Intraday patterns in stock prices traded on the NASDAQ Stock Exchange

Maximiliano Cacace

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Abstract

Patterns on stock returns have been documented over the past twenty years relating them to calendar time. We will use statistical theory of signal coherence in order to recognize such patterns on NASDAQ stock market intraday trading and we propose an alternative method for the recognition of such patterns. Based on the findings we propose an investment strategy and finally we back-test that strategy in a real life environment to compare it with a standard ‘buy and hold’ strategy.

maximiliano@madmax.com.ar
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1 Introduction

Many studies have been done in the past in order to recognize seasonalities and patterns on the stock markets. Those studies try to outperform a standard "buy and hold" strategy by proposing alternative strategies for low return periods. A typical "buy and hold" strategy is one that, for the studied period, gives just the variability of the its price as a return. As an example, whenever we say that a certain stock provided x% in the past year we refer to a "buy and hold strategy", buying that stock at the beginning of the year and selling it at the end.

A practical definition of "buy and hold" strategy is provided by investopedia\textsuperscript{1} website:

A passive investment strategy in which an investor buys stocks and holds them for a long period of time, regardless of fluctuations in the market. An investor who employs a buy-and-hold strategy actively selects stocks, but once in a position, is not concerned with short-term price movements and technical indicators.

The most commonly cited studies, which are known as "puzzles", are the Halloween effect or "indicator", the Turn-of-the-Month effect and the Monday effect. Many studies done on these puzzles have tried to identify explanations of the cause that generates these patterns or seasonalities, like vacations periods, the day of paychecks and more rationality in the processing of information during the weekends among others. Also they suggest ways to maximize profits and beat indexes or the standard "buy and hold" strategy.

Up until the arrival of the digital era, these studies used low frequency data and in many cases rudimentary computational and statistical tools. Also they focused on pattern recognition and seasonalities mostly in a year (Halloween effect), month (turn-of-the-month effect) or week (Mondays effect) time frames, even if the datasets comprised several years of observation.

An accurate definition of "seasonality" and "pattern" is presented by investopedia\textsuperscript{2} website:

Seasonality: A characteristic of a time series in which the data experiences regular and predictable changes which recur every calendar year. Any predictable change or pattern in a time series that recurs or

\textsuperscript{1}http://www.investopedia.com/terms/b/buyandhold.asp
\textsuperscript{2}http://www.investopedia.com/terms/s/seasonality.asp
http://www.investopedia.com/terms/p/pattern.asp
repeats over a one-year period can be said to be seasonal. Note that seasonal effects are different from cyclical effects, as seasonal cycles are contained within one calendar year, while cyclical effects (such as boosted sales due to low unemployment rates) can span time periods shorter or longer than one calendar year.

Pattern: In technical analysis, the distinctive formation created by the movement of security prices on a chart. It is identified by a line connecting common price points (closing prices, highs, lows) over a period of time. Chartists try to identify patterns to try to anticipate the future price direction. Also known as "trading pattern".

In this study we will use the words seasonalities and patterns indistinctly because we will do an analogy of yearly, monthly and weekly seasonalities and patterns to intraday ones. We will use high frequency data and two methods to try to recognize those patterns and be able to develop a strategy alternative to the standard "buy and hold" one. This will lead us to a back-testing phase where we can compare the results with other studies in this domain.
2 In-depth review

2.1 Seasonalities on intraday trading

The aim of this thesis is first to find, as has been done by others who were using wider time frames, seasonalities and patterns. However, we will use an intraday time-frame to try to find these patterns. In Section 4 we describe the two methods used to recognize this patterns. The first method is based on a parametric statistical model called Ramdomly Modulated Periodicity (RMP) followed by a Signal Coherence analysis developed by Hinich 2000. As an alternative to this method we propose the Average Compounded Relative Returns (ACRR) method.

In Section 5.1 and 5.2 we examine the empirical findings of the proposed methodologies and in order to exploit our recognized intraday pattern we propose a trading strategy with the aim of outperforming the standard "buy and hold" strategy. In the rest of Section 5, and in order to verify our findings, we back-test our developed intraday strategy not only to compare profits to the "buy and hold" strategy, but also to do an analogy with previous studies in this field.

The studies we took in consideration for contrasting our findings are the exhaustively studied seasonality: the Halloween effect, which is based on a popular saying "sell in may and go away" that anticipates a bear market with low return in stock prices. The Turn-of-the-Month effect, documented by Ariel (1987) where higher mean stock returns occur during the initial days of a trading month than during days later in the month. We will consider the first week of every month in order to compare these studies with our empirical results. And finally the Mondays effect, the acknowledgement of a lower return on Mondays relative to other weekdays an on average negative, which has been studied with limited success to explain the causes of this pattern. Maberly (1995) shows that financial practitioners were aware of the Monday effect as early as the late 1920s. (See Kelly, 1930.) Then, as now, the existence of negative returns on Mondays was a puzzling phenomenon.

This paper contributes to the existing literature in several ways. First it attempts to explore the intraday dynamics of major US equity markets using high frequency 1-minute data. Second we develop a strategy that exploits those dynamics. And third it provides an analogy to studies already done in this field. These conclusions are presented in Section 6.
2.2 Data

Time series have been filtered to maintain the 9:30am to 16:00pm market hours frame which means that no premarket data nor after hours quotes have been analyzed mainly because the daytraders cannot trade during those hours and they cannot hold positions overnight. The interest of this study is an intraday pattern recognition; a development of a strategy and its back-testing within the SEC regulatory frame.

