Université de Montréal

Essais en économie des contrats et des institutions

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Thèse présentée à la Faculté des études supérieures

en vue de l'obtention du grade de

PhilosophiæDoctor (Ph.D.)

en sciences économiques

Juillet 1999

Université de Montréal

Faculté des études supérieures

Cette thèse intitulée :

Essais en économie des contrats et des institutions

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Thèse acceptée le : Vendredi 24 septembre 1999.

Sommaire

La thèse étudie le comportement d'agents économiques liés par un accord de coopération. La méthode utilisée est basée sur la théorie des jeux et la théorie des contrats. Par la signature d'un contrat, l'adhésion à une norme sociale ou une institution, les agents expriment leur volonté d'améliorer leur situation économique par une meilleure allocation des ressources. Ils définissent les règles du jeu dans lequel ils vont interagir par la suite. Ces règles influencent leur comportement et, en bout de course, l'équilibre économique.

Le premier essai analyse l'organisation des activités de recherche et développement (R&D). Une innovation peut être soit produite à l'interne, soit achetée à l'externe par la firme qui l'utilise. Nous caractérisons le choix contractuel optimal en abordant les aspects de renégociation et de collusion entre les différentes parties.

Le second essai porte sur les accords informels de partage de risque au sein de la famille élargie dans les pays en voie de développement. De tels accords sont respectés sans qu'ils n'aient de reconnaissance juridique. Nous proposons un mécanisme de sanction sociale qui explique comment et pourquoi les plus riches subventionnent les plus pauvres. Nous analysons l'impact de ce mécanisme sur la forme de l'accord.

Le troisième essai étudie les accords de partage d'un fleuve entre riverains.

Nous caractérisons l'allocation optimale de l'eau. Nous identifions la distribution stable et équitable du surplus généré par une exploitation optimale du fleuve. Nous discutons ensuite de la mise en pratique de cette distribution dans différents environnements institutionnels.

Résumé

Le premier chapitre analyse dans les conditions pour lesquelles les activités de R&D devraient être produites par une unité de recherche indépendante ou par l'entreprise qui utilise l'innovation. Les deux types d'organisation sont courantes. Par exemple, de grandes firmes pharmaceutiques ont leur propre département de recherche. Elles signent également des alliances technologiques avec les entreprises de biotechnologies afin de développer des innovations.

La production d'une innovation met en relation deux agents : une unité de recherche et une firme qui utilise la nouvelle technologie ou vend le nouveau produit. L'unité de recherche détient une information privée sur les propriétés de l'innovation : le coût de son développement, les performances de la nouvelle technologie ou la profitabilité du nouveau produit. La R&D est organisée dans deux types de structures. Dans une structure intégrée, l'entreprise qui utilise l'innovation ou vend le nouveau produit finance et dirige les activités de recherche et développement. L'unité de recherche est alors une division de cette firme. La recherche se fait à l'interne. Dans une structure indépendante, l'entreprise achète l'innovation à l'unité de recherche. Le financement de la recherche se fait par une tierce partie, le partenaire financier. La gestion de la recherche et développement est déléguée à l'unité de recherche. La recherche se fait à l'externe.

La seule présence d'une asymétrie d'information ne permet pas de différencier les deux structures. La différence apparaît lorsque les agents peuvent renégocier

le contrat ou former une collusion. Dans la structure intégrée, les parties ont un intérêt commun à renégocier le contrat une fois que l'information est communiquée par l'unité de recherche à la direction. Dans la structure indépendante, la renégociation est écartée en délégant la prise de décision à la partie informée. Par contre, comme cette structure regroupe trois agents. L'unité de recherche contracte successivement avec un partenaire financier puis avec la firme qui utilise l'innovation. Une collusion entre deux agents au dépend d'un autre est donc envisageable. L'unité de recherche et la firme ont un intérêt commun de manipuler l'information transmise au partenaire financier.

Renégociation et collusion diminuent l'efficacité de l'organisation. Leur impact sur chacune des structures dépend des caractéristiques de la technologie. Nous montrons que la structure intégrée domine lorsqu'une innovation plus coûteuse à développer est aussi une technologie moins performante ou un produit moins profitable. La structure indépendante domine dans le cas contraire. Nous discutons ensuite de l'implication du résultat pour certaines industries. En particulier, notre modèle prédit que les innovations auront tendance à être produites à l'interne dans l'industrie pharmaceutique et à l'externe dans l'industrie des télécommunications et de l'informatique.

Le second chapitre porte sur les accords informels de partage de risque au sein de la famille élargie africaine. Ce système informel d'assurance sociale bénéficie à tout individu riscophobe. Une particularité intéressante de ce système est qu'il subsiste sans cadre légal. Un individu riche qui ne verse pas son dû à la famille ne sera aucunement inquiété par la justice. Qu'est-ce qui le motive à prendre à sa charge ses parents les plus pauvres ?

La solidarité familiale africaine est modélisée comme une norme sociale. C'est un contrat implicite imposé ex ante par l'autorité de la communauté pour améliorer le bien-être de tous. Son exécution est rendue possible ex post grâce à un mécanisme de sanction mutuelle basé sur le statut social. Tout individu qui ne se conforme pas à la norme social est sanctionné par les membres de la communauté qui la respectent. Cette sanction prend la forme d'une perte de statut social. Elle est d'autant plus intense qu'une proportion importante de la communauté suit la norme.

Nous supposons que chaque individu détient une information privée : la valeur qu'il affecte à son statut social. Certains agents accordent davantage d'importance à leur place dans la société que d'autres. La perte de statut social a un effet dissuasif sur un agent lorsque l'importance qu'il accorde à son statut social est suffisamment élevée. Les individus qui accordent peu de valeur à l'opinion des autres préféreront ne pas obéir à la norme.

Les agents souhaiteraient s'assurer complètement contre les aléas de leur revenu. Mais une telle norme requière un transfert élevé. Nombreux sont alors ceux qui, parmi les riches, ne vont pas respecter une norme trop exigeante. La défection de ces non-conformistes est coûteuse. Elle diminue la performance de la norme. La meilleure norme sociale maximise le partage de risque tout en minimisant la défection.

Nous montrons que la norme sociale est pratiquée dans un équilibre de Nash. La pleine assurance n'est possible seulement si tout le monde obéit à cette norme. Dans le cas contraire, la désobéissance ex post à la norme limite le partage de risque. Il peut même y avoir de la défection à l'équilibre. La norme apparaît alors comme un optimum de second rang contraint par la capacité de sanction mutuelle.

Les agents qui ne respectent pas la norme à l'équilibre de Nash bénéficient de l'assurance mutuelle informelle sans en subir les coûts. La norme sociale tolère parfois la présence de ces passagers clandestins. Nous montrons qu'on ne peut exclure les non-conformistes en leur laissant ex ante le libre choix d'adhérer ou non à la norme : il existe un équilibre parfait en sous-jeu avec une adhésion unanime à la norme ex ante et de la défection ex post.

Dans ce contexte, nous discutons de l'effet et de l'efficacité des politiques économiques. Une redistribution publique des revenus n'est pas nécessairement plus performante que la norme sociale pour partager le risque. Une telle politique génère ses propres coûts qui doivent être comparés aux coûts de la défection à la norme. Elle est néfaste à l'économie lorsqu'elle se substitue à une norme sociale plus performante pour atteindre un même objectif.

Le troisième chapitre porte sur le partage d'un fleuve entre riverains en situation de pénurie d'eau. Nous introduisons un modèle général qui s'applique au cas des fleuves internationaux.

Un fleuve est une ressource naturelle exploitée par plusieurs usagers. Sa particularité réside au fait que l'eau est inégalement distribuée le long du fleuve et qu'elle ne peut être transférée que de l'amont vers l'aval. Les usagers situés le long du fleuve extraient l'eau pour leur consommation personnelle. Ils s'échangent également un autre bien, la monnaie. Le modèle peut également être interprété en terme de pollution de l'eau.

L'exploitation en libre accès du fleuve est inefficace parce que les usagers situés en amont consomment trop d'eau. Les agents ont tous à gagner à redéfinir un mode de gestion du fleuve par un accord de coopération. Un tel accord spécifie une allocation de l'eau et un schéma de transferts entre agents.

Dans une première partie, nous identifions l'allocation optimale de l'eau. Nous n'obtenons pas toujours le résultat classique de l'exploitation optimale en propriété commune d'une ressource naturelle, à savoir, l'égalité des bénéfices marginaux des usagers. En effet, la rareté locale de l'eau peut contraindre sa répartition. Lorsqu'il y a trop peu d'eau en amont, cette égalité n'est pas réalisable car on ne peut faire remonter l'eau vers la source. Dans ce cas, les bénéfices marginaux des riverains sont égaux entre les contraintes de rareté. Mais entre deux contraintes, le bénéfice marginal diminue de l'amont vers l'aval.

Dans une deuxième partie, nous analysons les transferts de monnaie entre agents. Afin que l'accord de coopération soit accepté par tous, le schéma de transferts doit répondre à deux critères. Premièrement, il doit être tel que toute coalition d'agents préfère joindre l'accord de coopération global plutôt que de faire bande à part. Deuxièmement, il doit être perçu comme équitable.

Comme à tout schéma de transferts correspond un partage du bien-être de l'exploitation du fleuve, l'analyse des transferts peut se faire en terme de distribution du bien-être. Nous caractérisons le bien-être que peut se garantir une coalition arbitraire d'agents. Nous imposons une première contrainte sur la distribution de bien-être : elle doit assurer à tout groupe d'agents au moins ce qu'il peut se garantir. Autrement dit, elle doit appartenir au noyau du jeu.

Le jeu étant convexe, son noyau est composé de l'enveloppe convexe des vecteurs de contributions marginales. Nous introduisons un principe d'équité qui réduit le choix des distributions de bien-être. Une distribution de bien-être est considérée comme équitable lorsqu'aucun groupe d'agents n'obtient plus que ce qu'il aurait en l'absence des autres. Nous montrons qu'il n'existe qu'une seule distribution de bien-être qui soit à la fois stable (au sens où elle appartient au noyau) et équitable : la distribution incrémentale aval. Elle assigne à chaque agent sa contribution marginale à la coalition composée de ses prédécesseurs sur le fleuve.

Nous discutons ensuite de la décentralisation et de l'implémentation de la distribution de bien-être incrémentale aval. Une taxe Pigouvienne amène les usagers à extraire l'allocation optimale de l'eau. En reversant le montant collecté par un schéma de subventions forfaitaires, on établit la stabilité et l'équité. Nous montrons que, donner un droit de propriété à chaque agent sur la part de la ressource qu'il contrôle ne permet pas de converger vers la distribution de bien-être incrémentale aval dans une équilibre de marché concurrentiel. Enfin, nous proposons des règles de négociation qui permettent d'obtenir la distribution de bien-être désirée dans un équilibre parfait en sous-jeu. Ces règles donnent la priorité aux agents situés en aval. Elles pourraient êtres incluses dans une institution de gestion d'un fleuve international.

Remerciements

Je tiens à remercier mes directeurs de recherches, Michel Poitevin et Yves Sprumont pour leurs commentaires et leurs encouragements. Je remercie également tous ceux qui se sont donnés la peine de lire un des essais de ma thèse et qui m'ont fait part de leur commentaires. Je pense en particulier à Marcel Fafchamps, Gérard Gaudet, John Hartwick, André Martens, Jean-Philippe Platteau, Yves Richelle, Martin Ruckes et Michel Truchon. Enfin, au cours de mon doctorat, j'ai bénéficié de discutions fructueuses avec plusieurs de mes collègues et amis étudiants, notamment Karine Gobert, Louis Hotte, Paul Johnson, Marcel Rindisbacher, Nicolas Treich, Lars Vilhuber, Abdoul Aziz Wane et bien d'autres encore. Merci pour tout ces instants qui ont fait avancer mes connaissances.

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Introduction Générale

Les agents économiques, consommateurs ou firmes, interagissent dans des environnements complexes. Ils coordonnent leurs actions dans un cadre spécifié par un contrat dans le but d'améliorer leur bien-être. Par exemple, les activités de production sont organisées par une multitude de contrats de financement, d'impartition, de travail et de vente. Chaque agent détient une partie de l'information sur l'économie. Chaque joueur choisit parmi un ensemble de stratégies selon ses propres objectifs.

Le modèle de design de mécanisme à la Myerson (1979) donne un cadre d'analyse général à la théorie des contrats. Un mécanisme définit les règles du jeu dans lequel les agents vont interagir. Il est proposé par une autorité centrale, le principal. Il est ensuite accepté ou rejeté par les agents. Son exécution est supervisée par le principal. En ce sens, il peut être interprété comme un contrat. Le modèle de Myerson suppose que chaque agent détient une information privée sur ses préférences. Elle peut être utilisée à des fins stratégiques par les agents lors de l'exécution du contrat. Pour le principal, tout le problème réside à identifier le mécanisme qui maximise le bien-être de cette économie tout en prenant en compte cette asymétrie d'information.

Dans ce modèle, il est facile de caractériser le mécanisme optimal. D'après le principe de la révélation, on peut, sans perte de généralité, se limiter aux mécanismes directs révélateurs. Un mécanisme est direct s'il exige que chaque agent communique son information privé. Il est révélateur lorsque les agents sont incités à dire la vérité. Le mécanisme optimal est le meilleur mécanisme direct révélateur. Il maximise le bien-être sous la contrainte qu'il soit unanimement accepté et qu'il amène les agents à révéler de leur propre fait leur information. Le design de mécanisme a été appliquée à une multitude de problèmes économiques.

Le modèle de Myerson repose sur des hypothèses fortes. Premièrement, au cours du jeu, les agents ne peuvent, à aucun moment, renégocier les termes du contrat, et ce, même s'ils ont un intérêt mutuel à le faire. Deuxièmement, ils ne peuvent communiquer entre eux. Aucun groupe d'agent ne peut écrire son propre contrat, ni coordonner la divulgation de l'information. Troisièmement, tous les agents respectent les termes du contrats lors de son application. Aucun agent ne peut rompre sa relation avec ses partenaires.

D'un point de vue théorique, la thèse relâche certaines de ces hypothèses.

Le premier chapitre autorise la renégociation d'un contrat avant son exécution ainsi que la collusion entre agents lors de la divulgation de l'information. Il analyse la question de l'organisation optimale des activités de recherche et développement.

La production d'une innovation met en relation deux agents : une unité de recherche et une firme qui utilise la nouvelle technologie ou vend le nouveau produit. L'unité de recherche détient une information privé sur les propriétés de l'innovation : le coût de son développement, les performances de la nouvelle technologie ou la profitabilité du nouveau produit. La R&D est organisée dans deux types de structures. Dans une structure intégrée, l'entreprise qui utilise l'innovation ou vend le nouveau produit finance et dirige les activités de recherche et développement. L'unité de recherche est alors une division de cette firme. La recherche se fait à l'interne. Dans une structure indépendante, l'entreprise achète l'innovation à l'unité de recherche. Le financement de la recherche se fait par une tierce partie, le partenaire financier. La gestion de la recherche et développement est déléguée à l'unité de recherche. La recherche se fait à l'externe.

