

Cahier No.7512

MORE ON

PRICES versus QUANTITIES *

by

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February 1975

* This paper is essentially an articulated comment on Weitzman's "Prices versus Quantities" [1973] with which the reader should be acquainted. Financial support of the Département de l'Education du Québec is gratefully acknowledged.

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1. INTRODUCTION

The bulk of the literature on the theory of planning¹ has been devoted to the design of iterative exchanges of information between a center and decentralized economic agents (the periphery), and to the study of the properties of these schemes, essentially convergence towards a Pareto optimum (see for example Arrow Hurwicz [1960], Malinvaud [1967], Weitzman [1970] and Malinvaud [1973], Dreze - de la Vallée Poussin [1971] for planning with public goods, Heal [1973] for planning with increasing returns, Aoki [1971] for planning with externalities). However, these elegant formalizations of planning appear only as a very first step (eventhough necessary step) in the study of economic planning. In particular, the number of iterations in any realistic planning procedure is very limited, so that convergence towards a Pareto optimum without any idea of the speed of convergence, does not appear as a very strong property of a procedure.

Weitzman [1973] recently approached the problem of planning in a radically different way. Using a very simple model, he attempted to shed some light on the important question "what are the best instruments for planning: prices or quantities", question that existing models could not really deal with. Weitzman [1973] assumes that benefits and costs are random for the center and that costs are better known by the producers. Then he studies the trade-off between using prices that avoids big mistakes on the production side and using quantities that avoids big mistakes on the consumption side.

It is possible to link Weitzman's work to the existing literature in the following way. Suppose an iterative scheme is used but is stopped after a few iterations when a decision has to be taken. There is some remaining uncertainty for planners concerning benefits and costs of different allocations. Weitzman asks the question: what should planners do then?

Eventhough, Weitzman presented his model in a very symmetrical way, he then lost track of this symmetry, and, in some sense, exploited only half of his model. The purpose of this paper is to reveal a fundamental duality of his model and to make a clear distinction between two types of uncertainties with very different implications for the choice of planning instruments, first "genuine uncertainty" in the technology or in tastes, second subjective uncertainty of planners, i.e. information gaps.

In Section 2, we present our general model which includes Weitzman's model as a special case and we explain the symmetry of the problem. Section 3 neglects genuine uncertainty, and study the choice of instruments for planning with information gaps. The fundamental duality of the results is made explicit. Section 4 extends these results to planning with many agents. Finally, Section 5 introduces genuine uncertainty which becomes relevant when the center and the periphery have different expectations.

2. THE MODEL

Following Weitzman [1973], we first use a very crude model with only one commodity which can be produced in quantity q at cost $C(q)$ by the unique producer and benefit $B(q)$ ² for the unique consumer. The planning problem is to find and to implement the value q^* of the production which maximizes:

$$(2.1) \quad B(q) - C(q)$$

If the Center knows $B(\cdot)$ and $C(\cdot)$, he can solve:

$$(2.2) \quad B'(q^*) = C'(q^*)$$

and if

$$(2.3) \quad p^* = B'(q^*) = C'(q^*)$$

he can choose one of the following three equivalent options:

Option 1: Order the production q^* at the least cost and announce the consumption q^* to the consumer.

Option 2: Announce the price p^* to the producer, have him maximize profits $p^*q - C(q)$ and announce the producer's answer to the consumer.

Option 3: Announce the price p^* to the consumer, have him maximize his net benefits $B(q) - p^*q$ and announce the consumer's answer as production target to the producer.

The three options are equivalent in terms of results but clearly option 1 is the simplest one since it requires the least flows of information. When costs and benefits are random, the choice of an option or choice of planning instruments becomes much more interesting.

Suppose now that costs are uncertain for the planner and of the form $C(q, \theta_1, \theta_2)$ where the random variable θ_1 represents genuine technological uncertainty which exists also for the producer, for example weather uncertainty, and the random variable θ_2 , on the contrary, represents an information gap, i.e., θ_2 is known by the producer but random for the planner.