The primary data set consisted of 1-minute price quotes on the major NASDAQ index QQQQ from April 20th 2006 through April 19th 2007, totaling one year. After filtering the data for anomalies and eliminating undesired quotes, this data served as the basis for the pattern recognition and the strategy development. Summary statistics for 1-minute intraday returns are presented in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>95559</td>
<td>41.39475</td>
<td>2.741043</td>
<td>35.56</td>
<td>45.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>From 20-April-2006 to 19-April-2007</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A second time series dataset was used for the back-testing which consisted of 1-minute price quotes on the QQQQ index from October 3th 2005 through April 19th 2007, totaling one year. This is a bigger dataset that the first one because we want to see how well our strategy and our pattern recognition worked by comparing the results with a standard "buy and hold" strategy. Summary statistics for 1-minute intraday returns are presented in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>146060</td>
<td>41.2547</td>
<td>2.35455</td>
<td>35.56</td>
<td>45.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>From 03-Oct-2005 to 20-July-2007</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3: "Buy and Hold" Strategy Profits

<table>
<thead>
<tr>
<th>Symbol</th>
<th>3-Oct-2005</th>
<th>20-July-2007</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>QQQQ</td>
<td>$39.54</td>
<td>$50.05</td>
<td>26.53%</td>
</tr>
<tr>
<td>SUNW</td>
<td>$3.90</td>
<td>$5.32</td>
<td>36.41%</td>
</tr>
</tbody>
</table>

From 03-Oct-2005 to 20-July-2007

There is a third dataset that we used in order to compare our results in the previous stage. It is the time series of the most traded stock in the NASDAQ, SUNW. Table (3) shows a typical "buy and hold" strategy return for the period and symbols we studied.
3 Literature review

3.1 Halloween Indicator, "Sell in May and go away"

The Halloween Indicator is a phenomenon documented by Bouman and Jacobsen on a strong seasonal effect in stock returns based on a popular market saying "Sell in May and Go Away", in which the month of May signals the start of a bear market, so that investors are better off selling their stocks and holding cash.

They called this pattern then Halloween Indicator, because every October 31 the scary period for investors ends. Bouman and Jacobsen conclude that the anomaly in stocks returns becomes most noticeable when comparing May-October versus November-April periods. Then, they provide a way to exploit this anomaly through a simple strategy, just "Sell in May and go Away". This strategy suggests to invest in a value-weighted index like S&P500 during highest return periods (November-April) and investing in risk-free assets like T-bills during lower return periods (May-October).

Bouman and Jacobsen remark that³,

"Surprisingly, we find the Sell in May effect is present in 36 of the 37 countries in our sample. The effect tends to be particularly strong and highly significant in European countries, and also proves to be robust over time. Sample evidence shows that in a number of countries it has been noticeable for a very long time, and in the U.K. stock market, for instance, we have found evidence of a Sell in May effect as far back as 1694. We find no evidence that the effect can be explained by factors like risk, cross correlation between markets, or the January effect. We also try some alternative explanations . . . but none of them seems to provide an explanation for the puzzle (Bouman and Jacobsen 2002, 1618)."

In Bouman and Jacobsen's documentation there are some potential outliers that may raise questions to their conclusions, particularly for the U.S. equity returns. One is the October 1987 crash in world equity prices which the authors claim they took in consideration by including a dummy variable that did not change their results. In October 1987 U.S. stocks fell on average by over 20 percent. Another possible outlier is the Operation Desert Storm, the Kuwait invasion in

³The Halloween Indicator, "Sell in May and Go Away": Another Puzzle, Page 3
August 1990 which caused an increase in world oil prices. On August 1998 Russia announced its default, causing another crash in world financial markets, including the collapse of the hedge fund Long-Term Capital Management and the fall of U.S. stocks by over 15 percent on average.

Having analyzed 37 countries, developed and emerging markets, from the north and south hemispheres, for the period January 1970-August 1998, they conclude that monthly returns are larger during November-April periods than the rest of the year.

Their study concludes that,

"A simple strategy based on the saying would outperform a buy and hold portfolio in many countries . . . and would also be a lot less risky" (1619). "we are faced with the following problem: History and practice tells us that the old saying [Sell in May and go away] is right, while stock market logic tells us it is wrong. It seems that we have not yet solved this new puzzle" (1630).

Another author that had previously documented the same phenomenon, was Yale Hirsch in 1997 in his investment book Hirsch's Stock Trader's Almanac, an annual publication since 1968. He presents what he calls a Six-Month Switching strategy, which is similar to Bouman and Jacobsen's Halloween Indicator. Hirsch shows in a spreadsheet format, the returns of for his Six-Month Switching strategy for 1950-1996 periods on the Dow Jones Industrial Average (DJIA).

He back tested his Six-Month Switching strategy for the November-April period with a $10,000 investment in the DJIA, showing that it grew to $206,762. On the other hand he shows that the same investment of $10,000 on the DJIA, for the May-October period only grew to $17,272.

Even though Hirsh says, "Don't tell the big boys about this! Let's keep this one to ourselves (Hirsch 1997, 54)." His strategy was well known since the 1980's. If markets were efficient, once this information is well known there should no longer be any possibilities of arbitrage since then.

### 3.2 The Monday Effect

There have been shifts in the pattern for Monday returns—most notably in recent years. Monday returns for large-firm equities have been not only positive but also large relative to returns for other weekdays. Considerable research effort has been expended to explain these curious patterns.
Over a hundred years of trading activity, Monday returns for equity securities have been lower than the rest of the week. This applies across several kinds of assets as well as across different markets. This has been documented by market practitioners as well as by academics. Market practitioners have documented low Monday returns at least as early as 1920s even with rudimentary statistical tools or no databases. Kelly in 1930 suggests that the low return Mondays have its cause on individual investors weekend decision making process. He made a three-year statistical study and concludes that Mondays are the worst day to buy stocks. Fields in 1931 tests the DJIA against the conventional market wisdom of low-Saturday returns. He concludes that Saturday’s average closings are higher than the average of adjacent Friday and Monday closings. Merrill in 1966 compares the DJIA of Monday trading days against the rest of the weekdays for the period 1952 to 1965. He concludes that Monday returns are the lowest 43 percent against at least 50 percent of any other weekday. Also Cross in 1973 as he studies price changes for the S&P500 reports that the proportion of Fridays prices increases are higher than on Mondays for the period of 1953 to 1970.

