

Université de Montréal

**International Financial Asset Returns: Theory and Evidence**

**(Analyse des rendements financiers internationaux:  
théorie et évidence empirique)**

par

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Cette thèse intitulée:

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présentée par

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## Sommaire

Cette thèse présente une analyse de la détermination des rendements des actifs financiers internationaux. Notre objectif est de déterminer les facteurs qui influencent la valeur d'un actif financier qui se transige sur le marché international ainsi que la dynamique de cette valeur. Pour ce faire, nous nous concentrons, en particulier, sur les actions, les bons du Trésor et les devises.

Le premier chapitre est un survol des développements théoriques et empiriques récents dans le domaine de la finance internationale. Puisque les modèles d'évaluation d'actifs internationaux sont essentiellement des extensions des modèles financiers en économie fermée, nous présentons les faits empiriques ainsi que les différents modèles d'équilibre partiel et général dans les deux cadres économiques considérés. Dans ce chapitre, nous présentons aussi un petit modèle empirique simple qui tente de distinguer entre les différents types de chocs et leurs évolution sur les marchés de devises d'une part, et les marchés des actions d'autre part. Aux fins de cette analyse, nous utilisons des données mensuelles sur les États-Unis et le Japon.

Pour ce qui est des faits empiriques, nous soulignons les éléments suivants: 1) les premier et deuxième moments des rendements excédentaires des actifs sont prévisibles et cette prévisibilité montre l'existence possible d'une prime de risque variable sur ces marchés; 2) il n'est pas clair qu'il existerait une relation stable entre les premier et deuxième moments du rendement d'un actif; 3) en général, les modèles d'agent représentatif en équilibre général n'ont pas bien réussi à expliquer le niveau et la dynamique de ces primes de risque; 4) il existe des facteurs communs qui influencent les marchés des actions et des devises.

Dans le deuxième chapitre, nous mettons l'accent sur deux modèles empiriques

populaires d'évaluation d'actifs internationaux. Ces modèles ont été estimés par la méthode des moments généralisées et ont été jugé comme étant acceptables selon des tests de suridentification. Cependant, étant donné la faible puissance de ces tests contre des alternatives spécifiques, nous montrons que ces modèles sont rejetés selon des tests de stabilité des paramètres ou des tests d'orthogonalité. Par conséquent, nous proposons un nouveau modèle de type ARCH à facteurs qui minimise l'écart moyen des erreurs du modèle et qui, de plus, n'est rejeté par aucun des tests considérés.

Finalement, dans le troisième chapitre, nous proposons un nouveau modèle d'équilibre général d'évaluation d'actifs internationaux en se basant sur les faits observés dans les chapitres précédents. Ainsi, notre modèle monétaire incorpore deux généralisations par rapport aux modèles similaires existants. D'une part, nous généralisons la fonction d'utilité de l'agent représentatif en lui donnant des préférences de type aversion à la déception. Cette caractéristique relâche l'hypothèse de la séparabilité additive de la fonction d'utilité et permet de mieux capter les attitudes des agents vis-à-vis des petits risques. D'autre part, nous supposons que les processus de dotations dans notre économie contiennent de l'hétéroscédasticité. Ceci permet d'augmenter la variabilité du taux d'escompte stochastique dans le modèle et de mieux répliquer la variabilité des primes de risque dans les marchés considérés. Pour ce faire, nous proposons, pour chaque pays, des processus de Markov bivariés pour leurs taux de croissance de la consommation et de la masse monétaire. Nos résultats montrent certaines améliorations par rapport aux modèles existants et soulignent les complexités des interactions présentes entre les paramètres des préférences et des processus de dotations du modèle.

## Résumé

Au cours des quinze dernières années, la recherche en finance internationale, tout comme en finance en économie fermée, essaye d'expliquer la prévisibilité observée des premier et deuxième moments des rendements excédentaires des actifs. Malgré les nombreuses causes probables de ce phénomène, la majorité des chercheurs ont attribué cette prévisibilité à l'existence de primes de risque variables sur ces marchés. Cependant, les modèles d'équilibre général d'agent représentatif d'évaluation d'actifs qui ont tenté de formaliser cette explication n'ont pas eu beaucoup de succès à répliquer les faits stylisés.

Dans le premier chapitre de cette thèse, nous retraçons l'historique des développements concernant cette prévisibilité des rendements de plusieurs types d'actifs. Nous présentons, en premier lieu, les travaux empiriques qui ont mené à la constatation que les prix d'actifs ne suivent pas une marche aléatoire tel qu'on le croyait. Ces premières études ont été menées dans le cadre de la finance en économie fermée et ont éventuellement engendré des recherches semblables dans le domaine de la finance internationale. Par la suite, nous montrons comment plusieurs chercheurs ont essayé de traiter cette question à l'intérieur d'un cadre théorique plus complet qui se basait sur des fondements microéconomiques. Les modèles utilisés étaient donc essentiellement des modèles d'équilibre général avec agent représentatif et anticipations rationnelles. Nous documentons le fait que ces modèles ont largement échoué dans la reproduction de la dynamique et du niveau de la partie prévisible des rendements excédentaires. Nous soulignons que ces échecs sont le plus souvent attribués à la faible variabilité du taux d'escompte stochastique dans ces modèles et donc une très faible variabilité des primes de risque découlant du modèle. Face à ces échecs, d'autres chercheurs ont essayé de modifier le modèle de base, visant à mieux

caractériser, soit la fonction d'utilité, soit les processus de dotation. Nous documentons les résultats obtenus par ces modèles plus généraux et concluons que, malgré certaines améliorations, les modèles ne sont toujours pas en mesure d'expliquer tous les faits. Ensuite, dans le but de jeter les bases de notre modèle d'évaluation d'actifs internationaux, nous réalisons notre propre étude empirique afin d'établir les faits empiriques pertinents à partir d'une même base de données. Dans cette étude, nous établissons en particulier la prévisibilité de nos rendements excédentaires. De plus, à l'aide d'un petit modèle simple de type VAR, nous tentons de déterminer la dynamique des effets des chocs monétaires et réels sur le marché des actions d'une part et sur celui des devises d'autre part.

Dans le chapitre deux, nous nous attardons sur deux modèles économétriques populaires de finance internationale. Ces modèles sont ceux utilisés dans les études de Dumas et Solnik (1995) et de Ferson et Harvey (1993). Le premier est un modèle de type CAPM international qui relie l'espérance des rendements nominaux d'un actif à la covariance entre ces rendements et les changements dans le taux de change. Les coefficients du modèle sont supposés variables dans le temps et fonction d'un nombre de variables d'information qui sont, en fait, des variables financières du marché des actions des États-Unis. Le second modèle est un modèle de type APT international. Celui-ci rend l'espérance des rendements nominaux d'un actif fonction de quelques facteurs non observés. On permet aux coefficients sur ces facteurs de varier dans le temps.

Les deux modèles ci-haut sont estimés par la méthode des moments généralisés (GMM) et sont jugés acceptables selon le test de suridentification, appelé test-J. Cependant, nous savons que ce test a une faible puissance contre certaines hypothèses alternatives bien précises et donc, dans ce chapitre, nous nous demandons si ces modèles tiennent toujours lorsqu'ils sont soumis à d'autres tests de spécification plus puissants par rapport à certaines alternatives d'intérêt. Notamment, nous évaluons la performance de ces modèles vis-à-vis de la stabilité des paramètres du modèle et de l'orthogonalité des résidus par rapport à des variables contenant de l'information supplémentaire pour l'évaluation des actifs. Le test sur la stabilité des paramètres que

nous utilisons est le test Sup LM proposé par Andrews (1993) et qui détecte un changement structurel dans les données lorsque le point de rupture n'est pas connu. Le deuxième test, le test CS est celui développé par Newey (1985) et qui teste l'orthogonalité des résidus du modèle par rapport à différents variables d'information. Essentiellement, ce test permet de savoir si le modèle utilisé, qui est de forme réduite, fait usage de tout l'information pertinente pour l'évaluation des rendements.

Après avoir estimé ces deux modèles avec nos données mensuelles, nous appliquons les deux tests et constatons que ces modèles manquent de robustesse. En effet, dans les deux cas, nous rejetons l'hypothèse nulle de la stabilité d'un ensemble de paramètres du modèle. De plus, pour le modèle de Ferson et Harvey, nous trouvons que le modèle n'exploite pas toute l'information disponible.

En se basant sur ces faits, nous proposons un modèle empirique alternatif. Il s'agit toujours d'un modèle à facteurs, mais contrairement à Ferson et Harvey, nous supposons que les coefficients de ces facteurs sont des fonctions non linéaires de quelques variables instrumentales. En particulier, ces coefficients sont des ratios de la covariance entre les rendements de l'actif que nous voulons évaluer avec le facteur en question et de la variance du facteur. De plus, on fait l'hypothèse que ces deuxièmes moments suivent des processus ARCH. Après une estimation du modèle par GMM, nous le soumettons aux mêmes tests que les autres modèles et nous parvenons à la conclusion que notre modèle est plus robuste.

Dans le troisième chapitre nous présentons un nouveau modèle d'évaluation d'actifs internationaux. Il s'agit d'un modèle d'équilibre général à deux pays qui incorpore la monnaie en faisant usage de fonction de coûts de transaction.

Ayant constaté les raisons pour lesquelles des modèles semblables ont largement échoué dans le contexte d'une économie internationale, et s'inspirant du modèle de Bonomo et Garcia (1994) appliqué avec succès dans le cadre d'une économie fermée, nous proposons deux extensions au modèle de base. Ces deux modifications ont pour but d'augmenter la variabilité dans le temps de la prime de risque sur les marchés examinés. Ainsi, d'une part, nous généralisons les préférences

de l'agent représentatif en le dotant d'une aversion à la déception. Dans ce cas, la fonction d'utilité est de forme non espérée et permet de séparer le concept d'aversion au risque de la substitution intertemporelle de la consommation. De plus, elle relâche l'hypothèse de la séparabilité de l'utilité marginale à travers les états de la nature. Ceci implique que l'utilité marginale de la consommation dans un état favorable n'est plus indépendante du niveau de la consommation dans un mauvais état de la nature. Finalement, avec ce type de préférences, l'agent représentatif réagit différemment face aux petits et aux grands risques.

D'un autre côté, par rapport au modèle de base, nous enrichissons aussi notre processus des dotations afin d'inclure de l'hétéroscédasticité. Ainsi, nous supposons que les séries du taux de croissance de la consommation et du taux de croissance de la masse monétaire suivent un processus Markovien bivarié à deux régimes dans chaque pays. Les moyennes et les variances de ces séries sont, par conséquent, fonction de l'état de la nature et changent selon le régime en vigueur. Cette hypothèse est une généralisation de la formulation retenue dans les modèles de base qui supposent un processus autorégressif homoscedastique pour l'environnement exogène du modèle.

Étant donné la grande complexité du modèle, nous procédons de la façon suivante pour évaluer la performance de celui-ci lorsqu'il est appliqué au cas d'un monde composé du Japon et des États-Unis. En premier lieu, nous résolvons notre modèle pour obtenir des conditions d'Euler théoriques et nous intégrons, de façon analytique, les fonctions du processus de dotation dans ces conditions du premier ordre. Ensuite, nous estimons les paramètres de ces fonctions Markov bivariées pour chacun de ces pays. Ayant obtenu les valeurs de ces paramètres, nous les substituons dans les équations d'Euler. À cette étape, nous résolvons nos conditions du premier ordre numériquement et exprimons les différents rendements dans notre modèle en fonction de quelques variables endogènes qui changent de valeur selon l'état de la nature dans lequel on se trouve. Il nous est maintenant possible de tirer des erreurs dans les lois spécifiées pour les processus et de simuler des séries de rendements pour les actifs qui nous intéressent.



Avec ces séries simulées, nous obtenons des moments que nous comparons aux valeurs observés dans une échantillon de données réelles. De plus, nous testons la capacité de notre modèle à générer des rendements excédentaires prévisibles avec des instruments créés, eux-mêmes, par le modèle. Nos résultats montrent que notre modèle approxime relativement bien le niveau des moments des marchés des actions dans les deux pays. De plus, nous sommes capables de reproduire la prévisibilité des rendements excédentaires dans ces marchés ainsi que dans le marché des devises. Cependant, nous constatons que le modèle n'est pas aussi performant en ce qui concerne la reproduction des moments des variables dans le marché des devises. Nous concluons, donc, avec certaines suggestions et extensions qui pourraient être incorporées à des recherches futures.

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## **Chapter 1**

# **Recent Developments in International Financial Markets: A Survey**

### **1. 1 Introduction**

The driving force behind much of the research on financial markets in the last decade has been the realization that both first and second moments of asset returns appear predictable. Before this evidence was put forward, it was assumed that asset prices followed random walk processes and that expected returns were constant. Researchers, therefore, had focused mainly on analyzing the cross-sectional relationships between returns. Only after observing that the distribution of the error terms from asset-pricing models had leptokurtic ('fat') tails did studies carefully look at the time-series properties of returns. This heteroskedasticity in the data prompted an interest in the conditional moments of assets to consequently establish their predictability.

In order to explain the above, attention was duly focused on assumptions concerning information processing, the formation of expectations and risk considerations in models of asset pricing. For example, surveys were conducted to establish forecast error patterns of individuals, numerous tests were conducted to



examine the efficient market hypothesis, and theoretical models were built to show the existence of risk premiums in high risk markets. The results, however, are still mainly inconclusive. Although a large number of studies attributed the above observations to the existence of time-varying risk premiums, only a few theoretical models have produced equilibrium conditions, which, in empirical tests, come close to replicating some of the behavior of asset moments. On the other hand, it seems that the existence of peso problems, learning patterns and irrational expectations could equally well explain these facts.

In this chapter we review the new generation of asset-pricing models for foreign exchange and equity markets, concentrating only on the risk premium explanation. We expose some of the key findings and provide a comprehensive review of studies which shed light on the relation between foreign exchange and equity markets. We then conduct an empirical assessment of some of the facts documented earlier using a consistent data set, including an examination of the dynamic effects of real and nominal shocks on excess returns in equity and foreign exchange markets.

## **1. 2 Developments in Equity Markets**

### **1. 2. 1 Early Equity Market Models**

In the early 1970's literature, the most popular model was the Sharpe-Lintner-Black model (hereafter SLB). The main implications of this model were that expected returns are a positive linear function of the market beta (i.e. the covariance of the security's return with that of the market portfolio divided by the variance of the market portfolio return) and that this beta is the only measure of risk needed to explain the cross-section of expected returns.

The first tests of this model were generally supportive of the model implications. The main criticisms, however, started with Roll's critique (1977) which stated that proxies used for the market portfolio in tests of the SLB model were not appropriate. Next, it was noted that variables other than the market beta were capable of explaining the cross-section of expected returns. The main ones were earnings-to-price ratios (Basu (1977, 1983)), stock size (Banz(1981)), and book-to-market equity

value (Chan, Hamao, & Lakonishok (1991), Fama & French (1995,1996)). Basically, this implied either that these variables were correlated with the true beta and that the estimates themselves were noisy, or that the variables were acting as proxies for factors omitted from the model.

Another popular model was Ross'(1976) arbitrage pricing model. This was a multifactor asset-pricing model which assumed that expected returns were explained by a number of factors (as opposed to the SLB, which is a single factor model). For the case where factors were unobservable, the approach consisted of extracting common factors in returns using factor analysis and then testing whether loadings on these factors could explain expected returns. Later, Chen, Roll & Ross(1986) looked for observable economic variables to give these factors economic significance. Among the variables they examined, the growth rate of industrial production and differences in returns between long-term low-grade corporate bonds and long term government bonds were found to have good explanatory power. They concluded that their macroeconomic variables were either risk factors or that they proxied for such factors. However, Fama (1991) cautioned that tests of such models needed to be carefully checked for robustness. He explained that, since these models offered only vague predictions about the variables that mattered for expected returns, there was a danger of spurious relationships emerging between returns and factors.

The two types of models described above are partial equilibrium ones, in the sense that they focus mainly on the mean-variance efficiency of portfolios. Sometimes, however, it is better to have a more complete representation of the economic environment so that the link between various economic and financial variables is more clearly established. The intertemporal asset pricing models of Rubinstein (1976), Lucas (1978) and Breeden (1979), though simple in structure, fall in this latter category and their Euler equations have a rich structure. Thus, for example, Breeden's model describes the interaction between optimal consumption and portfolio investment decisions, implying a positive linear relation between expected returns and the growth rate of per capita consumption.

In this case, and unlike the previous two categories of models described, tests

of these models consisted of testing joint hypotheses on both the cross-sectional and the time-series predictions of the model. Based on these, it was generally noted that, despite their richer parameterization, the restrictions implied by these models were rejected. In fact, when such models were confronted with the SLB model (Mankiw & Shapiro (1986)) and with the multifactor models (Chen, Roll & Ross (1986)), they did not fare very well. Only tests for the main cross-sectional prediction of the model (stating that expected returns are a positive linear function of consumption beta) were usually supportive of the chosen specification.

Nevertheless, a number of interesting facts have emerged from the above studies and are now at the center of present research. Possibly, the most famous of these facts is the equity premium puzzle put forward by Mehra & Prescott (1985). This puzzle indicates that the representative consumer should have an unreasonably high level of risk aversion if we are to explain the large spread between expected returns on stocks and on low-risk securities. We will discuss some of the models that tried to solve this puzzle in section 1.3. However, first we summarize the time-series literature and document the stylized facts.

### 1. 2. 2 Time-series behavior of expected returns

Early work on time-series returns predictability found that only a very small portion of the variance of returns (less than 1%) could be explained by the variance of expected returns. Consequently, the hypothesis of constant expected returns and market efficiency was not rejected. This implied that the best forecast of a return was its historic mean. These tests, however, lacked statistical power (Fama (1991)). With the increasing availability of data at various frequencies, and with the development of new time-series analysis techniques, more recent studies found daily, weekly and monthly returns to be predictable from past returns. This obviously rejects the constant returns model. Moreover, it was noted that small-stock portfolio returns were more predictable than larger-stock portfolios.

Studies on longer horizon returns predictability increased after Shiller (1984) and Summers(1986) presented models in which stock prices take swings away from

fundamental values but in a large and slowly decaying manner. These models implied small autocorrelation in the shorter horizon returns and strong negative autocorrelation for the longer horizon returns. Still, although Fama & French (1988) and Poterba & Summers (1988) both find evidence for this kind of behaviour in the data, the literature is aware of the fact that tests on long horizon returns mean small samples and therefore low power.

Since future returns could be predicted from past ones, perhaps they could also be explained with other financial and economic variables. The studies that took this approach, and examined the predictability of excess returns that is, returns minus the yield on a risk-free rate), soon found a multitude of 'instruments' (or information variables) which were useful in this respect. There are the so-called 'technical' variables which derive only from the equity markets, such as dividend yields (defined as dividend to price ratio), earnings to price ratios, default spreads and term spreads. (See for instance Campbell (1987), Fama & French (1988,1989), Ferson & Harvey (1993), Bekaert & Hodrick (1992, 1993), Canova & Marrinan (1995)). Then, there are economic variables such as inflation rates and interest rates, changes in industrial production and changes in fiscal deficits. (Examples are Kaul (1987), Wasserfallen (1989), Breen, Glosten & Jagannathan (1989), Darrat (1990), Ferson & Harvey (1993)). Yet, in all of the above, R-squares of regressions of returns on any combination of instruments rarely exceeded 10 %, although it appears that this value increases with the length of holding periods for computing returns.

### 1. 2. 3 Time-Varying Second Moments

If one assumes expectations to be rational and markets to be efficient, then the fitted value of regressions of excess asset returns on various instruments can be interpreted as a risk premium to reward individuals for incurring additional risk in asset markets. In this case, risk considerations are important and it therefore becomes useful to examine the volatility of returns. Thus, we now turn to the issue of the predictability of the conditional variance of returns.

Time-series plots of equity returns clearly show volatility clustering in the data

with alternating periods of high and low variability. This implies a certain persistence in the conditional volatility series. Thus, when Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) specification many studies realized the importance of using this time-series modelling technique for characterizing the conditional variability of financial assets. We can write this model in its simplest form as:

$$\begin{aligned}
 R_t &= g(x_{t-1}; \gamma) + \varepsilon_t \\
 \varepsilon_t | I_{t-1} &\sim N(0, h_t) \\
 h_t &= \alpha_0 + \sum_{j=1}^p \alpha_j \cdot \varepsilon_{t-j}^2
 \end{aligned} \tag{1.1}$$

where  $R_t$  is the return on an equity,  $\varepsilon_t$  is the regression error and  $h_t$  is the conditional variance of this error term. The terminology is that the conditional variance follows an ARCH(p). Later, this model was generalized to the GARCH by Bollerslev (1986) where the approach consists of adding lagged own conditional variances to the third equation in the above system. A GARCH(p,q) is thus given by:

$$\begin{aligned}
 R_t &= g(x_{t-1}; \gamma) + \varepsilon_t \\
 \varepsilon_t | I_{t-1} &\sim N(0, h_t) \\
 h_t &= \alpha_0 + \sum_{j=1}^p \alpha_j \cdot \varepsilon_{t-j}^2 + \sum_{k=1}^q \beta_j \cdot h_{t-k}
 \end{aligned} \tag{1.2}$$

In equity markets, highly significant test statistics were reported for the  $\alpha$  and  $\beta$  coefficients in many such studies. See, for instance, French, Schwert and Stambaugh (1987) for daily returns, Chou (1988) for weekly returns, and Attanasio and Wadhvani (1989) for monthly and annual returns. Generally, the retained specifications were low-order GARCH(p,q) type models with the exception of ARCH(3) formulations employed for monthly stock returns as in Bodurtha and Mark (1991) and Attanasio (1991).

Initially, the innovation term in the returns equation was assumed to be distributed conditionally normally. Models with this normality assumption were

generally found to explain only part of the observed fat tails in the unconditional distributions of returns. To remedy this situation, Baillie and DeGennaro (1990) and De Jong, Kemna and Kloeck (1990) considered conditionally t-distributed errors instead. They found that their specifications provided a good description of daily and monthly US equity returns. They also indicated that inadequate modelling of the fat-tailed property could induce spurious connections in risk-return relations.

Despite this improvement, it remained that this category of models could not account for the negative correlation that exists between current returns and future volatility. This is known as the leverage effect and was suggested by Black (1976). It was found, however, that the exponential GARCH proposed by Nelson (1990), the so-called EGARCH, could explain this phenomenon.

In conclusion, versions of the ARCH specification have been successful at capturing fairly adequately the observed volatility clustering in the data. It was also noted that adding other information variables to the autoregressive terms in these equations sometimes helped improve predictability even further. The one major criticism that is made regarding these above models is that they lack a theoretical foundation. Despite this fact, they remain highly useful and are routinely applied in financial models.

#### **1. 2. 4 The Relation Between Conditional Mean and Variance**

The capital asset-pricing model implies that the risk premium is influenced by the second moments of the returns series. In order to model this relation empirically while simultaneously accounting for the predictability of the moments of returns, the basic model was crossbred with econometric methods used for estimating time-varying conditional moments.

This gave rise to the so-called ARCH-M class of models, which, with respect to the ARCH, has the additional property that the variance term directly enters the returns regression equation. Later, the GARCH-M specification was also developed to generalize the above basic framework. Examples of studies applying this methodology to equity returns are French, Schwert and Stambaugh (1987) and Baillie and

DeGennaro (1990).

Others estimated conditional moments by moving averages, such as Pindyck (1984) and Poterba and Summers (1986), by regressions on observable instruments, as in Campbell (1987) and Harvey (1991), or even by nonparametric methods, for instance, Gallant and Tauchen(1989), and looked at the nature of the relation between conditional mean and second moments. The conclusion to all of the above is that a consensus does not exist on the sign and the importance of this relation.

More specifically, French, Schwert and Stambaugh (1987) examined whether expected excess returns in the stock market were related to the ex-ante volatility of the stock market in an intertemporal context. Using daily US data for the 1928-1984 period they found monthly estimates of volatility. These showed large time-variation. They then decomposed this series into predictable and unpredictable components and showed that regressions of monthly excess returns on each of them implied a positive relation between returns and volatility. On the other hand, Campbell (1987) noted that although there is good evidence that conditional variances of excess returns change over time, the relation between this and the conditional mean is negative. However, he also added that this result was sensitive to the particular set of instruments used in his study. In another study, Baillie and DeGennaro (1990), using daily and monthly data and various GARCH-M specifications, found this relation to be weak. They also provided evidence as to the sensitivity of their results to different specifications chosen. Finally, Poterba & Summers (1986) showed that fluctuations in volatility could explain only a small fraction of the variation in stock prices.

This issue was analysed comprehensively in a recent study by Backus and Gregory (1993). The authors examined this relation in numerical versions of a dynamic asset-pricing theory and showed that it could be increasing, decreasing, flat, or nonmonotonic and that its shape depended on the stochastic structure of the specified economy, as well as on the preferences of agents.

### 1.3 Models of equity risk premiums

It became the objective of new equilibrium models to try and reproduce the predictability of returns and return variances as well as other facts mentioned above. Proponents of rational expectations adapted existing asset pricing models by making them conditional on the information available to agents at each time period. These constitute the new generation of conditional asset pricing models, i.e. conditional SLB, APT and consumption CAPM models. They are usually estimated by the generalized method of moments (GMM) and employ the 'instruments' described previously.

Bodurtha and Mark (1991) and Ng (1991) both found some support for using the conditional CAPM with time-varying first and second moments to explain the behavior of monthly returns for portfolios containing common stocks listed on the NYSE. In the first study conditional second moments were specified as ARCH(3), expected return as an autoregression, and estimation was carried out using GMM. In the latter, innovations were assumed to follow GARCH(1,1) processes and the conditional variance term was assumed to enter the conditional mean equation through an ARCH-M structure. This model was estimated by the maximum likelihood method.

Others used the conditional CAPM successfully in an international context. For instance, Harvey(1991) examined its validity for describing monthly returns for 17 countries. He conditioned returns and second moments on common instruments mostly derived from the US equity market and found that he could reject his model restrictions at the 5 % level for only 3 of those countries. Chan, Karolyi and Stulz (1992) also found that they could not reject a model combining the international version of the CAPM and a bivariate GARCH specification for daily data on the S&P500 and a major foreign index.

In contrast to the single-beta model just described, and despite its fairly good success, the multi-factor conditional APT model was often shown to be superior, specially in tests of international model specifications. Examples of studies belonging to this category are Ferson and Harvey (1993), Ferson and Korajczyk (1995), Fama and French (1993), Dumas (1994), Harvey, Solnik, and Zhou (1994), and King, Sentana and Wadhvani (1994). Whether the factors are latent or observable, large fractions of



the predictability of national equity returns were found to be explained with models including at least a world market factor and a foreign exchange factor. Similarly, the time-variation in the covariances between stock markets was found to be fairly adequately captured with a multivariate multi-beta model, with the risk premium on assets being a linear combination of risk premia associated with these factors.

Recently, however, MacKinlay (1995) warned against the automatic use of APT models since it is hard to distinguish between deviations from the CAPM induced by risk-related and non-risk variables. In addition, it has not yet been shown how these factors are driven by the stochastic behavior of earnings, profitability, or other fundamentals. Thus, a theoretical explanation is still lacking. Such an approach also ignores the effect of costs of investing in a foreign country and other information problems.

Finally, there is the category of full general equilibrium consumption CAPM models which suppose that risk is priced on the basis of the covariance of returns with the marginal rate of substitution between present and future consumption. Although these models are a more complete characterization of the economy than the ones described so far, they have often been rejected in tests. More specifically, the Euler equations of these models imply that the conditional expectation of discounted asset returns equals a constant. In this context, the key to reproducing the required behaviour of assets is specifying a suitable form for the stochastic discount rate, that is, the intertemporal marginal rate of substitution. This question was examined by Mehra and Prescott (1985) in the case of equity markets. They showed that for reasonable configurations of preference and endowment processes, an exchange economy equilibrium model could not reproduce the secular difference between the average return on stocks and that on Treasury bills (this is the so-called equity premium puzzle).

In addition, when Attanasio (1991) tested the traditional CAPM against the consumption CAPM within a nested framework, he found that, even with two different specifications of heteroskedasticity, ARCH and ARCH plus other instruments, the simple CAPM performed better than its alternative. He suggested that one reason for the failure of the latter is that the covariation between consumption growth and excess

returns is too low to account for the average risk premium unless a very high degree of risk aversion is retained.

Eventually, the basic consumption CAPM model was verified in two different directions. On the one hand, some authors tried to generalize the preference structure of the models, rendering the utility function either state or time non-separable. These include the works of Epstein and Zin (1989, 1991), Weil (1989), Constantinides (1990), Ferson and Constantinides (1991), and Campbell and Cochrane (1995). The first three authors introduced much more flexibility in their preference structure by allowing attitudes towards risk to vary independently from intertemporal substitutability in their models. The remaining studies focused instead on making the risk aversion coefficient time-varying. However, while these models were able to somewhat increase the variability of the resulting risk premium, this augmentation was still insufficient to replicate the stylized facts.

The other direction taken by recent research in trying to improve the basic consumption CAPM specification focused on the time-series of the endowment process in these models. Among these are the works of Rietz (1988), Cecchetti, Lam and Mark (1993) and Bonomo and Garcia (1996). Rietz (1988) argued that the risk premium might be a response to the time-varying risk of economic catastrophes. He therefore tried to model his consumption process to reflect the probability of a huge catastrophe occurring. While his model did produce a good deal of variability in the stochastic discount rate, it is questionable whether such an assumption is valid.

The other studies, Bonomo and Garcia (1996) and Cecchetti, Lam and Mark (1993), both proposed a bivariate Markov switching process for consumption and dividend growths series to describe the endowment component in their expected utility framework models. The first authors showed that, on the one hand, loosening the usual assumption that dividends equal consumption and, on the other hand, using a three-state bivariate Markov switching model where the means and variances changed with the state, yielded good results on real returns in such models. The second authors used a two-state homoskedastic specification with isoelastic preferences, with and without leverage effects. However, for both studies, the adopted endowment process still could

not fully explain the facts related to excess returns in equity markets.

Finally, in another study, Bonomo and Garcia (1993) went a step further and tried to integrate both generalized preferences and heteroskedastic exogenous processes in their model. This study has had the most success so far in matching observed moments in the data. Indeed, these authors showed that both features needed to be included in the basic CCAPM model to be able to approximately replicate the stylized facts.

In the next section we examine the extent of the success of conditional models in explaining returns in the foreign exchange market. After establishing the predictability of excess foreign exchange returns, we provide an overview of international asset pricing models that explicitly take this predictability into account. We also document the possible existence of common shocks to different financial markets.

## **1. 4 The Foreign Exchange Market**

### **1. 4. 1 Tests of the Unbiasedness Hypothesis and Excess Returns on Foreign Exchange**

The concept of forward market efficiency states that, given rational expectations, the forward rate should be an unbiased predictor of the future spot rate. Should these two differ, under risk neutrality, arbitragers would force them into equality and there would be no excess returns to speculation. However, there exist numerous studies that have documented that the forward rate is not an unbiased predictor of the future spot rate. This result seems robust to the frequency of data used, to the selected bilateral exchange rates, and to varying time spans considered in tests. The equation most frequently tested has been the following regression<sup>1</sup>:

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1. This equation can also be estimated in levels but, in this case, it is necessary to normalize both the dependent and independent variables by  $s_t$  to induce stationarity (Meese and Rogoff (1983)).

$$s_{t+1} - s_t = \alpha + \beta(f_t - s_t) + u_{t+1} \quad (1.1)$$

where  $s_t$  is the log of spot exchange rate at time  $t$ ,  $f_t$  is the log of forward rate,  $f_t - s_t$  is the forward premium, and  $u_{t+1}$  is the regression error. The test of forward market efficiency consists in testing the null hypothesis of  $\alpha = 0$  and  $\beta = 1$ .

There is an impressive number of studies which report a negative estimate of  $\beta$  for various currencies and with different data frequencies. Some recent studies which have confirmed this are Froot and Frankel (1989), Backus, Gregory and Telmer (1993), and Mark, Wu and Hai (1993). Others re-assessed the rejection of the unbiasedness hypothesis, this time in bivariate and multivariate VAR models (See, for instance, Baillie (1989) and Bekaert & Hodrick (1992)).

It was suggested that this result could be due to measurement errors (see Cornell (1989)), small sample bias, or incorrect data sampling. However, Bekaert and Hodrick (1993) showed that the above result is robust to most of the issues raised, as well as to missing data.

The finding that the forward rate is not an unbiased predictor of the future spot rate was attributed mainly to two explanations: either that agents do not have rational expectations, or that there exists a risk premium in this market. To be able to distinguish between these two alternatives, a number of surveys were conducted. These found, on the one hand, that expectations in the foreign exchange market were not entirely rational and that forecast errors differed according to the forecast horizon. On the other hand, they also revealed the presence of non-zero risk premia. It therefore seems that the two explanations are valid. For our part, in this study we will concentrate only on the latter explanation.

Consider the regression coefficient in equation (1.1) above. From the OLS properties, and if the estimator of the slope coefficient is unbiased, we have that

$$\bar{\beta} = \frac{Cov(f_t - s_t, s_{t+1} - s_t)}{Var(f_t - s_t)} \quad (1.2)$$

If expectations are rational, then the foreign exchange risk premium can be defined as  $rp_t \equiv f_t - E_t(s_{t+1})$ . In this case,  $E_t(s_{t+1}) - s_t = f_t - s_t - rp_t$  and since we have that  $s_{t+1} - s_t = E_t(s_{t+1}) - s_t + \varepsilon_{t+1}$ , the numerator of the OLS slope coefficient in (4.1) can be written as

$$\begin{aligned} Cov(f_t - s_t, E_t(s_{t+1}) - s_t) &= Var(f_t - s_t) - Cov(E_t(s_{t+1}) - s_t, rp_t) \\ &\quad - Var(rp_t) \end{aligned} \quad (1.3)$$

Defining

$$\beta_{rp} = \frac{Cov(E_t(s_{t+1}) - s_t, rp_t) + Var(rp_t)}{Var(f_t - s_t)}, \quad (1.4)$$

we have that  $\bar{\beta} = 1 - \beta_{rp}$ . Now, if  $\bar{\beta} < 1$ , then it must be the case that  $\beta_{rp} > 0$ . In addition, using Fama's decomposition and maintaining rational expectations,  $Cov(E_t(s_{t+1}) - s_t, rp_t) < 0$  and  $Var(rp_t) > Var(E_t(s_{t+1}) - s_t)$ . Consequently, models of the foreign exchange risk premium should take the above facts into account. In the next section we discuss various foreign exchange risk premium models. However, first we document other aspects of the predictability of excess returns in the foreign exchange market.

As with equity markets, an examination of the regression error term in equation (1.1) shows the presence of autocorrelation and a 'fat-tail' distribution. If expectations are rational, this indicates that there is information other than the forward premium which could be exploited to predict exchange rate changes. This was confirmed by many studies which examined the predictability of excess returns in the exchange market. That is, they considered the predictability of the term  $s_{t+1} - f_t$ . Hodrick (1987) and Engel (1995) provide an extensive survey of these studies. In particular, we mention the work of Campbell and Clarida (1987), Giovannini and Jorion (1987), Cumby (1988), and Bekaert and Hodrick (1992) on this topic.

The findings in these studies indicate that the list of information variables that can predict returns include past exchange rates, past forward premiums, interest rates

and other term spread variables. Interestingly, some studies also point out that instruments typically used to predict excess equity returns (see section 1.2) are also capable of explaining foreign exchange excess returns. This reinforces the view that there are a few factors which affect most financial assets so that there is common variation in expected returns of different securities. We will come back to this in more detail in sections 1.5 and 1.6. For the moment we summarize the literature on some models of risk premiums in foreign exchange markets.

#### 1.4.2 Foreign Exchange Risk Premium Models

The risk premium in the foreign exchange market is defined as  $E_t(s_{t+1} - f_t)$ . That is, risk averse individuals demand a premium for incurring risk in the open position as opposed to hedged ones. If indeed such a premium exists, it must be identical to the predictable component of the excess returns in this market. As mentioned before, there are numerous studies which document the empirical properties of this risk premium. (See, for example, Hansen and Hodrick (1983), Domowitz and Hakkio (1985), Hsieh (1988), Mark (1988), McCurdy and Morgan (1988), Baillie and Bollerslev (1989,1990), Canova and Marrinan (1993, 1995). The consensus is that this premium is highly persistent and has a fair amount of variability, with the variance also itself predictable.

