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Essais sur l'exploitation d'un stock commun de ressource naturelle par des agents hétérogènes

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RÉSUMÉ

Cette thèse est composée de trois chapitres portant sur l'exploitation d'une ressource naturelle commune par des agents hétérogènes. Les problèmes d'exploitation d'une ressource naturelle commune sont généralement modélisés par des jeux différentiels dans lesquels il est tenu compte des interactions stratégiques entre les agents et aussi de l'impact de leurs actions sur la dynamique de la ressource. Ce travail apporte une double contribution à la littérature portant sur ce sujet. Premièrement, nous introduisons une double asymétrie dans les deux modèles d'agents hétérogènes que nous présentons et nous caractérisons explicitement des équilibres avec des stratégies en boucle fermée. Deuxièmement, nous analysons les effets des deux types d'asymétries sur les équilibres qui en découlent, dans un contexte d'équilibres markoviens parfaits. Dans le premier chapitre, nous nous restreignons aux stratégies markoviennes linéaires (stratégies exprimées comme une proportion du stock courant de la ressource) et déterminons les conditions nécessaires d'utilisation de telles stratégies. Dans le second chapitre, nous supposons que les conditions établies au premier chapitre sont vérifiées et nous caractérisons explicitement des stratégies markoviennes linéaires pour un modèle de « Fish War » dans lequel interviennent deux groupes d'agents différant par leurs taux d'actualisation. Dans ce premier modèle, les agents se font la concurrence seulement pour l'exploitation de la ressource. Nous examinons par la suite l'impact du différentiel de taux d'actualisation et de la répartition des agents sur les équilibres obtenus. Dans le dernier chapitre, nous présentons un modèle d'exploitation d'une ressource naturelle commune avec des agents repartis en deux groupes et différant par leurs coûts marginaux. Dans ce second modèle, les agents se font la concurrence aussi bien pour l'exploitation de la ressource, que pour la vente de leur production sur le même marché. Nous examinons les effets du différentiel de coûts marginaux et de la répartition des agents sur les équilibres obtenus. Pour les deux

modèles, nous trouvons que les deux types d'asymétries affectent effectivement les équilibres obtenus.

De façon plus spécifique, dans le premier chapitre, nous caractérisons les conditions nécessaires permettant l'utilisation de stratégies markoviennes linéaires dans les jeux différentiels décrivant l'exploitation d'une ressource naturelle commune. Nous montrons que l'existence de tels équilibres est assujettie à l'existence d'une relation précise entre les éléments essentiels du modèle, notamment la fonction d'utilité des agents et la fonction de « dynamique naturelle » ou de reproduction de la ressource exploitée. Ainsi, pour une fonction d'utilité donnée, seule une famille spécifique de fonctions de reproduction est compatible avec l'utilisation de stratégies markoviennes linéaires. De même, lorsque la fonction de reproduction est connue, seule une famille particulière de fonctions d'utilité permet l'utilisation de stratégies linéaires.

Dans le second chapitre, nous étudions un « Fish War » dans lequel les agents impliqués se font la concurrence uniquement sur le marché de l'intrant, c'est-à-dire uniquement au cours de l'exploitation de la ressource. Ces agents sont repartis en deux groupes et diffèrent par leur taux d'actualisation. Nous examinons l'impact du différentiel de taux d'actualisation sur l'équilibre de ce jeu. Nous montrons alors qu'au niveau global, des augmentations du différentiel de taux d'actualisation et de la proportion d'agents avec le taux d'actualisation le plus élevé (les « gros »), augmentent l'extraction totale et diminuent le stock de ressource à l'état stationnaire. Cependant, au niveau individuel, l'impact de ces deux types d'asymétrie dépend de la comparaison de l'élasticité de l'utilité marginale à un. Pour ce qui est de l'asymétrie de « taille de groupe », lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à un, une augmentation de la proportion de « gros » agents dans l'industrie, tend à réduire (augmenter) le taux d'extraction des deux types d'agents. Ainsi, chercher à rendre l'industrie plus « homogène » en « gros » agents aura tendance à atténuer (exacerber) la « guerre » engendrée par la concurrence, lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à un. Concernant l'asymétrie de taux d'actualisation, une augmentation du différentiel augmente toujours le taux d'extraction des « gros » agents. Par contre, pour les « petits » agents, cette augmentation du différentiel de taux d'actualisation tend à réduire (augmenter) leur taux d'extraction lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à l'unité.

Dans le dernier chapitre, nous considérons deux groupes d'entreprises qui exploitent une ressource naturelle commune et en vendent la production sur le même marché. Dans ce cas, ces entreprises se font la concurrence aussi bien sur le marché de l'intrant que sur le marché de l'extrant. Deux entreprises représentant chaque groupe diffèrent l'une de l'autre par leurs coûts marginaux. Nous avons ainsi un groupe d'entreprises à bas coût marginal auxquelles nous feront référence en tant que « grosses » entreprises, et un groupe à haut coût marginal que nous présenterons comme étant les « petites » entreprises. Nous caractérisons explicitement les stratégies markoviennes d'équilibre de ce jeu, ainsi que les effets des deux types d'asymétrie sur les équilibres obtenus. Les stratégies d'équilibre sont caractérisées par trois intervalles de stocks de ressource sur lesquels les entreprises adoptent des comportements différents. En-deçà d'un certain stock-seuil, aucune entreprise ne produit. Entre ce stock-seuil et un second stock-seuil, les entreprises exploitent la ressource à des taux linéaires et croissants avec le stock de ressource. Au-delà de ce second stock-seuil, les entreprises exploitent la ressource à des taux constants et qui correspondent aux taux d'exploitation qu'elles auraient adoptés si elles se faisaient une concurrence statique à la Cournot. Nous trouvons que la présence d'asymétries induit des discontinuités dans la stratégie des grosses entreprises, et par conséquent dans le taux d'exploitation agrégé. Nous montrons aussi que le stock-seuil à partir duquel les petites entreprises commencent leur exploitation et le stock-seuil à partir duquel elles adoptent leur exploitation à la Cournot statique, sont tous les deux plus élevés lorsque ces entreprises sont en présence de

grosses entreprises (cas asymétrique) que lorsqu'elles sont toutes identiques (cas symétrique). Quant aux grosses entreprises, lorsque leur proportion dans l'industrie dépasse un certain seuil, le stock-seuil auquel elles commencent l'exploitation est plus élevé dans le cas symétrique que dans le cas asymétrique. En-deçà de ce seuil, ces grosses entreprises commencent leur exploitation à un stock-seuil plus bas que dans le cas symétrique. Le stock-seuil auquel elles adoptent leur comportement à la Cournot statique est, quant à lui, toujours plus bas dans le cas asymétrique que dans le cas symétrique où elles sont toutes de grosses entreprises. Nous trouvons aussi que ce modèle admet un ou trois états stationnaires selon la valeur du différentiel de coût marginal ou la répartition des entreprises. De plus, chacun de ces états stationnaires peut être obtenu en faisant varier les deux types d'asymétries.

Mots clés : ressource naturelle commune, oligopole, jeu différentiel, stratégie markovienne, équilibre en boucle fermée, agents hétérogènes, asymétrie.

SUMMARY

This dissertation is composed of three essays dealing with heterogeneous agents exploiting a natural resource owned in common. Problems of common pool resource harvesting are often modelled as differential games, which take into account the strategic interactions between the agents involved and the dynamics of the resource. Our present work brings two main contributions to this literature. Firstly, we introduce asymmetry among the agents and derive explicit closed-loop equilibrium strategies for the asymmetric model obtained. Secondly, we examine the impact of these asymmetries on the outcomes of the game, that is, on the individual strategies of the agents, on the aggregate extraction rate and on the equilibrium steady states, in the context of Markov perfect Nash equilibria. In the first chapter, we restrict attention to linear Markov strategies (strategies expressed as a constant proportion of the current resource stock) and derive necessary conditions for the use of such strategies. In the second chapter, we assume a model that satisfies the conditions derived in the first chapter and solve for explicit linear Markov strategies for a "fish war" involving two groups of agents of different sizes with different discount rates. In this fish war model, agents compete only on the input market. We then examine the impact of the discount rate differential and the distribution of the agents on the equilibrium outcomes of the game. In the third chapter, we present a common pool resource game with two groups of agents (firms), competing this time on both input and output markets. In this last chapter asymmetric firms differ by their marginal costs. We examine the effects of the marginal cost differential and the distribution of the two types of firms on the outcomes of the game. We find that, in both models, both types of asymmetries affect the outcomes of the games in important way.

To be more specific, in the first chapter, we derive necessary conditions for the existence of Markov perfect Nash equilibria in linear strategies for common pool

resource differential games. We show that for such strategies to be used, a precise relationship must be satisfied between the primitives of the model, namely the utility function of the agents and the growth function of the resource. Thus, for a given utility function, only a specific family of growth functions is compatible with the use of linear Markov strategies. Conversely, for a given growth function, only a precise family of utility functions allows the use of such strategies.

In the second chapter, we present a "fish war" between agents exploiting a common pool resource and divided into two groups of potentially different sizes. Agents are identical within a group but differ between groups by their discount rates. We examine the impact of discount rate and group size asymmetries on the outcomes of the game. We show that, at the industry level, increases in the discount rate differential and in the proportion of "big" agents (those with the larger discount rate), both increase the aggregate extraction rate and decrease the steady state stock level. However, at the individual level, the impacts depend on whether the elasticity of marginal utility is greater or smaller than unity. For the "size" asymmetry, when the elasticity of marginal utility is greater (smaller) than unity, an increase in the proportion of big firms tends to decrease (increase) the individual extraction rates of both types of agents. This means that, making the industry "more homogeneous" in big agents will tend to attenuate (exacerbate) the "fish war", if the elasticity of marginal utility is greater (smaller) than one. As for the discount rate asymmetry, an increase in the discount rate differential always increases the individual extraction rates of the agents with the larger discount rate. However, when the elasticity of marginal utility is greater (smaller) than unity, this increase in the discount rate differential decreases (increases) the extraction rates of the agents with the smaller discount rate.

In the last chapter, we consider two groups of firms harvesting a common pool resource and selling their production on the same output market. They therefore compete on both the input and the output markets. Representative firms of these

two groups (of potentially different sizes) differ from one another by their marginal costs. We then have a group of low marginal cost firms – referred to as "big" firms – and a group high marginal cost firms – referred to as "small" firms. We derive explicit Markov perfect equilibrium strategies and examine the effects of marginal cost differential and group size asymmetries on the outcomes of the game. The equilibrium strategies of the firms are characterized by three intervals of stocks over which they adopt different exploitation behavior. When the resource stock is less than a certain threshold, there is no exploitation at all. Above that threshold and below a second threshold, the firms exploit the resource at rates that are linear and increasing in the resource stock. From this second threshold on, the firms produce at the constant harvest rates they would adopt under a static Cournot game. We find that the presence of asymmetries induces discontinuities in the strategy of the big firms and consequently in the aggregate harvest rate. We also find that the small low cost firms begin exploiting the resource and revert to their static Cournot production at threshold resource stocks that are higher when they are in presence of big firms than when they are the only type in the industry. As for the big low cost firms, they begin exploiting the resource at a higher resource stock in the asymmetric case than in the symmetric case when their proportion in the industry is above some threshold, and at a lower resource stock when their proportion is below that threshold. They begin producing at their static Cournot harvest rate at a lower resource stock in the asymmetric setting than in the symmetric setting. We also find that the equilibrium outcomes admit one or three steady states depending on the range of the asymmetries. Moreover any of these steady states can be reached by varying the asymmetries.

Keywords: common pool resource, oligopoly, differential game, closed-loop equilibrium, Markov strategies, heterogeneous agents, asymmetry.

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À l'Éternel des Armées, par qui ce rêve devient réalité. Merci pour cette grâce !

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Montréal, le 1er mars 2006 Hervé Lohoues

INTRODUCTION GÉNÉRALE

Les agents économiques exploitant un stock commun de ressources naturelles ne sont pas nécessairement identiques. À titre d'exemple nous pouvons citer le cas d'une industrie de pêche dans laquelle interviennent de grosses multinationales au même titre que de petites entreprises locales ou tout simplement de petits pêcheurs. L'une des caractéristiques de ces multinationales est qu'elles disposent de plus de moyens logistiques et technologiques leur permettant d'avoir un certain avantage quant à la quantité de poissons pêchés, sur les petites entreprises locales et les petits pêcheurs intervenant dans cette industrie. Comme second exemple, nous pouvons aussi citer le cas de l'accès à une nappe aquifère par des intervenants hétérogènes. Parmi eux, l'on pourrait trouver de grandes multinationales d'embouteillage d'eau minérale ainsi que de petites entreprises locales, dont l'activité est essentiellement à but lucratif.

Dans cette thèse, nous utilisons une approche permettant de prendre en compte cette hétérogénéité. Nous séparons les agents en deux groupes sur la base d'une caractéristique intrinsèque permettant de les classer. Nous faisons ainsi l'hypothèse que les agents sont tous identiques au sein d'un même groupe et que le nombre d'agents peut être différent d'un groupe à l'autre. Par cette méthode, nous introduisons dans nos modèles deux types d'asymétries : (i) une asymétrie dite « intrinsèque » qui porte essentiellement sur la caractéristique qui fait la différence entre des agents représentatifs de chaque groupe et (ii) une asymétrie de « taille de groupe » qui concerne la répartition des agents dans chaque groupe. Un des objectifs de cette thèse est justement d'analyser l'impact de ces asymétries sur les équilibres du jeu différentiel résultant de la modélisation de cette situation.

Les jeux différentiels sont des modèles de jeux dynamiques servant à étudier des systèmes évoluant dans le temps, et dont la dynamique peut être décrite par des équations différentielles (Dockner et al. [11]). Ils sont ainsi devenus un outil privilégié en économie des ressources naturelles, lorsqu'il s'agit de décrire des interactions stratégiques entre plusieurs agents économiques exploitant une ressource naturelle commune dont l'évolution du stock, qui constitue dans ce cas l'état du système, est décrite par une équation différentielle.

Dans la formalisation des jeux différentiels, deux concepts de solutions sont le plus souvent utilisés : les équilibres de Nash en boucle ouverte (« open-loop Nash equilibria ») et les équilibres de Nash en boucle fermée (« closed-loop Nash equilibria ») pour lesquels l'attention n'est en général accordée qu'aux équilibres de Nash markoviens parfaits (« Markov perfect Nash equilibria »). Dans le concept des équilibres de Nash en boucle ouverte, la stratégie (dans notre cas, la quantité de ressource extraite par un agent à chaque instant) ne dépend que du temps et de l'état initial du système (le stock initial de la ressource). De ce fait, il s'agit d'un équilibre relativement plus facile à déterminer et servant souvent de référence (« benchmark ») pour des comparaisons à d'autres types d'équilibres (Fudenberg et Tirole [15]). Cependant, un équilibre avec des stratégies en boucle ouverte, ne sera en général plus un équilibre si pour une quelconque raison, le stock de ressource dévie du sentier d'équilibre. Les équilibres de Nash markoviens parfaits sont quant à eux, de par leur construction, des équilibres parfaits en sous-jeux, dont les stratégies dépendent de l'état du système à chaque instant. Ainsi contrairement aux équilibres de Nash en boucle ouverte, les équilibres de Nash markoviens parfaits demeurent des équilibres de Nash même pour des valeurs du stock qui s'écartent du sentier d'équilibre. Cependant, les équilibres de Nash markoviens parfaits, restent plus difficiles à caractériser analytiquement que les équilibres en boucle ouverte. Pour plus de détails sur les comparaisons entre ces deux types d'équilibres et leurs implications, voir par exemple Reinganum [30], Dockner et al. [10], Clemhout et Wan [7], Dockner et Sorger [12], et Long et al. [25].

Dans cette thèse, nous nous servons uniquement des équilibres de Nash markoviens parfaits pour caractériser nos équilibres. L'objectif dans ce cas est double :

non seulement caractériser analytiquement ces équilibres lorsque les asymétries présentées plus haut sont introduites, mais aussi et surtout déterminer les impacts que de telles asymétries pourraient avoir sur les équilibres des jeux différentiels modélisés.

Dans la littérature sur l'exploitation des ressources naturelles communes, dans les jeux différentiels utilisant des équilibres markoviens parfaits, les agents économiques sont généralement considérés comme identiques, souvent pour faciliter la caractérisation de la solution. Des exemples sont, entres autres, Dockner et al. [11] dans lequel les agents se font la concurrence uniquement au cours de l'exploitation de la ressource (l'intrant), et Karp [20] et Mason, Polasky [26] et Benchekroun [1], où les agents se font la concurrence aussi bien pendant l'exploitation, mais aussi au cours de la vente de leur production (l'extrant) sur le marché. Il existe aussi certains modèles qui considèrent des agents hétérogènes présentant un type d'asymétrie, notamment l'asymétrie intrinsèque. En effet, dans leur célèbre modèle formulé en temps discret et connu sous le nom de « Fish War », Levhari et Mirman [21] considèrent deux agents dont la différence se situe au niveau de leur taux d'actualisation. Plourde et Yeung [29] proposent une version en temps continue du « Fish War » de Levhari et Mirman, avec plusieurs agents, toujours différant par leurs taux d'actualisation. Ces deux modèles caractérisent leurs équilibres à l'aide de stratégies markoviennes linéaires. Dockner et Sorger [12] considèrent deux agents hétérogènes différant par leur fonction d'utilité et caratérisent leurs équilibres à partir de stratégies markoviennes plus complexes. Toutefois, dans aucun de ces modèles à agents hétérogènes, l'effet de la différence entre les agents n'est examiné.

Au mieux de nos connaissances, la prise en compte de l'hétérogénéité entre agents introduisant aussi bien une différence entre les agents (asymétrie intrinsèque), qu'une différence de nombre d'agents dans chaque groupe (asymétrie de taille de groupe) n'a jamais été abordée dans la littérature sur l'exploitation d'une ressource naturelle commune. Ces asymétries pourraient avoir des impacts sur les équilibres résultant des jeux différentiels considérés. Cette thèse se veut donc une contribution à la littérature à ces deux niveaux. En effet, nous y proposons deux modèles d'agents hétérogènes à asymétrie double, pour lesquels nous caractérisons des équilibres markoviens parfaits et nous analysons l'impact des deux types d'asymétries sur ces équilibres. Dans le premier chapitre, nous déterminons les conditions nécessaires d'utilisation d'une classe particulière de stratégies markoviennes : les stratégies markoviennes linéaires. Dans le second chapitre nous présentons un modèle de « Fish War » avec les deux types d'asymétries et dont l'asymétrie intrinsèque est caractérisée par la différence entre les taux d'actualisation des agents. Dans ce second chapitre, les agents se font la concurrence uniquement sur le marché de l'intrant. Ce modèle est amené à vérifier les conditions nécessaires déterminées au premier chapitre pour permettre l'utilisation de stratégies linéaires dans la caractérisation des équilibres. Dans le troisième chapitre nous présentons un modèle dans lequel les agents se font la concurrence aussi bien sur le marché de l'intrant que sur le marché de l'extrant, et qui introduit aussi les deux types d'asymétries. Cette fois, l'asymétrie intrinsèque est saisie à travers la différence entre les coûts marginaux des agents. De façon générale, nous trouvons que les deux types d'asymétries ont des impacts importants sur les stratégies individuelles, sur le taux d'exploitation agrégé et sur les états stationnaires qui découlent des modèles présentés.

De façon plus spécifique, dans le premier chapitre, nous caractérisons les conditions nécessaires permettant l'utilisation de stratégies markoviennes linéaires dans les jeux différentiels décrivant l'exploitation d'une ressource naturelle commune. De telles stratégies sont par exemple utilisées par Levhari et Mirman [21], Clemhout et Wan [6], Plourde et Yeung [29], Fischer and Mirman [13, 14], Long et Shimomura [24], et Dockner et al. [11]. Nous montrons que l'existence de tels équilibres est assujettie à l'existence d'une relation précise entre les éléments essentiels du modèle, notamment la fonction d'utilité des agents et la fonction de "dynamique naturelle" ou de reproduction de la ressource exploitée. Ainsi, pour une fonction d'utilité donnée, seule une famille spécifique de fonctions de reproduction est compatible avec l'utilisation de stratégies markoviennes linéaires. De même, lorsque la fonction de reproduction est connue, seule une famille particulière de fonctions d'utilité permet l'utilisation de stratégies linéaires.

Dans le second chapitre, nous étudions un « Fish War » dans lequel les agents impliqués se font concurrence uniquement sur le marché de l'intrant, c'est-à-dire uniquement au cours de l'exploitation de la ressource, comme c'est le cas notamment dans Levhari et Mirman [21] et Plourde et Yeung [29]. Ces agents sont repartis en deux groupes de tailles potentiellement différentes. Nous examinons l'impact du différentiel de taux d'actualisation (l'asymétrie intrinsèque dans ce cas) et de la répartition des agents (l'asymétrie de taille de groupe) sur l'équilibre de ce jeu. Nous montrons alors qu'au niveau global, des augmentations dans le différentiel de taux d'actualisation et dans la proportion d'agents avec le taux d'actualisation le plus élevé (les « gros »), augmentent l'extraction totale et diminuent le stock de ressource à l'état stationnaire. Cependant, au niveau individuel, l'impact de ces deux types d'asymétrie dépend de la comparaison de l'élasticité de l'utilité marginale à l'unité. Pour ce qui est de l'asymétrie de « taille de groupe », lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à un, une augmentation de la proportion de « gros » agents dans l'industrie, tend à réduire (augmenter) le taux d'extraction des deux types d'agents. Ainsi, chercher à rendre l'industrie plus « homogène » en « gros » agents aura tendance à atténuer (exacerber) la « guerre » engendrée par la concurrence, lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à un. Concernant l'asymétrie de taux d'actualisation, une augmentation du différentiel augmente toujours le taux d'extraction des « gros » agents. Par contre, pour les « petits » agents, cette augmentation du différentiel de taux d'actualisation tend à réduire (augmenter) leur taux d'extraction lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à l'unité.

Dans le dernier chapitre, nous considérons deux groupes d'entreprises qui exploitent une ressource naturelle commune et en vendent la production sur le même marché. Dans ce cas et contrairement au second chapitre, ces entreprises se font concurrence aussi bien sur le marché de l'intrant que sur le marché de l'extrant, comme cela est le cas dans Benchekroun [1]. Deux entreprises représentant chaque groupe diffèrent l'une de l'autre de par leurs coûts marginaux. Nous avons ainsi un groupe d'entreprises à bas coût marginal auxquelles nous feront référence en tant que « grosses » entreprises, et un groupe à haut coût marginal que nous présenterons comme étant les « petites » entreprises. Nous caractérisons explicitement les stratégies markoviennes de ce jeu dont le cas symétrique se réduit au modèle de Benchekroun [1], ainsi que les effets des deux types d'asymétries sur les équilibres obtenus. À l'instar de Benchekroun [1], les stratégies d'équilibre que nous trouvons sont caractérisées par trois intervalles de stocks de ressource sur lesquels les entreprises adoptent des comportements différents. En-deçà d'un certain stock-seuil, aucune entreprise ne produit. Entre ce stock-seuil et un second stock-seuil, les entreprises exploitent la ressource à des taux linéaires et croissants avec le stock de ressource. Au-delà de ce second stock-seuil, les entreprises exploitent la ressource à des taux constants et qui correspondent aux taux d'exploitation qu'elles auraient adoptés si elles se faisaient une concurrence statique à la Cournot. Toutefois, la présence d'asymétries induit des discontinuités dans la stratégie des grosses entreprises, et par conséquent dans le taux d'exploitation agrégé. Nous montrons aussi que le stock-seuil à partir duquel les petites entreprises commencent leur exploitation et le stock-seuil à partir duquel elles adoptent leur exploitation à la Cournot statique, sont tous les deux plus élevés lorsque ces entreprises sont en présence de grosses entreprises (cas asymétrique) que lorsqu'elles sont toutes identiques (cas symétrique). Quant aux grosses entreprises, lorsque leur proportion dans l'industrie dépasse un certain seuil, le stock-seuil auquel elles commencent l'exploitation est plus élevé dans le cas symétrique que dans le cas asymétrique. En-deçà de ce seuil, ces grosses entreprises commencent leur exploitation à un stock-seuil plus bas que dans le cas symétrique. Le stock-seuil auquel elles adoptent leur comportement à la Cournot statique est, quant à lui, toujours plus bas dans le cas asymétrique que dans le cas symétrique où elles sont toutes de grosses entreprises. Nous trouvons aussi que ce modèle admet un ou trois états stationnaires selon la valeur du différentiel de coût marginal (l'asymétrie intrinsèque dans ce cas) ou la répartition des entreprises (l'asymétrie de taille de groupe). Par ailleurs, chacun de ces états stationnaires peut être obtenu en faisant varier les deux types d'asymétries.

CHAPITRE 1

ON LIMITS TO THE USE OF LINEAR MARKOV STRATEGIES IN COMMON PROPERTY NATURAL RESOURCE GAMES

by Gérard Gaudet and Hervé Lohoues[1]

Abstract

We derive conditions that must be satisfied by the primitives of the problem in order for an equilibrium in linear Markov strategies to exist in common property natural resource differential games. These conditions impose restrictions on the admissible form of the natural growth function, given a benefit function, or on the admissible form of the benefit function, given a natural growth function.

