Université de Montréal

Trois Essais dans l'Analyse des Fluctuations Economiques

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Summary

The analysis and characterization of economic fluctuations constitutes one of the fundemental areas of research in macroeconomics. This thesis consists of three essays on the analysis of economic fluctuations. The main objective of this dissertation is to propose new ways of analyzing the persistence and cyclical properties of fluctuations in economic variables. The essays aim to improve our understanding on the nature of fluctuations by providing new desciptive techniques for the measurement and interpretation of economic fluctuations. In the first two essays, we propose a new approach that measures the persistence of economic fluctuations. The third essay relates to the analysis of periodicities in turning point chronologies.

In the first essay, we propose an alternative approach to the assesment of persistence in economic fluctuations. After criticizing the traditional approach which is based on looking at the effect of shocks on forecasts at different horizons, we propose instead to look at return (or crossing) probabilities as a way to assess the extent to which fluctuations should be taken as "permanent". Similar to the ruin problem of a gambler, the current fluctuation is viewed as an initial capital and we then measure the probability that the fluctuation not be cancelled by future fluctuations. We propose estimators of non-return (or non-crossing) probabilities and study their asymptotic distributions. Proposed estimators remain invariant with respect to monotonically increasing trasformations of data. The examination of quarterly U.S. postwar data shows that fluctuations in real GNP have probability 4/5 to persist at least one quarter and probability 1/2 to persist 10 years. There is no qualitative difference between "trend stationary" and "integrated processes": return probabilities simply tend to be somewhat larger for the second class of processes.

In the second essay, we extend our approach to the measurement of economic fluctuation persistence to the case where the observations are unequally spaced with random occurence dates. The basic difficulty in evaluating the persistence of irregular observations is the fact that the dates of the observations are random. We propose a notion of persistence (non-return) probability at different horizons which

is applicable in that case and define natural estimators for these probabilities. The proposed measure of persistence preserves invariance properties. Furthermore, it does not require arbitrary data discretization procedures which induce spurious persistence at seasonal frequencies. As an application, we consider the real time bid and ask quotes on USD/DM, USD/JY and JY/DM exchange rates in worldwide foreign exchange rates markets (Olsen & Associates, HFDF dataset). Speculators acting in these markets are viewed as gamblers who take decisions to buy or sell currencies by observing continuously (bid/ask) quotes. In these data, we find that fluctuations in exchange rates have less than 1/2 probability to survive more than five minutes. After 90 working days, these probabilities do not tend to zero which support the hypothesis that the market does have long memory.

In the third essay, we suggest applying spectral methods on business cycles characterized by turning point chronologies. Cyclical chronologies are viewed as a realization of a random variable over a discrete space, usually two states, resulting in an alternating sequence of expansions and recessions like the one produced by the NBER, which covers a sample of monthly observations starting in 1854. Applying spectral methods to such discrete processes provides an easy tool to assess the similarities and differences between alternative reference chronologies. Indeed, a formal comparison via coherence can inform us how the two chronologies are related. Moreover, using the algorithm proposed by Bry and Boschan, we can date peaks and troughs in a set of individual time series like the prices or monetary aggregates, allowing us to study the comovements between the process identified by the NBER chrononlogy and the turning point process associated with any individual series. Such analysis allows us to describe the association of cycles between different series measuring real activity and monetary aggregates in a very novel perspective. The spectral density estimates of business cycle chronologies show multiple pics within the conventional business cycle band. Furthermore, the Walsh-Fourier spectrum presents additional pics. The presence of multiple pics is then attributed to the caracteristics of cycles before and after World War II.

Résumé

L'analyse et la caractérisation des fluctuations économiques constituent des thèmes fondamentaux de la macro-économie. Les deux premiers essais de cette thèse abordent le problème de *persistance* des fluctuations économiques et le troisième essai concerne la *periodicité* des points de retournement des variables macro-économiques. Les définitions préliminaires de ces deux aspects des fluctuations économiques seront éclairantes pour encadrer le contenu des esssais et préciser l'envergure des contributions. Premièrement, une fluctuation économique est persistante si l'effet du changement ne disparait pas dans l'horizon de la prévision. Inversement, elle est non-persistante si l'effet du changement disparait dans un horizon proche. L'analyse de la persistance a une importance primordiale dans la compréhension de la nature des mouvements économiques ainsi que dans l'évaluation des politiques économiques. Deuxièmement, les théories de fluctuations économiques suggèrent que les économies de marché traversent des périodes d'expansions et de récessions récurrentes et non-périodiques. Autrement dit, les fluctuations d'une période d'expansion n'ont pas le même degré de persistance que celles qui se produisent au cours d'une récession. Un processus stochastique est périodique s'il existe une corrélation importante entre les valeurs de différents délais. L'analyse de la périodicité des points de retournement soulève l'importance des méchanismes d'impulsion-propagation et la stabilité des politiques macro-économiques.

Les contributions de cette thèse consistent en deux nouvelles approches. Dans les deux premiers essais nous proposons une approche *probabiliste* afin de mesurer le degré de persistance des fluctuations économiques : le premier de ces essais aborde le cas des processus en temps discret et le second le cas des processus en temps réel c.à.d. observations avec des dates irrégulières. Dans le troisième essai, nous proposons une approche *séquentielle* afin d'analyser la périodicité des points de retournement. Chacun de ces essais est constitué d'une section empirique qui utilisent des bases de données populaires et intéressantes. Les résultats empiriques obtenus sont riches et concluants et permettent des nouvelles interprétations des fluctuations

macro-économiques et financières.

Dans le premier chapitre, nous proposons une approche *probabiliste* afin de mesurer la persistance des fluctuations. Cette appoche amène deux contributions originales dans l'analyse de la persistance : l'un au niveau de la conceptualisation et l'autre au niveau de la mesure. La persistance (non-persistance) d'une fluctuation est considérée comme un phénomène de non-annulation (annulation) d'un changement à la hausse ou à la baisse du processus. Une fluctuation est donc persistante (non-persistante) si le processus ne retourne pas (retourne) à sa position initiale dans le future. Etant donné que les mouvements futures du processus ne sont pas observés, il sera préférable d'avoir une mesure probabiliste d'annulation (non-annulation) d'une fluctuation. La persistance (non-persistance) est mesurée en estimant la probabilité que la fluctuation n'ait pas été annulée (ait été annulée) par des fluctuations futures.

La majorité des débats concernant la persistance se préoccupent de savoir de combien le choc dans l'output agrégé va affecter les prévisions optimales dans un horizon infini (Nelson et Plosser, 1982). Si la réponse à cette question est zéro, les agrégats macro-économiques seront mieux représentés par des fluctuations autour d'une tendance déterministe. Néanmoins, si la réponse est non-nulle, il sera préférable de les représenter par une marche aléatoire additionnée d'une composante stationnaire. Par example, Campbell et Mankiw (1987) concluent qu'une innovation d'un pourcent dans le PNB réel change les prévisions de ce dernier de plus d'un pourcent. D'autre part, Clark (1987) et Watson (1986) conlcuent qu'un pourcentage du choc engendre moins d'un pourcentage de changement dans l'output. Les résultats empiriques qui découlent de ces études et plusieurs d'autres sont ambigus et il est fort probable que l'activité économique soit représentée par une classe de processus qui soit différente de celles que l'on a considérées jusqu'à maintenant, Christiano and Eichenbaum (1990). En particulier, la composante cyclique semble plus faible que la composante de marche aléatoire. En conséquence les fluctuations économiques ressemblent aux jeux de hasard (Fisher, 1925). Cette proposition a une importance capitale en ce qui concerne les prévisions et évaluations des politiques économiques. En effet les changements permanents d'aujourd'hui auront alors des effets amplifiés dans un futur lointain tandis que les changements temporaires auront des effets qui disparaitront graduellement dans un futur proche. Cette façon d'évaluer la permanence des chocs est équivalente à une simulation incompatible avec la structure du processus dans laquelle les valeurs de tous les chocs futurs sont fixées à zéro. Il est clair que si l'économie subit un changement structurel, il n'y aura aucune raison de croire que le modèle retenu expliquera pertinemment le processus de génération des chocs (Lucas, 1973).

L'approche probabiliste change la formulation de la quesiton traditionnelle. la question consiste à connaitre la probabilité de persistance d'une fluctuation durant l'horizon de prévision. Cette question s'inspire du *problème de la ruine* de la théorie classique des marches aléatoires. Similaire au problème de la ruine, les fluctuations économiques peuvent être vues comme les gains et les pertes d'un joueur de hasard qui débute le jeu avec un capital pouvant possiblement être négatif. La perte de cette richesse se termine avec la ruine du joueur. Cependant, les acquisitions de capital tout au long du jeu peuvent retarder ou accélérer la durée du jeu. Il s'agit donc d'un concept dynamique des fluctuations économiques et de leurs mécanismes d'auto-propagation : les fluctuations peuvent avoir des effets transitoires si les fluctuations futures réduisent suffisamment l'effet de la première impulsion ou bien elles peuvent avoir des effets permanents si le flux des fluctuations futures prolongent l'impact de l'impulsion initiale. Dans la suite, nous proposons des estimateurs de ces probabilités et étudions leurs distributions asymptotiques.

Les résultats empiriques concernant l'économie des E.U. d'après la Seconde Guerre Mondiale suggèrent qu'une fluctuation dans le PNB réel a une chance de plus de 4/5 de persister durant un trimestre et une chance de plus d'1/2 de persister durant dix ans. Les fluctuations autour de la tendance séculaire et autour de la moyenne ont des probabilités de persistance statistiquement similaires : ces probabilités ne dépassent pas 1/4 sur un horizon de dix ans. Nous fournissons des probabilités de persistance conditionnelles aux périodes d'expansions et de récessions. Nous examinons la persistance des fluctuations de plusieurs aggrégats macro-économiques incluant la consommation, l'investissement, les prix, l'inflation, le taux d'intérêt, l'emploi, le taux de chômage et les indicateurs financiers. Contrairement aux mesures de persistance traditionnelles, comme celle du rapport des variances proposée par Cochrane (1986), nos résultats révèlent différents degrés de persistance.

Le deuxième chapitre est une extension du premier essai. Nous proposons des mesures de persistance pour des observations qui ne sont pas équidistantes dans le temps. Cette situation est très courante pour des séries macro-économiques et en particulier pour des séries financières. La difficulté dans l'évaluation de la persistance des fluctuations non-équidistantes est que, pour un horizon de la prévision fixe le nombre de fluctuations futures est lui-même un processus aléatoire. Dans ce cas-ci, les estimateurs du premier chapitre ne sont plus applicables. Nous surmontons cette difficulté en introduisant des mesures de persistance qui ne dépendent pas de la durée des mouvements futures du processus. Nous proposons des estimateurs de ces probabilités et étudions leurs propriétés. Cette façon d'évaluer la persistance des fluctuations en temps réel soulève deux avantages qui valent la peine d'être mentionnés. D'abord, comme dans le cas des mesures de persistance en temps discret, les estimateurs demeurent invariants par rapport aux transformations monotoniquement croissantes du processus. Deuxièmement, par construction, ils ne nécéssitent aucune procédure de discrétisation des observations qui induisent des persistances fausses dans des fréquences saisonnières, Wasserfalen et Zimmerman (1985).

Les resultats empiriques de ce chapitre concernent le marché mondial des taux de change. Une des caractéristiques importante de ce marché est que les négociations se font sur un réseau télé-électronique ouvert 24h/24h à travers le monde. Les partipants du marché, les grandes banques internationales proposent des prix qui apparaissent instantanément sur les écrans électroniques. Les instants de communication (avec une précision de 1/60 de la minute) et les propositions de prix forment une information pour tous les participants potentiels du marché. Similaire au problème de la ruine mentionné ci-haut, les spéculateurs de ces marchés sont considérés comme des joueurs de hasard qui prennent des décisions d'achats ou de ventes des monnaies étrangères. La question d'intérêt est de savoir la probabilité de persistance d'une fluctuation dans le taux de change pour différents horizons de prévision. Nous considérons trois taux de change: dollar americain versus mark allemand, dollar versus yen japonais et mark versus yen pour la période allant du 1er octobre 1992 au 31 septembre 1993. Les resultats empiriques suggèrent qu'une fluctuation dans le taux de change dollar/mark a 1/10 chance de persister dans les cinq premiers minutes suivant la fluctuation. La probabilité de persistance atteint les 1/20 et 1/40 respectivement pour les taux de changes dollar/yen et mark/yen. Après les 90 journées du marché, ces probabilités ne s'annullent pas qui confirment l'hypothèse de la mémoire longue des taux de changes.

Dans le troisième chapitre, nous proposons d'analyser les fluctuations économiques à travers le spectre de la chronologie des points de retournement des séries temporelles. Notre objectif est de fournir une méthode qui sera capable d'identifier la périodicité des points de retournements. Un spectre d'un processus spécifie la contribution de chaque fréquence à la variance totale. La représentation spectrale des variables macro-économiques correctement désaisonnalisées a une pente décroissante aussi longtemps que la fréquence augmente (Granger, 1966). Un pic dans des basses fréquences indique une persistance à longue-terme alors qu'un pic dans la tranche des fréquences des cycles économiques correspondant aux périodes d'un jusqu'à dix ans (Burns et Mitchell, 1946) indique un comportement cyclique de l'activité économique. La majorité des variables macro-économiques ont des composantes cycliques, Sargent et Sims (1977). Dans ce troisième essai, les dates de retournement d'un cycle, c.à.d. les périodes d'expansions et de récessions sont considérées comme des variables connues, soit par exemple à partir des annonces publiques du "National Bureau of Economic Research" ou soit à partir d'un algorithme de datation des points de retournement, Bry et Boschan (1971). Nous construisons des variables binaires qui dépendent de l'état cyclique de l'économie : expansions ou récessions. Etant donné que chaque cycle a un sommet et un creux avec une distance d'au moins six mois de durée, ce processus binaire représente des séquences de "+1" et de "-1" - similaire aux circuits *on/off* - d'au moins six mois consécutifs avec des croisements de zéro qui corréspondent exactement aux dates des points de retournement prédéterminés. Ce processus localement linéaire et sans tendance fait l'objet du troisième essai.

Les densités spectrales des processus filtrés aux points de retournement sont estimées en utilisant les transformations de Walsh basées sur les fonctions qui sont des oscillations rectangulaires similaires aux fonctions sinusoidales utilisées pour des transformations de Fourier. Nous présentons les estimations spectrales de deux types de transformations, Walsh et Fourier comme deux méthodes complémentaires plutôt que compétitives. Toutefois, les transformations Walsh s'avèrent adéquates pour des processus discrets ou qualitatifs, Beauchamp (1984) et Stoffer (1991).

Les spectres des chronologies indiquent des pics multiples localisés dans une bande de fréquence plus large que celle suggérée par la théorie conventionnelle. A fortiori, les spectres de Walsh-Fourier présentent des pics additionnels. La présence de pics multiples est attribuée aux caractéristiques des cycles des périodes précédant et suivant la Seconde Guerre et aux mécanismes d'impulsion-propagation intrinsèques à chaque période. Nous évaluons les similarités et les différences entre les chronologies alternatives. Ensuite nous mesurons les mouvements communs entre les points de retournement déterminés par un algorithme formel et la chronologie proposée par les comités de sélection du "National Bureau".

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Chapter 1

Persistence Measures Based on Return Probabilities With Macroeconomic Applications

(in collaboration with Jean-Marie Dufour)

1.1 Introduction

Since the earlier 1980's, the authors have been opposed to classical business cycle theories which suggest that a given fluctuation in aggregate output will necessarily be reversed through trend. Instead, they have argued that the fluctuations are persistent and there is no sense in which recessions or expansions are temporary. The persistence of shocks in a time series is typically measured by looking at the effect of these shocks on the forecast of a given time series at long horizons. More precisely, the effect of a shock is deemed to be "permanent" if changes in forecasts associated with a shock do not tend to vanish as the forecast horizon increases. Therefore, the forecasts of all future values are modified by an amount which does not go to zero as the forecast horizon goes to infinity. It is easy to see that the same property holds for general integrated processes, see Beveridge and Nelson (1981).

This way of assessing the permanence of shocks is equivalent to a counter-factural simulation where the values of all future shocks are set to zero. Obviously, if the distribution of future shocks suddenly became degenerate at zero, this would represent a major structural change and there would be no reason to expect that the model considered be relevant (Lucas critique)[1]. In this paper, we propose a different "more realistic" way of assessing the permanent character of fluctuations or (shocks), which is based on the idea of return (or crossing) probabilities. More precisely, the persistence (non-persistence) of a fluctuation is considered as the non-annulation (annulation) of an upward or downward change in economic activity. We then measure the persistence (non-persistence) of a fluctuation by estimating the probability that it has not been cancelled (has been cancelled) by future fluctuations. The motivation behind this idea goes back to the classical ruin problem of a gambler in a random walk theory. The present fluctuation is viewed as the initial (possible negative) capital of an economy. We introduce dynamic in the time evolution of shocks by asking whether this initial capital (fluctuation) is or is not ruined by the future capital flows (fluctuations). More precisely, we propose to measure the permanence of shocks by computing the probability that a given fluctuation in a time series not be cancelled by future fluctuations.

The basic advantage of proposed estimators is the invariance with respect to monotonically increasing transformation of data. A well-known example of such transformation is the natural logarithm of GNP. Since the estimators of persistence probabilities are based upon the sign transformation of fluctuations, taking the logarithm of observations does not affect the value of estimated probabilities. Other persistence measures, for instance Cochrane's (1988) variance-ratio procedure and Campbell and Mankiw's (1987) impulse-response analysis which is based on parametric shock generating functions are definitely affected by increasing transforma-

[1]The critique is based on the following simple syllogism: "given that the structure of econometric model consists of optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models".

tions. Furthermore, unlike the impulse-response analysis, the return probabilities do not require model building and selecting procedures prior to the persistence analysis.

The results in this paper are closely related on the theory of random walks for integers, which use the frequency of level (or zero) crossings to test for a unit roots in a given time series. For example, Feller (1971), Granger and Hallman (1991) and Feuerverger et al. (1994) exploit the integer properties of a random walks by making use of a number of level (zero) crossings. It is also akin to various sign-based statistical methods in econometrics and time series. For further work and references in this area, the reader may consult Dufour (1981), Dufour, Lepage and Zeidan (1982), Dufour and Hallin (1991), and Campbell and Dufour (1991, 1995, 1997).

The persistence of fluctuation in macroeconomic aggregates have been subject of large literature. Majority of debates concerning the persistence aim to measure effect of a shock on the forecasting horizon. If a shock does not affect optimal forecasts of processes it was concluded that macroeconomic aggregates are well represented by fluctuations around deterministic time trend, otherwise they are represented by a random walk plus a stationary composant, Nelson and Plosser (1982). For instance, Campbell and Mankiw (1987) conclude that one percent innovation to GNP changes the forecast more than a percent. Clark (1987) and Watson (1986) conclude that a percent shock generates less than percent change in output. Testing for the presence of random walk Phillips (1987), Phillips and Perron (1988), Perron (1988, 1989a, 1989b) are stimulated numerous works, Campbell and Perron (1991), Delong et.al.(1992) among many others. Christiano and Eichenbaum (1991) argue that most of debate related to the persistence of fluctuations are inconclusive and it is possible that macroeconomic variables must be represented by another class of process never been considered.

The examination of quarterly U.S. postwar data shows that fluctuations in real GNP have probability 4/5 chance to persist at least one quarter and probability 1/2 to persist ten years. The fluctuations around the deterministic trend and demeaned first differences have statistically similar persistence probabilities. For the latters, the probability that a fluctuation would persist after ten years is less than a 1/4. The results concerning fluctuations in major macroeconomic aggregates, such as the consumption, investment, employment, inflation, interest rates, money and financial indicators and many others show different persistence when compared with the measure based variance-ratio proposed by Cochrane (1988). Confirming the asymmetry hypothesis of business cycles Neftçi (1984), fluctuations occuring during the periods of expansion are more persistent than those occuring during the recessions.

The Chapter is organized as follows. In section 1.2, we give a brief overview of the different approaches taken to analyze the persistence of macroeconomic fluctuations. The aim of section 1.3 is to provide motivation and prepare the ground for our persistence measures based on return probabilities. Section 1.4 outlines the persistence measures based on return probabilities. Section 1.5 is devoted to the asymptotic properties of our estimators. Section 1.6 presents the empirical results. Our conclusions are presented in Section 1.7.

1.2 Persistence Measures in Univariate Models

A measurement of the permanence of economic fluctuations is crucial to both the theoretical and practical point of view in macroeconomics. The highly persistent shocks to the economy imply that a substantially large part of a given shock would persist through time. This conflicts with traditional standings of both Keynesian and classical macroeconomic theories, where output fluctuations are temporary deviations from a slowly growing natural or equilibrium level of output. On the other hand, for policymakers, the implications of strong persistence in output fluctuations would call into questions the appropriateness of counter-cyclical policies. If the cyclical component is no matter and there is no steady trend to which the economy returns, attempts that the counter-cyclical policies are at best misguided. A fortiori, if the fluctuations are largely permanent, the costs and benefits of policy making are different than when the fluctuations are transitory.

The dichotomy between trend and cycle has played an important role in the analysis of macroeconomic fluctuations. According to the traditional view, fluctuations in output represent temporary deviations from trend. The traditional model can be written as

$$y_t = \gamma t + a(L)\epsilon_t \tag{1.2.1}$$

where y_t stands for log GNP, $a(L)$ is an infinite polynomial in the lag operator L such that $a(L)\epsilon_t$ is a covariance stationary process, γt is a deterministic trend and ϵ_t is the zero mean, serially uncorrelated innovation of y_t. We denote the variance of ϵ_t by σ_ϵ^2. Since y_t is a covariance stationary Σa_j^2 and σ_ϵ^2 are both finite. For convenience, let us assume that $\Sigma|a_j| < \infty$. As a result, an unexpected change in output today should not substantially change one's forecast of output in the future.

The reexamination of this traditional point of view was motivated in part by developments in the econometrics of nonstationary time series (Dickey and Fuller 1979) and the applications of new econometric techniques to macroeconomic data (Nelson and Plosser 1982). Specifically, Nelson and Plosser (1982) argue that output fluctuations modelled as a deviations around deterministic trend are misspecified and find that the long run character of output is well described as a stochastic trend or a random walk (typically with drift). Consider the first difference of log GNP

$$\Delta y_t = \mu + b(L)u_t \tag{1.2.2}$$

where $b(L)$ is an infinite polynomial in the lag operator L, μ is a drift term, $\Sigma|b_j| < \infty$ and u_t is a zero mean, serially uncorrelated innovation of y_t. Denote the variance of u_t by σ_u^2. In addition we must impose the requirement that $b(1) \neq 0$, that is an unit root in the polynomial $b(L)$. This suggest that a natural measure of the size of the random walk, or the unit root component is to the sum of the coefficients of present and past innovations u_t.

There are basically two interpretations related to $b(1)$ in determining the degree of persistence in y_t. According to the first interpretation, a measure of persistence centers on the response to u_t of the optimal forecast of y_t into the infinite horizon.

Let E_t denotes the time t expectations operator conditional on the information set containing the present and past innovations $\{u_t, u_{t-1}, u_{t-2}, ...\}$. Beveridge and Nelson (1981) show that

$$\lim_{h\to\infty}[E_t y_{t+h} - E_{t-1} y_{t+h}] = b(1)u_t. \tag{1.2.3}$$

This means that $b(1)$ completely characterizes the revisions in the long run forecast for y_t induced by the present innovation on y_t. If y_t is difference stationary, then $b(1) \neq 0$, so that u_t affects the forecast of the level y_t into the infinite future. On the other hand, if y_t can be represented as (1.2.1), an innovation to y_t should have no impact on our forecast about the level y_t into the infinite future. This interpretation rises to the question of how much should an innovation to y_t at time t, affects our predictions into infinite horizon.

The other common measure of persistence revolves around the fact that the long-run forecast of a difference stationary process is always changing over the time. According to Beverdige-Nelson decomposition (1.2.3), the time revision to the long-run forecast of y_t is the random variable $b(1)u_t$. A natural measure of the amount of variation in this random variable is its variance $[b(1)]^2\,\sigma_u^2$. If y_t is trend stationary, then fluctuations in u_t induce only transitory movements in y_t; that is the long-run forecast is deterministic. Consequently, the variance of the revision to the long-run forecast of a trend stationary random variable $[\overline{a}(L)]^2\sigma_\epsilon^2$ is zero.

In order to review the literature on the measures of persistence, let us suppose that y_t contains a unit root, so that it admits the representation

$$\begin{aligned}\Delta y_t &= \alpha + \psi(L)\eta_t, \\ &= \alpha + \sum_{j=0}^{\infty}\psi_j\eta_{t-j}\end{aligned} \tag{1.2.4}$$

where η_t is the white noise innovation to y_t, with variance σ_η^2. The impact of a shock in period t, η_t, on the change in y_{t+h}, that is Δy_{t+h} is ψ_{t+h}. The impact of the shock on the level of y_{t+h} is therefore $1 + \psi_1 + \cdots + \psi_h$. The ultimate impact of the shock on the level is the infinite sum of these moving average coefficients, defined as $\psi(1)$. The value $\psi(L)$ can then be taken as a measure of how persistent shocks to y are. When $\psi(L) = 0$, (1.2.4) is a trend stationary, since $\psi(L)$ must contain a factor $(1 - L)$, whereas $\psi(1) = 1$ for a random walk , since $\psi_j = 0$ for $j > 0$. Other positive values of $\psi(1)$ are possible for more general processes.

Difficulties arise in estimating $\psi(1)$ because it is an infinite sum of moving average coefficients requiring the estimation of an infinite number of coefficients. Various measures have thus been proposed in the literature to circumvent this problem. Two of the most popular being the *impulse-response* measure proposed by Campbell and Mankiw (1987) and the *variance ratio* measure proposed by Cochrane (1988).

Campbell and Mankiw (1987) present a measure of $\psi(1)$ based on approximating $\psi(L)$ by a ratio of finite order polynomials

$$\psi(L) = \frac{\theta(L)}{\phi(L)} \tag{1.2.5}$$

where $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator of order p and q respectively. Equation 1.2.4 is then interperted as the moving average representation or impulse-response function of Δy_t. To test the hypothesis that y_t is trend stationary, Campbell and Mankiw obtain both an unconstrained estimate of $\psi(L)$ and an estimate of $\psi(L)$ subject to the constraint $\psi(1) = 0$. From the equality (1.2.5), the measure $\psi(1)$ can then be calculated directly as $\psi(1) = \phi(1)^{-1}\theta(1)$.

Unlike the parametric structure of impulse-response analysis, Cochrane (1988) propose an alternative persistence measure based on the ratio of variances, defined as

$$V^h \equiv \frac{\sigma_h^2}{\sigma_1^2} = \frac{h^{-1}\text{Var}(y_t - y_{t-h})}{\text{Var}(y_t - y_{t-1})}. \tag{1.2.6}$$

This measure is based on the following argument. If y_t is a pure random walk, then the variance of its h-th differences will grow linearly with h:

$$\begin{aligned}\text{Var}(y_t - y_{t-h}) &= \text{Var}(y_t - y_{t-1}) + \cdots + \text{Var}(y_{t-h+1} - y_{t-h}), \\ &= \sum_{j=1}^{h} \text{Var}(y_{t-j+1} - y_{t-j}) = h\sigma_\eta^2,\end{aligned}$$

hence the ratio in (1.2.6) equals to unity. If, on the other hand, y_t is trend stationary, the variance of its h-th differences approaches a constant, this being twice the unconditional variance of the series

$$\begin{aligned}\mathrm{Var}(y_t - y_{t-h}) &= \mathrm{Var}\left(h\alpha + \sum_{j=0}^{\infty}\psi_j\eta_{t-j} - \sum_{j=0}^{\infty}\psi_j\eta_{t-j-h}\right), \\ &= \mathrm{Var}\left(\sum_{j=0}^{h-1}\psi_j\eta_{t-j} + \sum_{j=0}^{\infty}(\psi_{h+j}-\psi_j)\eta_{t-j-h}\right), \\ &= \sigma_\eta^2\sum_{j=0}^{h-1}\psi_j + \sigma_\eta^2\sum_{j=0}^{\infty}(\psi_{h+j}-\psi_j)\end{aligned}$$

which, as $h \to \infty$, tends to $2\sigma_\eta^2\sum_{j=0}^{\infty}\psi_j = 2\mathrm{Var}(y_t)$, hence (1.2.6) approaches zero. Cochrane (1988) suggests plotting a sample estimate of σ_h^2 as a function of h. In providing sample estimate of σ_h^2, Cochrane (1988, A3, p. 917) corrects for two possible small sample bias. First, the sample mean of first differences $y_{t+1} - y_t$ is used to estimate the drift term for all h differences. Second, a degrees of freedom correction $T/(T-h-1)$ is included. Without this $\mathrm{Var}\{y_{t+h+1} - y_t\}$ will decline towards zero as $h \to T$ because a variance cannot be taken with one observation. These corrections produce an estimator of $\mathrm{Var}\{y_{t+h+1} - y_t\}$ that is unbiased when applied to a pure random walk with drift. The actual formula used to compute the estimator from the sample $\{y_t\}_0^T$ is,

$$\widehat{\sigma_h^2} = \frac{T}{h(T-h)(T-h-1)}\sum_{j=h}^{T}[y_j - y_{j-h} - \frac{h}{T}(y_T - y_0)]^2. \quad (1.2.7)$$

From Cochrane (1988), the asymptotic standard error of $\widehat{\sigma_h^2}$ is $\sqrt{(4h/3T)}\widehat{\sigma_h^2}$. The variance ratio can then be estimated as $\widehat{V^h} = \widehat{\sigma_h^2}/\widehat{\sigma_1^2}$. If y_t is a random walk, the plot should be constant at σ^2, whereas if y_t is trend stationary the plot should decline towards zero. If fluctuations in y_t are partly permanent and partly temporary, so that the series can be modelled as a combination of random walk and stationary components, the plot of σ_h^2 versus h should settle down to the variance of the innovation to the random walk component.

Cochrane shows that the variance ratio (1.2.6) can also be written as

$$V^h = 1 + 2\sum_{j=1}^{h}(1 - \frac{j}{h+1})\rho_j \quad (1.2.8)$$

where ρ_j is the jth autocorrelation of first differences. To motivate the usefulness of V^h as a measure of persistence, Cochrane considers the limit of this variance ratio when h tends to infinity

$$V \equiv \lim_{h \to \infty} V^h = \frac{S_{\Delta y}(1)}{\sigma^2_{\Delta y}} \tag{1.2.9}$$

where $S_{\Delta y}(1)$ is the spectral density of first differences evaluated at frequency zero. Allowing $z = e^{i\omega}$ for $\omega \in [0, 2\pi]$ then $S_{\Delta y}(z) = \psi(z)\psi(z^{-1})\sigma^2_\eta$, consequently

$$V = |\psi(1)|^2 \frac{\sigma^2_\eta}{\sigma^2_{\Delta y}}. \tag{1.2.10}$$

Combining these results, we see that the trend stationarity $\psi(1) = 0$ and difference stationary $\psi(1) \neq 0$ time series can be distinguished by looking to the spectral density of Δy_t at frequency zero $(z = 1)$ is zero or not respectively.

The crucial identifying assumptions underlying this measure of persistence is the assumption that whatever value of h is choosen, the higher autocorrelations are of negligible importance. Campbell and Mankiw (1987a,b) show that the measures based on impulse-response functions and variance-ratio are related by relationship

$$\psi(1) = \sqrt{V/(1 - R^2)} \tag{1.2.11}$$

where R^2 is the fraction of the variance predictable from the knowledge of the history of the process. Thus the square root of Cochrane's measure V is a lower bound on Campbell and Mankiw's persistence measure $\psi(1)$: the more highly predictable Δy_t, the greater the difference between the two measures. Campbell and Mankiw (1987a,b) suggest computing an approximative estimates of $\psi(1)$ by remplacing R^2 with the square of the first sample autocorrelation of first differences.

The variance ratio can also be used to obtain an estimate of $\psi(1)$ from an unobserved components model (UCM). Campbell and Mankiw (1987) note that the assumption of independence between the trend and noise components implies that V can be written as a weighted average of the variance ratios of two components. If these are denoted V_z and V_u respectively, then

$$V = \lambda V_z + (1 - \lambda) V_u$$

where $\lambda = \sigma_v^2\sigma_1^2$. Since the UCM assumes that z_t is a random walk and u_t is stationary, $V_z = 1$ and $V_u = 0$, so that $V = \lambda = \sigma_v^2\sigma_1^2$.

Each of these approaches to estimating persistence has its advantages and drawbacks. Impulse-response functions have the advantage of using the well-known ARMA models. Cochrane (1988), however, criticises the use of such models in this context, because they are designed to capture short-run dynamics rather than the long-run correlations that are of interest here. The UCM have also been criticised on similar grounds, in that the identifying restrictions required to estimate long-run behaviour are themselves based on short-run dynamics. Furthermore, such models rule out highly persistent processes. Nelson (1988) has provided Monte Carlo evidence to suggest that they also have tendency to indicate that a series consists of cyclical variations around a smooth trend when the data is actually generated by a random walk, thus biasing estimates of $\psi(1)$ downwards. Nonetheless, UCM captures long-run correlations much better than ARMA models. The nonparametric measure provides only an approximate estimate of $\psi(1)$, is accompanied by large standard errors, and the window size h can be difficult to determine[2]. Neverthless, Cochrane (1988) argues that it is the only measure which is explicitly designed to model long-run dynamics and to isolate random walk components without being contaminated by short-run correlations.

Several studies have provided estimates of these alternative persistence measures for the real gross national product in the United States. These estimates vary considerably depending on the data set used and the estimation procedure adopted. On the basis of low-order ARIMA models estimated on the quarterly U.S. data over the period 1947-1985, Campbell and Mankiw (1987a) conclude that "a 1 percent innovation to real GNP should change one's forecast of GNP over a long horizon by over 1 percent". Harvey (1985) obtains a similar results using an UCM applied to annual data over the period 1948-1970. However, Clark (1987) and Watson (1986) have obtained substantially lower estimates of persistence using an UCM estimated

[2]Cochrane (1988) argues that for real GNP a good value for h is in the region of 20-30 years.

on a quarterly data set comparable to that employed by Campbell and Mankiw. In these studies, a 1 percent shock would lead to around a 0.6 percent change in output in the long-run. Cochrane (1988) using a nonparametric procedure also finds little evidence of persistence in GNP. The evidence on the persistence of aggregate output fluctuations in the U.S. is mixed and inconclusive, and as argued in Christiano and Eichenbaum (1989) the issues of whether real GNP is trend or difference stationary may be very difficult to resolve on the basis of the available post-war quarterly data.

The random walks on integer are able to discover several attractive futures then the random walks on the reals. The reason for this is the fact that any path between two levels must pass at least once through every intervening level. The problem is known as a level (or zero) crossings in statistical theory. Granger and Hallman (1991) consider the unit root tests on the numerous transformed series such as the sign or sinusoidal transformations. Consider a simple symmetrical random walk with unit step

$$y_t = y_{t-1} + \epsilon_t \qquad (1.2.12)$$

with $y_0 = 0$ and where ϵ_t is an independently, identically distributed series which follows the probabilities $p = pr\{\epsilon_t = 1\}$, $q = pr\{\epsilon_t = -1\}$ and $p = q = 1/2$. They consider the sign transformation of the series y_t,

$$r_t = \mathrm{sgn}(y_t) \qquad (1.2.13)$$

where $\mathrm{sgn}(y_t)$ is the sign function such that

$$\mathrm{sgn}(y) = \begin{cases} 1 & , \text{if } y > 0 \\ -1 & , \text{if } y < 0 \end{cases} \qquad (1.2.14)$$

and show that the Dickey-Fuller (DF) test of transformed series is proportional to the number of zero crossings of the original series y_t. Evaluating the empirical distributions of the DF and augmented DF test on sign transformation r_t of a Gaussian random walk y_t, they found that the hypothesis that the transformed series is I(1) against I(0) is usually rejected. Burridge and Guerre (1995) provides the generalization of this result to Brownian motion.

1.3 Macroeconomic Fluctuations, Gambling and Ruin Problem

The idea that GNP may contain a random walk goes back to Irving Fisher's "Monte Carlo Hypothesis", examined further by McCulloch (1975) which suggest that business cycles are nothing more than "Monte Carlo Cycles", these are the cycles superstitious gamblers believe they are discerning in their luck at casinos like the one at Monte Carlo. Following this hypothesis, the information about past fluctuations would be of no help to us in predicting the future values of output. Let us consider a casino gambler, like the one at Monte Carlo, playing with an initial capital, say z dollars. The game consists of a sequence of independent turns and in each turn the gambler wins or loses a dollar. Therefore, the gambler's capital evolves by unit step in upward and downward directions with probability p and q respectively. The game continues until the initial capital z either is reduced to zero (ruined) or has increased to some fixed quantity $\bar{z}$. Thus the walk is restricted on $(0, \bar{z})$. Clearly the case $p > q$ corresponds to a drift to the right when shocks from the left are more probable; when $p = q = \frac{1}{2}$, the random walk is called symmetric. The above phenomenon is known as the ruin problem in random walk theory, see for instance Feller (1971). The question of interest is to compute the ruin probability of the initial capital z, say q_z and the duration of the game. When $\bar{z} \rightarrow \infty$, that is playing against infinitely rich adversary, Feller (1971) gives the limiting ruin probabilities in the form

$$q_z = \begin{cases} 1 & \text{, if } p \leq q \\ (q/p)^z & \text{, if } p > q \end{cases} \tag{1.3.15}$$

which state that if $p \leq q$, the process starting at $z > 0$ will ever reach the origin and the probability of ever returning to its initial position is equals to $(q/p)^z$ when $p > q$, Feller (1971, p.127).

If the first trial results in success, the game continues as if the initial position had been $z+1$. The conditional expectation of the duration assuming success at the

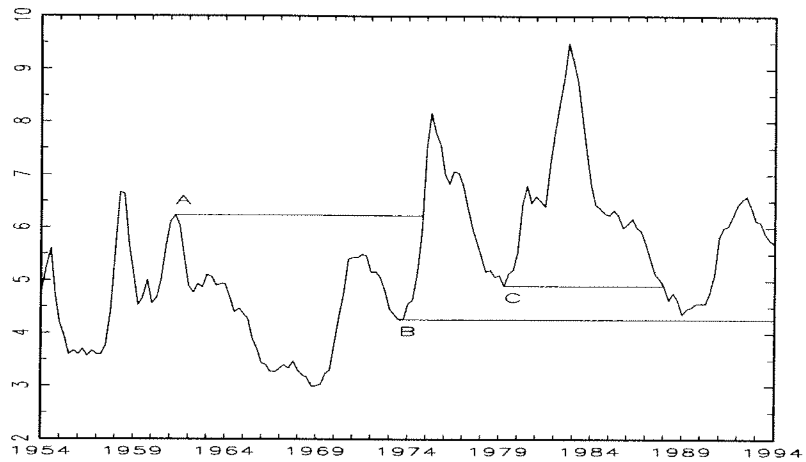

Figure 1.1: Fluctuations in U.S. Unemployment Rate: 1954:Q1-1994:Q4

first trial is therefore $D_{z+1} + 1$. This argument shows that the expected duration D_z satisfies the difference equation

$$D_z = pD_{z+1} + qD_{z-1} + 1 \tag{1.3.16}$$

for $0 < z < \bar{z}$, with boundary conditions $D_0 = 0$ and $D_{\bar{z}} = 0$. The solution of this nonhomogeneous difference equation is given in Feller (1971) by

$$D_z = \begin{cases} \frac{z}{q-p} - \frac{\bar{z}}{q-p}\frac{1-(q/p)^z}{1-(q/p)^{\bar{z}}} & \text{, if } p \neq q \\ z(\bar{z} - z) & \text{, if } p = q. \end{cases} \tag{1.3.17}$$

In the limiting case $\bar{z} \to \infty$, the game may go on forever for $p > q$ and it makes no sense to talk about its expected duration. When $p < q$, we get for expected duration $z(q-p)^{-1}$, but when the upward and downward moves are equally probable, $p = q$, the expected duration of the game is infinite.