Dans cet contexte, la structure intégrée est un "mécanisme direct". L'unité de recherche transmet son information à la firme qui décide ensuite de l'importance du projet de développement. Par contre, la structure indépendante est un "mécanisme indirect". La firme délègue la décision de production à l'unité de recherche. L'information est révélée une fois que l'innovation est développée par la taille de l'unité de production mise en place.

Le principe de la révélation nous dit que tout mécanisme indirect peut être reproduit par un mécanisme direct. Par conséquent, les deux types de mécanismes sont équivalents. La question ne se pose donc pas. Ce principe devient caduque lorsque l'on introduit la renégociation et la collusion.

Cet essai fait apparaître l'arbitrage entre communication et délégation dans une organisation. D'un côté, une organisation hiérarchique de la production engendre davantage de communication. La direction et la division de la firme ont alors l'opportunité de renégocier les termes du contrat avant son exécution. Dans le structure intégrée, les agents vont vouloir renégocier un fois que l'unité de recherche a transmis son information à la firme, avant que la décision soit prise. D'un autre côté, en délégant ses activités à plusieurs agents, l'organisation gère de multiples contrats. Elle s'expose à une collusion entre ses contractants. Dans la structure indépendante, l'unité de recherche et la firme peuvent s'entendre secrètement pour manipuler l'information au dépend du partenaire financier.

Renégociation et collusion réduisent l'efficacité de l'organisation. Elles limitent l'espace des contrats envisageables. Elles ajoutent des contraintes au programme de maximisation du bien-être des agents. Nous montrons que l'importance des coûts d'agence liés à la renégociation et à la collusion dépend de la technologie. Lorsqu'un innovation plus coûteuse à développer est aussi une technologie moins performante ou un produit moins profitable, la renégociation est un moindre mal par rapport à la collusion. La structure intégrée domine la structure indépendante. Les activités de recherche et développement seront regroupées au sein d'une même firme. Par contre, dans le cas contraire, les coûts d'agence liés à la collusion sont inférieurs à ceux liés à la renégociation. La structure indépendante domine. L'entreprise a plutôt intérêt à faire affaire avec plusieurs partenaires indépendants.

Dans le second chapitre, nous laissons la possibilité aux agents de renier leurs engagements. L'hypothèse est envisageable dans le cas d'accords informels qui, par définition, n'ont pas d'existence légale. Leur exécution n'est donc pas contrôlée par une administration judiciaire. Il revient aux agents eux-mêmes de faire en sorte que les contractants respectent leurs engagements.

La question est particulièrement pertinente pour les accords de partage de risques tels qu'observés en Afrique. Ce type de contrat implique qu'un individu ayant un revenu élevé le partage avec ses parents les plus pauvres. Tout agent riscophobe adhère volontiers à un tel contrat d'assurance mutuelle avant de connaître son revenu. Mais ensuite, quel est l'incitation pour un riche de partager son revenu ? Un transfert de la part d'un individu égoïste ayant un revenu élevé garanti n'est motivé que par une menace de sanction venant de ses pairs. Une punition doit dissuader l'agent de renier ses engagements.

Dans cet essai, le partage de risque informel au sein de la famille élargie africaine est modélisé comme une norme sociale. C'est un contrat implicite imposé par l'autorité de la communauté. L'objectif du principal est la maximisation du bien-être des agents. L'exécution du contrat est ensuite assurée par les agents eux-mêmes. Les agents font pression pour que chaque membre de la famille élargie respecte la norme sociale.

Le mécanisme de sanction mutuelle est basé sur le statut social. Nous supposons que chaque agent valorise l'opinion des autres. Un individu va respecter ses engagements pour conserver son rang dans la société. Le statut social peut également être interprété comme de la culpabilité ou de la honte. Cela peut être toute sorte de punition exercée par la communauté comme la perte du droit à l'héritage ou la sorcellerie.

La punition est supposée avoir un effet limité et hétérogène sur les agents. Chaque individu a son propre goût pour son statut social. Cette variable est une information privée. La menace de sanction a un effet dissuasif sur un agent lorsque l'importance qu'il accorde à son statut social est suffisamment élevé. Certains agents, peu soucieux de l'opinion des autres, préféreront ne pas obéir à la norme.

Dans un premier temps, nous montrons qu'il est rationnel pour des agents égoïstes d'obéir à la norme dans un équilibre de Nash. Ainsi, nous justifions la pratique de transferts venant des riches vers les pauvres. Ensuite, nous caractérisons la norme sociale. Elle apparaît comme la solution d'un arbitrage entre partage de risque ex ante et obéissance à la norme ex post. Idéalement, les agents souhaiteraient s'assurer complètement contre les aléas de leur revenu. Mais une telle norme requiert un transfert élevé de la part des riches. Nombreux sont alors ceux qui, parmi les riches, ne vont pas respecter une norme trop exigeante. La défection de ces non-conformistes est coûteuse. Elle diminue la performance de la norme. La meilleure norme sociale maximise le partage de risque tout en minimisant la défection.

Nous montrons que la pleine assurance est possible seulement lorsque tout le monde obéit à cette norme. Dans le cas contraire, la contrainte de conformité ex post limite le partage de risque ex ante. Il peut même y avoir de la défection à l'équilibre. Les agents qui ne respectent pas la norme bénéficient de l'assurance mutuelle informelle sans en subir les coûts. Il est donc parfois optimal de tolérer la présence de ces passagers clandestins.

Enfin, nous discutons de l'effet et de l'efficacité des politiques économiques. Une redistribution publique des revenus n'est pas nécessairement plus performante que la norme sociale pour partager le risque. Une telle politique génère ses propres coûts qui doivent être comparés aux coûts de la défection à la norme. Elle est néfaste à l'économie lorsqu'elle se substitue alors à une norme sociale plus performante pour atteindre un même objectif.

Dans le troisième chapitre, nous laissons aux agents la liberté de former des coalitions. Ils peuvent communiquer entre eux et, ainsi, signer leur propres accords de coopération. Une coalition d'agents ne va pas souscrire à un contrat global (incluant tous les agents) si elle peut obtenir mieux pour ses membres. Nous imposons une contrainte supplémentaire sur le contrat : il doit être accepté par toute coalition d'agents. Nous supposons également que les agents ont un souci d'équité.

Cet essai analyse un jeu simple d'information parfaite. Son originalité tient à sa structure. Elle reflète les contraintes physiques de la répartition des ressources en eau d'un fleuve. Nous supposons que l'eau est inégalement distribuée le long du fleuve et ne peut être transférée que de l'amont vers l'aval. Les agents consomment l'eau du fleuve et s'échangent un autre bien, la monnaie.

Un accord de coopération spécifie une allocation de l'eau et un schéma de transferts. L'allocation d'eau qui maximise le bien-être de tous les agents peut être caractérisée indépendamment des transferts. C'est le sujet de la première partie. De toute évidence, les agents ont intérêt à mettre en place une allocation optimale de l'eau. Mais l'adhésion des agents à un tel accord ne sera acquis qu'au prix d'un choix judicieux de transferts. Le schema de transferts doit répondre à deux critères. Premièrement, il doit être tel que toute coalition d'agents préfère joindre l'accord de coopération globale plutôt que de faire bande à part. Deuxièmement, il doit être perçu comme équitable.

Comme à tout shéma de transferts correspond un partage du bien-être de l'exploitation du fleuve, l'analyse des transferts peut se faire en terme de distribution du bien-être. Dans le cadre du jeu coopératif naturellement associé au modèle, nous caractérisons le bien-être que peut se garantir une coalition arbitraire d'agents. Nous imposons une première contrainte sur la distribution de bien-être : elle doit assurer à tout groupe d'agents au moins ce qu'il peut se garantir par lui-même. Autrement dit, elle doit appartenir au noyau du jeu, être stable.

Il se trouve que le jeu est convexe, de sorte que son noyau peut entièrement être caractérisé : c'est l'enveloppe convexe des vecteurs de contributions marginales. Il contient une infinité d'éléments. Afin de raffiner notre choix, nous imposons un second critère sur les distributions de bien-être : l'équité. Une distribution de bien-être est considérée comme équitable lorsqu'aucun groupe d'agents n'obtient plus que ce qu'il aurait en l'absence des autres. Nous identifions les distributions de bien-être du noyau qui survivent à ce critère. Nous montrons qu'une seule distribution est à la fois stable et équitable : la distribution incrémentale aval. Elle assigne à chaque agent sa contribution marginale à la coalition composée de ses prédécesseurs sur le fleuve. Nous discutons ensuite de la décentralisation de cette allocation par un mécanisme de taxes, par une affectation adéquate des droits de propriété dans un marché concurrentiel, et de l'implémentation par un jeu de négociation de la distribution de bien-être incrémentale aval.

Le thèse applique la théorie des jeux à l'organisation industrielle, l'économie du développement et l'économie de l'environnement. Elle apporte des réponses concrètes à des problèmes pertinents dans chacun de ces domaines des sciences économiques.

Chapitre 1

Organizational design of R&D activities

Abstract

This essay addresses the question of whether R&D should be carried out by an independent research unit or be produced in-house by the firm marketing the innovation. We define two contractual structures. In an independent structure, the firm that markets the innovation buys it from an independent research unit which is financed externally. In an integrated structure, the firm that markets the innovation also carries out and finances research leading to the innovation. We compare the two structures under the assumption that the research unit has some private information about the real cost of developing the new product. The sole presence of asymmetric information is not sufficient to differentiate the two structures. It is only when players can renegotiate and collude that a difference emerges. When an innovation costly to develop is also a less drastic technology or a product less valued by consumers, the integrated structure dominates. The independent structure dominates in the opposite case.

1.1 Introduction

Research and development activities take place in various organizational forms depending on who finances, creates, develops, produces and sells the innovation. A widely observed organizational form is in-house R&D. Innovation is created within the firm who then uses the new product or the new technology. Researchers-inventors are subject to an employment contract. The innovation is financed, managed and owned by the user firm.

Another organizational structure is "external R&D". Research and development activities are conducted by an independent firm whose objective is to create a new product or a new technology and then develop it with the user firm through a contractual agreement. Innovation is managed and owned by the independent research unit firm and financed by its financial partner, for example, a venture capitalist.

Both organizational structures are observed in many industries. Moreover, the same firm may employ both organizational forms. For example, consider the pharmaceutical industry. The innovation user is the drug firm while an independent research unit is a biotechnology firm. A drug firm like Merck is investing mainly in in-house R&D although some of its major rivals are outsourcing most of their research activities. Only 5% or so of Merck's research spending ends up outside the firm's laboratories. For other top drug companies however, the proportion of research done independently could reach 80%. Recently, American pharmaceutical companies moved from in-house R&D to independent R&D by increasing their research joint venture agreements. These research joint ventures are contractual agreements for developing, producing and selling a new medicine discovered by a biotech firm (Lerner and Merges, 1998). In 1994, 117 ventures between drug and biotechnology firms were signed, 70% more than the previous year[1].

[1] "The Economist", May 13th 1995, pp. 66-67, and May 24th 1997, pp. 59-60.

This empirical evidence highlights an important question. Why are the two organizational forms observed ? If one organization is more efficient than the other one, the inefficient organizational structure should not be observed in equilibrium. The objective of this paper is to provide some economic intuition based on contractual imperfections about the organizational choice of R&D activities.

The economic environment for the research and development activities and the eventual marketing of the innovation is characterized by two main features : uncertainty and informational asymmetries. When working on an innovation, a firm does not know for sure the result of its R&D activities. Research methodologies employed to discover an innovation (what Dosi (1988) calls "technology trajectories") can be specified ex ante but their outcome can hardly be perfectly predicted. For example, in the case of pharmaceutical industries, one favorite research methodology employed is "combinatorial chemistry" which consists in using arbitrary chemical reactions to generate millions of randomly shaped molecules. One of the new discovered molecules might just lead to the next drug. The discovery of a new drug depends on the success of the research process, and its properties (its safety, efficiency, cost effectiveness of treatment) are never known ex ante. Research and development are random activities and, therefore, constitute a risky investment.

Second, the marketing of an innovation is characterized by an asymmetric distribution of information. The value of an innovation depends on its properties such as the new technology's efficiency, the new product's quality or production. While this information is difficult to obtain before the innovation is developed, produced and sold, the research unit may have more information about the cost of bringing the innovation to the market, that is, when the innovation is transferred from its creator to its user. For example, in the pharmaceutical industry, coordination between researcher and factory designers is not easy. Clearly, bringing a new medicine to the market is not trivial and needs cooperation between agents which may not have the same information. A report states that mistakes in the development process increase costs by 40%[2]. Asymmetric information motivates the complexity of research joint venture agreements. Uncertainty and asymmetric information are two basic features of our model. But before describing our model, we review some of the relevant literature.

In the management literature, it is argued that in-house R&D may reduce problems associated with asymmetric information, and that better coordination between innovators, production and marketing departments is achieved within an organization. With its own research unit, a firm has the scientific expertise to evaluate new technologies and new products (Armour and Teece, 1979; Lampel, Miller and Floricel, 1996). This approach assumes that the objective of all units within the firm is to maximize the organization global profit. This may not be true if the units behave non cooperatively or opportunistically. A "selfish" research unit may not behave in accordance with the integrated organization's own interest. For example, a research unit may prefer not to reveal the true value (possibly low) of its discovery if its reward from the innovation does not provide it with such incentives. Hence, integrating the research unit within the user firm does not necessary solve the asymmetric information problem. The solution should be endogenous to the incentives provided by the organizational form, not by the adoption per se of an organizational form.

An incomplete contract approach to R&D management is developed in Aghion and Tirole (1994) in, what they call, a first attempt to open the "black box of innovation". They suppose that R&D is a random activity. Its success depends on an initial investment provided by the innovation user C and an effort supplied by the research unit RU. Since the innovation cannot be described ex ante, the contract can only specify the allocation of property rights when R&D is produced

[2] "The Economist", November 9th, 1996.

in-house. In that case, when R&D is produced in-house, the property right is allocated ex ante to C. When R&D is carried out by an independent research unit, RU owns the innovation and bargains ex post with C over licensing fees. The optimal organizational form of innovation activities depends on the marginal efficiency of RU's effort compared with the marginal efficiency of C's investment, on the ex ante bargaining power of the two parties and on C's financial constraint.

Recent papers pointed out that bureaucratic organizations perform poorly in innovating. In Dearden, Ickes and Samuelson (1990), a centralized structure has low incentives to adopt new technologies because of the ratchet effect. In Quian and Xu (1998), a soft budget constraint and an ex ante heavy evaluation process explain centralized organizations' failure in innovating. A bureaucracy makes mistakes by rejecting promising projects and delaying innovations. In-house R&D produces high cost and ex ante well-specified innovations, but is unable to subsidize less costly projects with higher uncertainty.

