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Essays in Resource Economics

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RÉSUMÉ

Cette thèse comporte trois essais en économie des ressources naturelles.

Le Chapitre 2 analyse les effets du stockage d'une ressource naturelle sur le bien-être et sur le stock de celle-ci, dans le contexte de la rizipisciculture. La rizipisciculture consiste à élever des poissons dans une rizière en même temps que la culture du riz. Je développe un modèle d'équilibre général, qui contient trois composantes principales : une ressource renouvelable à accès libre, deux secteurs de production et le stockage du bien produit à partir de la ressource. Les consommateurs stockent la ressource lorsqu'ils spéculent que le prix de cette ressource sera plus élevé dans le futur. Le stockage a un effet ambigu sur le bien-être, négatif sur le stock de ressource au moment où le stockage a lieu et positive sur le stock de ressource dans le futur.

Le Chapitre 3 étudie les effects de la migration de travailleurs qualifiés dans un modèle de commerce international lorsqu'il y a présence de pollution. Je développe un modèle de commerce à deux secteurs dans lequel j'introduis les questions de pollution et de migration dans l'objectif de montrer que le commerce interrégional peut affecter le niveau de pollution dans un pays composé de régions qui ont des structures industrielles différentes. La mobilité des travailleurs amplifie les effets du commerce sur le capital environnemental. Le capital environnemental de la région qui a la technologie la moins (plus) polluante est positivement (négativement) affecté par le commerce. De plus, je montre que le commerce interrégional est toujours bénéfique pour la région avec la technologie la moins polluante, ce qui n'est pas toujours le cas pour la région qui a la technologie la plus polluante.

Finalement, le Chapitre 4 est coécrit avec Yves Richelle. Dans ce chapitre, nous étudions l'allocation efficace de l'eau d'un lac entre différents utilisateurs. Nous considérons dans le modèle deux types d'irréversibilités : l'irréversibilité d'un investissement qui crée un dommage à l'écosystème et l'irréversibilité dans l'allocation des droits d'usage de l'eau qui provient de la loi sur l'eau (irréversibilité légale). Nous déterminons d'abord la valeur de l'eau pour chacun des utilisateurs. Par la suite, nous caractérisons l'allocation optimale de l'eau entre les utilisateurs. Nous montrons que l'irréversibilité légale entraîne qu'il est parfois optimal de réduire la quantité d'eau allouée à la firme, même s'il n'y a pas de rivalité d'usage. De plus, nous montrons qu'il n'est pas toujours optimal de prévenir le dommage créé par un investissement. Dans l'ensemble, nous prouvons que les irréversibilités entraînent que l'égalité de la valeur entre les utilisateurs ne tient plus à l'allocation optimale. Nous montrons que lorsqu'il n'y a pas de rivalité d'usage, l'eau non utilisée ne doit pas être considérée comme une ressource sans limite qui doit être utilisée de n'importe quelle façon.

Mots clés: Ressource renouvelable à accès libre, stockage, rizipisciculture, modèle dynamique à deux secteurs, migration, externalité provenant de la pollution, allocation de l'eau, irréversibilité, valeur de l'eau.

ABSTRACT

This thesis consists of three essays in resource economics.

Chapter 2 analyzes the effects of resource storage on welfare and on the resource stock, in the context of rice-fish culture. I develop a simple general equilibrium model, that has three central components: one open access renewable resource with logistic natural growth, two production sectors and storage of the good produced with the resource. Consumers store the resource when they speculate that the price of the resource will be higher in the future. Storage has an ambiguous effect on welfare, has a negative impact on resource stock at the period the storage takes place and has a positive impact for all following periods.

Chapter 3 examines the effects of migration of skilled workers in a model of interregional trade in the presence of pollution. I develop a two-sector model of trade that incorporates both pollution and migration issues to show that interregional trade can affect the pollution level of a country composed of regions with different industrial structures. The mobility of workers amplifies the effects of interregional trade on the environmental capital. The region with the less (more) polluting technology is affected positively (negatively) by trade. Migration doesn't affect the trade pattern. The region with the less polluting manufacturing industry always gains from trade. If the preferences over manufactures is relatively low, the region with the more pollutant technology can experience a loss from trade in the long run.

Finally, Chapter 4 is co-authored with Yves Richelle. In this chapter, we consider the problem of efficiently allocating water of a lake among different potential users. We consider two types of irreversibility: the irreversibility of an investment that creates a fixed damage to the ecosystem and the irreversibility of the right to use the resource that comes from the legislation (legislative irreversibility). First of all, we determine the value of water for users. Then, we characterize the optimal allocation of water among users. With legislative irreversibility, we show that it is sometimes optimal to reduce the amount of water allocated to the firm, even though there is no rivalry in use. Moreover, we show that it is not always optimal to prevent the damage created by the irreversible investment. We define the context, in which it is optimal to intervene to prevent the damage. Furthermore, with irreversibility, we prove that the marginal value of water at the efficient allocation for users is not equalized. Overall, we show that in the case of no rivalry in use, unused water should not be seen as a limitless resource to be used in any way whatever.

Keywords: Open access renewable resource, storage, rice-fish culture, two-sector dynamic model, migration, pollution externality, water allocation, irreversibility, value of water.

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Quand le dernier arbre aura été abattu,
Quand la dernière rivière aura été empoisonnée,
Quand le dernier poisson aura été péché,
Alors on saura que l'argent ne se mange pas.
- Geronimo

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CHAPITRE 1

INTRODUCTION

Notre dépendance aux ressources naturelles est considérable et les conséquences de nos activités sur l'environnement sont majeures. L'importance des répercussions sociales, économiques et environnementales de l'exploitation des ressources naturelles au Canada et ailleurs dans le monde expose nos propres responsabilités face à l'environnement.

Il est donc primordial de comprendre comment l'utilisation et l'évolution de cet important moteur de développement économique que sont les ressources naturelles, sont affectées par nos choix.

Prenons l'exemple du Canada dont le vaste territoire regorge de ressources naturelles. Ces richesses naturelles passent pas le zinc, l'or et le plomb du Yukon, par les mines de diamants du Territoire du Nord-ouest, par le hareng de la baie Fundy, par le pétrole de l'Alberta et par l'eau du Québec, pour ne nommer que celles-là. Considérant l'immensité et la diversité de ces ressources, il me semble indispensable de mieux comprendre les conséquences de leur exploitation.

Le premier volet de ma thèse tente de répondre en partie à ce questionnement. Plus précisément, ce volet, composé de deux articles, traite des ressources naturelles renouvelables.

Dans le chapitre intitulé *Effect of storage on the evolution of an open access renewable ressource: the case of rice-fish culture*, j'étudie l'effet du stockage sur l'évolution d'une ressource naturelle renouvelable en propriété commune. Mon modèle est appliqué au cas particulier de la rizipisciculture. La rizipisciculture consiste à élever des poissons dans une rizière en même temps que la culture du riz.[1] J'évalue si la rizipisciculture est bénéfique pour le bien-être des cultivateurs et pour le stock de ressource. Pour répondre à mes questionnements, je développe un modèle simple d'équilibre général. Le modèle a trois composantes centrales : une ressource naturelle renouvelable en propriété commune avec une croissance naturelle logistique, deux secteurs de production et la possi-

[1]Pour plus de détails, voir la Section 2.2.

bilité de stocker le bien qui est produit avec la ressource. Les consommateurs stockent la ressource lorsqu'ils spéculent que le prix de la ressource sera plus grand dans le futur. Le stockage a un effet ambigu sur le bien-être, un effet négatif ponctuel sur le stock de ressource et un effet positif sur le stock de ressource dans le long terme.

Dans le chapitre suivant, intitulé *Migration, trade and the environment*, je m'interroge sur l'interaction des marchés internationaux et de l'environnement. Depuis une vingtaine d'années, les canadiens ont vu leurs émissions de gaz à effet de serre (GHG) augmenter drastiquement. Cette augmentation est principalement causée par une forte croissance dans la production de pétrole et de gaz en Alberta. Pendant ce temps, d'autres provinces comme le Québec ont maintenu fixe ou diminué leurs émissions. Sachant que la production du pétrole albertain est majoritairement exportée, je me questionne sur l'effet qu'a le commerce international sur l'environnement au Canada. Pour étudier cette problématique, je développe un modèle de commerce à deux secteurs qui incorpore la problématique de la pollution et de la migration. Je considère un pays relativement petit par rapport au reste du monde, qui est composé de deux régions. La production dans un des secteurs crée une externalité négative sur le deuxième secteur par son effet sur le capital environnemental. Il y a deux types de travailleurs, qualifiés et non-qualifiés. Les travailleurs qualifiés sont les plus susceptibles de migrer, je fais donc l'hypothèse qu'ils sont mobiles entre les régions. J'introduis le commerce international dans ce modèle et j'évalue son impact sur la migration et le capital environnemental.

Dans un autre ordre d'idée, mais toujours ayant pour objectif d'analyser le rôle déterminant qu'ont nos choix sur les ressources naturelles, le second volet de la thèse porte sur la valorisation et l'allocation de l'eau. Ce volet est composé d'un seul chapitre co-écrit avec Yves Richelle, dont le titre est : *On the value and optimal allocation of water*.

De l'industrie de l'hydroélectricité à l'industrie de la fourrure, en passant par l'industrie forestière, l'eau a été un important levier pour le développement économique du Québec. Au fil du temps, ces multiples usages ont menés à certains conflits d'utilisation. Son statut particulier de *res communis* implique que par sa nature, l'eau appartient à tous les citoyens québécois et de ce fait est accessible et utilisable par tous. Puisqu'aucun prix ne peut être assigné à cette ressource, l'allocation des droits d'utilisation ne peut s'y baser. L'objectif de ce chapitre est d'étudier l'allocation optimale de l'eau d'un lac entre différents utilisateurs, notamment une firme, une municipalité et un écosystème. Ainsi, nous utilisons la valeur que chaque agent accorde à l'eau pour déterminer l'allocation optimale. Pour s'approcher le plus du contexte québécois, nous introduisons une irréversibilité législative qui consiste à allouer à une firme au moins autant d'eau dans le futur qu'elle en a aujourd'hui. Nous étudions également la possibilité que les activités de la firme endommagent de façon irréversible l'écosystème. Nous montrons que l'irréversibilité législative implique qu'il est parfois optimal de réduire la quantité d'eau allouée à la firme, même s'il n'y a pas de rivalité d'usage. De plus, nous montrons qu'il n'est pas toujours optimal de prévenir le dommage créé par les activités de la firme. De manière générale, ce chapitre montre que lorsqu'il n'y a pas de rivalité dans l'usage, l'eau non utilisée ne doit pas être considérée comme une ressource sans limite qui peut être utilisée inconsciemment.

Globalement, l'objet de cette thèse est de fournir des outils d'analyse qui visent à mieux comprendre les enjeux reliés à l'exploitation et l'allocation des ressources naturelles.

CHAPITRE 2

EFFECT OF STORAGE ON THE EVOLUTION OF AN OPEN ACCESS RENEWABLE RESOURCE: THE CASE OF RICE-FISH CULTURE

2.1 Introduction

Modern societies become more and more concerned about natural environment issues, in particular about resource sustainability. The question of sustainability is important for open access renewable resource, because property rights are not well defined and this makes them more vulnerable to over exploitation. It is then important to better understand the factors that affect the resources.

Open resources are "free" goods for firms that use them as production factor. Consequently, those firms produce more than if they would have paid to use the resources. The over exploitation by producers has a negative impact on resource stock and on production possibilities in the future. In a general equilibrium framework, savings might affect the production of the renewable resource through their effect on consumers' behavior. If an open access renewable resource is one of the production factors, then savings might affect its exploitation. Consumers can save either by using the traditional banking system or they can also save by storing the resource.

In this article, I am concerned about the latter way of saving. Although a basic comparison of the two types of savings is provided. Storing open resources might accelerate their exploitation in the short run as it is related by Milazzo (1998). In his article, he raises the issue of the "race to fish" that happened in Alaska in the early 1990's. Fishermen caught a very large amount of fish in a short period and froze the majority of their catch. This had an important negative effect on the current sable fish stock. Nevertheless, I think that storage might have a positive impact on the future stock of an open access renewable resource. Storage implies that the current harvest of the resource is larger than the current consumption and that at some point in the future, the harvest will be lower than consumption. The use of the storage might lead to a period of time, during which the resource would be less harvested, hence this might have a positive impact on the stock of the resource. Which of those two effects dominates determines the effective impact of storage on the resource stock. In my paper, I argue that storage might have a positive impact on the resource stock and on welfare. The principal objective of my paper is to construct a simple model of general equilibrium that shows the possible positive impact of storage on an open access renewable resource and on welfare. To illustrate my model, I consider the example of rice-fish farming.

The rice-fish farming is the combination of rice and fish cultures in the same rice field. All around the world, there are many farmers who have decided to store fish in their rice fields. Some argue that this type of culture has a positive impact on farmers' welfare and revenue. Since those benefits have not been established scientifically and the effect on stock of fish have not been studied, I think that this might be important to shed light on those effects. Many policies that give incentives to farmers to stock fish in their rice fields, have been experimented in the past and others will be experimented in the future. Having new insights into the impacts of rice-fish culture on fish stock, revenue and welfare seems to be important. Section 2.2 provides a general portrait of this particular culture.

I use a general equilibrium model which allows me to study the impact on welfare and revenue once prices adjust. My model has three central components : one open access renewable resource with logistic natural growth, two production sectors and storage of the good produced with the resource. Two goods can be produced and consumed, the harvest of the renewable resource and a consumption good. The production of the harvest good uses as production factors, labour and the resource, resource that the producers do not have to pay for because it is open access. The production of the consumption good uses only labour. The consumption good can be seen as an aggregate of all other goods. Consumers are allowed to store the harvest good. The open access resource can represent fish or aquatic animals that are stored in rice field, after the rice is harvested, farmers can either consume or sell fish stored in their field.

For tractability, I suppose that consumers can store harvest goods (fish) only in the first period. This might seem restrictive, however, even with this simple model I get good insights into the effects of storage on resource stock. My model can be seen as a proxy of the situation if consumers could store for ever. In my model, I do not consider storage cost. Even if there exists such cost in rice-fish farming, it is relatively small. The principal modification that farmers have to do in order to store fish in their fields is to provide deeper areas for fish. Moreover, those entry cost might be compensated by the reduction in costs due to the reduction in chemical pesticides and herbicides that is induced by the storage of fish.

The model leads to the following results. There exists a unique equilibrium in which consumers store the harvest good if the initial resource stock is larger than the steady state and do not store the harvest good otherwise. Consumers store the harvest good expecting an increase in the future price of the resource. Storage leads to an increase in the harvest production in the first period and hence to a negative impact on stock, however, it also implies a positive impact on the resource stock, through the diminution of harvest in the following period. Storage has a negative impact on stock only in the period after consumers store the harvest good and has a positive impact for the following periods. This creates downward pressures on the relative price of the harvest good in term of the consumption good. Storage is allowed only in one period, hence the steady state of the economy is not affected, however the economy reaches this state less rapidly compared to the situation without storage.

The effect on welfare is ambiguous, it is possible that the effects of storage on prices and income are such that welfare in the first two periods is lower with storage than without. However, welfare in the following periods increases due to the increase in the resource stock. This result implies that there might be a situation such that rice-fish culture is not welfare improving, hence policy makers might be careful when they promote this type of culture, if their objective is to maximize total welfare.

The characterization of the equilibria of this model allows to understand the way storage can affect the equilibrium behavior and the evolution of the open access renewable resource. This model modifies the model developed in Brander et Taylor (1998) by allowing storage. It provides an evaluation of the impact of storage on the evolution of the open access renewable resource.

In Section 2.2, I describe the rice-fish culture. Section 2.3 describes the economy. In Sections 2.3.2 and 2.3.3, I formalize the supply and the demand functions. The equilibrium is characterized and examined in Section 2.4. The effect of storage on the economy, in particular on welfare and resource stock is derived in Section 2.5. The conclusion and future extensions of the model are discussed in Section 2.6.

2.1.1 Literature Review

Even though this work concerns renewable resource, I have to mention one of the pioneering articles in this very large and growing literature in resource economics, which is Hotelling (1931). He develops the very well known Hotelling's rule, who points out the time path of non-renewable resource extraction that maximizes the value of the resource stock. He restricts his work to irreplaceable resource, although he notes that renewable resource extraction path might be important as well.

One important aspect of resources is the definition of their property rights. There is a wide range of property rights, going from open-access to private resources. Ostrom (1990) evaluates in which situations, governance of common property resources is a success or a failure. Tian (2000) suggests that if the costs of implementing property rights are larger than the benefits, property rights might be implemented. Recently, Copeland et Taylor (2009) endogenize the enforcement of property rights. By contrast, my model focuses on resource without any property rights.

There exist accumulated studies that deal with the issues of open access renewable resource. Gordon (1954) is a famous article in this literature. In its article, he studies the economic theory of common property natural resource utilization. He concentrates his attention on fishery. Since resource harvesters impose a negative externality on others, imperfect property rights might lead to excessive pressure on renewable resource stocks. Gordon (1954) points out that overexploitation is an important issue when these externalities are not internalized. The social problem of fisheries management is the main aim of Schaefer (1957). Brander et Taylor (1998) develop a general equilibrium model of renewable resource and population dynamics to shed light on the case of Easter Island. Interaction between openness and renewable resource management is an other important issue to discuss. Property rights can have important implications on the effect of trade on the environment and the welfare. Although this seems to be an important issue, the scope of the literature on trade in renewable resources appears to be relatively small. Chichilnisky (1994) and Brander et Taylor (1997) are two of the important articles of this small but still growing literature. They contradict the well-known result that trade is beneficial for all countries by showing that with open access renewable resources, resource exporting countries may lose from freer trade. Brander et Taylor (1997) show that trade might reduce income and natural capital. In a series of papers, Copeland et Taylor (1994, 1995, 1997, 1999) give insight into the effects of trade when an economy has an open access resource by studying a wide variety of situations. Copeland et Taylor (1997) investigate the trade-induced degradation hypothesis, Copeland et Taylor (1995) present a model with transboundary pollution and Copeland et Taylor (1999) deal with spatial separation.

Closer to my research, the literature on storage of an open access renewable resource is relatively modest, despite the importance of the issue. Many open access resources can be stored and not considering this possibility may lead to wrong predictions. Gaudet et Moreaux (2002) develop a model of extraction of a common property resource in a partial equilibrium framework. However, they focus their attention on non-renewable resources.

Kremer et Morcom (2000) investigate how government can rule out extinction of an open access renewable resource that is used to produce a storable good. The general equilibrium model presented in their paper suggests two policy recommendations. A government can either commit to tough enforcement if the resource stock falls below a targeted threshold, or use stockpiling of storable wildlife goods to reduce the risk of extinction. The storage decision is made by poachers in a competitive framework or by government. While this assumption seems to fit the case of the African elephant studied in their paper, it is not the correct assumption in the context of rice-fish culture. Bulte et al. (2003) question Kremer et Morcom (2000)s' results and argue that international organizations should hold stockpiles rather than the government.

Béné et Doyen (2000) analyze storage regulation in a fishery's production process in

a context of resource exportation and seasonal dephased oscillations. The aim of their model is the maintenance of the fishery's economical viability. Their study explores how policy makers can regulate storage to ensure the fishery's viability.
The literature on fish farming in a rice field is relatively rich. Halwart et Gupta (2004) provide a very complete review of rice-fish culture. Numbers of papers study the impact of this particular type of culture on public health, nutrition and society, however the economics of rice-fish culture has been neglected. Few papers study the economic benefits of raising fish in rice fields and the ones that do it are based on the evaluation of net return. Gupta et al. (1998), Lin et al. (1995) and Yan et al. (1995) find a positive net return, however the percentage of variation varies widely across rice fields, countries and years. Thongpan et al. (1992) study the case of Thailand culture and find a lower net return. Some authors suggest that fish-rice culture has economic benefits based on the observation that most of producers continue to combine those two cultures in the same rice field. However, this beneficial relation has not been demonstrated unequivocally. There are some countries where this practice has been abandoned, because the expected success has never materialized.
To the best of my knowledge, my paper is the first in the economic literature to develop a theoretical model to explore the effect of storage of an open access renewable resource on the evolution of resource stock and on welfare in a general equilibrium framework.

2.2 Rice-Fish Culture

"*There is rice in the field, fish in the water.*" This quotation, written during the Sukhothai period (700 years ago), reflects the combination of rice and fish cultures. The rice-fish culture is the growing of rice and fish together in the same field.[1] Fish in rice field is mostly implemented in Asia, but we can also find this type of culture in many African countries and in several South American countries. Even in North America and in Europe, there is rice-fish farming. Dots in Figure 2.1 represent locations where some farmers practice or have already practiced this particular culture.

[1] Note that fish means any aquatic animals (shrimp, crab, turtle, frog,...).

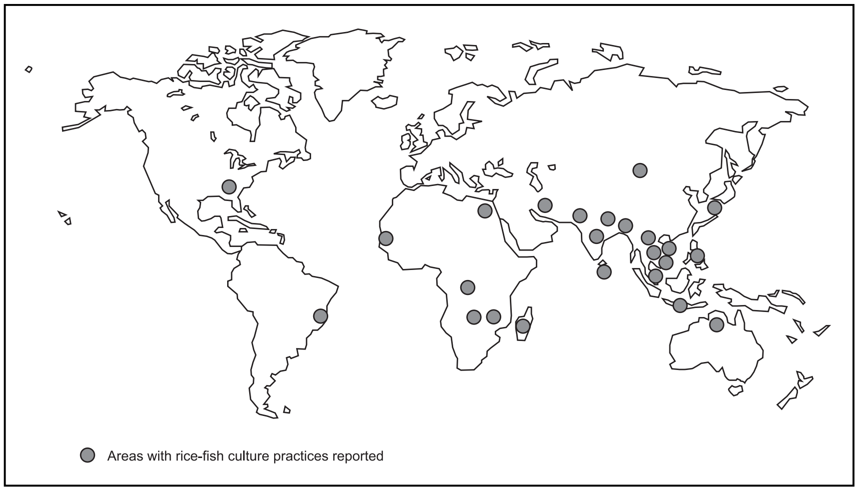

Figure 2.1: Countries where rice-fish farming is practiced or has already been practiced, Halwart et Gupta (2004)

This is a very old type of farming, the latest records trace it to China over 1700 years ago. Historians think that fish has become a product of the rice farmers unintentionally. Fish would have come into the rice field with flood water, stayed there, grew and reproduced within the duration of the rice farming cycle. Even if there is no evidence for the practice of rice-fish farming in India and other Asian countries, it might have been developed during this period of time in those regions.

Although the fish farming in a rice field is widely spread around the world, the proportion of this type of culture compared to rice farming without fish remains relatively low. In the 1950s, 28 countries on six continents practiced this integrated culture compared to more than a thousand that practiced only rice farming. Moreover, the fraction of farmers that store fish in their rice field has dropped significantly during the green revolution. The use of chemical pesticides and herbicides to increase rice production is perhaps one of the principal causes of this drop, because it is a major constraint for the success of rice-fish farming. Since the late 1980's global and governments' interest in rice-fish culture has been renewed. There is an increase in the promotion campaigns and in incentive policies in different countries.

2.2.1 Fish in Rice Field

This type of farming can be practiced in many different ways. Farmers can intentionally put fish in the rice field or fish can come with flood water. Fish and rice can grow at the same time either in the same field or side by side in separate compartments using the same water, or one after another.

For most of those ways, farmers have to carry several small modifications to their rice field before they start rice-fish farming. Providing deeper areas and refuge areas for fish and increasing the height of the dike are two examples of changes that have to be done before storing fish in the rice field. It is important to note that the costs of those modifications are relatively low. Once those adjustments are done, rice farmers buy fish on the market and store it in their rice field. There are many species available for raising in rice fields: tilapia, common carp and silver barb are among them. The fish that is stored is either native fish species caught in their natural environment or fish that comes from aquaculture, depending on whether the aquaculture is well developed in the country or not. In China, fish that is stored comes principally from aquaculture, which has been developed rapidly during the past decades. In this paper, I am more concerned about the storage of native fish that comes from lakes, rivers or oceans, as in northeast Brazil.[2]

Farmers have to provide supplemental feeding for fish as wheat bran and coconut. Those additional expenses might be compensated by the beneficial effect of fish on rice farming. Cagauan (1995) exposes different ways by which fish contributes in the nutrient cycle of the rice field ecosystem. The presence of fish may reduce the need to use pesticides and fertilizers, and hence reduces the expenses. Several studies also pointed out that rice-fish field has higher yields and healthier rice plants than rice field without fish. Farmers harvest fish once it is mature and they can either sell it or consume it. It is worth noting that fixed and variable costs of fish storage in the rice field seem relatively low. This fact is consistent with the assumption of the theoretical model presented in this paper, that there is no storage cost.

People in favour of combining rice and fish argue that it helps to improve nutrition and

[2]For the upcoming part of the paper, the term "fish" refers to "fish species caught in their natural environment"

combat poverty, by increasing the revenue and the production of farmers. Even if nutritional benefits have not been seriously proved, they are generally assumed. In remote areas, Demaine et Halwart (2001) find that an important fraction of the diet comes from rice fields fish. Rice-fish farming can be seen as a part of the solution to the poverty issues. It is also well known that rice-fish culture has a negative impact on pollution levels through the reduction of chemical pesticides and herbicides, and even if less pesticides are used, rice-fish culture appears to be a good way to control mosquitoes.

In the past few years, several governments have developed promotion campaigns and policies to give incentives to rice farmers to try the rice-fish culture. Madagascar, Brazil and the Philippines might be cited as examples. Some experimental policies have succeeded, however, some did not get the expected success as farmers had tried the combination of rice and fish, and after a short period of time they have decided not to store fish anymore. One perfect example of this kind of policy failure is the case of the Philippines in the early 1980's. After the government had decided to promote rice-fish culture, there was a peak of rice-fish farms in 1982 followed by a sudden decline of those fields.

When those policies are studied, it is usually done by evaluating the impact on health, pollution and revenue. However the impact on fish stock is never considered to be an important issue, even though this effect might be important. If storage of fish in rice field has a negative impact on the fish stock, rice-fish culture can affect the prices and welfare in the future through the fish stock degradation. My paper tries to fill the gap in the literature by modelling a general equilibrium model with an open access renewable resource that can be stored.

2.3 A General Equilibrium Model

The economy is closed. Two goods can be produced and consumed. The harvest from renewable resource, H, and the consumption good, M. There are two production factors: the stock of the renewable resource, S, and labour, L. The population is fixed and is equal to L. The horizon of the model is infinite. The structure of the market is sequential, hence consumers face a sequence of budget constraints. The decisions are made at the beginning of the first period.

2.3.1 Resource

2.3.1.1 Open Access Renewable Resource

The stock of the open access renewable resource at time t is defined by s_t . This resource can represent fish, wildlife stock, forest, or a combination of natural resources. In the case of interest, I consider fish as the resource. The natural growth rate of the resource, $G(s_t)$, is assumed to be a logistic function:

$$G(s_t) = rs_t\left(1 - \frac{s_t}{K}\right).$$

A logistic function is a common sigmoid curve, that can model the S-curve of growth of a population. For small initial levels of resource, growth is approximately exponential; then, as the resource becomes more abundant, its growth slows, and when it reaches the carrying capacity K, growth stops. The highest value that s_t can reach is K. The intrinsic growth rate, r, represents the proportional increase of the resource in one period when there is no congestion. For simplicity, I assume through the paper that the upper bound of r is $1/2$. If there is neither congestion nor harvesting and r reaches $1/2$, the growth rate of the resource would be 50%, which is relatively high for most natural resources considered in the paper, hence this assumption is not too strong. The evolution of the resource stock is defined by:

$$s_{t+1} - s_t = s_t r\left(1 - \frac{1}{K}s_t\right) - H_t, \tag{2.1}$$

where H_t denoted the quantity of resource harvested in period t. This is the discrete version of:

$$\frac{dS}{dt} = G(S(t)) - H(t) = rS(1 - \frac{S}{K}) - H(t)$$

based on Matsumoto (2002).

The resource is open access, which means that there is an inability to restrict access to the resource. According to Gordon (1954) the over-exploitation is the consequence of the inability to restrict access to the resource, which can be distinguished from common property.

Fish is not an official open access resource, because there are policies and laws to control fishery. However, as it is pointed out by the comparative analysis of sustainable fisheries law edited by "*Union internationale pour la conservation de la nature*" there are strong evidence that several fishermen all around the world defy those laws repeatedly and cause overfishing issues. Brazil is one of the countries that have to deal with those issues. Even fish resources that were considered to have a successful management are being overfished, as the Pink north coast shrimp. Therefore, open access assumption may not seem to me to be a strong assumption.

2.3.1.2 Private Resource

It is also important to note that once the resource is harvested and stored it becomes a private good. In the case of rice-fish culture, once a farmer stores fish in his field, property rights change. Fish is no longer an open resource, it becomes private. The growth rate of the resources that are stored is given by γ and I assume that it lies between zero and r. If $\gamma = 0$, then the harvest goods stored in first period do not grow and if γ equals the intrinsic growth rate the harvest good stored will grow at the same rate as the remaining resource stock would grow without congestion and harvesting.[3] The real growth rate of the resource in its natural environment (considering congestion) might be lower than the growth rate once the resource is stored. This possibility can be explained by the fact that the resource might be more protected once it is stored.[4]

2.3.2 Production

Every period the market for both goods is assumed to be perfectly competitive.

[3]For tractability, I consider that there is no congestion in the rice field.
[4]Rice-fish farmers might try to prevent fish from deceases and try to protect it from preys.

2.3.2.1 Consumption Sector

The production function is given by $M_t^p(L_{M,t}) = \sqrt{L_{M,t}}$. The production set of consumption good is convex and has non increasing returns to scale. Given prices and resource stock level, a producer chooses his supply by solving the following problem:

$$\max_{M_t^s, M_t^p, L_{M,t}} p_{M,t} M_t^s - w_{M,t} L_{M,t} \quad \text{s.t.} \quad M_t^s \leq M_t^p(L_{M,t}) = \sqrt{L_{M,t}}.$$

The supply of the consumption good is given by:

$$M_t^s = \frac{p_{M,t}}{2w_{M,t}}.$$

The competitive profit of this industry is given by:

$$\Pi_t = p_{M,t} M_t^s - w_{M,t} (M_t^s)^2 = \frac{(p_{M,t})^2}{4w_{M,t}}.$$

2.3.2.2 Harvest Sector

The harvest good is produced following a Schaefer production function:

$$H_t^p(L_{H,t}, s_t) = \alpha s_t L_{H,t}.$$

Even though the resource is one of the production factors of the harvest sector, producers do not have to pay for its use. Access to the resource is not restricted. For a given level of resource stock, the productivity of one unit of labour is constant and is equal to αs_t. As the resource stock increases productivity increases as well. Given prices and resource stock level, the producers' problem is the following:

$$\max_{H_t^s, H_t^p, L_{H,t}} p_{H,t} H_t^s - w_{H,t} L_{H,t} \quad \text{s.t.} \quad H_t^s \leq H_t^p(L_{H,t}, s_t) = \alpha s_t L_{H,t}.$$

The supply of the harvest good is given by:

$$H_t^s = \begin{cases} 0 & \text{if } p_{H,t} < \frac{w_{H,t}}{\alpha s_t} \\ H_t^p \geq 0 & \text{if } p_{H,t} = \frac{w_{H,t}}{\alpha s_t} \\ \infty & \text{if } p_{H,t} > \frac{w_{H,t}}{\alpha s_t}. \end{cases}$$

2.3.3 Consumers

There are L identical consumers in the economy. Every period, each consumer is endowed with one unit of labour. The firms' profits are divided equally among consumers.[5] Every consumer gets the same wage.[6] In the first period, consumers can store a fraction of the harvest good bought in order to consume or sell it in period 2. One unit stored in period 1 gives $(1+\gamma)$ units in period 2. Consumers' preferences in period t are represented by a Cobb-Douglas utility function:

$$u_t = u(h_t, m_t) = (h_t)^\beta (m_t)^{1-\beta},$$

with $\beta \in (0,1)$. The budget set is defined by the following budget constraints:

$$\begin{aligned} & p_{H,1}(h_1 + h_1^e) + p_{M,1}m_1 \leq w_1 + \pi_1, \\ & p_{H,2}h_2 + p_{M,2}m_2 \leq w_2 + \pi_2 + p_{H,2}(1+\gamma)h_1^e, \\ & p_{H,t}h_t + p_{M,t}m_t \leq w_t + \pi_t \quad \forall t \geq 3. \end{aligned}$$

The individual amount of harvest good stored in period 1 and the individual consumption of both goods in period t are denoted respectively by h_1^e, h_t, and m_t. The price system is composed of harvest good prices, $\{p_{H,t}\}_{t=1}^\infty$, consumption good prices, $\{p_{M,t}\}_{t=1}^\infty$, and wages, $\{w_t\}_{t=1}^\infty$. The sequential structure of the markets implies that, in every period,

[5]From Section 2.3.2.2, every period the equilibrium profits of the harvest sector are zero. Hence the consumer's revenue comes from labour and the profits of the consumption sector.

[6]From Section 2.3.4, if both productions take place in period t the wage denoted by w_t equals $w_{H,t} = w_{M,t}$. If only production j (j is either consumption good or harvest good) takes place in period t, the wage of a consumer is defined by $w_t = w_{j,t}$.

consumers face a budget constraint. Expenditures come from the harvest good and consumption good consumed. Note that in first period the expenditures are also composed of harvest good stored. In second period, consumers' revenue does not come exclusively from wage and profits, it also has a third component which is the product of the sale of the stored good.

Each consumer maximizes his lifetime utility function: [7]

$$\begin{aligned}
\mathbf{P} = \quad & \max_{(\{h_t, m_t,\}_{t=1}^{\infty}, h_1^e) \in D} \sum_{t=1}^{\infty} \sigma^t (h_t)^{\beta} (m_t)^{1-\beta} \\
& \text{s.t} \\
& p_{H,1}(h_1 + h_1^e) + p_{M,1} m_1 \leq w_1 + \pi_1 \\
& p_{H,2} h_2 + p_{M,2} m_2 \leq w_2 + \pi_2 + p_{H,2}(1+\gamma) h_1^e \\
& p_{H,t} h_t + p_{M,t} m_t \leq w_t + \pi_t \quad \forall \quad 3 \leq t \leq \infty \\
& h_1^e \geq 0 \\
& m_t \geq 0 \quad \forall t \\
& h_t \geq 0 \quad \forall t.
\end{aligned}$$

As in the classic dynamic consumption problem, in every period, consumers have to choose the quantity of consumption good and harvest good they demand. The choice of consumption is based on the comparison of the relative marginal benefit and the relative price. However, in this model, consumers have to take another decision in the first period. They have to choose the quantity of harvest good they want to store. A consumer decides to store a unit of harvest good if the cost of this unit is less than the benefit. The relative price of the harvest good in terms of the consumption good in period 1 reflects the cost of one unit of harvest good stored and the relative price in period 2 reflects the benefit.[8] If the marginal cost is higher than the marginal benefit consumers decide not to store. If the marginal cost equals the marginal benefit, then consumers are indifferent between storing or not. Finally, if the marginal cost is lower than the marginal benefit, consumers

[7] The discount factor is assumed to lie between zero and one.

[8] This is true under the assumption that $\sigma(1+\gamma) = 1$. This equality is assumed throughout the paper.

decide to use all their first period revenues to store and they implicitly decide not to consume in the first period.[9] Proposition 2.1 characterizes the solution to problem **P** which is the individual consumer demand.

Proposition 2.1.
For a strictly positive price vector,[10] *the individual consumer demand is given by:*

$$m_1 = \frac{1-\beta}{p_{M,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right); \quad h_1 = \frac{\beta}{p_{H,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right),$$
$$m_2 = \frac{1-\beta}{p_{M,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right); \quad h_2 = \frac{\beta}{p_{H,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right),$$
$$m_t = \frac{1-\beta}{p_{M,2}}(w_t+\pi_t); \quad h_t = \frac{\beta}{p_{H,t}}(w_t+\pi_t); \quad \forall t \geq 3,$$
$$h_1^e = \begin{cases} 0 & \text{if} \quad p_{H,1}/p_{M,1} > p_{H,2}/p_{M,2} \\ \in [0,(w_1+\pi_1)/p_{H,1}] & \text{if} \quad p_{H,1}/p_{M,1} = p_{H,2}/p_{M,2} \\ (w_1+\pi_1)/p_{H,1} & \text{if} \quad p_{H,1}/p_{M,1} < p_{H,2}/p_{M,2}. \end{cases}$$

The proof of Proposition 2.1 lies in the Appendix I.

2.3.4 Market Clearing Conditions

The equilibrium price system must clear the consumption good market, the harvest market and the labour market. In the consumption good market, the market clearing condition is:

$$M_t^s = Lm_t \quad \forall t.$$

[9] One might be surprised by the fact that consumers do not want to consume in the first period. I can get rid of this problem by imposing a minimum consumption of both goods in every period. Consumers will then choose the minimum consumption and use the rest of their income to store the harvest good. However, it is shown in Section 2.4.1 that, in equilibrium, consumers choose a strictly positive quantity of both good. It might be important to note that this result is driven by the linearity of the utility function in income.

[10] I restrict my attention to strictly positive prices because a null commodity's price leads to an infinite demand of that commodity.

In first period, consumers can store a fraction of the harvest good produced, hence the market clearing condition for consumption good in the first period is:

$$H_1^s = Lh_1 + Lh_1^e.$$

In the second period, if there is storage of harvest good, a fraction or the totality of the harvest good consumed comes from Lh_1^e. Combining the consumers supply, $L(1+\gamma)h_1^e$, and the producers supply, $H_2^s \geq 0$, provides the market clearing condition for the consumption good in the second period:

$$H_2^s + L(1+\gamma)h_1^e = Lh_2.$$

The market clearing condition in the harvest market for period $t \geq 3$ is:

$$H_t^s = Lh_t \quad \forall t \geq 3.$$

There are several conditions for the labour market to be in equilibrium. The first is the full employment condition for each period:

$$L_{H,t} + L_{M,t} = L \quad \forall t.$$

The labour supply is fixed to L and the demand of labour in the harvest sector and in the consumption sector are $L_{H,t}$ and $L_{M,t}$, respectively. It is assumed that labour is perfectly mobile among sectors, hence to ensure that the labour market in each sector is in equilibrium the following conditions on wages must be satisfied. If both goods are produced the wage in both sectors must be equal:

$$w_{M,t} = w_{H,t}.$$

In period t, if the harvest good is produced and the consumption good is not, then:

$$w_{M,t} \leq w_{H,t}.$$

In period t, if the harvest good is not produced and the consumption good is produced, then:

$$w_{M,t} \geq w_{H,t}.$$

2.4 Equilibrium

In this section, the general equilibrium and the steady state are characterized.

2.4.1 Equilibrium Analysis

The general equilibrium of the economy is fully described in the following proposition.

Proposition 2.2. *The equilibrium of the economy has the following properties:*

(i) the equilibrium is unique;

(ii) if:

$$0 < s_1 \leq \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right),$$

the equilibrium is characterized by:

$$h_1^{e*} = 0; \quad h_t^* = \frac{2\alpha\beta s_t^*}{1+\beta}; \quad H_t^{s*} = \frac{2\alpha\beta L s_t^*}{1+\beta}; \quad L_{H,t}^* = \frac{2\beta L}{1+\beta},$$
$$m_t^* = \sqrt{\frac{1-\beta}{1+\beta}\frac{1}{L}}; \quad M_t^{s*} = \sqrt{\frac{1-\beta}{(1+\beta)}L}; \quad L_{M,t}^* = \frac{1-\beta}{1+\beta}L,$$
$$p_{M,t}^* = w_{M,t}^*\sqrt{4\frac{1-\beta}{1+\beta}L}; \quad p_{H,t}^* = \frac{w_{H,t}^*}{\alpha s_t^*}; \quad w_t^* = w_{H,t}^* = w_{M,t}^*; \quad \forall t;$$

(iii) if:

$$\frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right) \leq s_1 \leq K,$$

for the first two periods, the equilibrium is characterized by $0 \leq h_1^{e*} \leq \alpha s_1$ *and:*[11]

$$
\begin{aligned}
&h_1^* = \frac{2\alpha\beta s_1 + (1-\beta)h_1^{e*}}{1+\beta}; \quad H_1^{s*} = \frac{2\alpha\beta s_1 + (1-\beta)h_1^{e*}}{1+\beta}L,\\
&m_1^* = \sqrt{\frac{1-\beta}{1+\beta}\left(1-\frac{h_1^{e*}}{\alpha s_1}\right)\frac{1}{L}}; \quad M_1^{s*} = \sqrt{\frac{1-\beta}{1+\beta}\left(1-\frac{h_1^{e*}}{\alpha s_1}\right)L},\\
&h_2^* = \frac{2\alpha\beta s_2^* + 2\beta(1+\gamma)h_1^{e*}}{1+\beta}; \quad H_2^{s*} = \frac{2\alpha\beta s_2^* - (1-\beta)(1+\gamma)h_1^{e*}}{1+\beta}L,\\
&m_2^* = \sqrt{\frac{1-\beta}{1+\beta}\left(1+\frac{(1+\gamma)h_1^{e*}}{\alpha s_2^*}\right)\frac{1}{L}}; \quad M_2^{s*} = \sqrt{\frac{1-\beta}{1+\beta}\left(1-\frac{h_1^{e*}}{\alpha s_2^*}\right)L},\\
&p_{M,1}^* = w_{M,1}^*\sqrt{4\frac{1-\beta}{1+\beta}\left(1-\frac{h_1^{e*}}{\alpha s_1}\right)L}; \quad p_{H,1}^* = \frac{w_{H,1}^*}{\alpha s_1}; \quad w_1^* = w_{H,1}^* = w_{M,1}^*,\\
&p_{M,2}^* = w_{M,2}^*\sqrt{4\frac{1-\beta}{1+\beta}\left(1+\frac{(1+\gamma)h_1^{e*}}{\alpha s_2^*}\right)L}; \quad p_{H,2}^* = \frac{w_{H,2}^*}{\alpha s_2^*},\\
&w_2^* = w_{H,2}^* = w_{M,2}^*,
\end{aligned}
$$

and for all other periods, the equilibrium is characterized by:

$$
\begin{aligned}
&h_t^* = \frac{2\alpha\beta s_t^*}{1+\beta}; \quad H_t^{s*} = \frac{2\alpha\beta L s_t^*}{1+\beta}; \quad L_{H,t}^* = \frac{2\beta L}{1+\beta},\\
&m_t^* = \sqrt{\frac{1-\beta}{1+\beta}\frac{1}{L}}; \quad M_t^{s*} = \sqrt{\frac{1-\beta}{(1+\beta)}L}; \quad L_{M,t}^* = \frac{1-\beta}{1+\beta}L,\\
&p_{M,t}^* = w_{M,t}^*\sqrt{4\frac{1-\beta}{1+\beta}L}; \quad p_{H,t}^* = \frac{w_{H,t}^*}{\alpha s_t^*}; \quad w_t^* = w_{H,t}^* = w_{M,t}^*.
\end{aligned}
$$

(iv) $s_{t+1}^* = s_t^*\left(1 + r - s_t^* r/K\right) - H_t^{s*} \quad \forall t.$

For the complete proof see the Appendix I.

Suppose that consumers cannot store the harvest good. Knowing that the relative price of the harvest good in period t is given by $1/\alpha s_t$, if the initial resource stock is lower (larger) than the steady state, the resource stock will increase (decrease) and hence the relative price of the harvest good will decrease (increase). For an initial resource stock

[11] The analytical expression of the quantity of harvest good stored is provided in the Appendix I.

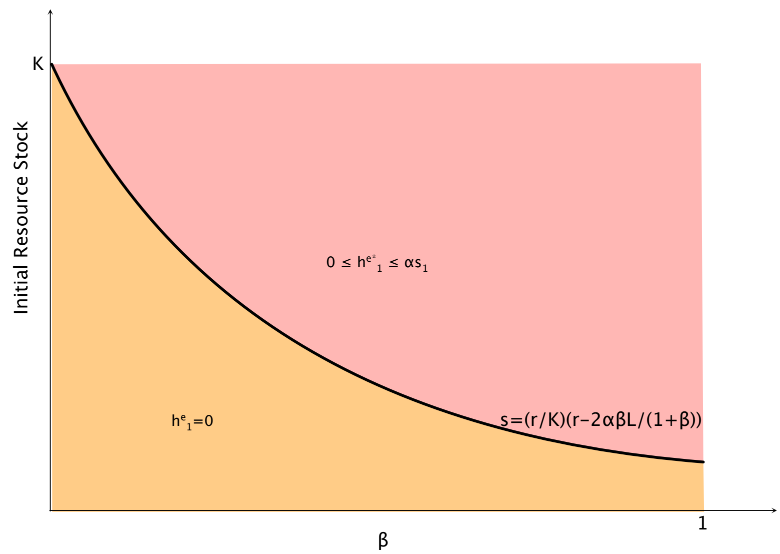

Figure 2.2: The equilibrium quantity of harvest good stored

larger than the steady state, the relative price is lower in first period than in second period, then consumers would want to stock harvest good if they could. For an initial resource stock lower than the steady state, the relative price is larger in first period than in second period, then consumers would not store harvest goods even if they could. If the initial resource stock is given by the steady state, the benefit of storing harvest good equals its cost and consumers are indifferent between storing or not. Then the cutoff curve between storing and not is given by:

$$\bar{s} = \left(r - \frac{2\alpha\beta L}{1+\beta} \right). \tag{2.2}$$

On this curve, the cost of storing the harvest good equals the benefit. Therefore, when consumers are allowed to store the harvest good, they are doing so only when the initial resource stock level lies above the steady state, $\bar{s}$.[12] For a low initial resource stock, there is no storage in equilibrium. It is interesting to note that there is some evidence

[12]Note that this analysis might be slightly different without the assumption that $\sigma(1+\gamma) = 1$. Without this assumption the cost and the benefit of storing the harvest good are different and this complicates the analysis.

that shows that when the fish stock is low, there is no rice-fish culture.[13]

In all periods after the second one, the allocations are described by the same equations, regardless of whether consumers have stored the harvest good or not in the first period. The equilibrium behaviour described by those equations is simply that consumers spend a fraction β of their revenue on the harvest good and a fraction $(1-\beta)$ on consumption good, hence both goods are produced every period. The price of a good equals its marginal cost and the wages in both sectors are equal.[14]

If consumers do not have access to storage, the equilibrium for all initial levels of resource stock is given by the characterization provided in point (ii) of Proposition 2.2.

The previous analysis gives insight into consumers' behaviour when the only way for saving is by storing the harvest good. The storage system can be seen as a banking system. It might be interesting to compare this type of saving to a more traditional one, which is the use of bonds. I will not repeat the previous general equilibrium analysis, nevertheless I study how consumers respond to the possibility of buying bonds, at the equilibrium price without storage in a partial equilibrium analysis. Proposition 2.3 describes the behaviour of consumers when they have access to a bond, b, between the first and the second period.

Proposition 2.3. *Under the assumption that prices are given by the equilibrium prices without storage of harvest good:*

$$\frac{p_{H,t}}{p_{M,t}} = \frac{1}{\alpha s_t}\sqrt{\frac{1+\beta}{1-\beta}\frac{1}{4L}},$$

consumer demand for bonds is given by:

$$b_1 = \begin{cases} \frac{w_1+\pi_1}{p_{H,1}} & \text{if} \quad s_1 < \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right) \\ \in \left[-\frac{w_2+\pi_2}{p_{H,2}(1+\gamma)}, \frac{w_1+\pi_1}{p_{H,1}}\right] & \text{if} \quad s_1 = \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right) \\ -\frac{w_2+\pi_2}{p_{H,2}(1+\gamma)} & \text{if} \quad s_1 > \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right). \end{cases}$$

[13] See Hong (2007)

[14] Even though the equilibria are described by the same equations, the equilibrium values are different, because the evolution of the resource stock is affected by the storage of the harvest good.

For the proof see Appendix I.

Propositions 2.2 and 2.3 show that the willingness to save is not driven by the willingness to smooth consumption across periods.[15] In the case of storage, if the initial level of resource stock is larger than the steady state, consumers speculate that the price of the harvest good will increase and store the resource. However, if the initial resource stock is lower, consumers do not want to store the harvest good, because the price of that good will decrease. The reasoning is quite similar to the one for bonds. If the initial resource stock lower (larger) than the steady state, the cost of bonds is lower (larger) than the benefit, hence consumers have a positive (negative) amount of bonds. Hence, having access to storage does have the same effect on consumers' behaviour than having access to traditional banking system, in the form of bonds.

2.4.2 Steady state

The steady-state variables are denoted by an upper bar. Next proposition characterizes the interior steady state of my economy.

Proposition 2.4. *Under the assumption that $r \geq 2\alpha\beta L/(1+\beta)$, the model has a unique interior steady state:*[16]

$$\bar{s} = \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right); \quad \bar{M} = \sqrt{\frac{1-\beta}{1+\beta}L}; \quad \bar{H} = \frac{2\alpha\beta L\bar{s}}{1+\beta},$$
$$\frac{\bar{p_H}}{p_M} = \frac{r(1+\beta)}{\alpha K(r(1+\beta) - 2\alpha\beta L)}\sqrt{\frac{1+\beta}{1-\beta}\frac{1}{4L}}.$$

Proof of Proposition 2.4

The interior steady state is given by the solution of the following system of equations:

$$s = s(r - \frac{sr}{K}) - H; \quad H = \frac{2\alpha\beta Ls}{1+\beta}; \quad L = \frac{H}{\alpha s} + M^2; \quad \frac{p_H}{p_M} = \frac{1}{\alpha s}\sqrt{\frac{1+\beta}{1-\beta}\frac{1}{4L}}.$$

[15] The result is due to the assumption that the utility function is linear in income.

[16] The model exhibits also a corner steady state at ($\bar{s} = 0$, $\bar{h^c} = 0$, $\bar{m^c} = 1/\sqrt{L}$), but for the upcoming part of the paper I focus on the interior steady state.

Under the assumption that $r \geq 2\alpha\beta L/(1+\beta)$ the resource stock and the harvest are positive at the steady state. *Q.E.D.*

Figure 2.3.a illustrates the determination of the resource stock at the steady state. The straight line and the curve represent, respectively, the harvest of the resource when there is no storage and the growth of the resource. At the intersection of the harvest function and the growth function, the growth is exactly offset by the harvest, hence the resource stock stays constant. The intersections represent the steady states of the economy. Those two curves intersect each other twice. There is, however, only one interior steady state.[17] At the steady state, the resource stock stays constant given prices and demands. If the resource stock is larger than the steady state, the harvest rate will be higher than the growth rate, hence the resource stock will shrink. As the resource stock falls the productivity in the harvest sector falls as well and the resource growth increases, those effects will lead the to steady state. If the resource stock is lower than the steady state, the growth rate is larger than the harvest rate, and hence the resource stock increases until it reaches the steady state. Figure 2.3.b shows that the steady state level of the resource stock is a decreasing function of consumers' preferences parameter, β.[18] As β increases, the harvest rate increases and the harvest function shifts counterclockwise. The larger the preference parameter over the harvest good is, the lower the steady level of resource stock is.