Our persistence concept is similar to gambler's ruin problem. Figure 1.1 illustrates the situation. The Figure plots the quarterly fluctuations in U.S. unemployment rate during the period 1954:Q1-1994:Q4. The series displays distinguishable fluctuations matching closely with the post-war business cycles. We retain three particular points in order to explain our persistence concept. First, just before the point A, a positive (upward) fluctuation in 1961:Q2 yields the unemployment rate to increase to the level of 6.2. This corresponds to the beginning of a longer expansion from the post-war II era. As the horizontal line from the point A indicates, the economy never returned to this unemployment rate until 1975 where the first oil-price shock in 1973 shows recessionnist effects. The two other points in the Figure 1.1, B and C, show the effects of the fluctuations occuring in 1973:Q3 and 1979:Q2. These fluctuations are caused largely by the OPEC oil price boosts. The difference between the two points is the in fact that the first oil-shock in 1973:Q3, the point B, is never cancelled, in other words, the economy is never returned to its unemployment rate at 1973:Q3, while the positive fluctuation just before 1979:Q2, the point C which correspond to the the second oil-price shock is cancelled earlier in 1988. Many authors, for example Perron (1988), suggest that the non-rejection of a unit-root hypothesis after second world-war is due in large part to the occurence of slowdown in growth after 1973, where the slope is changed.

The three fluctuations in unemployment rate occuring at the points A, B and C can be ordered according to the duration of these fluctuations. The negative fluctuation occuring at the point B persists longer than the fluctuation occuring at the point A and the latter persists longer than the fluctuation at the point C. This way of conceptualizing economic fluctuations is not new in economics. For instance, Romer (1990) provides *loss rules* when applied to data on industrial production data yield postwar business cycle dates that match the NBER reference dates as closely as possible. Following Romer's loss rules, the area under the horizontal line from the peak at the point A to the level crossing point at 1975, shows the cumulative output loss that has occured between that peak and the time when output returns to its previous peak level. Since, the unemployment rate has an inverted cycle, *i.e.* increasing during the business recessions and decreasing during business expansions,

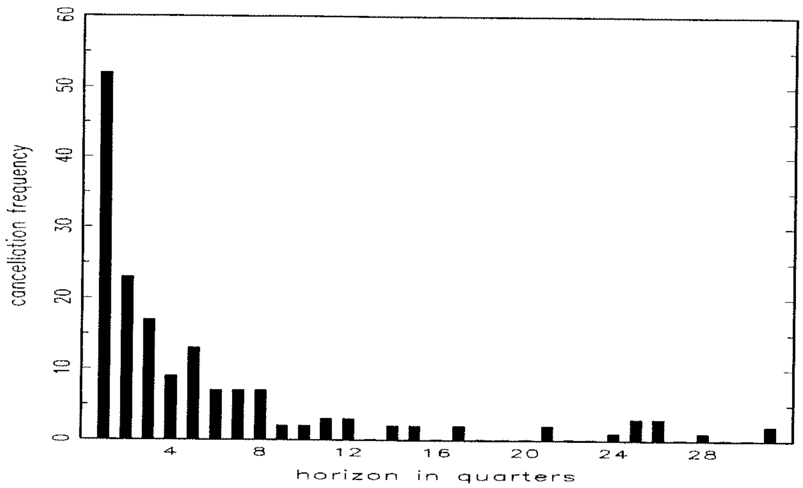

Figure 1.2: Cancellation Frequencies in Unemployment Rate Fluctuations

it would be convenient to speak about the output *gain* rather then the loss for that point. The area under the fluctuations beginning at the points B and C are then the cumulative losses in unemployment rate. Romer (1990) use the output loss rule to show the historical inconsistency of U.S. business cycles dating during the pre- and post-World War II areas. The inconsistency of historical reference dates would be examined further in Chapter 3.

In Figure 1.2, we count the number of fluctuations in U.S. unemployment rate which have been cancelled within h quarters. Among 163 fluctuations displayed in the Figure 1.1, more than fifty fluctuations are cancelled within a quarter. The number of cancelled fluctuations decrease as long as the forecast horizon get larger. For example, the number of cancelled fluctuations are less than five for each horizon longer than two years. This stimulates the computation of the probability that the current fluctuation is or is not cancelled within the fixed forecasting horizon.

1.4 Persistence Measures Based on Return Probabilities

Although the degree of persistence of shocks in economic time series has been the subject of intensive investigation over the last few decades, a variety of problems related on the persistence measure of economic fluctuations remain unexploited. One of these concerns the return problem (or zero crossing) of a unit shock. A fluctuation in a time series is a random deviation from some level, stochastically or deterministically determined by the intrinsic dynamic of a time series. In this section we define persistence (non-persistence) measures based on whether the sign of a fluctuation in a time series is reversed or not after h periods. Consider a real valued time series y_t. Between times $t-1$ and t, the value of y_t changes by $\Delta y_t = y_t - y_{t-1}$, and we are interested in knowing the probability that the fluctuation will be cancelled (or not) after h periods. This leads one to consider a return (or crossing) indicator,

$$r_t(h) = \text{sgn}[y_{t+h} - y_{t-1}] \cdot \text{sgn}[y_t - y_{t-1}] \tag{1.4.1}$$

where $\text{sgn}(y)$ is as defined in (1.2.14). Clearly $r_t(h) \leq 0$ entails that the fluctuation Δy_t has been cancelled at least once during the next h periods. In other words, the current fluctuation expressed in terms of unitary jump $\text{sgn}[y_t - y_{t-1}]$ will return to zero after h periods if the sign of h-periods ahead fluctuation $\text{sgn}[y_{t+h} - y_{t-1}]$ is the inverse of the sign of present fluctuation. Note that the first term on the right hand side of (1.4.1) is the sum of the h future shocks. Associated with the nature of current fluctuation in a time series, we can define several interesting return indicators similar to what is given in (1.4.1). For example, a large body of empirical literature related to the persistence debate examined the presence of a permanent component in the first differenced, as opposed to a trend removed time series. This approach suggests that the business cycles are of purely stochastic nature and there is no reason for the reversion to deterministic equilibrium path. The classical business cycle theories, instead, look at the residuals of a least square regression of the level of a series on time trend as a relevent data for the analysis of business cycles. These two competing theories can be written in terms of a return indicator in the following way. First, consider a trend removed series $\overline{y}_t = y_t - \alpha - \beta t$, where α is a constant and β is the slope of a time trend. The current fluctuation in trend removed series is $\overline{y}_t - \overline{y}_{t-1} = y_t - y_{t-1} - \beta$ is cancelled by the h periods ahead fluctuation $\overline{y}_{t+h} - \overline{y}_{t-1} = y_{t+h} - y_{t-1} - \beta(h+1)$ when the return indicator,

$$\overline{r}_t(h) = \text{sgn}[y_{t+h} - y_{t-1} - \beta(h+1)] \cdot \text{sgn}[y_t - y_{t-1} - \beta], \tag{1.4.2}$$

crosses zero for any horizons h. The coefficient of a time trend β in the expression (1.4.2)can be estimated consistently by least squares regression. It is worth noting that the crossing indicator of trend removed time series $\overline{r}_t(h)$ close up to the $r_t(h)$ when $\beta = 0$. On the other hand, a stochastic fluctuation for time series which contains a drift term μ, that is $\Delta y_t - \mu$ has the cancellation indicator given by

$$r_t^{\mu}(h) = \text{sgn}[y_{t+h} - y_{t-1} - \mu(h+1)] \cdot \text{sgn}[y_t - y_{t-1} - \mu] \tag{1.4.3}$$

for $h = 1, 2, \cdots$. The difference between (1.4.2) and (1.4.3) depends clearly on the assumption retained about the nature of the current fluctuation. Of course, numerous other return indicators can be produced. Instead, we would like to provide a crossing indicator counterpart of the most popular *half life of a shock*, which gives the lenght of time until the impact of a unit shock is half of its initial magnitude. For the difference stationary without drift, (1.4.1), we are looking for a unit shock $y_t - y_{t-1}$ that would be cancelled when future fluctuations discount half of the current shock. Therefore we consider, the half life of a return based on

$$r_t^{\frac{1}{2}}(h) = \text{sgn}[y_{t+h} - y_{t-1} - \frac{1}{2}(y_t - y_{t-1})] \cdot \text{sgn}[y_t - y_{t-1}]. \tag{1.4.4}$$

Given the above crossing indicators, it will be convenient to define the following probabilities:

$$c_t^-(h) = \Pr\{r_t(h) \leq 0\} \tag{1.4.5}$$

$$c_t^+(h) = \Pr\{r_t(h) > 0\} \tag{1.4.6}$$

for $h = 1, 2, \cdots$, where (-) refers to cancellation (or return to zero) and (+) refers to the persistence of a current fluctuation. It is easy to see that the probabilities (1.4.5) and (1.4.5) sum up to unity

$$c_t^-(h) + c_t^+(h) = 1. \tag{1.4.7}$$

The probabilities $c_t^-(h)$ and $c_t^+(h)$ give essentially the expected cancellation number of current fluctuation by future fluctuations measured in terms of unit of time. One of the disadventage in using these measures is the fact that they don't take an account the sign of the fluctuations capitalized within the forecasting period h. For example, when the current fluctuation have a positive sign, the cancellation probability would be positive if h periods ahead position of a time series is below of its initial at time t irrespective from pattern of the series during the forecast horizon. However, there are many situations where the sequence of positive (negative) shocks are followed by dramatic reversion, such as the panics (booms) in financial markets. Consequently we define the probability that the current fluctuation Δy_t be cancelled after h periods *exactly* for $h = 1, 2, \cdots$ by

$$d_t^-(h) = \Pr\{r_t(1) > 0, r_t(2) > 0, \cdots, r_t(h-1) > 0, r_t(h) \leq 0\} \tag{1.4.8}$$

and the probability that it "persists" for at least h periods is

$$d_t^+(h) = \Pr\{r_t(1) > 0, r_t(2) > 0, \cdots, r_t(h-1) > 0, r_t(h) > 0\}. \tag{1.4.9}$$

The probabilities (1.4.8) and (1.4.9) are the basic ones, because they consider the evolution of the cancellation indicators during the forecast period. For example, the cancellation probability $d_t^-(h)$ measures the joint probability that $y_{t+1} > y_{t-1}, y_{t+2} > y_{t-1}, \cdots, y_{t+h-1} > y_{t-1}, y_{t+h} \leq y_{t-1}$ which implies that the process is never returned until time $t + h - 1$ before returning at time $t + h$. Moreover, the position at time $t + h$ is below of its inital value at time $t - 1$.

The probability that the fluctuation Δy_t be cancelled at least once inside h periods is

$$D_t^-(h) \quad = \quad \Pr\{r_t(1) \leq 0\} + \Pr\{r_t(1) > 0, r_t(2) \leq 0\} + \cdots$$

$$+\Pr\{r_t(1) > 0, r_t(2) > 0, \cdots, r_t(h-1) > 0, r_t(h) \leq 0\} \quad (1.4.10)$$

From the definitions (1.4.8) and (1.4.9), the probabilities $d_t^-(h)$ and $d_t^+(h)$ are recursively related by

$$d_t^-(h) + d_t^+(h) = d_t^+(h-1), \quad h = 1, 2, \cdots \quad (1.4.11)$$

Rearranging the terms and proceeding back to horizon $h = 1$, we see that

$$\begin{aligned} d_t^+(h+1) &= d_t^+(h) - d_t^-(h+1) \\ &= d_t^+(1) - \sum_{k=2}^{h+1} d_t^-(k) \\ &= 1 - D_t^-(h+1). \end{aligned} \quad (1.4.12)$$

Starting from the above persistence (non-persistence) definitions and making the use of the fundemental probability law which relates the marginal distribution of a variable to its conditional distribution, we can easely extend our persistence (non-persistence) probability measures. For notational simplicity, let us define the vector valued return indicator.

$$\overline{r_t}(h) = \{r_t(1), r_t(2), \cdots, r_t(h-1), r_t(h)\}. \quad (1.4.13)$$

Therefore, the unconditional persistence (or non-persistence) probabilities can be rewritten as $d_t^-(h) = \Pr\{\overline{r_t}(h-1) > 0, r_t(h) \leq 0\}$ and $d_t^+(h) = \Pr\{\overline{r_t}(h-1) > 0, r_t(h) > 0\}$. Hence, the probability that a fluctuation has been cancelled by h-period ahead fluctuation given that it has not been cancelled during the $h-1$ periods is,

$$\begin{aligned} \Pr\{r_t(h) \leq 0 | \overline{r_t}(h-1) > 0\} &= \frac{\Pr\{r_t(h) \leq 0, \overline{r_t}(h-1) > 0\}}{\Pr\{\overline{r_t}(h-1) > 0\}} \\ &= \frac{d_t^-(h)}{d_t^+(h-1)} \end{aligned} \quad (1.4.14)$$

and similarly, the probability that a fluctuation has been persisted by h-period ahead fluctuation given that it has been persisted during the $h-1$ periods is,

$$\begin{aligned} \Pr\{r_t(h) > 0 | \overline{r_t}(h-1) > 0\} &= \frac{\Pr\{r_t(h) > 0, \overline{r_t}(h-1) > 0\}}{\Pr\{\overline{r_t}(h-1) > 0\}} \\ &= \frac{d_t^+(h)}{d_t^+(h-1)}. \end{aligned} \quad (1.4.15)$$

Using the recursive relationship between unconditional probabilities, we see that the conditional probabilities sum up to unity,

$$\frac{d_t^-(h)}{d_t^+(h-1)} + \frac{d_t^+(h)}{d_t^+(h-1)} = 1 \tag{1.4.16}$$

for $h = 2, 3, \ldots$. These conditional persistence (non-persistence) probabilities have an interesting interpretation as a duration dependence of the macroeconomic fluctuations. The duration dependence are traditionally analyzed by hasard functions, which is the conditional probability that a process will end after a duration of length h, given that it has not terminated earlier, Diebold and Rudebusch (1988, p.598). Economic literature has been focused on two duration specifications; (1) the constant hasard function, where the termination probability is constant through time, Hamilton (1989) ; and (2) increasing hazard function, where the termination probability increase with the age, Neftçi (1982). By analogy to the duration data analysis, we can argue that the constant conditional persistence (non-persistence) probability will provide a probability measure of how much a fluctuation is equally probable during the forecasting period and increasing persistence (non-persistence) probability will reflect that the persistence is more likely when the forecast horizon increases. As a related issue, we can investigate whether the conditional persistence probabilities of expansion and recession periods. The comparison between two conditional probabilities gives insights of how about the business cycles are symmetric, Neftçi (1982).

The simulated values of persistence (non-persistence) probabilities $c_t^-(h)$, $c_t^+(h)$, $d_t^-(h)$, $d_t^+(h)$ and $D_t^-(h)$ are presented in Table 1.1. The simulation experiments consist of 10000 independent replications of a series of lenght $T = 190$. This is approximatively equal to the number of quarterly observations in the majority of post war II U.S. macroeconomic time series analysed in the section 1.6. The included models are typical in standard time series analysis. We consider a noise, a pure random walk, a stationary AR(1) with drift, a random walk with drift, an AR(2) without intercept, an AR(2) with intercept and finally an ARMA(2,1) with drift. The models are parametrized as follows :

Table 1.1: Simulated Return Probabilities

	1	2	3	4	8	12	16	20	24	28	32
(1) $y_t = u_t$											
$c^-(h)$	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
$c^+(h)$	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667
$d^-(h)$	0.333	0.166	0.100	0.067	0.022	0.011	0.007	0.004	0.003	0.002	0.002
$d^+(h)$	0.667	0.500	0.400	0.334	0.200	0.143	0.111	0.091	0.077	0.067	0.059
$D^-(h)$	0.333	0.500	0.600	0.666	0.800	0.857	0.889	0.909	0.923	0.933	0.941
(2) $y_t = y_{t-1} + u_t$											
$c^-(h)$	0.250	0.304	0.333	0.353	0.392	0.411	0.422	0.430	0.437	0.441	0.445
$c^+(h)$	0.750	0.696	0.667	0.647	0.608	0.589	0.578	0.570	0.563	0.559	0.555
$d^-(h)$	0.250	0.125	0.078	0.055	0.022	0.013	0.008	0.006	0.005	0.004	0.003
$d^+(h)$	0.750	0.625	0.547	0.492	0.370	0.309	0.271	0.244	0.224	0.208	0.195
$D^-(h)$	0.250	0.375	0.453	0.508	0.630	0.691	0.729	0.755	0.776	0.792	0.805
(3) $y_t = 1.0 + 0.7y_{t-1} + u_t$											
$c^-(h)$	0.273	0.330	0.359	0.376	0.402	0.407	0.409	0.409	0.408	0.408	0.408
$c^+(h)$	0.727	0.670	0.641	0.624	0.598	0.593	0.591	0.591	0.592	0.592	0.592
$d^-(h)$	0.273	0.143	0.093	0.066	0.027	0.014	0.009	0.006	0.004	0.003	0.003
$d^+(h)$	0.727	0.584	0.491	0.425	0.277	0.205	0.162	0.134	0.114	0.099	0.088
$D^-(h)$	0.273	0.416	0.509	0.575	0.723	0.795	0.838	0.866	0.886	0.901	0.912
(4) $y_t = 1.0 + y_{t-1} + u_t$											
$c^-(h)$	0.134	0.150	0.155	0.157	0.159	0.159	0.159	0.159	0.159	0.159	0.159
$c^+(h)$	0.866	0.850	0.845	0.843	0.841	0.841	0.841	0.841	0.841	0.841	0.841
$d^-(h)$	0.134	0.039	0.015	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$d^+(h)$	0.866	0.827	0.812	0.806	0.801	0.800	0.800	0.800	0.800	0.800	0.800
$D^-(h)$	0.134	0.173	0.188	0.194	0.199	0.200	0.200	0.200	0.200	0.200	0.200
(5) $y_t = 1.98y_{t-1} - 0.99y_{t-2} + u_t$											
$c^-(h)$	0.038	0.060	0.080	0.099	0.168	0.234	0.298	0.360	0.421	0.480	0.535
$c^+(h)$	0.962	0.940	0.920	0.901	0.832	0.766	0.702	0.640	0.579	0.520	0.465
$d^-(h)$	0.038	0.026	0.023	0.021	0.018	0.018	0.017	0.017	0.016	0.016	0.015
$d^+(h)$	0.962	0.936	0.913	0.892	0.815	0.744	0.675	0.608	0.542	0.476	0.413
$D^-(h)$	0.038	0.064	0.087	0.108	0.184	0.256	0.325	0.392	0.458	0.523	0.587
(6) $y_t = 1.0 + 1.98y_{t-1} - 0.99y_{t-2} + u_t$											
$c^-(h)$	0.025	0.042	0.057	0.072	0.132	0.190	0.248	0.307	0.367	0.426	0.483
$c^+(h)$	0.976	0.959	0.943	0.928	0.868	0.810	0.751	0.693	0.633	0.574	0.517
$d^-(h)$	0.025	0.019	0.017	0.016	0.015	0.015	0.015	0.015	0.015	0.015	0.015
$d^+(h)$	0.976	0.957	0.940	0.923	0.862	0.801	0.741	0.681	0.620	0.560	0.500
$D^-(h)$	0.025	0.043	0.060	0.077	0.138	0.199	0.259	0.319	0.380	0.440	0.500
(7) $y_t = 1.0 + 1.98y_{t-1} - 0.99y_{t-2} + u_t - 0.99u_{t-1}$											
$c^-(h)$	0.025	0.046	0.064	0.082	0.146	0.207	0.267	0.326	0.384	0.442	0.499
$c^+(h)$	0.975	0.954	0.936	0.918	0.854	0.793	0.733	0.674	0.616	0.558	0.501
$d^-(h)$	0.025	0.022	0.020	0.019	0.017	0.016	0.016	0.016	0.015	0.015	0.015
$d^+(h)$	0.975	0.953	0.933	0.915	0.846	0.781	0.719	0.656	0.595	0.533	0.472
$D^-(h)$	0.025	0.047	0.067	0.085	0.154	0.219	0.281	0.344	0.405	0.467	0.527

(1) $y_t = u_t$

(2) $y_t = y_{t-1} + u_t$

(3) $y_t = 1.0 + 0.7y_{t-1} + u_t$

(4) $y_t = 1.0 + y_{t-1} + u_t$

(5) $y_t = 1.98y_{t-1} - 0.99y_{t-2} + u_t$

(6) $y_t = 1.0 + 1.98y_{t-1} - 0.99y_{t-2} + u_t$

(7) $y_t = 1.0 + 1.98y_{t-1} - 0.99y_{t-2} + u_t - 0.99u_{t-1}$

where the innovation term u_t is N(0,1). Each of the above model are initiated at zero. For each replication of the models (1)-(7) we then compute the sign crossing indicator $r_t(h)$ up to horizon $h = 32$. The return (non-return) probabilities given in (1.4.5) and (1.4.10) are then computed as the average of 10000 replications.

The persistence probabilities of noisy process when measured in terms of $c^+(h)$ are equal to 2/3 for all horizons while when measured in terms of $d^+(h)$ the persistence probabilities decrease quickly, half of $c^+(h)$ at $h = 4$ and approaches zero for larger horizons. The non-persistence probabilities $D_t^-(h)$ show that the fluctuations in noise process have 94.1 % probability of cancelling by $h = 32$. A fluctuation in a pure random walk without intercept, the model (2), persists with 3/4 chance in $h = 1$. The non-persistence probability $d_t^-(h)$ approaches zero very quickly. After $h = 32$, the persistence probability $d^+(h)$ is less then 1/5. The pure random walk becomes more persistent, when we add a constant term in the model (2). More precisely, a fluctuation in the random walk with intercept, model (4) persist 86.6 % probability in one horizon while when the forecasting horizon get larger, it persists 80.0 % probability. The non-persistence probabilities, $d^-(h)$ in the random walk with drift process is zero after $h = 4$. The stationary AR(1) model with drift, the model (3), have the return probabilities similar to those given for the noisy model but they are slightly more persistent. The difference between the model (1) and (3) comes from the value of autoregressive coefficient which is taken $\phi = 0.7$. When $\phi \rightarrow 0$ the return probabilities match the probabilities of noise process. The models (5)-(7) show the persistence (non-persistence) probabilities for different processes

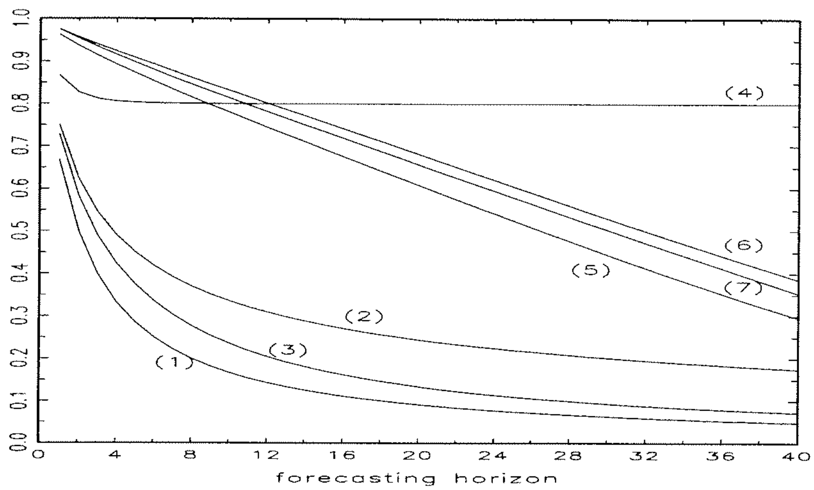

Figure 1.3: Simultated Persistence Probabilities: $d^{+}(h)$

having the autoregressive coefficients near the unit circle. For the AR(2) without intercept, the persistence (non-persistence) probabilities are lower (higher) than the AR(2) with intercept.

Figure 1.3 shows the persistence probabilities $d^{+}(h)$ for the models (1)-(7). As it can be expected a noise is the least persistent process of our experiments. A stationary AR(1) and I(1) are both decreasing persistence probabilities as long as the forecasting horizon gets larger. The hyperbolic shape of persistence probabilities for the noisy model, AR(1) and I(1) means that the change in persistence probability $d_t^{-}(h)$ decreases with the forecast horizon and for the series of lenght 190 considered here, they never reaches the zero probability at longer horizons. On the other hand, the change in persistence probabilities from one horizon to other is constant, for the models (5)-(7). More interestingly, we see that these models display more persistence at short horizons, say 8 to 12, than a pure random walk without drift (4). For the latter model, the change in persistence probabilities is constant after 8 forecasting horizons.

1.5 Statistical Inferences on Return Probabilities

The above measures are essentially based on the zero crossings of the transformed series $r_t(h)$ for different horizons h. We are concerned with statistical inference on the length of time that it takes a process to return to zero level, given that it crossed the zero at a particular previous time. Our parameters of interest in this persistence problem are the expected number of zero crossings per unit time and the variance of zero crossings per unit time. Distributional properties will be examined in the next Section. Note that the above probabilities may depend on t. However if the process Δy_t is strictly stationary, these probabilities will also be time invariant, so that the index may be dropped from $c_t^-(h)$, $c_t^+(h)$, $d_t^-(h)$, $d_t^+(h)$ and $D_t^-(h)$. Therefore the expected cancellation probability for (1.4.5) and (1.4.6) are given by

$$c^-(h) = E\{\delta^-[r_t(h)]\} \tag{1.5.1}$$

$$c^+(h) = E\{\delta^+[r_t(h)]\} \tag{1.5.2}$$

where $\delta^-[x]$ and $\delta^+[x]$ are defined by

$$\delta^-[x] = \begin{cases} 1 & \text{if } x \leq 0 \\ 0 & \text{if } x > 0, \end{cases}$$

$$\delta^+[x] = 1 - \delta^-[x] = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

A nice feature of the above measures is the fact that they can be estimated quite easily at least under the assumption that Δy_t is strictly stationary. In particular, unbiased estimators of $c^-(h)$ and $c^+(h)$ are given by:

$$\hat{c^-}(h) = \frac{1}{T-h} \sum_{t=1}^{T-h} \delta^-[r_t(h)] \tag{1.5.3}$$

$$\hat{c^+}(h) = \frac{1}{T-h} \sum_{t=1}^{T-h} \delta^+[r_t(h)] \tag{1.5.4}$$

for $h = 1, 2, 3, \ldots$. Similarly the expected number of cancellation and persistence of the present fluctuation after h periods exactly are respectively,

$$\begin{aligned} d^-(h) &= E\{\Delta_t^-(h)\} && (1.5.5) \\ d^+(h) &= E\{\Delta_t^+(h)\} && (1.5.6) \end{aligned}$$

where $\Delta_t^-(h)$ and $\Delta_t^+(h)$ are indicator functions defined by

$$\Delta_t^-(h) = \begin{cases} \delta^-[r_t(1)] & , \text{for } h = 1 \\ \{\prod_{k=1}^{h-1} \delta^+[r_t(k)]\}\ \delta^-[r_t(h)] & , \text{for } h > 1 \end{cases}$$

and

$$\Delta_t^+(h) = \begin{cases} \delta^-[r_t(1)] & , \text{for } h = 1 \\ \prod_{k=1}^{h} \delta^+[r_t(k)] & , \text{for } h > 1. \end{cases}$$

Similarly,

$$\begin{aligned} \hat{d^-}(h) &= \frac{1}{T-h} \sum_{t=1}^{T-h} \Delta_t^-(h) && (1.5.7) \\ \hat{d^+}(h) &= \frac{1}{T-h} \sum_{t=1}^{T-h} \Delta_t^+(h) && (1.5.8) \end{aligned}$$

for $h = 1, 2, 3, \ldots$, provide unbiased estimators of $d^-(h)$ and $d^+(h)$. Finally the expected probability that the fluctuation does not persist for h perods is

$$D^-(h) = E\{\sum_{k=1}^{h} \Delta_t^-(k)\} \qquad (1.5.9)$$

which can be estimated by

$$\hat{D^-}(h) = \sum_{t=1}^{T-h} \sum_{k=1}^{h} \Delta_t^-(k) \qquad (1.5.10)$$

In the remaining part of this section we look at the large sample distribution of the estimators of return probabilities. In order to obtain a compact form enough and avoid unnecessary repetition of formulas, we define a variable $\theta(h)$ for any fixed horizon h representing the return probabilities given in (1.4.5)-(1.4.10), that is

$$\theta(h) \in \{c^-(h),\ c^+(h),\ d^-(h),\ d^+(h),\ D^-(h)\}$$

and $\theta_t(h)$

$$\theta_t(h) \in \{\delta^-[r_t(h)],\ \delta^+[r_t(h)],\ \Delta_t^-(h),\ \Delta_t^+(h),\ \textstyle\sum_{k=1}^h \Delta_t^-(k)\ \}$$

where each element is a $(T-h)$ vector of functions $r_t(h)$ based on sign reversion between present and h period ahead fluctuation in the raw data and the unbiased estimators of these return probabilities given in (1.5.3) to (1.5.10) from a sample of lenght T by,

$$\widehat{\theta}_T(h) \in \{\widehat{c}^-(h),\ \widehat{c}^+(h),\ \widehat{d}^-(h),\ \widehat{d}^+(h),\ \widehat{D}^-(h)\}$$

We know, from the Section 1.4 that $\widehat{\theta}_T(h)$ are the unbiased estimators of return probabilities $\theta(h)$. Let us also define

$$\begin{aligned} w_T(h) &= \sqrt{T-h}(\widehat{\theta}_T(h) - \theta(h)) \\ &= \frac{1}{\sqrt{T-h}} \sum_{t=1}^{T-h} [\theta_t(h) - \theta(h)] \end{aligned} \tag{1.5.11}$$

The variance of the latter variable is

$$\begin{aligned} s_T(h) &= V\{w_T(h)\} \\ &= \frac{1}{T-h} \sum_{t=1}^{T-h} \sum_{s=1}^{T-h} E\{[\theta_t(h) - \theta(h)]\,[\theta_s(h) - \theta(h)]\}. \end{aligned} \tag{1.5.12}$$

Newey and West (1987) consider positive-semidefinite, heteroskedastic and autocorrelation consistent covariance estimator $\widehat{s}_T(h)$ which takes the form,

$$\widehat{s}_T(h) = \widehat{\gamma}_0 + 2\sum_{j=1}^{m} w(j,m)\widehat{\gamma}_j \tag{1.5.13}$$

with weights $w(j,m)$ equal to $[1 - j/(m+1)]$, for $j = 1, \cdots m$ and $m = 1, 2, \cdots$ and the empirical autocorrelation functions can be estimated consistently by

$$\widehat{\gamma}_j = \frac{1}{T-h-j} \sum_{t=1}^{T-h-j} [\theta_t(h) - \widehat{\theta}_T(h)][\theta_{t+j}(h) - \widehat{\theta}_T(h)] \tag{1.5.14}$$

Therefore the estimators of variances $\widehat{s}_T(h)$, are simply the weighted sums of autocovariance functions $\widehat{\gamma}_j$, with weights $w(j,m)$ which decline as j increases and they are numerically equal to 2π times an estimator of the spectral density of $\theta_t(h)-\theta(h)$, at frequency zero. Newey and West ((1987), Theorem 1) demonstrate positive semi-definiteness of these estimators for a substantially wide choice of weight functions $w(j,m)$ by making use of positive semi-definiteness of the sample autocovariance functions. Note that for fixed j, the weights $w(j,m)$ approach one as m grows. It is reasonable to expect that estimators $\widehat{s}_T(h)$ that are formed by smoothing sample autocovariances $\widehat{\gamma}_j$ with weights that approach one as m grows should be consistent if m is allowed to grow with the sample size T. Newey and West (1987, theorem 2) argue that under the following regularity conditions;

- *i)* $\{\theta_t(h)-\theta(h)\}$ is measurable in $\theta_t(h)$, for all $\theta(h)$ and continuously differentiable in $\theta(h)$ for all $\theta(h)$ in a neighborhood of $\theta^*(h)$ with probability one;

- *ii)* there is a measurable function $m(\theta(h))$ such that $sup_N|\theta(h)| < m(\theta(h))$ and $sup_N|\theta_t(h)| < m(\theta(h))$, where for some finite constant D, $\delta > 0$ and $r \geq 1$, such that for all $E[|\theta^*(h)|^{4(\delta+r)}] < D$;

- *iii)* the $\theta_t(h)$ is a mixing random sequence;

- *iv)* for all t, $E\{\theta_t(h)-\theta(h)\} = 0$, and $\sqrt{T-h}(\widehat{\theta}_T(h)-\theta(h))$ is bounded in probability;

- *v)* the weights $w(j,m)$ $j=1,2,\cdots,m$, $m=1,2,\cdots$ satisfy $|w(j,m)| < K$ for finite constant K and for each j, $\lim_{T\to\infty} w(j,m)=1$, if the window lenght m in smoothing autocorrelations is choosen to be a function of the sample size, $m(T)$, such that $\lim_{T\to\infty} m(T)=+\infty$ and $\lim_{T\to\infty} m(T)/T^{1/4}=0$, for large T

$$\widehat{\gamma}_0 + 2\sum_{j=1}^{m(T)} w(j,m(T))\widehat{\gamma}_j - s_T(h) \xrightarrow{P} 0. \qquad (1.5.15)$$

These assumptions require that $\theta_t(h)$ be dominated by a function of $\theta(h)$ that has a uniformly bounded second moment and by (*iii*) for a given horizon h, the dependence between observations go to zero as the distance between observations go to zero. In any given sample, it is of course necessary to choose m the number of autocorrelations to include in the estimation $\widehat{s}_T(h)$. Choosing a window size low may obscure cancellation probabilities manifested in higher autocorrelations. On the other hand, a large window size, may result in excessive cancellation, since as m approaches the sample size T, the estimator $\widehat{s}_T(h)$ approaches zero. Note that since the sample means have been removed from the data, $\widehat{s}_T(h)$ is identically zero at $h = T - 1$. Hence, while large m appears preferable, m must be small relative to the sample size. Consistency of the estimator $\widehat{s}_T(h)$ follows the assumptions (iv) and (v) which suggest to use a window size $m(T)$ growing more slowly than $T^{1/4}$.

If the random variable $\theta_t(h) - \theta(h)$ are i.i.d. sequences, it is easy to show that from the classical central limit theorem that, see for instance Davidson (1994, p.366)

$$\frac{w_T(h)}{\sqrt{\widehat{s}_T(h)}} \xrightarrow{D} N(0,1). \tag{1.5.16}$$

where the theoretical means in $w_T(h)$ being substituted by the unbiased estimators given in (1.5.3)-(1.5.10).

1.6 Macroeconomic Applications

In this section, we examine persistence probabilities of fluctuations in quarterly U.S. macroeconomic time series during the post War II period. The section is organized in two parts. First, in section 1.6.1, we discuss the persistence of fluctuations in gross national product. Second, in section 1.6.2, we analyze twenty individual time series including the consumption and investment expenditures, prices, inflation, money stock, interest rates, employment, wages, financial market indicators; stock prices and volume, productivity and capacity utilization rates. The data were obtained from the Citibase databank. The complete list of series and their full definitions are given in Table 1.18 at the end of section. The monthly observations are transformed to quarterly observations.

We report the estimated values of persistence probabilities $d^+(h)$ for forecasting horizon h spanning up to 40 quarters (10 years). The results from other probability measures (persistence or non-persistence) are not presented in order to avoid long tabulations. The Gauss program used to compute persistence probabilities is given at the end of section. The window size m in estimating the Newey-West heteroscedastic and autocorrelation consistent standard errors in expression (1.5.13) is choosen equal to eight quarters.

We examine the persistence probabilities of raw fluctuations as well as the persistence probabilities of fluctuations around linear time trend and the mean of fluctuations. The results are presented in following order. First, we provide a brief summary of debates for related variables. Secondly, we compare the persistence probabilities of raw fluctuations at short and long forecasting horizons. We also look at the persistence probabilities of detrended and demeaned fluctuations. Third, we compute persistence probabilities of raw fluctuations conditional to the business expansions and recessions identified by NBER dating committees. The details of our results are presented in corresponding Tables and Figures. The section terminates with the comparison of our results with those from variance-ratio estimators, Cochrane (1988).

1.6.1 Persistence Probabilities of U.S. GNP

Several papers have studied the persistence of GNP fluctuations, including Campbell and Mankiw (1986), Clark (1987), Cochrane (1988), Nelson and Plosser (1982), Quah (1986), Stock and Watson (1986) and Watson (1986). A major focus of these papers has been the extent to which GNP movements are well approximated by a process with unit root plus drift, as opposed to stationary movements around a time trend. Campbell and Mankiw (1988) and Nelson and Plosser (1982) both argue that if the random walk approximation is in fact reasonable, there are important implications for business cycle theory. This is because movements in random walks are permanent: a shock today has an infinitely long lived effect. The concept of a stationary natural rate, Campbell and Mankiw note, has little utility if a GNP shock is, on average, never offsets by a return to some trend rate of GNP. Nelson and Plosser (1982) suggest that monetary disturbances are unlikely to be an important source of GNP fluctuations, since monetary shocks are typically thought to have no permanent effect. Both papers conlude that if the random walk characterization is accurate, an implication is that fluctuations in GNP are unlikely to be driven by nominal demand shocks.

The results from these studies vary considerably depending on the data set used and the estimation procedure adopted. On the basis of low-order ARIMA models estimated on the quarterly U.S. data over the post Second War period, Campbell and Mankiw conclude that a 1 percent innovation to real GNP should change one's forecast of GNP over a long horizon by over 1 percent. Harvey (1985) obtains a similar result using an unobserved component model applied to annual data over the period 1948-1970. However Clark (1987) and Watson (1986) have obtained substantially lower estimates of persistence using an unobserved component model estimated on a quarterly data set comparable to that used by Campbell and Mankiw. In these studies, a 1 percent shock would lead to around a 0.6 percent change in output in the long run. Cochrane (1988), using a variance-ratio procedure also finds little evidence of persistence in GNP. Perron (1989) and (1993) argues that the GNP, as many macroeconomic variables is better constructed as stationary fluctuations around a breaking trend. The evidence on the persistence of aggregate output fluctuations is mixed and inconclusive and as argued in Christiano and Eichenbaum (1989) the issue of whether real GNP is trend or difference stationary may be very difficult to resolve on the basis of the available post-War quarterly data.

We give a particular attention to the per capita real GNP rather than the nominal GNP since movements in the aggregate output induced by the varying population and inflation will naturally persist and may obscure the persistence intrinsic to the market economy. However, we provide persistence probabilities of real and nominal fluctuations in GNP conforming to earlier studies. Table 1.2 presents the results. Several features of this table are noteworthy. First, the raw fluctuations in real per

Table 1.2: **Persistence Probabilities in U.S. GNP**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Real Per Capita GNP 1947:Q1 - 1994:Q3 [GNPQ/P16][b]													
(1)	0.847	0.809	0.722	0.694	0.555	0.478	0.460	0.441	0.440	0.426	0.418	0.409	0.393
	(0.025)	(0.027)	(0.040)	(0.046)	(0.060)	(0.071)	(0.074)	(0.079)	(0.081)	(0.080)	(0.080)	(0.079)	(0.076)
(2)	0.836	0.723	0.674	0.624	0.495	0.399	0.345	0.300	0.271	0.247	0.234	0.227	0.227
	(0.029)	(0.037)	(0.035)	(0.037)	(0.044)	(0.048)	(0.053)	(0.053)	(0.053)	(0.050)	(0.051)	(0.049)	(0.051)
(3)	0.836	0.739	0.679	0.624	0.495	0.388	0.328	0.294	0.271	0.247	0.228	0.227	0.227
	(0.030)	(0.036)	(0.033)	(0.036)	(0.047)	(0.052)	(0.053)	(0.052)	(0.053)	(0.051)	(0.049)	(0.050)	(0.052)
Real GNP 1947:Q1 - 1994:Q3 [GNPQ][c]													
(1)	0.884	0.835	0.802	0.747	0.676	0.669	0.672	0.682	0.675	0.667	0.658	0.656	0.647
	(0.030)	(0.037)	(0.045)	(0.054)	(0.068)	(0.068)	(0.068)	(0.069)	(0.069)	(0.068)	(0.069)	(0.069)	(0.069)
(2)	0.847	0.766	0.717	0.661	0.555	0.444	0.379	0.324	0.301	0.296	0.272	0.247	0.227
	(0.033)	(0.034)	(0.036)	(0.038)	(0.046)	(0.053)	(0.064)	(0.069)	(0.070)	(0.072)	(0.073)	(0.073)	(0.067)
(3)	0.847	0.771	0.722	0.667	0.560	0.449	0.385	0.324	0.307	0.302	0.272	0.247	0.227
	(0.033)	(0.034)	(0.036)	(0.038)	(0.046)	(0.053)	(0.063)	(0.069)	(0.070)	(0.072)	(0.073)	(0.073)	(0.067)
Nominal GNP 1946:Q1 - 1994:Q3 [GNP][d]													
(1)	0.959	0.938	0.916	0.905	0.898	0.896	0.893	0.891	0.888	0.886	0.883	0.880	0.877
	(0.020)	(0.025)	(0.034)	(0.039)	(0.041)	(0.042)	(0.042)	(0.042)	(0.042)	(0.043)	(0.043)	(0.043)	(0.044)
(2)	0.948	0.922	0.916	0.911	0.876	0.863	0.865	0.856	0.841	0.825	0.809	0.791	0.773
	(0.024)	(0.032)	(0.035)	(0.039)	(0.054)	(0.062)	(0.062)	(0.065)	(0.069)	(0.073)	(0.077)	(0.082)	(0.085)
(3)	0.959	0.911	0.901	0.895	0.855	0.846	0.848	0.839	0.824	0.807	0.790	0.772	0.747
	(0.016)	(0.032)	(0.037)	(0.040)	(0.059)	(0.064)	(0.065)	(0.067)	(0.072)	(0.075)	(0.079)	(0.084)	(0.088)

[a]Entries in (1) are the persistence probabilities of raw, (2) trend removed, (3) demeaned fluctuations. The numbers in paranthesis are heteroscedastic and autocorrelation consistent standard errors of estimates. Citibase mnemonics for the series are in brackets. The estimated trend regressions and the mean of fluctuations are:

[b]$y_t = 0.0134 + 7.2e - 05 \cdot t$ and $E(y_t - y_{t-1}) = 0.000$.