As electronic databases and statistical tools came to light, academics showed using these methods the differences across weekdays on equity returns. The S&P 500 has been tested with rigorous statistical methods by French in 1980 for the periods 1953 to 1977 and by Gibbons and Hess in 1981 for the period 1962 to 1978. In 1988 Lakonishok and Smidt report negative Monday returns for the DJIA from 1897 to 1986. Siegel in 1998 determines that if Monday returns had been equal to the average returns of the rest of the days from 1885 to 1997, the DJIA would be at twice its level by the end of 1997.

To show an extension of this effect internationally, Dubois and Louvet in 1996 make an examination on eleven indexes from nine countries from 1969 to 1992 in which they prove the existence of low Monday returns phenomenon. Tong in 2000 also extends this research on 23 European, Asian and North American markets with same results. On the emerging markets arena, Aggarwal and Rivoli in 1989 suggests that negative Monday returns and low Tuesday effects in four emerging Asian Markets are due to time lag between this markets and New York market.

Academics have used rigorous statistical analysis on different set of securities and period frames as well as different locations around the globe; on the other hand various practitioners document their findings of this market wisdom. All of them conclude that a significant difference exists across weekdays returns, particularly the existence of a significantly low return on Mondays.
3.3 Seasonal Pattern determination

Flexible Fourier Form (FFF) introduced by Gallant (1981, 1982) and proposed by Andersen and Bollerslev (1997, 1998) was found to be an efficient way of determining the seasonal pattern that allows a direct interaction between the level of the daily volatility and the shape of the intraday pattern. Their model is a good starting point for high-frequency volatility modelling in a coherent framework.

As Dacorogna et al. (1993) wrote: "The behaviour of a time series is called seasonal if it shows a periodic structure in addition to less regular movements".

One of the findings that Andersen and Bollerslev reach with their FFF model was that Monday appears the least volatile, while Thursdays and Fridays are the most volatile. This has been explained by Harvey and Huang 1991 as the result of macroeconomic news announcements, which are released mainly on these two days.

3.4 Hinich 2000 Autocoherence

Spectral analysis applied to a sample of data is an alternative to traditional regression methods with seasonal dummy variables for the detection of periodicities of the financial time series.

As a definition, the spectral analysis is a decomposition process of the original series into a set of mutually orthogonal cyclical components of different frequencies, where its spectrum is a plot of the signal amplitude against the frequency.

As an example, if it happens to be a white noise process, the spectrum will be flat, but statistically significant amplitudes at any given frequency can be taken to indicate evidence of a periodic behavior.

As Hinich(2000) states,

"A periodic signal can be perfectly predicted far into the future since it perfectly repeats every period. Nature does not produce perfect periodic signals. There is always some variation in the waveform over time for signals which are labeled as periodic but which are not truly deterministic."

The idea is to keep those frequency components that shows the least variation over time to detect the most important seasonalities. The measure of how much that waveform variates is called coherence.
As Hinich (2000) describes,

"This signal coherence function is very different from the coherence function between two stationary signals."

Hinich (2000) developed a way to measure the behavior of the Fourier amplitudes as a function of frequency. This type of spectrum is not the traditional power spectrum but a coherence spectrum, and is a normalized statistic independent of the height of the power spectrum at each frequency.

As this coherence is not the traditional coherence between two different signals, we call it "autocoherence", just because of the fact that there is only one source of data, one signal.

He applies this method to a digitized record of an acoustic signal generated by a boat in a bay in the Baltic Sea south of Stockholm, Sweden\(^4\).

\(^4\)A Statistical Theory of Signal Coherence, Page 259
4 Theoretical analysis

4.1 Randomly Modulated Periodicity

In a given plot of periodic data, there is always a certain intrinsic inter-periodic variability even if the data plot may look coherent as an ensemble. Yet, the most commonly sinusoidal functions used to model such data sets are by definition stable over time and display zero deviation from the sinusoid’s frequency. Therefore, these functions do not appear to be well suited to describe data sets with inter-periodic variability. This idea is validated by the fact that in nature, single sinusoidal functions with zero deviation (also called zero bandwidth in Fourier analysis) do not exist, because all natural events, from astrophysics, over molecular chemistry to genetics display intrinsic variability. Deterministic sinusoids with zero bandwidth are a purely mathematical construct that are convenient because they simplify modeling, but at the cost of accuracy of predictions. Therefore, it is important to find ways of describing periodic data, which can at a time reflect their cyclic nature and the irregularities associated with these cycles. One approach to this challenge is called Randomly Modulated Periodicity (RPM) and this was highlighted by Hinich in 2000.

By definition, a series sampled at regular intervals is said to exhibit random modulated periodicity if it can be expressed as:

\[ x(t) = a_0 + \frac{1}{K} \sum_{k=1}^{K} a_k \exp(i2\pi f_k t) + \frac{1}{K} \sum_{k=0}^{K} u_k(t) \exp(i2\pi f_k t) \]  

(1)

where \( f_k = k/T \) and \( u_{ik} \) (i=1,2) are jointly dependent zero mean random processes that are periodic block stationary and satisfy finite dependence. The signal \( x(t) \) can be expressed as the sum of a deterministic periodic component \( a(t) \), and a stochastic error term \( u(t) \). So (1) can be written as:

\[ x_m(t) = \sum_{t=0}^{T-1} x_m(\beta_m + t) \exp(-i2\pi f_k t) = a_k + U_m(k) \]  

(2)

The periodic component of \( a(t) \) is the mean component of \( x(t) \). In order to determine how stable the signal is at each frequency across the frames, the notion of signal coherence is employed. Signal coherence refers to the stability of a given signal over several frames and grossly corresponds to what is known as \( R^2 \) in regression analysis. Generally speaking, \( R^2 \) reflects the strength of dependency between two variables. However, in our specific case we are focusing on the stability
of a single signal over several frames within a single time series. Therefore, here we can talk about "autocoherence".

