VAL_MD_GS_QC_PCT	-0,007	-0,024	-0,087	1,000						
HOUSING_S_QC_PCT01	-0,042	-0,022	0,074	0,043	1,000					
HOUSING_STARTS_PCT01	-0,042	0,050	0,083	0,146	0,316	1,000				
BANK_RATE	0,485	0,042	-0,436	0,014	-0,067	-0,071	1,000			
DURABLE_GS_S_PCT	-0,079	0,192	-0,024	0,013	0,212	0,126	0,001	1,000		
GDP_PCT	0,061	-0,233	0,052	0,271	0,139	-0,102	-0,021	-0,068	1,000	
GDP_SEAS_ADJ_PCT	0,103	0,119	-0,131	0,283	0,267	0,161	-0,055	0,486	-0,009	1,000

We also used the correlogram test to check the correlation within observations. We notice that there is a strong dependence of order 1 within the observations of the probability of default series. This confirm the auto-regressive term in the models.

Correlogram		Sample: 1998:02 2002:12 Included observations: 59			
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
. |*******|	. |*******|	1 0.883	0.883	48.354	0.000
. |****** |	**| . |	2 0.734	-0.204	82.384	0.000
. |**** |	.*| . |	3 0.580	-0.097	103.97	0.000
. |*** |	. | . |	4 0.435	-0.048	116.33	0.000
. |** |	. | . |	5 0.319	0.027	123.10	0.000
. |** |	. | . |	6 0.233	0.022	126.79	0.000
. |** |	. |** |	7 0.220	0.238	130.13	0.000
. |** |	. | . |	8 0.219	-0.046	133.52	0.000
. |** |	. |*. |	9 0.262	0.189	138.45	0.000
. |** |	. | . |	10 0.310	0.016	145.51	0.000
. |*** |	. | . |	11 0.337	-0.031	154.01	0.000
. |*** |	. | . |	12 0.349	0.012	163.31	0.000
. |*** |	. | . |	13 0.340	0.041	172.36	0.000
. |** |	. | . |	14 0.324	0.003	180.75	0.000
. |** |	. | . |	15 0.283	-0.011	187.31	0.000
. |** |	. | . |	16 0.236	-0.027	191.98	0.000
. |*. |	. | . |	17 0.194	0.006	195.20	0.000
. |*. |	. | . |	18 0.158	0.004	197.40	0.000
. |*. |	.*| . |	19 0.121	-0.080	198.72	0.000
. |*. |	.*| . |	20 0.082	-0.059	199.34	0.000
. | . |	.*| . |	21 0.043	-0.066	199.51	0.000
. | . |	.*| . |	22 -0.005	-0.098	199.52	0.000
.*| . |	.*| . |	23 -0.062	-0.110	199.90	0.000
.*| . |	. | . |	24 -0.101	0.026	200.94	0.000

Descriptive Statistics of the final model for all variables

The table below shows the descriptive statistics of our model for all variables.

Dependent Variable: MEAN_EDF				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.17859	7.213466	1.411054	0.1653
SHIPM_INV	-0.934383	0.657346	-1.421449	0.1622
STOCK__INDEX_TSE_PC_TSE3	1.141988	0.282703	4.039530	0.0002
UNEMP	-0.000283	0.000123	-2.293732	0.0266
VAL_MFGED_GOODS_QBC_PCT	-0.103522	0.161380	-0.641481	■■■
BANK_RATE	0.184064	0.088301	2.084506	0.0430
DURABLE_GOODS_SALES_PCT	2.620609	0.700241	3.742439	0.0005
GDP_PCT	0.331805	0.241657	1.373040	0.1767
GDP_SEAS_ADJ_PCT	-5.069326	5.626295	-0.901006	■■■
HOUSING_STARTS_PCT01	0.184259	0.133013	1.385264	0.1730
HOUSING_STARTS_QBC_PCT02	-0.127463	0.085973	-1.482605	0.1453
AR(1)	0.985290	0.030205	32.62023	0.0000
R-squared	0.934160	Mean dependent var		1.849165
Adjusted R-squared	0.917700	S.D. dependent var		0.475979
S.E. of regression	0.136549	Akaike info criterion		-0.956857
Sum squared resid	0.820408	Schwarz criterion		-0.522853
Log likelihood	38.79200	F-statistic		56.75304
Durbin-Watson stat	1.611418	Prob(F-statistic)		0.000000