The present paper provides a complete contract approach to the organizational design of R&D activities. A contract can be written ex ante contingently on the innovation performance, namely, the development cost, production cost and market value of the innovation. We define two contractual structures. In an integrated structure, the innovation is produced in-house by the firm who then uses or markets it. This firm sets up its own research unit by financing a laboratory and hiring scientists. The contract signed between the firm and the members of the research unit is an employment contract. The manager of the firm has authority over the head of the research unit. He takes the main decisions about the development, production and marketing process of the innovation after considering the advice of its research unit. In an independent structure, the research unit is an independent firm. It is financed by a bank or a venture capitalist. The firm then sells the innovation to another firm by signing a joint-venture agreement or a technological alliance. The research unit installs the new process in a factory, or tests the new product for specific purposes. The user firm then operates the new technology, or produces and markets the new product. Transfers are then paid as prescribed by the financial and joint-venture contracts. The research unit pays back the bank and receives its share of the joint-venture's profit.

The two structures are equivalent when the agents can commit not to renegotiate and not to collude. In the integrated structure, the user firm insures partially the research unit against the uncertainty of the research process. The employment contract gives the research unit incentive to report the true value of the innovation to the manager of the firm. In the independent structure, partial insurance is provided by the financial partner. The research reveals the true value of the innovation by installing the new process or testing the new product. Hence, even under asymmetric information, we show that the two structures yield the same pay-off to the research unit and to the user firm.

The two structures perform differently when agents cannot commit not to renegotiate and not to collude. In the integrated structure, after the research unit reports the innovation quality but before the head of the firm decides the size of the development project, agents have incentive to renegotiate the employment contract. This renegotiation reduces the ex ante efficiency of the integrated structure. In the independent structure, the research unit contracts successively with the bank and the user firm. It may be tempted to secretly agree with the user firm, at the second contracting stage, not to behave as prescribed by the financial contract. That is, to misreport the size of the development project to the bank in order to pay the lower return. The collusion between the research unit and the user firm reduces the ex ante efficiency of the independent structure.

The relative efficiency of the two structures depends on the properties of the innovation. When an innovation costly to develop is also a less drastic technology or a product less valued by consumers, that is, when the marginal cost of developing the innovation is negatively correlated with its marginal profit, the integrated structure dominates the independent structure. The independent structure dominates in the opposite case. This paper therefore characterizes some forces which may explain the choice between external and in-house R&D.

The chapter is organized as follows. Section 1.2 presents the model. Section 1.3 analyses the two organizational structures in the case of symmetric information. In section 1.4, we introduce asymmetric information when agent can commit not to renegotiate and not to collude. We allow renegotiation and collusion in section 1.5. We compare the two strutures performance in section 1.6. Section 1.7 concludes the paper.

1.2 The model

Two agents, a research unit RU and the consumer of the new technology, firm C, coordinate their activities during the R&D process. At the research period, the research unit has access to a random research technology to produce an innovation. When investing I in research, RU obtains a high-quality innovation h with probability $p(I)$ and a low-quality innovation l ($l < h$) with probability $1 - p(I)$. We suppose p increasing and concave, with $p(0) = 0$, $p'(0) = \infty$, $lim_{I \to \infty} p(I) = 1$.

The innovation is marketed by firm C. To sell the innovation, RU and C must operationalize its production. This is the development phase. During that phase, RU incurs a development cost $D(q, \alpha)$ depending on the scale of project q and on the innovation quality α. We assume that D is increasing and convex in q and that total and marginal development costs are decreasing in α :

$$D_q(q, \alpha) > 0,\ D_{qq}(q, \alpha) > 0,\ D(q, h) < D(q, l),\ D_q(q, h) < D_q(q, l)\ \forall q > 0.$$

Following the development phase, C can start producing and marketing the product. C earns a profit $P(q, \alpha)$. The function P is assumed increasing and concave in q at least on $[0, \bar{q}]$ with $\bar{q}$ large.

$$P_q(q, \alpha) > 0, \; P_{qq}(q, \alpha) < 0, \; \forall 0 < q < \bar{q}.$$

This function is general in the way that it includes both process and product innovation. When the innovation is a new technology used by C, the new production cost depends on the innovation quality while the demand for the product may be unchanged. When the innovation is a new product marketed by C, the demand faced by C depends on the innovation quality. We will consider two cases for the effect of the innovation quality on P :

- *Case 1 : Major innovation.*

  A high quality innovation is less costly to develop and has a higher market value and/or is less costly to produce than a low quality one. Formally,

$$P(q, h) > P(q, l), \; P_q(q, h) > P_q(q, l) \; \forall 0 < q < \bar{q}.$$

- *Case 2 : Minor innovation.*

  A high quality innovation is less costly to develop but has a lower market value and/or is more costly to produce than a low-quality one. Formally,

$$P(q, h) < P(q, l), \; P_q(q, h) < P_q(q, l) \; \forall 0 < q < \bar{q}.$$

For an innovation quality $\alpha \in \{l, h\}$, after investing I, the R&D process generates a global profit gross of initial investment of

$$\pi(q, \alpha) = P(q, \alpha) - D(q, \alpha).$$

We denote by q^*_α the production level which maximizes the global profit $\pi(q, \alpha)$. We also denote by I^* the investment level which maximizes $p(I)\pi(q^*_h, h) + (1 - p(I))\pi(q^*_l, l) - I$.

It is assumed that $q^*_h \geq q^*_l$, that is, the optimal production for a high-quality innovation is always higher than for a low quality one.

RU's utility V depends on its income w net of development costs :

$$V(w, q, \alpha) = v(w - D(q, \alpha)).$$

We suppose that RU is risk averse, that is, v is increasing and concave ($v' > 0$, $v'' < 0$). The consumer firm C is risk neutral. Its utility U is linear in profits net of any payment w :

$$U(w, q, \alpha) = P(q, \alpha) - w.$$

The levels of investment and production depend on the organizational design of R&D activities. We define two types of organizations. In an *integrated structure*, R&D activities are conducted internally within firm C. Firm RU can be seen as a divison or a department of firm C. The relationship between RU and C can be modeled as a contract signed at the beginning of the research period between RU and C. This contract specifies an investment level I and an allocation $\{w_\alpha, q_\alpha\}_{\alpha=l}^{h}$ contingent on the innovation quality α, where w is a transfer of resources from C to RU. Firm C then invests and finances I in research, pays its research unit RU a wage w_α and produces q_α when the innovation quality is α. The players' expected utilities are :

- For RU : $E[V(w_\alpha, q_\alpha, \alpha)|I] = p(I)V(w_h, q_h, h) + (1 - p(I))V(w_l, q_l, l)$.
- For C : $E[U(w_\alpha, q_\alpha, \alpha)|I] - I = p(I)U(w_h, q_h, h) + (1 - p(I))U(w_l, q_l, l) - I$.

In an *independent* structure, RU is an autonomous firm, which implies that it must finance its research activities externally. A financial contract is signed at the beginning of the research period between RU and a bank or financial partner F. This contract specifies the investment I provided by F to RU and ex post repayments $\{R_\alpha\}_{\alpha=l}^{h}$ from RU to F contingent on the innovation quality α. After the research period and before the development period, RU sells its innovation to C who markets it. RU and C negotiate a joint-venture agreement which specifies the project size q_α and RU's wage or royalties w_α contingent on innovation quality α. Players' expected utilities are :

- For F : $E[R_\alpha|I] - I = p(I)R_h + (1-p(I))R_l - I$.
- For RU : $E[V(w_\alpha - R_\alpha, q_\alpha, \alpha)|I] = p(I)V(w_h - R_h, q_h, h) + (1-p(I))V(w_l - R_l, q_l, l)$.
- For C : $E[U(w_\alpha, q_\alpha, \alpha)|I] = p(I)U(w_h, q_h, h) + (1-p(I))U(w_l, q_l, l))$.

The objective of this paper is to study the optimal R&D organizational structure under various informational and commitment assumptions. We show that the relative efficiency of the two organizational structures depends on the informational structure as well as on the players ability to commit. The equilibrium allocations are characterized for the following cases :

- Symmetric information (Section 3).
- Asymmetric information with full commitment (Section 4).
- Asymmetric information without commitment (Section 5).

In each case, we compare the relative performance of the two structures. In all games, bargaining power is given to RU and F's and C's reservation utilities are normalized to zero.

1.3 Symmetric information

Before introducing asymmetric information in the model, it is useful to review the benchmark case where both players have full information about innovation quality. The full-information allocation corresponds to the first-best allocation.

1.3.1 The integrated structure

In an integrated structure, RU offers to C a R&D contract which specifies an allocation $\{I,\ \{w_\alpha, q_\alpha\}_{\alpha=l}^{h}\}$. The equilibrium allocation solves the following maximization program.

$\max_{I,\{w_\alpha,q_\alpha\}_{\alpha=l}^{h}} p(I)V(w_h, q_h, h) + (1-p(I))V(w_l, q_l, l)$ s/t

$$p(I)U(w_h, q_h, h) + (1-p(I))U(w_l, q_l, l) - I \ \geq\ 0 \ (RI^C)$$

We assume that firm C cannot default on the contract once it is signed. Its participation constraint only has to hold in expectation. The solution is characterized by the following relationships :

- $q_\alpha = q_\alpha^*$,
- $v(w_h - D(q_h, h)) = v(w_l - D(q_l, l))$,
- $I = I^*$.

First, the risk-neutral player C provides full insurance to the risk-averse player RU using ex post wages. Hence, risk sharing is optimal. Second, for each innovation quality, the production is at the efficient scale q_α^*. Third, the marginal benefits of the R&D investment equal its marginal costs. Investment is therefore efficient. Player C takes all risk and receives all benefits from a high-quality innovation. It then invests optimally.

1.3.2 The independent structure

In a independent structure, RU offers a financial contract $I^B, \{R_\alpha\}_{\alpha=l}^{h}\}$ and a development contract $\{w_\alpha^B, q_\alpha^B\}_{\alpha=l}^{h}\}$ to F and C, respectively, while RU invests I financed by the financier F. The equilibrium allocation solves the following maximization program.

$$\max_{\{I,\{R_\alpha,w_\alpha,q_\alpha\}_{\alpha=h}^{l}\}} p(I)V(w_h,q_h,h)+(1-p(I))V(w_l,q_l,l) \text{ s/t}$$

$$\begin{aligned} p(I)R_h+(1-p(I))R_l-I &\geq 0 \quad (IR^F) \\ U(w_\alpha,q_\alpha,\alpha) &\geq 0 \quad \forall\alpha=l,h \quad (IR_\alpha^C) \end{aligned}$$

The solution is characterized by the following relationships :

- $q_\alpha = q_\alpha^*$,
- $v(w_h - R_h - D(q_h,h)) = v(w_l - R_l - D(q_l,l))$,
- $P(q_\alpha,\alpha) - w_\alpha = 0$,
- $I = I^*$.

First, as in the integrated structure, RU's utility is equal in the two states of nature. The financial agent now provides full insurance to RU using ex post repayments. Second, for each innovation quality, production is at the efficient scale. Third, wages are defined by the binding participation constraints of firm C. Fourth, the initial investment in R&D is efficient since the financier F gets all the marginal benefit from a high-quality innovation.

It is easy to see that the two structures yield the same outcome. Under symmetric information, it is not possible to discriminate between the two structures. We now introduce asymmetric information.

1.4 Asymmetric Information with full commitment

We now assume that RU has private information about the quality of the innovation. Under such assumption, the interaction between RU and C in an integrated structure proceeds as follows. The research unit negotiates ex ante a contract with C. The investment level is then determined by C Ex post, RU communicates (not necessarily truthfully) the results of the research process to C. Firm C then chooses the production level. In the independent structure, RU finances externally its investment in research. After innovating, RU develops an application jointly with C. The application is then brought to the market.

The two structures are distinct in two important features. First, the contracting stage between RU and C occurs ex ante in the integrated structure and ex post in the independent structure. In the integrated structure, RU and C negotiate in a moral-hazard hidden-information environment. In the independent structure, RU and C play a signalling game[3].

Second, the communication process between the informed principal and the uninformed agent is different. In the integrated organization, the decision right over production belongs to C. This right cannot be credibly transferred from C to RU as the rule of law does not govern over such intrafirm transaction. For example, even if this right was transfered to RU, C could always repossess it because it has hierarchical authority over RU. In terms of communication, this amounts to RU sending a direct report, namely the innovation quality, to C. This information is used by C when it decides how much to produce and sell. This corresponds formally to a direct mechanism. In the independent organization, the decision right over production initially belongs to C. Since C and RU are independent firms, this right can be "sold" from C to RU : the judicial system can enforce such transaction. Formally, this amounts to RU sending an indirect message to C and to F by effectively choosing production. No communication needs to occur between RU and C after the contract is signed. This corresponds formally to an indirect mechanism. We now characterize the optimal allocation

[3]See Maskin and Tirole (1992) for a general framework of those game situations with an informed principal and common values.

under these two structures.

1.4.1 The integrated structure

In an integrated structure with asymmetric information and commitment, agents play the following game :

1. In the first stage, RU proposes a research and development contract $c_{RD} = \{I, \{w_{\hat{\alpha}}, q_{\hat{\alpha}}\}_{\hat{\alpha}=l}^{h}\}$ to C.

2. In the second stage, C accepts or rejects the contract.

3. In the third stage (if reached), RU observes the innovation quality α.

4. In the fourth stage, the contract is carried out ; that is RU selects a message $\hat{\alpha} \in \{l, h\}$ and then the innovation is developed, produced and sold while transfers are paid as prescribed by the contract.

This game has two important features. First, the environment we have chosen is one of hidden information : the contract is signed with the two agents' having the same information, but production is carried out just after RU has privately observed the state of nature. Note that the full-information allocation is not an equilibrium allocation of this game. With this allocation, RU's dominant strategy would be to pretend that innovation quality is low, thus reducing its development cost while maintaining its wage. Expecting this behaviour, C would refuse the full-information contract if ever offered.

It is easy to show that the equilibrium allocation $\{I^A, \{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^{h}\}$ is the solution to the following maximization problem.

(P_A) $\max_{I,\{w_\alpha,q_\alpha\}_{\alpha=l}^h} p(I)V(w_h,q_h,h)+(1-p(I))V(w_l,q_l,l))$ s/t

$$\begin{aligned} p(I)U(w_h,q_h,h)+(1-p(I))U(w_l,q_l,l))-I &\geq 0 & (IR^C) \\ V(w_h,q_h,h) &\geq V(w_l,q_l,h) & (IC_h) \\ V(w_l,q_l,l) &\geq V(w_h,q_h,l) & (IC_l) \end{aligned}$$

The equilibrium allocation is RU's preferred allocation among the set of allocations satisfying its incentive-compatibility constraints (IC_α) for each state of nature $\alpha \in \{l,h\}$ and C's participation or individual rationality constraint (IR^C). In the following proposition, we characterized the solution to the P_A maximization problem.

Proposition 1. *The equilibrium allocation of the integrated structure commitment game satisfies the following relationships.*

- $q_l^A < q_l^*, q_h^A = q_h^*,$
- $w_h^A - D(q_h^*,h) = w_l^A - D(q_l^A,h) > w_l^A - D(q_l^A,l),$
- $p'(I^A)\{\frac{V_h^A - V_l^A}{E[V_\alpha'^A|I^A]} + U_h^A - U_l^A\} = 1,$

where V_α^A and U_α^A are respectively RU and C equilibrium utility for an innovation quality $\alpha \in \{l,h\}$.