As with excess equity returns, the ARCH specification was judged to be well-suited to model the foreign exchange risk premium. Examples of studies who explored this route include Domowitz and Hakkio (1985) who used an ARCH(4) on monthly data for exchange rates relative to the US dollar, Hsieh (1988), who assumed an autoregression equation for the mean and an ARCH(12) for the variance, McCurdy and Morgan (1988), who modeled the variance as a GARCH(1,1), and others. These models were found to perform fairly well, although diagnostic checking showed that some leptokurtosis still remained in the data. Versions of the ARCH-in-mean specifications were also considered. Examples are Domowitz and Hakkio (1985), Pauly (1988), McCurdy and Morgan (1988), and Kendall (1989). In general, these were found to be less successful than models in the previous category. Multivariate GARCH models were also attempted. These were used to describe contemporaneous correlations

between a number of currencies. Examples are Baillie and Bollerslev (1990) and Engle and Rodrigues (1989). The first used weekly data and found favorable results whereas the latter rejected their model restrictions with monthly data. Finally, in related work, Mark (1988) combined the conditional CAPM with ARCH and ARCH-M formulations for the second moments. He found that the parameters in the variance equations of monthly data on four different exchange rates were significant.

The models described above were purely econometric and made only little use of international financial theory. Therefore, other studies tried to address the risk premium issue in a full general equilibrium optimization model. This was based on the idea that a more extensive theoretical articulation of the various existing economic relations would yield richer and more precise first-order conditions which, when estimated, would be able to explain the time-variation in the series of interest.

Mark (1985) was amongst the first who estimated such an Euler equation. Assuming a constant relative risk aversion utility function, he used the generalized method of moments (GMM) to jointly estimate the resulting first-order condition for four currencies relative to the US dollar. Mainly, he found that his estimate for the coefficient of relative risk aversion was too high and that when he used the forward premium and its lags as instruments, he rejected the overidentifying restrictions of the model.

Another study that took the same direction was by Kaminsky and Peruga (1990) who used monthly data on three currencies relative to the US dollar and a GARCH structure for their covariance matrix. While their model was judged acceptable overall, their estimated value for the coefficient of relative risk aversion was too high and diagnostics checking revealed that their estimated residuals were still correlated to information variables.

In the above cases, the models could neither capture the predictive power of the forward premium nor produce the variability of the risk premium. Consumption data was simply not variable enough to explain the high variance of ex ante excess returns for acceptable values for the coefficient of relative risk aversion. If nominal shocks are important in shaping returns, it becomes necessary to include the money growth process

as part of the exogenous environment of the economy. In this case, a natural starting point is the Lucas (1982) model. Examples of studies that try to explain the forward risk premium and are extensions of the Lucas (1982) model include Macklem (1991), Canova and Marrinan (1993) and Bekaert (1994).

In Macklem (1991), the author assumes that the foreign price level is exogenous and random so that exchange rates, interest rates and the risk premium are all functions of the stochastic processes of the domestic money supply, output and foreign prices. The assumed dynamics for these exogenous processes is taken to be a Markov process with a finite number of states and where transition probabilities between states is chosen such that the moments of the variables match those in the data. Once again, the results show that the obtained variance for the risk premium is extremely small, even with a coefficient of relative risk aversion of five.

The study by Canova and Marrinan (1993) is in the same vein as Macklem (1991). In this representative cash-in-advance model, the exogenous variables are assumed to be uncorrelated but are made to follow AR(1) processes with GARCH variances. The process for the money supply growth is estimated for the US and the rest of the world. However, the parameters of the government spending process are obtained differently. Based on a numerical algorithm, they are calibrated so as to reproduce moment values for the model risk premium that are as close as possible as their values in the data. Results, in this case, show that the authors can match fairly well the moments of the risk premium in their data and that this result is mainly due to the assumed time-varying variance for their government spending.

The final example in this category which will be described here is Bekaert (1994). This author's work combines elements of both of the above authors. Like Macklem (1991), he uses Markov chains to characterize the driving processes in his Svensson-based model. However, unlike the latter, Bekaert does not take foreign prices as exogenous to the model. Instead, he uses both domestic and foreign consumption and money growth processes for his exogenous environment which he allows to be correlated. Similar to the calibration methodology used by Canova and Marrinan (1993), the utility function parameters are estimated such that moments of different



variables in the model are close to their values in the data. The model is estimated for the US and Japan and its results indicate, once again, that the generated risk premium has a very small variance.

In such models, both preference and endowment parameters influence the level and the dynamics of the stochastic discount factor. Since studies that concentrated on improving the endowment process specifications did not fare very well, it was suggested that perhaps a richer parameterization of agents' preferences could improve the model predictions. With the usually assumed expected additively-separable utility model, the risk premium is proportional to the variance of the change in consumption generated by the gamble. Therefore, for gambles that are small relative to the agent's consumption, this risk premium is small<sup>2</sup>. Instead, if utility is not separable over states, the risk premium will be proportional to the standard deviation of the change in consumption. This, in turn, implies a larger value for the risk premium since the agents are more averse to smaller gambles. Finally, if the utility function is not time-separable, the coefficient of relative risk aversion becomes time-varying and will adjust depending on the difference between present and habitual consumption levels.

Backus, Gregory and Telmer (1993) followed the latter line of reasoning and introduced habit persistence in their model preferences. Using GMM, they then estimated their Euler equation jointly for four different exchange rates. However, despite the generalization in preferences, these authors also reject their model restrictions. Once again, given that their model depended only on consumption data, their model could simply not generate the required variability for their risk premium. Other studies that introduce habit persistence in their models and find similar results include Constantinides (1990) and Campbell and Cochrane (1995).

Finally, a study that used a state non-separable utility function in an international general equilibrium consumption asset-pricing model was the paper by Bekaert, Hodrick and Marshall (1997). The specific type of generalized preferences that these authors made use of is known as 'disappointment aversion' type preferences<sup>3</sup>. Choosing once again a VAR process with homoskedastic errors for their

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2. See Epstein and Zin (1989, 1991).

exogenous variables their model was neither able to match the variability of the risk premium observed in the data, nor explain the predictability of excess returns using their generated forward premium.

To conclude this section, it seems that international equilibrium asset-pricing models have fared poorly in explaining the properties of the risk premium and of excess returns. While incorporating either richer exogenous driving processes or more generalized preferences have helped improve the results, they have not been fully successful. One natural extension of the above literature is to try to include both these extension features in the model simultaneously. We propose just such a model in chapter three of this thesis.

### **1.5 Relations Between Equity and Currency Markets**

We have seen that there are time-varying risk premiums in both equity and foreign exchange markets, and that, so far, no general equilibrium model has yet offered a full explanation for this phenomenon in the international case. In particular, none of the latter models were able to produce sufficient variability in the stochastic discount rate for reasonable preference parameters. This has been the case despite efforts by some authors to either generalize preferences or characterize the exogenous environment of the economy more appropriately.

Bonomo and Garcia's (1993) study has shown that, in the case of the closed economy model, including generalized preferences and heteroskedastic driving processes simultaneously substantially improves the results. In chapter three of this thesis we follow the lead of these authors and propose an international asset pricing model that also generalizes preferences and the exogenous environment simultaneously. However, it is first necessary to investigate whether risk factors are similar in asset markets. More specifically, we are interested in finding out whether the same fundamentals affect equity and foreign exchange markets, and whether these effects are felt in the same manner. This is the topic of this section.

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Few papers have discussed the relation between exchange rates and stock

3. See Gul (1991).

market prices in any detail. Although some theoretical models exist, such as Gavin (1989) and Manuelli and Peck (1990), they were not empirically tested. From the intuition behind the portfolio balance approach to exchange rates, and from the application of financial models to forward contract pricing, some insight has been provided into the nature of the empirical relation between these two series. Nonetheless, a systematic study of such an association is yet to be undertaken.

General ideas about this issue come from studies such as Obstfeld (1985) which provide very broad associations of the two series of interest. Thus, in discussing the shocks which affected exchange rates in the 1977-1982 period, Obstfeld maintains that one way of assessing the relative importance of goods market shocks against the money market shocks is by looking at the correlation which exists between nominal exchange rates and nominal stock prices. He conjectures this correlation to be highly positive if the shock is a local money market shock, and low, and even negative, in the case of real shocks.

For example, an increase in money supply will decrease interest rates and increase inflation creating a greater demand for equities (thus increasing stock prices) and cause currency depreciation. On the other hand, a positive shock to aggregate demand will generally cause currency depreciation whereas the effect on stock prices is ambiguous. Similarly, a positive shock to aggregate supply will tend to increase stock prices, but will have an ambiguous effect on the exchange rate.

Obstfeld calculates correlations between percentage changes in effective exchange rates and stock price indexes for the US, Japan, and Germany with monthly data for the period 1976-1985. This reveals monetary shocks to have been more important for the US, contrary to Japan, where real shocks seem to have dominated. (The correlation is insignificantly negative for Germany). He also finds the correlation coefficients to have been unstable over time since their value changes significantly once his sample is split to two.

Finally, a comparison of standard deviations for the two series studied reveals the stock market to have been at least twice as volatile as exchange rates for the 1976-1985 period, except for Japan which was found to have comparable volatilities.

In addition to the Obstfeld study, more recently, some other empirical studies analyze, directly and indirectly, aspects of the relation between exchange rates and stock returns. These are elaborated in the following sections.

The first of these is a paper by Jorion (1991). This study asks whether foreign exchange risk is priced in the stock market. Since many multinational firms devote substantial resources to active hedging of foreign exchange risk, the answer to this question would be of much consequence to them for what it entails for their cost of capital.

Foreign exchange risk can be divided into risk on net monetary assets and on real assets. Whereas a firm's foreign monetary assets are clearly fully exposed to exchange risk, it is also true that the real assets of the firm are also subjected to this risk via real shocks in the economy. Consequently, one would expect correlations between the stock market and foreign exchange rates to be shaped according to the nature of the shocks in the economy. More specifically, if these shocks affect various industries differently, such a correlation is sure to vary according to industry. Furthermore, if there are common shocks both to the stock market and to exchange rates, and if pervasive sources of risk are priced in the stock market according to Ross' APT, then exchange risk should not be separately priced in the equity market.

Two versions of the APT model are used to test the above propositions. The first is a two-factor model with a market index and the component of the exchange rate which is orthogonal to the market. The second uses the six economic factors used in Chen, Roll and Ross (1986) as well as the component of exchange rate orthogonal to all the previous factors. The dependent variable is excess returns on value-weighted industry portfolios.

Test results using monthly data for the US for the 1971-1987 period indicate that substantial differences exist between the correlation of stock returns of industries relative to exchange risk exposure. Nevertheless, it is shown that excess expected returns in stocks are not systematically related to exchange risk. Thus, foreign exchange risk is already present in the pervasive factors used in the APT.

In the Jorion (1991) study constant coefficients are assumed in the model despite the existence of ample evidence on time-varying risk premia. Perhaps results would have been different had these facts been taken into account. Indeed, the study by Korajczyk and Viallet (1992) acknowledges that both forward premia and equity risk premia vary over time and asks whether there is a relation between the two.

The basic premise of this model is the Lucas (1982) intertemporal asset pricing model which is used to describe equilibrium excess returns in the forward market. This is combined to an equilibrium version of the APT. The hybrid version features the Lucas equation having a benchmark portfolio which is a linear combination of portfolios that imitate movements of common factors in the economy. The intuition behind this formulation is provided by Korajczyk and Viallet (1989) in that there exists significant correlation between equity benchmark portfolios and exchange rate movements indicating the incidence of common shocks to exchange rates and stock returns.

Eight exchange rates vis-a-vis the US dollar are studied. The data are monthly, extending from January 1974 to December 1988. First, returns on forward contracts are examined to verify that they exhibit forecastable components. This is found to be the case. Next, estimates of excess returns on the benchmark portfolio (i.e. factor returns) are obtained from returns on a large cross-section of international equity markets using the method of asymptotic principal components.

As a preliminary exercise, these returns are then regressed on the eight forward premia, lagged one period. Test results support the point of view that factor risk premia are ex-ante related to instruments which can predict excess forward returns. It is now possible to regress excess forward returns of each country on a constant, excess returns on five factor-imitating portfolios, and the forward premium. In these regressions, factor betas are made to depend on the forward premium to induce time-variation. In addition, a dummy is included in the regression to divide the sample into two.

Test results reject the constant beta hypothesis and also yield significant correlation of the dependent variable with factor returns. However, because the hypothesis that the intercept and the forward premium coefficients are zero is rejected,

the authors indicate that there still remains a time-varying component in excess forward returns not explained by the factor returns. This could either mean that there are sources of risk in forward markets not present in equity markets, or that the constructed reference portfolio fails to reflect the true benchmark returns.

According to this study, equity markets help explain forward excess returns. But is the reverse also true? In Roll (1992) the author tries to answer the question of why stock price indexes exhibit disparate behaviour when they are compared across countries. He explains this disparity essentially by three items: the technical construction of the index, the country's industrial structure, and the behaviour of exchange rates.

The study uses daily data for 24 countries over the three-year period of 1988 to 1991. The dollar-denominated return on a given country's national stock market is explained in an OLS regression by the concurrent relative change in the local exchange rate, seven broad industry return indexes, and a dummy variable to capture the Monday effect in the data. One daily lead and one daily lag of each variable, except the dummy, are also included to correct for spurious autocorrelation resulting from asynchronous world market trading effects.

The error term in the regression equation is diagnosed and found to exhibit both heteroskedasticity and leptokurtosis. Accordingly, each time-series model is also estimated using a GARCH(1,1) specification. Almost all the countries studied were found to have a significant 'alpha' parameter whereas seventeen also showed the 'beta' parameter to be significant as well. Nevertheless, the GARCH results for coefficients, t-statistics, and adjusted R-squares showed much similarity to the OLS results: both, global industry indexes and exchange rates are capable of explaining a fair proportion of stock price movements, although industry factors have the more explanatory power.

What are the implications of such a finding? With returns expressed in local currency, one would expect part of the index volatility to be induced by monetary phenomena, such as changes in anticipated inflation. If exchange rates are entirely a monetary phenomenon, converting returns to dollar-denominated series should eliminate this volatility.

Roll shows that this is not the case for the majority of countries studied. Large differences in volatility still remain even after nominal and inflation differences are taken into account.

One explanation for this is that real exchange rate volatility also affects stock index volatility. In the words of Roll(1992), 'occasional exogenous changes in exchange rates has a direct, immediate and causative influence on local nominal equity prices'. On the other hand, it is also possible that global industry shocks affect local equity index returns. This induces an effect in exchange rates via an eventual intervention on this market by the monetary authorities. If such an intervention is habitual and effective, it would surely be anticipated by market participants creating the illusion of concurrent changes in the two series studied.

The logical conclusion to the above sequencing of models is extending the framework to include 2-way causality. This can be elegantly done with a VAR structure and this is precisely the tool used in the next model which is a paper by Bekaert and Hodrick(1992).

In order to evaluate equity investment opportunities for the US investor in both local and foreign equity markets, each country's excess equity returns are converted to dollars. This makes the series examined comparable. In so doing, forward excess returns are found to be integrated into the VAR creating an opportunity for testing propositions on the mutual predictability of these returns. Consequently, two-country VAR(1)s are constructed for each of the countries along with the US. Variables included are the US and foreign country excess equity returns, the relevant forward excess return, two dividend yield variables, and the forward premium.

Data used are monthly, sampled at each end of the month, for the period 1981-1989. Each system is estimated by OLS and heteroskedasticity consistent standard deviations are reported for tests of the parameters. Results show that dividend yield variables have predictive power for excess returns in the foreign exchange market. Similarly, forward premia can predict excess returns in the equity market.

In conclusion to this section we can say that there is cause for believing that

there exist both common and idiosyncratic shocks in both markets. It is, therefore, specially interesting to try to study the importance of these shocks for one market or the other, and to document the dynamic response of series of interest to these shocks. The next section presents an empirical analysis of the facts examined so far.

## **1. 6 Empirical Analysis and Dynamics**

From the preceding sections, we document a number of stylized facts. These are that: 1) excess returns and their variances are predictable with different information variables, 2) it is unclear whether own variances can help predict excess returns, 3) foreign exchange excess returns are highly persistent and somewhat variable, 4) excess equity returns are less persistent and more volatile, 5) information variables of one market can help predict the risk premium in the other, and finally, 6) there are possibly idiosyncratic and common shocks to these two markets.

In this section we conduct our own empirical analysis for the US and Japan. We are mainly interested in 1) assessing the predictability of excess returns in the equity and foreign exchange markets in these countries, 2) distinguishing between the sources of shocks to each market, and 3) understanding the dynamics of excess returns with respect to these shocks.

### **1. 6. 1 Description of the data**

The data is monthly, spanning the period from January 1983 till February 1994, for a total of 134 observation points. The holding period is 90 days. From the IFS databank we obtained spot exchange rates and 90-day forward rates, quoted as dollars per yen and sampled at the end of each month, returns on 90-day euroyen and eurodollar deposits, the 30-day eurodollar return, industrial production indexes and changes in the consumer price index for each country, as well as the 90-day US treasury bill return which we assume to be the risk-free rate. To construct equity returns we used MSCI country and world indices. Finally, from CITIBASE, we obtained the 30-day dividend yield on the S&P 500 index and the US junk bond spread (defined as the difference in the returns on Moody's BAA and AAA rated bonds).



Without loss of generality, we will take the viewpoint of a US investor who has access to the local equity and money markets as well as to the corresponding markets in Japan. Thus, we will consider a two-country model where excess returns are taken relative to the 90-day US treasury-bill rate. To obtain foreign exchange return portfolios, returns on yen deposits were compounded by the exchange rate variation (relative to the US dollar) over the period of the investment.

We also use the US junk bond spread and the US dividend yield, both taken in excess of the 30-day return on a Eurodollar deposit. All three will be considered instruments of the US equity market and will be referred to as instruments.

### **1. 6. 2 Summary Statistics**

In Table 1.1, we report means and standard errors for monthly equity returns, risk-free rates, excess returns, instruments, the forward premium, and the change in the exchange rate, for 90-day holding periods. These returns are expressed in percentage per annum. In addition, we also report annualized inflation rates and growth rates in industrial production in both countries over the period considered.

The table shows that unconditional means of excess equity returns were 7.6% per year for the US and 6.4% for Japan. As for the excess foreign exchange returns, it averaged 1.3% over this period. This implies that a U.S. investor who invested in the Japanese money market every month for 183 months, each time holding the investment for 90 days, obtained a return of 1.3% beyond and above what he would have gained had he invested monthly in 90-day U.S. treasury-bills during the same period. Notice also, from the table, that the standard deviations of excess returns and the growth rate of the exchange rate were quite high, specially compared with the variation in the forward premium, inflation rates and the change in industrial production for the two countries.

Table 1.1: Summary Statistics

	Mean	Std. Error
$R_{t+1}^{US}$	15.478	29.825
$R_{t+1}^{JAP}$	12.576	39.704
$i_t^{US}$	7.836	3.120
$i_t^{JAP}$	6.123	2.140
$R_{t+1}^{US} - i_t^{US}$	7.649	30.377
$R_{t+1}^{JAP} - i_t^{JAP}$	6.395	40.031
$(F_t - S_t)/S_t$	3.093	3.294
$(S_{t+1} - S_t)/S_t$	4.385	25.507
$(S_{t+1} - F_t)/S_t$	1.263	26.708
$\pi_t^{US}$	5.930	3.294
$\pi_t^{JAP}$	3.109	2.307
$dY_t^{US}$	2.500	9.461
$dY_t^{JAP}$	3.613	15.957

Table 1.2: Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4
$R_{t+1}^{US} - i_t^{US}$	0.629	0.230	-0.093	-0.039
$R_{t+1}^{JAP} - i_t^{JAP}$	0.675	0.356	0.057	0.067
$(F_t - S_t)/S_t$	0.840	0.745	0.706	0.657
$(S_{t+1} - S_t)/S_t$	0.729	0.412	0.132	0.057
$(S_{t+1} - F_t)/S_t$	0.752	0.465	0.204	0.125
$\pi_t^{US}$	0.992	0.976	0.957	0.937
$\pi_t^{JAP}$	0.954	0.906	0.856	0.811
$dY_t^{US}$	0.405	0.248	0.225	0.113
$dY_t^{JAP}$	-0.405	0.077	0.312	-0.115

Table 1.2 contains autocorrelations over four lags for selected variables from the previous table. As expected, we see a fair amount of autocorrelation in these financial series since we have overlapping data. Nevertheless, except for the forward premium, the persistence in these series practically disappears by the fourth lag. As for the dynamics of the change in industrial production, for the US there is some positive autocorrelation in the data while, for Japan, the evolution of this series is more erratic. Finally, we observe that inflation series are very highly persistent and indicate possible nonstationarity. Indeed, when tested for unit roots using both the Phillips-Perron test and the augmented Dickey-Fuller tests, we cannot reject the presence of a unit root in these series. We consequently difference these series before using them in econometric estimation and testing.

We now turn our attention to the issue of predictability of excess returns and run OLS regressions of excess returns on various instruments. Three information sets are used for this purpose. The first is mainly a collection of US equity instruments and is a subset of the instrument set used in the works of Harvey (1991) and Dumas and Solnik (1995). It contains a constant, a dummy for the month of January, the excess US junk bond spread, the excess US dividend yield and the 30-day eurodollar rate. The second instrument set is similar to that used in Bekaert and Hodrick (1992). This includes lags of excess equity returns and the excess bilateral foreign exchange return, the excess US dividend yield and the forward premium. Finally, the third instrument set contains only economic variables. These are a constant and changes in inflation rates and in real industrial production in the two countries. This last set was chosen so as to have a rough idea of the distinct effects of real and nominal shocks on the different excess returns.

Tables 1.3.a to 1.3.c report the results of these regressions based on heteroskedasticity-consistent standard errors. We also include the R-squared and the Durbin-Watson statistics for these regressions.

These tables indicate that all three instrument sets have some predictive power for excess returns. Amongst the first set of instruments, the US 30-day T-bill rate is useful for forecasting returns in all categories, while the dividend yield seems to be of use for the equity markets only. In the second set, the lagged US excess equity return is

seen to have predictive power for all excess returns while lagged foreign excess returns seem to be useful for both the Japanese equity market and the foreign exchange excess returns.

### 1.3.a Regression of Excess Returns on Instrument Set 1

xs return	Significant Variables	R-squared	DW
US equity	jan, div(0,2,3), ius(0,2,3)	0.73	1.77
JAP equity	div(1,3), ius(1,3)	0.17	0.69
Exchange	junk((0), ius(0))	0.25	0.61

Significance is at the 5% level. jan denotes the dummy for the month of January. ius is the 30-day US T-bill rate and div is the dividend yield on the S&P500 in excess of ius. The numbers in the parentheses indicate the lags which are significant for that variable. For instance, ius(0) means that the contemporaneous value of the short term interest rate is significant at the 5% level.

### 1.3.b Regression of Excess Returns on Instrument Set 2

xs return	Significant Variables	R-squared	DW
US equity	USeq(1,3), fp(0)	0.52	1.88
JAP equity	USeq(1), JAPeq(1,3), xsfx(1,2,3), fp(1)	0.58	1.88
Exchange	USeq(3), xsfx(1,3), div(2)	0.66	1.89

See notes on previous table. USeq(j) denotes lag j of the US excess equity, JAPeq(j) denotes lag j of the Japanese excess equity, xsfx(j) is the jth lag on excess foreign exchange returns, fp is the forward premium and div is the dividend yield on the S&P500 in excess of the 30-day US T-bill rate.

### 1.3.c Regression of Excess Returns on Instrument Set 3

xs return	Significant Variables	R-squared	DW
US equity	dpiJAP(1), dyUS(2,7)	0.24	0.79
JAP equity	dpiJAP(9), dpiUS(0)	0.16	0.58
Exchange	dpiJAP(3,4), dyJAP(5), dyUS(6,7,8))	0.30	0.58

See notes on previous table. dpiUS and dyUS denote the change in the US inflation rate and in the US industrial production respectively.

Finally, from Table 1.3.c, it seems that excess equity and foreign exchange

returns are affected by both real and nominal shocks. In the first case, Japanese nominal shocks in the recent past, and US real shocks in the more distant past influence these returns.

The same situation applies for the excess foreign exchange returns, but with the added effect of Japanese real shocks. In opposition, the Japanese equity market seems to be mainly driven by nominal shocks originating in both countries. From here it is easy to see that the type and dynamics of shocks to the equity and foreign exchange markets are somewhat different<sup>4</sup>.

### 1. 6. 3 A VAR Model and Impulse Responses

Thus, the above regressions establish, for our data set, the predictability of excess returns and could be interpreted as evidence for the existence of risk premia in these markets. Interestingly, all three instrument sets have some explanatory power. Indeed, there seems to be both common and idiosyncratic shocks to each of the markets considered. It would therefore be interesting, given this information, to see whether we can say something more about the dynamics of real and nominal shocks on these excess returns. For this purpose, we set up a simple VAR model with 7 variables. This includes, for each country, the change in the inflation rate, the change in industrial production and excess equity returns, as well as the excess foreign exchange return for the bilateral exchange rate between these two countries. In essence, we want to examine whether the proxies that we use for real and nominal shocks in the economy contain information for the behavior of excess returns above and beyond the information present in the financial markets.

To choose the appropriate lag for our VAR, we started at 8 lags and tested downwards. In comparing the VAR(4) against the VAR(3), the LR statistic stood at 102.74 with a p-value of 0.0001 indicating that we should adopt the VAR(4) as our

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4. We should point out that, since our data is overlapping, the dynamics of returns resulting from various shocks could be a little different than if non-overlapping data were used. This issue is specially relevant in applications using variables from Instrument sets 1 and 3 where the regression autocorrelation is high.

system. We therefore estimated a fourth-order VAR and calculated orthogonalized impulse-response functions. At the end of this paper we present the plots of these impulse responses over 24 periods with respect to real and nominal shocks.

#### *The effects of real shocks*

The response of US excess equity returns to real US shocks is quite strongly positive, attaining a peak increase of about 4.5% by the fourth month after the shock. After that, it takes another three quarters for the returns to gradually come back to their initial level. On the other hand, the effect of the same shock on Japanese excess returns is somewhat different. Indeed, these returns decrease by about 2.5% by the end of the first month, to bounce back to plus 2% by the end of the first year. After this, while there is some oscillation in these returns, it takes around two years before the effect of the shock finally dissipates. As for the reaction of excess foreign exchange returns, after a sharp increase of around 2.5% by the end of the first quarter, these returns gradually and smoothly decline to their initial level by about a year and a half later.

The reaction of these markets to a real shock in the Japanese economy also produces quite different results. In the case of the excess US equity returns, an initial drop of 1% is followed by a peak of about 1.5% 2 months later and a drop back to negative 1% another three months later. Returns then gradually climb back to their initial level by about a year later. The Japanese excess returns' reaction, however, is much more short-lived. Three quarters after the occurrence of the shock, its effect has all but disappeared. In the meantime, however, there is some oscillation between minus and plus 2% in these returns. Finally, the reactions of foreign exchange excess returns to a real Japanese shock are milder than for the equity markets. An initial drop of about 1% is followed by some narrow oscillations around zero soon after.

#### *The effects of nominal shocks*

The effect of an acceleration in US inflation produces very similar patterns of behaviour in the excess equity markets of the two countries. They both drop by about 3% 3 to 4 months after the shock and gradually climb back to their initial level another three quarters later. On the other hand, excess returns in the foreign exchange market

oscillate sharply between plus and minus 1% and settle around zero by about 3 quarters after the shock.

Surprisingly, a nominal Japanese shock adversely affects excess equity returns in the US more than it does in Japan. While, in both cases, the response is short-lived, with the effect of the shock disappearing a little after a quarter of its occurrence, this response is twice as strong in the US than in Japan. In the case of the foreign exchange market, however, the effect of the shock is milder but longer-lived. It takes three quarters before the shock is fully absorbed and excess returns exhibit oscillations with a 1% band over this period.

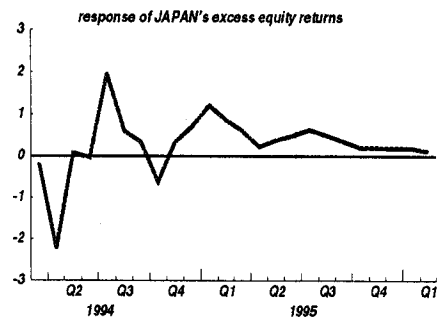
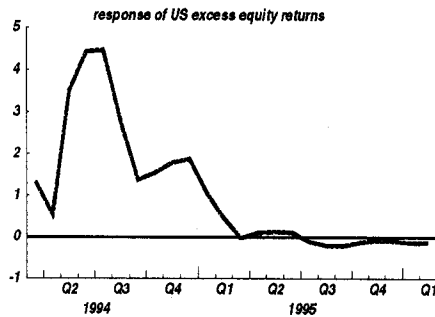
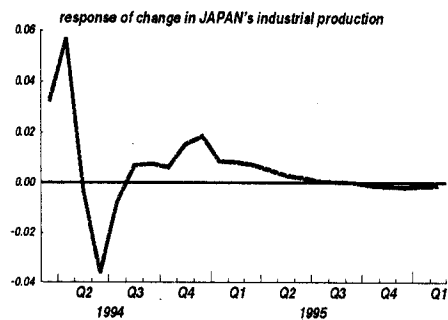
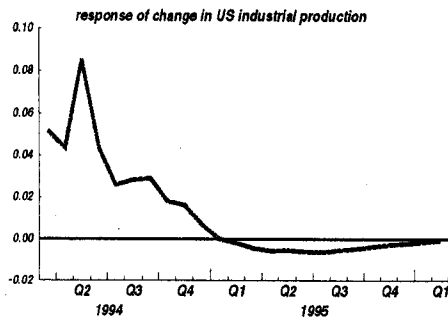
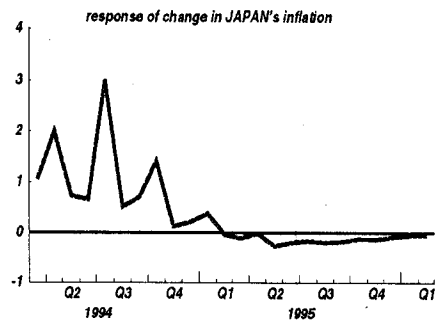
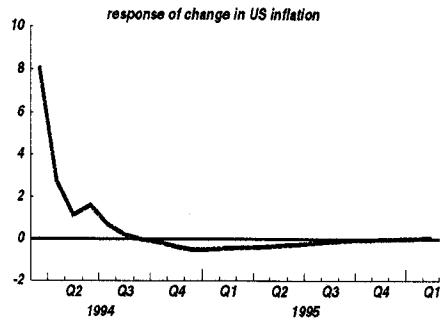
On the whole then, we observe that equity markets are more affected by real and nominal shocks in the two countries than is the exchange market. In addition, it seems that real shock effects have a slightly longer duration on these markets than have the effects of nominal shocks.

As expected, in the case of equity markets, a real US shock generally increases excess returns, reflecting technological advances. However, a shock to Japanese industrial production does not produce as clear-cut effects on these markets as returns oscillate between the positive and negative sides. On the other hand, nominal exogenous positive shocks generally decrease the returns in these markets. As for the foreign exchange market, the one disturbance that has the strongest effect on excess returns is the real US shock. The reaction to this shock is a strong (3%) and persistent increase in these returns. In all other cases, the effect on returns is reflected in short-lived oscillations in the 1% interval.

Based on the above analysis, we can therefore state that there are common shocks to both the equity and the foreign exchange market, but that these shocks have different dynamics in these markets. This does not exclude the presence of idiosyncratic shocks as well. Naturally, the above is true to the extent that the maintained hypothesis concerning the implicit causality structure in our atheoretical VAR is true. In addition, the above is valid to the extent that our proxies for real and nominal shocks are valid. Nevertheless, since other studies have also arrived at similar conclusions, in chapter three of this thesis we will present an international asset pricing model where real and

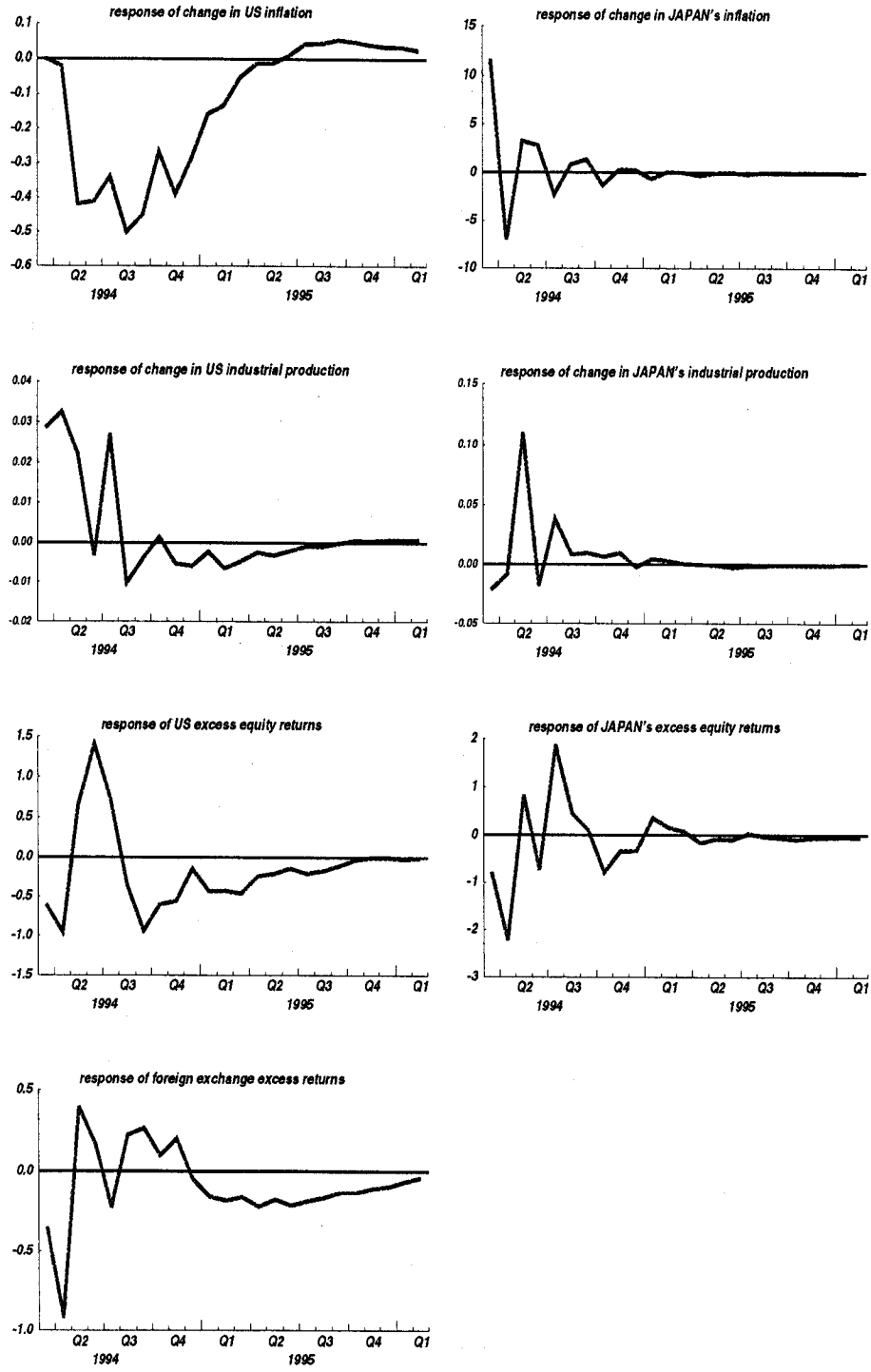
nominal shocks will play a role.