[c]$y_t = 973.5 + 21.5 \cdot t$ and $E(y_t - y_{t-1}) = 21.657$.

[d]$y_t = -1084.35 + 31.2 \cdot t$ and $E(y_t - y_{t-1}) = 33.913$.

capita GNP have more than 4/5 chance to persist after one quarter and approximately 2/5 chance to persist after 10 years. The persistence probabilities of raw fluctuations in real GNP are much more higher. After 10 years, the probability that a fluctuation in real GNP persist is more than 2/3. The fluctuations around a linear time trend and the mean level have statistically similar persistence probabilities. For the real per capita GNP series, these fluctuations persist at 83.6 % probability after one quarter and 22.7 % probability after ten years. When using the real GNP, the last figures remains almost similar: a fluctuation in real GNP persist at 84.7 % probability after one quarter and 22.7 % probability in ten years forecasting horizon. The fluctuations in both series have more than 1/2 chance to persist in one year horizon. Obviously, the persistence probabilities of nominal fluctuations in GNP are much more higher than the fluctuations in real GNP since they include fluctuations in population and prices.

Many recent papers discuss the implications of the assumption that macroeconomic variables are subject to two different probability distribution functions, one of which applies in business expansions and the other in recessions. The switches from the former to the latter (or vice versa) are supposed to occur suddenly at random time points and be unobserved; they can only be inferred from the data according to some model and prediction rules. For example, Neftçi (1982) splits the data on the composite index of leading indicators for 1948-1970 into downturn and upturn regimes. He smoothes the historical frequency distribution of the monthly changes in the index with a month centered moving average to estimate the probability distributions separately for the two regimes. His formula for assessing the probability of recessions is recursive and dynamic in that it inludes the previous month's outcome and cumulates the probabilities from zero at the start of each expansion to 100 % at the end. It also involves a prior probability distribution based on the assumption that the likelihood of a downturn increases slightly in each month as the expansion ages. The assumption that the life of an expansion is a declining function of its duration has long been questioned. Hamilton (1989)'s econometric analysis ex-

Table 1.3: **Persistence During the Business Cycles** [a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Real Per Capita GNP 1947:Q1 - 1994:Q3 [GNPQ/P16]													
Exp	0.839	0.800	0.742	0.715	0.617	0.553	0.541	0.517	0.518	0.504	0.496	0.488	0.472
	(0.030)	(0.036)	(0.045)	(0.051)	(0.058)	(0.073)	(0.075)	(0.083)	(0.085)	(0.084)	(0.085)	(0.084)	(0.082)
Rec	0.893	0.857	0.607	0.571	0.214	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.051)	(0.047)	(0.077)	(0.094)	(0.104)	(0.047)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Real GNP 1947:Q1 - 1994:Q3 [GNPQ]													
Exp	0.913	0.887	0.874	0.835	0.799	0.793	0.791	0.800	0.794	0.788	0.782	0.783	0.776
	(0.028)	(0.032)	(0.035)	(0.046)	(0.055)	(0.056)	(0.057)	(0.057)	(0.057)	(0.057)	(0.058)	(0.058)	(0.058)
Rec	0.714	0.536	0.393	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.070)	(0.075)	(0.100)	(0.098)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

[a] Exp and Rec denotes expansions and recessions.

tends the Neftçi's model. He applies a nonlinear iterative filter to quarterly growth rates in real GNP for 1952-1984 and reports the maximum likelihood estimates of the parameters of the underlying process. He argues that one possible outcome of the use of Markov switching regression to infer regime changes might have been an identification of above-trend and below-trend growth phases. However, he shows that the data are separated by positive growth periods and negative growth periods (recessions). The dates of the switches determined by this method agree quite well with the NBER's business cycle chronologies.

In Table 1.3, we present persistence probabilities conditional to the business cycles. We see that a fluctuation in real per capita GNP occuring in business expansions have 1/2 chance to persist after ten years. On the other hand, the probability that a fluctuation occuring in business recessions persists more than three years is statistically equal to zero. The conditional probabilities of the fluctuations in real GNP are more pronounced. For example, fluctuations occuring during the business expansions have more than 3/4 chance to persist after ten years while fluc-

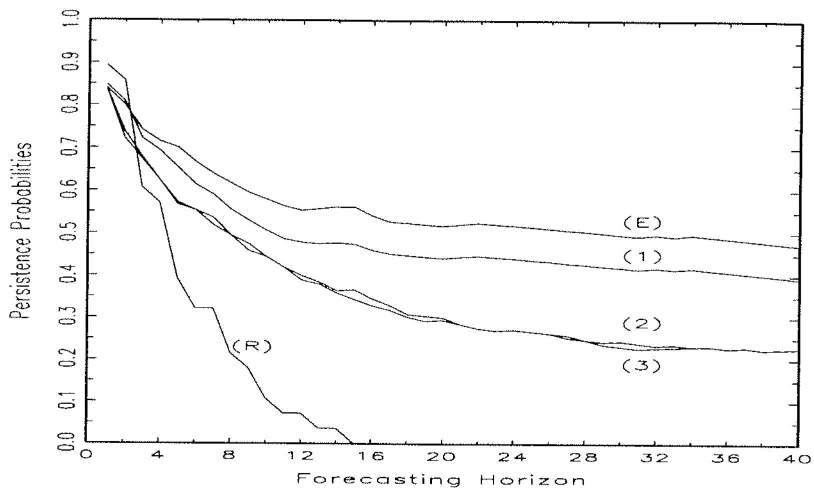

Figure 1.4: Persistence Probabilities in Real Per Capita GNP

tuations during the business recessions reaches the zero probability within the two years. These results support the hypothesis of the asymmetry of business fluctuations which suggests that the expansions are much longer than recessions during the post-War II period.

Figure 1.4 summarizes the typical pattern of persistence probabilities of fluctuations in real per capita GNP. In this figure, we show superimposed persistence probabilities of (1) raw, (2) detrended and (3) demeaned fluctuations as well as the persistence probabilities conditional to expansions (E) and recessions (R). The whole cycle persistence probabilities are high during the short forecasting horizons, but they decline slowly for larger horizons. The raw fluctuations persist at 1/2 probability for longer horizons. Once the drift (or time trend) is dropped, persistence probabilities are about 1/4 after ten years. It appears no quantitative difference between trend or difference stationary fluctuations. Return probabilities associated with fluctuations (around deterministic time trend and those around stochastic time trend) appear to have similar persistence probabilities. The overall picture of return probabilities during the cyclical regimes were also plotted in the bottom panel of Figure 1.4. Typically, they show quickly decaying shape when the return probabilities are conditioned to business recessions. After one year, they are zero probability. On the other hand, conditional to business expansions, the fluctuations persist at a half probability after 2-3 years.

1.6.2 Some Stylized Facts in U.S. Macroeconomic Time Series

Consumption and Investment

In this section, we examine the persistence of the fluctuations in total consumption expenditures which include the expenditures on durable, nondurable and services. It is known that real consumer expenditures on durable goods have large fluctuations, most of which correspond well to business cycles. In contrast, consumption of nonduarble goods and services shows relatively small fluctuations, see Moore and Zarnowitz (1994). Table 1.4 provides the persistence probabilities in real consumption and real investment expenditures. We eliminate the effects of population growth by dividing both series by the total civil population. The raw fluctuations in consumption expenditures are highly persistent: 92.1 % after one quarter and only 82.1 % after ten years. Once the time trend or the mean of fluctuations is removed persistence probabilities becomes 75-80 % after one quarter but fall quickly to 30 % after ten years. In per capita terms, the second entry in Table 1.4, the consumption expenditures appears to be less persistent at long forecasting horizon. For instance, the raw fluctuations in consumption have least than 2/3 chance to survive after ten years while demeaned or detrended fluctuations persist with 1/9 chance.

The third and forth entries of Table 1.4 report persistence probabilities in real fixed investment. All of these probabilities are less then the probabilities for the fluctuations in consumption expenditures. These results confirm permanent income hypothesis which suggests that fluctuations in real consumption is more persistent than those in investment. The probability that a raw fluctuation in real per capita investment expenditures persist after one quarter is 86.8 %. In per capita terms, the raw fluctuations have approximately the same persistence probabilities as with detended or demeaned fluctuations.

The results in Table 1.5 show the persistence probabilities of consumption and investment expenditures during the business cycles. In per capita terms, the fluctuations in consumption expenditures during the business expansions have 94.4 % chance to persist after one quarter. Looking at the long forecasting horizon, we see that in per capita terms, the fluctuations in consumption are more persistent during the expansion periods. On the other hand, the persistence probabilities of fixed investment expenditures during the recessions are higher at short forecasting horizons than expansions and reach the zero probability after 16 quarters. The Figure 1.5 plots the patterns of persistence probabilities reported in this section.

Prices and Inflation

The degree of persistence in inflation has several important macroeconomic implications. Recent economic theory suggests that changes in the persistence of inflation may result from changes in monetary policymakers' preferences for inflation or from changes in monetary regime. Ball (1990) constructs a time-consistent model of central bank behavior in which temporary and exogeneous macroeconomic shocks trigger temporary inflations that become persistent and long lasting only if weak policymakers accommodate these shocks. A change from high and persistent inflation to low and stable inflation only occurs when a new strong policymaker is willing to incur temporary output losses in order to reduce the inflation under strong policymakers. A low and less persistent inflation rate results because these policymakers do not accommodate temporary macroeconomic shocks. Alogoskoufis and Smith (1991) use an aggregate demand-supply macroeconometric model with price-setting firms and staggered wage-setting to show that the persistence in inflation will be

Table 1.4: **Consumption and Investment**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Real Consumption: 1947:Q1 - 1994:Q4 [GCQ]													
(1)	0.921	0.862	0.840	0.840	0.814	0.810	0.817	0.830	0.826	0.822	0.830	0.826	0.821
	(0.019)	(0.034)	(0.043)	(0.044)	(0.052)	(0.053)	(0.052)	(0.051)	(0.052)	(0.052)	(0.053)	(0.053)	(0.053)
(2)	0.779	0.714	0.691	0.674	0.563	0.464	0.411	0.380	0.359	0.344	0.346	0.335	0.305
	(0.032)	(0.038)	(0.039)	(0.040)	(0.051)	(0.066)	(0.072)	(0.077)	(0.078)	(0.083)	(0.083)	(0.084)	(0.083)
(3)	0.795	0.709	0.681	0.663	0.557	0.464	0.411	0.380	0.353	0.344	0.352	0.329	0.305
	(0.032)	(0.036)	(0.039)	(0.040)	(0.050)	(0.065)	(0.071)	(0.076)	(0.077)	(0.081)	(0.081)	(0.082)	(0.081)
Real Per Capita Consumption: 1947:Q1 - 1994:Q4 [GCQ/P16]													
(1)	0.868	0.804	0.755	0.722	0.667	0.620	0.611	0.626	0.641	0.638	0.642	0.639	0.636
	(0.030)	(0.037)	(0.041)	(0.046)	(0.057)	(0.066)	(0.070)	(0.068)	(0.067)	(0.068)	(0.069)	(0.069)	(0.069)
(2)	0.795	0.714	0.665	0.620	0.481	0.369	0.326	0.251	0.210	0.190	0.170	0.148	0.113
	(0.035)	(0.036)	(0.040)	(0.044)	(0.051)	(0.059)	(0.065)	(0.065)	(0.060)	(0.055)	(0.053)	(0.049)	(0.047)
(3)	0.800	0.725	0.665	0.620	0.475	0.369	0.326	0.251	0.210	0.190	0.176	0.148	0.113
	(0.032)	(0.034)	(0.040)	(0.044)	(0.051)	(0.059)	(0.065)	(0.065)	(0.060)	(0.055)	(0.054)	(0.049)	(0.047)
Real Investment: 1947:Q1 - 1994:Q4 [GIFQ]													
(1)	0.858	0.788	0.745	0.684	0.492	0.380	0.314	0.298	0.305	0.301	0.308	0.316	0.325
	(0.026)	(0.038)	(0.039)	(0.042)	(0.054)	(0.057)	(0.065)	(0.064)	(0.064)	(0.061)	(0.061)	(0.061)	(0.061)
(2)	0.884	0.810	0.761	0.706	0.519	0.335	0.291	0.234	0.204	0.184	0.145	0.142	0.126
	(0.024)	(0.029)	(0.032)	(0.038)	(0.046)	(0.052)	(0.051)	(0.054)	(0.054)	(0.053)	(0.043)	(0.041)	(0.036)
(3)	0.874	0.799	0.739	0.684	0.503	0.374	0.326	0.287	0.240	0.202	0.164	0.123	0.126
	(0.026)	(0.033)	(0.038)	(0.041)	(0.047)	(0.054)	(0.055)	(0.059)	(0.060)	(0.061)	(0.055)	(0.052)	(0.052)
Real Per Capita Investment: 1947:Q1 - 1994:Q4 [GIFQ/P16]													
(1)	0.868	0.799	0.734	0.701	0.503	0.346	0.286	0.228	0.210	0.202	0.189	0.194	0.199
	(0.026)	(0.033)	(0.039)	(0.037)	(0.053)	(0.053)	(0.057)	(0.060)	(0.061)	(0.058)	(0.059)	(0.060)	(0.060)
(2)	0.895	0.804	0.745	0.695	0.508	0.346	0.286	0.263	0.246	0.196	0.164	0.123	0.119
	(0.024)	(0.030)	(0.032)	(0.038)	(0.044)	(0.048)	(0.047)	(0.050)	(0.051)	(0.045)	(0.039)	(0.039)	(0.040)
(3)	0.889	0.799	0.739	0.690	0.508	0.346	0.286	0.257	0.240	0.184	0.151	0.123	0.113
	(0.022)	(0.028)	(0.032)	(0.038)	(0.042)	(0.049)	(0.047)	(0.051)	(0.048)	(0.045)	(0.042)	(0.037)	(0.038)

[a] Entries in (1) are the persistence probabilities of raw, (2) trend removed, (3) demeaned fluctuations. The estimated trend regressions and the mean of fluctuations are: $485.1 + 15.1 \cdot t$ and 14.9 for real consumption, $0.0076 + 5.5e - 5 \cdot t$ and 5.5e-5 for real p.c. consumption, $131.6 + 3.5 \cdot t$ and 1.5e-5 fir real investment, $0.0019 + 1.2e - 5 \cdot t$ and 3.88 for real p.c. investment.

Table 1.5: **Consumption and Investment During Business Cycles**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Real Consumption: 1947:Q1 - 1994:Q4 [GCQ]													
Exp	0.944	0.913	0.887	0.887	0.871	0.868	0.872	0.877	0.873	0.870	0.881	0.877	0.873
	(0.018)	(0.028)	(0.040)	(0.040)	(0.045)	(0.046)	(0.046)	(0.046)	(0.047)	(0.048)	(0.048)	(0.048)	(0.049)
Rec	0.786	0.571	0.571	0.571	0.500	0.500	0.519	0.560	0.560	0.560	0.560	0.560	0.560
	(0.070)	(0.056)	(0.056)	(0.056)	(0.071)	(0.071)	(0.066)	(0.072)	(0.072)	(0.072)	(0.072)	(0.072)	(0.072)
Real Per Capita Consumption: 1947:Q1 - 1994:Q4 [GCQ/P16]													
Exp	0.883	0.832	0.794	0.780	0.735	0.689	0.676	0.685	0.704	0.703	0.709	0.708	0.706
	(0.028)	(0.037)	(0.042)	(0.044)	(0.055)	(0.065)	(0.072)	(0.069)	(0.067)	(0.068)	(0.069)	(0.070)	(0.070)
Rec	0.786	0.643	0.536	0.393	0.286	0.250	0.259	0.280	0.280	0.280	0.280	0.280	0.280
	(0.046)	(0.084)	(0.100)	(0.065)	(0.074)	(0.068)	(0.070)	(0.079)	(0.079)	(0.079)	(0.079)	(0.079)	(0.079)
Real Investment: 1947:Q1 - 1994:Q4 [GIFQ]													
Exp	0.846	0.783	0.744	0.698	0.542	0.430	0.365	0.342	0.352	0.348	0.358	0.369	0.381
	(0.030)	(0.042)	(0.046)	(0.047)	(0.058)	(0.063)	(0.071)	(0.071)	(0.070)	(0.067)	(0.066)	(0.065)	(0.066)
Rec	0.929	0.821	0.750	0.607	0.214	0.107	0.037	0.040	0.040	0.040	0.040	0.040	0.040
	(0.059)	(0.083)	(0.089)	(0.109)	(0.077)	(0.036)	(0.032)	(0.033)	(0.033)	(0.033)	(0.033)	(0.033)	(0.033)
Real Per Capita Investment: 1947:Q1 - 1994:Q4 [GIFQ/P16]													
Exp	0.858	0.776	0.725	0.704	0.542	0.391	0.331	0.267	0.246	0.239	0.224	0.231	0.238
	(0.032)	(0.039)	(0.044)	(0.041)	(0.054)	(0.058)	(0.065)	(0.067)	(0.068)	(0.065)	(0.067)	(0.068)	(0.068)
Rec	0.929	0.929	0.786	0.679	0.286	0.107	0.037	0.000	0.000	0.000	0.000	0.000	0.000
	(0.059)	(0.059)	(0.101)	(0.130)	(0.105)	(0.036)	(0.031)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

[a] Exp and Rec denotes expansions and recessions.

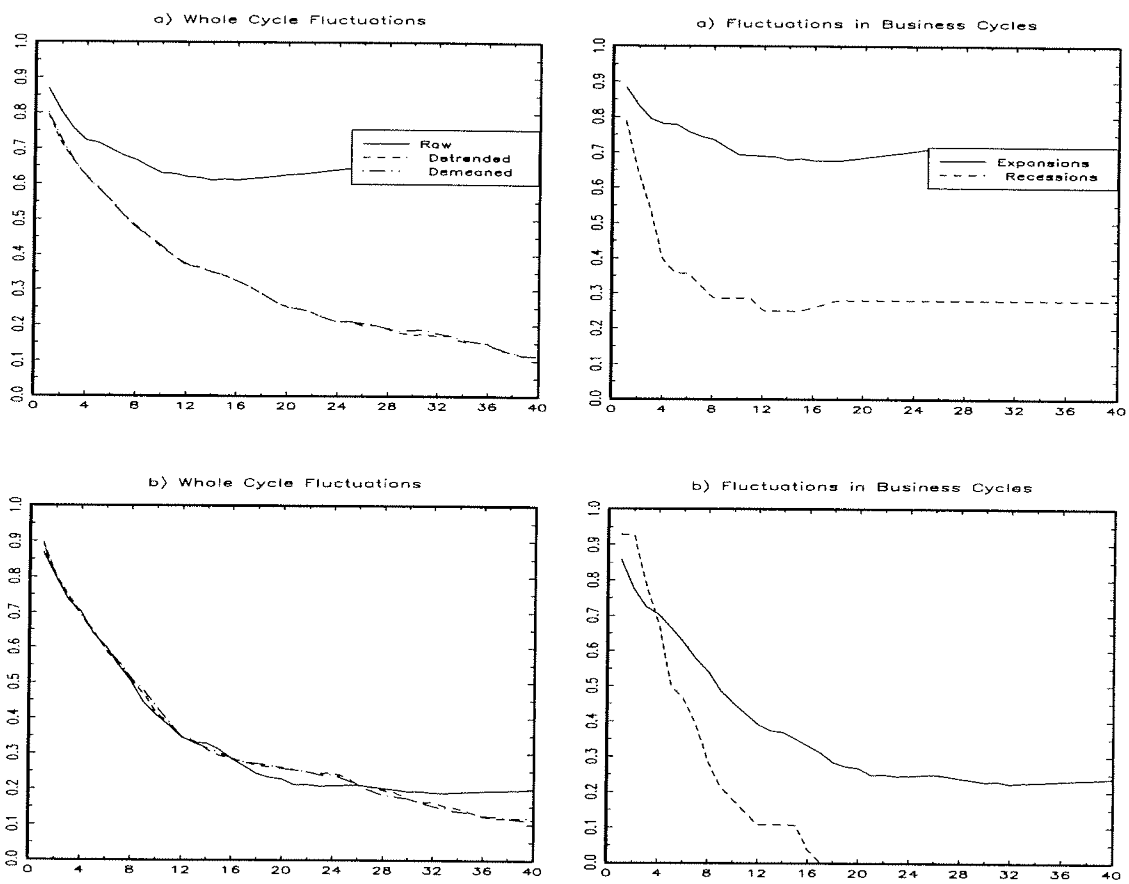

Figure 1.5: (a) Consumption, (b) Investment

positively related to monetary accommodation of price shocks. Barsky (1987) shows that if inflation is persistent, then ex-post Fisher effects (which refers to the hypothesis that changes in expected inflation should be reflected in nominal interest rates) are more likely to be observed. Therefore, if inflation has become less persistent in the 1980's, the relationship between interest rates and inflation has changed, moreover the lagged values of inflation will have less predictive power for future inflation.

The results are presented in Table 1.6 and left panels of Figure 1.6 . The inflation is measured by the percent change in the implict price deflator of GNP (nominal GNP divided by real GNP). The raw fluctuations in inflation have 4/5 chance to persist after a quarter. At long forecasting horizon raw fluctuations in inflation rate have 1/5 chance to persist. The fluctuations around linear time trend and the mean level have almost similar probabilities. We also consider the persistence probabilities in consumer and producer prices. The persistence probabilities of fluctuations in consumer prices are higher than the persistence probabilities of producer prices for all forecasting horizons. This suggests that producer prices are more volatile than consumer prices.

The Tables 1.7 and the right hand plots in Figure 1.6 show the persistence probabilities conditional on business cycles. The fluctuations in inflation rate are likely persistent up to two quarters when the economy stays in a recessionist regime. After two quarters, fluctuations occuring in business expansions are more persistent then those occuring during the business recessions. More precisely, the probability that a fluctuation occuring in business expansion persists after two quarters is 73.7 % and in business recessions 75.0 % while after ten years fluctuations in inflation rate are more persistent if the economy is in expansion, 21.5 % against 16.0 %. The same figures are also valid for consumer prices but with certain lags. The fluctuations in consumer prices during a business recessions have higher persistence probabilities than expansions up to four quarters. On the other hand, the persistence probabilities in producer price fluctuations conditional on business expansions are higher than the probabilities when fluctuation occur during the business recessions. It should be noted that, the conditional persistence probabilities in inflation rate are much more lower than the consumer and producer prices.

Money and Interest Rates

Shocks to the quantity of money or other measures of Federal Reserve policy have long been suspected to influence output. Starting from the quantity-theoretic proposition, changes in the stock of money are the main determinant of changes in nominal income. Given significant lags in wage and price adjustments, high and low growth rates in the quantity of money lead to corresponding fluctuations in real economic activity relative to secular trends. Long periods of positive growth rates produce business slowdowns and long periods of negative monetary growth rates lead to depression (Friedman and Schwarz, 1963). Alternative models emphasize the role of real variables and minimize or ignores the influence of monetary shocks : Kydland and Prescott (1983), Long and Plosser (1983), King and Plosser (1984). There are basicly two reasons for this challenge. First, using annual data, Nelson and Plosser (1982) suggest that real disturbances are the primary source of the variance of real output. Second, King and Plosser's measure of high powered money (monetary base unadjusted for changes in required reserve ratios) shows a weak relation to real activity. They find that much of the relation between money and economic activity is between bank deposits and real output. This relationship is interpreted as reverse causation from economic activity to money. Growth of high powered money contributes little to fluctuations in real output.

We examine the fluctuations in real money stock (M2) as an indicator of real money supply fluctuations. The choice seems to be common to many contributions. The results are presented in Table 1.8. The raw fluctuations in real money balances seem to be highly persistent. They have 90.9 % probability to persist after one quarter and more than fifty percent probability after ten years. Fluctuations around deterministic time trend and constant mean level of fluctuations in money balances

Table 1.6: **Prices and Inflation**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Inflation: 1947:Q1 - 1994:Q3 (Annual Growth Rate of GNP deflator [GNP/GNPQ]) [b]													
(1)	0.805	0.739	0.683	0.604	0.466	0.374	0.312	0.283	0.241	0.228	0.227	0.220	0.205
	(0.034)	(0.036)	(0.036)	(0.040)	(0.051)	(0.046)	(0.042)	(0.038)	(0.044)	(0.044)	(0.045)	(0.044)	(0.046)
(2)	0.811	0.723	0.667	0.588	0.444	0.356	0.294	0.265	0.222	0.209	0.214	0.200	0.185
	(0.035)	(0.039)	(0.037)	(0.041)	(0.050)	(0.049)	(0.045)	(0.043)	(0.045)	(0.044)	(0.043)	(0.042)	(0.045)
(3)	0.805	0.734	0.667	0.615	0.461	0.374	0.312	0.277	0.222	0.215	0.221	0.227	0.219
	(0.032)	(0.037)	(0.034)	(0.038)	(0.049)	(0.048)	(0.047)	(0.046)	(0.050)	(0.052)	(0.052)	(0.052)	(0.052)
Consumer Prices: 1947:Q1 - 1994:Q4 [PUNEW] [c]													
(1)	0.932	0.921	0.915	0.904	0.869	0.866	0.863	0.860	0.856	0.853	0.849	0.852	0.848
	(0.028)	(0.030)	(0.033)	(0.038)	(0.055)	(0.056)	(0.056)	(0.057)	(0.057)	(0.057)	(0.058)	(0.059)	(0.059)
(2)	0.958	0.942	0.936	0.920	0.891	0.877	0.857	0.830	0.808	0.791	0.780	0.787	0.795
	(0.018)	(0.023)	(0.025)	(0.032)	(0.047)	(0.052)	(0.059)	(0.067)	(0.072)	(0.076)	(0.079)	(0.077)	(0.078)
(3)	0.958	0.942	0.936	0.920	0.891	0.877	0.857	0.830	0.808	0.791	0.780	0.787	0.795
	(0.018)	(0.023)	(0.025)	(0.032)	(0.047)	(0.052)	(0.059)	(0.067)	(0.072)	(0.076)	(0.079)	(0.077)	(0.078)
Producer Prices: 1946:Q1 - 1994:Q4 [PW] [d]													
(1)	0.861	0.808	0.766	0.749	0.674	0.645	0.615	0.600	0.585	0.575	0.564	0.572	0.587
	(0.033)	(0.047)	(0.053)	(0.057)	(0.074)	(0.078)	(0.085)	(0.088)	(0.090)	(0.090)	(0.091)	(0.091)	(0.089)
(2)	0.871	0.829	0.812	0.796	0.738	0.699	0.665	0.646	0.608	0.581	0.558	0.528	0.516
	(0.034)	(0.040)	(0.045)	(0.049)	(0.064)	(0.072)	(0.079)	(0.080)	(0.086)	(0.091)	(0.094)	(0.099)	(0.098)
(3)	0.876	0.829	0.802	0.785	0.711	0.694	0.648	0.634	0.608	0.575	0.552	0.528	0.523
	(0.030)	(0.040)	(0.046)	(0.050)	(0.069)	(0.075)	(0.084)	(0.085)	(0.089)	(0.094)	(0.096)	(0.099)	(0.099)

[a] Entries in (1) are the persistence probabilities of raw, (2) trend removed, (3) demeaned fluctuations. The estimated trend regressions and the mean of fluctuations are:

[b] $y_t = 2.7084 + 0.0157 \cdot t$ and $E(y_t - y_{t-1}) = -0.0251$.

[c] $y_t = -3.795 + 0.672 \cdot t$ and $E(y_t - y_{t-1}) = 0.6708$.

[d] $y_t = 3.286 + 0.563 \cdot t$ and $E(y_t - y_{t-1}) = 0.5273$.

Table 1.7: **Prices and Inflation During Business Cycles**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Inflation: 1948:Q1 - 1994:Q3 (Annual Growth Rate of GNP deflator [GNP/GNPQ])													
Exp	0.790	0.737	0.690	0.604	0.513	0.404	0.333	0.298	0.248	0.241	0.240	0.232	0.215
	(0.040)	(0.038)	(0.038)	(0.042)	(0.058)	(0.053)	(0.049)	(0.043)	(0.049)	(0.047)	(0.046)	(0.045)	(0.047)
Rec	0.893	0.750	0.643	0.607	0.214	0.214	0.192	0.200	0.200	0.160	0.160	0.160	0.160
	(0.077)	(0.089)	(0.072)	(0.077)	(0.124)	(0.124)	(0.116)	(0.124)	(0.124)	(0.133)	(0.133)	(0.133)	(0.133)
Consumer Prices: 1947:Q1 - 1994:Q4 [PUNEW]													
Exp	0.920	0.907	0.906	0.899	0.877	0.874	0.878	0.870	0.866	0.862	0.858	0.862	0.857
	(0.033)	(0.036)	(0.037)	(0.040)	(0.050)	(0.051)	(0.051)	(0.051)	(0.051)	(0.052)	(0.053)	(0.054)	(0.054)
Rec	0.964	0.964	0.929	0.893	0.786	0.786	0.741	0.760	0.760	0.760	0.760	0.760	0.760
	(0.032)	(0.032)	(0.062)	(0.088)	(0.163)	(0.163)	(0.160)	(0.167)	(0.167)	(0.167)	(0.167)	(0.167)	(0.167)
Producer Prices: 1946:Q1 - 1994:Q4 [PW]													
Exp	0.867	0.824	0.787	0.767	0.698	0.665	0.638	0.627	0.610	0.599	0.587	0.597	0.615
	(0.039)	(0.048)	(0.054)	(0.060)	(0.077)	(0.082)	(0.090)	(0.093)	(0.096)	(0.096)	(0.098)	(0.097)	(0.094)
Rec	0.821	0.714	0.643	0.643	0.536	0.536	0.481	0.440	0.440	0.440	0.440	0.440	0.440
	(0.063)	(0.085)	(0.100)	(0.100)	(0.147)	(0.147)	(0.139)	(0.140)	(0.140)	(0.140)	(0.140)	(0.140)	(0.140)

[a] Exp and Rec denotes expansions and recessions.

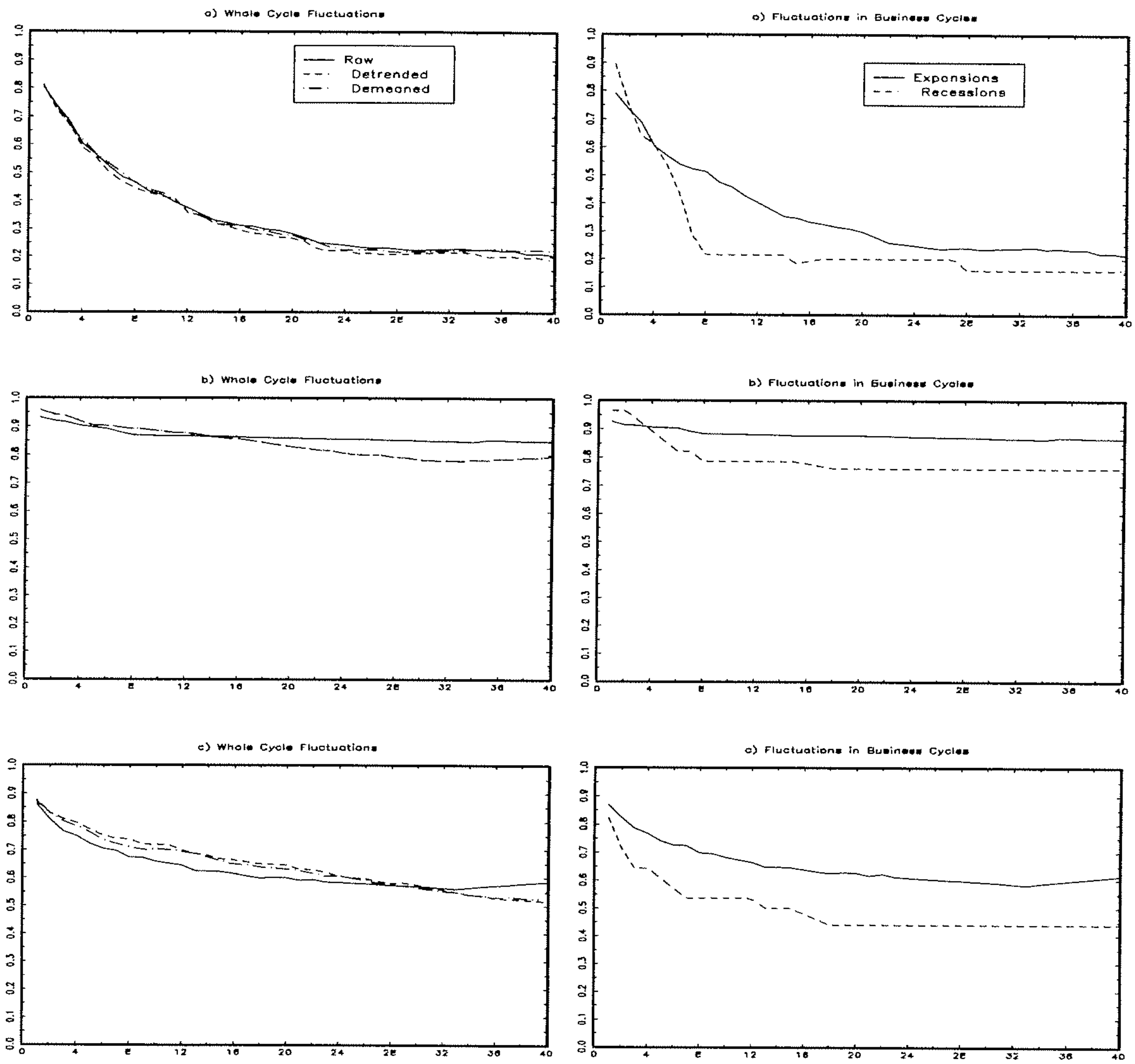

Figure 1.6: (a) Inflation, (b) Consumer Prices, (c) Producer Prices

have surprisingly more persistent than the raw fluctuations from 3 quarters horizon until four years. After four years, raw fluctuations persist more than the detrended and demeaned fluctuations. The situation can also be surveyed from the left panel of Figure 1.7.

The changes in the nominal interest rates are read as signals of changes in the stock of money and inflation. Their adjustments to inflation were sluggish and incomplete to suggest that people treated the nominal rates as if they were appropriate proxies for the real rates. Monetarists opposed to the idea that monetary intervention were seen as causing interest rates to deviate from their equilibrium levels so that they fail to coordinate saving and investment decisions. McCulloch (1977, 1981) argues that business fluctuations are associated with unanticipated changes in the interest rates.

We examine the characteristics of fluctuations in short term nominal interest rates (three-month bill; yields on actively traded issues adjusted to constant maturities) and corporate bond yield (Moody's AAA). In Table 1.8, we see that a raw fluctuation in interest rates has 85.3 % chance to persist in one quarter. At ten years horizon, persistence probability of a raw fluctuation is about 14.6 %. The detrended and demeaned fluctuations show similar features, they approach zero probability at ten years horizon. The probability that fluctuation around the mean or time trend persists after ten years is 6.0 %. The overall pictures of persistence probabilities are displayed in the second row of Figure 1.7. Corporate Bond Yield seems to be more persistent than the short term interest rates at long horizons. The raw fluctuations persist at 30.5 % probability after ten years while the persistence probabilities of demeaned and detrended fluctuations are 13.9 %. The persistence probabilities conditional to cyclical regimes are plotted in the right panels of Figure 1.7. Table 1.8 provides associated return probabilities. The fluctuations occuring during the expansions seem to be more persistent in the long term.

Table 1.8: **Money and Interest Rates**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Real Money Stock (M2): 1948:Q1 - 1994:Q4 [FM2DQ] [b]													
(1)	0.909	0.870	0.804	0.749	0.631	0.571	0.544	0.515	0.521	0.528	0.535	0.536	0.524
	(0.021)	(0.029)	(0.041)	(0.050)	(0.073)	(0.082)	(0.087)	(0.093)	(0.092)	(0.093)	(0.093)	(0.094)	(0.093)
(2)	0.882	0.865	0.815	0.798	0.687	0.589	0.550	0.497	0.442	0.396	0.335	0.311	0.279
	(0.026)	(0.029)	(0.038)	(0.041)	(0.060)	(0.077)	(0.082)	(0.085)	(0.084)	(0.083)	(0.084)	(0.084)	(0.086)
(3)	0.925	0.865	0.810	0.765	0.693	0.600	0.561	0.473	0.448	0.403	0.361	0.344	0.313
	(0.022)	(0.030)	(0.038)	(0.046)	(0.057)	(0.073)	(0.080)	(0.085)	(0.086)	(0.088)	(0.093)	(0.092)	(0.090)
Short Term Interest Rates: 1947:Q1 - 1994:Q4 [FYGM3] [c]													
(1)	0.853	0.746	0.686	0.642	0.470	0.363	0.263	0.216	0.186	0.178	0.157	0.148	0.146
	(0.026)	(0.039)	(0.039)	(0.042)	(0.050)	(0.054)	(0.053)	(0.053)	(0.051)	(0.048)	(0.042)	(0.040)	(0.040)
(2)	0.863	0.725	0.638	0.604	0.426	0.313	0.200	0.164	0.144	0.123	0.107	0.077	0.060
	(0.025)	(0.037)	(0.038)	(0.041)	(0.049)	(0.052)	(0.049)	(0.044)	(0.043)	(0.038)	(0.033)	(0.027)	(0.024)
(3)	0.858	0.720	0.654	0.620	0.437	0.324	0.211	0.181	0.156	0.129	0.119	0.090	0.060
	(0.028)	(0.039)	(0.039)	(0.042)	(0.051)	(0.052)	(0.052)	(0.049)	(0.047)	(0.042)	(0.038)	(0.030)	(0.024)
Bond Yield (Corporate): 1947:Q1 - 1994:Q4 [FYBAAC] [d]													
(1)	0.842	0.751	0.686	0.642	0.508	0.441	0.366	0.351	0.335	0.325	0.321	0.323	0.305
	(0.028)	(0.031)	(0.036)	(0.040)	(0.051)	(0.058)	(0.065)	(0.067)	(0.069)	(0.070)	(0.069)	(0.070)	(0.069)
(2)	0.853	0.741	0.691	0.647	0.530	0.430	0.343	0.322	0.287	0.227	0.201	0.161	0.139
	(0.028)	(0.035)	(0.039)	(0.038)	(0.047)	(0.054)	(0.059)	(0.063)	(0.063)	(0.060)	(0.058)	(0.056)	(0.050)
(3)	0.832	0.741	0.665	0.604	0.497	0.408	0.320	0.298	0.269	0.239	0.208	0.181	0.139
	(0.030)	(0.033)	(0.039)	(0.043)	(0.048)	(0.053)	(0.057)	(0.058)	(0.057)	(0.057)	(0.057)	(0.057)	(0.053)

[a] Entries in (1) are the persistence probabilities of raw, (2) trend removed, (3) demeaned fluctuations. The estimated trend regressions and the mean of fluctuations are:

[b] $y_t = 725.1 + 12.2 \cdot t$ and $E(y_t - y_{t-1}) = 9.385$.

[c] $y_t = 1.32 + 0.04 \cdot t$ and $E(y_t - y_{t-1}) = 0.025$.

[d] $y_t = 2.662 + 0.053 \cdot t$ and $E(y_t - y_{t-1}) = 0.032$.