Proposition 1 states first that there is underproduction for a low quality innovation and optimal production for a high quality innovation ; second, that the integrated structure cannot share risk efficiently between the two agents as the wage difference is constrained by the high-innovation incentive-compatibility constraint ; third, that investment is determined by the marginal benefit of a high-quality innovation shared between the two agents.

This is the usual result in hidden-information games. Under the symmetric-information optimal allocation, RU has incentives to report a low-quality innovation when it knows that the innovation quality is high. The incentive-compatibility constraint for a high-quality innovation is therefore not satisfied. Production for a low-quality innovation is distorted and the wage difference is increased in order to satisfy IC_h. We now study the independent structure with commitment.

1.4.2 The independent structure

In an independent structure with asymmetric information and commitment, agents play the following game :

1. In the first stage, RU proposes a financial contract $c_F = \{I, \{R(q_\alpha)\}_{\alpha=l}^{h}\}$ to F.

2. In the second stage, F accepts or rejects the contract.

3. In the third stage (if reached), RU observes the innovation quality α.

4. In the fourth stage, RU proposes a development contract $c_D = \{w(q_\alpha), q_\alpha\}_{\alpha=l}^{h}$ to C.

5. In the fifth stage, C accepts or rejects the contract.

6. In the sixth stage (if reached), the contract is carried out ; that is, RU implements the production level $q_{\hat{\alpha}} \in \{q_l, q_h\}$ with C and it is observed by F ; the innovation is then developed, produced and sold while transfers are paid as prescribed by the contract.

The commitment game has two important features. First, while the financial contract is signed and carried out in a hidden-information environment, the development contract is negotiated in an adverse-selection environment. Second, the development project size $q_{\hat{\alpha}}$ is observable by all the players, that is, this indirect message is publicly sent to C and F. In other words, it is as if the principal can commit to send the same message to the two agents. Note that the full-information allocation is not an equilibrium allocation of this game : with this allocation, RU's dominant strategy would be to pretend that innovation quality is low thus reducing its development costs while maintaining its wage net of financial cost. Expecting this behaviour, C and F would refuse the full-information contract proposal.

The equilibrium allocation $\{I^B, \{R^B(q_\alpha), w^B(q_\alpha), q_\alpha^B\}_{\alpha=l}^h\}$ is the solution to the following maximization problem.

(P_B) $\max_{I,\{R(q_\alpha),w(q_\alpha),q_\alpha\}_{\alpha=l}^h} p(I)V(w(q_h)-R(q_h), q_h, h)+(1-p(I))V(w(q_l)-R(q_l), q_l, l))$ s/t

$$\begin{aligned} p(I)R(q_h) + (1-p(I))R(q_l) - I &\geq 0 & (IR^F) \\ U(w(q_h), q_h, h) &\geq 0 & (IR_h^C) \\ U(w(q_l), q_l, l) &\geq 0 & (IR_l^C) \\ V(w(q_h) - R(q_h), q_h, h) &\geq V(w(q_l) - R(q_l), q_l, h) & (IC_h) \\ V(w(q_l) - R(q_l), q_l, l) &\geq V(w(q_h) - R(q_h), q_h, l) & (IC_h) \end{aligned}$$

The equilibrium allocation is RU's preferred allocations among the set of allocations satisfying F's individual rationality constraint (IR^F), RU's incentive compatibility constraints (IC_α) and C's individual rationality constraint (IR_α^C) for each state of nature $\alpha \in \{l, h\}$. Note that the timing of the game requires that C's participation constraint must be satisfied for each state of nature, which implies that C is unable to share risk with the research unit. F's participation constraint, however, must be satisfied only in expectation, and thereby provides room for explicit insurance.

Since the same message is sent to F and C, incentive-compatibility constraints are similar to those of the previous integrated structure game. The two incentive-compatibility constraints provide incentives to reveal its information because RU's private information is publicly disclosed. In the following proposition, we characterized the solution to the P_B maximization problem.

Proposition 2. *The equilibrium allocation of the independent structure commitment game satisfies the following relationships :*

- $q_h^B = q_h^A = q_h^*$, $q_l^B = q_l^A < q_l^*$,
- $w^B(q_h) - R^B(q_h) = w_h^A$, $w^B(q_l) - R^B(q_l) = w_l^A$,
- $I^B = I^A$,
- $P(q_\alpha^B, \alpha) - w^B(q_\alpha) = 0, \ \forall \alpha \in \{l, h\}$,
- $w^B(q_h) - R^B(q_h) - D(q_h^B, h) = w^B(q_l) - R^B(q_l) - D(q_l^B, h) > w^B(q_l) - R^B(q_l) - D(q_l^B, l)$.

Proposition 2 states the equivalence between the two structures. Formally, it shows that the two maximization programs are equivalent. Risk sharing is provided by the risk-neutral bank via the financial contract. As in the previous game, truthfull revelation implies underproduction for a low-quality innovation. As the report is public, incentive-compatibility constraints are similar to those of the previous game. The financial partner is therefore able to provide partial insurance to the research unit as the consumer firm did in the integrated-structure commitment game. RU receives the same transfer from its R&D activity for the same production level and, therefore, the same utility level as in the integrated structure for all states of nature. All participation constraints are binding, thus no rents are allocated to agents. The high-quality innovation benefit is shared between RU and F in the same way as it was between RU and C in the previous subsection, which implies the same level of investment. The intuition for the equivalence of the two organizational structures is the following. Since informational reports must be the same to C and F, it is as if these two agents were the same. C and F can then provide insurance to RU as efficiently in the independent structure as C can in the integrated structure.

The two structures are equivalent when agents can fully commit to the terms of the contracts. Hence, asymmetric information only cannot help to explain the existence of different organizational forms of R&D activities. In the next section, we characterize and compare the two structures in a no-commitment game.

1.5 Asymmetric information with no commitment

The removal of the commitment assumption introduces the possibility for players to renegotiate the initial contract and the possibility for a subset of players to collude to extract rents from a third party. Under our assumptions, renegotation is likely to affect the allocation under the integrated structure, while collusion becomes a distinct possibility in the independent structure.

There are two potential instances in which players may want to renegotiate a contract. First, the arrival of information may create some opportunity for renegotiation. In the integrated structure, players may therefore want to renegotiate immediately after RU observes the state of nature but before it chooses a message. In that case, renegotiation, called interim renegotiation, would occur after stage 3 but before stage 4. In the independent structure, RU may want to renegotiate with F after observing the state of nature between stages 3 and 4. In a similar environment, Beaudry and Poitevin (1995) point out that allowing for interim renegotiation does not affect the equilibrium allocation of the game (see also Holmström and Myerson, 1983 ; Maskin and Tirole, 1992). The reason is that, before selecting an element in the menu of the outstanding contract, an offer to renegotiate is simply cheap talk which has no effect on the allocation. Allowing for interim renegotiation would therefore not change the results.

Second, the actual selection by RU of an element in the menu of the outstanding contract may also create some opportunity for renegotiation. This is called ex-post renegotiation. Players could renegotiate after RU has selected an element in the menu but before actions are actually executed. In the integrated structure, renegotiation would therefore occur at stage 4 after the message is sent to firm C but before the innovation is developed and produced. In the independent structure, information is conveyed to C and F by the actual production of the innovated good, that is, RU communicates indirectly its information to its partner C through the action it executes. After the production project is developed, there is no room for renegotiation. Therefore, in the independent structure, the indirect mechanism is a commitment device not to renegotiate (See Beaudry and Poitevin, 1994; Caillaud, Jullien and Picard, 1995, for a discussion on this issue). For the above reasons, we restrict ourselves to ex post renegotiation.

In the independent structure, a principal contracts successively with two agents. The principal may be tempted to secretly agree with the agent at the second contracting stage, not to behave as prescribed by the first contract. More specifically, the research unit could secretly agree with firm C not to reveal the level of production implemented to F. RU could then select the lower financial contract payment in the menu by lying, with C's approval, on the innovation quality. The way such collusion is modelled in our paper is similar to that in Laffont and Martimort (1997). We allow the principal to include a report manipulation function in the development contract which specifies the message sent to F for each level of production. To make the analysis interesting, we therefore have to assume that F cannot observe the production level.

1.5.1 The integrated structure

In an integrated structure, when agents cannot commit, they play the following game :

1. In the first stage, RU proposes a research and development contract $c_{RD} = \{I, \{w_\alpha, q_\alpha\}_{\alpha=l}^{h}\}$ to C.
2. In the second stage, C accepts or rejects the contract.
3. In the third stage (if reached), RU observes the innovation quality α.
4. In the fourth stage, the contract is carried out ; that is, RU selects a message $\hat{\alpha} \in \{l, h\}$.
   (a) RU proposes a contract $c_r = (w, q)$.
   (b) C accepts or rejects the contract offer. If it is rejected, the contract c_{RD} remains the outstanding contract. If c_r is accepted, it becomes the outstanding contract. The innovation is then developed, produced, and sold while transfers are paid as prescribed by the outstanding contract.

We characterize the equilibrium allocations that are not renegotiated along the equilibrium path, namely, renegotiation-proof allocations. There are allocations that can be supported by equilibrium strategies that do not involve any renegotiation along the equilibrium path.

Clearly, the integrated structure equilibrium allocation $\{I^A, \{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^{h}\}$ derived in the previous section is not renegotiation-proof. Suppose that the innovation is drastic and that $\alpha = h$. Consider the following actions in stages 4 and 4.a : RU selects the report $\alpha = l$ and then offers the renegotiation allocation (w, q) with $q = q_{lh}^*$, where $q_{lh}^* = \text{argmax}_q\{P(q, l) - D(q, h)\}$, and $w = P(q_{lh}^*, l) - [P(q_l^A, l) - w_l]$. C always accepts this renegotiation offer. Compared to the status quo, its utility is the same if it believes that the innovation quality is l and higher if it believes that the innovation quality is h. Therefore, C accepts this renegotiation offer for any beliefs. RU utility would then be :

$$V(w,q,l) = v(w_h^A - D(q_h^A, h) + P(q_{lh}^*, l) - D(q_{lh}^*, h) - \pi(q_l^A, l)).$$

This utility is higher than that obtained without renegotiating. Hence, the equilibrium allocation of the previous section is not renegotiation-proof.

A renegotiation-proof allocation must satisfy the following inequalities.[4]

$$\begin{aligned}
&V(w_h, q_h, h) \geq \max_{(w,q)}\{V(w,q,h) \ s/t \\
&U(w,q,h) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, h) \\
&U(w,q,l) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, l)\} \ \forall \hat{\alpha} = l,h \qquad (RP_h^{\hat{\alpha}}) \\
&V(w_l, q_l, l) \geq \max_{(w,q)}\{V(w,q,l) \ s/t \\
&U(w,q,h) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, h) \\
&U(w,q,l) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, l)\} \ \forall \hat{\alpha} = l,h \qquad (RP_l^{\hat{\alpha}})
\end{aligned}$$

These constraints are more stringent than the usual incentive-compatibility constraints, and therefore, they represent generalized incentive-compatibility constraints that incorporate the possibility of ex post renegotiation. Each constraint $RP_\alpha^{\hat{\alpha}}$ implies that, given a status-quo position $(w_{\hat{\alpha}}, q_{\hat{\alpha}})$, C only accepts those renegotiation offers that increase its utility regardless of its beliefs. They are called surely-acceptable renegotiation offers. Suppose that constraint $RP_\alpha^{\hat{\alpha}}$ is satisfied at a status-quo position $(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A)$. For any offer that RU prefers to $(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A)$, there exists a belief for C such that it is worse off under the new offer than under the status-quo position. When assigned with this belief, C simply rejects the offer of RU. If an allocation satisfies these constraints, it is not possible for RU to increase its utility by selecting a message $\hat{\alpha} \in \{l,h\}$ and then offer a surely-acceptable renegotiation. It is in this sense that the renegotiation-proof constraints represent generalized incentive-compatibility constraints.

[4]This is shown formally in Beaudy and Poitevin (1995).

In Proposition 3, we characterize one such allocation as an equilibrium allocation, namely, the allocation that yields RU the highest expected utility.

Proposition 3. *The allocation $\{I^A, \{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^h\}$ that solves the following maximization problem is an equilibrium allocation.*

(P_R) $\max_{\{I,\{w_\alpha,q_\alpha\}_{\alpha=l}^h\}} p(I)V(w_h, q_h, h) + (1-p(I))V(w_l, q_l, l)$ s/t

$$p(I)U(w_h, q_h, h) + (1-p(I))U(w_l, q_l, l) - I = 0$$

$$\begin{aligned} & V(w_h, q_h, h) \geq \max_{(w,q)}\{V(w,q,h) \ s/t \\ & U(w,q,h) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, h) \\ & U(w,q,l) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, l)\} \ \forall \hat{\alpha} = l, h \qquad (RP_h^{\hat{\alpha}}) \\ & V(w_l, q_l, l) \geq \max_{(w,q)}\{V(w,q,l) \ s/t \\ & U(w,q,h) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, h) \\ & U(w,q,l) \geq U(w_{\hat{\alpha}}^A, q_{\hat{\alpha}}^A, l)\} \ \forall \hat{\alpha} = l, h \qquad (RP_l^{\hat{\alpha}}) \end{aligned}$$

Proposition 4. *The solution to the P_R maximization problem satisfies the following relationships :*

- *For major innovations,*
  $q_h^A = q_h^*$, $q_l^A = q_l^*$,
  $w_h^A - D(q_h^*, h) = w_l^A - D(q_l^*, h) + P(q_{lh}^*, l) - D(q_{lh}^*, h) - [P(q_l^*, l) - D(q_l^*, h)]$.
- *For minor innovations,*
  $q_h^A = q_h^*$, $q_l^A < q_l^*$
  $w_h^A - D(q_h^*, h) = w_l^A - D(q_l^A, h) + \pi(q_h^*, h) - \pi(q_l^A, h)$.
- $p'(I^A)\{(\frac{V_h^A - V_l^A}{E[V_\alpha'^A|I^A]} + U_h^A - U_l^A\} = 1$.

With commitment, underproduction was chosen by RU in equilibrium in order to satisfy the incentive-compatibility constraints without taking too much risk. The effect of renegotiation is to limit the amount of underproduction, thereby imposing more risk on RU. When the innovation is major, no distortion in q_l can be used ex ante to induce truth-telling. In the other case, some underproduction for the low-quality innovation is renegotiation-proof. It therefore arises to mitigate the risk allocated to RU.

In all cases, the more stringent renegotiation-proof constraint is RP_h^l, that is, when the innovation quality is high and RU announces a low-quality innovation. In order to satisfy the binding renegociation-proof constraint, the research unit takes on more risk by increasing the wage gap. Finally, investment is defined by the usual first-order condition and depends on risk sharing. We now move to the independent structure game.