We noticed that the *probability* that the coefficient equal zero is too high for two variables: Gross Domestic Product (Seasonally Adjusted) and Value of Goods of Own Manufacturing. We concluded that these variables are not significant. We repeat the process of elimination of variables until we find "good variables" at an acceptable level of confidence.

The most significant variables, which has a low correlation among each other and high R-square are:

1. Unemployment rate
2. Bank rate
3. Shipm_inventory
4. TSE300

The following table show the descriptive statistics for each of our explicative variables.

	BANK_RATE	SHIPM_INV	TSE300	UNEMP
Mean	4.590517	1.811552	0.000559	14823.21
Median	5.000000	1.820000	0.007418	14851.75
Maximum	6.000000	1.960000	0.118437	15850.20
Minimum	2.250000	1.630000	-0.201991	13669.90
Std. Dev.	1.189960	0.087636	0.058835	522.4017

The second model

$mean\_edf = f(\,ar(1)\,,\, bank\_rate(-1),\, shipm\_inv(2), stock\_\_index\_tse\_pc\_tse3(-1),$
$\quad unemp(-2))$

Instead of three models, we are proposing a single model to forecast the EDF.

Descriptive Statistics of the final model with its most significant variables

The Table below shows the statistic of our regression.

Dependent Variable: MEAN_EDF				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
BANK_RATE(-1)	0.179483	0.096534	1.859271	0.0688
SHIPM_INV(-2)	-0.917444	0.513574	-1.786392	0.0800
STOCK__INDEX_TSE_PC_TSE3(-1)	-0.920908	0.303169	-3.037609	0.0038
UNEMP(-2)	0.000217	6.53E-05	3.315351	0.0017
AR(1)	0.938561	0.051607	18.18669	0.0000
R-squared	0.893060	Mean dependent var		1.919094
Adjusted R-squared	0.884672	S.D. dependent var		0.463122
S.E. of regression	0.157276	Akaike info criterion		-0.776583
Sum squared resid	1.261523	Schwarz criterion		-0.595748
Log likelihood	26.74433	Durbin-Watson stat		1.485999

The model's R-square is 89 percent, indicating that the model fits the data reasonably well at an acceptable level of confidence. In addition, the model variables are statistically significant and have signs consistent with economic theory.

For example, the coefficient of the TSE300 in our model is negative, indicating a negative correlation between the average default and TSE300. This is quite intuitive as an increase in TSE300 partially reflect a good economy and in a good economy environment, the probability of default should normally decreased.

The coefficient of the Bank Rate is positive, meaning a positive correlation between the Bank Rate and Probability of Default. Normally as bank rate increases, it becomes more expensive for firm to pay their debt.. Investment which has a positive Net Present Value or Internal Rate of Return higher than cost of Capital, may no longer profitable with a higher bank rate. Thus increasing the probability of default. To survive an increase in Bank Rate, firms must should have higher rate of return than cost of capital.

The coefficient of the Auto-regressive terms is positive and is almost equal to 1, meaning that today observation is strongly influenced by yesterday's observation.