4.1.1 Forecasting

The advantage of this method is that it can forecast the seasonal time series out of sampling of the same signal (dataset). In 2002, Li and Hinich pointed out that seasonal ARMA models fail at correctly forecasting periodic events with intrinsic variability because these models are based on most recent seasonal patterns. Here, we propose instead to base predictions only on the least fluctuating sample components.

The mean frame is computed from the non overlapping frames and is subtracted from each frame. We compute the Fourier transform for the mean frame and for each residual frame.

The coherence spectrum is then computed from the amplitudes that are significant. Those significant amplitudes are, in other words, the most coherent part of the mean frame. The inverse Fourier transform is computed on the most coherent part of this mean frame.

Then we forecast using a VAR on the amplitudes of the non-zeroed components of the Fourier transform of the residual frames.

4.2 Signal Coherence Spectrum

A periodic function is an idealization of a real periodic process. This ideal periodic function of period T can be written as a sum of weighted sinc and cosine functions whose frequencies are integer multiples of the fundamental frequency 1/T. These frequencies are called Fourier frequencies. The sum of the amplitudes is called a Fourier transform of the periodic function.

Each amplitude of the Fourier transform of a real periodic process (which is not a perfectly periodic process) is a constant plus a zero mean random time series process. This zero mean process may or may not be stationary and is the part that makes the original signal variate randomly from period to period.

A measure of this variation of the amplitudes from period to period is called coherence spectrum, which is very different from the traditional power spectrum.

This method is widely explained by Melvin Hinich and Serletis in their "Ran-

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5Developed by Melvin J. Hinich "A Statistical Theory of Signal Coherence" and used by Melvin J. Hinich and Apostolos Serletis in "Randomly Modulated Periodic Signals in Alberta’s Electricity Market"; by Chris Brooks, Melvin J. Hinich and Douglas M. Patterson in "Intraday
domly Modulated Periodic Signals in Alberta’s Electricity Market work. Hinich (2000) introduced this measure of the modulation relative to the underlying periodicity which he called Signal Coherence Spectrum. For each Fourier frequency $f_k = k/T$ the value of the coherence signal coherence spectrum is

$$
\gamma_x(k) = \frac{|a_k|^2}{\sqrt{|a_k|^2 + \sigma_u^2(k)}}
$$

where $a_k = a_{1k} + ia_{2k}$ is the amplitude of the $k$th sinusoid written in complex variable form, $i = \sqrt{-1}$, $\sigma_u^2(k) = E|U(k)|^2$ and

$$
U_m(k) = \sum_{t=0}^{T-1} u_m(t) \exp(-i2\pi f_k t)
$$

is the discrete Fourier transform (DFT) of the modulation process $u_k(t) = u_{1k}(t) + iu_{2k}(t)$ written in complex variable form.

Each $\gamma_x(k)$ is in the $(0,1)$ interval. If $a_k = 0$ then $\gamma_x(k) = 0$. If $U(k) = 0$ then $\sigma_x(k) = 1$. The Signal Coherence measures the amount of 'wobble' in each frequency component of the signal $x(t)$ about its amplitude when $a_k > 0$. The amplitude standard deviation (AMS) is

$$
\rho_x(k) = \frac{|a_k|}{\sigma_u(k)}
$$

for frequency $f_k$. Thus,

$$
\gamma_x^2(k) = \frac{\rho_x^2(k)}{\rho_x^2(k) + 1}
$$

is a monolitical increasing function of this signal-to-noise ratio. Inverting this relationship, it follows that

$$
\rho_x^2(k) = \frac{\gamma_x^2(k)}{1 - \gamma_x^2(k)}
$$

An AMS of 1.0 equals a signal coherence of 0.71 and an AMS of 0.5 equals a signal coherence of 0.45. To estimate the Signal Coherence, $\gamma_x(k)$, suppose that we know the fundamental period and we observe the signal over $M$ such periods. The $m$th period is \( \{x((m-1)T+t); t = 0, ..., T-1\} \). The estimator of $\gamma(k)$ introduced

by Hinich (2000) is

\[
\hat{\gamma}_x(k) = \sqrt{\frac{|\hat{A}_k|^2}{|\hat{A}_k|^2 + \hat{\sigma}_u^2(k)}}
\]

where,

\[
\bar{X}(k) = \frac{1}{M} \sum_{m=1}^{M} X_m(k)
\]

is the sample mean of the DFT,

\[
X_m(k) = \sum_{t=0}^{T-1} x((m-1)T + t) \exp(-i2\pi f_k t)
\]

and

\[
\hat{\sigma}_u^2(k) = \frac{1}{M} \sum_{m=1}^{M} |X_m(k) - \bar{X}(k)|^2
\]

is the sample variance of the residual discrete Fourier transform, \(X_m(k) - \bar{X}(k)\). This estimator is consistent as \(M \to \infty\) and if the modulations have a finite dependence of span \(D\) then the distribution of

\[
Z(k) = \frac{M \bar{X}(k)^2}{N \hat{\sigma}_u^2(k)}
\]

is asymptotically chi-squared with two degrees-of-freedom and a non-centrality parameter

\[
\lambda_k = \frac{M}{N} \rho_x^2(k) = \frac{M \gamma_x^2(k)}{N (1 - \gamma_x^2(k))}
\]

as \(M \to \infty\) see Hinich and Wild (2001). These \(\chi^2_2(\lambda_k)\) variates are approximately independently distributed over the frequency band when \(D << N\). If the null hypothesis for frequency \(f_k\) is that \(\gamma_x(k) = 0\) and thus its AMS is zero, then \(Z(k)\) is approximately a central chi-squared statistic. Thus \(Z(k)\) can be used to falsify the null hypothesis that \(\gamma_x(k) = 0\). The tests across the frequency band are approximately independent distributed tests. The use of transformation to the
Z(k)'s is the only straightforward way to put statistical confidence on the signal coherence point estimates\(^6\).