1.5.2 The independent structure

In an independent structure, when agents can collude, they play the following game :

1. In the first stage, RU proposes a financial contract $c_F = \{I, \{R_{m(q_\alpha)}\}_{\alpha=l}^h\}$ to F.
2. In the second stage, F accepts or rejects the contract.
3. In the third stage (if reached), RU observes the innovation quality α.
4. In the fourth stage, RU proposes to C a development contract $c_D = \{w(q_\alpha), q_\alpha, m(q_\alpha)\}_{\alpha=l}^h$ which includes a secret manipulation report function $m(q_\alpha)$.
5. In the fifth stage, C accepts or rejects the contract.
6. In the sixth stage (if reached), the contract is carried out : RU implements the production level $q_{\hat{\alpha}}$ and the corresponding report $m(q_{\hat{\alpha}})$ is sent to F ; the

innovation is then developed, produced and sold while transfers are paid as prescribed by the contract.

We model collusion between RU and C as a secret report manipulation function :

$$m(q_{\hat{\alpha}}): \ \{q_l, q_h\} \rightarrow \{l, h\}.$$

This function defines a report $m(q_{\hat{\alpha}})$ for each level of production $q_{\hat{\alpha}}$. For an indirect message $q_{\hat{\alpha}} \in \{q_l, q_h\}$ selected by RU, C and RU secretly agree to report the message $m(q_{\hat{\alpha}})$ to F. This collusive agreement allows RU to report, possibly, different direct and indirect messages $m(q_{\hat{\alpha}})$ and $q_{\hat{\alpha}}$ to F and C respectively. Moreover, using the report manipulation function, RU can coordinate its communication activity with C. Even if C is asked by F to report the innovation quality or production level implemented, the development contract tells C to act as prescribed by the report manipulation function.

Clearly, the equilibrium allocation with commitment is not an equilibrium of this game. Assume that $\alpha = h$. If RU proposes the collusion agreement $m^B(q_\alpha) = l$ for all $q_\alpha \in \{q_l, q_h\}$, that is, to report a low-quality innovation to F regardless of the indirect message $q_{\hat{\alpha}}$, then RU's ex post utility is $v(\pi(q_h^*, h) - R_l^B)$. Since $R_h^B > R_l^B$, RU's payoff is increased when RU and C secretly agree to report $m(q_{\hat{\alpha}}) = l$ when $\alpha = h$. Hence, this allocation is not robust to a collusive agreement between RU and C, that is, it is not collusion-proof. Expecting the collusion, F would refuse to sign this contract.

In order to find the equilibrium allocation, we proceed by backward induction. We first consider the development-contracting "subgame" starting at stage 3. We provide necessary and sufficient conditions for a development contract to be an equilibrium of this subgame for any given financial contract. We derive the equilibrium financial contract offered in stage 1.

Lemma 1. *For a given financial contract* $\{\bar{I}, \{\bar{R}(q_\alpha)\}_{\alpha=l}^{h}\}$*, an allocation* $\{w^B(q_\alpha), q_\alpha^B, m^B(\alpha)\}_{\alpha=l}^{h}$ *is an equilibrium allocation of the development-contracting subgame if and only if it solves the following maximization problem.*

(P_{CD}) $\max_{\{w(q_\alpha), q_\alpha, m(q_\alpha)\}_{\alpha=l}^{h}} p(\bar{I})V(w(q_h) - \bar{R}_{m(q_h)}, q_h, h)) + (1 - p(\bar{I}))V(w(q_l) - \bar{R}_{m(q_l)}, q_l, l)$ s/t

$$\begin{array}{rcll} U(w(q_h), q_h, h) & \geq & 0 & (IR_h^C) \\ U(w(q_l), q_l, l) & \geq & 0 & (IR_l^C) \\ V(w(q_h) - \bar{R}_{m(q_h)}, q_h, h) & \geq & V(w(q_l) - \bar{R}_{m(q_l)}, q_l, h) & (IC_h) \\ V(w(q_l) - \bar{R}_{m(q_l)}, q_l, l) & \geq & V(w(q_h) - \bar{R}_{m(q_h)}, q_h, l) & (IC_h) \end{array}$$

The effect of the report manipulation is captured in the incentive-compatibility constraints. They state that RU proposes a secret report agreement that provides it incentives to reveal its information to player C. In Lemma 2, we characterize the equilibrium allocation of this subgame.

Lemma 2. *For any financial contract* $\{\bar{I}, \{\bar{R}_{m(q_\alpha)}\}_{\alpha=l}^{h}\}$*, the equilibrium development allocation of the contracting subgame satisfies the following relationships :*

- $P(q_\alpha^B, \alpha) - w_\alpha = 0, \forall \alpha \in \{l, h\}$.

- *For major innovations,*
  $$q_l^B = q_l^*,\ q_h^B = \begin{cases} q_h^* & if \quad \pi(q_l^*, l) \geq P(q_h^*, h)) - D(q_h^*, l) \\ q_h^S & otherwise \end{cases}$$
  with $q_h^S > q_h^*$ *such that* $\pi(q_l^*, l) = P(q_h^S, h)) - D(q_h^S, l)$ *;*

- *For minor innovations,*
  $$q_h^B = q_h^*,\ q_l^B = \begin{cases} q_l^* & if \quad \pi(q_h^*, h) \geq P(q_l^*, l) - D(q_l^*, h) \\ q_l^S & otherwise \end{cases}$$
  with $q_l^S < q_l^*$ *such that* $\pi(q_h^*, h) = P(q_l^S, l) - D(q_l^S, h)$ *;*

- $m^B(.)$ *is such that* $m^B(q_\alpha) = argmin_\alpha\{\bar{R}_\alpha\}, \forall q_\alpha \in \{q_l, q_h\}$.

Equilibrium wages are still defined by C's ex post binding participation constraints. In order to satisfy the incentive-compatibility constraints, overproduction occur for the high-quality innovation. The agents RU and C secretly agree to report to F the innovation quality associated with the lower repayment. Note that the development contract does not depend on the given financial contract. We now solve for the equilibrium of the whole.

Proposition 5. *The equilibrium financial contract is such that :*

- $R_h^B = R_l^B = I^B$,
- $p'(I^B)\frac{V_h^B - V_l^B}{E[V_\alpha'^B | I^B]} = 1$.

The main consequence of collusion is that F cannot provide any insurance since financial repayments are the same in each state of nature. This financial contract can therefore be interpreted as a debt contract in which the initial amount lent I^B must be paid back at the end of the R&D process. Since RU takes all the research risk, investment is therefore determined by the incremental value of a high-quality innovation compared to low-quality one. Since the equilibrium financial contract is a debt contract, no information needs to be revealed to F. The development contract is therefore negotiated in a two-agent signalling environment. C's individual rationality constraints are binding. When the binding incentive constraint is that for a low (high) quality innovation, RU may overproduce (underproduces) when innovation quality is high (low) in order to satisfy this constraint. In the next section, we endogenize the organizational choice of R&D activities by comparing the performance of the two structures.

1.6 Performance of the two structures without commitment

Suppose now that the organizational choice of R&D activities is endogenized. RU's decision as to whether produce an innovation in an integrated structure or in an independent structure depends on its expected utility under each structure. To make the comparison simple, we introduce the following assumption on the innovation profit.

Assumption 1. *There exists* $\lambda \in (0, \frac{\pi(q_h^*,h)+D(q_l^*,h)}{P(q_l^*,h)}]$ *such that* $P(q,l) = \lambda P(q,h)$, $\forall q \in [0,\bar{q}]$.

It says first that that low-quality innovation profit is a linear transformation of the high-quality innovation profit. This characterization gives us a simple interpretation of so-called innovation drasticity. A major innovation is represented by $\lambda < 1$, while a minor innovation is the case $\lambda > 1$. When $\lambda = 1$, the innovation quality does not effect the profit. Such an innovation is called neutral. Second, the gap between marginal profits in case of minor innovation should not be too high. This condition guarantees that production is not distroted in equilibrium for minor innovations. Note that the upper bound on λ depends on optimal productions. However, as long as $q_l^* < q_h^*$, this upper bound exceeds 1 and therefore a minor innovation is allowed. [5]

Proposition 6. *For major (minor) innovations, the integrated (independent) structure dominates. For a neutral innovation, both structures are equivalents.*

Proposition 6 states that, the choice of structure depends on the technology. The intuition of this result can be given in terms of the effects of contractual

[5] Note that assumption 1 is used *only* to prove that the independent structure dominates for minor innovations such that λ is "not too high".

imperfections of the extent of risk sharing provided to RU. Agency costs come from the fact that the contract is used to insure RU against the risk of innovation. The achievement of this goal conflicts with asymmetric information and non-commitment. To understand the result, it is useful to assess the relative impact of these factors on risk-sharing. Suppose first that C's revenues are independent of the quality of innovation ($\lambda = 1$ in the example). As shown in the proposition, both organizational structures are equivalent. Collusion and renegotiation have the same effects on the extent of risk-sharing as all risk is shifted to firm RU.

Now suppose that a low-quality innovation generates slightty higher marginal revenues than a high-quality one ($\lambda > 1$). In the independent structure, this improves risk sharing as the difference between V_h and V_l shrinks. This difference shrinks since increasing λ increases the profitability of the low-quality innovation. Since firm C gets its reservation value in each state and financing is achieved through debt, firm RU gets all benefits from such increase, thus increasing its expected payoff and improving risk-sharing. In the integrated structure, increasing λ makes the renegotiation-proof constraint less stringent and therefore allows for some distortions in q_l to improve risk-sharing. But this implies that RU cannot appropriate the whole surplus generated by the increase in λ. The independent structure then domiates the integrated structure.

Now suppose that a low-quality innovation generates slightly lower marginal revenues than a high-quality one ($\lambda < 1$). In the independent structure, this worseur risk sharing as the difference between V_h and V_l increases. It increases since decreasing λ decreases the profitability of the low-quality innovation. As in the previous case, firm RU supports the full loss from such decrease, thus reducing its expected payoff and suffering more risk. In the integrated structure, risk-sharing is unaffected as all increase in risk is supported by firm C. Firm RU, however, still supports the full loss in the profitability of the low-quality innovation. The integrated structure then dominates the independent structure.

The difference between the two cases stems from the effect of technology on the amount of risk in the venture. From a situation where there is no revenue risk ($\lambda = 1$), increasing λ reduces the total risk, that is, the risk of $P(q, \alpha) - D(q, \alpha)$. In the independent structure, RU supports all risk. It can therefore gain from the increase in revenue as well as from the reduction in risk. The independent structure is then optimal. When λ is decreased, the opposite holds. Total risk is increased. In the independent structure, RU support all this extra risk while also losing from the loss in revenue. The integrated structure is then optimal.

This result has a testable implication. The innovation must be major (minor) for in-house (independent) R&D. For instance, consider the R&D process in the pharmaceutical industry described in the introduction. Development activities consist in testing the new drug. The development process starts from toxicology analyses and goes through clinical trials on animals, human volunteers and then patients (small samples and then large samples). The molecule must be patented before entering in the trial process. The patent-protection lasts twenty years and the trial process can take several years.[6] Saving time during the development phase is therefore particularly important. Every day saved on trial is an extra day of patent-protection saved. The trial period of an innovation costly to develop is long and therefore lowers its patent-protection and, finally, the gross profit of the pharmaceutical compagny. This corresponds to the case of major innovation. Our model predicts that the R&D activities are more efficiently organized in-house.

For a technological innovation, it is often the case that when the cost to install a new technology is high, the saving on production cost is high. Consider the information-technology industry. Suppose that a firm can reduce its costs by using a more efficient communication network. A new telecommunication network is costly to install but can treat a lot of information very quickly. An improvement

[6]The Economist, February 21st, 1998, Tapon and Calsby (1996).

of the existing network is cheap to install but it is usually less efficient. In this case, the innovation is minor. Another example is the computer industry. When a new version of an existing software or system is adopted by a firm, the costs incurred by the research unit (mostly the training of the user firm's employers) is low. When the software or the system is very different, and therefore needs more training, the saving on production costs could be very high. Again innovation is minor. In these two cases, our model predict that the innovation should be produced by an independent firm.

These two examples seem to fit with stylized facts. Most research in the pharmaceutical industry tends to be produced in-house. And most research in information technology seems to be produced by independent firms. Our model rests its explanation of these facts on different contractual imperfections. Integrated firms tend to be inefficient because it is easy to deviate from an initial plan, which we model as renegotiation here. Independent firms incur agency costs when seeking external financing, which we model as collusion here. We believe that a different formulation for these agency costs would still yield a tradeoff between the two structures, albeit maybe different. The advantage of our modelling assumptions is that it yields a definitive tradeoff which seems to broadly fit some stylized facts.

1.7 Conclusion

This essay studies the optimal structure of R&D activities in a model with a random research process, asymmetric information about its outcome and heterogeneity in agents' attitude toward risk. We prove that, while the two structures are equivalent in a full-commitment world, a tradeoff emerges when players are allowed to renegotiate and collude. In the integrated structure, RU has incentives to renegotiate after the report is made. It has to take more risk by proposing ex-post efficient production levels and by increasing the wage difference. In the independent structure, RU is tempted to secretly agree with C to manipulate the message sent to F. The financial contract must then be a debt contract, and F cannot provide any insurance to RU. However, RU can mitigate the risk taken by distorting the production level.

We find that the integrated structure dominates the independent struture when the marginal cost of developing the innovation is negatively correlated with its marginal revenue, that is, when an innovation which is cheap to develop creates a more drastic process innovation or a product innovation with a higher market value. The independent structure performs better than the integrated structure in the opposite case. This result provides a testable implication of our model. Our approach explains how the organizational structure of R&D activities depends on the technological properties of innovations for each industry.

Chapitre 2

Income-sharing within extended families as a social norm

Abstract

In this essay, african income-sharing within extended families is modeled as a social norm implemented by the community authority and then mutually enforced by agents through individuals' heterogeneous valuation of social status. It is explained how norm obedience could be observed in a Nash equilibrium. Full income-sharing is implemented if and only if there is full obedience to this norm. Otherwise, the norm solves a trade-off between risk-sharing and mutual enforcement. Partial risk-sharing is achieved and some norm disobedience arise in equilibrium as a second best. This approach helps to understand the effects and efficiency of public policy in developing countries.

"Quiconque a été au chômage en Afrique
sait à quel point la fameuse solidarité
africaine sert surtout à dilater
l'égo social de parents fortunés"

Axelle Kabou
"Et si l'Afrique refusait le développement ?"
Editions l'Harmattan, Paris, 1991.

2.1 Introduction

In Sub-Saharian Africa, rich family members provide to the basic needs of their poor relatives. They protect against income shortfall, disease, or unemployment. Mutual assistance arrangements allow people to share risk in an informal way. All risk averse agents benefit ex ante from a behavior that smoothes their future consumption. But ex post, an agent with a high secure revenue has to share his income even if he does not ever benefit from the mutual assistance arrangement. How come this agent has incentives to fulfill his duties ?

This paper proposes a mutual enforcement mechanism which explains the practice of an ex post income-sharing within an enlarged family. This mechanism is based on the agents' valuation of "reputation" *à la Akerlof (1980).* Reputation can be interpreted as social status, popularity, esteem, shame or guilt. Because they care about others opinion, agents may conform to an income-sharing social norm. This mechanism is limited by peer pressure capacity. Therefore some family members would prefer to disobey the norm even if they are punished by their relatives.