Actual, Fitted, Residual Graph

The following graph shows the Actual, Fitted, Residual Graph. We can noticed how "good" the model fits the data.
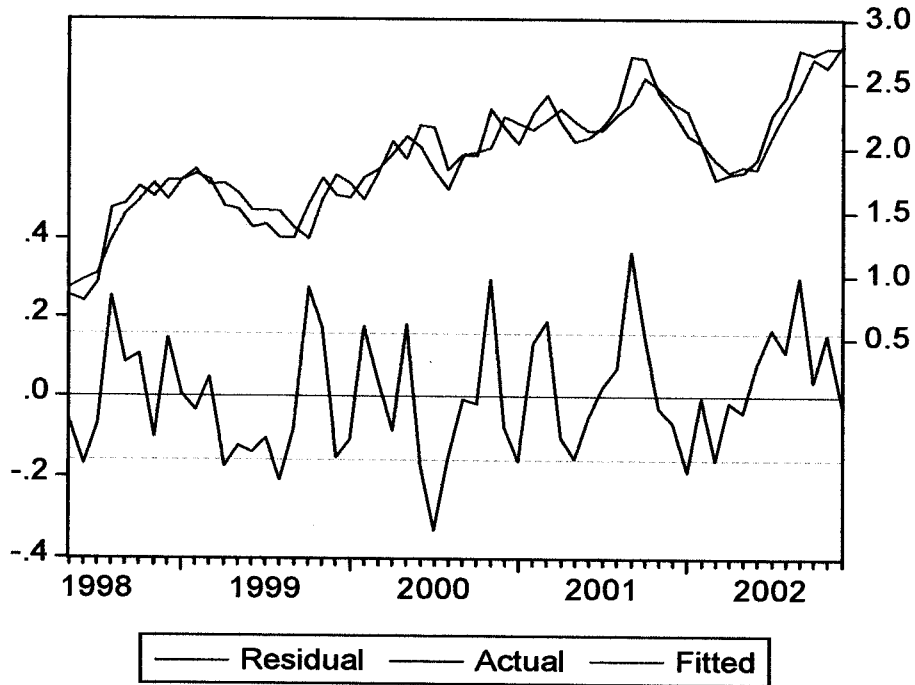
The second graph represents the actual observation and a sequence of one-step ahead forecasts. Data from 1998 up to 2001 and the model is used to forecast observation for 2002. The red line represents the actual observation and the blue line represents the forecasts. We can see that the model forecasts very well.

one-step ahead forecasts
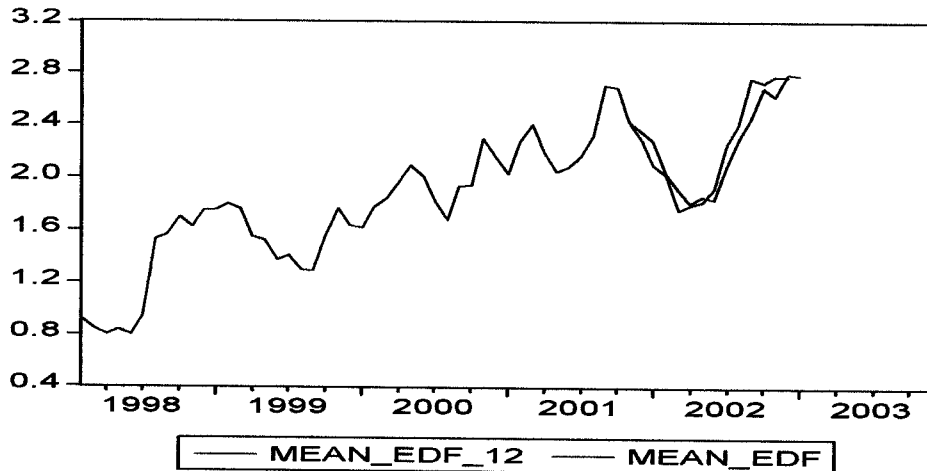
Forecast for January 2003.

The third graph shows an estimation of the probability of default for January 2003.