2.5 The Effect of Storage on the Economy

2.5.1 Resource Stock Evolution

In this section, I compare the evolution of the resource stock when consumers store the harvest good in equilibrium to the evolution of the resource stock when consumers

[17] Although there is a steady state at zero, for the upcoming sections of the paper, the steady state refers to the interior solution.

[18] Note that in the upcoming part of the paper, I assume that for all $0 < \beta < 1$ there exists a positive steady state, that is, $r \geq 2\alpha\beta L/(1+\beta)$.

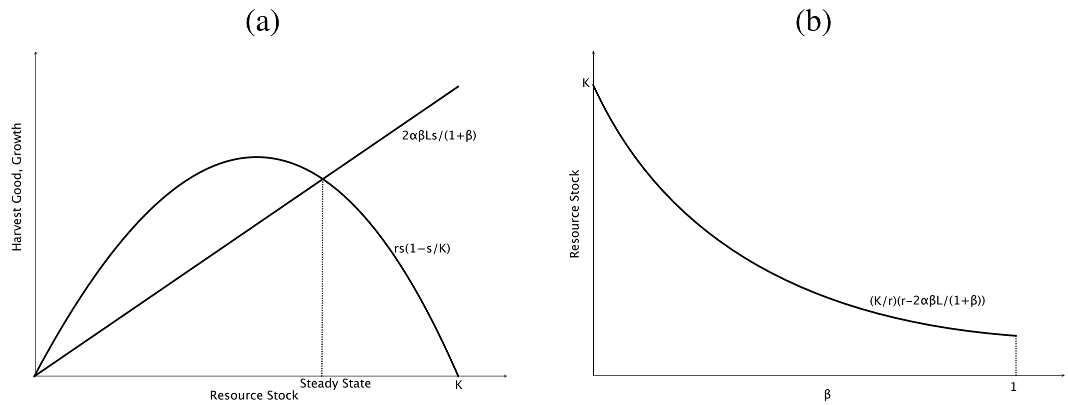

Figure 2.3: Resource stock at the steady state

cannot store the harvest good.[19] First, Proposition 2.5 establishes a general result.

Proposition 2.5. *If s_t is larger (lower) than the steady state,*

$$s_{t+1} = s_t(1 + r - s_t r/K - 2\alpha\beta L/(1+\beta)),$$

is also larger (lower) than the steady state.

Proof of Proposition 2.5

Given an initial resource stock, s_t, define the difference between the resource stock in the following period and the steady state by:

$$f(s_t) = s_{t+1} - s_t = s_t\left(1 + r - \frac{s_t r}{K} - \frac{2\alpha\beta L}{1+\beta}\right) - \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right).$$

This quadratic function has zeroes at $\bar{s}$ and at (K/r), and is concave. Hence, for $s_t \geq \bar{s}$ ($s_t \leq \bar{s}$), $f(s_t) \geq 0$ ($f(s_t) \leq 0$), this means that the resource stock in the following period is also larger (lower) than $\bar{s}$, when:

$$s_{t+1} = s_t\left(1 + r - \frac{s_t r}{K} - \frac{2\alpha\beta L}{1+\beta}\right).$$

Q.E.D.

[19]Since storage takes place only when the initial resource stock is larger than $\bar{s}$, I consider, for the analysis of the effect of storage on resource stock, only initial resource stock larger than the steady state.

I denote by $\tilde{s}_2$ and $\tilde{s}_3$ the resource stock in periods two and three respectively if consumers do not have access to storage:

$$\tilde{s}_2 = s_1\left(1+r-\frac{s_1 r}{K}-\frac{2\alpha\beta L}{1+\beta}\right); \quad \tilde{s}_3 = \tilde{s}_2\left(1+r-\frac{\tilde{s}_2 r}{K}-\frac{2\alpha\beta L}{1+\beta}\right).$$

From Proposition 2.5, those two levels of resource stock are larger than the steady state level.

If consumers store the harvest good, the resource stock in second and third period is:

$$s_2^* = \tilde{s}_2 - \frac{1-\beta}{1+\beta}Lh_1^{e*}; \quad s_3^* = s_2^*\left(1+r-\frac{s_2^* r}{K}-\frac{2\alpha\beta L}{1+\beta}\right)+(1+\gamma)\frac{1-\beta}{1+\beta}Lh_1^{e*}.$$

It is clear that storage has a negative impact on the resource stock in period 2. Between period 1 and two the resource stock decreases more rapidly compared to the situation where consumers store the harvest good.

Using Mathematica, I show that even if there is more harvest with storage than without, the equilibrium resource stock in period 2, s_2^*, is still larger than the steady state. From Proposition 2.5 and $s_2^* > \bar{s}$:

$$s_2^*\left(1+r-\frac{s_2^* r}{K}-\frac{2\alpha\beta L}{1+\beta}\right) > \frac{K}{r}\left(r-\frac{2\alpha\beta L}{1+\beta}\right).$$

Hence the resource stock in period three is also larger than the steady state:

$$\begin{aligned} s_3^* &= s_2^*\left(1+r-\frac{s_2^* r}{K}-\frac{2\alpha\beta L}{1+\beta}\right)+(1+\gamma)\frac{1-\beta}{1+\beta}Lh_1^{e*} \\ &> s_2^*\left(1+r-\frac{s_2^* r}{K}-\frac{2\alpha\beta L}{1+\beta}\right) > \frac{K}{r}\left(r-\frac{2\alpha\beta L}{1+\beta}\right) = \bar{s}. \end{aligned}$$

Even if consumers cannot store the harvest good after period three, the storage has an impact on the resource stock in the following periods.

Next proposition characterizes the positive impact of storage on the resource stock in period three.

Proposition 2.6. *Storage has a beneficial effect on the resource stock in period three: $s_3^* > \tilde{s}_3$.*

Proof of Proposition 2.6

$$\begin{aligned}
s_3^* &= s_2^*\left(1+r-\frac{s_2^* r}{K}-\frac{2\alpha\beta L}{1+\beta}\right)+(1+\gamma)\frac{1-\beta}{1+\beta}Lh_1^{e*} \\
&= \left(\tilde{s}_2-\frac{1-\beta}{1+\beta}Lh_1^{e*}\right)\left(1+r-\frac{r}{K}\left(\tilde{s}_2-\frac{1-\beta}{1+\beta}Lh_1^{e*}\right)-\frac{2\alpha\beta L}{1+\beta}\right) \\
&\qquad +(1+\gamma)\frac{1-\beta}{1+\beta}Lh_1^{e*} \\
&= \tilde{s}_3+\tilde{s}_2\frac{r}{K}L\left(\frac{1-\beta}{1+\beta}\right)h_1^{e*}-\frac{1-\beta}{1+\beta}Lh_1^{e*}\left(1+r-\frac{s_2^* r}{K}-\frac{2\alpha\beta L}{1+\beta}\right) \\
&\qquad +(1+\gamma)\frac{1-\beta}{1+\beta}Lh_1^{e*} \\
&= \tilde{s}_3+\frac{r}{K}L\left(\frac{1-\beta}{1+\beta}\right)h_1^{e*}\left[\tilde{s}_2-\frac{K}{r}\left(1+r-\frac{s_2^* r}{K}-\frac{2\alpha\beta L}{1+\beta}-1-\gamma\right)\right] \\
&= \tilde{s}_3+\frac{r}{K}L\left(\frac{1-\beta}{1+\beta}\right)h_1^{e*}\left[\tilde{s}_2-\frac{K}{r}\left(r-\frac{2\alpha\beta L}{1+\beta}\right)+\frac{K}{r}\left(\frac{s_2^* r}{K}+\gamma\right)\right] \\
&> \tilde{s}_3.
\end{aligned}$$

The last inequality holds because it has been established that $\tilde{s}_2 > \bar{s}$. *Q.E.D.*

Knowing that $s_3^* > \tilde{s}_3$ implies that storage has a positive impact on the resource stock of all the following periods. This is shown in the following proposition.

Proposition 2.7. *If $s_t > s_t'$, then $s_{t+1} > s_{t+1}'$ for all $t \geq 3$.*

Proof of Proposition 2.7

Given the resource stock, s_t, the combination of the evolution of the resource stock provided in Section 2.3.1.1 and the equilibrium harvest function defined in Proposition 2.2 characterizes the resource stock in period $t+1$:

$$s_{t+1} = s_t\left(1+r-\frac{s_t r}{K}-\frac{2\alpha\beta L}{1+\beta}\right).$$

The resource stock in period $t+1$ is increasing for $s_t \in [0,K]$, because $\partial s_{t+1}/\partial s_t > 0$ for all $s_t \in [0, K]$. *Q.E.D.*

Figure 2.4 illustrates one possible path for the resource stock. It shows that storage has a first negative impact on the resource stock in period 2 and a second effect that is positive and larger than the first one. Hence the overall effect of storage on the resource evolution is positive.[20]

Coming back to the main example of this paper, rice-fish culture, this result suggests that even if storing fish in rice fields had a direct negative impact on the fish stock in lakes and rivers, it might have a positive effect in the long run.

2.5.2 Welfare

Storage might affect welfare. In this section, I show that it is not clear whether storage has a welfare improving effect or not. The equilibrium utility of period 1, when storage is allowed is given by:[21]

$$u_1^* = \beta^\beta (1-\beta)^{1-\beta} \left(\underbrace{\frac{1}{\alpha s_1} \left(\sqrt{4L \frac{1-\beta}{1+\beta} \left(1 - \frac{h_1^{e*}}{\alpha s_1} \right)} \right)^{-1}}_{\frac{p_{H,1}}{p_{M,1}}^*} \right)^{1-\beta} \underbrace{\left(\frac{2\alpha\beta s_1}{1+\beta} - \frac{2h_1^{e*}\beta}{1+\beta} \right)}_{\text{income component}}.$$

Storage has two effects on the equilibrium utility level of period 1, one effect via the price and one effect via the income component. When there is more storage in equilibrium, the relative price of the harvest good in term of the consumption good increases. This is principally due to the reduction in production of the consumption good. Available income for consumption is negatively affected by the equilibrium quantity of harvest good stored. Less income is left for current consumption. The following proposition proves that the effect of storage on income is larger than its effect on price.

[20]This analysis is done under the assumption that consumers can store the harvest good only during the first period. Allowing storage in more than one period might change this result.

[21]Note that h_1^{e*} stands for the equilibrium storage of harvest good. The analytical expression of h_1^{e*} is provided in the Appendix I.

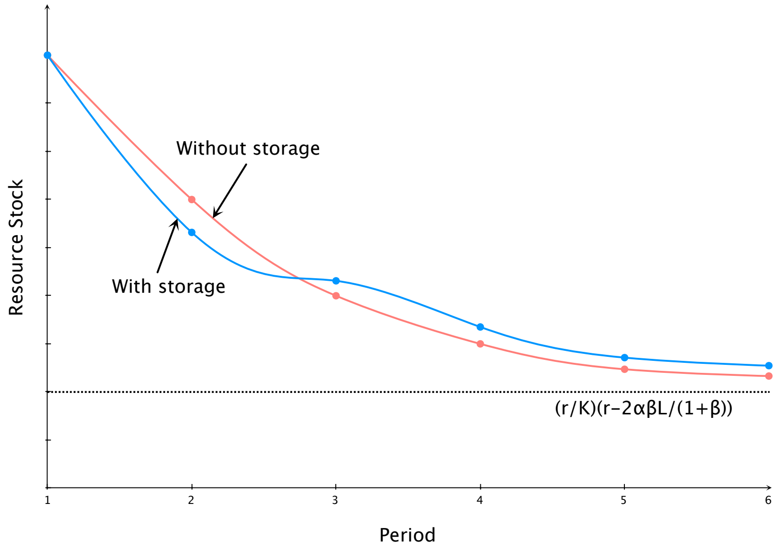

Figure 2.4: Resource stock evolution

Proposition 2.8. *The overall effect of the equilibrium quantity of storage on the utility level in period 1 is negative.*

For proof see Appendix I.

The effect in second period is ambiguous. The equilibrium utility of period 2 when storage is allowed is given by:[22]

$$u_2^* = \beta^{\beta}(1-\beta)^{1-\beta}\left(\underbrace{\frac{1}{\alpha s_2^*}\left(\sqrt{4L\frac{1-\beta}{1+\beta}\left(1+(1+\gamma)\frac{h_1^{e*}}{\alpha s_2^*}\right)}\right)^{-1}}_{\frac{p_{H,2}}{p_{M,2}}^{*}}\right)^{1-\beta}\underbrace{\left(\frac{2\alpha\beta s_2^*}{1+\beta}+(1+\gamma)\frac{2h_1^{e*}\beta}{1+\beta}\right)}_{\text{income component}}.$$

It is important to note that s_2^* depends on the equilibrium quantity of harvest good stored. The storage has a negative impact on the resource stock in period 2. Proposition 2.9 summarizes the effect of storage on the equilibrium utility level in period 2.

Proposition 2.9. *In period 2, storage has a positive effect on income, however, its effect on price is ambiguous. Therefore the overall effect of storage on the equilibrium utility,*

[22]Note that $s_2^* = s_1\left(1+r-s_1 r/K-2\alpha\beta L s_1/(1+\beta)\right)-(1-\beta)h_1^{e*}L/(1+\beta)$.

u_2^* is ambiguous as well.

Proof see Appendix I.

The storage increases the supply of harvest good in period 2, this increase of supply affects negatively the price. However the storage also affects negatively the resource stock in period 2. The reduction of the resource stock has a negative impact on the production of the harvest good, and hence has a positive impact on price. Which of those two effects dominates determines the overall impact of storage on price in period 2. If storage has a positive impact on the price ratio in period 2, u_2^* is larger with storage than without. However, if the price ratio in period 2 is affected negatively by storage, u_2^* might be lower with storage.

For all other periods, the effect of storage on the equilibrium utility level is positive through its positive impact on the resource evolution after period three:

$$u_t^* = \beta^\beta (1-\beta)^{(1-\beta)} \left(\frac{1}{\alpha s_t^*} \sqrt{\frac{1+\beta}{4L(1-\beta)}} \right)^{(1-\beta)} \frac{2\alpha\beta s_t^*}{1+\beta},$$

$$\frac{\partial u_t^*}{\partial h_1^{e*}} = \beta^\beta (1-\beta)^{(1-\beta)} \left(\sqrt{\frac{1+\beta}{4L(1-\beta)}} \right)^{(1-\beta)} \frac{2\beta}{1+\beta} \beta (\alpha s_t^*)^{\beta-1} \frac{\partial s_t^*}{\partial h_1^{e*}} > 0.$$

Therefore the effect of storage on total welfare is ambiguous, storage is not always welfare improving.

Coming back to my rice-fish farming example, it is possible that storing fish in rice field has a negative impact on farmers welfare. This may explain why some promoting policies failed. However a majority of researchers think that rice-fish culture has a positive impact on population, as it is poured out by Lin et al. (1995). In this paper, the economic benefits of rice-fish farming are related. In the Philippines, they estimate that the adoption of rice-fish farming technology can generate an additional 23% more farm income by raising fish as well. These results highlight that even though storage seems to have a beneficial impact on welfare, policy makers must be careful when they develop a policy to promote storage of resources.

2.6 Conclusion

I develop a simple general equilibrium model, that gives insights into the effect of storing resources. I restrict my attention to open-access renewable resources. In particular the storage of fish in rice field. The rice-fish farming is a type of culture that one can find all around the world. During the last decades, some policy makers aim to promote this type of farming. I characterize the unique equilibrium of my model and evaluate the impact of storage on resource stock evolution and welfare. I find that storing the harvest good has a negative impact on the resource stock in the period right after the storage and has a positive impact on the resource stock in the following periods. The impact of storage on the total welfare is ambiguous.

This paper does not provide a rule that tells us the conditions under which storage improve the total welfare, however, this paper shows that storing the resources is not always beneficial. It could be interesting to calibrate the model for different areas where one can find rice-fish culture and evaluate if storing fish is beneficial for farmers. For tractability I assume that consumers can store harvest good only in the first period. I think that it might be interesting to relax this assumption.

CHAPITRE 3

MIGRATION, TRADE AND THE ENVIRONMENT

3.1 Introduction

Canadians are among the highest per capita emitters of greenhouse gas (GHG). With only 0.5% of the world's population, Canada contributes about 2% of the total global greenhouse gas emissions. The impact of GHG emissions and more generally of climate change on Canada has been studied by several specialists. A consensus on this issue seems, however, difficult to reach. Some economic sectors might be negatively affected by climate change, whereas others might see no difference or might even be positively affected. Even in some particular sectors the effects are unclear. For example, agricultural productivity might be improved through longer and warmer growing seasons, but it can also be reduced through the more severe and frequent droughts or unusually wet years. There is no evidence whether climate change affects more one province or another. Nevertheless, what seems clear is that climate change and GHG emissions will significantly affect Canada.

Energy, transport, and industrial process are the sectors that contribute the most to Canadian GHG emissions. The last two decades have seen a rise in GHG emissions principally due to the increase in oil and gas production and the number of motor vehicles.[1]

This rise is largely caused by the increasing production of petroleum in Alberta. Alberta's GHG emissions have surpassed Ontario's, the previous most polluting province in Canada. In 2010, Alberta's GHG emissions accounted for 34% of the national total of GHG emissions.[2] With only 11% of the Canadian population, Alberta has the highest GHG per capita.[3] Provinces contribute differently to Canadian GHG emissions, depending on their economic activities. Relying on abundant hydroelectricity Quebec shows

[1] Since 2005, total Canadian GHG emissions have decreased.

[2] Canada's total greenhouse gas (GHG) emissions in 2010 were 692 megatonnes (Mt) of carbon dioxide equivalent

[3] GHG per capita in Alberta is around 0.00006317=235Mt/3,724,834.

stable emissions, that represent around 11% of total Canadian emissions. In Quebec, GHG per capita is about 6 times lower than in Alberta.[4]

Migration of workers should be taken in consideration, when the impact of trade on pollution is studied. In Canada, interprovincial migration has risen, principally due to the increasing economic opportunities in Western provinces. Many Canadians migrate from east to west every year. In 2006, Alberta gained 62,291 people, while Quebec lost 13,574 persons. Quebec lost people to other provinces almost every year between 1987 and 2006. This migration might be affected by trade and might have an impact on Canadian emissions. Migration is affected by relative wages, that in turn are affected by comparative advantages and industrial structure.

In this paper, I develop a two-sector model of trade that incorporates both pollution and migration issues to show that interregional trade can affect the pollution level of a country composed of regions with different industrial structures. The different industrial structures of regions imply that trade affects relative wages, that in turn have an impact on the mobility of the labour, and hence on the comparative advantages of regions. Therefore, the change in trade flows between regions has an impact on pollution.

I consider an infinite horizon model that includes two almost identical regions. In each region, there are two types of workers, skilled and unskilled. Since more educated people are more likely to move, I assume that skilled workers are mobile across regions and unskilled are not. The other production factor is the environmental capital, that represents an aggregate of all natural resources in a region. The environmental capital is given at any moment in time, but may change over time. The evolution of the environmental capital depends on pollution flows and on the regeneration of natural resources. Each region has its own environmental capital, with the same regeneration rate and natural steady state. Therefore, the difference between the environmental capital in both regions is solely determined by the flows of pollution. Production technology in both sectors is different in its sensitivity to environmental capital and in its effect on natural resources. The manufacturing sector, not affected by the stock of environmental capital, creates pollution, which affects negatively the productivity of the agricultural sector. The agri-

[4] GHG per capita in Quebec is around 0.00001019=80Mt/7,905,679.

cultural sector is affected by the stock of environmental capital, through its effect on natural resources. Production in the manufacturing sector imposes a negative externality on the agricultural sector through its effect on the environmental capital. The flow of pollution created by the manufacturing sectors is different between the two regions. One region uses a more polluting technology than the other region does (i.e., green energy vs brown one). I introduce migration in the model to evaluate its impact on trade and pollution.

The model leads to the following results. Migration alone reduces the gap between the environmental capital of regions with different industrial structures. Trade has an opposite effect, it affects positively the environmental capital in the region with the least polluting technology (Region 2) and negatively in the region with the smallest initial environmental capital (Region 1). Migration and trade together increase the gap between the environmental capital of regions with different industrial structures, even more than trade alone does. Migration doesn't affect the pattern of trade. Region 2 always gains from trade, but if the preference over the manufactures is low, region 1 can experience a loss from trade, in the long run. With a strong demand for agriculture, the increase of the production of manufactures induced by trade in region 1 leads to a deterioration of its environmental capital, which in turn degrades its terms of trade.[5]

In Section 3.2, I propose a brief review of the important papers of the literature. Section 3.3 describes the theoretical model. The different temporary and stationary equilibria are described and examined in Section 3.4. The effects of migration on trade and environmental capital are discussed in Section 3.5.

3.2 Literature Review

The literature about the effects of international trade on the environment has been growing in the last twenty years. For instance, Copeland et Taylor (1994) and Copeland et Taylor (1995) examine theoretically how trading opportunities affect world pollution. Whereas, Grossman et Krueger (1993) study those effects from an empirical point of

[5] As in Copeland et Taylor (1999).

view. Brander et Taylor (1997) show that trade might reduce income and natural capital. However, most articles ignore the long run effects of pollution on the productivity on an environmentally sensitive production. Copeland et Taylor (1999) is one of the few papers that deal with this issue. With a two-sector, two-country model, they show that pollution can be a motive for trade by separating incompatible industries. One of the two sectors pollutes and affects the environmental capital through its pollutants. The second sector uses the environmental capital as one of its production factors. Since the first sector harms the second one, this production externality can be reduced if the two industries can move away from each other. At the steady state, at least one country is specialized, either in the manufacturing industry or in the agricultural industry. Just as in my model, there is no transboundary pollution, no direct disutility of pollution and no regulation. By contrast, my model includes migration and heterogeneous workers. As a similar result to mine, they show that trade is not always welfare improving, it can lead to a reinforcing process of environmental degradation.

The literature on international trade and the environment that deals with migration is relatively modest, despite the importance of the issue. Kondoh (2006), Kondoh (2007) and Karp et Paul (2007) are among the small number of papers that contribute to this limited literature. To analyze the welfare effects of international migration in the presence of transboundary pollution, Kondoh (2006) modifies the model of Copeland et Taylor (1999). He assumes that the abatement technology is superior in the home country than in the foreign country. His focus is on the international difference in the quality of abatement technology. He assumes that labour is perfectly mobile across sectors as well as across countries. Whereas, in my model, there are two types of workers: skilled workers, that are mobile across regions, and unskilled workers, that are mobile across sectors. In his framework, if there is no trade, the real wage rate of the home country is higher than that of the foreign country, hence workers tend to move from the foreign country to the home country. The gains from migration in both sectors are ambiguous and are significantly different than in my paper. With trade, there is a real wage difference between countries only if both countries are specialized, therefore, migration occurs only in this case. Migration increases the production of the polluting good, therefore, increases the level of world pollution. Most results are driven by the linearity of the production function and the assumption that there is only one type of workers.

My paper is also related to Kondoh (2007), that develops a two-country, two-sector model with skilled and unskilled labour. In his model, only the foreign country pollutes. In contrast to my model, he assumes that pollution abatement technology of the home country is perfect. In the foreign country, the abatement technology is positively affected by skilled workers. It is hypothesized that the positive effect of improvement in pollution abatement technology, brought about by an increase in skilled workers overwhelms the negative effect of the expanded production of the smokestack manufacturing industry. Domestic pollution comes exclusively from the foreign country. The author supposes that labour is perfectly mobile across countries and sectors and skilled workers can work as unskilled workers. Those assumptions about workers are quite different to mine and lead to very different results. When the two countries trade, migration occurs only if the wage differential still persists after trade and tends to increase the pollution level. His assumptions on abatement technologies and the absence of the dynamic aspect relating to natural recovery of environmental capital are crucial to get his results on welfare and on the pollution level. It is worth noting that I don't make those assumptions in my model.

In contrast to my model, some papers of this literature study workers' adjustment costs, as in Karp et Paul (2007). In their small open economy with two sectors, where the pollutants of one sector affect the other sector, workers face an adjustment cost when they change from one sector to the other. The objective of their paper is to study the joint dynamics of labour reallocation, therefore there is no direct results about the effects on pollution.

My paper is not necessarily innovative in the techniques used, but it is relatively new in the subject addressed. Few papers have modelled an economy with migration, trade and environmental issues. The existing papers impose very strong and restrictive assumptions, that I don't impose. In my paper, I aim to study how the introduction of migration in a model of trade and environment effects pollution.

3.3 Model with regional differences in polluting externalities ($\lambda_1 > \lambda_2$)

The horizon is infinite and time is continuous. I consider a country composed of two regions, $i = \{1,2\}$. There are three production factors: skilled (L_s) and unskilled (L_u) workers, and environmental capital. Since more educated people are more likely to move, I assume that skilled workers are mobile across regions and unskilled are not.[6] The environmental capital, K^i, that represents the aggregate of all natural resources of a region, is given at any moment t, but may evolve over time. Changes in environmental capital in region i is caused by flows of pollution in that region, Z^i, and by the recovery rate of the environment, g:

$$\frac{\partial K^i}{\partial t} = g\left(\bar{K} - K^i\right) - Z^i,$$

where $\bar{K}$ is the natural level of environmental capital. Each region has its own environmental capital, with the same regeneration rate and natural steady state. Therefore, the difference between K in both regions is determined by the pollutants. I assume that there is no transboundary pollution.

There are two sectors: the manufacturing sector (M), not affected by the stock of environmental capital, and the agricultural sector (A), affected by the stock of the environmental capital. Both regions have the same production technologies in each sector. Skilled and unskilled workers are needed in the production of manufactures. Skilled and unskilled workers are not perfectly substitutable. I assume for simplicity that the production function in the manufacturing sector is a Cobb-Douglas:

$$M_i = \left(L_s^i\right)^{\frac{1}{2}} \left(L_{u,M}^i\right)^{\frac{1}{2}}.$$

The environmental capital and the unskilled workers are required in the production of agriculture:

$$A^1 = F(K^1)L_{u,A}^1 = \left(K^1\right)^{\varepsilon} L_{u,A}^1; \quad A^2 = F(K^2)L_{u,A}^2 = \left(K^2\right)^{\varepsilon} L_{u,A}^2.$$

[6] Workers who migrate from region i to region j keep their wage in region j. I do not explore the possibility that workers who migrate from region i to region j send their wage back to region i.

The sensitive environmental capital sector represents the aggregation of all sectors that are affected by the environmental capital. The effect on one particular sector can be positive or negative, however the overall effect is assumed to be negative. Agriculture is a perfect example to illustrate the ambiguous effect of a variation in the environmental capital. The increase in the temperature can have a direct positive impact on Quebec's agriculture by transforming uncultivable fields in cultivable ones, however, it could also have a negative impact by its effect on mosquitos,..., and on natural catastrophes, (Bélanger et Bootsma (2003)).

The manufacturing sector pollutes. The flow of pollutants in region $i = \{1,2\}$, is given by Z^i. Pollution is a by-product:

$$Z^i = \lambda^i M^i.$$

The difference between the two regions is determined by λ^i. If $\lambda^1 > \lambda^2$, the effect of manufacturing production on the environmental capital is larger in region 1 than in region 2. This assumption reflects the fact that region 1 uses a more pollutant technology than region 2 does, (i.e., brown energy vs green one). Data from Statistics Canada[7] shows that manufacturing industries in Alberta use less green energy than Quebec. Quebec uses more hydroelectricity than Alberta does and this type of energy has a smaller impact on the environmental capital than other sources. Hence, I assume that one unit of manufacture produced in region 1 affects more the environmental capital than in region 2.

In each region, L_u and L_s are the endowments of unskilled and skilled labour respectively and are fixed. Note that since skilled labour is mobile, we only have one aggregate full-employment condition.

I assume that pollutants have no direct impact on consumers' utility. The utility of a representative agent is given by:

$$U = b_m \ln M + b_a \ln A,$$

where b_m and b_a are the shares of spending on M and A ($b_m + b_a = 1$). I assume that

[7]Reference: Report No. 57-003-X, StatCan (2012)

consumers in both regions have the same utility.

As a benchmark, autarky is first studied.[8] The individual effects of interregional trade and then migration are discussed in Sections 3.4.2 and 3.4.3 To evaluate how interregional trade affects the pattern of migration and the environmental capital, the equilibrium with migration and trade is lastly examined.

3.4 Equilibrium

I am interested in the characterization of the steady state equilibrium. For that purpose I characterize first the temporary equilibrium for given environmental capital. An allocation is a temporary equilibrium if it solves the firms' maximization profit problem, if it solves the consumers' utility maximization problem, and if all markets (manufacturing market, agricultural market, and labour market) are cleared. The environmental capital evolves over time according to its law of motion. Firms' and consumers' decisions do not affect current environmental capital. However, they have an indirect impact on future environmental capital through their influences on the laws of motion. The economy reaches the stationary equilibrium, when the environmental capital is stable in both regions. Therefore, the steady state equilibrium is the allocation that satisfies all the temporary equilibrium conditions and at which the environmental capital in both regions stays constant over time.

I can characterize the steady state equilibrium that way because firms' and consumers' decisions do not affect the current period state variables. The only link between periods, the environmental capital, is not a choice variable.

I assume that initially both regions are endowed with L_s units of skilled workers. I normalize the price of agriculture to 1.

3.4.1 Autarky

In this section, I describe the equilibrium when skilled workers are not mobile across regions and when there is no trade between regions.

[8] The autarky is a case in which there is neither mobility of skilled workers nor trade between regions.

The firm's profit maximization problem in the manufacturing sector in region $i = 1,2$ is:

$$\max_{L_s^i, L_{u,M}^i} p^i \left(L_s^i\right)^{\frac{1}{2}} \left(L_{u,M}^i\right)^{\frac{1}{2}} - w_s^i L_s^i - w_u^i L_{u,M}^i. \tag{3.1}$$

The first order conditions (FOC) for this sector are:

$$\frac{p^i}{2}\left(\frac{L_s^i}{L_{u,M}^i}\right)^{\frac{1}{2}} = w_u^i \quad \text{and} \quad \frac{p^i}{2}\left(\frac{L_{u,M}^i}{L_s^i}\right)^{\frac{1}{2}} = w_s^i. \tag{3.2}$$

The firm's profit maximization problem in the agricultural sector in region $i = 1,2$ is:

$$\max_{L_{u,A}^i} \left(K^i\right)^{\varepsilon} L_{u,A}^i - w_u^i L_{u,A}^i. \tag{3.3}$$

The first order condition is:

$$\left(K^i\right)^{\varepsilon} = w_u^i. \tag{3.4}$$

In region $i = 1,2$, the consumers' utility maximization problem under budget constraint is:

$$\max_{M^i, A^i} b_m \ln M^i + b_a \ln A^i \quad \text{s.t} \quad p^i M^i + A^i = I^i, \tag{3.5}$$

where I^i represents the consumers' income. The income is uniquely composed of wages, $I^i = w_u^i L_u + w_s^i L_s^i$.

The solution of the preceding maximization problem gives the consumers' demand in region $i = 1,2$:

$$M^i = \frac{b_m I^i}{p^i}; \quad A^i = b_a I^i. \tag{3.6}$$

The mobility of unskilled workers across sectors implies that the wage of unskilled workers in both sectors within a region is equal:

$$\frac{p^i}{2}\left(\frac{L_s^i}{L_{u,M}^i}\right)^{\frac{1}{2}} = w_u^i = \left(K^i\right)^{\varepsilon} \Rightarrow p^i = 2\left(K^i\right)^{\varepsilon}\left(\frac{L_{u,M}^i}{L_s^i}\right)^{\frac{1}{2}}. \tag{3.7}$$

The combination of first order conditions in the manufacturing sector and (3.7) provides the following condition on wage of skilled workers:

$$w_s^i = \quad \frac{p^i}{2}\left(\frac{L_{u,M}^i}{L_s^i}\right)^{\frac{1}{2}} = \left(K^i\right)^{\varepsilon}\left(\frac{L_{u,M}^i}{L_s^i}\right)^{\frac{1}{2}}\left(\frac{L_{u,M}^i}{L_s^i}\right)^{\frac{1}{2}} = \left(K^i\right)^{\varepsilon}\frac{L_{u,M}^i}{L_s^i}. \tag{3.8}$$

Under the assumptions that both regions have initially the same endowments of workers and that skilled workers are not mobile across regions, in equilibrium each region has L_s skilled workers. The full employment conditions (FEC) imply that all skilled workers are hired in the manufacturing sector, $L_s^1 = L_s^2 = L_s$, and:

$$L_u = L_{u,M}^i + L_{u,A}^i, \quad i = 1,2.$$

The market clearing conditions (MCC) ensure that the demand equals the supply in all markets. In autarky, the demand of a good in region $i = 1,2$ must equals the supply of that good in that particular region. This condition has to hold for every good in the economy. The MCC in manufacturing market is:

$$\left(L_s^i\right)^{\frac{1}{2}}\left(L_{u,M}^i\right)^{\frac{1}{2}} = \frac{b_m\left(w_u^i L_u + w_s^i L_s^i\right)}{p^i}, \tag{3.9}$$

and in agricultural market, the MCC is:

$$\left(K^i\right)^{\varepsilon} L_{u,A}^i = b_a\left(w_u^i L_u + w_s^i L_s^i\right). \tag{3.10}$$

Replacing $L_{u,A}^i$, w_u^i, w_s^i, and L_s^i in the MCC condition in the agricultural sector provides the equilibrium allocation of unskilled workers among sectors:

$$\begin{aligned}
\left(K^i\right)^{\varepsilon} L^i_{u,A} &= b_a\left(w^i_u L_u + w^i_s L^i_s\right)\\
\left(K^i\right)^{\varepsilon}\left(L_u - L^i_{u,M}\right) &= b_a\left(w^i_u L_u + w^i_s L_s\right)\\
\left(K^i\right)^{\varepsilon}\left(L_u - L^i_{u,M}\right) &= b_a\left(w^i_u L_u + \left(K^i\right)^{\varepsilon}\frac{L^i_{u,M}}{L_s}L_s\right)\\
\left(K^i\right)^{\varepsilon}\left(L_u - L^i_{u,M}\right) &= b_a\left(\left(K^i\right)^{\varepsilon}L_u + \left(K^i\right)^{\varepsilon}\frac{L^i_{u,M}}{L_s}L_s\right)\\
L^i_{u,M} &= \frac{\frac{1}{2}b_m L_u}{\frac{1}{2}b_m + b_a}.
\end{aligned}$$

Then, the equilibrium price system is given by:

$$\left(p^i, w^i_s, w^i_u\right) = \left(2\left(\frac{L_u}{L_s}\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}}\left(K^i\right)^{\varepsilon}, \frac{L_u}{L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\left(K^i\right)^{\varepsilon}, \left(K^i\right)^{\varepsilon}\right)$$

and the equilibrium allocation by:

$$\begin{aligned}
&(L^i_s, L^i_{u,M}, L^i_{u,A}, A^i, M^i) =\\
&\left(L_s, L_u\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}, L_u\frac{b_a}{\frac{1}{2}b_m + b_a}, L_u\frac{b_a}{\frac{1}{2}b_m + b_a}\left(K^i\right)^{\varepsilon}, (L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}}\right).
\end{aligned}$$

The allocation of unskilled workers across sectors is the same in both regions. The difference in λ does not affect the allocation of workers. Therefore, production of manufactures in both regions are equal. However, the production of agriculture does not depend only on workers, but also on the environmental capital, hence the agricultural production varies from one region to the other.

The regional pollution depends on the manufacturing production and on λ. Under the assumption that $\lambda^1 > \lambda^2$:

$$Z^1 = \lambda^1 M^1 = \lambda^1 (L_s)^{\frac{1}{2}} \left(\frac{\frac{1}{2} b_m L_u}{\frac{1}{2} b_m + b_a} \right)^{\frac{1}{2}} > \lambda^2 (L_s)^{\frac{1}{2}} \left(\frac{\frac{1}{2} b_m L_u}{\frac{1}{2} b_m + b_a} \right)^{\frac{1}{2}} = \lambda^2 M^2 = Z^2.$$

Even though the production of manufactures is the same in both regions, the flows of pollution in region 1 is larger than in region 2.

Steady state

For this economy to be at a steady state, the temporary equilibrium conditions must be satisfied and the environmental capital must be stable. The evolution of the environmental capital over time is given by:[9]

$$\frac{\partial K^i(MA, MA; a)}{\partial t} = g \left(\bar{K} - K^i \right) - Z^i = g \left(\bar{K} - K^i \right) - \lambda^i M^i.$$

Therefore, the steady state environmental capital is:[10]

$$\frac{\partial K^i(MA, MA; a)}{\partial t} = 0$$
$$\Rightarrow \mathscr{K}^i(MA, MA; a) = \bar{K} - \frac{\lambda^i}{g} M^i = \bar{K} - \frac{\lambda^i}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} \left(\frac{\frac{1}{2} b_m}{\frac{1}{2} b_m + b_a} \right)^{\frac{1}{2}}.$$

Under the assumption that $\lambda^1 > \lambda^2$, the steady sate environmental capital is strictly larger in region 2 than in region 1, $\mathscr{K}^1(MA, MA; a) < \mathscr{K}^2(MA, MA; a)$. Even though manufacturing production depends neither on the environmental capital nor on λ, region 1 produces more pollutants as a by-product than region 2. The lower steady state environmental capital in region 1 reflects the larger impact of manufacturing production on the environment. If one region pollutes more than the other, then the impact on the productivity of the agricultural sector is larger in the former region (with the larger λ). The

[9]For the upcoming part of the paper, $X^i(\cdot, \cdot; \cdot)$ refers to the variable X in region i. The first two arguments describe the production in region 1 and 2 respectively (M refers to the production of manufactures and A refers to the production of agriculture), and the last argument indicates the type of regime (autarky, interregional trade, migration, or interregional trade and migration).

[10]$\mathscr{K}$ is the environmental capital at the steady state equilibrium. I assume that $\mathscr{K}^i(MA, MA; a) > 0$.

production of manufactures is the same in both regions, however, more environmental sensitive goods are produced in region 2. The relative price of manufactures is larger in region 2 than in region 1.

3.4.2 Interregional Trade

To evaluate how trade between regions affects the benchmark model, I now introduce interregional trade into the model, assuming that skilled workers are not mobile across regions. An allocation is a temporary equilibrium under the same condition as the autarky case. The firms' maximization profit problem, the consumers' utility maximization problem, and the MCC in labour market are identical to the benchmark model. However, the MCC in the manufacturing and agricultural industries are sensibly different. The total consumption of a particular good in both regions must equal the total amount of that good produced in both regions. For an allocation to be a steady state equilibrium, all the temporary equilibrium conditions must hold and the environmental capital in both regions should be constant in time.

If both goods can be traded between the two regions, then the price, p, must be equalized across regions.

Without trade, the two regions produce both goods in equilibrium, however trade allows regions to be specialized in equilibrium. Without mobility of skilled workers and under the FEC, both regions produce manufactures in equilibrium, however, it is possible that one of the two regions does not produce agriculture. The two possibilities are studied below, first the diversified and then the specialized case.

3.4.2.1 Both regions are diversified.

I consider first the possibility that, in equilibrium, both goods are produced in both regions.

Temporary equilibrium

The firms' profit maximization problem and the consumers' utility maximization are identical to (3.1), (3.3), and (3.5). Therefore, the FOC are given by (3.2), (3.4), and

(3.6).

The wage of unskilled workers in each region is $w_u^i = \left(K^i\right)^{\varepsilon}$ and the wage of skilled workers is given by (3.8).

The full employment conditions are the same as the autarky case, therefore, at the temporary equilibrium: $L_s^1 = L_s^2 = L_s$ and $L_{u,M}^i + L_{u,A}^i = L_u$ for $i = 1,2$.

With trade between regions, the market clearing conditions imply that the total supply of each good equals the total demand. The total supply in the manufacturing sector is the sum of manufactures produced in both regions:

$$M_s = M_s^1 + M_s^2 = L_s \frac{p}{2} \left(\frac{1}{\left(K^1\right)^{\varepsilon}} + \frac{1}{\left(K^2\right)^{\varepsilon}} \right).$$

The total supply in the agricultural sector is:

$$A_s = A_s^1 + A_s^2 = \left(L_u - \left(\frac{p}{2\left(K^1\right)^{\varepsilon}} \right)^2 \right) \left(K^1\right)^{\varepsilon} + \left(L_u - \left(\frac{p}{2\left(K^2\right)^{\varepsilon}} \right)^2 \right) \left(K^2\right)^{\varepsilon}.$$

The total demand of manufactures (agriculture) is the sum of manufactures (agriculture) demanded in region 1 and 2:

$$M_d = \frac{b_m\left(I^1 + I^2\right)}{p}; \quad A_d = b_a\left(I^1 + I^2\right),$$

where,

$$I^i = w_u^i L_u^i + w_s^i L_s = \left(K^i\right)^{\varepsilon} L_u + \frac{p}{2} L_s \left(\frac{p}{2\left(K^i\right)^{\varepsilon}} \right), \quad i = 1,2.$$

The market clearing condition in the agricultural sector is:

$$\begin{aligned}
&A_d = A_s \\
&b_a\left(\left(K^i\right)^{\varepsilon} L_u + \frac{p}{2} L_s \left(\frac{p}{2\left(K^i\right)^{\varepsilon}} \right) + \left(K^i\right)^{\varepsilon} L_u + \frac{p}{2} L_s \left(\frac{p}{2\left(K^i\right)^{\varepsilon}} \right) \right) \\
&= \left(L_u - \left(\frac{p}{2\left(K^1\right)^{\varepsilon}} \right)^2 \right) \left(K^1\right)^{\varepsilon} + \left(L_u - \left(\frac{p}{2\left(K^2\right)^{\varepsilon}} \right)^2 \right) \left(K^2\right)^{\varepsilon}.
\end{aligned}$$

The preceding equality determines the interregional price, p:

$$p = 2\left(\frac{L_u}{L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\frac{\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}}{\frac{1}{\left(K^1\right)^{\varepsilon}} + \frac{1}{\left(K^2\right)^{\varepsilon}}}\right)^{\frac{1}{2}}. \tag{3.11}$$

The combination of (3.7) and (3.11) provides the allocation of unskilled workers in region $i = 1,2$:

$$\begin{aligned} L^i_{u,M} &= \left(\frac{p}{2\left(K^i\right)^{\varepsilon}}\right)^2 L_s \\ &= L_u \frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\frac{\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}}{\frac{1}{\left(K^1\right)^{\varepsilon}} + \frac{1}{\left(K^2\right)^{\varepsilon}}}\left(\frac{1}{\left(K^i\right)^{\varepsilon}}\right)^2. \end{aligned} \tag{3.12}$$

It is clear that the region with the lowest environmental capital allocates more unskilled workers to the manufacturing sector. Therefore, this region produces more manufactures. From (3.4), (3.8), and (3.11), the wages of skilled and unskilled workers in region $i = 1,2$ are respectively:

$$w^i_s = \frac{L_u}{L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\frac{\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}}{\frac{1}{\left(K^1\right)^{\varepsilon}} + \frac{1}{\left(K^2\right)^{\varepsilon}}}\frac{1}{\left(K^i\right)^{\varepsilon}}; \quad w^i_u = \left(K^i\right)^{\varepsilon}.$$

For the upcoming part of the paper, I assume that the region with the less polluting manufacturing industry, region 2, has a larger endowment of environmental capital than the other region (i.e., initially $K^1 < K^2$).

Some conditions are necessary for this allocation to be an equilibrium. The allocation of unskilled workers in each sector and in each region must lies between 0 and L_u:

$$0 < L^1_{u,M} < L_u; \quad 0 < L^2_{u,M} < L_u; \quad 0 < L^1_{u,A} < L_u; \quad 0 < L^1_{u,A} < L_u.$$

From (3.12), $L^i_{u,M} > 0$ for $i = 1,2$, therefore $L^i_{u,A} = L_u - L^i_{u,M} < L_u$ for $i = 1,2$. Since $L^1_{u,M} > L^2_{u,M}$, the restrictive condition is $L^1_{u,M} < L_u$. If $L^1_{u,M} < L_u$, then $L^2_{u,M} < L_u$ and $L^i_{u,A} > 0$ for $i = 1,2$.

Under the assumption that initially $K^1 < K^2$, the condition for this allocation to be a temporary equilibrium is:[11]

$$L^1_{u,M} = L_u \frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a} \frac{\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}}{\frac{1}{\left(K^1\right)^{\varepsilon}} + \frac{1}{\left(K^2\right)^{\varepsilon}}} \left(\frac{1}{(K^1)^{\varepsilon}}\right)^2 < L_u$$

$$\Rightarrow \frac{1}{2}b_m \left(K^2\right)^{\varepsilon} < b_a \left(K^1\right)^{\varepsilon} + \left(\frac{1}{2}b_m + b_a\right) \left(\frac{\left(K^1\right)^{\varepsilon}}{(K^2)^{\varepsilon}}\right)^2 \left(K^2\right)^{\varepsilon}$$

$$\Rightarrow K^2 < \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{1/\varepsilon} K^1. \tag{3.13}$$

If the endowment of the environmental capital in region 2 is larger than in region 1, does it necessarily imply that this relation holds in the future? Proposition 3.1 proves it.

Proposition 3.1. *Under the assumption that Condition (*3.13*) holds, if $K^1 < K^2$, then $\left(K^1\right)' \leq \left(K^2\right)'$.*

Proof of Proposition 3.1

Under the assumption that Condition (3.13) is satisfied, if $K^1 < K^2$, then there are more (less) manufactures produced in region 1 (region 2) with trade than in autarky:[12]

$$(L_s)^{\frac{1}{2}} \left(L_u \frac{\frac{1}{2}b_m}{b_a + \frac{1}{2}b_m} \frac{\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}}{\frac{1}{\left(K^1\right)^{\varepsilon}} + \frac{1}{\left(K^2\right)^{\varepsilon}}} \left(\frac{1}{(K^1)^{\varepsilon}}\right)^2\right)^{\frac{1}{2}} > (L_s)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m L_u}{b_a + \frac{1}{2}b_m}\right)^{\frac{1}{2}}.$$

Therefore, the higher (lower) bound of the evolution of the environmental capital with free trade when both regions are diversified, is the rate of environmental capital in region 1 (region 2) in autarky.

Since $\partial K^1(MA,MA;t)/\partial t$ and $\partial K^2(MA,MA;t)/\partial t$ exist for all K^1 and K^2 strictly larger than zero, K^1 and K^2 are continuous functions of time. Therefore, for K^1 to become larger than K^2, it would first have to be equal to K^2. However, if K^1 equals K^2, then the variation of the environmental capital in region 2 would be larger than the variation in

[11] Condition (3.13) represents simply the positive section of a quadratic equation.

[12] The proof for region 2 is straightforward.

region 1 and K^1 would become lower than K^2 again. This implies that $\left(K^1\right)' \leq \left(K^2\right)'$.
Q.E.D.

From Proposition 3.1, as long as Condition (3.13) holds, the environmental capital in region 1 remains lower than the one in region 2.

It is important to note that even if Condition (3.13) is not satisfied anymore, the environmental capital remains larger in region 2 than in region 1. It is shown in the next section that if along the transition path, inequality (3.13) does not hold anymore, then the equilibrium is characterized by the specialized allocation. Proposition (3.17) shows that the specialized allocation is also characterized by more (less) manufactures produced in region 1 (region 2) with trade than without.

The equilibrium price is strictly lower (larger) than the autarky price in region 2 (region 1). Therefore, region 2 imports manufactures and exports agriculture and region 1 the inverse. Not only that manufacturing industries in region 1 produce more than in region 2 in equilibrium, but they produce more than they do in autarky.[13] Therefore, more (less) pollution is produced as a by-product in region 1 (region 2), with interregional trade than without and region 1 is still the more polluting region. The aggregate output of manufactures increases with free trade and remains larger as the environmental capital evolves. Even though trade doesn't affect directly the aggregate production of agriculture, it affects it through its impacts on the environmental capital. The direction of the change of the aggregate production of agriculture depends on $\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}$. For trade to have a positive impact, the increase in the environmental capital in region 2 should compensate the reduction in region 1. Otherwise, if trade leads to a very important environmental degradation, the aggregate production possibility set might become smaller with trade than without. Therefore, trade is not always welfare improving.[14]

Steady state

At the steady state, the economy must satisfy all the previous temporary equilibrium conditions. Moreover, the environmental capital in both regions should be constant,

[13]Manufacturing output in region 2 is less than in autarky.

[14]This result is also obtained in Copeland et Taylor (1999).

$\partial K^i(MA,MA;t)/\partial t = 0$ for $i = 1,2$, with: [15]

$$\frac{\partial K^i(MA,MA;t)}{\partial t} = g\left(\bar{K} - K^i\right) - \lambda^i \left(L_s\right)^{\frac{1}{2}} \left(L_u\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} \left(\frac{(K^{-i})^{\varepsilon}}{(K^i)^{\varepsilon}}\right)^{\frac{1}{2}} = 0. \tag{3.14}$$

In Figure 3.1, the purple and the blue curves represent respectively:

$$\partial K^1(MA,MA;t)/\partial t = 0 \quad \text{and} \quad \partial K^2(MA,MA;t)/\partial t = 0.$$

The reflection of $\partial K^1(MA,MA;t)/\partial t = 0$ with respect to the 45 degree line is illustrated by the dashed purple curve. It is worth noting that the curve that represents $\partial K^2(MA,MA;t)/\partial t = 0$ lies above this dashed purple curve. For a diversified steady state equilibrium to exist, both curves should intersect in the blue area, which represents the area that satisfies Condition (3.13) and $K^1 < K^2$. Figure 3.1 represents one of the possible ways both curves intersect. The other possibilities are studied in Section 3.4.2.3 as well as the existence and the characterization of diversified steady state equilibrium.

Proposition 3.2. *If a specialized steady state exists, then the environmental capital in region 1 (region 2) is lower (larger) with interregional trade than without.*

Proof of Proposition 3.2

[15] For $i = 1$, Equation (3.14) becomes:

$$K^2 = \left(\frac{g}{\lambda}\right)^{\frac{2}{\varepsilon}} \left(\frac{1}{L_s L_u}\right)^{\frac{1}{\varepsilon}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - K^1\right)^{\frac{2}{\varepsilon}} K^1.$$

Those two functions are continuous, increasing if $K^i < \varepsilon\bar{K}/(2+\varepsilon)$ and concave if $K^i < 2\varepsilon\bar{K}/(2+\varepsilon)$. The function $\partial K^i(MA,MA;t)/\partial t = 0$ crosses the 45 degree line at $K^i = \mathscr{K}^i(MA,MA;a)$. The functions $\partial K^1(MA,MA;t)/\partial t = 0$ and $\partial K^2(MA,MA;t)/\partial t = 0$ cross the line $K^2 = (((1/2)b_m + b_a)/\,(1/2)b_m)^{1/\varepsilon} K^1$ at $K^1 = \bar{K} - \left(\lambda^1/g\right)(L_u)^{1/2}(L_s)^{1/2}$ and $K^2 = \bar{K} - \left(\lambda^2/g\right)(L_u)^{1/2}(L_s)^{1/2}\left(\frac{1}{2}b_m/\left(\frac{1}{2}b_m + b_a\right)\right)$ respectively. In Section 3.4.2.2 those two quantities are shown to be respectively $\mathscr{K}^1(M,MA;t)$ and $\mathscr{K}^2(M,MA;t)$. The function $\partial K^1(MA,MA;t)/\partial t = 0$ ($\partial K^2(MA,MA;t)/\partial t = 0$) passes through $(0,0)$ and $(\bar{K},0)$ ($(0,\bar{K})$).