[1]We wish to thank Rabah Amir and Ngo Van Long for their useful comments on an earlier draft.

1.1 Introduction

For some differential games, it can be shown that there exist equilibrium decision rules that are linear in the current value of the state variables. These types of strategies, called linear Markov strategies, are attractive because of their simplicity and ease of interpretation. They also greatly facilitate the computation of the equilibrium and of its properties. For these reasons, the analysis is often restricted to this class of equilibria, when they exist. Notable examples in common property resource games are Clemhout and Wan [6], Lehvari and Mirman [21], Plourde and Yeung [29], Fischer and Mirman [13, 14], Long and Shimomura [24] and Dockner *et al.* [11].

The use of linear Markov strategies is however limited by the restrictions that must be imposed on the primitives of the model in order for such an equilibrium to exist. In this paper, we derive necessary conditions to the use of linear Markov strategies in natural resource differential games.

The model is presented in section 1.2. In section 1.3, we derive restrictions that must be imposed on the natural growth function, given the frequently assumed constant elasticity utility function. In section 1.4, we derive restrictions that must be imposed on the utility function, given a specific natural growth function. In section 1.5, we briefly discuss an extension to the case where benefit is derived from the remaining resource stock as well as the flow of consumption. We end with some concluding remarks in section 1.6.

1.2 The model

Consider a natural resource that is commonly owned and exploited by n economic agents. Denote by $x(t)$ the stock of the resource at time t and by $c_i(t)$ the rate of harvest of agent i, $i = 1, \ldots, n$. If $g(x(t))$ is the natural growth function of the resource stock, then the state variable $x(t)$ evolves according to the differential equation

$$\dot{x}(t) = g(x(t)) - \sum_{i=1}^{n} c_i(t). \tag{1.1}$$

It is assumed that agent i derives an instantaneous net benefit $u(c_i(t))$ from his harvest, with $u'(c_i(t)) > 0$ and $u''(c_i(t)) < 0$.

By assumption, we restrict attention to equilibria in stationary linear Markov strategies. Stationary Markov strategies in this context are decision rules that specify an agent's harvest rate as a function of the current resource stock : $c_i(t) = \phi_i(x(t))$. A linear strategy for agent i is a strategy of the form $\phi_i(x(t)) = \delta_i x(t)$, with $\delta_i > 0$ a constant.

An equilibrium in linear Markov strategies, if it exists, will necessarily have the property that a best response of agent i to linear strategies being played by each of his $n-1$ rivals is also a linear strategy. The question then is : What are the minimal restrictions that need to be put on the primitives of the problem (the natural growth function $g(x(t))$ and the utility function $u(c_i)$) in order for this property to be satisfied ?

At equilibrium it will be the case that, taking as given the vector of decision rules $\phi_j(x) = \delta_j x$, $j \neq i$ of his $(n-1)$ rivals, agent i's own decision rule, $c_i = \phi_i(x)$, maximizes

$$\int_0^{\infty} e^{-r_i t} u(c_i) dt \tag{1.2}$$

subject to

$$\dot{x} = g(x) - c_i - x \sum_{j \neq i} \delta_j \tag{1.3}$$

$$x(0) = x_0 \quad \text{given} \tag{1.4}$$

$$c_i \geq 0, \quad \lim_{t \to \infty} x(t) \geq 0, \tag{1.5}$$

where r_i is agent i's discount rate.

The current value Hamiltonian associated to this problem is

$$H(x, c_i, \lambda_i) = u(c_i) + \lambda_i[g(x) - c_i - x \sum_{j \neq i} \delta_j], \tag{1.6}$$

where λ_i is the shadow value of the resource stock for agent i.

An equilibrium must satisfy, for $i = 1 \ldots, n$, the following set of necessary conditions, in addition to (1.3) and (1.4) :

$$[u'(c_i) - \lambda_i]c_i = 0, \quad u'(c_i) - \lambda_i \leq 0, \quad c_i \geq 0 \tag{1.7}$$

$$\frac{\dot{\lambda}_i}{\lambda_i} = r_i - g'(x) + \sum_{j \neq i} \delta_j \tag{1.8}$$

$$\lim_{t \to \infty} e^{-r_i t} \lambda_i x = 0, \quad \lim_{t \to \infty} e^{-r_i t} \lambda_i \geq 0 \quad \lim_{t \to \infty} x(t) \geq 0. \tag{1.9}$$

Assume $\phi_i(x) = \delta_i x$ to be a solution, with $\delta_i > 0$. Then, for any $x > 0$, it will be the case that $\dot{c}_i = \delta_i \dot{x}$ and hence

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{x}}{x}. \tag{1.10}$$

It also follows that (1.3) can be rewritten as

$$\frac{\dot{x}}{x} = \frac{g(x)}{x} - \sum_{j \neq i} \delta_j - \delta_i. \tag{1.11}$$

Furthermore, from (1.7) and (1.8), along an interior solution,

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\eta(c_i)} \left[g'(x) - \sum_{j \neq i} \delta_j - r_i \right], \tag{1.12}$$

where $\eta(c_i)$ is the elasticity of marginal utility[1], given by

$$\eta(c_i) = \left[-\frac{c_i u''(c_i)}{u'(c_i)} \right]. \tag{1.13}$$

[1]The reciprocal, $1/\eta(c_i)$, can be interpreted as the instantaneous elasticity of intertemporal substitution.

Therefore, substituting from (1.11) and (1.12) into (1.10), we find that the following condition must be satisfied in order for $c_i = \delta_i x$ to be a best response :

$$\frac{1}{\eta(\delta_i x)}\left[g'(x) - \sum_{j\neq i}\delta_j - r_i\right] - \left[\frac{g(x)}{x} - \sum_{j\neq i}\delta_j - \delta_i\right] = 0, \tag{1.14}$$

where δ_i and the δ_j's are constants that remain to be determined.

It follows that, for any given utility function $u(c_i)$, the growth function $g(x)$ must satisfy the following first-order linear differential equation in x :

$$xg'(x) - \eta(\delta_i x)g(x) = \left[\sum_{j\neq i}\delta_j + r_i\right]x - \left[\sum_{j\neq i}\delta_j + \delta_i\right]\eta(\delta_i x)x. \tag{1.15}$$

Alternatively, given a growth function $g(x)$, the marginal utility function $u'(c_i)$ must satisfy the following first-order linear differential equation in c_i :

$$\left[\frac{g(c_i/\delta_i)}{(c_i/\delta_i)} - \sum_{j\neq i}\delta_j - \delta_i\right]c_i u''(c_i) + \left[g'(c_i/\delta_i) - \sum_{j\neq i}\delta_j - r_i\right]u'(c_i) = 0. \tag{1.16}$$

1.3 Admissible growth functions, given a utility function

Typically, in this type of problem, attention is restricted to the class of utility functions that exhibit a constant elasticity of marginal utility. Denoting by $\theta > 0$ this elasticity, the utility function may then take the form :

$$u(c_i) = \frac{c_i^{1-\theta}}{1-\theta} \tag{1.17}$$

or

$$u(c_i) = \ln c_i, \tag{1.18}$$

which is the limiting case of (1.17) for $\theta = 1$.[2]

[2] A more general representation of this utility function is $u(c_i) = a(c_i^{1-\theta})/(1-\theta) + b$ or $u(c_i) = a \ln c_i + b$ for $\theta = 1$. In the present context, there is no loss of generality in setting $a = 1$ and $b = 0$.

In that case, $\eta(c_i) = \theta$, a constant, and (1.15) has as a unique general solution :

$$g(x) = \begin{cases} \left[r_i - \theta\delta_i + (1-\theta)\sum_{j\neq i}\delta_j\right]\dfrac{x}{1-\theta} + kx^\theta & \text{if } \theta \neq 1 \\ (r_i - \delta_i)\, x \ln x + kx & \text{if } \theta = 1\ , \end{cases} \tag{1.19}$$

where k is the constant of integration.

Therefore, given a utility function of the form (1.17) or (1.18), a decision rule of the form $\phi_i(x) = \delta_i x$ will be a best response to decision rules of the form $\phi_j(x) = \delta_j x$, $j \neq i$, on the part of i's $n-1$ rivals, only if the growth function is of the form :

$$g(x) = \begin{cases} \alpha x + \beta x^\theta & \text{if } \theta \neq 1 \\ \alpha x + \beta x \ln x & \text{if } \theta = 1\ . \end{cases} \tag{1.20}$$

Substituting from (1.20) and (1.17) or (1.18) into (1.15), we get the following system of n equations :

$$\begin{cases} \theta\delta_i + (\theta-1)\sum_{j\neq i}\delta_j - r_i - (\theta-1)\alpha = 0 & \text{if } \theta \neq 1 \\ \delta_i = r_i - \beta & \text{if } \theta = 1 \end{cases} \tag{1.21}$$

which determines the constant equilibrium values of δ_i, $i = 1, \ldots, n$. In particular, with identical agents (i.e., $r_i = r$, $i = 1, \ldots, n$), the symmetric equilibrium is given by

$$\delta = \begin{cases} \dfrac{r + \alpha(\theta-1)}{n\theta - (n-1)} & \text{if } \theta \neq 1 \\ r - \beta & \text{if } \theta = 1\ . \end{cases} \tag{1.22}$$

The class of functions in (1.20) exhibits desirable properties for a natural growth function when the parameter values are restricted to $\alpha \geq 0$, $\beta < 0$ and $\theta \geq 1$ (or $\alpha < 0$, $\beta > 0$ and $0 < \theta < 1$). It is then strictly concave, with $g(0) = 0$ and

$g(\bar{x}) = 0$, where $\bar{x} = (-\alpha/\beta)^{\frac{1}{\theta-1}}$ in the case of $\theta > 1$ (or $0 < \theta < 1$) and $\bar{x} = e^{-\alpha/\beta}$ in the case of $\theta = 1$.[3] The stock level $\bar{x}$ constitutes a stable steady-state in the absence of harvesting of the resource and captures the idea of the natural carrying capacity of the environment.

A major drawback however is that unless we restrict the growth function to $\beta = 0$, it must depend explicitly on a parameter of the utility function, namely θ, if the decision rule $\phi_i(x) = \delta_i x$ is to be a best response to $\phi_j(x) = \delta_j x$, $j \neq i$. This also means that unless $\beta = 0$ is imposed, heterogeneity over the θ's is not admissible, since the growth function $g(x)$ must be common to all agents, by the very nature of the problem.

1.4 Admissible utility functions, given a growth function

Conversely, consider the case where the growth function is known to be of one of the forms in (1.20), with α, β and θ being known exogenous parameters. Then, from (1.16), we have that the elasticity of marginal utility must be given by :

$$\eta(c_i) = \frac{-c_i u''(c_i)}{u'(c_i)} = \begin{cases} \dfrac{\alpha + \theta\beta\left(\dfrac{c_i}{\delta_i}\right)^{\theta-1} - \sum_{j\neq i}\delta_j - r_i}{\alpha + \beta\left(\dfrac{c_i}{\delta_i}\right)^{\theta-1} - \sum_{j\neq i}\delta_j - \delta_i} & \text{if } g(x) = \alpha x + \beta x^{\theta} \\[2ex] \dfrac{\alpha + \beta + \beta\ln\left(\dfrac{c_i}{\delta_i}\right) - \sum_{j\neq i}\delta_j - r_i}{\alpha + \beta\ln\left(\dfrac{c_i}{\delta_i}\right) - \sum_{j\neq i}\delta_j - \delta_i} & \text{if } g(x) = \alpha x + \beta x\ln x\ . \end{cases} \tag{1.23}$$

It follows that a decision rule of the form $\phi_i(x) = \delta_i x$ can be a best response to decision rules of the form $\phi_j(x) = \delta_j x$, $j \neq i$, on the part of i's $n-1$ rivals only if

[3]Imposing $\alpha \geq 0$, $\beta \leq 0$ and $\theta \geq 1$ (or $\alpha < 0$, $\beta > 0$ and $0 < \theta < 1$) in fact guarantees the sufficiency of conditions (1.7), (1.8) and (1.9). Note that when $\alpha = \beta = 0$, we have the case of a non renewable resource.

$\eta(c_i)$ is of the following form :

$$\eta(c_i) = \begin{cases} \dfrac{A + \theta B c_i^{\theta-1}}{C + B c_i^{\theta-1}} & \text{if } g(x) = \alpha x + \beta x^{\theta} \\ \dfrac{D + E \ln c_i}{F + E \ln c_i} & \text{if } g(x) = \alpha x + \beta x \ln x. \end{cases} \tag{1.24}$$

Hence the utility function will be of the form :

$$u(c_i) = a \int^{c_i} e^{-\int^z \frac{\eta(s)}{s} ds} dz + b \tag{1.25}$$

and the marginal utility function of the form :

$$u'(c_i) = a e^{-\int^{c_i} \frac{\eta(s)}{s} ds}, \tag{1.26}$$

where $a > 0$ and $\eta(c_i)$ must be given by (1.24). Strict concavity is assured by imposing $\eta(c_i) > 0$ in (1.24).

This class of utility functions of course includes as a special case that specified in (1.17) whenever $A = \theta C$ (with $b = 0$ and $a = 1/(1-\theta)$), or that specified in (1.18) whenever $D = F$ (with $b = 0$ and $a = 1$).

Substituting from (1.24) into (1.23), we find that the constant equilibrium solution for δ_i, $i = 1, \ldots, n$, must satisfy, if $g(x) = \alpha x + \beta x^{\theta}$:

$$\begin{aligned} &\frac{A}{B}\left(1 - \theta\frac{C}{A}\right)\beta\delta_i^{1-\theta} - \left(\theta\delta_i + (\theta-1)\textstyle\sum_{j\neq i}\delta_j - r_i - (\theta-1))\alpha\right) = 0 \\ &\frac{C}{A} - \frac{\alpha - \sum_{j\neq i}\delta_j - \delta_i}{\alpha - \sum_{j\neq i}\delta_j - r_i} = 0 \end{aligned} \tag{1.27}$$

and, if $g(x) = \alpha x + \beta x \ln x$:

$$\delta_i = r_i - \beta + (D - F). \tag{1.28}$$

In particular, with identical agents, the symmetric equilibrium value of δ will be given, if $g(x) = \alpha x + \beta x^\theta$, by :

$$\begin{aligned} &\frac{A}{B}\left(1 - \theta\frac{C}{A}\right)\beta\delta^{1-\theta} - [n\theta - (n-1)]\delta - r - (\theta - 1))\alpha = 0 \\ &\frac{C}{A} - \frac{\alpha - n\delta}{\alpha - (n-1)\delta - r} = 0 \end{aligned} \tag{1.29}$$

and, if $g(x) = \alpha x + \beta x \ln x$, by :

$$\delta = r - \beta + (D - F). \tag{1.30}$$

Notice that if $A = \theta C$ and $D = F$, in which case, as noted above, the utility function is of the constant elasticity form (1.17) and (1.18) respectively, then (1.27) and (1.28) reduce to (1.21) and (1.29) and (1.30) reduce to (1.22). But the admissible class of utility functions is wider than the constant elasticity class. Again, however, an important drawback is that the parameters of the utility function depend explicitly on the exogenous parameters of the growth function, namely α, β and θ.

1.5 An extension

It is sometimes appropriate to have utility depend not only on the flow of consumption of the resource, but also directly on the stock, because of the flow of amenities it may provide.[5] To capture this, assume that agent i derives an instantaneous benefit $u(c_i(t), x(t))$ from those two sources, with $u_c(c_i(t), x(t)) > 0$, $u_x(c_i(t), x(t)) > 0$, $u_{cc}(c_i(t), x(t)) < 0$ and $u_{xx}(c_i(t), x(t)) < 0$. Then the equivalent of (1.15) is :

$$\begin{aligned} &xg'(x) - [\eta(\delta_i x, x) - \xi(\delta_i x, x)]g(x) = \\ &\quad \left[\sum_{j\neq i}\delta_j + r_i\right]x - \left[\sum_{j\neq i}\delta_j + \delta_i\right][\eta(\delta_i x, x) - \xi(\delta_i x, x)]x - S(\delta_i x, x)x, \end{aligned} \tag{1.31}$$

[5]We thank Ngo Van Long for suggesting this extension.

where

$$\eta(c_i, x) = -\frac{c_i u_{cc}(c_i, x)}{u_c(c_i, x)} \quad \text{and} \quad \xi(c_i, x) = \frac{x u_{cx}(c_i, x)}{u_c(c_i, x)}$$

are respectively the elasticity of marginal utility of c_i with respect to c_i and x and

$$S(\delta_i x, x) = \frac{u_x(\delta_i x, x)}{u_c(\delta_i x, x)}$$

is the marginal rate of substitution between c_i and x. Given the function $u(c_i(t), x(t))$, the function $g(x)$ must satisfy the first-order linear differential equation (1.31) if $c_i = \delta_i x$ is to be a best response.

Consider, as an example, the following version of the constant elasticity utility function, with $\sigma > 0$ and $\theta \neq 1$:

$$u(c_i, x) = x^\sigma \frac{c_i^{1-\theta}}{1-\theta} \tag{1.32}$$

Then $\eta(\delta_i x, x) = \theta$, $\xi(\delta_i x, x) = \sigma$, $S(\delta_i x, x) = \sigma\delta_i/(1-\theta)$ and (1.31) has as a general solution :

$$g(x) = \left[r_i - (\theta - \sigma + \frac{\sigma}{1-\theta})\delta_i + (1-\theta+\sigma)\sum_{j\neq i}\delta_j\right]\frac{x}{1-\theta+\sigma} + kx^{\theta-\sigma} \tag{1.33}$$

where k is the constant of integration. Hence the growth function has to be of the form :

$$g(x) = \alpha x + \beta x^{\theta-\sigma} \tag{1.34}$$

In particular, if $\sigma = \theta$, then admissible functions must be of the form

$$g(x) = \alpha x + \beta.$$

In that case, the utility function is homogeneous of degree one and hence marginal utility of consumption can be written $u_c(c_i/x, 1)$, which becomes a constant when we set $c_i = \delta_i x$. Therefore, from the first-order condition for the maximization of the Hamiltonian, $\lambda(t)$ must be a constant. It then follows directly from condition (1.8) that $g'(x)$ must be a constant as well. This will be true of any utility function that is homogeneous of degree one in c and x.

The constant equilibrium values of δ_i, $i = 1, \ldots, n$ are obtained as the solution to the following system of n equations :

$$\left[(\theta - \sigma) + \frac{\sigma}{1-\theta}\right]\delta_i + (\theta - \sigma - 1)\sum_{j \neq i}\delta_j - r_i - (\theta - \sigma - 1)\alpha = 0. \qquad (1.35)$$

Setting $r_i = r$, the symmetric equilibrium is given by :

$$\delta = \frac{r + (\theta - \sigma - 1)\alpha}{n(\theta - \sigma) - (n-1) + \sigma/(1-\theta)}.$$

The limiting case of (1.32) for $\theta = 1$, $\sigma > 0$, provides an example of the fact that for $g(x)$ to satisfy condition (1.31), though necessary, is not sufficient for the best response to be linear. The utility function is then given by :

$$u(c_i, x) = x^\sigma \ln c_i, \qquad (1.36)$$

and $\eta(\delta_i x, x) = 1$, $\xi(\delta_i x, x) = \sigma$ and $S(\delta_i x, x) = \sigma\delta_i[\ln x + \ln \delta_i]$. The solution to the differential equation (1.31) is then :

$$g(x) = \left[\frac{r_i}{\sigma} + (1 - \ln\delta_i)\delta_i + \sum_{j \neq i}\delta_j\right] x - \delta_i x \ln x + k x^{1-\sigma},$$

which means that admissible growth functions must be of the form :

$$g(x) = \alpha x + \beta x \ln x + \gamma x^{1-\sigma}.$$

Substituting for this form of growth function into the differential equation (1.31), we find that it will be satisfied only if δ_i, $i = 1, \ldots, n$, solves :

$$(1 - \sigma + \sigma\ln\delta_i)\delta_i - \sigma\sum_{j \neq i}\delta_j - r_i + \sigma\alpha + \beta + \sigma(\delta_i + \beta)\ln x = 0, \qquad (1.37)$$

which involves x. Clearly, for agent i's best response to be linear in x further requires that $\delta_i = -\beta > 0$, with, in addition, $\alpha = [r_i - n\sigma + \sigma \ln(-\beta)]/\sigma$. This in turn requires $r_i = r$ for all i, since it is inherent to the problem that the growth function is common to all agents.

1.6 Concluding remarks

The above results place in proper perspective models that rely on linear Markov strategies to study the competition over a common property resource, by showing that the parameters of the utility function and of the growth function cannot be chosen independently of one another, but must satisfy a precise relationship. Assigning specific numerical values to the parameters of functional forms that happen to satisfy this relationship — for instance a unit elasticity of marginal utility or a linear growth function — sometimes tends to obscure this fact.

The papers cited in the introduction all assume specific functional forms for the utility function and the growth function that happen to jointly satisfy this necessary relationship. Clemhout and Wan [6] and Plourde and Yeung [29] both assume a logarithmic utility function as in (1.18) — hence an elasticity of marginal utility equal to one — combined with a growth function as in (1.20) with $\theta = 1$. Lehvari and Mirman [21] assume a discrete time version of the same growth function, also combined with a logarithmic utility function. This also applies to Fischer and Mirman [13, 14], although in those cases the growth functions allow for interaction between two types of resources. Long and Shimomura [24] assume the utility function to be homogeneous of degree $h > 0$ (or the log of such a function). Such a utility function exhibits a constant elasticity of $1 - h$ (or of 1 in the case of the logarithmic version). Their growth function is assumed homogeneous of degree one, which in effect means a function such as that in (1.20) with $\beta = 0$. Dockner *et al.* [11] (chapter 12) study an exemple with a constant elasticity function of the form (1.17) and a growth function as in (1.20) with $\theta \neq 1$.

CHAPITRE 2

GROUP AND SIZE ASYMMETRIES IN A FISH WAR

by Hervé Lohoues

Abstract

We present a "fish war" between agents exploiting a common property resource and divided into two groups of potentially different sizes. Agents are identical within a group but differ between groups by their discount rates. We examine the impact of discount rate and group size asymmetries on the outcomes of the game. We show that, at the industry level, increases in the discount rate differential and in the proportion of "big" agents (those with the larger discount rate), both increase the aggregate extraction rate and decrease the steady state stock level. However, at the individual level, the impacts depend on whether the elasticity of marginal utility is greater or less than unity. For the "size" asymmetry, when the elasticity of marginal utility is greater (smaller) than unity, an increase in the proportion of big firms tends to decrease (increase) the individual extraction rates of both types of agents. This means that, making the industry "more homogeneous" in big agents will tend to attenuate the "fish war", if the elasticity of marginal utility is greater than one. On the contrary, when the elasticity of marginal utility is less than unity, this will exacerbate the "fish war". As for the discount rate asymmetry, an increase in the discount rate differential always increases the individual extraction rates of the agents with the larger discount rate. However, when the elasticity of marginal utility is greater (smaller) than unity, this increase in the discount rate differential decreases (increases) the extraction rates of the agents with the smaller discount rate.

2.1 Introduction

Exploitation of a common property resource most often involves agents that are not necessarily identical. They can often be divided into two (or more) groups of different sizes, within which agents are identical. In that case, both intrinsic differences between representative agents from each group and the size of the groups matter. We will refer to the former as "intrinsic asymmetry" and to the latter as "size asymmetry".

In real life, examples of this kind of two-sided asymmetry abound. A first example is the case of fisheries, where it is common to find one or a few "big" multinational fishing firms competing with many "small" local firms, or simply local fishermen exploiting the fishing grounds for subsistence. Another example is the existence of a small number of big water bottling firms exploiting an aquifer which is also used by many small firms or individuals.

The situations described above can be modelled as a "fish war", first developed in the seminal paper by Levhari and Mirman [21] to describe the economic implications inherent to fishing conflicts. In a larger sense, this analysis can be extended to any other duopolistic and oligopolistic situation in which many players have access to the same replenishable (or exhaustible) natural resource stock owned in common. In the literature, this is referred to as a common property resource exploitation game. As a differential game, this model has two basic features. The first concerns the strategic aspect : each of the players will take into account the actions of the other participants. The second concerns the dynamics of the resource stock : the future size or rate of growth is affected by the actions undertaken by the players.

We will assume the industry to be composed of two types of agents differing by their discount rates. A first type exhibits a high discount rate and the other type, a low discount rate. We will refer to these agents as the "big" agents and "small" agents respectively. To take the example of the fisheries, we can assume that the big multinational firms have access to alternative investment opportunities that have a higher rate of return. They use this rate of return to discount the yields over time of their fishing activities. Conversely, the small local fishermen have a low rate of return on alternative investments.