**Graph 1.1: Impulse Responses to a Real US Shock**

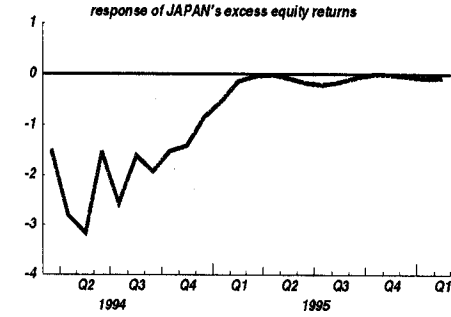
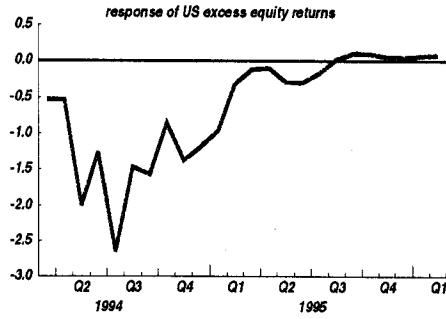
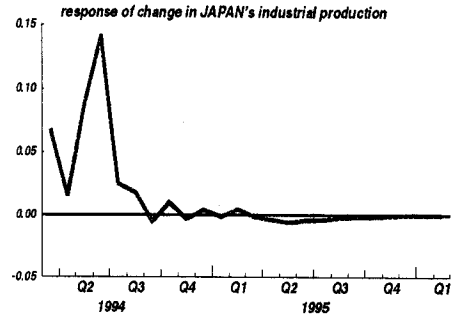
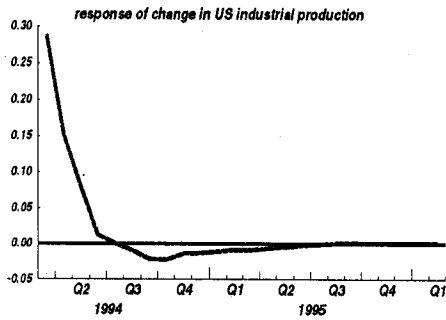
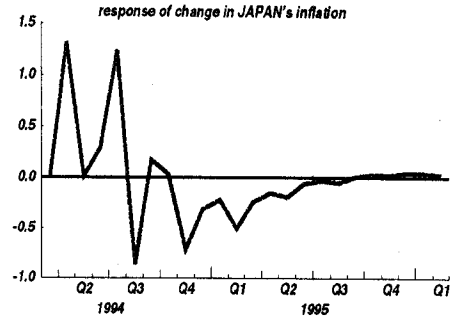
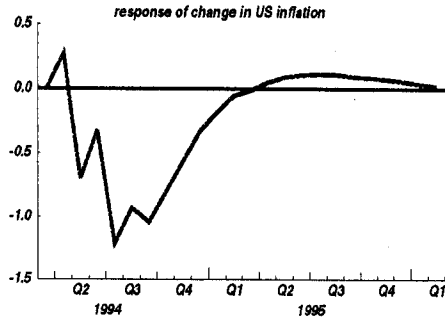




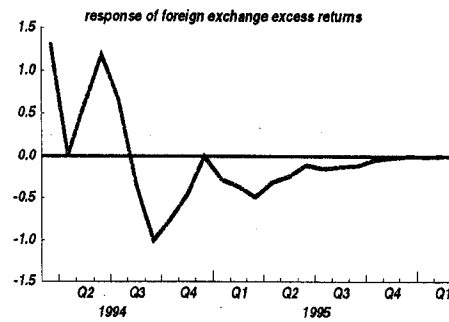
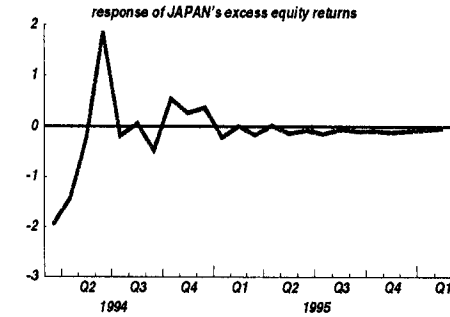
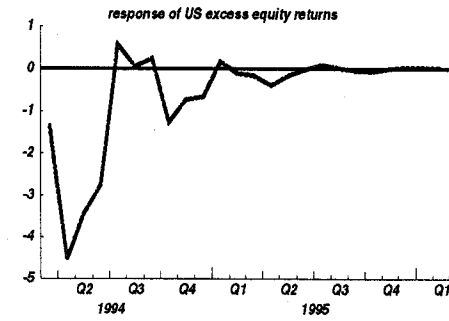
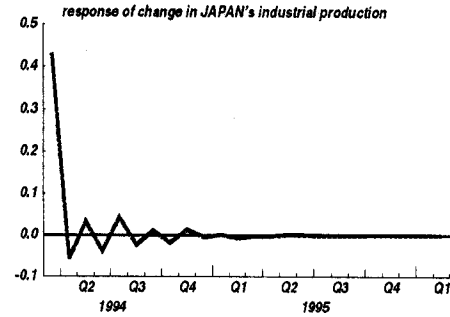
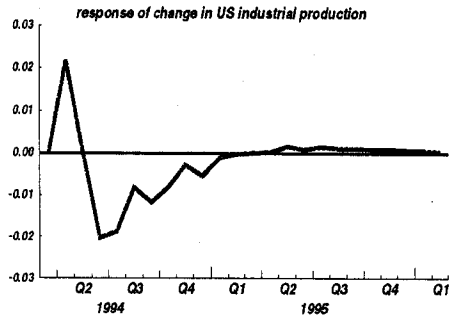
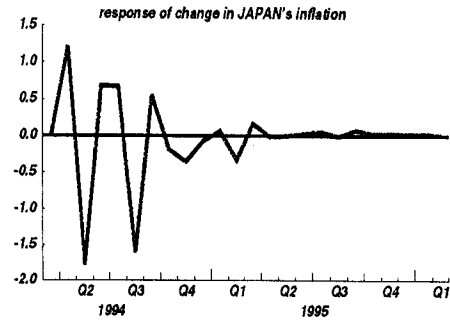
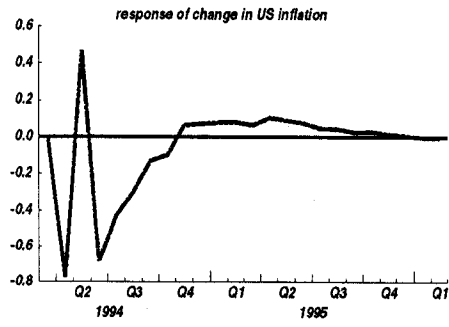
Graph 1.2: Impulse Responses to a Real Japanese Shock



**Graph 1.3: Impulse Responses to a Nominal US Shock**



**Graph 1.4: Impulse Responses to a Nominal Japanese Shock**



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## Chapter 2

# Empirical Models of International Asset Pricing

### 2.1 Introduction

Evidence of returns predictability in both equity and foreign exchange markets has led researchers to model risk premia as time-varying and a number of conditional versions of the CAPM and APT models have been tested with some success. In the international finance literature, early unconditional international asset pricing models, such as Stulz (1981) and Adler and Dumas (1983), were re-formulated and tested in their conditional form. This approach was similar to that adopted in the field of domestic finance models. Amongst these studies, Harvey (1991), Ferson and Harvey (1993), Dumas and Solnik (1995), Ferson and Korajczyk (1992) and others, found evidence in favor of time-varying risk premia in both equity and foreign exchange markets.

Mainly, in all these models conditional moments at time  $t$  are modelled as linear projections on various economic or financial instruments whose values are

known at time  $t-1$ . The estimation technique used is the generalized method of moments procedure and the main criterion used to assess the goodness of fit of the model is the  $J$ -test for overidentifying restrictions( see Hansen (1982)).

There also exist other conditional models of asset pricing which follow more of the ARCH tradition. That is, conditional moments are specified as projections only on lags of squared returns. This formulation stresses the importance of taking into account the observed heteroskedasticity in the volatilities and covariances of returns. The estimation method used is either maximum likelihood or, again, generalized method of moments. An example of a paper in this class of models using GMM is the conditional domestic CAPM of Bodurtha and Mark (1991).

To allow for time-varying risk premia certainly yields more sophisticated asset pricing models, but the search for adequate model specifications is obviously more delicate. In particular, the dynamics of predictable returns needs to be scrutinized seriously as misspecification could be costly in terms of pricing error. In fact, to our knowledge, there has been no systematic attempt to investigate whether either of the two, the instrument method or the ARCH method, performs better than the other in predicting expected returns. Another important aspect to realize about these models is that the  $J$ -test which is frequently used to assess the overall validity of the formulation chosen has low power against particular local misspecification alternatives and is therefore not well suited for uncovering systematic mispricing due to specification errors in the model. Studies by Newey (1985) and Ghysels and Hall (1990a,b) provide examples of such situations where the test has low power.

We begin with a rigorous examination of both the CAPM model of Dumas and Solnik (1995) and the APT model of Ferson and Harvey (1993), as well as various extensions of these models. Using our own data set, we first re-estimate these models and subject them to diagnostics tests on the parameters and on the

residuals of the models.

Essentially, the test on the parameters is the Andrews (1993) test for structural change with unknown change point. We show that for most of the conditional international asset pricing models that we estimated, we reject parameter stability over time. This means that even though these models cannot be rejected on the basis of the overidentifying restrictions  $J$ -test, they are not very useful for consistently predicting the conditional first and second moments of equity and foreign exchange returns over time because of parameter variation which is left unspecified.

The test on residuals, on the other hand, checks whether the model residuals are orthogonal to certain specific alternatives. We use this to analyze the orthogonality of the residuals from the model that uses one approach, say the instrument approach, to the informational content present in variables typically used in the other approach (in this case, the ARCH approach). We find that the Dumas and Solnik (1995) models are generally not rejected against the alternatives tested as opposed to the Ferson and Harvey (1993) models where we find that unexploited information remains in variables not used in the estimation; these variables being the ones typically used in the ARCH approach.

Given this evidence for the presence of pertinent information in the autoregressive elements of the model, we next propose a factor ARCH specification to explain international returns. With respect to all three tests described above, that is, the  $J$ -statistic criterion, the stability test, as well as, to some extent, the orthogonality test, our model is shown to perform well. Finally, we conclude our analysis by examining and comparing estimated pricing errors.

The paper is divided in the following manner. Section 2.2 presents the Dumas and Solnik(1995) and the Ferson and Harvey (1993) models in some detail. Section

2.3 includes a discussion on the validity of the  $J$ -statistic test for structural change in the above models, an explanation of the Andrews (1993) Sup LM test, some remarks on the asymptotic local power of GMM tests and an exposition of the form of optimal tests against specific alternatives. In section 2.4, we describe our data set and report estimation and diagnostics test results for the conditional CAPM and APT models. Section 2.5 exposes our model and includes estimations and test results. Section 2.6 covers the discussion on pricing errors. The last section concludes.

## 2.2 International Conditional Asset Pricing Models

What differentiates international financial theory from its domestic counterpart is essentially the presence of different nations in the former framework. Thus much depends on the definition of the concept of nation adopted. A most useful definition is one attributed initially to Solnik (1974) and where a nation is referred to as a zone of common purchasing power unit. This means that individuals of one zone use a different price index to deflate their monetary investment earnings than those in another zone. Naturally, were the hypothesis of purchasing power parity (PPP) to hold at all times, this distinction would not arise. But it is now well documented by numerous empirical studies that PPP holds at best in the long run, if at all. Furthermore, deviations from PPP are shown to be significant, of long duration and highly random.

An important such international asset pricing model is that of Adler and Dumas (1983). This is a theoretical intertemporal model of utility maximization where investors' real returns vary due to the existence of various nations, that is, the presence of various zones of common purchasing power unit. It is also assumed that the  $N$  risky security prices of the model, as well as the price index of each country, follow stationary Ito processes and that preferences are homothetic. The resulting equilibrium condition is therefore an international CAPM which relates



expected nominal returns of each asset to its covariance with inflation for each country and its covariance with the market return. In addition, if one makes the assumption that, in the short run, inflation risk expressed in local currency is negligible, the covariance terms with inflation can be replaced by covariances of nominal returns with exchange rate changes. This is the starting point for the conditional model found in Dumas and Solnik (1995) which is given by

$$E[r_{jt}|\Omega_{t-1}] = \sum_{i=1}^L \lambda_{i,t-1} \text{cov}[r_{jt}, r_{n+i,t}|\Omega_{t-1}] + \lambda_{m,t-1} \text{cov}[r_{jt}, r_{mt}|\Omega_{t-1}] \quad (1)$$

The total number of assets is  $m = n + L + 1$  that is,  $n$  equity portfolios,  $L$  currency deposits (other than the measurement currency), and a world portfolio. Furthermore,  $r_{jt}$  is the nominal return on asset  $j$ , ( $j = 1, \dots, m$ ) in excess of the risk-free rate of the measurement currency country,  $r_{n+i,t}$  is the excess return on the  $i$ th currency deposit ( $i = 1, \dots, L$ ), and  $r_{mt}$  is the excess nominal return on the world portfolio. Thus, both exchange rate risk and market risk are conditionally priced in this model. These prices are the time-varying coefficients,  $\lambda_{k,t-1}$ ,  $k = i, m$ .

In order to write this model in a more parsimonious way, Dumas and Solnik estimate a restricted form of equation (1). Since the first-order condition of any portfolio choice problem can be written as

$$E[M_t r_{jt} | \Omega_{t-1}] = 0 \quad (2)$$

where  $M_t$  is the intertemporal marginal rate of substitution of returns, they define  $u_t$  as the unanticipated component of the relative intertemporal marginal rate of substitution and write it as

$$u_t = 1 - \frac{M_t}{E[M_t | \Omega_{t-1}]} \quad (3)$$

with the property that

$$E[u_t | \Omega_{t-1}] = 0. \quad (4)$$

Then, equation (2) implies the condition that

$$E[r_{jt}|\Omega_{t-1}] = E[r_{jt}u_t|\Omega_{t-1}], \quad j = 1, 2, \dots, m. \quad (5)$$

Substituting  $E[r_{jt}|\Omega_{t-1}]$  for its expression in (1), and given (4), we have the expression for  $u_t$  given by:

$$u_t = - \left[ \sum_{i=1}^L \lambda_{i,t-1} E(r_{n+i,t}|\Omega_{t-1}) + \lambda_{m,t-1} E(r_{mt}|\Omega_{t-1}) \right] + \sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t} + \lambda_{m,t-1} r_{mt} \quad (6)$$

At this point, the model still has a high number of parameters and is quite nonlinear. Empirical estimation is therefore still cumbersome. To simplify further, Dumas and Solnik further restrict equation (6) to give

$$u_t = \lambda_{0,t-1} + \sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t} + \lambda_{m,t-1} r_{mt} \quad (7)$$

and estimate the model represented by equations (4), (5) and (7). The advantage of this particular formulation is that one does not need to explicitly specify the expected nominal returns. However, this same advantage can be perceived as a drawback. Given that the purpose is to test the conditional CAPM, it should be interesting to discriminate between the informational content contributed by each component of the model to the predictability of returns. With a linear projection, as in equation (7), it is no longer possible to make explicit and test the specification of the expected nominal returns separately from the market prices. In addition, much of the nonlinearity of the model is reduced which could prove to be critical. We will be returning to this point later on.

Since market prices are time-varying, Dumas and Solnik make them linearly dependent upon a number of information instruments which are known at time  $t - 1$ . These are mainly US equity market instruments and were found to have good informational content for the predictability of nominal returns (expressed in dollars) of many countries in the study by Harvey (1991). This set of instruments is denoted  $Z_{t-1}$  and is assumed to contain all relevant past information. Therefore:

$$\begin{aligned} \lambda_{0,t-1} &= -Z_{t-1}\delta \\ \lambda_{i,t-1} &= Z_{t-1}\phi_i \\ \lambda_{m,t-1} &= Z_{t-1}\phi_m \end{aligned} \quad (8)$$

The model is estimated by GMM for the period March 1970 till December 1991 using monthly data with the US dollar as the measurement currency. The countries considered are Germany, Japan, United Kingdom and the United States. The equity assets are the country equity indexes, the currency deposits are DM, Yen and Pound deposits, and the exchange rates are the bilateral spot exchange rates of each country with the US. The instruments include a constant, the lagged excess world return, the 30-day return on a Eurodollar deposit, the difference in yield between Moody's BAA and AAA rated bonds, the excess dividend yield on the S&P500 index, and a dummy for the month of January. These are the same instruments used in Harvey (1991) except for the 30-day Eurodollar rate which replaces the excess yield on the 90-day US T-bill.

Estimation results show that exchange risk premia are significant and time-varying with several of the  $\phi$  and  $\delta$  coefficients having high t-statistics. Also, based on the chi-squared test of the overidentifying restrictions provided by the moment conditions of the model, the authors conclude that their parsimonious representation is a satisfactory description of the international CAPM.

By relaxing the constraints imposed by Dumas and Solnik, one obtains a model similar to Ferson and Harvey's (1993) model, which is an international APT model where national equity markets are related to global risk factors. In Ferson and Harvey (1993), the expected risk premia are conditional and are linearly related to the variables constituting the information set  $\Omega$  at time  $t - 1$  whereas the conditional betas measure the sensitivity to the global risk factors and are linearly dependent on local information variables. Although the paper reports results for estimations carried out with various numbers of factors, we will concentrate only on their 2-factor model. In this case, the world market portfolio and an aggregate of exchange rates are taken as the two factors underlying the behavior of assets based upon the theoretical justification provided by Adler and Dumas (1983).

The model is given by:

$$E[r_{jt}|\Omega_{t-1}] = \sum_{k=1}^K \beta_{jk}(\Omega_{t-1})E[f_{kt}|\Omega_{t-1}], \quad (9)$$

where  $f_{kt}$  designates a factor,  $E[f_{kt}|\Omega_{t-1}]$  is the expected excess return of that factor, and  $\beta_{jk}(\Omega_{t-1})$  are the conditional betas of the expected returns. Information available to investors at time  $t - 1$  is in  $\Omega_{t-1}$  and includes global information variables as well as local ones.

Estimation of the above model is carried out for eighteen countries using monthly data extending from 1970 to 1989. Returns are in excess of the 30-day T-bill rate. The study concludes that the addition of the second factor, that is the exchange rates aggregate, shows a modest improvement over the single-factor alternative and that most of the predictability in expected returns is related to global risk premia.

Since the data extends over three different currency regimes; a fixed exchange rate period from 1970:02 till 1973:02, a dirty float from 1973:03 till 1980:12, and a more flexible float period afterwards, the authors regress pricing errors for each country on dummy variables representing each of these periods for the single-factor case and the five-factor case. They find no important misspecification related to currency regimes. Nevertheless, for both the Dumas and Solnik and Ferson and Harvey specifications, we will show that stability of coefficients proves to be an issue even if the models are judged acceptable according to the usual overidentification restrictions tests. In the next section, we present tests for structural stability and for misspecification against selected alternatives. These are applied in the sections after, in addition to the usual tests, to assess the validity of various models.

### 2.3 Diagnostics beyond overidentifying restrictions

Usually, to assess the goodness of fit of an asset pricing model that is estimated by GMM, Hansen's overidentifying restrictions test is carried out. While the test is an overall diagnostic, it is not an omnibus test against misspecification. This is because it is constructed as a test against general local misspecification alternatives and therefore has low power against some particular forms of misspecification.

In fact, Newey (1985) shows that the overidentifying restrictions test, or the  $J$ -test, which is distributed as non-central chi-squared variables under a sequence of local alternatives, has zero non-centrality parameter for some non-zero misspecification directions. In another study, Ghysels and Hall (1990b) formally show that the  $J$ -test has no power against local alternatives characterized by time-varying parameters, which means that such tests are not well suited for examining dynamic specification errors such as those related to parameter variation through time. In fact, the authors show that while the parameters that appear in the moment conditions are assumed fixed in estimations, the corresponding restrictions are not imposed in the above test. That is, if the parameters appearing in these moment restrictions are truly time-varying, but they are estimated imposing fixed coefficients instead, the overidentifying restrictions test will tend not to reject the model.

We can therefore see that in order to have a better idea about the validity of these models, one should proceed by examining explicitly both the stability of model parameters, and the soundness of the chosen specification against particular alternatives which are judged to be of possible pertinence for the model in question.

In what follows, we will explain two such explicit tests. These are the tests we will consequently use, in addition to the  $J$ -statistic, to undertake the model

diagnostics described above on various asset pricing models. We also include a section discussing model pricing errors. In fact, once the models are estimated, pricing errors are easy to calculate. An examination of these is of course valuable as well for establishing overall model validity.

### 2.3.1 Testing the Parameters: a test for structural change

To test for structural change in the context of GMM, one must test the null hypothesis of constant parameters explicitly. Let us denote the parameter set as  $\gamma_t$  and let the alternative of a one-time change in the value of  $\gamma_t$  occur at the time  $\pi T$ , where  $T$  is the sample size and  $\pi \in (0, 1)$ . The parameter vector  $\gamma_t$  either contains all coefficients of the model or a subset. The latter case is referred to as a "partial" test of structural change since only a subset of parameters are tested. The null and alternative of the test are then formulated as

$$H_0 : \gamma_t = \gamma_0$$

$$H_1 : \gamma(\pi) : \begin{cases} \gamma_1(\pi), & \text{for } t = 1, 2, \dots, \pi T \\ \gamma_2(\pi), & \text{for } t = \pi T + 1, \dots, T \end{cases}$$

From Andrews and Fair (1988), for the case where  $\pi$  is known, it is possible to construct Wald, LM, or LR-like statistics to test  $H_0$  against  $H_1$ . However, when  $\pi$  is unknown, or known to belong to a subset  $\Pi$  of  $(0, 1)$ , one can calculate test statistics, for instance, based on the supremum of  $W_T(\pi)$ ,  $LM_T(\pi)$  or  $LR_T(\pi)$ , for  $\pi \in \Pi$  as proposed by Andrews (1993). Amongst these test statistics, we will concentrate only on the Sup LM test as it only requires estimation of the model under the null, which considerably reduces computation costs. This test statistic is a quadratic form based on the score function obtained from the minimization of the GMM criterion function evaluated at a given restricted estimator. The quadratic form is assigned a weight matrix such that the statistic has a chi-squared distribution under the null for each fixed  $\pi$ . The statistic is therefore calculated for each  $\pi$  and the maximal value is designated as the Sup LM test.<sup>1</sup>

<sup>1</sup>For the full definition of the statistic and more details, we refer the reader to Andrews (1993).

This Sup LM test has been used by Ghysels (1994) to test the stability of the model coefficients of the conditional CAPM of Harvey (1991), the conditional APT of Ferson and Korajczyk (1992), and the nonlinear APT of Bansal and Viswanathan (1993). The results showed, among other things, that most of the linear projection parameters of these models were unstable.

### 2.3.2 Testing the Residuals: orthogonality tests

To test for the orthogonality of the model residuals to particular information variables, we will use an optimal GMM test, as described in Newey (1985) and Tauchen (1985). Here, optimality refers to the fact that the value of the non-centrality parameter of the test statistic under the local alternative is maximal for all misspecification directions, and that the test has the smallest possible degrees of freedom amongst those with the aforementioned property. It should be noted that the  $J$ -test has a non-centrality parameter which is the largest amongst GMM tests with  $r-q$  degrees of freedom (where  $r$  is the number of orthogonality conditions and  $q$  is the dimension of the parameter vector to be estimated) and for all misspecification directions. However, Newey shows that when particular alternatives are chosen, one can find test statistics which have, in addition to the largest possible value of the non-centrality parameter, degrees of freedom smaller than  $r-q$ . Such tests have therefore more power than the  $J$ -test.

Since asset pricing models choose only certain information variables from a set of such variables to assess the predictability of expected returns, it is always interesting to check whether those instruments not used in the estimation could have increased the explanatory power of the model had they been used. In the context of a model estimated via GMM, this amounts to testing whether other orthogonality conditions could have been used in the estimation. The test is described as follows:

Given the moment conditions  $f_{1t}(b)$  for  $t = 1, 2, \dots, T$ , at the true parameter

value  $b_0$  we have that  $E[f_{1t}(b_0)] = 0$  and  $b$  is then a consistent estimator of  $b_0$ .

Using the sample moments we can define the terms

$$g_{1T}(b) = (1/T) \sum_{t=1}^T f_{1t}(b) \quad \text{and} \quad S_{11,T} = (1/T) \sum_{t=1}^T f_{1t}(b) f_{1t}(b)' \quad (10)$$

then the GMM estimator  $b_T$  is the parameter set that minimizes the quadratic form

$$\Phi(b_T) = g_{1T}(b)' [S_{11,T}]^{-1} g_{1T}(b) \quad (11)$$

Now, if  $f_{2t}(b)$  is an  $(l \times 1)$  vector of orthogonality conditions which were not used in the estimation and which we suspect should have been included in the model, an optimal GMM statistic can be formulated to test for the omission of these moments. This statistic is given by

$$CS = T[L_T g_T(b_T)]' [Q_T]^{-1} [L_T g_T(b_T)] \quad (12)$$

and is distributed as a  $\chi^2$  with  $l$  degrees of freedom. The matrices which define this test are detailed as follows<sup>2</sup>

$$g_T(b) = [g_{1T}(b)' \ g_{2T}(b)']', \quad \text{with} \quad g_{2T}(b) = (1/T) \sum_{t=1}^T f_{2t}(b) \quad (13)$$

and  $L_T = [0 : I_l]$ , so that  $L_T g_T(b_T)$  is a linear combination of the estimated sample moments  $g_T(b_T)$ .

In addition,

$$S_{ij,T} = (1/T) \sum_{t=1}^T f_{it}(b) f_{jt}(b)', \quad i = 1, 2, \quad j = 1, 2 \quad (14)$$

$$H_{iT} = (1/T) \sum_{t=1}^T \partial f_{it}(b_T) / \partial b, \quad i = 1, 2 \quad \text{and} \quad B_T = [H'_{1T} [S_{11,T}]^{-1} H_{1T}]^{-1} H'_{2T} \quad (15)$$

Finally,

$$Q_T = S_{22,T} - S_{21,T} [S_{11,T}]^{-1} H_{1,T} B_T - B_T' H_{1,T}' [S_{11,T}]^{-1} S_{12,T} + H_{2T} B_T \quad (16)$$

Thus, for a given significance level, a large value of the statistic means rejection of the null in favor of the specified alternative moments.

<sup>2</sup>For a more detailed exposition of the test, we refer the reader to Newey (1985) and Bodurtha and Mark (1991)



### 2.3.3 Comparing Ex-Post Pricing Errors

A fairly easy and natural way of assessing overall model validity is by examining ex-post pricing errors of various models. A  $T \times 1$  vector of pricing errors,  $e_j$ , for returns on an asset  $j$  is defined as the difference between the observed excess returns data for that asset and the returns predicted by the estimated model. For a single observation this is given by:

$$e_{jt} = r_{jt} - E[r_{jt}|\Omega_{t-1}] \quad (17)$$

where  $r_{jt}$  is the excess return on asset  $j$  at time  $t$ , and  $E[r_{jt}|\Omega_{t-1}]$  is the estimated excess return at time  $t$  implied by the model.

There are two typical measures which are useful for evaluating such errors. These are the absolute mean error and the root mean squared error. For an asset  $j$ , the absolute mean error is expressed as:

$$AME_j = (1/T) \sum_{t=1}^T |e_{jt}|, \quad (18)$$

and the root mean squared error is given by:

$$RMSE_j = \sqrt{1/(T-1) \sum_{t=1}^T (e_{jt} - ME_j)^2} \quad (19)$$

where

$$ME_j = (1/T) \sum_{t=1}^T e_{jt} \quad (20)$$

By comparing these measures across models, in addition to their autocorrelations, one can determine the accuracy of a model's in-sample performance relative to the other.

## 2.4 Data, Estimations and Test Results

In this section, we test the Dumas and Solnik (1995) and the Ferson and Harvey (1993) models after re-estimating them with our own data set. Where feasible,

the data and instruments were kept as similar to the original model as possible to allow for meaningful comparisons. A subsection is devoted to each model.

#### 2.4.1 The Dumas and Solnik Model

First, we estimate and test the Dumas and Solnik (1995) model. The data is basically the same if only for a longer holding period; 30-day holding period data being unavailable to us, we work with data on 90-day investment periods. Having said this, our data is therefore monthly, measured in US dollars, for Germany, Japan, the UK and the US, spanning the period of September 1978 till February 1994. Equity returns are constructed using MSCI country indexes (with dividend reinvestment) and returns on the foreign exchange market are calculated using returns on a currency deposit compounded by the exchange rate variation relative to the US dollar. The instruments used are a constant, the lagged excess world return, a dummy for the month of January, the excess US junk bond spread, the excess US dividend yield and the 30-day return on a Eurodollar deposit. Excess returns are taken with respect to the 90-day US T-bill rate.

The estimations and testing are carried out using GMM and the Sup LM tests. It should be noted that with respect to the Sup LM test, 'full' testing indicates that all the model parameters are tested as opposed to 'partial' testing where only selected parameters are tested.<sup>3</sup> The econometric specification for the 4-country model of Dumas and Solnik (1995) that we estimate is given as follows:

$$u_t = \lambda_{0,t-1} + \sum_{i=1}^3 \lambda_{i,t-1} r_{n+i,t} + \lambda_{m,t-1} r_{mt}$$

with

$$\begin{aligned} \lambda_{0,t-1} &= -Z_{t-1}\delta \\ \lambda_{i,t-1} &= Z_{t-1}\phi_i \\ \lambda_{m,t-1} &= Z_{t-1}\phi_m \end{aligned}$$

<sup>3</sup>The interval adopted for the LM tests here and in the rest of the paper is (0.2, 0.8). Significance of the test in the tables is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%. Note that for parameter numbers greater than 20, asymptotic critical values for Sup LM test were obtained by extrapolation. Nevertheless, these are probably good approximations since the increase in the critical values with the number of parameters is quite linear.

and where equations (4) and (5) are conditioned relative to  $Z_{t-1}$ .

Defining  $h_{jt}$  as the unanticipated error term from equation (5), that is  $h_{jt} = r_{jt} - (r_{jt}u_t)$ , we obtain 4 residuals for each of the 4 countries' equity portfolios, 3 residuals for the currency deposits (DM, Yen and Pound deposits), and a residual for the world portfolio. These, in addition to  $u_t$ , constitute the residual set to which  $Z_{t-1}$  is orthogonal, and together form the orthogonality conditions which are then exploited in the GMM estimation. Therefore, this model has a total of 30 parameters and 54 orthogonality conditions. This means there are 24 degrees of freedom for the  $J$ -statistic.

Estimation results show a value of 20.75 for this statistic with a P-value of 65%, compared with 28.80 and a P-value of 23% obtained by Dumas and Solnik. According to this criterion, our data yields an even better fit than the original study and the model cannot be rejected. However, once explicit stability tests are carried out on the model and the Sup LM test applied, the hypothesis of stable coefficients is rejected at the 5% level, both when all the parameters of the model are jointly tested, and also when only the six parameters of  $\lambda_0$  are tested. Table 2.1 summarizes these results.

At this point, it is interesting to see if this rejection is due to the specific choice of instruments above given that the Ghysels (1994) study found the parameters of certain instruments to be specially unstable regardless of the model specification tested. Accordingly, we divide the instrument set into subsets of five instruments each: Instrument set A includes a constant, the lagged excess world return, a January dummy, the excess junk spread, and the excess dividend yield. Set B contains the first four instruments of set A as well as the short term rate. Finally, set C contains the first three instruments of set A, the excess dividend yield and the short term rate. We then estimate the 4-country model with each of these sets and test again for stability. For each of these cases, total parameter count is 25

and the number of orthogonality conditions is 45, implying 20 degrees of freedom for the  $J$ -statistic. Also, since the number of instruments is now five, this is also the number of parameters in  $\lambda_0$ .

The results are reported in Table 2.2. They indicate that regardless of the set of instruments employed, and despite high P-values of the  $J$ -statistic, parameter stability is strongly rejected when the full parameter vector is tested. It therefore does not seem as though the particular instrument combination used is the cause of model rejection.

We also investigate whether the rejection is caused by the behavior of excess returns of one of the countries in particular or if it is a phenomenon general to all countries. This is carried out by estimating and testing the stability of 2-country versions of the above model (that is, with the US and another country) with the three instrument sets A, B, and C. The results are found in Table 2.3 and indicate that except for the UK in the case of set A and Japan in the case of set B, parameter stability is rejected in the remaining seven cases.

From Tables 2.1-2.3 we notice that partial parameter stability is also majoritarily rejected at the 5% level. It is interesting to see whether the particular linear constraint imposed by Dumas and Solnik is the cause of this, given that it does not allow the manifestation of the nonlinear dynamics in returns which is implied by the main model found in equation (1). We therefore replace equation (7) with equation (6) and test again. For this purpose, however, we need to impose a specific formulation for the expressions of the conditional expected returns and we choose to model these as linear projections on the instruments. The projection equations for  $\lambda_{i,t-1}$  and  $\lambda_{m,t-1}$  remain unchanged. Again, we use the three instrument sets A, B, and C defined above.

The econometric model estimated in this case is therefore

$$u_t = - \left[ \sum_{i=1}^L \lambda_{i,t-1} E(r_{n+i,t} | Z_{t-1}) + \lambda_{m,t-1} E(r_{mt} | Z_{t-1}) \right] + \sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t} + \lambda_{m,t-1} r_{mt}$$

with

$$\begin{aligned} E(r_{n+i,t} | Z_{t-1}) &= Z_{t-1} \delta_i \\ E(r_{mt} | Z_{t-1}) &= Z_{t-1} \delta_m \\ \lambda_{i,t-1} &= Z_{t-1} \phi_i \\ \lambda_{m,t-1} &= Z_{t-1} \phi_m \end{aligned}$$

along with equations (4) and (5), again conditional to  $Z_{t-1}$ .

Here, the total number of parameters is 40 and a full LM test is undertaken. The results are tabulated in Table 2.4. Once again, P-values for the overidentification restrictions test are very high and the model would not have been rejected on the basis of this statistic alone. Nevertheless, the stability test overwhelmingly rejects invariance of coefficients over time indicating that the added nonlinearity is insufficient to improve the model specification. For the sake of comparison, we also estimated and tested the 2-country versions of the above model (see Table 2.5). The conclusions remain unchanged.

Next, we go back to the constrained Dumas and Solnik model (equations 4,5 & 7) and examine the orthogonality conditions for each country pair. We test whether each of the residuals of the model is orthogonal to various lags of the model returns. These lags are grouped into subsets and are designated as alternative instrument sets  $AZ_{t-1}^1, AZ_{t-1}^2, AZ_{t-1}^3$ , and  $AZ_{t-1}^4$ . The first one includes lags 1, 2 and 3 of the excess equity returns of the country paired with the US, the second includes lags 1, 2 and 3 of the excess returns to a deposit made in the currency of the country other than the US, the third set is comprised of lags 1, 2 and 3 of the excess equity returns of the US, and the last includes lags 2 and 3 of the world excess returns as well as lag 1 of the US excess equity returns. Orthogonality tests against  $AZ_{t-1}^1, AZ_{t-1}^2$  and  $AZ_{t-1}^3$  revealed very high P-values in all cases and for all the residuals, varying from 0.95 to 1.00 and implying that the model could not be rejected against these alternatives. Nevertheless, for the

last subset,  $AZ_{t-1}^4$ , low P-values for a few of the residuals indicated that there might be some misspecification present. The results against this last alternative instrument set are tabulated in Table 2.8 for all three country pairs. They show that only for the Germany-US and Japan-US pairs is the issue of misspecification a concern. Despite this fact, in general, one can conclude that the Dumas and Solnik model fares well against all four alternative instrument subsets.

#### 2.4.2 The Ferson and Harvey Model

We now return to equation (1) which is the most general form of this international asset pricing and can therefore be seen as a multi-beta (APT) model if the  $\lambda$  coefficients are interpreted as risk premia (see equation (9)). In fact, the 2-country version of such a model is then closely comparable to the international APT model of Ferson and Harvey (1992) which is estimated by GMM. This is the model examined next.

We estimate the international APT 2-factor 2-country model, with the US as one of the countries and either Germany, Japan or the UK as the other. The two factors are the excess return on the world portfolio and the excess return on the bilateral exchange rate. Instead of this latter factor, Ferson and Harvey use an aggregate of exchange rates, as explained previously. However, we chose to adopt the bilateral exchange rate because it is probably more helpful in capturing the evolution of the various shocks in the economies of the nations involved. This is specially valid in a two-country case model where one country's instruments predominate in the information set, in this case, the US's.

We start by estimating a constant beta version of the APT with all time-variation captured by the conditional risk premia. This is based on the conclusion of the Ferson and Harvey study proper, that most of the dynamics is found in the risk premia rather than in the betas.

The Ferson and Harvey (1993) econometric model specification is described as

$$\varepsilon_t = (u_{1t} \ u_{2t} \ u_{3t}) = \begin{pmatrix} (r_t - Z_{t-1}\delta)' \\ (f_t - Z_{t-1}\gamma)' \\ (u_{2t}u'_{2t}\beta - f_t u_{1t})' \end{pmatrix}, \quad (21)$$

$$E(\varepsilon_t | Z_{t-1}) = 0, \quad (22)$$

where  $f_t$  is the  $2 \times 1$  vector of factors,  $r_t$  is the excess equity return of the country considered other than the US, and  $Z_{t-1}$  is a 5-instrument information set (either A, B or C defined previously). The residuals  $u_{2t}$  and  $u_{3t}$  are both of dimension  $1 \times 2$  as they include a term for each of the two factors in the model.

Table 2.6 contains the results of GMM estimations and the various LM tests for each of the 2-country multi-beta models. The number of parameters, in each case, is as follows: total 17, delta 5, gamma 10, beta 2. The instrument sets are successively A, B, and C. We can see that the P-values for these models are high varying from 39 to 99 percent and that for Japan and the UK, the model is not rejected at the 5% level in 2 cases. This provides us with valuable information in selecting the desirable instrument set for each country pair in the context of these types of models. Nevertheless, we also note that, for the rejected cases, much of the instability is coming from the  $\gamma$  coefficients which are the linear projections of the factors on the instruments chosen.