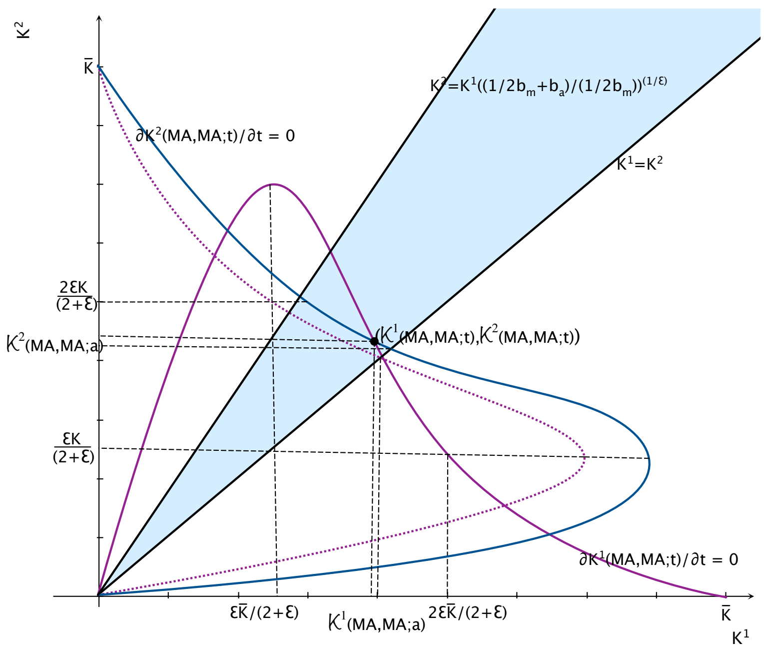

Figure 3.1: Diversified steady state equilibrium with trade

If $\left(K^1\right)^{\varepsilon} < \left(K^2\right)^{\varepsilon} < \left(K^1\right)^{\varepsilon} \left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)$:

$$\frac{\partial K^1(MA,MA;t)}{\partial t} < \frac{\partial K^1(MA,MA;a)}{\partial t},$$

and

$$\frac{\partial K^2(MA,MA;t)}{\partial t} > \frac{\partial K^2(MA,MA;a)}{\partial t},$$

hence, $\partial K^1(MA,MA;t)/\partial t = 0$ lies at the left hand side of $\partial K^1(MA,MA;a)/\partial t = 0$ and $\partial K^2(MA,MA;t)/\partial t = 0$ lies above $\partial K^2(MA,MA;a)/\partial t = 0$.

Therefore, if a diversified steady state exists:

$$\mathscr{K}^1(MA,MA;t) < \mathscr{K}^1(MA,MA;a) \quad \text{and} \quad \mathscr{K}^2(MA,MA;t) > \mathscr{K}^2(MA,MA;a).$$

Q.E.D.

3.4.2.2 Specialization

I consider now the case, where in equilibrium, one of the two regions is specialized in the manufacturing sector.

Temporary equilibrium

As it is mentioned above, both regions should produce manufactures at the equilibrium. The firms' profit maximization problem and the FOC in the manufacturing sector are given by (3.1) and (3.2). One of the two regions does not produce agriculture. In the region that produces agriculture, the firm's profit maximization problem and the FOC of that sector are given by (3.3) and (3.4). In the region that doesn't produce agriculture, the wage of unskilled workers in the agricultural sector must be lower than in the manufacturing sector. Otherwise, unskilled workers would want to work in the agricultural sector instead of in the manufacturing sector. Hence, this region would not be specialized. Proposition 3.3 shows that only region 1 can be specialized under the assumption that initially the environmental capital is larger in region 2 than in region 1.

Proposition 3.3. *If the initial environmental capital is larger in region 2 than in region 1, then only region 1 can be specialized.*

Proof of Proposition 3.3

Let's assume that region 2 does not produce agriculture and region 1 does. From (3.2) and (3.4), the wage of unskilled workers in the manufacturing sector should be larger than in the agricultural sector in region 2:[16]

$$\left(w_u^2\right)_M = \frac{p}{2}\left(\frac{L_s}{L_u}\right)^{\frac{1}{2}} \geq \left(K^2\right)^{\varepsilon} = \left(w_u^2\right)_A .$$

The wage of unskilled workers in region 1 would be determined by (3.7):[17]

$$w_u^1 = \frac{p}{2}\left(\frac{L_s}{L_{u,M}^1}\right)^{\frac{1}{2}} = \left(K^1\right)^{\varepsilon} .$$

[16] The firms' profit maximization problems are given by (3.1) and (3.3).
[17] The firms' profit maximization problems are given by (3.1) and (3.3).

Therefore,

$$\frac{p}{2}\left(\frac{L_s}{L_u}\right)^{\frac{1}{2}} \geq \left(K^2\right)^{\varepsilon} > \left(K^1\right)^{\varepsilon} = \frac{p}{2}\left(\frac{L_s}{L_{u,M}^1}\right)^{\frac{1}{2}}.$$

This inequality implies that $L_{u,M}^1 > L_u$, which is impossible. This condition is never satisfied.[18] Hence, region 2 always produces both goods. The only region that can be specialized in manufacture is region 1. *Q.E.D.*

If region 1 is specialized, from the FEC, $L_{u,M}^1 = L_u$ and $L_s^1 = L_s$. Hence, the manufacturing production in region 1 is $M_s^1 = (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}}$. One condition for this allocation to be an equilibrium is that the wage of unskilled workers in the manufacturing sector should be larger than in the agricultural sector, from (3.2) and (3.4):[19]

$$\left(w_u^1\right)_M = \frac{p}{2}\left(\frac{L_s}{L_u}\right)^{\frac{1}{2}} \geq \left(K^1\right)^{\varepsilon} = \left(w_u^1\right)_A. \tag{3.15}$$

The wage of unskilled workers in region 2 is determined by (3.7):[20]

$$w_u^2 = \frac{p}{2}\left(\frac{L_s}{L_{u,M}^2}\right)^{\frac{1}{2}} = \left(K^2\right)^{\varepsilon} \quad \Rightarrow \quad L_{u,M}^2 = \left(\frac{p}{2\left(K^2\right)^{\varepsilon}}\right)^2 L_s.$$

The FEC in region 2 are $L_u = L_{u,M}^2 + L_{u,A}^2$ and $L_s^2 = L_s$.

The market clearing conditions are very similar to the ones in the diversified case, however production in each sector in both regions differs. The total supply of each good is given by:

$$M^s = (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} + L_s \frac{p}{2\left(K^2\right)^{\varepsilon}}; \quad A^s = \left(L_u - \left(\frac{p}{2\left(K^2\right)^{\varepsilon}}\right)^2 L_s\right)\left(K^2\right)^{\varepsilon}.$$

From the consumers' utility maximization problem (3.5), the total demand of each good

[18] This result holds under the assumption that both regions have the same labour allocation, $L_u^1 = L_u^2 = L_u$.

[19] The firms' profit maximization problems are given by (3.1) and (3.3).

[20] The firms' profit maximization problems are given by (3.1) and (3.3).

is given by:[21]

$$M_d = b_m (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} + \frac{b_m}{2} \frac{p}{2(K^2)^{\varepsilon}} L_s + \frac{b_m}{p} (K^2)^{\varepsilon} L_u,$$
$$A_d = b_a p (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} + b_a \frac{p}{2} \frac{p}{2(K^2)^{\varepsilon}} L_s + b_a (K^2)^{\varepsilon} L_u.$$

The equalization of the total demand of manufactures and the total supply determines the equilibrium price:

$$M_d = M_s$$
$$b_m (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} + \frac{b_m}{2} \frac{p}{2(K^2)^{\varepsilon}} L_s + \frac{b_m}{p} (K^2)^{\varepsilon} L_u = (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} + L_s \frac{p}{2(K^2)^{\varepsilon}}$$
$$b_m L_u \left(K^2\right)^{\varepsilon} = b_a p (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} + p \frac{p}{2(K^2)^{\varepsilon}} L_s \left(\frac{1}{2} b_m + b_a\right). \tag{3.16}$$

The combination of (3.15) and (3.16) gives the condition for this allocation to be an equilibrium:

$$\frac{1}{2} b_m \left(K^2\right)^{\varepsilon} \geq b_a \left(K^1\right)^{\varepsilon} + \left(\frac{1}{2} b_m + b_a\right) \left(\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^2 \left(K^2\right)^{\varepsilon}. \tag{3.17}$$

It is worth noting that Conditions (3.17) and (3.13) are mutually exclusive and that Condition (3.17) implies that $K^1 < K^2$.
From (3.15) and the assumption that initially $K^1 < K^2$, the allocation of unskilled workers in region 2 is lower than in region 1:

$$L_{u,M}^2 = \left(\frac{p}{2(K^2)^{\varepsilon}}\right)^2 L_s < L_u = L_{u,M}^1.$$

Manufacturing production is larger in region 1 than in region 2, therefore, region 1 pollutes more than region 2. Proposition 3.4 describes the relation between the production with interregional trade and without when the specialization case is an equilibrium.

[21] The incomes are $I^1 = p (L_s)^{\frac{1}{2}} L_u^{\frac{1}{2}}$ and $I^2 = \frac{p}{2} \frac{p}{2(K^2)^{\varepsilon}} L_s + \left(K^2\right)^{\varepsilon} L_u$.

Proposition 3.4. *Under Condition (*3.17*), the manufacturing production in region 1 (region 2) is larger (lower) with interregional trade than without.*

Proof of Proposition 3.4

In region 1, the manufacturing production is larger with interregional trade than without:

$$(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}} > (L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}.$$

In region 2, the manufacturing production with interregional trade is lower than in autarky if:

$$p < \left(\frac{L_u}{L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}} 2\left(K^2\right)^{\varepsilon}.$$

This condition is always satisfied when this allocation is an equilibrium, otherwise Condition (3.17) does hold. *Q.E.D.*

Therefore, region 1 (region 2) pollutes more (less) with trade than without. From Proposition 3.4, it is straightforward to prove that:

$$\partial K^1(M,MA;t)/\partial t < \partial K^1(MA,MA;a)/\partial t,$$
$$\partial K^2(M,MA;t)/\partial t > \partial K^2(MA,MA;a)/\partial t.$$

By the same arguments used to prove Proposition 3.1, it is clear that the environmental capital in region 1 always remains lower than in region 2.

As in the diversified case, the equilibrium price is strictly lower (larger) than the autarky price in region 2 (region 1), hence region 2 (region 1) imports manufactures (agriculture) and exports agriculture (manufactures). The qualitative results of the diversified case that concern the production of manufactures hold in the specialized case. Trade doesn't induce instantaneous change in the production of agriculture in region 2, but due to the shut down of the agricultural industries in region 1, the aggregate production falls. However, the evolution of the environmental capital in that region conducts to an increase of the agricultural production in region 2. In the long run, this increase might compensate the reduction caused by the shutdown in region 1.

Steady state

At the steady state, the economy satisfies all the temporary equilibrium conditions and the environmental capital should be stable. The evolution of the environmental capital over time is given by:

$$\frac{\partial K^1(M,MA;t)}{\partial t} = g\left(\bar{K}-K^1\right) - Z^1 = g\left(\bar{K}-K^1\right) - \lambda^1 (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}},$$
$$\frac{\partial K^2(M,MA;t)}{\partial t} = g\left(\bar{K}-K^2\right) - Z^2 = g\left(\bar{K}-K^2\right) - \lambda^2 (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right).$$

Therefore, if the steady state equilibrium exists, it is characterized by:

$$\frac{\partial K^1(M,MA;t)}{\partial t} = 0 \Rightarrow \mathscr{K}^1(M,MA;t) = \bar{K} - \frac{\lambda^1}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}},$$
$$\frac{\partial K^2(M,MA;t)}{\partial t} = 0 \Rightarrow \mathscr{K}^2(M,MA;t) = \bar{K} - \frac{\lambda^2}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right).$$

The steady state equilibrium exists if $\left(\mathscr{K}^1(M,MA;t), \mathscr{K}^2(M,MA;t)\right)$ satisfies Condition (3.17), i.e.:

$$\bar{K} - \frac{\lambda^2}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{b_a+\frac{1}{2}b_m}\right) \geq \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \frac{\lambda^1}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}}\right).$$

In figure 3.2, the green and the purple curves represent one case with a specialized equilibrium and one without, respectively . The intersection of the green $\partial K^1(M,MA;t)/\partial t = 0$ and $\partial K^2(M,MA;t)/\partial t = 0$ satisfies Condition (3.17), therefore it is a specialized steady state equilibrium. Contrary to the green case, the intersection of the purple $\partial K^1(M,MA;t)/\partial t = 0$ and $\partial K^2(M,MA;t)/\partial t = 0$ does not satisfy Condition (3.17). Hence, in this particular case, there exists no specialized steady state equilibrium. It is worth noting that it does not mean that there exists no steady state with trade. As it is shown in Section 3.4.2.3, in this purple case, at the steady state both regions are diversified. Before proceeding with the complete characterization of the steady state equilibrium, Proposition 3.5 establishes the impact of interregional trade on the environmental capital in both regions.

Proposition 3.5. *If a specialized steady state exists, the environmental capital in region 1 (region 2) is lower (larger) with interregional trade than without and the environmental capital in region 1 is lower than in region 2.*

The proof of Proposition 3.5 is in the Appendix II.

3.4.2.3 Steady state

In this section, I combine the analysis of the diversified and specialized cases to characterize the steady state equilibrium with interregional trade. As in the temporary equilibrium, at the steady state, region 1 can be either specialized or diversified. I first prove the existence of a steady state equilibrium in Proposition 3.6. Once the existence is established, Proposition 3.7 shows that the steady state is not always unique. The economy can have a unique diversified steady state (DSSE), a unique specialized steady state equilibrium (SSSE), or the economy can have multiple steady states. The set of multiple equilibria is composed of one (SSSE) and at least one (DSSE). I end this section by describing parameters that are helpful in the determination of the steady state.

Proposition 3.6. *With interregional trade, a steady state equilibrium always exists.*

For the proof see the Appendix II.

Once the existence of a steady state equilibrium is established, the characterization of this equilibrium should be studied. This economy with interregional trade does not reach necessarily a unique steady state. For some parameter values, the economy has multiple steady states. I do not characterize exhaustive sets of parameters for which the economy has a unique (DSSE) or (SSSE), or has multiple steady state equilibria. However, I prove that such parameters do exist.

Proposition 3.7. *With interregional trade, the economy has either a unique (DSSE), a unique (SSSE) or multiple steady state equilibria.*

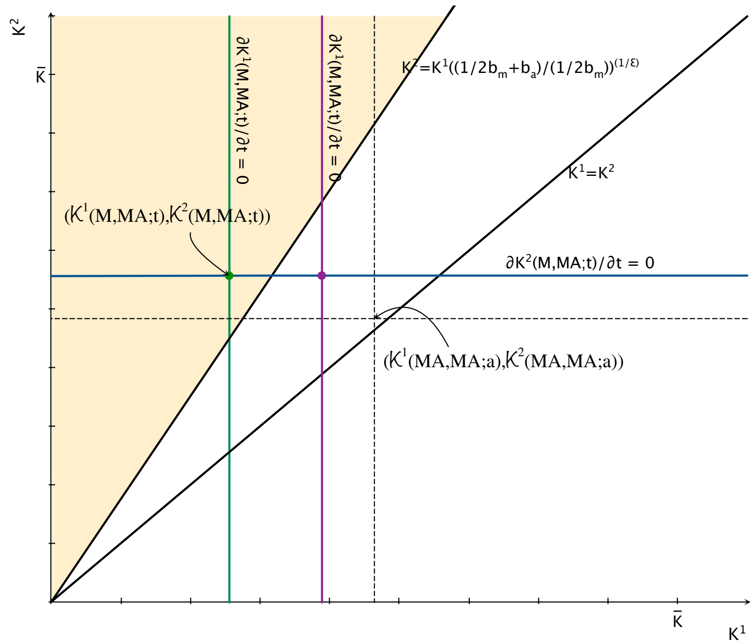

Figure 3.2: Specialized steady state equilibrium with trade

Appendix II provides the complete proof of Proposition 3.7.

The parameters λ^1, λ^2, and b_m are important to determine whether the economy has a (SSSE) and/or a (DSSE). The parameter b_m reflects the preference of consumers toward manufactures. This parameter has two opposite effects. On one side, it affects positively the production of manufactures in region 2, hence it affects negatively $\mathscr{K}^2(M,MA;t)$. The smaller $\mathscr{K}^2(M,MA;t)$ is, the smaller the chance that Condition (3.17) holds. On the other hand, it has an impact on Conditions (3.13) and (3.17), the larger the preference over manufactures is, the hardiest it is to satisfied Condition (3.13) and the easiest it is to satisfy (3.17). The parameter λ represents the force of the polluting externalities. The closer the effect of manufacturing production on environmental capital is between regions, the larger are the chance that the economy has a (DSSE).

It has been showed that production of manufactures increases with trade in region 1 and falls in region 2. It is easy to see that region 1 produces more manufactures in the specialized case that in the diversified. In region 2, it is the inverse. Since the pollution in each region is closely and positively related to the production of manufactures the qualitative results that concerned the production of manufactures hold for the pollution. Therefore, the difference between the environmental capital in region 1 and in region 2 increases with trade and is larger when region 1 is specialized.

3.4.3 Migration

What would happen if workers were mobile across regions? Would they stay in their region of origin or not? What are the forces that drive the decision to stay or to migrate? What are the effects of migration on pollution? This section addresses an answer to those questions.

The impact of migration on the benchmark model is studied by introducing mobility of skilled workers into the model, assuming that there is no trade between regions. The equilibrium conditions of the autarky case in addition to the condition of equality of skilled workers' utility between regions are the conditions for an allocation to be a temporary equilibrium. Aside from the preceding conditions, the steady state equilibrium requires also that the environmental capital in both regions remains unchanged over time. Without trade, in equilibrium it is not possible that all skilled workers stay in the same region. Suppose that there is no skilled workers in region i, therefore there would be no production of manufactures in this region. Without trade unskilled workers left in the region would consume only agriculture. An allocation where consumers consume only one of the two goods is not an equilibrium. Therefore, in equilibrium, there must be skilled workers in both regions and hence, both regions must produce both goods.

Temporary equilibrium

Firms' decisions in both sectors and consumers' decisions are the same as the autarky case and are described by (3.1), (3.2), (3.3), (3.4), (3.5), and (3.6). Mobility of unskilled workers across sectors implies equation (3.7) and the wage of skilled workers in region $i = 1,2$ is described by (3.8). The full employment condition of unskilled workers imposes that the sum of unskilled workers in both sectors equals L_u in both regions, as the autarky case:

$$L_u = L^i_{u,M} + L^i_{u,A}, \quad i = 1,2.$$

The FEC of skilled workers is different from the autarky, where skilled workers are not mobile across regions. With mobility across regions, the sum of skilled workers working in both regions must equal the total endowment of skilled workers in the economy, $2L_s$:

$$2L_s = L_s^1 + L_s^2.$$

Without trade between regions, the conditions that ensure that the demand equals the supply in manufacturing and agricultural sectors are the same as the autarky case: (3.9) and (3.10). Therefore, the equilibrium allocation of unskilled workers across sectors within a region is the same as in autarky:

$$L_{u,M}^i = L_u \frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}; \quad L_{u,A}^i = L_u \frac{b_a}{\frac{1}{2}b_m + b_a}. \tag{3.18}$$

The allocation of unskilled workers across sectors within a region is independent of the amount of skilled labour and the environmental capital.
The combination of (3.7), (3.8), and (3.18) provides the wage of skilled workers and the relative price of manufactures in region $i = 1,2$:

$$w_s^i = \frac{L_u}{L_s^i} \frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a} \left(K^i\right)^{\varepsilon}; \quad p^i = 2\left(\frac{L_u}{L_s^i}\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} \left(K^i\right)^{\varepsilon}.$$

Skilled workers have the possibility to migrate from one region to the other. They decide their working region by comparing the welfare they would get in both regions. They chose to migrate to region 1 if:

$$V^1(w_s^1, p^1) \geq V^2(w_s^2, p^2),$$

where $V^i(w_s^i, p^i) = \ln(w_s^i) - b_m \ln(p^i) + C$.[22]
At the equilibrium, the allocation of skilled workers across regions is determined by the condition that the indirect utility function of skilled workers in both regions must be

[22] $C = b_m \ln(b_m) + b_a \ln(b_a)$

equal:

$$
\begin{aligned}
V^2(w_s^2, p^2) &= V^1(w_s^1, p^1) \\
\ln\left(\frac{L_u}{L_s^2}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(K^2\right)^{\varepsilon}\right) - b_m \ln\left(2\left(\frac{L_u}{L_s^2}\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\left(K^2\right)^{\varepsilon}\right) & \\
= \ln\left(\frac{L_u}{L_s^1}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(K^1\right)^{\varepsilon}\right) - b_m \ln\left(2\frac{L_u}{L_s^1}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\left(K^1\right)^{\varepsilon}\right) & \\
\ln\left(\left(K^2\right)^{\varepsilon}\right) - \ln\left(L_s^2\right) - b_m \ln\left(\left(K^2\right)^{\varepsilon}\right) + \frac{1}{2}b_m \ln\left(L_s^2\right) & \\
= \ln\left(\left(K^1\right)^{\varepsilon}\right) - \ln\left(L_s^1\right) - b_m \ln\left(\left(K^1\right)^{\varepsilon}\right) + \frac{1}{2}b_m \ln\left(L_s^1\right) & \\
b_a \ln\left(\left(K^2\right)^{\varepsilon}\right) - \left(\frac{1}{2}b_m + b_a\right)\ln\left(L_s^2\right) &= b_a \ln\left(\left(K^1\right)^{\varepsilon}\right) - \left(\frac{1}{2}b_m + b_a\right)\ln\left(L_s^1\right) \\
b_a \ln\left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right) &= \left(\frac{1}{2}b_m + b_a\right)\ln\left(\frac{L_s^2}{2L_s - L_s^2}\right) \\
\ln\left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}} &= \ln\left(\frac{L_s^2}{2L_s - L_s^2}\right) \\
\left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}} &= \frac{L_s^2}{2L_s - L_s^2} \\
2L_s\left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}} &= L_s^2\left(1 + \left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}\right).
\end{aligned}
$$

The allocation of skilled workers in region 2 is given by:

$$
L_s^2 = \frac{2L_s\left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}}{1 + \left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}} = \frac{2L_s}{1 + \left(\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}}, \tag{3.19}
$$

and in region 1 is given by:

$$L_s^1 = \frac{2L_s \left(\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}}{1+\left(\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}}. \tag{3.20}$$

Under the assumption that initially $K^1 < K^2$ and the endowment of skilled workers in both regions is L_s, the mobility of skilled workers leads to a migration from region 1 to region 2, $L_s^2 > L_s^1$. From (3.19) and (3.20) the temporary equilibrium wage of skilled workers and price in both regions can be characterized:

$$w_s^2 = \frac{L_u}{2L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(K^2\right)^{\varepsilon}\left(1+\left(\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}\right); \quad p^2 = 2\left(K^2\right)^{\varepsilon}\left(\frac{L_{u,M}^2}{L_s^2}\right)^{\frac{1}{2}},$$

$$w_s^1 = \frac{L_u}{2L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(K^1\right)^{\varepsilon}\left(1+\left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m+b_a}}\right); \quad p^1 = 2\left(K^1\right)^{\varepsilon}\left(\frac{L_{u,M}^1}{L_s^1}\right)^{\frac{1}{2}}.$$

If $K^1 < K^2$, there would be more manufactures produced in region 2 than in region 1, because the allocation of unskilled workers across sectors is the same in both regions and there is more skilled workers in region 2, $L_s^1 < L_s^2$. It is important to note that even though region 2 produces more manufactures than region 1, there might be less pollution in region 2 than in region 1, since $\lambda^1 > \lambda^2$.
Is it sufficient for K^2 to remain larger than K^1 that the environmental capital in region 2 is larger that in region 1 initially? Proposition 3.8 addresses this question.

Proposition 3.8. *If $K^1 < K^2$, then the environmental capital in region 1 always remains lower than in region 2.*

Proof of Proposition 3.8
Since $\partial K^1(MA,MA;m)/\partial t$ and $\partial K^2(MA,MA;m)/\partial t$ exist for all K^1 and K^2 strictly larger than zero, K^1 and K^2 are continuous functions of time. Therefore, for K^1 to become larger than K^2, it would first be equal to K^2. However, if K^1 equals K^2, then

$L_s^1 = L_s^2 = L_s$. Hence, the manufacturing production would be the same in both regions and the production of pollution as a by-product would be lower in region 2 than in region 1. Therefore, the variation of the environmental capital is lower in region 1 than in region 2:

$$\frac{\partial K^1(MA,MA;m)}{\partial t} < \frac{\partial K^2(MA,MA;m)}{\partial t}.$$

Hence, the environmental capital in region 1 remains lower than in region 2. *Q.E.D.*

Steady state

The evolution of the environmental capital in each region is described by:

$$\frac{dK^1(MA,MA;m)}{dt} = g\left(\bar{K} - K^1\right) - \lambda^1 \left(2L_s\right)^{\frac{1}{2}} \left(L_u\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} \left(1 + \left(\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m + b_a}}\right)^{-\frac{1}{2}},$$

$$\frac{dK^2(MA,MA;m)}{dt} = g\left(\bar{K} - K^2\right) - \lambda^2 \left(2L_s\right)^{\frac{1}{2}} \left(L_u\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} \left(1 + \left(\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{\frac{b_a}{\frac{1}{2}b_m + b_a}}\right)^{-\frac{1}{2}}.$$

At the steady state equilibrium all the temporary equilibrium conditions must be satisfied and the environmental capital in both regions must be stable:

$$\partial K^1(MA,MA;m)/\partial t = 0 \quad \text{and} \quad \partial K^2(MA,MA;m)/\partial t = 0.$$

The following proposition establishes the relation between the environmental capital in region 1 and in region 2.

Proposition 3.9. *With migration, the economy reaches a unique steady state. At this steady state, the environmental capital in region 1 is lower than in region 2.*

Proof of Proposition 3.9

The function $\partial K^1(MA, MA; m)/\partial t = 0$ passes through:

$$\left(\bar{K} - \frac{\lambda^1}{g} (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}} \left(\frac{b_m}{\frac{1}{2} b_m + b_a} \right)^{\frac{1}{2}}, 0 \right) \quad \text{and} \quad \left(\mathscr{K}^1(MA, MA; a), \mathscr{K}^1(MA, MA; a) \right),$$

and the function $\partial K^2(MA, MA, m)/\partial t = 0$ passes through:

$$\left(0, \bar{K} - \frac{\lambda^2}{g} (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}} \left(\frac{b_m}{\frac{1}{2} b_m + b_a} \right)^{\frac{1}{2}} \right) \quad \text{and} \quad \left(\mathscr{K}^2(MA, MA; m), \mathscr{K}^2(MA, MA; m) \right).$$

In the interval $[0, \bar{K}]$, the function $\partial K^i(MA, MA; m)/\partial t = 0$ is continuous, crosses the 45 degrees line only once and is strictly increasing with respect to K^i , $i = 1, 2$. Hence, both functions cross one with each other only once as it is illustrated in Figure 3.3. Therefore, there exists a unique steady state equilibrium. Since $\mathscr{K}^1(MA, MA; a) < \mathscr{K}^2(MA, MA; a)$, the functions $\partial K^1(MA, MA; m)/\partial t = 0$ and $\partial K^2(MA, MA; m)/\partial t = 0$ cross above the 45 degree line, therefore:

$$\mathscr{K}^1(MA, MA; m) < \mathscr{K}^2(MA, MA; m).$$

Q.E.D.

Once the relation between the environmental capital in region 1 and in region 2 is established, I specify the impact of migration on the environmental capital in both periods in Proposition 3.10.

Proposition 3.10. *At the steady state, the environmental capital in region 1 (region 2) is larger (lower) with migration than without.*

Proof of Proposition 3.10

If $K^1 < K^2$, more (less) manufactures are produced in region 2 (region 1) with the mo-

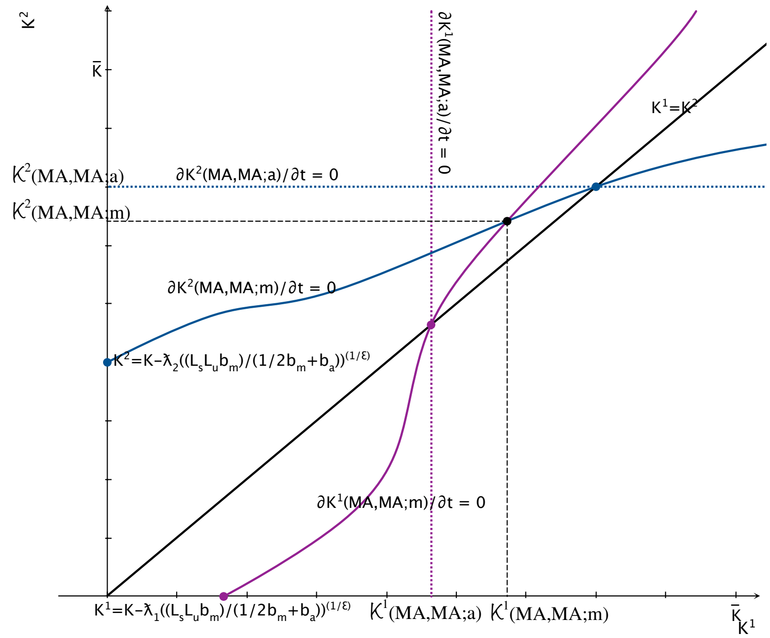

Figure 3.3: Steady state with migration

bility of skilled workers than without, therefore:

$$\frac{\partial K^1(MA,MA;a)}{\partial t} < \frac{\partial K^1(MA,MA;m)}{\partial t}; \quad \frac{\partial K^2(MA,MA;a)}{\partial t} > \frac{\partial K^2(MA,MA;m)}{\partial t}.$$

For $K^1 < K^2$, $\partial K^1(MA,MA;a)/\partial t = 0$ lies at the left of $\partial K^1(MA,MA;m)/\partial t = 0$ and $\partial K^2(MA,MA;a)/\partial t = 0$ lies above $\partial K^2(MA,MA;m)/\partial t = 0$. Therefore:

$$\mathscr{K}^1(MA,MA;a) < \mathscr{K}^1(MA,MA;m) \quad \text{and} \quad \mathscr{K}^2(MA,MA;a) > \mathscr{K}^2(MA,MA;m).$$

Q.E.D.

At the steady state, there are more skilled workers in region 2 than in region 1 and the allocation of unskilled workers between sectors is the same in both regions, therefore more manufactures are produced in region 2 than in region 1. Even though more manufactures are produced in region 2 than in region 1, the environmental capital remains larger in the former region. More (less) manufactures is produced in region 2 (region

1) with the mobility of workers than without, this is why the steady state environmental capital in region 2 (region 1) is lower (larger) with the mobility compared to the autarky case.

It is worth noting that interregional trade and migration have opposite effects on the environmental capital. In region 2 (region 1), trade affects positively (negatively) and migration affects negatively (positively) the environmental capital. Migration reduces the gap between the environmental capital and interregional trade increases it. Which of the migration and the trade effects would be stronger when there are interregional trade and migration in the economy? Section 3.4.4 addresses this question.

3.4.4 Interregional Trade and Migration

I introduce migration and interregional trade in the benchmark model to evaluate how the environmental capitals are affected.

An allocation is a temporary equilibrium with mobility of skilled workers and interregional trade if firms maximize their profit, if consumers maximize their utility and if all markets are cleared. When skilled workers are mobile, the FEC become:

$$2L_s = L_s^1 + L_s^2; \quad L_u = L_{u,M}^i + L_{u,A}^i, \quad i = 1,2.$$

The MCC in the manufacturing and agricultural sectors imply that the total consumption of a particular good in both regions equals the total production in both regions. Note that since both goods can be traded, the relative price of manufactures must be the same in both regions, p.

In addition to the temporary equilibrium, the steady state equilibrium requires the environmental capital in both regions to be stable.

Under the assumption that initially the environmental capital in region 1 is lower than in region 2, Proposition 3.11 characterizes the steady state equilibrium, whereas Proposition 3.12 established its existence. The proof of each proposition is provided in the Appendix II.

Proposition 3.11. *With interregional trade and migration, if initially $K^1 < K^2$:*

- *if $\frac{1}{2}b_m > b_a$, region 1 is specialized in the production of manufactures and region 2 is diversified;*
- *if $\frac{1}{2}b_m = b_a$, region 1 is specialized in the production of manufactures and region 2 is specialized in the production of agriculture;*
- *if $\frac{1}{2}b_m < b_a$, region 1 is either diversified or specialized in the production of manufactures and region 2 is specialized in the production of agriculture.*

Proposition 3.12. *With migration and interregional trade, a steady state equilibrium always exists. Moreover:*

- *if $\frac{1}{2}b_m > b_a$, at the unique steady state, region 1 is specialized in the production of manufactures and region 2 is diversified;*
- *if $\frac{1}{2}b_m = b_a$, at the unique steady state, region 1 is specialized in the production of manufactures and region 2 is specialized in the production of agriculture;*
- *if $\frac{1}{2}b_m < b_a$, the steady state is not necessarily unique. Region 1 is either diversified or specialized in the production of manufactures and region 2 is specialized in the production of agriculture.*

Proposition 3.13 states that the mobility of skilled workers and interregional trade affects negatively the stationary environmental capital in region 1, whereas the effect is positive in region 2.

Proposition 3.13. *The stationary environmental capital in region 1 (region 2) is lower (larger) with mobility of skilled workers and interregional trade than without.*

For the proof of Proposition 3.13 see the proof of Proposition 3.11.
The stationary equilibrium is characterized by one of the three cases described in the proof of Proposition 3.11 depending on parameters and the initial endowments of environmental capital in both regions. Regardless of which of the three cases is the stationary equilibrium, the stationary environmental capital in region 1 (region 2) is lower (larger) with migration and interregional trade compared to the autarky situation.

3.5 Comparison

The current environmental capital is essential in the determination of the trade pattern. The region with the largest environmental capital has a comparative advantage in the production of agriculture. It has been shown in Section 3.4.2 that regardless whether regions are diversified or specialized, the introduction of interregional trade in the autarky model increases the gap between the environmental capital in region 1 and in region 2:[23]

$$\mathscr{K}^1(\cdot,\cdot;t) < \mathscr{K}^1(MA,MA;a) < \mathscr{K}^2(MA,MA;a) < \mathscr{K}^2(\cdot,\cdot;t).$$

Region 1 has a comparative advantage in the manufacturing sector, due to its lower endowment of environmental capital.

With migration, skilled workers move to the region with the highest welfare. Let's shed light on the reasons why region 2 is the wealthiest region in this context. Since unskilled workers are mobile across sectors, they should earn the same wage in both sectors. From (3.4), the wage of unskilled workers is simply K^{ε}, thus, those workers earn more in region 2 than in region 1. Moreover, from (3.2), the wage of unskilled workers depends on price and on labour ratio. It is shown in Section 3.4.3 that the allocation of unskilled workers is the same in both regions and is constant over time. Under the assumption that initially both regions have the same amount of skilled workers, the larger environmental capital of region 2 is reflected in an higher price in that region. In consequence, skilled workers are more valuable in region 2 than in region 1. This is reflected in their wage. The welfare of skilled workers does not depend only on their wage, but to a lesser extent, on price. Consequently, the welfare of skilled workers is larger in region 2 than in region 1. For that reason, skilled workers migrate in the direction of the wealthiest region in term of environmental capital, which is region 2. The flow of skilled workers leads to an increase in the production of manufactures, therefore, affecting negatively the environmental capital in region 2. However, the effect is not strong enough to invert the inequality between the environmental capital in region 1 and 2. The introduction of

[23] $\mathscr{K}^i(\cdot,\cdot;t)$ denotes either the specialized, or the diversified steady state environmental capital with interregional trade.

migration in the autarky model reduces the gap between the environmental capitals:

$$\mathscr{K}^1(MA,MA;a) < \mathscr{K}^1(MA,MA;m) < \mathscr{K}^2(MA,MA;m) < \mathscr{K}^2(MA,MA;a).$$

With the mobility of skilled workers, even though region 2 produces more manufactures, it remains the region with the largest environmental capital.

In summary, migration reduces the gap between environmental capital in region 1 and in region 2 and interregional trade increases it. How those two opposite effects interact one with each other when there is free trade and skilled workers are mobile?

Interregional trade affects relative wages, hence it has an impact on workers' decision to move from one region to the other. With interregional trade, skilled workers in region 1 are no longer interested in moving into region 2. The large environmental capital of that region is no longer enough to incite them to move. Trade opens perspectives in region 1, which give incentive to skilled workers from region 2 to move to region 1.

Proposition 3.14 reveals how interregional trade and mobility of skilled workers jointly affect the environmental capital.

Proposition 3.14. *The effect of interregional trade is amplified by the mobility of skilled workers:*[24]

$$\mathscr{K}^1(\cdot,\cdot;tm) < \mathscr{K}^1(\cdot,\cdot;t) < \mathscr{K}^1(MA,MA;a) < \mathscr{K}^1(MA,MA;m) <$$
$$\mathscr{K}^2(MA,MA;m) < \mathscr{K}^2(MA,MA;a) < \mathscr{K}^2(\cdot,\cdot;t) < \mathscr{K}^2(\cdot,\cdot;tm).$$

The proof of Proposition 3.14 is provided in the Appendix II.

In both regions, the effect of trade is amplified by the mobility of skilled workers. This surprising result is partly explained by the comparative advantages of the two regions. Let's consider the situation with trade before migration takes place. It is shown in Section 3.4.2 that the region with the largest environmental capital has a comparative advantage in agricultural production, thus in equilibrium this region is a net importer of

[24] The first two arguments of $\mathscr{K}^1(\cdot,\cdot;t)$ and $\mathscr{K}^1(\cdot,\cdot;tm)$ are not specified because this relation holds for all the equilibrium cases of trade, and trade and migration.

manufactures. Prior to migration, both regions have the same amount of skilled workers, consequently region 1 should have more unskilled workers working in the manufacturing sector than region 2. Therefore, the wage of skilled (unskilled) workers in region 1 is larger (smaller) than in region 2. The welfare of skilled workers is based on price and on wage. Trade equalizes the price between regions, thus, skilled workers only take the wages into account when they compare their welfare in their own region to the welfare they would get in the other region. Since skilled workers earn more in region 1 than in region 2, skilled workers from region 2 want to move to region 1. Migration increases the relative advantage of region 1 in the production of manufactures. Unskilled workers in region 1 move from the agricultural sector to the manufacturing sector, therefore, more manufactures are produced in that region. In region 2, there is not only a flow of skilled workers out of the region, there is also a movement of unskilled workers from the manufacturing sector to the agricultural sector. Hence, the production of manufactures declines in region 2. Compared to the migration model without trade, the environmental capital in region 2 is positively affected by migration when there is free trade. In region 1, the effects of migration go in opposite directions.

In region 2, trade (with or without migration) has a positive impact on the environmental capital through its negative effect on the production of manufactures. Therefore, the productivity of the agricultural sector gain from trade. That region always gains from trade. This is not the case of region 1, which can suffer from an environmental degradation, as in Copeland et Taylor (1999). Even though there is no disutility cost from pollution, it can be harmful in the long run, through its effect on the production possibilities. In region 1, trade leads to an increase of manufacturing production, and therefore, a degradation of the environmental capital. When the preferences over manufactures are low and agriculture are still produced in region 1 in equilibrium, that region might suffer from a loss of income due to the degradation of the environment.

Trade pattern is not affected by the mobility of skilled workers. The lower the environmental capital is, the larger the comparative advantage in the manufacturing sector is. The region with the lowest environmental capital produces more manufactures and becomes an exporter in that sector. Although the quantity exchanged might change with migration, region 1 (region 2) still exports (imports) manufactures and imports (exports) agriculture.

3.6 Conclusion

I develop a two-sector model of trade that incorporates both pollution and migration issues. This model gives insights into the effects of migration on the results of a trade and environment model. In a context of trade, I prove that the mobility of workers affects the pollution level of regions, that have different industrial structures. In the model, there is one polluting industry that harms one of the production factors, the environmental capital, of the environmental sensitive industry. I find that the effects of migration and interregional trade on the environmental capital are diametrically different. Migration reduces the gap between the environmental capitals and trade increases it. The trade pattern is not affected by the mobility of skilled workers, the more polluting region exports manufactures and the other region exports agriculture. As in Copeland et Taylor (1999), the more polluting region can lose from trade in the long run if the preferences over the polluting good is low. The presence of migration amplifies the possibility of potential loss from trade.

Some of my results rely on the assumption that the initial endowments of environmental capital is larger in region 2 than in region 1. One could wonder what happens if this assumption is relaxed. First, the results are not perfectly symmetric. As a result of the more pollutant technology used in region 1, even if the initial environmental capital in region 1 is larger than in region 2, it might end up being smaller at the steady state equilibrium. This is always the case in autarky and with migration. With trade, region 1 would initially produce less manufactures than region 2. If the difference between the environmental capital in both regions is relatively small and the technology in both regions is quite different, then the economy might end up in one of the steady state equilibria described in this current work. Whereas, if both regions have similar technology and K^1 is far from K^2, then the economy might reach other steady state equilibria with larger environmental capital in region 1. From my perspective, this assumption is not too strong in the context. However, it could be interesting to investigate what the outcomes of the model are when the environmental capital in region 1 is initially larger than in region 2.

It could be interesting to extend the model by letting the two-region country trade with the rest of the world. Results about the environmental capital might change with international trade. Throughout the paper, I assume that migrants keep their money in their working regions. However, it happens that they send a large amount of their money back to their region of origin. I think it might be interesting to examine also this possibility.

CHAPITRE 4

ON THE VALUE AND OPTIMAL ALLOCATION OF WATER

4.1 Introduction

Water has a strong historic presence in the economic development of the province of Quebec. From the fur industry to the hydroelectric industry passing through the forest industry, the rivers and lakes of Quebec have been and are still an important economic lever. Multiple uses of this resource have led to different conflicts of uses. The laws that rule the status of water have evolved over time trying to resolve the conflicts between the different users. The aim of the current paper is not to provide a retrospective of the status of water in Quebec, whereas one can find an interesting review of the principal highlights in Richelle et Thibaudin (2011). The most recent legislation, *Loi affirmant le caractère collectif des ressources en eau et visant à renforcer leur protection (LRQ, chapitre C-6.2)*, states, principally, that water is a collective resource (*res communis*).[1] This law gives the population of the province of Quebec the exclusive right to use the resource and names the State its unique keeper. The particularity of a *res communis* is that no one can possess the resource, and hence, no price can be assigned to it. Only fees, that are managed by the keeper of the resource, can be determined. Those fees are designed to capture only the cost of restoration, management and use of the water. In contrast with prices, fees don't reflect the value of the water. If water had a price based on the willingness to pay of potential users, only the ones that value the most the resource would use it. Consequently, some conflicts of uses might be resolved. In this context of no price for water, it is relevant to ask: how should we allocate the right to use the resource? We suggest an allocation of water based on the maximization of the welfare induced by the different uses of water. This allocation is socially optimal. The welfare of water directly comes from the value that users give to the water. The evaluation of the value of water is a challenging task. Part of the difficulties lies in the mismatch between

[1] From now on, *Law of Water*

the marginal value and the use value, as it is pointed out by Smith (1776). The value of a standard economic good is determined by the marginal value, which is based on the relative rarity. If we assume that water is a standard good, its value is derived from what it is possible to get in exchange. Is water a standard economic good? The answer to this question is really important, in the view of the fact that it has an important impact on the optimal allocation of the resource. Once the value of water is established, we can tackle the issues of water allocation.

Water is used by a variety of different users: municipalities, firms, local residents, ecosystems, etc. Some investments might be needed by some users before using the resource, e.g., water purification plant, road construction. Other investments are made to improve the productivity of water, e.g., repair of leaky pipes. Part of those investments leads to destruction, degradation and fragmentation of natural ecosystems. If the destruction and degradation cannot be altered, the investments create in fact irreversible damages. The size of the investment is not always a good indicator of its impact on the ecosystem, i.e., even small investments that lead to fragmentation of ecological areas can cause major environmental damages. In their analysis of the degrading effects of fragmentation of the ecosystems, Haddad et al. (2015) show that fragmentation is an important issue. In particular, they show that 70% of the global forest cover are affected by fragmentation. The project of a new oil port in Cacouna is another good example of investment that is irreversible and that affects the ecosystem regardless of its size. Environmentalists and biologists argue that regardless of the size of an oil port, once the habitat of beluga is affected, this species would be in danger of extinction. These investments known as irreversible developments may lead to a loss of biodiversity or to animal extinction. To take this issue into account, we incorporate in the analysis of the value and the allocation of water, an investment that irreversibly damages the ecosystem regardless of its size. The damage depends neither on the level of the net investment nor on the level of the firm production, and creates a lump sum reduction of welfare.

The model also encompasses how legislation affects the optimal allocation of water. In particular, we introduce a form of legislative irreversibility that is inspired by the *Law of Water*, which states that the current allocation of water to the firm binds its future allocation. Based on the *Law of water*, once the State gives the right to use a certain amount of water, x, to a firm, in every future period this firm will have the right to use at least x units of water. Obviously, this legislation creates an irreversibility. By choosing the allocation of water in the current period, the decision maker chooses the lower bound amount of water that some users have the right to use in the future. Legislative irreversibility should be considered by anyone who plans to study the value of water and the optimal allocation of this resource.

We develop a two-period analytical model to shed light on the optimal allocation of water of a lake among different potential users, namely: a firm, a municipality, and an ecosystem. First of all, we determine the value of water for the different users of the economy. Then, we characterize the optimal allocation of water among users in various situations. The optimal allocation in the benchmark model without irreversibility is analyzed first. As we should expect the marginal value is equal across users. The solution of this basic case, particularly when there is no rivalry in use, highlights the fact that increasing the allocation to one particular user, doesn't necessarily increase the social benefit. In the case of no rivalry in use, unused water should not be seen as a limitless resource to be used in any way whatever.

Next, we suppose that net investment irreversibly damages the ecosystem and creates a lump sum welfare loss. We show that it is not always optimal to prevent the damage, moreover we characterize the range of welfare losses for which the decision maker intervenes to prevent the damage. To approach the reality, we introduce the irreversibility of the right to use the resource (legislative irreversibility). With legislative irreversibility, we show that it is sometimes optimal to reduce the amount of water allocated to the firm, even though there is no rivalry in use. In other words, it might be optimal to limit the access to water of the firm even if its marginal value is zero and there is unused water in the lake. Furthermore, with irreversibility, we prove that the equalization of the marginal value of water between users doesn't hold.

In Section 4.2, we discuss the principal characteristics of this particular good, that is water. Section 4.3 sets the basis of the theoretical model. The optimal value and allocation of water is analyzed in Section 4.4. In Section 4.5, we discuss our principal results.

4.1.1 Literature review

The literature on the economic value of water and the optimal allocation of this resource is relatively modest, despite the importance of the issues. Ambec et Sprumont (2002) and Ambec et Ehlers (2008) are among the few papers that study rigorously the problem of efficiently sharing water among a group of agents. They provide an analytical framework to evaluate how to allocate the water of a river among a group of satiable agents located along the river. Gaudet et al. (2006) evaluate the optimal paths of production and water usage by two sectors (agricultural and non renewable resource) that share a limited amount of water. In comparison with our analysis, neither the irreversibility of investment nor the legislative irreversibility is considered. Despite their importance, it appears that optimal allocation of water and irreversibility have seldom been analyzed together.

The current paper is also related to the literature on the evaluation of the value of water. Hanemann et al. (2006) and Ward (2007) review the economic concepts related to water. They agree on the importance of determining the value of water, however, they do not provide any analytical model. Gibbons (1986) provides a framework for understanding water values and demand for water in various sectors in the United States. She studies different methods of estimating the value of water. Gibbons (1986) is an illustrative study of water values that results in a heterogeneous set of values which are not comparable between sectors. Furthermore, most of her estimations are based on price data. Whereas, in our model, no price can be assigned to water. Dachraoui et Harchaoui (2004) estimate the private value of water for self-supplied firms through the diminution of cost induced by the reduction of inputs caused by an extra unit of water. This method is appropriate when capital (labour) and water are substitutes, but is unsuitable when capital (labour) and water are complements. They estimate that for the majority of the 36 industries considered capital (labour) and water are substitutes and for 12 industries out of 36, capital (labour) and water are complements. Thus, for one third of the industries the method used underestimates the value of water for firms.

Our paper is also related to the literature on non market economic valuation of environmental resources. For instance, Young (2004) evaluates a monetary value on goods and services provided by water. The aim of Young (2004) is to apply non market methods for measuring benefits and costs of particular public policies relating to water. By contrast, the aim of our model is no to assign monetary measures of individuals' preferences for outcomes of policy proposals or events.

Because our model incorporates an irreversible damage created by the net investment, it is relevant to mention the vast literature of option value. The notion of option value introduced by the pioneering paper of Weisbrod (1964) has broadly evolved over time. This paper was the first of a long series: Cicchetti et FreemanIII (1971), Fisher et al. (1972), Arrow et Fisher (1974), Henry (1974), Hanemann (1989), Fisher (2001), Fisher et Narain (2003), Zhao (2011), and many more. More recently, option value theory has been used to study the timing of investment in the context of climate change: Fisher et Narain (2003), Fisher (2001), and Pindyck (2002). In particular Fisher et Narain (2003) show that an irreversible decision or action has to clear a higher hurdle to pass a cost-benefit analysis.

To the best of our knowledge, this paper is the first in the economic literature to develop a theoretical model that analyses the value of water and the optimal allocation of water in the presence of irreversibility.

4.2 Water: a particular economic good

In this section, we outline the characteristics of the water, as an economic good, that are important for our analysis.

4.2.1 Legal aspect

Over time, water has been used in very different usages: navigation, fur industry, forest industry, and hydroelectricity. The multiplication of the uses has led to potential conflicts. Laws have evolved over time to resolve the conflicts between the different users. Richelle et Thibaudin (2011) provide a brief summary of the evolution of the importance of water for Quebec together with the evolution of the laws that rule the competing uses of water. In the present paper, we focus our attention on the current legislation, which is *Loi affirmant le caractère collectif des resources en eau et visant à renforcer leur protection (LRQ, chapter C-6.2)*.