We will restrict attention to equilibria in stationary linear Markov strategies. A Markov perfect Nash equilibrium (MPNE) is an equilibrium of a dynamic game in which players adopt decision rules that are contingent upon the current state of the game. MPNEs can be tedious to derive explicitly and are therefore often restricted to those in which decision rules are linear in the current value of the state variable, if such equilibria exist. Linear strategies greatly facilitate the computation of the equilibrium and of its properties and have been used by a number of authors to deal with common property resource games.[1]

Our model is closely related to those of Levhari and Mirman [21] and Plourde and Yeung [29]. Levhari and Mirman assume there are two players and use a recursive dynamic Cournot-Nash approach to solve their duopolistic discrete time dynamic game. Plourde and Yeung present a continuous time version of the Levhari and Mirman's model with n players. In both models, the agents differ from one another by their discount rates. This means that these models examine only a one-sided asymmetry, corresponding to what we call here the "intrinsic" asymmetry. Our model also includes this "intrinsic" asymmetry between the two types of agents, who differ in their discount rates. In addition, we introduce a potential "size" asymmetry by assuming that the number of each of the two types of agents may differ. Our aim is to examine how these two types of asymmetries affect the outcomes of the game. More precisely, we address the impact of the discount

[1] See for instance, Clemhout and Wan [6], Lehvari and Mirman [21], Plourde and Yeung [29], Fischer and Mirman [13, 14], Long and Shimomura [24] and Dockner *et al.* [11]. Gaudet and Lohoues [16] establish necessary conditions for the use of such strategies.

rate differential and the impact of an increase in the proportion of big agents on the individual and aggregate equilibrium extraction rates and on the steady state resource stock level, when it exists.

We show that both the size and intrinsic asymmetries do affect the equilibrium individual and aggregate strategies. At the industry level, increases in the proportion of big agents and in the discount rate differential, both increase the aggregate extraction rate and decrease the steady state stock. However, at the individual level, the impact depends on the asymmetry examined and on whether the elasticity of marginal utility is greater or less than unity. It will be shown that, for the size asymmetry, when the elasticity of marginal utility is greater (smaller) than unity, an increase in the proportion of big firms tends to decrease (increase) the individual extraction rates of both types of players. This means that, making the industry "more homogeneous" in big agents will tend to attenuate the "fish war" when the elasticity of marginal utility is greater than one. On the contrary, when the elasticity or marginal utility is less than unity, this will exacerbate the "fish war". As for the intrinsic asymmetry, an increase in the discount rate differential always increases the individual extraction rates of the big agents. However, when the elasticity of marginal utility is greater (smaller) than unity, an increase in the discount rate differential tends to decrease (increase) the extraction rates of the small agents.

The remainder of the paper is organized as follows. The model is presented and solved in section 2.2. In section 2.3, we analyze how both types of asymmetries affect the equilibrium outcomes of the game. We conclude by summing up our main findings in section 2.4.

2.2 The model

Consider a natural resource that is commonly owned and exploited by n economic agents divided into two groups : a group of n_b "big" agents and a group of n_s

"small" agents. They are identical within a group but asymmetric between groups. A representative member from a given group k, $k = s, b$ has a discount rate r_k. We assume that $r_b \geq r_s$, that is the big agents have larger discount rate than the small agents. The agents compete by choosing the extraction rates that maximize the present value of their flows of discounted benefits.

Denote by $x(t)$ the stock of the resource at time t and by $c_i(t)$ the rate of harvest of a given agent i, $i = 1, \ldots, n$. We assume that agent i derives an instantaneous net benefit $u(c_i(t))$ from his harvest, with $u(c_i)$ being of the form

$$u(c_i) = a\frac{c_i^{1-\theta}}{1-\theta} + b, \tag{2.1}$$

where θ is a strictly positive constant such that $\theta \neq 1$, $a \neq 0$, and b a positive constant. The limiting case of $\theta = 1$ is $u(c_i) = a \ln c_i + b$.[2] The specification in (2.1) gives an isoelastic utility function, very often used in economics, for which the elasticity of marginal utility is given by $\eta(c_i) = -c_i u''(c_i)/u'(c_i) = \theta$.[3] We have that $u'(c_i) > 0$ and $u''(c_i) < 0$, that is $u(.)$ is an increasing and strictly concave function of the extraction rate. We will hereafter, without loss of generality, assume $a = 1$.

The natural growth function of the resource is taken to be of the following form :

$$g(x) = \alpha x - \beta x^{\sigma}, \tag{2.2}$$

where assumptions on parameters σ, α and β will be made such that $g(x)$ is concave.[4].

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[2]This is the utility function used by Plourde and Yeung [29] and Levhari and Mirman [21] with $b = 0$.

[3]The reciprocal, $1/\eta(c_i) = 1/\theta$, can be interpreted as the instantaneous elasticity of intertemporal substitution.

[4]The function $g(x)$ is concave either with $\sigma \geq 1$ and $\alpha \geq \beta \geq 0$ or with $\sigma \leq 1$ and $\alpha \leq \beta \leq 0$. It is important however that α and β not differ in sign, in order for the maximum sustainable yield, given by $g'(x_{MSY}) = 0$, and the carrying capacity, given by $g(K) = 0$, $K \neq 0$, both exist and be positive.

By assumption, we restrict attention to equilibria in stationary linear Markov strategies. Stationary Markov strategies in this context are decision rules that specify an agent's harvest rate as a function of the current resource stock : $c_i(t) = \phi_i(x(t))$. A linear strategy for agent i is a strategy of the form $\phi_i(x(t)) = \delta_i x(t)$, where $\delta_i > 0$ is a constant that represents the harvesting effort of agent i. In that case, taking as given the vector of decision rules $\phi_j(x) = \delta_j x$, $j \neq i$ of his $(n-1)$ rivals, agent i's own decision rule, $c_i = \phi_i(x)$, maximizes

$$\int_0^{\infty} e^{-r_i t} u(c_i) dt \tag{2.3}$$

subject to

$$\dot{x} = g(x) - c_i - x \sum_{j \neq i} \delta_j \tag{2.4}$$

$$x(0) = x_0 \quad \text{given} \tag{2.5}$$

$$c_i \geq 0, \quad \lim_{t \to \infty} x(t) \geq 0. \tag{2.6}$$

The current value Hamiltonian associated to this problem is

$$H_i(x, c_i, \lambda_i) = u(c_i) + \lambda_i [g(x) - c_i - x \sum_{j \neq i} \delta_j] \ , \tag{2.7}$$

where λ_i is the shadow value of the resource stock for agent i.

An equilibrium must satisfy, for $i = 1 \ldots, n$, the following set of necessary conditions, in addition to (2.4) and (2.5) :

$$[u'(c_i) - \lambda_i] c_i = 0, \quad u'(c_i) - \lambda_i \leq 0 \ , \quad c_i \geq 0 \tag{2.8}$$

$$\frac{\dot{\lambda}_i}{\lambda_i} = r_i - g'(x) + \sum_{j \neq i} \delta_j \tag{2.9}$$

$$\lim_{t \to \infty} e^{-r_i t} \lambda_i x = 0, \quad \lim_{t \to \infty} e^{-r_i t} \lambda_i \geq 0, \quad \lim_{t \to \infty} x(t) \geq 0. \tag{2.10}$$

Since $u(c_i)$ and $g\,(x)$ are concave, $H_i(x, c_i, \lambda_i)$ is also concave in c_i and x. This guarantees that the necessary conditions are also sufficient.

Assume $\phi_i(x) = \delta_i x$ to be a solution. Then, for any $x > 0$, it will be the case that $\dot{c}_i = \delta_i \dot{x}$ and hence

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{x}}{x}. \tag{2.11}$$

It also follows that (2.4) can be rewritten as

$$\frac{\dot{x}}{x} = \frac{g(x)}{x} - \sum_{j \neq i} \delta_j - \delta_i. \tag{2.12}$$

Furthermore, from (2.8) and (2.9), along an interior solution,

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\eta(c_i)} \left[g'(x) - \sum_{j \neq i} \delta_j - r_i \right]. \tag{2.13}$$

Therefore, substituting from (2.12) and (2.13) into (2.11), we find that the following condition must be satisfied in order for $c_i = \delta_i x$ to be a best response :

$$\left[g'(x) - \sum_{j \neq i} \delta_j - r_i \right] - \eta(\delta_i x) \left[\frac{g(x)}{x} - \sum_{j \neq i} \delta_j - \delta_i \right] = 0, \tag{2.14}$$

where δ_i and the δ_j's are constants that remain to be determined.

With the specifications in (2.1) and (2.2), this last condition simplifies to :

$$\theta \delta_i - (1 - \theta) \sum_{j \neq i} \delta_j - r_i + (1 - \theta)\, \alpha - \beta\, (\sigma - \theta)\, x^{\sigma - 1} = 0. \tag{2.15}$$

In order for the best response of agent i to be linear, and hence the δ_i's to be constant, we must therefore impose at least one of the following three restrictions :

$$\sigma = \theta \quad \text{or} \quad \beta = 0 \quad \text{or} \quad \sigma = 1. \tag{2.16}$$

Under one or the other of those assumptions, condition (2.15) becomes

$$\theta \delta_i - (1 - \theta) \sum_{j \neq i} \delta_j = r_i - (1 - \theta)\, \gamma \tag{2.17}$$

where

$$\gamma = \begin{cases} \alpha & \text{if either } \sigma = \theta \text{ or } \beta = 0 \\ \alpha - \beta & \text{if } \sigma = 1 \end{cases} . \tag{2.18}$$

Now let us be more specific about the conditions in (2.16) which deal with conditions for the use of linear strategies in our model. If $\sigma = \theta$ (and then $\gamma = \alpha$), there exists a specific relationship between the utility function of the agents and the growth function of the resource. In such a case, when $\theta > 1$, the condition for concavity of $g(x)$ (see footnote 4) implies that $\alpha \geq \beta \geq 0$. When $\theta < 1$, we must have $\alpha \leq \beta \leq 0$. In addition, in the case where strict concavity of the growth function is required, e.g. for the existence of a steady state, we must have $\beta \neq 0$.

If $\sigma \neq \theta$ (and then $\gamma = \alpha - \beta$), it is necessary that either $\beta = 0$ or $\sigma = 1$ in order to ensure the existence of an equilibrium with linear strategies. In other words, if $\sigma \neq \theta$, the growth function must necessarily be linear in order for the δ_i's to be constant. When the growth function is linear, there exists no steady state.

The constants δ_i's are determined by solving the system of equations in (2.17) which, when taking into account the fact that there are two types of agents, can be rewritten in the following matrix form :

$$\begin{bmatrix} 1-(1-\theta)\,n_s & -(1-\theta)\,n_b \\ -(1-\theta)\,n_s & 1-(1-\theta)\,n_b \end{bmatrix} \begin{bmatrix} \delta_s \\ \delta_b \end{bmatrix} = \begin{bmatrix} r_s-(1-\theta)\,\gamma \\ r_b-(1-\theta)\,\gamma \end{bmatrix}, \tag{2.19}$$

the solutions to which are :

$$\delta_s = \frac{r_s - (1-\theta)\left[\gamma - (r_b - r_s)\,n_b\right]}{1-(1-\theta)\,(n_s+n_b)} \tag{2.20}$$

and

$$\delta_b = \frac{r_b - (1-\theta)\left[\gamma + (r_b - r_s)\,n_s\right]}{1-(1-\theta)\,(n_s+n_b)}. \tag{2.21}$$

Finally, the expression of the equilibrium individual extraction rate for a representative agent of a given group $k = s, b$, $k \neq l$, $l = s, b$ is

$$c_k^* = \phi_k(x) = \left[\frac{r_k - (1-\theta)\left[\gamma + (r_k - r_l)\, n_l\right]}{1 - (1-\theta)(n_s + n_b)}\right] x. \tag{2.22}$$

At the industry level, the equilibrium aggregate extraction rate is given by

$$Q^* = n_s \phi_s(x) + n_b \phi_b(x) = \Delta x,$$

where Δ is the aggregate harvesting effort :

$$\Delta = \frac{\left[r_s - (1-\theta)\gamma\right](n_s + n_b) + (r_b - r_s)\, n_b}{1 - (1-\theta)(n_s + n_b)}. \tag{2.23}$$

We will assume that, the parameters of the problem and the numbers of agents are such that the extraction efforts are strictly positive.

These strategies can be used to derive the dynamic behavior of the resource stock. In particular, the steady state stock level, when it exists,[5] is given by :

$$\bar{x} = \left[\frac{\alpha - (n_s + n_b)\, r_s - (r_b - r_s)\, n_b}{\beta\left[1 - (1-\theta)(n_s + n_b)\right]}\right]^{\frac{1}{\theta - 1}}. \tag{2.24}$$

For identical agents, that is, when $r_b = r_s$, the individual harvesting efforts are identical and equal to

$$\delta = \frac{r - (1-\theta)\gamma}{1 - (1-\theta)\, n} \tag{2.25}$$

and the industry harvesting effort is

$$\Delta_{id} = \frac{\left[r - (1-\theta)\gamma\right] n}{1 - (1-\theta)\, n}. \tag{2.26}$$

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[5]A finite steady state exists only in the case where $\sigma = \theta$ and $g(x)$ is strictly concave, that is, in addition to the conditions in footnote 4, we must also have $\beta \neq 0$, $\sigma \neq 0$ and $\sigma \neq 1$. Moreover, for $\theta < 1$, it must be verified that $\alpha > \sum_{i=1}^{n} r_i$. These assumptions ensure that the non-cooperative solution implies convergence of the resource stock to a strictly positive steady state level $\bar{x}$.

The symmetric case steady state stock level, when it exists, is :

$$\bar{x}_{id} = \left[\frac{\alpha - nr}{\beta\left[1-(1-\theta)\,n\right]}\right]^{\frac{1}{\theta-1}} \tag{2.27}$$

2.3 Effects of the asymmetries

Notice from (2.20) and (2.21) that the difference in the individual harvesting efforts is exactly equal to the differential in the discount rates, no matter the value of θ and no matter the number of agents. In particular :

$$\delta_b - \delta_s = r_b - r_s \geq 0. \tag{2.28}$$

That is, the more important the discount rate asymmetry, the larger the effort differential. This also means that agents with higher discount rates will harvest at higher rates.

We now turn to the analysis of equations (2.20), (2.21), (2.23) and (2.24) to examine how both discount rate and size asymmetries affect the players' strategies and the resource dynamics at equilibrium. To do this, let $\varepsilon = r_b - r_s$ and $\rho = n_b/n$ be respectively the discount rate differential and the proportion of big agents in the industry. For the sake of the discussion, normalize the total number of firms to one ($n = 1$). Then, (2.20), (2.21), (2.23) and (2.24) become respectively :

$$\delta_s = \frac{r_s - (1-\theta)\left[\gamma - \rho\varepsilon\right]}{\theta}, \tag{2.29}$$

$$\delta_b = \frac{r_b - (1-\theta)\left[\gamma + (1-\rho)\varepsilon\right]}{\theta}, \tag{2.30}$$

$$\Delta = \frac{r_s - (1-\theta)\,\gamma + \rho\varepsilon}{\theta}, \tag{2.31}$$

and

$$\bar{x} = \left[\frac{\alpha - r_s - \rho\varepsilon}{\beta\theta}\right]^{\frac{1}{\theta-1}}. \tag{2.32}$$

2.3.1 Impacts of "size" asymmetry

Consider first the impacts of the "size" asymmetry. Differentiating (2.29) to (2.32) with respect to ρ, we get :

$$\frac{\partial \delta_s}{\partial \rho} = \frac{\partial \delta_b}{\partial \rho} = \frac{(1-\theta)}{\theta}\varepsilon, \tag{2.33}$$

$$\frac{\partial \Delta}{\partial \rho} = \frac{1}{\theta}\varepsilon \geq 0, \tag{2.34}$$

and

$$\frac{\partial \bar{x}}{\partial \rho} = \frac{\varepsilon}{(1-\theta)\,\theta\beta}\left[\frac{\alpha - r_s - \rho\varepsilon}{\beta\theta}\right]^{\frac{1}{\theta-1}-1} \leq 0. \tag{2.35}$$

The sign of the derivatives in (2.33) only depend on whether θ is greater or smaller than one. However the sign of the derivative in (2.34) is always positive and does not depend on that comparison. Also, when the steady state exists (see footnote 5 and comments on conditions (2.16)), $(1-\theta)$ and β are always of opposite signs. Therefore, the sign of the derivative in (2.35) is always negative. Notice that the signs of parameters α and β do not matter in the determination of the signs of these derivatives, only the sign of $(1-\theta)$ leads the results.

At the industry level, equations (2.34) and (2.35) suggest that an increase in the relative number of big agents increases the total amount of resource extracted and decreases the steady state resource stock, no matter the value of θ. This means that making the industry more homogeneous in big agents is "resource consuming", whereas making it more homogeneous in small agents (decreasing ρ) conserves the resource.

At the individual level, equation (2.33) shows that the impact of the size asymmetry is the same for all the agents no matter their type and depends on whether θ is greater or less than one. When the elasticity of marginal utility exceeds unity, an increase in the relative size of the group of big firms tends to reduce the individual extraction rates of both types of players, resulting in a attenuation of the "fish war". When the elasticity of marginal utility is less than one, we have the opposite effect : this increase tends to raise both individual extraction rates, resulting in an exacerbation of the "fish war". To see the intuition behind this, recall that the elasticity of marginal utility is also the inverse of the intertemporal elasticity of substitution. Then, for instance, when the elasticity of marginal utility is close to zero (an extreme case of when $\theta < 1$), the utility function is nearly linear and the elasticity of intertemporal substitution is very large. In such conditions, increasing the number of players who are more "impatient" tends to induce the individuals to extract more resource today than tomorrow. In other terms, an exacerbation of the "fish war" will occur. For $\theta > 1$, an extreme case is when the elasticity of marginal utility approaches infinity, then the elasticity of substitution is near zero. This means that the players will decrease their extraction rate today to have more resource left for extraction tomorrow. Also, the impact is the same for the small agents since they are being squeezed out by the big players.

2.3.2 Impacts of "intrinsic" asymmetry

Now, consider the impacts of the "intrinsic" or discount rate asymmetry by differentiating (2.29) to (2.32) with respect to ε, keeping r_s constant :[6]

$$\frac{\partial \delta_s}{\partial \varepsilon} = \left(\frac{1-\theta}{\theta}\right)\rho, \tag{2.36}$$

$$\frac{\partial \delta_b}{\partial \varepsilon} = 1 + \left(\frac{1-\theta}{\theta}\right)\rho \geq 0, \tag{2.37}$$

$$\frac{\partial \Delta}{\partial \varepsilon} = \frac{1}{\theta}\rho \geq 0, \tag{2.38}$$

and

$$\frac{\partial \bar{x}}{\partial \varepsilon} = \frac{\rho}{(1-\theta)\,\theta\beta}\left[\frac{\alpha - r_s - \rho\varepsilon}{\beta\theta}\right]^{\frac{1}{\theta-1}-1} \leq 0. \tag{2.39}$$

[6]We will assume here that the change in ε is due to a change in r_b, keeping r_s constant. An equivalent analysis can be carried out, *mutatis mutandis*, keeping r_b constant rather than r_s.

The sign of the derivative in (2.36) depends on whether θ is greater or less than one. However, the signs of the derivatives in (2.37) and (2.38) are always positive and that of (2.39) is always negative, no matter the sign of $(1-\theta)$.

At the industry level, equations (2.38) and (2.39) imply that an increase in the discount rate differential always increases the aggregate extraction effort and decreases the steady state stock, no matter the value of θ. This means that the larger the discount rate differential, the more the resource is currently extracted by the industry and the less the steady state stock. Then, like the size asymmetry, an increase in the intrinsic asymmetry is "resource consuming".

At the individual level, an increase in the discount rate differential always increases the individual extraction effort of the big agents, no matter whether θ is greater or smaller than one. When $\theta > 1$, then $0 \leq \partial\delta_b/\partial\varepsilon \leq 1$: the effect of a change in the discount rate differential is less than one. When $\theta < 1$, then $\partial\delta_b/\partial\varepsilon \geq 1$: the effect is greater than one. As for the small agents, an increase in the discount rate differential decreases their individual extraction effort when $\theta > 1$ and increases it when $\theta < 1$.

To explain these results, assume the increase in the discount rate differential to be due to an increase in the big agents' discount rate, the discount rate of the small agents remaining constant. This tends to raise the extraction effort of the big agents, no matter what is the value of θ : since their discount rate is higher, they put a greater weight on current versus future extraction. As for the small agents, since $\partial\delta_s/\partial\varepsilon = \partial\delta_b/\partial\varepsilon - 1$ by the definition of ε, they will decrease (increase) their equilibrium effort whenever $\partial\delta_b/\partial\varepsilon \leq 1$ ($\partial\delta_b/\partial\varepsilon \geq 1$), which, as just seen, occurs whenever $\theta > 1$ ($\theta < 1$). The explanation is similar if the change in ε is due to a change in r_s keeping r_b constant.

2.4 Concluding summary

We have presented a model of extraction of a common property resource or "fish war" where we have two types of players, with potentially different numbers of each type. We have shown that both the "intrinsic" and "size" asymmetries do affect the outcomes of the game in important ways. Whereas the impacts on the individual behavior of the players depend on whether the elasticity of marginal is greater of less than unity, the global impacts (on the aggregate extraction effort and on the steady state stock level) do not depend on this distinction.

At the industry level, the impact on the aggregate extraction effort, of increasing either the discount rate differential or the proportion of the agents with the larger discount rate, is always positive. This means that the larger the discount rate differential or the more important the proportion of big agents, the greater the total rate of extraction and the lower the steady state resource stock.

However, the impacts of both types of asymmetries on the players' extraction rates, taken individually, depend on whether the elasticity of marginal utility is greater or smaller than unity. For the size asymmetry, the impact is the same for all the agents no matter their types. When the elasticity of marginal utility is greater (smaller) than unity, an increase in the relative size of the group of big firms tends to decrease (increase) the individual extraction rates of both types of players, resulting in an attenuation (exacerbation) of the "fish war".

For the intrinsic asymmetry, the impacts on the individual extraction efforts depend on the agents' type. For the players with the larger discount, an increase in the discount rate differential always increases their extraction effort. As for the small agents, when the elasticity of marginal utility is greater (smaller) than unity, an increase in the discount rate differential, decreases (increases) their individual rate of extraction.

This analysis has assumed that the agents differ by their discount rate. It has also assumed that the number of each type of agents is exogenously given. This leaves room for further research involving both other forms of asymmetry and endogenously determined numbers of agents.

CHAPITRE 3

ASYMMETRIES IN A COMMON POOL RESOURCE OLIGOPOLY

by Hervé Lohoues

Abstract

We consider two groups of firms harvesting a common pool resource and selling their production on the same output market. They therefore compete on both the input and the output markets. Representative firms of these two groups (of potentially different sizes) differ from one another by their marginal costs. We then have a group of low marginal cost firms – referred to as "big" firms – and a group high marginal cost firms – referred to as "small" firms. We derive explicit Markov perfect equilibrium strategies and examine the effects of marginal cost differential and group size asymmetries on the outcomes of the game. The equilibrium strategies of the firms are characterized by three intervals of stocks over which they adopt different exploitation behavior. When the resource stock is less than a certain threshold, there is no exploitation at all. Above that threshold and below a second threshold, the firms exploit the resource at rates that are linear and increasing in the resource stock. From this second threshold on, the firms produce at the constant harvest rates they would adopt under a static Cournot game. We find that the presence of asymmetries induces discontinuities in the strategy of the big firms and consequently in the aggregate harvest rate. We also find that the small low cost firms begin exploiting the resource and revert to their static Cournot production at threshold resource stocks that are higher when they are in presence of big firms than when they are the only type in the industry. As for the big low cost firms, they begin exploiting the resource at a higher resource stock in the asymmetric case than in the symmetric case when their proportion in the industry is above some threshold, and at a lower resource stock when their share is below that threshold. They begin producing at their static Cournot harvest rate at a lower resource stock in the asymmetric setting than in the symmetric setting. We also find that the equilibrium outcomes admit one or three steady states

depending on the range of the asymmetries. Moreover any of these steady states can be reached by varying the asymmetries.

3.1 Introduction

Studies of the economic dynamics of common pool resource exploitation typically assume that the economic agents exploiting the resource are all identical. Yet, in many situations, the heterogeneity of the agents is an inescapable characteristic of the problem. Think for example of the case of fisheries, where it is common to find a number of big multinational fishing firms competing with many small local fishermen for the exploitation of a common fishing ground. These big firms have access to large scale technologies and consequently face considerably lower marginal costs than the small local fishermen. Similarly, aquifers are often shared by a few large capacity users — for instance big bottling firms — and many small capacity users. In such cases, it seems important to take into account the heterogeneity of the agents in order to properly characterize the equilibrium. It is the purpose of this paper to introduce some form of heterogeneity into a common pool resource model and to analyze the impact of this heterogeneity on the equilibrium outcome of the dynamic game being played by the agents.

More precisely, we consider the exploitation of a renewable resource stock by a finite number of two types of agents : a low marginal cost type, which we will call the "big firms" for short, and a high marginal cost type, which we will call the "small firms". The total number of firms, which we will assume fixed, will thus be divided into two groups of firms, identical within groups but different across groups. The distribution of the fixed number of firms between the two groups will be allowed to vary. Thus both the cost differential between the representative agents from each group and the relative size of the groups will be important parameters. We will occasionally refer to the former as the"cost asymmetry" and to the latter as the "size asymmetry".