Similarly, benefits are now uncertain and of the form $B(q, \eta_1, \eta_2)$ where the random variable η_1 represents tastes uncertainty which exists also at the level of the consumer while the random variable η_2 represents an information gap, i.e., η_2 is known by the consumer but random for the planner.

The first point we want to emphasize is that genuine randomness (θ_1, η_1) is irrelevant for the choice of instruments, as long as the planner and the decentralized agents have the same expectations about this randomness. This should be obvious since in this case there is no gain in a decentralized decision.

In the next Section, we reformulate Weitzman's results³ and their dual counterpart.

3. PRICES VERSUS QUANTITIES FOR PLANNING WITH INFORMATIONAL GAPS

We consider in this Section the case where $\theta_1 \equiv \eta_1 \equiv 0$, or the case where planners and periphery have the same expectations about technological uncertainty (the results would be identical). The two polar cases are then Weitzman case ($\eta_2 \equiv 0$) and Weitzman dual case ($\theta_2 \equiv 0$). These polar cases reduce the instrument choice to a choice between two options. Indeed, in

Weitzman case, option 1 and option 3 are identical in terms of results. Moreover, option 1 dominates option 3 in terms of information flows. That is the reason why Weitzman's discussion is reduced to a choice between option 1 and option 2. Let us formulate the choice of instruments in a more precise way.

Option 1

The planner obtains the optimal quantity instrument \hat{q} by maximizing expected gain:

$$(3.1) \quad \max_q E [B(q, \eta_2) - C(q, \theta_2)]$$

where E is the expected operator of the planner.

$$(3.2) \quad E B_1(\hat{q}, \eta_2) = E C_1(\hat{q}, \theta_2)$$

Option 2

When price p is announced to the producer, he maximizes his profit:

$$(3.3) \quad p q - C(q, \theta_2)$$

giving first order condition:

$$(3.4) \quad p = C_1(q, \theta_2)$$

i.e., the reaction function $h(p, \theta_2)$ which satisfies

$$(3.5) \quad p = C_1(h(p, \theta_2), \theta_2)$$

The optimal price announcement to the producer is obtained from maximizing:

$$(3.6) \quad E [B(h(p, \theta_2), \eta_2) - C(h(p, \theta_2), \theta_2)]$$

giving first order condition:

$$(3.7) \quad E [B_1(h(\tilde{p}, \theta_2), \eta_2) h_1(\tilde{p}, \theta_2)] = E [C_1(h(\tilde{p}, \theta_2), \theta_2) h_1(\tilde{p}, \theta_2)]$$

Using (3.5), we obtain:

$$(3.8) \quad \tilde{p} = \frac{E [B_1(h(\tilde{p}, \theta_2), \eta_2) h_1(\tilde{p}, \theta_2)]}{E [h_1(\tilde{p}, \theta_2)]}$$

The producer will answer with

$$(3.9) \quad \tilde{q} \equiv h(\tilde{p}, \theta_2)$$

which is transmitted to the consumer.

Option 3

The price p is now announced to the consumer who maximizes his benefit

$$(3.10) \quad B(q, \eta_2) - p q$$

which yields first order condition :

$$(3.11) \quad B_1(q, \eta_2) = p$$

from which we deduce a reaction function $k(p, \eta_2)$ which satisfies

$$(3.12) \quad B_1(k(p, \eta_2), \eta_2) = p$$

The optimal price announcement to the consumer is obtained from maximizing:

$$(3.13) \quad E [B(k(p, \eta_2), \eta_2) - C(k(p, \eta_2), \theta_2)]$$

i.e.:

$$(3.14) \quad E [B_1(k(\overset{\circ}{p}, \eta_2), \eta_2) k_1(\overset{\circ}{p}, \eta_2)] = E [C_1(k(\overset{\circ}{p}, \eta_2), \theta_2) k_1(\overset{\circ}{p}, \eta_2)]$$

Using (3.12), we obtain:

$$(3.15) \quad \overset{\circ}{p} = \frac{E [C_1(k(\tilde{p}, \eta_2), \theta_2) k_1(\overset{\circ}{p}, \eta_2)]}{E [k_1(\overset{\circ}{p}, \eta_2)]}$$

The consumer will answer with:

$$(3.16) \quad \overset{\circ}{q} = k(\overset{\circ}{p}, \eta_2)$$

which is transmitted to the producer.