This law has been voted in 2009. Some provisions of this law are worth our attention. First, the law confirms not only the *res communis* status of water, but goes further by affirming that water becomes now a collective resource, that is, water is now part of the heritage of Quebec society. Consequently, the appropriation of this resource is formally forbidden.[2] Furthermore, the law introduces the principles of user pays. However, it is only two years after the legislation took effect that the fees have been determined. It is worth noting that the fees are determined only to cover the cost of managing the resource, therefore there is no price associated with the resource. By asserting the exclusivity of usage by the population of Quebec, this law implies that water is neither public property nor private property. The law stipulates that the management of the resource is the responsibility of the State, in other words, the State becomes the keeper of water. The last articles of the law that ought to be mentioned are articles 33 to 38. These articles stipulate that users of water will have at their disposal during the next 10 years at least the same amount of water that they have in the current period. This piece of legislation is relatively important in the study of our model since it creates irreversible situations. We refer to this type of irreversibility as legislative irreversibility.

4.2.2 Value of water

The *res communes* property implies that water cannot be owned by anyone, as a result, water cannot be exchanged on the market and no price can be determined. With price, conflicts could be resolved more easily based on the willingness to pay of potential users. Without any price, how could water be allocated among users? Should the allocation be organized in a hierarchy as the ecologists suggest? We don't think so. The impossibility to assign a price to water doesn't imply that it is impossible to determine its value. The allocation based the maximization of the welfare of water is the solution that we prone. Users are those that value the most the resource. Moreover, this allocation is the optimal

[2]Except under very special conditions specified in the law.

one. But, this alternative raises an important question: how can we determine the welfare of water and what are its components?

First, we claim that water is an economic good. Water is indeed a good with particular characteristics, but even so it is an economic good. Let's shed light on the particularities of water. One really important characteristic of water is that in contrast to most economic goods, the use of water does not destroy it, it simply transforms it, and in turn this transformed water can be used. It is important to note that it is not the same water that is used twice, the initial water and its transformation should be considered as two different economic goods. The warming and the addition of pollution are examples of such transformations.

By way of illustration, let us consider a firm that uses as an input water pumped in a lake and rejects it one degree warmer. Water that enters and that gets out of the plant should be considered as two different economic goods. A user that needs cold water cannot utilize the water that comes out of the plant, but can utilize the water that enters the plant.

Another important property of water is its mobility. The flow of a stream carries the water to lakes or in rivers. Consequently, the use of water in a particular stream might affect the use of water in another stream in the future. Water upstream and downstream a river should not be considered as the same economic good.

The last feature that is worth mentioning is time. Time can directly transform water. For instance, time can reduce pollution of a lake through settling water. If water is subject to transformations through time, then we should consider different economic goods.

The private and the social marginal value of a private good coincide. In the following section, we argue that water is a private good, consequently the private value is also the social value.

4.2.3 Water, a private good

A public good is both non-excludable and non-rival. To understand the reason why water is not a public good, clarifications about rivalry should be made. There are two types of rivalry: rivalry in consumption and rivalry in use. There is non-rivalry in consumption, if the consumption by one consumer doesn't prevent the consumption by other consumers at a given moment. One unit of water consumed at a given moment in time, cannot be consumed by other users at the same time. Therefore, water is rival in consumption. On the other hand, water can be non-rival in use. If there is enough water in a lake, there might be several users at the same time. The simultaneous uses of the water of a lake simply reflect that there is enough water in the lake for all users, and does not reflect the non-rivalry in consumption.

Consequently, water is a private good. For that reason, we consider the private value of water for different users in the analytical model.

4.3 The model

Consider two discrete time periods, $t = 1,2$. There is a lake with a fixed amount of water within a period, Θ_t. The amount of water can vary from one period to the other. Three agents want to use the water of the lake: a firm, a municipality and an ecosystem. The allocation of water among the firm, the municipality and the ecosystem is denoted by A_t^f, A_t^m, and A_t^e, respectively. Given the allocation of water, the amount of water that is actually used by each of the three agents in period t is given by: Q_t^f, Q_t^m and Q_t^e. To make it clear, an agent does not have to use the totality of its allocation of water. Therefore, the allocation of water is the upper bound of the amount of water used by each agent.

For simplicity, we assume that water can be used by any of the three agents without any transformation.

In Sections 4.3.1, 4.3.2, and 4.3.3, we evaluate the marginal value of water for the three agents.

4.3.1 The firm

A price-taking firm produces a consumption good. Every period the market for that good is assumed to be perfectly competitive.

The consumption good is produced using two intermediate inputs, y_t and z_t. The first intermediate input, y_t, is produced using capital, K_t, and labour, L_t. The second intermediate input, z_t, is produced using water, Q_t^f, factors that are perfect complements of water, X_t^c, and capital, K_t^s. The technologies F and H represent the production function of y_t and z_t, respectively:

$$y_t = F(L_t, K_t),$$
$$z_t = H(\min\{Q_t^f, f(X_t^c)\}, K_t^s).$$

The assumption of perfect complementarity of Q_t^f and X_t^c implies that optimally, the firm will combine the two inputs such that $Q_t^f = f(X_t^c)$. Consequently, z_t can be rewritten as $z_t = H(Q_t^f, K_t^s)$. There are two types of capital, K_t and K_t^s. K_t is the classical stock of capital, e.g., machinery, tools and buildings. This stock of capital has a direct positive effect on the production of the consumption good. Its depreciation rate, $0 < \delta < 1$, is assumed to be constant. The capital stock evolves as follows (where I is the investment):

$$K_2 = (1-\delta)K_1 + I.$$

The other type of capital, K_t^s, affects the productivity of water. Its depreciation rate, $0 < \delta^s < 1$, is also assumed to be constant. We have:

$$K_2^s = (1-\delta^s)K_1^s + I^s.$$

We make the distinction between both types of capital to distinguish between an investment I that increases the capacity of production and an investment I^s that increases the productivity of water.

The consumption good is produced using the two intermediate inputs, z_t and y_t. The quantity produced is given by:

$$G(z_t, y_t) = G(H(Q_t^f, K_t^s), F(L_t, K_t)).$$

Assumption 4.1 regroups the assumptions about the technologies.

Assumption 4.1.

A1. $f(\cdot)$ *is invertible with* $f^{-1}(\cdot) = g(\cdot)$, $g(\cdot)$ *is convex, and* $\lim_{Q_t^f \to 0} g'(\cdot) \neq \infty$.

A2. The technologies G, H, and F satisfy the Inada conditions.

A3. The Hessian of G, H, and F are negative definite, as a result those functions are strictly concave in their arguments.

A4. F is homogeneous of degree 1.

A5. $G_{yz} > 0$.

A6. $H_{K^sQ}H_{K^s} - H_{K^sK^s}H_Q > 0$.[3]

To alleviate the notation, we use the following simplification. For a function $f(x_1,x_2)$, f_{x_1}, f_{x_2}, $f_{x_1x_1}$, $f_{x_2x_2}$, and $f_{x_1x_2}$ stand for $\partial f(x_1,x_2)/\partial x_1$, $\partial f(x_1,x_2)/\partial x_2$, $\partial^2 f(x_1,x_2)/\partial x_1^2$, $\partial^2 f(x_1,x_2)/\partial x_2^2$, and $\partial^2 f(x_1,x_2)/\partial x_1 \partial x_2$, respectively.

The *res communis* status of water implies that there is no price associated with the resource.[4] The prices of the consumption good, the labour, the complement investment, the substitute investment, and the complement inputs are represented respectively by p_t, w_t, r, r^s and r_t^c. The decision maker gives to the firm the right to use at most A_t^f units of water in period t. In other words, in period t, the firm has at its disposal A_t^f units of water. The Lagrange multiplier of its profit maximization problem associated with this constraint represents the marginal value of water for the firm. In other words, the marginal value of water for firm is the marginal profit of relaxing the constraint, $Q_t^f \leq A_t^f$.

[3]The explanation of this assumption is provided in the Appendix III.

[4]There are fees that cover the cost of managing the resource. Because those fees are negligible, we do not include them in the analysis.

The profit maximization problem of the firm is given by:

$$\max_{\{\{Q_t^f, L_t, X_t^c\}_{t=1,2}, I, K_2, I^s, K_2^s\}} \sum_{t=1,2} \Big(p_t G(H(Q_t^f, K_t^s), F(L_t, K_t)) - w_t L_t - r_t^c X_t^c \Big) - rI - r^s I^s \tag{4.1}$$

$$\begin{aligned} \text{s.t.} \quad & X_t^c = g(Q_t^f) \\ & K_2 = (1-\delta)K_1 + I, \quad K_1 > 0 \quad \text{given} \\ & K_2^s = (1-\delta^s)K_1^s + I^s, \quad K_1^s > 0 \quad \text{given} \\ & Q_t - A_t^f \leq 0 \quad (\lambda_t^f), \quad t = 1,2. \end{aligned}$$

The maximization problem (4.1) can be rewritten by:[5]

$$\begin{aligned} \max_{\{\{Q_t^f, L_t\}_{t=1,2}, K_2, K_2^s\}} \sum_{t=1,2} & \Big(p_t G(H(Q_t^f, K_t^s), F(L_t, K_t)) - w_t L_t - r_t^c g(Q_t^f) \Big) \\ & - r(K_2 - (1-\delta)K_1)) - r_1^s (K_2^s - (1-\delta^s)K_1^s)) \\ & + \lambda_1^f (A_1^f - Q_1^f) + \lambda_2^f (A_2^f - Q_2^f). \end{aligned} \tag{4.2}$$

The first order conditions of (4.2) are given by:

$$(Q_t^f): \quad p_t G_z H_Q - r_t^c g_Q - \lambda_t^f = 0, \quad t = 1,2, \tag{4.3}$$

$$(L_t): \quad p_t G_y F_L - w_t = 0, \quad t = 1,2, \tag{4.4}$$

$$(K_2): \quad p_2 G_y F_K - r = 0, \tag{4.5}$$

$$(K_2^s): \quad p_2 G_z H_{K^s} - r^s = 0, \tag{4.6}$$

and the Kuhn-Tucker conditions of (4.2) are given by:

$$\lambda_t^f (Q_t^f - A_t^f) = 0 \quad \lambda_t^f \geq 0 \quad A_t^f - Q_t \geq 0, \quad t = 1,2.$$

[5]The assumption that G, F and H satisfy the Inada conditions implies that every input is essential to the production. The combination of the Inada conditions and the assumption that the limit of the first derivative of $g(\cdot)$ isn't positive infinity as the amount of water used by the firm approaches 0 implies that all the inputs are strictly positive at the optimum.

We denote by $\bar{Q}_t^f$, the amount of water that maximizes the profit of the firm without any constraint on the amount of water that can be used. The Inada conditions of the technology G ensure that there exists such $\bar{Q}_t^f$. When $A_t^f \geq \bar{Q}_t^f$, the solution of the profit maximisation problem is interior, i.e., $Q_t^f = \bar{Q}_t^f$ and the marginal value of water for the firm, λ_t^f, is zero. Otherwise, the optimum is achieved at a corner solution and the optimum amount of water used by the firm, Q_t^f, equals A_t^f. In that case, the marginal value of water for the firm is given by the expression of λ_t^f obtained by (4.3) evaluated at the solution of (4.2): [6]

$$\lambda_t^f = p_t G_z H_Q - r_t^c g_Q > 0. \tag{4.7}$$

Consequently, for a given amount of water allocated to the firm, A_t^f, the amount of water actually used by the firm is the minimum between A_t^f and $\bar{Q}_t^f$:

$$Q_t^f = \min\{A_t^f, \bar{Q}_t^f\}.$$

For an amount of water used by the firm, x, the marginal value of water for the firm is defined by:

$$\lambda_t^f(x) = \begin{cases} 0 & \text{if} \quad x \geq \bar{Q}_t^f \\ p_t G_z(H(x, K_t^s), F(L_t, K_t)) H_Q(x, K_t^s) - r_t^c g_Q(x) & \text{if} \quad x < \bar{Q}_t^f. \end{cases}$$

Consequently, in period t, given an allocation of water, A_t^f, the use value of the water for the firm is given by:

$$\Pi_t = \int_0^{Q_t^f} \lambda_t^f(x)dx = \int_0^{Q_t^f} [p_t G_z(H(x, K_t^s), F(L_t, K_t)) H_Q(x, K_t^s) - r_t^c g_Q(x)]\, dx, \tag{4.8}$$

with $Q_t^f = \min\{A_t^f, \bar{Q}_t^f\}$.

[6] In other words, Q_1^f, Q_2^f. L_1, L_2, K_2, and K_2^s are the optimal quantity chosen by the firm. Henceforth, when we refer to (4.7), we implicitly consider (4.3) evaluated at the solution of (4.2).

We define the use value of water for the firm by the sum of Π_1 and Π_2:

$$\Pi = \sum_{t=1,2} \Pi_t.$$

It is worth noting that the use value of water for the firm equals its profit. It raises the important point that even though the marginal value of water is zero, the use value of water isn't zero.

4.3.1.1 Comparative statics

This section provides a series of propositions about the comparative statics. Throughout this section, we assume that $A_t^f \leq \bar{Q}_t^f$, otherwise a variation of the firm allocation of water has no effect on the optimal variables. The proofs of Propositions 4.1, 4.2, 4.3, and 4.4 are in the Appendix III.
Proposition 4.1 establishes the impact of the allocation of water on the optimal L_2, K_2, and K_2^s.

Proposition 4.1. *The firm allocation of water in period two has a positive effect on L_2 and K_2, whereas its effect on K_2^s is ambiguous.*

An increase of the allocation of water to the firm, A_2^f, has two opposite effects on the capital that affects the productivity of water, K_2^s. On one hand, an increase of A_2^f leads to a substitution effect. The firm reduces the capital K_2^s relatively to its use of water. On the other hand, an increase of A_2^f has a level effect, that affects positively K_2^s. Which of those two effects dominates determines the overall effect of an increase of A_2^f on K_2^s.
Even though, the global effect of A_2^f on K_2^s is unknown, we prove that A_2^f affects positively the intermediate input, z_2. Which in turn, induces the firm to choose a larger amount of labour and capital, L_2 and K_2.
Next proposition specifies the impact of the prices of the substitute investment, the complement investment, the labour, and the consumption good on labour and on the capital stocks.

Proposition 4.2.

(i) The price of the substitute investment, r^s, affects negatively K_2^s, K_2, and L_2;

(ii) the price of the complement investment, r, affects negatively K_2 and K_2^s, whereas its effect on L_2 is ambiguous;

(iii) the price of labour, w, affects negatively L_2 and K_2^s, whereas its effect on K_2 is ambiguous;

(iv) the price of the consumption good, p_2, affects positively K_2^s, K_2, and L_2.

When r_s increases, the firm cannot substitute water for K_2^s, because the amount of water used by the firm is bounded by the amount of water allocated to the firm. Therefore, an increase of r^s affects negatively the intermediate good z_2. This reduction of z_2 induces the firm to reduce the labour and the capital, L_2 and K_2. As expected, an increase of r leads to a reduction of the classical stock of capital, K_2, whereas the effect on labour is ambiguous. The change of r has two opposite effect on L_2: a substitution effect and a level effect. Which of these effects dominates determines the global effect of a variation of r on L_2. Even though the effect of a change in r on labour is ambiguous, the overall effect on the intermediate input, y_2, is negative. Consequently, r affects negatively K_2^s. An increase of w_2 leads to a reduction of labour, L_2, whereas the effect on the classic stock of capital, K_2, is ambiguous. The change of w_2 has two opposite effect on K_2: a substitution effect and a level effect. Which of these effects dominates determines the global effect of a variation of w_2 on K_2. Even though the effect of a change in w_2 on labour is ambiguous, the overall effect on the intermediate input, y_2, is negative. Consequently, w_2 affects negatively K_2^s. An increase in the value of the consumption good, p_2, induces the firm to produce more, and therefore, it brings the firm to increase its inputs, L_2, K_2, and K_2^s.

So far, we have analyzed the impact of different parameters on the labour and the different types of capital. The following step is to evaluate the impact of those parameters on the marginal value of water for the firm in period two, λ_2^f.

In Proposition 4.3, we proceed with the analysis of the effects of the factor prices on λ_2^f.

Proposition 4.3. *All factor prices have a negative impact on λ_2^f, whereas the price of the consumption good, p_2, affects positively the marginal value of water for the firm in period two.*

From Proposition 4.2, an increase of either one of r, r^s, and w_2 leads to a reduction of both intermediate goods. Consequently, the production of the consumption good is reduced. In other words, the firm produces less with the same amount of water, therefore the marginal value of water falls. The effect of r_2^c on the marginal value of water is direct. Because we restrict our attention on cases where $A_2^f \leq \bar{Q}_2^f$, even though r_2^c increases the firm continues to use the same amount of water. However, the cost of each unit of water used is more important, hence the value of water falls. An increase of p_2 leads to an increase of both intermediate goods. As a result, the production of the consumption good increases. The firm produces more with the same amount of water. Consequently, the marginal value of water raises.

Finally, Proposition 4.4 establishes the impact of the firm allocation of water on the marginal value of water.

Proposition 4.4. *The marginal value of water for the firm is decreasing in its amount of water used.*

Proposition 4.4 highlights an important feature of the value of water for the firm. As it is illustrated in Figure 4.1, the marginal value of water for the firm decreases as its use of water increases. Figure 4.1 illustrates also the difference between the marginal value and the use value of water. If the amount of water allocated to the firm is $\hat{A}_t^f$, the amount of water used by the firm is $\hat{Q}_t^f$, the marginal value of water is given by $\hat{\lambda}_t^f$ and the use value, (4.8), is represented by the blue area.[7]

4.3.2 The municipality

Every period, the municipality is allocated various types of resources, among which is water. We denote by Q_t^m the amount of water used by the municipality in period $t = 1, 2$.

[7]No assumption is made on the third derivative of the production functions, thus, the marginal value of water can take various shapes. Figure 4.1 illustrates a convex marginal value of water for the firm.

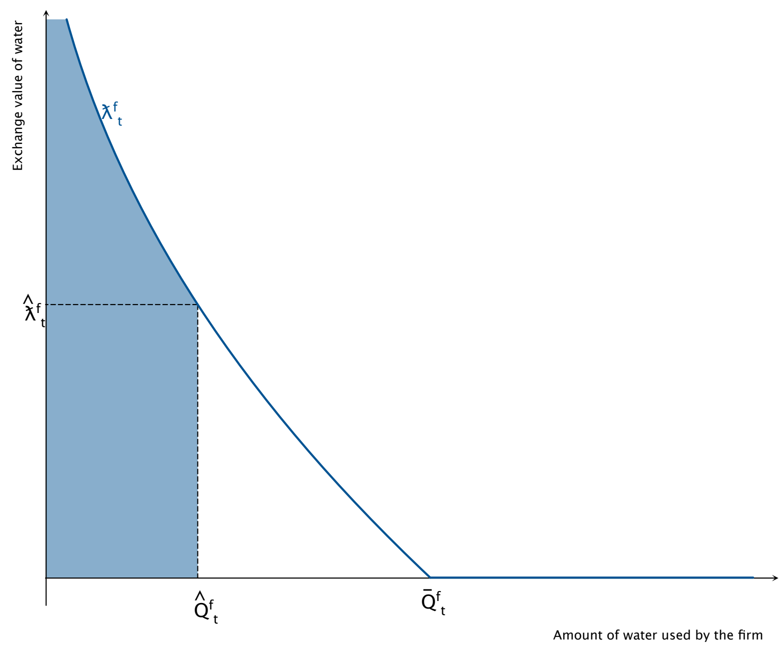

Figure 4.1: The value of water for the firm

Apart from water, all the resources are gathered together and are represented by M_t. The municipality has to choose the optimal allocation of its resources, $\{Q_t^m, M_t\}_{t=1,2}$, among its citizens. We define the value function of the municipality by $\upsilon(Q_t^m, M_t)$. We assume that the value function, $\upsilon(\cdot,\cdot)$, is concave in its arguments. The decision maker gives to the municipality the right to use at most A_t^m units of water in period t, i.e., $Q_t^m \leq A_t^m$. The marginal value of water for the municipality, λ_t^m, is defined as the marginal benefit of relaxing the constraint $Q_t^m \leq A_t^m$.

We make the assumption that for a given M_t, there exists a level of water that maximizes the value function of the municipality without any constraint on the amount of water that can be used, which we denote by $\bar{Q}_t^m$. When the allocation of water to the municipality is larger than $\bar{Q}_t^m$, the municipality uses $\bar{Q}_t^m$ and the marginal value of water is zero, i.e., $\lambda_t^m = 0$. Thus, as for the firm, there exists an amount of water above which the marginal value of water is zero. If the allocation of water to the municipality is smaller than $\bar{Q}_t^m$, then the municipality uses A_t^m. In that case, the marginal value of water for the municipality is given by:

$$\lambda_t^m = \upsilon_Q(A_t^m, M_t).$$

Consequently, for a given amount of water allocated to the municipality, A_t^m, the amount of water actually used by the municipality is the minimum between A_t^m and $\bar{Q}_t^m$:

$$Q_t^m = \min\{A_t^m, \bar{Q}_t^m\}.$$

For an amount of water used by the municipality, x, the marginal value of water for the municipality is defined by:

$$\lambda_t^m(x) = \begin{cases} 0 & \text{if} \quad x \geq \bar{Q}_t^m \\ \upsilon_Q(x, M_t) & \text{if} \quad x < \bar{Q}_t^m. \end{cases}$$

The concavity assumption of $\upsilon(\cdot,\cdot)$ implies that the marginal value of water for the municipality is decreasing in x.[8] In period t, given an allocation of water A_t^m, the use value of water for the municipality is given by:

$$\Upsilon_t = \int_0^{Q_t^m} \lambda_t^m(x)dx = \int_0^{Q_t^m} \upsilon_Q(x, M_t)dx, \tag{4.9}$$

with $Q_t^m = \min\{A_t^m, \bar{Q}_t^m\}$. The use value of water for the municipality for both periods is simply defined by the sum of Υ_1 and Υ_2:

$$\Upsilon = \sum_{t=1,2} \Upsilon_t.$$

The use value Υ is the monetary equivalent of the welfare of the municipality.

[8]The curvature of the marginal value of water for the municipality in period t depends on the third derivative of $\upsilon(\cdot,\cdot)$ with respect to the amount of water used. Since no assumption is made on the third derivative, the marginal value of water for the municipality can take various shapes.

4.3.3 The ecosystem

The ecosystem is a particular user of water, indeed neither preference system nor profit function can be assigned to the ecosystem. At first sight, it is not an easy task to attribute a value of water to this user. We base our analysis on the *Millennium Ecosystem Assessment (MA)* called for by the United Nations Secretary-General in 2000.[9] The *MA* assesses the consequences of ecosystem change for human well-being. They conceptualize the ecosystem as an input/output model. The relation between the input (water, sun,...) and output (ecosystem services) is exclusively technical. For that reason, the marginal value of water assigned to the ecosystem can be derived using a similar method as the one used for the municipality.

The ecosystem produces services that are denoted by S. The ecosystem services are produced using water, Q_t^e, and other inputs that are gathered together and are represented by E_t. The inputs are allocated in the ecosystem, in order to maximize the ecosystem services. We make the assumption that the utility function associated to the ecosystem services is increasing in S. As a result, the allocation of inputs that maximizes the ecosystem services, maximizes also the utility function. We define the value function associated to the ecosystem services by $\phi(Q_t^e, E_t)$. We make the assumption that the value function, $\phi(\cdot,\cdot)$, is concave in its arguments. In period t, the decision maker allocates A_t^e units of water to the ecosystem, i.e, $Q_t^e \leq A_t^e$. The marginal value of water for the ecosystem, λ_t^e, is defined as the marginal benefit of relaxing the constraint, $Q_t^e \leq A_t^e$.

As for the firm and the municipality, we assume that for a given E_t, there exists a level of water that maximizes the ecosystem services without any constraint, which we denote by $\bar{Q}_t^e$. When the allocation of water to the ecosystem is larger than $\bar{Q}_t^e$, the ecosystem uses $\bar{Q}_t^e$ and the marginal value of water is zero. If the allocation of water to the municipality is smaller than $\bar{Q}_t^e$, then the ecosystem uses A_t^e. In that case, the marginal value of water for the ecosystem is given by:

$$\lambda_t^e = \phi_Q(A_t^e, E_t), \quad t = 1, 2.$$

[9] www.maweb.org

Therefore, for a given amount of water allocated to the ecosystem, A_t^e, the amount of water actually used by the municipality is the minimum between A_t^e and $\bar{Q}_t^e$, that is:

$$Q_t^e = \min\{A_t^e, \bar{Q}_t^e\}.$$

Given the amount of water used by the ecosystem, x, the marginal value of water for the ecosystem is defined by:

$$\lambda_t^e(x) = \begin{cases} 0 & \text{if} \quad x \geq \bar{Q}_t^e \\ \phi_Q(x, E_t) & \text{if} \quad x < \bar{Q}_t^e \end{cases}.$$

The concavity of $\phi(\cdot,\cdot)$ implies that the marginal value of water for the ecosystem services is decreasing in the amount of water used by the ecosystem.[10] The use value of water for the ecosystem in period t is given by:

$$\Phi_t = \int_0^{Q_t^e} \lambda_t^e(x)dx = \int_0^{Q_t^e} \phi_Q(x, E_t)dx, \tag{4.10}$$

with $Q_t^e = \min\{A_t^e, \bar{Q}_t^e\}$. The use value of water for the ecosystem for both periods is simply defined by the sum of Φ_1 and Φ_2:

$$\Phi = \sum_{t=1,2} \Phi_t.$$

The use value Φ is the monetary equivalent of the welfare from the ecosystem services.

4.4 The optimal value of water

According to environmentalists, there exist investments and economic developments that, regardless of their sizes, lead to irreversible degradation and destruction of the natural ecosystem. While several examples of this type of investment are provided in Section 4.1, we now provide a formal structure to analyze the issue. More specifically,

[10]The curvature of the marginal value of water for the ecosystem services in period t depends on the third derivative of $\phi(\cdot,\cdot)$ with respect to the amount of water.

we build a model where the firm's net investments create an irreversible damage to the ecosystem. In this setting, we characterize the optimal allocation of water among the different users and determine the optimal intervention of the decision maker given the damage irreversibility.

Following the discussion in Section 4.2, the model also encompasses how legislation affects the optimal allocation of water. In particular, we introduce a form of legislative irreversibility that is inspired by the Law of Water, which states that the current allocation of water to the firm binds its future allocation.

Our analysis proceeds in several steps. In Section 4.4.1, we study the simplest case, which is the case without any form of irreversibility. Then, in Sections 4.4.2 and 4.4.3, we analyze legislative irreversibility and damage irreversibility, separately. At last, in Section 4.4.4, we analyze the combination of the two types of irreversibility.

Given an allocation of water among the different users, $\{A_t^f, A_t^m, A_t^e\}$, we define the welfare of water in period t by the sum of the value in use of water for each agent:

$$\mathscr{W}_t = \Pi_t + \Upsilon_t + \Phi_t, \quad t = 1,2.$$

Combining the preceding definition with (4.8), (4.9), and (4.10) implies that:

$$\mathscr{W}_t = \int_0^{Q_t^f} \lambda_t^f(x)dx + \int_0^{Q_t^m} \lambda_t^m(x)dx + \int_0^{Q_t^e} \lambda_t^e(x)dx, \quad t = 1,2,$$

with $Q_t^i = \min\{A_t^i, \bar{Q}_t^i\}$, $i = f, m, e$. The welfare of water for both periods is defined by the sum of $\mathscr{W}_1$ and $\mathscr{W}_2$:

$$\mathscr{W} = \sum_{t=1,2} \mathscr{W}_t.$$

Prior to this analysis, one remark has to be discussed. It concerns the composition of the welfare of water. The decision maker chooses the allocation of water among the different users by maximizing the welfare of water. Υ and Φ represent the use value associated with the use of water by the municipality and the ecosystem, respectively, whereas Π

represents the firm use value of water, which doesn't take in consideration the use value of water for the consumers. Before going further into the analysis, let us provide some clarifications. In the consumption good market, the use value of water is the addition of the use value for the consumers and for the firm. Assuming that the consumers use value of water is independent of the production of the firm, the decision maker can consider only Π in its analysis of the allocation of water. This assumption can be satisfied in various contexts, e.g., when the consumption good is partly imported and the local firm is small relatively to the rest of the world. In that context, the production of the local firm has no impact on the total consumption and the price. As a result, the allocation of water among the different users has no impact on the consumer use value. Therefore the decision maker can consider only Π. Moreover, we make the assumption that there is no distortion in the labour market.

4.4.1 No irreversibility

We evaluate the marginal value of water and the optimal allocation of water among the firm, the municipality and the ecosystem without irreversibility. To do so, we maximize the welfare of water:

$$\begin{aligned} \max_{\{A_t^f, A_t^m, A_t^e\}} \sum_{t=1,2} & \left[\int_0^{Q_t^f} \lambda_t^f(x)dx + \int_0^{Q_t^m} \lambda_t^m(x)dx + \int_0^{Q_t^e} \lambda_t^e(x)dx \right] \quad (4.11) \\ \text{s.t.} \quad & Q_t^f + Q_t^m + Q_t^e \leq \Theta_t \quad t = 1,2 \\ & Q_t^i = \min\{A_t^i, \bar{Q}_t^i\} \quad t = 1,2 \quad i = f,m,e. \end{aligned}$$

It is worth noting that without irreversibility, at the optimum, $A_t^i \leq \bar{Q}_t^i$ for $t = 1,2$ and $i = f,m,e$. Hence, (4.11) can be rewritten as:

$$\begin{aligned} \max_{\{A_t^f, A_t^m, A_t^e\}} \sum_{t=1,2} & \left[\int_0^{A_t^f} \lambda_t^f(x)dx + \int_0^{A_t^m} \lambda_t^m(x)dx + \int_0^{A_t^e} \lambda_t^e(x)dx \right] \\ \text{s.t.} \quad & A_t^f + A_t^m + A_t^e \leq \Theta_t \quad t = 1,2. \end{aligned}$$

The first order conditions and the Kuhn-tucker conditions are given by:

$$(A_t^i): \quad \lambda_t^i(A_t^i) - \mu_t = 0 \quad i = f, m, e \quad t = 1, 2,$$
$$\mu_t(\Theta_t - A_t^f - A_t^m - A_t^e) = 0, \quad \mu_t \geq 0, \quad \Theta_t - A_t^f - A_t^m - A_t^e \geq 0.$$

In period t, if $\bar{Q}_t^f + \bar{Q}_t^m + \bar{Q}_t^e \leq \Theta_t$, then there is no rivalry in use. Hence, every user can use the amount of water he wants, consequently, the optimal allocation is characterized by $\{A_t^f, A_t^m, A_t^e\} = \{\bar{Q}_t^f, \bar{Q}_t^m, \bar{Q}_t^e\}$. Therefore, the marginal value of water at the optimal allocation is zero. It is important to note that it does not mean that water is worth nothing for the agents. The welfare of water is the area under the curve λ_t^f, λ_t^m, and λ_t^e. Figure 4.2 illustrates this case with simple linear values of water.[11] The welfare of water is represented by the grey area of Figure 4.2.

If $\bar{Q}_t^f + \bar{Q}_t^m + \bar{Q}_t^e > \Theta_t$, then there is rivalry in use and the optimal allocation, $\{A_t^f, A_t^m, A_t^e\}$, is characterized by the equalization of the marginal value of water for each agent:

$$\lambda_t^f(A_t^f) = \lambda_t^m(A_t^m) = \lambda_t^e(A_t^e) > 0.$$

As a result, the marginal value of water is strictly positive. Figure 4.3 illustrates this situation with linear marginal values of water.[12] The total social value of water is represented by the grey area of Figure 4.3.

We denote by $\{A_t^{f,(4.11)}, A_t^{m,(4.11)}, A_t^{e,(4.11)}\}_{t=1,2}$, the optimal allocation without irreversibility and the amount of water used by the different users induced by this optimal allocation of water is denoted by $\{Q_t^{f,(4.11)}, Q_t^{m,(4.11)}, Q_t^{e,(4.11)}\}_{t=1,2}$.

The solution of this basic case, particularly when there is no rivalry in use, highlights the fact that increasing the allocation to one particular user, doesn't necessarily increase

[11] In Figures 4.2 and 4.3, the marginal value of water for the firm is larger than the marginal value of water for the municipality, which in turn is larger than the marginal value of water for the ecosystem. This figure illustrates one possible relation between the values of water, whereas in our analytical analysis, no assumption is made on this relationship.

[12] In Figures 4.2 and 4.3, the marginal value of water for the firm is larger than the marginal value of water for the municipality, which in turn is larger than the marginal value of water for the ecosystem. This figure illustrates one possible relation between the values of water, whereas in our analytical analysis, no assumption is made on this relationship.

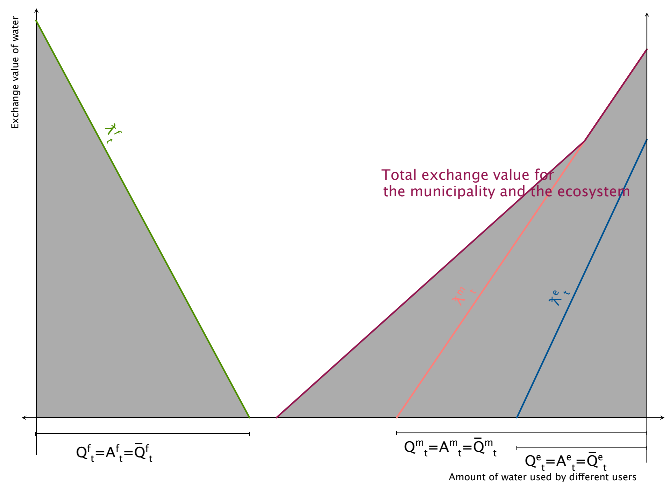

Figure 4.2: Optimal allocation of water without rivalry in use

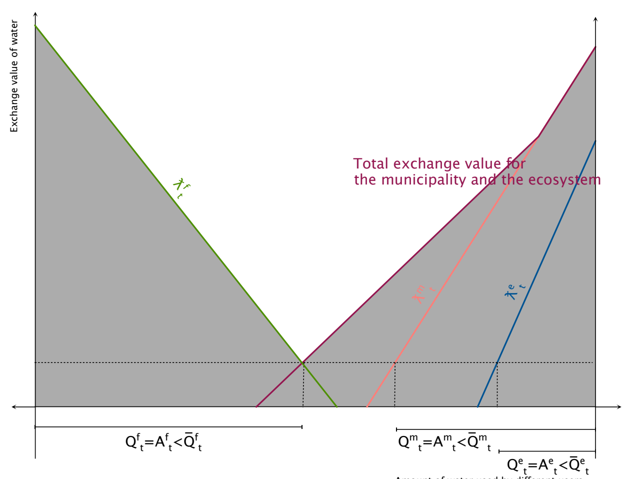

Figure 4.3: Optimal allocation of water with rivalry in use

the social benefit. In the case of no rivalry in use, unused water shouldn't be seen as a limitless resource to be used in any way whatever.

One remark has to be made upon this case. This basic situation shows evidence that the optimal allocation among different users is based on the marginal value of water, not on the use value. And most importantly, even though the marginal value of water is null, agents value water.

Turning next to the legislative irreversibility, we show that even though the marginal value of water is zero in the first period, at the optimum the decision maker might want to limit the access to water.

4.4.2 Legislative irreversibility

The current amount of water allocated to the firm binds its future allocation. The *Law of Water* states that once the keeper of the water gives the right to a firm to use a given amount of water, the firm should have the right to use at least the same amount in the future.[13] In other words, the current legislation creates an irreversibility that should be taken into account at the optimal allocation. Once a water-taking permit is issued, the decision maker cannot change the authorization for a predetermined period of time.

The legislative irreversibility adds to the maximization of the welfare of water (4.11) the constraint $A_1^f \leq A_2^f$. Thus, the optimal allocation of water is given by the solution of the following maximization:

$$\begin{aligned} \max_{\{A_t^f, A_t^m, A_t^e\}_{t=1,2}} \sum_{t=1,2} & \left[\int_0^{Q_t^f} \lambda_t^f(x)dx + \int_0^{Q_t^m} \lambda_t^m(x)dx + \int_0^{Q_t^e} \lambda_t^e(x)dx \right] \qquad (4.12) \\ \text{s.t.} \quad & Q_t^f + Q_t^m + Q_t^e \leq \Theta_t, \quad t = 1,2 \\ & A_1^f \leq A_2^f \\ & Q_t^i = \min\{A_t^i, \bar{Q}_t^i\} \quad t = 1,2 \quad i = f,m,e. \end{aligned}$$

The solution of (4.12) and the amount of water used by the different users induced by

[13] The *Law of Water* states that the water-taking permits are valid for a period of 10 years.

the solution of (4.12) are denoted respectively by:

$$\{A_t^{f(4.12)}, A_t^{m(4.12)}, A_t^{e(4.12)}\}_{t=1,2} \quad \text{and} \quad \{Q_t^{f(4.12)}, Q_t^{m(4.12)}, Q_t^{e(4.12)}\}_{t=1,2}.$$

It is worth noting that the amount of water allocated to the firm in period two is linked to its allocation in the first period. For that reason, it might be optimal to reduce the allocation of water to the firm in the first period in order to restrain its rights to use water in the second period. The decision maker reduces the amount of water allocated to the firm in period one as long as the welfare loss in that period is compensated by the welfare gain in period two. On the other hand, if the welfare loss in period one is relatively large compared to the gain in period two, it might be optimal to allocate to the firm more than $\bar{Q}_2^f$ units of water in period two, in order to let it uses more water in period one. The amount of water allocated to the firm in period two is the only one that might be larger than $\bar{Q}_2^f$ at the optimum. There is no benefit in allowing the municipality, the ecosystem and the firm in period one, to use more than their respective $\bar{Q}$. Therefore, $Q_t^i = A_t^i \leq \bar{Q}_t^i$, $t = 1,2$ and $i = m,e$, and $Q_1^f = A_1^f \leq \bar{Q}_1^f$.

Maximization problem (4.12) is the constrained version of (4.11). If the solution of the unconstrained maximization problem (4.11) satisfies the inequality constraint $A_1^{f(4.11)} \leq A_2^{f(4.11)}$, then, $A_t^{i(4.12)} = A_t^{i(4.11)}$ for $i = f,m,e$ and $t = 1,2$. We concentrate our attention to the other case, i.e., $A_1^{f(4.11)} > A_2^{f(4.11)}$.

First, we provide an example to highlight some important features generalized in Proposition 4.5. We consider an example with the marginal value of water in period one equals to zero, i.e., $Q_1^{i(4.11)} = \bar{Q}_1^i$ for $i = f,m,e$. If for any reason $A_1^{f(4.11)} > A_2^{f(4.11)}$, the legislative irreversibility implies that the allocation of water, $\{A_t^{f,(4.11)}, A_t^{m,(4.11)}, A_t^{e,(4.11)}\}_{t=1,2}$, is no longer achievable.[14] Indeed, with legislative irreversibility if the firm has the right to use $A_1^{f(4.11)}$ in period one, its allocation of water in period two cannot be limited to $A_2^{f(4.11)}$. With legislative irreversibility, the allocation of water to the firm is the same in both periods and lies between $A_2^{f(4.11)}$ and $A_1^{f(4.11)}$. As a result, even though the

[14]There are several reasons why $A_1^{f(4.11)}$ might be larger than $A_2^{f(4.11)}$, among them there is the possibility that the marginal value of water in first period is larger than in period two *ceteris paribus*, or the possibility that $\Theta_1 > \Theta_2$ *ceteris paribus*

marginal value of water is zero, it might be optimal to restrict its usage. In this example, the marginal value of water for the firm in period one (two) is larger (smaller) with legislative irreversibility than without. If the decision maker believes that the marginal value of water would be larger in the future, the current marginal value of water becomes larger. Consequently, the marginal value of water for the firm in period one is not zero any more. Moreover, the marginal value of water is no longer equal among users within a period. Proposition 4.5 generalizes this example.

Proposition 4.5. *With legislative irreversibility, if $A_1^{f(4.11)} > A_2^{f(4.11)}$:*

- *at the optimum, the marginal value of water for the firm is different from the marginal value of water for the municipality and the ecosystem:*

$$\lambda_1^{f(4.12)} \neq \lambda_1^{m(4.12)} = \lambda_1^{e(4.12)}; \quad \lambda_2^{f(4.12)} \neq \lambda_2^{m(4.12)} = \lambda_2^{e(4.12)};$$

- *the marginal value of water for the firm is larger (smaller) in period one (two) and the marginal value of water for the municipality and for the ecosystem is smaller (larger) in period one (two):*

$$\begin{aligned} &\lambda_1^{f(4.12)} \geq \lambda_1^{f(4.11)}; \quad \lambda_1^{m(4.12)} \leq \lambda_1^{m(4.11)}; \quad \lambda_1^{e(4.12)} \leq \lambda_1^{e(4.11)}, \\ &\lambda_2^{f(4.12)} \leq \lambda_2^{f(4.11)}; \quad \lambda_2^{m(4.12)} \geq \lambda_2^{m(4.11)}; \quad \lambda_2^{e(4.12)} \geq \lambda_2^{e(4.11)}. \end{aligned}$$

Proof of Proposition 4.5

To prove Proposition 4.5, we proceed with the characterization of the optimal allocation of water, $\{A_t^{f(4.12)}, A_t^{m(4.12)}, A_t^{e(4.12)}\}_{t=1,2}$. Four different allocations characterize the solution of (4.12): (4.13), (4.14), (4.15), and (4.16). To refer to one specific allocation we use the notation $\{A_t^{fj}, A_t^{mj}, A_t^{ej}\}_{t=1,2}$ for $j = (4.13), (4.14), (4.15), (4.16)$. To refer to the amount of water used by the different users induced by this specific optimal allocation, we use the notation $\{Q_t^{fj}, Q_t^{mj}, Q_t^{ej}\}_{t=1,2}$ for $j = (4.13), (4.14), (4.15), (4.16)$.
If $A_2^{f(4.11)} = \bar{Q}_2^f < A_1^{f(4.11)}$, there is no benefit from lowering the amount of water allocated to the firm in period one. If the decision maker reduces the allocation of water in

period one, then this reduction creates a loss of welfare in period one and doesn't create a gain in period two. Consequently, the optimal allocation of water is given by:

$$A_1^f = A_2^f = A_1^{f(4.11)}; \quad A_t^i = A_t^{i(4.11)} \quad t = 1,2 \quad i = m,e. \tag{4.13}$$

Given the optimal allocation of water, the amount of water used by the different users is given by:

$$Q_1^f = A_1^f; \quad Q_2^f = \bar{Q}_2^f; \quad Q_t^i = A_t^i \quad t = 1,2 \quad i = m,e.$$

Even though, the firm has the right to use $A_1^f > \bar{Q}_2^f$ units of water in period two, it uses only $\bar{Q}_2^f$. In period two, the difference between the amount of water allocated to the firm and the amount of water used by the firm, $A_1^f - \bar{Q}_2^f$, is useless to the firm. As a consequence, the decision maker can allocate to the municipality and the ecosystem $\bar{Q}_2^m$ and $\bar{Q}_2^e$ unit of water, respectively.[15] It is worth noting that in that case, the legislative irreversibility affects neither the marginal value of water nor the welfare at the optimal allocation. One example of the context in which the conditions for this solution to be optimal hold is when in period one the marginal value of water for the firm is larger than in period two and when there is no rivalry in use in period two. Figure 4.4.a illustrates this example.[16]

If $A_2^{f(4.11)} < A_1^{f(4.11)} < \bar{Q}_2^f$, the solution is characterized by:

$$\begin{aligned}
&A_t^i < \bar{Q}_t^i \quad t = 1,2 \quad i = f,m,e; \quad A_1^f = A_2^f, \\
&A_t^f + A_t^m + A_t^e = \Theta_t \quad t = 1,2, \\
&\lambda_1^f(A_1^f) - \lambda_1^m(A_1^m) + \lambda_2^f(A_1^f) = \lambda_2^m(A_2^m) = \lambda_2^e(A_2^e), \\
&\lambda_1^f(A_1^f) + \lambda_2^f(A_1^f) - \lambda_2^m(A_2^m) = \lambda_1^m(A_1^m) = \lambda_1^e(A_1^e).
\end{aligned} \tag{4.14}$$

[15] Note that even if $\sum_{i=f,m,e} A_2^i$ is larger than Θ_2, an allocation of water is achievable if $\sum_{i=f,m,e} Q_2^i \leq \Theta_2$.

[16] Without loss of generality, we represent the marginal value of water for the municipality and the ecosystem as a single curve in Figure 4.4.

Therefore,

$$\lambda_1^f(A_1^f)+\lambda_2^f(A_1^f)=\lambda_1^m(A_1^m)+\lambda_2^m(A_2^m)=\lambda_1^e(A_1^e)+\lambda_2^e(A_2^e).$$

Given the optimal allocation of water, every user uses exactly the amount of water allocated to them, i.e., $Q_t^i = A_t^i$ for $t=1,2$ and $i=f,m,e$. The first important remark about this solution is that the one period equality of the marginal value of water among the different users doesn't hold anymore. Moreover, at the allocation (4.14), the marginal value of water for the firm in period one (two) is larger (smaller) in comparison of the optimal allocation without legislative irreversibility. The inverse relationship holds for the municipality and the ecosystem. If the marginal value of water for the municipality or for the ecosystem is larger in period two than in period one and there is rivalry in use in both periods, the conditions for this case to be the solution hold. Figure 4.4.b illustrates this example.

If $A_2^{f(4.11)} < \bar{Q}_2^f \leq A_1^{f(4.11)}$, it is not always optimal to reduce the allocation of water to the firm in period one. The welfare loss in period one caused by the limitation of the allocation of water to the firm might not be compensated by the welfare gain in period two. Before determining a condition under which it is optimal to limit the allocation of water to the firm in period one, we characterize the optimal allocation in both situations. If the decision maker wants to reduce the allocation of water to the firm in period one, the optimal allocation is characterized by:

$$\begin{aligned} &A_1^f \in [A_2^{f(4.11)}, \bar{Q}_2^f]; \quad A_2^f = A_1^f, \\ &\lambda_t^m(A_t^m) = \lambda_t^e(A_t^e); \quad A_t^f + A_t^m + A_t^e = \Theta_t \quad t=1,2. \end{aligned} \tag{4.15}$$

We refer to the welfare associated with the allocation (4.15) as $\mathscr{W}^{(4.15)}$:

$$\begin{aligned} \mathscr{W}^{(4.15)} = &\int_0^{A_1^{f(4.15)}} \lambda_1^f(x)dx + \int_0^{A_1^{m(4.15)}} \lambda_1^m(x)dx + \int_0^{A_1^{e(4.15)}} \lambda_1^e(x)dx \\ &+ \int_0^{A_1^{f(4.15)}} \lambda_2^f(x)dx + \int_0^{A_2^{m(4.15)}} \lambda_2^m(x)dx + \int_0^{A_2^{e(4.15)}} \lambda_2^e(x)dx. \end{aligned}$$

(a) Optimal allocation characterized by (4.13) (b) Optimal allocation characterized by (4.14)

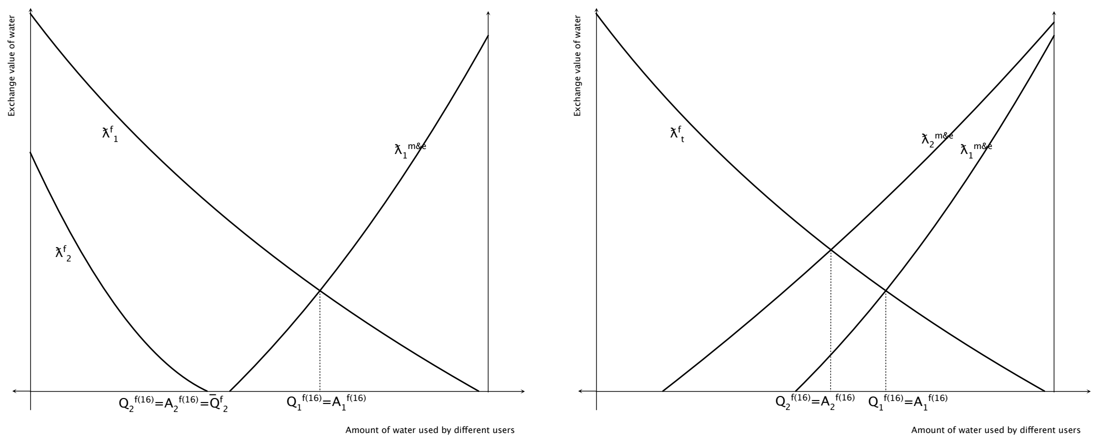

Figure 4.4: Legislative irreversibility, allocations (4.13) and (4.14)

If the decision maker doesn't limit the allocation of water to the firm in period one, the optimal allocation is characterized by:

$$\begin{aligned} &A_1^i = A_1^{i(4.11)} \quad i = f,m,e; \quad A_2^f = A_1^f, \\ &\lambda_2^m(A_2^m) = \lambda_2^e(A_2^e); \quad \bar{Q}_2^f + A_2^m + A_2^e = \Theta_2. \end{aligned} \tag{4.16}$$

We refer to the welfare associated with the allocation (4.16) as $\mathscr{W}^{(4.16)}$:

$$\begin{aligned} \mathscr{W}^{(4.16)} &= \sum_{t=1,2}\sum_{i=f,m,e}\int_0^{A_2^{i(4.16)}} \lambda_t^i(x)dx \\ &= \int_0^{A_1^{f(4.11)}} \lambda_1^f(x)dx + \int_0^{A_1^{m(4.11)}} \lambda_1^m(x)dx + \int_0^{A_1^{e(4.11)}} \lambda_1^e(x)dx \\ &\quad + \int_0^{\bar{Q}_2^f} \lambda_2^f(x)dx + \int_0^{A_2^{m(4.16)}} \lambda_2^m(x)dx + \int_0^{A_2^{e(4.16)}} \lambda_2^e(x)dx. \end{aligned}$$

To determine whether it is optimal to limit the amount of water allocated to the firm in period one, we compare $\mathscr{W}^{(4.15)}$ to $\mathscr{W}^{(4.16)}$. It is optimal to reduce the allocation of water to the firm in period one if $\mathscr{W}^{(4.15)} \geq \mathscr{W}^{(4.16)}$, that is:

$$\mathscr{W}^{(4.15)} - \mathscr{W}^{(4.16)} = \underbrace{- \int_{A_1^{f(4.15)}}^{A_1^{f(4.11)}} \lambda_1^f(x)dx + \int_{A_1^{m(4.11)}}^{A_1^{m(4.15)}} \lambda_1^m(x)dx + \int_{A_1^{e(4.11)}}^{A_1^{e(4.15)}} \lambda_1^e(x)dx}_{\Delta_1 < 0}$$
$$\underbrace{- \int_{A_1^{f(4.15)}}^{\bar{Q}_2^f} \lambda_2^f(x)dx + \int_{A_2^{m(4.11)}}^{A_2^{m(4.15)}} \lambda_2^m(x)dx + \int_{A_2^{e(4.11))}}^{A_2^{e(4.15)}} \lambda_2^e(x)dx}_{\Delta_2 > 0} \geq 0.$$

The reduction of the allocation of water to the firm leads to a welfare loss in period one, which we denote by Δ_1, whereas, it also leads to a welfare gain in period two defined by Δ_2.