The situations described above will be modelled as an oligopolistic differential game in which two groups of firms, identical within groups, have access to the same renewable natural resource pool, which they exploit in common. They then sell their harvest on the same output market. We restrict attention to non cooperative equilibria in stationary Markov strategies. A stationary Markov strategy is a decision rule that is contingent only upon the current state of the game. In our context, it specifies the firm's extraction rate as a function of the current stock of the resource.

A number of authors have analyzed the problem of the exploitation of a common pool resource in a differential game framework. Amongst them, Dockner *et al.* [11], Dockner and Sorger [12], Fischer and Mirman [13, 14], Gaudet and Lohoues [16], Levhari and Mirman [21], Plourde and Yeung [29], consider cases where the agents involved compete only for the exploitation of the resource, but do not compete on the output market. In those papers, the benefit functions of the agents depend on their own production only and not on that of their rivals. In this paper, the agents compete in the output market as well as in the exploitation of the resource, as in Benchekroun [1], Karp [20] and Mason and Polasky [26]. Those authors also assume benefit functions that depend not only on the agents' own production, but also, through the market demand, on the production of their rivals. However, they assume identical agents when comes the time to derive equilibrium strategies. We will allow for heterogeneous agents.

Indeed, exploitation of a common pool natural resource may involve heterogeneous agents that incur different operating costs, depending on their intrinsic exploitation capability. In real life, examples of this kind of heterogeneity among agents abound. An example is the case of fisheries, where it is common to find one or a few "big" multinational fishing firms competing with many "small" local firms, or simply local fishermen exploiting the fishing grounds for subsistence. These multinationals which generally use bigger boats or more efficient techniques consequently incur lower marginal costs than the small firms or the fishermen. Another example is the existence of a small number of big water bottling firms exploiting an aquifer which is also used by many small firms or individuals.

Our model is more closely related to that of Benchekroun [1], in that, as in Benchekroun, we consider a renewable natural resource characterized by a concave growth function which is approximated by two linear segments. As in Benchekroun also, we consider that all the firms sell the product of their harvest on the same output market, characterized by a downward sloping demand function. Benchekroun assumes two identical players exploiting the resource at zero marginal cost, and focuses on the effects on the equilibrium resource stock of a unilateral restriction of the exploitation of one firm and the corresponding adjustment in the rival's exploitation. We assume a finite number of firms split into two groups and differentiated by their marginal costs. We focus on the effects of the cost asymmetry and the group size asymmetry on the individual strategies and the aggregate harvest rate, as well as on the dynamics of the resource stock and its steady states.

To do this, we derive a Markov perfect Nash equilibrium of our asymmetric model, compare the corresponding equilibrium strategies to those that arise in the symmetric model, and examine how the two types of asymmetries affect the steady states of the game. More precisely, we compare the situation where both types of firms coexist to the situations where either the small firms or the big firms are the only actors exploiting the resource. We also fully characterize, in terms of the parameters representing the cost asymmetry and the group size asymmetry, the types and the number of steady states obtained.

We find that, as in Benchekroun [1], the equilibrium strategies of the firms are characterized by three intervals of stocks over which they adopt different exploitation behavior. When the resource stock is less than a certain well identified threshold, there is no exploitation at all. Above that threshold and below a second threshold, the firms exploit the resource at rates that are linear and increasing in the resource stock. From this second threshold on, the firms produce at the constant harvest rates they would adopt under a static Cournot game. However, the presence of asymmetries induces discontinuities in the strategy of the big firms and consequently in the aggregate harvest rate. We also find that the small low cost firms begin exploiting the resource and revert to their static Cournot production at threshold resource stocks that are higher when they are in presence of big firms than when they are the only type in the industry. As for the big low cost firms, they begin exploiting the resource at a higher resource stock in the asymmetric case than in the symmetric case when their proportion in the industry is above some threshold, and at a lower resource stock when their share is below that threshold. They begin producing at their static Cournot harvest rate at a lower resource stock in the asymmetric setting than in the symmetric one. We also find that the equilibrium outcomes admit one or three steady states depending on the range of the asymmetries. Moreover any of these steady states can be reached by varying the asymmetries.

The remainder of the paper is organized as follows. The model is presented and solved in Sections 3.2 and 3.3, respectively. In Section 3.4, we compare the outcomes of the asymmetric model to those obtained with the symmetric models when the firms are all identical, either big or small. In Section 3.5, we analyze the type and the number of steady states obtained given the range of values of the asymmetries. We conclude in Section 3.6.

3.2 The model

Consider a natural resource that is commonly owned and exploited by n firms divided into two groups : a group of n_b "big" firms and a group of n_s "small" firms, with $n_s + n_b = n$. They are identical within a group but differ between groups by their (constant) marginal costs. The representative member from a given group i,

$i = s, b$, has a marginal cost w_i. We will assume that $w_s \geq w_b$. Hence, the big firms have a marginal cost advantage over the small firms.

Let $x(t)$ denote the stock of the resource at time t and $q_k(t)$ the rate of harvest of a given firm k, $k = 1, \ldots, n$. The inverse demand function for the output is

$$P(Q) = a - bQ, \tag{3.1}$$

where a and b are two positive constants. We assume that $a - w_i > 0, \;\; i = s, b$.

As in Benchekroun [1], we assume that the natural growth function of the resource takes the form :

$$g(x) = \begin{cases} \delta x & \text{for } \; x \leq x_{\max}/2 \\ \delta \left(x_{\max} - x\right) & \text{for } \; x > x_{\max}/2 \end{cases}, \tag{3.2}$$

where δ and $x_{\max}$ are positive parameters reflecting the characteristics of the ecosystem. δ represents the *intrinsic growth rate* of the resource and $x_{\max}$, the *carrying capacity* of the ecosystem. We assume that the intrinsic growth rate of the resource satisfies :

$$\delta > \frac{(n^2 + 1)\, r}{2}, \tag{3.3}$$

where r is the discount rate, assumed the same for all the firms. This condition is needed to guarantee convergence of the resource stock to strictly positive steady state levels, as will become clear in due course.

We restrict attention to equilibria in stationary Markov strategies. Stationary Markov strategies in this context are decision rules that specify a firm's harvest rate as a function of the current resource stock : $q_k(t) = \phi_k(x(t))$. Firm k takes the strategies of its $(n-1)$ rivals as given in choosing its own decision rule, $q_k = \phi_k(x)$ in order to maximize the present value of its instantaneous profits :

$$J_k = \int_0^{\infty} e^{-rt} \left\{ \left[P\left(q_k + \sum_{l \neq k} \phi_l(x) \right) - w_k \right] q_k \right\} dt \tag{3.4}$$

subject to

$$\dot{x} = g(x) - q_k - \sum_{l \neq k} \phi_l(x) \;, \tag{3.5}$$

$$q_k \geq 0, \quad x(t) \geq 0. \tag{3.6}$$

An n-tuple of strategies $(\phi_1(x), ..., q_k, ...\phi_n(x))$ constitutes a Markov Perfect Nash Equilibrium if, for every possible initial condition $x(0) = x_0$, it simultaneously solves the above problem for $k = 1, 2, ..., n$. Since the firms are identical within each group, it suffices to find a pair of Markov strategies $(\phi_s(x), \phi_b(x))$ which gives an n-tuple composed of n_s decision rules $\phi_s(x)$ and n_b decision rules $\phi_b(x)$ that satisfies this property.

3.3 Characterization of an equilibrium

In this section, we characterize a Markov perfect Nash equilibrium for this non-cooperative differential game. The following proposition provides such an equilibrium when not all the firms are of the same type.

Proposition 3.1. *Assume $0 < n_i < n$ and let $\phi_i(x)$, $i = s, b$, denote the following harvesting strategy :*

$$\phi_i(x) = \begin{cases} 0 & \text{for } x \in [0, x_{1s}) \\ f_i(x) \equiv \alpha(x - x_{1i}) & \text{for } x \in [x_{1s}, x_{2s}) \\ q_i^c & \text{for } x \in [x_{2s}, x_{\max}] \end{cases} \tag{3.7}$$

where, for $i, j = s, b$, $i \neq j$,

$$\alpha = \frac{n+1}{n^2}\left(\delta - \frac{r}{2}\right), \tag{3.8}$$

$$q_i^c = \frac{1}{(n+1)b}\left[a - w_i - n_j(w_i - w_j)\right], \tag{3.9}$$

$$x_{1i} = \frac{\left[\delta - (n^2+1)\frac{r}{2}\right]}{(n+1)^2 b\delta\left(\delta - \frac{r}{2}\right)}\left[a - w_i + n_j(w_i - w_j)\frac{[(2+n)\delta - r]}{n(\delta - r)}\right], \tag{3.10}$$

$$x_{2i} = \frac{(n^2+1)}{(n+1)^2 b\delta}\left[a - w_i + n_j(w_i - w_j)\frac{[2\delta - (n^2+1)r]}{n(n^2+1)(\delta - r)}\right]. \tag{3.11}$$

The n-tuple $(\phi_s, ..., \phi_s, \phi_b ..., \phi_b)$ *composed of* n_s *decision rules* $\phi_s(x)$ *and* n_b *decision rules* $\phi_b(x)$ *constitutes a Markov Perfect Nash Equilibrium.*

Proof. See Appendix A.

Note that these strategies are such that there is no interval of resource stock over which only one type of firm produces. The level of stock above which both types of firms begin harvesting the resource is x_{1s}. It is given by (3.10), with $i = s$. When the resource stock is smaller than x_{1s}, neither type produces.

Note also that, as in Benchekroun [1], q_i^c is the static Cournot equilibrium quantity of a firm of type i. Hence, when the resource stock is sufficiently large (above a certain threshold), the Markov Perfect Nash Equilibrium consists in both types of firms simultaneously producing their static Cournot quantity. This threshold level of stock is the same for each type of firm and is given by x_{2s}, obtained from (3.11) by setting $i = s$.

In order to guarantee that $x_{1b} > 0$, $x_{2b} \leq \frac{x_{\max}}{2}$ and $x_{1s} < x_{2s}$, we will assume :

$$\xi < \frac{n_s}{n}(w_s - w_b) < \frac{\delta - r}{[(2+n)\delta - r]}(a - w_b). \tag{3.12}$$

where

$$\xi = \max \left\{ \begin{array}{c} \frac{(n^2+1)(\delta-r)}{2\left[\delta - \frac{(n^2+1)}{2}\right]}\left[(a - w_b) - \frac{(n+1)^2 b}{(n^2+1)}\frac{\delta x_{\max}}{2}\right], \\ \frac{(n+1)\left[\delta - \frac{(n+1)}{2}\right](w_s - w_b) - n(\delta - r)(a - w_b)}{\left[\delta - \frac{(n^2+1)}{2}\right]} \end{array} \right\}. \tag{3.13}$$

As shown in Appendix B, we will then have :

$$0 < x_{1b} \leq x_{1s} < x_{2s} \leq x_{2b} < \frac{x_{\max}}{2}. \tag{3.14}$$

The resulting equilibrium strategies are illustrated in Figure 3.1, along with a possible corresponding aggregate production, $\Phi(x) = n_s\phi_s(x) + n_b\phi_b(x)$.

A number of additional implications about the individual strategies can be drawn from Proposition 3.1. Firstly, both the strategies $\phi_s(x)$ and $\phi_b(x)$ are non decreasing functions of the resource stock, with the same non-negative slopes on each of the three distinct intervals over which they are defined. These slopes are given, for $i = s, b$, by :

$$\phi_i'(x) = \begin{cases} 0 & \text{for } \; x \in [0, x_{1s}) \\ \alpha > 0 & \text{for } \; x \in [x_{1s}, x_{2s}) \\ 0 & \text{for } \; x \in [x_{2s}, x_{\max}] \end{cases} \quad . \tag{3.15}$$

They are independent of the marginal cost differential and of the distribution of both types of firms.

Secondly, the strategy $\phi_s(x)$ is a continuous function of x over $[0, x_{\max}]$, whereas $\phi_b(x)$ exhibits jumps at both x_{1s} and x_{2s}, unless all the firms in the industry are identical (i.e., $w_s = w_b$). Indeed, at x_{1s} we have :[1]

$$\phi_b(x_{1s}^-) = 0 \quad \text{and} \quad \phi_b(x_{1s}^+) = \alpha\,(x_{1s} - x_{1b})\,,$$

and hence

$$\phi_b(x_{1s}^+) - \phi_b(x_{1s}^-) = \alpha\,(x_{1s} - x_{1b}) = \frac{\left[\delta - (n^2+1)\,\frac{r}{2}\right]}{n^2 b\,(\delta - r)}\,(w_s - w_b)\,. \tag{3.16}$$

At x_{2s}, we have

$$\phi_b(x_{2s}^-) = \alpha\,(x_{2s} - x_{1b}) \quad \text{and} \quad \phi_b(x_{1s}^+) = q_b^c = \phi_b(x_{2b}) = \alpha\,(x_{2b} - x_{1b})\,,$$

and hence

$$\phi_b(x_{2s}^+) - \phi_b(x_{2s}^-) = \alpha\,(x_{2b} - x_{2s}) = \frac{n-1}{n^2 b}\frac{\left(\delta - \frac{r}{2}\right)}{(\delta - r)}\,(w_s - w_b)\,. \tag{3.17}$$

The jumps observed in $\phi_b\,(x)$ at x_{1s} and x_{2s} are proportional to the marginal cost differential. In both cases, the larger the cost differential, the larger the jump.

Thirdly, the difference in the harvest rate of the two types of firms, $\phi_b(x) - \phi_s(x)$,

[1]We adopt the following notations : $\phi_b(z^-) = \lim\limits_{x \to z, x < z} \phi_b(x)$ and $\phi_b(z^+) = \lim\limits_{x \to z, x > z} \phi_b(x)$. The function $\phi_b(x)$ is continuous at z if $\phi_b(z^-) = \phi_b(z^+)$. Otherwise, $\phi_b(x)$ exhibits a jump at $x = z$.

is proportional to the cost differential, $w_s - w_b$, and given by :

$$\phi_b(x) - \phi_s(x) = \begin{cases} 0 & \text{for } x \in [0, x_{1s}) \\ \frac{\left[\delta - \left(n^2+1\right)\frac{r}{2}\right]}{n^2 b(\delta - r)} (w_s - w_b) & \text{for } x \in [x_{1s}, x_{2s}) \\ \frac{1}{b}(w_s - w_b) & \text{for } x \in [x_{2s}, x_{\max}] \end{cases} . \quad (3.18)$$

The big firms have a higher harvest rate as a consequence of their cost advantage.

Finally and not surprisingly, when $w_s = w_b$ the strategies described in Proposition 3.1 reduce to those found in Benchekroun [1].

At the aggregate level, the overall harvest rate, $\Phi(x) = n_s\phi_s(x) + n_b\phi_b(x)$, is given by :

$$\Phi(x) = \begin{cases} 0 & \text{for } x \in [0, x_{1s}) \\ F(x) \equiv n\alpha(x - \bar{x}_1) & \text{for } x \in [x_{1s}, x_{2s}) \\ Q^c & \text{for } x \in [x_{2s}, x_{\max}] \end{cases} , \quad (3.19)$$

where

$$\bar{x}_1 = \frac{n_s}{n}x_{1s} + \frac{n_b}{n}x_{1b} = \frac{\left[2\delta - \left(n^2+1\right)r\right]}{(n+1)^2 b\delta(2\delta - r)}\left[a - w_s + \frac{n_b}{n}(w_s - w_b)\right], \quad (3.20)$$

and

$$Q^c = \frac{n}{(n+1)b}\left[a - w_s + \frac{n_b}{n}(w_s - w_b)\right]. \quad (3.21)$$

Note that when (3.3) holds, we also have that $\delta > (n+1)\frac{r}{2}$ and therefore,

$$n\alpha - \delta = \frac{1}{n}\left[\delta - (n+1)\frac{r}{2}\right] > 0, \quad (3.22)$$

which means that the slope of $\Phi(x)$ is greater than that of $g(x)$ over the entire interval $[x_{1s}, x_{2s})$. Like for the individual strategies, the slopes of $\Phi(x)$ over any of the three intervals over which it is defined are independent of the marginal cost differential and of the distribution of the firms between the two types. These slopes are 0, $n\alpha$ and 0 over $[0, x_{1s})$, $[x_{1s}, x_{2s})$ and $[x_{2s}, x_{\max}]$, respectively. A possible representation of $\Phi(x)$ is depicted in Figure 3.1.[2]

The equilibrium aggregate production exhibits jumps at x_{1s} and x_{2s}, due to the jumps in the big firms' equilibrium strategy at these two particular resource stock levels. The size of those jumps is given by :

$$\Phi(x_{1s}^{+}) - \Phi(x_{1s}^{-}) = n_b \alpha \left(x_{1s} - x_{1b} \right) = \frac{\left[\delta - \left(n^2 + 1 \right) \frac{r}{2} \right]}{n \left(\delta - r \right) b} \frac{n_b}{n} \left(w_s - w_b \right), \qquad (3.23)$$

and

$$\Phi(x_{2s}^{+}) - \Phi(x_{2s}^{-}) = n_b \alpha \left(x_{2b} - x_{2s} \right) = \frac{n-1}{n} \frac{\left(\delta - \frac{r}{2} \right)}{\left(\delta - r \right) b} \frac{n_b}{n} \left(w_s - w_b \right). \qquad (3.24)$$

At both x_{1s} and x_{2s}, for any given $0 < n_b < n$, the size of the jump is proportional to both the marginal cost differential, $w_s - w_b$ and the proportion of big firms, n_b/n. Thus discontinuities in the aggregate production will always be observed unless $w_s = w_b$, as is the case in Benchekroun [1] (see Figure 3.2). For any given cost differential, the greater the number of big firms, the greater the jump in aggregate production.

When $w_s = w_b$, there will always be either one or three steady states, as shown in Benchekroun [1]. This is also the case when $w_s > w_b$. However, when $w_s > w_b$, steady states may occur at the stock levels for which there is a jump in the aggregate harvest. The resource growth function $g(x)$ and the aggregate harvest function $\Phi(x)$ then cross at a point of discontinuity in $\Phi(x)$. We will call these "irregular" steady states, to distinguish them from the "regular" steady states, for which $\Phi(x)$ and $g(x)$ cross within a continuous segment of $\Phi(x)$.

Before analyzing in more detail the different steady-state configurations in Section 3.5, we first consider in the next section the effects of the distribution of firms between the two types on their individual equilibrium strategies and on the aggregate outcome.

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[2]We come back later to a discussion of all the possible representations of $\Phi(x)$ when we discuss the possible steady states, in Section 3.5.

3.4 Effects of the Distribution of Firms

In this section, we assume the marginal costs differential to be positive and given. We focus on the effect of having the two types of firms coexisting, rather than having all the firms of the same type. To do this, we compare the equilibrium strategies derived in Proposition 3.1, where $0 < n_b < n$, to those that arise when all firms are of the same type, with either $n_b = n$ or $n_s = n$. We will use the superscript "o" to denote situations where all the firms are of the same type.

From Proposition 3.1, and in accord with Benchekroun [1], the individual strategies when all the firms are of the same type ($w_s = w_b$), are, for $i = s, b$:

$$\phi_i^o(x) = \begin{cases} 0 & \text{for } \; x \in [0, x_{1i}^o) \\ f_i^o(x) \equiv \alpha\,(x - x_{1i}^o) & \text{for } \; x \in [x_{1i}^o, x_{2i}^o) \\ q_i^{co} & \text{for } \; x \in [x_{2i}^o, x_{\max}] \end{cases} , \tag{3.25}$$

where,

$$\alpha = \frac{n+1}{n^2}\left(\delta - \frac{r}{2}\right),$$

$$q_i^{co} = \frac{1}{(n+1)\,b}\left[a - w_i\right], \tag{3.26}$$

$$x_{1i}^o = \frac{\left[\delta - (n^2+1)\,\frac{r}{2}\right]}{(n+1)^2\,b\delta\left(\delta - \frac{r}{2}\right)}\left[a - w_i\right], \tag{3.27}$$

$$x_{2i}^o = \frac{(n^2+1)}{(n+1)^2\,b\delta}\left[a - w_i\right]. \tag{3.28}$$

Setting, $w_s = w_b = 0$, $n = 2$, and $x_{\max} = 1$, these strategies reduce exactly to those derived by Benchekroun.

When $n_i = n$, so that all the firms are identical, with marginal cost w_i, x_{1i}^o is the level of the resource stock beyond which the firms choose to harvest at a positive rate and x_{2i}^o is that beyond which they choose their static-Cournot harvest rate.

We further assume that $x^o_{2s} \geq x^o_{1b}$, which requires :

$$w_s - w_b \leq \frac{n^2}{(n^2+1)} \frac{\delta}{\left(\delta - \frac{r}{2}\right)} (a - w_b) \,. \tag{3.29}$$

Hence the marginal cost differential is assumed bounded from above.

To better understand the implications of the asymmetry in the distribution of firms between the two types of firms, we compare the equilibrium strategies in three cases, all with $w_s > w_b$:

(i) Comparison of the equilibrium strategies when $n_b = n$ and when $n_s = n$;

(ii) Comparison of the equilibrium strategies of the small firms when $n_s < n$ and when $n_s = n$;

(iii) Comparison of the equilibrium strategies of the big firms when $n_b < n$ and when $n_b = n$.

We present those comparisons in the next three subsections and then discuss briefly the effect of the distribution of the types of firms on the aggregate harvest rate.

The detailed calculations required to make each of those comparisons are presented in Appendix C.

3.4.1 Comparing $n_b = n$ and $n_s = n$

Contrary to the case where $0 < n_i < n$, $i = b, s$, if either $n_s = n$ or $n_b = n$, the equilibrium strategies for both types of firms, and therefore the aggregate harvesting rate, are everywhere continuous functions of the resource stock. Hence the jumps in the harvesting rates are due strictly to the simultaneous presence of both types of firms.

The equilibrium strategies and aggregate harvesting rates for $n_s = n$ and $n_b = n$ are juxtaposed in Figure 3.2.[3] Note that $x^o_{1s} < x^o_{1b}$. This means that if $n_s = n$, so that there are only small firms, the threshold level of stock at which the firms begin

[3]For the detailed calculations behind Figure 3.2, see Appendix C.

exploiting the resource is lower than if $n_b = n$, so that all the firms are big firms. Put differently, with identical firms, the lower the marginal cost, the higher the level of stock at which the firms begin exploiting the resource. In fact $x^o_{1b} - x^o_{1s}$ is proportional to $w_s - w_b$, as can be verified from equation (3.99) in Appendix C.

The decision not to harvest is a decision to invest in the resource by leaving it in place in order to generate growth. The reason why identical low cost firms (the big firms) tend to begin harvesting at a higher level of the stock than identical high cost firms (the small firms) is that, because of their lower cost of exploitation, each one of them values investing in the resource more than their high cost counterpart.

For the same reason, it is also the case that $x^o_{2s} < x^o_{2b}$ (see equation (3.100) in Appendix C). This means that identical low cost firms ($n_b = n$) will revert to their static Cournot strategy at a higher level of the stock than identical high cost firms ($n_s = n$). As a result, there exists a stock level, which we denote x^o_0 in Figure 3.2, such that $\phi^o_b(x) < \phi^o_s(x)$ for $x^o_{1s} \leq x < x^o_0$ and $\phi^o_b(x) > \phi^o_s(x)$ for $x > x^o_0$. This is unlike the case with both types of firms coexisting ($0 < n_i < n$). In that case, as can be seen from equation (3.18) and is illustrated in Figure 3.1, the harvesting rate of the big firms ($\phi_b(x)$) is higher than the harvesting rate of the small firms ($\phi_s(x)$) for all stock levels for which production is positive. From (3.25), we see that the stock level x^o_0 is given by $x^o_0 = x^o_{1b} + q^{co}_s/\alpha$.

3.4.2 Comparing $0 < n_s < n$ and $n_s = n$

In Figure 3.3, we illustrate the harvest strategy of the small firms when some of their rivals are big low cost firms ($0 < n_s < n$) to their strategy when they are the only firms operating in the industry ($n_s = n$).

As derived in Appendix C, equation (3.104), and illustrated in Figure 3.3, we will have $x_{1s} > x^o_{1s}$. That is, if all n firms are small firms, each one of them begins harvesting the resource at a lower stock level than if they are sharing the common resource pool with $n-n_s$ big firms. Thus each high cost firm has a higher equilibrium valuation of investing in the common resource pool if some of its rivals are low cost firms than if all its rivals are also high cost firms. This is because each of them is aware that a positive harvest rate by the small firms immediately brings about competition from the big low cost firms for their share of the common resource pool.

Similarly, $x_{2s} > x_{2s}^o$ (Equation (3.105), Appendix C), which means that the threshold level of stock at which a small firm reverts to its static Cournot output is lower if all its rivals are also small firms than if some of them are big low cost firms.

Furthermore, the static Cournot harvest rate of the high cost firm in the symmetric case ($n_s = n$), given by equation (3.26) with $i = s$, will be higher than that in the asymmetric case ($0 < n_s < n$), given by equation (3.9) with $i = s$.[4] As a result, for all $x > 0$, $\phi_s(x) \leq \phi_s^o(x)$, with $\phi_s(x) < \phi_s^o(x)$ for all $x > x_{1s}^o$. Thus the individual harvest rate of the small firm is lower in the presence of low cost firms amongst its rivals than it is otherwise, as illustrated in Figure 3.3.