Table 1.9: **Money and Interest Rates During Business Cycles** [a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Real Money Stock (M2): 1948:Q1 - 1994:Q4 [FM2DQ]													
Exp	0.911	0.879	0.814	0.761	0.662	0.592	0.556	0.528	0.536	0.545	0.554	0.556	0.541
	(0.024)	(0.030)	(0.045)	(0.054)	(0.073)	(0.081)	(0.086)	(0.094)	(0.093)	(0.093)	(0.093)	(0.094)	(0.093)
Rec	0.893	0.821	0.750	0.679	0.464	0.464	0.481	0.440	0.440	0.440	0.440	0.440	0.440
	(0.046)	(0.065)	(0.072)	(0.107)	(0.129)	(0.129)	(0.140)	(0.124)	(0.124)	(0.124)	(0.124)	(0.124)	(0.124)
Short Term Interest Rates: 1947:Q1 - 1994:Q4 [FYGM3]													
Exp	0.840	0.745	0.688	0.648	0.471	0.358	0.270	0.233	0.197	0.188	0.164	0.154	0.151
	(0.030)	(0.043)	(0.043)	(0.046)	(0.060)	(0.066)	(0.058)	(0.061)	(0.057)	(0.054)	(0.047)	(0.045)	(0.045)
Rec	0.929	0.750	0.679	0.607	0.464	0.393	0.222	0.120	0.120	0.120	0.120	0.120	0.120
	(0.039)	(0.085)	(0.087)	(0.092)	(0.127)	(0.158)	(0.138)	(0.100)	(0.100)	(0.100)	(0.100)	(0.100)	(0.100)
Bond Yield (Corporate): 1947:Q1 - 1994:Q4 [FYBAAC]													
Exp	0.827	0.745	0.688	0.642	0.516	0.470	0.385	0.363	0.359	0.348	0.343	0.346	0.325
	(0.031)	(0.036)	(0.042)	(0.046)	(0.061)	(0.065)	(0.075)	(0.077)	(0.077)	(0.078)	(0.077)	(0.079)	(0.078)
Rec	0.929	0.786	0.679	0.643	0.464	0.286	0.259	0.280	0.200	0.200	0.200	0.200	0.200
	(0.040)	(0.087)	(0.078)	(0.092)	(0.064)	(0.093)	(0.089)	(0.095)	(0.088)	(0.088)	(0.088)	(0.088)	(0.088)

[a] Exp and Rec denotes expansions and recessions.

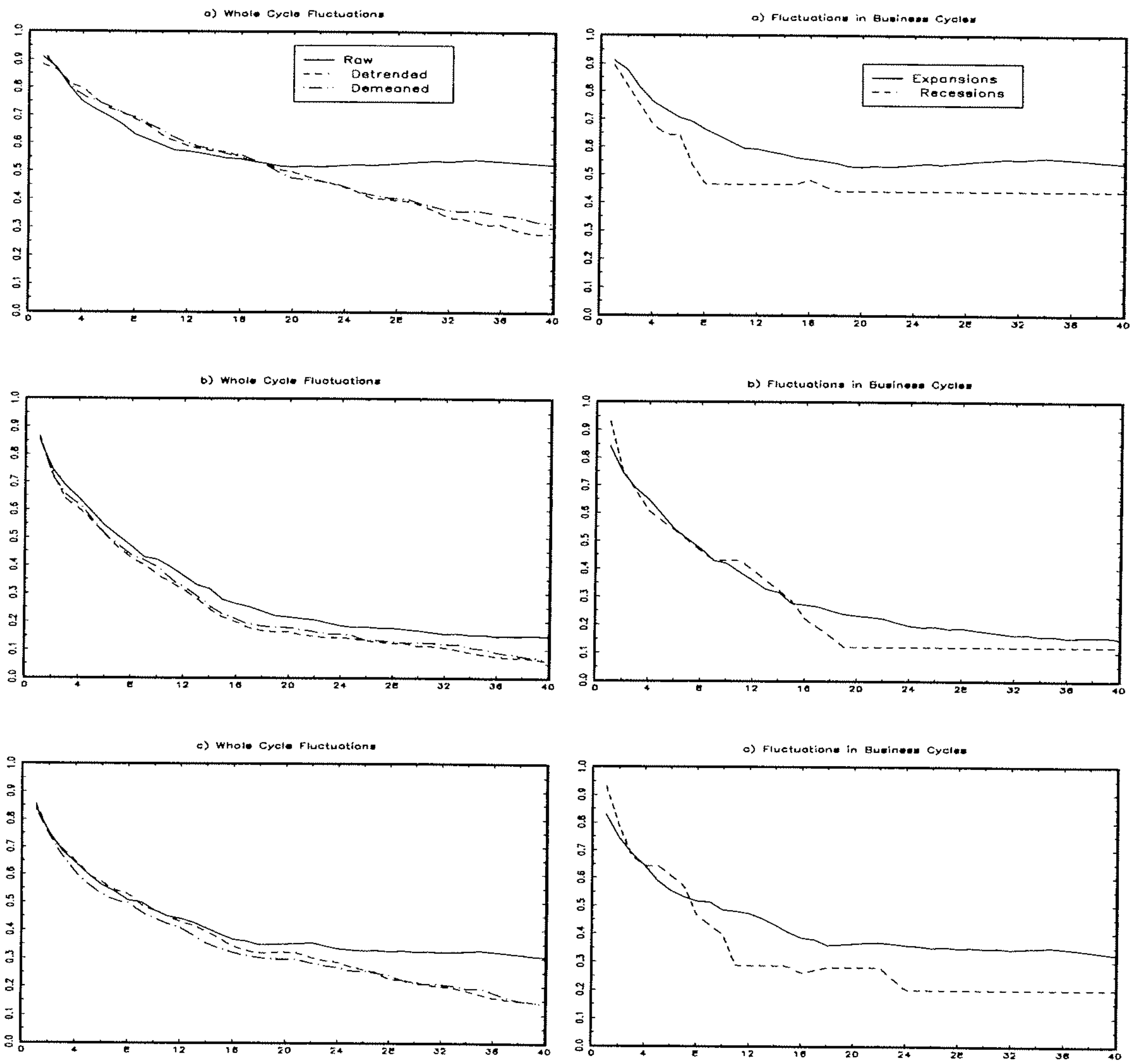

Figure 1.7: (a) Money, (b) Interest Rates, (c) Bond Yield

Labor Market and Employment

Real wages discussed in this section are money wages deflated by the consumer price index, not by the producer price index of current output of labor. The wage in terms of consumables is of prime interest to workers, whereas the wage in terms of the products of work (product wage rate) is of prime interest to firms. Producer prices tend to vary more than consumer prices, if money wages are less flexible than producer prices but more flexible than CPI, then the real wages deflated by PPI could turn to be counter-cyclical and the CPI deflated real wages procyclical.

Table 1.10 shows the persistence probabilities in employment, unemployment rate, real and nominal wages fluctuations. The raw fluctuations in employment have 93.7 % chance to persist after one quarter. In the long term the probability that a current fluctuation does not return is more than 1/2. The similarities between detrended and demeaned fluctuations are again apparant. The persisence probability of demeaned or detrended fluctuations is less than 1/4 over long horizon (10 years) while in the short-run fluctuations around the mean seem to be more persistent.

Financial Markets

A number of recent models of stock market behavior yield the prediction of stock returns, far from being unpredictable, should exhibit negative autocorrelation over long time horizons, that they should be mean reverting. These models use the techniques to examine time dependence in expected returns, or equivalently, the presence of temporary components in stock prices. Fama and French (1988a) and Poterba and Summers (1988) found some evidence that lagged returns forecast future returns. Richardson (1989) and others have argued that the apparent forecast power of lagged returns is statistically insignificant. However Fama and French (1988b) find that other variables, and the divident/price ratio in particular, are strong and statistically significant predictors of future returns.

A striking feature of financial indicators is that they lead the business cycles, Moore and Zarnowitz (1994). The Table 1.13 and right hand panel of Figure 1.9

Table 1.10: **Labor Markets and Employment**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Employment: 1947:Q1 - 1994:Q4 [LPNAG]													
(1)	0.937	0.873	0.824	0.775	0.639	0.603	0.611	0.626	0.617	0.607	0.597	0.587	0.576
	(0.021)	(0.030)	(0.039)	(0.049)	(0.078)	(0.084)	(0.083)	(0.084)	(0.084)	(0.084)	(0.083)	(0.083)	(0.082)
(2)	0.911	0.841	0.771	0.717	0.590	0.486	0.400	0.357	0.311	0.294	0.277	0.277	0.245
	(0.024)	(0.029)	(0.032)	(0.039)	(0.048)	(0.062)	(0.071)	(0.070)	(0.068)	(0.067)	(0.071)	(0.073)	(0.065)
(3)	0.895	0.825	0.771	0.701	0.585	0.480	0.383	0.333	0.299	0.282	0.283	0.265	0.245
	(0.024)	(0.030)	(0.034)	(0.043)	(0.050)	(0.061)	(0.072)	(0.070)	(0.069)	(0.070)	(0.072)	(0.070)	(0.064)
Unemployment Rate: 1947:Q1 - 1994:Q4 [LHMUR]													
(1)	0.821	0.745	0.700	0.654	0.529	0.424	0.374	0.308	0.252	0.215	0.168	0.157	0.146
	(0.032)	(0.036)	(0.036)	(0.041)	(0.052)	(0.060)	(0.061)	(0.061)	(0.061)	(0.058)	(0.051)	(0.052)	(0.053)
(2)	0.821	0.720	0.688	0.648	0.523	0.424	0.361	0.322	0.273	0.215	0.176	0.181	0.179
	(0.032)	(0.036)	(0.037)	(0.042)	(0.052)	(0.063)	(0.063)	(0.062)	(0.063)	(0.059)	(0.060)	(0.061)	(0.062)
(3)	0.821	0.739	0.706	0.667	0.529	0.424	0.381	0.315	0.266	0.215	0.168	0.157	0.146
	(0.032)	(0.033)	(0.033)	(0.039)	(0.052)	(0.061)	(0.061)	(0.063)	(0.062)	(0.058)	(0.051)	(0.052)	(0.053)
Real Wages: 1947:Q1 - 1994:Q4 [LEHM/PUNEW]													
(1)	0.821	0.725	0.686	0.663	0.623	0.570	0.543	0.520	0.503	0.491	0.478	0.458	0.470
	(0.027)	(0.039)	(0.047)	(0.050)	(0.057)	(0.068)	(0.077)	(0.083)	(0.084)	(0.083)	(0.083)	(0.086)	(0.085)
(2)	0.800	0.709	0.676	0.652	0.596	0.525	0.486	0.456	0.443	0.429	0.415	0.406	0.397
	(0.032)	(0.038)	(0.040)	(0.044)	(0.050)	(0.064)	(0.071)	(0.072)	(0.070)	(0.067)	(0.066)	(0.069)	(0.068)
(3)	0.795	0.704	0.676	0.647	0.590	0.525	0.480	0.444	0.431	0.417	0.403	0.394	0.384
	(0.032)	(0.039)	(0.041)	(0.044)	(0.050)	(0.061)	(0.070)	(0.073)	(0.070)	(0.068)	(0.066)	(0.069)	(0.069)
Nominal Wages: 1947:Q1 - 1994:Q4 [LEHM]													
(1)	0.958	0.958	0.952	0.952	0.951	0.950	0.949	0.947	0.946	0.945	0.943	0.942	0.940
	(0.020)	(0.020)	(0.024)	(0.024)	(0.024)	(0.024)	(0.025)	(0.025)	(0.025)	(0.025)	(0.025)	(0.026)	(0.026)
(2)	0.916	0.899	0.894	0.882	0.852	0.849	0.834	0.807	0.796	0.785	0.792	0.800	0.808
	(0.032)	(0.038)	(0.039)	(0.042)	(0.054)	(0.058)	(0.066)	(0.075)	(0.080)	(0.084)	(0.082)	(0.081)	(0.082)
(3)	0.932	0.910	0.904	0.893	0.863	0.849	0.846	0.825	0.814	0.791	0.792	0.800	0.808
	(0.028)	(0.034)	(0.036)	(0.040)	(0.050)	(0.062)	(0.065)	(0.071)	(0.077)	(0.083)	(0.083)	(0.082)	(0.084)

[a] Entries in (1) are the persistence probabilities of raw, (2) trend removed, (3) demeaned fluctuations. The estimated trend regressions and the mean of fluctuations are : $36009.496 + 392.103 \cdot t$ and 372.82 for employment, $4.284 + 0.012 \cdot t$ and 0.0004 for unemployment rate, $0.0659 + 0.0001 \cdot t$ and 0.0001 for real wages, $-0.762 + 0.061 \cdot t$ and 0.058 for nominal wages.

Table 1.11: **Labor Markets and Employment During Business Cycles** [a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Employment: 1947:Q1 - 1994:Q4 [LPNAG]													
Exp	0.938	0.870	0.844	0.824	0.742	0.715	0.723	0.733	0.725	0.717	0.709	0.700	0.690
	(0.024)	(0.039)	(0.044)	(0.049)	(0.069)	(0.078)	(0.077)	(0.078)	(0.078)	(0.078)	(0.079)	(0.079)	(0.078)
Rec	0.929	0.893	0.714	0.500	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.035)	(0.058)	(0.083)	(0.105)	(0.068)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Unemployment Rate: 1954:Q1 - 1994:Q4 [LHMUR]													
Exp	0.793	0.712	0.659	0.613	0.489	0.380	0.341	0.282	0.217	0.190	0.143	0.130	0.115
	(0.035)	(0.037)	(0.043)	(0.049)	(0.056)	(0.064)	(0.064)	(0.063)	(0.061)	(0.062)	(0.052)	(0.052)	(0.051)
Rec	1.000	0.955	0.955	0.909	0.773	0.682	0.571	0.474	0.474	0.368	0.316	0.316	0.316
	(0.000)	(0.048)	(0.048)	(0.055)	(0.088)	(0.116)	(0.109)	(0.128)	(0.128)	(0.145)	(0.159)	(0.159)	(0.159)
Real Wages: 1947:Q1 - 1994:Q4 [LEHM/PUNEW]													
Exp	0.821	0.745	0.706	0.692	0.658	0.603	0.568	0.548	0.535	0.522	0.507	0.485	0.500
	(0.033)	(0.042)	(0.049)	(0.051)	(0.055)	(0.070)	(0.080)	(0.089)	(0.087)	(0.086)	(0.086)	(0.091)	(0.089)
Rec	0.821	0.607	0.571	0.500	0.429	0.393	0.407	0.360	0.320	0.320	0.320	0.320	0.320
	(0.060)	(0.104)	(0.118)	(0.123)	(0.152)	(0.171)	(0.177)	(0.182)	(0.195)	(0.195)	(0.195)	(0.195)	(0.195)
Nominal Wages: 1947:Q1 - 1994:Q4 [LEHM]													
Exp	0.969	0.969	0.969	0.969	0.968	0.967	0.966	0.966	0.965	0.964	0.963	0.962	0.960
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)
Rec	0.893	0.893	0.857	0.857	0.857	0.857	0.852	0.840	0.840	0.840	0.840	0.840	0.840
	(0.071)	(0.071)	(0.099)	(0.099)	(0.099)	(0.099)	(0.100)	(0.102)	(0.102)	(0.102)	(0.102)	(0.102)	(0.102)

[a] Exp and Rec denotes expansions and recessions.

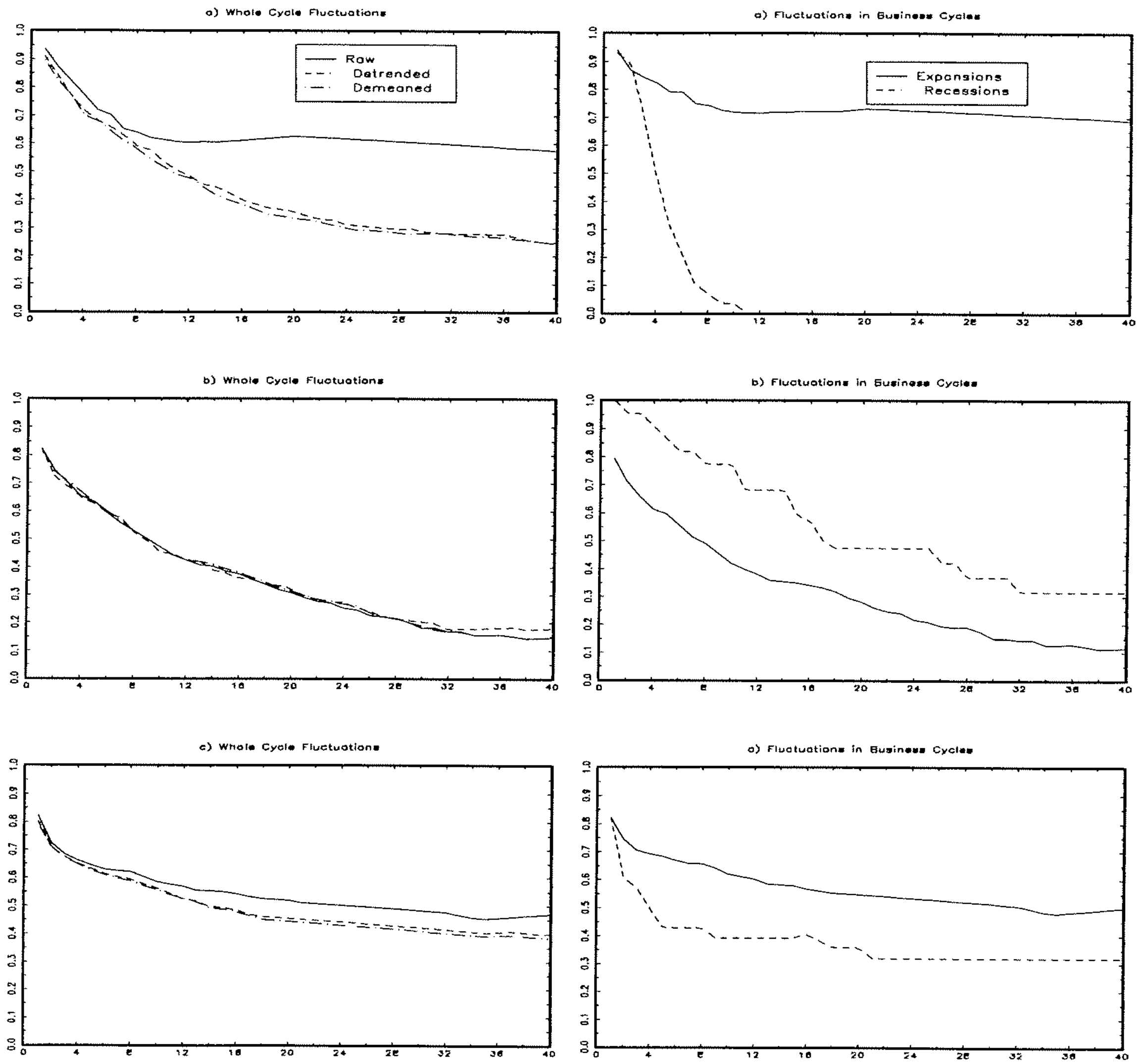

Figure 1.8: (a) Employment, (b) Unemployment Rate, (c) Real Wages

Table 1.12: **Financial Markets**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
S&P Common Stock Price Index: 1947:Q1 - 1994:Q4 [FSPCOM] [b]													
(1)	0.832	0.735	0.654	0.599	0.475	0.425	0.394	0.404	0.389	0.368	0.365	0.348	0.331
	(0.026)	(0.035)	(0.040)	(0.048)	(0.059)	(0.063)	(0.069)	(0.070)	(0.069)	(0.070)	(0.072)	(0.068)	(0.066)
(2)	0.842	0.762	0.697	0.647	0.536	0.453	0.423	0.433	0.431	0.411	0.415	0.400	0.377
	(0.029)	(0.035)	(0.043)	(0.051)	(0.061)	(0.071)	(0.077)	(0.077)	(0.078)	(0.080)	(0.081)	(0.080)	(0.079)
(3)	0.826	0.757	0.686	0.663	0.552	0.508	0.491	0.503	0.497	0.479	0.484	0.471	0.457
	(0.034)	(0.039)	(0.048)	(0.048)	(0.063)	(0.069)	(0.072)	(0.072)	(0.074)	(0.076)	(0.077)	(0.078)	(0.078)
Common Stock Prices: Dow Jones: 1947:Q1 - 1994:Q4 [FSDJ] [c]													
(1)	0.821	0.725	0.654	0.588	0.454	0.391	0.343	0.339	0.323	0.301	0.302	0.290	0.272
	(0.031)	(0.034)	(0.042)	(0.047)	(0.052)	(0.057)	(0.063)	(0.067)	(0.065)	(0.066)	(0.066)	(0.064)	(0.061)
(2)	0.811	0.714	0.649	0.626	0.492	0.425	0.360	0.363	0.341	0.331	0.333	0.303	0.272
	(0.031)	(0.036)	(0.043)	(0.046)	(0.051)	(0.052)	(0.061)	(0.063)	(0.062)	(0.062)	(0.063)	(0.060)	(0.060)
(3)	0.837	0.735	0.676	0.642	0.552	0.508	0.474	0.480	0.479	0.466	0.465	0.439	0.404
	(0.028)	(0.038)	(0.043)	(0.050)	(0.057)	(0.063)	(0.068)	(0.068)	(0.069)	(0.071)	(0.071)	(0.072)	(0.075)
NYSE Volume: 1947:Q1 - 1994:Q4 [FSVOL] [d]													
(1)	0.721	0.587	0.479	0.455	0.383	0.363	0.331	0.333	0.335	0.344	0.352	0.342	0.338
	(0.029)	(0.031)	(0.029)	(0.031)	(0.038)	(0.040)	(0.045)	(0.045)	(0.045)	(0.045)	(0.046)	(0.045)	(0.045)
(2)	0.832	0.746	0.686	0.668	0.617	0.609	0.600	0.602	0.605	0.620	0.635	0.645	0.629
	(0.034)	(0.049)	(0.063)	(0.065)	(0.076)	(0.079)	(0.084)	(0.085)	(0.087)	(0.085)	(0.083)	(0.083)	(0.089)
(3)	0.805	0.762	0.702	0.690	0.617	0.615	0.606	0.608	0.623	0.632	0.648	0.652	0.642
	(0.038)	(0.048)	(0.061)	(0.063)	(0.080)	(0.082)	(0.086)	(0.088)	(0.086)	(0.086)	(0.084)	(0.086)	(0.090)

[a] Entries in (1) are the persistence probabilities of raw, (2) trend removed, (3) demeaned fluctuations. The estimated trend regressions and the mean of fluctuations are:

[b] $y_t = -46.7053 + 1.820 \cdot t$ and $E(y_t - y_{t-1}) = 2.328$.

[c] $y_t = -212.1269 + 13.474 \cdot t$ and $E(y_t - y_{t-1}) = 19.017$.

[d] $y_t = -1162.934 + 23.333 \cdot t$ and $E(y_t - y_{t-1}) = 32.986$.

Table 1.13: **Financial Markets During Business Cycles** [a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
S&P Common Stock Price Index: 1947:Q1 - 1994:Q4 [FSPCOM]													
Exp	0.840	0.752	0.688	0.635	0.510	0.457	0.426	0.432	0.415	0.391	0.388	0.369	0.349
	(0.029)	(0.038)	(0.041)	(0.046)	(0.060)	(0.068)	(0.075)	(0.076)	(0.075)	(0.075)	(0.077)	(0.073)	(0.070)
Rec	0.786	0.643	0.464	0.393	0.286	0.250	0.222	0.240	0.240	0.240	0.240	0.240	0.240
	(0.066)	(0.095)	(0.080)	(0.083)	(0.091)	(0.107)	(0.113)	(0.113)	(0.113)	(0.113)	(0.113)	(0.113)	(0.113)
Common Stock Prices: Dow Jones: 1947:Q1 - 1994:Q4 [FSDJ]													
Exp	0.827	0.733	0.681	0.616	0.484	0.411	0.358	0.349	0.331	0.304	0.306	0.292	0.270
	(0.031)	(0.036)	(0.042)	(0.044)	(0.052)	(0.058)	(0.066)	(0.070)	(0.068)	(0.069)	(0.070)	(0.066)	(0.062)
Rec	0.786	0.679	0.500	0.429	0.286	0.286	0.259	0.280	0.280	0.280	0.280	0.280	0.280
	(0.058)	(0.041)	(0.058)	(0.061)	(0.103)	(0.103)	(0.104)	(0.108)	(0.108)	(0.108)	(0.108)	(0.108)	(0.108)
NYSE Volume: 1947:Q1 - 1994:Q4 [FSVOL]													
Exp	0.704	0.565	0.450	0.428	0.342	0.318	0.284	0.288	0.289	0.297	0.306	0.292	0.286
	(0.030)	(0.039)	(0.035)	(0.036)	(0.042)	(0.044)	(0.050)	(0.050)	(0.051)	(0.051)	(0.052)	(0.051)	(0.050)
Rec	0.821	0.714	0.643	0.607	0.607	0.607	0.593	0.600	0.600	0.600	0.600	0.600	0.600
	(0.056)	(0.060)	(0.077)	(0.083)	(0.083)	(0.083)	(0.080)	(0.083)	(0.083)	(0.083)	(0.083)	(0.083)	(0.083)

[a]Exp and Rec denotes expansions and recessions.

show the persistence probabilities in S&P composite stock price index, Dow Jones industrial average and NYSE reported share volume conditional to business cycle regimes. The fluctuations in stock prices occuring in business expansions persist more likely than those occuring during the business recessions. In contrast, the fluctuations in shared volume in New York Stok Exchange Market have more chance to persist during business recessions than business expansions. A fortiori, stock prices, measured either by S&P index or by Dow Jones average have decreasing persistence probabilities as long as the prediction horizon gets larger. The fluctuations in shared volume have 3/5 chance to persist after five years, while the persistence probabilities stabilizes at around 1/5 after five years.

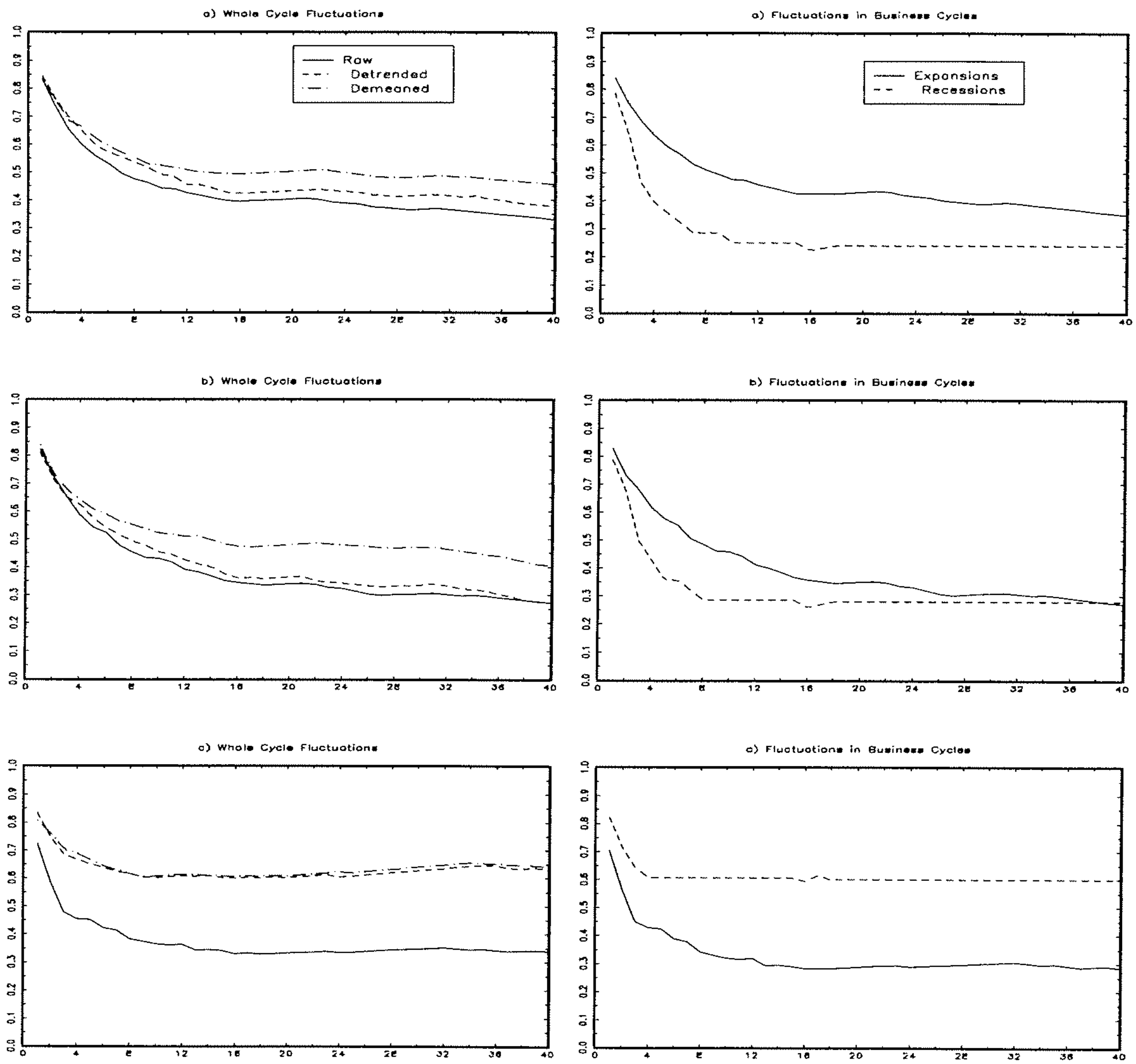

Figure 1.9: (a) S&P Prices, (b) Dow Jones, (c) NYSE Volume

Production and Productivity

The literature investigating the trend properties of economic time series has focused almost entirely on variables such as GNP or aggregate industrial production. In this section, we consider three key variables in the industrial production sector: industrial production index, capacity utilization rate and the productivity of labor. It has been stressed that the fluctuations in aggregate industrial production show greater persistence than dissagregated industrial production indices. Instead of considering the industries individually, we prefer to give a general idea about the persistence degree of the fluctuations in the total industry. Two other related quantities are also included. First, we consider a very procyclical series, capacity utilization rate in manufacturing industry. Second, we consider the labor productivity, by dividing the industrial production by the employee-hours in nonagricultural sectors. This gives a rough measure of the industrial production per employee-hour. Table 1.14 and Figure 1.10 present the results from our estimates.

In the remaining part of this section, we compare the performance of our persistence measure based on return probabilities of current fluctuation with the variance-ratio measure proposed by Cochrane (1988). The comparison is established via the ranking of series according to the extent that they persist at long horizons. Since, both persistence measures are invariant with respect to the monotonic transformation of the series, the persistence ranking is a way that provides an idea about the divergence and similarities between measures. In analyzing the ranking of series, we have dropped, the nominal quantities, such as the GNP in current prices and the wages in manufacturing industry. Population growth was also eliminated by considering only the per capita measure of consumption and investment expenditures. Part A of the Table 1.16 reports the ranking based on return probabilities of eighteen series, up to ten years. The part B reports the ranking based on Cochrane variance ratio procedures. The variance ratio at one quarter horizon are not ranked since they are all equal to unity for all series.

Regarding the return probabilities $d^+(h)$, Table 1.16 shows that the real per

Table 1.14: **Production and Productivity**[a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Industrial Production: 1947:Q1 - 1994:Q4 [IP] [b]													
(1)	0.858	0.767	0.697	0.668	0.514	0.430	0.423	0.433	0.443	0.448	0.434	0.432	0.424
	(0.030)	(0.042)	(0.048)	(0.052)	(0.074)	(0.078)	(0.079)	(0.078)	(0.078)	(0.079)	(0.078)	(0.079)	(0.079)
(2)	0.816	0.730	0.681	0.631	0.481	0.402	0.326	0.281	0.240	0.227	0.208	0.181	0.172
	(0.025)	(0.031)	(0.030)	(0.035)	(0.045)	(0.054)	(0.059)	(0.058)	(0.053)	(0.052)	(0.050)	(0.048)	(0.050)
(3)	0.816	0.730	0.681	0.631	0.481	0.402	0.326	0.281	0.240	0.227	0.208	0.181	0.172
	(0.025)	(0.031)	(0.030)	(0.035)	(0.045)	(0.054)	(0.059)	(0.058)	(0.053)	(0.052)	(0.050)	(0.048)	(0.050)
Capacity Utilization Rate: 1947:Q1 - 1994:Q4 [IPXMCA] [c]													
(1)	0.844	0.768	0.723	0.656	0.492	0.394	0.287	0.263	0.227	0.201	0.161	0.119	0.122
	(0.026)	(0.031)	(0.037)	(0.038)	(0.049)	(0.051)	(0.055)	(0.058)	(0.054)	(0.049)	(0.041)	(0.036)	(0.036)
(2)	0.849	0.773	0.723	0.661	0.492	0.394	0.287	0.263	0.221	0.201	0.161	0.119	0.122
	(0.026)	(0.031)	(0.037)	(0.039)	(0.051)	(0.053)	(0.056)	(0.058)	(0.054)	(0.050)	(0.042)	(0.034)	(0.035)
(3)	0.860	0.778	0.728	0.661	0.497	0.406	0.292	0.269	0.233	0.208	0.168	0.126	0.129
	(0.025)	(0.029)	(0.036)	(0.037)	(0.049)	(0.052)	(0.056)	(0.059)	(0.056)	(0.051)	(0.043)	(0.038)	(0.038)
Labor Productivity: 1947:Q1 - 1994:Q4 [IP/LPHMU] [d]													
(1)	0.842	0.741	0.654	0.588	0.437	0.346	0.297	0.304	0.305	0.313	0.321	0.323	0.331
	(0.033)	(0.037)	(0.043)	(0.048)	(0.065)	(0.068)	(0.072)	(0.072)	(0.071)	(0.071)	(0.071)	(0.072)	(0.072)
(2)	0.811	0.714	0.660	0.599	0.432	0.358	0.286	0.269	0.246	0.202	0.201	0.206	0.192
	(0.030)	(0.034)	(0.037)	(0.041)	(0.055)	(0.056)	(0.063)	(0.060)	(0.058)	(0.055)	(0.055)	(0.056)	(0.054)
(3)	0.811	0.714	0.660	0.599	0.437	0.358	0.286	0.269	0.246	0.196	0.195	0.200	0.185
	(0.030)	(0.034)	(0.037)	(0.041)	(0.054)	(0.056)	(0.063)	(0.060)	(0.058)	(0.054)	(0.053)	(0.055)	(0.053)

[a] Entries in (1) are the persistence probabilities of raw, (2) trend removed, (3) demeaned fluctuations. The estimated trend regressions and the mean of fluctuations are:

[b] $y_t = 14.9434 + 0.512 \cdot t$ and $E(y_t - y_{t-1}) = 0.513$.

[c] $y_t = 83.950 - 0.0196 \cdot t$ and $E(y_t - y_{t-1}) = 0.003$.

[d] $y_t = 0.2673 + 0.0016 \cdot t$ and $E(y_t - y_{t-1}) = 0.002$.

Table 1.15: **Production and Productivity During Business Cycles** [a]

	Forecasting Horizon												
	1	2	3	4	8	12	16	20	24	28	32	36	40
Industrial Production: 1947:Q1 - 1994:Q4 [IP]													
Exp	0.858	0.758	0.706	0.686	0.594	0.510	0.500	0.507	0.521	0.529	0.515	0.515	0.508
	(0.035)	(0.049)	(0.053)	(0.057)	(0.073)	(0.078)	(0.080)	(0.079)	(0.078)	(0.079)	(0.079)	(0.080)	(0.081)
Rec	0.857	0.821	0.643	0.571	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.041)	(0.041)	(0.057)	(0.077)	(0.053)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Capacity Utilization Rate: 1947:Q1 - 1994:Q4 [IPXMCA]													
Exp	0.823	0.739	0.686	0.632	0.497	0.401	0.306	0.282	0.239	0.209	0.162	0.119	0.123
	(0.031)	(0.036)	(0.042)	(0.044)	(0.058)	(0.057)	(0.064)	(0.067)	(0.063)	(0.056)	(0.046)	(0.041)	(0.041)
Rec	0.964	0.929	0.929	0.786	0.464	0.357	0.185	0.160	0.160	0.160	0.160	0.120	0.120
	(0.035)	(0.055)	(0.055)	(0.086)	(0.098)	(0.085)	(0.062)	(0.051)	(0.051)	(0.051)	(0.051)	(0.056)	(0.056)
Labor Productivity: 1947:Q1 - 1994:Q4 [IP/LPHMU]													
Exp	0.827	0.745	0.662	0.610	0.484	0.391	0.338	0.342	0.345	0.355	0.366	0.369	0.381
	(0.037)	(0.042)	(0.048)	(0.052)	(0.069)	(0.075)	(0.080)	(0.080)	(0.078)	(0.078)	(0.078)	(0.079)	(0.078)
Rec	0.929	0.714	0.607	0.464	0.179	0.107	0.074	0.080	0.080	0.080	0.080	0.080	0.080
	(0.042)	(0.060)	(0.085)	(0.105)	(0.063)	(0.048)	(0.056)	(0.057)	(0.057)	(0.057)	(0.057)	(0.057)	(0.057)

[a] Exp and Rec denotes expansions and recessions.

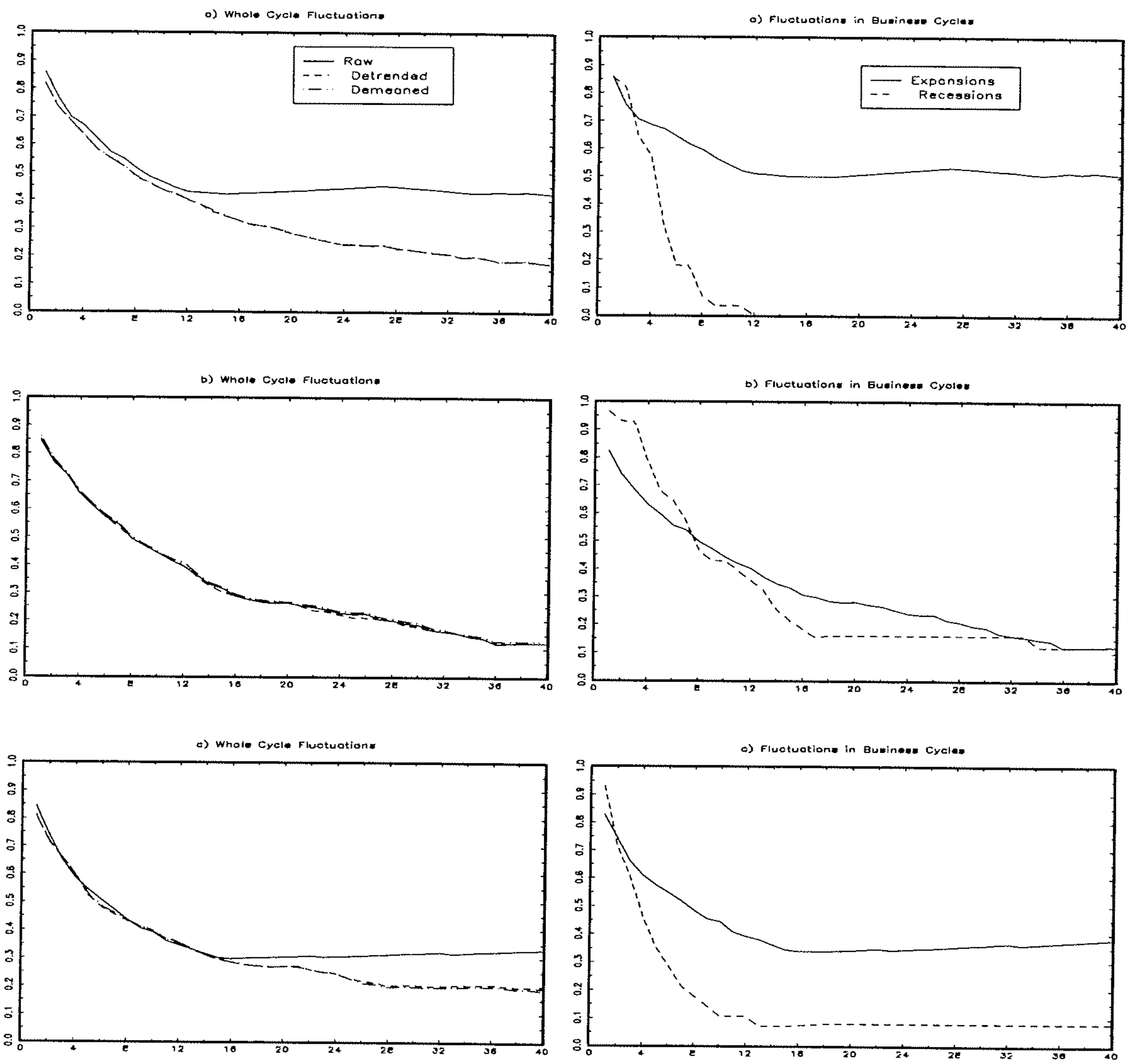

Figure 1.10: (a) Industrial Production, (b) Capacity Utilization, (c) Labor Productivity

capita consumption, prices (consumer and producer prices), real money stock (M2), employment are the most persistent series of our sample. On the other hand, we see that inflation, short term interest rates, unemployment rate, labor productivity and financial indicators, S&P stock price index, Dow Jones industrial index and NYSE volume and labor productivity are the least persistent series. With the exception of the fixed real investment, the ranking of series seems approximately stable up to ten years horizon. In the case of real per capita investment expenditures, the series being in 5 to 7th position, ranked after the consumption expenditures, up to one year horizon, fall dramatically to the end of the ranking during the business cycle frequencies (2 to 6 years) and reaches 15th position at 10 years horizon. The real per capita GNP and the industrial production stays in the upper middle of the ranking for all horizon. The inflation appears to be less persistent than the unemployement rate up to 6 years horizon, but at long term, fluctuations in inflation persist more than the unemployement rate.