Orthogonality tests are also carried out for each of the model residuals with respect to the same alternatives as described in the previous section. Tables 2.9, 2.10 and 2.11 report P-values for the three country pairs for the tests that were carried out against the first three of the four alternative subsets. In them there is some indication of misspecification as all three tables include results which are insignificant at the 5 % level. More specifically, this misspecification seems to be concentrated around the residual from the projection equation for the exchange rate factor, specially with the alternative instrument set  $AZ_{t-1}^2$ . This indicates

that the projection equation for this factor should have included a number of own lags. Furthermore, test results against the fourth set,  $AZ_{t-1}^4$ , which are not tabulated, proved to be more dramatic as all P-values for all the residuals and for all the country pairs hovered around values of  $10^{-4}$ . Clearly, this is a strong indication of the presence of misspecification against the alternative represented by lags 2 and 3 of world excess equity returns and lag 1 of excess US equity returns.

In conclusion to this section, we see that although the models were judged satisfactory according to the  $J$ -criterion, they are either unstable over time, or contain misspecification, or exhibit both symptoms. Having pin-pointed the variables which still contain pertinent information, and understanding the importance of avoiding instruments which could lead to unstable model coefficients, it seems appropriate to propose a factor-ARCH model which formulates conditional moments as functions of own lagged variables. This is the model presented in section 2.5.

## 2.5 Autoregressive Factor Models

It seems apparent, at this point, that simple linear projection equations with constant betas do not adequately describe the process of conditional excess returns. On the one hand, the instruments used proved to yield unstable parameter values in linear projection equations, and, on the other hand, the models did not take into account certain nonlinear phenomena observed for financial and monetary series, such as persistence in volatility. We propose a factor-ARCH model which takes both these facts into account. Following along the lines of Bodurtha and Mark (1991), our model avoids external instruments altogether. In addition, the autoregressive components in the variance and covariance terms of the model will help capture much of the observed behavior of volatility in financial and monetary series.



Our purely autoregressive factor ARCH model is given by:

$$E[r_{jt}|\Omega_{t-1}] = \sum_{k=1}^K \beta_{jk}(\Omega_{t-1})E[f_{kt}|\Omega_{t-1}], \quad (23)$$

where

$$E[f_{kt}|\Omega_{t-1}] = \sum_{l=1}^4 \alpha_{kl} f_{k,t-l}, \quad f_{kt} = r_{xt}, r_{mt} \quad (24)$$

and

$$\beta_{jk}(\Omega_{t-1}) = \frac{Cov[r_{jt}, f_{kt}|\Omega_{t-1}]}{Var[f_{kt}|\Omega_{t-1}]} \quad (25)$$

We adopt an AR(4) for the expected value of the factors because, in a separate regression, this formulation yielded significant coefficients and the residuals did not appear to be correlated. In addition, it is a relatively parsimonious representation.

The remaining econometric specification of the model is given as follows. The unanticipated components of factors, that is, excess foreign exchange returns ( $r_{xt}$ ) and excess market returns ( $r_{mt}$ ), are given by:

$$u_{xt} = r_{xt} - E[r_{xt}|Z_{t-1}] \quad (26)$$

$$u_{mt} = r_{mt} - E[r_{mt}|Z_{t-1}] \quad (27)$$

and that for excess equity returns as:

$$u_{jt} = r_{jt} - E[r_{jt}|Z_{t-1}] \quad (28)$$

where  $Z_{t-1}$  is the set of information variables available to the investor.

From here, we obtain that

$$\begin{aligned} \text{Var}[r_{xt}|Z_{t-1}] &= E[u_{xt}^2|Z_{t-1}] \\ \text{Var}[r_{mt}|Z_{t-1}] &= E[u_{mt}^2|Z_{t-1}] \\ \text{Cov}[r_{xt}, r_{jt}|Z_{t-1}] &= E[u_{xt}u_{jt}|Z_{t-1}] \\ \text{Cov}[r_{mt}, r_{jt}|Z_{t-1}] &= E[u_{mt}u_{jt}|Z_{t-1}] \end{aligned} \quad (29)$$

We assume the conditional variance to be an ARCH(1) and the conditional covariance an autoregression of order one. The unanticipated components of these elements are given by

$$\begin{aligned}
 \eta_{xt} &= u_{xt}^2 - E[u_{xt}^2 | Z_{t-1}] \\
 \eta_{mt} &= u_{mt}^2 - E[u_{mt}^2 | Z_{t-1}] \\
 \eta_{xjt} &= u_{xt}u_{jt} - E[u_{xt}u_{jt} | Z_{t-1}] \\
 \eta_{mjt} &= u_{mt}u_{jt} - E[u_{mt}u_{jt} | Z_{t-1}]
 \end{aligned} \tag{30}$$

therefore we obtain,

$$\begin{aligned}
 E[u_{xt}^2 | Z_{t-1}] &= \delta_{x0} + \delta_{x1}(u_{x,t-1}^2) \\
 E[u_{mt}^2 | Z_{t-1}] &= \delta_{m0} + \delta_{m1}(u_{m,t-1}^2) \\
 E[u_{xt}u_{jt} | Z_{t-1}] &= \delta_{xj0} + \delta_{xj1}(u_{x,t-1}u_{j,t-1}) \\
 E[u_{mt}u_{jt} | Z_{t-1}] &= \delta_{mj0} + \delta_{mj1}(u_{m,t-1}u_{j,t-1})
 \end{aligned} \tag{31}$$

The model thus implies the following orthogonality conditions:

$$E(u_{xt}Z_{t-1}^1, u_{mt}Z_{t-1}^1, \eta_{xt}Z_{t-1}^2, \eta_{mt}Z_{t-1}^2, \eta_{xjt}Z_{t-1}^3, \eta_{mjt}Z_{t-1}^3, u_{jt}Z_{t-1}^3) = 0 \tag{32}$$

where

$Z_{t-1}^1$  includes a constant,  $r_{xt}$  lagged one period,  $r_{mt}$  lagged one period,  
 $Z_{t-1}^2$  includes a constant,  $u_{xt}^2$  lagged one period,  $u_{mt}^2$  lagged one period,  
 $Z_{t-1}^3$  includes a constant,  $u_{xt}u_{jt}$  lagged one period,  $u_{mt}u_{jt}$  lagged one period.

There are, therefore, 18 parameters and 21 moment conditions implying 3 overidentifying restrictions. Estimation and full parameter LM test results are found in Table 2.7. For all cases results are very satisfactory both according to the  $\chi^2$  statistic and with respect to stability. The  $\chi^2$  P-values are at 99% for the UK and

Germany and at 58% for Japan. As for stability, none of the models is rejected even at the 10% level.

Next, we apply the CS test to our model to check the validity of our residuals against certain alternatives. This time, the alternative subsets are comprised of the instruments used in the models of Dumas and Solnik (1995) and Ferson and Harvey (1993). The first of these subsets is denoted XSJUNK and includes 3 lags of the excess US junk bond spread, the second, XSDIV, contains 3 lags of the excess US dividend yield, and the last, STRATE, includes 3 lags of the 30-day Eurodollar return.

Results of the CS tests are found in Tables 2.12, 2.13 and 2.14. They indicate that, generally speaking, the model is robust against the external instrument alternatives. Nevertheless, they also reveal that it will be suitable to modify the specification of the covariance terms to include possibly more own lags, since P-values of covariance residual tests turn out to be generally lower than 5 %.

## 2.6 Pricing Errors

In this section, we examine pricing errors of the equity portfolios for the Dumas and Solnik constrained model, the Ferson and Harvey model and the factor-ARCH model, in the case of two countries. For the first model, the pricing error is determined from equation (5). As defined in section 4.1, this is given by the variable  $h_{jt}$  which we re-define here as  $e_{jt}$  and write it as

$$e_{jt} = r_{jt} - (r_{jt}u_t) \quad (33)$$

For the Ferson and Harvey model,  $e_{jt}$  is given by:

$$e_{jt} = r_{jt} - \sum_{k=1}^2 \beta_{jk} E[f_{kt} | \Omega_{t-1}] \quad (34)$$

and for the factor-ARCH, this is given by

$$e_{jt} = r_{jt} - \sum_{k=1}^2 \frac{Cov[r_{jt}, f_{kt} | \Omega_{t-1}]}{Var[f_{kt} | \Omega_{t-1}]} \sum_{l=1}^4 \alpha_{kl} f_{k,t-l} \quad (35)$$

as in equation (28). From these we obtain the absolute mean and the root mean square errors which are tabulated in Table 2.15. From these we can see both the absolute mean error and the root mean square error yield qualitatively similar results. Quantitatively, the Ferson and Harvey and the factor-ARCH values are generally close. Also, for all three country pairs, the Dumas and Solnik model has the highest pricing error statistics. For instance, the absolute mean error varies between 5.38 % per month (this is the case of Germany and the US estimated with instrument set C) and 8.72 % per month (the UK-US country pair estimated with instrument set A). Similarly, the root mean squared error ranges from 8.16 % to 12.72 % p.m. for the same two cases. Amongst the remaining models, the Ferson and Harvey model yields the smallest statistics for the UK-US pair with 4.92 % p.m. for the absolute mean and 6.22 % for the root mean squared error. However, for Germany and the US and Japan and the US, the factor-ARCH model outperforms the others with absolute mean errors of 4.87 % for the first and 5.57 % for the second. To summarize the findings from this table, we have that, in 2 cases out of 3, the factor-ARCH yields the best results. The Ferson and Harvey model performs the best in the remaining case.

We also calculated 12 autocorrelations for each of the  $e_{jt}$ . From amongst these we selected lags 1, 2, 3, 6, and 12 which we report in Table 2.16. Ljung-Box white noise tests were also run on the 12 autocorrelations, the P-values of which we also included in the same table. The results show that, generally, all the autocorrelations are low and that only in two cases is the null hypothesis of white noise autocorrelations rejected at the 5% level. These are the Dumas and Solnik model results for the Germany-US pair, once estimated with instrument set A, and another time, with instrument C.

## 2.7 Conclusion

In conclusion, we can say that based on the  $J$ -test criterion, we would not have rejected any of the models. What allows us to distinguish the more desirable model is the explicit testing for stability on the one hand, and for orthogonality of errors, on the other. As it turns out, the factor-ARCH formulation seems to be the surest to adopt amongst the three models. It was found to be sound structurally, to hold well versus misspecification against various specific alternatives, and yielded pricing errors which could not be rejected when tested for white noise. In opposition, the Dumas and Solnik model, although more parsimonious, was shown to be structurally unstable, and the Ferson and Harvey model, although relatively more stable, exhibited misspecification against some alternatives.

In choosing an econometric description for the international conditional asset pricing model, one has the option of using the latent approach, leaving first or second moments of returns unspecified, or, one can parameterize these moments more particularly. The decision to use either a linear or a nonlinear formulation should be directed according to observed facts and empirical regularities. Thus, although the first option has the advantage of being parsimonious and is easier to handle from a numerical point of view, it seems important, given the outcome of the tests above, not to ignore the observed persistence in volatility and not to oversimplify by using linear structures. In this respect, the factor-ARCH formulation, with second moments specified nonlinearly, albeit in an ad hoc manner, seems to be the more adequate and viable specification for the purpose described above.

Table 2.1  
Dumas and Solnik model - equations (4), (5) and (7)  
GER,JAP,UK,US; six instruments

MODEL	$\chi^2$ (P-value)	Deg. Freedom	Full Sup LM Test	Partial Sup LM Test
Our data	20.75(0.65)	24	108.5 ***	20.76
D&S data	28.80(0.23)	24	N.A.	N.A.

Significance of tests is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%.

Sup LM critical values, all parameters 1%=65, 5%=58, 10%=55

Sup LM critical values,  $\lambda_0$  parameters 1%=24.3, 5%=19.6, 10%=17.6

Table 2.2  
Dumas and Solnik model - equations (4), (5) and (7)  
GER,JAP,UK,US; five instruments in each set

Instrument Set	$\chi^2$ (P-value)	Deg. Freedom	Full Sup LM	Partial Sup LM
SET A	17.67(0.61)	20	66.4 ***	17.7 *
SET B	10.22(0.96)	20	80.9 ***	10.2
SET C	25.62(0.18)	20	106.8 ***	25.6 ***

Significance of tests is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%.

Sup LM critical values, all parameters 1%=57, 5%=50.5, 10%=47.1

Sup LM critical values,  $\lambda_0$  parameters 1%=21.9, 5%=17.9, 10%=15.6

Table 2.3  
Dumas and Solnik model - equations (4), (5) and (7)  
The US and another country; five instruments in each set

Model	Instrument set	$\chi^2$ (P-value)	Deg. Freedom	Full LM	Partial LM
GER,US	A	9.78(0.46)	10	61.03 ***	27.8 ***
	B	10.99(0.36)		48.1 ***	19.9 **
	C	10.01(0.44)		91.1 ***	57.7 ***
JAP,US	A	10.94(0.36)	10	59.3 ***	19.2 **
	B	6.63(0.76)		37.9 **	24.2 ***
	C	10.33(0.41)		49.2 ***	19.8 **
UK,US	A	2.07(0.99)	10	25.5	12.8
	B	8.32(0.60)		41.3 ***	20.4 **
	C	7.62(0.67)		111.9 ***	70.0 ***

*SET A contains: a constant, lag 1 of world equity excess returns, a january dummy, excess junk bond spread, excess dividend yield; SET B contains: a constant, lag 1 of world equity excess returns, a january dummy, excess junk bond spread, short term rate; SET C contains: a constant, lag 1 of world equity excess returns, a january dummy, excess dividend yield, short term rate*

Significance of tests is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%.

Sup LM critical values, all parameters 1%=40.1, 5%=34.3, 10%=31.7

Sup LM critical values,  $\lambda_0$  parameters 1%=21.9, 5%=17.9, 10%=15.6

Table 2.4  
Dumas and Solnik model - equations (4), (5) and (6)  
GER, JAP, UK, US; five instruments in each set

Instrument Set	$\chi^2$ (P-value)	Degrees of Freedom	Full Sup LM Test
SET A	16.7(0.89)	25	862.6 ***
SET B	5.5(0.99)	25	304.4 ***
SET C	18.2(0.83)	25	1222.6 ***

*For the definitions of the instrument sets, see Table 2.3.*

Significance of tests is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%.  
Sup LM critical values, all parameters 1%=83.5, 5%=75.2, 10%=71.8

Table 2.5  
Dumas and Solnik model - equations (4), (5) and (6)  
The US and another country; five instruments in each set

Model	Instrument set	$\chi^2$ (P-value)	Deg. freedom	Full Sup LM Test
GER, US	A	14.01(0.52)	15	119.2 ***
	B	23.08(0.08)		119.2 ***
	C	21.79(0.11)		179.2 ***
JAP, US	A	11.34(0.73)	15	116.2 ***
	B	6.41(0.97)		70.4 ***
	C	13.54(0.56)		407.4 ***
UK, US	A	17.63(0.28)	15	176.9 ***
	B	3.24(0.58)		36.5
	C	15.41(0.42)		104.7 ***

*For the definitions of the instrument sets, see Table 2.3.*

Significance of tests is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%.  
Sup LM critical values, all parameters 1%=47.8, 5%=41.9, 10%=39.0

Table 2.6  
Ferson and Harvey model - equations (18) and (19)  
The US and another country; five instruments in each set; constant betas

Model	Inst. set	$\chi^2$ (P-value)	DF	Sup LM			$\delta$	$\gamma$	$\beta$
				Full					
GER, US	A	5.04(0.75)	8	128.5 ***	24.9 ***	121.6 ***	65.4 ***		
	B	7.69(0.43)		40.9 **	9.4	34.4 ***	8.0		
	C	2.82(0.95)		85.4 ***	33.8 ***	80.3 ***	29.7 ***		
JAP, US	A	3.07(0.93)	8	30.3	3.5	15.0	15.1 **		
	B	3.17(0.92)		53.0 ***	15.2	42.1 ***	22.2 ***		
	C	5.30(0.72)		32.2	10.5	17.9	13.4 **		
UK, US	A	3.68(0.88)	8	39.3 **	12.4	36.4 ***	7.2 *		
	B	8.51(0.39)		31.3	8.3	20.5	3.4		
	C	1.73(0.99)		120.7 ***	32.9 ***	116.9 ***	77.9 **		

*For the definitions of the instrument sets, see Table 2.3.*

Significance of tests is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%.  
Sup LM critical values, all parameters 1%=43.3, 5%=37.5, 10%=34.5  
Sup LM critical values,  $\delta$  parameters 1%=21.9, 5%=17.9, 10%=15.6  
Sup LM critical values,  $\gamma$  parameters 1%=32.0, 5%=26.4, 10%=24.0  
Sup LM critical values,  $\beta$  parameters 1%=15.1, 5%=11.3, 10%=9.6

Table 2.7  
Factor ARCH model

The US and another country; Instrument sets are  $Z_{t-1}^1, Z_{t-1}^2, Z_{t-1}^3$

MODEL	$\chi^2$ (P-value)	Deg. Freedom	Full Sup LM Test
GER,US	0.01(0.99)	3	34.6
JAP,US	1.94(0.58)	3	31.8
UK,US	0.02(0.99)	3	20.9

Significance of tests is reported as: \* at 10%, \*\* at 5%, \*\*\* at 1%.  
Sup LM critical values, all parameters 1%=44.1, 5%=38.8, 10%=36.1

Table 2.8  
CS Tests for the Dumas and Solnik Model

Two-Country Cases; Alternative instrument set is  $AZ_{t-1}^4$

RESIDUAL	INSTR. SET	GERMANY-US	JAPAN-US	UK-US
P-value				
$u_t$	A	0.999	0.000	0.998
$h_{jt}$		0.999	0.000	1.000
$h_{US,t}$		0.999	0.998	0.996
$h_{wt}$		0.118	1.000	0.999
$hcd_{jt}$		0.995	0.999	0.891
$u_t$	B	0.991	0.999	0.999
$h_{jt}$		0.991	0.828	1.000
$h_{US,t}$		0.995	1.000	0.998
$h_{wt}$		0.999	0.992	0.988
$hcd_{jt}$		0.020	0.997	0.991
$u_t$	C	0.999	0.977	0.989
$h_{jt}$		0.951	0.886	0.994
$h_{US,t}$		0.999	0.930	0.998
$h_{wt}$		0.958	0.999	0.999
$hcd_{jt}$		0.943	0.990	0.999

For the definitions of the instrument sets, see Table 2.3. Residual  $u_t$  is defined in equation (7), the  $h$  error terms are from equation (5):  $h_{jt}$  is for equity returns of country  $j$ ,  $h_{US,t}$  is for US equity returns,  $h_{wt}$  is for world equity returns, and  $hcd_{jt}$  is for returns on a deposit in the currency of country  $j$ .



Table 2.9  
 CS Tests for the Ferson and Harvey Model  
 Case of the US and Germany

RESIDUAL	INSTR. SET	$AZ_{t-1}^1$	$AZ_{t-1}^2$	$AZ_{t-1}^3$
P-value				
$u_{1t}$	A	0.51	0.74	0.72
$u_{2xt}$		0.17	0.00	0.07
$u_{2wt}$		0.26	0.06	0.77
$u_{3xt}$		1.00	1.00	1.00
$u_{3wt}$		1.00	1.00	1.00
$u_{1t}$		B	0.53	0.86
$u_{2xt}$	0.14		0.00	0.26
$u_{2wt}$	0.48		0.25	0.60
$u_{3xt}$	1.00		1.00	1.00
$u_{3wt}$	1.00		1.00	1.00
$u_{1t}$	C		0.60	0.57
$u_{2xt}$		0.03	0.00	0.27
$u_{2wt}$		0.93	0.12	0.35
$u_{3xt}$		1.00	1.00	1.00
$u_{3wt}$		1.00	1.00	1.00

*For the definitions of the instrument sets, see Table 2.3. Alternative instrument sets are denoted  $AZ_{t-1}^1$ ,  $AZ_{t-1}^2$ , and  $AZ_{t-1}^3$ .*

*The first contains lags 1, 2, and 3 of excess equity returns for Germany,  $AZ_{t-1}^2$  contains lags 1, 2, and 3 of excess returns to a DM deposit, and  $AZ_{t-1}^3$  contains lags 1, 2, and 3 of excess US equity returns.*

Table 2.10  
CS Tests for the Ferson and Harvey Model  
Case of the US and Japan

RESIDUAL	INSTR. SET	$AZ_{t-1}^1$	$AZ_{t-1}^2$	$AZ_{t-1}^3$
P-value				
$u_{1t}$	A	0.56	0.44	0.40
$u_{2xt}$		0.14	0.00	0.16
$u_{2wt}$		0.75	0.55	0.63
$u_{3xt}$		1.00	1.00	1.00
$u_{3wt}$		1.00	1.00	1.00
$u_{1t}$	B	0.35	0.32	0.03
$u_{2xt}$		0.00	0.00	0.04
$u_{2wt}$		0.45	0.41	0.05
$u_{3xt}$		1.00	1.00	1.00
$u_{3wt}$		1.00	1.00	1.00
$u_{1t}$	C	0.67	0.42	0.25
$u_{2xt}$		0.84	0.00	0.05
$u_{2wt}$		0.65	0.55	0.35
$u_{3xt}$		1.00	1.00	1.00
$u_{3wt}$		1.00	1.00	0.00

*For the definitions of the instrument sets, see Table 2.3. Alternative instrument sets are denoted  $AZ_{t-1}^1$ ,  $AZ_{t-1}^2$ , and  $AZ_{t-1}^3$ .*

*The first contains lags 1, 2, and 3 of excess equity returns for Japan,  $AZ_{t-1}^2$  contains lags 1, 2, and 3 of excess returns to a Yen deposit, and  $AZ_{t-1}^3$  contains lags 1, 2, and 3 of excess US equity returns.*

Table 2.11  
 CS Tests for the Ferson and Harvey Model  
 Case of the US and UK

RESIDUAL	INSTR. SET	$AZ_{t-1}^1$	$AZ_{t-1}^2$	$AZ_{t-1}^3$
P-value				
$u_{1t}$	A	0.04	0.05	0.91
$u_{2xt}$		0.47	0.00	0.02
$u_{2wt}$		0.08	0.73	0.92
$u_{3xt}$		1.00	1.00	1.00
$u_{3wt}$		1.00	1.00	1.00
$u_{1t}$		B	0.11	0.06
$u_{2xt}$	0.90		0.00	0.16
$u_{2wt}$	0.19		0.30	0.39
$u_{3xt}$	1.00		1.00	1.00
$u_{3wt}$	1.00		1.00	1.00
$u_{1t}$	C		0.32	0.10
$u_{2xt}$		0.01	0.00	0.00
$u_{2wt}$		0.78	0.36	0.24
$u_{3xt}$		1.00	1.00	1.00
$u_{3wt}$		1.00	1.00	1.00

*For the definitions of the instrument sets, see Table 2.3. Alternative instrument sets are denoted  $AZ_{t-1}^1$ ,  $AZ_{t-1}^2$ , and  $AZ_{t-1}^3$ .*

*The first contains lags 1, 2, and 3 of excess equity returns for the UK,  $AZ_{t-1}^2$  contains lags 1, 2, and 3 of excess returns to a £ deposit, and  $AZ_{t-1}^3$  contains lags 1, 2, and 3 of excess US equity returns.*

Table 2.12  
CS Tests for Purely Autoregressive Model  
 Case of the US and Germany

RESIDUAL	XSJUNK	XSDIV	STRATE
CS Statistic $\chi^2$ (P-value)			
>Returns			
$u_{xt}$	1.06(0.787)	0.74(0.863)	0.80(0.849)
$u_{mt}$	0.37(0.947)	0.38(0.944)	0.34(0.952)
$u_{jt}$	4.05(0.256)	2.24(0.524)	2.474(0.480)
Variances			
$\eta_{xt}$	4.03(0.258)	4.30(0.231)	3.59(0.309)
$\eta_{mt}$	1.64(0.650)	1.71(0.635)	2.04(0.565)
Covariances			
$\eta_{xjt}$	5.68(0.129)	4.79(0.188)	6.37(0.095)
$\eta_{mjt}$	151.92(0.000)	101.07(0.000)	192.0(0.000)

*XSJUNK means lags 1,2,3 of excess junk bond spread,  
 XSDIV means lags 1,2,3 of excess dividend yield,  
 STRATE means lags 1,2,3 of Eurodollar rate.*

Table 2.13  
CS Tests for Purely Autoregressive Model  
 Case of the US and Japan

RESIDUAL	XSJUNK	XSDIV	STRATE
CS Statistic $\chi^2$ (P-value)			
>Returns			
$u_{xt}$	0.055(0.996)	0.031(0.998)	0.093(0.993)
$u_{mt}$	0.360(0.948)	0.211(0.976)	0.057(0.996)
$u_{jt}$	10.404(0.015)	8.133(0.043)	3.618(0.306)
Variances			
$\eta_{xt}$	3.161(0.367)	3.334(0.342)	3.100(0.376)
$\eta_{mt}$	0.878(0.831)	0.960(0.811)	1.170(0.760)
Covariances			
$\eta_{xjt}$	62.981(0.000)	57.351(0.000)	31.123(0.000)
$\eta_{mjt}$	347.728(0.000)	313.341(0.000)	212.299(0.000)

*For the definitions of XSJUNK, XSDIV & STRATE see Table 2.12.*

Table 2.14  
CS Tests for Purely Autoregressive Model  
Case of the US and the UK

RESIDUAL	XSJUNK	XSDIV	STRATE
CS Statistic $\chi^2$ (P-value)			
Returns			
$u_{xt}$	0.959(0.811)	1.186(0.756)	0.877(0.831)
$u_{mt}$	0.018(0.999)	0.056(0.997)	0.008(0.999)
$u_{jt}$	6.731(0.081)	9.150(0.027)	9.292(0.026)
Variances			
$\eta_{xt}$	1.338(0.720)	1.532(0.675)	1.566(0.667)
$\eta_{mt}$	2.412(0.491)	3.360(0.339)	2.750(0.432)
Covariances			
$\eta_{xjt}$	12.608(0.006)	10.050(0.018)	14.222(0.003)
$\eta_{mjt}$	603.313(0.000)	283.365(0.000)	770.338(0.000)

*For the definitions of XSJUNK, XSDIV & STRATE see Table 2.12.*

Table 2.15  
Pricing Errors Statistics

COUNTRIES	MODEL	ABS MEAN	RMS ERROR
GER,US	D&S - Inst. A	5.4424	8.1760
	D&S - Inst. B	5.6043	9.0030
	D&S - Inst. C	5.3804	8.1594
	F&H - Inst. A	4.9365	6.5350
	F&H - Inst. B	4.8731	6.4839
	F&H - Inst. C	4.9130	6.5321
	Factor-ARCH	4.8718	6.4808
JAP,US	D&S - Inst. A	6.1112	9.4168
	D&S - Inst. B	6.2691	10.1762
	D&S - Inst. C	6.6673	11.2405
	F&H - Inst. A	5.7550	7.3630
	F&H - Inst. B	5.6307	7.3434
	F&H - Inst. C	5.6935	7.3105
	Factor-ARCH	5.5671	7.2881
UK,US	D&S - Inst. A	8.7422	12.7205
	D&S - Inst. B	6.9369	10.4199
	D&S - Inst. C	6.6939	9.6600
	F&H - Inst. A	4.9233	6.2219
	F&H - Inst. B	5.0887	6.2303
	F&H - Inst. C	5.1957	6.4217
	Factor-ARCH	5.3778	7.6048

*ABS MEAN is absolute mean error, RMS ERROR is root mean square error.*

Table 2.16  
Autocorrelations and White Noise Test

COUNTRIES	MODEL	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_6$	$\rho_{12}$	P-value LB
GER,US	D&S - Inst. A	-0.0088	-0.0814	0.1570	0.0507	-0.0965	0.036
	D&S - Inst. B	0.0349	-0.0332	0.1473	0.0966	-0.0839	0.298
	D&S - Inst. C	-0.0999	-0.1277	0.1622	0.0421	-0.0779	0.001
	F&H - Inst. A	-0.0577	0.0002	0.0972	0.0063	-0.0902	0.328
	F&H - Inst. B	-0.0400	-0.0028	0.0961	0.0208	-0.1085	0.283
	F&H - Inst. C	-0.0428	-0.0001	0.0969	0.0132	-0.0846	0.355
	Factor-ARCH	-0.0064	0.0116	0.1179	0.0104	-0.0720	0.251
JAP,US	D&S - Inst. A	-0.0713	-0.0469	0.0464	0.0007	0.0138	0.959
	D&S - Inst. B	-0.0917	-0.0404	0.0219	0.0210	0.0100	0.797
	D&S - Inst. C	-0.1081	-0.0297	0.0264	0.0011	-0.0199	0.885
	F&H - Inst. A	0.0689	-0.0442	0.0540	-0.0149	0.0712	0.553
	F&H - Inst. B	0.0837	-0.0582	0.0407	-0.0152	0.0762	0.496
	F&H - Inst. C	0.0735	-0.0599	0.0433	-0.0218	0.0653	0.634
	Factor-ARCH	0.0660	-0.0694	0.0433	-0.0280	0.0704	0.636
UK,US	D&S - Inst. A	0.0617	0.0238	-0.1594	-0.0610	-0.0454	0.374
	D&S - Inst. B	0.0022	-0.0922	-0.1206	-0.0738	-0.0303	0.392
	D&S - Inst. C	0.0143	-0.0044	-0.1555	-0.0882	-0.0348	0.346
	F&H - Inst. A	-0.0965	-0.1155	-0.0717	-0.0804	-0.1273	0.328
	F&H - Inst. B	-0.0629	-0.1180	-0.0724	-0.0791	-0.1270	0.435
	F&H - Inst. C	-0.0327	-0.0911	-0.0498	-0.0586	-0.1348	0.587
	Factor-ARCH	-0.0581	-0.0702	-0.0359	0.0303	-0.0505	0.987

*Autocorrelations 1,2,3,6 and 12 are in the designated  $\rho$ 's; LB test refers to the LJUNG-BOX white noise test*

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## Chapter 3

# Modeling the Risk Premium in International Financial Markets.

### 3.1 Introduction

Much of the research on financial markets in the last decade concentrated on explaining the observed predictability of excess asset returns. From this literature, a large number of empirical studies attributed this phenomenon to the presence of time-varying risk premiums in these markets. However, the typical theoretical models of asset pricing that also adhered to this view were not very successful in producing equilibrium conditions that replicated the actual behaviour of asset moments in empirical tests<sup>5</sup>.

Typical (rational expectations) asset pricing Euler equations imply that the conditional expectation of discounted asset returns equals a constant. In this context, the key to reproducing the required behaviour of assets is specifying a suitable form for the stochastic discount rate, that is, the intertemporal marginal rate of substitution. This question was examined by Mehra and Prescott (1985) in the case of equity markets. They showed that for reasonable configurations of preference and endowment

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5. See, for instance, Canova and Marrinan (1995) and Bekaert and Hodrick (1997).

processes, an exchange economy equilibrium model could not reproduce the secular difference between the average return on stocks and that on Treasury bills (this is the so-called equity premium puzzle).

In order to address this empirical issue, it was suggested that the constraints imposed by expected utility theory were not consistent with observed behaviour. In particular, in the standard discounted expected utility theory, marginal utility in period  $t$  is independent of the level of consumption in any other time period. However, it appears more reasonable to assume that marginal utility in a particular time period depends on the consumption level realized in previous time periods, as well as on other variables such as leisure. Research on this idea consequently proceeded in three different directions.

On one side, Epstein and Zin (1989, 1991) and Weil (1989) introduced the notion of non-expected utility preferences which loosens the hypothesis of independence of marginal utility of consumption across different states. In this case, marginal utility of consumption in a good state is allowed to be influenced by the consumption level in a bad state. In addition, these new generalized preferences, contrary to the isoelastic preferences case, also permit the distinct notions of risk aversion and intertemporal substitution to be disentangled. This implies that a risk-averse consumer will not necessarily see consumption in different periods as highly complementary goods. The advantage of both of these properties of non-expected utility preferences is that they allow greater flexibility in the intertemporal marginal substitution function thereby inducing more variation in it. Indeed, when applied to US equity data, these studies found that their results provided some improvement on the expected utility framework outcomes. However, these gains were still not sufficient to solve the equity premium puzzle.

Another line of research proceeded to resolve the empirical anomalies by focusing on time non-separability of preferences instead. Studies by Constantinides (1990), Ferson and Constantinides (1991), and Campbell and Cochrane (1995) used the idea of habit formation to address this issue. That is, marginal utility of current consumption increases when consumption attains the level the individual is accustomed

to consuming. Similarly, when current consumption drops relative to the habitual level, risk aversion increases inducing the agent to increase saving in the face of a possible further decline in consumption. Thus, with this set-up, risk aversion is time-varying. In addition, similar to the non-expected utility case, these preferences also relax the link between intertemporal substitution and risk aversion.

However, although habit formation was found to exhibit improved dynamics of returns over the isoelastic preferences case, it could not match the high levels of equity risk premiums unless very high risk aversion was assumed.

Benartzi and Thaler (1995) explain that while habit formation preferences stress the asymmetry between gains and losses they rely too heavily on consumption which, in actual data, has very low variability. A better method to adopt would be to assume that investors have preferences over returns rather than consumption. Using Kahneman and Tversky's (1979) prospect theory, they develop the notion of myopic loss aversion preferences. In this case, agents are assumed to be more sensitive to reductions in their well-being than increases. Furthermore, they are assumed to take risk more willingly only if they evaluate the performance of their portfolios infrequently.

According to this theory, the equity premium is affected by a combination of loss aversion and by frequent portfolio evaluations. Using simulation techniques, the authors show that this premium would fall if the evaluation frequency decreased. While this is a possible explanation for the existence of the puzzle, empirical studies of consumption asset pricing model have yet to assess the performance of these types of preferences with actual data.

Since the intertemporal marginal rate of substitution is affected both by endowment and preference parameters, other studies concentrated on the time-series of the endowment process to try to solve the puzzle. Among these are the works of Rietz (1988), Bonomo and Garcia (1996) and Cecchetti, Lam and Mark (1993) for equity markets.

For instance, Rietz (1988) argued that the risk premium might be a response to the time-varying risk of economic catastrophes. However, for this hypothesis to be true,

the probability of being faced with a much greater catastrophe than the 1929 stock market crash should exist. In addition, this catastrophe should affect only stocks and not bonds.

Both Bonomo and Garcia (1994) and Cecchetti, Lam and Mark (1993) proposed a bivariate Markov switching process for consumption and dividend growths series to describe the endowment component in an expected utility framework model. The first authors showed that, on the one hand, loosening the usual assumption that dividends equal consumption and, on the other hand, using a three-state bivariate Markov switching model where the means and variances changed with the state, yielded good results on real returns in such models. The second authors used a two-state homoskedastic specification with isoelastic preferences, with and without leverage effects. However, for both studies, the adopted endowment process could not fully explain the facts related to excess returns in equity markets.

A natural extension to the above discussion is postulating a model which simultaneously integrates generalized preferences on the one hand, and heteroskedastic driving processes for the endowment series on the other. Such an exercise was undertaken in the study by Bonomo and Garcia (1993). In this closed economy study, the authors showed that in order for a model to reproduce magnitudes of US equity and risk-free rate moments that are close to actual data, it needed to have generalized preferences and non-linear data-generating processes simultaneously. More specifically, the agents were assumed to have a particular type of non-expected utility function resulting from disappointment aversion type preferences, and the bivariate consumption and dividend growth process was made to follow a three-state Markov regime-switching process where both the means and variances were state-dependent. The authors stressed the fact that disappointment aversion preferences coupled with a joint random walk endowment process could produce only half the magnitude of the average US equity risk premium estimated using the previous model. Similarly, isoelastic preferences combined with a bivariate three-state Markov switching model yielded an even lower value for the average equity risk premium and a very high value for the mean of the risk-free rate.

The results of the Bonomo and Garcia (1993) study highlights the importance of the complex interaction between attitudes towards risk, intertemporal substitution and the heteroskedastic nature of endowment shocks. It is therefore clear that first-order risk aversion, alone, cannot match the moments of returns. Having heteroskedastic data-generating-processes for the endowment series is therefore necessary to reproduce the dynamics of real and excess returns in equity markets.

For the international returns case an added complication arises in that moments of excess returns on the foreign exchange market should also be explained. So far, international models of consumption asset pricing have fared very poorly in trying to reproduce the dynamics of international returns data. These include, for instance, cash-in-advance type models where money is introduced in CCAPM models through a cash-in-advance constraint. That is, money is required to buy goods but financial markets are not accessible when agents are trading in the goods' markets. Examples of these kinds of models are Lucas (1982), Svensson (1985), Labadie (1989), and Giovannini and Labadie (1991). The conclusion that arises from these models is that while inflation risk does affect the stochastic properties of asset returns, when preferences are isoelastic, the resulting variability in the stochastic discount rate is still too small to match asset returns dynamics.