If there exists $A_1^f \in [A_2^{f(4.11)}, \bar{Q}_2^f]$, such that the welfare loss in period one is totally compensated by the gain in period two, the decision maker reduces the amount of water allocated to the firm in period one compared to the optimal allocation of the unconstraint maximisation problem (4.11). The optimal allocation is characterized by (4.15). The equality of the marginal value of water among the different users doesn't hold anymore. Moreover, at the allocation (4.15), the marginal value of water for the firm in period one (two) is larger (smaller) compared to the optimal allocation without legislative irreversibility (4.11). The inverse relationship holds for the municipality and the ecosystem. Figure 4.5.a illustrates this case.[17] The welfare loss is represented by the green and the blue-green area, whereas the gain is represented by the blue and the blue-green area. This case might happen if the marginal value of water for the municipality and the firm is larger in period two than in period one and if the marginal value of water for the firm is slightly smaller in period two than in period one.

If for all $A_1^f \in [A_2^{f(4.11)}, \bar{Q}_2^f]$, the welfare loss in period one is larger than the gain in period two, it is not optimal to reduce the amount of water allocated to the firm in period one compared to the optimal allocation of the unconstrained maximisation problem (4.11). The optimal allocation is characterized by (4.16). The equality of the marginal value of

[17]Without loss of generality, we represent the marginal value of water for the municipality and the ecosystem as a single curve in Figure 4.5.

(a) Optimal allocation characterized by (4.15) (b) Optimal allocation characterized by (4.16)

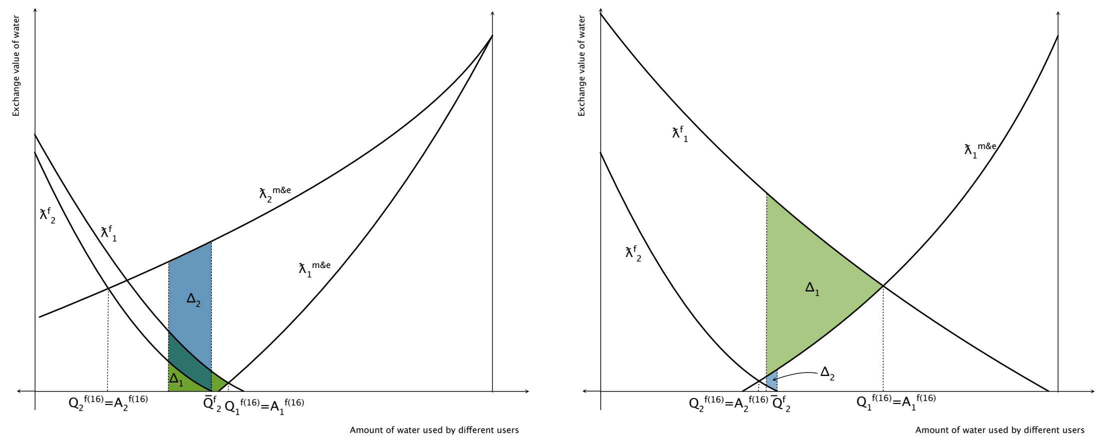

Figure 4.5: Legislative irreversibility, allocations (4.15) and (4.16)

water among the different users in period two doesn't hold any more, however, in period one the marginal value of water doesn't change. At the allocation (4.16), the marginal value of water for the firm in period two is smaller compared to the optimal allocation without legislative irreversibility. The inverse relationship holds for the municipality and the ecosystem. Figure 4.5.b represents this case. There does not exist $A_1^f \in [A_2^{f(4.11)}, \bar{Q}_2^f]$ such that the welfare loss in period one, represented by the green area, is lower than welfare gain in period two represented by the blue area. This case might happen if the marginal value of water for firm in period one is really larger than in period two. *Q.E.D.*

In the following sections, we analyze the impact of an irreversible damage to the ecosystem created by the net investment of the firm on the optimal allocation of water . In Section 4.4.3, we assume that there is no legislative irreversibility. This assumption is relaxed in Section 4.4.4.

4.4.3 Investment

We suppose that a positive net investment, $K_2 - (1-\delta)K_1 > 0$, creates an irreversible damage to the ecosystem. The damage is fixed, in other words, it depends neither on the level of the net investment nor on the level of production. We suppose that the damage is independent of the level of the net investment to represent investments that affect the ecosystem regardless of their size as discussed in Section 4.1. Some types of investment affect in an irreversible way the ecosystem, as the project of new oil port in Cacouna. Regardless of the size of an oil port, once the habitat of beluga is affected, this species would be in danger. Damage that depends solely on the size of the investment would not model properly this reality.

As long as the investment is larger than the capital depreciation, δK_1, the ecosystem suffers the damage. The damage creates a lump sum reduction of welfare denoted by D. If $I > \delta K_1$, then regardless of the level of investment there is a damage. On the other hand, if $I \leq \delta K_1$, there is no damage. We define the damage function by:

$$\mathscr{D} = \begin{cases} D & \text{if} \quad I > \delta K_1 \\ 0 & \text{if} \quad I \leq \delta K_1. \end{cases}$$

It is important to note that from the *Law of Water*, the manager of the water has no taxing power. To prevent the damage, his only way is to implement a change in the investment of the firm through a modification of its allocation of water. In that section, we characterize the range of welfare losses created by the damage that induce a modification of the optimal allocation of water.

The decision maker chooses the allocation of water among the firm, the municipality and the ecosystem by maximizing the welfare:

$$\begin{aligned} \max_{\{A_t^f, A_t^m, A_t^e\}} \sum_{t=1,2} & \left[\int_0^{Q_t^f} \lambda_t^f(x)dx + \int_0^{Q_t^m} \lambda_t^m(x)dx + \int_0^{Q_t^e} \lambda_t^e(x)dx \right] - \mathscr{D} \qquad (4.17) \\ \text{s.t.} \quad & Q_t^f + Q_t^m + Q_t^e \leq \Theta_t, \quad t = 1,2 \\ & Q_t^i = \min\{A_t^i, \bar{Q}_t^i\} \quad t = 1,2 \quad i = f,m,e. \end{aligned}$$

The solution of (4.17) and the amount of water used by the different users induced by the solution of (4.17) are denoted respectively by:

$$\{A_t^{f(4.17)}, A_t^{m(4.17)}, A_t^{e(4.17)}\}_{t=1,2} \quad \text{and} \quad \{Q_t^{f(4.17)}, Q_t^{m(4.17)}, Q_t^{e(4.17)}\}_{t=1,2}.$$

The decision maker has to decide whether he intervenes or not. From (III.5), $dK_2/dA_2^f > 0$, hence to prevent the damage the decision maker has to limit the allocation of water to the firm. The decision maker intervenes only if the welfare loss induced by the damage is larger than the welfare loss induced by his intervention.

If the decision maker doesn't intervene, the optimal allocation of water among users is characterized by $\{A_t^{f,(4.11)}, A_t^{m,(4.11)}, A_t^{e,(4.11)}\}_{t=1,2}$.

It is worth mentioning that if the investment induced by the allocation of water $A_2^{f(4.11)}$ is smaller than the depreciation of the capital, then there is no damage. In other words, if the net investment induced by the optimal allocation $A_2^{f(4.11)}$ is negative, then the decision maker doesn't have to intervene to prevent to damage, because there is no damage. We restrict our attention to cases where the optimal allocation induces a positive level of net investment.

The allocation of water that leads to the prevention of the damage is noted by $\tilde{A}_2^f$ and is defined as the level of allocation of water that induces $I = \delta K_1$. Since the amount of water allocated to the firm has a positive impact on K_2:

$$\tilde{A}_2^f \leq A_2^{f(4.11)}.$$

To prevent the damage, the decision maker allocates $\tilde{A}_2^f$ units of water to the firm.

In order to characterize a set of welfare losses created by the net investment of the firm that induce the decision maker to intervene by reducing the allocation of water to the firm in period two, we determine the smallest of these losses. We define $\tilde{D}$ as the welfare loss induced by the damage such that the decision maker is indifferent between intervening to prevent the damage and not. In order to evaluate $\tilde{D}$ in that situation, we should first analyze the optimal allocation of water if the decision maker intervenes to prevent the damage.

Proposition 4.6. *If the decision maker intervenes to prevent the damage,*

- *in period 1, the optimal allocation of water is characterized by:*

$$\{A_1^{f(4.11)}, A_1^{m(4.11)}, A_1^{e(4.11)}\};$$

- *in period 2, the optimal allocation of water is characterized by:*

$$A_2^f = \tilde{A}_2^f; \quad A_2^m + A_2^e \leq \Theta_2 - \tilde{A}_2^f; \quad \lambda_2^m(A_2^m) = \lambda_2^e(A_2^e).$$

Proof of Proposition 4.6

The prevention of the damage has no impact on the first period, therefore the solution of (4.11) is the optimal allocation.

In period 2, to prevent the damage the net investment should be equal to the depreciation of the capital. In order to induce this level of investment the allocation of water to the firm is set to $\tilde{A}_2^f$.

If $\tilde{A}_2^f + \bar{Q}_2^m + \bar{Q}_2^e \leq \Theta_t$, then $A_2^i = \bar{Q}_2^i$ for $i = m, e$. Consequently, $A_2^m + A_2^e \leq \Theta_2 - \tilde{A}_2^f$.

If $\tilde{A}_2^f + \bar{Q}_2^m + \bar{Q}_2^e > \Theta_t$, then $A_2^i < \bar{Q}_2^i$ for $i = m, e$ and $\lambda_2^m(A_2^m) = \lambda_2^e(A_2^e)$. Consequently, $A_2^m + A_2^e = \Theta_2 - \tilde{A}_2^f$. *Q.E.D.*

To avoid any confusion, we denote respectively by $\tilde{A}_2^m$ and $\tilde{A}_2^e$ the allocation of water to the municipality and to the ecosystem when the decision maker intervenes to prevent the damage.

Proposition 4.7. *If the decision maker intervenes to prevent the damage,*

- *in period 2, the marginal value of water for the firm is different from the marginal value of water for the municipality and the ecosystem;*

- *in period 2, the intervention leads to an increase of the marginal value of water for the firm and to a decrease of the marginal value of water for the municipality and the ecosystem.*

Proof of Proposition 4.7

The proof of Proposition 4.7 comes directly from the proof of Proposition 4.6.

If the decision maker doesn't intervene to prevent the damage, from the solution of (4.11), the marginal value of water among users are equal.

If the decision maker intervenes to prevent the damage, from Proposition 4.6, he reduces the amount of water allocated to the firm and increases the amount of water allocated to the municipality and the ecosystem. As a result, the marginal value of water for the firm increases and the marginal value of water for the municipality and the ecosystem falls. Consequently, the marginal value of water among users are not equal anymore. *Q.E.D.*

In the following step, we evaluate the welfare when the decision maker doesn't intervene and when he intervenes to prevent the damage. If he doesn't intervene, the welfare is given by:

$$\mathscr{W} = \sum_{t=1,2} \sum_{i=f,m,e} \int_0^{Q_t^{i(4.11)}} \lambda_t^i(x)dx - D. \tag{4.18}$$

We refer to this welfare as $\mathscr{W}^{(4.18)}$. If the decision maker intervenes, the welfare is given by:

$$\mathscr{W} = \sum_{i=f,m,e} \int_0^{Q_1^{i(4.11)}} \lambda_1^i(x)dx + \sum_{i=f,m,e} \int_0^{\tilde{A}_2^i} \lambda_2^i(x)dx. \tag{4.19}$$

We refer to this welfare as $\mathscr{W}^{(4.19)}$. In that situation, $\tilde{D}$ is the level of damage such that $\mathscr{W}^{(4.18)} = \mathscr{W}^{(4.19)}$, that is:

$$\begin{aligned}
\tilde{D} &= \sum_{t=1,2} \sum_{i=f,m,e} \int_0^{Q_t^{i(4.11)}} \lambda_t^i(x)dx - \sum_{i=f,m,e} \int_0^{Q_1^{i(4.11)}} \lambda_1^i(x)dx - \sum_{i=f,m,e} \int_0^{\tilde{A}_2^i} \lambda_2^i(x)dx \\
&= \sum_{i=f,m,e} \int_0^{Q_2^{i(4.11)}} \lambda_2^i(x)dx - \sum_{i=f,m,e} \int_0^{\tilde{A}_2^i} \lambda_2^i(x)dx \\
&= \int_{\tilde{A}_2^f}^{Q_2^{f(4.11)}} \lambda_2^i(x)dx - \int_{Q_2^{m(4.11)}}^{\tilde{A}_2^m} \lambda_2^i(x)dx - \int_{Q_2^{e(4.11)}}^{\tilde{A}_2^e} \lambda_2^i(x)dx.
\end{aligned} \tag{4.20}$$

If $D < \tilde{D}$, the decision maker doesn't intervene to prevent the damage, otherwise he intervenes to prevent the damage.

Parameters that affect the intervention

Every parameter that affects the marginal value of water for the firm, for the municipality or for the ecosystem in period two affects also $\tilde{D}$.

An increase of every parameter that has a positive impact on the marginal value of water for the municipality or on the marginal value of water for the ecosystem reduces $\tilde{D}$. Consequently, the decision maker intervenes to prevent damages that create smaller welfare loss. Figure 4.6 illustrates an increase in the marginal value of water for the municipality.[18] The addition of the dark-blue and the pale-green areas represents $\tilde{D}$ prior to the increase of the marginal value of water for the municipality. The dark-blue area represents $\tilde{D}$ after the increase in the marginal value of water for the municipality.

The impact of a variation of the input prices on the marginal value of water for the firm has been calculated in Section 4.3.1.1. The variation of the marginal value of water for the firm that is induced by a variation of r, w_2, r^s, r_2^c, and p_2, is represented respectively by (III.17), (III.18), (III.19), (III.20), and (III.21). The marginal value of water for the firm is negatively affected by r, w_2, r^s, and r_2^c, whereas p_2 has a positive impact on λ_2^f. The level of capital that leads to the prevention of the damage, δK_1, is not affected by the variation of input prices. Does it imply that the allocation of water that induces the firm to choose the level of capital that leads to the prevention of the damage is not affected? The answer to that question is no. Proposition 4.8 establishes the impact on the intervention of the decision maker of a variation of r, w_2, r^s, r_2^c, and p_2.

Proposition 4.8. *The input prices affect negatively $\tilde{D}$, whereas p_2 has a positive impact on $\tilde{D}$:*

$$\frac{d\tilde{D}}{dr_2^c} < 0; \quad \frac{d\tilde{D}}{dr} < 0; \quad \frac{d\tilde{D}}{dr^s} < 0; \quad \frac{d\tilde{D}}{dw_2} < 0; \quad \frac{d\tilde{D}}{dp_2} > 0.$$

Proof of Proposition 4.8

The price of the inputs that are perfect complements of water has no impact on K_2, L_2, and K_2^s. Consequently, the choice of A_2^f is not affected by the variation of r_2^c. However,

[18] An increase in the marginal value of water for the ecosystem can similarly be illustrated.

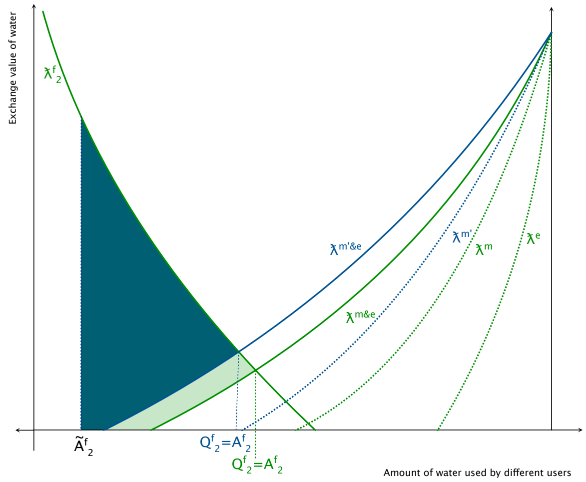

Figure 4.6: Impact of an increase of the value of water for the municipality

the value of water for the firm is negatively affected. Figure 4.7.a illustrates those effects. If r_2^c increases, the range of welfare losses induced by the damage for which the decision maker intervenes increases as well.

From (III.10), (III.11), and (III.12), we know that r, w_2 and r^s affect negatively K_2. Therefore, an increase of r, w_2 or r^s induces the decision maker to choose a larger $\tilde{A}_2^f$. Moreover, from (III.17), (III.18), and (III.19), the impact of these input prices on the value of water for the firm in period two is negative. Consequently, the range of welfare losses induced by the damage for which the decision maker intervenes increases. The decision maker intervenes for smaller losses of welfare induced by the damage. Figure 4.7.b illustrates these effects.

In Figures 4.7.a and 4.7.b, the addition of the dark-blue and the pale-green areas represents the minimum damage for which the decision maker intervenes prior to the increase of one of the input prices. The dark-blue area represents the minimum damage for which the decision maker intervenes to prevent the damage after the increase of one of the input prices.

From (III.13), the impact of p_2 on the capital K_2 is positive. Therefore, an increase of p_2 induces the decision maker to reduce the amount of water allocated to the firm to prevent the damage, $\tilde{A}_2^f$. From (III.21), the marginal value of water for the firm is also affected

positively by a change of p_2. Figure 4.7.c illustrates these effects. The range of losses of welfare induced by the damage for which the decision maker intervenes is reduced.
In Figure 4.7.c, the green area represents the minimum damage for which the decision maker intervenes prior to the increase of p_2. The dark-blue area represents the minimum damage for which the decision maker intervenes to prevent the damage after the increase of p_2. *Q.E.D.*

Impact of I^s

We must question ourselves on the impact of the investment that affects the productivity of water, I^s, on the decision maker's intervention. In order to prevent the damage, does the decision maker allocate more or less water to the firm when the firm can invest in K^s? Does the investment in K^s induce the decision maker to intervene more or less often to prevent the damage? To address these questions, we analyze the behaviour of the firm when this investment is so expensive that the firm doesn't invest in capital K^s, therefore $K_2^s = (1-\delta^s)K_1^s$. Proposition 4.9 addresses these questions.

Proposition 4.9. *The investment in K^s affects positively $\tilde{D}$.*

Proof of Proposition 4.9
It is worth mentioning that the level of investment needed to prevent the damage, δK^1, is independent of the investment in capital K^s. There exists a unique z_2 such that $I = \delta K^1$. Because two inputs compose z_2, Q_2 and K_2^s, the investment in K^s induces the decision maker to allocate a smaller amount of water to the firm.
If the firm doesn't invest in the capital that affects the productivity of water, then each unit of water is less productive. Consequently, the marginal value of water is negatively affected. From (III.16), the impact of K_2^s on λ_2^f is positive, therefore if there is no investment in K^s, then the marginal value of water is smaller. Consequently, $\tilde{D}$ is positively affected by the investment in K^s. *Q.E.D.*

In Figure 4.8, the green (blue) area represents $\tilde{D}$ without (with) the investment in K^s. The investment in K^s induces the decision maker to intervene only for larger welfare

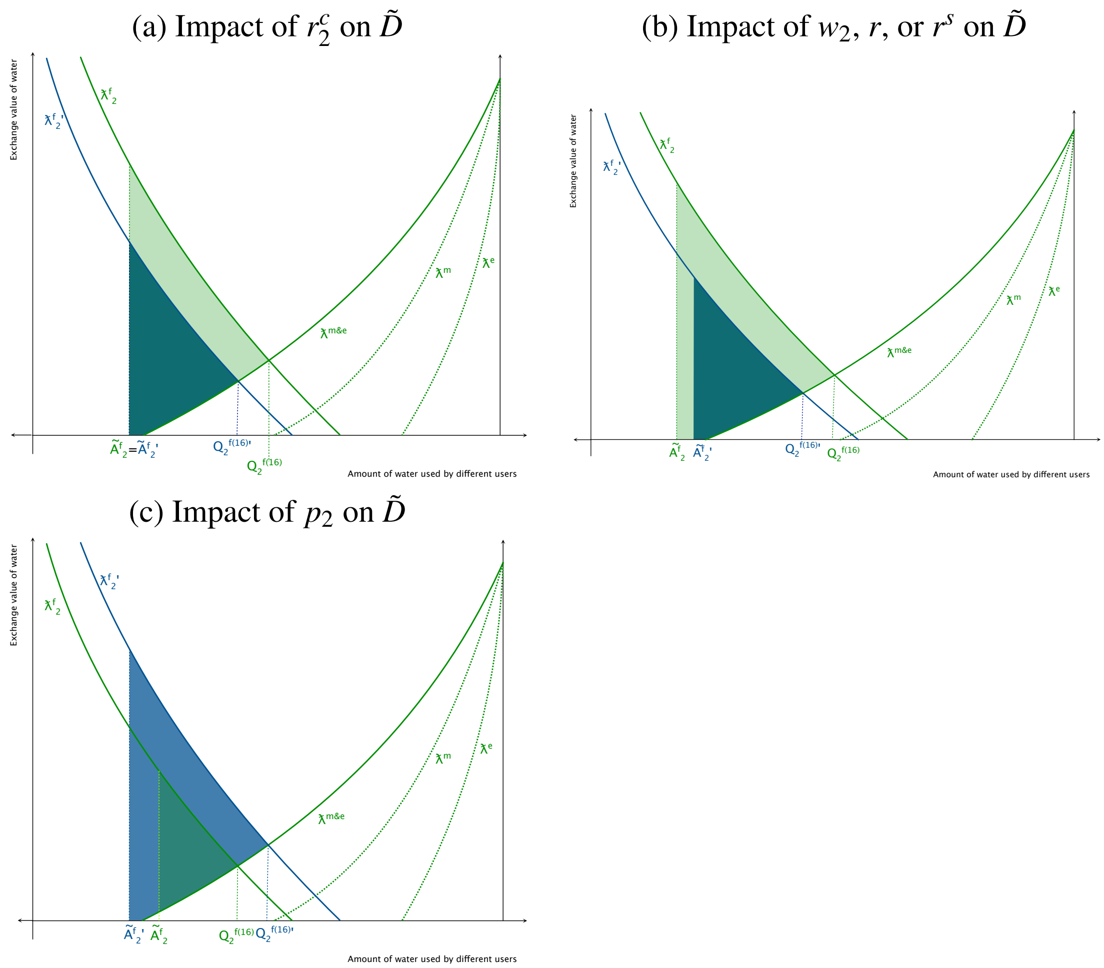

Figure 4.7: Impact r_2^c, r^s, w_2, r, and p_2 on $\tilde{D}$

losses created by the damage. In other words, the range of losses of welfare induced by the damage for which the decision maker intervenes is reduced.

4.4.4 Investment and legislative irreversibility

How does the presence of the legislative irreversibility introduced in Section 4.4.2 affect the intervention of the decision maker to prevent a fixed irreversible damage caused by the net investment of the firm that creates a lump sum reduction of welfare as in Section 4.4.3? To address this question, we determine the impact of the legislative irreversibility on the level of loss created by the damage that induces the decision maker to be indifferent between intervening or not, which is $\tilde{D}$.

First, we specify the maximization of the welfare taking into account the fixed damage function, $\mathscr{D}$, and the condition that the allocation of water to the firm in period 1 is the lower bound of its allocation in period 2.

The decision maker chooses the allocation of water among the firm, the municipality, and the ecosystem by maximizing the welfare:

$$\begin{aligned} \max_{\{A_t^f, A_t^m, A_t^e\}} \sum_{t=1,2} & \left[\int_0^{Q_t^f} \lambda_t^f(x)dx + \int_0^{Q_t^m} \lambda_t^m(x)dx + \int_0^{Q_t^e} \lambda_t^e(x)dx \right] - \mathscr{D} \qquad (4.21) \\ \text{s.t.} \quad & A_1^f < A_2^f \\ & Q_t^f + Q_t^m + Q_t^e \leq \Theta_t \\ & Q_t^i = \min\{A_t^i, \bar{Q}_t^i\} \quad t = 1,2 \quad i = f,m,e. \end{aligned}$$

The solution of (4.21) and the amount of water used by the different users induced by the solution of (4.21) are denoted respectively by:

$$\{A_t^{f(4.21)}, A_t^{m(4.21)}, A_t^{e(4.21)}\}_{t=1,2} \quad \text{and} \quad \{Q_t^{f(4.21)}, Q_t^{m(4.21)}, Q_t^{e(4.21)}\}_{t=1,2}.$$

As in Section 4.4.3, the decision maker has to decide whether he intervenes to prevent the damage or not. He intervenes if the welfare loss induced by the damage is larger than the welfare loss induced by his intervention. Moreover, as in Section 4.4.2, the

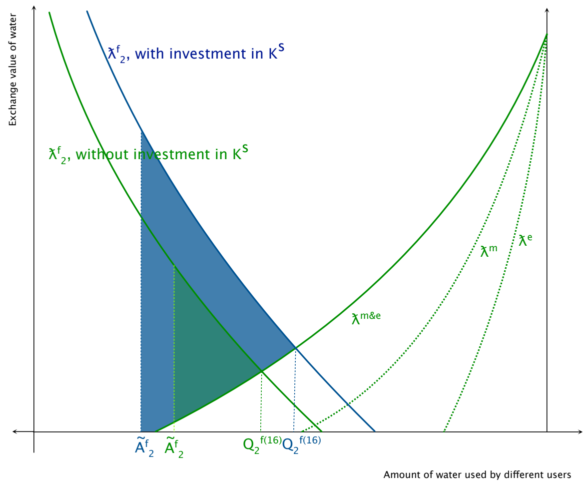

Figure 4.8: Impact of the presence of the investment in K^s on $\tilde{D}$

decision maker has to decide whether he limits the allocation of water in period one or not. By contrast with Section 4.4.3, with the legislative irreversibility, the intervention of the decision maker might affect the optimal allocation of water in the first period.

Before determining the impact of the legislative irreversibility on $\tilde{D}$, we characterize the solution of (4.21).

If the decision maker doesn't intervene, the allocation of water among users is characterized by the optimal allocation of (4.12), i.e., $\{A_t^{f(4.12)}, A_t^{m(4.12)}, A_t^{e(4.12)}\}_{t=1,2}$.

The presence of the legal irreversibility doesn't affect the amount of water the decision maker allocates to the firm to prevent the damage. In Section 4.4.3, this amount of water has been denoted by $\tilde{A}_2$. As in Section 4.4.3 we restrict our attention to cases where the optimal allocation induces a positive level of net investment, with legislative irreversibility the condition becomes $A_t^{f(4.12)} > \tilde{A}_2$. Proposition 4.10 specifies the optimal allocation of water among users if the damage is prevented.

Proposition 4.10. *If the decision maker intervenes to prevent the damage, the solution of (4.21) is characterized by:*

- *if* $A_1^{f(4.11)} \leq \tilde{A}_2^f$*, the optimal allocation is characterized by the solution (4.17);*

- *if $A_1^{f(4.11)} > \tilde{A}_2^f$, the optimal allocation is characterized by:*

$$A_1^f = A_2^f = \tilde{A}_2^f; \quad \lambda_t^m(A_t^m) = \lambda_t^e(A_t^e),$$
$$\tilde{A}_2^f + A_t^m + A_t^e = \Theta_t \quad t = 1,2.$$

Proof of Proposition 4.10
If $A_1^{f(4.11)} \leq \tilde{A}_2^f$, the optimal allocation with and without legal irreversibility are identical. If the decision maker wants to prevent the damage, he doesn't have to restrict the amount of water allocated to the firm in period 1. In that case, the solution of (4.21) is given by the solution of (4.17).
If $A_1^{f(4.11)} > \tilde{A}_2^f$, to prevent the damage, the net investment should be equal to the depreciation of the capital. In order to induce this level of investment the allocation of water to the firm in period 2 is set to $\tilde{A}_2^f$. Due to the legislative irreversibility, the amount of water allocated to the firm in period 1 should be limited to $\tilde{A}_2^f$ as well.
If $\tilde{A}_t^f + \bar{Q}_t^m + \bar{Q}_t^e \leq \Theta_t$, then $A_t^i = \bar{Q}_t^i$ for $i = m, e$ and $t = 1,2$. Consequently, $A_t^m + A_t^e \leq \Theta_2 - \tilde{A}_2^f$.
If $\tilde{A}_t^f + \bar{Q}_t^m + \bar{Q}_t^e < \Theta_t$, then $A_t^i < \bar{Q}_t^i$ for $i = m, e$ and $t = 1,2$, and $\lambda_t^m(A_t^m) = \lambda_t^e(A_t^e)$. Consequently, $A_t^m + A_t^e = \Theta_2 - \tilde{A}_2^f$. To avoid any confusion, we denote by $\tilde{A}_1^m$ and $\tilde{A}_1^e$ the allocation of water to the municipality and the ecosystem when the decision maker intervenes to prevent the damage. *Q.E.D.*

Proposition 4.7 establishes that without legislative irreversibility, if the decision maker intervenes to prevent the damage, the equalization of the marginal value of water between users doesn't hold in period 2. The intervention leads to an increase (decrease) in the marginal value of water for the firm (the municipality and the ecosystem). Moreover, in period 1, Proposition 4.7 states that the intervention has no effect on the optimal allocation and hence, on the marginal value of water. These results are slightly different with legislative irreversibility. Proposition 4.11 specifies those differences. This proposition establishes that legislative irreversibility has no impact on the effect of the intervention on the marginal value of water in period 2, in contrast with the first period.

Proposition 4.11. *If the decision maker intervenes to prevent the damage,*

- *the legislative irreversibility has no impact on the effect of the intervention on the marginal value of water in period 2;*
- *in period 1, if* $A_1^{f(4.11)} > \tilde{A}_2^f$*, the intervention leads to an increase in the marginal value of water for the firm and to a decrease of the marginal value of water for the municipality and the ecosystem. Consequently, the marginal value of water for the firm is different from the marginal value of water for the municipality and the ecosystem;*
- *in period 1, if* $A_1^{f(4.11)} \leq \tilde{A}_2^f$*, the intervention has no impact on the marginal value of water.*

Proof of Proposition 4.11

Suppose that the decision maker intervenes to prevent the damage.

Period 2

The allocation of water among users is the same regardless the presence of legislative irreversibility. Therefore, the presence of legislative irreversibility has no impact on the effect of the intervention on the value of water.

Period 1

If $A_1^{f(4.11)} \leq \tilde{A}_2^f$, the allocation of water is characterized by the solution of (4.17). From Proposition 4.7, the intervention has no impact.

If $A_1^{f(4.11)} > \tilde{A}_2^f$, from Proposition 4.10, the amount of water allocated to the firm (the municipality and the ecosystem) with the legislative irreversibility is lower (larger) than without. Therefore, the marginal value of water for the firm (the municipality and the ecosystem) increases (decreases). Since, without legislative the marginal value of water is equal among users, with legislative irreversibility, the equality does hold. *Q.E.D.*

The following proposition establishes the effect of legislative irreversibility on the welfare loss created by the damage for which the decision maker is indifferent between intervening and not intervening.

Proposition 4.12. *The effect on $\tilde{D}$ of legislative irreversibility is:*

- *null if $A_1^{f(4.11)} \leq \tilde{A}_2^f$;*
- *positive if $A_2^{f(4.11)} = \bar{Q}_2^f < A_1^{f(4.11)}$ or if $\quad \tilde{A}_2^f < A_1^{f(4.11)} \leq A_2^{f(4.11)}$;*
- *ambiguous if otherwise.*

Proof of Proposition 4.12

If the decision maker intervenes to prevent the damage, the legislative irreversibility is an issue and the welfare is given by:

$$\mathscr{W} = \sum_{t=1,2} \sum_{i=f,m,e} \int_0^{\tilde{A}_t^i} \lambda_t^i(x)dx. \tag{4.22}$$

We refer to this welfare as $\mathscr{W}^{(4.22)}$. Since $\tilde{A}_1^i > Q_1^{i(4.11)}$ for $i = m, e$ and $\tilde{A}_2^f < Q_1^{f(4.11)}$, $\mathscr{W}^{(4.22)}$ is smaller than $\mathscr{W}^{(4.19)}$, which is the welfare when the decision maker intervenes to prevent the damage without legislative irreversibility. In other words, when the decision maker intervenes to prevent the damage, the welfare is always smaller with legislative irreversibility than without.

Case $A_1^{f(4.11)} \leq \tilde{A}_2^f$

From Proposition 4.10, the optimal allocation of (4.21) is characterized by the optimal allocation of (4.17). Therefore, legislative irreversibility is not an issue and it has no impact on $\tilde{D}$.

Case $\tilde{A}_2^f < A_1^{f(4.11)} \leq A_2^{f(4.11)}$

If the decision maker doesn't intervene to prevent the damage, the legislative irreversibility is not an issue and the welfare is given by $\mathscr{W}^{(4.18)}$.

The combination of the preceding remark and the fact that $\mathscr{W}^{(4.22)}$ is smaller than $\mathscr{W}^{(4.19)}$ implies that legislative irreversibility has a positive effect on $\tilde{D}$.

Case $A_2^{f(4.11)} = \bar{Q}_2^f < A_1^{f(4.11)}$

If the decision maker intervenes to prevent the damage, the welfare is given by $\mathscr{W}^{(4.22)}$ which is smaller than $\mathscr{W}^{(4.19)}$.

If the decision maker doesn't intervene to prevent the damage, the allocation of water among users is the same regardless the presence of legislative irreversibility and the welfare is given by $\mathscr{W}^{(4.18)}$.

Consequently, legislative irreversibility leads to an increase of $\tilde{D}$.

Case $A_2^{f(4.11)} < \bar{Q}_2^f \leq A_1^{f(4.11)}$

If the decision maker doesn't intervene to prevent the damage, the optimal allocation is characterized by either (4.14), (4.15), or (4.16) and the welfare is characterized by:

$$W = \sum_{t=1,2} \sum_{i=f,m,e} \int_0^{A_t^{ij}} \lambda_t^f(x)dx - D \quad j = (4.14), (4.15), (4.16). \tag{4.23}$$

We refer to this welfare as $\mathscr{W}^{(4.23)}$. We evaluate the level of welfare loss that induces the decision maker to be independent between intervening and not, that is $\tilde{D}$. In that situation, $\tilde{D}$ is the welfare loss created by the damage that induces the equality between $\mathscr{W}^{(4.23)}$ and $\mathscr{W}^{(4.22)}$, that is:

$$\tilde{D} = \sum_{i=f,m,e} \int_{\tilde{A}_1^i}^{A_1^{ij}} \lambda_1^f(x)dx + \sum_{i=f,m,e} \int_{\tilde{A}_2^i}^{A_2^{ij}} \lambda_t^f(x)dx \quad j = (4.14), (4.15), (4.16). \tag{4.24}$$

The first argument of (4.24) represents the welfare loss in period 1 caused by the intervention of the decision maker to prevent the damage in the presence of legislative irreversibility. It is worth noting that without legislative irreversibility, there is no such loss in the first period. The second argument of (4.24) summarizes two opposite effects of the intervention on the welfare in period 2. First, the augmentation of the allocation of water to the municipality and the ecosystem from A^{ij} for j=(4.14), (4.15), (4.16) to $\tilde{A}_2^i$ for $i = m, e$ has a positive impact on the welfare, whereas the reduction of the allocation of water to the firm from A^{fj} for j=(4.14), (4.15), (4.16) to $\tilde{A}_2^f$, affects negatively the welfare.

In the following, we prove that the effect of legislative irreversibility on $\tilde{D}$ is ambiguous. In order to do so, we compare (4.20) to (4.24). The difference between (4.20) and (4.24) is defined as $\Delta_{\tilde{D}}$:

$$\Delta_{\tilde{D}} = -\sum_{i=f,m,e} \int_{\tilde{A}_1^i}^{A_1^{ij}} \lambda_1^f(x)dx - \sum_{i=f,m,e} \int_{A_2^{i(4.11)}}^{A_2^{ij}} \lambda_t^f(x)dx \quad j = (4.14), (4.15), (4.16).$$

Since for j=(4.14), (4.15), (4.16),

$$\begin{array}{lll} A_1^{f(j)} \leq A^{f(4.11)}; & A_1^{m(j)} \geq A_1^{m(4.11)}; & A_1^{e(j)} \geq A_1^{e(4.11)}, \\ A_2^{f(j)} > A_2^{f(4.11)}; & A_2^{m(j)} < A_2^{m(4.11)}; & A_2^{e(j)} < A_2^{e(4.11)}, \end{array}$$

the sign of $\Delta_{\tilde{D}}$ is undetermined. If $\Delta_{\tilde{D}} > 0$, the legislative irreversibility has a positive effect on $\tilde{D}$, and negative otherwise. *Q.E.D.*

We have proved that the presence of legislative irreversibility has an ambiguous effect on the range of losses created by the damage for which the decision maker intervenes.

4.5 Conclusion

We develop a two-period model to analyze the value and the optimal allocation of water in the context of sharing water from a lake among different users in the presence of legislative irreversibility and irreversible damage.

The optimal allocation in the benchmark model without irreversibility is analyzed first. If there is neither legislative irreversibility nor irreversible damage, then the marginal value is equal across users and might be zero.

Then, we analyze both types of irreversibility separately. With legislative irreversibility, it is sometimes optimal to reduce the amount of water allocated to the firm, even though there is unused water in the lake. The optimal amount of water allocated to the firm in the first period falls, whereas it raises in the second period. The inverse relationship holds for the municipality and the ecosystem. At the optimal allocation of water, the marginal value is not equal across users in both periods. When investment irreversibly damages the ecosystem, our work suggests that it is not always optimal to prevent the damage. To prevent the damage, the allocation of water to the firm falls in the second period. Consequently, the equalization of marginal value between users doesn't hold. The damage has no impact on the value and the optimal allocation of water in period 1. At last, to approach the reality, we study both types of irreversibility together. We establish that the legislative irreversibility has an ambiguous impact on the intervention to prevent the damage. Contrary to the case without legislative irreversibility, when the damage is prevented, the amount of water allocated to the firm falls in both periods. Consequently, in both periods, the equalization of the marginal value between users doesn't hold.

Finally, we see several extensions of the model that would further our understanding of the optimal allocation of water among different users. First, it would be worthwhile to extend the model by introducing uncertainty about the total amount of water in the lake. This uncertainty would add realism into the model. In the current analysis, we consider investments that affect the ecosystem regardless of their sizes, to illustrate that the magnitude of investments is not always a good indicator of their impacts on the ecosystem. Some projects create irreversible damages to the ecosystem that are independent of their sizes. Nevertheless, it would be interesting to consider the possibility that investments might also create variable damages. In order to do so, we would need to generalize the damage function, by adding to the lump sum welfare loss created by the damage a variable component. It would be interesting to address those possibilities in future research.

CHAPITRE 5

CONCLUSION

L'object du premier volet, composé des Chapitres 2 et 3, de cette thèse est d'étudier comment l'utilisation et l'évolution des ressources naturelles sont affectés par nos choix. Le Chapitre 2 a pour objectif de donner un aperçu des effets du stockage de ressources naturelles renouvelables sur celles-ci. Je montre que dans le court terme le stockage des ressources a un effet négatif sur le stock des ressource, tandis que dans le long terme, l'effet est plutôt positif. De plus, je montre que l'effet sur le bien-être est ambigu. Le Chapitre 3 vise à approfondir les connaissances des effets de la migration sur le commerce et l'environnement. Un des résultats importants du chapitre est que la présence de la mobilité des travailleurs qualifiés tend à amplifier l'effet potentiellement négatif du commerce sur la région la plus polluante.

L'objectif du deuxième volet est d'analyser la valorisation et l'allocation des ressources, en particulier celles de l'eau. L'un des messages principaux de ce dernier volet est que l'eau non utilisée ne doit pas être considérée comme une ressource sans limite qui peut être utilisée inconsciemment.

De manière générale, dû à l'importance dans nos vies et à l'immensité des ressources qui nous entourent, il est primordial d'améliorer notre compréhension de l'impact de nos choix sur ces dernières. Cette thèse s'inscrit dans la lignée d'articles qui visent à comprendre l'incidence que nous avons sur ces richesses que sont les ressources naturelles.

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Annexe I

Appendix to Chapter 2

Proof of Proposition 2.1

Note first that problem **P** is the maximization of a continuous function on a compact set, hence by the Weiertrass theorem, the set of solutions is non-empty and convex. Moreover, an allocation x^0 is solution to **P** if and only if there exists

$$\mu^0 = (\mu_1, \mu_1^m, ..., \mu_1^h, ..., \lambda_1, ...)$$

such that (x^0, μ^0) is solution of problem **C**, Truchon (1987):

$$\begin{aligned}
\mathbf{C} = \max_{\{h_t, m_t,\}_{t=1}^{\infty}, h_1^e} & (h_1)^\beta (m_1)^{1-\beta} + \lambda_1 (w_1 + \pi_1 - p_{H,1}(h_1 + h_1^e) - p_{M,1} m_1) + \sigma[(h_2)^\beta (m_2)^{1-\beta} \\
& + \lambda_2 (w_2 + \pi_2 + p_{H,2}(1+\gamma) h_1^e - p_{H,2} h_2 - p_{M,2} m_2)] \\
& + \sum_{t=3}^{\infty} \sigma^{t-1} [(h_t)^\beta (m_t)^{1-\beta} + \lambda_t (w_t + \pi_t - p_{H,t} h_t - p_{M,t} m_t)] \\
& + \mu_1 (h_1^e) + \sum_{t=1}^{\infty} \mu_t^h h_t + \sum_{t=1}^{\infty} \mu_t^m m_t.
\end{aligned}$$

The first order conditions of Problem **C** are given by:

$$(h_t) : \beta \left(\frac{m_t}{h_t} \right)^{1-\beta} + \mu_t^h - \lambda_t p_{H,t} = 0 \quad \forall t, \tag{I.1}$$

$$(m_t^c) : (1-\beta) \left(\frac{h_t}{m_t} \right)^{\beta} + \mu_t^m - \lambda_t p_{M,t} = 0 \quad \forall t, \tag{I.2}$$

$$(m_1^e) : -\lambda_1 p_{H,1} + \lambda_2 \sigma (1+\gamma) p_{H,2} + \mu_1 = 0, \tag{I.3}$$

and the Kuhn-Tucker conditions of Problem **C** are given by:

$$
\begin{aligned}
&\mu_1 h_1^e = 0; \quad \mu_1 \geq 0; \quad h_1^e \geq 0,\\
&\mu_t^h h_t = 0; \quad \mu_t^h \geq 0; \quad h_t \geq 0 \quad \forall t,\\
&\mu_t^m m_t = 0; \quad \mu_t^m \geq 0; \quad m_t \geq 0 \quad \forall t,\\
&\lambda_1 (w_1 + \pi_1 - p_{H,1}(h_1 + h_1^e) - p_{M,1} m_1) = 0; \quad \lambda_1 \geq 0;\\
&\qquad w_1 + \pi_1 - p_{H,1}(h_1 + h_1^e) - p_{M,1} m_1 \geq 0,\\
&\lambda_2 (w_2 + \pi_2 + p_{H,2}(1+\gamma) h_1^e - p_{H,2} h_2 - p_{M,2} m_2) = 0; \quad \lambda_2 \geq 0;\\
&\qquad w_2 + \pi_2 + p_{H,2}(1+\gamma) h_1^e - p_{H,2} h_2 - p_{M,2} m_2 \geq 0,\\
&\lambda_t (w_t + \pi_t - p_{H,t} h_t - p_{M,t} m_t) = 0; \quad \lambda_t \geq 0;\\
&\qquad w_t + \pi_t - p_{H,t} h_t - p_{M,t} m_t \geq 0 \quad \forall t \geq 3.
\end{aligned}
$$

It is important to note that the instantaneous utility function is continuous, concave on $\mathbf{R}_+^2$ and differentiable if $h_t > 0$ and $m_t > 0$. It is not differentiable if at least one variable equals zero. Therefore, the objective function is concave and continuous on $\mathbf{R}_+^\infty$, but is non-differentiable if at least one variable is nil. The Lagrangian associated with problem **P** is not differentiable if at least one variable is nil, hence two steps are required to determine all the candidates for the solution to this problem. The first step is the determination of all allocations that satisfy the Kuhn-Tucker conditions of the Lagrangian associated with problem **P** among the ones such that the Lagrangian function is differentiable. For those allocations, the Kuhn-Tucker conditions are necessary and sufficient. The second step is to determine the candidates for the solution of this problem among all the allocations such that the Lagrangian function is not differentiable, and hence the Kuhn-Tucker conditions are not necessary.

Among the allocations such that the Lagrangian function is differentiable, $h_t > 0$ and $m_t > 0$ for all t. From the complementary conditions, $\mu_t^h = \mu_t^m = 0$, and hence $(I.1)$ and $(I.3)$ lead to:

$$
-\beta \left(\frac{m_1}{h_1}\right)^{1-\beta} + \sigma(1+\gamma)\beta \left(\frac{m_2}{h_2}\right)^{1-\beta} + \mu_1 = 0, \tag{I.4}
$$

$(I.1)$ and $(I.2)$ lead to :

$$\frac{p_{H,t}}{p_{M,t}} = \frac{\beta}{1-\beta}\left(\frac{m_t}{h_t}\right). \tag{I.5}$$

Every period, the budget constraint must be satisfied with equality. The assumption that the budget constraint is not satisfied with equality in period $T \geq 1$ implies that $\lambda_T = 0$. From $(I.1)$ and $(I.2)$:

$$\beta\left(\frac{m_T}{h_T}\right)^{1-\beta} = (1-\beta)\left(\frac{h_T}{m_T}\right)^{\beta} = 0.$$

This equality cannot hold for $h_T > 0$ and $m_T > 0$. The combination of equation $(I.5)$ and the budget constraints gives, for a given h_1^e, the following solution to problem **C**:

$$\begin{aligned} m_1 &= \frac{1-\beta}{p_{M,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right); \quad h_1 = \frac{\beta}{p_{H,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right), \\ m_2 &= \frac{1-\beta}{p_{M,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right); \quad h_2 = \frac{\beta}{p_{H,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right), \\ m_t &= \frac{1-\beta}{p_{M,2}}(w_t+\pi_t); \quad h_t = \frac{\beta}{p_{H,t}}(w_t+\pi_t); \quad \forall t \geq 3. \end{aligned}$$

The combination of $(I.4)$ and $(I.5)$ leads to:

$$\beta\left(\frac{1-\beta}{\beta}\frac{p_{H,1}}{p_{M,1}}\right)^{1-\beta} - \sigma(1+\gamma)\beta\left(\frac{1-\beta}{\beta}\frac{p_{H,2}}{p_{M,2}}\right)^{1-\beta} = \mu_1. \tag{I.6}$$

If $(p_{H,1}/p_{M,1})^{1-\beta} = \sigma(1+\gamma)(p_{H,2}/p_{M,2})^{1-\beta}$, from $(I.6)$, $\mu_1 = 0$. Hence, $m_1^e \geq 0$. To satisfy the budget constraints and the non negative conditions, $h_1^e \leq w_1/p_{M,1}$. The value of the utility evaluated at any of those allocations is given by:

$$\begin{aligned} &\sum_{t=1}^{\infty}\sigma^{t-1}\left(\frac{\beta}{p_{H,t}}\right)^{\beta}\left(\frac{1-\beta}{p_{M,t}}\right)^{1-\beta}(w_t+\pi_t) + (1-\beta)^{1-\beta}\beta^{\beta}\left(\left(\frac{p_{H,2}}{p_{M,2}}\right)^{1-\beta} - \left(\frac{p_{H,1}}{p_{M,1}}\right)^{1-\beta}\right)h_1^e \\ &= \sum_{t=1}^{\infty}\sigma^{t-1}\left(\frac{\beta}{p_{H,t}}\right)^{\beta}\left(\frac{1-\beta}{p_{M,t}}\right)^{1-\beta}(w_t+\pi_t). \end{aligned}$$

If $(p_{H,1}/p_{M,1})^{1-\beta} > \sigma(1+\gamma)(p_{H,2}/p_{M,2})^{1-\beta}$, from $(I.6)$, $\mu_1 > 0$. Hence, $m_1^e = 0$. The value of the lifetime utility is given by:

$$\sum_{t=1}^{\infty} \sigma^{t-1} \left(\frac{\beta}{p_{H,t}}\right)^{\beta} \left(\frac{1-\beta}{p_{M,t}}\right)^{1-\beta} (w_t + \pi_t).$$

Note that if $(p_{H,1}/p_{M,1})^{1-\beta} < \sigma(1+\gamma)(p_{H,2}/p_{M,2})^{1-\beta}$, there does not exist an allocation among the ones such that the Lagrangian function is differentiable that respects the Kuhn-Tucker conditions. Therefore, from $(I.6)$, μ_1 should be negative.

The second step of the proof is to find among all points such that the Lagrangian function is not differentiable the candidate(s) for the solution of problem **C**. It is never optimal for consumers to consume only one of the two available goods within a period. A consumer that is doing so would be strictly better off reallocating his revenue between the two available goods. If a consumer sets his consumption of both goods to zero for period t, $t \geq 2$, he is not using his revenue at all and he is getting no benefit. This consumer would be better off if he uses his revenue for consumption. Hence, for periods larger than two, consumers demand a strictly positive quantity of both goods. However, it might be optimal for a consumer not to consume in the first period. During this period, he can store the harvest good. The only allocation remaining among all allocations such that the Lagrangian function is not differentiable is $h_1 = 0$, $m_1 = 0$, $h_t > 0$ and $m_t > 0$ for all $t \geq 2$. Note that all budget constraints must be satisfied with equality, otherwise there exist profitable deviations.