3.4.3 Comparing $0 < n_b < n$ and $n_b = n$

In Figure 3.4, we display the strategy of the big firm in the asymmetric case ($0 < n_b < n$), where some of its rivals are small high cost firms, and in the symmetric case ($n_b = n$,) where all its rivals are also big firms.

As already mentioned, in the asymmetric case, there is a jump in the harvest rate of the individual low cost firm at both x_{1s} and x_{2s}. Such jumps do not occur in the symmetric case. Also, such jumps in the harvest rate never occurred in the case of the high cost firm.

[4] To see why this is the case, assume $n_s = n = 2$. We have upward sloping reaction curves because demand and cost are linear. The symmetric equilibrium rate of production will be $q_1^{co} = q_2^{co} = q^{co}$. Now assume that firm 2 is replaced by a lower cost firm. The reaction curve of 1's rival then shifts up, resulting in an equilibrium rate of production for firm 1 of $q_1^c < q^{co}$. Note that we are dealing with strategic substitutes here, since both demand and costs are linear.

Another important difference between the low and high cost firms is that, whereas $x_{1s} > x^o_{1s}$, so that the low cost firm begins exploiting the resource at a lower stock level in the symmetric case than in the asymmetric case, it is not necessarily the case that $x^o_{1b} < x_{1s}$. Indeed, from Appendix C, Equation (3.110), we get that :

$$x_{1s} - x^o_{1b} \geq 0 \Longleftrightarrow \frac{n_b}{n} \geq \frac{1}{1 + \frac{(n+1)\delta}{\delta - r}} \equiv \rho^o. \tag{3.30}$$

This means that in the asymmetric case, if the proportion of big firms, n_b/n, is larger than a certain threshold, which we denote ρ^o, each big firm begins exploiting the resource at a stock level that is higher than if there were only big firms.[5] Moreover, because the big firms start exploiting the resource at the same stock level as the small firms, and at a higher rate (because of their cost advantage), there results a jump in their equilibrium strategy. When the proportion of big firms is smaller than ρ^o, each begins exploiting the resource at a lower stock level than if they found themselves in the symmetric case, with only big firms as rivals, although they still begin exploiting at the same stock level as the small firms.

As for the threshold level of stock at which the big low cost firm reverts to its static Cournot output, we always have $x_{2s} < x^o_{2b}$ (Equation (3.118), Appendix C). That is, this threshold is always lower in the asymmetric case ($0 < n_b < n$) than in the symmetric case ($n_b = n$), whatever the proportion of big firms in the asymmetric case.

Moreover, the static Cournot harvest rate of the big low cost firm in the asymmetric case, given by equation (3.9) with $i = b$, will be greater than that in the symmetric case, given by equation (3.26) with $i = b$. As a result, for all $x > x_{1s}$, $\phi_b(x) > \phi^o_b(x)$. Thus, as illustrated in Figure 3.4, the individual harvest rate of the big firm is greater in the presence of small high cost firms amongst its rivals than it is otherwise, for all stock levels for which the big firm produces in the asymmetric

[5] This is the case depicted in Figure 3.4.

case.

3.4.4 The Aggregate Harvest Rates

From (3.21) and (3.26), and using the fact that $a - w_b = (a - w_s) + (w_s - w_b)$, it is easy to show, as seen in Figure 3.5, that :

$$nq_s^{co} \leq Q^c \leq nq_b^{co}. \tag{3.31}$$

The inequality in (3.31) implies that when the resource is relatively abundant ($x \geq x_{2b}^o$), that is, on the portion of resource stock where all the firms produce at their static Cournot levels, the aggregate harvest rate is the largest when there are only big low cost firms in the industry, and the smallest when there are only small high cost firms. The aggregate harvest rate in the asymmetric case lies between the two.

For stock levels between x_{2s}^o and x_{2b}^o the comparison is not monotonic.

When $x_{1s}^o \leq x \leq x_{2s}^o$, the direction of the inequalities in (3.31) is reversed and we have

$$n\phi_b^o(x) \leq \Phi(x) \leq n\phi_s^o(x). \tag{3.32}$$

That is, for stock levels between x_{1b}^o and x_{2s}^o, the aggregate harvest rate is the largest with the identical small high cost firms, and the smallest with the identical big low cost firms. In the asymmetric case the aggregate harvest rate again lies between the two.

For stock levels too small ($x \leq x_{1s}^o$), there is no harvest at all in any of the three cases examined.

3.5 Steady States

We now turn to the analysis of how both cost and size asymmetries affect the steady-state stocks. We focus only on the asymmetric model in which both types of firms coexist.[6]

In order to do this, it is convenient to introduce the idea of a mean preserving marginal cost differential. Let ε denote this mean preserving marginal cost differential. Then :

$$\varepsilon = w_s - w_b, \quad w_s = \bar{w} + \frac{\varepsilon}{2} \quad \text{and} \quad w_b = \bar{w} - \frac{\varepsilon}{2}, \tag{3.33}$$

where $\bar{w}$ is the mean marginal cost ($\bar{w} = (w_s + w_b)/2$), assumed constant.

Also, let $\rho = n_b/n$ be the proportion of big firms in the industry.

Using these notations, individual (ϕ_s and ϕ_b) and aggregate (Φ) equilibrium strategies can be rewritten as :

	Intervals		
Strategies	$[0, x_{1s})$	$[x_{1s}, x_{2s})$	$[x_{2s}, x_{\max}]$
$\phi_s(x)$	0	$f_s(x) \equiv \alpha(x - x_{1s})$	q_s^c
$\phi_b(x)$	0	$f_b(x) \equiv \alpha(x - x_{1b})$	q_b^c
$\Phi(x)$	0	$F(x) \equiv n\alpha(x - \bar{x}_1)$	Q^c

where

$$x_{1s} = \frac{\left[\delta - (n^2+1)\frac{r}{2}\right]}{(n+1)^2 b\delta\left(\delta - \frac{r}{2}\right)} \left\{a - \bar{w} + \left[\frac{[(2+n)\delta - r]}{(\delta - r)}\rho - \frac{1}{2}\right]\varepsilon\right\}, \tag{3.34}$$

$$x_{1b} = \frac{\left[\delta - (n^2+1)\frac{r}{2}\right]}{(n+1)^2 b\delta\left(\delta - \frac{r}{2}\right)} \left\{a - \bar{w} + \left[\frac{1}{2} - \frac{[(2+n)\delta - r]}{(\delta - r)}(1-\rho)\right]\varepsilon\right\}, \tag{3.35}$$

$$\bar{x}_1 = (1-\rho)x_{1s} + \rho x_{1b} = \frac{\left[\delta - (n^2+1)\frac{r}{2}\right]}{(n+1)^2 b\delta\left(\delta - \frac{r}{2}\right)} \left[a - \bar{w} + \left(\rho - \frac{1}{2}\right)\varepsilon\right], \tag{3.36}$$

$$q_s^c = \frac{n}{(n+1)b}\left[\frac{a - \bar{w}}{n} - \left(\frac{1}{2n} + \rho\right)\varepsilon\right], \tag{3.37}$$

$$q_b^c = \frac{n}{(n+1)b}\left\{\frac{a - \bar{w}}{n} + \left[\frac{1}{2n} + (1-\rho)\right]\varepsilon\right\}, \tag{3.38}$$

[6] See Benchekroun [1] for the analysis of the existence and the number of steady states in the symmetric case, given the value of the intrinsic growth rate.

$$Q^c = \frac{n}{(n+1)\,b}\left[a - \bar{w} + \left(\rho - \frac{1}{2}\right)\varepsilon\right], \tag{3.39}$$

$$x_{2s} = \frac{(n^2+1)}{(n+1)^2\,b\delta}\left\{a - \bar{w} + \left[\frac{[2\delta - (n^2+1)\,r]}{(n^2+1)\,(\delta - r)}\rho - \frac{1}{2}\right]\varepsilon\right\}, \tag{3.40}$$

and

$$x_{2b} = \frac{(n^2+1)}{(n+1)^2\,b\delta}\left\{a - \bar{w} + \left[\frac{1}{2} - \frac{[2\delta - (n^2+1)\,r]}{(n^2+1)\,(\delta - r)}(1-\rho)\right]\varepsilon\right\}. \tag{3.41}$$

From equations (3.34) to (3.41), and as was illustrated in Section 3.4, we can see both size and cost asymmetries have an impact not only on the individual and aggregate amounts of resource extracted at equilibrium, but also on the level of stock at which the firms begin harvesting the resource, and the stock level at which they begin harvesting the resource at their respective Cournot quantities. Now, we will examine the impact of these asymmetries on the steady states.

3.5.1 Type and Number of Steady States

We analyze the number and the type of steady states, given the range of values of both asymmetries, focusing on the corresponding steady-state stocks. As in Benchekroun [1], we find that there are either one or three steady-state stocks.[7] More precisely, we always have one steady-state stock (we denote by x^*) over the interval $[x_{1s}, x_{2s}]$, and either two steady states (x^{**} and x^{***}) or no steady state over the interval $[x_{2s}, x_{\max}]$.

For ease of exposition, we will refer to steady states at which there is a jump in the aggregate harvest rate as "irregular" steady states, and those at which there is no jump as "regular" steady states. For irregular steady-state stocks, we do not have $\Phi(x) - g(x) = 0$ as this would be the case at a regular steady-state stock. Over the interval $[x_{1s}, x_{2s}]$ the steady-state stock can be either regular (when $x^* \in (x_{1s}, x_{2s})$) or irregular (when $x^* = x_{1s}$ or $x^* = x_{2s}$). The steady states over the

[7] In some particular cases we will have two steady state stocks.

interval $[x_{2s}, x_{\max}]$, when they exist, are both regular. All these possibilities are depicted in Figures 3.6 to 3.11.

In what follows, we define the conditions under which we have regular or irregular steady states, and when we have one or three steady states.

Firstly, let us define the stock level $\tilde{x}$ which is such that $F(\tilde{x}) = g(\tilde{x})$. The stock level $\tilde{x}$ corresponds to the intersection of the ascending part of $g(x)$ with the increasing segment of $\Phi(x)$ (Recall that $\Phi(x)$ coincides with the function $F(x)$ over the interval $[x_{1s}, x_{2s})$). We have

$$\tilde{x} = \frac{n\alpha}{(n\alpha - \delta)} \bar{x}_1. \tag{3.42}$$

Secondly, let us denote by Δ_1, Δ_2 and Δ_3, the following terms :

$$\Delta_1 = \tilde{x} - x_{1s}, \quad \Delta_2 = \tilde{x} - x_{2s} \quad \text{and} \quad \Delta_3 = g\left(x_{\max}/2\right) - Q^c. \tag{3.43}$$

Over the interval $[0, x_{1s})$, there is no possible steady state since there is no production.

Over the interval $[x_{1s}, x_{2s}]$, there is a unique steady-state stock x^* which is such that :

- $x^* = x_{1s}$ if $\Delta_1 \leq 0$, that is, we have an irregular steady-state stock at x_{1s} ;
- $x^* = x_{2s}$ if $\Delta_2 \geq 0$, meaning that we have an irregular steady-state stock at x_{2s} ;
- $x^* = \tilde{x} \in (x_{1s}, x_{2s})$, if $\Delta_1 > 0$ and $\Delta_2 < 0$, which means that we have a regular steady-state stock at $\tilde{x}$.

Over the interval $[x_{2s}, x_{\max}]$, we have either two regular steady states at x^{**} and x^{***} or no steady state at all.

- When $\Delta_3 < 0$, we have no steady state.
- When $\Delta_3 \geq 0$, we have two regular steady states at x^{**} and x^{***} defined as

solutions of the equation $g(x) - Q^c = 0$.[8] Then, we have

$$x^{**} = \frac{Q^c}{\delta}, \tag{3.44}$$

and

$$x^{***} = x_{\max} - \frac{Q^c}{\delta}. \tag{3.45}$$

3.5.1.1 Regular or Irregular Steady State at x^*?

We have a regular steady-state stock at x^* when $x_{1s} < \tilde{x} < x_{2s}$ (i.e. when $\Delta_1 > 0$ and $\Delta_2 < 0$) and an irregular steady-state stock otherwise. In particular, we have an irregular steady-state stock at x_{1s} (i.e. $x^* = x_{1s}$) when

$$\Delta_1 \leq 0 \Leftrightarrow \varepsilon \geq \frac{a - \bar{w}}{\frac{1}{2} + \tau\rho} \equiv E(\rho), \tag{3.46}$$

and an irregular steady-state stock at x_{2s} (i.e. $x^* = x_{2s}$) when

$$\Delta_2 \geq 0 \Leftrightarrow \varepsilon \geq \frac{a - \bar{w}}{\frac{1}{2} + \tau\rho} \equiv E(\rho), \tag{3.47}$$

where

$$\tau = \frac{\delta - (n^2 + 1)\frac{r}{2}}{n(\delta - r)} > 0. \tag{3.48}$$

Note that both irregular steady state cases are delimited by the same locus defined by equation $\varepsilon = E(\rho)$ (see Figures 3.12 and 3.13). Above that curve, the corresponding values of ρ and ε lead to one or the other of these two irregular steady-state stocks. In other words, above (and all along) this curve we have an irregular steady state (either x_{1s} or x_{2s}) and below, we have a regular steady state. See Appendix D for the derivations leading to (3.46) and (3.47).

[8] When $\Delta_3 = 0$, we have a particular case in which $Q^c = \delta x_{\max}/2$, and then $x^{**} = x^{***} = x_{\max}/2$. In such a case, we have one steady state stock over the interval $[x_{2s}, x_{\max}]$, and thus two steady state stocks overall.

3.5.1.2 One or Three Steady States?

Since we always have one steady-state stock over the interval $[x_{1s}, x_{2s}]$, to verify whether we have one or three steady-state stocks is equivalent to verifying for what range of values of ρ and ε we have either no steady state or two steady states, over the interval $[x_{2s}, x_{\max}]$. This in turn is equivalent to verifying whether $\Delta_3 < 0$ or $\Delta_3 \geq 0$. In particular, the case of $\Delta_3 = 0$ gives an equation $\varepsilon = E_3(\rho)$ which delimits the (ρ, ε)-space for these two cases (see Figures 3.12 and 3.13 for the graphs and Appendix D for the derivations).

To be more specific, we have that

$$\Delta_3 \geq 0 \quad \Leftrightarrow \quad \varepsilon \leq E_3(\rho) \equiv \frac{(n+1)\,b}{n}\left[g\left(x_{\max}/2\right) - Q^{co}_{1/2}\right]\frac{1}{\left(\rho - \frac{1}{2}\right)}, \qquad (3.49)$$

where

$$Q^{co}_{1/2} = \frac{n\,(a - \bar{w})}{(n+1)\,b}. \qquad (3.50)$$

Note that $Q^{co}_{1/2}$ is the static Cournot aggregate production when both types firms are either in equal number (i.e. $\rho = \frac{1}{2}$) or when all firms are identical like in Benchekroun [1], with a marginal cost equal to $\bar{w}$ (i.e. $\varepsilon = 0$ and $w_i = \bar{w}$). Q^c is greater than $Q^{co}_{1/2}$ when the big firms outnumber the small firms (i.e. $\rho > 1/2$).

Equation (3.49) implies that, in the (ρ, ε)-space, below the locus defined by equation $\varepsilon = E_3(\rho)$, our model admits three steady-state stocks and above it, only one steady-state stock. However, the shape of the curve corresponding to equation $\varepsilon = E_3(\rho)$ depends on the sign of $\left[g\left(x_{\max}/2\right) - Q^{co}_{1/2}\right]$. This sign in turn depends on how the parameter δ is compared to a particular value δ^o that only depends on parameters relating to the demand and growth functions but not on the asymmetries present in the model. Indeed, we have that

$$\left[g\left(\frac{x_{\max}}{2}\right) - Q^{co}_{1/2}\right] \geq 0 \quad \Leftrightarrow \quad \delta \geq \frac{2n}{(n+1)\,b}\frac{(a-\bar{w})}{x_{\max}} \equiv \delta^o. \qquad (3.51)$$

3.5.2 Asymmetries and Steady States

Using (3.51), we can distinguish two cases : the case where $\delta > \delta^o$ and that where $\delta < \delta^o$. When the intrinsic growth rate δ of the resource is larger than the particular value δ^o defined in (3.51), we have $\left[g\left(x_{\max}/2\right) - Q^{co}_{1/2}\right] > 0$ and the corresponding graph is presented in Figure 3.12. The case $\delta < \delta^o$ which implies that $\left[g\left(\frac{x_{\max}}{2}\right) - Q^{co}_{1/2}\right] < 0$ is depicted in Figure 3.13.[9]

In the symmetric version of our model, (i.e. when all the firms are identical) the case $\delta > \delta^o$ would lead to three regular steady-state stocks, whereas when $\delta < \delta^o$ we would have only one regular steady state.

In our asymmetric model, however, in both cases, it will always be possible to get to all the combinations of steady states depicted in Figures 3.6 to 3.11. This is possible because of the presence of the asymmetries. Indeed, as can be seen in Figures 3.12 and 3.12, in both cases, the curves derived in the preceding subsection divide the (ρ, ε)-space in four quadrants :

- Quadrant I : This quadrant corresponds to the possibility presented in Figure 3.6. There are three regular steady-state stocks : $x^* = \tilde{x}$, x^{**} and x^{***}, two of which are stable ($\tilde{x}$ and x^{***}). Which stable steady-state stock the game will lead to depends on the initial resource stock x_0. If $x_0 < x^{**}$, the resource stock will converge to $\tilde{x}$, and when $x_0 > x^{**}$ the resource stock will converge to x^{***}. In this last case, the strategies in which each type plays its Cournot quantity is sustainable and can be played indefinitely (at x^{***}).
- Quadrant II : When (ρ, ε) falls in this quadrant, there is only one regular and stable steady state ($x^* = \tilde{x}$), as illustrated in Figure 3.7. In this case, the equilibrium resource stock converges to $\tilde{x}$.
- Quadrant III : This corresponds to the cases where there is one irregular

[9] In fact, to be more rigourous, the two curves presented in Figures 3.12 and 3.13 must be completed by two other curves that take into account the assumptions in (3.12) which impose $x_{1b} > 0$ and $x_{2b} < x_{\max}/2$. However, we willingly ignore these last two curves to focus on those partitionning the (ρ, ε)-space from conditions relating to the type and the number of steady states.

steady-state stocks as in Figures 3.9 and 3.11. In Figure 3.9, the steady-state stock is x_{2s}, which is stable but irregular, because of the jump that occurs in the aggregate harvest rate at that steady-state stock. The equilibrium resource stock will always converge to x_{2s}, but the aggregate steady-state harvest rate will depend on the level of the initial stock. If $x_0 > x_{2s}$, the aggregate output will be the Cournot quantity, which the firms can extract indefinitely in a sustainable manner by all playing their individual Cournot quantities. However, if the $x_0 < x_{2s}$, the overall output is less than the aggregate Cournot quantity. In Figure 3.11, the steady-state stock is x_{1s} which is a stable but irregular steady-state stock. In that figure, the equilibrium resource stock always converges to x_{1s}. However, for $x_0 < x_{1s}$, there will be no harvest.

– In Quadrant IV, we have the cases where there are three steady states, but with one irregular. These cases are those presented in Figures 3.8 and 3.10. In Figure 3.8, the steady-state stocks are : x_{2s} (irregular), x^{**} and x^{***}. The steady-state stocks x_{2s} and x^{***} are both stable and the steady state that will be reached by the resource stock depends on the initial stock. If $x_0 > x^{**}$, the resource stock will converge to x^{***}. However, if $x_0 < x^{**}$, the equilibrium resource stock will converge to the irregular steady-state stock x_{2s}. In Figure 3.10, the steady states stocks are : x_{1s} (irregular), x^{**} and x^{***} where x_{1s} and x^{***} are both stable. The steady state to which the resource stock will converge depends on the level of the initial stock. If $x_0 > x^{**}$ the resource stock will converge to x^{***}. If $x_0 < x^{**}$, it converges to the (irregular) steady-state stock x_{1s}.

In each of the two cases (when $\delta > \delta^o$ or $\delta < \delta^o$), by varying either ρ or ε keeping the other constant, it is possible to move from one quadrant to another. For instance in Figure 3.12, for a given cost asymmetry (marginal cost differential), it is possible to move from Quadrant I to IV or III, or from I to II etc. only by modifying the size asymmetry (proportion of big firms). Similarly, controlling for the size asymmetry, it is possible, for instance, to move from Quadrant I to II or III, or from I to IV.

This last point can have policy implications. Since in our model, the differences in cost and size are exogenously fixed, these asymmetries can be used as policy instruments to reach a previously fixed goal in terms of the situations described in our four quadrants. We have shown with our model that it is possible to reach any of these quadrants by controlling for both types of asymmetries, in any of the two cases $\delta > \delta^o$ or $\delta < \delta^o$, even though these are out of the control of a policy maker.

3.6 Conclusion

We have presented a model of oligopolistic exploitation of a common pool resource where we have two types of firms differing in their marginal costs, with potentially different numbers of firms in each group. We have shown that both the marginal cost differential (cost asymmetry) and the distribution of firms (size asymmetry) do affect the outcomes of the game in important ways. This is seen by comparing the equilibrium outcomes obtained in the asymmetric case to those in the symmetric case, where all the firms are all identical, either big or small. We have also examined the implications for the steady states.

We have found that in our model, in addition to the "usual" strategic interactions due to the presence of other players (either of the same type or of different type), there are three other forces in action that lead to the behavior observed : (i) the common property effect which makes the big firms begin exploiting the resource at the same stock level as the small firms, which, in an identical-firm industry, would have started exploiting the resource at a lower stock level ; (ii) the strategic effect that makes the small firm anticipate an earlier entry of the big firms, that will harvest at a higher rate, given their cost advantage ; (iii) the demand effect, as a result of which the small firms begin their exploitation at a higher stock to avoid a decrease in the price of the output due to the higher harvest rate of the big firms. Compared to the symmetric case, the combination of these asymmetry-related forces finally results in : (a) discontinuities in the big firm harvest rate, and consequently in the aggregate outcome ; (b) a forward shift in the beginning of the exploitation by the small firms and in the beginning of their static Cournot behavior as well. That is, in presence of big low cost firms, the small firms will begin exploiting the resource or start playing their Cournot quantity at higher stock level than if they were the only type of firms in the industry ; (c) a forward shift (when they are relatively more) or a backward shift (when relatively less) in the beginning of the exploitation by the big low cost firms and a forward shift in the beginning of their Cournot behavior when they are in presence of small high cost firms than when they are all identical. We have also found that the equilibrium outcomes admit one or three steady states depending on the range of values of the cost differential and the proportion of big firms. Moreover, any of these steady states can be reached by controlling the asymmetries. In such a case, our model can lead to interesting policy issues for a policy maker if it is possible to control the asymmetries.

Our analysis has assumed that the agents differ by their marginal costs and that the number of each type of firms is exogenously given. This leaves room for further research involving both other forms of asymmetry and endogenously determined numbers of firms.

3.7 APPENDIX

APPENDIX A : Proof of proposition 3.1

This proof is a constructive one. Let us consider the strategies $\phi_i(x)$, $i = s, b$, proposed in (3.7). For the vector $(\phi_s, ..., \phi_s, \phi_b, ..., \phi_b)$ to constitute a Markov perfect Nash equilibrium, we need to show that there exists two value functions $V_s(x)$ and $V_b(x)$, defined over $[0, x_{\max}]$, such that, the guess $\phi_k(x)$ is solution of the following Hamilton-Jacobi-Bellman equation :

$$rV_k(x) = \underset{q_k}{Max}\left\{\left[a - w_k - b\left(q_k + \sum_{l\neq k}\phi_l(x)\right)\right]q_k + V_k'(x)\left[g(x) - q_k - \sum_{l\neq k}\phi_l(x)\right]\right\}. \tag{3.52}$$

We start by checking for interior solutions, that is, for $x \in [x_{1s}, x_{2s})$ and $x \in [x_{2s}, x_{\max}]$. Maximization of the right-hand side of (3.52) gives the following first order condition,

$$a - w_k - 2bq_k - b\sum_{l\neq k}\phi_l(x) - V_k'(x) = 0. \tag{3.53}$$

When considering the fact that there are two groups of firms and that at equilibrium, $q_i = \phi_i(x)$, from (3.53) we get the following system of equations, for $i, j = s, b$, $i \neq j$:

$$b(n_i + 1)\phi_i(x) + bn_j\phi_j(x) = a - w_i - V_i'(x), \tag{3.54}$$

from which we derive the following solution, for $i, j = s, b$, $i \neq j$:

$$\phi_i(x) = \frac{1}{b(n+1)}\left\{a - w_i - V_i' - n_j\left[\left(V_i' - V_j'\right) + (w_i - w_j)\right]\right\}. \tag{3.55}$$

If we designate by $\Phi(x) = \sum_k \phi_k(x)$ and by $\Lambda(x) = \sum_k V_k(x)$ the aggregate equilibrium harvest rate and value function, respectively, we also have :

$$b\left(n+1\right)\Phi\left(x\right)=n\left(a-w_i\right)+n_j\left(w_i-w_j\right)-\Lambda'\left(x\right). \tag{3.56}$$

At equilibrium, the Hamilton-Jacobi-Bellman equation in (3.52) becomes, for $i,j=s,b,\ i\neq j$:

$$rV_i\left(x\right)=\left[a-w_i-b\Phi\left(x\right)\right]\phi_i(x)+V_i'\left(x\right)\left[g(x)-\Phi\left(x\right)\right] \tag{3.57}$$

where

$$\Phi\left(x\right)=n_i\phi_i(x)+n_j\phi_j(x). \tag{3.58}$$

By substituting the ϕ_i's from (3.55) and (3.58) into (3.57), we get the following system of differential equations, for $i,j=s,b,\ i\neq j$:

$$\begin{aligned}\left(n+1\right)^2 brV_i\left(x\right) &= \left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]^2 \\ &\quad +V_i'\left(x\right)\left\{b\left(n+1\right)^2 g\left(x\right)-\left(n^2+1+2n_j\right)\left(a-w_i\right)-2n_in_j\left(w_i-w_j\right)\right\} \\ &\quad +V_j'\left(x\right)\left\{2n_j\left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]\right\} \\ &\quad +\left[n_iV_i'\left(x\right)+n_jV_j'\left(x\right)\right]^2\end{aligned} \tag{3.59}$$

To solve (3.59), when $x\in[x_{1s},x_{2s})$ and $x\in[x_{2s},x_{\max}]$, we use the "undetermined coefficients technique" to determine the value functions $V_s\left(x\right)$ and $V_b\left(x\right)$, for each of these two intervals over which the ϕ_i's are interior solutions.