### 4.3 The Average Compounded Relative Return, (ACRR)

As an alternative to the signal coherence method, we propose the analysis of the compounded returns relative to the opening price for the day.

There is always a highest and lowest price during the day for a specific stock. If these prices seem to happen at around the same time every day, we would say that this is the pattern followed by the "way" that stock is traded.

Regardless of the volatility of the stock, its Opening price (the price at which it is first traded at market opening, 9:30AM) will capture the adjustments due to news on that particular stock, so big movements will happen normally before market opens or after market close. We suspect that once these adjustments are captured by the stock price, its price moves solely by popularity and intraday traders.

The goal is to figure out the way this stock moves by identifying the specific time at which highest and lowest prices normally happens within a day. If there is a specific period of the day where this normally happens, then we will try to profit this anomaly.

To do this it is necessary to calculate the compounded return (to the minute), of the price in respect to the initial price for the day (the opening price). The idea then is to compare each day and try to find similarities between them. For that we propose then,

\[
R(t) = \frac{1}{d} \sum_{i=1}^{d} \ln(X_d(t)) - \ln(O_d)
\]

Where \(O(d)\) is the Opening price of the day \(d\), \(X_d(t)\) is the price at minute \(t\) of day \(d\), and \(R_d(t)\) is the return in respect to the opening price of minute \(t\) on day \(d\).

Then we need to aggregate every minute return of the year and we can plot the average returns of each minute in respect to the initial price of each day.

This per minute aggregate will provide an averaged behavior of the stock relative to a starting point, (the opening price of it at 9:30am). As day traders cannot hold positions overnight, it is assumed that this stock behavior restarts again every day.

\(^6\)Explanation of this method taken from Melvin J. Hinich and Apostolos Serletis "Randomly Modulated Periodic Signals in Alberta's Electricity Market*.

Maximiliano Cacace

Université de Montreal
5 Empirical findings

5.1 RMP in NASDAQ-100 QQQQ index

NASDAQ-100 index time series was used, namely QQQQ closing prices and volumes from April 20th 2006 to April 19th 2007. Figure (1) shows a section of the volume time series, and Figure (2) shows a section of the per minute closing price for the QQQQ symbol over the same period. Volume has a daily cycle but it is difficult to see it on Figure (1) as there are so many days on the same chart. A more deeply study on daily volumes can show this pattern. As for price cycles, which is our interest in this research, Figure (2) does not provide any a priori information.

Figure 1: A section of volume traded on QQQQ
Signal coherence spectral analysis was applied to the QQQQ closing prices and volumes time series, using the FORTRAN 95 'Spectrum.for' program developed by Hinich and available at his web page, www.la.utexas.edu/~hinich. In doing so, the minutely QQQQ closing prices and the minutely traded volume data were first detrended by fitting an AR(12) model to each time series. The AR(12) filter is used to make the data have a flat spectrum; it is a linear transformation and thus it does not create nor destroy coherence. The residuals of the fitted model are then analyzed for the presence of a randomly modulated periodicity with a fundamental period of one day (390 minutes). An AR fit is a linear operation that cannot create signal coherence. Indeed signal coherence can only be reduced by an improperly applied detrended method. The prices were first used to calculate compounded returns.

The Signal Coherence of the minutely QQQQ volume time series is shown in Figure (3). Only harmonic 390 and 90 minutes has a coherence greater than 0.55 all the others are even lower than 0.5. These period components are not coherent and of little use for forecasting.
The Signal Coherence of the minutely QQQQ returns shown in Figure (4) shows limited evidence of coherence, having no significant coherence periodicities at all. None of the harmonics has a coherence greater than 0.3. The plot of the Signal Coherence spectrum in Figure (4) shows that the standard methods for fitting a Fourier expansion of the daily cycle will not contribute much to a forecast of the prices.
We conclude at this point that the Hinich 2000 method is not of use for the recognition of patterns in the QQQQ intraday returns nor for predicting its future prices. One could not develop an intraday strategy using this method that could maximize a "buy and hold" strategy for the NASDAQ-100 index.

5.2 The Average Compounded Relative Return, (ACRR) method

The data analyzed was provided by Opentick for the QQQQ symbol from April 20th 2006 to April 19th 2007. This quotes has been filtered to maintain the 9:30am to 16:00pm market hours frame. No premarket data nor after hours has been analyzed mainly because the daytraders and institutional traders cannot trade during those hours and they cannot hold positions overnight or at least they are obliged to liquidate positions due to a difference between Day and Overnight Buying Powers\(^7\).

As we explained earlier, the Compounded Relative Return is calculated at every minute with respect to the opening price for that day. By doing so, one can have a measure of how good the stock is doing at any particular minute. That

\(^7\)See NASD Daytrader Regulations.
Compounded Relative Return for each particular minute is then averaged in order to plot how a stock will "behave" in a typical day.

Doing a graphical analysis one can see that the maximum ACRR - Averaged Compounded Relative Return will happen at minute #159, and the minimum at minute #339. After this period there is another highest ACRR at minute #365. To be more precise we did this by sorting this values in STATA, and we concluded that we should enter a short position around 12:09PM and exit that position at 15:09PM. Conversely we could enter a long position at 15:09PM and exit before market close around 15:49PM.