The income-sharing norm is endogenized as an informal rule implemented by elders, the traditional community authority. Parents are assumed altruistic with their children. The elders' objective is to maximize the norm obedients' expected utility by sharing risk via ex post transfers. Optimally, elders want their children to share fully their income. However, full income sharing may be too demanding and may induce too much norm disobedience. The full income-sharing norm will be implemented by elders if and only if everyone obey it. Otherwise, the norm solves a trade-off between risk sharing and ex post enforcement. Partial income-sharing with some norm desobedience may be implemented as a second best. In other words, it may be ex ante efficient to design a norm that will not be fulfilled by some family members.

Nonconformists benefit from the informal insurance without paying the cost. They are subsidized by norm obedients and reduce the norm efficiency. They cannot be excluded by letting them decide ex ante not to enter into the mutual insurance network : There exists a subgame perfect equilibrium where all agents, conformists and nonconformists, adhere ex ante to the income-sharing agreement.

A redistributive policy does not necessary perform better than the income-sharing norm because of high administrative costs in LDCs. These costs should be compared with the proportion of nonconformity to the equivalent income-sharing norm. When they are sufficiently low, a redistributive policy supplemented by the income-sharing norm increases agents'expected utility.

The current chapter proceeds as follow. Section 2.2 motivates our approach. Section 2.3 describes the model. Section 2.4 analyzes income-sharing norm practice in equilibrium. Section 2.5 endogenizes the norm as an informal institution. Section 2.6 analyses income-sharing as an informal contract. Section 2.7 gives some comparative static properties. Section 2.8 discusses norm efficiency and public policy. Section 2.9 concludes the paper.

2.2 Motivation

Two explanations for self-enforcement have been proposed. The first one is altruism. If an agent cares about his relative's utility, he gets vicarious satisfaction from increasing his relative's consumption by giving him a private transfer (see e.g. Dearden and Ravallion (1988)). Although altruism might be relevant to motivate transfers between parents and children, strong altruistic links between cousins can less easily explain the importance of inter-vivos transfers.

The second motive is given by the repeated game literature. Income-sharing is interpreted as a cooperative equilibrium of an infinitely repeated game where risk averse agents earn a random revenue at each period. Agent's self-interest remain into the mutual insurance arrangement by making transfers as long as he expects to benefit from it. This "reciprocity without commitment" has been described in LDC's by Fafchamps (1992) and formalized by Coate and Ravallion (1993) ; Ligon, Thomas and Worral (1997). However, reciprocity does not explain why agents with a relative secure revenue subsidize poor relatives with low future opportunities.

This kind of behavior is often observed in African societies. Indeed, Fafchamps (1995) points out that people suffering from incurable disease, physical or mental handicap, are not excluded from the mutual assistance network. Mahieu (1990) finds that most of Abidjan university employees give high private transfers to their family. Lucas and Stark (1985) observe that migrant remittances are not generally paid back. They argue that private transfer from Botswana migrants may be motivated by a "social asset" such as the relationship with family and friends. Even if inter-vivos transfers should be partly motivated by a combination of altruism and reciprocity, empirical evidence suggests that they also arise because people want to conform to an informal rule publicly known as welfare improving.

Platteau (1996, 1997) observes that, first, able men have refused promotion because the material benefit may not compensate their relative claim. Second, an individual"s unshared capital accumulation would be generally disapproved by the community.[1] People would not disregard individual saving and investment if agents were altruistic or if saving behavior constitutes a rational outcome of the repeated contract. They disapprove a person who does not fulfill his duty by keeping her temperary monetary gain. As a consequence, money received from public aid or from a credit program is often partly misappropriated from the financed activity to be redistributed within the community.[2] Furthermore, a credit program can be rejected by a community because of inconsistency with sharing obligations (see Lewis, 1978 ; Ghatak and Guinnane, 1998).

The anthropological literature points out that human's behavior is strongly influenced by informal rules in African traditional societies (e.g. Guyer, 1981). These rules are designed and enforced within the traditional organization, the extended family or lineage. This organization is composed by households which descent from a common ancestor. They recognize the authority of some particular elders. Community leaders define informal rules like the income-sharing norm. Hence, households are not the unique economic decision unit, but rather only a subset of an often quite large extended family. Therefore, as argued in Grimard (1997), Mahieu (1990), Gastellu (1980), the household's behavior should be analyzed in conjunction with its kinship group. Whereas these rules are not legally stated, the whole community enforces it.

The enforcement of a social norm requires at least three elements. First,

[1]Platteau reports the case of a prosperous fisherman who was disapproved by the community for arresting children of poor families who seized a few fish from his stock oiled up on the beach in Senegal.

[2]For instance, Ndione (1992) reports that, in a credit project in the suburb of Dakar, 22.6% of the total amount of loan was investing in "social assets" rather than in the project it was earmarked for.

community members have to be informed : They must know that agent X failed to obey to the rule. They must be able to punish norm disobedient. Third, the penalty mechanism must be credible : People should have incentives ex-post to punish norm disobedient.

The diffusion of the information about members' behavior concerning informal rules occurs during frequent visits and meetings between relatives. Traditional ceremonies like baptisms, weddings, funerals, religious events, gives the agents the opportunity to advertise the gifts.[3] There is a "common knowledge norm" about the gift that must be done contingently on the donor's and the recipient kinship and fortune.[4] The "Griots" advertises people's behavior by singing the donor's praises (Azam, 1995). This facts are evidences that, contrary to the altruism approach, the diffusion of information about transfers within the community matter.

An agent may accept to reduce its consumption by sharing its income because he may come to desire esteem from its relatives.[5]. Penalties are applied via social status or reputation. This good is produced and allocated by the community members. A person's capacity to lower other agents'social status depends on his conformity to the norm. In equilibrium, norm obedients keep their social status and punish norm desobedients by reducing their social status. The person's

[3]Lund and Fafchamps (1997) finds that rituals have a larger impact on private transfers in rural Philippines than unemployment or crop shocks.

[4]For instance, a poor Senegalese woman interviewed by Ndione (1992) sets out her transfer behavior (my translation) : *"I benefit from the network established by my family.[...] I am often invited to ceremonies, weddings, baptism, funerals and I always make a gift to my host. My partner gives me always twice the gift when he have to make his gift back. The value of the gift and the timing of the payment depend on the quality of the relationship. When it is a close relative, I can delay the payment. [...] when I need money, I visit my relations. But it do not happen often because my relatives do not wait until there are in demand. It is how you recognize real relatives."*

[5]Note that this approach is closely related to altruism : It is assumed that that a person cares about someone else's opinion about him (or her) rather than someone else's feelings.

reputation is therefore affected not only by his conformity to the norm, but also by community members'attitude with regard to the norm. The more people conform to the norm, the higher is the loss of social status for norm desobedients.

The idea that an agent satisfaction is influenced by other persons' opinion about his behavior goes back to earlier economist's debates on agent utility concept. Bentham in 1789, cited by Becker (1974), mentioned about 15 basic kinds of pleasures and pains, among others "the pleasures... of being on good term with him or them". Becker (1974) formalized this concept by including "basics wants or commodities" in an agent utility function. These "goods" are produced with others agents actions such as their consumption's choices or their opinion. Indirectly, the agent's satisfaction about "basics wants or commodities" are influenced by other agents' actions.

Akerlof (1980) has used Becker's generalized utility function in his theory of social custom. He assumes that a person's utility is affected by his reputation within his community. Deviations from social customs are punished by loss of reputation. The reputation loss depends on other agents' obedience to the custom. Akerlof shows how stable inefficient customs are sustainable in LDCs' labor markets. My approach is based on Akerlof's concepts and applied to income-sharing.[6] As in Akerlof, the agent individual utility is affected by his reputation within the community. This reputation depends on others' behavior. However, there are at least two important differences between Akerlof's analysis and this paper. First, I introduce heterogeneity in the agent's reputation valuation. Hence, we could have

[6]Recently, Akerlof's model has been applied in several economic contexts. Corneo and Jeanne (1997) explores conformity to consumers' behavior when the private value of a good depends on other consumers' purchases. Lindbeck, Nyberg and Weibull (1997) assumes that people suffer from a disutility of not working by an amount decreasing with economic equilibrium unemployment. Other papers (Kandel and Lazear, 1992, Rob and Zemsky, 1997) model workers'peer pressure and mutual monitoring within firms.

partial obedience to social custom in equilibrium. Second, I explain not only social custom persistence but also how and why a norm came into being. Consequently, the social custom is efficient in equilibrium.

The enforcement mechanism has several alternative interpretation. First, it could be emotions such as guilt or shame. These feelings could be modeled as utility losses that depends on other agents' morality in regard to the norm (Kandel and Lazear, 1992, Elster, 1998). The larger is the fraction of the population adhering to the norm, the more intensely it is felt by the individual. Shame is a social sanction involved by conformists. It is external pressure that requires observability. Guilt is internal pressure increasing with the proportion of norm obedience. Second, the penalty might be a material loss controlled by the community. The nonconformists could be excluded from common-pool resources (land, forest, fisheries, water). They could loose their inheritance (Hoddinott, 1994). In many African rural societies, the property right of land is allocated by the village council (e.g. Schmitz, 1993) which might disinherit norm desobedients. The probability of being disinherited should depend on the level of norm conformity within the community. Third, as argued by Platteau (1996), witchcraft could be a form of social justice.

The norm requires not only that agents share their income but also that they punish norm desobedients. Such behavior is rational ex post even if sanctioning is costly as long as norm obedience and mutual punishment are decided simultaneously. The set up fits with Elster's definition of a social norm (Elster, 1989) :

"When there is a norm to do X, there is usually a "meta-norm" to sanction people who fail to do X, perhaps even a norm to sanction people who fail to sanction people who fail to do X. As long as the cost of expressing disapproval is less than the cost of receiving disapproval for not expressing it, it is one's rational self-interest to express it."

In the case of income-sharing, a poor asks for transfer and disapproves of those who do not give. A rich who shares his income wants the other rich to give in order to reduce his individual transfer. He exerts pressure to induce his rich relatives to share their income. Consequently, both poor and rich norm conformists have ex-post incentives to punish nonconformists.

The income-sharing norm is endogenized as an informal redistribution rule implemented by elders. This assumption is based on the traditional gerontocratic nature of African societies. In those oral based societies, rules and agreements are not formally written. They remain in the most experienced people's memory, the old generation members. Elders define unwritten rules and judge agent behavioral conformity to these rules. Indeed, the oral tradition of African societies implies its gerontocratic nature (e.g. Koulibaly, 1997).

This assumption is not restrictive. Since the norm is the best informal risk-sharing arrangement, it could endogenize by making alternative assumptions. First, it should be implemented in any democratic community. Suppose that the norm is proposed the community authority. It will be unanimously accepted by all members. Second, it could emerge from an evolutionary process. We now introduce the model.

2.3 The model

Consider an overlapping generation economy where agents live two periods. Each generation interacts with the preceding generation at the first period and with the following generation at the second period. An extended family or a community is composed by a continuum of individuals. Each individual utility $U(C, R, \theta) = u(C) + \theta R$ is a function of consumption C, reputation R and an agent's characteristic θ which represents the agent's taste for his reputation. The parameter θ balances the individual preferences for consumption and the social preferences for community judgement : Agents with an higher (lower) θ are more (less) affected by their relatives opinion. This taste for reputation may be interpreted as the physical or psychological distance from the agent's family. It is privately observed by agents. All agents are equally risk averse : u is increasing and concave ($u' > 0$ and $u'' < 0$). The agent's taste for reputation θ is distributed within the extended family in $\Theta = [\underline{\theta}, \bar{\theta}]$ at each generation according to a publicy known cumulative F. The cumulative first and second derivatives are respectively denoted f and f'. The following assumption is made on θ's distribution[7] : $f'(\theta) \geq -\frac{f(\theta)}{\theta}$.

An agent's income is drawn from an i.i.d. random walk y. With probability p, an agent earns a wage from a formal job or an output from a production activity : $y = \bar{y}$. He is considered as "rich". Otherwise, he is "poor" and he has only access to an informal job where $y = \underline{y}$. There is a (possibly large) gap between $\bar{y}$ and $\underline{y}$ ($\bar{y} > \underline{y}$).

An income-sharing norm affects both consumption and social status. Formally, it defines the transfer α that must be given by a rich to the poor contingent on the economic environment and a punishment mechanism which affects individual's social status. A rich has to transfer α to his poor relatives and thus consume $C = \bar{y} - \alpha$. A poor receives the transfer δ if he practices the income-sharing norm and consume $C = \underline{y} + \delta$. The transfer is the same for all the rich and therefore α does not depend on the agents' own taste for reputation θ. The conformity to the income-sharing norm affects individual's social status : if an individual follows the social norm, he does not suffer from a loss of reputation. His reputation is

[7]If θ is interpreted as the physical or psychological distance from family, it is natural to suppose that, if the family core location is normalised at $\theta = \bar{\theta}$, the proportion of the family members decreases with the distance between θ and $\bar{\theta}$. This assumption is also satisfied for uniform distributions. It is made for technical convenience.

therefore normalized to 0. Otherwise, he is punished by his relatives who obey the norm. Let r be one agent punishment capacity. The total reputation loses depends on this fixed individual sanction r and on the equilibrium proportion of norm obedient. It is assumed that the extended family or the community is large enough so that the ex post proportion of rich and poor can be correctly approximated by, respectively, p and $1-p$; and that θ is ex post distributed according to F in Θ among the rich population. Agents ex post utility is now derived.

A poor agent receives a positive transfer δ and does not suffer from a loss of reputation. His self-interest is to follow the norm. His ex post utility is :

$$U(\underline{y}, 0, \theta) = u(\underline{y} + \delta)$$

Let us denote μ the proportion of the rich family members who practice the income-sharing norm.[8]

An agent who obeys (disobeys) the income-sharing norm when he is rich will be referred to a conformist (nonconformist). A rich conformist consumes $\bar{y} - \alpha$ and does not suffer from a loss of social status. His utility is :

$$U(\bar{y} - \alpha, 0, \theta) = u(\bar{y} - \alpha).$$

An nonconformist consumes all his income $\bar{y}$. He is sanctioned by the conformists, that is, his $1-p$ poor relatives and the $p\mu$ rich conformists. His reputation[9] is

[8] If $\mu = 0$, no transfers are given and the income-sharing norm is therefore not implemented.

[9] The model is consistent with a bequest motive to remit. When the social sanction is disinheritance, θr is interpreted as the agent θ's value of his bequest (θ represents the surface of land and r, the future profit from land per unit of surface). Assume that agent is conformist, he gets the bequest with probability 1. Otherwise, the probability of receiving the future bequest is a function of the community's opinion. The agent θ expects to have his bequest with a probability given by the proportion of nonconformists, $1-[1-p+\mu p]$, and nothing otherwise. His expected valuation of bequest is $(1-[1-p+\mu p])\theta r$. It is straightforward to show that this interpretation gives an equivalent analysis of the income-sharing norm.