– **For** $x\in[x_{1s},x_{2s})$

Let us start by the interval $[x_{1s},x_{2s})$, over which both strategies are strictly increasing functions. To solve (3.59) for $x\in[x_{1s},x_{2s})$, we use a "guess and verify" method, by guessing a quadratic form for the value functions, i.e. for $i,j=s,b$, $i\neq j$:

$$V_i\left(x\right)=\frac{A_i}{2}x^2+B_ix+C_i, \tag{3.60}$$

where A_s, A_b, B_s, B_b, C_s and C_b are parameters to be determined.

Replacing this guess in (3.59) gives, for $i, j = s, b$:

$$\begin{aligned}
&\left\{b(n+1)^2 A_i\left(\delta-\frac{r}{2}\right)+\left(n_i A_i+n_j A_j\right)^2\right\} x^2 \\
&+\left\{b(n+1)^2 B_i(\delta-r)-A_i\left[\left(n^2+1+2n_j\right)\left(a-w_i\right)+2n_i n_j\left(w_i-w_j\right)\right]\right. \\
&\left.+2n_j A_j\left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]+2\left(n_i A_i+n_j A_j\right)\left(n_i B_i+n_j B_j\right)\right\} x \\
&+\left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]^2-B_i\left[\left(n^2+1+2n_j\right)\left(a-w_i\right)+2n_i n_j\left(w_i-w_j\right)\right] \\
&+2n_j B_j\left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]+\left(n_i B_i+n_j B_j\right)^2-(n+1)^2 brC_i \\
=\;& 0
\end{aligned} \tag{3.61}$$

Since (3.61) must hold for any $x \in [x_{1s}, x_{2s})$, this imposes all the coefficients of this second degree polynomial to be zero. Then, we get the following system of equations, for $i, j = s, b$:

$$b(n+1)^2 A_i\left(\delta-\frac{r}{2}\right)+\left(n_i A_i+n_j A_j\right)^2=0 \tag{3.62}$$

$$\begin{aligned}
&b(n+1)^2 B_i(\delta-r)-A_i\left[\left(n^2+1+2n_j\right)\left(a-w_i\right)+2n_i n_j\left(w_i-w_j\right)\right] \\
&+2n_j A_j\left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]+2\left(n_i A_i+n_j A_j\right)\left(n_i B_i+n_j B_j\right) \\
=\;& 0
\end{aligned} \tag{3.63}$$

$$\begin{aligned}
&\left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]^2-B_i\left[\left(n^2+1+2n_j\right)\left(a-w_i\right)+2n_i n_j\left(w_i-w_j\right)\right] \\
&+2n_j B_j\left[\left(a-w_i\right)-n_j\left(w_i-w_j\right)\right]+\left(n_i B_i+n_j B_j\right)^2-(n+1)^2 brC_i \\
=\;& 0
\end{aligned} \tag{3.64}$$

from which we obtain the values of the unknowns A_i, B_i and C_i specified in (3.60). That is, for $i, j = s, b, i \neq j$:

$$A_i = A_j = A = -\frac{(n+1)^2 b}{n^2}\left(\delta-\frac{r}{2}\right) \tag{3.65}$$

$$B_i = -\frac{A}{(n+1)^2 b\delta}\left[\left(n^2+1\right)(a-w_i)+n_j(w_i-w_j)\frac{\left[2\delta-\left(n^2+1\right)r\right]}{n(\delta-r)}\right] \quad (3.66)$$
$$= \frac{\left(\delta-\frac{r}{2}\right)}{n^2\delta}\left[\left(n^2+1\right)(a-w_i)+n_j(w_i-w_j)\frac{\left[2\delta-\left(n^2+1\right)r\right]}{n(\delta-r)}\right]$$

and

$$C_i = \frac{1}{(n+1)^2 br}\left\{\left[(a-w_i)-n_j(w_i-w_j)\right]^2+(n_iB_i+n_jB_j)^2 \right. \quad (3.67)$$
$$\left. -(a-w_i)\left[\left(n^2+1\right)B_i+2n_j(B_i-B_j)\right]-2n_j(w_i-w_j)(n_iB_i+n_jB_j)\right\}$$

Some helpful relations :

$$B_i - B_j = \frac{A}{(n+1)^2 b}\frac{\left(n^2-1\right)}{(\delta-r)}(w_i-w_j) \quad (3.68)$$
$$= -\frac{\left(n^2-1\right)}{n^2}\frac{\left(\delta-\frac{r}{2}\right)}{(\delta-r)}(w_i-w_j)$$

$$n_iB_i+n_jB_j = -\frac{A\left(n^2+1\right)}{(n+1)^2 b\delta}\left[n(a-w_i)+n_j(w_i-w_j)\right] \quad (3.69)$$
$$= \frac{\left(n^2+1\right)}{n^2}\frac{\left(\delta-\frac{r}{2}\right)}{\delta}\left[n(a-w_i)+n_j(w_i-w_j)\right]$$

$$B_i-(a-w_i) = \frac{\left[2\delta-\left(n^2+1\right)r\right]}{2n^2\delta}\left[(a-w_i)+n_j(w_i-w_j)\frac{(2\delta-r)}{n(\delta-r)}\right] \quad (3.70)$$

$$(B_i-B_j)+(w_i-w_j) = (w_i-w_j)\frac{\left[2\delta-\left(n^2+1\right)r\right]}{2n^2(\delta-r)} \quad (3.71)$$

When the assumption in (3.3) holds and when $w_s - w_b > 0$, we have :

$$A < 0, \quad B_b > B_s > 0 \quad (3.72)$$

For $i, j = s, b,\ i \neq j$, let us define the functions $W_i : [0, x_{\max}] \longmapsto \mathbb{R}$ and $f_i : [0, x_{\max}] \longmapsto \mathbb{R}$ such that :

$$W_i(x) \equiv \frac{A}{2}x^2 + B_i x + C_i \quad (3.73)$$

and

$$f_i(x) \equiv \frac{1}{b(n+1)} \left[-Ax + (a - w_i - B_i) - n_j (w_i - w_j + B_i - B_j)\right], \qquad (3.74)$$

and the stock levels

$$\begin{aligned} x_{1i} &\equiv \frac{1}{-A} \left[B_i - (a - w_i) + n_j (B_i - B_j + w_i - w_j)\right] \qquad (3.75) \\ &\equiv \frac{1}{-A} \frac{\left[2\delta - (n^2+1) r\right]}{2n^2\delta} \left[(a - w_i) + n_j (w_i - w_j) \frac{(2+n)\delta - r}{n(\delta - r)}\right] \end{aligned}$$

which are such that $f_i(x_{1i}) = 0$. In other terms,

$$f_i(x) \equiv \frac{-A}{b(n+1)} \left[x - x_{1i}\right] = \alpha (x - x_{1i}). \qquad (3.76)$$

Note that

$$x_{1i} - x_{1j} = \frac{(n+1)}{-A} \frac{\left[2\delta - (n^2+1) r\right]}{2n^2(\delta - r)} (w_i - w_j), \qquad (3.77)$$

and

$$f_i(x) - f_j(x) = \frac{A}{b(n+1)} \left[x_{1i} - x_{1j}\right]. \qquad (3.78)$$

The stock levels x_{2i} are defined such that $W_i'(x_{2i}) = 0$. We have

$$x_{2i} = \frac{B_i}{-A} \qquad (3.79)$$

from which we can get :

$$x_{2i} - x_{2j} = \frac{B_i - B_j}{-A}. \qquad (3.80)$$

It will be demonstrated in Appendix B that :

$$0 < x_{1b} \leq x_{1s} < x_{2s} \leq x_{2b} \leq \frac{x_{\max}}{2}, \qquad (3.81)$$

and,

$$f_b(x) \geq f_s(x), \text{ for all } x \geq x_{1b}. \tag{3.82}$$

To sum up, for $x \in [x_{1s}, x_{2s})$, V_b and V_s coincide with W_b and W_s, respectively and are continuously differentiable over (x_{1s}, x_{2s}). Then, by construction, for $x \in [x_{1s}, x_{2s})$, the strategies $\phi_b(x)$ and $\phi_s(x)$ are the restrictions of $f_b(x)$ and $f_s(x)$, respectively, over this interval.

– **For** $x \in [x_{2s}, x_{\max}]$

For $x \geq x_{2s}$, the guessed strategies are interior solutions and are both constant functions :

$$\phi_s(x) = q_s^c, \quad \phi_b(x) = q_b^c, \quad \Phi(x) = n_s q_s^c + n_b q_b^c. \tag{3.83}$$

Over this interval, (3.57) becomes, for $i, j = s, b$, $i \neq j$:

$$rV_i(x) = \left[a - w_i - b\left(n_i q_i^c + n_j q_j^c\right)\right] q_i^c + V_i'(x)\left[\delta x - \left(n_i q_i^c + n_j q_j^c\right)\right]. \tag{3.84}$$

We guess that $V_i(x)$ is a constant function and get

$$V_i(x) = \frac{1}{r}\left[a - w_i - b\left(n_i q_i^c + n_j q_j^c\right)\right] q_i^c.$$

The parameters q_i^c are obtained by using (3.55), from which we get,

$$q_i^c = \frac{1}{b(n+1)}\left[a - w_i - n_j(w_i - w_j)\right], \tag{3.85}$$

and consequently, for $x \in [x_{2s}, x_{\max}]$,

$$V_i(x) = \frac{\left[a - w_i - n_j(w_i - w_j)\right]^2}{(n+1)^2 br} \equiv Y_{2i}(x). \tag{3.86}$$

To sum up, for $x \in [x_{2s}, x_{\max}]$, V_b and V_s coincide with Y_{2b} and Y_{2s} respectively and are continuously differentiable over $(x_{2s} x_{\max}]$. Then, by construction, for $x \in (x_{2s}, x_{\max}]$, the strategies $\phi_b(x)$ and $\phi_s(x)$ are constant functions, and are equal to

q_b^c and q_s^c, respectively.

– **For** $x \in [0, x_{1s})$.

For $x \in [0, x_{1s})$, we have corner solutions. When, $\phi_s(x) = \phi_b(x) = \Phi(x) = 0$, (3.57) becomes

$$rV_i(x) = \delta x V_i'(x). \tag{3.87}$$

A solution of (3.87) that is continuous at x_{1s} is

$$V_i(x) = W_i(x_{1s}) \left[\frac{x}{x_{1s}}\right]^{\frac{r}{\delta}} \equiv Y_{1i}(x). \tag{3.88}$$

APPENDIX B : Comparison of the x_{ki}'s

We show here that,

$$0 < x_{1b} \leq x_{1s} < x_{2s} \leq x_{2b} \leq \frac{x_{\max}}{2}. \tag{3.89}$$

First, from (3.10) with $i = b$, to have $x_{1b} > 0$, it must be true that :

$$\frac{n_s}{n}(w_s - w_b) < \frac{\delta - r}{[(2+n)\delta - r]}(a - w_b). \tag{3.90}$$

Second, to have $x_{2b} < x_{\max}/2$, from (3.11) with $i = b$, we must verify that :

$$\frac{n_s}{n}(w_s - w_b) > \frac{(n^2+1)(\delta - r)}{2\left[\delta - \frac{(n^2+1)}{2}\right]}\left[(a - w_b) - \frac{(n+1)^2 b}{(n^2+1)}\frac{\delta x_{\max}}{2}\right]. \tag{3.91}$$

Third, from (3.77) with $i = s$ and $j = b$ we have

$$x_{1s} - x_{1b} = \frac{[2\delta - (n^2+1)r]}{b(n+1)(2\delta - r)(\delta - r)}(w_s - w_b) \geq 0, \tag{3.92}$$

from which we deduce that $x_{1s} \geq x_{1b}$.

Fourth, from (3.80) and (3.68) with $i = s$ and $j = b$, we can write

$$x_{2s} - x_{2b} = -\frac{(n^2-1)}{(n+1)^2 b(\delta - r)} (w_s - w_b) \leq 0, \tag{3.93}$$

which shows that $x_{2b} \geq x_{2s}$.

Fifth, from (3.75) and (3.79), using (3.71) with $i = s$ and $j = b$, we get

$$x_{2s} - x_{1s} = \frac{2n^2}{(n+1)^2 b(2\delta - r)} \left\{ (a - w_s) - \frac{n_b}{n}(w_s - w_b) \frac{[2\delta - (n^2+1) r]}{2n(\delta - r)} \right\}, \tag{3.94}$$

which implies that, to have $x_{2s} > x_{1s}$, it must be true that :

$$\frac{n_b}{n}(w_s - w_b) < \frac{2n(\delta - r)}{[2\delta - (n^2+1) r]} (a - w_s), \tag{3.95}$$

or written as in (3.91), it must be verified that

$$\frac{n_s}{n}(w_s - w_b) > \frac{(n+1)\left[\delta - \frac{(n+1)}{2}\right](w_s - w_b) - n(\delta - r)(a - w_b)}{\left[\delta - \frac{(n^2+1)}{2}\right]}. \tag{3.96}$$

To sum up, in order for (3.89) to be verified, it must be true that :

$$\xi < \frac{n_s}{n}(w_s - w_b) < \frac{\delta - r}{[(2+n)\delta - r]} (a - w_b), \tag{3.97}$$

where

$$\xi = \max \left\{ \begin{array}{c} \frac{(n^2+1)(\delta - r)}{2\left[\delta - \frac{(n^2+1)}{2}\right]} \left[(a - w_b) - \frac{(n+1)^2 b}{(n^2+1)} \frac{\delta x_{\max}}{2}\right], \\ \frac{(n+1)\left[\delta - \frac{(n+1)}{2}\right](w_s - w_b) - n(\delta - r)(a - w_b)}{\left[\delta - \frac{(n^2+1)}{2}\right]} \end{array} \right\}. \tag{3.98}$$

APPENDIX C : Derivations for Section 3.4

Comparing $n_b = n$ and $n_s = n$

From (3.27), (3.28), and (3.26) we get :

$$x_{1b}^o - x_{1s}^o = \frac{\left[2\delta - \left(n^2+1\right)r\right]}{\left(n+1\right)^2 b\delta\left(2\delta - r\right)}\left[w_s - w_b\right] > 0. \tag{3.99}$$

$$x_{2b}^o - x_{2s}^o = \frac{\left(n^2+1\right)}{\left(n+1\right)^2 b\delta}\left[w_s - w_b\right] > 0. \tag{3.100}$$

$$q_b^{co} - q_s^{co} = \frac{1}{\left(n+1\right)b}\left[w_s - w_b\right] > 0. \tag{3.101}$$

Also,

$$\frac{x_{2s}^o}{x_{1b}^o} = 1 + \frac{\delta n^2}{\delta - \left(n^2+1\right)\frac{r}{2}} - \frac{w_s - w_b}{a - w_b}\left[1 + \frac{\delta n^2}{\delta - \left(n^2+1\right)\frac{r}{2}}\right]. \tag{3.102}$$

This implies that

$$x_{2s}^o \geq x_{1b}^o \Longleftrightarrow w_s - w_b \leq \frac{n^2}{\left(n^2+1\right)}\frac{\delta}{\left(\delta - \frac{r}{2}\right)}\left(a - w_b\right). \tag{3.103}$$

Comparing $0 < n_s < n$ and $n_s = n$

From (3.10), (3.11), (3.27), (3.28), with $i = s$, we get :

$$x_{1s} - x_{1s}^o = \frac{\left[2\delta - \left(n^2+1\right)r\right]}{\left(n+1\right)^2 b\delta\left(2\delta - r\right)}\frac{\left[\left(2+n\right)\delta - r\right]}{\left(\delta - r\right)}\frac{n_b}{n}\left(w_s - w_b\right) > 0, \tag{3.104}$$

$$x_{2s} - x_{2s}^o = \frac{\left[2\delta - \left(n^2+1\right)r\right]}{\left(n+1\right)^2 b\delta\left(\delta - r\right)}\frac{n_b}{n}\left(w_s - w_b\right) > 0. \tag{3.105}$$

From (3.9) and (3.26), with $i = s$, it comes

$$q_s^c - q_s^{co} = -\frac{n}{\left(n+1\right)b}\frac{n_b}{n}\left(w_s - w_b\right) < 0. \tag{3.106}$$

Comparing : $0 < n_b < n$ and $n_b = n$

From (3.10), (3.11), (3.27), (3.28), with $i = b$, we get :

$$x_{1b} - x_{1b}^o = -\frac{\left[2\delta - \left(n^2+1\right)r\right]}{\left(n+1\right)^2 b\delta\left(2\delta - r\right)}\frac{\left[\left(2+n\right)\delta - r\right]}{\left(\delta - r\right)}\frac{n_s}{n}\left(w_s - w_b\right) < 0, \tag{3.107}$$

$$x_{2b} - x_{2b}^{o} = -\frac{\left[2\delta - \left(n^2+1\right) r\right]}{\left(n+1\right)^2 b\delta\left(\delta - r\right)} \frac{n_s}{n} \left(w_s - w_b\right) < 0. \tag{3.108}$$

From (3.9) and (3.26), with $i = b$, it comes

$$q_b^c - q_b^{co} = \frac{n}{\left(n+1\right) b} \frac{n_s}{n} \left(w_s - w_b\right) > 0. \tag{3.109}$$

Since x_{1s} and x_{2s} are the two important threshold stock levels for the big firm strategies in the asymmetric case, we need to compare them to the corresponding thresholds of the symmetric case : x_{1b}^{o} and x_{2b}^{o}. We have :

$$x_{1s} - x_{1b}^{o} = \frac{\left[2\delta - \left(n^2+1\right) r\right]}{\left(n+1\right)^2 b\delta\left(2\delta - r\right)} \left(w_s - w_b\right) \left[\frac{n_b}{n} \frac{\left[\left(2+n\right)\delta - r\right]}{\left(\delta - r\right)} - 1\right], \tag{3.110}$$

from which we get :

$$x_{1s} - x_{1b}^{o} \geq 0 \Longleftrightarrow \frac{n_b}{n} \geq \frac{1}{1 + \frac{(n+1)\delta}{\delta - r}} \equiv \rho^{o}. \tag{3.111}$$

We have

$$\rho^{o} \equiv \frac{1}{1 + \frac{(n+1)\delta}{\delta - r}} = \frac{\delta - r}{\left(2+n\right)\delta - r}. \tag{3.112}$$

We also have

$$x_{2s} - x_{2b}^{o} = \frac{\left(n^2+1\right)}{\left(n+1\right)^2 b\delta} \left(w_s - w_b\right) \left[\frac{n_b}{n} \frac{\left[2\delta - \left(n^2+1\right) r\right]}{\left(n^2+1\right)\left(\delta - r\right)} - 1\right]. \tag{3.113}$$

Then,

$$x_{2s} - x_{2b}^{o} \geq 0 \Longleftrightarrow \frac{n_b}{n} \frac{\left[2\delta - \left(n^2+1\right) r\right]}{\left(n^2+1\right)\left(\delta - r\right)} \geq 1 \tag{3.114}$$

or

$$x_{2s} - x_{2b}^{o} \geq 0 \Longleftrightarrow \frac{n_b}{n} \left[1 - \frac{\left(n^2-1\right)}{\left(n^2+1\right)} \frac{\delta}{\left(\delta - r\right)}\right] \geq 1, \tag{3.115}$$

which gives

$$x_{2s} - x_{2b}^o \geq 0 \Longleftrightarrow \frac{n_b}{n} \geq \frac{1}{1 - \frac{(n^2-1)}{(n^2+1)}\frac{\delta}{(\delta-r)}} \equiv \rho_2^o. \tag{3.116}$$

However, since $\frac{n_b}{n} < 1$, and that

$$\rho_2^o \equiv \frac{(n^2+1)(\delta-r)}{[2\delta-(n^2+1)r]} = \frac{1}{1-\frac{(n^2-1)}{(n^2+1)}\frac{\delta}{(\delta-r)}} \geq 1, \tag{3.117}$$

then the first inequality in (3.116) cannot happen and then finally, we will always have

$$x_{2s} - x_{2b}^o \leq 0. \tag{3.118}$$

APPENDIX D : Derivations for Section 3.5

Regular or Irregular steady-state stock at $x^* = x_{1s}$?

We have an irregular steady-state stock at x_{1s} if $\Delta_1 \equiv \tilde{x} - x_{1s} \leq 0$. Putting $\Delta_1 = 0$ allows to define the locus that splits the (ρ, ε)-space into two parts : one in which $x^* = x_{1s}$ i.e. we have an irregular steady state which is x_{1s} and one in which $x^* = \tilde{x}$, i.e. a regular steady state (see Figures 3.12 and 3.13).

Note that :

$$\frac{n\alpha}{(n\alpha-\delta)} = \frac{(n+1)\left(\delta-\frac{r}{2}\right)}{\left[\delta-(n+1)\frac{r}{2}\right]} = 1 + \frac{n\delta}{\left[\delta-(n+1)\frac{r}{2}\right]}.$$

We can simplify (3.42), using (3.36) to get :

$$\tilde{x} = \frac{\left[\delta-(n^2+1)\frac{r}{2}\right]}{\left[\delta-(n+1)\frac{r}{2}\right](n+1)b\delta}\left[a-\bar{w}+\left(\rho-\frac{1}{2}\right)\varepsilon\right]. \tag{3.119}$$

Equalizing this last expression with (3.34), we get :

$$\frac{\left[a-\bar{w}+\left(\rho-\frac{1}{2}\right)\varepsilon\right]}{\left[\delta-(n+1)\frac{r}{2}\right]} = \frac{\left\{a-\bar{w}+\left[\frac{[(2+n)\delta-r]}{(\delta-r)}\rho-\frac{1}{2}\right]\varepsilon\right\}}{(n+1)\left(\delta-\frac{r}{2}\right)}$$

from which we deduce

$$\Delta_1 \equiv \tilde{x} - x_{1s} \leq 0 \quad \Leftrightarrow \quad \left\{ \frac{1}{2} + \left[\frac{(n+1)}{n} \frac{\left(\delta - (n+1)\frac{r}{2}\right)}{\delta - r} - 1 \right] \rho \right\} \varepsilon \geq (a - \bar{w})$$

or

$$\Delta_1 \equiv \tilde{x} - x_{1s} \leq 0 \quad \Leftrightarrow \quad \left\{ \frac{1}{2} + \left[\frac{\delta - (n^2+1)\frac{r}{2}}{n(\delta - r)} \right] \rho \right\} \varepsilon \geq (a - \bar{w}).$$

Finally, we get

$$\Delta_1 \equiv \tilde{x} - x_{1s} \leq 0 \quad \Leftrightarrow \quad \varepsilon \geq \frac{(a - \bar{w})}{\frac{1}{2} + \tau_1 \rho} \equiv E_1(\rho), \tag{3.120}$$

where

$$\tau_1 \equiv \frac{1}{n} \frac{\delta - (n^2+1)\frac{r}{2}}{(\delta - r)} > 0. \tag{3.121}$$

Equation (3.120) shows that in the (ρ, ε)-space, above and all along the locus depicted by the equation $\varepsilon = E_1(\rho)$, we have $\Delta_1 \leq 0$, that is we have an irregular steady-state stock which is x_{1s}. Below that locus, we have a regular steady-state stock which is $\tilde{x}$.

Regular or Irregular steady-state stock at $x^* = x_{2s}$?

Following the same steps as previously, we have an irregular steady-state stock at x_{2s} if $\Delta_2 \equiv \tilde{x} - x_{2s} \geq 0$. The limiting case $\Delta_2 = 0$ will allow us to define the locus that splits the (ρ, ε)-space in two parts : one in which $x^* = x_{2s}$ i.e. we have an irregular steady state which is x_{2s} and one in which $x^* = \tilde{x}$, i.e. a regular steady state (see Figures 3.12 and 3.13).