As in the single-country models, some studies tried to modify the basic expected utility structure in order to have more complex driving processes. For instance, Canova and Marrinan (1993, 1995) used heteroskedastic endowment processes in the representative agent cash-in-advance model. They found that while the resulting excess asset returns were variable, heteroskedastic and serially correlated, the magnitudes of these second moments were considerably smaller than those observed in actual data.

An attempt was also made at integrating generalized preferences in a two-country monetary model. The study by Bekaert, Hodrick and Marshall (1997) uses non-expected utility preferences in a two-country model to explain excess returns in the international market. More specifically, the authors use disappointment aversion in order to characterize agents' preferences. With this set-up agents are more sensitive to

a disappointing state of the endowment than to an elating one so that a fairly small amount of uncertainty in the agents' exogenous environment can lead to potentially large fluctuations in the stochastic discount rate. While preferences are different than in a typical consumption CAPM model, the authors maintain the assumption of a homoskedastic driving process in their economy. Finally, in this model, the incentive to hold money comes from the assumption that while real consumption is costly, these transaction costs are alleviated by holding real money balances.

Once again the authors conclude that while first-order aversion substantially increases the variance of risk premiums this increase is still insufficient to match the excess return predictability in the data. They therefore conclude that excess return predictability of financial assets cannot be explained only by modifying the preference assumptions of the model and that learning, peso problems and Markov switching models may be more suitable frameworks for examining the problem at hand.

Extrapolating on the above discussion, in this paper, we adopt the basic framework of Bekaert, Hodrick and Marshall (1997) but modify the endowment processes to allow for multivariate regime-switching, as in Bonomo and Garcia(1993). We empirically examine the relative roles of the preference and endowment parameters, as well as their various combinations, for the simultaneous resolution of the equity and risk-free puzzles in an international context. We show that these parameters interact together in a complex way and that they influence returns in a non-monotonic fashion. In general, our results present some improvements over the BHM model. In particular, we are able to generate the predictability of excess returns in addition to the mean levels of excess equity returns. Nevertheless, we are unable to produce as satisfactory results for the moments in the foreign exchange market. On the basis of our experiments we therefore suggest some possible extensions to our model in the final sections of the paper. These would presumably improve the above results even further.

### **3. 2 The Structure of Preferences**

A representative agent is endowed with disappointment aversion preferences and maximizes intertemporal utility over home and foreign goods. This generalized

preference structure was axiomatized by Gul (1991) to be the most restrictive formulation that is consistent with Allais' paradox and which also includes expected utility theory as a special case. Therefore, the corresponding utility function is not the usual expected utility function, but rather the recursive functional form of Epstein and Zin (1989) which was developed to accommodate such generalized preferences in an infinite horizon setting.

As explained above, the reason for adopting these preferences is as follows. An unattractive feature of the Von Neumann-Morgenstern expected utility preferences is that relative risk aversion and intertemporal substitutability are intertwined; that is, the coefficient of relative risk aversion is the inverse of the elasticity of intertemporal substitution. This arises from the assumption of time and state separability that expected utility theory imposes. Thus, the respective roles of risk attitudes and intertemporal substitution cannot be fully exploited in consumption-savings and portfolio choice equations in the typical consumption asset pricing model. It has been suggested that this inflexibility of the preference structure has been a possible cause for the failure of such models. On the other hand, since generalized preferences permits these two distinct aspects of preferences to be disentangled, it is a desirable feature to have in an intertemporal general equilibrium model.

Secondly, disappointment aversion preferences display first-order risk aversion. This means that even when faced with a lottery close to perfect certainty, agents are substantially risk averse. In contrast, individuals having expected utility preferences (which exhibit second-order risk aversion instead) behave as if they are risk neutral in the face of the same lottery. With the latter specification, the equilibrium conditional variance of next period's consumption is small.

From these explanations we can see the importance of having first-order risk aversion for the purposes of obtaining higher variability in the stochastic discount rate. This, in turn, implies larger movements in expected asset returns.

### 3. 2. 1 Recursive Utility

The Epstein and Zin recursive utility is a CES function of the form

$$W(c_0, \mu[V(m)]) = \left\{ c_0^\rho + \beta(\mu[V(m)])^\rho \right\}^{1/\rho} \quad (1)$$

where  $c_0$  is consumption in period zero,  $V(\cdot)$  is the probability measure for future utility,  $m$  is an element of the admissible lottery space, and  $\mu$  is a certainty equivalent measure for random future utility (i.e. it is a weighted mean). The form of  $\mu$  is decided by the type of preferences assumed. In our case, the certainty equivalent function for disappointment aversion preferences is implicitly defined as:

$$\frac{\mu(P)^\alpha}{\alpha} \equiv \frac{1}{KK} \left( \int_{(-\infty, \mu(P))} \frac{z^\alpha}{\alpha} dP(z) + A \int_{(\mu(P), \infty)} \frac{z^\alpha}{\alpha} dP(z) \right) \quad (2)$$

where  $KK = A \cdot \text{prob}(z > \mu) + \text{prob}(z \leq \mu)$ ,  $A \leq 1, \alpha < 1$ . When  $A$  is less than one, the elation region is downweighted relative to the disappointment region, that is, outcomes that are below the certainty equivalent are more heavily weighted than those that are above  $\mu(P)$ . Notice that for  $A = 1$  and  $\alpha = \rho$ , the above equation becomes the usual expected utility certainty equivalent function definition. Thus, using this framework, one can explicitly test the expected utility model conclusions to its generalized counterpart. We also point out that when  $A = 1$  and  $\alpha \neq \rho$ , we obtain Kreps-Porteus preferences which are of the non-expected utility type and were used by Weil (1989).

### 3. 3 The Two-Country Model

Our starting point is the economic framework described in Bekaert, Hodrick and Marshall<sup>6</sup> (1997) which we detail in this section. As mentioned above, the main difference between our study and theirs is that our endowment processes and solution methods for the model Euler equations differ. These modifications will be discussed in sections 3.4 and 3.5.

Let  $C_t^x$  denote the representative agent's consumption of the good produced in country  $x$  at time  $t$  and  $C_t^y$  his consumption of the good produced in country  $y$  at the

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6. Hereafter (BHM)



same time period. We suppose that consuming involves real transaction costs. These are given by  $\psi_t^x$  for good  $x$  and  $\psi_t^y$  for good  $y$ . However, agents can reduce these costs by holding real money balances. Let  $M_{t+1}^x$  and  $M_{t+1}^y$  be the amounts of currencies of countries  $x$  and  $y$  respectively acquired by the agent at time  $t$  and held until time  $t+1$ . Defining  $P_t^x$  as the price of good  $x$  and  $P_t^y$  as that of good  $y$ , the transaction cost equations, evaluated in units of  $x$  and  $y$  respectively, are defined as:

$$\psi_t^x \equiv \gamma (C_t^x)^\nu \left( \frac{M_{t+1}^x}{P_t^x} \right)^{1-\nu} \quad (3)$$

and

$$\psi_t^y \equiv \zeta (C_t^y)^\xi \left( \frac{M_{t+1}^y}{P_t^y} \right)^{1-\xi} \quad (4)$$

where  $\nu > 1, \gamma > 0, \xi > 1, \zeta > 0$ .

In addition to the currencies, agents can also hold  $n$  capital assets. We designate  $z_{i,t+1}$  as the real value of the agent's investment in asset  $i$  which pays off  $R_{i,t+1}$  at time  $t+1$ . Finally, we denote  $W_t$  as the agent's wealth at the beginning of period  $t$  and  $J_t$  as the information available to him in the same time period. With this structure and with the retained preferences, the maximum value function  $V(W_t, J_t)$  can be written as:

$$V(W_t, J_t) = \max_{C_t^x, C_t^y, M_{t+1}^x, M_{t+1}^y, \{z_{i,t+1}\}} \left\{ \left( [C_t^x]^\delta [C_t^y]^{1-\delta} \right)^\rho + \beta \left( \mu [P_{V(W_{t+1}, J_{t+1})} | J_t] \right)^\rho \right\}^{\frac{1}{\rho}} \quad (5)$$

where  $0 < \delta < 1, \rho < 1$ . This maximizing is subject to the individual's budget constraint.

Letting  $S_t$  denote the exchange rate, this budget constraint (evaluated in units of good  $x$ ) is given by:

$$C_t^x + \psi_t^x + \frac{S_t P_t^y}{P_t^x} (C_t^y + \psi_t^y) + \sum_{i=1}^n z_{i,t+1} + \frac{M_{t+1}^x + S_t M_{t+1}^y}{P_t^x} \leq W_t \quad (6)$$

with the wealth being defined as:

$$W_t = \sum_{i=1}^N R_{i,t} z_{i,t} + \frac{M_t^x + S_t M_t^y}{P_t^x} \quad (7)$$

Solving the agent's problem, that is, maximizing the value function subject to the budget constraint, the wealth and transaction cost equations, as well as the market clearing conditions, we obtain the following set of Euler equations<sup>7</sup>:

$$E_t \{ I_A(Z_{t+1}) [Z_{t+1}^\alpha - 1] \} = 0 \quad (8)$$

$$E_t \left\{ I_A(Z_{t+1}) \left[ Z_{t+1}^\alpha \frac{R_{i,t+1}}{R_{t+1}} \right] \right\} = E_t \{ I_A(Z_{t+1}) \} \quad (9)$$

$$\forall i = x, y, 1, \dots, N$$

with

$$Z_{t+1} \equiv \left[ \beta \left( \frac{C_{t+1}^x}{C_t^x} \right)^{\rho\delta-1} \left( \frac{C_{t+1}^y}{C_t^y} \right)^{\rho(1-\delta)} \left( \frac{1 + \Psi_{1t}^x}{1 + \Psi_{1t+1}^x} \right) R_{t+1} \right]^{\frac{1}{\rho}} \quad (10)$$

Here  $\Psi_{it}^x$  is the derivative of the function  $\Psi_t^x$  with respect to its  $i$ th argument,  $R_{t+1}$  is the real return on the market portfolio and  $I_A(Z)$  is an indicator function defined as:

$$I_A(Z) = \begin{cases} A & \text{if } Z \geq 1 \\ 1 & \text{if } Z < 1 \end{cases} \quad (11)$$

Finally, the exchange rate is obtained as

$$S_t = \frac{P_t^x}{P_t^y} \left( \frac{1 + \Psi_{1t}^x}{1 + \Psi_{1t}^y} \right) \left( \frac{C_t^x}{C_t^y} \right) \left( \frac{1 - \delta}{\delta} \right) \quad (12)$$

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7. For the derivations of these equations see Appendix 3.A.

Equation (8) is the Euler equation for the market portfolio. Substituting the expression for  $Z_{t+1}$  and introducing the  $I_B(Z)$  indicator function notation as

$$I_B(Z) = \begin{cases} 0 & \text{if } Z < 1 \\ Z & \text{if } Z \geq 1 \end{cases} \quad (13)$$

the market portfolio equation becomes

$$\begin{aligned} E_t \left\{ \left[ \beta^{\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^x}{C_t^x} \right)^{(\rho\delta-1)\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^y}{C_t^y} \right)^{(1-\delta)\frac{\alpha}{\rho}} \left( \frac{1+\psi_{1t}^x}{1+\psi_{1t+1}^x} \right)^{\frac{\alpha}{\rho}} \right] \left( R_{t+1}^{\frac{\alpha}{\rho}} \right) - 1 \right\} + \\ (A-1)E_t \left\{ I_B \left( \left[ \beta^{\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^x}{C_t^x} \right)^{(\rho\delta-1)\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^y}{C_t^y} \right)^{(1-\delta)\frac{\alpha}{\rho}} \left( \frac{1+\psi_{1t}^x}{1+\psi_{1t+1}^x} \right)^{\frac{\alpha}{\rho}} \right] \left( R_{t+1}^{\frac{\alpha}{\rho}} \right) - 1 \right) \right\} = 0 \end{aligned} \quad (14)$$

Similarly, the Euler equation for any asset  $i$  return is given by:

$$\begin{aligned} E_t \left\{ \left[ \beta^{\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^x}{C_t^x} \right)^{(\rho\delta-1)\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^y}{C_t^y} \right)^{(1-\delta)\frac{\alpha}{\rho}} \left( \frac{1+\psi_{1t}^x}{1+\psi_{1t+1}^x} \right)^{\frac{\alpha}{\rho}} \right] \left( R_{t+1}^{\frac{\alpha}{\rho}} \right) R_{i,t+1} \right\} + \\ (A-1)E_t \left\{ I_B \left( \left[ \beta^{\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^x}{C_t^x} \right)^{(\rho\delta-1)\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^y}{C_t^y} \right)^{(1-\delta)\frac{\alpha}{\rho}} \left( \frac{1+\psi_{1t}^x}{1+\psi_{1t+1}^x} \right)^{\frac{\alpha}{\rho}} \right] \left( R_{t+1}^{\frac{\alpha}{\rho}} \right) R_{i,t+1} - 1 \right) \right\} \\ = 1 \end{aligned} \quad (15)$$

In particular, the real return to holding the currencies of countries  $x$  and  $y$  are defined as:

$$R_{x,t+1} \equiv \left( \frac{P_t^x}{P_{t+1}^x} \right) \left( \frac{1}{1+\psi_{2t}^x} \right), \quad R_{y,t+1} \equiv \left( \frac{S_{t+1}P_t^x}{S_t P_{t+1}^x} \right) \left( \frac{1}{1+\psi_{2t}^y} \right) \quad (16)$$

$$(19) \quad \ln \left( \frac{M_{l,t+1}^f}{M_{l,t}^f} \right) = \mu_{M,l}^f + \omega_{M,l}^f \varepsilon_{M,l,t+1}^f$$

and

$$(18) \quad \ln \left( \frac{C_{l,t+1}^f}{C_{l,t}^f} \right) = \mu_{C,l}^f + \omega_{C,l}^f \varepsilon_{C,l,t+1}^f$$

$l, t = x, y$ , we have that:

The bivariate process for each country is comprised of the growth rates of the logarithms of real consumption and money supply. Thus, for  $s_{t+1} = j$  and for country

$l$  is given by  $\Phi^l$  with elements  $[P_{lj}^l] = [P^l(s_{t+1} = j | s_t = i)]$  for  $i, j = 1, 2$ . The stationary transition probability matrix for the probability of passing from one state to the next for country at time  $t$  englobes all previous periods' information. The stationary transition one of two values (1 or 2) and follows a Markov process so that information available depending on the value taken by the state variable  $s_{t+1}$ . This state variable can take endowment growth process  $g_{l,t} = x, y$ , with means and variances that change Thus, for each country, we assume the existence of a bivariate exogenous

is no correlation between the endowment processes of the two countries. (1) each country's real dividend equals a proportion of its consumption and (2) that there For this first experiment we use the simplest possible extension. We assume that

endowment process will help improve the model fit. purpose is to see whether explicitly allowing for changing means and variances in the In this section we explain the exogenous environment of the economy. Our

### 3.4 The Endowment Process

$$(17) \quad \ln \left( \frac{x_{l,t}}{1} \right) = \ln \left( \frac{1 + \psi_{2l}^t}{1} \right), \quad \ln \left( \frac{y_{l,t}}{1} \right) = \ln \left( \frac{1 + \psi_{2l}^t}{1} \right)$$

functions of the marginal cost functions and are given by Finally, the continuously compounded nominal interest rates in this economy are

The vector of residuals  $\varepsilon_{t+1}^l$  in the above equations are assumed to have a joint normal distribution with mean zero and correlation matrix  $\rho^l$  so that the process  $g^l$  is also normally distributed with mean  $\mu_j^l$  and variance-covariance matrix  $\Omega_j^l$  with non-zero off-diagonal terms. Thus we have that:

$$\begin{bmatrix} \varepsilon_{t+1}^{C,l} \\ \varepsilon_{t+1}^{M,l} \end{bmatrix} \sim N(0, \rho^l) \quad \text{with} \quad \rho^l = \begin{bmatrix} 1 & \rho^{CM,l} \\ \rho^{CM,l} & 1 \end{bmatrix} \quad (20)$$

so that  $g_{t+1}^l \sim N(\mu_j^l, \Omega_j^l)$  with

$$g_{t+1}^l = \begin{bmatrix} \ln\left(\frac{C_{t+1}^l}{C_t^l}\right) \\ \ln\left(\frac{M_{t+1}^l}{M_t^l}\right) \end{bmatrix} \quad \text{and} \quad \Omega_j^l = \begin{bmatrix} (\omega_j^{C,l})^2 & \omega_j^{CM,l} \\ \omega_j^{CM,l} & (\omega_j^{M,l})^2 \end{bmatrix} \quad (21)$$

### 3.5 Data and Solution Method

The purpose of this paper is to attempt to explain excess returns in the Japanese and US equity markets, as well as the return on the USD/Yen exchange rate. We therefore require data on stock returns, spot and forward exchange rates, interest rates, as well as money supply and real consumption for both countries. In order to be able to make our results comparable to those of BHM, we try to obtain data similar to theirs and make use of some of their calibrated parameters.

We obtain monthly equity returns data for the US and Japan from Morgan Stanley Country Indexes and convert these to quarterly returns. Our monthly interest rate data for the US and Japan are LIBOR 90-day Eurodollar and Euroyen rates respectively and are obtained from the International Financial Statistics (IFS) data base. From the same source we also obtain end-of-period spot and 90-day forward dollar/yen

exchange rates.

With regards to the exogenous environment, we obtain quarterly consumption and money supply data from the OECD and divide by population to obtain per capita data. Our quarterly US consumption is defined to be real US consumption of non-durables and services. However, for lack of comparable data, we define Japanese consumption as the sum of real total government and private consumption. Naturally, this series is bound to be smoother than one describing only non-durables and services data for Japan. However, as pointed out by BHM, consumption data is, at any rate, only an approximation to endowment because it excludes locally produced goods that get exported to the rest of the world and includes imported goods. Finally, we use US and Japanese quarterly deseasonalised nominal M1 data to characterize money supplies.

Although our exogenous process parameters are estimated, a few other parameters in the model are calibrated. These are the parameters related to preferences and the transaction cost technology parameters. With regards to taste parameters we experiment with a range of possible values as given by the intervals:  $A \in \{1.0, 0.75, 0.5, 0.25\}$ ,  $\alpha \in \{-1, -2\}$ ,  $\delta = \{0.5, 0.8\}$  and  $\rho \in \{-1, -3, -6, -9\}$ . Finally, similar to BHM, we fix the subjective quarterly discount rate  $\beta$  to  $(0.96)^{0.25}$ .

To provide values for our transaction cost function parameters, we borrow the calibrated values for these from BHM. The authors calculated them by applying linear regression on the model's implications for money demand which were derived from equation (16). Thus, for the US, we set  $\gamma = 0.001$  and  $v = 4.35$ , while, for Japan,  $\zeta = 0.017$  and  $\xi = 2.01$ .

Next we describe a brief overview of our solution method of which the intuition is the following<sup>8</sup>. The model implies expressions for the first and second moments of asset returns. These can be written as functions of a few state-dependent endogenous variables. Given particular preference and estimated endowment process parameter values, we numerically solve for the values of these endogenous variables in the different states. Using these in conjunction with the laws of motion specified in the

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8. We refer the reader to appendix 3.B for the details.

endowment process we can then simulate different financial returns series. From here we obtain first and second moments which can subsequently be compared to those obtained from actual data for the same series.

In somewhat more detail, at first we estimate two different versions of two-state bivariate Markov switching processes for each of the US and Japan using maximum likelihood. The estimation period is 1975q1 to 1995q4. Model A is a two-mean, single variance model while, Model B also allows the variances to change with the state. At this stage, we do not try to describe the particular specification in this class of models that best fits the data. Rather, we are simply trying to assess whether introducing the simplest form of heteroskedasticity helps to improve the model fit. Tables 3.1 and 3.2 summarize the values of these estimates.

**Table 3.1: Bivariate two-state Markov Estimation Results for the US**

	<i>Model A</i> <i>Two means, one variance</i>		<i>Model B</i> <i>Two means, two variances</i>	
	State 1	State 2	State 1	State 2
$\mu_j^{C, US}$	0.5784 (9.80)	0.2560 (3.94)	0.5872 (10.96)	0.2532 (3.05)
$\mu_j^{M, US}$	0.8403 (6.86)	2.1759 (6.13)	0.8607 (5.65)	2.1016 (11.46)
$\omega_j^{C, US}$	0.3808 (15.31)		0.3380 (9.28)	0.4802 (9.38)
$\omega_j^{M, US}$	0.7823 (21.90)		0.9139 (8.60)	1.0895 (9.79)
$P_{11}^{US}$	0.838 (24.55)		0.877 (15.2)	
$P_{22}^{US}$	0.880 (13.04)		0.906 (17.7)	
$\rho^{US}$	0.779 (17.26)		0.715 (11.6)	
US llf	-11.5		2.4	

From these tables we can see<sup>9</sup> that the means of consumption and money supply

growths are quite different in the two states. Thus, average US consumption growth in state 1 is almost twice its value in state 2 whereas, in Japan, the reverse is true. Average money growth, however, exhibits an identical pattern in both countries. Notice that both models yield similar values for the means.

Turning now to the value of the maximized log likelihood function for these models, it is clear that model B, with the state-dependent variances, is the preferred model for both countries. More specifically, state 2 is the more volatile regime with Japan exhibiting a much greater difference in variances across the two states than the US. Overall, then, state 1 is characterized by low means and low variances, while state 2 is the opposite.

Table 3.2: Bivariate two-state Markov Estimation Results for Japan

	Model A Two means, one variance		Model B Two means, two variances	
	State 1	State 2	State 1	State 2
$\mu_{C,JAP}^f$	0.4713 (4.79)	0.9956 (8.26)	0.3021 (3.24)	0.8635 (7.24)
$\mu_{M,JAP}^f$	0.6040 (5.41)	2.6514 (22.66)	0.7864 (6.31)	1.7237 (7.90)
$\omega_{C,JAP}^f$	0.6547 (12.23)		0.3809 (5.52)	0.6413 (9.36)
$\omega_{M,JAP}^f$	0.7268 (24.34)		0.5333 (5.24)	1.5186 (9.89)
$P_{JAP}^{P11}$	0.662 (7.37)		0.916 (19.4)	
$P_{JAP}^{P22}$	0.768 (12.07)		0.733 (9.09)	
$\rho_{JAP}$	-0.0585 (0.03)		0.052 (0.44)	
JAP III	-111.8		-56.378	

These tables also report the values of the correlation coefficients between the

9. Although one cannot apply the usual t-test in these types of models, the large values of these statistics for most of the estimated parameters indicate that they may be significant.



growth rates of consumption and money supply for the US and Japan. In general, the dynamics are quite different for each country. Thus, in the case of the US, the growth rate in consumption and money exhibit a high degree of correlation over time. This is not the case for Japan where it would seem that the different variables evolve almost independently of each other.

It seems then that there are two quite distinct regimes with different means and variances for both countries. However, as mentioned above, it should be kept in mind that we have imposed a two state model here and that it would have been more rigorous to have carried out various specification tests in order to choose the appropriate number of states.

Substituting these estimated parameter values for their expressions in the equilibrium Euler equations, and selecting values for the preference parameters from the specified grids above, we solve for the values of the state-dependent endogenous variables. These are solved using numerical techniques. All asset returns can then be expressed as functions of these state-dependent endogenous variables. Thus, the market portfolio real return is given by:

$$R_{t+1} = \left( \frac{\lambda(S_{t+1}^t)}{\lambda(S_t^t) + 1} \right) \left( \frac{1 + \psi_x^I(S_{t+1}^t)}{1 + \psi_x^I(S_t^t)} \right) \exp(\mu_{C,x}(S_{t+1}^t) + \omega_{C,x}(S_{t+1}^t)) \varepsilon_{C,x}^{t+1} \quad (22)$$

where  $\lambda_t$  is ratio of aggregate invested wealth at time  $t$  to real consumption inclusive of transaction costs in the same period,  $\psi_x^I$  is the time  $t$  derivative of the cost function of country  $x$  with respect to its first argument,  $\mu_{C,x}^{t+1}$  is the mean of consumption in country  $x$  the time  $t+1$  and  $\omega_{C,x}^{t+1}$  is its standard deviation for the same time period.

As for real equity returns for the two countries, remember that we have assumed that each country's real dividend is a proportion of its consumption. For our empirical analysis we obtained an estimate of this proportion by regressing each country's real dividend series on consumption over our usual data span. For the US, this estimate is 0.2 while for Japan, it is 0.03. Consequently, real equity returns for country  $x$  (the US) and  $y$  (Japan) are respectively given by:

The substantial literature that exists in the field of asset pricing, and, more generally, of international asset pricing, has noted that excess returns are predictable in almost all types of assets and for many different countries<sup>10</sup>. Moreover, when continuously compounded excess equity returns or excess forward returns are regressed

model's performance.

Let us briefly summarize some relevant stylized facts that will help us assess the

### 3.6 Stylized Facts

Accordingly, given the laws of motion specified in the endowment process, we can simulate these various returns and calculate moments.

$$(26) \quad \frac{S_t}{1+t_x} = \frac{F_t - S_t}{1+t_y}$$

condition. That is

while the forward premium can be obtained from the covered interest rate parity

$$(25) \quad \left( \frac{1 + \psi_x^2(S_t^t)}{1} \right)^{t_x} = \left( \frac{1 + \psi_y^2(S_t^t)}{1} \right)^{t_y} \quad \text{and} \quad \left( \frac{1 + \psi_x^2(S_t^t)}{1} \right)^{t_x} = \left( \frac{1 + \psi_y^2(S_t^t)}{1} \right)^{t_y}$$

as

consumptions. Finally, the nominal risk-free rates, simply compounded, can be written where  $\phi_x^t$  and  $\phi_y^t$  are the respective ratios of country equity prices to their

$$(24) \quad R_{y,t+1} = \left( \frac{\phi_y^t(S_t^t)}{\phi_y^t(S_{t+1}^t) + 0.03} \right) \left( \frac{1 + \psi_x^2(S_t^t)}{1 + \psi_x^2(S_{t+1}^t)} \right) \exp(\mu_{C,x}(S_{t+1}^t) + \omega_{C,x}(S_{t+1}^t)) e_{C,x}^{t+1}$$

and

$$(23) \quad R_{x,t+1} = \left( \frac{\phi_x^t(S_t^t)}{\phi_x^t(S_{t+1}^t) + 0.2} \right) \left( \frac{1 + \psi_x^2(S_t^t)}{1 + \psi_x^2(S_{t+1}^t)} \right) \exp(\mu_{C,x}(S_{t+1}^t) + \omega_{C,x}(S_{t+1}^t)) e_{C,x}^{t+1}$$

The period for which we calculate our stylized facts extends from 1983q1 to 1994q2. This time span was chosen so as to exclude the transition period of 1978-1982 during which the monetary authorities changed their targeted policy variable. Indeed, during this period, the growth of money supply was a policy instrument for most countries and was kept relatively smooth. On the other hand, interest rates were left to fluctuate freely and to reflect nominal shocks in the economy. Since, in our theoretical

All figures are annualized quarterly returns in percentage..

Series	mean	std. error
$R_{US}^{t+1}$	14.996	25.386
$R_{JAP}^{t+1}$	12.700	35.970
$R_{US}^t$	7.287	2.391
$R_{JAP}^t$	5.459	1.623
$R_{US}^{t+1} - R_{US}^t$	7.709	25.374
$R_{JAP}^{t+1} - R_{JAP}^t$	7.241	36.3612
$(F^t - S^t) / S^t$	1.908	2.112
$(S^{t+1} - S^t) / S^t$	9.515	25.520
$(S^{t+1} - F^t) / S^t$	7.612	25.928

Table 3.3: Descriptive Statistics on sample data

on a forward premium of that country, the slope coefficient is normally negative. These facts are not consistent with the joint hypotheses of market efficiency, rational expectations and risk neutrality. However, if agents are assumed to be rational but not risk neutral, they will demand a premium for incurring additional risk in these markets. In this case, predictable excess returns can be interpreted as being equal to the risk premium in that market.

Define the forward premium as  $fp^t = (F^t - S^t) / S^t$ , with  $F^t$  as the dollar/yen 90-day forward rate and  $S^t$  as the dollar/yen spot exchange rate. Excess equity returns for the US and Japan are given respectively by  $R_{US}^{t+1} - R_{US}^t$  and  $R_{JAP}^{t+1} - R_{JAP}^t$ . Similarly, excess forward returns are defined as  $(S^{t+1} - F^t) / S^t$ .

11. It is important to note that, when compared to statistics calculated over a much larger time span, our returns are substantially higher. Thus, for example, the mean of the 3-month US interest rate over a hundred year period has been reported in some studies to be around 1%, whereas it is approximately 7% in our data. Similarly, the mean of the 3-month Japanese interest rate is generally less than one percent annually, compared with 5.5% in our data sample. Equity returns reported here are also typically higher than when calculated over a longer time period. These facts indicate that our particular sample is representative of a rather exceptional set of economic circumstances. This should be kept in mind in comparing model results to these statistics.

Dependent Variable	Constant $c$	Slope Coeff. $b$	$R^2$	Std. err of Dep. var	Std. Estimate
$R_{US}^{t+1} - r_t^{US}$	10.151	-1.280	0.114	25.374	25.514
$R_{JAP}^{t+1} - r_t^{JAP}$	-2.607	5.161	0.090	36.361	35.080
$(S_{t+1} - F^t) / S_t$	13.400	-3.033	0.061	25.928	25.407

Table 3.4: Regression Results with actual data

Table 3.4 reports the results of the regressions of excess returns on a constant and the forward premium. As expected, regression R-squares of 6 per cent and more indicate some predictability in these returns. Notice also that the coefficient on the forward premium is generally negative. The results are somewhat different for Japan. However our regressions for this country are in concordance with most empirical work which suggests that the slope is positive and that the  $R^2$  is lower than for other country excess returns.

Table 3.3 contains some descriptive statistics on our chosen sample data. From here we can see that the means of the various excess returns are of the order of 7%. When these are then regressed on different financial instruments, the fitted value can be interpreted as the risk premium associated with these markets.<sup>11</sup>

the dynamics of assets during this 1978-1982 period. This would have made it very difficult for the model to match the moments and generated extremely small fluctuations in the different nominal measures for this transmitted into the economy via the quantity equation, our model would have model, all nominal innovations are generated in the money supply process and get

### 3.7 Model Results

The purpose of our model is to explain the behavior of excess and nominal returns in different markets, notably, the equity, the risk-free and the exchange rate markets. This is a considerable task since, not only are we attempting to explain the different levels of the variables concerned, but we are also concerned with the dynamic interactions between different financial variables.

Typically, previous studies in international finance were predominantly concerned with explaining excess return predictability. This meant that, as long as the moments of excess returns were reproduced, we did not care if these models could match the means and variances of the different nominal returns series. However, a good model should be able to explain most of the major empirical facts it is confronted with. As Weil (1989) has pointed out, it is wrong to focus only on excess returns because, in conjunction with the risk premium puzzle, there exists the risk-free rate puzzle. That is, for reasonable values of the coefficient of relative risk aversion, the expected utility model's predicted value for the risk-free rate will substantially surpass the level observed in the sample. Thus, separating the concepts of aversion to risk and of intertemporal substitution, and integrating heteroskedasticity in the exogenous process of the model will have consequences for both the risk premium as well as the risk-free rate. We therefore feel that, in order to understand the full usefulness of an international asset-pricing model, it is necessary to examine its implications for the resolution of both puzzles.

The drawback is that such an undertaking requires us to examine a very large number of statistics and to indulge in a considerable amount of scrutiny and analysis. Therefore, so as to get an overall sense of the strengths and weaknesses of our model and not to get entangled in details, we focus on two general measures. First, we examine the extent to which our generated forward premium can explain the dynamic facts of various generated excess returns with respect to predictability. This criterion should mostly capture the dynamic dimension of our model. Second, we compare moments of nominal returns obtained from the model to those documented in Table 3 above. In this case, we are mainly concerned with matching levels.

### 3. 7. 1 Implications of the Model for Excess Return Predictability

In order to assess the goodness of fit of our model through time we examine the extent to which the model can predict excess returns. These results also permit us to gauge our model performance relative to others. This will be discussed in detail in section 3.7.3.

For a given combination of preference parameters and for each of the returns of interest, we generate 200 series of 63 observations<sup>12</sup> with our model. We then perform 200 regressions of each of the different excess returns on the forward premium. From these we collect various statistics, notably, the constant, the slope and the R-squared value, and calculate their distributions. We then report the median values of these distributions for particular parameter combinations in Tables 3.5-3.7 below<sup>13</sup> and compare these to the corresponding statistics tabulated in Table 3.4. In addition to the above, and to give a better picture of the model outcomes, we also report the p-value of the sample data statistic in the model-generated statistic distributions.

The most striking conclusion that emerges from these tables is that for no one parameter combination does the model perform well for all the assets simultaneously. Similarly, it is not generally the case that the two-variance endowment specification is better than the single-variance case. This implies that the interaction between preference and endowment parameters is nonlinear and that a particular asset-pricing model cannot explain excess return variations in different markets. Nevertheless, most of the results are encouraging.

Consider Table 3.5 which documents the regression outcomes for the US excess equity returns. The particular parameter combination that comes closest to reproducing regression statistics close to the ones obtained in the sample data is the case where  $\alpha = -1$ ,  $\rho = -3$ ,  $\delta = 0.5$ , and  $A = 1$ , and where a single-variance specification is assumed for the endowment processes. While the generated-data slope is of the wrong sign, the p-value (45.5%) of the actual data regression slope in the distribution of slopes

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12. It would have been better to have generated 1000 series instead of the 200 we use. But, since we find that results do not differ much when the number of series is increased, we keep this number at 200. The number of observations corresponds to that in our actual sample data.

13. We refer the reader to Appendix 3.C for the full set of results.

indicates that the median is quite close to the value of -1.28. Similarly, the median value of the constant is quite close to the actual-data estimated constant of 10.151 since the latter is situated at the 59th percentile in the distribution of generated-data regression constants.

Table 3.5: Selected Results for US Excess Equity Returns; Median of Distribution & P-values

	Model A		Model B	
	A=1	A=0.5	A=1	A=0.5
$\alpha = -1$	5.013	6.417	2.183	0.377
$p = -3$	(68.5)	(66.5)	(88.5)	(99.5)
$\delta = 0.8$	2.887	2.147	10.352	11.453
	(25.5)	(28.5)	(3.5)	(-)
R-squared	0.0089	0.0068	0.0035	0.0037
	(99.5)	(-)	(-)	(-)
$\alpha = -1$	7.711	5.634	1.563	5.614
$p = -3$	(59.0)	(73.0)	(70.5)	(61.5)
$\delta = 0.5$	0.269	2.631	16.216	8.700
	(45.5)	(33.5)	(29.0)	(35.5)
R-squared	0.0079	0.0097	0.0045	0.0058
	(99.0)	(98.0)	(-)	(99.5)
$\alpha = -1$	-8.263	-1.737	15.452	15.502
$p = -9$	(77.0)	(72.5)	(40.0)	(39.5)
$\delta = 0.8$	5.279	4.640	5.662	5.274
	(85.5)	(87.0)	(84.0)	(85.0)
R-squared	0.0505	0.0455	0.0558	0.0507
	(-)	(0.5)	(1.0)	(1.5)
b				
c				

Model A is the two-mean, one-variance model while Model B is the two-mean, two-variance specification. The reported medians for c, b, and R-squared are the respective medians of the distributions of c, b, and R-squared obtained from the 200 regressions. The values in parentheses are the p-values of the actual data statistic in the relevant generated distribution.

However, while this particular model-generated forward premium is able to explain fairly well the mean of the US excess returns over the time period considered, its variability relative to that of the excess returns is substantially lower than that indicated by the actual-data regression R-squared. That is, the median of the distribution of the generated-data regression R-squared is only 0.0079 relative to the 0.114 value obtained in the sample regression. Incidentally, the sample R-squared stands at the 99th percentile in the R-squared distribution.

With regard to the role of preference parameters, a few comments are in order.

First, it is interesting to note that in the above case, Krepes-Porteus type preferences ( $\alpha \neq \rho$  and  $A = 1$ ) perform better than disappointment aversion preferences ( $\alpha \neq \rho$  and  $A > 1$ ). Thus, when  $A = 0.5$ , and the remaining parameters are unchanged, the regression statistics are generally further from the sample-data estimated statistics. However, this is not a fact that is consistent across other parameter combinations. For example, for  $\rho = -9$  and  $\delta = 0.8$ , the results somewhat improve going from  $A = 1$  to the case of  $A = 0.5$ , regardless of the endowment process. Second, when  $\delta = 0.8$ , substitutability of present utility relative to the certainty equivalent of future utility, does worse for the single-variance endowment case but does better when the second Markov specification (Model B) is assumed. Third, unlike the case in BHM, varying the aggregation parameter  $\delta$  in the composite good seems to have a considerable effect on the regression outcomes. While it is very hard to say what this parameter value should be theoretically, looking at the first and second rows of Table 3 we can see that results are better when  $\delta = 0.5$  regardless of the endowment process chosen.