5.3 Back-testing the strategy

The strategy developed is based on a one year period from April 20th 2006 to April 19th 2007. This strategy may be applied to any single day and provide, in average, positive expected returns. For a better testing, it was applied in a wider period starting October 3rd 2005 and ending July 20th 2007. By doing the period covered was almost two years, with two "Sell in May and Go Away" factors, and several Monday effect and Turn-of-the-Month seasonalties.

We did this study by month, by week and by day of the week, to see if this
strategy works better under certain circumstances.

Comparing this strategy with a cumulated profit of 32.34% against a "buy and hold" strategy buying in October 3rd 2005 at $39.54 and selling in July 20th 2007 at $50.03 with a cumulated profit of 26.53%; one can see that the strategy developed with ACRR method outperformed the "buy and hold" strategy by 5.81.

Even though a better performance was expected, this strategy may be fine-tuned manually by an experienced daytrader to achieve better results by choosing the right moment to get in/out of a position. Also, a moving average may be used to choose the highest price.

5.3.1 Monthly Net Profit & Loss

One can see that the results are consistent with the Halloween factor. From May to October our strategy performs great mostly because it has a considerable component for shorting stocks during the day, (An average of 178.8 minutes for short positions against an average of 40.1 minutes for long positions), and market is not that performing during those months. From October on we start having not that enthusiastic results, with some big negative months.
<table>
<thead>
<tr>
<th>Performance</th>
<th>All Trades</th>
<th>Long Trades</th>
<th>Short Trades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Net Profit</td>
<td>$11.44</td>
<td>$4.16</td>
<td>$7.28</td>
</tr>
<tr>
<td>Gross Profit</td>
<td>$58.91</td>
<td>$18.10</td>
<td>$40.81</td>
</tr>
<tr>
<td>Gross Loss</td>
<td>-$47.47</td>
<td>-$13.94</td>
<td>-$33.53</td>
</tr>
<tr>
<td>Commission</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Profit Factor</td>
<td>1.24</td>
<td>1.30</td>
<td>1.22</td>
</tr>
<tr>
<td>Cumulated Profit</td>
<td>32.34%</td>
<td>11.19%</td>
<td>19.01%</td>
</tr>
<tr>
<td>Max. Drawdown</td>
<td>-6.22%</td>
<td>-2.53%</td>
<td>-5.38%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.62</td>
<td>0.44</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Start Date: 10/3/2005
End Date: 7/20/2007

| Total # of Trades                | 859        | 421         | 438          |
| Percent Profitable               | 53.08%     | 55.58%      | 50.68%       |
| # of Winning Trades              | 456        | 234         | 222          |
| # of Losing Trades               | 403        | 187         | 216          |
| Average Trade                    | 0.03%      | 0.03%       | 0.04%        |
| Average Winning Trade            | 0.31%      | 0.19%       | 0.44%        |
| Average Losing Trade             | -0.28%     | -0.18%      | -0.37%       |
| Ratio avg. Win / avg. Loss       | 1.11       | 1.06        | 1.19         |
| Max. conseq. Winners             | 7          | 9           | 5            |
| Max. conseq. Losers              | 9          | 10          | 7            |
| Largest Winning Trade            | 2.33%      | 0.95%       | 2.33%        |
| Largest Losing Trade             | -2.14%     | -1.03%      | -2.14%       |
| # of Trades per Day              | 1.31       | 0.64        | 0.67         |
| Avg. Time in Market              | 110.8 min  | 40.1 min    | 178.8 min    |
| Avg. Bars in Trade               | 109.6      | 39.9        | 176.7        |
| Profit per Month                 | 1.31%      | 0.49%       | 0.81%        |
| Max. Time to Recover             | 143.00 days| 147.00 days | 182.00 days  |
| Average MAE                      | 0.29%      | 0.19%       | 0.37%        |
| Average MFE                      | 0.33%      | 0.22%       | 0.44%        |
| Average ETD                      | 0.30%      | 0.19%       | 0.40%        |
| Turn Around                      | 28.82%     | 10.74%      | 18.09%       |

From October 3rd 2006 to July 29th 2007
5.3.2 Weekly Net Profit & Loss

It was not possible to prove the Turn-of-the-Month effect as no consistence was found on the first week of the month positives or negative returns. As we defined above, we consider the first week of the month to evaluate the Turn-of-the-Month effect, and we could not see any pattern in our strategy returns on the aggregate of the first 5 days of each month. We suspect that part of this is due to the fact that the expected phenomenon is marginally positive returns, while our strategy profits mostly from negative returns.
5.3.3 Daily Net Profit & Loss

On the other hand we agree with the day-of-the-week effect as we could see a significant variation when aggregating the profits per day of the week. Table (4) shows that our strategy performed a lot better on Mondays than on any other day of the week, again, because of its big component in shorting stocks.

Table 5: Per day-of-the-week performance

<table>
<thead>
<tr>
<th>Day of Week</th>
<th>Avg. Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>14.27%</td>
</tr>
<tr>
<td>Tuesday</td>
<td>7.42%</td>
</tr>
<tr>
<td>Wednesday</td>
<td>2.67%</td>
</tr>
<tr>
<td>Thursday</td>
<td>0.59%</td>
</tr>
<tr>
<td>Friday</td>
<td>3.88%</td>
</tr>
</tbody>
</table>

03-Oct-2005 to 20-Jul-2007
5.3.4 MAE/MFE and Profit protection

Average maximum adverse excursion which represents the worst loss level a trade reached. Average maximum favorable excursion which represents the best profit level a trade reached. This information could be used in order to set a profit protection technique so that a specific position does never go over a certain profit/loss threshold. This type of analysis, known as Maximum Adverse Excursion (MAE) and Maximum Favorable Excursion (MFE)\(^8\).