Turning to ranking based on Cochrane's variance-ratio procedure, we see that the above results does not hold in general. The prices (both consumer and producer), real money balances, employement have again the most persistent fluctuations. The rankings between consumption and investment expenditures have been reversed; at short horizon, investments are more persistent than consumption, but after 3 years horizon, the fluctuations in consumption persist more than those in investment, contradicting the permanent income hypothesis. Another interesting results is the propagation mechanism related to the fluctuations in financial indicators. The stock prices and volume having relatively transitory fluctuations at short horizon exhibit a high volatility in their persistence rankings and becomes the most persistent series. The situation can be explained by the presence of long memory in financial indicators. It is worth to note that the same property is not valid for the rankings of Table 1.16.

In Table 1.17, we turn our attention to the ranking of persistence probabilitites $d^{+}(h)$ conditional to the business cycle regimes; expansions and recessions identified by the quarterly NBER turning point chronologies. We do not provide similar rankings for the variance-ratio procedures. The ranking in Table 1.17 shows comparative persistence of fluctuations during business cycles. There are few series for which the ranking are equal during the expansions and recession periods. For example, the real per capita GNP is ranked between 6th and 10th place during the expansions while during recessions, the rankings changes between 6th and 18th.

1.7 Conclusion

The evaluation of the persistence of economic fluctuations (shocks) is an important research area in macroeconomics. Most controversy has focused on whether macroeconomic aggregates are better approximated by fluctuations around deterministic trend, or by a random walk plus a stationary component. The empirical results from these studies are ambiguous and it seems probable that the macroeconomic aggregates belong to a much wider class of processes than previously considered. In this paper, we have proposed persistence (non-persistence) measures which depend on the probability of sign reversion (non-reversion) between present and h periods ahead fluctuations in a given time series. In other terms, we measure the persistence (non-persistence) of a fluctuation by estimating the probability that a current fluctuation is not cancelled (is cancelled) by future fluctuations. This way of assessing the permanence to the economic fluctuations is entirely new in time series analysis. The basic contribution of this paper is the association of a probability measure to the evolution of fluctuations in forecasting horizon. More precisely, the persistence (non-persistence) measures proposed in this paper attribute a special attention to the flows of future fluctuations. The future effect of today's impulse in a given time series can be reduced or aggravated in a given horizon. Consequently, the persistence or non-persistence probabilities of a time series may be low or high depending on the flow of future fluctuations.

Hencefort, the rejection or non-rejection of permanence or transitory components

Table 1.16: **Persistence Ranking of Raw Fluctuations** [a]

Series	Forecasting Horizon 1	2	3	4	8	12	16	20	24	28	32	36	40
A. Ranking Based on Return Probabilities													
Real Per Capita GNP	9	4	8	7	7	7	7	7	8	8	8	8	8
Real P.C. Consumption	4	6	5	5	3	3	3	2	2	2	2	2	2
Real P.C. Investment	5	7	6	6	11	18	17	17	17	16	15	15	15
Inflation	17	14	14	14	15	14	14	15	15	14	14	14	14
Consumer Prices	2	1	1	1	1	1	1	1	1	1	1	1	1
Producer Prices	6	5	4	3	2	2	2	4	4	4	4	4	3
Real Money Stock	3	3	3	4	5	5	5	6	5	5	5	5	5
Short Term Int. Rate	8	11	12	13	14	16	18	18	18	18	18	17	17
Bond Yield	11	10	11	12	10	8	11	10	10	11	11	11	12
Employment	1	2	2	2	4	4	4	3	3	3	3	3	4
Unemployment Rate	16	12	9	11	8	11	10	13	14	15	16	16	16
Real Wages	14	17	13	9	6	6	6	5	6	6	6	6	6
S&P Stock Prices	13	15	17	15	13	10	9	9	9	9	9	9	11
Dow Jones	15	16	15	16	16	13	12	11	12	13	13	13	13
NYSE Volume	18	18	18	18	18	15	13	12	11	10	10	10	9
Industrial Production	7	9	10	8	9	9	8	8	7	7	7	7	7
Capacity Utilization	10	8	7	10	12	12	16	16	16	17	17	18	18
Labor Productivity	12	13	16	17	17	17	15	14	13	12	12	12	10
B. Ranking Based on Cochrane's Variance Ratio													
Real Per Capita GNP	-	11	10	10	9	7	7	9	9	9	9	9	10
Real P.C. Consumption	-	16	14	13	11	10	8	10	10	10	10	10	11
Real P.C. Investment	-	6	6	6	7	8	12	13	16	17	17	17	17
Inflation	-	10	11	12	17	18	18	18	17	16	16	16	15
Consumer Prices	-	1	1	1	1	1	1	1	1	1	1	1	1
Producer Prices	-	5	4	4	2	2	2	2	2	2	2	2	2
Real Money Stock	-	3	3	3	3	3	3	4	4	4	4	4	6
Short Term Int. Rate	-	14	17	17	16	14	14	14	15	15	15	15	16
Bond Yield	-	9	7	7	6	6	6	6	6	6	7	8	8
Employment	-	2	2	2	4	5	5	5	5	5	5	6	5
Unemployment Rate	-	4	5	5	8	9	11	11	11	11	11	12	13
Real Wages	-	12	12	9	5	4	4	3	3	3	3	3	3
S&P Stock Prices	-	17	15	15	15	12	9	7	7	7	6	5	4
Dow Jones	-	15	16	16	14	13	10	8	8	8	8	7	7
NYSE Volume	-	18	18	18	18	17	15	15	14	12	12	11	9
Industrial Production	-	7	8	8	10	11	13	12	13	14	14	14	14
Capacity Utilization	-	8	9	11	12	15	17	17	18	18	18	18	18
Labor Productivity	-	13	13	14	13	16	16	16	12	13	13	13	12

[a]The ranking of a series is high if the persistence probability is high.

Table 1.17: **Persistence Ranking During Business Cycles** [a]

Series		Forecasting Horizon 1	2	3	4	8	12	16	20	24	28	32	36	40
Real Per Capita GNP	Exp	10	6	6	6	7	7	7	7	8	8	8	7	8
	Rec	9	6	14	11	15	16	17	18	17	17	17	17	17
Real P.C. Consumption	Exp	4	4	4	3	3	3	3	3	3	3	2	2	2
	Rec	17	17	16	17	12	11	8	8	7	7	7	7	7
Real P.C. Investment	Exp	7	7	7	7	9	15	15	17	15	15	15	15	14
	Rec	8	4	4	4	11	15	15	15	16	15	15	15	15
Inflation	Exp	17	15	10	17	11	12	14	13	14	14	14	14	15
	Rec	10	10	10	9	14	13	12	11	10	12	12	11	11
Consumer Prices	Exp	2	1	1	1	1	1	1	1	1	1	1	1	1
	Rec	3	1	3	2	1	1	1	1	1	1	1	1	1
Producer Prices	Exp	5	5	5	4	4	4	4	4	4	4	4	4	4
	Rec	15	14	11	7	4	4	4	4	4	3	3	3	3
Real Money Stock	Exp	3	2	3	5	5	6	6	6	5	5	5	5	5
	Rec	11	8	5	5	8	5	5	5	5	4	4	4	4
Short Term Int. Rate	Exp	8	11	12	10	17	17	18	18	18	18	16	16	16
	Rec	4	11	7	8	5	6	10	13	13	13	13	12	12
Bond Yield	Exp	11	12	11	11	10	9	10	10	10	11	11	11	11
	Rec	5	9	8	6	7	9	7	7	11	10	10	10	10
Employment	Exp	1	3	2	2	2	2	2	2	2	2	3	3	3
	Rec	7	5	6	14	18	17	16	17	15	16	16	18	18
Unemployment Rate	Exp	16	17	17	15	14	16	12	15	17	17	18	17	18
	Rec	1	2	1	1	2	2	3	3	3	5	6	6	6
Real Wages	Exp	15	10	9	8	6	5	5	5	6	7	7	8	7
	Rec	13	18	15	13	9	7	6	6	6	6	5	5	5
S&P Stock Prices	Exp	9	9	13	12	12	10	9	9	9	9	9	10	10
	Rec	18	16	18	18	10	12	11	10	9	9	9	9	9
Dow Jones	Exp	13	16	15	14	15	11	11	11	12	12	13	13	13
	Rec	16	15	17	16	13	10	9	9	8	8	8	8	8
NYSE Volume	Exp	18	18	18	18	18	18	17	14	13	13	12	12	12
	Rec	14	12	12	10	3	3	2	2	2	2	2	2	2
Industrial Production	Exp	6	8	8	9	8	8	8	8	7	6	6	6	6
	Rec	12	7	9	12	17	18	18	16	18	18	18	16	16
Capacity Utilization	Exp	14	14	14	13	13	13	16	16	16	16	17	18	17
	Rec	2	3	2	3	6	8	13	12	12	11	11	13	13
Labor Productivity	Exp	12	13	16	16	16	14	13	12	11	10	10	9	9
	Rec	6	13	13	15	16	14	14	14	14	14	14	14	14

[a] Exp and Rec denotes expansions and recessions. The ranking of a series is high if the persistence probability is high.

in a given time series may be responded in terms of probability measures. Namely, it is possible to say how much today's fluctuation is more likely persistent (transient) in finite forecasting horizon. Our persistence probabilities are not only simple to compute but they are also quantitatively invariant with respect to monotonically increasing transformation of the data. This property is particularly important, when one wish to compare the persistence degree of economic aggregates. Traditional dissatisfaction with the conventional methods, for instance from the impulse-response functions by Campbell and Mankiw (1988) and the variance ratio procedures by Cochrane (1988) leads the probabilistic approach more attractive.

We found that a today's fluctuation in real GNP has more than 1/2 chance to persist after one year and less than 2/5 chance to persist after ten years. We do not find a quantitative difference between the fluctuations around linear time trend and the mean of fluctuations. For the latters, fluctuations have nearly 1/2 chance to persist after one year and less than 1/4 chance after ten years. The fluctuations in real GNP appear to be more persistent when the economy is in business expansions. The probability that a fluctuation which occurs during the business recessions persist after three years is statistically equal to zero. Regarding the individual time series, we found different persistent degrees than the variance-ratio procedures in the sense that, the persistence rankings altered considerably.

Table 1.18: **Citibase Data Definitions**[a]

M	F	P	N	Definition
FM2DQ	M	1948:1-1995:1	565	Money Supply M2 (Bil. 1987 $) BCI-106.
FSDJ	M	1947:1-1995:2	578	Common Stock Prices: Dow Jones Industrial Average.
FSPCOM	M	1947:1-1995:2	578	S&P Common Stock Price Index: Composite (1941-43=100).
FSVOL	M	1947:1-1995:1	577	Stock Market: NYSE Reported Share Volume (Mil. of Shares, NSA).
FYBAAC	M	1947:1-1995:1	577	Bond Yield: Moody's BAA Corporate (% Per Annum).
FYGM3	M	1947:1-1995:1	577	Interest Rate: U.S. Treasury Bills, Sec. Mkt. 3mo. (% Per Annum, NSA).
GCQ	Q	1947:1-1994:4	192	Personal Consumption Expenditures (Bil. 1987 $) (T.1.2).
GIFQ	Q	1947:1-1994:4	192	Gross Private Domestic Investment: Fixed Investment (Bil. 1987 $) (T.1.2).
GNP	Q	1946:1-1994:3	195	Gross National Product, Total.
GNPQ	Q	1947:1-1994:3	191	Gross National Product, (Bil. 1987 $) (T.10).
IP	M	1947:1-1995:1	577	Industrial Production: Total Index (1987=100, SA).
IPXMCA	M	1948:1-1995:1	565	Capacity Utilization Rate: Manufacturing Total (% of Capacity, SA) (FRB).
LEHM	M	1947:1-1995:1	577	Average Hr. Earnings of Prod. Workers: Manufacturing ($, SA).
LHMUR	M	1954:1-1995:1	493	Unemployment Rate: Men, 20 Yeas & Over (%, SA).
LPHMU	M	1947:1-1995:1	577	Employee-hours in nonagric. est. (Bil. hours, SAAR).
LPNAG	M	1947:1-1995:1	577	Employees on Nonagr. Payrolls, Total (Thous., SA).
P16	M	1947:1-1994:12	576	Population: Total Civilian Noninstitutional, (Thous., NSA).
PUNEW	M	1947:1-1995:1	577	CPI-U: All Items (1982-84=100, SA).
PW	M	1946:1-1995:1	589	Producer Price Index: All Commodities (1982=100, NSA).

[a]The series was obtained from 26 September 1995 release of Citibase. The Citibase mnemonics are in the first column. Nobs. is the number of observations. The abbreviations are; FRB: Board of Governors of the Federal Reserve System, SA: Seasonally Adjusted, NSA: Not Seasonally Adjusted.

Gauss Code For Persistence Probabilities

```
PROC(1) = persist(x,hmax,m,dfact,staton,state);
local   w,cneg,cpos,dneg,dpos,cumd,r,h,j,cn,cp,dn,dp,dcn,sdc,dure;

@
persistence probabilities and standard deviations, dufour & sarlan (1996).
 ~ypnosis
 . w } = persist(x,hmax,m,dfact,staton,state);

input requirements
y        : (n.1) vector of data.
hmax     : forecasting horizon
m        : window width in smoothing Newey-West variance estimator
dfact    : constant, deflating the h-period fluctuation by a factor "dfact".
state    : (n.1) vector of binary state variable. The vector takes values 1 if data
           are in one state, say expansion and 0 if data are in other state, say
           recession.
staton   : activates the state conditionning. if staton = 1, the state variable is  "on"
           otherwise the state variable is "off".

outputs
           W (hmax,11) matrix of persistence probabilities and their heteroscedastic and
           autocorrelation consistent standard errors.
           The columns of the  matrix W contain the following quantities.
W[.,1:2]         :  c-(h) and its standard deviation, for h=1,2,...,hmax.
W[.,3:4]         :  c+(h) and its standard deviation, for h=1,2,...,hmax.
W[.,5:6]         :  d-(h) and its standard deviation, for h=1,2,...,hmax.
W[.,7:8]         :  d+(h) and its standard deviation, for h=1,2,...,hmax.
W[.,9:10]        :  D-(h) and its standard deviation, for h=1,2,...,hmax.
W[.,11]          :  c-(h) and its standard deviation, for h=2,3,...,hmax.
@

@       window weights                                              @

 ` = 1-seqa(1,1,m)/(m+1);

@       initialize the estimators                                   @

cneg = zeros(hmax,2);
cpos = zeros(hmax,2);
dneg = zeros(hmax,2);
dpos = zeros(hmax,2);
cumd = zeros(hmax,2);
dure = zeros(hmax,1);

h = 1; do while h <= hmax;

@       compute the return indicator r(h)                           @


r   = (lagn(x,-h)-lag1(x)-(h+1)*dfact).*(x-lag1(x)-dfact);
r = packr(r);

@       check for state variable                                    @

if staton == 1;
r = selif(r,trimr(state,1,h));
endif;
@       compute the persistence indicators                          @
cn   = r .le 0;                    @ non persistence c-(h) @
cp   = r .gt 0;                    @ persistence     c+(h) @
r = rows(cn);
 f h == 1;
un   = cn;                         @ non persistence d-(1) @
dp   = cp;                         @ persistence     d+(1) @
sdc = dn;
dcn = sdc;                         @ non persistence D-(1) @
else;
dn   = dp[1:r].*cn;                @ non persistence d-(h) @
```

```
dp  = dp[1:r].*cp;                @ persistence     d+(h) @
sdc = sdc[1:r]+dn;
dcn = sdc;                        @ non persistence D-(h) @
endif;

@       mean and variance of persistence indicators     @

cneg[h,.] = meanc(cn)~vcx(cn);
cpos[h,.] = meanc(cp)~vcx(cp);
dneg[h,.] = meanc(dn)~vcx(dn);
dpos[h,.] = meanc(dp)~vcx(dp);
cumd[h,.] = meanc(dcn)~vcx(dcn);
dure[h]   = sumc(dp);

@       newey-west standard errors                      @

j = 1; do while j <= m;
cneg[h,2] = cneg[h,2]+2*w[j]*
            sumc((cn[1:r-j]-cneg[h,1]).*(cn[1+j:r]-cneg[h,1]))/r;
cpos[h,2] = cpos[h,2]+2*w[j]*
            sumc((cp[1:r-j]-cpos[h,1]).*(cp[1+j:r]-cpos[h,1]))/r;
dneg[h,2] = dneg[h,2]+2*w[j]*
            sumc((dn[1:r-j]-dneg[h,1]).*(dn[1+j:r]-dneg[h,1]))/r;
dpos[h,2] = dpos[h,2]+2*w[j]*
            sumc((dp[1:r-j]-dpos[h,1]).*(dp[1+j:r]-dpos[h,1]))/r;
cumd[h,2] = cumd[h,2]+2*w[j]*
            sumc((dcn[1:r-j]-cumd[h,1]).*(dcn[1+j:r]-cumd[h,1]))/r;
j = j + 1; endo;
h = h + 1; endo;

cneg[.,2] = sqrt(cneg[.,2]/r);
cpos[.,2] = sqrt(cpos[.,2]/r);
dneg[.,2] = sqrt(dneg[.,2]/r);
dpos[.,2] = sqrt(dpos[.,2]/r);
cumd[.,2] = sqrt(cumd[.,2]/r);
w = cneg~cpos~dneg~dpos~cumd~dure;
retp(w);
endp;

PROC (0) = prt1(w);
"PERSISTENCE PROBABILITIES and STANDARD DEVIATIONS";
"";
"   h      c-    (sc-)    c+    (sc+)    d-    (sd-)    d+    (sd+)    D-    (sD-)";
"  -----  -----  -----  -----  -----  -----  -----  -----  -----  -----  -----";
format /rdn 7,3;
seqa(1,1,rows(w))~w[.,1:10];
endp;

PROC (0) = prt2(w);
"DEPENDENCE PROBABILITIES ";
"   h     dd+    ";
"  -----  -----  ";
format /rdn 7,3;
seqa(2,1,rows(w)-1)~w[2:rows(w),11]./w[1:rows(w)-1,11];
endp;

PROC (2) = detrend(x);
local n,z,b;
n = rows(x);
z = ones(n,1)~seqa(1,1,n);
b = invpd(z'z)*(z'x);
retp(b, x-z*b);
endp;

PROC (1) = demean(x);
retp(meanc(packr(x-lag1(x))));
endp;
```

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Chapter 2

Nonparametric Persistence Analysis of Irregularly Spaced Observations with Application to High Frequency Foreign Exchange Rates

(in collaboration with Jean-Marie Dufour)

2.1 Introduction

The objective of this essay is to analyze the persistence of fluctuations in unequally spaced observations. In earlier work (Dufour and Sarlan, 1996), we proposed to measure the persistence of economic fluctuations in discrete time series by looking at the probability that a given fluctuation not be cancelled by subsequent fluctuations (over different horizons). According to this approach, persistence is measured by the probability that the sign of present and future fluctuations are reversed (or not) at different forecasting horizons. We argue that this way of assessing the permanence of economic fluctuations is more realistic than traditional persistence measures where future shocks are set to zero, which can be irrelevant under the structure of an econometric model.

We extend our analysis to the case where the fluctuations are unequally spaced with random dates. This situation arises frequently with macroeconomic and financial observations. In this context, the results in Dufour and Sarlan (1996) are not directly applicable because the numbers and dates of observations are themselves random processes. Instead, we propose a notion of persistence (non-return) probability at different horizons and define natural estimators for these probabilities. Proposed measure of persistence is invariant with respect to monotonically increasing transformation of data. Furthermore, it does not require arbitrary data discretisation procedures.

A striking example of such observations is microdata on quotes in worldwide foreign exchange market. The distributional characteristics of daily and lower frequency returns in this market have been the subject of a very large literature. Now the increasing availability of tick-by-tick data has stimulated interest in analysing intraday variation in returns and volatility. The popular example of this data is the Olsen & Associates "HFDF93" data set compiled from the interbank Reuters network for the period October 1, 1992 to September 29, 1993. These data include tick-by-tick bid and ask quotes for the Deutsche Mark - U.S. Dollar, Japanese Yen - U.S. Dollar, and Japanese Yen - Deutsche Mark currencies. The interbank foreign exchange market has an interesting feature which consists in allowing investement opportunities around the world. For instance, when American markets (say New York) are closed, speculators look at quotes from Tokyo and Frankfurt markets. The worldwide market stays open 24/24 hours and 7/7 days. The observations (bid/ask) are electronically recorded up to the nearest second. The time distance between observations are then unevenly spaced. Similar to our probabilistic approach to the persistence of economic fluctuations (Dufour and Sarlan, 1996) speculators acting in worldwide foreign exchange markets are viewed as gamblers who take decisions to buy or sell currencies by looking regularly at bid and ask quotes. A fluctuation of an exchange rate (initial capital invested in the game) persists at a given horizon d if the price level does not return to its current level before d periods in the future.

We found that fluctuations in USD/DM have a probability of barely 1/10 to survive more than five minutes. On the other hand the exchange rates USD/JY and JY/DM are much less volatile in the sense that five-minute survival probabilities of fluctuations are 2/10 and 4/10. After 90 working days, these probabilities do not tend to zero which support the hypothesis that the market does have a long memory : Ding, Granger and Engle (1993). We also consider conditional probability measures. First, by conditioning on the sign of fluctuations, we obtain a measure of asymmetry in exchange rate movements. Typically, the USD/DM rate appears more persistent in response to positive fluctuations, while the USD/JY and JY/DM rates are more persistent to negative fluctuations. On the other hand, conditioning on the size of fluctuations provides measures related to the reversion horizon of small or big fluctuations. Our results support the market efficiency hypothesis, in the sense that for all currency rates in our dataset, big jumps seem to be more persistent than small jumps .

The chapter is organized as follows. In Section 2.2, we define probability measures of persistence for unequally spaced observations. We provide probability measures conditional on the sign and size of fluctuations. Section 2.3 deals with estimation of persistence probabilities. In section 2.4, we apply our theoretical results to USD/DM, USD/JY and JY/DM exchange rates. Section 2.5 concludes this chapter.

2.2 Persistence Measures of Unequally Spaced Observations

The measurement of persistence in real time data requires the definition of some preliminary concepts. Consider a sequence $\{y_{t_j}, j = 0, 1, \cdots, N\}$ observed at irregularly spaced time points $t_0, t_1, \cdots, t_j, \cdots, t_N$, where N is the total number of observations up to time t_N, with the convention $t_0 = 0$. Let us first define the random variable $\xi(t_j, t_{j+k})$ as the product of the latest fluctuation $y_{t_j} - y_{t_{j-1}}$ with the k observation ahead fluctuation $y_{t_{j+k}} - y_{t_{j-1}}$:

$$\xi(t_j, t_{j+k}) \equiv (y_{t_j} - y_{t_{j-1}})(y_{t_{j+k}} - y_{t_{j-1}}), \text{ for } k = 1, 2, \cdots \tag{2.2.1}$$

for $j = 1, 2, \cdots, N-k$. Depending on whether a given fluctuation $y_{t_j} - y_{t_{j-1}}$ is positive or negative, we will say that the latter has persisted up to time t_{j+k} if the observed values of the process up to time t_{j+k} are all strictly above or below the initial value at time t_{j-1}. The time distance $t_{j+k} - t_j$ between each future observation and the latest one, measures the persistence horizon of the fluctuation $y_{t_j} - y_{t_{j-1}}$. On the other hand, a negative sign of $\xi(t_j, t_{j+k})$ indicates that the fluctuation $y_{t_j} - y_{t_{j-1}}$ has been cancelled by subsequent fluctuations at the horizon $t_{j+k} - t_j$ or earlier. Formally, we will say the fluctuation $y_{t_j} - y_{t_{j-1}}$ has persisted up to horizon d if

$$\xi(t_j, t_{j+k}) > 0, \text{ for all } k \text{ such that } t_{j+k} - t_j \leq d \ ; \tag{2.2.2}$$

we shall denote by $S(t_j, d)$ the latter event. On the other hand, we will say it has been cancelled at horizon d if

$$\begin{aligned} &\xi(t_j, t_{j+l}) > 0, \text{ for } l = 1, 2, \cdots, k-1, \text{ and } \xi(t_j, t_{j+k}) \leq 0, \\ &\qquad \text{for some } k \text{ such that } t_{j+k} - t_j \leq d, \end{aligned} \tag{2.2.3}$$

an event we shall denote by $C(t_j, d)$. In (2.2.2) and (2.2.3), the time horizon d is a measure of duration that can be defined in terms of seconds, minutes, hours or days. Note the timings t_{j+k} and the number of the observations which satisfy $0 < t_{j-1} - t_{j+k} \leq d$ are random, a feature which differentiate this setup from the one considered in Dufour and Sarlan (1996). We wish to study the probabilities that the events $S(t_j, d)$ and $C(t_j, d)$ do or do not occur for different horizons d. Specifically the probability that a fluctuation $y_{t_j} - y_{t_{j-1}}$ persists for at least d periods is

$$\begin{aligned} p(t_j, d) &= \mathrm{P}[S(t_j, d)] \\ &= \mathrm{P}[\xi(t_j, t_{j+k}) > 0, \text{ for all } k \text{ such that } t_{j+k} - t_j \leq d] \end{aligned} \tag{2.2.4}$$

and the probability that $y_{t_j} - y_{t_{j-1}}$ be cancelled at horizon d or earlier is

$$\begin{aligned} q(t_j, d) &= \mathrm{P}[C(t_j, d)] \\ &= \mathrm{P}[\xi(t_j, t_{j+l}) > 0, \text{ for } l = 1, 2, \cdots, k-1, \\ &\quad \text{and } \xi(t_j, t_{j+k}) \leq 0, \text{ for some } k \text{ such that } t_{j+k} - t_j \leq d] \\ &= 1 - \mathrm{P}[S(t_j, d)] = 1 - p(t_j, d). \end{aligned} \tag{2.2.5}$$

In earlier work (Dufour and Sarlan, 1996), we studied the persistence (or cancellation) probabilities of fluctuations given that they have persisted for a given period and we related the resulting conditional probabilities to the duration dependence of macroeconomic fluctuations. Similarly if we start from the probabilities of the events $S(t_j, d)$ and $C(t_j, d)$, it would be interesting to study conditional persistence (cancellation) probabilities for unequally spaced observations. For instance, the probability that a fluctuation at time t_j persists for d periods given that it has persisted up to time t_{j-1} can be written by conditioning the event $S(t_j, d)$ upon $S(t_{j-1}, d)$, *i.e.*

$$\mathrm{P}[S(t_j, d) | S(t_{j-1}, d)] = \frac{\mathrm{P}[S(t_{j-1}, d) \cap S(t_j, d)]}{\mathrm{P}[S(t_{j-1}, d)]} \tag{2.2.6}$$

and for cancellation probabilities

$$\begin{aligned} \mathrm{P}[C(t_j, d) | S(t_{j-1}, d)] &= \frac{\mathrm{P}[S(t_{j-1}, d) \cap C(t_j, d)]}{\mathrm{P}[S(t_{j-1}, d)]} \\ &= 1 - \mathrm{P}[S(t_j, d) | S(t_{j-1}, d)] \end{aligned} \tag{2.2.7}$$

More generally, consider any (measurable) function $g(\cdot)$ of past fluctuations:

$$z_{t_j} = g(y_{t_j} - y_{t_{j-1}}, y_{t_{j-1}} - y_{t_{j-2}}, \cdots, y_{t_1} - y_{t_0}) \tag{2.2.8}$$

where $g(\cdot)$ may be a continous or discontinous. Many types of conditioning can be considered. For instance, it may be of interest to determine the persistence probabilities of positive (as opposed to negative) fluctuations, hence giving insight on the asymmetry of fluctuations. This can be interpreted as conditioning on the sign of fluctuations. Another possibility would consist in considering the size of fluctuations to see whether large jumps are more likely to persist than small jumps.

This issue is closely related to mean reverting properties of macroeconomic time series.

Formally, the sign of a fluctuation $y_{t_j} - y_{t_{j-1}}$ is

$$z_{t_j} = \mathrm{sgn}(y_{t_j} - y_{t_{j-1}}) \tag{2.2.9}$$

for $j = 1, 2, \cdots, N$, where

$$\mathrm{sgn}(x) = \begin{cases} 1 & , \text{if } x > 0 \\ 0 & , \text{if } x = 0 \\ -1 & , \text{if } x < 0. \end{cases} \tag{2.2.10}$$

The effect of the sign of $y_{t_j} - y_{t_{j-1}}$ on persistence may then be measured by the conditional probabilities

$$\mathrm{P}[S(t_j, d) | z_{t_j} = x] = \frac{\mathrm{P}[S(t_j, d) \cap \{z_{t_j} = x\}]}{\mathrm{P}[z_{t_j} = x]} \tag{2.2.11}$$

where $x = -1, 0$ or 1.

Similarly, to study the effect of size on persistence, it is intuitively attractive to measure the latter by absolute relative changes

$$z_{t_j} = |y_{t_j} - y_{t_{j-1}}| / y_{t_j} \tag{2.2.12}$$

for $j = 1, 2, \cdots, N$. The probabilities of $S(t_j, d)$ and $C(t_j, d)$ may then be evaluated conditional on $z_{t_j} > z$, yielding:

$$\mathrm{P}[S(t_j, d) | z_{t_j} > z] = \frac{\mathrm{P}[\{z_{t_j} > z\} \cap S(t_j, d)]}{\mathrm{P}[z_{t_j} > z]} \tag{2.2.13}$$

and

$$\mathrm{P}[C(t_j, d) | z_{t_j} > z] = \frac{\mathrm{P}[\{z_{t_j} > z\} \cap C(t_j, d)]}{\mathrm{P}[z_{t_j} > z]}. \tag{2.2.14}$$

Of course, it is also possible to replace $z_{t_j} > z$ by $z_{t_j} \leq z$ in (2.2.13)-(2.2.14).

2.3 Estimation

Denote the collection of time points at which observations are available by $T = \{t_j : j = 0, 1, \cdots, N, t_0 = 0\}$, where $N + 1$ is the total number of observations and define the indicator

$$\delta[x] = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases} \tag{2.3.1}$$

Than the event $\delta[\xi(t_j, t_{j+k})] = 1$ means that a (non-zero) fluctuation has persisted up to time t_{j+k} (at least), while $\delta[\xi(t_j, t_{j+k})] = 0$ means it has been cancelled at time t_{j+k} (or before). Now let

$$T(t_j, d) = \{\tau \in T : 0 < \tau - t_j \leq d\}, \quad j = 0, 1, \cdots, N \tag{2.3.2}$$

and

$$\Delta^S(t_j, d) = \prod_{\tau \in T(t_j, d)} \delta[\xi(t_j, \tau)]. \tag{2.3.3}$$

Then the event $\Delta^S(t_j, d) = 1$ means that the fluctuation $y_{t_j} - y_{t_{j-1}}$ is non-zero ($y_{t_j} - y_{t_{j-1}} \neq 0$) and has persisted for at least d periods (up to horizon d). Similarly, the event $\Delta^C(t_j, d) = 1$, where

$$\Delta^C(t_j, d) = 1 - \Delta^S(t_j, d) = \prod_{\tau \in T(t_j, d)} (1 - \delta[\xi(t_j, \tau)]), \tag{2.3.4}$$

means the fluctuation $y_{t_j} - y_{t_{j-1}}$ is zero or has been cancelled at the horizon d (or before). The set of points t_j for which we can evaluate $\Delta^S(t_j, d)$ and $\Delta^C(t_j, d)$ is

$$T(d) = \{t_j \in T : \exists \tau \in T(t_j, d)\} \tag{2.3.5}$$

and we denote by $N(d) = \text{card}(T(d))$ the number of elements in $T(d)$. The sets $T(t_j, d)$ and $T(d)$ are random.

Let us now assume the probabilities $p(t_j, d)$ and $q(t_j, d)$ do not depend on t_j. For example this will be the case if the process $y_{t_j} - y_{t_0}$ is strictly stationary as a process on the non-negative integers $Z_0 = \{0, 1, 2, \cdots\}$. Then we can write

$$p(d) = p(t_j, d), \quad q(d) = q(t_j, d) \tag{2.3.6}$$

and it is natural to estimate the persistence probability $p(d)$ by

$$\widehat{p}(d) = \frac{1}{N(d)} \sum_{t_j \in T(d)} \Delta^S(t_j, d) \tag{2.3.7}$$

and the cancellation probability $q(d)$ by

$$\widehat{q}(d) = \frac{1}{N(d)} \sum_{t_j \in T(d)} \Delta^C(t_j, d). \tag{2.3.8}$$

Similarly, under an appropriate stationarity assumption, the conditional probabilities in (2.2.11) or (2.2.13)-(2.2.14) do not depend on t_j and they can be estimated by the corresponding sample analogues. For instance, the estimator of persistence given $z_{t_j} > z$, as in (2.2.13), is

$$\widehat{p}(d|z_{t_j} > z) = \frac{\widehat{p}(d, z_{t_j} > z)}{\widehat{p}(z_{t_j} > z)}. \tag{2.3.9}$$

where

$$\widehat{p}(d, z_{t_j} > z) = \frac{1}{N(d)} \sum_{t_j \in T(d)} \Delta^S(t_j, d)\delta[z_{t_j} - z] \tag{2.3.10}$$

and

$$\widehat{p}(z_{t_j} > z) = \frac{1}{N+1} \sum_{t_j \in T} \delta[z_{t_j} - z]. \tag{2.3.11}$$

We can estimate the probability of persistence given $z_{t_j} \leq z$ by

$$\widehat{p}(d|z_{t_j} \leq z) = \frac{\widehat{p}(d, z_{t_j} \leq z)}{\widehat{p}(z_{t_j} \leq z)} \tag{2.3.12}$$

where

$$\widehat{p}(d, z_{t_j} \leq z) = \frac{1}{N(d)} \sum_{t_j \in T(d)} \Delta^S(t_j, d)(1 - \delta[z_{t_j} - z]) \tag{2.3.13}$$

and

$$\widehat{p}(z_{t_j} \leq z) = \frac{1}{N} \sum_{t_j \in T} (1 - \delta[z_{t_j} - z]) = 1 - \widehat{p}(d|z_{t_j} > z) \tag{2.3.14}$$

Other types of conditional probabilities such as those in (2.2.11) or (2.2.14) can be estimated in a similar way.

2.4 Applications : Interbank Foreign Exchange Markets

The empirical results of this paper deal with the around-the-clock trading in the interbank foreign exchange (FX) market. Understanding the basic characteristics of this market has been the subject of an increasing literature in recent years. However, the distributional properties of exchange rates negociated on this market are less known and seem to offer great challenges to econometricians. The main reason is the availability of tick-by-tick data. In this section, we look at the persistence of fluctuations in the level of exchange rate. A large body of literature is concerned with persistence issues of higher moments particularly on the investigation of potential sources of exchange rates return volatilities. As we shall see further in the following sub-section, these studies sample the unequally spaced observations at equally spaced time instants. In this section, we summarize discussions surrounding the interbank foreign exchange market, describe briefly our data set and interpret the results.

2.4.1 Discussions on High Frequency Exchange Rate Data

The literature using high frequency exchange rate data has become increasingly popular, with most of the attention given to the occurence times of price negociations. For instance, in Bollerslev and Domowitz's [1993] model, the quote arrival times measure the activity level of the market and approximate the arrival of information. They include the lagged values of the quote arrival number and the duration between trades on the conditional volatility of DM/USD exchange rate returns spanned at five minute intervals and find statistically negligible market activity effect. Muller et al [1990] and Dacarogna [1993] present an empirical law which states that the mean absolute price changes over a time interval are proportional to a power of interval size. They find that the price change distributions become increasingly leptokurtic with decreasing time intervals and argue that it is impossible to identify the power of the interval size associated with the characteristic exponent of stable distributions. Goodhard and Figliuoli [1991] documented the first-order autocorrelation in the data especially after jumps in the level of exchange rate, the changing level of activity throughout a 24-hours as well as the time aggregation effects. Goodhard et al. [1993] used eight weeks of data to estimate a GARCH-M model of the sterling-dollar exchange rate. They conclude that news effects to the conditional variance equation had a significant effect. Andersen and Bollerslev [1994] examined the seasonality and volatility persistence for both intraday stock market returns and intraday foreign exchange data. The conclude that conventional seasonal adjustment techniques are less useful on the analysis of high frequency data. They provide detailed summary statistics for five minutes returns and absolute returns. Intraday heteroskedasticity patterns are illustrated using a plot of the average absolute returns for each of the 288 intervals associated with the 24-hours trading day. Eddelbuttel and McCurdy (1997) investigates the impacct of the frequency of general and currency-specified news headlines from the Reuters screen on intraday exchange rate changes.

The probabilistic approach to the persistence of exchange rate fluctuations is not new in the literature of financial economics. For instance, Krasker [1980] argues that market may indeed be concerned with a major disaster with low probability each period that would completely destroy the value of all stocks. In studies of foreign exchange market efficieny, the possibility of such a disaster has been called "peso problem" referring to the fluctuation in the peso forward rate in anticipation of a devaluation that did not occur in the sample period, see also Lewis [1988], Evans and Lewis [1992], Kaminsky [1993] and Lewis [1994].

Computing the return probabilities of high frequency data allows two remarkable advantages. First they do not require *ad hoc* procedures for sampling the observations at equally spaced dates : see for example Wasserfalen and Zimmermann [1985] for a discussion of the bias inherent to periodic sampling procedure of unequally spaced data. On the other hand, Muller et al. (1990), Andersen and Bollerslev (1994), Melvin and Yin (1996), DeGennaro and Shrieves (1995) among many others argue that data contain strong intraday seasonality patterns. Classical measures of persistence, such as those based on autocorrelation functions of asset returns exhibit spurious persistence at these seasonal frequencies. Second, they are invariant with respect to any monotonic nonlinear transformation of observations. For instance, Granger et al. [1993], Andersen and Bollerslev [1995] among many others, document different degrees of persistence of data similar to those used in this paper. They show that real valued powers of fluctuations in prices have different persistences accross their first and higher moments.

2.4.2 Data Issues

The data studied in this paper consists of interbank price negociations (bid/ask) on U.S. Dollars in terms of Deutsch Marks, U.S. Dollars in terms of Japanese Yens and Deutsch Marks in terms of Japanese Yens for a period of a year, from October 1, 1992 to September 30, 1993[1]. One of the striking features of the interbank foreign exchange market is the absence of a specific market place. Instead price propositions (bid/ask) are carried out by means of a worldwide communication network. The communication network stays open without interruption twenty-four hours a day including weekends and holidays. The arrival of quotes is, of course, irregular, depending for example on the opening and closing hours of local markets around the world. The communication network records price negociations individually (one-by-one) on electronic market bulletins up to the nearest second. The total number of price records exceeds thousands of propositions each day. Care is needed in interpreting pricing behavior of banking institutions. Goodhart and Figliuoli [1991] state that the quotes are the price advertisements at which the banks are willing to deal, but may not be representative of true transaction prices. For the potential biases inherent in intraday quotation data for the examination of pricing behavior, see Goodhard and Figliuoli [1991, p. 25-26] and Goodhart et al. [1993]. The

[1]The data have been collected by Reuters and provided by the Swiss instititute Olsen and Associates. More about this dataset may be found in Dacarogna et.al. [1993].

main entries in the dataset consist of the following quantities: *i)* the record time t_j measured up to the nearest second in Greenwich Mean Time, *ii)* the bid prices b_{t_j}, *iii)* the ask prices a_{t_j}, and *iv)* the country and institution codes of recorded prices. Following the notation from the survey by Guillaume et al. [1994] we express the exchange rate at time t_j by the arithmetic mean of bid and ask prices, that is

$$y_{t_j} = \frac{b_{t_j} + a_{t_j}}{2}, \text{ for } j = 1, \cdots, N \tag{2.4.1}$$

where N is the total number of prices in worldwide exchange rate markets. Two advantages of our persistence measures become immediately obvious. First, our persistence measures are invariant with respect to monotonically increasing transformation of the data, hence taking the logarithm of b_{t_j} and a_{t_j} and constructing the logarithmic middle price, see Guillaume [1994], as a price of a foreign currency at time t is not necessary. Secondly, unlike the other studies working on high frequency FX data we are no deal with time scale discretisation. For instance, Muller et al. [1990] advocate interpolating the data from adjacent ticks (bid/ask), or Wasserfallen and Zimmermann [1985] use the most recent price y_{t_j} before time instant t.