Let $\Delta(i/j)$ the comparative advantage of option i over option j , defined as the difference between the expected gains of the two options

$$(3.17) \quad \text{Example: } \Delta(2/1) = E [B(\tilde{q}(\theta_2), \eta_2) - C(\tilde{q}(\theta_2), \theta_2)] \\ - E [B(\hat{q}, \eta_2) - C(\hat{q}, \theta_2)]$$

To obtain analytical results we approximate costs and benefits with quadratic approximations around \hat{q} , and we assume that uncertainty affects only marginal cost and benefit.

$$(3.18) \quad C(q, \theta_2) = C(\hat{q}, \theta_2) + [C' + \alpha(\theta_2)] (q - \hat{q}) + \frac{C''}{2} (q - \hat{q})^2$$

$$(3.19) \quad B(q, \eta_2) = B(\hat{q}, \eta_2) + [B' + \beta(\eta_2)] (q - \hat{q}) + \frac{B''}{2} (q - \hat{q})^2$$

$$(3.20) \quad \text{with } E(\alpha(\theta_2)) = E(\beta(\eta_2)) = 0, \quad \theta_2 \text{ and } \eta_2 \text{ independent.}$$

$$\text{with } C' = E[C_1(\hat{q}, \theta_2)], \quad B' = E[B_1(\hat{q}, \eta_2)]$$

Moreover C'' and B'' are not random.

Let us denote:

$$(3.21) \quad \sigma^2 = E [\alpha(\theta_2)^2]$$

$$(3.22) \quad s^2 = E [\beta(\eta_2)^2]$$

Simple manipulations⁴ give then:

$$(3.23) \quad \Delta(2/1) = \frac{\sigma^2}{2C''^2} (B'' + C'')$$

$$(3.24) \quad \Delta(3/1) = - \frac{s^2}{2B''^2} (B'' + C'')$$

and therefore:

$$(3.25) \quad \Delta(2/3) = \left(\frac{\sigma^2}{2C''^2} + \frac{s^2}{2B''^2} \right) (B'' + C'')$$

The crucial expression is $B'' + C''$ i.e. the difference of curvature between benefits and costs.

Weitzman [1973] who implicitly assumed $s^2 = 0$ was reduced to a comparison between (1) and (2). But observe that everything he says in favor of quantities ($(B'' + C'') < 0$) is more generally in favor of using prices on the consumption side (see Section 4, Weitzman [1973]). The choice is between using prices on the production sector and using prices in the consumption sector. Option 1 is always dominated by option 2 or option 3 and this is natural since it uses less information. On the other hand, option 1 is always better than one of the two others. If costs associated to the necessary flows of information in the different options were formalized, option 1 might become the best one.

Another way to express the robustness of option 1 is to assume that curvatures are random. Consider the simplified following case:

$$(3.26) \quad C(q, \theta_2) = C(\hat{q}, \theta_2) + [C' + \alpha(\theta_2)] (q - \hat{q}) + \frac{C''}{2a(\theta_2)} (q - \hat{q})^2$$

$$(3.27) \quad B(q, \eta_2) = B(\hat{q}, \eta_2) + [B' + \beta(\eta_2)] (q - \hat{q}) + \frac{B''}{2b(\eta_2)} (q - \hat{q})^2$$

$$\text{with } E(a(\theta_2)) = E(1/a(\theta_2)) = 1$$

$$E(b(\eta_2)) = E(1/b(\eta_2)) = 1$$

$$\alpha(\theta_2) \text{ independent from } a(\theta_2)$$

$$\beta(\eta_2) \text{ independent from } b(\eta_2)$$

$$\delta^2 = E[a(\theta_2) - E a(\theta_2)]^2$$

$$\lambda^2 = E[b(\eta_2) - E b(\eta_2)]^2$$

We then easily obtain:

$$\Delta(2/1) = \frac{\sigma^2}{2C''^2} [B'' (1 + \delta^2) + C'']$$

$$\Delta(3/1) = -\frac{s^2}{2B''^2} [B'' + C'' (1 + \lambda^2)]$$

Since $B'' < 0$ and $C'' > 0$, it is then clear that if uncertainty in the curvatures is large enough $[\delta^2 > -\frac{C'' + B''}{B''}]$ and $\lambda^2 > -\frac{B'' + C''}{C''}]$ we may have simultaneously:

$$\Delta(2/1) < 0 \quad \text{and} \quad \Delta(3/1) < 0$$

i.e., the direct quantity instrument becomes the best instrument. Note also that in this case, the magnitude of σ^2 and s^2 matters in the comparison of options 2 and 3.