Every period, consumers have to choose their consumption of the consumption good and the harvest good. During the first period, consumers have to take another decision: they have to choose the fraction of the harvest good that they want to store. Consumers may choose not to consume in the first period in order to store the maximum amount of harvest good. Suppose that consumers decide not to consume and store all the harvest good they bought in the first period, $h_1 = m_1 = 0$ and $h_1^e = (w_1 + \pi_1)/p_{H,1}$. Consumers have to choose h_t and m_t for all $t \geq 2$. It is never optimal to choose either $h_t = 0$ or $m_t = 0$, then at the solution $h_t > 0$ and $m_t > 0$. The consumer chooses $\{h_t, m_t\}_{t=2}^{\infty}$ in order to maximize :

$$\max_{(\{h_t,m_t,\}_{t=2}^{\infty})\in D} \sum_{t=2}^{\infty} \sigma^t (h_t)^{\beta} (m_t)^{1-\beta}$$

s.t

$$p_{H,2}h_2 + p_{M,2}m_2 = w_2 + \pi_2 + p_{H,2}(1+\gamma)\frac{w_1+\pi_1}{p_{H,1}}$$

$$p_{H,t}h_t + p_{M,t}m_t = w_t + \pi_t \quad \forall 3 \le t \le \infty$$

$$m_t \ge 0 \quad \forall t$$

$$h_t \ge 0 \quad \forall t.$$

All optimal allocations involve positive consumptions of both goods. Since this problem has the right properties to ensure that the Kuhn-Tucker conditions of the Lagrangian problem associated with this maximization are necessary and sufficient. Therefore, the only candidate for the solution to problem **P** among all the allocations such that the lifetime utility function is not differentiable is:

$$h_1^e = \frac{w_1+\pi_1}{p_{H,1}}; \quad m_1 = 0; \quad h_1 = 0,$$
$$m_2 = \frac{1-\beta}{p_{M,2}}\left(w_2+\pi_2+(1+\gamma)\frac{w_1+\pi_1}{p_{H,1}}\right),$$
$$h_2 = \frac{\beta}{p_{H,2}}\left(w_2+\pi_2+(1+\gamma)\frac{w_1+\pi_1}{p_{H,1}}\right),$$
$$m_t = \frac{1-\beta}{p_{M,t}}(w_t+\pi_t); \quad h_t = \frac{\beta}{p_{H,t}}(w_t+\pi_t) \quad \forall t \ge 3.$$

The lifetime utility value evaluated at this allocation is given by:

$$\sigma\left(\frac{\beta}{p_{H,2}}\right)^{\beta}\left(\frac{1-\beta}{p_{M,2}}\right)^{1-\beta}\left(\frac{w_1+\pi_1}{p_{H,1}}\right) + \sum_{t=2}^{\infty}\sigma^{t-1}\left(\frac{\beta}{p_{H,t}}\right)^{\beta}\left(\frac{1-\beta}{p_{M,t}}\right)^{1-\beta}(w_t+\pi_t).$$

All the candidates for the solution to problem **P** are found. To determine which ones are solutions, the value of the lifetime utility is compared.
If $(p_{H,1}/p_{M,1})^{1-\beta} < \sigma(1+\gamma)(p_{H,2}/p_{M,2})^{1-\beta}$, the only candidate is the allocation with

consumers that allocate all their first period revenue to storage. Hence, this is the solution.

If $(p_{H,1}/p_{M,1})^{1-\beta} = \sigma(1+\gamma)(p_{H,2}/p_{M,2})^{1-\beta}$, all candidate solutions provide the same utility, and therefore, are all solutions. If $(p_{H,1}/p_{M,1})^{1-\beta} > (p_{H,2}/p_{M,2})^{1-\beta}$, the candidate with consumers that are not storing provides the highest utility value, and hence this is the solution. Under the assumption that $\sigma(1+\gamma) = 1$, the preceding conditions become the comparison of:

$$\frac{p_{H,1}}{p_{M,1}} \quad \text{and} \quad \frac{p_{H,2}}{p_{M,2}}.$$

Q.E.D.

Proof of Proposition 2.2

First, I partition the set of all allocations in eight parts. Three characteristics distinguish the parts of the partition: allocations with or without storage, allocations with or without consumption in first the period and allocations with or without production of the harvest good in the second period. Table I.I represents this partition.

	Harvest good stocked	Consumption, $t=1$	Harvest good produced, $t=2$
P_1	√	√	√
P_2	√		√
P_3	√	√	
P_4	√		
P_5		√	√
P_6			√
P_7		√	
P_8			

Table I.I: Partition of allocations

Note that the equilibrium is neither in P_7 nor in P_8. The allocations of those parts are characterized by no consumption of the harvest good in period 2. From Proposition 2.1, those allocations do not satisfy the consumer demand.

Allocations of P_6 are not equilibria. From Proposition 2.1, consumers do not want to consume in period 1 only when they store the harvest good.

Furthermore, the equilibrium is neither in P_2 nor in P_4. Suppose that an allocation in those parts is an equilibrium, then the demand and the production of consumption good are nil. From Proposition 2.1, the price of that good should be nil, $p_{M,1} = 0$. However, at that price, consumers are not demanding zero consumption good, but an infinite quantity. Hence, the allocation is not an equilibrium.

The equilibrium lies either in P_1, P_3 or P_5.

Let's first evaluate if there exists in P_5 a candidate for the equilibrium. In the allocations of this part there is no storage. From Proposition 2.1, the condition under which consumers do not store the harvest good is:

$$\frac{p_{H,1}}{p_{M,1}} \geq \frac{p_{H,2}}{p_{M,2}}.$$

The consumer demand is given by:

$$m_t = \frac{1-\beta}{p_{M,2}}(w_t + \pi_t); \quad h_t = \frac{\beta}{p_{H,t}}(w_t + \pi_t); \quad h_1^e = 0 \quad \forall t.$$

From Subsections 2.3.2.1 and 2.3.2.2:

$$M_t^s = \frac{p_{M,t}}{2w_{M,t}}; \quad \Pi_t = \frac{(p_{M,t})^2}{4w_{M,t}}; \quad \frac{p_{H,t}}{w_t} = \frac{1}{\alpha s_t} \quad \forall t.$$

The market clearing conditions are given by:

$$H_t^s = \frac{\beta L}{p_{H,t}}(w_t + \pi_t); \quad \frac{p_{M,t}}{w_{M,t}} = 2L\frac{1-\beta}{p_{M,t}}(w_t + \pi_t); \quad w_t = w_{M,t} = w_{H,t} \quad \forall t.$$

The conditions on prices, profits and the market clearing conditions lead to the following equilibrium price:

$$\frac{p_{M,t}}{w_t} = \sqrt{4\frac{1-\beta}{1+\beta}L}.$$

The consumer demand of harvest good is:

$$Lh_t = \frac{2\alpha\beta L s_1}{1+\beta}.$$

From (2.1):

$$s_2 = s_1\left(1 + r - \frac{rs_1}{K} - \frac{2\alpha\beta L}{1+\beta}\right).$$

The condition for this allocation to be an equilibrium is:

$$\frac{1}{\alpha s_1}\left(\sqrt{4\frac{1-\beta}{1+\beta}L}\right)^{-1} \geq \sigma(1+\gamma)\frac{1}{\alpha s_2}\left(\sqrt{4\frac{1-\beta}{1+\beta}L}\right)^{-1},$$

and it implies that $s_2 \geq s_1$. Therefore:

$$s_1 \leq \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right) = \bar{s}.$$

The allocation is an equilibrium under the assumption that:

$$0 < s_1 \leq \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right),$$

and is characterized by:

$$\begin{aligned}
&h_1^e = 0; \quad h_t = \frac{2\alpha\beta s_t}{1+\beta}; \quad H_t^s = \frac{2\alpha\beta L s_t}{1+\beta}; \quad L_{H,t} = \frac{2\beta L}{1+\beta},\\
&m_t = \sqrt{\frac{1-\beta}{1+\beta}\frac{1}{L}}; \quad M_t^s = \sqrt{\frac{1-\beta}{(1+\beta)}L}; \quad L_{M,t} = \frac{1-\beta}{1+\beta}L,\\
&p_{M,t} = w_{M,t}\sqrt{4\frac{1-\beta}{1+\beta}L}; \qquad p_{H,t} = \frac{w_{H,t}}{\alpha s_t}; \qquad w_t = w_{H,t} = w_{M,t},\\
&s_{t+1} = s_t\left(1 + r - \frac{rs_t}{K} - \frac{2\alpha\beta}{1+\beta}\right); \quad \forall t.
\end{aligned}$$

For this allocation to be an equilibrium, the upper bound of the initial resource stock is given by (2.2). If s_1 is larger than (2.2), consumers want to store the harvest good, and hence the allocation doesn't satisfy the consumer demand.

Let's find the allocation in P_1 that might be an equilibrium. There is storage in the allocations of this part. From Proposition 2.1, the condition under which consumers store the harvest good is:

$$\left(\frac{p_{H,1}}{p_{M,1}}\right)^{1-\beta} \leq \left(\frac{p_{H,2}}{p_{M,2}}\right)^{1-\beta}.$$

The consumer demand is given by:

$$\begin{aligned}
m_1 &= \frac{1-\beta}{p_{M,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right); \quad h_1 = \frac{\beta}{p_{H,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right), \\
m_2 &= \frac{1-\beta}{p_{M,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right); \quad h_2 = \frac{\beta}{p_{H,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right), \\
m_t &= \frac{1-\beta}{p_{M,2}}(w_t+\pi_t); \quad h_t = \frac{\beta}{p_{H,t}}(w_t+\pi_t); \quad \forall t \geq 3, \\
h_1^e &\in \left[0, \frac{w_1+\pi_1}{p_{H,1}}\right].
\end{aligned}$$

In P_1, every period, both goods are produced. Therefore, from Subsections 2.3.2.1 and 2.3.2.2:

$$M_t^s = \frac{p_{M,t}}{2w_{M,t}}; \quad \Pi_t = \frac{(p_{M,t})^2}{4w_{M,t}}; \quad \frac{p_{H,t}}{w_{H,t}} = \frac{1}{\alpha s_t}.$$

The equilibrium price system must clear the harvest market:

$$\begin{aligned}
H_1^s &= \frac{\beta L}{p_{H,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right)+Lh_1^e, \\
H_2^s &= \frac{\beta L}{p_{H,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right)-(1+\gamma)Lh_1^e, \\
H_t^s &= \frac{\beta L}{p_{H,t}}\left(w_t+\pi_t\right) \quad \forall t \geq 3,
\end{aligned}$$

the labour market:

$$w_t = w_{M,t} = w_{H,t} \quad \forall t,$$

and the consumption good market:

$$\frac{p_{M,1}}{w_{M,1}} = 2L\frac{1-\beta}{p_{M,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right),$$
$$\frac{p_{M,2}}{w_{M,2}} = 2L\frac{1-\beta}{p_{M,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right),$$
$$\frac{p_{M,t}}{w_{M,t}} = 2L\frac{1-\beta}{p_{M,t}}\left(w_t+\pi_t\right) \quad \forall t \geq 3.$$

Market clearing conditions and conditions on prices lead to:

$$\frac{p_{M,1}}{w_1} = \sqrt{4\left(\frac{1-\beta}{1+\beta}\right)\left(1-\frac{h_1^e}{\alpha s_1}\right)L},$$
$$\frac{p_{M,2}}{w_2} = \sqrt{4\left(\frac{1-\beta}{1+\beta}\right)\left(1+(1+\gamma)\frac{h_1^e}{\alpha s_2}\right)L},$$
$$\frac{p_{M,t}}{w_t} = \sqrt{4\left(\frac{1-\beta}{1+\beta}\right)L} \quad \forall t.$$

To be an equilibrium, this allocation must satisfy the equality condition on price ratio between period 1 and period 2:

$$\frac{1}{\alpha s_1}\left(\sqrt{4\left(\frac{1-\beta}{1+\beta}\right)\left(1-\frac{h_1^e}{\alpha s_1}\right)L}\right)^{-1} = \frac{1}{\alpha s_2}\left(\sqrt{4\left(\frac{1-\beta}{1+\beta}\right)\left(1+(1+\gamma)\frac{h_1^e}{\alpha s_2}\right)L}\right)^{-1},$$

this equation can be rewritten as:

$$s_1(\alpha s_1 - h_1^e) = s_2(\alpha s_2 + (1+\gamma)h_1^e). \tag{I.7}$$

The quantity of harvest good produced in period 1 is:

$$H_1^s = L\left(\frac{2\alpha\beta s_1}{1+\beta} + \frac{1-\beta}{1+\beta}h_1^e\right),$$

and hence the resource stock in period 2 is given by:

$$s_2 = s_1\left(1+r-\frac{rs_1}{K}\right) - L\left(\frac{2\alpha\beta s_1}{1+\beta} + \frac{1-\beta}{1+\beta}h_1^e\right). \tag{I.8}$$

The combination of $(I.7)$ and $(I.8)$ determines the equilibrium quantity of storage and the resource stock in the second period. The system of equations has two solutions. Nevertheless, only one is positive, lower than the maximal amount of harvest good the economy can produce and leads to a positive production of harvest good in period 2.

I denote the two solutions of the system of equations by h_f^e and h_s^e. Those two solutions depend on s_1 and on all the other parameters. It is important to note that h_f^e and h_s^e might equal zero if:

$$s_1 = \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right) \quad \text{or} \quad s_1 = \frac{K}{r}\left(2+r-\frac{2\alpha\beta L}{1+\beta}\right).$$

Note that $(K/r)(2+r-2\alpha\beta L/(1+\beta))$ is never lower than K for $0<\beta<1$. Therefore, I do not consider it for the upcoming part of the proof. The first condition is lower than K for $0<\beta<1$. Under the conditions on the parameters assumed throughout the paper, when

$$s_1 = \frac{K}{r}\left(r-\frac{2\alpha\beta L}{1+\beta}\right),$$

h_s^e equals zero and h_f^e does not equal zero. The two solutions, h_f^e and h_s^e, might reach the maximal amount of harvest good that can be produced, αs_1, if:

$$s_1 = \frac{K}{r}(1+r-\alpha L) \quad \text{or} \quad s_1 = \frac{K}{r}(2+r-\alpha L+\gamma).$$

Those two functions are always larger than the carrying capacity under the assumption that $0 \leq \gamma \leq r$. Hence, both solutions do not equal αs_1 for any combinations of $0 < s_1 \leq K$ and $0 \leq \gamma \leq r$. They are either larger or lower than the maximal amount of harvest good that can be produced in period 1.

At $s_1 = 0$, both solutions are equal to s_1 and αs_1. I do not consider $s_1 = 0$, because this case is not interesting. Both solutions are continuous for the set of parameters consid-

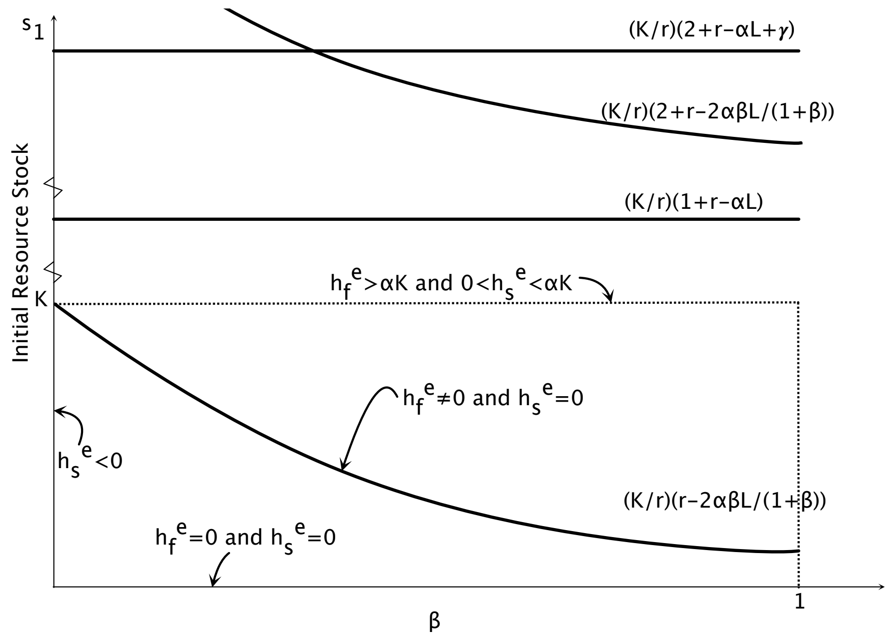

Figure I.1: Harvest good stored

ered. If $s_1 = K$, the value of h_f^e is larger than αK. The value of h_s^e, at $s_1 = K$, is larger than zero and lower than αK. The solution h_s^e is lower than zero if $\beta = 0$. All the preceding analysis, summarized in Figure I.1, implies that for $0 < s_1 \leq K$ and $0 < \beta < 1$, the first solution is always larger than the maximal amount of the production in the harvest sector in period 1, and therefore cannot be an equilibrium quantity. The preceding analysis also implies that the second solution is positive and lower than the maximal production in the harvest sector. Despite the preceding analysis, I cannot readily conclude that this allocation is an equilibrium. I still have to show that the production in the harvest sector is positive in period 2. To prove it, the analysis is quite similar to the previous one.
The production in the second period is given by:

$$H_2^s(h_1^e) = \frac{2\alpha\beta L s_2}{1+\beta} - (1+\gamma)\frac{1-\beta}{1+\beta} L h_1^e.$$

This function, evaluated at $h_1^e = h_s^e$, equals zero if s_1 is zero and if s_1 equals either $\tilde{s_1}$ or

$\hat{s_1}$.[1] Both, $\tilde{s_1}$ and $\hat{s_1}$, are larger than the carrying capacity. The function $H_2^s(h_1^e)$, evaluated at $h_1^e = h_s^e$, is a continuous function of all parameters and is positive at $s_1 = K$. Therefore, $H_2^s(h_s^e) > 0$ for $0 < s_1 \leq K$ and $0 < \beta < 1$.

This part of the partition has an equilibrium allocation if:

$$K(r - 2\alpha\beta L/(1+\beta)) \leq s_1 \leq K,$$

and its equilibrium storage is given by:

$$\begin{aligned} h_1^{e*} =& (s_1(K(4L^2\alpha^2(-1+\beta)\beta - (1+\beta)^2(2+\gamma) + 2L\alpha(1+\beta)(1+\beta\gamma) - \\ & r(1+\beta)(1+2L\alpha(-1+\beta)+\beta+\gamma+\beta\gamma)) + \\ & (1+\beta)(rs_1(1+2L\alpha(-1+\beta)+\beta+\gamma+\beta\gamma) + \\ & \sqrt{}(r^2 s1^2(1+\beta)^2(1+\gamma)^2 - 2Krs_1(1+\beta)(r(1+\beta)(1+\gamma)^2 + \\ & (1+\beta)(1+\gamma)(2+\gamma) - 2L\alpha(1+\beta\gamma(2+\gamma))) + K^2(r^2(1+\beta)^2(1+\gamma)^2 + \\ & (1+\beta)^2(2+\gamma)^2 - 4L\alpha(1+\beta)(2+\gamma)(1+\beta\gamma) + 4L^2\alpha^2(1+\beta^2\gamma(2+\gamma)) - \\ & 2r(1+\beta)(-(1+\beta)(1+\gamma)(2+\gamma) + 2L\alpha(1+\beta\gamma(2+\gamma)))))))) / \\ & (2KL(-1+\beta)(1+L\alpha(-1+\beta)+\beta+\gamma+\beta\gamma)). \end{aligned}$$

There exists an allocation in P_3 that can be an equilibrium. There is storage in the allocations of this part. From Proposition 2.1 the condition under which consumers store the harvest good is:

$$\frac{p_{H,1}}{p_{M,1}} = \frac{p_{H,2}}{p_{M,2}}.$$

In period 1, the consumer demand is given by:

$$\begin{aligned} & m_1 = \frac{1-\beta}{p_{M,1}}(w_1 + \pi_1 - p_{H,1}h_1^e); \quad h_1 = \frac{\beta}{p_{H,1}}(w_1 + \pi_1 - p_{H,1}h_1^e), \\ & h_1^e \in \left[0, \frac{w_1+\pi_1}{p_{H,1}}\right], \end{aligned}$$

[1]These values depend on parameters and are characterized in the Mathematica program of this proof.

and in the following periods, the consumer demand is given by:

$$m_2 = \frac{1-\beta}{p_{M,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right); \quad h_2 = \frac{\beta}{p_{H,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right),$$
$$m_t = \frac{1-\beta}{p_{M,2}}(w_t+\pi_t); \quad h_t = \frac{\beta}{p_{H,t}}(w_t+\pi_t) \quad \forall t \geq 3.$$

From Subsection 2.3.2.1, the supply of the consumption good in period t is given by:

$$M_t^s = \frac{p_{M,t}}{2w_{M,t}},$$

and the profit of this industry is:

$$\Pi_t = \frac{(p_{M,t})^2}{4w_{M,t}}.$$

In allocations of this part, there is no production in the harvest sector in period 2. Therefore, from the full employment condition $M_2^s = \sqrt{L}$. From Subsection 2.3.2.2:

$$\frac{p_{H,t}}{w_t} = \frac{1}{\alpha s_t} \quad \forall t \setminus \{2\},$$
$$\frac{p_{H,2}}{w_{H,2}} \leq \frac{1}{\alpha s_2}. \tag{I.9}$$

The market clearing conditions are given by:

$$H_1^s = \frac{\beta L}{p_{H,1}}\left(w_1+\pi_1-p_{H,1}h_1^e\right)+Lh_1^e,$$
$$H_2^s = \frac{\beta L}{p_{H,2}}\left(w_2+\pi_2+(1+\gamma)p_{H,2}h_1^e\right)-(1+\gamma)Lh_1^e = 0,$$
$$H_t^s = \frac{\beta L}{p_{H,t}}\left(w_t+\pi_t\right) \quad \forall t \geq 3,$$

$$\frac{p_{M,1}}{w_{M,1}} = 2L\frac{1-\beta}{p_{M,1}}\left(w_1 + \pi_1 - p_{H,1}h_1^e\right), \tag{I.10}$$

$$\frac{p_{M,2}}{w_{M,2}} = 2L\frac{1-\beta}{p_{M,2}}\left(w_2 + \pi_2 + (1+\gamma)p_{H,2}h_1^e\right) = 2\sqrt{L}, \tag{I.11}$$

$$\frac{p_{M,t}}{w_{M,t}} = 2L\frac{1-\beta}{p_{M,t}}\left(w_t + \pi_t\right),$$

$$w_t = w_{M,t} = w_{H,t} \quad \forall t \setminus \{2\}; \quad w_2 = w_{M,2} \geq w_{H,2}.$$

The conditions on prices and $(I.10)$ lead to the following equilibrium price:

$$\frac{p_{M,1}}{w_1} = \sqrt{4\frac{1-\beta}{1+\beta}\left(1 - \frac{h_1^e}{\alpha s_1}\right)L}.$$

The conditions on prices, profits, and $(I.11)$ imply that:

$$\frac{p_{H,2}}{w_2} = \frac{2\beta}{(1-\beta)(1+\gamma)h_1^e}.$$

The consumers' condition for this allocation to be an equilibrium is given by the equality of the price ratios between period 1 and period 2:

$$\frac{1}{\alpha s_1}\left(\sqrt{4\frac{1-\beta}{1+\beta}\left(1 - \frac{h_1^e}{\alpha s_1}\right)L}\right)^{-1} = \frac{\beta}{(1-\beta)(1+\gamma)\sqrt{L}h_1^e}.$$

This equation has two solutions, but only one is positive:

$$h_1^{e*} = \frac{2\alpha\beta s_1\left(-\beta + \sqrt{(1+\gamma)^2 - \beta^2\gamma(2+\gamma)}\right)}{(1-\beta^2)(1+\gamma)^2}.$$

This amount of storage is positive and lower than αs_1, the highest quantity of the harvest good that can be produced in period 1. The equilibrium level of the resource stock in period 2 is determined by:

$$s_2^* = s_1\left(1 + r - \frac{rs_1}{K} - \frac{2\alpha\beta L}{1-\beta}\right) - h_1^{e*}L.$$

Two conditions ought to be satisfied for this allocation to be an equilibrium: condition (I.9) and $w_2 = w_{M,2} \geq w_{H,2}$:

$$\frac{p_{H,2}}{w_2} \leq \frac{p_{H,2}}{w_{H,2}} \leq \frac{1}{\alpha s_2}.$$

Under the assumptions on parameters taken throughout the paper, the preceding inequality cannot be satisfied. Therefore, there is no equilibrium allocation in P_3.

So far, I have proved that there is no equilibrium allocation in six out of eight parts of the partition. Moreover, it has been proved that in parts, P_1 and P_5, there is a unique allocation that is an equilibrium. The allocation in P_1 is an equilibrium if and only if:

$$\frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right) \leq s_1 \leq K,$$

and the allocation in P_5 is an equilibrium if an only if:

$$0 < s_1 \leq \frac{K}{r}\left(r - \frac{2\alpha\beta L}{1+\beta}\right).$$

It is worth noting that if $s_1 = (K/r)(r - 2\alpha\beta L/(1+\beta))$, both allocations boil down to the same expression. As a result, the conditions under which both allocations are equilibria are mutually exclusive and jointly exhaustive. Therefore, there exists a unique equilibrium.

The Mathematica program used to prove Proposition 2.2 can be provided upon demand.

Q.E.D.

Proof of Proposition 2.3

The consumers' problem is given by:

$$\begin{aligned}
\mathbf{C} \quad = \max_{(\{h_t, m_t,\}_{t=1}^{\infty}, b_1)} & (h_1)^{\beta}(m_1)^{1-\beta} + \lambda_1(w_1 + \pi_1 - b_1 - p_{H,1}h_1 - p_{M,1}m_1) \\
& + \sigma[(h_2)^{\beta}(m_2)^{1-\beta} + \lambda_2(w_2 + \pi_2 + (1+\gamma)b_1 - p_{H,2}h_2 - p_{M,2}m_2)] \\
& + \sum_{t=3}^{\infty} \sigma^{t-1}[(h_t)^{\beta}(m_t)^{1-\beta} + \lambda_t(w_t + \pi_t - p_{H,t}h_t - p_{M,t}m_t)] \\
& + \mu_1(b_1) + \sum_{t=1}^{\infty} \mu_t^h h_t + \sum_{t=1}^{\infty} \mu_t^m m_t.
\end{aligned}$$

From the proof of Proposition 2.1 and assuming that

$$\frac{p_{H,t}}{w_t} = \frac{1}{\alpha s_t} \quad \text{and} \quad \frac{p_{M,t}}{w_t} = \sqrt{\frac{4L(1-\beta)}{(1+\beta)}},$$

the results are straightforward. *Q.E.D.*

Proof of Proposition 2.8

When storage is allowed, the equilibrium quantity of the harvest good and the equilibrium quantity of consumption good are:

$$m_1^* = (1-\beta)\left(\frac{1}{\alpha s_1}\left(\sqrt{4L\frac{1-\beta}{1+\beta}\frac{\alpha s_1 - h_1^{e*}}{\alpha s_1}}\right)^{-1}\right)\left(\frac{2\alpha s_1 - 2h_1^{e*}}{1+\beta}\right); \quad h_1^* = \beta\left(\frac{2\alpha s_1 - 2h_1^{e*}}{1+\beta}\right).$$

In equilibrium with storage, u_1 is given by:

$$u_1^* = \beta^{\beta}(1-\beta)^{1-\beta}\left(\frac{1}{\alpha s_1}\left(\sqrt{4L\frac{1-\beta}{1+\beta}\frac{\alpha s_1 - h_1^{e*}}{\alpha s_1}}\right)^{-1}\right)^{1-\beta}\left(\frac{2\alpha s_1 - 2h_1^{e*}}{1+\beta}\right).$$

If storage is not allowed, the equilibrium utility level is given by:

$$\tilde{u}_1 = \beta^{\beta}(1-\beta)^{1-\beta}\left(\frac{1}{\alpha s_1}\left(\sqrt{4L\frac{1-\beta}{1+\beta}}\right)^{-1}\right)^{1-\beta}\left(\frac{2\alpha s_1}{1+\beta}\right).$$

The quantity of harvest good stored cannot be negative, therefore $u_1^* < \tilde{u}_1$:

$$-h_1^{e*} < 0$$

$$\left(\frac{\alpha s_1 - h_1^{e*}}{\alpha s_1}\right)^{\frac{1+\beta}{2}} < 1$$

$$\left(\left(\sqrt{\frac{\alpha s_1 - h_1^{e*}}{\alpha s_1}}\right)^{-1}\right)^{1-\beta}\left(\frac{\alpha s_1 - h_1^{e*}}{\alpha s_1}\right) < 1$$

$$\frac{\beta^{\beta}(1-\beta)^{1-\beta}\left(\frac{1}{\alpha s_1}\left(\sqrt{4L\frac{1-\beta}{1+\beta}\frac{\alpha s_1 - h_1^{e*}}{\alpha s_1}}\right)^{-1}\right)^{1-\beta}\left(\frac{2\alpha s_1 - 2h_1^{e*}}{1+\beta}\right)}{\beta^{\beta}(1-\beta)^{1-\beta}\left(\frac{1}{\alpha s_1}\left(\sqrt{4L\frac{1-\beta}{1+\beta}}\right)^{-1}\right)^{1-\beta}\left(\frac{2\alpha s_1}{1+\beta}\right)} < 1$$

$$\frac{u_1^*}{\tilde{u}_1} < 1.$$

Q.E.D.

Proof of Proposition 2.9

The income effect is positive:

$$\frac{\partial\left(\frac{2\alpha\beta s_2^*}{1+\beta} + (1+\gamma)\frac{2h_1^{e*}\beta}{1+\beta}\right)}{\partial h_1^{e*}} = -\frac{2\alpha\beta}{1+\beta}\frac{1-\beta}{1+\beta}L + (1+\gamma)\frac{2\beta}{1+\beta} > 0.$$

The direct effect of storage on income is larger than its indirect effect through its impact on the resource stock in period 2.

The effect of storage on price is ambiguous, the sign of the following expression is uncertain:

$$\partial\left(\frac{1}{\alpha s_2^*}\left(\sqrt{4L\frac{1-\beta}{1+\beta}\left(1+(1+\gamma)\frac{h_1^{e*}}{\alpha s_2^*}\right)}\right)^{-1}\right)^{1-\beta} / \partial h_1^{e*}$$

$$= (1-\beta)\left(\left(\frac{p_{H,2}}{p_{M,2}}\right)^*\right)^{1-\beta}\frac{1-\beta}{1+\beta}L\left(\frac{1}{s_2^*} - \left(\frac{p_{H,2}}{p_{M,2}}\right)^* 4(1+\gamma)\left(1+\frac{h_1^{e*}L(1-\beta)}{s_2^*(1+\beta)}\right)\right).$$

In period 2, if storage has a positive impact on the price ratio, the effect on the utility is positive as well. However, if the effect on price is negative, the overall effect of storage is ambiguous. *Q.E.D.*

Annexe II

Appendix to Chapter 3

Proof of Proposition 3.5

At the steady state, the environmental capital in region 1 is lower than in region 2:

$$\left(\frac{\frac{1}{2}b_m}{b_a+\frac{1}{2}b_m}\right)<1 \quad \text{and} \quad \lambda^1>\lambda^2 \Rightarrow \lambda^1>\lambda^2\left(\frac{\frac{1}{2}b_m}{b_a+\frac{1}{2}b_m}\right)$$
$$\Rightarrow \mathscr{K}^1(M,MA;t)<\mathscr{K}^2(M,MA;t).$$

At the steady state, the environmental capital in region 1 (region 2) is lower (larger) with interregional trade than without:

$$\begin{aligned}
\mathscr{K}^1(M,MA;t) &= \bar{K}-\frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}} \\
&< \bar{K}-\frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}=\mathscr{K}^1(MA,MA;a); \\
\mathscr{K}^2(M,MA;t) &= \bar{K}-\frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right) \\
&> \bar{K}-\frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}=\mathscr{K}^2(MA,MA;a).
\end{aligned}$$

Q.E.D.

Proof of Proposition 3.6

Suppose that there exists no (SSSE), then:

$$\mathscr{K}^2(M,MA;t)<\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{1/\varepsilon}\mathscr{K}^1(M,MA;t).$$

From the autarky analysis, $\mathcal{K}^1(MA,MA;a)$ is smaller than $\mathcal{K}^2(MA,MA;a)$.
As shown in the Section 3.4.2.1, the function $\partial K^1(MA,MA;t)/\partial t = 0$ passes through:[1]

$$\left(\mathcal{K}^1(M,MA;t), \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{1/\varepsilon} \mathcal{K}^1(M,MA;t)\right),$$

and

$$\left(\mathcal{K}^1(MA,MA;a), \mathcal{K}^1(MA,MA;a)\right),$$

and the function $\partial K^2(M,MA;t)/\partial t = 0$ passes through:[2]

$$\left(\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{1/\varepsilon} \mathcal{K}^2(M,MA;t), \mathcal{K}^2(M,MA;t)\right)$$

and

$$\left(\mathcal{K}^2(MA,MA;a), \mathcal{K}^2(MA,MA;a)\right).$$

Both functions are continuous, hence, as it is illustrated in Figure II.1.a, they cross in the blue area. Therefore, there exists a diversified steady state equilibrium, which is $\left(\mathcal{K}^1(MA,MA;t), \mathcal{K}^2(MA,MA;t)\right)$. If there exists no (SSSE), then there exists a diversified steady state equilibrium (DSSE).

Q.E.D.

Next step is to prove that if there exists no (DSSE), then a (SSSE) exists. As it is illustrated in Figure II.1.b, if there exists no (DSSE), then:

$$\partial K^1(MA,MA;t)/\partial t = 0 \quad \text{and} \quad \partial K^2(MA,MA;t)/\partial t = 0,$$

don't cross in the blue area. It is important to note that those two functions pass respectively through:

$$\left(\mathcal{K}^1(MA,MA;a), \mathcal{K}^1(MA,MA;a)\right) \quad \text{and} \quad \left(\mathcal{K}^2(MA,MA;a), \mathcal{K}^2(MA,MA;a)\right),$$

[1] Those points are denoted in Figure II.1 by A and B, respectively.
[2] Those points are denoted in Figure II.1 by C and D, respectively.

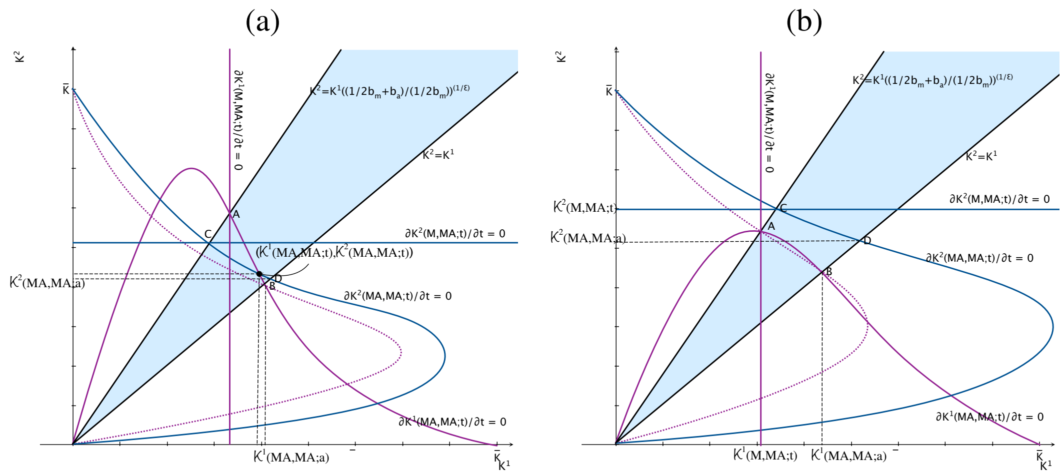

Figure II.1: Existence of a steady state with trade

and that $\mathscr{K}^1(MA,MA;a) < \mathscr{K}^2(MA,MA;a)$. Therefore, $\partial K^1(MA,MA;t)/\partial t = 0$ lies below $\partial K^2(MA,MA;t)/\partial t = 0$ in the blue area and:

$$\mathscr{K}^2(M,MA;t) > \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{1/\varepsilon} \mathscr{K}^1(M,MA;t).$$

This inequality implies that there exists a (SSSE).

Proof of Proposition 3.7

Let's first prove that it is possible that there is a unique (SSSE).

If $\mathscr{K}^1(MA,MA;a) \leq \varepsilon\bar{K}/(2+\varepsilon)$, then:

$$\mathscr{K}^1(M,MA;t)\left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \leq \mathscr{K}^1(MA,MA;t).$$

The combination of the preceding inequality and:

$$\mathscr{K}^1(MA,MA;a) < \mathscr{K}^2(MA,MA;a) < \mathscr{K}^2(M,MA;t),$$

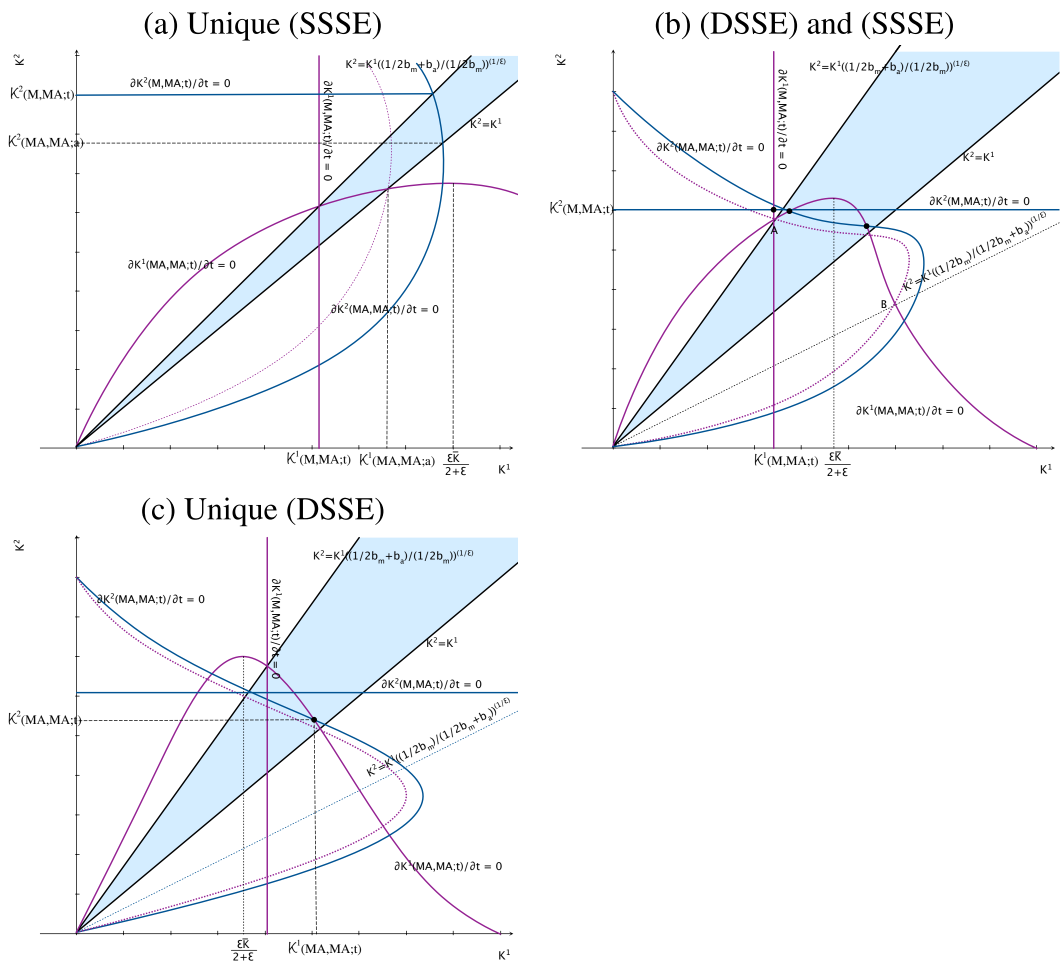

Figure II.2: Uniqueness or multiplicity of steady state with trade implies that there exists a (SSSE):

$$\mathscr{K}^1(M,MA;t) < \mathscr{K}^2(M,MA;t)\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{\varepsilon}}.$$

The assumption that $\mathscr{K}^1(MA,MA;a) \leq \varepsilon\bar{K}/(2+\varepsilon)$ implies that:

$$\partial K^1(MA,MA;t)/\partial t = 0 \quad \text{and} \quad \partial K^2(MA,MA;t)/\partial t = 0,$$

don't cross one with each other in the blue area of Figure II.2.a. Therefore, in this particular case, a (DSSE) doesn't exist.

Let's prove that specialized and diversified steady state equilibria can coexist.

If $\partial K^1(MA,MA;t)/\partial t = 0$ passes through:[3]

$$\left(\mathscr{K}^1(M,MA;t), \left(\frac{\frac{1}{2}b_m+b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}\mathscr{K}^1(M,MA;t)\right),$$

and:

$$\left(\bar{K}-\frac{\lambda^1}{g}(L_u)^{\frac{1}{2}}(L_s)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right), \left(\bar{K}-\frac{\lambda^1}{g}(L_u)^{\frac{1}{2}}(L_s)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)\right)\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{\varepsilon}}\right),$$

then, the curve $\partial K^1(MA,MA;t)/\partial t = 0$ and its reflection with respect to the 45 degree line (the dashed purple curve in Figure II.2.b) cross at the same coordinates:

$$K^2 = K^1\left(\frac{\frac{1}{2}b_m+b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \quad \text{and} \quad K^2 = K^1\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{\varepsilon}}.$$

Since $\lambda^2 < \lambda^1$:

$$\mathscr{K}^2(M,MA;t) > \mathscr{K}^1(M,MA;t)\left(\frac{\frac{1}{2}b_m+b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}},$$

[3] Those two points are represented in Figure II.2.b as A and B respectively.

therefore, a (SSSE) exists.

Suppose that both regions are relatively similar, which means that λs are close one with each other, $\lambda^1 = \lambda^2 + \gamma$. The smallest γ is, the closest the curve:

$$\partial K^2(MA, MA; t)/\partial t = 0,$$

is to the dashed purple curve. Adding the assumption that $\mathscr{K}^1(M, MA; t) < \varepsilon\bar{K}/(2+\varepsilon) < \mathscr{K}^1(MA, MA; a)$ implies that the curve $\partial K^1(MA, MA; t)/\partial t = 0$ reaches a maximum that lies between $K^2 = K^1$ and $K^2 = K^1\left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)$, as it is illustrated in Figure II.2.b.

The parameter γ can be chosen such that $\partial K^2(MA, MA; t)/\partial t = 0$ crosses at least the curve $\partial K^1(MA, MA; t)/\partial t = 0$ once and that Condition (3.13) is satisfied. As it is illustrated in Figure II.2.b, there might exist two diversified steady state equilibria.[4]

The last part of the proof shows that a diversified steady state equilibrium might be unique, as it is illustrated in Figure II.2.c. Let's assume that $\mathscr{K}^1(M, MA; t) \geq \varepsilon\bar{K}/(2+\varepsilon)$ and:

$$\mathscr{K}^1(M, MA; t) > \left(\bar{K} - \frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)\right)\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{\varepsilon}}.$$

I continue to assume that both regions are relatively similar, $\lambda^1 = \lambda^2 + \gamma$ and I choose γ very small such that:

$$\mathscr{K}^2(M, MA; t) < \mathscr{K}^1(M, MA; t)\left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right).$$

Therefore a (SSSE) doesn't exist, however a (DSSE) does exist. *Q.E.D.*

[4] Note that γ can also be chosen such that there is only one (DSSE) or no (DSSE).

Proof of Proposition 3.11

Note first that each region can be either specialized in manufactures, specialized in agriculture, or diversified. Manufactures and agriculture must be produced in at least one region.

Therefore there exist seven cases that can characterized the equilibrium. The seven cases are identified by the vector:

$$(\text{production in region 1}, \text{production in region 2}; tm).$$

For example, $(MA, M; tm)$ means that region 1 is diversified, region 2 is specialized in the production of manufactures, and that there is free trade and workers are mobile.

To prove Proposition 3.11, I consider all seven cases. For each case, first I characterize the temporary equilibrium (given the endowments of environmental capital), then I impose the environmental capital stationary condition to describe the steady state equilibrium. Three cases out of seven characterize the temporary and the steady state equilibria under particular conditions.[5] The conditions under which each case is a temporary equilibrium are mutually exclusive and cover the entire set of parameters, therefore an equilibrium exists. The conditions under which the four other cases are temporary equilibria contradict the initial assumptions.[6] Therefore, those cases are not temporary equilibria. They are not steady state equilibria either. To be a steady state equilibrium, an allocation must satisfy all the temporary equilibrium conditions and the environmental capital must be stable. Therefore, these cases are put aside.

First, I consider the three cases, $(M, MA; tm)$, $(M, A; tm)$, and $(MA, A; tm)$.

i. Region 1 is specialized in the production of manufactures and region 2 is diversified, $(M, MA; tm)$

Region 1 is specialized in the production of manufactures. Firms' decisions in the manufacturing sector are described by (3.1) and (3.2). Agricultural firm's profit maximization problem is given by (3.3). Every unskilled worker in region 1 works in the manufac-

[5] $(M, MA; tm)$, $(M, A; tm)$, and $(MA, A; tm)$

[6] $(MA, MA; tm)$,$(MA, M; tm)$, $(A, M; tm)$, and $(A, MA; tm)$

turing sector, hence in equilibrium $L_{u,M}^{1}=L_{u}$. The salary of unskilled workers in the manufacturing sector should be larger than in the agricultural sector:

$$\frac{p}{2}\left(\frac{L_{s}^{1}}{L_{u}}\right)^{\frac{1}{2}}=w_{u}^{1}\geq(K^{1})^{\varepsilon}. \tag{II.1}$$

Region 2 is diversified. Firms' decisions are characterized by (3.1), (3.2), (3.3), and (3.4). The mobility of unskilled labour across sectors within region 2 implies that:

$$\frac{p}{2}\left(\frac{L_{s}^{2}}{L_{u,M}^{2}}\right)^{\frac{1}{2}}=(K^{2})^{\varepsilon}. \tag{II.2}$$

Mobility of skilled workers implies that their wage is equal between regions:[7]

$$\begin{aligned} V(w_{s}^{1},p) &= V(w_{s}^{2},p) \\ \ln(w_{s}^{1})-b_{m}\ln(p)+C &= \ln(w_{s}^{2})-b_{m}\ln(p)+C \\ w_{s}^{1} &= w_{s}^{2}. \end{aligned}$$

The condition on the wage of skilled workers and (3.2) imply that the wage of unskilled workers must also equalize across regions:

$$\frac{p}{2}\left(\frac{L_{u}}{L_{s}^{1}}\right)^{\frac{1}{2}}=\frac{p}{2}\left(\frac{L_{u,M}^{2}}{L_{s}^{2}}\right)^{\frac{1}{2}} \quad\Rightarrow\quad \frac{L_{u}}{L_{s}^{1}}=\frac{L_{u,M}^{2}}{L_{s}^{2}} \quad\Rightarrow\quad w_{u}^{1}=w_{u}^{2}. \tag{II.3}$$

From (3.4) and $(II.1)$, this case is an equilibrium only if the environmental capital in region 1 is weakly lower than in region 2. Under the assumption that initially $K^{1}<K^{2}$, this condition is satisfied. It is shown below that this inequality holds along the transition path and at the steady state equilibrium.

Consumers' maximization problem is given by (3.5). From (3.4), (3.6), and $(II.3)$, the

[7] $C=b_{m}\ln(b_{m})+b_{a}\ln(b_{a})$

total demand of each good is determined by:

$$M^d = \frac{b_m}{p} L_u (K^2)^\varepsilon + b_m L_s \frac{p}{2\left(K^2\right)^\varepsilon}; \quad A^d = 2 b_a L_u \left(K^2\right)^\varepsilon + b_a L_s p \frac{p}{2\left(K^2\right)^\varepsilon}.$$

The price that clears the market is determined by equalizing the total demand and the total supply of manufactures:

$$\begin{aligned} M^s &= M^d \\ L_s \frac{p}{\left(K^2\right)^\varepsilon} &= \frac{b_m}{2p} L_u \left(K^2\right)^\varepsilon + b_m L_s \frac{p}{2\left(K^2\right)^\varepsilon} \\ p &= 2 \left(\frac{L_u}{L_s}\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2} b_m}{\frac{1}{2} b_m + b_a}\right)^{\frac{1}{2}} \left(K^2\right)^\varepsilon. \end{aligned} \tag{II.4}$$

From (3.4), (3.6), (II.3), (II.4), and the FEC, the allocation of skilled workers across regions is given by:

$$L_s^1 = L_s \frac{\frac{1}{2} b_m + b_a}{\frac{1}{2} b_m}; \quad L_s^2 = L_s \frac{\frac{1}{2} b_m - b_a}{\frac{1}{2} b_m}.$$

Using (II.2) and the allocation of skilled workers across regions, the allocation of unskilled workers is given by:

$$L_{u,M}^2 = L_s^2 \left(\frac{p}{2\left(K^2\right)^\varepsilon}\right)^2 = L_u \frac{\frac{1}{2} b_m - b_a}{\frac{1}{2} b_m + b_a}.$$

For region 2 to be diversified, the allocation of skilled and unskilled workers in the manufacturing sector in that region must be strictly positive, $L_s^2 > 0$ and $L_{u,M}^2 > 0$. Those conditions hold only if $\frac{1}{2} b_m > b_a$. Hence, the condition for this allocation to be a temporary equilibrium is that $\frac{1}{2} b_m > b_a$.

It is worth noting that, in equilibrium, there are more skilled workers in region 1 than in region 2, $L_s^2 < L_s^1$, and there are less unskilled workers in the manufacturing sector in region 2 than in region 1, $L_{u,M}^2 < L_{u,M}^1 = L_u$. These results imply that less manufactures are produced in region 2 than in region 1, $M_s^1 > M_s^2$. Therefore, region 1 produces more

pollution as a by-product than region 2, $Z^1 = \lambda^1 M_s^1 > Z^2 = \lambda^2 M_s^2$.
In equilibrium, there are less unskilled workers that work in the manufacturing sector in region 2 with mobility of skilled workers and interregional trade than in autarky. Therefore, the production of manufactures in region 2 is lower than in autarky. In region 1, all the unskilled workers are in the manufacturing sector and $L_s^1 > L_s$, hence the manufacturing production is larger than in autarky. These results imply that:

$$\frac{dK^2(M,MA;tm)}{dt} < \frac{dK^2(MA,MA;a)}{dt} \quad \text{and} \quad \frac{dK^1(MA,MA;a)}{dt} < \frac{dK^1(M,MA;tm)}{dt}. \qquad \text{(II.5)}$$

Under the assumption that initially $K^1 < K^2$, the case $(M,MA;tm)$ is an temporary equilibrium and from $(II.5)$, this inequality holds for future periods. Therefore, this allocation remains an equilibrium along the transition path.
Steady state
At the stationary equilibrium, all the temporary equilibrium conditions must be satisfied and the environmental capital in both regions must be stable:

$$\frac{\partial K^1(M,MA;tm)}{\partial t} = g\left(\bar{K} - K^1\right) - \lambda^1 (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{2}} = 0,$$
$$\frac{\partial K^2(M,MA;tm)}{\partial t} = g\left(\bar{K} - K^2\right) - \lambda^2 (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m - b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m - b_a}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} = 0.$$

At the steady state, region 2 reaches a higher environmental capital than that of region 1:

$$\mathscr{K}^1(M,MA;tm) = \bar{K} - \frac{\lambda_1}{g} (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{2}},$$
$$\mathscr{K}^2(M,MA;tm) = \bar{K} - \frac{\lambda_2}{g} (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m - b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m - b_a}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}}.$$

This allocation is a long run equilibrium. Because the condition that the environmental capital in region 1 is lower than in region 2 is satisfied:

$$\mathscr{K}^1(M,MA;tm) < \mathscr{K}^2(M,MA;tm).$$

ii. Region 1 is specialized in the production of manufactures and region 2 is specialized in the production of agriculture, $(M,A;tm)$

As in the case $(M,MA;tm)$, region 1 is specialized in the production of manufactures, therefore, firms' decisions are characterized by the same equations.