The total number of price propositions during this period is impressive: $N = 1,463,896$ for USD/DM, $N = 566,709$ for USD/JY and $N = 158,416$ for DM/JY. The observations on Fridays also include such bias because during the weekends, particularly after 21h00 on Friday until 21h00 Sunday there are relatively few price propositions. These observations are systematically excluded from the sample and the time scale of the remaining observations are rescaled in such a way that after 21h00 Friday the first second becomes 21h00 Sunday.

2.4.3 Empirical Results

Tables 2.1 to 2.3 present the persistence probabilities of fluctuations in USD/DM, USD/JY and JY/DM exchange rates respectively with five persistence probabilities in columns. In the first columns, we indicate the forecasting horizon of fluctuations. They span periods beginning from five minutes (300 seconds) to three months (90 working days of 86400 seconds). The forecasting horizons d are considered as the integer multiples of a minute (60 seconds). This is just for practical purposes. The footnotes at the bottom of Tables provide statistics used in the estimation of probabilities: the number of observations N, the probability of observing positive or negative fluctuations and median of fluctuation sizes. Since zero fluctuations die out immediately with probability one, the sum of up and down fluctuations do not sum to unity exactly.

In the second columns of Tables 2.1 to 2.3, we show the unconditional persistence probabilities. For instance, the fluctuations in the USD/DM exchange rate have 10.6 percent chance to persist after five minutes in the future. In other word, the probability that a level of USD/DM do not return back within the five minutes is about 1/10. After one hour (3600 seconds), the persistence probability is only 2.8 percent. We can state that many fluctuations in the level of USD/DM exchange rate occur at horizons shorter than an hour. For example, the fluctuations surviving more than a working day (86400 seconds) are noticeably low. They reach almost zero probability for horizons longer than three months. Similarly, from the second column of Table 2.2, we notice that fluctuations in USD/JY exchange rate have 19.1 percent and from the Table 2.3 fluctuations in JY/DM rate have 42.1 percent probability to persist after five minutes. These unconditional probabilities show that short term (five minutes) speculations have at least 1/10 chance to be cost-effective given that investors are communicated the value of ascending (positive return) currency. We shall return to this topic.

The fluctuations in USD/DM persist with probability less than 1.0 percent after six working hours. In the case of USD/JY, 1.0 percent persistence is reached at 24 hours horizon and for JY/DM it takes 20 working days that a fluctuation in any sign has a probability of persistence less than a percent. These figures are closely related to the memory content of exchange rate data. As a general tendency, fluctuations in more active markets have a small chance to persist at long horizons.

Some special features of these fluctuations are in our interest. First, we consider the sign of fluctuations as providing a measure of asymmetry for exchange rate movements. Since the exchange rate data measure the value of a parent currency in terms of its competitor, positive fluctuations in relative price mean that investing on parent currency yields non-negative profits against its competitor. Inversely, negative returns mean the gain of the competitor. If the continuous time process is difference stationary, the probability of observing positive return would be equal to observing the negative return. However, the probability that an upward or downward fluctuations persist for a given horizon d may not be equal if the returns in holding the parent (alternative) currency are higher for longer periods than holding the alternative (parent) currency. Formally the sign of a fluctuation $y_{t_j} - y_{t_{j-1}}$ is computed

$$z_{t_j} = \text{sgn}(y_{t_j} - y_{t_{j-1}}) \tag{2.4.2}$$

for $j = 1, 2, \cdots, N$, where

$$\text{sgn}(x) = \begin{cases} 1 & , \text{if } x > 0 \\ 0 & , \text{if } x = 0 \\ -1 & , \text{if } x < 0. \end{cases} \tag{2.4.3}$$

Excluding the zero fluctuations which die out immediately, we found that investing on U.S. dollar against deustch mark yields slightly more than investing for deutsch marks. For example, the probability that a positive fluctuation (positive return for U.S. dollars in investing against deutsch marks) persist five minutes is 12.4 percent, while the probability that a negative fluctuation (positive return for deutsch marks in investing against U.S. dollars) is only 11.8 percent. The short term (five minutes) difference between the persistence probabilities of up and down behavior of data can be related to the presence of a trend in USD/DM exchange rates. This trend still be alive for periods longer than three months. The reverse fact holds for USD/JY and JY/DM rates, where the persistence of negative fluctuations is more likely than positive fluctuations.

Whereas the fact that the conditioning on the sign of fluctuations can be related to the non-symmetrical behavior around zero, conditioning on the size of fluctuations can be related to the reversion horizon of fluctuations with different sizes. The persistence horizon of fluctuations may be short or long depending upon the size of fluctuations, while the size of fluctuations may be small or big depending upon the scope of underlying shocks. To study the effect of size on persistence, it is intuitively attractive to measure the latter by absolute relative changes

$$z_{t_j} = |y_{t_j} - y_{t_{j-1}}|/y_{t_j} \tag{2.4.4}$$

The parameter z in expression (2.2.12) specifies the size of fluctuation and it can be estimated separately from the sample. We assume that the parameter z is the sample median of observed returns. Of course, other measures such as a quadratic one

$$z_{t_j} = (y_{t_j} - y_{t_{j-1}})^2 \tag{2.4.5}$$

could be considered; see for instance Ding et al. [1993], Granger and Ding [1994] who studied the memory properties of exchange rate returns, and Dacarogna [1993]. The parsimony in the fluctuation size are then obtained at the cost of losing high resolution for fluctuation size. Clearly, when z is the median fluctuation size, the size-conditioned probabilities become two times the joint probabilities. For the other letter values of relative fluctuation size, say the hinges or eights of exchange rates returns, we obtain four or eight times the joint persistence (non-persistence) probabilities as conditional persistence (non-persistence) probabilities. Our results over three exchange rates in our dataset suggest that upper-median sized fluctuations are almost two times more persistent than lower-median sized fluctuations. At longer forecast horizons, say 90 working days, the persistence probabilities approach steadily zero suggesting that the market remembers the fluctuations occured three months ago with propability less than one percent. The overall results are plotted in Figure 2.1. The horizontal axis show 24 hours forecasting horizons in fifteen minutes intervals. The unconditional (raw), positive, negative, upper- and lower median sized conditioning are superimposed. The decreasing patterns of persistence probabilities are clear.

Table 2.1: **Persistence Probabilities of USD/DM** *

Horizon d	$p(d)$	$p(d\|z_{t_j} > 0)$	$p(d\|z_{t_j} < 0)$	$p(d\|z_{t_j} \geq m)$	$p(d\|z_{t_j} < m)$
		$z_{t_j} = \text{sgn}(y_{t_j} - y_{t_{j-1}})$		$z_{t_j} = \frac{\|y_{t_j} - y_{t_{j-1}}\|}{y_{t_j}}$	
5 minutes	0.1057	0.1236	0.1183	0.1613	0.0500
10 minutes	0.0749	0.0876	0.0838	0.1156	0.0341
15 minutes	0.0607	0.0714	0.0677	0.0944	0.0271
30 minutes	0.0416	0.0487	0.0465	0.0651	0.0181
45 minutes	0.0332	0.0389	0.0371	0.0523	0.0142
1 hour	0.0284	0.0334	0.0317	0.0448	0.0121
2 hours	0.0199	0.0236	0.0220	0.0316	0.0082
3 hours	0.0164	0.0196	0.0180	0.0262	0.0067
4 hours	0.0142	0.0170	0.0156	0.0228	0.0056
5 hours	0.0127	0.0152	0.0140	0.0205	0.0050
6 hours	0.0116	0.0137	0.0127	0.0186	0.0045
12 hours	0.0086	0.0105	0.0092	0.0139	0.0033
24 hours	0.0061	0.0076	0.0064	0.0098	0.0025
2 days	0.0042	0.0055	0.0040	0.0066	0.0017
3 days	0.0034	0.0046	0.0032	0.0054	0.0014
4 days	0.0030	0.0041	0.0026	0.0047	0.0012
5 days	0.0026	0.0038	0.0022	0.0041	0.0011
6 days	0.0024	0.0035	0.0020	0.0038	0.0010
7 days	0.0022	0.0033	0.0018	0.0035	0.0009
8 days	0.0021	0.0031	0.0017	0.0033	0.0008
9 days	0.0020	0.0029	0.0017	0.0032	0.0008
10 days	0.0019	0.0028	0.0015	0.0030	0.0008
20 days	0.0013	0.0020	0.0009	0.0021	0.0005
30 days	0.0011	0.0019	0.0006	0.0018	0.0004
60 days	0.0006	0.0012	0.0001	0.0010	0.0002
90 days	0.0004	0.0010	0.0000	0.0007	0.0001

*N = 1439816, pr$\{\text{sgn}(y_{t_j} - y_{t_{j-1}}) > 0\} = 0.4405$, pr$\{\text{sgn}(y_{t_j} - y_{t_{j-1}}) < 0\} = 0.4330$ and $m = 1.2931$.

Table 2.2: **Persistence Probabilities of JY/USD** *

Horizon d	$p(d)$	$p(d\|z_{t_j} > 0)$	$p(d\|z_{t_j} < 0)$	$p(d\|z_{t_j} \geq m)$	$p(d\|z_{t_j} < m)$
		$z_{t_j} = \text{sgn}(y_{t_j} - y_{t_{j-1}})$		$z_{t_j} = \frac{\|y_{t_j} - y_{t_{j-1}}\|}{y_{t_j}}$	
5 minutes	0.1910	0.2213	0.2258	0.2727	0.1094
10 minutes	0.1376	0.1593	0.1629	0.2012	0.0741
15 minutes	0.1129	0.1307	0.1336	0.1666	0.0592
30 minutes	0.0792	0.0920	0.0935	0.1188	0.0397
45 minutes	0.0636	0.0738	0.0752	0.0962	0.0311
1 hour	0.0546	0.0631	0.0646	0.0828	0.0263
2 hours	0.0378	0.0439	0.0446	0.0577	0.0179
3 hours	0.0302	0.0350	0.0357	0.0465	0.0139
4 hours	0.0257	0.0296	0.0306	0.0397	0.0117
5 hours	0.0227	0.0261	0.0271	0.0352	0.0102
6 hours	0.0205	0.0235	0.0245	0.0319	0.0092
12 hours	0.0143	0.0159	0.0177	0.0221	0.0066
24 hours	0.0104	0.0112	0.0131	0.0160	0.0047
2 days	0.0074	0.0078	0.0096	0.0115	0.0034
3 days	0.0060	0.0058	0.0082	0.0092	0.0028
4 days	0.0051	0.0048	0.0071	0.0078	0.0024
5 days	0.0044	0.0040	0.0063	0.0067	0.0021
6 days	0.0039	0.0034	0.0057	0.0059	0.0018
7 days	0.0035	0.0031	0.0052	0.0053	0.0017
8 days	0.0032	0.0025	0.0049	0.0048	0.0015
9 days	0.0030	0.0022	0.0048	0.0045	0.0015
10 days	0.0028	0.0021	0.0046	0.0043	0.0014
20 days	0.0019	0.0012	0.0034	0.0029	0.0010
30 days	0.0017	0.0008	0.0032	0.0025	0.0009
60 days	0.0014	0.0006	0.0026	0.0020	0.0007
90 days	0.0011	0.0001	0.0024	0.0016	0.0005

*N = 556962, $\text{pr}\{\text{sgn}(y_{t_j} - y_{t_{j-1}}) > 0\} = 0.4350$, $\text{pr}\{\text{sgn}(y_{t_j} - y_{t_{j-1}}) < 0\} = 0.4195$ and $m = 1.8458$.

Table 2.3: **Persistence Probabilities of JY/DM** *

Horizon d	$p(d)$	$p(d\|z_{t_j} > 0)$	$p(d\|z_{t_j} < 0)$	$p(d\|z_{t_j} \geq m)$	$p(d\|z_{t_j} < m)$
		$z_{t_j} = \mathrm{sgn}(y_{t_j} - y_{t_{j-1}})$		$z_{t_j} = \frac{\|y_{t_j} - y_{t_{j-1}}\|}{y_{t_j}}$	
5 minutes	0.4207	0.4734	0.4742	0.5720	0.2694
10 minutes	0.3399	0.3809	0.3845	0.4694	0.2103
15 minutes	0.2948	0.3297	0.3342	0.4107	0.1794
30 minutes	0.2228	0.2488	0.2530	0.3149	0.1311
45 minutes	0.1870	0.2082	0.2129	0.2662	0.1081
1 hour	0.1655	0.1840	0.1887	0.2369	0.0944
2 hours	0.1219	0.1330	0.1415	0.1755	0.0685
3 hours	0.1012	0.1097	0.1181	0.1461	0.0564
4 hours	0.0891	0.0961	0.1045	0.1288	0.0494
5 hours	0.0798	0.0857	0.0939	0.1158	0.0439
6 hours	0.0728	0.0774	0.0865	0.1056	0.0401
12 hours	0.0522	0.0530	0.0645	0.0756	0.0288
24 hours	0.0372	0.0359	0.0478	0.0544	0.0201
2 days	0.0265	0.0224	0.0371	0.0389	0.0141
3 days	0.0220	0.0172	0.0323	0.0320	0.0121
4 days	0.0194	0.0141	0.0296	0.0282	0.0107
5 days	0.0177	0.0117	0.0280	0.0256	0.0098
6 days	0.0164	0.0103	0.0265	0.0237	0.0091
7 days	0.0148	0.0087	0.0247	0.0213	0.0084
8 days	0.0139	0.0082	0.0230	0.0196	0.0081
9 days	0.0132	0.0077	0.0220	0.0187	0.0077
10 days	0.0127	0.0069	0.0217	0.0180	0.0074
20 days	0.0102	0.0038	0.0192	0.0144	0.0060
30 days	0.0081	0.0008	0.0175	0.0110	0.0053
60 days	0.0072	0.0000	0.0162	0.0095	0.0049
90 days	0.0065	0.0000	0.0147	0.0087	0.0044

*N = 156296, pr{sgn($y_{t_j} - y_{t_{j-1}}$) > 0} = 0.4434, pr{sgn($y_{t_j} - y_{t_{j-1}}$) < 0} = 0.4446 and $m = 1.5616$.

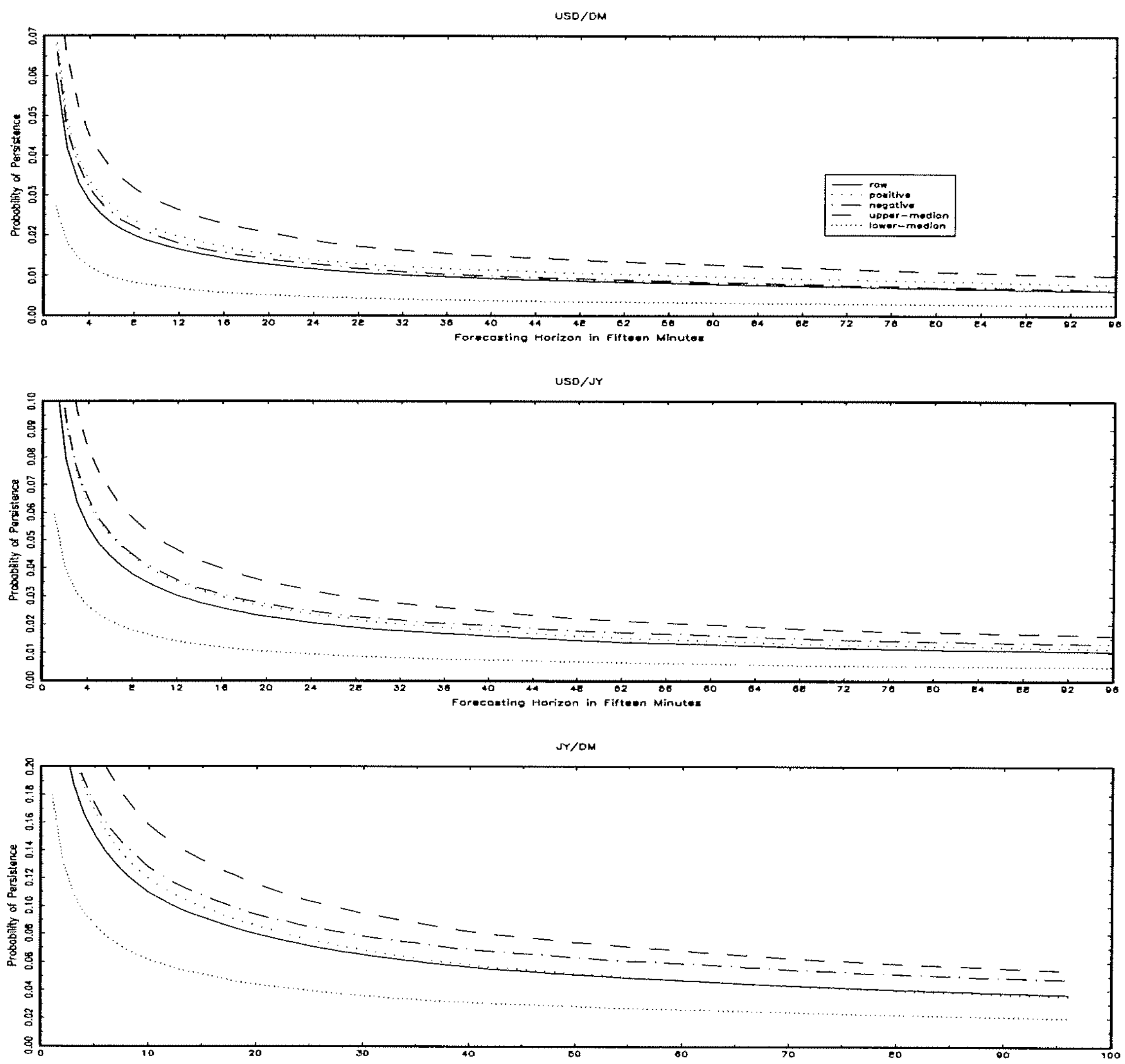

Figure 2.1: Persistence Probabilities in HF Foreign Exchange Rates

2.5 Conclusion

The probabilistic approach to the persistence measures of economic fluctuations in Dufour and Sarlan (1996) is extended to the case where observations are unequally spaced through time. We have proposed estimators for these observations and applied then to the second-by-second interbank foreign exchange rate fluctuations in USD/DM, USD/JY and JY/DM for a period from October 1st, 1992 to September 30th, 1993.

Among the million price propositions, we observed that only 1/10 of them persist at the five minutes horizon in the case of USD/DM, 2/10 in the case of USD/JY and 4/10 in the case of JY/DM fluctuations. At long forecasting horizons, say 90 working days, the persistence probabilities do not reach zero supporting the hypothesis that the market contains a long memory, Ding et al. (1993). We are also considered conditional probabilities of persistence. First, by conditioning on the sign of fluctuations, we obtain a measure of asymmetry in exchange rate movements. The USD/DM rate appears more persistent in response to positive fluctuations, while the USD/JY and JY/DM rates are more persistent to negative fluctuations. Second, conditioning on the size of fluctuations provide measures related to the reversion horizon of small or big fluctuations. Our results support the market efficiency hypothesis, in the sense that for all currenies big jumps seem to be more persistent than small jumps .

It is worth to note that the persistence probabilities proposed in this paper can explain the volatility of stock prices only if persistence (non-cancellation) probabilities change substantially from period to period. The results obtained in this paper are useful both for practicioners working on worldwide markets who would like to know which price can reveal important business opportunities and also for theoreticians who are interested in knowing the distributional properties of future prices.

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Chapter 3

Spectral Analysis of Business Cycle Chronologies

(in collaboration with Eric Ghysels)

3.1 Introduction

The measurement of business cycle phenomena has been a very active area of research since at least the thirties, when Burns, Mitchell and Tinbergen proposed a variety of statistical methods to examine macroeconomic data. To some, the phenomenon of business cycles was one of regimes, like expansions and recessions, which led to the work by Burns and Mitchell (1946) who proposed to study business cycles via chronologies. To others, when discussing cyclical phenomena, the picture that came to mind was a sine wave with its regular and recurrent pattern. This led to the more modern techniques of spectral analysis initiated in econometrics by Hannan (1960), Granger and Hatanaka (1964) and Nerlove (1964). See also Sargent (1987) for a fairly extensive coverage of business cycle phenomena and spectral decompositions. Business cycle chronologies and spectral analysis of time series have been largely independent developments, as they were techniques associated with two very different views about modeling business cycles. In this paper, we propose to pair the two developments. Indeed, we suggest to apply spectral methods not to the data directly but instead to time series consisting of business cycle chronologies. A business cycle chronology, such as the one produced by the National Bureau of Economic Research (NBER) which covers a sample of monthly observations starting in 1854, can be viewed as a realization of a random variable over a discrete space, usually two states, resulting in an alternating sequence of expansions and recessions. Such a time series, when stationary, has a spectral representation allowing us to take advantage of tools developed over the last several decades but hitherto not exploited[1].

Before we get into the technicalities about how to apply spectral methods to such discrete processes, let us explain what the advantages could be. First, they provide an easy tool to assess the similarities and differences between alternative reference chronologies. Indeed, two chronologies may be different not only in the dating of peaks and troughs but also in the number of recessions and expansions. A formal comparison via the coherence can inform us how the two chronologies are related. Second, using an algorithm such as the one proposed by Bry and Boschan (1971), we can date peaks and troughs in a set of individual time series like interest rates, money supply, etc., allowing us to study the comovements between the process identified by the NBER chronology and the turning point process associated with any individual series. Such is an alternative way to describe the association of cycles between different series measuring real activity and monetary aggregates. Third, while it is true that by focusing on business cycle phases instead of the actual series like GNP much information is thrown out, it should be noted that applying spectral analysis to chronologies aims at investigating nonlinear properties of the data instead of the linear ones. Namely, the spectral methods are applied to time series reflecting

[1]Hatanaka (1964) proposed an approach somewhat similar to ours when he estimated the spectral density of a zigzagged pattern of the U.S. business cycles with a discrete-valued triangular pattern, taking its maximum values at the business cycle peaks and its minimum values at the troughs. Moore and Zarnowitz (1986) also displayed such zigzagged patterns to show the matching time of reference chronologies for four countries.

duration of cycles, regime switches and turning points. A related issue is the critical dependence of empirical stylized facts regarding business cycles on the detrending of the data. The methods we propose put emphasis on turning points instead of trends, which has certain advantages.

Applying spectral methods to the rectangular processes of two states requires some technical discussions. The well-known Fourier transform based on sinusoidal functions is one of at least two ways to proceed and compute a spectral decomposition of a series within a class of orthogonal functions. An alternative approach consists of a frequency-based analysis of time series via the Walsh-Fourier transform based on Walsh functions, which are similar to trigonometric functions, except that they take rectangular shapes[2]. Both Fourier and Walsh-Fourier representations have their merits in the analysis of discrete-valued time series and will be used throughout the paper. There are, however, some clear advantages to using the Walsh-Fourier analysis for decomposing chronologies. Section 3.3.2 includes a brief introduction to the Walsh-Fourier analysis. We discuss the univariate spectral analysis of two alternative U.S. reference cycle chronologies given by the NBER and Romer (1992). Several major individual chronologies are also considered. The reference dates of the latter were selected by the Bry and Boschan (1971) dating algorithm for cyclical turning points.

Proposing a new fancy time series technique should not be an aim itself. Its use should instead be justified when it sheds a different light on some outstanding business cycle facts. Let us therefore briefly describe some of the results obtained. The univariate spectral analysis of the NBER and the Romer chronologies reveal a double peak in the spectrum for cycles between two and six years. Such heterogeneity suggests that not all cycles are alike and that probably different sources of impulses and propagation mechanisms may be at work. This result holds before WWII as well as in the post-WWII era. It should parenthetically be noted that a standard

[2]References regarding Walsh functions and their use include Ahmed and Rao (1975); Kohn (1980a,b); Moretttin (1981) and Stoffer (1987, 1990, 1991).

application of spectral methods to raw data such as industrial production series does *not* reveal such double peaks. There is also a peak at the seasonal frequency before the WWII. When we compute the coherency of the NBER and the Romer chronologies, we find that it averages to about 0.95 in the post-WWII era, yet only to 0.79 before WWII. Outside the business cycle frequency band, the two chronologies do not match very closely, as the average coherencies at high and low frequencies are at most 50 to 60 percent.

Studying comovements among individual series also yields interesting insights about business cycle comovements. Using Walsh coherencies, we first compare pre- and post-WWII cycles and find striking differences in the cyclical behavior of prices, bond yields and the stock market across the two eras. The pattern of comovements between industrial production and the NBER reference cycle also shows dramatic changes at the short end of the spectrum around seasonal frequencies. A comparable exercise involving standard applications of spectral methods again shows significantly different results.

Besides comparisons of pre- and post-WWII eras, we also investigate the stylized facts for the latter period for a larger set of series. Alternative detrending methods tend to affect business cycle frequencies differently, as noted by Canova (1991) for instance, in his elaborate study of detrending and stylized facts. As a result, some important empirical evidence regarding business cycle behavior critically depends on detrending methods. The approach via spectral decompositions of chronologies has the advantage that it does not depend in any direct way on detrending[3]. We therefore study post-WWII business cycle facts, via coherence analysis, through the spectral representation of several chronologies associated with a set of major economic time series.

In Section 3.3.1 and 3.3.2, we review some of the basic tools of the Fourier and Walsh-Fourier analysis used in the remainder of the paper. Section 3.5 is devoted

[3]Obviously, some algorithms for selecting turning points proceed according to a certain trend specification.

to the spectral decomposition of the basic reference chronologies. Section 3.6 covers comovements between individual series, including a separate study of pre- and post-WWII eras as well as a review of stylized facts since WWII. Conclusions appear in section 3.7.

3.2 The Use of Spectral Analysis in Econometrics

The forerunners of modern spectral analysis were Fourier series fitting techniques, which assumed a series contained important deterministic cycles of known period, and the periodograms, which assumed the same model but the components had periods that need to be determined. These models were used by economists, despite the considerable computing costs, the best examples being the works by Moore (1914) and Beveridge (1921, 1922). An account of these and other early applications can be found in Cargill (1974). The main objective of this work was to search for cycles in data with the hope that cycles of similar periods in pairs of series would indicate relationships between these series, an example being sunspots and rainfall and hence wheat prices. In a sense the search for cycles was too successful, for instance Beveridge found evidence of over twenty in his long English wheat price series. This unlikely multiplicity of cycles brought the basic model into some disrepute and undoubtedly this was partly responsible for Yule developing the alternative autoregressive and moving average models in the late 1929s and early 1930s. The resulting tension between the time-domain and frequency-domain approaches lasted until quite recently. Now there is a better understanding of the reason for the periodogram giving evidence of too many apparent cycles. It is explained by the low correlation between estimates at adjacent frequencies and the fact that it is an inconsistent estimator of the theoretical spectrum. Smoothing procedures now used to estimate spectra circumvent these problems.

The link between Fourier series, the periodogram and modern spectral methods was pointed out by Davis (1941). By 1959 spectral methods still had not been applied to economic data. However, in that year Morgenstern initiated a project to investigate the usefulness of spectral methods in economics. The project was supervised by Tukey, who had developed the interpretation of cross-spectral techniques, and was staffed by Hatanaka, Granger and Godfrey. The first report of this project was published in Granger (1961) and the complete report resulted in the book by Granger and Hatanaka (1964). At the same time Nerlove was using these techniques to study seasonal adjustment problems, and Hannan (1960) had previously worked on the same problem.

Nerlove (1964) considered the effects of the Bureau of Labor Statistics method of seasonal adjustment on seventy-five U.S. employment, unemployment and labor force series. The most surprising finding was that the adjusted series often had spectra with dips at seasonal frequencies, suggesting in a sense that the adjustment procedure had removed too much. Nerlove (1964) also examined the cross spectrum between the adjusted and unadjusted series and found that the gain at nonseasonal frequencies was usually substantially lower than one, particularly at higher frequencies. This suggest that the higher-frequency components could have been disrupted.

Empirical studies in econometrics appear to go through phases where different techniques become particularly popular. Initially single-equation regressions were dominant but were then replaced by the more appropriate simultaneous equation models. In the late 1960's and very early 1970s spectral methods became popular and probably more papers were published using these techniques than using the more classical simultaneous models. By the mid 1970s time-domain time-series techniques came into vogue, due to the appearance of the influential book by Box and Jenkins (1970). Nold (1972) produced a bibliography of applications of spectral methods in economics covering much of the most active period, listing 101 papers by 68 different authors. Recently, spectral techniques have largely been out of favour by applied econometricians although they are still used as one of the bundle of empirical techniques available for analysis of time-series data. The theoretical aspects of the frequency domain representations remain i mportant when the properties of these various techniques are considered.

The obvious features of a univariate, power spectrum that can be easily noted are any peaks, such as at the seasonal frequencies, $2\pi k/12, k = 1, 2, ..., 6$, for monthly data and any shape that is complicated compared to the simple shapes that arise from a white noise or first-order autoregressive and moving average models. Economic fluctuations are often characterized as alternating periods of expansion and depression, known as the business cycle. An obvious application of spectral techniques was to investigate these fluctuations. It should be emphasized that the business cycle is not regular, or deterministic, and so corresponds to several frequency band. The business cycle corresponds to rather low frequencies at least 12 months whole cycle length and so estimation of this component is difficult even with monthly or quarterly data, unless very long series are available. The situation is little improved by considering a number of different series from the same economy, as this provides little extra information; most parts of the economy are inclined to move together at low frequencies. Although some evidence was found for certain low-frequency components being especially important, in general all low frequencies were usually observed to be important for the levels of major economic variables, and so the business cycle component did not prove to be special or outstanding. The relative importance of low-frequency components compared to all higher-frequency components was found so frequently that a spectrum that steadily declined from low to higher frequencies, except possibly at seasonal frequencies, was called *typical spectral shape* in Granger (1966). Unfortunately there are a number of different time-domain models that produce such a spectral shape, including AR(1) with a parameter near one and integrated models of order d, where d can be a fraction and which includes the random walk model.

Because it is difficult to estimate the spectrum at very low frequencies it is also difficult to distinguish between these models using the estimated power spectrum of the original series. Sometimes it is easier to distinguish between some of these models by looking at the spectrum, of the first differenced series. The typical spectral shape was found so frequently that it was used as a method of evaluating a large-scale econometric model by Howrey (1971, 1972). The Klein-Goldberger and Wharton econometric models were used to produce simulated data and the spectra of these data compared to the typical shape. In general the models passed this not particularly stringent test. The typical spectral shape is of course an oversimplification and actual spectra may have other discernable properties, as Nerlove (1971) found in a study of U.S. price series. He also found difficulty in interpreting these properties.

The other obvious use of the power spectrum is to investigate the relevance of a particular model suggested by a theory. For example, a number of economic theories suggests that the change in particular series should be white noise, so that the spectra of these changes will be flat over all frequencies if the theory is correct. This procedure was used by Sargent (1972) to test rational expectations for forward interest rates, by Granger and Morgenstern (1963, 1970) to test the random walk theory for stock market prices and Labys and Granger (1970) to test the same theory for commodity prices. The method was found useful and occasionally some slight deviations from the predicted spectral shape were found. It would be possible to use a similar method to test other specific time-domain models.

Potentially the most important technique available in the early period was the cross spectrum and the functions derived from it, the coherence and the phase and gain diagrams. The coherence measures the strength of relationships (squared correlation coefficient) between corresponding frequency components in the two series. As components with different frequencies are necessarily uncorrelated for jointly stationary series, the coherence thus totally measures the (second-order) strength of relationships between the series and has the added advantage that in theory its value is not altered by applications of the same linear filters to the individual series. The gain essentially represents the regression coefficient of the frequency component of one series on the corresponding frequency component of the other. In the case where one series is leading the other, the phase diagram can be used to measure this lead. The cross spectrum may also be used to identify or select time-domain models. Engle (1976) has used this approach to specify the relationship between housing investment and interest rates. He found that the distributed lag weights change sign as would be predicted by an accelerator-type model. Thus spectral spectral methods may be useful in a first exploratory look at economic data to pick acceptable models.

Many other studies have applied spectral techniques to economic time series. A brief summary will be given of results in two areas, the term structure of interest rates and the evaluation of leading indicators. The interest rates charged on loans depend in part on their maturity. Sargent (1968) found that coherences were generally high, particularly between rates of similar maturity, and that in general the longer rates lead the shorter rates with the lead increasing as the differences in maturity increase. Granger and Rees (1968) using British data found similar coherence results but with the lags reversed.

The timing of the long swings in the macroeconomy is very irregular. The prediction of turning points, the upturns and downturns, is of considerable interest to governments and companies. One method of prediction is to find series that consistently lead at the turns and the NBER has suggested many such leading indicators and also an index of these indicators. Another application of spectral techniques is the evaluation of leading indicators. A possible way of evaluating the performance of such indicators, in terms of their consistency and the extend of their lead, is by examining the coherence and phase diagrams at low frequencies from the cross-spectrum between the indicator series and a measure of the state of the economy such as the index of industrial production. Hatanaka (1964) and Hymans (1973) found that the indicators did lead, in that the phase diagrams indicate such a lead, but the coherences are often lower than might be hoped for and the leads are less than those suggested by the NBER. These indicators include for instance, housing starts, new business formation, changes in business inventories, new orders for consumer goods and materials, productivity, capacity utilization, average workweek, bond prices, real money supply and monetary growth rates. Hymans point out that the NBER's index of leading indicators could be improved by a better choice of the weights, with some of the present components given zero weight. Rather similar results have been found by Neftç (1979) using time-domain methods. The main criticism of these papers is that the NBER chooses series that lead at turning points and these series do not necessarily lead at other parts of the cycle, whereas the studies mentioned assume a constant lead throughout the cycle.

Simulated business cycle patterns were also suggested in the earlier literature of U.S. business cycles. Hatanaka (1969) estimated the spectral density of a *zigzag* pattern of U.S. business cycles where the discrete-valued triangular pattern takes its hypothetical maximum values at the business cycle peaks and its minimum values at the troughs. He found roughly strong coherence between industrial production index and triangular series obtained from the NBER chronology. Moore and Zarnowitz (1986) also display such zigzag patterns to show the matched timing of the reference chronologies of four countries.

Another domain where spectral methods have been adapted is the frequency domain version of the factor analytic models implemented by combining spectral analysis and factor analysis. A variety of economic applications of the frequency domain factor model have been suggested including the Sargent and Sims (1977) macroeconomic model, Geweke's (1977) model of production and Singleton's (1980) model of the term structure of interest rates. In each case, both the economic questions asked and the estimation methods are novel. Recently, Stock and Watson (1990) examined the business cycle properties of numerous monthly U.S. economic time series from 1959 to 1988 by means of spectral techniques. They measure the comovements of each individual time series with a reference series, such as the Index of Coincident Indicators. They also provide a new lists of leading indicators based on predictive contents for overall economic activity.

3.3 Fourier and Walsh-Fourier Analysis

3.3.1 Frequency Domain Analysis of Stationary Processes

Spectral analysis is well covered in many textbooks of time series analysis. Its conventional use involves Fourier transforms of weakly stationary processes. For a univariate time series y_t which has time invariant mean and autocovariances $\gamma_y(h) = Cov(y_t, y_{t-h})$, one has two fundamental relationships: (1) the Cramer representation

$$y_t = \int_{-\pi}^{\pi} e^{it\omega} dz(\omega) \tag{3.3.1}$$

where

$$\begin{aligned} E(dz(\omega)\overline{dz(\lambda)}) &= 0, \quad \text{for } \omega \neq \lambda \\ &= f_y(\omega)d\omega, \quad \text{for } \omega = \lambda \end{aligned} \tag{3.3.2}$$

where $f_y(\omega)$ is the spectrum of the process y_t and (2) the spectral representation of the autocovariances is

$$\gamma_y(h) = \int_{-\pi}^{\pi} e^{ih\omega} f_y(\omega) d\omega. \tag{3.3.3}$$

Equations (3.3.1) and (3.3.3) both involve Fourier transforms. The first equation states that a stationary process can be thought of as a noncountably infinite sum of uncorrelated components, and the second equation provides the spectral representation $f_y(\omega)$ of the autocovariance structure of the process. Roughly, the representation theorems say that we may think of a stationary time series as being formed by the random superposition of sine and cosine waveforms,

$$y_t = \sum_{j=1}^{q} (A(j)\cos(2\pi\omega_j t) + B(j)\sin(2\pi\omega_j t)) \tag{3.3.4}$$

where $\omega_1, ..., \omega_q$ are different frequencies measured in cycles per unit time and the $A(j)$'s and $B(j)$'s are mutually uncorrelated random variables with $Var\{A(j)\} = Var\{B(j)\} = \sigma_j^2$. This implies that the total variance in the time series is $Var\{y_t\} = \sum_{j=1}^{q} \sigma_j^2$, and it can be decomposed into components σ_j^2 corresponding to sinusoidal waveforms at various frequencies of oscillation. That is in fact what is in (3.3.3).

When a sample of size T of the series y_t is available, we can compute the cosine transform

$$C_y(\omega_j) = T^{-1/2} \sum_{t=1}^{T-1} y_t \cos(2\pi\omega_j t) \tag{3.3.5}$$

which is essentially the correlation of the data y_t with cosines and the sine transform

$$S_y(\omega_j) = T^{-1/2} \sum_{t=1}^{T-1} y_t \sin(2\pi\omega_j t) \tag{3.3.6}$$

which is essentially the correlation of y_t with sines. The frequencies ω_j is equal to the inverse of a period, $\omega_j = j/T$, that is j cycles per T time points; $1 \leq j \leq T/2$. Typically the Fourier periodogram of the data

$$I_y^F(\omega_j) = C_y^2(\omega_j) + S_y^2(\omega_j) \tag{3.3.7}$$

is computed and a plot of $I_y^F(\omega_j)$ versus ω_j is inspected for peaks. The idea here is that $I_y^F(\omega_j)$ will be large when the time series y_t contains harmonic components near the frequency ω_j. The periodogram is essentially the squared correlation of the data with the sine and cosine waves that oscillate at frequency ω_j.

The usefulness of spectral techniques is increased when several series are considered. If x_t and y_t are both stationary and they are also second-order jointly stationary so that the cross-covariances $\gamma_{xy}(h) = Cov(x_t, y_{t-h})$ are not time dependent, then expanding the notation (3.3.1) in an obvious way, the bivariate version of (3.3.2) is

$$\begin{aligned} E(dz_x(\omega)\overline{dz_y(\lambda)}) &= 0, \quad \text{for } \omega \neq \lambda \\ &= f_{xy}(\omega)d\omega, \quad \text{for } \omega = \lambda \end{aligned} \tag{3.3.8}$$

where $f_{xy}(\omega)$ is the cross-spectrum. The equation (3.3.3) becomes

$$\gamma_{xy}(h) = \int_{-\pi}^{\pi} e^{ih\omega} f_{xy}(\omega)d\omega. \tag{3.3.9}$$

As $\gamma_{xy}(h) \neq \gamma_{xy}(-h)$ in general, $f_{xy}(\omega)$ will not be a real function of frequency. A more convenient pair of functions is the coherence, defined by

$$K_{xy}^F(\omega) = \frac{|f_{xy}(\omega)|^2}{f_x(\omega)f_y(\omega)} \tag{3.3.10}$$

where F denotes Fourier. The coherence $K_{xy}^{F}(\omega)$ measures the strength of relationship between corresponding frequency components in the series x_t and y_t. A good introduction to the spectral (Fourier) analysis of time series can be found in Priestley (1981) who provide a comprehensive treatment of spectral methods.

3.3.2 Walsh-Fourier Analysis of Discrete-Valued Time Series

Recently, attention has been focused on the Walsh-Fourier analysis of real time stationary time series. Kohn (1980a,b) laid the groundwork by showing that many of the results concerning the decomposition of stationary time series using trigonometric functions have their Walsh function analogs, although Morettin (1974) had obtained limit theorems for the Walsh-Fourier transform of stationary time series. Statistical data analysis via the Walsh-Fourier transform can be found in Ott and Kronmal (1976) where the transform is used in classification problems for strictly binary data and in Stoffer (1988), where an analysis of variance based on the Walsh-Fourier transform is used to assess the effect of maternal alcohol consumption on neaonatal sleep-state cycling. Further applications of Walsh spectral analysis can be found in Ahmed and Rao (1975).