Then equalizing (3.119) and (3.40), we get :

$$\frac{\left[\delta - (n^2+1)\frac{r}{2}\right]}{\left[\delta - (n+1)\frac{r}{2}\right]} \left[a - \bar{w} + \left(\rho - \frac{1}{2}\right) \varepsilon \right] = \frac{(n^2+1)}{(n+1)} \left\{ a - \bar{w} + \left[\frac{\left[2\delta - (n^2+1) r\right]}{(n^2+1)(\delta - r)} \rho - \frac{1}{2} \right] \varepsilon \right\},$$

from which we deduce that $\Delta_2 \equiv \tilde{x} - x_{2s} \geq 0$ if and only if :

$$\left\{\frac{1}{2}n(n-1)\delta + \left[-n(n-1)\delta + (n^2-1)\left(\delta - (n+1)\frac{r}{2}\right)\frac{\delta}{\delta - r}\right]\rho\right\}\varepsilon$$
$$\geq \{n(n-1)\delta\}(a - \bar{w}),$$

and after further simplifications,

$$\Delta_2 \equiv \tilde{x} - x_{2s} \geq 0 \quad \Leftrightarrow \quad \left\{\frac{1}{2} + \left[\frac{(n+1)}{n}\frac{\delta - (n+1)\frac{r}{2}}{\delta - r} - 1\right]\rho\right\}\varepsilon \geq (a - \bar{w}),$$

or

$$\Delta_2 \equiv \tilde{x} - x_{2s} \geq 0 \quad \Leftrightarrow \quad \left\{\frac{1}{2} + \left[\frac{\delta - (n^2+1)\frac{r}{2}}{n(\delta - r)}\right]\rho\right\}\varepsilon \geq (a - \bar{w}),$$

and finally

$$\Delta_2 \equiv \tilde{x} - x_{2s} \geq 0 \quad \Leftrightarrow \quad \varepsilon \geq \frac{(a - \bar{w})}{\frac{1}{2} + \tau_2\rho} \equiv E_2(\rho), \tag{3.122}$$

where

$$\tau_2 \equiv \frac{1}{n}\frac{\delta - (n^2+1)\frac{r}{2}}{(\delta - r)} > 0. \tag{3.123}$$

Equation (3.122) shows that in the (ρ, ε)-space, above and all along the locus depicted by the equation $\varepsilon = E_2(\rho)$, we have $\Delta_2 \geq 0$, that is we have an irregular steady-state stock which is x_{2s}. Below that locus, we have a regular steady-state stock which is $\tilde{x}$.

Remark :

We note that, $\tau_2 = \tau_1$. This implies that $E_2(\rho) = E_1(\rho)$. In other terms, the locus depicted by equations $\varepsilon = E_1(\rho)$ and $\varepsilon = E_2(\rho)$ coincide. Above this common locus we have an irregular steady-state stock, either x_{1s} or x_{2s} and below, we have a regular steady-state stock $\tilde{x}$. However, above that locus, it is not possible to make the distinction between the irregular steady-state stocks x_{1s} and x_{2s}. We will then refer to that locus as being depicted by equation $\varepsilon = E(\rho)$, where $E(\rho) = E_1(\rho) =$

$E_2(\rho)$ and then the corresponding τ is given by $\tau = \tau_1 = \tau_2$.

Two or No steady-state stock Over Interval $[x_{2s}, x_{\max}]$?

We have two (regular) steady-state stocks over $[x_{2s}, x_{\max}]$ if $\Delta_3 \equiv g(x_{\max}/2) - Q^c \geq 0$. The limiting case $\Delta_3 = 0$ allows us to define the locus that splits the (ρ, ε)-space in two parts : one in which there are two steady states over $[x_{2s}, x_{\max}]$ and one in which there is no steady state at all.[10] (see Figures 3.12 and 3.13).

Using (3.39) and (3.26), we can rewrite Q^c as

$$Q^c = Q^{co}_{1/2} + \frac{n}{(n+1)b}\left(\rho - \frac{1}{2}\right)\varepsilon,$$

where

$$Q^{co}_{1/2} = \frac{n(a - \bar{w})}{(n+1)b}.$$

$Q^{co}_{1/2}$ is the static Cournot aggregate production when both types firms are either in equal number (i.e. $\rho = \frac{1}{2}$) or when all firms are identical as in Benchekroun's model, with a marginal cost equal to $\bar{w}$. Then, we can rewrite :

$$\Delta_3 = g\left(\frac{x_{\max}}{2}\right) - Q^{co}_{1/2} - \frac{n}{(n+1)b}\left(\rho - \frac{1}{2}\right)\varepsilon.$$

Consequently,

$$\Delta_3 \geq 0 \quad \Leftrightarrow \quad \varepsilon \leq \frac{(n+1)b}{n}\left[g\left(\frac{x_{\max}}{2}\right) - Q^{co}_{1/2}\right]\frac{1}{\left(\rho - \frac{1}{2}\right)} \equiv E_3(\rho).$$

The shape of the locus depicted by equation $\varepsilon = E_3(\rho)$ depends on the sign of $\left[g\left(\frac{x_{\max}}{2}\right) - Q^{co}_{1/2}\right]$. Below that locus, there are two (regular) steady-state stocks over $[x_{2s}, x_{\max}]$ and above, no steady-state stock. Along that locus, both steady-state stocks coincide.

[10]Note that along that locus, we have a particular case in which both steady states stock coincide in one.

Note that,

$$\left[g\left(\frac{x_{\max}}{2}\right) - Q^{co}_{1/2}\right] \geq 0 \quad \Leftrightarrow \quad \delta \geq \frac{2n}{(n+1)\,b}\frac{(a-\bar{w})}{x_{\max}} \equiv \delta^{o}.$$

3.8 Figures

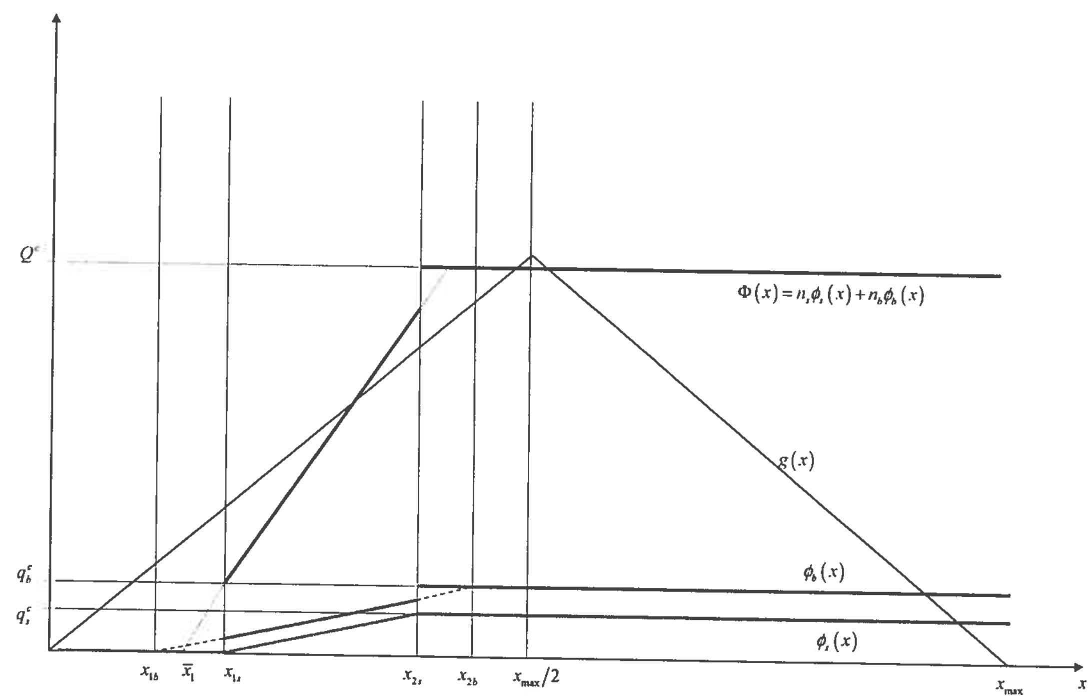

FIG. 3.1 – Equilibrium Strategies and Aggregate Outcome when $0 < n_s, n_b < n$

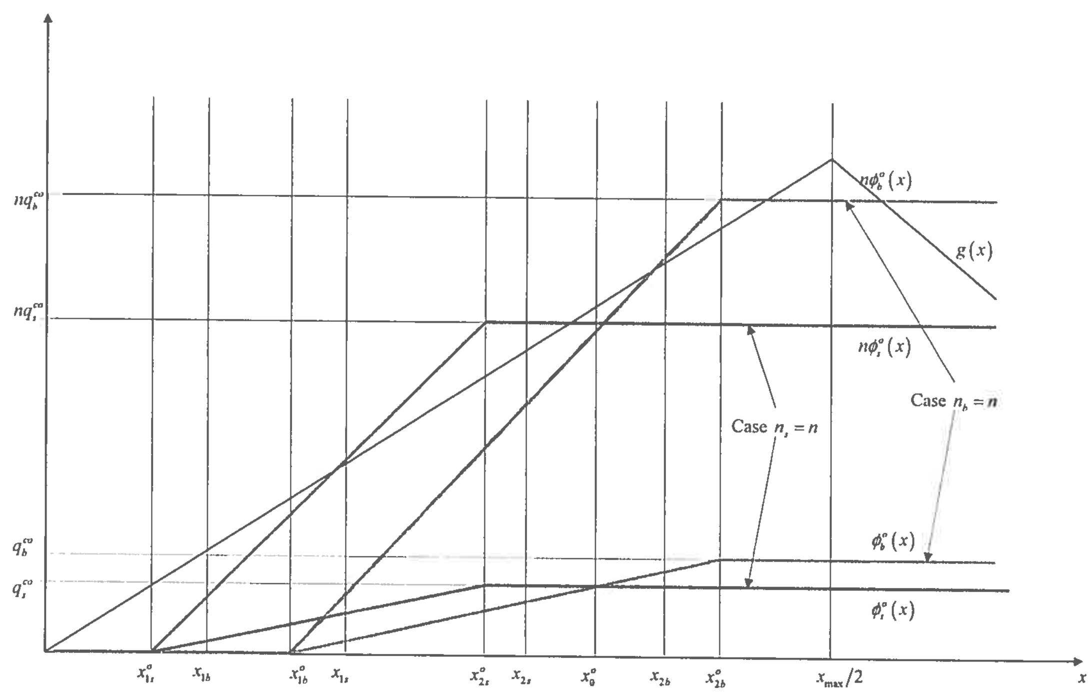

FIG. 3.2 – Equilibrium Strategies and Aggregate Outcomes when $n_s = n$ and $n_b = n$

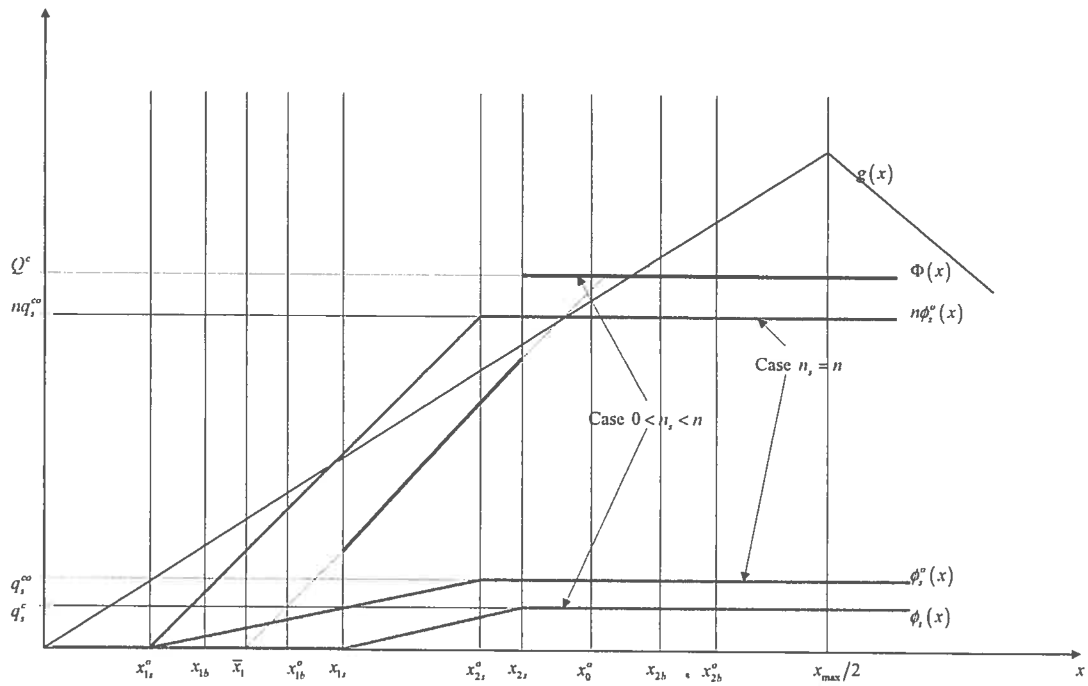

FIG. 3.3 – Equilibrium Strategies and Aggregate Outcomes when $0 < n_s < n$ and $n_s = n$

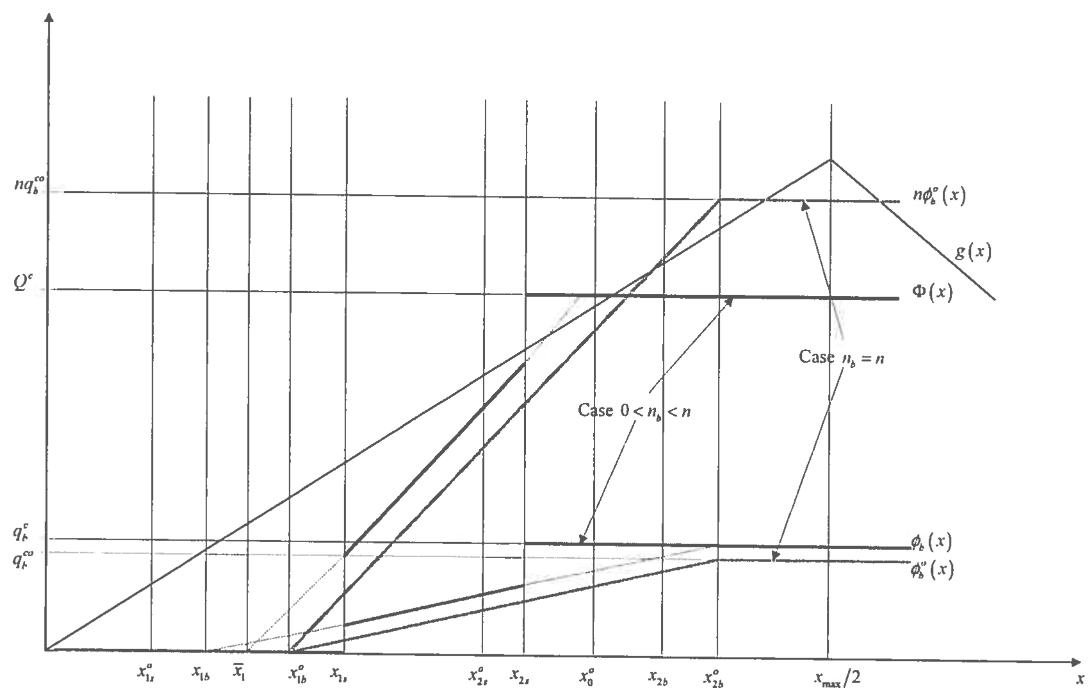

FIG. 3.4 – Equilibrium Strategies and Aggregate Outcomes when $0 < n_b < n$ and $n_b = n$

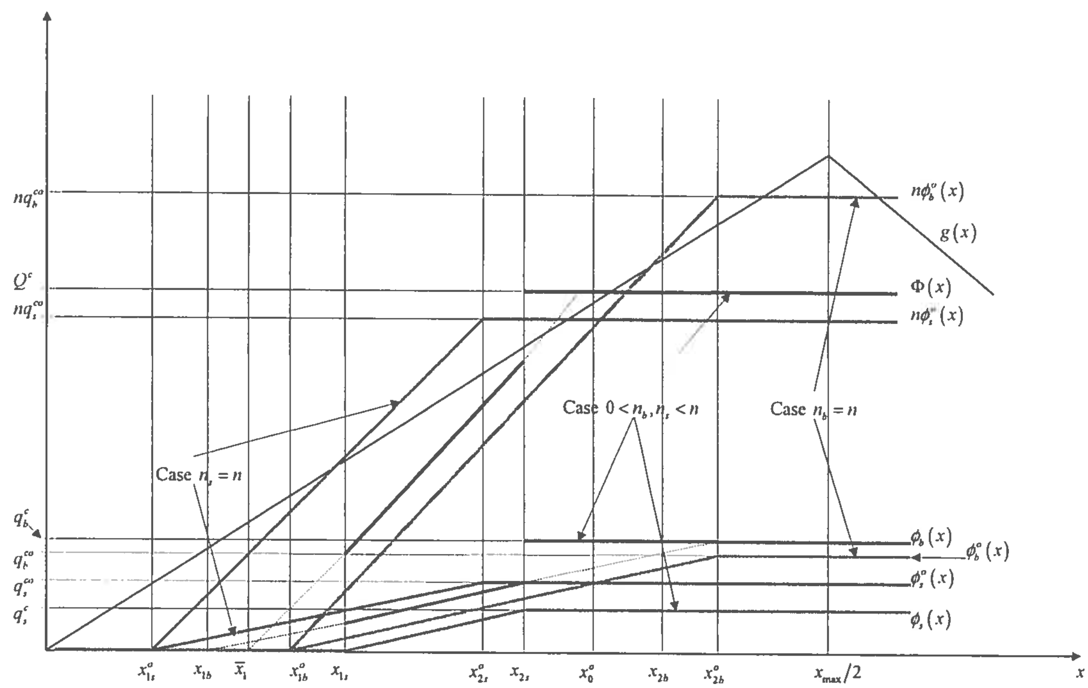

FIG. 3.5 – Equilibrium Strategies and Aggregate Outcomes when $0 < n_b < n$, $n_s = n$ and $n_b = n$

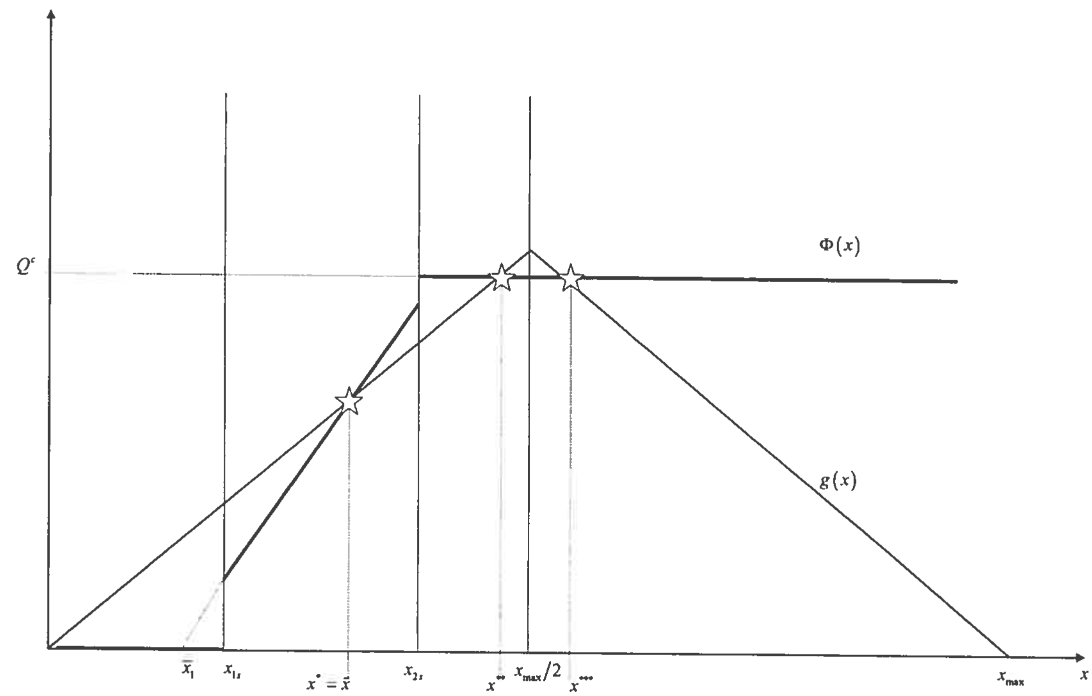

FIG. 3.6 – Steady States when $x_{1s} < \tilde{x} < x_{2s}$ and $g\left(x_{\max}/2\right) \geq Q^c$. Three Steady States (All Regular)

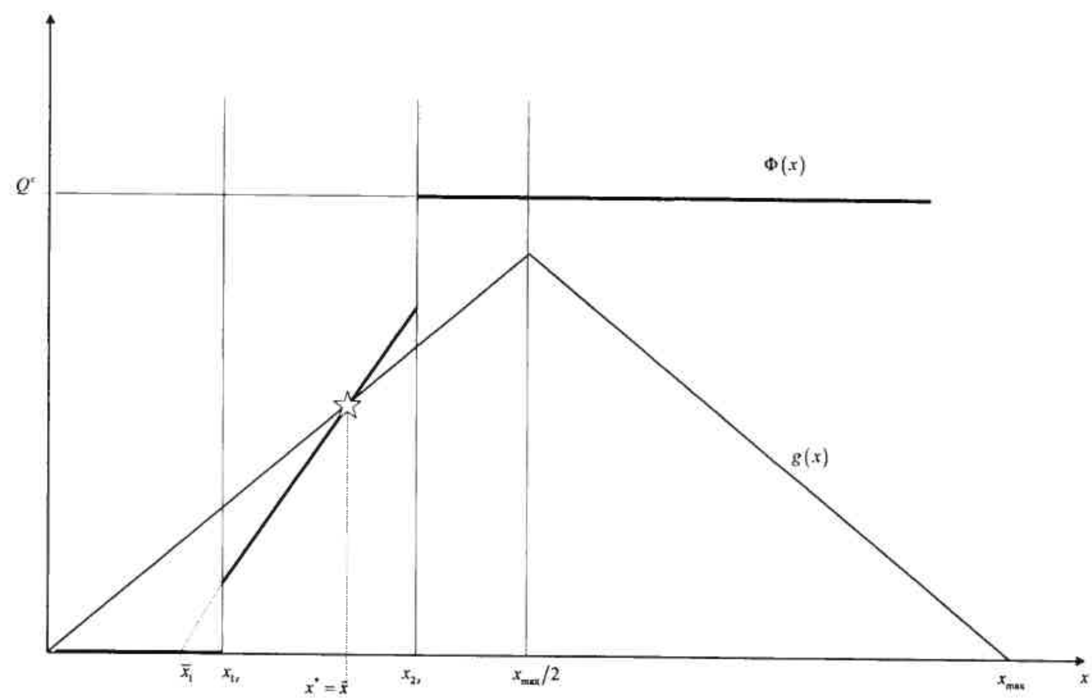

FIG. 3.7 – Steady States when $x_{1s} < \tilde{x} < x_{2s}$ and $g(x_{\max}/2) < Q^c$. One Steady State (Regular)

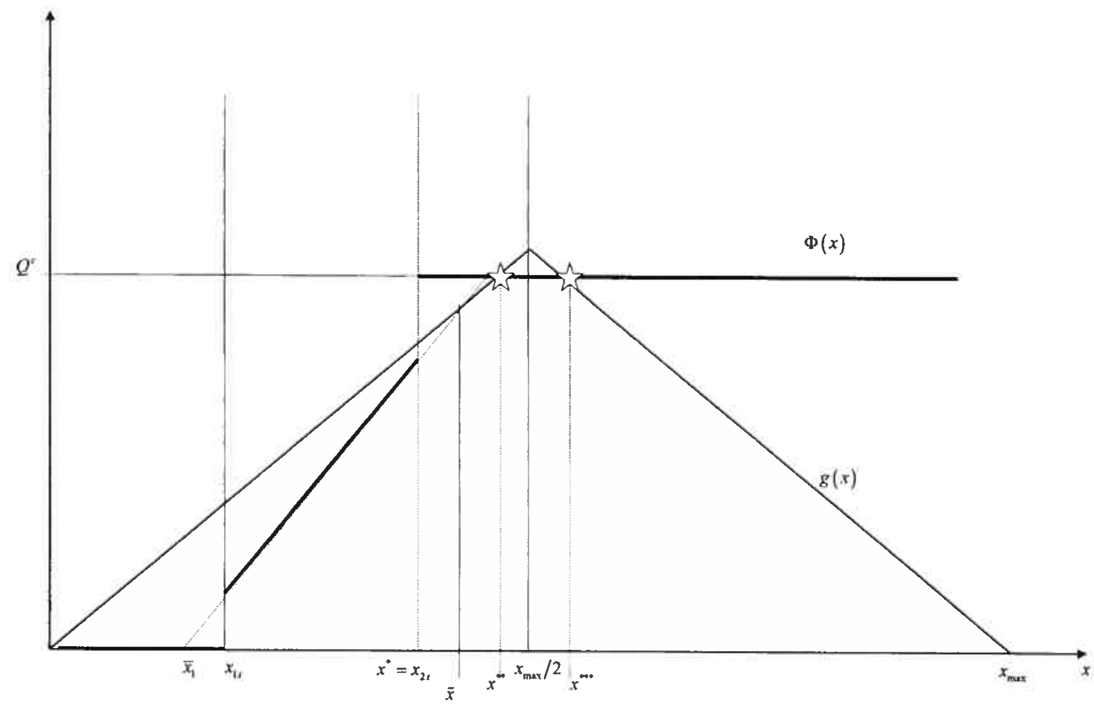

FIG. 3.8 – Steady States when $\tilde{x} \geq x_{2s}$ and $g\left(x_{\max}/2\right) \geq Q^c$. Three Steady States (One Irregular)