Region 2 is specialized in the production of agriculture. Agricultural firms' decisions are characterized by (3.3) and (3.4). Manufacturing firm's profit maximization problem is given by (3.1). From the full employment conditions, in equilibrium, every unskilled worker should work in the agricultural sector, $L^2_{u,A} = L_u$ and no skilled worker should be in region 2, $L^2_s = 0$. Therefore, the mobility of unskilled workers across sectors within region 1 implies the following condition:

$$\frac{p}{2}\left(\frac{2L_s}{L_u}\right)^{\frac{1}{2}} = w^1_u \geq \left(K^1\right)^{\varepsilon}. \tag{II.6}$$

To ensure that all skilled workers are in region 1, their welfare in region 1 should be weakly larger than in region 2:[8]

$$\begin{gathered} V(w^2_s,p) \leq V(w^1_s,p) \\ \ln(w^2_s) - b_m \ln(p) + C \leq \ln(w^1_s) - b_m \ln(p) + C \\ w^2_s \leq w^1_s. \end{gathered}$$

In both regions, consumers' demands are characterized by (3.5) and (3.6). Hence, the total demand of each good is given by:

$$M^d = \frac{b_m}{p}\left(L_u\left(K^2\right)^{\varepsilon} + p(L_u)^{\frac{1}{2}}(2L_s)^{\frac{1}{2}}\right); \quad A^d = b_a\left(L_u\left(K^2\right)^{\varepsilon} + p(L_u)^{\frac{1}{2}}(2L_s)^{\frac{1}{2}}\right).$$

The market clearing condition requests the equality between the total demand of manu-

[8] $C = b_m \ln(b_m) + b_a \ln(b_a)$

factures and the total supply, that is provided exclusively in region 1. The equilibrium price is given by:

$$\frac{b_m}{p}\left(L_u\left(K^2\right)^{\varepsilon}+p(L_u)^{\frac{1}{2}}(2L_s)^{\frac{1}{2}}\right)=(L_u)^{\frac{1}{2}}(2L_s)^{\frac{1}{2}}$$

$$p=\frac{b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^2\right)^{\varepsilon}.$$

The equilibrium price determines the wages, in particular the wage of unskilled workers in region 1:

$$w_u^1=\frac{\frac{1}{2}b_m}{b_a}\left(K^2\right)^{\varepsilon}. \tag{II.7}$$

From $(II.6)$ and $(II.7)$, this case is an equilibrium only if the ratio of environmental capital in region 1 and 2 is larger than $b_a/\frac{1}{2}b_m$:

$$\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\geq\frac{b_a}{\frac{1}{2}b_m}. \tag{II.8}$$

I show now that if $\frac{1}{2}b_m>b_a$, the manufacturing firm in region 2 has a profitable deviation, that is, it would want to produce instead of shutting down the production. At the prices characterized above, the manufacturing profit in region 2 would be:

$$\Pi_M^2=\frac{b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^2\right)^{\varepsilon}\left(L_{u,M}^2\right)^{\frac{1}{2}}\left(L_s^2\right)^{\frac{1}{2}}-\left(K^2\right)^{\varepsilon}\left(L_{u,M}^2\right)-\frac{\frac{1}{2}b_m}{b_a}\left(\frac{L_u}{2L_s}\right)\left(K^2\right)^{\varepsilon}\left(L_s^2\right).$$

Is $(L_{u,M}^2=0,L_s^2=0)$ the optimal allocation for the manufacturing firm in region 2? The following maximization problem shows that if $\frac{1}{2}b_m>b_a$, it is optimal for the firm to produce $\left(L_{u,M}^2>0,L_s^2>0\right)$:[9]

$$\max_{L_{u,M}^2,L_s^2}\Pi_M^2+\chi_s^+\left(2L_s-L_s^2\right)+\chi_u^+\left(L_u-L_{u,M}^2\right)+\chi_s^-\left(L_s^2\right)+\chi_u^-\left(L_{u,M}^2\right).$$

[9]For the upcoming part of the paper, I use χ for Lagrange multipliers.

The first order conditions are:

$$\frac{\frac{1}{2}b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^2\right)^{\varepsilon}\left(\frac{L_s^2}{L_{u,M}^2}\right)^{\frac{1}{2}}+\chi_u^- = \left(K^2\right)^{\varepsilon}+\chi_u^+,$$

$$\frac{\frac{1}{2}b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^2\right)^{\varepsilon}\left(\frac{L_s^2}{L_{u,M}^2}\right)^{-\frac{1}{2}}+\chi_s^- = \frac{\frac{1}{2}b_m}{b_a}\left(\frac{L_u}{2L_s}\right)\left(K^2\right)^{\varepsilon}+\chi_s^+.$$

From the FOC and the Kuhn-Tucker conditions, the solution of this maximization problem is given by:

$$\begin{aligned} &L_s^2 = \frac{2L_s}{L_u}L_{u,M}^2 && \text{if} \quad \frac{1}{2}b_m = b_a,\\ &L_s^2 = 2L_s; \quad L_{u,M}^2 = L_u && \text{if} \quad \frac{1}{2}b_m \geq b_a,\\ &L_s^2 = L_{u,M}^2 = 0 && \text{if} \quad \frac{1}{2}b_m \leq b_a. \end{aligned}$$

This case is an equilibrium only if $\frac{1}{2}b_m \leq b_a$. Combining this result with $(II.8)$ implies the following condition:

$$1 \leq \frac{b_a}{\frac{1}{2}b_m} \leq \frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}. \tag{II.9}$$

If the current environmental capital satisfies the preceding condition, this case is an equilibrium. Since there are no production of manufactures in region 2, no pollution takes place in that region. The production of manufactures in region 1 (region 2) is larger (lower) with interregional trade and mobility of skilled workers than in autarky, hence the inequality $(II.5)$ holds in this case too. The environmental capital in region 2 remains larger than in region 1, however, it is possible that the ratio becomes lower than $b_a/\frac{1}{2}b_m$. If this happens, then this case is no longer an equilibrium and from the proof of Proposition 3.12, case (MA,A;tm) becomes the temporary equilibrium.

Steady state

If this case remains a temporary equilibrium along the transition path, then the steady state environmental capital in region 2 would be larger than in region 1, $\mathscr{K}^2(M,A;tm) >$

$\mathscr{K}^1(M,A;tm)$:

$$\frac{dK^1(M,A;tm)}{dt} = g\left(\bar{K} - K^1\right) - \lambda^1 (2L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} = 0$$
$$\Rightarrow \mathscr{K}^1(M,A;tm) = \bar{K} - \frac{\lambda^1}{g} (2L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}},$$
$$\frac{dK^2(M,A;tm)}{dt} = g\left(\bar{K} - K^2\right) = 0 \Rightarrow \mathscr{K}^2(M,A;tm) = \bar{K}.$$

From ($II.5$), the environmental capital in region 1 (region 2) is lower (larger) with inter-regional trade and mobility of skilled workers than in autarky.

One necessary condition for this allocation to be a long run equilibrium is that:[10]

$$1 \leq \left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \leq \frac{\bar{K}}{\bar{K} - \frac{\lambda^1}{g} (L_u)^{\frac{1}{2}} (2L_s)^{\frac{1}{2}}}.$$

Proposition 3.12 gives more details about the steady state equilibrium.

iii. Region 1 is diversified and region 2 is specialized in the production of agriculture, $(MA,A;tm)$

Region 1 is diversified as the autarky case, therefore, firms' decisions are described by (3.1), (3.2), (3.3), and (3.4).

As in the case $(M,A;tm)$, region 2 is specialized in the production of agriculture, hence, firms' decisions are characterized by the same equations.

The mobility of unskilled labour across sectors within region 1, the FEC, and the FOC (3.2) and (3.4) imply that the equality of the wage of unskilled workers in both sectors:

$$\frac{p}{2}\left(\frac{2L_s}{L^1_{u,M}}\right)^{\frac{1}{2}} = \left(K^1\right)^{\varepsilon}. \tag{II.10}$$

As in the case $(M,A;tm)$, the mobility of skilled workers across regions implies that, $w^2_s \leq w^1_s$.

Consumers' decisions and demands are described by (3.5) and (3.6). Using ($II.10$), the

[10] If $\frac{1}{2}b_m = b_a$, this inequality always holds.

total demand of each good is given by:

$$M^d = \frac{b_m}{p} L_u \left(\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon} \right) + \frac{1}{2} b_m L_s \frac{p}{\left(K^1\right)^{\varepsilon}},$$
$$A^d = b_a L_u \left(\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon} \right) + b_a p L_s \frac{p}{\left(K^1\right)^{\varepsilon}}.$$

The price that clears the manufacturing market is given by:

$$M^d = M^s$$
$$\frac{b_m}{p} L_u \left(\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon} \right) + \frac{1}{2} b_m L_s \frac{p}{\left(K^1\right)^{\varepsilon}} = L_s \frac{p}{\left(K^1\right)^{\varepsilon}}$$
$$2 \left(\frac{L_u}{2L_s} \right)^{\frac{1}{2}} \left(\frac{\frac{1}{2} b_m}{\frac{1}{2} b_m + b_a} \right)^{\frac{1}{2}} \left(\frac{\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}} \right)^{\frac{1}{2}} \left(K^1\right)^{\varepsilon} = p. \tag{II.11}$$

From (II.10) and (II.11), the allocation of unskilled workers in region 1 is determined by:

$$L^1_{u,M} = L_u \frac{\frac{1}{2} b_m}{\frac{1}{2} b_m + b_a} \left(1 + \frac{\left(K^2\right)^{\varepsilon}}{(K^1)^{\varepsilon}} \right).$$

Since region 1 is diversified, there must be unskilled workers in both sectors:

$$L^1_{u,M} = L_u \frac{\frac{1}{2} b_m}{\frac{1}{2} b_m + b_a} \left(1 + \frac{\left(K^2\right)^{\varepsilon}}{(K^1)^{\varepsilon}} \right) < L_u \Rightarrow \frac{(K^2)^{\varepsilon}}{(K^1)^{\varepsilon}} < \frac{b_a}{\frac{1}{2} b_m}. \tag{II.12}$$

If $\left(K^2\right)^{\varepsilon} / \left(K^1\right)^{\varepsilon} < 1$, at the equilibrium price described below, the manufacturing firm in region 2 has a profitable deviation, that is, it would not want to shut down. The profit in region 2 in the manufacturing sector would be:

$$\begin{aligned}\Pi^2_M =& 2 \left(\frac{L_u}{2L_s} \right)^{\frac{1}{2}} \left(\frac{\frac{1}{2} b_m}{\frac{1}{2} b_m + b_a} \right)^{\frac{1}{2}} \left(1 + \frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}} \right)^{\frac{1}{2}} \left(K^1\right)^{\varepsilon} \left(L^2_{u,M}\right)^{\frac{1}{2}} \left(L^2_s\right)^{\frac{1}{2}} - \left(K^2\right)^{\varepsilon} L^2_{u,M} \\ & - \frac{L_u}{2L_s} \frac{\frac{1}{2} b_m}{\frac{1}{2} b_m + b_a} \left(\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon} \right) L^2_s.\end{aligned}$$

Is $(L_{u,M}^2 = 0, L_s^2 = 0)$ the optimal allocation for the manufacturing firm in region 2? The following maximization problem shows that if $\left(K^2\right)^{\varepsilon} < \left(K^1\right)^{\varepsilon}$, it is optimal for the manufacturing firm to produce $(L_{u,M}^2 > 0, L_s^2 > 0)$:

$$\max_{L_{u,M}^2, L_s^2} \Pi_M^1 + \chi_s^+ \left(2L_s - L_s^2\right) + \chi_u^+ \left(L_u - L_{u,M}^2\right) + \chi_s^- \left(L_s^2\right) + \chi_u^- \left(L_{u,M}^2\right).$$

The first order conditions are:

$$\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} \left(1 + \frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{1}{2}} \left(K^1\right)^{\varepsilon} \left(\frac{L_s^2}{L_{u,M}^2}\right)^{\frac{1}{2}} + \chi_u^- = \left(K^2\right)^{\varepsilon} + \chi_u^+,$$

$$\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} \left(1 + \frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{1}{2}} \left(K^1\right)^{\varepsilon} \left(\frac{L_s^2}{L_{u,M}^2}\right)^{-\frac{1}{2}} + \chi_s^-$$
$$= \frac{L_u}{2L_s} \frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a} \left(\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}\right) + \chi_s^+.$$

The solution of this maximization problem is:

$$\begin{aligned} &0 < L_s^2 \leq 2L_s; \quad 0 < L_{u,M}^2 \leq L_u && \text{if} \quad \left(K^2\right)^{\varepsilon} < \left(K^1\right)^{\varepsilon}, \\ &L_s^2 = L_{u,M}^2 = 0 && \text{if} \quad \left(K^2\right)^{\varepsilon} \geq \left(K^1\right)^{\varepsilon}. \end{aligned}$$

Combining this solution with $(II.12)$ implies that this case is an equilibrium only if:

$$1 \leq \frac{(K^2)^{\varepsilon}}{(K^1)^{\varepsilon}} < \frac{b_a}{\frac{1}{2}b_m}. \tag{II.13}$$

If the environmental capital satisfies the preceding condition, then this case is an equilibrium. As in the case $(M,A;tm)$, there is no production of manufactures in region 2, therefore, no pollution takes place in that region and the evolution of the environmental capital in region 2 is larger than in region 1. Manufacturing production in region 1 (region 2) is larger (lower) than in autarky. The inequality $(II.5)$ holds in this case too. The environmental capital in region 2 remains larger than in region 1, however, it is possible that the ratio of environmental capital becomes larger than $b_a/\frac{1}{2}b_m$. If it happens, this case is no longer an equilibrium, but as it is shown in Proposition 3.12, case $(M,A;tm)$ becomes the temporary equilibrium.

Steady state

If this allocation remains a temporary equilibrium along the transition path, the stationary equilibrium is given by:

$$\frac{dK^2(MA,A;tm)}{dt} = g\left(\bar{K} - K^2\right) = 0 \Rightarrow \mathscr{K}^2(MA,A;tm) = \bar{K},$$
$$\frac{dK^1(MA,A;tm)}{dt} = g\left(\bar{K} - K^1\right) - \lambda^1 \left(L_u\right)^{\frac{1}{2}} \left(2L_s\right)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}} \left(1 + \frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}\right)^{\frac{1}{2}} = 0.$$

Since no manufactures are produced in region 2:

$$\mathscr{K}^2(MA,A;tm) > \mathscr{K}^1(MA,A;tm).$$

Moreover, the manufacturing production in region 1 (region 2) is larger (lower) than in the autarky equilibrium, hence the stationary environmental capital is lower (larger) than in autarky.

Two remarks ought to be made. First, the conditions under which each case is a temporary are mutually exclusive and cover the entire set of parameters. Second, in the three preceding cases, the environmental capital in region 1 remains lower than that in region 2 along the transition path and at the steady state. This implies that if initially the environmental capital is lower in region 1 than in region 2, then one of the three cases is the equilibrium and K^1 remains lower than K^2.

More details about the steady state is given in Proposition 3.12.

I show now that for the other four cases to be temporary equilibria, the environmental capital in region 1 has to be weakly larger than that in region 2. From the first part of the proof, it is clear that this condition never holds. Therefore, those cases are neither temporary nor stationary equilibria.

iv. Both regions are diversified, $(MA,MA;tm)$

The production patterns in both regions are the same as in the autarky case, hence firms' decisions in both sectors are described by (3.1), (3.2), (3.3), and (3.4). Consumers' decisions are characterized by (3.5) and (3.6). Mobility of unskilled workers across sectors within a region implies (3.7) and the wage of skilled workers in region $i = 1,2$ is characterized by (3.8).

Manufacturing production takes place in both regions, therefore, there are skilled workers in both. Since skilled workers are mobile across regions their utility must be equal:[11]

$$\begin{aligned} V(w_s^1, p) &= V(w_s^2, p) \\ \ln(w_s^1) - b_m \ln(p) + C &= \ln(w_s^2) - b_m \ln(p) + C \\ \Rightarrow w_s^1 &= w_s^2. \end{aligned} \tag{II.14}$$

The condition that wage of skilled workers in both regions must be equal combined with (3.2) and (3.7) imply that this case characterizes an equilibrium only if both regions have the same environmental capital: $K^1 = K^2$.

It has been shown above that if initially $K^1 < K^2$, then the environmental capital in region 1 remains strictly lower that in region 2 along the transition path and at the steady state. Therefore, this case can never be either a temporary equilibrium or a stationary equilibrium.

v. Region 1 is diversified and region 2 is specialized in the production of manufactures, $(MA, M; tm)$

In region 1, both goods are produced and the firms' decisions are characterized by (3.1), (3.2), (3.3), and (3.4). In region 2, only manufactures are produced. Manufacturing firm's decisions are characterized by (3.1) and (3.2). Agricultural firm's maximization profit problem is given by (3.3), no production takes place in this sector if $w_u^2 \geq \left(K^2\right)^{\varepsilon}$. From (3.1), (3.3), and $w_u^2 \geq \left(K^2\right)^{\varepsilon}$, the wage of unskilled workers is lower in the agricultural sector than in the manufacturing sector:

$$\frac{p}{2}\left(\frac{L_s^2}{L_u}\right)^{\frac{1}{2}} = w_u^2 \geq \left(K^2\right)^{\varepsilon}. \tag{II.15}$$

[11] $C = b_m \ln(b_m) + b_a \ln(b_a)$

The mobility of unskilled labour across sectors within region 1 implies (3.7). Both regions produce manufactures, hence, with the mobility of skilled workers, their utility must be equal between regions in equilibrium as it is shown by $(II.14)$, therefore, $w_s^1 = w_s^2$.

The condition that $w_s^1 = w_s^2$, (3.1), (3.7), and $(II.15)$ imply that in equilibrium the environmental capital in region 1 should be lower than in region 2, $K^2 \leq K^1$.

This allocation is an equilibrium only if the environmental capital of region one is weakly larger than the environmental capital of region two. As the case $(MA, MA; tm)$, this case is neither a temporary nor a stationary equilibrium under the assumption that initially $K^1 < K^2$.

vi. Region 1 is specialized in the agricultural production and region 2 is specialized in the agricultural production, $(A, M; tm)$

Region 1 is specialized in agricultural production. Agricultural firm's decisions are characterized by (3.3) and (3.4). Manufacturing firm's maximization profit problem is given by (3.1).

Region 2 is specialized in manufacturing production. Manufacturing firm's decisions are characterized by (3.1) and (3.2). Agricultural firm's maximization profit problem is given by (3.3) and the condition for agricultural production to be nil is $w_u^2 \geq \left(K^2\right)^{\varepsilon}$. Therefore, the mobility of unskilled workers across sectors within a region and (3.2) imply that the wage of unskilled workers is larger in manufacturing sector than in agricultural sector:

$$\frac{p}{2}\left(\frac{2L_s}{L_u}\right)^{\frac{1}{2}} \geq \left(K^2\right)^{\varepsilon}. \tag{II.16}$$

Since there is no manufacturing production in region 1, every skilled worker should live in region 2. To ensure that skilled workers have no incentive to move to region 1, the wage of skilled workers should be weakly larger in region 2 than in region 1:[12]

$$\begin{gathered} V(w_s^1, p) \leq V(w_s^2, p) \\ \ln(w_s^1) - b_m \ln(p) + C \leq \ln(w_s^2) - b_m \ln(p) + C \\ w_s^1 \leq w_s^2. \end{gathered}$$

Consumers' maximization problem is given by (3.5). Using (3.6), the total demand of each good is:

$$M^d = \frac{b_m}{p}\left(L_u\left(K^1\right)^{\varepsilon} + p\left(L_u\right)^{\frac{1}{2}}\left(2L_s\right)^{\frac{1}{2}}\right); \qquad A^d = b_a\left(L_u\left(K^1\right)^{\varepsilon} + p\left(L_u\right)^{\frac{1}{2}}\left(2L_s\right)^{\frac{1}{2}}\right).$$

With interregional trade, the market clearing conditions impose that the total demand of manufactures equals the total supply:

$$M^d = \frac{b_m}{p}\left(L_u\left(K^1\right)^{\varepsilon} + p\left(L_u\right)^{\frac{1}{2}}\left(2L_s\right)^{\frac{1}{2}}\right) = \left(L_u\right)^{\frac{1}{2}}\left(2L_s\right)^{\frac{1}{2}} = M^s$$

$$p = \frac{b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^1\right)^{\varepsilon}. \tag{II.17}$$

Combining ($II.16$) and ($II.17$) provides the following equilibrium condition:

$$\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}} \leq \frac{\frac{1}{2}b_m}{b_a}. \tag{II.18}$$

I show now, that if $\frac{1}{2}b_m > b_a$, the manufacturing firm in region 1 has a profitable deviation, that is, it wants to produce. From (3.2), (3.4), and ($II.17$), the profit in region 1 in manufacturing sector is:

$$\Pi_M^1 = \frac{b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^1\right)^{\varepsilon}\left(L_{u,M}^1\right)^{\frac{1}{2}}\left(L_s^1\right)^{\frac{1}{2}} - \left(K^1\right)^{\varepsilon}L_{u,M}^1 - \frac{1}{2}\frac{b_m}{b_a}\left(\frac{L_u}{2L_s}\right)\left(K^1\right)^{\varepsilon}L_s^1.$$

Is $(L_{u,M}^1 = 0, L_s^1 = 0)$ the optimal allocation for the manufacturing firm of region 1? The

[12] $C = b_m \ln(b_m) + b_a \ln(b_a)$

following maximization problem shows that if $\frac{1}{2}b_m > b_a$, it is optimal for the firm to choose $(L^2_{u,M} > 0, L^2_s > 0)$:

$$\max_{L^1_{u,M}, L^1_s} \Pi^1_M + \chi^+_s \left(2L_s - L^1_s\right) + \chi^+_u \left(L_u - L^1_{u,M}\right) + \chi^-_s \left(L^1_s\right) + \chi^-_u \left(L^1_{u,M}\right).$$

The first order conditions are:

$$\frac{\frac{1}{2}b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^1\right)^{\varepsilon}\left(\frac{L^1_s}{L^1_{u,M}}\right)^{\frac{1}{2}} + \chi^-_u = \left(K^1\right)^{\varepsilon} + \chi^+_u,$$
$$\frac{\frac{1}{2}b_m}{b_a}\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(K^1\right)^{\varepsilon}\left(\frac{L^1_s}{L^1_{u,M}}\right)^{-\frac{1}{2}} + \chi^-_s = \frac{\frac{1}{2}b_m}{b_a}\left(\frac{L_u}{2L_s}\right)\left(K^1\right)^{\varepsilon} + \chi^+_s.$$

The FOC and the Kuhn-Tucker conditions give the solution of the maximization problem:

$$\begin{aligned}
&L^1_s = \frac{2L_s}{L_u}L^1_{u,M} && \text{if} \quad \frac{1}{2}b_m = b_a,\\
&L^1_s = 2L_s; \quad L^1_{u,M} = L_u && \text{if} \quad \frac{1}{2}b_m \geq b_a,\\
&L^1_s = L^1_{u,M} = 0 && \text{if} \quad \frac{1}{2}b_m \leq b_a.
\end{aligned}$$

Therefore, from $(II.18)$ the case $(A, M; tm)$ is an equilibrium if the environmental capital is lower in region 2 than in region 1:

$$\frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}} \leq \frac{\frac{1}{2}b_m}{b_a} \leq 1.$$

As the case (MA,MA;tm), this case is neither a temporary nor a stationary equilibrium under the assumption that initially $K^1 < K^2$.

vii. Region 1 is specialized in the production of agriculture and region 2 is diversified $(A, MA; tm)$

Region 1 is specialized in agricultural production. Agricultural firm's decisions are characterized by (3.3) and (3.4). Manufacturing firm's maximization profit problem is given by (3.1).

Region 2 is diversified, hence, production decisions are described by the same equations as in the autarky case, (3.1), (3.2), (3.3) and (3.4). Since no manufacture is produced in region 1, from the full employment condition, every skilled worker should be in region 2, $L_s^2 = 2L_s$. From (3.7), the mobility of unskilled labour across sectors within region 2 implies the following condition:

$$\frac{p}{2}\left(\frac{2L_s}{L_{u,M}^2}\right)^{\frac{1}{2}} = \left(K^2\right)^{\varepsilon}. \tag{II.19}$$

In equilibrium, skilled workers should have no incentive to move from region 2 to region 1. Skilled workers' utility should be larger in region 2 than in region 1:[13]

$$\begin{gathered} V(w_s^1, p) \leq V(w_s^2, p) \\ \ln(w_s^1) - b_m \ln(p) + C \leq \ln(w_s^2) - b_m \ln(p) + C \\ w_s^1 \leq w_s^2. \end{gathered}$$

Consumers' decisions are characterized by (3.5) and (3.6). Using $(II.19)$, the total demand of each good depends on $L_{u,M}^2$:

$$\begin{aligned} M^d &= \frac{1}{2} b_m L_u \left(1 + \frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)\left(\frac{2L_s}{L_{u,M}^2}\right)^{\frac{1}{2}} + \frac{1}{2} b_m \left(L_{u,M}^2\right)^{\frac{1}{2}} \left(2L_s\right)^{\frac{1}{2}}, \\ A^d &= b_a L_u \left(\left(K^1\right)^{\varepsilon} + \left(K^2\right)^{\varepsilon}\right) + 2 b_a \left(K^2\right)^{\varepsilon} L_{u,M}^2. \end{aligned}$$

The allocation of unskilled workers between sectors in region 2 is determined by the

[13] $C = b_m \ln(b_m) + b_a \ln(b_a)$

MCC:

$$
\begin{aligned}
M^d &= M^s \\
\frac{1}{2}b_m L_u\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)\left(\frac{2L_s}{L_{u,M}^2}\right)^{\frac{1}{2}}+\frac{1}{2}b_m\left(L_{u,M}^2\right)^{\frac{1}{2}}\left(2L_s\right)^{\frac{1}{2}} &= \left(L_{u,M}^2\right)^{\frac{1}{2}}\left(2L_s\right)^{\frac{1}{2}} \\
\frac{1}{2}b_m L_u\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)\left(\frac{2L_s}{L_{u,M}^2}\right)^{\frac{1}{2}} &= \left(\frac{1}{2}b_m+b_a\right)\left(L_{u,M}^2\right)^{\frac{1}{2}}\left(2L_s\right)^{\frac{1}{2}} \\
L_u\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right) &= L_{u,M}^2.
\end{aligned}
$$

The case $(A,MA;tm)$ is characterized by the diversity of the production in region 2, therefore, there must be unskilled workers in both sectors, that is ensured by the following condition:

$$
L_u\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)<L_u \Rightarrow \frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}}>\frac{\frac{1}{2}b_m}{b_a}.
$$

With the allocation of unskilled workers in region 2, we can determine the price and the wages of the economy in equilibrium. At this price system, if $(K^1)^{\varepsilon}<(K^2)^{\varepsilon}$, the manufacturing firm in region 1 has a profitable deviation. At those prices, it would want to produce. At the given prices, the manufacturing profit in region 1 is:

$$
\begin{aligned}
\Pi_M^1 =& 2\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{\frac{1}{2}}\left(K^2\right)^{\varepsilon}\left(L_{u,M}^1\right)^{\frac{1}{2}}\left(L_s^1\right)^{\frac{1}{2}}-\left(K^1\right)^{\varepsilon}L_{u,M}^1 \\
&-\frac{L_u}{2L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(\left(K^1\right)^{\varepsilon}+\left(K^2\right)^{\varepsilon}\right)L_s^1.
\end{aligned}
$$

Is $\left(L_{u,M}^1=0, L_s^1=0\right)$ the optimal allocation for the manufacturing firm in region 1? The following maximization problem shows that if $\left(K^2\right)^{\varepsilon}<\left(K^1\right)^{\varepsilon}$, it is optimal for the manufacturing firm to choose $\left(L_{u,M}^1>0, L_s^1>0\right)$:

$$
\max_{L_{u,M}^1, L_s^1}\Pi_M^1+\chi_s^+\left(2L_s-L_s^1\right)+\chi_u^+\left(L_u-L_{u,M}^1\right)+\chi_s^-\left(L_s^1\right)+\chi_u^-\left(L_{u,M}^1\right).
$$

The first order conditions are:

$$\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{\frac{1}{2}}\left(K^2\right)^{\varepsilon}\left(\frac{L_s^1}{L_{u,M}^1}\right)^{\frac{1}{2}}+\chi_u^-=(K^1)^{\varepsilon}+\chi_u^+,$$

$$\left(\frac{L_u}{2L_s}\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)^{1-\alpha}\left(K^2\right)^{\varepsilon}\left(\frac{L_s^1}{L_{u,M}^1}\right)^{-\frac{1}{2}}+\chi_s^-$$
$$=\frac{L_u}{2L_s}\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\left(1+\frac{\left(K^1\right)^{\varepsilon}}{\left(K^2\right)^{\varepsilon}}\right)\left(K^2\right)^{\varepsilon}+\chi_s^+.$$

The FOC and the Kuhn-Tucker conditions give the solution of the maximization problem:

$$\begin{array}{lll} 0<L_s^1\leq 2L_s; \quad 0<L_{u,M}^1\leq L_u & \text{if} & \left(K^1\right)^{\varepsilon}<\left(K^2\right)^{\varepsilon}, \\ L_s^1=L_{u,M}^1=0 & \text{if} & \left(K^1\right)^{\varepsilon}\geq\left(K^2\right)^{\varepsilon}. \end{array}$$

The case $(A,MA;tm)$ could be an equilibrium only if the environmental capital is lower in region 2 than in region 1. Therefore, this case is neither a temporary nor a stationary equilibrium under the assumption that initially $K^1<K^2$. *Q.E.D.*

Proof of Proposition 3.12

If $\frac{1}{2}b_m>b_a$, see Part i of the proof of Proposition 3.11.

If $\frac{1}{2}b_m=b_a$, see Parts ii and iii of the proof of Proposition 3.11.

Otherwise, from Parts ii and iii of the proof of Proposition 3.11, at the steady state, region 1 is either diversified or specialized in the manufacturing sector and region 2 is specialized in the production of agriculture.

When region 1 is diversified, the evolution of the environmental capital is denoted by $dK^1(MA,A;tm)/dt=0$. This function is continuous and passes through:

$$\left(\bar{K}-\frac{\lambda^1}{g}2\left(L_s\right)^{\frac{1}{2}}\left(L_u\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}},\bar{K}-\frac{\lambda^1}{g}2\left(L_s\right)^{\frac{1}{2}}\left(L_u\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\right),$$

and

$$\left(\bar{K}-\frac{\lambda^1}{g}\left(2L_s\right)^{\frac{1}{2}}\left(L_u\right)^{\frac{1}{2}},\left(\bar{K}-\frac{\lambda^1}{g}2\left(L_s\right)^{\frac{1}{2}}\left(L_u\right)^{\frac{1}{2}}\right)\left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}\right).$$

The functions $dK^1(MA,A;tm)/dt=0$ and $dK^2(MA,A;tm)/dt=0$ can cross at most twice. Let's consider the functions:

$$f(K^1)=g\left(\bar{K}-K^1\right)\quad\text{and}\quad h(K^1)=\frac{\lambda^1}{g}\left(2L_s\right)^{\frac{1}{2}}\left(L_u\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\left(1+\frac{(\bar{K})^{\varepsilon}}{(K^1)^{\varepsilon}}\right)^{\frac{1}{2}}.$$

Those two functions are decreasing. Function $f(K^1)$ is linear and function $h(K^1)$ is convex. As illustrated in Figure II.3.d, $f(K^1)$ and $h(K^1)$ can intersect one with each other at most twice.

As it is illustrated in Figure II.3.a, if:

$$\bar{K}\left(\frac{\frac{1}{2}b_m}{b_a}\right)^{\frac{1}{\varepsilon}}\leq\bar{K}-\frac{\lambda^1}{g}\left(2L_s\right)^{\frac{1}{2}}\left(L_u\right)^{\frac{1}{2}},$$

the intersection between $dK^1(MA,A;tm)/dt=0$ and $dK^2(MA,A;tm)/dt=0$ satisfies condition $(II.13)$. However, the intersection between $dK^1(M,A;tm)/dt=0$ and $dK^2(M,A;tm)/dt=0$ doesn't satisfy condition $(II.9)$. Therefore, at the steady state, region 1 is always diversified.

Since $dK^1(MA,A;tm)/dt=0$ intersects $dK^2(MA,A;tm)/dt=0$ at most twice, and that $dK^1(MA,A;tm)/dt=0$ crosses $K^2=K^1$ and $K^2=\left(b_a/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}}$ only once, there is a unique steady state.

Region 1 is specialized in the manufacturing sector at the steady state, if:

$$\bar{K}-\frac{\lambda^1}{g}2\left(L_s\right)^{\frac{1}{2}}\left(L_u\right)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m+b_a}\right)^{\frac{1}{2}}\leq\bar{K}\left(\frac{\frac{1}{2}b_m}{b_a}\right)^{\frac{1}{\varepsilon}}.$$

There would be no intersection between:

$$dK^1(MA,A;tm)/dt = 0 \quad \text{and} \quad dK^2(MA,A;tm)/dt = 0,$$

that satisfies $(II.13)$.

However, $dK^1(M,A;tm)/dt = 0$ intersects $dK^2(M,A;tm)/dt = 0$ such that $(II.9)$ is satisfied. Figure II.3.b represents this case.

There might be multiple equilibria, if:

$$\bar{K} - \frac{\lambda^1}{g}(2L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}} < \bar{K}\left(\frac{\frac{1}{2}b_m}{b_a}\right)^{\frac{1}{\varepsilon}} < \bar{K} - \frac{\lambda^1}{g}2(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{2}},$$

Region 1 is either diversified or specialized in the manufacturing sector. As it is illustrated in Figure II.3.c, the intersection between:

$$dK^1(M,A;tm)/dt = 0 \quad \text{and} \quad dK^2(M,A;tm)/dt = 0,$$

satisfies $(II.9)$. The two representations of $dK^1(MA,A;tm)/dt = 0$ (purple and green) show that the economy might also have a diversified steady state equilibrium. Therefore, there is one steady state where both regions are specialized and at most two steady states with diversification of production in region 1. *Q.E.D.*

Proof of Proposition 3.14

The relations between the environmental capital in autarky, with trade and with migration come from Sections 3.4.1, 3.4.2, and 3.4.3.

To compare the environmental capital with free trade with the one with free trade and mobility of workers, I proceed in several steps. First, I construct the phase diagrams for both situations. Then, for all initial endowments of environmental capital, I determine the steady state of the economy in each situation, that is free trade with the mobility of workers and free trade without mobility of workers. Finally, I compare the equilibria that can occur with free trade to the ones that can occur with free trade and migration.

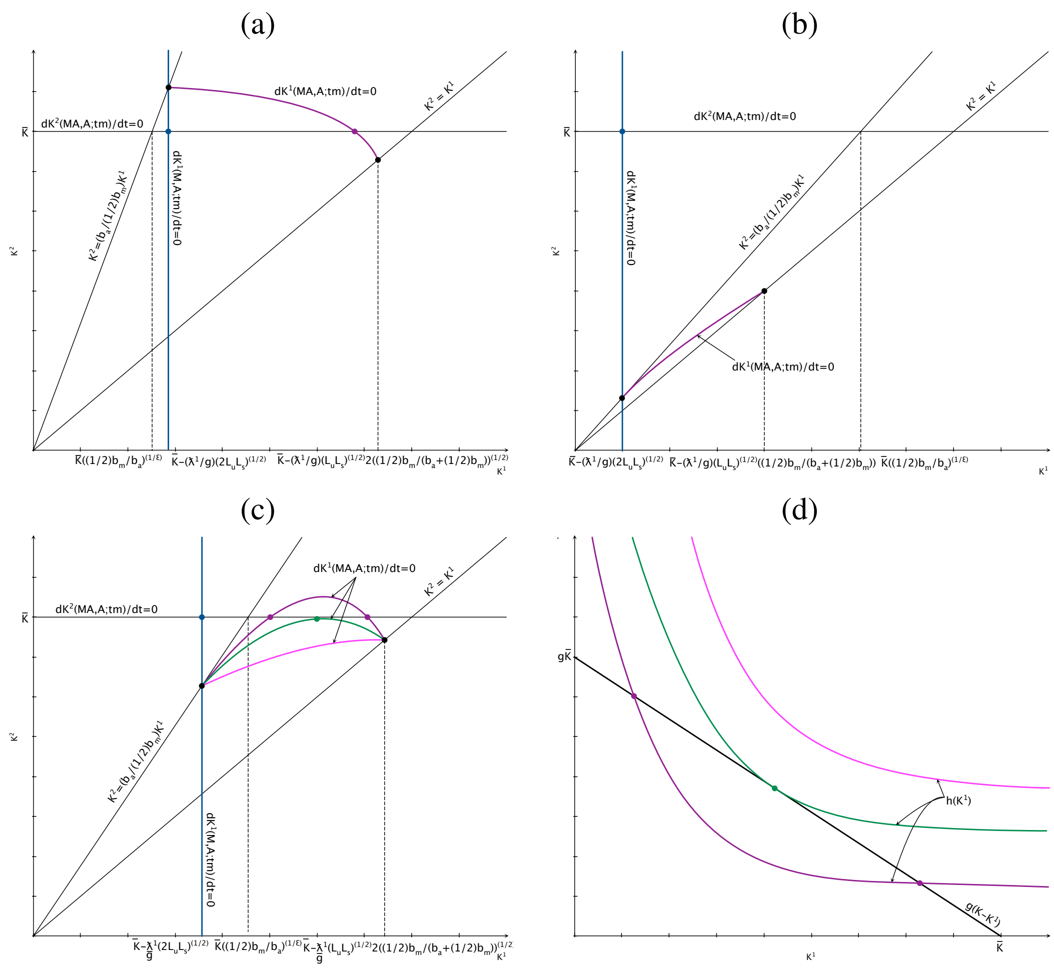

Figure II.3: Uniqueness and multiplicity of steady state with trade and migration

To do so, given a fixed endowments of environmental capital, I determine the possible steady states to which the economy can converge when workers are not mobile, and then compare to the possible steady states that ca be reached, given the same initial endowments, when workers are mobile.

1. Phase diagrams, Interregional trade

To study the phase diagrams of the evolutions of environmental capital with free trade, four cases ought to be considered: a unique SSSE, a unique DSSE, one SSSE and two DSSE, and one SSSE and one DSSE. Figure II.4 summarizes the analysis. If the initial endowments of environmental capital lie in the purple area, the steady state equilibrium is $(M,MA;t)$. If the initial endowments of environmental capital lie in the turquoise blue area, the steady state equilibrium is $(MA,MA;t)$. If the initial endowments of environmental capital lie in the green area, it is ambiguous.

If there is a *unique specialized steady state equilibrium,* $(M,MA;t)$*,* the phase diagram of the evolution of the environmental capital is illustrated in Figure II.4.a.

First, I prove that once region 1 is specialized in equilibrium, it remains specialized along the transition path until the economy reaches the specialized steady state, $(M,MA;t)$.[14] If region 1 is specialized and:

$$K^1 < \bar{K} - \frac{\lambda^1}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}},$$

even if in both regions, the environmental capital increases, the economy doesn't cross the line $\left(K^2\right)^{\varepsilon} = \left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)\left(K^1\right)^{\varepsilon}$ and region 1 remains specialized. If region 1 is specialized and:

$$K^1 > \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \frac{\lambda^2}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)\right),$$

even if in both regions, the environmental capital decreases, the economy doesn't cross the line $\left(K^2\right)^{\varepsilon} = \left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)\left(K^1\right)^{\varepsilon}$ and region 1 remains specialized. Starting

[14] In other words, I prove that once $\left(K^2\right)^{\varepsilon} \geq \left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)\left(K^1\right)^{\varepsilon}$, this inequality holds for ever.

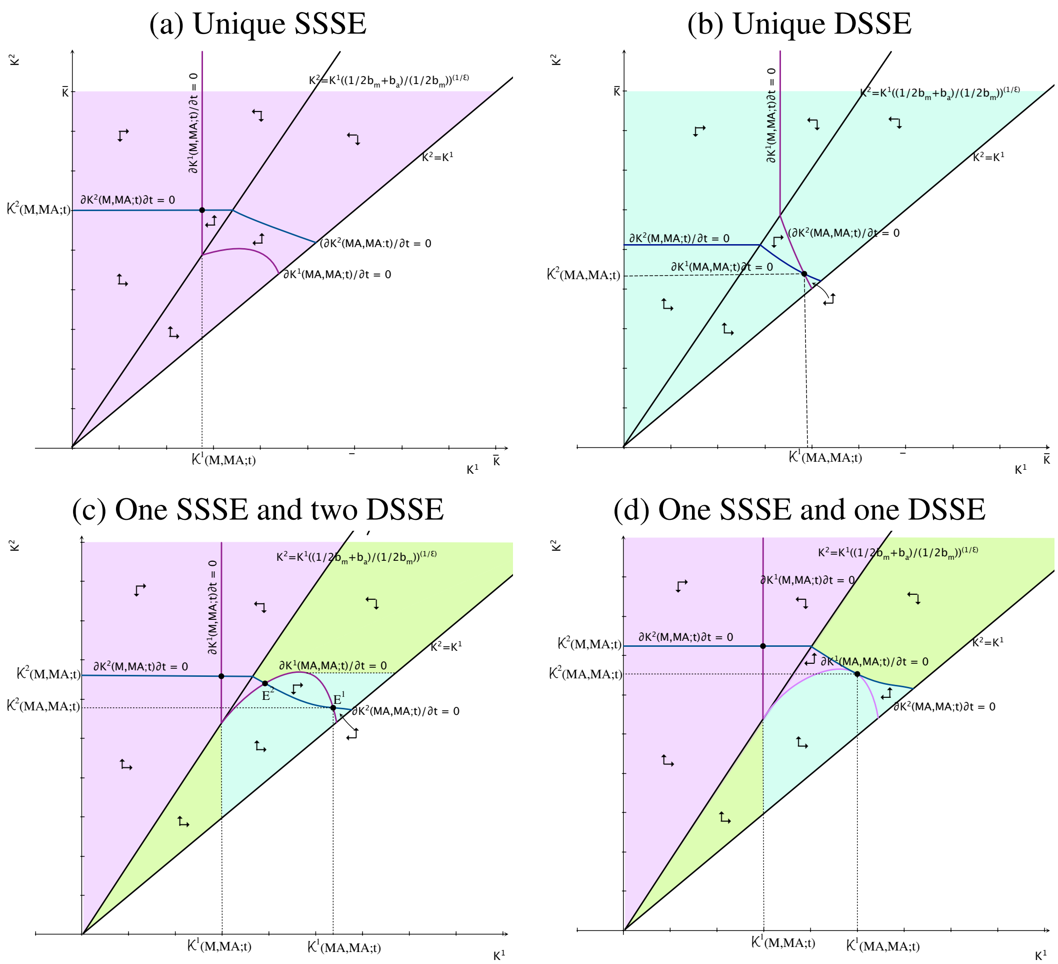

Figure II.4: Phase diagrams, interregional trade with the condition that a specialized steady state equilibrium exists, I show that if:

$$K^1 < \bar{K} - \frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}},$$

then the slope of the transition path is larger than $\left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)$ and if:

$$K^1 > \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{\varepsilon}}\left(\bar{K} - \frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)\right),$$

the slope of the transition path is smaller than $\left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)$:[15]

$$\begin{aligned}
\bar{K} - \frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right) &\geq \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}\left(\bar{K} - \frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\right) \\
\bar{K} - K^2 - \frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right) &\geq \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}\left(\bar{K} - K^2\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{\varepsilon}} - \frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\right) \\
\bar{K} - K^2 - \frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right) &\geq \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}\left(\bar{K} - K^1 - \frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\right) \\
g\left(\bar{K} - K^2\right) - \lambda^2(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right) &\geq \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}\left(g\left(\bar{K} - K^1\right) - \lambda^1(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\right) \\
\frac{\partial K^2(M,MA;t)}{\partial t} &\geq \frac{\partial K^1(M,MA;t)}{\partial t}\left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}.
\end{aligned}$$

Therefore, if $K^1 < \bar{K} - (\lambda^1/g)(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}$, then:

$$\frac{\partial K^2(M,MA;t)}{\partial K^1(M,MA;t)} \geq \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}},$$

[15] For K^2 to become strictly smaller than $\left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}}K^1$, K^2 should first be equal to $\left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}}K^1$. I prove that for $K^2 = \left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}}K^1$ the economy doesn't change from a specialized temporary equilibrium to the diversified one. Therefore, it is also true for $K^2 > \left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}}K^1$.

and if:

$$K^1 > \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)\right),$$

then:

$$\frac{\partial K^2(M,MA;t)}{\partial K^1(M,MA;t)} \leq \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}.$$

The slope of the evolution of the environmental capital is larger than the slope of the line that separates the two cases if:

$$K^1 < \bar{K} - \frac{\lambda^1}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}},$$

and smaller if:

$$K^1 > \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \frac{\lambda^2}{g}(L_s)^{\frac{1}{2}}(L_u)^{\frac{1}{2}}\left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right)\right).$$

Therefore, once the temporary equilibrium is $(M,MA;t)$, region 1 remains specialized until the economy reaches the specialized stationary equilibrium. This result also holds for the cases with multiple steady state equilibria that are studied below.

Regardless of the initial endowments of environmental capital, the economy reaches this unique stable steady state equilibrium. If the initial endowments of environmental capital are such that $\left(K^2\right)^{\varepsilon} < \left(\left(\frac{1}{2}b_m + b_a\right)/\frac{1}{2}b_m\right)\left(K^1\right)^{\varepsilon}$, region 1 is first diversified and as the environmental capital evolves it becomes specialized. Otherwise, region 1 remains specialized along the transition path.

It is worth reminding that if the initial endowments of environmental capital are such that $K^1 \leq K^2$, this inequality remains true along the transition path. Before becoming larger, K^1 would equal K^2, and if so, the variation of the environmental capital in region 2 in absolute value would always be larger than in region 1. Therefore, K^2 remains bigger than K^1. This is true in all the following cases with trade.

If there is a *unique diversified steady state equilibrium,* $(MA, MA; t)$, Figure II.4.b illustrates that regardless of the initial endowments of environmental capital, the economy reaches the unique diversified steady state equilibrium. Note that this steady state equilibrium is stable.

If the initial endowments of environmental capital are such that:

$$\left(K^2\right)^{\varepsilon} < \left(\left(\frac{1}{2}b_m + b_a\right) / \frac{1}{2}b_m\right)\left(K^1\right)^{\varepsilon},$$

the initial temporary equilibrium is characterized by $(MA, MA; t)$, otherwise it is characterized by $(M, MA; t)$. In both cases, the economy ends up at the diversified steady state, $(MA, MA; t)$, but might move from one case to the other along the transition path.

If there are *one specialized steady state equilibrium,* $(M, MA; t)$, *and two diversified steady state equilibria,* $(MA, MA; t)$, the phase diagram of the evolution of the environmental capital is illustrated in Figure II.4.c.

From the previous analysis, the economy reaches the temporary specialized equilibrium, it remains there along the transition path and reaches the specialized steady state equilibrium.

From Figure II.4.c, one can see that E^2 is an unstable steady state equilibrium. Hence, I don't consider it in my analysis. In this two diversified steady state equilibria case, $(MA, MA; t)$ refers to the stable diversified steady state equilibrium with trade, E^1.

If the initial endowments of environmental capital lie in the green areas, the steady state equilibrium is ambiguous. Those areas are examined later on.

If there are *one specialized steady state equilibrium,* $(M, MA; t)$, *and one diversified steady state equilibrium,* $(MA, MA; t)$, the phase diagram of the evolution of the environmental capitals is illustrated in Figure II.4.d.

From the preceding analysis, once the economy reaches the temporary specialized equilibrium, it remains there along the transition path and reaches the specialized steady state equilibrium.

If the initial endowments of environmental capital lie in the green areas, the steady state equilibrium is ambiguous.

2. Phase diagrams, Interregional trade and migration

The analysis of the phase diagrams of the evolutions of environmental capital with free trade and migration is similar to the situation with free trade only. If $b_a > \frac{1}{2}b_m$, fours cases ought to be considered: $(M,A;tm)$ is the unique steady state equilibrium, $(MA,A;tm)$ is the unique steady state equilibrium, there are three steady states: one $(M,A;tm)$ and two $(MA,A;tm)$, and the last case with two steady states: one $(M,A;tm)$ and one $(MA,A;tm)$. Figure II.5 summarizes the analysis. If the initial endowments of environmental capital lie in the purple area, the steady state equilibrium is characterized by $(M,A;tm)$. If the initial endowments of environmental capital lie in the turquoise blue area, the steady state equilibrium is characterized by $(MA,A;tm)$. If the initial endowments of environmental capital lies in the pink area, the steady state equilibrium is characterized by $(M,MA;tm)$. If the initial endowments of environmental capital lie in the green area, the steady state equilibrium is ambiguous.

If $b_a = \frac{1}{2}b_m$ ($b_a < \frac{1}{2}b_m$), there is only one case: $(M,A;tm)$ ($(M,MA;tm)$). Figure II.6 summarizes the analysis.

First, assume that $b_a > \frac{1}{2}b_m$.

If $(M,A;tm)$ *is the unique steady state equilibrium* the phase diagram of the evolution of the environmental capital is illustrated in Figure II.5.a.

First, I prove that once both regions are specialized in equilibrium, they remain specialized along the transition path until the economy reaches the specialized steady state equilibrium, $(M,A;tm)$.[16] If both regions are specialized and:

$$K^1 \geq \bar{K} - \frac{\lambda^1}{g} 2^{\frac{1}{2}} (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}},$$

then in region 1, the environmental capital decreases and in region 2, it increases. Hence, the preceding inequality remains true. It is not so evident if both regions are specialized and:

$$K^1 < \bar{K} - \frac{\lambda^1}{g} 2^{\frac{1}{2}} (L_u)^{\frac{1}{2}} (L_s)^{\frac{1}{2}},$$

[16] In other words, I prove that once $K^2 \geq \left(b_a/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}} K^1$, this inequality holds for ever.