4. PLANNING WITH MANY AGENTS

Consider first two simple cases. Suppose that there is still one consumer but n_1 producers. With different simplifying assumptions⁵, we obtain:

$$(4.1) \quad \Delta(2/1) = \frac{1}{n_1} \frac{B'' \sigma^2(n_1)}{2C''^2} + \frac{\sigma^2(n_1)}{2C''}$$

where $\sigma^2(n_1)$ increases eventually with n_1 . For non extreme values of C'' it is clear that the first term of the right hand side of (4.1) becomes negligible as n_1 grows and option 2 dominates option 1. The use of price to decentralize production assures efficiency of production for this price. The overall level of production may be inappropriate but the first effect dominates as n_1 becomes large.

Symmetrically⁶, if there is one producer and n_2 consumers, we obtain:

$$(4.2) \quad \Delta(3/1) = -\frac{1}{n_2} \frac{C'' s^2(n_2)}{2B''^2} - \frac{s^2(n_2)}{2B''}$$

and similarly when n_2 is large, option 3 dominates option 1. Consumption is then efficient but may be not at the right level.

When we have simultaneously n_1 producers and n_2 consumers, the planning problem becomes much more complex. Suppose that we use prices to decentralize production. Then we obtain an output $q = \sum_{i=1}^{n_1} q_i$ which must be allocated among consumers. This can be done in many different ways. Similarly, if we decentralize demand with prices, we obtain a desired level of production which then must be allocated among producers.

Since the price instrument becomes very efficient when the number of agents increase, a good method is certainly to use prices to decentralize both production and demand and then use a rationing process if they do not match exactly. With large numbers and independent risks, the rationing will become negligible by the law of large numbers. However, macro risks exist for which the last remark does not apply. This is a promising area for further research.

5. PLANNING UNDER UNCERTAINTY

The consideration of genuine uncertainty becomes relevant when the center and decentralized agents have different expectations about this uncertainty. For example, suppose that the center knows the objective probability distribution of the weather or has the best subjective probability distribution (in terms of incorporating all the available information), and decentralized farmers have their own subjective distributions.

Consider the simple case where there is no uncertainty in benefits and only one producer. Let $\mu_1(\theta_1)$ be the center's probability distribution over the technological uncertainty, while $m(\theta_1)$ is the producer's subjective distribution over this uncertainty. Let $\mu_2(\theta_2)$ be the center's probability distribution over the uncertainty which formalizes the information gap between the center and the producer.

Option 1

The center computes the optimal quantity with the following program:

$$\max_q B(q) - \int C(q, \theta_1, \theta_2) \mu_1(\theta_1) \mu_2(\theta_2) d\theta_1 d\theta_2$$

which gives the first order condition

$$B_1(q) = \int C_1(q, \theta_1, \theta_2) \mu_1(\theta_1) \mu_2(\theta_2) d\theta_1 d\theta_2$$

Option 2

When price p is announced to the producer, he maximizes his profit:

$$pq - \int C(q, \theta_1, \theta_2) m(\theta_1) d\theta_1$$

$$\text{or } p = \int C_1(q, \theta_1, \theta_2) m(\theta_1) d\theta_1$$

which corresponds to the reaction function $h(p, \theta_2)$ satisfying:

$$(5.1) \quad p = \int C_1(h(p, \theta_2), \theta_1, \theta_2) m(\theta_1) d\theta_1$$