As for the role of the endowment processes, it is interesting to note that except for the last row of the Table, results generally deteriorate when moving from the Model A specification to the Model B one. This indicates that there is sufficient variability in the model already so that, unlike what some studies claim, more variability in the endowment process is not called for.

Table 3.6 documents results for the case of the Japanese excess return regression statistics. Interestingly, the parameter combination which exhibited the best results for the previous case (that is,  $\alpha = -1$ ,  $\rho = -3$ ,  $\delta = 0.5$ ,  $A = 1$ ) turns out to yield the best results for the Japanese case as well. More specifically, the model-generated median value for the constant (-2.838), as well as the R-squared (0.097), are remarkably close to the actual-data regression constant and the R-squared values in Table 4. The median of the generated-data regression slopes, however, is not so close as the other statistics. With a value of 16.297 it is quite different from the 5.161 value that we would like to have replicated. In fact, the latter places at only the 3rd percentile in the distribution of slopes for this parameter combination. The best result for the slope is



obtained for the same parameter combination but for  $\delta = 0.8$  and with Model B endowment specifications. However, in this case, the rest of the statistics are very different from the desired values.

Table 3.6: Selected Results for Japanese Excess Equity Returns; Median of Distribution & P-values

	Model A		Model B	
	A=1	A=0.5	A=1	A=0.5
$\alpha = -1$	-10.705	-9.198	18.680	2.25
$\rho = -3$	(86.5)	(86.0)	(1.0)	(-)
$\delta = 0.8$	16.555	14.522	3.840	14.929
	(1.0)	(2.0)	(56.5)	(-)
R-squared	0.10785	0.0961	0.0005	0.00372
	(41.5)	(44.0)	(-)	(-)
$\alpha = -1$	-2.838	-3.264	-16.409	-17.207
$\rho = -3$	(52.5)	(61.5)	(93.0)	(93.5)
$\delta = 0.5$	16.297	16.888	55.426	52.054
	(3.0)	(3.5)	(0.5)	(2.0)
R-squared	0.0970	0.1106	0.0369	0.0568
	(45.5)	(36.0)	(93.5)	(75.0)
$\alpha = -2$	-6.211	-5.119	43.893	30.166
$\rho = -3$	(74.5)	(64.0)	(0.5)	(2.0)
$\delta = 0.8$	17.156	15.513	-41.089	-15.620
	(4.0)	(4.5)	(95.5)	(83.0)
R-squared	0.1002	0.0967	0.0275	0.0030
	(42.0)	(45.5)	(95.5)	(-)

In the case of Japan, changing from Kreps-Porteus preferences to disappointment aversion does not alter model results much when Model A endowment processes are assumed. Nevertheless, with Model B processes and for  $\delta = 0.8$ , all the statistics seem to be affected when the type of preferences changes. In addition, when  $\alpha$  passes from -1 to -2, thereby increasing the representative agent's aversion to risk, with Model B endowments, results change substantially. This change is even more dramatic when  $A = 1$ .

Another aspect of preferences that merits some discussion is the value assumed for  $\delta$ . This parameter affects mainly the constant in the case of Model A, but in the case of Model B, it influences the outcomes for all the statistics.

Comparing the results across different endowment processes, we can see that inducing more variability in this leads to important changes in results. For example, in row one, we can see that for  $A = 1$  the value of the constant increases from -10.705 to 18.68, the slope decreases from 16.297 to 3.84, and the R-squared drops from 0.1 to 0.0005. Except for the slope value, the changes are equally important for the case of  $A = 0.5$ . These changes are even more dramatic when the coefficient of relative risk aversion is increased.

In summary, then, similar to the US case, the single-variance endowment specification, in conjunction with Kreps-Porteus preferences, works better in having the model-generated forward premium explain the model-generated Japanese excess returns.

We turn now to the results for the foreign exchange excess returns which are tabulated in Table 3.7. Unlike the previous excess returns, the best outcomes for this case are obtained when the Model B endowment specification is used. More specifically, while it is difficult to distinguish between the better type of preferences, the case where  $\alpha = -1$ ,  $\rho = -9$  and  $\delta = 0.5$  seems to yield the best results.

For the case of disappointment aversion preferences, the value of the model-generated constant stands at 8.263 compared with 13.4 in the sample-data regression constant value. In the case of the slope, the model yields -2.44 which is quite close to the negative 3.033 obtained from the sample data regression. Finally, the model R-squared is an impressive 0.1291 relative to the lower bound of 0.061 provided by the sample data regression fit. Interestingly, however, and despite the closeness of the values obtained to the desired quantities, the p-values of the sample-data statistics in the respective model-generated distributions are not in the centers of these density functions.

Concerning the preference structure, comparing the first and the third rows of Table 3.7 we can see that decreasing the value of  $p$  strongly affects the regression statistics. In fact, not only do the constants increase in value and the slope values decrease, but the R-squared values move from practically zero to values close to 0.5. This indicates that it is not attitudes towards risk that play a pivotal role in improving the dynamic relation between the forward premium and excess returns, but rather, the degree of intertemporal substitutability of goods across different time periods. Indeed, looking at Tables 50-53 in Appendix C, changing the value of  $\alpha$  from -1 to -2 changes results only slightly. More interestingly, the adopted endowment process plays a very important role for this particular market. Thus, for the same set of preference parameter values, the two-variance endowment specification always yields better results than the single-variance case.

	Model A		Model B	
	A=1	A=0.5	A=1	A=0.5
$\alpha = -1$	-2.286	-1.814	3.737	0.996
$p = -3$	(-)	(-)	(97.5)	(99.0)
$\delta = 0.8$	1.507	1.188	1.080	1.197
	(1.5)	(0.5)	(11.5)	(-)
R-squared	0.0160	0.0110	0.0008	0.0004
	(91.5)	(95.0)	(-)	(-)
$\alpha = -1$	4.368	4.416	8.063	8.263
$p = -9$	(96.5)	(97.0)	(87.5)	(90.0)
$\delta = 0.5$	-1.884	-1.900	-2.391	-2.440
	(11.5)	(9.0)	(28.0)	(21.0)
R-squared	0.0495	0.0527	0.1080	0.1291
	(63.0)	(49.0)	(29.5)	(22.5)
$\alpha = -1$	3.495	3.590	6.538	6.311
$p = -9$	(99.0)	(-)	(93.0)	(95.5)
$\delta = 0.8$	-1.193	-1.203	-1.285	-1.270
	(-)	(-)	(0.5)	(-)
R-squared	0.5322	0.5237	0.4762	0.5048
	(0.5)	(-)	(4.5)	(3.0)

Table 3.7: Selected Results for Excess Foreign Exchange Returns; Median of Distribution & P-values

In summary, results in this section show the complex interplay of preference and endowment process parameters. We have shown that, in general, outcomes are more sensitive, by order of importance, to the assumed degree of intertemporal substitution (as determined by  $\rho$ ) to the choice of the aggregation parameter  $\delta$ , and to attitudes towards risk (as captured by the parameter  $\alpha$ ). These parameters seem to have a stronger influence on the results than do the type of preferences adopted. Furthermore, their effect becomes even stronger when more variability is assumed in the endowment process.

We have also shown that, while a particular parameter combination yields satisfactory results for the dynamics of excess equity market regressions, another combination of parameters is preferred for that of the excess foreign exchange market case. In particular, the latter requires a smaller elasticity of substitution (and, therefore, a more negative value of  $\rho$ ) in order to improve the results. Similarly, while Model A preferences seem to produce better results for equity markets, the foreign exchange market regressions require the two-variance specification to yield better outcomes. This indicates that perhaps assumptions that influence specifically the exchange rate in the model need to be re-examined. Section 3.7.3 will provide some leads into those assumptions that might need to be modified.

So far, we put the emphasis on assessing the usefulness of our model for explaining excess returns and we have concluded that our model has some merits. But how does it perform with respect to matching moments for nominal returns? This is the topic of the next section.

### 3.7.2 Implications of the Model for Nominal Returns

A second way of evaluating our model performance is by comparing model-obtained nominal return moments to those obtained from the sample data. That is, we are trying to see whether our model can address the risk premium and the risk-free rate puzzles simultaneously.

Thus, for a particular parameter combination, and for each of the 200 generated

series, we compute the mean and the standard error. We then calculate the 'empirical' distribution of the means and that of the standard errors and check whether their medians are close to the statistics obtained from the sample data. In addition, to give a better sense of the model performance, we report the p-values of the actual sample data moments relative to the generated empirical distributions. If the model is a good representation of reality, the actual sample data mean and standard deviation should fall somewhere in these generated empirical distributions. Tables 3.8-3.11 below show the medians and p-values, of the generated distributions.

Overall, while results are mostly satisfactory, no one preference and endowment combination can closely reproduce most of the means and the standard deviations observed in the sample data. In particular, magnitudes of the mean and the standard deviation of the Japanese interest rate, the growth rate of the exchange rate (DELS in the tables) and the foreign exchange excess returns (xsFX in the tables) are never attained. As we saw in section 3.7.1, here too, the combination that gives generally good results for the excess equity returns undervalues the moments for foreign exchange returns. Similarly, the model that performs well for the latter yields much higher values for the returns in the equity markets than those calculated from the sample data.

Having said this, let us now turn to Tables 3.8 and 3.9 and discuss the results in more detail. These tables report the medians of the distributions of the mean and the standard deviation respectively for each of the different returns. To facilitate the exposition, those values that come closest to the ones reported in Table 3.3 have been highlighted in bold letters.

If we were to look only at excess equity returns, we would conclude that the preferred parameter and endowment combination in the regression experiments also yields the desired means of these returns. This is the case even with a coefficient of relative risk aversion of 2. Thus, simply disjoining attitudes towards risk and aspects of intertemporal substitutability, and making use of the appropriate value of the aggregation parameter is already sufficient to produce this result.

Table 3.8: Medians of Distributed Means of Returns

	Model A		Model B	
	A=1	A=0.5	A=1	A=0.5
$\alpha = -1$				
$\rho = -3$				
$\delta = 0.8$				
RUS	15.084	15.058	18.293	17.760
RJAP	15.161	15.115	25.104	23.970
IUS	5.026	5.060	3.983	4.077
IJAP	1.389	1.390	1.374	1.375
xSRUS	10.069	10.058	14.305	13.675
xSRJAP	13.770	13.725	23.730	22.595
FP	1.507	1.521	1.088	1.127
DELS	1.565	1.520	3.845	3.795
xsFX	0.087	0.027	2.749	2.667
$\alpha = -1$				
$\rho = -3$				
$\delta = 0.5$				
RUS	10.899	10.482	13.813	13.921
RJAP	8.838	8.810	14.251	14.120
IUS	2.750	2.815	2.522	2.646
IJAP	1.331	1.335	1.318	1.321
xSRUS	8.068	7.634	11.295	11.266
xSRJAP	7.507	7.473	12.934	12.798
FP	0.604	0.629	0.515	0.565
DELS	1.692	1.567	3.775	3.848
xsFX	1.071	0.924	3.253	3.289
$\alpha = -1$				
$\rho = -9$				
$\delta = 0.8$				
RUS	75.329	76.651	94.354	94.074
RJAP	95.171	97.829	233.391	236.978
IUS	27.465	27.987	24.549	24.720
IJAP	1.443	1.443	1.444	1.444
xSRUS	47.638	49.338	69.015	67.901
xSRJAP	93.727	96.384	231.949	235.534
FP	10.550	10.763	9.363	9.429
DELS	1.577	1.558	3.884	3.753
xsFX	-8.954	-9.094	-5.583	-5.326
$\alpha = -2$				
$\rho = -9$				
$\delta = 0.5$				
RUS	22.774	22.170	28.741	28.129
RJAP	24.960	24.375	52.370	50.814
IUS	6.921	7.187	5.993	6.484
IJAP	1.408	1.410	1.409	1.411
xSRUS	15.802	14.980	22.731	21.710
xSRJAP	23.551	22.964	50.960	49.405
FP	2.265	2.375	1.893	2.082
DELS	1.671	1.519	4.140	3.537
xsFX	-0.607	-0.862	2.294	1.611

RUS and RJAP denote returns in the US and Japanese equity markets while xSRUS and xSRJAP are the excess returns in these markets. IUS and IJAP are the respective nominal interest rates. FP is the forward premium, DELS is the growth rate of the exchange rate and xsFX is the excess foreign exchange return.

Table 3.9: Medians of Distributed Standard Deviations of Returns

	Model A	Model B
$\alpha = -1$	$A=1$	$A=0.5$
$\alpha = -1$	RUS	27.418
$\alpha = -1$	RJAP	27.805
$\alpha = -1$	!US	37.602
$\alpha = -1$	!JAP	37.602
$\alpha = -1$	!US	0.518
$\alpha = -1$	!JAP	0.176
$\alpha = -1$	xSRUS	27.766
$\alpha = -1$	xSRJAP	27.766
$\alpha = -1$	FP	37.586
$\alpha = -1$	DELS	0.144
$\alpha = -1$	xSFX	8.721
$\alpha = -1$	RUS	17.626
$\alpha = -1$	RJAP	17.486
$\alpha = -1$	!US	0.972
$\alpha = -1$	!JAP	0.178
$\alpha = -1$	xSRUS	17.511
$\alpha = -1$	xSRJAP	17.468
$\alpha = -1$	FP	0.360
$\alpha = -1$	DELS	4.951
$\alpha = -1$	xSFX	4.921
$\alpha = -1$	RUS	15.389
$\alpha = -1$	RJAP	10.439
$\alpha = -1$	!US	0.557
$\alpha = -1$	!JAP	0.172
$\alpha = -1$	xSRUS	15.380
$\alpha = -1$	xSRJAP	10.426
$\alpha = -1$	FP	0.201
$\alpha = -1$	DELS	5.077
$\alpha = -1$	xSFX	5.059
$\alpha = -1$	RUS	98.751
$\alpha = -1$	RJAP	146.480
$\alpha = -1$	!US	11.033
$\alpha = -1$	!JAP	0.185
$\alpha = -1$	xSRUS	95.289
$\alpha = -1$	xSRJAP	146.457
$\alpha = -1$	FP	4.447
$\alpha = -1$	DELS	4.857
$\alpha = -1$	xSFX	6.937
$\alpha = -1$	RUS	27.203
$\alpha = -1$	RJAP	32.155
$\alpha = -1$	!US	1.185
$\alpha = -1$	!JAP	0.181
$\alpha = -1$	xSRUS	26.972
$\alpha = -1$	xSRJAP	32.140
$\alpha = -1$	FP	0.431
$\alpha = -1$	DELS	4.796
$\alpha = -1$	xSFX	4.842
$\alpha = -2$	RUS	50.316
$\alpha = -2$	RJAP	83.679
$\alpha = -2$	!US	2.575
$\alpha = -2$	!JAP	0.182
$\alpha = -2$	xSRUS	49.752
$\alpha = -2$	xSRJAP	83.656
$\alpha = -2$	FP	1.036
$\alpha = -2$	DELS	9.365
$\alpha = -2$	xSFX	9.673
$\alpha = -1$	RUS	144.328
$\alpha = -1$	RJAP	379.710
$\alpha = -1$	!US	16.864
$\alpha = -1$	!JAP	0.185
$\alpha = -1$	xSRUS	141.725
$\alpha = -1$	xSRJAP	379.692
$\alpha = -1$	FP	6.813
$\alpha = -1$	DELS	8.417
$\alpha = -1$	xSFX	11.796

RUS and RJAP denote returns in the US and Japanese equity markets while xSRUS and xSRJAP are the excess returns in these markets. !US and !JAP are the respective nominal interest rates. FP is the forward premium. DELS is the growth rate of the exchange rate and xSFX is the excess foreign exchange return.

Looking at Tables 3.10 and 3.11, it is clear that no particular parameter combination generates nominal returns moment distributions that contain the corresponding sample statistic for all the returns simultaneously. Similarly, one can easily discern the existence of the dichotomy between those combinations of parameters that yield good results for nominal equity returns on the one hand and those distributions.

The above facts are confirmed when one examines the p-values of sample data statistics for all the above nominal returns in their respective model-generated distributions. The above facts are confirmed when one examines the p-values of sample data statistics for all the above nominal returns in their respective model-generated distributions.

not obtain good outcomes for the risk-free rates, specially for Japan. regression experiments (see Tables 15-16 and 24-25 in the Appendix). Finally, we do parameter and endowment combination in the foreign exchange excess returns foreign exchange market. The latter is the case even when we use the preferred do not quite match their variability. We are also unable to produce good statistics for the countries considered and are able to reproduce fairly adequately their predictability, we Thus, in essence, while we match the means of excess equity returns for the parameter and endowment combination in the foreign exchange excess returns foreign exchange market. The latter is the case even when we use the preferred do not quite match their variability. We are also unable to produce good statistics for the countries considered and are able to reproduce fairly adequately their predictability, we

3-6 times lower than in the data. The statistics for the remaining elements of the foreign exchange market are also these returns, which are 7 and 5 times smaller than their respective values in the sample The model produces a mean of only 1% and a standard deviation of just 5% for data. The statistics for the remaining elements of the foreign exchange market are also these returns, which are 7 and 5 times smaller than their respective values in the sample

also not particularly good. respectively. Finally, results for the excess returns in the foreign exchange market are returns, they are about 4 and 9 times smaller for the US and Japanese interest rates these are approximately half to one third of their values for excess and nominal equity combination also undervaluates the standard deviations of the series of interest. While sample data. In addition, from Table 3.9, we can see that this same parameter are of the order of 2.8% and 1.3% compared with 7.3% and 5.5% respectively in the the data. Similarly, the means of US and Japanese interest rates generated by the model Japanese equity returns stand at 10.9% and 8.8% compared with the 15% and 12.7% in the means of the equity and interest rate returns. For instance, the means of the US and On the negative side, however, this same scenario produces low magnitudes for



that do a little better in the exchange market. Again, these are simply more formal confirmations of the conclusions reached earlier.

Table 3.10: P-values of Medians of Distributed Means of Returns

	Model A	Model B
	A=1	A=0.5
$\alpha = -1$	RUS 47.5	48.5
$\rho = -3$	RJAP 20.0	25.0
$\delta = 0.8$	IUS -	-
	IJAP -	-
	FP -	95.0
	DELS -	-
$\alpha = -1$	RUS 97.0	97.5
$\rho = -3$	RJAP 97.0	96.5
$\delta = 0.5$	IUS -	-
	IJAP -	-
	FP -	98.5
	DELS -	-
$\alpha = -1$	RUS -	-
$\rho = -9$	RJAP -	-
$\delta = 0.8$	IUS -	-
	IJAP -	-
	FP -	96.5
	DELS -	-
$\alpha = -2$	RUS 0.5	1.5
$\rho = -9$	RJAP -	2.0
$\delta = 0.5$	IUS 92.0	66.0
	IJAP -	-
	FP -	-
	DELS -	51.0
		99.0
		98.5
		85.5
		34.5

For definitions of variables, see the notes for Table 3.9.

On the whole, results from this section reveal the following generally consistent facts: 1) equity return statistics are closer to their sample moments when  $p = -3$  while results are better for the foreign exchange market with a lower degree of intertemporal substitutability, 2) Model B endowment processes produce more satisfactory outcomes for the moments of the foreign exchange market and the standard deviations of equity markets, while Model A seems better for the means of nominal and excess equity markets, and 3) it is difficult to state that the type of preferences chosen makes a big difference to the results.

	Model A					Model B				
	A=1					A=0.5				
	RUS	RJAP	IUS	IJAP	FP	RUS	RJAP	IUS	IJAP	FP
$\alpha = -1$	86.0	97.0	-	-	-	88.5	97.0	-	-	-
$\delta = 0.8$	43.5	56.0	-	-	-	44.0	46.0	-	-	-
$p = -3$	43.5	56.0	-	-	-	44.0	46.0	-	-	-
$\alpha = -1$	89.5	99.5	-	-	-	95.5	99.5	-	-	-
$\delta = 0.5$	53.5	84.0	-	-	-	57.0	83.5	-	-	-
$p = -3$	53.5	84.0	-	-	-	57.0	83.5	-	-	-
$\alpha = -1$	-	-	-	-	-	-	-	-	-	-
$\delta = 0.8$	-	-	-	-	-	-	-	-	-	-
$p = -9$	-	-	-	-	-	-	-	-	-	-
$\alpha = -1$	34.5	59.5	-	-	-	40.0	65.0	-	-	-
$\delta = 0.5$	9.0	28.5	-	-	-	7.0	26.5	-	-	-
$p = -9$	9.0	28.5	-	-	-	7.0	26.5	-	-	-
DELS	-	-	-	-	-	-	-	-	-	-

Table 3.11: P-values of Medians of Distributed Standard Deviations of Returns

### 3. 7. 3 Model Performance Relative to BHM and Discussion

With respect to excess returns predictability our model performs much better than BHM. For all parameter combinations considered, and for regressions similar to ours, these authors obtain only negligible R-squared values. In addition, their model-generated slope coefficients are much smaller in magnitude than those obtained with their sample data. While they show that the variances of ex ante risk premiums do increase as first-order risk aversion increases (that is, as  $A$  decreases), given that the model-generated slopes are non-monotonic in  $A$ , this does not necessarily lead to increases in the slope magnitudes. On the basis of these results, these authors conclude that a risk-based explanation for the predictability of excess asset returns is not sufficient.

Based on our own results we concur with BHM in saying that simply increasing the degree of aversion to small lotteries is insufficient to explain the predictability of excess returns. Indeed, this dimension of preferences seems to be the least important factor in enhancing the model performance. Intertemporal substitutability, overall attitudes towards risk, the extent of goods' aggregation and the amount of heteroskedasticity in the exogenous environment all contribute, in a non-linear way, to the outcomes of the model. We were able to get these additional insights into the workings of the international asset-pricing model by following a different strategy than BHM. By adopting Markov-switching endowments in our model, integrating these in our Euler equations and obtaining full analytical forms for these first-order conditions, we were able to shed some light on the respective roles of more parameters in the model than those considered by BHM. Thus, we were better able to understand the role of the aggregation parameter  $\delta$ , that of the general aversion parameter  $\alpha$ , as well as the interactions between these and other preference and endowment parameters. Finally, the added dimension that our model has relative to BHM with respect to the role of increased heteroskedasticity in the endowment process has allowed us to confirm that increased variability in these is not always desirable.

Most importantly, we have shown that while it is possible to obtain satisfactory results with respect to the predictability and the matching of first moments of excess

equity returns with a specific combination of preference and endowment parameter values, model assumptions need to be modified before similar results can be obtained for excess foreign exchange returns. In our view, a crucial assumption in the model is that of purchasing power parity (PPP). This determines the level of the exchange rate in the model. Yet, numerous empirical studies have shown that this parity does not hold in the data for relatively short spans like ours. If PPP did not hold, agents in the different countries would evaluate their consumption bundles differently because of the varying relative prices of goods in their respective countries. This would result in a different value for the exchange rate, and therefore of excess foreign exchange returns. In turn, this has implications for the means of the growth rate of the exchange rate and of the excess foreign exchange returns.

However, our results also indicate that the variability of the exchange rate in our model needs to be substantially increased. While this is somewhat achieved by increasing the amount of heteroskedasticity in the exogenous process, it is still not enough to match the amounts observed in the data. We feel that there are three explanations for this. First, it may well be the case that a three-state representation for the endowment process would yield better results than the two-state specification. This is bound to increase the variability of the money growth processes, and, therefore, of all the nominal returns. Second, even if the joint money and consumption growth process is better characterized by a three-state time-series, it remains that all the shocks that are driving the returns do not necessarily originate from these two sources. Fiscal shocks may be just as important in shaping the behavior of returns. For the moment, there is no fiscal side to our two-country economies. Yet, the study by Canova and Marrinan (1995) has shown that variation in the volatility of fiscal aggregates in their model substantially contributes to generating variability in their simulated returns series. Third, there is evidence<sup>14</sup> that the premium in the exchange market is quite distinct from that in equity markets. This implies that the various types of structural uncertainties in the model apply to these markets in different ways. For the moment, our model evaluates all returns in the same fashion, through a rich, time-varying, but identical, stochastic discount rate. Perhaps, a heterogeneous agent model is needed to

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14. See chapter one of this thesis.

differently evaluate these returns and to create the desired risk premium in the exchange market. For instance, in a habit-formation type model, agents in the two countries can be made to have different attitudes towards risk. This could be accomplished by specifying state and country-dependent coefficients of relative risk aversion.

### 3. 8 Conclusion

In this paper we use Markov-switching endowment processes in the two-country transaction cost model of Bekaert, Hodrick and Marshall (1997) with disappointment aversion type preferences. We analytically integrate our assumed processes in the Euler equations of the model, solve these numerically for the various shadow prices and the endogeneous money velocities, and using these, simulate series of financial returns for various assets.

We show that many factors, interacting in a non-linear fashion, need to be united in order for an international asset-pricing model of this type to yield good results. By good results we mean that the model can approximately reproduce, on the one hand, the observed predictability of excess returns using the forward rate, and, on the other, match the means and the standard deviations of various nominal returns. We show that simply changing preferences from Kreps-Porteus to disappointment aversion is not enough. Nor is it sufficient to only change the endowment process. In fact, there seems to be a fundamental difference between outcomes for equity markets and those for the foreign exchange market regardless of the combination of parameters retained.

We therefore conclude that such a model cannot simultaneously explain returns in all the considered markets. In particular, while it is possible to obtain satisfactory results for the excess equity markets, some model assumptions need to be modified in order to also approximate the levels and the dynamics of excess foreign exchange returns.

With its present set of assumptions, it may well be the case that our results would improve if we used better data for Japanese consumption, estimated our preference parameters, used a three-state heteroskedastic endowment process and integrated fiscal shocks to the model. Nevertheless, we feel that we would not be able

to reconcile outcomes for the equity markets with those of foreign exchange markets unless the purchasing power parity assumption was abandoned or some form of agent heterogeneity was considered. These aspects are left open for future research.

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## Synthèse

Cette thèse est une étude à la fois théorique et empirique de la détermination des rendements des actifs financiers internationaux. Nous avons essayé de déterminer les facteurs qui influencent la valeur et la dynamique des actions, des bons du Trésor et les devises dans un cadre international, d'estimer et de tester ces relations dans des modèles de forme réduite et, finalement, de construire un modèle théorique complet qui formalise les idées explorées précédemment.

Étant donné le lien qui existe entre les modèles d'évaluation d'actifs financiers en économie fermée et ouverte, dans le premier chapitre nous avons présenté un survol des progrès théoriques et empiriques dans ces deux cadres d'analyse. Ensuite, utilisant une même banque de données, nous avons confirmé les faits stylisés découlant d'études empiriques faisant usage de modèles et de données très différents. Ainsi, nous avons montré que les premier et deuxième moments des rendements excédentaires des actifs financiers internationaux sont prévisibles, ce qui implique l'existence possible d'une prime de risque variable sur ces marchés. De plus, à l'aide d'un petit modèle simple, nous avons essayé de déterminer si les primes du marché de devises d'une part, et celles des marchés des actions d'autre part, étaient engendrées par différents facteurs, et si la dynamique de ces chocs variaient d'un marché à l'autre.

Dans le deuxième chapitre nous nous sommes concentrés sur deux modèles empiriques très utilisés et récents d'évaluation d'actifs internationaux. Ces modèles sont de forme réduite et caractérisent les rendements excédentaires en fonction de un ou plusieurs facteurs. La variabilité des primes de risque vient du fait que les coefficients du modèle sont supposés variables dans le temps et fonction de diverses variables. Bien que ces modèles sont acceptables selon un test de suridentification, nous avons montré que lorsqu'ils sont soumis à d'autres tests de spécifications plus puissants par rapport à certaines hypothèses alternatives d'intérêt, ils sont rejetés. Les tests

utilisés à ces fins sont celui de Andrews (1993) pour le changement structurel et le test d'orthogonalité des résidus de Newey (1985). Par conséquent, nous avons proposé un nouveau modèle économétrique de type ARCH à facteurs qui n'est rejeté par aucuns des tests considérés.

Dans le troisième chapitre, nous avons souligné que les modèles d'équilibre général n'ont pas eu beaucoup de succès à expliquer le niveau et la dynamique des primes de risque observées. La grande difficulté de ces modèles est de générer une variabilité suffisante du taux d'escompte stochastique. Pour répondre à ce problème, nous avons proposé un nouveau modèle théorique d'évaluation d'actifs internationaux. Notre modèle d'équilibre général à deux pays incorpore à la fois des préférences généralisées (de type aversion à la déception) et de l'hétéroscédasticité dans les processus de dotation. La première modification relâche l'hypothèse de la séparabilité additive de la fonction d'utilité et permet de mieux capter les attitudes des agents vis-à-vis des petits risques. La deuxième extension est de supposer un modèle Markov bivariés à deux régimes pour le processus de dotation de chaque pays, ce qui introduit plus de variabilité dans les taux de croissance de la consommation et de la masse monétaire de ces pays. Une des contributions de notre étude à la littérature est d'intégrer ces deux généralisations en même temps dans le cadre d'un modèle à deux pays.

Une comparaison approfondie des moments et des régressions des séries de rendements excédentaires simulées à partir de notre modèle d'une part et ceux des séries de données en échantillon d'autre part, montre que notre modèle peut reproduire relativement bien le niveau des moments des rendements excédentaires sur le marché des équités, ainsi que la prévisibilité observée des rendements excédentaires de tous les marchés considérés. Nos résultats sont donc meilleurs que ceux obtenus dans des études similaires précédentes. Cependant, nous avons aussi constaté qu'il faudrait utiliser des valeurs différentes sur certains paramètres selon que nous analysons le rendement des devises ou celui des actions. Ceci nous porte à croire que certaines hypothèses qui affectent le taux de change dans le modèle devraient être modifiées.

### Appendix 3.A

This section explains the details of the derivation of the first order conditions (8) and (9) of our optimization problem. It is parallel to the derivation in Epstein and Zin (1989, 1991).

Let  $q$  be a random variable with probability distribution  $P_q$  which has a certainty equivalent of  $\mu[P_q]$ . Then, this certainty equivalent function is implicitly defined through the relation:

$$\int \phi\left(\frac{\eta}{\mu[P_q]}\right) d(P_q) \equiv 0 \quad (\text{A.1})$$

with  $\phi$  defined as:

$$\phi(Z) \equiv I_A(Z) \left( \frac{Z^\alpha - 1}{\alpha} \right) \quad (\text{A.2})$$

The equation (A.1) is equivalent to (2) in the text. We also note that  $\phi$  is a continuous, increasing and concave function, with  $\phi(1) = 0$ .

Denote  $\hat{W}_t$  as the amount of real aggregate wealth that is invested by the representative agent in period  $t$ . This amount is invested in each of the two currencies as well as in the  $n$  capital assets. Then the budget constraint implies that

$$\hat{W}_t = \sum_{i=1}^N z_{i,t+1} \quad (\text{A.3})$$

The function  $V$  is linearly homogeneous in  $W_t$ , so that  $V(W_t, J_t) = v(J_t)W_t$ . Similarly defining  $w_{i,t+1} = z_{i,t+1}/\hat{W}_t$ , the value function in equation (5) can be written as:

$$v(J_t)W_t = \underset{c_t^x, c_t^y, \{w_{i,t+1}\}_{i=1}^N}{\text{MAX}} \left\{ \left( [c_t^x]^\delta [c_t^y]^{1-\delta} \right)^p + \beta \hat{W}_t^p (\mu(\mathcal{P}))^p \right\}^{\frac{1}{p}} \quad (\text{A.4})$$

subject to the constraint that

$$\sum_{i=1}^N w_{t,i+1} = 1 \quad (\text{A.5})$$

and where  $\mu(\mathcal{P})$  is defined as

$$\mu(\mathcal{P}) = \mu \left[ \begin{array}{c} P \\ V(J_{t+1}) \sum_{i=1}^N w_{i,t+1} R_{i,t+1} \end{array} \middle| J_t \right] \quad (\text{A.6})$$

Given the optimally chosen portfolio weights  $w_{i,t+1}^*$  for each asset  $i$ , the first order conditions with respect to  $c_t^x$  and  $c_t^y$  are:

$$\rho \delta [c_t^x]^{\rho \delta - 1} [c_t^y]^{\rho(1-\delta)} + \beta \rho \hat{w}_t^{\rho-1} \frac{\partial \hat{w}_t}{\partial c_t^x} [\mu_t^*]^{\rho} = 0 \quad (\text{A.7})$$

and

$$\rho(1-\delta) [c_t^x]^{\rho \delta} [c_t^y]^{(1-\delta)\rho-1} + \beta \rho \hat{w}_t^{\rho-1} \frac{\partial \hat{w}_t}{\partial c_t^y} [\mu_t^*]^{\rho} = 0 \quad (\text{A.8})$$

where

$$\mu_t^* \equiv \mu \left[ \begin{array}{c} P \\ V(J_{t+1}) \sum_{i=1}^N w_{i,t+1}^* R_{i,t+1} \end{array} \middle| J_t \right] \quad (\text{A.9})$$

Notice that, given  $w_{i,t+1}^*$ , the choice of  $c_t^x$  and  $c_t^y$  determines  $M_{t+1}^x/P_t^x$  and  $M_{t+1}^y/P_t^y$  since we have that

$$M_{t+1}^x/P_t^x = m^x(c_t^x, c_t^y) \quad (\text{A.10})$$

$$M_{t+1}^y/P_t^y = m^y(c_t^x, c_t^y). \quad (\text{A.11})$$

From the definition of the cost functions, and using Euler's theorem for homogeneous

functions, we also have that:

$$m^x(c_t^x, c_t^y) \cdot \psi_{2t}^x = \psi_t^x - c_t^x \psi_{1t}^x \quad (\text{A.12})$$

$$m^y(c_t^x, c_t^y) \cdot \psi_{2t}^y \left[ \frac{S_t P_t^y}{P_t^x} \right] = (\psi_t^y - c_t^y \psi_{1t}^y) \left[ \frac{S_t P_t^y}{P_t^x} \right] \quad (\text{A.13})$$

Substituting these terms for their expressions in the partial derivatives of the Euler equations, collecting terms and simplifying, we obtain two distinct expressions for  $\beta(\hat{w}_t^*, \mu^*)$ . These are given by:

$$\beta(\hat{w}_t^*, \mu^*)^p = \frac{\rho^{(1-\delta)} [c_t^x]^{\rho\delta} [c_t^y]^{(1-\delta)\rho-1} \left[ w_t - c_t^x (1 + \psi_{1t}^x) - \left[ \frac{S_t P_t^y}{P_t^x} \right] c_t^y (1 + \psi_{1t}^y) \right]}{\left[ \frac{S_t P_t^y}{P_t^x} \right] (1 + \psi_{1t}^x)} \quad (\text{A.14})$$

$$\beta(\hat{w}_t^*, \mu^*)^p = \frac{\rho\delta [c_t^x]^{\rho\delta-1} [c_t^y]^{\rho(1-\delta)} \left[ w_t - c_t^x (1 + \psi_{1t}^x) - \left[ \frac{S_t P_t^y}{P_t^x} \right] c_t^y (1 + \psi_{1t}^y) \right]}{1 + \psi_{1t}^x} \quad (\text{A.15})$$

Equating these two terms, one obtains equation (12) which is the definition of the equilibrium exchange rate.