$-(1-p+p\mu)r$. His utility is :

$$U(\bar{y}, -(1-p+p\mu)r, \theta) = u(\bar{y}) - \theta(1-p+p\mu)r.$$

The family budget constraint states that the total gift $(1-p)\delta$ received by the poor is shared between conformists. For $\mu \neq 0$, transfers given (α) and received (δ) must satisfy the following constraint :

$$\mu p \alpha = (1-p)\delta. \tag{2.1}$$

We now introduce the sequence of actions in the model. Each agent plays the following game :

1. He observes his type θ and he learns the social norm α from the last generation.
2. He observes his random income $y \in \{\underline{y}, \bar{y}\}$.
   (a) He is rich $(y = \bar{y})$ with probability p.
   (b) He is poor $(y = \underline{y})$ with probability $1-p$.
3. He chooses between :
   (a) being conformist (sharing income and punishing nonconformists)
   (b) being nonconformist.
4. He gets his pay-off.
5. He designs and teaches a new norm α to the next generation.

An individual has to make the following choices. First, after observing his income, he chooses to be conformist or nonconformist. Then, he designs the next generation norm. The second action, the norm design, is not affected by the first action, agents conformity behavior. Hence, one agent decision regarding the next generation norm design does not depend on his conformity to the current norm. Therefore, we can derive the population norm obedience for a given transfer α (section 2.4) before endogenizing the next generation income-sharing norm (section 2.5).

2.4 The practice of an income-sharing norm

Given the other agents' behavior, an agent decides to be conformist if his utility is higher. Because there is a continuum of agents in this economy, a single player weight is nil. Therefore, one agent strategy does not affect significantly the equilibrium proportion of conformists. Hence, for each agent, the other rich players strategy is captured by the proportion of rich conformists μ. We now derive the Nash equilibrium of this game defined by a proportion of conformists μ^* such that, given the other's behavior μ^*, each individual maximizes his utility.

Suppose that all agents in the economy are expecting a proportion of rich conformists μ. A rich person of type θ decides to be a conformist if and only if :

$$U(\bar{y}-\alpha, 0, \theta) \geq U(\bar{y}, -[1-p+p\mu]r, \theta).$$

This condition can be rewritten :

$$\theta \geq \frac{u(\bar{y}) - u(\bar{y}-\alpha)}{[1-p+p\mu]r}.$$

An individual obeys the code of behavior if his taste for reputation is bigger than what he gains in utility from consuming all his income per unit of reputation loss. We can now characterize the critical agent or taste $\tilde{\theta}$ such that all rich individuals with a higher taste choose to conform to the norm and those with lower taste choose not to. Before doing that, we need to introduce new notation. Let $\mu_{\tilde{\theta}}$ be the minimum proportion of conformists that convinces the $\theta = \bar{\theta}$ type agent to obey the norm :

$$u(\bar{y} - \alpha) = u(\bar{y}) - \bar{\theta}[1 - p + p\mu_{\bar{\theta}}]r.$$

It is assumed that the sanction by the poor alone does not induce the higher θ type agent to be a conformist and therefore $\mu_{\bar{\theta}} > 0$. Let $\mu_{\underline{\theta}}$ be the minimum proportion of conformists which convince the $\theta = \underline{\theta}$ type agent to follow the norm :

$$u(\bar{y} - \alpha) = u(\bar{y}) - \underline{\theta}[1 - p + p\mu_{\underline{\theta}}]r.$$

Hence, we have :

$$\mu_{\bar{\theta}} = \frac{u(\bar{y}) - u(\bar{y} - \alpha)}{\bar{\theta} r p} - \frac{1-p}{p},$$
$$\mu_{\underline{\theta}} = \frac{u(\bar{y}) - u(\bar{y} - \alpha)}{\underline{\theta} r p} - \frac{1-p}{p}.$$

Since $\bar{\theta} > \underline{\theta}$, then $\mu_{\bar{\theta}} < \mu_{\underline{\theta}}$. Note that $\mu_{\underline{\theta}}$ does not exist if agent $\underline{\theta}$ does not obey the norm for $\mu = 1$:

$$u(\bar{y} - \alpha) < u(\bar{y}) - \underline{\theta}[1 - p + p]r.$$

That is if the community enforcement capacity is lower than $\underline{r}$ defined as :

$$r < \underline{r} = \frac{u(\bar{y}) - u(\bar{y} - \alpha)}{\underline{\theta}}.$$

In this case, we define $\mu_{\underline{\theta}} = 0$.

The taste $\tilde{\theta}$ of the agent indifferent between obeying and disobeying the norm is :

$$\tilde{\theta}(\mu) = \begin{cases} \underline{\theta} & if \quad \mu \geq \mu_{\underline{\theta}} \\ \frac{u(\bar{y}) - u(\bar{y} - \alpha)}{[1 - p + p\mu]r} & if \quad \mu_{\underline{\theta}} \geq \mu \geq \mu_{\bar{\theta}} \\ \bar{\theta} & if \quad \mu \leq \mu_{\bar{\theta}} \end{cases} . \tag{2.2}$$

The function $\tilde{\theta}$ characterizes the rich best reply for a common expected proportion of rich conformists μ. If μ is higher than $\mu_{\underline{\theta}}$, then all the rich decide to be conformist. If μ is lower than $\mu_{\bar{\theta}}$, then they all choose to be nonconformist. For $\mu : \mu_{\underline{\theta}} \geq \mu \geq \mu_{\bar{\theta}}$, the best reply for the rich with a reputation taste higher than $\tilde{\theta}(\mu)$ is to be conformist. For those with a reputation taste lower than $\tilde{\theta}(\mu)$, the best strategy is to be nonconformist. It is shown in the Appendix that the function $\tilde{\theta}(\mu)$ is decreasing and convex in $[\mu_{\bar{\theta}}, \mu_{\underline{\theta}}]$. This function is illustrated in Figure 1 when $\mu_{\underline{\theta}}$ does not exist, that is $r < \underline{r}$, and in Figure 2 otherwise by the plain line.

It is easy to derive the proportion of conformists for a given indifferent agent $\tilde{\theta}$. Since f is the density of agents' taste for reputation θ within the rich population share, the proportion of rich conformists for a critical taste $\tilde{\theta}$ is :

$$\mu(\tilde{\theta}) = \int_{\tilde{\theta}}^{\bar{\theta}} f(\theta)d\theta,$$

which can be rewritten :

$$\mu = 1 - F(\tilde{\theta}). \tag{2.3}$$

The relation (2.3) between μ and $\tilde{\theta}$ is represented by the dotted line in figures 1 and 2 for a taste θ uniformly distributed in Θ.

The Nash equilibrium is defined by a proportion of rich conformists μ^* such that, given the others' behavior μ^*, each individual maximizes his utility. In other words, at μ^*, each agent plays his best reply to other agent strategies which are represented by μ^*. Nash equilibrium is computed using equations (2.2) and (2.3). The rich best reply $\tilde{\theta}(\mu)$ stated in (2.1) defines the critical taste $\tilde{\theta}$ for a proportion of rich conformists μ. Equation (2.2) yields the proportion of rich conformist μ for each critical taste $\tilde{\theta}$. In equilibrium, the rich best reply function evaluated at μ^* equalizes the equilibrium proportion of conformists. The equilibrium μ^* of this game is defined by :

$$\mu^* = 1 - F(\tilde{\theta}(\mu^*)),$$

which can be summarized by :

$$\mu^* = \begin{cases} 1 & if \quad \mu^* \geq \mu_{\underline{\theta}} \\ 1 - F(\frac{u(\bar{y}) - u(\bar{y} - \alpha)}{[1 - p + p\mu^*]r}) & if \quad \mu_{\underline{\theta}} \geq \mu^* \geq \mu_{\bar{\theta}} \\ 0 & if \quad \mu^* \leq \mu_{\bar{\theta}} \end{cases} . \tag{2.4}$$

Mathematically, an equilibrium is a fixed point of the right-hand side of (2.4). This function gives the best reply proportion of conformists for each proportion of conformists. Since this function is an increasing continuous function of μ mapping the unit interval $[0, 1]$ into itself, there exists at least one equilibrium proportion of conformists. In fact, the equilibrium μ_3^* where no rich follow the norm ($\mu_3^* = 0$) always exists. Depending on the economic environment, and specifically on the enforcement capacity r, there may be two more equilibria μ_1^* and μ_2^*. In figures 1 and 2, equilibrium points are located where the two curves are intersecting. There is one equilibrium μ_1^* with a high proportion of conformists and one equilibrium μ_2^* with a low proportion of conformists. If $r > \underline{r}$, then the enforcement capacity is high enough to make $\mu_1^* = 1$: There exists an equilibrium where all the rich conform to the norm. Otherwise, some rich will disobey the norm in equilibrium.

We can identify two main features from Figures 1 and 2. First, positive transfers from the rich to the poor can be observed in a Nash equilibrium. Second, conformity to the norm can co-exist with some nonconformity. This result is explained by the agent's heterogeneous taste for reputation. Since some agents are less affected by their reputation, individuals are not equal with respect of the income-sharing norm enforcement. Consequently, nonconformists have higher ex post pay-offs than conformists.

Multiplicity of equilibrium raises the problem of equilibrium selection. One may ask which norm practice is expected to be observed from a given income-sharing norm in a given economic environment. I argue that, because μ_2^* is unstable, we can naturally eliminate this equilibrium from the set of plausible equilibria. In the model, the stability concept could be interpreted as follows. Assume that a Nash equilibrium is reached by the following tâtonnement or learning process. Starting from an initial common expected proportion of rich conformists μ_0, agents choose their best strategy. Then, given the best reply proportion of conformists, they readjust their behavior and decide whether to remain conformist or not until they reach a Nash equilibrium. If $\mu_0 \neq \mu_2^*$ then norm practice converges toward one of the stable equilibria. More precisely, if $\mu_0 < \mu_2^*$, the learning process converges toward μ_3^* and otherwise it converges toward μ_1^*. Moreover, μ_2^* is not robust to a local perturbation induced by a subset of agents of positive mesure deviations. Suppose that a measurable subset of agents deviates from the Nash equilibrium μ_2^* and that they readjust their behavior by playing the same tâtonnement process starting from the out of equilibrium norm conformity. Only one of the two stables equilibrium, μ_3^* or μ_1^*, would be reached, not μ_2^*. There are two only stable equilibrium, one, μ_1^*, with a high proportion of conformists (possibly full obedience to the norm), the other, μ_3^*, with no practice of the income-sharing norm. We now endogenize the income-sharing norm.

2.5 The income-sharing norm design

The income-sharing norm is designed by the preceding generation and then taught to the next generation. It is assumed that agents are equally altruistic for their children. When designing a norm, an elder's objective is to maximize the expected utility from future norm obedience. Formally, elders choose the transfer given[10] α which maximizes the sum of the next generation conformists' expected utility discounted by the altruism factor. However, each generation defines a new norm and this decision is not affected by the current norm or its conformity. The elders' influence is therefore bounded to the next generation. Hence, the elders'

[10] Note that they can equivalently define the corresponding transfer received δ.

objective is reduced to the next generation expected utility.[11]

When implementing an income-sharing norm, elders expect to reach the corresponding equilibrium μ^*. For a given α, a conformist's ex ante expected utility is :

$$U^C(\alpha) = pu(\bar{y} - \alpha) + (1-p)u(\underline{y} + \frac{\alpha p \mu^*}{1-p}).$$

Since μ_2^* is unstable and there is no norm practice at μ_0^*, elders naturally expected to reach the equilibrium μ_1^*. Therefore, the expected equilibrium is a stable strictly positive equilibrium $\mu^* = \mu_1^*$. Formally, the following property is satisfied for local stable[12] interior equilibrium μ^* :

$$f(\tilde{\theta}(\mu^*))\tilde{\theta}'(\mu^*) < 1.$$

The transfer is defined contingently on economic environment parameters. The amount given α depends on the rich' s aid capacity $\bar{y}$, on the needs of the poor $\underline{y}$, the proportion of rich, the agent's preferences and punishment capacity : $\alpha = \alpha(\bar{y}, \underline{y}, p, r)$. An income-sharing norm is full income-sharing defined by :

$$\bar{y} - \alpha^F = \underline{y} + \frac{p\mu^*\alpha^F}{1-p}$$

$$\iff \alpha^F = \frac{\bar{y} - \underline{y}}{\frac{p\mu^*}{1-p} + 1}.$$

The income-sharing transfer solves the following maximization program :

[11]The model can be extended by allowing for intergeneration example effects. That is, let what a generation does influence what the next generation feels its duty is. Individuals which descent from nonconformists would not learn about the income-sharing norm. Indeed, they would be excluded from the mutual insurance system. The proportion of conformist should therefore decrease. However, it could be compensated by the population growth.

[12]When considering the dynamic tâtonnement process $\mu_t = 1 - F(\tilde{\theta}(\mu_{t-1}))$, the equilibrium μ^* is locally stable if $\frac{d\mu_t}{d\mu_{t-1}}|_{\mu^*} < 1$.

$$Max_{\alpha} pu(\bar{y}-\alpha)+ \quad (1-p)u(\underline{y}+\tfrac{\alpha p\mu^*}{1-p}) \quad subject\ to$$

$$\mu^* = \begin{cases} 1 & if \quad \mu^* \geq \mu_{\underline{\theta}} \\ 1-F(\frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu^*]r}) & if \quad \mu_{\underline{\theta}} \geq \mu^* \geq \mu_{\bar{\theta}} \\ 0 & if \quad \mu^* \leq \mu_{\bar{\theta}} \end{cases}$$

Propositions 1 and 2 characterize the solution α^* of this maximization problem .

Proposition 1. *Full income-sharing is implemented if and only if there is full obedience to the full income-sharing norm.*

This proposition identifies two features. First, first best risk-sharing can be reached by an income-sharing norm if mutual enforcement capacity is high enough ($r > u(\bar{y}) - u(E[y])$). That means that punishment r must be high enough to induce all agents to share all their income. Second, full income-sharing as the best income-sharing norm is not robust to some agent disobedience. When there is too much heterogeneity in agents' taste for reputation, the income-sharing norm fails to reach the first best. If some family member disobeys to a full income-sharing norm, a conformist's expected utility is maximized with partial income-sharing. [13]

Proposition 2. *If full income-sharing is not implementable, then α^* is defined by :*

$$u'(\underline{y}+\delta^*)[\mu^* + \alpha^* \frac{d\mu^*}{d\alpha^*}] = u'(\bar{y}-\alpha^*),$$

with $\frac{d\mu^*}{d\alpha^*} = -\frac{f(\tilde{\theta}(\mu))\frac{u'(\bar{y}-\alpha^*)}{[1-p+p\mu^*]r}}{1-f(\tilde{\theta}(\mu^*))p\frac{u(\bar{y})-u(\bar{y}-\alpha^*)}{[1-p+p\mu^*]^2 r}}$ *and* $\delta^* = \frac{\alpha^* p\mu^*}{1-p}$.