The red line represents the actual observation and the blue line represents the forecasts. We can see that the model forecasts very well
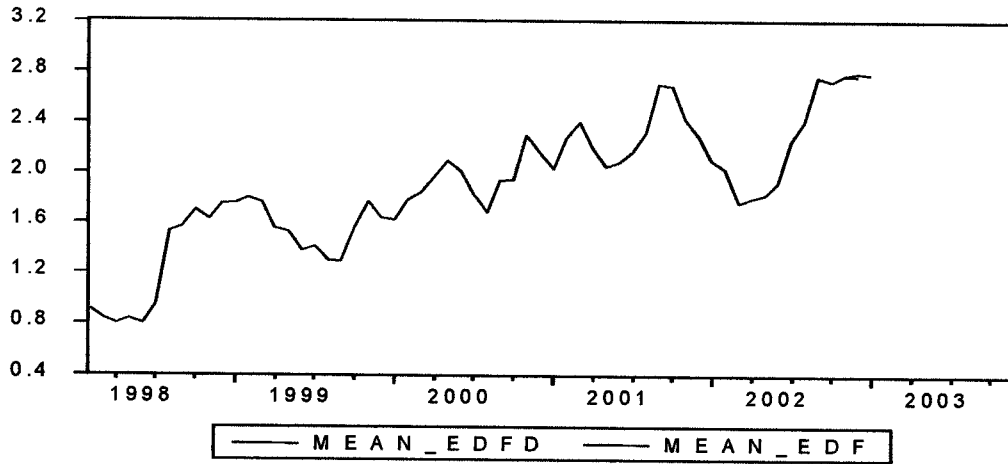
Back Testing

The following graph shows a back testing of our model. The red line represents the actual observation and the blue line represents the forecasted values. For the back testing, we have compared the actual data for the months of January, February, March and April 2003. We can observed that forecasted values are not far from the actual observations.
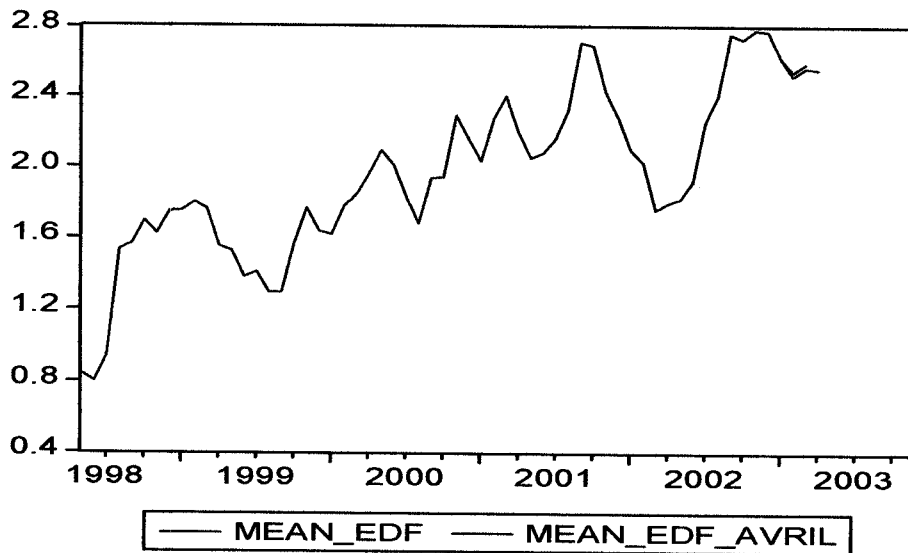
Conclusion

Once we have our model to forecast the probability of default for a year, we can stress test our models to evaluate the impact of macroeconomic variables on the probability of default.

The following scenarios were studied:

- Consensus
- Worst_Case 2 sigma
- Worst_Case 3 sigma
- Recession 1982.

EDF under different scenarios.

The following table represented the EDF under the different scenarios.