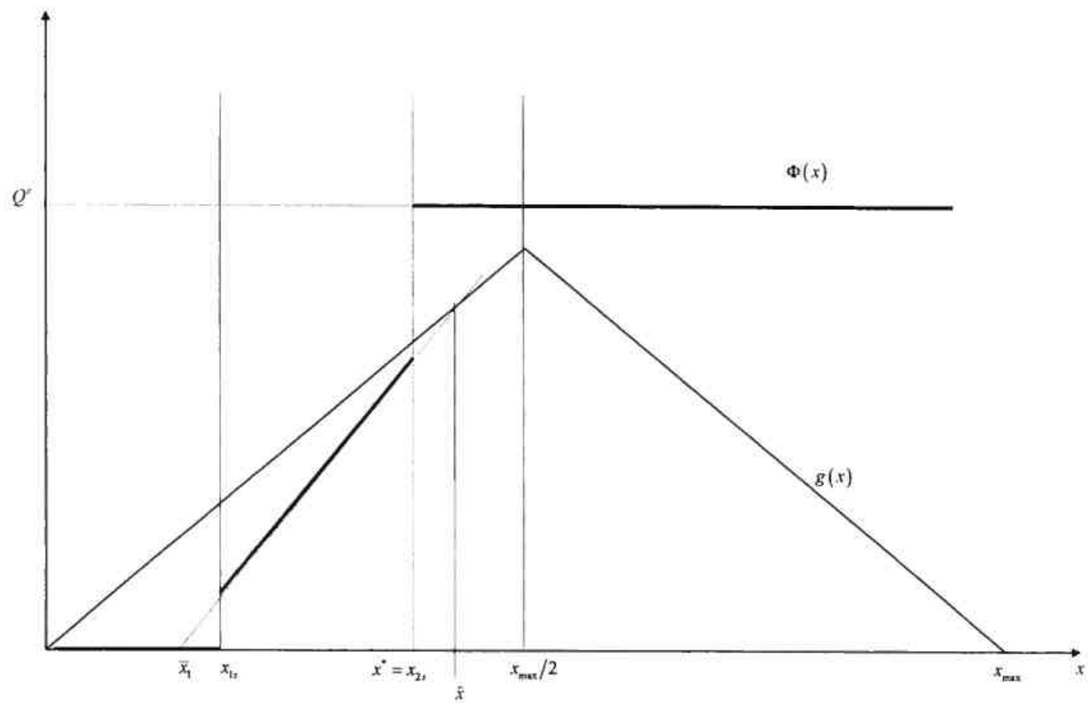

FIG. 3.9 – Steady States when $\tilde{x} \geq x_{2s}$ and $g(x_{\max}/2) < Q^c$. One Steady State (Irregular)

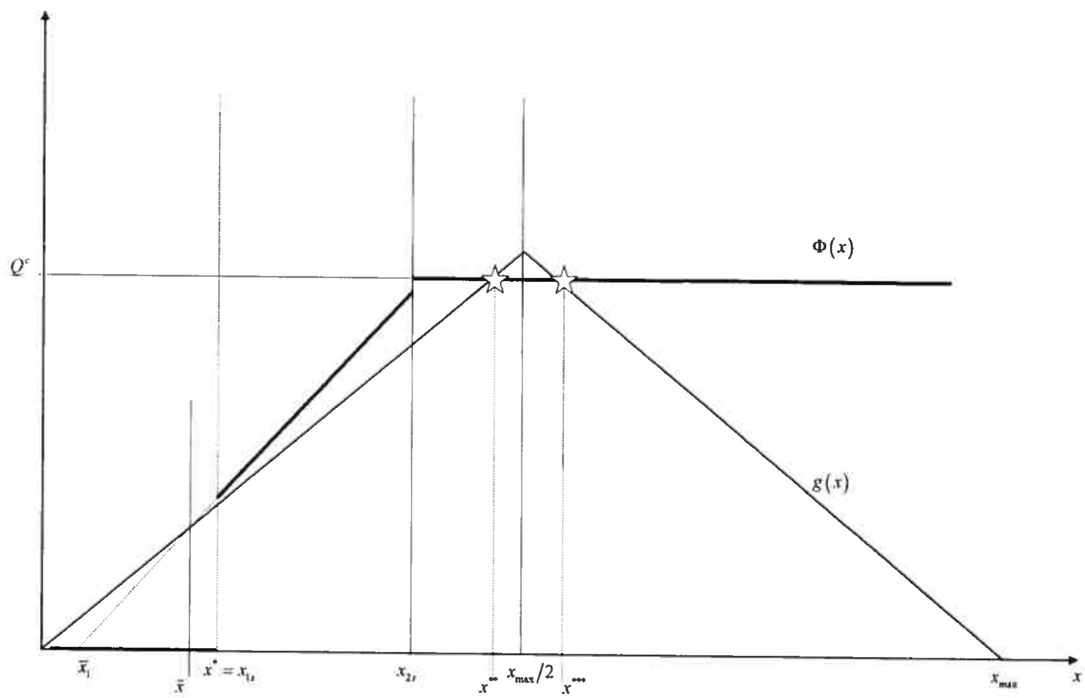

FIG. 3.10 – Steady States when $\tilde{x} \leq x_{1s}$ and $g\,(x_{\max}/2) \geq Q^c$. Three Steady States (One Irregular)

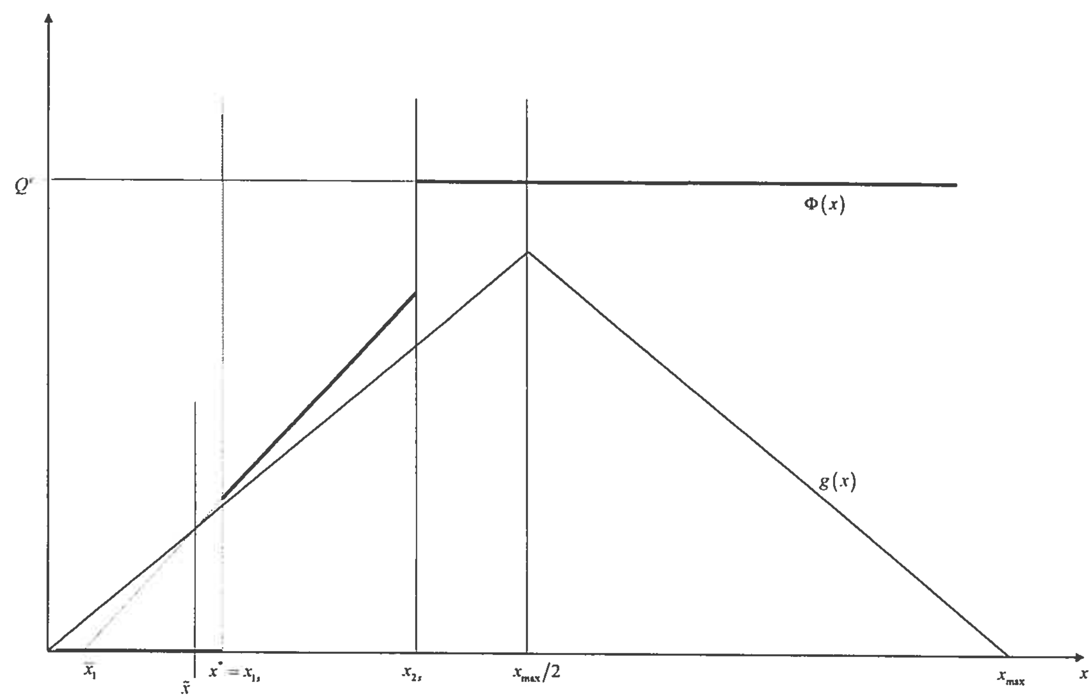

FIG. 3.11 – Steady States when $\tilde{x} \leq x_{1s}$ and $g(x_{\max}/2) < Q^c$. One Steady State (Irregular)

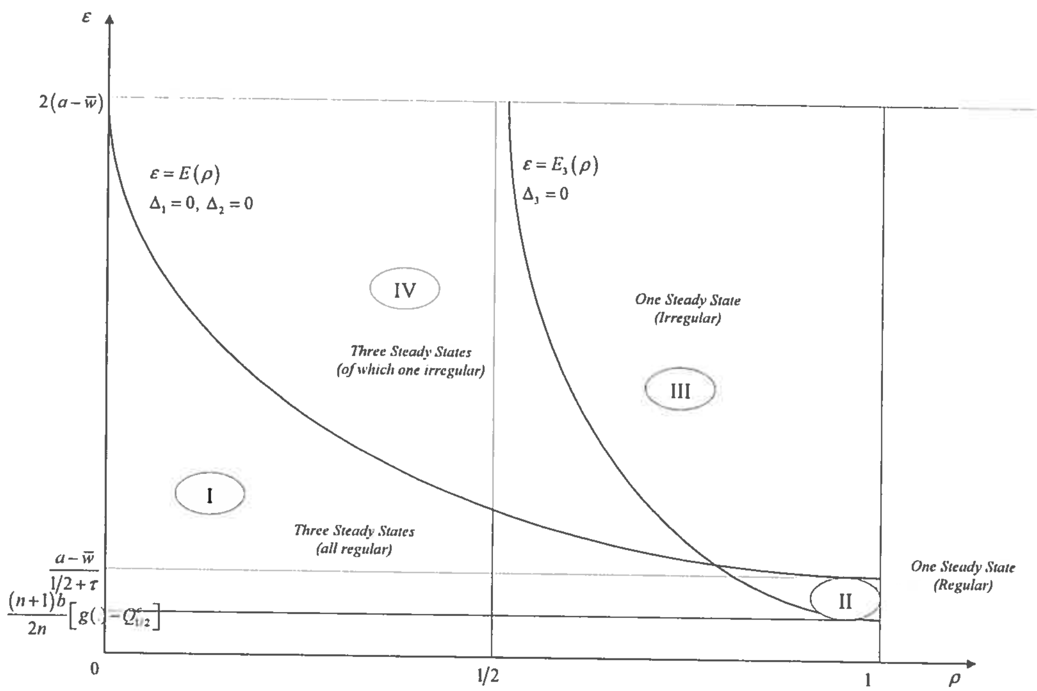

FIG. 3.12 – Steady States in the (ρ, ε)-Space when $\delta > \delta^o$

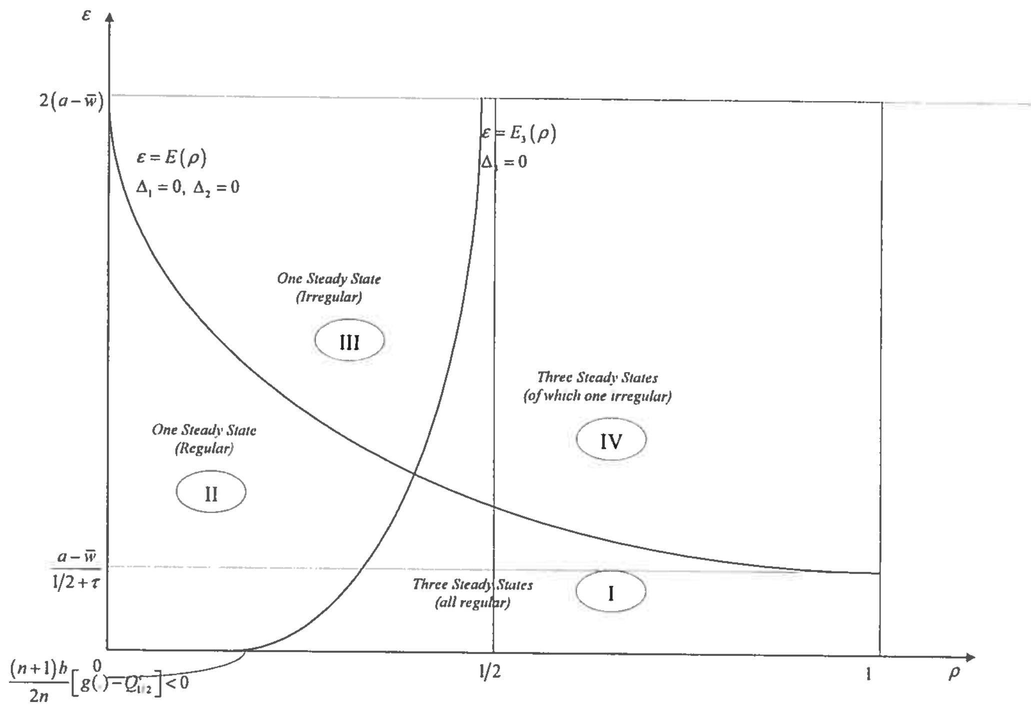

FIG. 3.13 – Steady States in the (ρ, ε)-Space when $\delta < \delta^o$

CONCLUSION GÉNÉRALE

Dans cette thèse, nous avons présenté deux modèles d'extraction d'une ressource naturelle commune par des agents hétérogènes. Pour prendre en compte cette hétérogénéité, nous avons séparé les agents en deux groupes au sein desquels ils sont respectivement identiques. Par cette méthode, nous avons introduit dans nos modèles deux types d'asymétries : (i) une asymétrie dite « intrinsèque » qui porte essentiellement la caractéristique qui fait la différence entre des agents représentatifs de chaque groupe et (ii) une asymétrie de « taille de groupe » qui concerne la répartition des agents dans chaque groupe. Nous avons montré que ces deux types d'asymétrie avaient des impacts notables sur les équilibres markoviens parfaits que nous avons caractérisés.

Dans le premier chapitre, nous avons déterminé les conditions nécessaires permettant l'utilisation de stratégies markoviennes linéaires dans les jeux différentiels décrivant l'exploitation d'une ressource naturelle commune. Nous avons démontré que l'existence de tels équilibres est assujettie à l'existence d'une relation précise entre les éléments essentiels du modèle, notamment la fonction d'utilité des agents et la fonction de "dynamique naturelle" ou de reproduction de la ressource exploitée. Ainsi, pour une fonction d'utilité donnée, seule une famille spécifique de fonctions de reproduction est compatible avec l'utilisation de stratégies markoviennes linéaires. De même, lorsque la fonction de reproduction est connue, seule une famille particulière de fonctions d'utilité permet l'utilisation de stratégies linéaires.

Dans le second chapitre, nous avons étudié un « Fish War » dans lequel les agents impliqués se font la concurrence uniquement sur le marché de l'intrant, c'est-à-dire uniquement au cours de l'exploitation de la ressource. Dans ce premier modèle à agents hétérogènes, les agents représentatifs des deux groupes diffèrent par leur taux d'actualisation. Nous avons ainsi examiné l'impact du différentiel de taux d'actualisation et de la répartition des agents sur l'équilibre de ce jeu. Nous avons alors découvert qu'au niveau global, des augmentations dans le différentiel de taux d'actualisation et dans la proportion d'agents avec le taux d'actualisation le plus élevé (les « gros »), augmentent l'extraction totale et diminuent le stock de ressource à l'état stationnaire. Cependant, au niveau individuel, l'impact de ces deux types d'asymétrie dépend de la comparaison de l'élasticité de l'utilité marginale à un. Pour ce qui est de l'asymétrie de « taille de groupe », lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à un, une augmentation de la proportion de « gros » agents dans l'industrie, tend à réduire (augmenter) le taux d'extraction des deux types d'agents. Ainsi, chercher à rendre l'industrie plus « homogène » en « gros » agents aura tendance à atténuer (exacerber) la « guerre » engendrée par la concurrence, lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à un. Concernant l'asymétrie de taux d'actualisation, une augmentation du différentiel augmente toujours le taux d'extraction des « gros » agents. Par contre, pour les « petits » agents, cette augmentation du différentiel de taux d'actualisation tend à réduire (augmenter) leur taux d'extraction lorsque l'élasticité de l'utilité marginale est supérieure (inférieure) à l'unité.

Dans le dernier chapitre, nous avons présenté deux groupes d'entreprises qui exploitent une ressource naturelle commune et en vendent la production sur un même marché. Dans ce cas et contrairement au second chapitre, ces entreprises se font concurrence aussi bien sur le marché de l'intrant que sur le marché de l'extrant. Deux entreprises issues des deux groupes se distinguent par leurs coûts marginaux, les « grosses » entreprises représentant celles avec le plus haut coût marginal et les « petites », celles avec le plus bas coût marginal. Les stratégies d'équilibre sont caractérisées par trois intervalles de stocks de ressource sur lesquels les entreprises adoptent des comportements différents. En-deçà d'un certain stock-seuil, aucune entreprise ne produit. Entre ce stock-seuil et un second stock-seuil, les entreprises exploitent la ressource à des taux linéaires et croissants avec le stock de ressource. Au-delà de ce second stock-seuil, les entreprises exploitent la ressource à des taux constants et qui correspondent aux taux d'exploitation qu'elles auraient adoptés si elles se faisaient une concurrence à la Cournot statique. Nous avons trouvé que la présence d'asymétrie induit des discontinuités dans la stratégie des grosses entreprises, et par conséquent dans le taux d'exploitation agrégé. Nous avons aussi montré que le stock-seuil à partir duquel les petites entreprises commencent leur exploitation et le stock-seuil à partir duquel elles adoptent leur exploitation à la Cournot statique, sont tous les deux plus élevés lorsque ces entreprises sont en présence de grosses entreprises (cas asymétrique) que lorsqu'elles sont toutes identiques (cas symétrique). Quant aux grosses entreprises, lorsque leur proportion dans l'industrie dépasse un certain seuil, le stock-seuil auquel elles commencent l'exploitation est plus élevé dans le cas symétrique que dans le cas asymétrique. En-deçà de ce seuil, ces grosses entreprises commencent leur exploitation à un stock-seuil plus bas que dans le cas symétrique. Le stock-seuil auquel elles adoptent leur comportement à la Cournot statique est, quant à lui, toujours plus bas dans le cas asymétrique que dans le cas symétrique où elles sont toutes de grosses entreprises. Nous avons aussi montré que ce modèle admet un ou trois états stationnaires selon la valeur du différentiel de coût marginal ou la répartition des entreprises. Par ailleurs, chacun de ces états stationnaires peut être obtenu en faisant varier les deux types d'asymétries.

Cette thèse a permis de montrer que la prise en compte de l'asymétrie a un impact sur les équilibres pouvant en résulter. Nous avons proposé deux méthodes permettant de prendre en compte l'hétérogénéité. Il serait donc intéressant dans une future recherche d'examiner d'autres types d'asymétries. Par ailleurs, dans les deux modèles, nous supposons le nombre d'agents exogènes. D'autres pistes de recherche consisteraient donc à examiner comment rendre endogène(s) le(s) nombre(s) de joueurs et déterminer comment les équilibres sont affectés par de tels changements.

BIBLIOGRAPHIE

[1] Benchekroun, H. (2003). “Unilateral Production Restrictions in a Dynamic Duopoly”, *Journal of Economic Theory* 111, 214-239.

[2] Baland, J.-M. and Platteau, J.-P. (1997).“Wealth Inequality and Efficiency in the Commons, i : The Unregulated Case”. *Oxford Economic Papers* 49, pp. 451-482.

[3] Brooks, R., M. Murray, S. Salant and J. C. Weise (1999). “When is the Standard Analysis of the Common Property Extraction Under Free Access Correct ? A Game-Theoretic Justification for Non-Game-Theoretic Analyses”. *The Journal of Political Economy*, vol. 107, no 4, pp. 843-858.

[4] Bulow, J., J. D. Geanakoplos and P. D. Klemperer (1985). "Multimarket Oligopoly : Startegic Substitutes and Complements”. *Journal of Political Economy*, 93, pp. 488-511.

[5] Clemhout, S. and H. Wan Jr (1974). “A Class of Trilinear Differential Games”.*Journal of Optimization Theory and Applications*, Vol. 14, pp. 419-424.

[6] Clemhout S. and H. Y. Wan (1985). “Dynamic Common Property Resources and Environmental Problems”. *Journal of Optimization Theory and Applications*, Vol. 46, 471-481.

[7] Clemhout S. and H. Y. Wan (1994). “Differential Games - Economic Applications”. in Aumann, R. J. and S. Hart, eds. *Handbook of Game Theory, II.* pp. 801-825.

[8] Cornes, R., C. Mason and T. Sandler (1986). “The Commons and the Optimal Number of Firms”, *Quarterly Journal of Economics, 100,* pp. 641-646.

[9] Dasgupta, P. and G. M. Heal (1979). *Economic Theory and Exhautible Resources.* Cambridge Univ. Press, Cambridge.

[10] Dockner, E., G. Feichtinger and S. Jorgensen (1985). "Tractable Classes of Nonzero Sum Open-loop Differential Games : Theory and Examples". *Journal of Optimization Theory and Applications*, Vol. 45, pp. 179-197.

[11] Dockner, E., S. Jorgensen, N. V. Long and G. Sorger (2000). *Differential Games in Economics and Management Science,* Cambridge University Press, Cambridge.

[12] Dockner, E. and G. Sorger (1996). "Existence and Properties of Equilibria for a Dynamic Game on Productive Assets", *Journal of Economic Theory* 71, 209-227.

[13] Fischer, R. and L. J. Mirman (1992). "Strategic dynamic interactions : Fish wars", *Journal of Economic Dynamics and Control*, Vol. 16, 267-287.

[14] Fischer, R. and L. J. Mirman (1996). "The Compleat Fish Wars : Biological and Dynamic Interactions", *Journal of Environmental Economics and Management*, Vol. 30, 34-42.

[15] Fudenberg, D. and J. Tirole (1991). *Game Theory*, Cambridge, MA, MIT Press.

[16] Gaudet, G. and H. Lohoues (2004). "On Limits to the use of Linear Markov Strategies in Common Property Natural Resource Games", mimeo, Université de Montréal.

[17] Gaudet, G., and S. Salant (1991). "Increasing the Profits of a Subset of Firms in Oligopoly Models with Strategic Substitutes", *American Economic Review,* Vol. 81, No. 3, pp. 655-665.

[18] Hardin, G. (1968). "The Tragedy of Commons". *Science*, 162, pp. 1243-1248.

[19] Kamien, M. and N. Schwartz (1991). *Dynamic Optimization : The Calculus of Variations and Optimal Control in Economics and Management.* Amsterdam : North-Holland.

[20] Karp, L. (1992). "Social Welfare in a Common Property Oligopoly". *International Economic Review*, vol. 33, issue 2, pp. 353-372.

[21] Levhari, D. and L. J. Mirman (1980). "The Great Fish War : An example using a dynamic Cournot-Nash Solution", *The Bell Journal of Economics*, Vol. 11, no. 11, 322-334.

[22] List, J. A. and C. Mason (2001). "Optimal Institutional Arrangements for a Transboundary Pollutants in a Second-Best World : Evidence from a Differential Game with Asymmetric Players". *Journal of Environmental Economics and Management*, 42, pp. 277-296.

[23] Lohoues, H. (2005). "Group and Size Asymmetries in a Fish War", mimeo, Université de Montréal.

[24] Long, N. V. and K. Shimomura (1998). "Some Results on Markov Equilibria of a Class of Differential Games", *Journal of Economic Behavior and Organization*, Vol. 33, 557-566.

[25] Long, N. V., K. Shimomura and H. Takahashi (1999). "Comparing Open-Loop with Markov Equilibria in a class of Differential Games, *The Journal of the Japanese Economic Association*, Vol. 50, pp 457-469.

[26] Mason, C. and S. Polasky (1997). "The Optimal Number of Firms in the Commons : A Dynamic Approach". *The Canadian Journal of Economics*, vol. 30, issue 4b, pp. 1143-1160.

[27] Novschek, W. (1985). "On the Existence of Cournot Equilibrium", *Review of Economic Studies*, 52, pp. 85-98.

[28] Olson, M. (1965). *The logic of the collective action.* Harvard University Press, Cambridge, Mass.

[29] Plourde, C. and D. Yeung (1989). "Harvesting of a Transboundary Replenishable Fish Stock : A Noncooperative Solution", *Marine Resource Economics*, Vol. 6, 57-70.

[30] Reinganum, J. F. (1982). "A Class of Differential Games for which the Open-Loop and Closed-Loop Equilibria Coincide". *Journal of Optimization Theory and Applications*, Vol. 36, pp. 253-262.

[31] Tirole, J. (1988). *The Theory of Industrial Organization.* The MIT Press, Cambridge, Mass.

[32] Yeung, D. (1998). "A Class of Differential Games which admits a Feedback Solution with Linear Value Functions". *European Journal of Operational Research*, Vol. 107, pp. 737-754.