The aformentioned works demonstrate that the Walsh-Fourier transform can be a powerful tool in the statistical analysis of spectra. Hence it is of considerable importance that Walsh-Fourier theory for statistical analysis of time series data and in particular processes with a limited number of levels such as discrete-valued and categorical procceses (*square waveforms*), be extended at least to the point of development of the statistical theory of Fourier (trigonometric) analysis for *sinusoidal waveforms*. In this section, we give a brief account of the Walsh-Fourier theory for stationary time series. Specific details and references may be found in Kohn (1980a,b), Morettin (1981) and Stoffer (1987).

The Walsh functions are similar in some respects to sinusoidal harmonics used in Fourier analysis and the same basic ideas in Section 3.3.1 are valid in Walsh-Fourier spectral analysis. There are a number of formal definitions of Walsh functions, eg., Ahmed and Rao (1975), Beauchamp (1984), Kohn (1980a), Morettin (1981), but we adopt Stoffer (1991)'s definition which give more insight about their generation. Unlike their sinusoidal counterparts, the Walsh functions are square waveforms that form a complete orthonormal sequence on $[0, 1)$ and take only two values -1 and $+1$ (up and down). Although the various definitions of the Walsh functions lead to different orderings, we shall be interrested primarly in the Walsh or sequency ordering since this ordering is comparable to the frequency ordering of sines and cosines. The *sequency ordered* Walsh functions will be denoted by $W(t, \lambda)$, where $t = 0, 1, 2, ...; 0 \leq \lambda < 1$. In Figure 3.1 the first eight continuous sequency ordered Walsh functions, $W(t, \lambda)$ for $t = 0, 1, 2, ..., 7$, were superimposed on Fourier harmonics. On the top panel of Figure 3.1, $W(0, \lambda)$ and $\cos(0)$ always take the value $+1$, and makes no sign changes or zero crossings. The first Walsh function $W(1, \lambda)$ makes one sign change when $\lambda = 1/2$[4]. Similarly, for example $W(4, \omega)$ switches four times on the interval $0 \leq \omega \leq 1$ from $+1$ to -1 at $1/8$, then from -1 to $+1$ at $3/8$, from $+1$ to -1 at $5/8$ and, finally, from -1 to $+1$ at $7/8$. As it can be easely seen from the Figure 3.1 the fundemental difference between sinusoids and Walsh functions is that the latter are aperiodic. This is in contrast to the sinusoids $\cos(2\pi t\omega)$ and $\sin(2\pi t\omega)$ for $t = 1, 2, ...$ which are characterized by their frequency of oscillation t in terms of the complete cycles they make in the interval $0 \leq \omega < 1$. The frequency parameter t in sinusoidals may be interpreted as one half the number of zero crossings or sign changes that a sinusoidals make per unit time (Harmuth (1969)). For example, when $t = 4$ as above, the Fourier harmonics $\cos(2\pi t\omega)$ and $\sin(2\pi t\omega)$ each cross zero eight times. Roughly speaking, while frequency is inversely related to the length of a full cycle, a sequency is inversely related to the length of half a cycle.

Suppose that a sample of lenth $T = 2^p$, p is a positive integer, is available. The discrete valued Walsh functions are calculated via the Hadamard matrix $H(p)$, which is defined to be the symmetric orthogonal $(T.T)$ matrix whose (u, v)th element

[4]For sine functions the zero crossings at $\lambda = 0$ is counted but not the one at $\lambda = 1$.

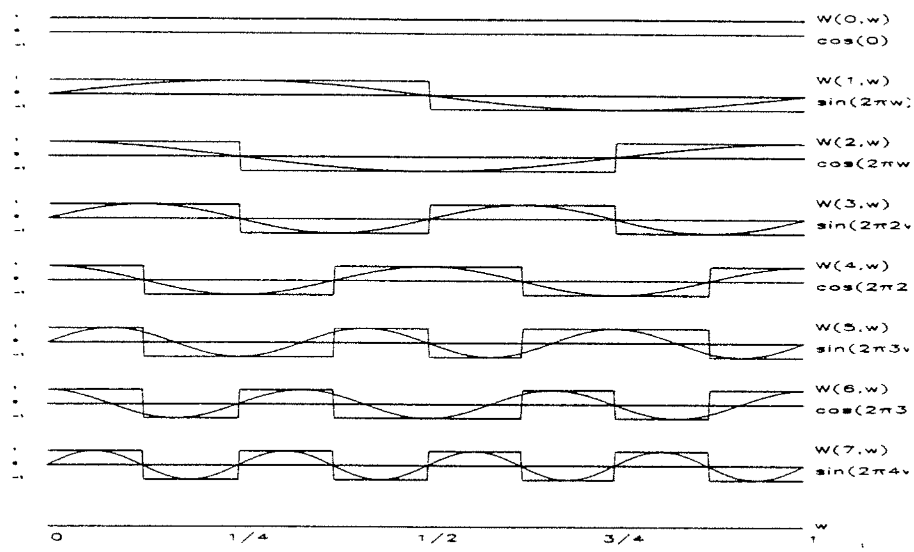

Figure 3.1: Walsh and Fourier Harmonics

$u, v = 0, 1, ..., T-1$, $\phi(u, v)$ is equal to $\Pi_{j=0}^{p-1}(-1)^{u_j v_j}$ where the binary representations of u and v are given by $(u_{p-1}, u_{p-2}, \cdots, u_0)$ and $(v_{q-1}, v_{q-2}, \cdots, v_0)$ respectively, u_j and v_j are either 0 or 1. For example, with $T = 8$, the (1,5) th element of $H(3)$ is $\phi(5, 1) = (-1)^{0+0+1} = -1$, whereas the (3,7) element is $\phi(3, 7) = (-1)^{0+1+1} = 1$. The Hadamard matrix gives the discrete Walsh functions as rows (or columns) in what is called natural or *Hadamard ordering*. To obtain the Walsh functions in *sequency order*, we can reorder the rows of $H(p)$ according to the number of sign changes. We denonte the sequency or Walsh ordered Hadamard matrix by $H_W(p)$. An alternate method of obtaining $H_W(p)$ from $H(p)$ exists but they involve counting the sign changes and they are not efficient procedures. We shall discuss fast Walsh-ordered Hadamard transform (FWT) of the data. Consider the recursive generation of the Hadamard matrix by setting initially $H(0) = 1$ and then processing by

$$H(k+1) = \begin{pmatrix} H(k) & H(k) \\ H(k) & -H(k) \end{pmatrix} \tag{3.3.1}$$

for $k = 1, 2, \ldots, (p-1)$. The Walsh-ordered Hadamard matrix $H_W(p)$ can be computed by

$$H_W(p) = \Pi_{i=1}^{p} H_i(p) \cdot B \tag{3.3.2}$$

where,

$$H_i(p) = \begin{pmatrix} F_s & & & & 0 \\ & G_s & & & \\ & & \cdot & \cdot & \cdot \\ & & & F_s & \\ 0 & & & & G_s \end{pmatrix}, \quad s = 2^{i-1}, \tag{3.3.3}$$

with $F_s = \begin{pmatrix} I_s & I_s \\ I_s & -I_s \end{pmatrix}$, $G_s = \begin{pmatrix} I_s & -I_s \\ I_s & I_s \end{pmatrix}$ and I_s being the $(s \times s)$ identity matrix. The matrix B in (3.3.2) bit reverses the order of the matrix $H(p)$. Namely, matrix B counts the number of sign changes in each row (column) of the $H(p)$ and then reorders the rows (columns) to obtain $H_W(p)$. As an example, suppose that $p = 3$, that is $N = 8$. The Walsh ordered Hadamard matrix is,

$$\begin{aligned} H_W(3) &= \Pi_{i=1}^{3} H_i(p) \cdot B \\ &= \begin{pmatrix} F_1 & 0 & 0 & 0 \\ 0 & G_1 & 0 & 0 \\ 0 & 0 & F_1 & 0 \\ 0 & 0 & 0 & G_1 \end{pmatrix} \cdot \begin{pmatrix} F_2 & 0 \\ 0 & G_2 \end{pmatrix} \cdot F_4 \cdot B \\ &= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{pmatrix} \end{aligned}$$

where the matrix B is given by,

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3.3.4)$$

Let $y(0), y(1), ..., y(T-1)$ by a sample of length $T = 2^p$, $p > 0$ integer, from a weakly stationary time series $\{y(t); t = 0, \mp 1, \mp 2, ...\}$ with absolutely summable autocovariance function $\gamma_y(h) = Cov\{y(t), y(t-h)\}, h = 0, \mp 1, \mp 2, ...$ We assume for instant that the constant mean value of $y(t)$ is zero. Let $W(t, \lambda)$ be the t th Walsh function in sequency order and let

$$d_T(\lambda) = T^{-1/2} \sum_{t=0}^{T-1} y(t) W(t, \lambda), \quad 0 \leq \lambda < 1 \quad (3.3.5)$$

be the finite (or discrete) Walsh-Fourier transform of the data. The *logical covariance* of $y(t)$ (Kohn (1980a) is defined to be

$$\tau_y(j) = T^{-1} \sum_{t=0}^{T-1} \gamma_y(j \oplus k - k) \quad (3.3.6)$$

where by $(j \oplus k)$, is the *dyadic addition* of j and k [5]. It can be shown (Kohn 1980a) that the variance of $d_T(\lambda)$ is given by

$$Var\{d_T(\lambda)\} = \sum_{j=0}^{T-1} \tau_y(j) W(j, \lambda). \quad (3.3.7)$$

Taking the limit as $T \to \infty$ in (3.3.7), we have that $Var\{d_T(\lambda)\} \to g_y(\lambda)$, where

$$g_y(\lambda) = \sum_{j=0}^{\infty} \tau_y(j) W(j, \lambda), \quad 0 \leq \lambda < 1 \quad (3.3.8)$$

[5]The dyadic addition of j and k, where $j = \sum_{i=0}^{q} j_i 2^i$ and $k = \sum_{i=0}^{q} k_i 2^i$ is given by $j \oplus k = \sum_{i=0}^{q} |j_i - k_i| 2^i$. Note that, in dyadic addition $1 \oplus 1 = 0 = 0 \oplus 0$ and $1 \oplus 0 = 1 = 0 \oplus 1$. For example, when $j = 5$ and $k = 3$, the addition yields to $(101) \oplus (011) = (100) = 6$.

is called the *Walsh-Fourier spectral density* of $y(t)$. We note that $g_y(\lambda)$ exists since the absolute summability of $\gamma_y(h)$ implies the absolute summability of $\tau_y(j)$. Specifically, Kohn (1980, Lemma a) shows that if

$$\lim_{t\to\infty} \sum_{|j|<2^t} \left(1 - \frac{|j|}{2^t}\right) |\gamma_y(j)| < \infty \tag{3.3.9}$$

then $\sum_{j=0}^{\infty} |\tau_y(j)| < \infty$ and $g_y(\lambda)$ is well defined. If $y(0), y(1), ..., y(T-1)$ is a sample of length $T = 2^p$, the finite transform (3.3.5) is calculated for $\lambda_T = s/T$, for $s = 0, 1, ..., T-1$. Since the discrete Walsh functions are symmetric in their arguments for $T = 2^p$, that is

$$W(t, s/T) = W(s, t/T) \quad (s, t = 0, 1, ..., T-1) \tag{3.3.10}$$

the value of λ_T in the finite Walsh-Fourier transform corresponds to sequency. As with the usual Fourier analysis, if the mean of the series is unknown, the only sequency of the form $\lambda_T = s/T$ for which the transform cannot be evaluated is at the zero ($s = 0$) sequency. To see this, let $\theta = E\{y(t)\}$, all t, and note that for $t = 0, 1, ..., T-1$,

$$\frac{1}{T} \sum_{t=0}^{T-1} W(t, s/T) = \delta_0^s \tag{3.3.11}$$

where δ is the Kronecker delta (see Kohn, 1980a, Lemma 1). It is clear from (3.3.11) that the mean-centered transform will be the uncentered transform except at ($s = 0$), and in particular

$$\begin{aligned} E\{d_T(s/T)\} &= T^{-1/2} \sum_{t=0}^{T-1} \theta W(t, s/T) \\ &= T^{-1/2} \theta \delta_0^s \end{aligned} \tag{3.3.12}$$

for $s = 0, 1, ..., T-1$. Kohn (1980a, Corollary 3) gives the following useful results on the convergence of the second moment of the finite Walsh-Fourier transform under condition (3.3.9). Let the binary representation of λ_T be finite. If $\lambda_T \oplus \lambda \to 0$ as $T = 2^p \to \infty$, then

$$E\{d_T^2(\lambda_T)\} \to g_y(\lambda). \tag{3.3.13}$$

In general, the asymptotic covariance of the Walsh-Fourier transform at two distinct sequencies is not zero (Kohn, 1980s, Theorem 3). However, if the binary representation of $\lambda_{1,T}$ and $\lambda_{2,T}$ are both finite and $|\lambda_{1,T} - \lambda_{2,T}| \geq T^{-1}$ with $(\lambda_{i,T} \oplus \lambda_T) \to 0$, $i = 1, 2$ as $T = 2^p \to \infty$, then

$$E\{d_T(\lambda_{1,T})d_T(\lambda_{2,T})\} \to 0. \tag{3.3.14}$$

Various authors have been established central limit theorems for the finite Walsh-Fourier transform under a wide range of conditions, see for instance, (Kohn (1980a), Morettin (1983), Stoffer (1985).

The basic result is that, under appropriate conditions (that typically include some *mixing conditions* -loosely, events that occur far apart in time are nearly independent), $d_T(\lambda_T) \xrightarrow{D} N(0, g_y(\lambda))$ as $T \to \infty$; that is, the large sample distribution of the transform $d_T(\lambda_T)$ is Normal with mean zero and variance $g_y(\lambda)$ given in (3.3.8). Under these conditions, if $\{\lambda_{1,T}, ..., \lambda_{S,T}\}$ is a collection of S sequencies that are all chosen close to sequency of interest, λ, such that $|\lambda_{j,T} - \lambda_{k,T}| \geq T^{-1}$ for $j \neq k$, and $(\lambda_{j,T} \oplus \lambda) \to 0$ for $j = 1, 2, ..., S$, then as $T \to \infty$

$$\sum_{j=1}^{S} d_T^2(\lambda_{j,T}) \xrightarrow{D} g_y(\lambda)\chi_S^2. \tag{3.3.15}$$

where χ_S^2 denotes a chi-squared distributionb with S degrees of freedom. From this we deduce that $S^{-1}\sum_{j=1}^{S} d_T^2(\lambda_{j,T})$ is an estimate of $g_y(\lambda)$ having variance $2g_y^2(\lambda)/S$. If we let $S \to \infty$ as $T \to \infty$ with $S/T \to 0$, then the estimate is a mean squared consistent estimate of $g_y(\lambda)$ on $(0 < \lambda < 1)$. Hence a consistent estimate of the Walsh-Fourier sprectrum $g_y(\lambda)$, is simply the average of the Walsh periodogram (3.3.17) at sequencies near the sequencies of interest.

Like the usual trigonometric spectrum estimate, the consistent estimate of the Walsh-Fourier spectrum can be obtained by smoothing the periodogram ordinates. The fast Walsh-Fourier transform (FWT) corresponding to equation (3.3.5) for the data vector $Y = (y(0), \cdots, y(T-1))$ is computed as

$$\begin{aligned} d_T(\lambda_T) &= T^{-1/2} H_W(p) \cdot Y \\ &= T^{-1/2} H_i(p) \cdot B \cdot Y. \end{aligned} \tag{3.3.16}$$

where $\lambda_T = (0/T, 1/T, \cdot, (T-1)/T)'$. The vector of periodogram ordinates can be obtained by squaring each element of $d_T(\lambda_T)$. Let $I_y^W(s) = d_T^2(s/T)$ denote the s'th periodogram ordinate $s = 0, 1, ...N-1$. It is seen that

$$I_y^W(s) = \sum_{j=0}^{T-1} \hat{\tau}_y(j) W(j, s/T) \tag{3.3.17}$$

where

$$\hat{\tau}_y(j) = \frac{1}{T} \sum_{k=0}^{T-1} y(k) y(j \oplus k). \tag{3.3.18}$$

Employing relationships (3.3.10) and (3.3.11), we may write

$$\hat{\tau}_y(j) = \frac{1}{T} \sum_{s=0}^{T-1} I_y^W(s) W(s, j/T). \tag{3.3.19}$$

Thus, for large T, the quickest way to compute $\hat{\tau}_y(j)$ is to use the fast Walsh-Fourier transform twice, once to compute $I_y^W(s)$ and once to compute the logical covariance.

In the empirical framework, we make use of both fast Fourier transform FFT and the fast Walsh-Fourier transform FWT. In order to make the two techniques comparable, we use Harmuth (1969)'s definition of sequency[6]. Harmuth (1969) introduced the term sequency to describe generalized frequency to distinguish functions, like Walsh functions, that are not necessarily periodic. In this approach, the frequency parameter n in sinusoidals is interpreted as one half the number of zero crossings or sign changes per unit time. So the term sequency will simply denote the half of frequency. This concept can be applied to aperiodic as well as periodic functions and the definitions of Harmuth -sequency coincides with that of frequency when applied to sinusoidal functions[7]. As previously mentioned, the discrete Walsh function $W(t, \lambda)$ makes t zero-crossings (or switches) per unit time, and its corresponding sequency value is t; the Harmuth sequency of that function is $t/2$ if t is even, and $(t+1)/2$ if t is odd (recall that Harmuth sequency and frequency coincide for sinusoids). To differentiate between the two definitions of sequency one will note that

[6] See, Stoffer (1991) for a similar treatment of Walsh-Fourier spectrum.

[7] The reader may easely convert back and forth between the two measures since the Harmuth sequency is two times the frequency.

two Walsh functions $W(2t, \lambda)$ and $W(2t-1, \lambda)$ for $t = 1, 2, \ldots$ each have an Harmuth sequency of t; this is also true for sinusoids, that is, $\cos(2\pi t\lambda)$ and $\sin(2\pi t\lambda)$ each have a frequency of t. Thus the Walsh-Fourier periodogram given in (3.3.17) can be modified to look like the Fourier periodogram by setting,

$$I_y^*(\lambda_j) = I_y^W(2j-1) + I_y^W(2j) \qquad (3.3.20)$$

where λ_j represents the Harmuth sequency. Since $\sum_j I_y^*(\lambda_j) = 2\sum I_y^F(\lambda_j)$ excluding $\lambda_j = 0$ and $\lambda_j = 1/2$, we can superimpose $I_y^*(\lambda_j)/2$ and $I_y^F(\lambda_j)$ and make the comparison between Fourier and Walsh-Fourier periodograms, see Stoffer (1991). Note that, the Fourier comparable Walsh spectrum of $y(t)$ would be the average of $I_y^*(\lambda_j)$ at Harmuth sequency (or frequency) λ_j. This spectrum can be denoted by $g_y^*(\lambda)$.

A useful measure of the degree of association between two time series $x(t)$ and $y(t)$ is coherency

$$K_{xy}^*(\lambda) = \frac{|g_{xy}^*(\lambda)|^2}{g_x^*(\lambda)g_y^*(\lambda)} \qquad (3.3.21)$$

where $g_x^*(\lambda)$ and $g_y^*(\lambda)$ are the Walsh-Fourier spectra of series $x(t)$ and $y(t)$ respectively and $g_{xy}^*(\lambda)$ is the *cross-spectrum* of two series, all are measured on the Harmuth sequency λ. The cross-spectrum is related to the covariance of the Walsh-Fourier transform of series $x(t)$ and $y(t)$ similarly to its Fourier counterpart expressed in (3.3.9). Thus, coherency is a sequency dependent measure of correlation and analogous to the usual correlation inequality $-1 \leq K_{xy}^*(\lambda) \leq 1$. Remember that in the trigonometric (Fourier) case, the cross-spectra is complex-valued and hence squared coherency rather than coherency is measured. That is one advantage of working in the real-valued Walsh-Fourier domain. Consequently the Walsh-Fourier coherence $K_{xy}^*(\lambda)$ measures not only a strength of association but also its sign.

Beauchamp (1984, Chapter 3) provides numerous comparisons between Walsh-Fourier and Fourier spectral analysis. He finds that Fourier analysis is most relevant for smooth-varying time series, while the Walsh-Fourier analysis is more suitable for series with sharp discontinuities and a limited number of discrete realizations.

In the remainder of this Chapter, we will use both types of spectral methods as complementary rather than as substitutes.

3.4 Business Cycle Chronologies and Their Transformation

Bry and Boschan (1971) present the possibility of using computer programs to simulate the NBER's procedure. This method consists essentially, in first identifying major cyclical swings in an economic time series, then sketching out the neighbourhoods of their extrema throughout a set of *ad hoc* filters and finally narrowing the search for turning points to specific calendar dates. The results are remarkably encouraging in the sense that the dates selected formerly by the NBER's specialist are, in general, reproduced by the programmed procedures. This stepwise filtering procedure is necessary because most economic time series contain short quick waves which make it difficult to implement any direct selection mechanism of cyclical extrema. The first filter from which turning points are determined is a 12-month moving average filter. The MA(12) filter eliminates the fluctuations of subcyclical duration or of very shallow amplitude of the seasonally adjusted data. The rule for selecting turning points is this: any month whose value is higher than those of the five preceding months and the five following months is regarded as the date of a tentative peak; analogously, the month whose value is lower than the five values on either side of it is regarded as the date of a tentative trough. These tentative turns are tested for compliance with a set of constraint rules concerning alternation of phases and duration of phases and cycles. The second step in the process is the determination of tentative cyclical turns on the Spencer curve. Bry and Boschan argue that Spencer curve's turns tend to be closer to those of the unsmoothed data than are those of MA(12) filter. Basically, the program search, in the neighbourhood (defined as plus or minus five months) of the turns established on the twelve-month moving average, for like turns on the Spencer curve. That is, in the neighbourhood of peaks, it search for the highest of the eleven points on the Spencer curve; in the neighbourhood of troughs, for the lowest. The Spencer curve turns thus located are then subjected to several tests. A turn is rejected when it is (1) less than six months from either end of the series; (2) one of a pair of like turns less than fifteen months apart; or (3) one of a pair of like turns without an intervening opposite turn. The accepted turns in the Spencer curve provide the basis for the next step in the search for turns in the unsmoothed data. In this step, the series is smoothed by a MA(3) to MA(6) filter. The exact number of months of these filters depends on the time that the cyclical component take to exceed the irregular component. The method of deriving turning points in these short-term MA filters is practically the same as that for the Spencer curve. The last step of the procedure is to find the peak and trough values in the unsmoothed, seasonally adjusted data which correspond to the short-term MA turns previously established. This search is analogous to the previous ones. The program establishes the highest values in the unsmoothed data within a span of plus or minus five months from the peak in the short-term moving average curve; similarly, the lowest value of the unsmoothed data in the neighbourhood of MA troughs is established. Any turns not complying with the rules having been eliminated, the remaining ones are the final programmed turning points of the series. It should be noted that the computer program does not utilize directly any information on the amplitude of cycles. The only way in which amplitude plays a role is that the moving averages, especially the initial twelve-month moving average, tend to iron out minor swings (trough only if they are also brief). But there is no specification of amplitude minima because setting them would involve problems that would greatly complicate the program. One major difficulty is that the typical amplitude of a series changes over time, so that standards derived from an earlier period may be entirely inappropriate in a later one. The program's disregard for amplitudes makes the good agreement between programmed and traditional specific cycles even more remarkable, because amplitudes are among the factors considered in the selecting turns by traditional methods although no minimum amplitudes are prescribed. Consistent with practice at the NBER, the dates produced by Bry-Boschan algorithm use the level of the series rather than the detrended series. Thus recessions correspond to sequences of absolute declines in a series. This will be important when interpreing the changes in the pre- and post-War average period of oscillation.

The analysis of Romer (1992) starts from the observation that there appear to be inconsistencies between the determination of NBER dates before and after World War II. Romer proposed an algorithm that chooses postwar turning points which match the NBER dates, but produces a chronology quite different both in terms of length of cycles and number of recessions and expansions. The binary series obtained from the Romer chronology will be denoted b_t^R.

Table 3.1 reports the average length of whole cycle duration in peak-to-peak (PP), trough-to-trough (TT) formats as well as the average length of recessions (PT) and the expansions (TP)[8]. A distinction is made between the pre-WWII and

[8]The data sources are as follows. The NBER and Romer chronologies were taken from Romer (1992). The pre-WWII series were obtained from Watson (1992). The mnemonics indicate the NBER, BCD, ID numbers. *m01585* is the pig iron production, (1877:1 - 1941:12), *SPPRWARR* is the S&P common stock price index, (1871:1 - 1940:12), *m04010* is wholesale price index, (1890:1 - 1940:12), *m13024* is the RR bond yields, (1857:1 - 1940:12). The post-WWII series were obtained from Citibase. The Citibase mnemonics are preserved to identify the series. The whole definition of series are as follows. *IP* is the industrial production index, total, (1947:1 - 1993:8). The *FSPCOMPR* is the Real S&P's common stock price index, (1947:1 - 1993:8). It is obtained by $FSPCOMR = FSPCOM/PUNEW$, where *PUNEW* is all items CPI-U (sa). *PW* is the Producer Price Index for all commodities (1946:1 - 1993:8). *FYBAAC* is the Bond Yield, Moody's BAA corporate, percentage per annum, (1947:1 - 1993:8). The price Inflation *INFLP* (1948:1 - 1993:8) is defined as $INFLP = 100*log(PUNEW/t/PUNEW_{t-12})$. The wage inflation *INFLW* (1947:1 - 1993:8) is $INFLW = 100*log(LEHM_t/LEHM_{t-12})$, where *LEHM* is the average hourly earnings in manufacturing industry. The nominal short term interest rate *FYGM3* is the U.S. treasury bills (1947:1 - 1993:8). The Real short-Term interest rate *FYGM3R* (1948:1 - 1993:8) is the difference between series *FYGM3* and *INFLP*. *LHMUR* is the unemployment rate, me, 20 years and over, percentage, sa, (1948:1 - 1993:8). *LHOURS* is the Man-hours of Employed Labour Force, (1947:1 - 1993:7). Finally the labor productivity *LPROD* (1947:1 - 1993:7) is the ratio between *IP* and

post-WWII data series for two reasons: (1) in some cases, the series involved are not exactly the same over those two eras, and (2) there has been much discussion about the distinct business cycle patterns.[9]

It is not uncommon to apply spectral analysis to transformed series instead of the raw data x_t. Perhaps the best example is that of seasonal adjustment, where the spectral properties of x_t^{SA} instead of x_t are studied. Such transformations are, at least in principle, designed to extract from the raw series those features of the data that are of interest to the researcher. We essentially apply a similar principle here in a different context. Namely, let us construct a binary time series b_t, where:

$$b_t \equiv CH(X_{t-k}, \ldots, X_t, \ldots X_{t+l}, t) \tag{3.4.1}$$

$$b_t \in \{-1, 1\} \, \forall t \tag{3.4.2}$$

$$X_t = (x_t, y_t, z_t, \ldots) \, . \tag{3.4.3}$$

For convention, the rectangular patterns are scaled according to whether the economy is in expansion, $b_t = +1$ or in recession, $b_t = -1$.[1] Note that the function CH, generating a chronology, may be a function of several series when X_t is multivariate. Such is the case, for instance, with the NBER chronology, designed by dating committees gathering evidence from a multitude of series. The function mapping X_t into b_t may also vary through time, hence $CH(\cdot, t)$, since the dating committees may change the *modus operandi* of defining the phases of the business cycle. Moreover, producing a chronology may involve future as well as past observations, hence the leads and lags appearing in (3.4.1). The algorithm proposed by Bry and Boschan (1971) is another example where a specific rule applies to a single series, i.e., $b_t \equiv CH(x_{t-k}, \ldots, x_t, \ldots x_{t+l}, t)$. Yet, another example is the algorithm proposed by Hamilton (1989) based on a Markov switching regime framework. Figure 3.2 displays the business cycle rectangular patterns b_t for a variety of series ranging from

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LHOURS.

[9]Some of the most recent papers on this subject include Diebold and Rudebusch (1992), Romer (1992), Watson (1992), among many others.

[1]The values +1 and −1 are arbitrary and the procedures we use are invariant to scaling.

Table 3.1: **Average Phase Durations in Months** [a]

Series	Sample Period	P-T-P	T-P-T	P-T	T-P
NBER (entire)	1905:3 - 1990:7	55.4	51.2	14.6	40.4
NBER (prewar)	1896:1 - 1938:6	45.0	44.7	19.0	25.7
NBER (postwar)	1948:1 - 1990:8	63.4	56.7	11.0	51.5
Romer (entire)	1905:3 - 1990:7	52.4	53.8	12.7	40.0
Romer (prewar)	1896:1 - 1938:6	40.7	41.4	12.4	29.0
Romer (postwar)	1948:1 - 1990:8	63.3	57.4	12.4	50.2
Pig Iron Production	1878:4 - 1938:6	44.5	44.8	15.0	29.3
S&P Common Stock Price	1872:8 - 1939:7	42.3	42.8	18.5	23.8
M0401X	1890:10 - 1939:8	46.8	47.3	19.8	27.5
RR Bond Yields	1857:12 - 1940:11	43.3	44.0	20.1	23.2
Industrial Production	1948:7 - 1991:3	63.4	57.0	37.1	19.8
Real S&P Price Index	1949:7 - 1990:10	41.5	45.1	19.2	25.9
Producer Price Index	1948:9 - 1990:10	63.2	62.8	13.2	50.0
Bond Yield	1948:4 - 1990:10	51.1	51.7	24.5	26.6
Price Inflation	1949:9 - 1993:2	45.6	46.3	23.6	21.8
Wage Inflation	1949:12 - 1992:11	36.6	36.8	20.0	16.8
Short Term Int. Rate (Nominal)	1949:2 - 1989:3	53.5	55.8	16.9	36.7
Short Term Int. Rate (Real)	1949:11 - 1993:4	42.0	42.2	21.6	20.1
Unemployment Rate	1949:11 - 1992:6	56.9	57.0	37.1	19.8
Man-hours of Labor	1948:5 - 1991:8	62.6	62.7	16.4	46.4
Labor Productivity	1948:6 - 1991:4	43.9	45.7	16.3	29.0

[a] P-P : Peak to Peak, T-T : Trough to Trough, P-T : Peak to Trough, T-P : Trough to Peak.

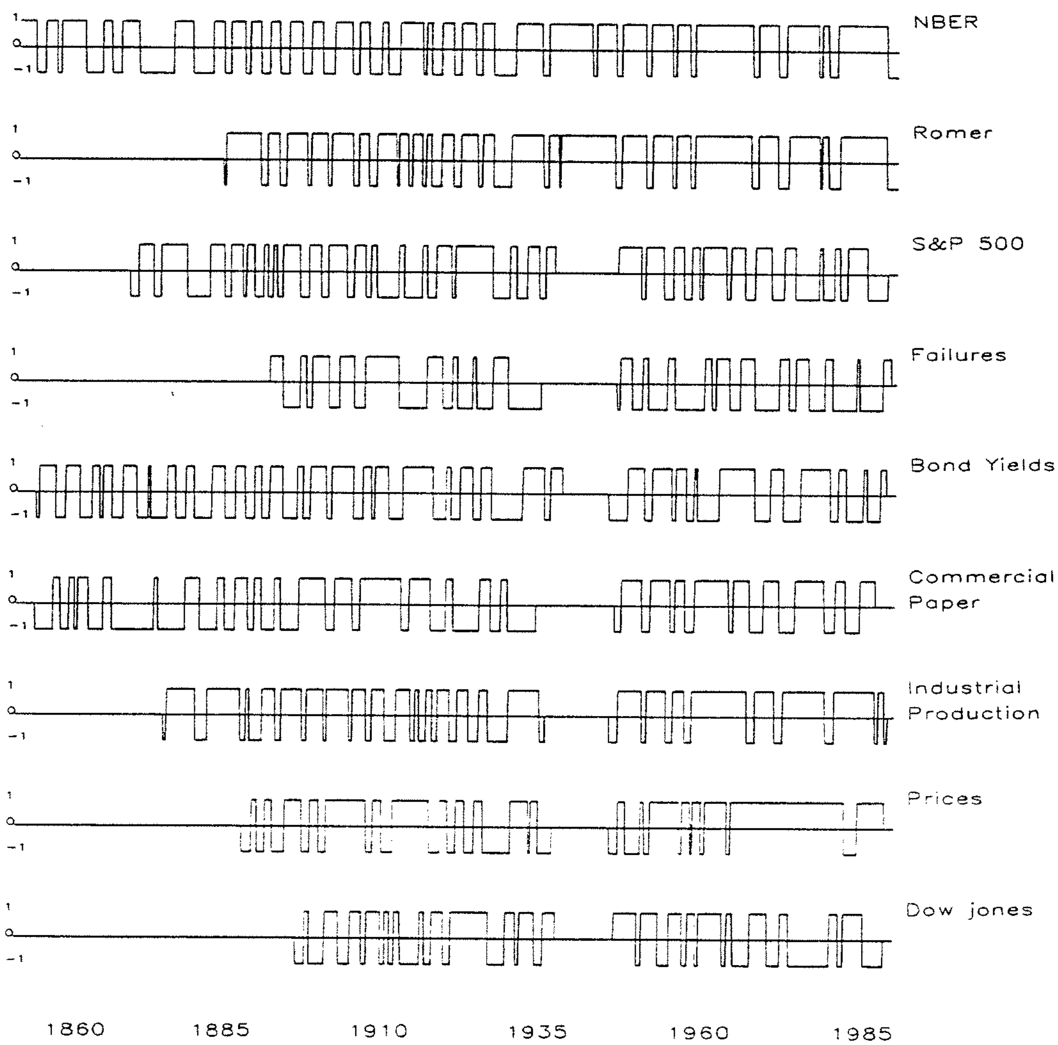

Figure 3.2: Historical Plots of U.S. Business Cycles

the NBER and the Romer chronologies to several major individual series generated by the Bry and Boschan dating algorithm for turning points. The rectangularization of NBER chronology according to (3.4.2) reproduces the time series appearing at the top of Figure 3.2. We denote, this rectangular pattern by b^{NBER}.

The switching points with zero crossings from +1 to −1 correspond to downturns and from −1 to +1 represent upturns. Hence, the length between zero-crossings reflects the durations of cycles.

3.5 Spectral Decomposition of NBER and Romer Chronologies

We would now like to investigate several features of the series b_t. First, in analogy with standard spectral analysis, we would like to decompose the square wave pattern of series b_t in orthogonal harmonic components. We expect, of course, that the business cycle frequencies will be dominant in the spectral shape, yet other cycles may be revealed as well. Moreover, inside the business cycle frequency band certain patterns of interest may appear. Second, we would also like to study comovements across chronologies using a frequency-domain representation. Such analysis serves two purposes: namely, (1) to examine competing chronologies, like the NBER and the alternative proposed by Romer (1992) for instance, and (2) to study relationships between different series of economic activity through their chronology transforms. For instance, one may investigate the stock market and cyclical comovements with b_t series obtained from the Dow Jones and the NBER. It is clear that spectral analysis enables us to formally assess the similarities and differences between two chronologies of the business cycle. Such comparisons are generally not straightforward, since chronologies may not only differ with respect to the location of a turning point, but may also involve a different number of recessions and expansions. Furthermore, when the b_t series of say the NBER and the Dow Jones are examined, it is clear that we apply spectral analysis in the context of a regime switching framework. Namely, we by-pass the linear properties of the series through the $CH(\cdot,\cdot)$ filter and study the association of business cycle phase patterns across series.

When the b_t series is weakly stationary, then the fundamental spectral representation theorem tells us we can apply Fourier transforms of the series and compute cos $C_b(\omega)$ and sin transform $S_b(\omega)$ and proceed as usual to compute a spectral decomposition. Yet, as the b_t series is a square-waved series, an alternative spectral decomposition may be considered as well. To approximate square waves, one can replace the sine and cosine functions by so-called Walsh functions.

Such functions, which will be discussed shortly, are displayed in Figure 3.1 on top of the standard Fourier harmonic functions. The spectral analysis based on such functions is called the Walsh-Fourier analysis and is, to our knowledge, new in terms of econometric applications.

Obviously, one may expect Walsh functions, appearing in Figure 3.1, to be a better approximation to decompose the patterns displayed in Figure 3.2 for the various macroeconomic chronologies. Let us first investigate whether there are noticable differences between the Fourier spectrum and the Walsh-Fourier spectrum. As illustrated in Figure 3.2, the data have different lengths, and hence to utilize the Fourier and Walsh-Fourier transformations, the length of the series were truncated to the nearest power. For the sake of comparability between sinusoidal and asinusoidal waves, the spectral density estimates were computed using and asinusoidal window generator, namely a tent-type kernel with eleven equally weighted periodogram ordinates in the frequency domain.

The sample periods in Figures 3.3 and 3.4 consist of either 512 monthly observations or, in the case of the entire sample of reference chronologies, 1,024. Thus, any arbitrary padding schemes were avoided by the truncation of the series, as both sample sizes are integer powers of 2. The spectral density ordinates $f(\omega_j)$ were decomposed over the following three nonoverlapping business cycle bands: for the number of observations $N = 512$ $(p = 9)$, the frequency domain was decomposed as $j = 1, \ldots, 7$, $j = 8, \ldots, 21$, $j = 22, \ldots, 42$. Hence, these bands are centered at peri-

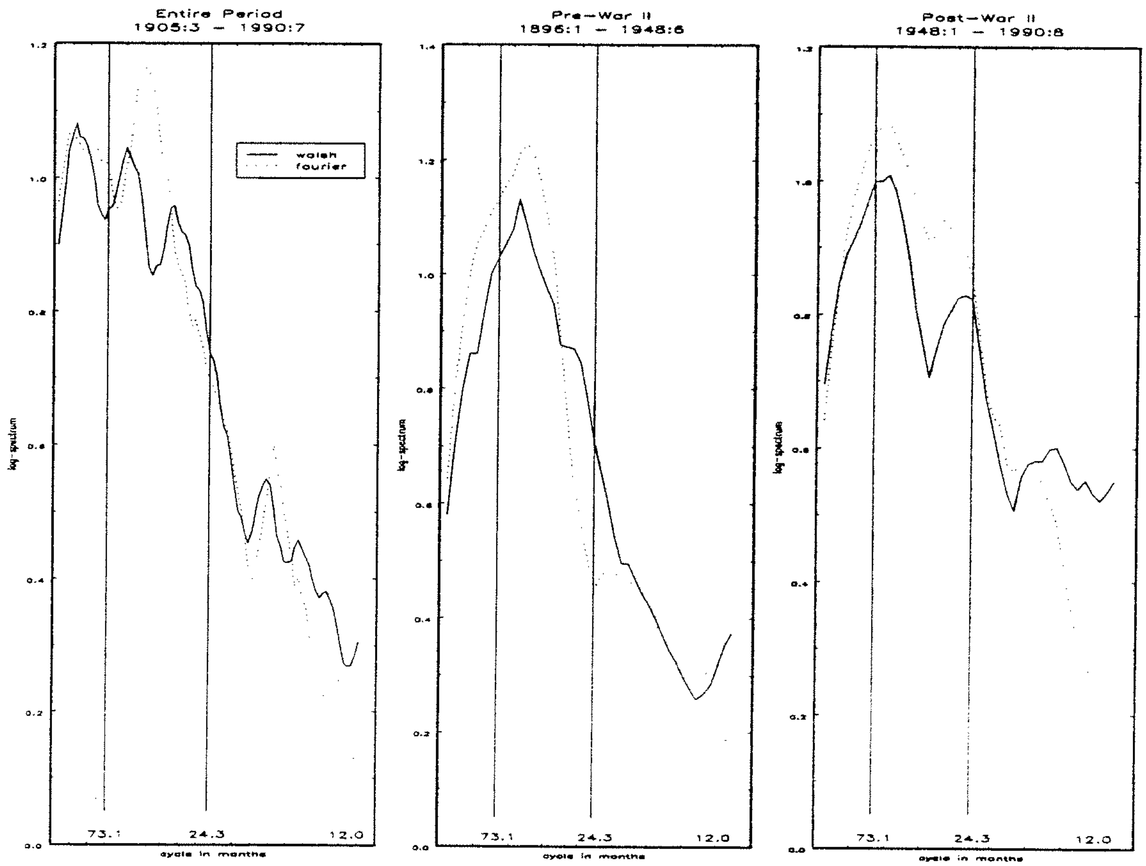

Figure 3.3: Spectral Decomposition of NBER Chronology

odicities of 189.6, 38.5 and 16.6 months respectively. The first band ranges between periodicities of 512 to 73.1 months and is thus the band in which long oscillations occur. The remaining bands are considered as business cycle oscillations and higher frequencies. When $N = 1,024$ ($p = 10$), then the bands considered were quite similar to the previous ones by setting $j = 1, \ldots, 14$, $j = 15, \ldots, 42$, $j = 43, \ldots, 85$ with averaged periodicities of 237, 39.3 and 16.6 months.