Substituting equation (A.15) in equation (A.4) one can also derive a simple expression for  $v(J_t)$ . This is thus given by:

$$v(J_t) = \frac{\delta c^x(J_t)^{\rho\delta-1} c^y(J_t)^{\rho(1-\delta)}}{(1 + \psi_{1t}^x)} \quad (\text{A.16})$$

We now move on to the explanation of the derivation of the first-order conditions with respect the portfolio weights. Thus, given optimal choices for consumption, we must maximize the value function  $\mu[P_{V(w_{t+1}, J_{t+1})}]$ . We should, therefore, maximize:

$$\begin{aligned} & \text{MAX}_{M_{t+1}^x, M_{t+1}^y, \{z_{i,t+1}\}_{i=1}^n} \\ & E \left[ \phi \left( \frac{v(J_{t+1}) \left[ \frac{M_{t+1}^x}{P_{t+1}^x} \right] + \left[ \frac{S_{t+1} P_{t+1}^y}{P_{t+1}^x} \right] \left[ \frac{M_{t+1}^y}{P_{t+1}^y} \right] + \sum_{i=1}^N z_{i,t+1} R_{i,t+1}}{\mu^*} \right) \middle| J_t \right] \end{aligned} \quad (\text{A.17})$$

subject to the constraint that  $\sum_{i=1}^N w_{i,t+1} = 1$  and where  $\mu^*$  is defined as  $\mu^* = \hat{W}_t^* \mu^*$ . Consequently, the first-order conditions with respect to  $\{z_{i,t+1}\}_{i=1}^n$ ,  $M_{t+1}^x$  and  $M_{t+1}^y$  are respectively:

$$E \left[ \phi' \left( \frac{v(J_{t+1}) W_{t+1}^*}{\mu^*} \right) v(J_{t+1}) R_{i,t+1} \middle| J_t \right] = \lambda_t \quad i = 1, \dots, n \quad (\text{A.18})$$

$$E \left[ \phi' \left( \frac{v(J_{t+1}) W_{t+1}^*}{\mu^*} \right) v(J_{t+1}) \frac{1}{P_{t+1}^x} \middle| J_t \right] = \lambda_t \left[ \frac{1}{P_t^x} + \frac{\Psi_{2t}^x}{P_t^x} \right] \quad (\text{A.19})$$

$$E \left[ \phi' \left( \frac{v(J_{t+1}) W_{t+1}^*}{\mu^*} \right) v(J_{t+1}) \frac{S_{t+1}}{P_{t+1}^x} \middle| J_t \right] = \lambda_t \left[ \frac{S_t}{P_t^x} + \frac{S_t \Psi_{2t}^y}{P_t^x} \right] \quad (\text{A.20})$$

with  $\lambda_t$  as the Lagrange multiplier associated with the budget constraint.

Now define  $R_{x,t+1}$  and  $R_{y,t+1}$  as in equations (15) in the text and difference (A.18) and (A.19) to obtain:

$$E \left[ \phi' \left( \frac{v(J_{t+1}) W_{t+1}^*}{\mu^*} \right) v(J_{t+1}) (R_{i,t+1} - R_{j,t+1}) \middle| J_t \right] = 0 \quad i, j = x, y, 1, \dots, n \quad (\text{A.21})$$

To write this equation only in terms of observable data, first eliminate  $W_{t+1}^*$  and  $\mu^*$  from the expression. To do so, first define the term  $\hat{W}_t^H$  as

$$\hat{W}_t^H \equiv \hat{W}_t + \left( \frac{M_{t+1}^x}{P_t^x} \Psi_{2t}^x \right) + \left( \frac{S_t M_{t+1}^y}{P_t^x} \Psi_{2t}^y \right) \quad (\text{A.22})$$

Also, re-define the portfolio weights as

$$w_{x,t+1}^H \equiv \left( \frac{M_{t+1}^x}{P_t^x} (1 + \psi_{2t}^x) \right) / \hat{W}_t^H \quad (\text{A.23})$$

$$w_{y,t+1}^H \equiv \left( \frac{S_t M_{t+1}^y}{P_t^x} (1 + \psi_{2t}^y) \right) / \hat{W}_t^H \quad (\text{A.24})$$

$$w_{i,t+1}^H \equiv z_{i,t+1} / \hat{W}_t^H \quad (\text{A.25})$$

so that  $W_{t+1} = \hat{W}_t^H R_{t+1}$  and where the return on the optimally invested wealth is given by

$$R_{t+1} \equiv \sum_{i=1}^n w_{i,t+1}^H R_{i,t+1} + w_{x,t+1}^H R_{x,t+1} + w_{y,t+1}^H R_{y,t+1} \quad (\text{A.26})$$

Finally, define  $\mu_t^H$  as  $\mu_t^H \equiv \mu_t^* / \hat{W}_t^H$ . Now, we have that  $\mu_t^H = \mu^* (\hat{W}_t / \hat{W}_t^H)$ . With these transformations, equation (A.21) can be re-written as:

$$E \left[ \phi' \left( \frac{v(J_{t+1}) R_{t+1}}{\mu_t^H} \right) v(J_{t+1}) (R_{i,t+1} - R_{j,t+1}) \middle| J_t \right] = 0 \quad i, j = x, y, 1, \dots, n \quad (\text{A.27})$$

Using equations (A.14), (12), the above new definitions and Euler's theorem, we obtain another simple expression for the value function. This is given by

$$v(J_{t+1}) = \left[ \frac{\rho \delta [C_{t+1}^x]^{\rho \delta - 1} [C_{t+1}^y]^{\rho(1-\delta)}}{(1 + \psi_{1t+1}^x)} \right]^{\frac{1}{\rho}} \cdot \left( \frac{1}{W_{t+1}} \right)^{\frac{\rho-1}{\rho}} \quad (\text{A.28})$$



Similarly, we have that

$$\mu_t^H = \frac{\rho \delta [c_t^x]^{\rho \delta - 1} [c_t^y]^{\rho(1-\delta)}}{\beta(1 + \psi_{1t}^x) [\hat{W}_t^H]^{\rho - 1}} \quad (\text{A.29})$$

Using (A.27), (A.28) and (A.29), we obtain the Euler equation form:

$$E \left[ \phi'(Z_{t+1})(Z_{t+1}) \left( \frac{1}{R_{t+1}} \right) (R_{i,t+1} - R_{j,t+1}) \middle| J_t \right] = 0 \quad i, j = x, y, 1, \dots, n \quad (\text{A.30})$$

with  $Z_{t+1}$  defined as in equation (10) of the text.

Finally, using this last definition and the expression for  $\mu_t^H$ , we have that  $\mu[P_{Z_{t+1}} | J_t] = 1$ . This, together with (A.1) imply the other Euler equation given by:

$$E[\phi(Z_{t+1}) | J_t] = 0 \quad (\text{A.31})$$

Finally, for any further details we refer the reader to the unpublished technical appendix of the BHM study from where we reproduce the above formulas.

### Appendix 3.B

#### Euler Equation for Market Portfolio

The real return to holding the market portfolio is given by

$$R_{t+1} = \frac{\hat{W}_{t+1} + \overline{C_{t+1}}}{\hat{W}_t} \quad (\text{B.1})$$

where  $\hat{W}_t$  is aggregate invested wealth and  $\overline{C_{t+1}}$  is the real time t+1 payoff to this investment. Using Euler's theorem and defining  $\lambda_{t+1} = \hat{W}_{t+1}/C_{t+1}$ , we have that

$$\frac{\overline{C_{t+1}}}{C_t} = \left( \frac{C_{t+1}^x}{C_t^x} \right) \left( \frac{1 + \psi_{1t+1}^x}{1 + \psi_{1t}^x} \right) \quad (\text{B.2})$$

so that the market return is written

$$R_{t+1} = \left( \frac{\lambda_{t+1} + 1}{\lambda_t} \right) \cdot \left( \frac{C_{t+1}^x}{C_t^x} \right) \left( \frac{1 + \psi_{1t+1}^x}{1 + \psi_{1t}^x} \right). \quad (\text{B.3})$$

Replacing terms for their expressions and knowing that, at equilibrium, all output must be consumed or spent as transaction costs, the market portfolio equation (14), simplified, is written as

$$\begin{aligned} & E_t \left\{ \beta^{\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^x}{C_t^x} \right)^{\alpha \delta} \left( \frac{C_{t+1}^y}{C_t^y} \right)^{(1-\delta) \frac{\alpha}{\rho}} \left( \frac{\lambda_{t+1} + 1}{\lambda_t} \right)^{\frac{\alpha}{\rho}} - 1 \right\} + \\ & (A-1) E_t \left\{ I_B \left( \beta^{\frac{\alpha}{\rho}} \left( \frac{C_{t+1}^x}{C_t^x} \right)^{\alpha \delta} \left( \frac{C_{t+1}^y}{C_t^y} \right)^{(1-\delta) \frac{\alpha}{\rho}} \left( \frac{\lambda_{t+1} + 1}{\lambda_t} \right)^{\frac{\alpha}{\rho}} - 1 \right) \right\} = 0 \end{aligned} \quad (\text{B.4})$$

Now re-writing the various variables in terms of state-dependent expressions, for  $s_t = i$  and  $s_{t+1} = j$ , the above equation becomes

$$\frac{\alpha}{\beta^{\frac{\alpha}{\rho}}} \cdot E_t \left\{ \exp \left( a_j^x + a_j^y \right) \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho}} - 1 \right\} + (A-1) E_t \left[ I_B \left[ \beta^{\frac{\alpha}{\rho}} \exp \left( b_j^x + b_j^y \right) \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho}} - 1 \right] \right] = 0 \quad (\text{B.5})$$

where the abbreviations stand for the following:

$$\begin{aligned} a_j^x &= \alpha \delta \mu_j^{C,x} + \frac{\alpha^2 \delta^2 [\omega_j^{C,x}]^2}{2} \\ a_j^y &= (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y} + \left( \frac{(1-\delta)^2 \alpha^2}{2\rho^2} \right) [\omega_j^{C,y}]^2, \\ b_j^x &= \alpha \delta \left( \mu_j^{C,x} + \omega_j^{C,x} \varepsilon_{t+1}^{C,x} \right) \\ b_j^y &= \frac{(1-\delta)\alpha}{\rho} \left( \mu_j^{C,y} + \omega_j^{C,y} \varepsilon_{t+1}^{C,y} \right) \end{aligned} \quad (\text{B.6})$$

We now define  $z_j$  as

$$z_j = \alpha \delta \omega_j^{C,x} \varepsilon_{t+1}^{C,x} + \left( \frac{(1-\delta)\alpha}{\rho} \right) \omega_j^{C,y} \varepsilon_{t+1}^{C,y} \quad (\text{B.7})$$

with marginal density  $f(z_j)$  given by

$$f(z_j) = \frac{1}{\sqrt{2\pi} \cdot \sigma_j} \cdot \exp \left( \frac{-z_j^2}{2\sigma_j^2} \right) \quad (\text{B.8})$$

and variance

$$\sigma_j^2 = \left( \alpha \delta \omega_j^{C,x} \right)^2 + \left( \left( \frac{(1-\delta)\alpha}{\rho} \right) \omega_j^{C,y} \right)^2. \quad (\text{B.9})$$

Replacing these terms in equation (22) and after some algebraic manipulations, the

market portfolio equation becomes

$$\begin{aligned}
 & (\beta^{\alpha/\rho}) \cdot \sum_{j=0}^K p_{ij} \left[ \exp\left(a_j^x + a_j^y\right) \left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho}} \right] \\
 & \quad + \\
 & (A-1) \cdot \sum_{j=0}^K p_{ij} \left\{ \kappa_j \int_{B(i,j)}^{\infty} [c_j \exp(z_j) - 1] f(z_j) dz_j \right\} \\
 & \quad = 0
 \end{aligned} \tag{B.10}$$

where the state dependent constant  $c_j$  is given by

$$c_j = \beta^{\frac{\alpha}{\rho}} \exp\left(\alpha \delta \mu_j^{C,x} + (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y}\right) \left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho}} \tag{B.11}$$

and the term  $\kappa_j$  is defined as

$$\kappa_j = \left(\frac{(1-\delta)\alpha}{\rho} \mu_j^{C,y}\right), \tag{B.12}$$

and where  $B(i, j)$  is the lower limit for  $z_j$  and is given by

$$B(i, j) = -\frac{\alpha}{\rho} \log(\beta) - \alpha \delta \mu_j^{C,x} - (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y} - \frac{\alpha}{\rho} \log\left(\frac{\lambda_j + 1}{\lambda_i}\right). \tag{B.13}$$

### Euler Equations for Equity Portfolios

Following the above steps and defining  $\varphi_{t+1}^x$  and  $\varphi_{t+1}^y$  as the respective ratios of country equity prices to their dividends, both evaluated in real terms, it can be shown that the country x equity portfolio can be written in terms of state-dependent variables as:

$$\begin{aligned}
& (\beta^{\alpha/\rho}) \cdot \sum_{j=0}^K p_{ij} \left( \exp(a_j^x + a_j^y) \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho} - 1} \left( \frac{\varphi_j^x + 1}{\varphi_i^x} \right) \right) \\
& \quad + \\
& (A-1) \cdot \sum_{j=0}^K p_{ij} \left\{ \kappa_j \int_{B(i,j)}^{\infty} [c_j^x \exp(z_j) - 1] f(z_j) dz_j \right\} \\
& \quad = 1
\end{aligned} \tag{B.14}$$

where the state dependent constant  $c_j^x$  is given by

$$c_j^x = \beta^{\frac{\alpha}{\rho}} \exp\left(\alpha \delta \mu_j^{C,x} + (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y}\right) \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho} - 1} \left( \frac{\varphi_j^x + 1}{\varphi_i^x} \right). \tag{B.15}$$

The country y equity portfolio equation, in turn, is given by

$$\begin{aligned}
& (\beta^{\alpha/\rho}) \cdot \sum_{j=0}^K p_{ij} \left( \exp(a_j^x + a_j^y) \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho} - 1} \left( \frac{\varphi_j^y + 1}{\varphi_i^y} \right) \right) \\
& \quad + \\
& (A-1) \cdot \sum_{j=0}^K p_{ij} \left\{ \kappa_j \int_{B(i,j)}^{\infty} [c_j^y \exp(z_j) - 1] f(z_j) dz_j \right\} \\
& \quad = 1
\end{aligned} \tag{B.16}$$

where the state dependent constant  $c_j^y$  is given by

$$c_j^y = \beta^{\frac{\alpha}{\rho}} \exp\left(\alpha \delta \mu_j^{C,x} + (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y}\right) \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho} - 1} \left( \frac{\varphi_j^y + 1}{\varphi_i^y} \right) \tag{B.17}$$

Euler Equations for Currency Portfolios

The Euler equation for the return on holding the country x currency is defined next. Given the cost function in equation (3), its derivative with respect to the first and second arguments are  $\psi_{1t}^x = \gamma v(v_t^x)^{\gamma-1}$  and  $\psi_{2t}^x = \gamma(1-v)(v_t^x)^\gamma$ , where  $v_t^x$  is the money velocity of country x at time t. Using these definitions, define the term  $V_{ij}^x$  as:

$$V_{ij}^x = \left( \frac{v_i^x}{v_j^x} \right) \left( \frac{1 + \psi_{1i}^x}{1 + \psi_{1j}^x} \right) \left( \frac{1}{1 + \psi_{2i}^x} \right) \quad (\text{B.18})$$

Given the definition of returns to currency in equation (16) and using the above equations, the country x currency holdings Euler equation is written

$$\begin{aligned} & (\beta^{\alpha/\rho}) \cdot \sum_{j=0}^K p_{ij} \left( \exp(m_j^x + a_j^y) \left( \frac{\lambda_j + 1}{\lambda_t} \right)^{\frac{\alpha}{\rho} - 1} (V_{ij}^x) \right) \\ & + \\ (A-1) \cdot \sum_{j=0}^K p_{ij} & \left\{ h_j^x \int_{B(i,j)}^{\infty} c_j^{cx} \exp \left( z_j + \frac{[-2\alpha\delta\omega_j^{C,x} \omega_j^{M,x} \rho^{CM,x}] z_j}{\sigma_j^2} \right) f(z_j) dz_j \right\} \\ & - \\ (A-1) \cdot \sum_{j=0}^K & p_{ij} \left\{ \kappa_j \int_{B(i,j)}^{\infty} f(z_j) dz_j \right\} \\ & = 1 \end{aligned} \quad (\text{B.19})$$

where the abbreviated terms are:

$$m_j^x = \exp \left( \alpha \delta \mu_j^{C,x} - \mu_j^{M,x} + \frac{\alpha^2 \delta^2 \left[ \omega_j^{C,x} \right]^2 + \left[ \omega_j^{M,x} \right]^2 - 2\alpha \delta \omega_j^{C,x} \omega_j^{M,x} \rho^{CM,x}}{2} \right), \quad (\text{B.20})$$

$$h_j^x = \kappa_j \exp \left( \frac{\left( -\kappa_j \omega_j^{M,x} \rho^{CM,x} \right)^2}{\sigma_j^2} \right) \quad (\text{B.21})$$

and where the state dependent variable  $c_j^{cx}$  is given by

$$c_j^{c,x} = \beta^{\frac{\alpha}{\rho}} \left( \exp \left( \alpha \delta \mu_j^{C,x} + (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y} - \mu_j^{M,l} + \frac{(\omega_j^{M,x})^2 [1 - (\rho^{CM,x})^2]}{2} \right) v_{ij}^x \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho} - 1} \right) \quad (\text{B.22})$$

As for the country y currency portfolio Euler equation, given the cost function in equation (3), its derivative with respect to the first argument is

$$\psi_{1t}^y = \zeta \xi (v_t^y)^{\xi - 1} \quad (\text{B.23})$$

and its derivative with respect to the real money balances is

$$\psi_{2t}^y = \zeta (1 - \xi) (v_t^y)^{\xi} \quad (\text{B.24})$$

where  $v_t^y$  is the money velocity of country y at time t. Using these definitions, as before, define the term  $v_{ij}^y$  as:

$$v_{ij}^y = \left( \frac{v_i^y}{v_j^y} \right) \left( \frac{1 + \psi_{1i}^y}{1 + \psi_{1j}^y} \right) \left( \frac{1}{1 + \psi_{2i}^y} \right) \quad (\text{B.25})$$

Given the definition of returns to the currency y in equation (15) and using the above expressions, the country y currency holdings Euler equation is written

$$\begin{aligned} & (\beta^{\alpha/\rho}) \cdot \sum_{j=0}^K p_{ij} \left( \exp(m_j^y + a_j^x) \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho} - 1} (V_{ij}^y) \right) \\ & + \\ & (A-1) \cdot \sum_{j=0}^K p_{ij} \left\{ h_j^y \int_{B(i,j)}^{\infty} c_j^{cy} \exp \left( z_j + \frac{[-2\alpha(1-\delta)\omega_j^{C,y} \omega_j^{M,y} \rho^{CM,y}] z_j}{\rho \sigma_j^2} \right) f(z_j) dz_j \right\} \quad (\text{B.26}) \\ & - \\ & (A-1) \cdot \sum_{j=0}^K p_{ij} \left\{ \kappa_j \int_{B(i,j)}^{\infty} f(z_j) dz_j \right\} \\ & = 1 \end{aligned}$$

where the abbreviated terms are:

$$m_j^y = \exp \left( (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y} - \mu_j^{M,y} + \frac{(1-\delta)^2 \alpha^2 [\omega_j^{C,y}]^2 + [\omega_j^{M,y}]^2 - 2\alpha(1-\delta) \omega_j^{C,y} \omega_j^{M,y} \rho^{CM,y}}{2\rho^2} \right) \quad (\text{B.27})$$

and

$$h_j^y = \kappa_j \exp \left( \frac{(-2\alpha\delta \omega_j^{C,x} \omega_j^{M,y} \rho^{CM,y})^2}{2\sigma_j^2} \right) \quad (\text{B.28})$$

and where the state dependent variable  $c_j^{cy}$  is given by

$$c_j^{cy} = \beta^{\frac{\alpha}{\rho}} \exp \left( \alpha\delta \mu_j^{C,x} + (1-\delta) \frac{\alpha}{\rho} \mu_j^{C,y} - \mu_j^{M,y} + \frac{(\omega_j^{M,y})^2 [1 - (\rho^{CM,y})^2]}{2} \right) \cdot V_{ij}^y \cdot \left( \frac{\lambda_j + 1}{\lambda_i} \right)^{\frac{\alpha}{\rho} - 1} \quad (\text{B.29})$$

Given the estimated and calibrated coefficients, we can now solve the system of two equations ( $i = 1, 2$ ) for endogenous state-dependent  $\lambda$  terms numerically.



## Appendix 3.C

Table 12: Distribution of Generated Means of Returns, Model B,  $\delta=0.8$ 

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -1$ $\delta = 0.8$	RUS	11.021	11.429	11.656	11.697
	RJAP	11.880	12.188	12.402	12.690
	iUS	2.522	2.614	2.722	2.827
	iJAP	1.297	1.297	1.297	1.297
	xsRUS	8.473	8.707	8.872	8.876
	xsRJAP	10.584	10.886	11.113	11.378
	FP	0.528	0.563	0.616	0.657
	DELS	3.805	3.935	3.989	4.007
	xsFX	3.267	3.347	3.348	3.375

Table 13: Distribution of Generated Means of Returns, Model A,  $\delta=0.8$ 

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	15.084	15.707	15.058	15.333
	RJAP	15.161	16.109	15.115	15.693
	iUS	5.026	5.080	5.060	5.116
	iJAP	1.389	1.390	1.390	1.390
	xsRUS	10.069	10.563	10.058	10.228
	xsRJAP	13.770	14.721	13.725	14.305
	FP	1.507	1.526	1.521	1.545
	DELS	1.565	1.730	1.520	1.690
	xsFX	0.087	0.240	0.027	0.137
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	33.294	32.189	32.532	32.704
	RJAP	37.834	37.719	36.242	39.039
	iUS	11.289	11.382	11.435	11.367
	iJAP	1.427	1.427	1.428	1.428
	xsRUS	21.891	20.829	21.169	21.273
	xsRJAP	36.407	36.292	34.814	37.612
	FP	4.024	4.062	4.084	4.056
	DELS	1.569	1.438	1.527	1.802
	xsFX	-2.450	-2.650	-2.559	-2.249
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	75.329	75.788	76.651	79.695
	RJAP	95.171	96.383	97.829	101.534
	iUS	27.465	27.556	27.987	27.909
	iJAP	1.443	1.443	1.443	1.443
	xsRUS	47.638	48.048	49.338	52.685
	xsRJAP	93.727	94.940	96.384	100.091
	FP	10.550	10.584	10.763	10.730
	DELS	1.577	1.459	1.558	1.677
	xsFX	-8.954	-8.889	-9.094	-8.948

Table 14: Distribution of Generated Means of Returns, Model B, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	18.293	18.723	17.760	17.799
	RJAP	25.104	24.547	23.970	24.224
	iUS	3.983	4.033	4.077	4.122
	iJAP	1.374	1.375	1.375	1.3755
	xsRUS	14.305	14.682	13.675	13.676
	xsRJAP	23.730	23.172	22.595	22.848
	FP	1.088	1.109	1.127	1.145
	DELS	3.845	3.860	3.795	3.569
	xsFX	2.749	2.736	2.667	2.423
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	38.432	37.225	37.640	39.740
	RJAP	74.220	73.810	73.761	75.455
	iUS	9.417	9.524	9.464	9.517
	iJAP	1.427	1.427	1.426	1.427
	xsRUS	28.891	28.045	27.645	30.488
	xsRJAP	72.796	72.385	72.335	74.031
	FP	3.262	3.310	3.283	3.297
	DELS	3.949	3.692	3.775	3.814
	xsFX	0.634	0.468	0.621	0.542
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	94.354	94.663	94.074	97.058
	RJAP	233.391	228.439	236.978	235.726
	iUS	24.549	25.716	24.720	24.807
	iJAP	1.444	1.444	1.444	1.445
	xsRUS	69.015	68.257	67.901	71.029
	xsRJAP	231.949	226.993	235.534	234.283
	FP	9.363	9.841	9.429	9.466
	DELS	3.884	3.781	3.753	4.174
	xsFX	-5.583	-5.959	-5.326	-5.038

Table 15: Distribution of Generated Means of Returns, Model A, delta=0.5

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	10.899	10.889	10.482	10.494
	RJAP	8.838	8.852	8.810	8.827
	iUS	2.750	2.790	2.815	2.872
	iJAP	1.331	1.332	1.335	1.336
	xsRUS	8.068	8.092	7.634	7.659
	xsRJAP	7.507	7.521	7.473	7.491
	FP	0.604	0.621	0.629	0.652
	DELS	1.692	1.741	1.567	1.556
	xsFX	1.071	1.098	0.924	0.864
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	16.042	16.623	16.556	16.283
	RJAP	16.052	16.383	16.519	16.096
	iUS	5.137	5.183	5.232	5.280
	iJAP	1.391	1.392	1.393	1.394
	xsRUS	10.926	11.436	11.313	10.956
	xsRJAP	14.660	14.990	15.126	14.703
	FP	1.551	1.569	1.589	1.608
	DELS	1.598	1.654	1.714	1.584
	xsFX	0.066	0.085	0.137	0.002
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	26.504	26.020	26.785	26.853
	RJAP	30.113	29.196	30.721	29.674
	iUS	9.212	9.312	9.337	9.401
	iJAP	1.420	1.421	1.421	1.421
	xsRUS	17.370	16.665	17.385	17.283
	xsRJAP	28.693	27.775	29.300	28.252
	FP	3.188	3.231	3.329	3.265
	DELS	1.741	1.631	1.475	1.595
	xsFX	-1.475	-1.579	-1.707	-1.668

Table 16: Distribution of Generated Means of Returns, Model B, delta=0.5

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	13.813	14.266	13.921	13.571
	RJAP	14.251	14.160	14.120	13.818
	iUS	2.522	2.586	2.646	2.720
	iJAP	1.318	1.319	1.321	1.323
	xsRUS	11.295	11.656	11.266	10.818
	xsRJAP	12.934	12.840	12.798	12.495
	FP	0.515	0.542	0.565	0.596
	DELS	3.775	3.958	3.848	3.642
	xsFX	3.253	3.398	3.289	3.040
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	20.301	20.526	20.518	20.924
	RJAP	29.734	29.870	29.544	30.629
	iUS	4.728	4.805	4.832	4.912
	iJAP	1.392	1.392	1.392	1.393
	xsRUS	15.500	15.806	15.724	16.128
	xsRJAP	28.340	28.479	28.153	29.234
	FP	1.386	1.413	1.425	1.457
	DELS	3.659	3.720	3.736	3.737
	xsFX	2.266	2.342	2.295	2.265
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	33.151	34.088	33.725	34.171
	RJAP	64.740	63.114	64.190	64.159
	iUS	8.734	8.896	8.774	8.798
	iJAP	1.423	1.424	1.425	1.424
	xsRUS	25.270	25.481	24.921	25.323
	xsRJAP	63.316	61.687	62.768	62.734
	FP	2.983	3.049	3.006	3.014
	DELS	3.933	3.963	3.711	3.599
	xsFX	1.089	1.042	0.826	0.647

Table 17: Distribution of Generated Means of Returns, Model A, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	13.266	13.038	13.537	13.481
	RJAP	13.455	13.315	13.661	13.658
	iUS	3.852	3.905	4.006	4.067
	iJAP	1.367	1.369	1.370	1.372
	xsRUS	9.327	9.112	9.547	9.456
	xsRJAP	12.089	11.948	12.290	12.286
	FP	1.036	1.059	1.103	1.124
	DELS	1.623	1.684	1.797	1.797
	xsFX	0.577	0.561	0.633	0.704
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	24.207	24.125	24.827	24.911
	RJAP	27.636	27.877	27.977	28.597
	iUS	7.155	7.197	7.326	7.416
	iJAP	1.410	1.411	1.411	1.412
	xsRUS	17.386	16.937	17.505	17.491
	xsRJAP	26.227	26.466	26.569	27.185
	FP	2.359	2.380	2.429	2.466
	DELS	1.538	1.613	1.727	1.586
	xsFX	-0.787	-0.790	-0.686	-0/876

Table 18: Distribution of Generated Means of Returns, Model B, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	14.831	15.253	15.641	14.415
	RJAP	20.428	20.869	21.666	20.167
	iUS	2.675	2.777	2.874	2.997
	iJAP	1.337	1.340	1.342	1.345
	xsRUS	12.164	12.518	12.812	11.420
	xsRJAP	19.094	19.530	20.325	18.823
	FP	0.568	0.609	0.648	0.698
	DELS	3.935	3.862	4.037	3.773
	xsFX	3.366	3.269	3.505	3.074
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	27.298	28.001	27.016	27.017
	RJAP	54.906	52.307	51.413	53.550
	iUS	5.153	5.274	5.514	5.450
	iJAP	1.399	1.402	1.403	1.405
	xsRUS	22.108	22.773	21.643	21.628
	xsRJAP	53.503	50.900	50.010	52.147
	FP	1.548	1.602	1.691	1.667
	DELS	4.009	3.871	3.578	3.677
	xsFX	2.478	2.306	1.874	2.088

Table 19: Distribution of Generated Means of Returns, Model A,  $\delta=0.5$ 

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	14.406	14.406	14.841	14.850
	RJAP	13.968	14.066	14.310	14.202
	iUS	4.111	4.193	4.309	4.407
	iJAP	1.375	1.377	1.379	1.381
	xsRUS	10.256	10.201	10.509	10.425
	xsRJAP	12.593	12.688	12.930	12.820
	FP	1.141	1.174	1.221	1.260
	DELS	1.652	1.469	1.692	1.609
xsFX	0.500	0.287	0.501	0.346	
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	22.774	22.782	22.170	22.196
	RJAP	24.960	24.683	24.375	24.489
	iUS	6.921	7.089	7.187	7.303
	iJAP	1.408	1.409	1.410	1.411
	xsRUS	15.802	15.934	14.980	14.912
	xsRJAP	23.551	23.274	22.964	23.077
	FP	2.265	2.335	2.375	2.421
	DELS	1.671	1.498	1.519	1.439
xsFX	-0.607	-0.789	-0.862	-0.969	

Table 20: Distribution of Generated Means of Returns, Model B,  $\delta=0.5$ 

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	17.904	18.977	18.403	19.066
	RJAP	25.211	25.676	26.224	26.531
	iUS	3.510	3.626	3.753	3.872
	iJAP	1.370	1.372	1.375	1.376
	xsRUS	14.556	15.263	14.671	15.146
	xsRJAP	23.845	24.307	24.851	25.156
	FP	0.894	0.941	0.994	1.041
	DELS	3.760	3.852	3.865	3.933
xsFX	2.917	2.891	2.912	2.865	
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	28.741	28.777	28.129	29.231
	RJAP	52.370	51.041	50.814	53.484
	iUS	5.993	6.355	6.484	6.470
	iJAP	1.409	1.410	1.411	1.412
	xsRUS	22.731	22.582	21.710	22.695
	xsRJAP	50.960	49.625	49.405	52.072
	FP	1.893	2.029	2.082	2.068
	DELS	4.140	3.813	3.537	4.041
xsFX	2.294	1.886	1.611	2.131	

Table 21: Distribution of Generated Standard Devs. of Returns, Model A, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	17.626	18.382	18.211	17.567
	RJAP	17.486	18.674	17.763	17.715
	iUS	0.972	0.992	1.007	1.024
	iJAP	0.178	0.178	0.178	0.178
	xsRUS	17.511	18.373	18.070	17.520
	xsRJAP	17.468	18.658	17.748	17.698
	FP	0.360	0.367	0.375	0.381
	DELS	4.951	5.090	4.992	5.278
	xsFX	4.921	5.027	4.937	5.237
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	39.088	38.361	37.915	37.943
	RJAP	49.962	48.970	47.253	50.292
	iUS	1.935	1.934	1.932	1.912
	iJAP	0.183	0.183	0.183	0.183
	xsRUS	38.771	38.040	37.653	37.691
	xsRJAP	49.944	48.953	47.233	50.274
	FP	0.735	0.735	0.734	0.725
	DELS	4.719	4.964	4.460	5.122
	xsFX	4.818	5.082	4.628	5.274
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	98.751	96.545	96.458	106.692
	RJAP	146.480	145.957	146.961	152.913
	iUS	11.033	11.042	11.133	11.068
	iJAP	0.185	0.185	0.185	0.185
	xsRUS	95.289	93.836	92.142	103.547
	xsRJAP	146.457	145.939	146.946	152.892
	FP	4.447	4.447	4.487	4.462
	DELS	4.857	4.859	4.736	5.311
	xsFX	6.937	6.924	7.112	7.323

Table 22: Distribution of Generated Standard Devs. of Returns, Model B, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -1$ $\delta = 0.8$	RUS	16.541	17.848	18.780	18.241
	RJAP	17.604	18.931	19.321	18.773
	iUS	0.574	0.641	0.730	0.830
	iJAP	0.168	0.168	0.170	0.170
	xsRUS	16.471	17.751	18.658	18.096
	xsRJAP	17.592	18.918	19.310	18.762
	FP	0.215	0.248	0.288	0.333
	DELS	8.245	8.989	9.005	8.996
	xsFX	8.180	8.932	8.946	8.914

Table 23: Distribution of Generated Standard Devs. of Returns, Model B, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	27.805	28.703	27.418	25.502
	RJAP	37.602	38.017	34.446	35.793
	iUS	0.518	0.520	0.523	0.528
	iJAP	0.176	0.176	0.176	0.176
	xsRUS	27.766	28.669	27.373	25.467
	xsRJAP	37.586	38.001	34.428	35.778
	FP	0.144	0.144	0.145	0.147
	DELS	8.721	8.726	8.513	8.423
	xsFX	8.721	8.726	8.507	8.415
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	59.326	56.313	56.582	62.171
	RJAP	118.714	112.238	119.769	124.867
	iUS	3.666	3.631	3.599	3.565
	iJAP	0.183	0.189	0.183	0.183
	xsRUS	58.585	56.038	56.329	62.054
	xsRJAP	118.693	112.219	119.735	124.846
	FP	1.469	1.456	1.447	1.432
	DELS	8.513	8.619	8.687	8.165
	xsFX	8.846	8.997	9.071	8.518
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	151.637	144.080	144.328	156.528
	RJAP	390.757	359.169	379.710	404.895
	iUS	16.817	16.720	16.864	16.761
	iJAP	0.185	0.185	0.185	0.185
	xsRUS	145.618	137.613	141.725	150.783
	xsRJAP	390.742	359.154	379.692	404.879
	FP	6.806	6.576	6.813	6.789
	DELS	8.901	8.509	8.417	9.501
	xsFX	12.215	11.808	11.796	12.619



Table 24: Distribution of Generated Standard Devs. of Returns, Model A, delta=0.5

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	15.389	14.833	14.419	14.495
	RJAP	10.439	10.373	10.357	10.201
	iUS	0.557	0.576	0.593	0.609
	iJAP	0.172	0.172	0.172	0.173
	xsRUS	15.380	14.828	14.402	14.463
	xsRJAP	10.426	10.363	10.344	10.186
	FP	0.201	0.207	0.216	0.223
	DELS	5.077	4.743	5.154	4.946
	xsFX	5.059	4.695	5.110	4.893
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	18.893	20.810	19.930	20.421
	RJAP	19.419	19.767	21.177	19.728
	iUS	0.676	0.685	0.695	0.706
	iJAP	0.178	0.178	0.178	0.178
	xsRUS	18.848	20.781	19.852	20.357
	xsRJAP	19.403	19.753	21.161	19.709
	FP	0.207	0.213	0.217	0.222
	DELS	5.192	4.910	4.971	5.088
	xsFX	5.178	4.877	4.946	5.058
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	32.512	30.531	33.849	32.483
	RJAP	40.076	38.859	41.923	40.531
	iUS	1.665	1.669	1.657	1.655
	iJAP	0.182	0.182	0.182	0.182
	xsRUS	32.297	30.374	33.653	32.285
	xsRJAP	40.056	38.840	41.904	40.512
	FP	0.633	0.630	0.627	0.624
	DELS	5.266	4.738	4.746	5.295
	xsFX	5.349	4.809	4.851	5.383

Table 25: Distribution of Generated Standard Devs. of Returns, Model B, delta=0.5

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	23.828	24.060	24.112	23.769
	RJAP	22.937	21.527	23.802	22.343
	iUS	0.338	0.359	0.384	0.416
	iJAP	0.169	.169	0.169	0.170
	xsRUS	23.797	24.033	24.092	23.704
	xsRJAP	22.925	21.516	23.791	22.330
	FP	0.078	0.091	0.106	0.124
	DELS	8.566	8.598	8.572	8.195
	xsFX	8.556	8.582	8.549	8.174
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	31.888	31.673	33.368	31.861
	RJAP	48.104	47.053	46.169	45.790
	iUS	0.909	0.888	0.869	0.849
	iJAP	0.178	0.178	0.178	0.178
	xsRUS	31.920	31.646	33.311	31.783
	xsRJAP	48.089	47.039	46.155	45.775
	FP	0.332	0.321	0.311	0.301
	DELS	8.226	8.561	8.523	8.411
	xsFX	8.314	8.606	8.595	8.456
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	49.424	50.098	49.153	50.201
	RJAP	105.755	98.968	103.159	99.181
	iUS	3.171	3.140	3.112	3.065
	iJAP	0.183	0.183	0.183	0.183
	xsRUS	49.147	49.616	48.741	49.730
	xsRJAP	105.738	98.946	103.143	99.163
	FP	1.270	1.259	1.249	1.226
	DELS	8.566	8.823	8.299	8.271
	xsFX	8.839	9.093	8.674	8.618