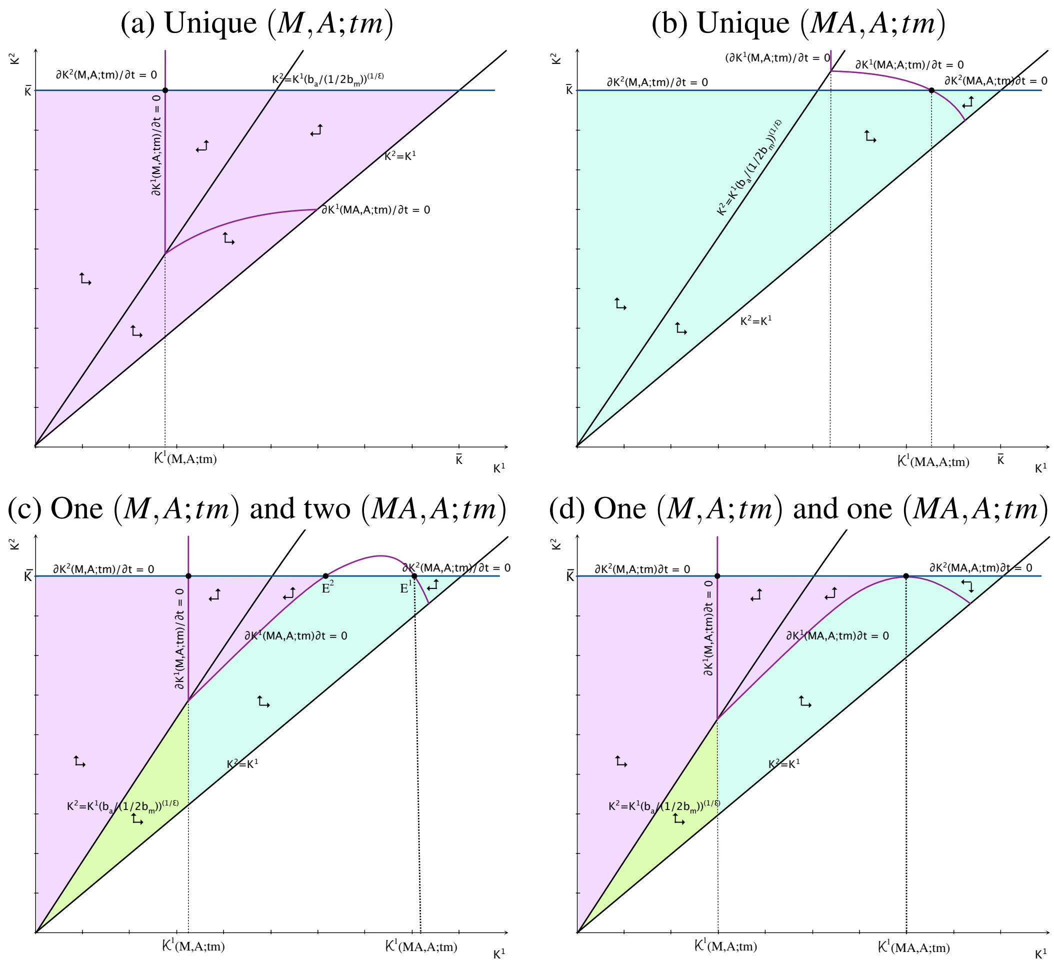

Figure II.5: Phase diagrams, interregional trade and migration, $b_a > \frac{1}{2}b_m$

(a) Unique $(M,MA;tm)$, $b_a < \frac{1}{2}b_m$ (b) Unique $(M,A;tm)$, $b_a = \frac{1}{2}b_m$

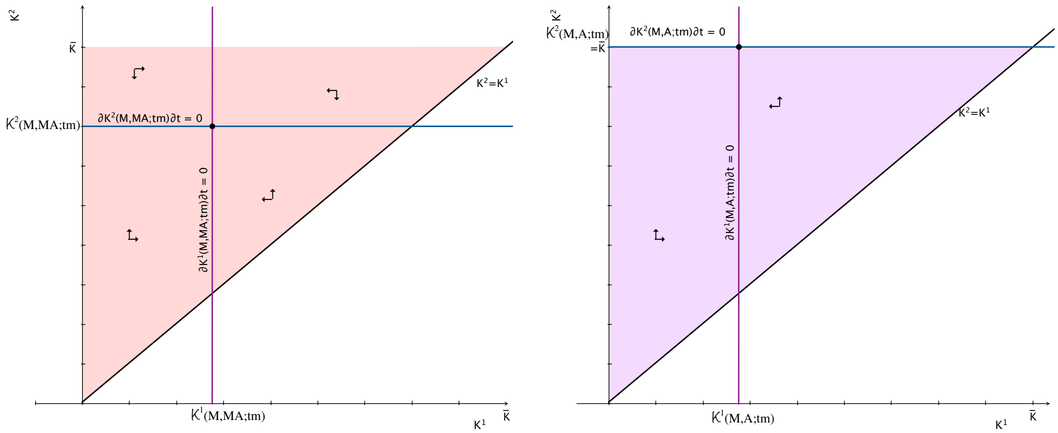

Figure II.6: Phase diagrams, interregional trade and migration, $b_a \leq \frac{1}{2}b_m$

because the environmental capital increases in both regions. Let's prove that in this case the temporary equilibrium remains $(M,A;tm)$. Starting with the condition that a specialized steady state equilibrium exists, I show that if:

$$K^1 < \bar{K} - \frac{\lambda^1}{g} 2^{\frac{1}{2}} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}},$$

the slope of the transition path is larger than $\left(b_a/\frac{1}{2}b_m\right)$:[17]

$$\begin{aligned}
\bar{K} &> \left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \frac{\lambda^1}{g} 2^{\frac{1}{2}} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}}\right) \\
\bar{K} - K^2 &> \left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \left(\frac{\frac{1}{2}b_m}{b_a}\right)^{\frac{1}{\varepsilon}} K^2 - \frac{\lambda^1}{g} 2^{\frac{1}{2}} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}}\right) \\
g\left(\bar{K} - K^2\right) &> \left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \left(g\left(\bar{K} - K^1\right) - \frac{\lambda^1}{g} 2^{\frac{1}{2}} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}}\right)
\end{aligned}$$

[17] For K^2 to become strictly smaller than $\left(b_a/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}} K^1$, K^2 should first be equal to $\left(b_a/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}} K^1$. If I prove that for $K^2 = \left(b_a/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}} K^1$, the economy doesn't change from a specialized temporary equilibrium to the diversified one, then this it is also true for $K^2 > \left(b_a/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}} K^1$.

$$\frac{\partial K^2(M,A;tm)}{\partial t} > \frac{\partial K^1(M,A;tm)}{\partial t}\left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}$$
$$\frac{\partial K^2(M,A;tm)}{\partial K^1(M,A;tm)} > \left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}}.$$

The slope of the evolution of the environmental capital is larger than the slope of the line that separates the two cases. Therefore, once the temporary equilibrium is $(M,A;tm)$, both regions remain specialized until the economy reaches the specialized stationary equilibrium. This result holds also for the cases with multiple steady state equilibria that are studied below.

Regardless of the initial endowments of environmental capital, the economy reaches this unique stable steady state equilibrium. If the initial endowments of environmental capital are such that $K^2 < \left(b_a/\frac{1}{2}b_m\right)^{\frac{1}{\varepsilon}} K^1$, region 1 is first diversified and as the environmental capital evolves, it becomes specialized. Otherwise, region 1 remains specialized along the transition path. Note that region 2 is always specialized.

If the initial endowments of environmental capital are such that $K^1 < K^2$, this inequality remains true along the transition path. Before becoming larger, K^1 would equal K^2, and if so, the variation of the environmental capital in region 2 in absolute value would always be larger than in region 1. Therefore, K^2 remains larger than K^1. This result holds for all the cases with trade and migration.

If $(MA,M;tm)$ *is the unique steady state equilibrium*, Figure II.5.b illustrates that regardless of the initial endowments of environmental capital, the economy reaches the unique steady state equilibrium $(MA,M;tm)$. Note that this steady state equilibrium is stable.

If the initial endowments of environmental capital are such that:

$$\left(K^2\right)^{\varepsilon} < \left(b_a/\frac{1}{2}b_m\right)\left(K^1\right)^{\varepsilon},$$

the initial temporary equilibrium is characterized by $(MA,M;tm)$, otherwise it is characterized by $(M,A;tm)$. In both cases, the economy ends up in the diversified steady state, $(MA,A;tm)$, but might move from one case to the other along the transition path.

If there are *three steady state equilibria: one* $(M,A;tm)$ *and two* $(MA,A;tm)$, the phase diagram of the evolution of the environmental capital is illustrated in Figure II.5.c.

From the previous analysis, we one can see that once the economy reaches the temporary equilibrium $(M,A;tm)$ it remains there along the transition path and reaches the specialized steady state equilibrium, $(M,A;tm)$.

From Figure II.5.c, one can see that E^2 is an unstable steady state equilibrium, therefore, it is not considered in my analysis. Hence, $(MA,A;tm)$ refers to the stable diversified steady state equilibrium, E^1.

If the initial endowments of environmental capital lie in the green areas, the steady state equilibrium is ambiguous. Those areas are examined later on.

If there are *two steady state equilibria: one* $(M,A;tm)$ *and one* $(MA,A;tm)$, the phase diagram of the evolution of the environmental capital is illustrated in Figure II.5.d.

As the preceding case, once the economy reaches the temporary specialized equilibrium $(M,A;tm)$, it remains there along the transition path and reaches the specialized steady state equilibrium.

If the initial endowments of environmental capital lie in the green areas, the steady state equilibrium is ambiguous.

Assume that $b_a \leq \frac{1}{2}b_m$.

The analysis is quite simple. If $b_a < \frac{1}{2}b_m$ ($b_a = \frac{1}{2}b_m$), the only temporary and stationary equilibrium is $(M,MA;tm)$, ($(M,A;tm)$). Figure II.6.a (Figure II.6.b) illustrates this case.

3. Setting the comparisons

If there exists initial endowments of environmental capital such that the economy reaches a steady state equilibrium x when trade is free and a steady state equilibrium y when there are free trade and migration, then x and y can be compared. However, if such initial endowments of environmental capital don't exist, then the comparison between x and y is not relevant. In the following part of the proof, I prove that the only comparison that is not relevant is $(M,MA;t)$ and $(MA,A;tm)$.

If $(M,A;tm)$ ($(M,MA;tm)$) is the unique steady state equilibrium with trade and migration, any of the four cases with trade can occur. Therefore $(M,A;tm)$ and $(M,MA;tm)$ can be compared to $(MA,MA;t)$ and $(M,MA;t)$. For environmental capital endow-

ments that lie in the purple area of Figure II.5, the economy with free trade and migration reaches the steady state $(M,A;tm)$ and the economy with free trade reaches either $(MA,MA;t)$ or $(M,MA;t)$. For environmental capital endowments that lie in the pink area of Figure II.6, the economy with free trade and migration reaches the steady state $(M,MA;tm)$ and the economy with free trade reaches either $(MA,MA;t)$ or $(M,MA;t)$.
If $(MA,A;tm)$ is the unique steady state equilibrium with trade and migration, the only case with trade is a unique diversified steady state equilibrium, $(MA,MA;t)$. If $(MA,A;tm)$ is the unique steady state equilibrium, then:

$$\bar{K} < \left(\frac{b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \lambda^1 2^{\frac{1}{2}} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}}\right),$$

which implies,

$$\bar{K} - \frac{\lambda^2}{g} (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}} \left(\frac{\frac{1}{2}b_m}{\frac{1}{2}b_m + b_a}\right) < \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{\varepsilon}} \left(\bar{K} - \lambda^1 (L_s)^{\frac{1}{2}} (L_u)^{\frac{1}{2}}\right).$$

Therefore, the only possible case with trade is a unique $(MA,MA;t)$. Hence, $(MA,A;tm)$ can be compared to $(MA,MA;t)$.
So far, it has been established that I should compare $(M,A;tm)$ to $(MA,MA;t)$ and $(M,MA;t)$, $(M,MA;tm)$ to $(MA,MA;t)$ and $(M,MA;t)$, and $(MA,A;tm)$ to $(MA,MA;t)$. Therefore, what remains to be done is to figure out if the comparison between $(MA,A;tm)$ and $(M,MA;t)$ is relevant. In other words, is it possible for an economy that reaches $(MA,A;tm)$ when trade is free and workers are mobile to reach $(M,MA;t)$ when trade is free but workers aren't mobile?
The answer to that question is no. When the steady state with trade and migration is unique and characterized by $(MA,A;tm)$, it is shown that an economy doesn't reach $(M,MA;t)$ at the steady state with trade. The only two cases with trade and migration left are one $(M,A;tm)$ and two $(MA,A;tm)$, and one $(M,A;tm)$ and one $(MA,A;tm)$.
Considering the case one $(M,A;tm)$ and two $(MA,A;tm)$, the possible cases with trade

is one $(M,MA;t)$ and two $(MA,MA;t)$, and a unique $(MA,MA;t)$.[18] It is shown below that for any given (K^1,K^2):

$$\left(\frac{\partial K^1(MA,MA;t)}{\partial t}\right) > \left(\frac{\partial K^1(MA,A;tm)}{\partial t}\right),$$

and

$$\left(\frac{\partial K^2(MA,MA;t)}{\partial t}\right) < \left(\frac{\partial K^2(MA,A;tm)}{\partial t}\right),$$

therefore, $\partial K^1(MA,MA;t)/\partial t = 0$ lies above $\partial K^1(MA,A;tm)/\partial t = 0$. Moreover, the function $\partial K^2(MA,MA;t)/\partial t = 0$ lies below $\partial K^2(MA,A;tm)/\partial t = 0$.

The set of initial endowments of environmental capital from which the economy reaches for sure the steady state equilibrium with trade and migration $(MA,A;tm)$ is contained in the set of initial endowments of environmental capital from which the economy reaches for sure the steady state equilibrium with trade $(MA,MA;t)$. This result is illustrated in Figure II.7, the turquoise blue area of Figure II.7.a is contained in the turquoise blue area of Figure II.7.b. The only other set of environmental capitals from which the economy goes to $(MA,A;tm)$ with free trade and migration and from which the economy is susceptible to reach $(M,MA;t)$ is the intersection of the green areas of Figure II.7.a and Figure II.7.b. Knowing that:

$$\frac{\partial K^1(MA,MA;t)}{\partial t} > \frac{\partial K^1(MA,A;tm)}{\partial t} \quad \text{and} \quad \frac{\partial K^2(MA,MA;t)}{\partial t} < \frac{\partial K^2(MA,A;tm)}{\partial t},$$

the economy cannot reach $(M,MA;t)$ with trade and $(MA,M;tm)$ with trade and migration from any initial endowments of environmental capital in the intersection of both green areas. The slope of the transition path is larger with trade and migration than with trade, therefore from initial endowments of environmental capital in the intersection of both green areas if the economy reaches $(M,MA;t)$ with trade, then it should reach $(M,A;tm)$ with trade and migration.

[18] The following analysis is also true for the case one $(M,A;tm)$ and one $(MA,A;tm)$.

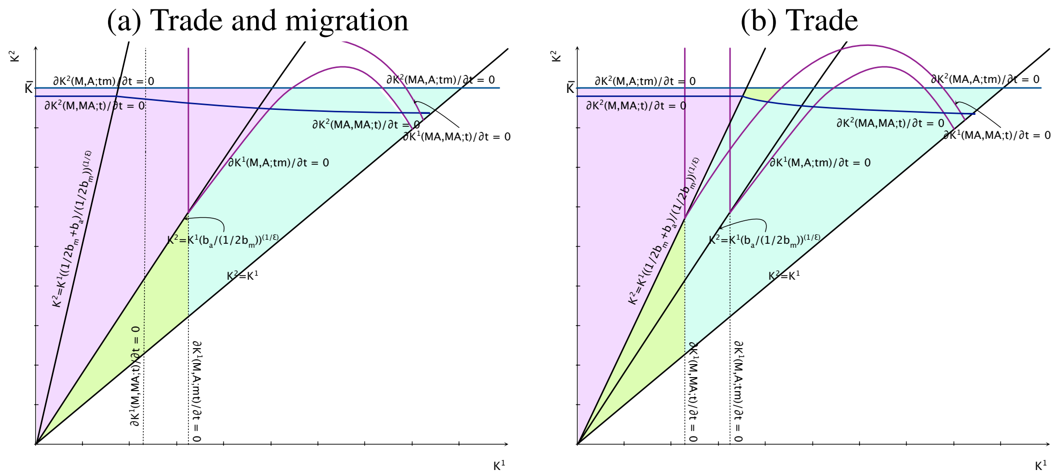

Figure II.7: Transition path

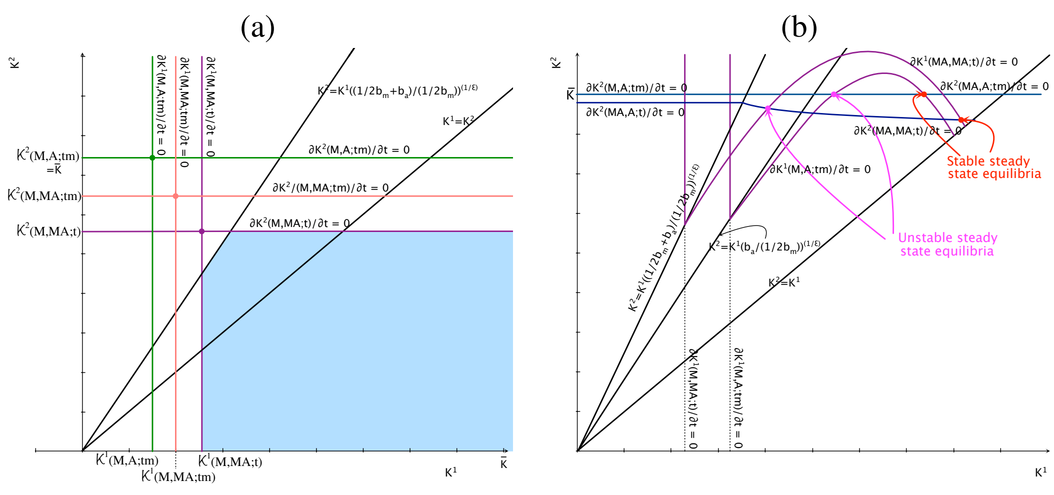

Figure II.8: Comparison

4. Comparisons

Comparison between $(M,MA;t)$ *and* $(M,MA;tm)$

Under the assumptions that $(M,MA;t)$ and $(M,MA;tm)$ are steady state equilibria and that $\frac{1}{2}b_m > b_a$:

$$\begin{aligned} 1 < \left(\frac{\frac{1}{2}b_m + b_a}{\frac{1}{2}b_m}\right)^{\frac{1}{2}} \quad &\Rightarrow \quad \frac{\partial K^1(M,MA;t)}{\partial t} > \frac{\partial K^1(M,MA;tm)}{\partial t} \\ 0 > -\frac{1}{4}b_m^2 - b_a\left(\frac{1}{2}b_m - b_a\right) \quad &\Rightarrow \quad \frac{\frac{1}{2}b_m}{\left(\frac{1}{2}b_m + b_a\right)} > \frac{\frac{1}{2}b_m - b_a}{\left(\frac{1}{2}b_m\right)^{\frac{1}{2}}\left(\frac{1}{2}b_m + b_a\right)^{\frac{1}{2}}} \\ &\Rightarrow \quad \frac{\partial K^2(M,MA;t)}{\partial t} < \frac{\partial K^2(M,MA;tm)}{\partial t}. \end{aligned}$$

The function $\partial k^1(M,MA;t)/\partial t$ equals zero at $K^1 = \mathscr{K}^1(M,MA;t)$. For all K^1 larger (smaller) than $K^1 = \mathscr{K}^1(M,MA;t)$, $\partial k^1(M,MA;t)/\partial t$ is negative (positive).

Since $\partial K^1(M,MA;t)/\partial t > \partial K^1(M,MA;tm)/\partial t$, the function $\partial K^1(M,MA;tm)/\partial t$ is negative for all $K^1 \geq \mathscr{K}^1(M,MA;t)$. Therefore:

$$K^1 = \mathscr{K}^1(M,MA;tm) < \mathscr{K}^1(M,MA;t).$$

The function $\partial k^2(M,MA;tm)/\partial t$ equals zero at $K^2 = \mathscr{K}^2(M,MA;tm)$. For all K^2 larger (smaller) than $\mathscr{K}^2(M,MA;tm)$, $\partial k^2(M,MA;tm)/\partial t$ is negative (positive). Since

$$\frac{\partial K^2(M,MA;tm)}{\partial t} > \frac{\partial K^2(M,MA;t)}{\partial t},$$

the function $\partial K^2(M,MA;t)/\partial t$ is negative for all $K^2 \geq \mathscr{K}^2(M,MA;tm)$.

Therefore, $\mathscr{K}^2(M,MA;t) < \mathscr{K}^2(M,MA;tm)$. This result is illustrated in Figure II.8.a.

Comparison between $(MA,MA;t)$ *and* $(M,MA;tm)$

In Section 2.4.2, it is shown that under the assumption that a diversified steady state exists: $\mathscr{K}^1(M,MA;t) < \mathscr{K}^1(MA,MA;t)$ and $\mathscr{K}^2(M,MA;t) > \mathscr{K}^2(MA,MA;t)$. Therefore, $(\mathscr{K}^1(MA,MA;t), \mathscr{K}^2(MA,MA;t))$ lies somewhere in the blue area of Figure II.8.a. Combining those inequalities with the comparison between $(M,MA;t)$ and $(M,MA;tm)$

implies that:

$$\mathscr{K}^1(M,MA;tm) < \mathscr{K}^1(MA,MA;t) \quad \text{and} \quad \mathscr{K}^2(MA,MA;t) < \mathscr{K}^2(M,MA;tm).$$

Therefore:[19]

$$\mathscr{K}^1(M,MA;tm) < \mathscr{K}^1(\cdot;\cdot;t) \quad \text{and} \quad \mathscr{K}^2(\cdot,\cdot;t) < \mathscr{K}^2(M,MA;tm).$$

Comparison between $(MA,MA;t)$ and $(MA,A;tm)$

Under the assumptions that $(MA,MA;t)$ and $(MA,A;tm)$ are steady state equilibria and that $\frac{1}{2}b_m < b_a$:

$$0 < \frac{\left(K^2\right)^{\varepsilon}}{\left(K^1\right)^{\varepsilon}} + 2 \quad \Rightarrow \quad \frac{\partial K^1(MA,MA;t)}{\partial t} > \frac{\partial K^1(MA,A;tm)}{\partial t}.$$

Therefore, the function $\partial K^1(MA,MA;t)/\partial t = 0$ lies above $\partial K^1(MA,A;tm)/\partial t = 0$. The comparison of the environmental capital in region 2 is straightforward. As it is illustrated in Figure II.8.b, the environmental capital in region 1 (region 2) at the stable steady state equilibrium is larger (smaller) with trade than with trade and migration.[20]

The comparison between $(M,MA;t)$ and $(M,A;tm)$ is direct, as the comparison between $(MA,MA;t)$ and $(M,A;tm)$. *Q.E.D.*

[19] Note that the unstable steady states are not considered.

[20] In Figure II.8.b, only the case with two (MA,MA,t) and two $(MA,A;tm)$ is illustrated, but the analysis for the other possible cases is the same and leads to the same results.

Annexe III

Appendix to Chapter 4

Proof of Proposition 4.1

To determine the impact of A_2^f on L_2, K_2, and K_2^s, we first study the impact of z_2 on the optimal choice of K_2 and L_2. In order to do so, we totally differentiate the first order conditions (4.4) et (4.5) with respect to K_2, L_2, and z_2:

$$\underbrace{\begin{pmatrix} p_2 G_{yy}(F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yy}(F_L)^2 + p_2 G_y F_{LL} \end{pmatrix}}_{\Delta_{KL}} \begin{pmatrix} dK \\ dL \end{pmatrix} = \begin{pmatrix} -\, p_2 G_{yz} F_K dz \\ -\, p_2 G_{yz} F_L dz \end{pmatrix}.$$

The concavity assumption of G and F ensures that $|\Delta_{KL}|$ is strictly positive:

$$\begin{aligned} |\Delta_{KL}| = & p_2^2 \Big[(G_{yy})^2 \left((F_L)^2 (F_K)^2 - (F_L F_K)^2 \right) \\ & + G_{yy} G_y \left((F_L)^2 F_{KK} + (F_K)^2 F_{LL} - 2 F_L F_K F_{LK} \right) \\ & + (G_y)^2 \left(F_{LL} F_{KK} - (F_{LK})^2 \right) \Big] > 0. \end{aligned}$$

It is worth mentioning that the concavity of F also implies that Σ_F is negative:

$$\Sigma_F = (F_L)^2 F_{KK} + (F_K)^2 F_{LL} - 2 F_L F_K F_{LK} = \begin{pmatrix} -F_L & F_K \end{pmatrix} \begin{pmatrix} F_{KK} & F_{LK} \\ F_{LK} & F_{LL} \end{pmatrix} \begin{pmatrix} -F_L \\ F_K \end{pmatrix} < 0.$$

From the Cramer rule:

$$\frac{dK_2}{dz_2} = \frac{-1}{|\Delta_{KL}|} \begin{vmatrix} p_2 G_{yz} F_K & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} \\ p_2 G_{yz} F_L & p_2 G_{yy}(F_L)^2 + p_2 G_y F_{LL} \end{vmatrix} = \frac{-p_2^2 G_{yz} G_y \left(F_K F_{LL} - F_L F_{LK} \right)}{|\Delta_{KL}|}. \tag{III.1}$$

The assumption of homogeneity of degree 1 of F implies that F_L is homogeneous of degree 0 and F_{LK} is strictly positive:

$$F_L L + F_K K = F \quad \Rightarrow F_K = \frac{F}{K} - F_L \frac{L}{K},$$
$$F_{LL} L + F_{LK} K = 0 \quad \Rightarrow F_{LK} K = -F_{LL} L > 0.$$

Therefore,

$$\begin{aligned} F_K F_{LL} - F_L F_{LK} &= (F/K) F_{LL} - F_L (F_{LL} L/K + F_{LK}) \\ &= (F/K) F_{LL} - \frac{F_L}{K} (F_{LL} L + F_{LK} K) = \frac{F F_{LL}}{K} < 0. \end{aligned}$$

Consequently, under the assumption of homogeneity of degree 1 and the assumption that the cross derivative of G is positive, $G_{yz} > 0$, an increase of z_2 leads to an increase of K_2, i.e., $dK_2/dz_2 > 0$. Correspondingly,

$$\begin{aligned} \frac{dL_2}{dz_2} &= \frac{-1}{|\Delta_{LK}|} \begin{vmatrix} p_2 G_{yy} (F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yz} F_K \\ p_2 G_{yy} F_K F_L + p_2 G_y F_{LK} & p_2 G_{yz} F_L \end{vmatrix} \\ &= \frac{-p_2^2 G_{yz} G_y \left(F_L F_{KK} - F_K F_{LK} \right)}{|\Delta_{LK}|} > 0. \end{aligned} \tag{III.2}$$

We can combine these two results and determine the impact of a variation of z on the optimal level of y that is chosen by the firm:

$$\frac{dy_2}{dz_2} = F_K \frac{dK_2}{dz_2} + F_L \frac{dL_2}{dz_2} = -\frac{p_2^2 G_y G_{yz} \Sigma_F}{|\Delta_{LK}|} > 0.$$

Let us proceed to the analysis of the impact of a variation of y_2 on K_2^s. The total derivative of (4.6) implies that:

$$\frac{dK_2^s}{dy_2} = -\frac{G_{yz} H_{K^s}}{G_{zz} \left(H_{K^s} \right)^2 + G_z H_{K^s K^s}}.$$

The concavity of G and H, and the assumption that $G_{yz} > 0$ ensure that $dK_2^s/dy_2 > 0$. Before proceeding with the evaluation of the impact of a variation of the amount of water allocated to the firm, A_2^f, on the optimal choice of K_2, L_2 et K_2^s, we define by Ω_2 the Hessian matrix associated with the first order conditions (4.4), (4.5), and (4.6):

$$\Omega_2 = \begin{bmatrix} p_2 G_{yy} (F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yz} F_K H_{K^s} \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yy} (F_L)^2 + p_2 G_y F_{LL} & p_2 G_{yz} F_L H_{K^s} \\ p_2 G_{yz} F_K H_{K^s} & p_2 G_{yz} F_L H_{K^s} & p_2 G_{zz} (H_{K^s})^2 + p_2 G_z H_{K^s K^s} \end{bmatrix}.$$

The determinant of Ω_2 is strictly negative:

$$\begin{aligned} |\Omega_2| =& p_2 G_z H_{K^s K^s} |\Delta_{KL}| + p_2^3 (G_y)^2 G_{zz} (H_{K^s})^2 \left(F_{LL} F_{KK} - (F_{LK})^2 \right) \\ &+ P_2^3 G_y (H_{K^s})^2 \left(G_{zz} G_{yy} - (G_{yz})^2 \right) \Sigma_F < 0. \end{aligned}$$

The impact of a variation of the amount of water allocated to the firm on K_2^s is determined using the Cramer rule:

$$\frac{dK_2^s}{dA_2^f} = -\frac{|\Omega_{A_2^f K_2^s}|}{|\Omega_2|},$$

with,

$$\Omega_{A_2^f K_2^s} = \begin{bmatrix} p_2 G_{yy} (F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yz} F_K H_Q \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yy} (F_L)^2 + p_2 G_y F_{LL} & p_2 G_{yz} F_L H_Q \\ p_2 G_{yz} F_K H_{K^s} & p_2 G_{yz} F_L H_{K^s} & p_2 G_{zz} H_{K^s} H_Q + p_2 G_z H_{QK^s} \end{bmatrix}.$$

The sign of the determinant of $\Omega_{A_2^f K_2^s}$ is ambiguous:

$$\begin{aligned} |\Omega_{A_2^f K_2^s}| =& p_2 G_z |\Delta_{LK}| H_{QK^s} + p_2^3 G_{zz} H_Q \left(F_{LL} F_{KK} - (F_{LK})^2 \right) \\ &+ p_2^3 G_y H_Q H_{K^s} \left(G_{zz} G_{yy} - (G_y z)^2 \right) \Sigma_F . \end{aligned}$$

Therefore, the impact of the amount of water allocated to the firm on the optimal level of K_2^s is ambiguous. For the upcoming analysis, it is useful to rewrite the determinant of

$\Omega_{A_2^f K^s}$ by:

$$|\Omega_{A_2^f K_2^s}| = |\Omega| \frac{H_Q}{H_{K^s}} + p_2 G_z |\Delta_{LK}| \left(\frac{H_{QK^s}}{H_Q} - \frac{H_{K^sK^s}}{H_{K^s}} \right) H_Q.$$

Therefore,

$$\frac{dK_2^s}{dA_2^f} = -\frac{H_Q}{H_{K^s}} - \frac{p_2 G_z |\Delta_{LK}| (H_{QK^s} H_{K^s} - H_{K^sK^s} H_Q)}{H_{K^s} |\Omega_2|}. \tag{III.3}$$

Using $(III.3)$, we can evaluate the variation of z that follows the variation of water allocated to the firm:

$$\begin{aligned} \frac{dz_2}{dA_2^f} =& H_Q \frac{dQ_2^f}{dA_2^f} + H_{K^s} \frac{dK_2^s}{dA_2^f} \\ =& -\frac{p_2 G_z |\Delta_{LK}| (H_{QK^s} H_{K^s} - H_{K^sK^s} H_Q)}{|\Omega_2|} > 0. \end{aligned} \tag{III.4}$$

The Assumption A6 ensures that an increase of water used by the firm induces an increase of services produced by that water.

The impact of a variation of the amount of water allocated to the firms on K_2 and L_2 is determined by the combination of (III.1), (III.2), and (III.4):

$$\frac{dK_2}{dA_2^f} = \frac{dK_2}{dz_2} \frac{dz_2}{dA_2^f} = \frac{p_2^3 G_y G_z G_{yz} \left(F_{LL} F_K - F_{LK} F_L\right) \left(H_{QK^s} K_{K^s} - H_{K^sK^s} H_Q\right)}{|\Omega_2|}, \tag{III.5}$$

$$\frac{dL_2}{dA_2^f} = \frac{dK_2}{dz_2} \frac{dz_2}{dA_2^f} = \frac{p_2^3 G_y G_z G_{yz} \left(F_L F_{KK} - F_{LK} F_K\right) \left(H_{QK^s} K_{K^s} - H_{K^sK^s} H_Q\right)}{|\Omega_2|},$$

which are positive under the assumptions taken throughout the paper. *Q.E.D.*

Proof of Proposition 4.2

First, we determine the impact of r on K_2^s using the Cramer rule:

$$\frac{dK_2^s}{dr} = \frac{|\Omega_{rK_2^s}|}{|\Omega_2|},$$

with,

$$\Omega_{rK_2^s} = \begin{bmatrix} p_2 G_{yy}(F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & 1 \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yy}(F_L)^2 + p_2 G_y F_{LL} & 0 \\ p_2 G_{yz} F_K H_{K^s} & p_2 G_{yz} F_L H_{K^s} & 0 \end{bmatrix}.$$

Therefore,

$$\frac{dK_2^s}{dr} = \frac{p_2^2 G_y G_{yz} H_{K^s}(F_L F_{LK} - F_K F_{LL})}{|\Omega_2|} < 0. \tag{III.6}$$

Next, we determine the impact of w_2 on K^s:

$$\frac{dK_2^s}{dw_2} = \frac{|\Omega_{w_2 K_2^s}|}{|\Omega_2|},$$

with,

$$\Omega_{w_2 K_2^s} = \begin{bmatrix} p_2 G_{yy}(F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & 0 \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yy}(F_L)^2 + p_2 G_y F_{LL} & 1 \\ p_2 G_{yz} F_K H_{K^s} & p_2 G_{yz} F_L H_{K^s} & 0 \end{bmatrix}.$$

Therefore,

$$\frac{dK_2^s}{dw_2} = \frac{p_2^2 G_y G_{yz} H_{K^s}(F_K F_{LK} - F_L F_{KK})}{|\Omega_2|} < 0. \tag{III.7}$$

We analyse the impact of r^s on K_2^s:

$$\frac{dK_2^s}{dr^s} = \frac{|\Omega_{r^s K_2^s}|}{|\Omega_2|},$$

with,

$$\Omega_{r^s K_2^s} = \begin{bmatrix} p_2 G_{yy} (F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & 0 \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yy} (F_L)^2 + p_2 G_y F_{LL} & 0 \\ p_2 G_{yz} F_K H_{K^s} & p_2 G_{yz} F_L H_{K^s} & 1 \end{bmatrix}.$$

Therefore,

$$\frac{dK_2^s}{dr^s} = \frac{|\Delta_{LK}|}{|\Omega_2|} < 0. \tag{III.8}$$

We prove that p_2 affects positively K_2^s. Using the Cramer rule, the impact of p_2 on K_2^s is defined by:

$$\frac{dK_2^s}{dp_2} = \frac{|\Omega_{p_2 K_2^s}|}{|\Omega_2|},$$

with,

$$\Omega_{p_2 K_2^s} = \begin{bmatrix} p_2 G_{yy} (F_K)^2 + p_2 G_y F_{KK} & p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & -G_y F_K \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & p_2 G_{yy} (F_L)^2 + p_2 G_y F_{LL} & -G_y F_L \\ p_2 G_{yz} F_K H_{K^s} & p_2 G_{yz} F_L H_{K^s} & -G_z H_{K^s} \end{bmatrix}.$$

Therefore,

$$\frac{dK_2^s}{dp_2} = \frac{(G_y)^2 G_{yz} H_{K^s} \Sigma_F - G_z H_{K^s} |\Delta_{LK}|}{|\Omega_2|} > 0. \tag{III.9}$$

Then, we evaluate the impact of factor prices on K_2 using the Cramer rule. We first determine dK_2/dr:

$$\frac{dK_2}{dr} = \frac{|\Omega_{rK_2}|}{|\Omega_2|},$$

with:

$$\Omega_{rK_2} = \begin{bmatrix} 1 & p_2G_{yy}F_LF_K + p_2G_yF_{LK} & p_2G_{yz}F_KH_{K^s} \\ 0 & p_2G_{yy}(F_L)^2 + p_2G_yF_{LL} & p_2G_{yz}F_LH_{K^s} \\ 0 & p_2G_{yz}F_LH_{K^s} & p_2G_{zz}(H_{K^s})^2 + p_2G_zH_{K^sK^s} \end{bmatrix}.$$

Therefore,

$$\begin{aligned} \frac{dK_2}{dr} =& \frac{p_2^2}{|\Omega_2|}\Big[\Big(G_{yy}G_{zz} - (G_{yz})^2\Big)(F_LH_{K^s})^2 + G_zG_{yy}(F_L)^2H_{K^sK^s} \\ &+ G_yG_{zz}F_{LL}(H_{K^s})^2 + G_yG_zF_{LL}H_{K^sK^s}\Big] < 0. \end{aligned} \tag{III.10}$$

We determine dK_2/dw_2:

$$\frac{dK}{dw_2} = \frac{|\Omega_{w_2K_2}|}{|\Omega_2|},$$

with,

$$\Omega_{w_2K_2} = \begin{bmatrix} 0 & p_2G_{yy}F_LF_K + p_2G_yF_{LK} & p_2G_{yz}F_KH_{K^s} \\ 1 & p_2G_{yy}(F_L)^2 + p_2G_yF_{LL} & p_2G_{yz}F_LH_{K^s} \\ 0 & p_2G_{yz}F_LH_{K^s} & p_2G_{zz}(H_{K^s})^2 + p_2G_zH_{K^sK^s} \end{bmatrix}.$$

Therefore,

$$\begin{aligned} \frac{dK_2}{dw_2} =& \frac{p_2^2}{|\Omega_2|}\Big[(G_{yz})^2F_LF_K(H_{K^s})^2 - G_{yy}G_{zz}F_LF_K(H_{K^s})^2 - G_zG_{yy}F_LF_KH_{K^sK^s} \\ &- G_yG_{zz}F_{LK}(H_{K^s})^2 - G_yG_zF_{LK}H_{K^sK^s}\Big]. \end{aligned} \tag{III.11}$$

We calculate the impact of r^s on K_2:

$$\frac{dK_2}{dr^s} = \frac{|\Omega_{r^sK_2}|}{|\Omega_2|},$$

with:

$$\Omega_{r^sK_2} = \begin{bmatrix} 0 & p_2G_{yy}F_LF_K + p_2G_yF_{LK} & p_2G_{yz}F_KH_{K^s} \\ 0 & p_2G_{yy}(F_L)^2 + p_2G_yF_{LL} & p_2G_{yz}F_LH_{K^s} \\ 1 & p_2G_{yz}F_LH_{K^s} & p_2G_{zz}(H_{K^s})^2 + p_2G_zH_{K^sK^s} \end{bmatrix}.$$

As a result,

$$\frac{dK_2}{dr^s} = \frac{p_2^2G_yG_{yz}H_{K^s}(F_LF_{LK} - F_KF_{LL})}{|\Omega_2|} < 0. \quad \text{(III.12)}$$

We also evaluate the impact of p_2 on K_2:

$$\frac{dK_2}{dp_2} = \frac{|\Omega_{p_2K_2}|}{|\Omega_2|},$$

with:

$$\Omega_{p_2K_2} = \begin{bmatrix} -G_yF_K & p_2G_{yy}F_LF_K + p_2G_yF_{LK} & p_2G_{yz}F_KH_{K^s} \\ -G_yF_L & p_2G_{yy}(F_L)^2 + p_2G_yF_{LL} & p_2G_{yz}F_LH_{K^s} \\ -G_zH_{K^s} & p_2G_{yz}F_LH_{K^s} & p_2G_{zz}(H_{K^s})^2 + p_2G_zH_{K^sK^s} \end{bmatrix}.$$

Consequently,

$$\frac{dK_2}{dp_2} = \frac{p_2^2G_y(F_KF_{LL} - F_LF_{LK})\left((G_zG_{yz} - G_yG_{zz})(H_{K^s})^2 - G_yG_zH_{K^sK^s}\right)}{|\Omega_2|} > 0. \quad \text{(III.13)}$$

At last, we determine the impact of factor prices on L_2 using the Cramer rule. We begin with the impact of r:

$$\frac{dL_2}{dr} = \frac{|\Omega_{rL_2}|}{|\Omega|},$$

with:

$$\Omega_{rL_2} = \begin{bmatrix} p_2 G_{yy} (F_K)^2 + p_2 G_y F_{KK} & 1 & p_2 G_{yz} F_K H_{K^s} \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & 0 & p_2 G_{yz} F_L H_{K^s} \\ p_2 G_{yz} F_K H_{K^s} & 0 & p_2 G_{zz} (H_{K^s})^2 + p_2 G_z H_{K^s K^s} \end{bmatrix}.$$

Therefore,

$$\begin{aligned} \frac{dL_2}{dr} =& \frac{p_2^2}{|\Omega_2|} \Big[- \left(G_{yy} G_{zz} - (G_{yz})^2 \right) F_L F_K (H_{K^s})^2 - G_z G_{yy} F_L F_K H_{K^s K^s} \\ & - G_y G_{zz} F_{LK} (H_{K^s})^2 - G_y G_z F_{LK} H_{K^s K^s} \Big]. \end{aligned}$$

We determine the impact of w_2 on L_2:

$$\frac{dL_2}{dw_2} = \frac{|\Omega_{w_2 L_2}|}{|\Omega_2|},$$

with,

$$\Omega_{w_2 L_2} = \begin{bmatrix} p_2 G_{yy} (F_K)^2 + p_2 G_y F_{KK} & 0 & p_2 G_{yz} F_K H_{K^s} \\ p_2 G_{yy} F_L F_K + p_2 G_y F_{LK} & 1 & p_2 G_{yz} F_L H_{K^s} \\ p_2 G_{yz} F_K H_{K^s} & 0 & p_2 G_{zz} (H_{K^s})^2 + p_2 G_z H_{K^s K^s} \end{bmatrix}.$$

As a result,

$$\begin{aligned} \frac{dL_2}{dw_2} =& \frac{p_2^2}{|\Omega_2|} \Big[\left(G_{yy} G_{zz} - (G_{yz})^2 \right) (F_K H_{K^s})^2 + G_z G_{yy} (F_K)^2 H_{K^s K^s} \\ & + G_y G_{zz} F_{KK} (H_{K^s})^2 + G_y G_z F_{KK} H_{K^s K^s} \Big] < 0. \end{aligned}$$

Then, we determine the impact of r^s on L_2:

$$\frac{dL_2}{dr^s} = \frac{|\Omega_{r^s L_2}|}{|\Omega_2|},$$

with,

$$\Omega_{r^sL_2} = \begin{bmatrix} p_2G_{yy}(F_K)^2 + p_2G_yF_{KK} & 0 & p_2G_{yz}F_KH_{K^s} \\ p_2G_{yy}F_LF_K + p_2G_yF_{LK} & 0 & p_2G_{yz}F_LH_{K^s} \\ p_2G_{yz}F_KH_{K^s} & 1 & p_2G_{zz}(H_{K^s})^2 + p_2G_zH_{K^sK^s} \end{bmatrix}.$$

Then,

$$\frac{dL_2}{dr^s} = \frac{p_2^2G_yG_{yz}H_{K^s}(F_KF_{LK} - F_{KK}F_L)}{|\Omega_2|} < 0.$$

At last, we evaluate the impact of p_2 on L_2:

$$\frac{dL_2}{dp_2} = \frac{|\Omega_{p_2L_2}|}{|\Omega_2|},$$

with:

$$\Omega_{p_2L_2} = \begin{bmatrix} p_2G_{yy}(F_K)^2 + p_2G_yF_{KK} & -G_yF_K & p_2G_{yz}F_KH_{K^s} \\ p_2G_{yy}F_LF_K + p_2G_yF_{LK} & -G_yF_L & p_2G_{yz}F_LH_{K^s} \\ p_2G_{yz}F_KH_{K^s} & -G_zH_{K^s} & p_2G_{zz}(H_{K^s})^2 + p_2G_zH_{K^sK^s} \end{bmatrix}.$$

Therefore,

$$\frac{dL_2}{dp_2} = \frac{p_2^2G_y(F_LF_{KK} - F_KF_{LK})\left((G_zG_{yz} - G_yG_{zz})(H_{K^s})^2 - G_yG_zH_{K^sK^s}\right)}{|\Omega_2|} > 0. \quad \text{(III.14)}$$

Q.E.D.

Proof of Proposition 4.3

First, we determine the impact of K_2^s on λ_2^f. To do so we combine (4.3) and (4.6):

$$\lambda_2^f = r^2\frac{H_Q}{H_{K^s}} - r_2^cg_Q. \quad \text{(III.15)}$$

The positive impact of K_2^s on the value of water for the firm is shown by derivating equation (III.15) with respect of K_2^s:

$$\frac{d\lambda_2^f}{dK_2^s} = \frac{r^s \left(H_{QK^s} H_{K^s} - H_{K^s K^s} H_Q\right)}{\left(H_{K^s}\right)^2} > 0. \tag{III.16}$$

The impact of r on λ_2^f can be written as the combination of $(III.6)$ and $(III.16)$:

$$\begin{aligned}\frac{d\lambda_2^f}{dr} =& \frac{d\lambda_2^f}{dK_2^s}\frac{dK_2^s}{dr}\\ =& \frac{p_2^2 r^s G_{yz} G_y \left(F_L F_{LK} - F_K F_{LL}\right)\left(H_{QK^s} H_{K^s} - H_Q H_{K^s K^s}\right)}{H_{K^s}|\Omega_2|}\\ =& \frac{p_2^3 G_y G_z G_{yz} \left(F_L F_{LK} - F_K F_{LL}\right)\left(H_{QK^s} H_{K^s} - H_Q H_{K^s K^s}\right)}{|\Omega_2|} < 0.\end{aligned} \tag{III.17}$$

Next, the impact of w_2 on λ_2^f can be written as the combination of $(III.7)$ and $(III.16)$:

$$\begin{aligned}\frac{d\lambda_2^f}{dw_2} =& \frac{d\lambda_2^f}{dK_2^s}\frac{dK_2^s}{dw_2}\\ =& \frac{p_2^2 r^s G_y G_{yz} \left(F_K F_{LK} - F_L F_{KK}\right)\left(H_{QK^s} H_{K^s} - H_Q H_{K^s K^s}\right)}{H_{K^s}|\Omega_2|}\\ =& \frac{p_2^3 G_y G_z G_{yz} \left(F_K F_{LK} - F_L F_{KK}\right)\left(H_{QK^s} H_{K^s} - H_Q H_{K^s K^s}\right)}{|\Omega_2|} < 0.\end{aligned} \tag{III.18}$$

The effect of r^s on λ_2^f can be written as the combination of $(III.8)$ and $(III.16)$:

$$\begin{aligned}\frac{d\lambda_2^f}{dr^s} =& \frac{d\lambda_2^f}{dK_2^s}\frac{dK_2^s}{dr^s}\\ =& \frac{r^s \left(H_{QK^s} H_{K^s} - H_Q H_{K^s K^s}\right)}{\left(H_{K^s}\right)^2}\frac{|\Delta_{LK}|}{|\Omega_2|} < 0.\end{aligned} \tag{III.19}$$

The impact of r_2^c on the marginal value of water in period 2 can be directly calculate from (4.3), that is:

$$\frac{d\lambda_2^f}{dr_2^c} = -g_Q < 0. \tag{III.20}$$

The impact of p_2 on λ_2^f can be written as the combination of (III.9) and (III.16):

$$\begin{aligned}\frac{d\lambda_2^f}{dp_2} =& \frac{d\lambda_2^f}{dK_2^s}\frac{dK_2^s}{dp_2} \\ =& \frac{r^s\left(H_{QK^s}H_{K^s} - H_Q H_{K^sK^s}\right)\left(\left(G_y\right)^2 G_{yz} H_{K^s}\Sigma_F - G_z H_{K^s}|\Delta_{LK}|\right)}{\left(H_{K^s}\right)^2|\Omega_2|} > 0. \qquad \text{(III.21)}\end{aligned}$$

Q.E.D.

Proof of Proposition 4.4

We first prove that the impact of A_1^f on λ_1^f is negative. The combination of (4.3) and (4.4) implies that:

$$\begin{aligned}\frac{d\lambda_1^f}{\partial A_1^f} =& p_2 G_{zz}(H_Q)^2 + p_2 G_z H_{QQ} - r_1^c g_{QQ} - \frac{(p_2 G_{yz} F_L H_Q)^2}{p_2 G_{yy}(F_L)^2 + p_2 G_y F_{LL}} \\ =& \frac{p_2}{G_{yy}(F_L)^2 + G_y F_{LL}}\left[G_{yy}G_{zz}(F_L H_Q)^2 + G_y G_{zz}(H_Q)^2 F_{LL}\right. \\ & \left. + G_z G_{yy}(F_L)^2 H_{QQ} + G_y G_z F_{LL} H_{QQ} - (G_{yz}F_L H_Q)^2\right] - r_1^c g_{QQ} \\ =& \frac{p_2}{G_{yy}(F_L)^2 + G_y F_{LL}}\left[(G_{yy}G_{zz} - G_{yz})(F_L H_Q)^2 + G_y G_{zz}(H_Q)^2 F_{LL}\right. \\ & \left. + G_z G_{yy}(F_L)^2 H_{QQ} + G_y G_z F_{LL} H_{QQ}\right] - r_1^c g_{QQ} < 0.\end{aligned}$$

Before proceeding with the analysis of the impact of A_2^f on λ_2^f, it is worth mentioning that the concavity of H implies that:

$$\Sigma_H = H_{QQ}(H_{K^s})^2 + H_{K^sK^s}(H_Q)^2 - 2H_Q H_{K^s} H_{QK^s} < 0.$$

To determine $d\lambda_2^f/dA_2^f$, we calculate the total derivative of (III.15):

$$\begin{aligned}
\frac{d\lambda_2^f}{dA_2^f} =& r_s \frac{H_{QQ}H_{K^s} - H_{QK^s}H_Q}{(H_{K^s})^2} - r_2^c g_{QQ} + \frac{d\lambda_2^f}{dK_2^s}\frac{dK_2^s}{dA_2^f} \\
=& \frac{r^s}{(H_{K^s})^2}\left(\frac{H_{QQ}(H_{K^s})^2 + H_{K^sK^s}(H_Q)^2 - 2H_QH_{K^s}H_{QK^s}}{H_{K^s}}\right. \\
& \left. - \frac{p_2G_z|\Delta_{LK}|\left(H_{QK^s}H_{K^s} - H_QH_{K^sK^s}\right)^2}{H_{K^s}|\Omega_2|}\right) - r_2^c g_{QQ} \\
=& \frac{r^s}{(H_{K^s})^3|\Omega_2|}\left[p_2G_z|\Delta_{LK}|\left(\Sigma_H H_{K^sK^s} - (H_{QK^s}H_{K^s} - H_QH_{K^sK^s})^2\right)\right. \\
& + \Sigma_H p_2^3\left(G_{zz}(H_{K^s})^2(G_y)^2\left(F_{LL}F_{KK} - (F_{LK})^2\right)\right. \\
& \left.\left. + G_y(H_{K^s})^2\left(G_{zz}G_{yy} - (G_{yz})^2\right)\Sigma_F\right)\right] - r_2^c g_{QQ} \\
=& \frac{r^s}{(H_{K^s})^3|\Omega_2|}\left[p_2G_z|\Delta_{LK}|(H_{K^s})^2(H_{QQ}H_{K^sK^s} - (H_{QK^s})^2)\right. \\
& + \Sigma_H p_2^3\left((G_y)^2G_{zz}(H_{K^s})^2\left(F_{LL}F_{KK} - (F_{LK})^2\right)\right. \\
& \left.\left. + G_y(H_{K^s})^2\left(G_{zz}G_{yy} - (G_{yz})^2\right)\Sigma_F\right)\right] - r_2^c g_{QQ} < 0.
\end{aligned}$$

Consequently, the marginal value of water for firm in period t is decreasing in A_t^f. *Q.E.D.*