BNC's ratings	EDF	Consensus	Worst case 2 sigma	Worst case 3 sigma	Recession 1982
1	0,03%	0,0297%	0,0377%	0,0412%	0,0328%
1,5	0,09%	0,0891%	0,1130%	0,1235%	0,0984%
2	0,14%	0,1386%	0,1758%	0,1921%	0,1530%
2,5	0,20%	0,1981%	0,2512%	0,2744%	0,2186%
3	0,28%	0,2773%	0,3516%	0,3842%	0,3060%
3,5	0,41%	0,4060%	0,5149%	0,5625%	0,4481%
4	0,55%	0,5446%	0,6907%	0,7546%	0,6010%
4,5	0,86%	0,8516%	1,0800%	1,1799%	0,9398%
5	1,23%	1,2179%	1,5446%	1,6875%	1,3442%
5,5	1,82%	1,8021%	2,2854%	2,4969%	1,9889%
6	3,21%	3,1784%	4,0309%	4,4039%	3,5079%
6,5	6,13%	6,0696%	7,6976%	8,4098%	6,6988%
7	8,19%	8,1093%	10,2844%	11,2360%	8,9499%
7,5	11,99%	11,8719%	15,0561%	16,4493%	13,1025%
8	18,37%	18,1891%	23,0677%	25,2021%	20,0746%
8,5	28,14%	27,8628%	35,3360%	38,6056%	30,7510%
9	100%	100%	100%	100%	100%
9,5	100%	100%	100%	100%	100%
10	100%	100%	100%	100%	100%
EDF average	2,78%	2,75%	3,49%	3,81%	3,03%
% change		-0,98%	25,57%	37,19%	9,28%

A *Worst-Case* scenario was studied for this trimester. Under this scenario, we move each macro-economic variables to a worst-case value. This value is based on its standard deviation and its last observed value. In each model, the economic variables are taken at their Worst-Case value and the respectively probability of default was estimated. Using this estimated probability default and the Credit Exposure and Loss given Default provided by BNC, we estimated the Expected Loss and the Economic Capital for the next year.

These values is then compared with the Actual Reserves(Expected Loss) and Capital held by the BNC. In both phases, BNC is over capitalised stating that even in worst situation, BNC has enough cash or cash equivalent to offset huge losses.

Due to confidentiality, imposed by management, I'm not allowed to present actual result.

The following table shows the values of the economic variables under the Worst-Case and the respectively Expected Loss and Capital. However it should be note that I'm not allowed to present the real value of the Expected Loss and Capital calculated under this scenario.

The Worst Case scenarios of the three models(Phase1)

Phase 1: Model 1

table 1

Model 1 : % of volume on the watch									
Macro-ecnomic	Lag (month)	Effet sur Watchlist	last	value					
Shipment to inventory ratio for finished	0	-	1,78	June	1,81	1,81	1,30	1,30	1,04
% change in the Canadian Bank	1	-	0,00	02-	4,60	3,60	-	-	-8,59%
% change in Can. Durable goods	2	-	0,72	June	1,05	-	-	-	-2,31%
% change in Canadian	0	-	1,21	Juin	1,28	-	-	-	4,02%
% change in housing starts in	1	-	7,53	Aug	-	-	-	-	-12,92%
Result									
% of volume of watch	Corporat		xxxx	02-	xxxx	xxxx	xxxx	xxxx	xxxx
	Commerici		xxxx	02-	xxxx	xxxx	xxxx	xxxx	xxxx

The table 1 shows the percentage of volume estimated to be on the watch list for the corporate portfolio and commercial portfolio.(xxxx was normally real numbers).

Phase 1: Model 2
Table 2

Macro-ecnomic	Lag (month)	effect on EDF	last observed value						
Model 2 ; average EDF for loans graded 3 to									
% change in Canadian durable goods	0	-	0,72%	June	0,46%	2,36%	-4,94%	-3,04%	3,43%
% change in seasonally adjusted Canadian	3	-	0,14%	June	0,22%	0,21%	-0,73%	-0,74%	-1,00%
% change in housing starts in	1	-	5,92%	Aug 02	-0,57%	-0,69%	-	-	-14,43%
% change in exchange rate (CAN vs	0	+	1,42%	Aug 02	-0,13%	0,22%	1,98%	2,33%	-0,84%
% change in number of persons employed in	0	-	-1,66%	02-sept	0,38%	0,10%	-2,52%	-2,80%	-3,27%
Result									
Average EDF for loans rated 3 to 6,5			xxxx	Aug 02	xxxx	xxxx	xxxx	xxxx	xxxx

The table 2 shows the values of all the economic under different scenarios.