The optimal price announcement to the producer is obtained from:

$$\text{Max}_p \int B(h(p, \theta_2)) \mu_2(\theta_2) d\theta_2 - \int C(h(p, \theta_2), \theta_1, \theta_2) \mu_1(\theta_1) \mu_2(\theta_2) d\theta_1 d\theta_2$$

or

$$\begin{aligned} & \int B_1(h(\tilde{p}, \theta_2)) h_1(\tilde{p}, \theta_2) \mu_2(\theta_2) d\theta_2 \\ & = \int C_1(h(\tilde{p}, \theta_2), \theta_1, \theta_2) h_1(\tilde{p}, \theta_2) \mu_1(\theta_1) \mu_2(\theta_2) d\theta_1 d\theta_2 \end{aligned}$$

But

$$\begin{aligned} & \int C_1(h(\tilde{p}, \theta_2), \theta_1, \theta_2) h_1(\tilde{p}, \theta_2) \mu_1(\theta_1) \mu_2(\theta_2) d\theta_1 d\theta_2 \\ & = \int [\int C_1(h(\tilde{p}, \theta_2), \theta_1, \theta_2) m_1(\theta_1) d\theta_1] h_1(\tilde{p}, \theta_2) \mu_2(\theta_2) d\theta_2 \\ & + \int [\int C_1(h(\tilde{p}, \theta_2), \theta_1, \theta_2) (\mu_1(\theta_1) - m_1(\theta_1)) d\theta_1] h_1(\tilde{p}, \theta_2) \end{aligned}$$

$$\mu_2(\theta_2) d\theta_1 d\theta_2$$

$$\begin{aligned}
\text{Let } D_1(\theta_2) &= \int C_1(h(\tilde{p}, \theta_2), \theta_1, \theta_2) (\mu_1(\theta_1) - m_1(\theta_1)) d\theta_1 \\
&\int C_1(h(\tilde{p}, \theta_2), \theta_1, \theta_2) h_1(\tilde{p}, \theta_2) \mu_1(\theta_1) \mu_2(\theta_2) d\theta_1 d\theta_2 \\
&= \tilde{p} \int h_1(\tilde{p}, \theta_2) \mu_2(\theta_2) d\theta_2 + \int D_1(\theta_2) h_1(\tilde{p}, \theta_2) \mu_2(\theta_2) d\theta_2
\end{aligned}$$

We finally obtain:

$$(5.2) \quad \tilde{p} = \frac{\int [B_1(h(\tilde{p}, \theta_2)) - D_1(\theta_2)] h_1(\tilde{p}, \theta_2) \mu_2(\theta_2) d\theta_2}{\int h_1(\tilde{p}, \theta_2) \mu_2(\theta_2) d\theta_2}$$

As above, we use quadratic approximations:

$$C(q, \theta_1, \theta_2) = C(\hat{q}, \theta_1, \theta_2) + (C' + \alpha_1(\theta_1) \alpha_2(\theta_2)) (q - \hat{q}) + \frac{C''}{2} (q - \hat{q})^2$$

$$B(q) = B(\hat{q}) + B'(q - \hat{q}) + \frac{B''}{2} (q - \hat{q})^2$$

where $\int \alpha_2(\theta_2) \mu_2(\theta_2) d\theta_2 = 0$, $\int \alpha_1(\theta_1) \mu_1(\theta_1) d\theta_1 = 1$.

Then (5.1) becomes:

$$\begin{aligned}
p &= C' + \alpha_2(\theta_2) \int \alpha_1(\theta_1) m(\theta_1) d\theta_1 + C'' (q - \hat{q}) \\
(5.3) \text{ or } h(p, \theta_2) &= \hat{q} + \frac{p - C' - \alpha_2(\theta_2) \int \alpha_1(\theta_1) m(\theta_1) d\theta_1}{C''}
\end{aligned}$$

$$\text{and } h_1(p, \theta_2) = \frac{1}{C''}$$

Substituting in (5.2) yields:

$$\tilde{p} = \int [B_1(h(\tilde{p}, \theta_2)) - D_1(\theta_2)] \mu_2(\theta_2) d\theta_2$$

$$B_1(h(\tilde{p}, \theta_2)) = B' + B''(h(\tilde{p}, \theta_2) - \hat{q})$$

Using (5.3)