Let $d_A(\omega)$ and $d_B(\omega)$ denote the Walsh-Fourier transforms of the rectangular cyclical pattern of b_1 and b_2. The sample coherency between b_1 and b_2 was computed according to (2.15), where the cross-spectral estimation $\hat{f}_{12}(\omega)$ was obtained by averaging the values of the product $d_1(m/N)d_2(m/N)$ over values of m in the neighborhood of ω. Similarly, $\hat{f}_{11}(\omega)$ and $\hat{f}_{22}(\omega)$ were obtained by averaging the Walsh periodograms $d_1^2(m/N)$ and $d_2^2(m/N)$ over values of m in the neighborhood of ω.

Figure 3.3 displays both types of spectral densities for the NBER chronology over three samples. Recall that there are restrictions in choosing a sample length ($N = 2^p$ where p is an integer) for computing Walsh-Fourier spectra. We selected the following samples: (1)1905:3-1990:7, (2) 1896:1-1938:6, (3)1948:1-1990:8. The first sample is approximately the "entire sample," though the earlier part of the chronology was deleted[2]. The second sample covers the pre-WWII era and, finally, the last sample covers the post-WWII era. The Fourier and Walsh-Fourier spectra for each of the three samples appear in Figure 3.3. For the chronology proposed by Romer, the spectra are reported in Figure 3.4. The two curves in each of the plots match fairly closely, yet there are some significant differences worth noting. Namely, the Walsh-Fourier spectrum of the NBER and Romer series gives rise to extra spectral peaks in the business cycle frequency range.[3] The frequency band of

[2]We chose to delete the 19th century part of the chronology to have sample size matching $N = 1024$ data points ($p = 10$). The earlier part was deleted, as there is greater uncertainty regarding the location of turning points [see, e.g., Diebold and Rudebusch (1989) for discussion]. This choice of entire sample also allowed for a direct comparison of the NBER and Romer chronologies.

[3]Stoffer (1991) also reports and discusses peaks uncovered by Walsh-Fourier which do not appear

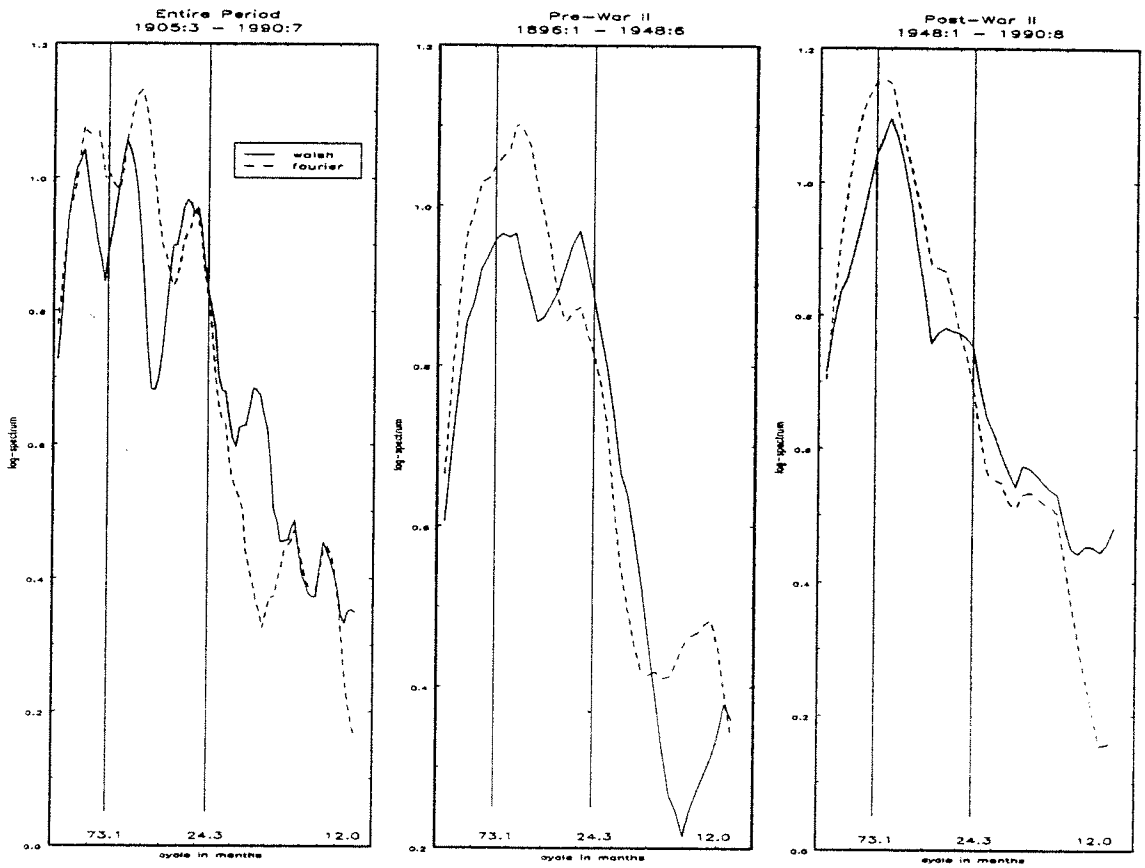

Figure 3.4: Spectral Decomposition of Romer Chronology

cycles of two to six years long are identified via two vertical lines appearing in each plot. The economic interpretation of several peaks in business cycle bands will be discussed below. The differences appear to be in the number of spectral peaks and their location, which means that the average period of oscillation in rectangular b_t patterns differ remarkably if one approximates them via Fourier or Walsh functions. For the entire sample of the NBER series in Figure 3.3, we notice three distinct peaks in the Walsh-Fourier spectrum for cycles of two years and more. The Fourier spectrum on the other hand, has only a single peak located almost exactly at a dip in the Walsh-Fourier spectrum. A similar exercise applied to the Romer chronology, which appears in Figure 3.4, confirms this finding. In fact, with the Romer chronology, the three peaks in the Walsh-Fourier spectrum are much stronger. This seems to indicate that there is a certain degree of heterogeneity in business cycle patterns uncovered by the Walsh-Fourier analysis which remains concealed with the Fourier spectrum. The heterogeneity, suggested by the Walsh-Fourier spectrum, can be attributed to at least two sources. As the sample includes both pre- and post-WWII observations one may expect heterogeneity in business cycle lengths to emerge because of the distinct character of business cycles before and after World War II. Another source of heterogeneity can be explained in the context of the impulse propagation framework introduced by Frisch (1933) and Slutsky (1937). There are different views regarding the nature of shocks and their propagation mechanism. This leads to the question, as noted, for instance, by Blanchard and Watson (1986), whether all business cycles are alike.

The first possible source of heterogeneity, namely the pre- versus the post-WWII eras having different characteristics can be investigated by simply studying the subsamples. Let us therefore focus on the separate pre- and post-WWII samples. The Fourier and Walsh-Fourier spectra once again do not entirely agree on some critical issues. In particular, for the Romer pre-WWII spectra, we notice again a double-dip pattern with the Walsh-Fourier spectrum, not revealed by the Fourier spectrum.

in the Fourier spectra.

This is also the case with post-WWII NBER chronology. Several other observations emerge from the pre- and post-WWII comparison. We notice a very different spectral shape for the two eras, particularly with the Walsh-Fourier analysis, but also with the standard spectral representation. Moreover, one also observes significant differences between the NBER and Romer chronologies. Indeed, before WWII, we found a double-dip pattern with Walsh-Fourier applied to the Romer chronology, but no such pattern emerged with the NBER chronology. Both post-WWII chronologies suggest two distinct peaks in the business cycle frequency bands, though this is more evident for the NBER chronology. Hence, the only chronology not exhibiting a double-peak is the NBER one before WWII. The double peak spectrum emerging from our analysis suggests that the heterogeneity does not only appear to be related to the so-called stabilization hypothesis after WWII. There is indeed evidence of a mixture of business cycle patterns, relatively long cycles of over five years and cycles that are much shorter, that is, less than three years. The advantage of spectral decompositions is to uncover such peaks. We cannot, of course, from this univariate decomposition derive the sources of shocks and propagation mechanism which generate the heterogeneity.

The double peak obtained from applying spectral methods to chronologies does *not* appear in conventional spectral analysis. Indeed, in Figure 3.5, we computed the spectrum of the log first-differenced industrial production series for three different sample and series configurations. The first two spectra cover the pre-WWII era for two alternative measures, one related to Pig Iron production, the other consisting of an index proposed by Miron and Romer (see data sources in the Appendix). The third spectrum applies to the post-WWII era. Clearly, none of the plots display patterns uncovered by the spectral analysis of chronologies. It is also worth noting that at the end of the frequency domain plotted in Figures 3.3 and 3.4, we observe a peak at yearly cycles for the entire sample as well as the pre-WWII sample, particularly for the Romer chronology. The appearance of such a peak at the seasonal frequency is related to the observations made in Ghysels (1991, 1992) and regarding

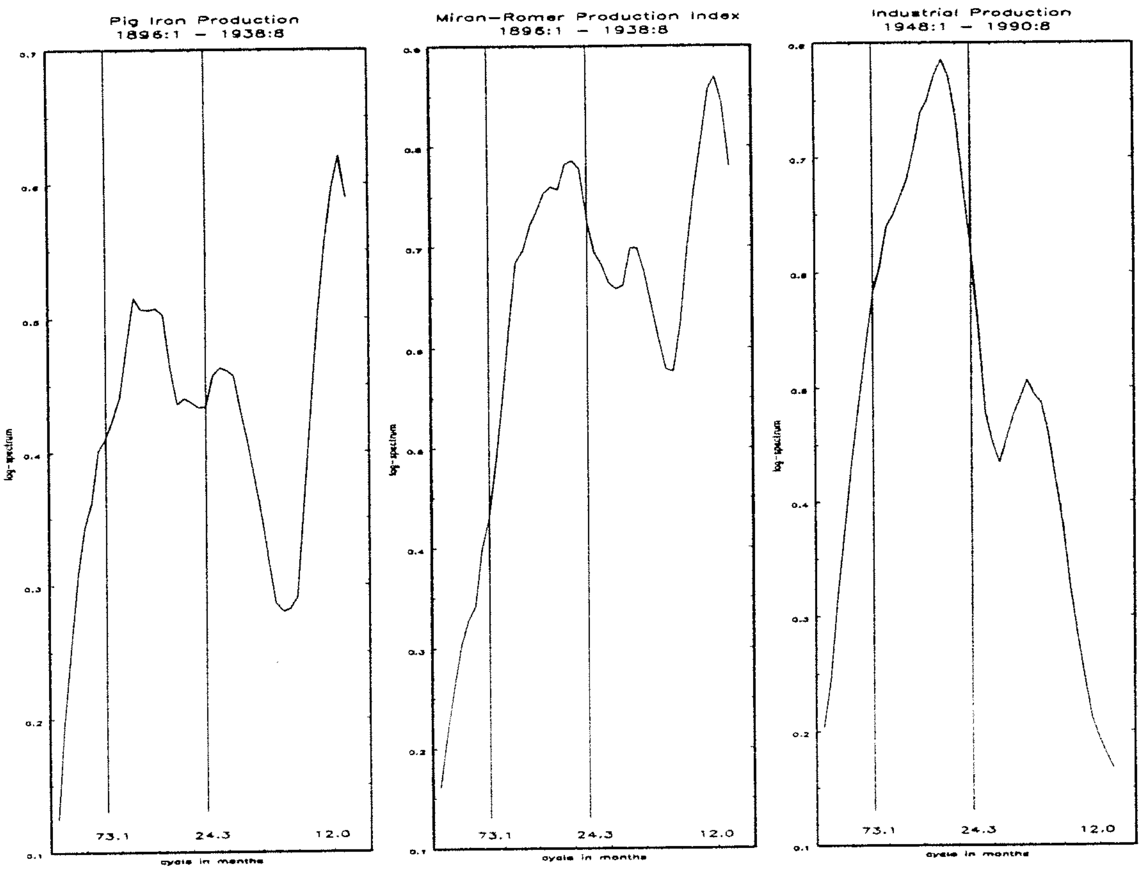

Figure 3.5: Fourier-Spectrum of Growth Rates in Industrial Production

Table 3.2: **Average Coherencies Between b^N and b^R**

Spectra	over 73.1 months	24.4 to 64.0 months	23.3 to 12.1 months
	Entire Sample, 1905:3-1990:7		
Walsh-Fourier	0.82	0.85	0.77
Fourier	0.81	0.87	0.67
	Pre-WWII Sample, 1896:1-1938:6		
Walsh-Fourier	0.63	0.80	0.45
Fourier	0.68	0.79	0.47
	Post-WWII Sample, 1948:1-1990:8		
Walsh-Fourier	0.95	0.95	0.89
Fourier	0.97	0.96	0.78

the nonuniform distribution of turning points throughout the calendar year. Namely, it suggests that the propensity of the economy to emerge from a recession or end an expansion is calendar-dependent. Obviously, the peak which emerges is not as dominant as those in the business cycle frequency band, yet it is clearly present in almost all the plots. This finding, which is essentially obtained via nonparametric methods, i.e., spectral methods, complements the nonparametric duration analysis discussed in Ghysels (1991).

The spectral plots in Figure 3.3 suggest differences between the NBER and Romer chronologies. We can measure the association of the two chronologies via the multivariate spectral analysis discussed in section 3.3.2. In particular, we can compute the coherence between two chronologies. In Table 3.1, we report the average coherencies over different frequency bands. Clearly, after WWII, the two spectra are much in agreement, as the coherencies run on average at 0.95 and higher in the business cycle frequency band. However, before WWII, the two spectra are substantially more in disagreement with a coherency of 80% or less. Over the entire sample, the coherency is below 90%. Outside the business cycle band, there is far less agreement. In fact, at the low and high frequencies, there appears to be less than 60% to 50% coherence. These measures quantify much of the discussion regarding the differences between the two alternative chronologies. It may also be worth noting that this time the results obtained from the standard spectral methods appear to be in agreement with the Walsh-Fourier coherencies.

3.6 Comovements Between Individual Series

A key characteristic of the business cycle is that fluctuations are common across sectors of the economy. We turn our attention here to a set of individual series covering real activity, prices and financial indicators. We will be interested in a pre- and post-WWII comparison of comovements as well as a more detailed study of post-WWII series, since a wider range of data series are available for this era. Unlike reference chronologies, which are the output of some $CH(\cdot, t)$ procedure defined in (3.4.1), there is no direct turning point chronology available for individual series. Hence, for each series we need to construct a binary business cycle phase series. Like Watson (1992), we opted to use the Bry and Boschan (1971) algorithm to date business cycle phases. The merit of this method is that it reproduces the NBER chronology quite accurately. All chronologies appearing in the remainder of this section will be based on the Bry-Boschan algorithm. A first subsection will be devoted to pre- and post-WWII comparisons. while a second section covers the post-WWII era.

3.6.1 Business Cycle Comovements Before and After WWII

Comparing business cycle features before and after WWII has been the subject of many research papers. A very incomplete list of the most recent papers includes Moore and Zarnowitz (1986), Romer (1992), Diebold and Rudebusch (1992) and Watson (1992). The question whether there has been a fundamental change in the nature of business cycles has been vigorously debated among economists for several reasons, particularly with respect to the success of postwar stabilization policies. A comparison of both eras is limited to a relatively small set of series, as there are not many matching pairs of uniformly defined or approximately similar series. A total of eight series were considered similar to those studied by Watson (1992). Sources of all the data series are described in the Appendix, while Figure 3.2 displays the binary processes extracted via the Bry-Boschan algorithm for a subset of those series. We focus our attention on four series of broad measures of economic activity, namely the industrial production (IP) turning point series, denoted b_t^{IP}, the S&P common stock price index b_t^{SP}, the producer price index b_t^{PP} and bond yields b_t^{BY}. Figure 3.6 shows the coherency among these four individual series as well as their coherency with the NBER reference cycle. The coherency was computed before and after WWII so that each plot in Figure 3.6 has two curves. The frequency band of cycles of two to six years are again marked on each plot. It is worth recalling from section 3.3.2 that the Walsh-Fourier cross-spectrum is real, unlike the Fourier cross-spectrum. Consequently, the Walsh-Fourier coherency can assume negative values, a clear advantage over its Fourier counterpart which will be discussed shortly, as it reveals the magnitude as well as the sign of comovements.

The last row of plots in Figure 3.6 shows the coherency of b_t^{IP}, b_t^{PP}, b_t^{BY} and b_t^{SP} respectively with the NBER reference cycle. Among the four individual series, b_t^{PP} shows a most dramatic change in cyclical pattern. After WWII, there was virtually no cyclical pattern in prices, while before the war, prices moved strongly pro-cyclical. Kydland and Prescott (1990) also noted the change in price level business cycle patterns, yet they claimed that prices moved countercyclical after WWII. Our results do not support such a view of post-WWII price behavior. The bond yield chronology also displays very different patterns across the two samples with two strong and distinct peaks after WWII, including one of a short-cycle comovement with the NBER series (under two years). At the short end of the spectrum, we also notice a sharp change of b_t^{IP} and b_t^{N} comovements with a strong seasonal coherency before WWII, which virtually disappeared in the last forty years. It is also interesting and not surprising to note that b_t^{IP} and b_t^{PP} show the same dramatic change in coherency as b_t^{N} and b_t^{PP} do. The stock market was strongly negatively correlated across the

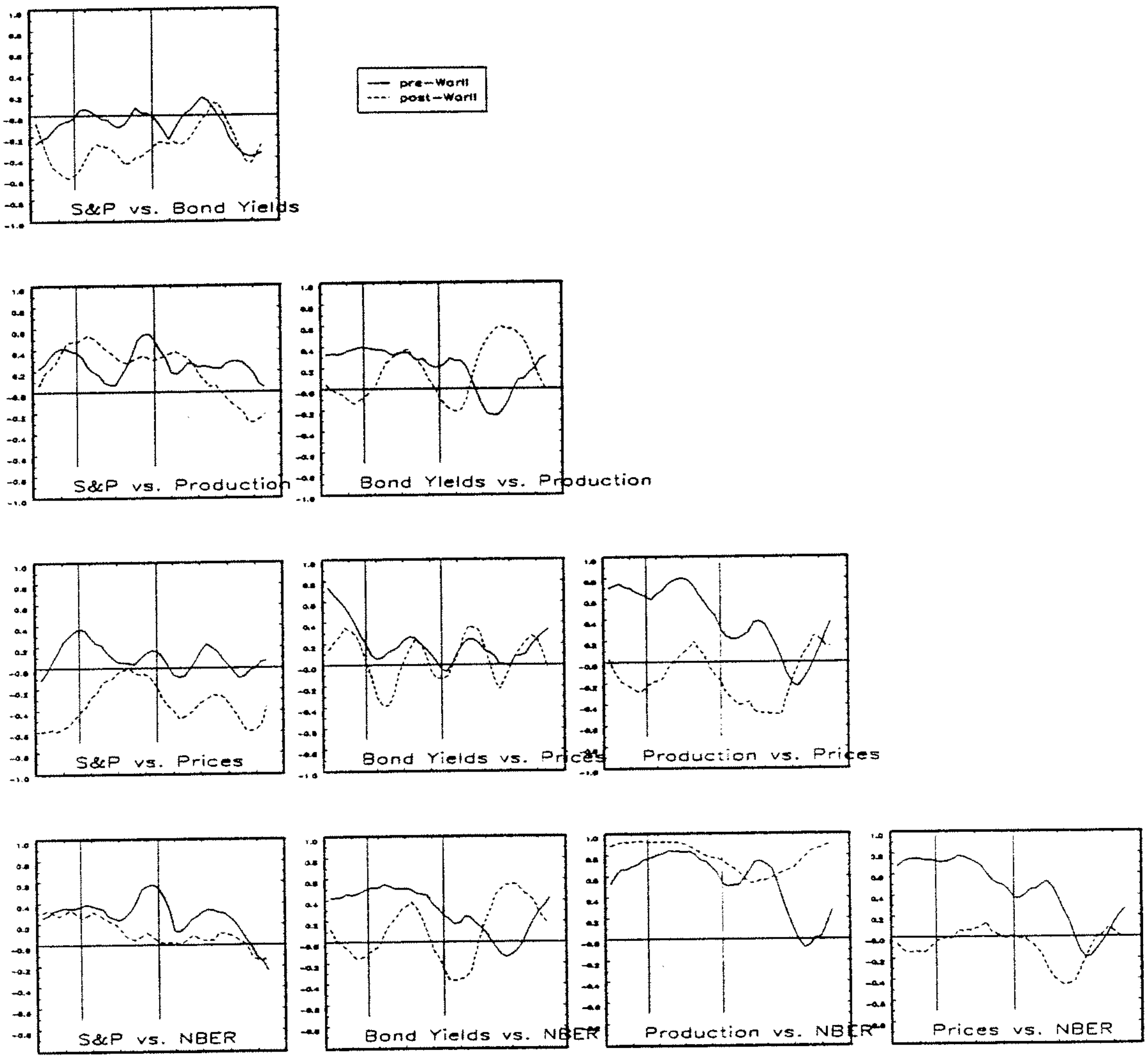

Figure 3.6: Walsh Coherencies: Pre- and Post-WWII Comparison

frequency domain with prices, but this is no longer the case since WWII. Bond yields and the stock market also appear negatively related across all frequencies before the war but little remains since.

Let us briefly turn our attention to coherencies obtained from conventional Fourier transforms of the raw (first-differenced) data. Those appear in Figure 3.7 except for the four plots at the bottom of Figure 3.6 it takes the same structure. The top panel covering S&P and bond yield coherencies shows both very strong (of course always positive by construction) coherencies before and after WWII between both series. This differs significantly from the findings in Figure 3.6. Moving to the next row, we notice that bond yields and production show almost no coherency in the business cycle frequency bond. Before WWII, there appear to be two peaks, one similar to the Walsh-Fourier chronology analysis result and is located at the seasonal frequencies but the other is at the low frequencies. The next and final row displays comovements with prices. Overall, there appear in this case not as strong differences with the chronology-based coherencies apart for the sign changes of course.

3.6.2 Coherency Since WWII

Stylized facts of business cycle comovements over the post-WWII era have been documented by a large variety of authors, sometimes using quite diverse statistical methods and data transformations for detrending, seasonal adjustment, etc. In general, one analyzes the timing relation between various series and some reference series, usually real GNP, by means of cross-correlation coefficients. There exist more complicated procedures, however, such as VAR impulse response analysis, common factor and index models. Documenting stylized facts is quite sensitive to prefiltering data. Such prefiltering occurs either when detrending or seasonally adjusting series. For instance, Canova (1991) shows in detail that a multitude of key stylized facts in business cycle analyses are inconclusive because of prefiltering

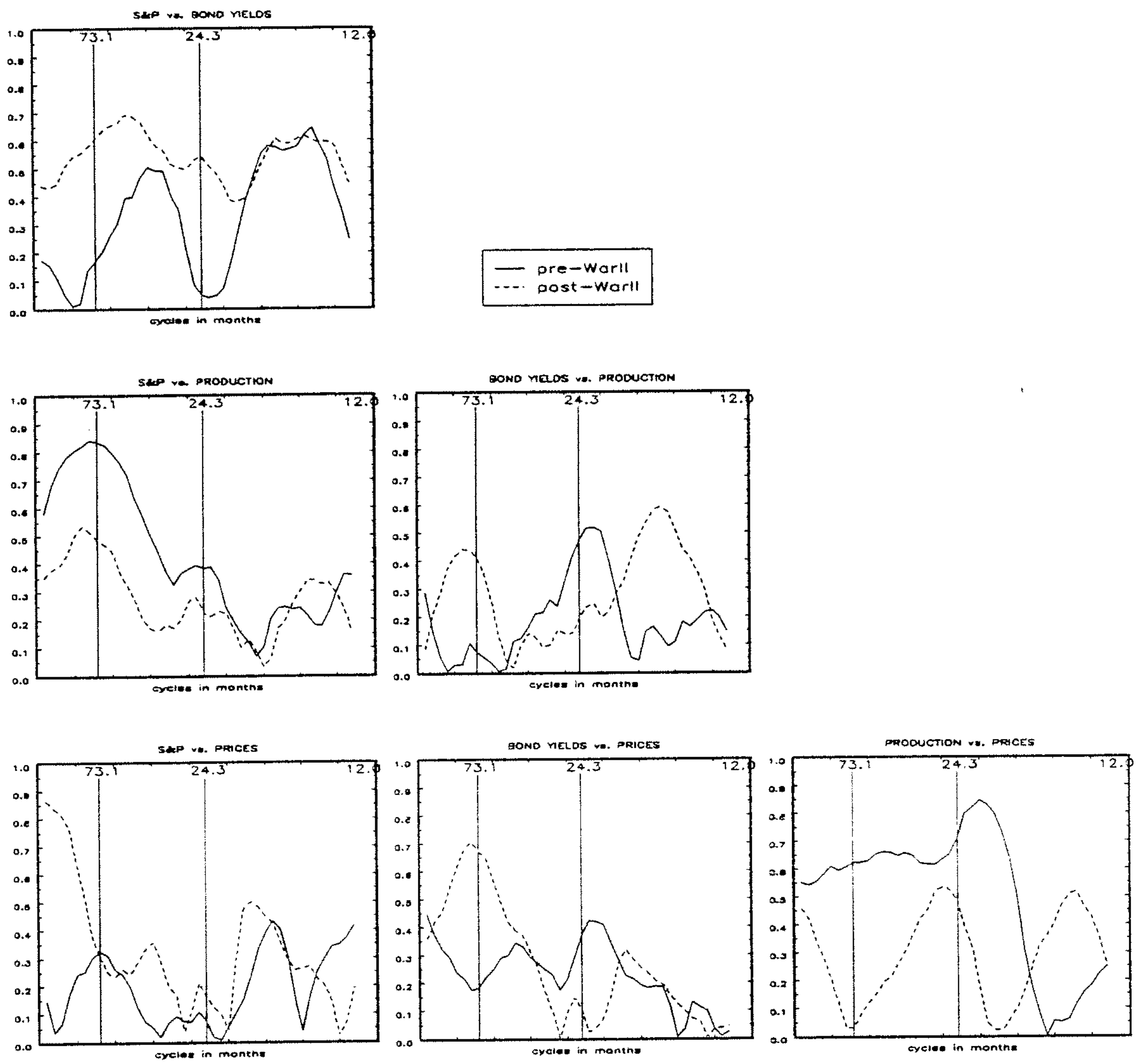

Figure 3.7: Coherencies From First Differenced Data: Pre- and Post WWII Comparison

effects[4]. There probably is less disagreement regarding the location of turning points, particularly for the post-WWII era, than there is regarding the specification of the secular component of macroeconomic time series. Therefore, we suggest to use the Walsh-Fourier coherency methods here as an alternative tool of studying post-WWII business cycle features.

Over the typical business cycle, it is claimed that employment varies substantially, while the determinants of labor supply, like real wages and real interest rates, vary only slightly [see, e.g., Mankiw (1989)]. The Walsh-Fourier coherencies between the NBER and individual series plotted in Figure 3.8 confirm this finding to a large degree, except for the comovements between real interest rates and the NBER chronology. They appear indeed important, compared to the de facto zero coherence between NBER and real wages at all frequencies of the spectrum[5]. In contrast to the real wage, we observe strong procyclicality of labor productivity. It was already noted that the price level, measured via the PPI, is neither procyclical nor countercyclical. Also, in Figure 3.8, we notice that the unemployment rate is strongly countercyclical, yet there appears to be a sharp (positive) peak at the seasonal frequency. Hours worked and real wages are typically found to have low correlation. Figure 3.9 shows a zigzag coherence pattern which decomposes the low correlation in a sharp positive peak around the seasonal frequency as well as a large dip in the business cycle frequency band. Unemployment and real wages also show mostly a positive coherency, as would be expected, but again labor productivity and the real wage are basically uncorrelated across frequencies. Finally, inflation against the nominal interest rate as well as against the real interest rate also yields some peculiar patterns. Inflation and real interest rates show a strong negative correlation in the business cycle frequency band. For the nominal rate, there are two sharp

[4]Several other papers have raised this question, including Singleton (1988), Cogley (1990) and Ghysels, Lee and Siklos(1993).

[5]It is often claimed that real wages are procyclical. While they are for certain business cycle frequencies, they also appear negatively correlated with the reference cycle over other business cycle frequencies.

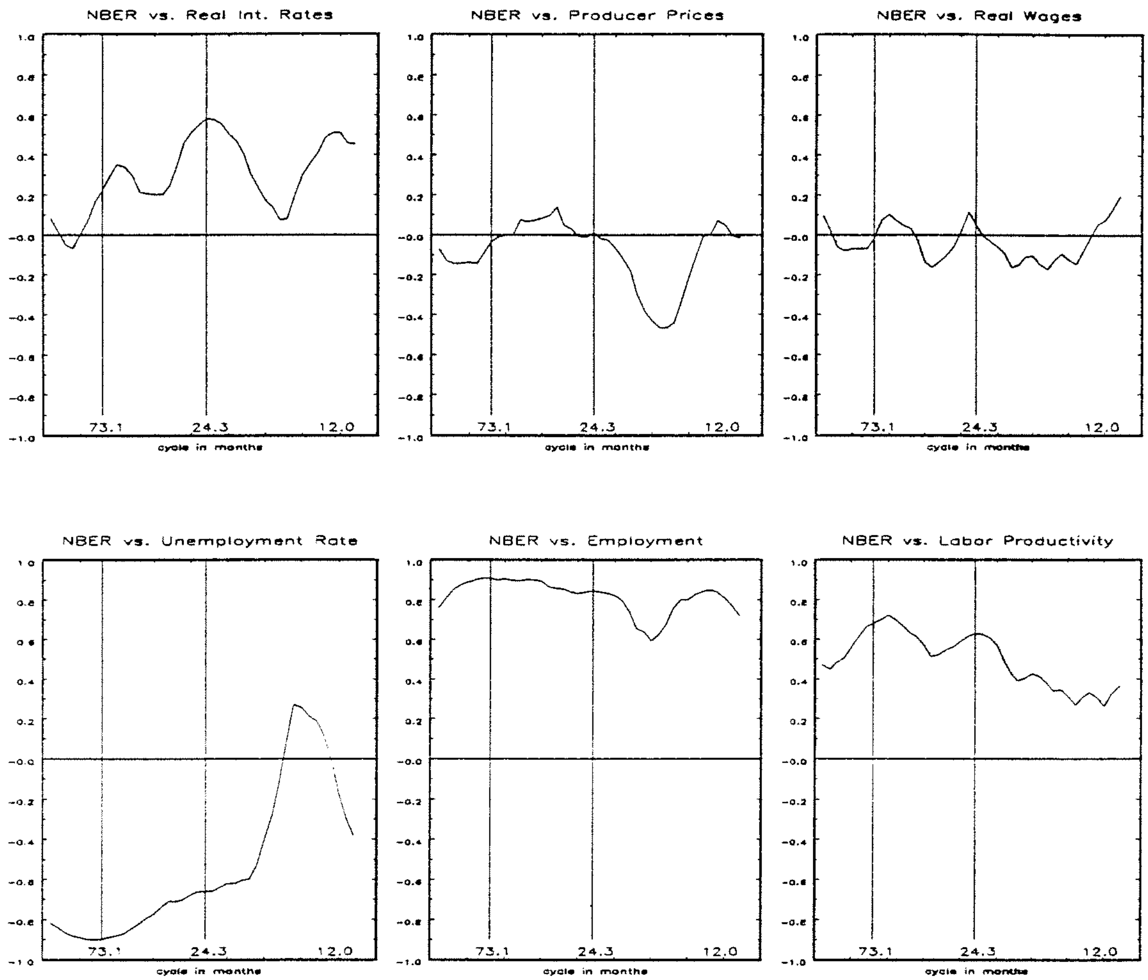

Figure 3.8: Some Stylized Facts with Walsh-Fourier Coherency: Post-WWII

positive peaks decomposing the comovements.

We also computed the standard spectral decompositions from the raw series. Figure 3.10 is comparable to the previous figure. The differences between the plots in Figures 3.9 and 3.10 are clearly not as important as in some of the other cases covered before.

3.7 Conclusion

In this paper, we have introduced spectral methods as a tool for analyzing business cycle chronologies. It is a fairly convenient way to examine the nature of comovements across the chronologies of different series, and it is also an ideal tool to compare competing chronologies of reference or other cycles. We uncovered interesting features regarding (1) the relation between the NBER and Romer chronology, (2) the nature of pre- and post-WWII business cycle fluctuations and (3) some stylized facts with respect to the post-WWII era.

Of course, as with any application of spectral analysis, one can only rely on it as a method for decomposing observed series in orthogonal cycles. It does not readily yield economic interpretations of the decomposition. But if one is only paying attention to stylized facts, there are some clear advantages to pairing spectral methods with chronologies.

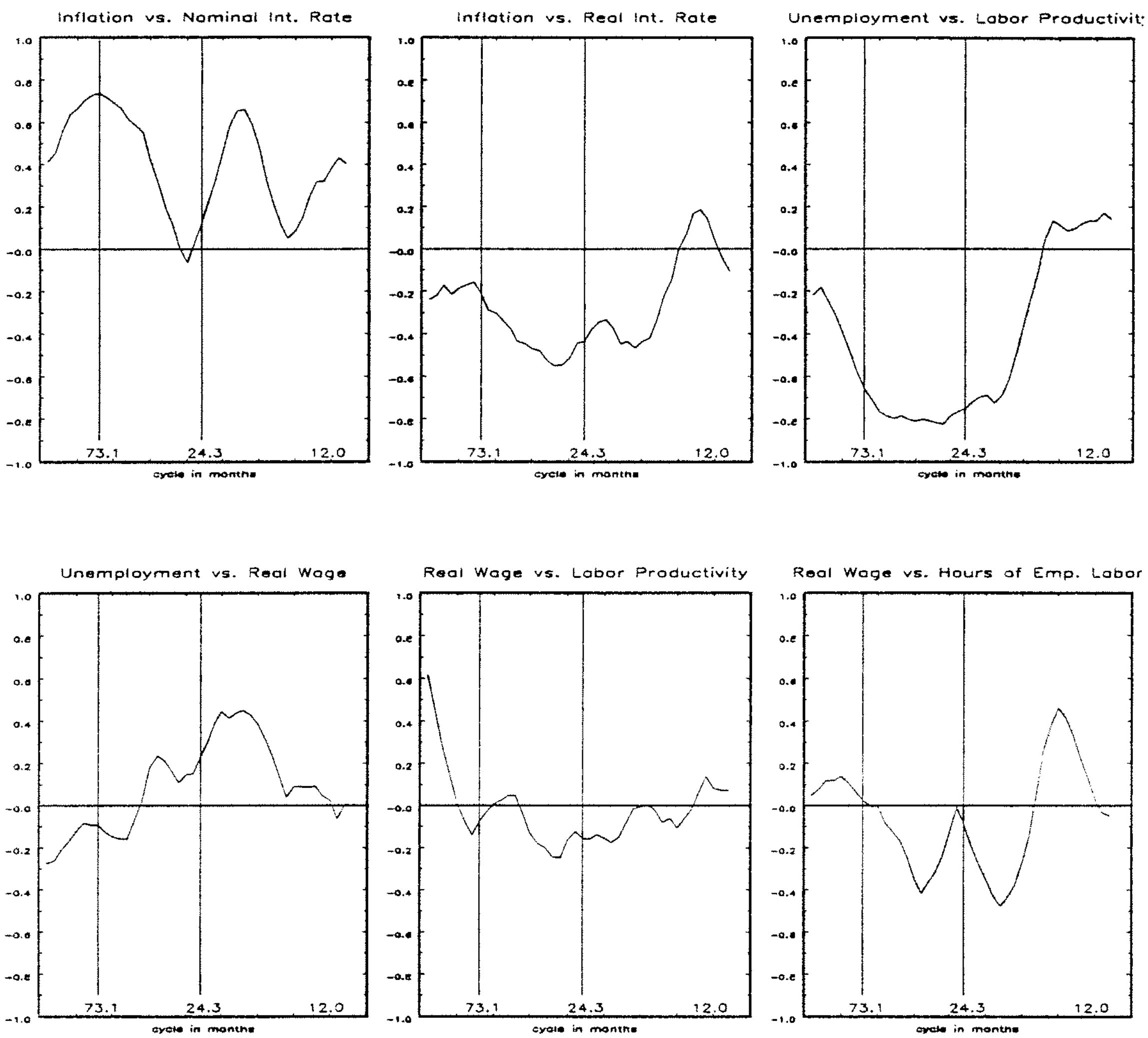

Figure 3.9: Comovements - Individual Series: Chronology-Based Spectrum

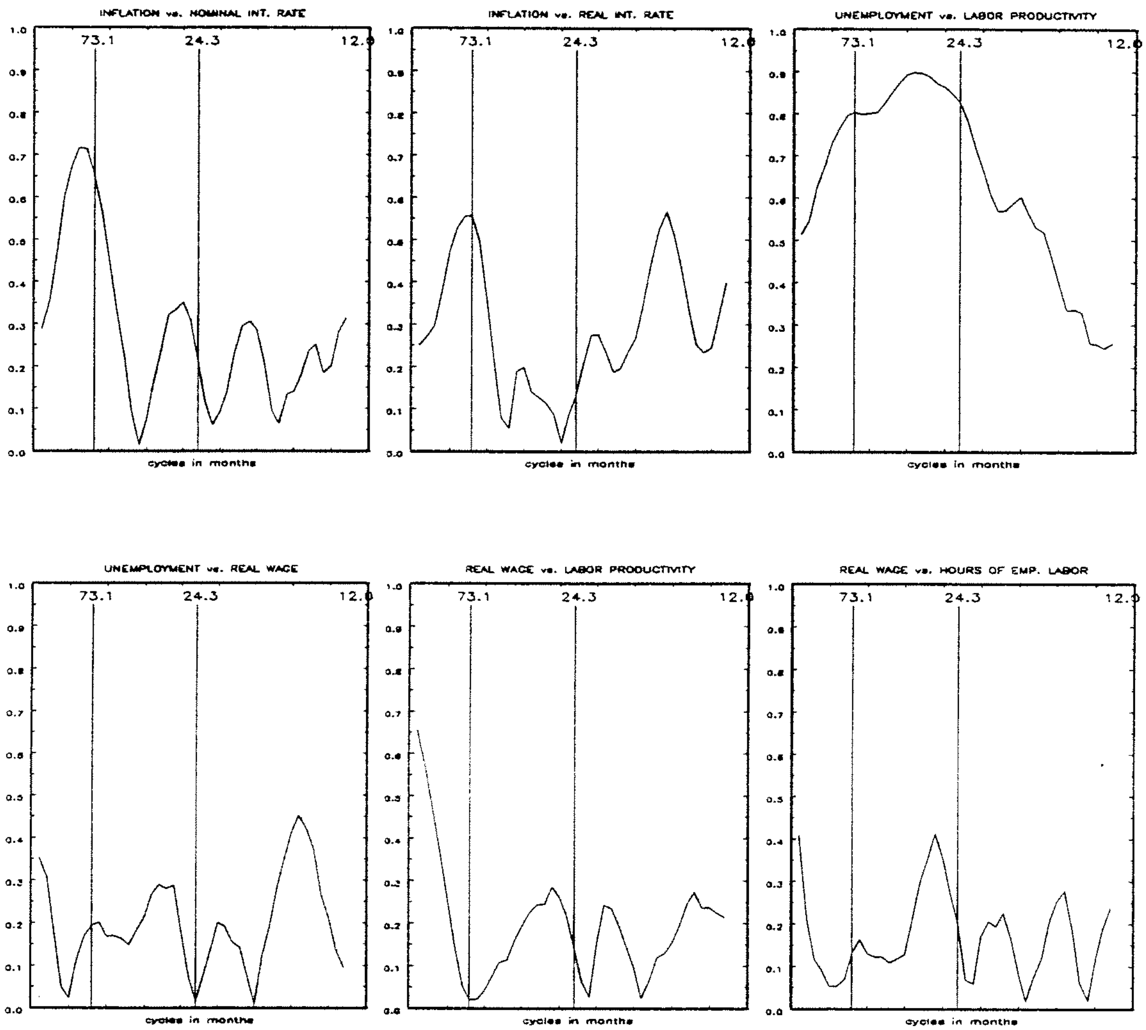

Figure 3.10: Comovements - Individual Series: Fourier Spectrum

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