For example, for the percentage change in exchange rate, the last observed value was 1.42 % and under the Worst-Case scenario was 1.98%. The "Effect on EDF" column gives the correlation sign of the variable with EDF. Following with our example, an increase in Exchange rate will increase the EDF. The last row " Average EDF for loans rated 3 to 6,5 " normally shows that actual EDF which I'm not allow to present.

Phase 1: Model 3

Table 3

Macro-ecnomic variables	Lag (month)	Effet sur EDF	last observed value						
model 2: Average EDF for loans rated 1 to 2,5									
% change in housing starts in Canada	1	-	5,92%	Aug 02	-0,57%	-0,69%	-20,31%	-20,42%	-14,43%
Exchange rate	2	+	1,57	Aug 02	1,52	1,52	1,,77	1,77	1,27
Result									
Average EDF for loans rated 1 to 2,5			xxxx	Aug 02	xxxx	xxxx	xxxx	xxxx	xxxx

As table 2, the economic variables are shows and its corresponding values under different scenarios.

Estimated Expected Loss and Economic Capital

Effects on BNC's credit portfolio.

Secteur	Expected Loss Initial	W_ Case	Capital Initial	W_ Case	Difference	Excess of Capital disponible(2003)
		12 months		12 months		
Commercial	100 $	135,10 $	100 $	110,90 $	10,90 $	10 $
% Change		35,10%		10,90%		
Corporate	100 $	128,10 $	100 $	109,10 $	9,10 $	10 $
% Change		28,10%		9,10%		

Final Words

As mentioned before the effectiveness of any statistical model is subject to change and potential deterioration over time, and therefore it is essential to develop an adequate system to continuously monitor and validate the model performance. It is anticipated that BNC would continue to expand and maintain the econometric models for stress testing its various portfolios. This project is only meant as a starting point for the bank in its quest for more quantitative methodologies to measure and manage credit risk and its interaction with the economic environment.

The Expected Loss and Capital using the phase two model.

The table below show the value of the variables used in the final model and its corresponding value under the Worst Case scenario. For example, the value of Unemployment rate is 7,66 % under the Worst-Case scenario.

Phase 2: changes of variables under "Worst-Case" Scenario

Changes of economic variables under "Worst Case" scenario

Economic Variables	Last observed value		Worst Case - 12 month	
	month	Value	Value	% change
Shipm_inventory	Sept 02	1 151,8 millions $	1007,7 millions $	-1,87%
Unemployment rate	Jan 03	7,00 %	7,66 %	9,44%
Bourse canadienne (TSX 300)	Feb 03	6 478 points	4 426 points	-31,67%
Bank Rate	Jan 03	5,58 %	8,21 %	47,03%

Just for comparison, the following table shows the variables and their average values. We can notice that the average values is less than the corresponding Worst-Case values. For example, for Unemployment rate, its average 7,09 % while its value under Worst-Case scenario is 7,66%.

Phase 2: Average changes of economic variables

Average change of the economic variables

Economic Variables	Last observed value		Average -12 month	
	month	Value	value	% change
Shipm_inventory	Sept 02	1 151 800 millions $	1 184 059 millions $	2,80%
Unemployment rate	Jan 03	7,00 %	7,09	1,33%
Bourse canadienne (TSX 300)	Feb 03	6 478 points	6 990 points	7,92%
Taux d'intérêt canadien	Jan 03	5,58 %	5,52 %	-1,12%

The final table shows the estimated Expected Loss and Economic Capital for year 2003. (These numbers are not the actual numbers). According to these numbers BNC is over capitalised. It means that BNC has enough cash or cash equivalent to offset losses even under the Worst-Case scenarios.

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