$$= B' + B'' \frac{p - C' - \alpha_2(\theta_2) \int \alpha_1(\theta_1) m(\theta_1) d\theta_1}{C''}$$

$$\int B_1(h(\tilde{p}, \theta_2)) \mu_2(\theta_2) d\theta_2 = C'$$

$$\text{because } \int \alpha_2(\theta_2) \mu_2(\theta_2) d\theta_2 = 0$$

Therefore

$$q(\theta_2) = \hat{q} - \frac{\int D_1(\theta_2) \mu_2(\theta_2) d\theta_2 + \alpha_2(\theta_2) \int \alpha_1(\theta_1) m(\theta_1) d\theta_1}{C''}$$

Let $D_1 = \int D_1(\theta_2) \mu_2(\theta_2) d\theta_2$ be the expected marginal cost evaluation bias

Let $D_2 = \int \alpha_1(\theta_1) m(\theta_1) d\theta_1$. $D_2 - 1$ is the uncertainty bias

$$\tilde{q}(\theta_2) = \hat{q} - \left[\frac{D_1 + \alpha_2(\theta_2) D_2}{C''} \right]$$

Hence, if $\int \alpha_2^2(\theta_2) \mu_2(\theta_2) d\theta_2 = \sigma^2$

$$\Delta(2/1) = \frac{D_1^2 + D_2^2 \sigma^2}{2C''^2} [B'' + C'' \zeta]$$

$$\text{with } \zeta = 2 \frac{\sigma^2 D_2}{D_1^2 + \sigma^2 D_2^2} - 1$$

The information bias is defined as:

$$\delta = \zeta - 1 = 2 \left[\frac{\sigma^2 D_2}{D_1^2 + \sigma^2 D_2^2} - 1 \right]$$

Suppose the center thinks that the producer does not make on average any marginal cost bias ($D_1 = 0$). Then, $\delta > 0$ if the producer underestimates uncertainty ($D_2 < 1$). Then, the weight of C'' in the sign of $\Delta(2/1)$ is enhanced, i.e., the use of prices instead of quantities is favoured.

On the contrary, if the producer exaggerates uncertainty ($D_2 > 1$, $\delta < 0$), the use of quantities is favoured. More interestingly, if the expected valuation of uncertainty by the producer is correct ($D_2 = 1$) but the center thinks that the producer does not evaluate correctly the expected marginal cost ($D_1 \neq 0$) whatever the direction of this mistake, δ is negative, which means that the use of quantities is favoured.

In principle, to compute $\Delta(2/1)$, the center should know $m(\theta_1)$. Therefore $m(\theta_1)$ is better interpreted as the probability distribution attributed by the center to the producer, rather than the true one.

CONCLUSION

Eventhough, it is true that in any given situation, a large number of agents is not enough to guarantee the superiority of prices, I think it is fair to say that in the framework of our model prices become superior to quantities when the number of agents is large. However, we certainly should not stop here. One crucial feature of planning under uncertainty, the incentive question, has been ignored so far.

In a subsequent paper, we hope to focus our attention on the following question: what can be said about the choice quantities versus prices as far as incentives are concerned?

FOOTNOTES

- ¹. See Heal [1973] for a comprehensive introduction.
- ². $B''(q) < 0$, $C''(q) > 0$, $B'(0) > C'(0)$, $B'(q) < C'(q)$ for q large enough.
- ³. In our general formulation, Weitzman dealt with the case $\theta_1 \equiv 0$, $\eta_2 \equiv 0$.
As we said above the consideration of η_1 was useless. Therefore, we will refer to Weitzman case as $\theta_1 \equiv 0$, $\eta_1 \equiv 0$, $\eta_2 \equiv 0$. We will refer to Weitzman dual case for $\theta_1 \equiv 0$, $\theta_2 \equiv 0$, $\eta_1 \equiv 0$.
- ⁴. For a detail of these manipulations, see Weitzman [1973] or Section 5.
- ⁵. See Weitzman [1973]. The assumptions are essentially identical independent cost functions. Any degree of dependence would lessen the "number effect".
- ⁶. The assumptions should be here identical independent benefit functions.

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