Table 26: Distribution of Generated Standard Devs. of Returns, Model A, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	16.119	15.397	15.694	15.886
	RJAP	15.486	15.276	15.757	16.228
	iUS	0.796	0.828	0.862	0.904
	iJAP	0.176	0.176	0.176	0.176
	xsRUS	15.974	15.337	15.688	15.788
	xsRJAP	15.469	15.259	15.746	16.214
	FP	0.293	0.308	0.323	0.341
	DELS	5.119	5.096	5.320	5.383
	xsFX	5.083	5.083	5.285	5.301
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	30.171	29.539	30.582	31.866
	RJAP	36.851	37.011	38.013	39.079
	iUS	1.131	1.127	1.127	1.135
	iJAP	0.181	0.181	0.181	0.181
	xsRUS	30.064	29.456	30.480	31.696
	xsRJAP	36.829	36.992	37.994	39.059
	FP	0.408	0.404	0.404	0.405
	DELS	4.923	4.942	4.979	5.117
	xsFX	4.950	4.992	5.044	5.161

Table 27: Distribution of Generated Standard Devs. of Returns, Model B, delta=0.8

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	24.871	24.584	24.479	23.336
	RJAP	30.813	32.434	32.227	30.298
	iUS	0.411	0.394	0.381	0.384
	iJAP	0.171	0.172	0.172	0.172
	xsRUS	24.859	24.546	24.450	23.312
	xsRJAP	30.796	32.418	32.212	30.284
	FP	0.120	0.106	0.093	0.090
	DELS	9.034	8.447	9.067	8.359
	xsFX	9.049	8.462	9.068	8.357
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	46.560	46.574	46.885	45.306
	RJAP	86.263	85.958	81.144	80.672
	iUS	2.465	2.452	2.464	2.407
	iJAP	0.181	0.181	0.181	0.181
	xsRUS	46.166	46.221	46.223	44.859
	xsRJAP	86.245	85.939	81.128	80.649
	FP	1.002	0.994	0.997	0.972
	DELS	8.537	8.897	7.633	8.189
	xsFX	8.767	9.128	7.887	8.490

Table 28: Distribution of Generated Standard Devs. of Returns, Model A, delta=0.5

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	18.776	18.493	18.451	18.329
	RJAP	17.130	17.330	17.194	16.942
	iUS	0.562	0.583	0.607	0.632
	iJAP	0.176	0.176	0.177	0.177
	xsRUS	18.756	18.456	18.437	18.275
	xsRJAP	17.111	17.315	17.181	16.925
	FP	0.168	0.177	0.188	0.200
	DELS	5.048	4.796	5.259	4.442
	xsFX	5.022	4.775	5.243	4.434
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	29.049	30.056	27.203	28.600
	RJAP	33.056	35.075	32.155	31.704
	iUS	1.182	1.192	1.185	1.184
	iJAP	0.180	0.180	0.181	0.181
	xsRUS	28.857	29.953	26.972	28.425
	xsRJAP	33.038	35.055	32.140	31.685
	FP	0.432	0.436	0.431	0.429
	DELS	5.135	4.851	4.796	4.542
	xsFX	5.186	4.903	4.842	5.591

Table 29: Distribution of Generated Standard Devs. of Returns, Model B, delta=0.5

	median	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	30.223	33.524	31.155	34.156
	RJAP	39.984	40.653	40.180	41.885
	iUS	0.783	0.753	0.714	0.668
	iJAP	0.176	0.176	0.176	0.176
	xsRUS	30.122	33.407	31.095	34.047
	xsRJAP	39.970	40.635	40.165	41.869
	FP	0.292	0.275	0.253	0.229
	DELS	8.406	9.807	8.317	8.791
	xsFX	8.417	9.843	8.368	8.818
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	50.316	47.627	44.122	49.095
	RJAP	83.679	82.602	84.988	88.929
	iUS	2.575	2.545	2.502	2.452
	iJAP	0.182	0.182	0.182	0.181
	xsRUS	49.752	47.250	43.541	48.501
	xsRJAP	83.656	82.579	84.966	88.910
	FP	1.036	1.027	1.006	0.984
	DELS	9.365	8.403	8.512	9.233
	xsFX	9.673	8.604	8.766	9.329

Table 30: P-Value Performance for Median, Model A, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	47.5	43.5	48.5	46.0
	RJAP	20.0	15.5	25.0	20.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	-	-	-	-
	RJAP	-	-	-	-
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	-	-	-	-
	RJAP	-	-	-	-
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-

Table 31: P-Value Performance for Median, Model B, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	22.0	20.5	20.5	27.5
	RJAP	6.0	2.5	3.5	4.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	95.0	96.5	97.0	98.5
	FP	-	-	-	-
	xsFX	94.5	94.5	97.0	98.0
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	-	-	-	-
	RJAP	-	-	-	-
	iUS	3.5	1.5	5.0	0.5
	iJAP	-	-	-	-
	DELS	96.5	98.5	95.0	97.5
	FP	-	-	-	-
	xsFX	97.0	98.5	97.0	98.0
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	-	-	-	-
	RJAP	-	-	-	-
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	96.5	98.0	97.0	97.0
	FP	-	-	-	-
	xsFX	98.5	99.0	99.5	-

Table 32: P-Value Performance for Median, Model A, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	97.0	98.5	97.5	95.5
	RJAP	97.0	96.0	96.5	96.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	34.5	35.0	32.5	32.5
	RJAP	16.5	16.0	11.0	17.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	-	-	-	-
	RJAP	0.5	-	-	-
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-

Table 33: P-Value Performance for Median, Model B, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	62.0	57.0	60.0	58.5
	RJAP	36.0	33.5	34.0	30.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	98.5	99.0	98.0	98.0
	FP	-	-	-	-
	xsFX	95.5	93.5	98.0	98.0
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	9.0	7.0	11.0	7.0
	RJAP	-	1.5	-	1.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	97.5	97.0	95.5	97.0
	FP	-	-	-	-
	xsFX	97.0	96.0	95.0	96.0
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	-	-	-	-
	RJAP	-	-	-	-
	iUS	8.0	10.5	9.0	4.0
	iJAP	-	-	-	-
	DELS	96.0	96.0	98.0	97.0
	FP	-	-	-	-
	xsFX	96.5	97.5	98.5	97.0



Table 34: P-Value Performance for Median, Model A, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	76.0	77.0	68.0	74.5
	RJAP	39.5	42.5	37.0	35.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	-	-	-	-
	RJAP	-	-	0.5	0.5
	iUS	99.0	95.5	90.5	81.0
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-

Table 35: P-Value Performance for Median, Model B, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	51.5	48.5	44.5	55.0
	RJAP	7.5	7.0	5.0	7.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	96.0	96.5	97.5	95.0
	FP	-	-	-	-
	xsFX	94.0	94.0	96.0	94.0
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	-	1.0	-	-
	RJAP	-	-	-	-
	iUS	-	98.5	99.0	97.5
	iJAP	-	-	-	-
	DELS	98.0	97.5	97.0	98.5
	FP	82.5	76.5	75.5	77.5
	xsFX	96.5	96.5	97.0	98.0

Table 36: P-Value Performance for Median, Model A, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	60.0	58.5	52.5	52.0
	RJAP	27.5	38.5	29.5	31.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	0.5	0.5	1.5	0.5
	RJAP	-	1.0	2.0	1.0
	iUS	92.0	79.0	66.0	44.0
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-

Table 37: P-Value Performance for Median, Model B, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	26.5	16.5	21.5	14.0
	RJAP	2.0	0.5	0.5	0.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	97.5	96.0	99.0	98.5
	FP	-	-	-	-
	xsFX	96.5	95.5	97.5	97.0
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	0.5	0.5	-	-
	RJAP	-	-	-	-
	iUS	92.5	86.5	85.5	83.0
	iJAP	-	-	-	-
	DELS	99.0	98.0	98.5	96.0
	FP	51.0	40.5	34.5	29.0
	xsFX	99.0	97.0	98.5	96.0

Table 38: P-Value Performance for Std. Dev., Model A, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	86.0	86.5	88.5	84.0
	RJAP	97.0	97.0	97.0	97.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	7.5	4.5	8.0	11.5
	RJAP	15.5	20.5	20.5	15.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	98.5	99.5
	FP	-	-	-	-
	xsFX	-	-	98.5	99.5
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	-	-	-	-
	RJAP	0.5	-	-	-
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	99.5	99.5	-	99.5
	FP	1.0	-	-	0.5
	xsFX	99.5	99.5	-	99.5

Table 39: P-Value Performance for Std. Dev., Model B, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	RUS	44.0	39.0	43.5	50.0
	RJAP	46.0	45.0	56.0	50.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	90.5	93.5	94.0	96.0
	FP	-	-	-	-
	xsFX	91.0	93.5	94.0	96.0
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	RUS	1.5	1.5	2.0	-
	RJAP	0.5	0.5	1.0	1.0
	iUS	3.5	1.5	5.0	1.5
	iJAP	-	-	-	-
	DELS	94.0	95.5	93.0	95.5
	FP	-	-	-	-
	xsFX	94.5	95.5	93.0	95.5
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	RUS	-	-	-	-
	RJAP	-	-	-	-
	iUS	-	-	-	0.5
	iJAP	-	-	-	-
	DELS	91.5	94.5	94.0	92.5
	FP	-	0.5	-	0.5
	xsFX	91.0	94.5	93.0	89.5

Table 40: P-Value Performance for Std. Dev., Model A, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	89.5	93.5	95.5	91.5
	RJAP	99.5	-	-	99.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	99.5	98.0	-	-
	FP	-	-	-	-
	xsFX	99.5	98.0	-	-
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	80.5	70.0	72.5	73.5
	RJAP	95.0	95.5	92.0	95.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	99.5	-	-
	FP	-	-	-	-
	xsFX	-	99.5	-	-
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	22.5	27.0	26.0	25.0
	RJAP	38.5	40.5	38.0	40.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	-	-
	FP	-	-	-	-
	xsFX	-	-	-	-

Table 41: P-Value Performance for Std. Dev., for Std. Dev, Model B, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	RUS	57.0	52.0	53.5	53.5
	RJAP	83.5	85.5	84.0	87.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	95.0	92.5	95.0	98.0
	FP	-	-	-	-
	xsFX	95.0	93.0	95.0	98.0
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	RUS	31.0	31.0	31.0	31.0
	RJAP	33.0	29.0	26.5	29.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	94.5	93.0	90.5	92.5
	FP	-	-	-	-
	xsFX	94.5	93.0	90.5	92.5
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	RUS	3.0	6.0	3.5	2.5
	RJAP	1.0	1.5	0.5	2.0
	iUS	5.0	6.0	9.0	4.5
	iJAP	-	-	-	-
	DELS	92.5	93.0	94.0	93.5
	FP	-	-	-	-
	xsFX	93.0	93.0	94.0	93.5

Table 42: P-Value Performance for Std. Dev., Model A, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	90.5	92.0	83.5	88.0
	RJAP	99.0	99.0	99.0	98.0
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	99.5	98.5	-	-
	FP	-	-	-	-
	xsFX	99.5	98.5	-	-
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	30.5	33.5	30.0	24.5
	RJAP	45.0	47.0	45.0	38.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	98.5	-	-	99.5
	FP	-	-	-	-
	xsFX	98.5	-	-	99.5

Table 43: P-Value Performance for Std. Dev., Model B, delta=0.8

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	RUS	52.5	50.5	52.5	56.0
	RJAP	62.0	58.0	58.5	63.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	95.0	91.5	94.0	92.5
	FP	-	-	-	-
	xsFX	95.0	93.0	94.0	92.5
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	RUS	11.5	9.0	10.5	13.0
	RJAP	2.5	5.0	4.5	5.0
	iUS	38.0	36.0	35.0	48.0
	iJAP	-	-	-	-
	DELS	94.5	95.0	96.0	94.5
	FP	-	-	-	-
	xsFX	94.5	95.5	96.0	94.5

Table 44: P-Value Performance for Std. Dev., Model A, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	78.0	77.5	75.5	82.0
	RJAP	94.5	99.0	96.5	95.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	99.5	99.5	-	-
	FP	-	-	-	-
	xsFX	99.5	99.5	-	-
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	34.5	36.0	40.0	39.5
	RJAP	59.5	54.5	65.0	65.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	-	-	99.5	99.5
	FP	-	-	-	-
	xsFX	-	-	99.5	-

Table 45: P-Value Performance for Std. Dev., Model B, delta=0.5

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	RUS	37.5	27.0	37.0	31.0
	RJAP	43.0	39.0	39.5	31.5
	iUS	-	-	-	-
	iJAP	-	-	-	-
	DELS	94.5	90.5	96.5	94.0
	FP	-	-	-	-
	xsFX	94.5	90.5	96.5	94.5
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	RUS	7.0	11.5	9.0	9.0
	RJAP	2.5	3.0	4.0	1.5
	iUS	26.5	26.5	28.5	36.0
	iJAP	-	-	-	-
	DELS	95.5	93.0	95.0	92.0
	FP	-	-	-	-
	xsFX	95.5	93.0	95.0	92.0



Table 46: Distribution of Generated Means of Regression Statistics, Model A, delta=0.8

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	<i>xsRUS</i>	c	5.013	5.634	6.417	5.836
		b	2.887	2.348	2.147	2.432
		R-squared	0.00889	0.00794	0.00680	0.00777
	<i>xsRJAP</i>	c	-10.705	-10.427	-9.198	-9.133
		b	16.555	15.784	14.522	14.830
		R-squared	0.10785	0.09495	0.09609	0.10223
	<i>xsFX</i>	c	-2.286	-2.356	-1.814	-2.240
		b	1.507	1.442	1.188	1.577
		R-squared	0.01601	0.01588	0.01106	0.01476
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	<i>xsRUS</i>	c	-7.710	-2.967	-8.048	-8.038
		b	7.021	5.584	6.958	6.794
		R-squared	0.01656	0.01219	0.01639	0.01753
	<i>xsRJAP</i>	c	67.877	73.765	63.109	70.102
		b	-8.142	-9.531	-7.257	-8.427
		R-squared	0.01265	0.01788	0.01231	0.01540
	<i>xsFX</i>	c	3.918	4.429	3.842	4.632
		b	-1.638	-1.736	-1.623	-1.732
		R-squared	0.05730	0.06652	0.06601	0.06154
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	<i>xsRUS</i>	c	-8.263	-4.666	-1.737	-10.256
		b	5.279	4.938	4.640	5.859
		R-squared	0.05047	0.05012	0.04546	0.05080
	<i>xsRJAP</i>	c	166.112	160.387	170.212	174.442
		b	-6.896	-6.563	-7.061	-7.138
		R-squared	0.04127	0.03473	0.04295	0.03940
	<i>xsFX</i>	c	3.495	3.215	3.590	3.388
		b	-1.193	-1.164	-1.203	-1.168
		R-squared	0.53219	0.53824	0.52372	0.513

Table 47: Distribution of Generated Means of Regression Statistics, Model B,  $\delta=0.8$ 

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	<i>xsRUS</i>	c	2.183	0.849	0.377	-0.255
		b	10.352	11.057	11.453	12.200
		R-squared	0.00347	0.00362	0.00368	0.00419
	<i>xsRJAP</i>	c	18.680	12.750	2.251	-1.885
		b	3.840	7.839	14.929	21.839
		R-squared	0.00051	0.00115	0.00372	0.00778
	<i>xsFX</i>	c	3.737	2.374	0.996	-0.347
		b	-1.080	-0.010	1.197	2.413
		R-squared	0.00075	0.00019	0.00039	0.00160
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	<i>xsRUS</i>	c	9.729	9.672	10.499	6.786
		b	5.391	5.153	4.477	6.004
		R-squared	0.01819	0.01792	0.01769	0.02067
	<i>xsRJAP</i>	c	129.158	132.170	131.944	136.203
		b	-17.108	-18.796	-18.499	-18.474
		R-squared	0.048542	0.04849	0.05101	0.04235
	<i>xsFX</i>	c	6.969	7.427	7.439	7.387
		b	-2.087	-2.213	-2.166	-2.134
		R-squared	0.11806	0.11808	0.10816	0.12021
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	<i>xsRUS</i>	c	15.452	14.792	15.502	15.368
		b	5.662	4.959	5.274	5.462
		R-squared	0.05581	0.05173	0.05071	0.05235
	<i>xsRJAP</i>	c	362.791	339.439	369.420	374.305
		b	-13.921	13.203	-13.762	-13.459
		R-squared	0.04771	0.05669	0.05577	0.05052
	<i>xsFX</i>	c	6.538	6.324	6.311	6.866
		b	-1.285	-1.272	-1.270	-1.310
		R-squared	0.47621	0.49121	0.50480	0.450411

Table 48: Distribution of Generated Means of Regression Statistics, Model A, delta=0.5

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	<i>xsRUS</i>	c	7.711	7.114	5.634	7.040
		b	0.269	1.276	2.631	0.238
		R-squared	0.00791	0.00661	0.00966	0.00885
	<i>xsRJAP</i>	c	-2.838	-2.868	-3.264	-2.209
		b	16.297	16.625	16.888	14.774
		R-squared	0.09697	0.10283	0.11058	0.09840
	<i>xsFX</i>	c	-1.593	-1.472	-1.540	-1.351
		b	3.835	3.973	3.738	3.457
		R-squared	0.02594	0.02541	0.02477	0.02458
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	<i>xsRUS</i>	c	2.472	4.032	3.652	3.558
		b	5.207	3.920	4.187	3.911
		R-squared	0.00351	0.00167	0.00257	0.00239
	<i>xsRJAP</i>	c	-12.122	-11.501	-14.246	-14.067
		b	17.787	16.477	18.588	17.898
		R-squared	0.03477	0.03185	0.03646	0.03775
	<i>xsFX</i>	c	-1.917	-2.277	-2.463	-2.192
		b	1.296	1.479	1.634	1.313
		R-squared	0.00274	0.00427	0.00496	0.00441
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	<i>xsRUS</i>	c	-7.776	-1.518	-5.854	-6.003
		b	7.842	6.059	6.874	7.001
		R-squared	0.02024	0.01515	0.01739	0.01691
	<i>xsRJAP</i>	c	53.873	59.204	58.389	58.881
		b	-8.021	9.645	-9.013	-9.315
		R-squared	0.01852	0.02512	0.01972	0.02012
	<i>xsFX</i>	c	4.368	4.281	4.416	4.717
		b	-1.884	-1.899	-1.900	-1.905
		R-squared	0.04952	0.06169	0.05272	0.05872

Table 49: Distribution of Generated Means of Regression Statistics, Model B, delta=0.5

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	<i>xsRUS</i>	c	1.563	5.012	5.614	4.792
		b	16.216	8.718	8.700	7.986
		R-squared	0.00452	0.00521	0.00576	0.00725
	<i>xsRJAP</i>	c	-16.409	-15.548	-17.207	-15.329
		b	55.426	52.489	52.054	46.105
		R-squared	0.03685	0.04419	0.05679	0.06059
	<i>xsFX</i>	c	4.972	-5.447	-5.196	-4.611
		b	16.033	15.758	14.709	12.490
		R-squared	0.02280	0.02859	0.02688	0.03228
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	<i>xsRUS</i>	c	5.873	8.167	3.828	3.994
		b	5.831	4.706	6.852	6.778
		R-squared	0.00765	0.00858	0.00989	0.00904
	<i>xsRJAP</i>	c	59.998	63.310	62.342	62.420
		b	-24.759	-24.706	-24.544	-23.498
		R-squared	0.03297	0.03211	0.02414	0.02437
	<i>xsFX</i>	c	8.191	8.810	8.811	9.063
		b	-4.459	-4.829	-4.843	-4.922
		R-squared	0.03393	0.03181	0.03041	0.03165
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	<i>xsRUS</i>	c	9.612	10.204	9.277	7.394
		b	5.408	4.086	4.501	5.178
		R-squared	0.01751	0.01508	0.01464	0.01798
	<i>xsRJAP</i>	c	119.465	115.057	117.924	113.351
		b	-18.672	-17.929	-19.346	-17.207
		R-squared	0.04286	0.05046	0.04772	0.04702
	<i>xsFX</i>	c	8.063	7.907	8.263	7.466
		b	-2.391	-2.405	-2.440	-2.299
		R-squared	0.10796	0.10259	0.12910	0.10599

Table 50: Distribution of Generated Means of Regression Statistics, Model A,  $\delta=0.8$ 

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	<i>xsRUS</i>	c	6.099	6.076	6.924	6.242
		b	2.917	2.061	1.674	2.141
		R-squared	0.00879	0.00965	0.00961	0.00936
	<i>xsRJAP</i>	c	-6.211	-5.714	-5.119	-5.381
		b	17.156	16.338	15.513	15.367
		R-squared	0.10016	0.10119	0.09669	0.10448
	<i>xsFX</i>	c	-1.589	-1.955	-1.846	-1.526
		b	2.170	2.187	2.192	1.930
		R-squared	0.01386	0.01978	0.01968	0.01528
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	<i>xsRUS</i>	c	-7.614	-6.354	-5.512	-6.756
		b	11.082	7.379	9.514	10.052
		R-squared	0.01859	0.05955	0.01463	0.01428
	<i>xsRJAP</i>	c	48.539	102.592	48.531	49.888
		b	-9.655	-8.095	-9.363	-9.647
		R-squared	0.01056	0.03787	0.01048	0.00959
	<i>xsFX</i>	c	3.991	3.225	3.983	4.490
		b	-1.991	-1.347	-1.943	-2.149
		R-squared	0.02973	0.28850	0.02753	0.02724

Table 51: Distribution of Generated Means of Regression Statistics, Model B,  $\delta=0.8$ 

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	<i>xsRUS</i>	c	8.034	5.460	2.276	0.368
		b	5.425	8.610	11.572	15.593
		R-squared	0.00664	0.00616	0.00309	0.00359
	<i>xsRJAP</i>	c	43.893	40.978	30.166	5.862
		b	-41.089	-37.724	-15.620	17.661
		R-squared	0.02745	0.01500	0.00304	0.00277
	<i>xsFX</i>	c	9.328	8.359	6.346	1.230
		b	-11.102	-8.828	-5.383	2.246
		R-squared	0.02162	0.01435	0.00379	0.00052
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	<i>xsRUS</i>	c	10.676	9.261	8.917	8.422
		b	6.451	7.372	6.317	6.887
		R-squared	0.02234	0.02104	0.01828	0.01998
	<i>xsRJAP</i>	c	85.781	89.207	85.985	83.815
		b	-21.034	-22.448	-21.315	-19.992
		R-squared	0.05360	0.05147	0.06111	0.05277
	<i>xsFX</i>	c	6.570	6.919	6.362	6.399
		b	-2.801	-2.939	-2.774	-2.787
		R-squared	0.09684	0.10085	0.11997	0.11163

Table 52: Distribution of Generated Means of Regression Statistics, Model A, delta=0.5

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	<i>xsRUS</i>	c	7.328	6.943	8.430	6.426
		b	2.878	2.336	1.388	3.122
		R-squared	0.00205	0.00219	0.00299	0.00293
	<i>xsRJAP</i>	c	-14.422	-14.282	-14.765	-15.932
		b	23.554	22.552	22.749	22.308
		R-squared	0.04987	0.05607	0.05580	0.06524
	<i>xsFX</i>	c	-2.911	-2.563	-2.929	-2.922
		b	2.998	2.405	2.640	2.359
		R-squared	0.00986	0.00770	0.00910	0.01090
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	<i>xsRUS</i>	c	-6.581	-6.344	-4.348	-4.172
		b	9.815	9.506	7.803	8.019
		R-squared	0.01964	0.01654	0.01539	0.01786
	<i>xsRJAP</i>	c	47.359	45.613	47.562	45.887
		b	-11.122	-10.378	-10.523	-9.235
		R-squared	0.01851	0.01510	0.02110	0.01567
	<i>xsFX</i>	c	4.359	3.991	4.231	4.060
		b	-2.245	-2.101	-2.095	-2.160
		R-squared	0.03820	0.03986	0.04317	0.03814

Table 53: Distribution of Generated Means of Regression Statistics, Model B, delta=0.5

	xs Return	Statistic	A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	<i>xsRUS</i>	c	5.895	6.101	2.895	-0.298
		b	7.730	9.499	10.540	12.768
		R-squared	0.00810	0.01029	0.00991	0.01084
	<i>xsRJAP</i>	c	49.431	52.694	55.707	55.687
		b	-28.070	-31.187	-30.240	-28.479
		R-squared	0.04790	0.04349	0.03347	0.02379
	<i>xsFX</i>	c	8.034	9.803	9.119	8.509
		b	-6.188	-6.798	-6.517	-5.830
		R-squared	0.04703	0.03947	0.03680	0.02461
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	<i>xsRUS</i>	c	7.962	7.341	7.876	5.610
		b	7.462	6.144	5.983	6.873
		R-squared	0.02053	0.02120	0.01775	0.02186
	<i>xsRJAP</i>	c	87.909	86.856	90.699	95.326
		b	-18.679	-19.438	-19.753	-20.227
		R-squared	0.04756	0.05286	0.04709	0.05194
	<i>xsFX</i>	c	7.503	6.866	7.256	7.8767
		b	-2.744	-2.656	-2.795	-2.796
		R-squared	0.08577	0.09744	0.09179	0.08222



Table 54: P-value Performance, Model A,  $\delta=0.8$ , US Equity Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	68.5	66.0	66.5	67.0
$\rho = -3$	b	25.5	26.0	28.5	27.0
$\delta = 0.8$	R-squared	99.5	99.5	-	99.0
$\alpha = -1$	c	82.5	77.5	84.5	79.0
$\rho = -6$	b	2.5	4.0	4.5	6.0
$\delta = 0.8$	R-squared	-	99.5	-	99.5
$\alpha = -1$	c	77.0	75.5	72.5	81.0
$\rho = -9$	b	-	0.5	0.5	-
$\delta = 0.8$	R-squared	85.5	88.5	87.0	86.5
$\alpha = -2$	c	66.0	67.5	62.5	67.0
$\rho = -3$	b	32.0	32.0	35.5	29.5
$\delta = 0.8$	R-squared	99.0	99.0	99.0	99.0
$\alpha = -2$	c	89.5	87.5	88.0	87.0
$\rho = -6$	b	3.0	3.5	3.0	3.0
$\delta = 0.8$	R-squared	99.5	-	-	-

Table 55: P-value Performance, Model B,  $\delta=0.8$ , US Equity Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	88.5	95.0	99.5	-
$\rho = -3$	b	3.5	-	-	-
$\delta = 0.8$	R-squared	-	-	-	-
$\alpha = -1$	c	50.5	51.0	49.5	57.0
$\rho = -6$	b	10.0	13.5	13.5	12.5
$\delta = 0.8$	R-squared	96.0	99.0	96.5	97.5
$\alpha = -1$	c	40.0	36.5	39.5	38.5
$\rho = -9$	b	1.0	0.5	1.5	1.0
$\delta = 0.8$	R-squared	84.0	86.0	85.0	85.5
$\alpha = -2$	c	57.5	62.5	71.0	99.5
$\rho = -3$	b	38.0	34.5	22.0	-
$\delta = 0.8$	R-squared	-	-	-	-
$\alpha = -2$	c	47.5	52.0	56.0	58.5
$\rho = -6$	b	8.0	7.5	10.0	7.0
$\delta = 0.8$	R-squared	94.0	97.5	95.5	94.5

**Table 56: P-value Performance, Model A, delta=0.5, US Equity Excess Returns**

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	c b R-squared	59.0 45.5 99.0	66.0 38.0 99.5	73.0 33.5 98.0	66.5 43.0 96.0
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	c b R-squared	85.5 4.5 -	76.5 9.5 -	72.0 16.5 -	75.5 15.0 -
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	c b R-squared	85.5 5.0 99.5	80.5 6.0 99.0	82.5 4.5 98.0	84.0 3.5 -
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	c b R-squared	64.5 29.0 -	62.0 32.0 99.5	58.0 34.0 99.5	67.0 27.0 -
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	c b R-squared	86.5 4.5 99.0	87.0 7.0 99.5	87.0 5.5 99.5	88.5 3.5 99.5

**Table 57: P-value Performance, Model B, delta=0.5, US Equity Excess Returns**

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.5$	c b R-squared	70.5 29.0 -	62.5 34.0 -	61.5 33.0 -	61.5 35.5 99.5
$\alpha = -1$ $\rho = -6$ $\delta = 0.5$	c b R-squared	58.0 30.5 99.0	59.5 25.0 -	65.5 26.5 -	59.0 28.0 -
$\alpha = -1$ $\rho = -9$ $\delta = 0.5$	c b R-squared	52.0 14.5 97.0	50.0 17.5 95.0	52.0 15.0 97.0	56.0 9.5 97.5
$\alpha = -2$ $\rho = -6$ $\delta = 0.5$	c b R-squared	65.0 22.5 96.5	62.5 27.0 98.0	70.0 23.0 99.5	72.5 20.5 99.5
$\alpha = -2$ $\rho = -9$ $\delta = 0.5$	c b R-squared	58.0 8.0 95.0	61.5 10.5 97.0	58.5 7.5 95.0	60.0 9.5 94.0

Table 58: P-value Performance, Model A, delta=0.8, Japanese Equity Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	c b R-squared	86.5 1.0 41.5	85.0 2.0 44.5	86.0 2.0 44.0	83.0 2.5 44.5
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	c b R-squared	- 98.5 99.5	1.0 98.0 99.5	0.5 97.5 -	- 99.0 -
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	c b R-squared	- 99.0 83.5	- - 89.0	- 99.5 81.5	- - 82.5
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	c b R-squared	74.5 4.0 42.0	69.0 3.0 43.0	64.0 4.5 45.5	68.5 3.5 42.0
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	c b R-squared	- 96.0 99.5	- 98.5 99.5	- 95.5 -	- 97.0 -

Table 59: P-value Performance, Model B, delta=0.8, Japanese Equity Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$ $\rho = -3$ $\delta = 0.8$	c b R-squared	1.0 56.5 -	1.0 31.5 -	- - -	26.5 - -
$\alpha = -1$ $\rho = -6$ $\delta = 0.8$	c b R-squared	- 97.5 79.5	- 99.5 78.0	- 97.0 80.5	- 97.5 82.0
$\alpha = -1$ $\rho = -9$ $\delta = 0.8$	c b R-squared	- - 77.0	- 99.0 68.5	- 99.0 76.5	- 99.0 80.5
$\alpha = -2$ $\rho = -3$ $\delta = 0.8$	c b R-squared	0.5 95.5 99.5	1.0 95.0 -	2.0 83.0 -	0.5 0.5 -
$\alpha = -2$ $\rho = -6$ $\delta = 0.8$	c b R-squared	- 98.5 72.5	- 99.0 74.5	- 99.0 73.5	- 98.5 75.0

**Table 60: P-value Performance, Model A, delta=0.5, Japanese Equity Excess Returns**

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	52.5	56.5	61.5	43.5
$\rho = -3$	b	3.0	5.0	3.5	4.0
$\delta = 0.5$	R-squared	45.5	42.5	36.0	49.5
$\alpha = -1$	c	97.0	95.0	92.5	92.5
$\rho = -6$	b	1.0	-	-	-
$\delta = 0.5$	R-squared	-	-	-	98.0
$\alpha = -1$	c	0.5	-	-	0.5
$\rho = -9$	b	99.0	98.0	98.0	99.5
$\delta = 0.5$	R-squared	99.0	97.5	99.5	-
$\alpha = -2$	c	94.0	93.5	91.5	97.0
$\rho = -6$	b	-	0.5	0.5	-
$\delta = 0.5$	R-squared	91.0	84.0	80.5	73.5
$\alpha = -2$	c	-	0.5	-	0.5
$\rho = -9$	b	97.0	97.5	98.5	97.5
$\delta = 0.5$	R-squared	99.0	99.5	98.0	99.0

**Table 61: P-value Performance, Model B, delta=0.5, Japanese Equity Excess Returns**

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	93.0	92.0	93.5	89.0
$\rho = -3$	b	0.5	0.5	2.0	2.0
$\delta = 0.5$	R-squared	93.5	87.5	75.0	69.0
$\alpha = -1$	c	1.0	0.5	-	0.5
$\rho = -6$	b	95.0	95.5	96.0	94.0
$\delta = 0.5$	R-squared	90.5	93.5	96.0	96.0
$\alpha = -1$	c	0.5	1.0	-	-
$\rho = -9$	b	98.5	95.5	97.5	98.5
$\delta = 0.5$	R-squared	80.5	75.0	79.5	85.0
$\alpha = -2$	c	-	-	-	0.5
$\rho = -6$	b	97.5	96.0	97.0	94.0
$\delta = 0.5$	R-squared	82.5	83.0	90.0	97.0
$\alpha = -2$	c	-	0.5	-	-
$\rho = -9$	b	97.0	97.5	99.0	99.5
$\delta = 0.5$	R-squared	78.0	74.5	79.0	79.0

Table 62: P-value Performance, Model A, delta=0.8, Foreign Exchange Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	-	99.5	-	-
$\rho = -3$	b	1.5	1.0	0.5	-
$\delta = 0.8$	R-squared	91.5	92.5	95.0	89.5
$\alpha = -1$	c	98.0	98.5	98.0	97.0
$\rho = -6$	b	4.5	4.5	4.5	6.5
$\delta = 0.8$	R-squared	52.5	44.5	46.5	50.0
$\alpha = -1$	c	99.0	99.5	-	99.0
$\rho = -9$	b	-	-	-	-
$\delta = 0.8$	R-squared	0.5	1.5	-	1.0
$\alpha = -2$	c	99.5	-	45.5	-
$\rho = -3$	b	2.5	1.5	-	2.0
$\delta = 0.8$	R-squared	91.5	91.5	1.0	91.5
$\alpha = -2$	c	97.5	97.0	-	96.5
$\rho = -6$	b	21.0	19.0	12.0	17.0
$\delta = 0.8$	R-squared	77.0	78.5	80.5	86.5

Table 63: P-value Performance, Model B, delta=0.8, Foreign Exchange Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	97.5	99.5	99.0	-
$\rho = -3$	b	11.5	0.5	-	-
$\delta = 0.8$	R-squared	-	-	-	-
$\alpha = -1$	c	88.0	87.0	90.5	90.5
$\rho = -6$	b	14.0	18.5	16.0	12.0
$\delta = 0.8$	R-squared	21.5	24.0	25.0	27.0
$\alpha = -1$	c	93.0	97.0	95.5	91.0
$\rho = -9$	b	0.5	-	-	0.5
$\delta = 0.8$	R-squared	4.5	4.0	3.0	2.5
$\alpha = -2$	c	77.0	81.5	91.5	-
$\rho = -3$	b	88.0	85.0	66.5	-
$\delta = 0.8$	R-squared	89.5	95.0	-	-
$\alpha = -2$	c	94.0	94.0	95.0	93.5
$\rho = -6$	b	41.0	46.5	39.5	40.5
$\delta = 0.8$	R-squared	33.5	31.0	23.0	31.5

Table 64: P-value Performance, Model A, delta=0.5, Foreign Exchange Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	99.5	-	-	-
$\rho = -3$	b	3.0	2.0	1.5	3.0
$\delta = 0.5$	R-squared	82.5	83.0	80.5	85.0
$\alpha = -1$	c	-	-	-	-
$\rho = -6$	b	-	-	-	-
$\delta = 0.5$	R-squared	-	-	-	-
$\alpha = -1$	c	96.5	99.0	97.0	98.0
$\rho = -9$	b	11.5	10.0	9.0	11.0
$\delta = 0.5$	R-squared	63.0	49.0	56.0	52.5
$\alpha = -2$	c	-	-	-	-
$\rho = -6$	b	-	0.5	-	-
$\delta = 0.5$	R-squared	99.5	99.5	-	98.0
$\alpha = -2$	c	99.0	97.5	97.5	98.0
$\rho = -9$	b	25.0	21.5	27.0	24.5
$\delta = 0.5$	R-squared	72.0	71.5	66.5	70.0

Table 65: P-value Performance, Model B, delta=0.5, Foreign Exchange Excess Returns

		A=1.0	A=0.75	A=0.50	A=0.25
$\alpha = -1$	c	99.0	99.5	-	-
$\rho = -3$	b	1.0	1.0	1.5	0.5
$\delta = 0.5$	R-squared	94.0	84.0	83.0	74.5
$\alpha = -1$	c	82.5	78.0	73.0	75.0
$\rho = -6$	b	74.5	77.0	75.0	75.0
$\delta = 0.5$	R-squared	71.5	74.5	78.0	75.0
$\alpha = -1$	c	87.5	87.5	90.0	88.0
$\rho = -9$	b	28.0	25.0	21.0	23.5
$\delta = 0.5$	R-squared	29.5	29.0	22.5	28.5
$\alpha = -2$	c	85.0	78.0	80.5	77.0
$\rho = -6$	b	82.0	85.5	88.0	76.5
$\delta = 0.5$	R-squared	63.0	71.5	73.5	84.5
$\alpha = -2$	c	94.0	92.0	92.0	87.5
$\rho = -9$	b	38.5	38.5	40.0	43.5
$\delta = 0.5$	R-squared	35.5	35.0	33.5	39.0