MAE is calculated as the greatest difference between the entry price and the worst price that goes against a position before the position is closed. MFE is calculated as the greatest difference between the entry price and the best price that goes in the favor of a position before the position is closed.

---

Dots in red show positions that ended with a loss but were at one time profitable. This clearly shows that the strategy can be manually tuned so that already acquired profits do not end in losses. During the life of a position, an experienced daytrader can manually quit a position if he sees a trend reversal.

One can dissect this information for a better understanding of which trades (long/short) has the worst adverse/favorable excursions in order to fine-tune the strategy, but this exceeds the scope of this research.

5.3.5 SUNW results

As an interesting verification in the quest to find consistency in our results, we run this strategy back-testing on the SUNW symbol. SUNW represent the most traded stock in the NASDAQ stock market, with a daily average volume of over
$40 millions$\

To our surprise, this stock provided a 53.49% profit using the proposed strategy compared to a "buy and hold" buying at $3.9 and selling at $5.32 with a profit of 36.41%, outperforming the latest in 17.08%. We can see that the maximum cumulated returns overcome an amazing 70%, which means that if we had stopped trading at that month we would have almost doubled the profits made by the standard "buy and hold" strategy.

![NinjaTrader Cumulated Profit Report, 03/10/2006 - 20/07/2007](image)

**Figure 11: Cumulated profits**

The risk is lower than in a "buy and hold" strategy, not only because we are in the market 219.3 minutes out of 390 per day but also because we don't absorb pre/after market volatilities.

To see the consistency of the Halloween effect ("Sell in May and go Away" saying) with our findings, we can compare QQQQ and SUNW Cumulated Profits, and we have almost the same tendency that could be drawn for the same period of time, which coincides with the May to October period.

If our strategy is fine-tuned taking into consideration the periods where cumulated profits starts declining, surprisingly in accordance with the Halloweenen

---

<table>
<thead>
<tr>
<th>Performance</th>
<th>All Trades</th>
<th>Long Trades</th>
<th>Short Trades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Net Profit</td>
<td>$1.94</td>
<td>$0.64</td>
<td>$1.30</td>
</tr>
<tr>
<td>Gross Profit</td>
<td>$12.43</td>
<td>$4.22</td>
<td>$8.21</td>
</tr>
<tr>
<td>Gross Loss</td>
<td>-$10.49</td>
<td>-$3.58</td>
<td>-$6.91</td>
</tr>
<tr>
<td>Commission</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Profit Factor</td>
<td>1.18</td>
<td>1.18</td>
<td>1.19</td>
</tr>
<tr>
<td>Cumulated Profit</td>
<td>53.49%</td>
<td>17.41%</td>
<td>30.74%</td>
</tr>
<tr>
<td>Max. Drawdown</td>
<td>-17.51%</td>
<td>-6.65%</td>
<td>-21.00%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.42</td>
<td>0.48</td>
<td>0.31</td>
</tr>
<tr>
<td>Start Date</td>
<td>10/3/2005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>End Date</td>
<td>7/20/2007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total # of Trades</td>
<td>857</td>
<td>417</td>
<td>440</td>
</tr>
<tr>
<td>Percent Profitable</td>
<td>61.03%</td>
<td>63.79%</td>
<td>58.41%</td>
</tr>
<tr>
<td># of Winning Trades</td>
<td>523</td>
<td>266</td>
<td>257</td>
</tr>
<tr>
<td># of Losing Trades</td>
<td>334</td>
<td>151</td>
<td>183</td>
</tr>
<tr>
<td>Average Trade</td>
<td>0.05%</td>
<td>0.04%</td>
<td>0.07%</td>
</tr>
<tr>
<td>Average Winning Trade</td>
<td>0.49%</td>
<td>0.33%</td>
<td>0.66%</td>
</tr>
<tr>
<td>Average Losing Trade</td>
<td>-0.64%</td>
<td>-0.48%</td>
<td>-0.77%</td>
</tr>
<tr>
<td>Ratio avg. Win / avg. Loss</td>
<td>0.77</td>
<td>0.7</td>
<td>0.86</td>
</tr>
<tr>
<td>Max. conseq. Winners</td>
<td>11</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Max. conseq. Losers</td>
<td>5</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Largest Winning Trade</td>
<td>5.59%</td>
<td>2.36%</td>
<td>5.59%</td>
</tr>
<tr>
<td>Largest Losing Trade</td>
<td>-3.30%</td>
<td>-1.52%</td>
<td>-3.30%</td>
</tr>
<tr>
<td># of Trades per Day</td>
<td>1.31</td>
<td>0.64</td>
<td>0.67</td>
</tr>
<tr>
<td>Avg. Time in Market</td>
<td>111.5 min</td>
<td>40.1 min</td>
<td>179.2 min</td>
</tr>
<tr>
<td>Avg. Bars in Trade</td>
<td>109.7</td>
<td>39.8</td>
<td>175.9</td>
</tr>
<tr>
<td>Profit per Month</td>
<td>2.01%</td>
<td>0.75%</td>
<td>1.25%</td>
</tr>
<tr>
<td>Max. Time to Recover</td>
<td>353.00 days</td>
<td>268.00 days</td>
<td>364.00 days</td>
</tr>
<tr>
<td>Average MAE</td>
<td>0.61%</td>
<td>0.44%</td>
<td>0.78%</td>
</tr>
<tr>
<td>Average MFE</td>
<td>0.67%</td>
<td>0.50%</td>
<td>0.81%</td>
</tr>
<tr>
<td>Average ETD</td>
<td>0.61%</td>
<td>0.46%</td>
<td>0.76%</td>
</tr>
<tr>
<td>Turn Around</td>
<td>45.47%</td>
<td>16.61%</td>
<td>28.86%</td>
</tr>
</tbody>
</table>

From October 3rd 2006 to July 20th 2007
Figure 12: SUNW vs QQQQ Cumulated profits

indicator where the markets starts a bullish trend, results would indeed be even more profitable.

Also consistent with day-of-the-week effect, this strategy performed with SUNW far better on Mondays than on any other day of the week, results expected as with QQQQ. The only big difference we found was on Wednesday, in which our strategy outperformed Tuesdays, whereas with QQQQ Wednesdays was even worse than Fridays.
### Table 7: Per day-of-the-week performance

<table>
<thead>
<tr>
<th>Day of Week</th>
<th>Avg. Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>19.38%</td>
</tr>
<tr>
<td>Tuesday</td>
<td>11.56%</td>
</tr>
<tr>
<td>Wednesday</td>
<td>18.22%</td>
</tr>
<tr>
<td>Thursday</td>
<td>-6.71%</td>
</tr>
<tr>
<td>Friday</td>
<td>2.93%</td>
</tr>
</tbody>
</table>

03-Oct-2005 to 20-Jul-2007

At this point, instead of fine-tuning this strategy we consider it would be appropriate to run the whole pattern recognition model again and back-test for this particular stock in order to have a better base strategy and then proceed to fine-tune it.
6 Final thoughts

If we consider a comparison of our strategy in a period other than the one we used for its conception, and we compare its return with the typical buy and hold strategy we find in a dissected period the following results.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Pre ACRR period</th>
<th>ACRR Period</th>
<th>Post ACRR period</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACRR strategy</td>
<td>8.37%</td>
<td>22.31%</td>
<td>-1.85%</td>
</tr>
<tr>
<td>Buy&amp;Hold strategy</td>
<td>7.84%</td>
<td>5.88%</td>
<td>10.85%</td>
</tr>
</tbody>
</table>

ACRR Period is 20-Apr-2006 to 19-Apr-2007

As we can see when we compare ACRR periods the proposed strategy largely outperforms a typical buy and hold strategy, (22.31% for the ACRR strategy against a mere 5.88%). But when we try to extrapolate the method’s results to other periods we are not that lucky. For the Pre ACRR period which goes from Oct 3th 2005 to April 20 2006 returns are pretty much the same; and for the Post ACRR period from April 19 2007 to July 20 2007 there is a gap of 11% in favor to the Buy and Hold strategy.

This means that our methodology is good for finding a strategy can only be used in a period nearby the one we used for the strategy development. We also note that as long as the tendency does not change, the resulting strategy mostly outperform the buy and hold one.

For this reason the constant analysis and reformulation of a strategy has to be done more frequently, specially if a tendency reversal is acknowledged.

We also note that under the efficient market hypothesis, this method will adapt as long as there is a marked tendency. A further research may show the frequency at which adjustment to the strategy should be made and how far in the future it could be applied. This forecasting plays a key role for a day trader to maximize his returns.
7 Conclusions

Our findings using Signal Coherence analysis did not provide positive coherence in any frequency. It was not possible to do any kind of forecast and no strategy could be developed. We did see some consistency in our results for the coherence on stock prices with Hinich and Serletis results for electricity prices in Alberta. Much can be explained by the fact that prices time series are stochastic processes. On the other hand, our ACRR model provided us with the information about highest and lowest intraday returns necessary to develop a strategy which we back-tested.

During the back-tests we searched for consistency with other well documented phenomena. We could not verify Turn-of-the-Month effect. The first week of every month seem to have no pattern. We suspect that part of this is due to the fact that the expected phenomenon is marginally positive returns, while our strategy profits mostly from negative returns.

Consistency was found with the Halloween Effect "Sell in May and Go Away", by which the month of May signals the start of a bear market, so that investors are better off selling their stocks. As our strategy is heavy weighted in shorting stocks, we expect to have large returns during a bear market rather than in a bullish one. Therefore, we agree with Bouman and Jacobsen’s study and we profit from this anomaly in our strategy. However, it could also be fine-tuned.

We found consistency in Day-of-the-Week effect, Mondays have by far the biggest cumulated profit for our proposed strategy than any other day of the week. This is mainly because our strategy has a big component on short positions, and Mondays is well know for giving the lowest returns. This phenomenon has been verified by a wide number of researchers, from Kelly, Merrill, Cross, French, Gibbons and Hess, Siegel and Tong among others.

Following this reasoning, our strategy could also be further fine-tuned using this information. One thing that we would like to note is the fact that we haven’t taken into consideration transaction costs and commissions. The reason for that is the fact that, as our strategy could be further refined to send limit orders instead of market ones. This means that commissions can be offset and also generate revenues as ECNs pay liquidities when neither hitting the bid nor the asking side of the offer. Lastly we would like to mention that the point of testing our strategy in a wider period is to see if a generalization can be done so that our strategy applies forever in any given period.

Not necessarily our strategy will at least match the buy and hold one, it could perform worse and also give some negative returns. Specially as one would imagine
that if normal returns are positives, shorting the stock most of the time would not be a good idea. Much fine-tuning attempts has been done in order to see if there is still room to get better returns in a per period basis. This include running again the whole process of pattern determination and its back-testing. We leave this open for further research.

The fact that we could choose the same period for the back-testing as the one we used for the pattern finding is left for a further analysis and fine tuning also. This is mostly because a daytrader will try to redo this analysis in a monthly or weekly basis to see if there has been a change in the way the stock is traded, specially when big movement or trend changes happens. So this segmentation of the data used for developing is also part of fine tuning process which is left open.

To conclude, within the scope of this work we have achieved limited success in proving the second hypothesis (ACRR model). However it would be necessary to conduct wider testing over a longer time frame and using the fine tuning mentioned above, to prove the model using wider variables and within other environments.
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