First, Proposition 2 tells us that if there is full obedience ($\mu^* = 1$) but the full risk-sharing is not implementable ($r < u(\bar{y}) - u(E[y])$), the transfer made is the higher transfer accepted by the agent which is the less affected by his reputation (otherwise, we would have $\frac{d\mu^*}{d\alpha^*} = 0$, therefore $\alpha^* = \alpha^F$). If the best

[13] This property is an empirical implication of the model : Full income-sharing cannot be observed with some nonconformity.

norm implies full obedience, the transfer will be increased as long as every family member obeys the norm. Second, the first-order condition characterizes the trade-off between risk-sharing by ex post income-sharing and incentive to obey the norm. Remember that the transfer objective is to share risk ex ante by redistributing ex post the revenue. In a full-enforcing world, an optimal risk-sharing rule equalize individual's marginal utilities in each state of nature. Here, risk-sharing is constraint by punishment capacity. The equality between marginal utility in the two states of nature is bounded by the mutual enforcement constraint. The income-sharing norm equalizes rich donors' marginal utility to poor recipients' marginal utility adjusted by the losses generated by the nonconformist' s behavior. This term reflects the fact that the utility lost in the good state of nature does not compensate one to one the utility earned in the bad state of nature. It is interpreted as follows. If the rich have to give one more unit of consumption, the poor would receive only μ^* units for a constant proportion of conformists. Moreover an increase of α makes the norm less attractive so that the proportion of conformists μ^* decreases (For a stable interior equilibrium, we have $\frac{d\mu^*}{d\alpha} < 0$). The transfer recieved is reduced below μ^*. We now investigate whether nonconformists can be excluded by letting them decide ex ante not to enter into the mutual insurance network.

2.6 The income-sharing norm as an informal contract

Suppose now that agents have ex ante the choice to adhere or not to the norm. The income-sharing norm can be interpreted as an informal insurance contract offered by elders to the young. An agent is punished only if he does not respect the "contract he signed". If he rejects the contract, on one hand, he does not benefit from it if he is poor. But, on an other hand, he is not sanctioned and he does not have to make the transfer if he is rich. Each agent plays the following game.

1. He observes his type θ and he learns the informal insurance contract α from the last generation.
2. He chooses between :
   (a) accepting the contract.
   (b) refusing the contract.
3. He observes his random income $y \in \{\underline{y}, \bar{y}\}$.
   (a) He is rich ($y = \bar{y}$) with probability p.
   (b) He is poor ($y = \underline{y}$) with probability $1 - p$.
4. If he accepted the contract, he chooses between :
   (a) being conformist (sharing income and punishing nonconformists)
   (b) being nonconformist.
5. He gets his pay-off.
6. He designs and teaches a new informal insurance contract α to the next generation.

In this new game, I add a new stage where each agent decides to accept or not to share ex-post his income before observing it. If the agent refuses the informal mutual assistance arrangement, he gets his random income and quits the game. A subgame perfect equilibrium of the game is characterized in Proposition 9.

Proposition 3. *There exists a (subgame) perfect equilibrium of the game where all agents accept the informal insurance contract.*

Proposition 3 states that nonconformists cannot necessarily be self-excluded from the mutual insurance system. Due to lack of legal enforcement, an agent adheres to the norm without committing himself to fulfill his duties ex post. Opportunist behavior could then be observed at the equilibrium. These agents are free riders since they benefit from the informal contract without paying the cost. They are subsidized by conformists and reduce the efficiency of the informal contract.

This result should seem surprising for readers familiar with the mechanism design literature. The mechanism design approach shows that, by offering a menu of contract, a private information can be costly revealed (e.g. Fudenberg and Tirole, 1993, Chap. 7). Here, the unobserved variable is agent's taste for social status. The result does not hold because private information is related to the enforcement of the contract. Suppose that elders offer a menu of transfers given $\alpha(\theta)$ and received $\delta(\theta)$ contingently on each agent's taste for reputation θ. All agent would choose the best risk-sharing arrangement, that is the higher pair of transfers, whatever his taste for reputation. In other words, nonconformists would report an high θ, benefit of a high payout δ following unsuccessful outcome without giving anything otherwise. Consequently, a complex menu of contracts is as efficient as an uniform rule for risk sharing proposes within the community. The community authority would prefer use the more simple instrument for risk-sharing proposes. We now study the comparative static properties of the income-sharing norm.

2.7 Comparative statics

Comparative static properties are derived in the Appendix when agent's taste for reputation is uniformly distributed in $[1, 2]$. First, we derive the evolution of norm obedience when the transfer α is exogenous fixed and there is partial norm conformity ($\mu^* < 1$) :

$$\frac{d\mu^*}{dr} > 0, \ \frac{d\mu^*}{d\bar{y}} > 0, \ \frac{d\mu^*}{d\underline{y}} = 0, \ \frac{d\mu^*}{dp} < 0.$$

Second, we analyse the evolution of the transfer given α^* :

$$\frac{d\alpha^*}{dr} > 0, \ \frac{d\alpha^*}{d\bar{y}} > 0, \ \frac{d\alpha^*}{d\underline{y}} < 0, \ \frac{d\alpha^*}{dp} < 0.$$

If agents' enforcement capacity r decreases, norm obedience decreases for α constant. The transfer α^* required must be reduced to minimize loses generated by nonconformity. Consider now a reduction of the rich' s income $\bar{y}$. Suppose, for instance, that civil servant wages are reduced by a structural adjustment policy. For a fix transfer α, norm obedience is less attractive. The private transfer α^* is reduced to give the rich enough incentives to obey to the norm and for better risk sharing. Consider an increase of $\underline{y}$ through, for instance, more public aid given to the poor. For a fixed transfer α, a change in the poor revenue does not affect norm obedience. However, for better risk-sharing and to increase norm conformity, the transfer required α^* is reduced.

An increase in the proportion of rich p has two effects on the income-sharing norm. First, the proportion of people who sanctions nonconformists $1 - p + p\mu^*$ decreases. The punishment is less dissuasive and norm conformity decreases for a constant transfer α. In order to makes enough persons obey the norm, the transfer given α^* must be reduced. Second, the total pie collected from the rich increases whereas the number of slices the pie is cut decreases. That is, for the same transfer received by a poor δ, if norm conformity is constant, the amount required to the rich is reduced. These two effects decrease the private transfer α^*. We now link income-sharing efficiency with public policy.

2.8 Public policy

My approach of private transfers should shed light on the debate about the efficiency of LDCs' nonmarket institutions. One of the issues is the efficiency of informal insurance arrangements compared to a public or private insurance system. For Besley (1995) , informal risk sharing responds to uncertainties in the legal system, low level of human capital and poor development of infrastructures facilitating communication. Informal institutions exploit a comparative advantage in monitoring and enforcement capacity. For Cox and Jimenez (1990), many social objectives are already being met through private transfers.[14] For Arnott and Stiglitz (1991), informal risk sharing is generally a dysfunctional crowding out with a formal insurance system. The reason is that informal risk sharing doesn't internalize the market imperfection caused by moral hazard.

In this section, I show first that a redistributive policy does not necessary share risk more efficiency than the income-sharing norm. I provide a sufficient condition for the implementation of income redistribution. Second, I explain how a low cost redistributive policy increases the efficiency of the income-sharing norm and, finally, agents' expected utility.

Whereas the income-sharing norm is limited by the community enforcement capacity, public income redistribution can be legally enforced. The government is able to make all the rich pay an income tax τ to finance an individual subsidy σ to the poor. Furthermore, public income redistribution pools the risk by sharing it not only within enlarged family but also between enlarged families. However, in order to implement it, the government must observe individuals' income. In less developed economies, information extraction, monitoring and public fund

[14] They estimate that if unemployment insurance systems were introduced in the Philippines, private transfers would fall so much that the intended beneficiaries of the program would hardly be better off (Cox and Jimenez, 1995).

administration can be very costly compared to individuals' income. These costs are usually represented in regulation theory (Laffont and Tirole, 1993) by the shadow cost of public funds λ : For each dollar given to the poor, the rich have to be taxed $1+\lambda$ dollars ($\lambda > 0$). The state budget constraint is :

$$\frac{p\tau}{1+\lambda} = (1-p)\sigma.$$

Using equation 1, it is easy to compare the relative performance of formal and informal insurance systems. For an equal transfer received by poor $\sigma = \delta^*$, the rich contribution τ is lower than α^* if :

$$\frac{1-p}{p}(1+\lambda) < \frac{1-p}{\mu^* p}.$$

A formal insurance system is more efficient that the equivalent income-sharing norm if :

$$\lambda < \frac{1-\mu^*}{\mu^*}. \tag{2.5}$$

Public income redistribution shares risk at a lower cost than the income-sharing norm when administrative costs are lower than the ratio of rich nonconformists over rich conformists. If this condition is not satisfied, high administrative costs may offset the benefit of a redistribution policy. When norm obedience is close to be complete, the implementation of a redistribution policy is justified by very low public costs or strong inequality between income-sharing networks. The proportion of free riding or nonconformity in the rich population $1-\mu^*$ measures the inefficiency of the income-sharing norm.

Suppose now that a income redistribution policy is implemented. The comparative static analysis shows that the transfer required by the norm α^* decreases. Therefore, the norm inefficiency $1-\mu^*$ decreases when there is partial norm-obedience. Moreover, in all cases (full or partial norm-obedience), the constraint in the maximization program of the income-sharing norm design is relaxed. Then, if condition (2.5) is satisfied, the conformist's expected utility, and therefore the nonconformist's one, is higher. Finally, we find that public income redistribution increases the efficiency of the income-sharing norm when administrative cost are sufficiency low. Otherwise, it may be costly crowding out for risk-sharing within extended families.

2.9 Conclusion

In this essay, African income-sharing practice is modeled as a social norm implemented by the old generation and mutually enforced by agents through individual's heterogeneous valuation on social status. It is explained how a partial or full norm-obedience could be observed at the equilibrium. The norm is then endogenized. It shares fully the risk if and only if there is full obedience to this norm. Otherwise, it solves a trade-off between risk-sharing and mutual enforcement. We could have partial risk-sharing and some nonconformity in equilibrium as a second best. Nonconformists cannot be excluded by letting them decide ex ante not to enter into the mutual insurance system. A redistributive policy may share risk at a higher cost than the income-sharing norm. Indeed, the effect of public transfers can be completely offset by changes in private transfers. People would be worse-off with a public policy that smooth consumption less efficiently than the norm. But when administrative costs are low, a redistributive policy supplemented by private transfers increases agents'welfare.

Recent papers on financial institutions in developing countries focus on the group lending properties to avoid information asymmetries (see Ghatak and Guinname, 1998, for a review). Groups are able to monitor and enforce contractual arrangement because agents know each others and can exert mutual sanctions more costly than legal sanctions. By lending to groups instead of lending to a single person, a bank can exploit informal rule's relative advantage on monitoring and enforcement. A natural extension of our model would be to allow an individual's income to depend on its work effort. When effort is observed by individuals, the norm could include an effort behavior. This effort level is then implemented through peer monitoring and mutual enforcement. As in group lending contracts, the income-sharing norm perform better than an institutional insurance because of its relative advantage in monitoring. When effort is unobservable, the income-sharing norm is constraint not only by mutual enforcement capacity but also by individuals' incentives on effort. However, contrary to Arnott and Stiglitz (1991), we may find scope for efficient risk-sharing via ex post income-sharing.

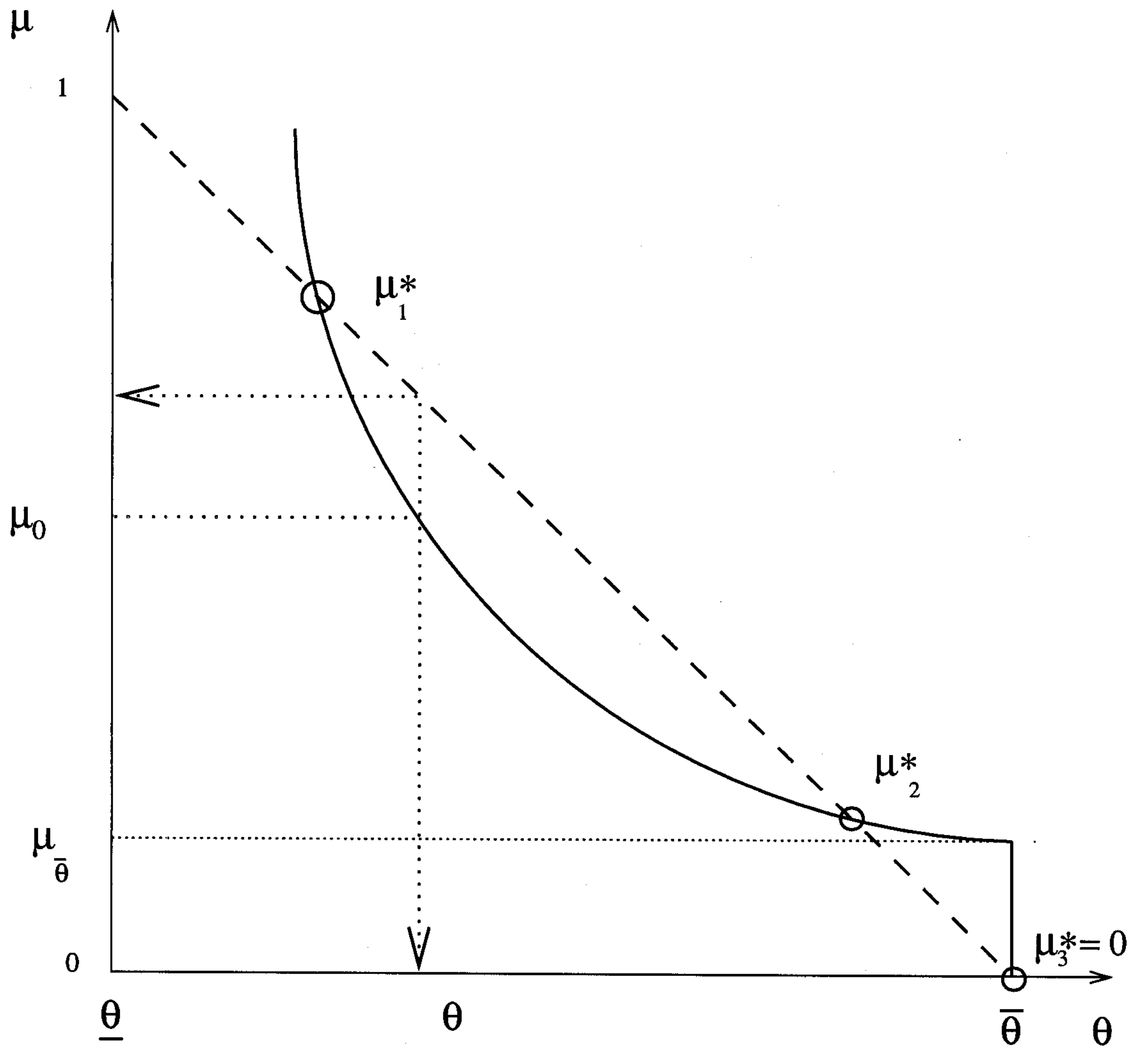

Figure 1

$$\text{———————} : \quad \tilde{\theta}(\mu) = \begin{cases} \frac{u(\bar{y}) - u(\bar{y}-\alpha)}{[1-p+p\mu]r} & if \quad 1 \geq \mu \geq \mu_{\bar{\theta}} \\ \bar{\theta} & if \quad \mu \leq \mu_{\bar{\theta}} \end{cases}$$

$$\text{– – – – – – – –} : \quad \mu = 1 - F(\tilde{\theta})$$

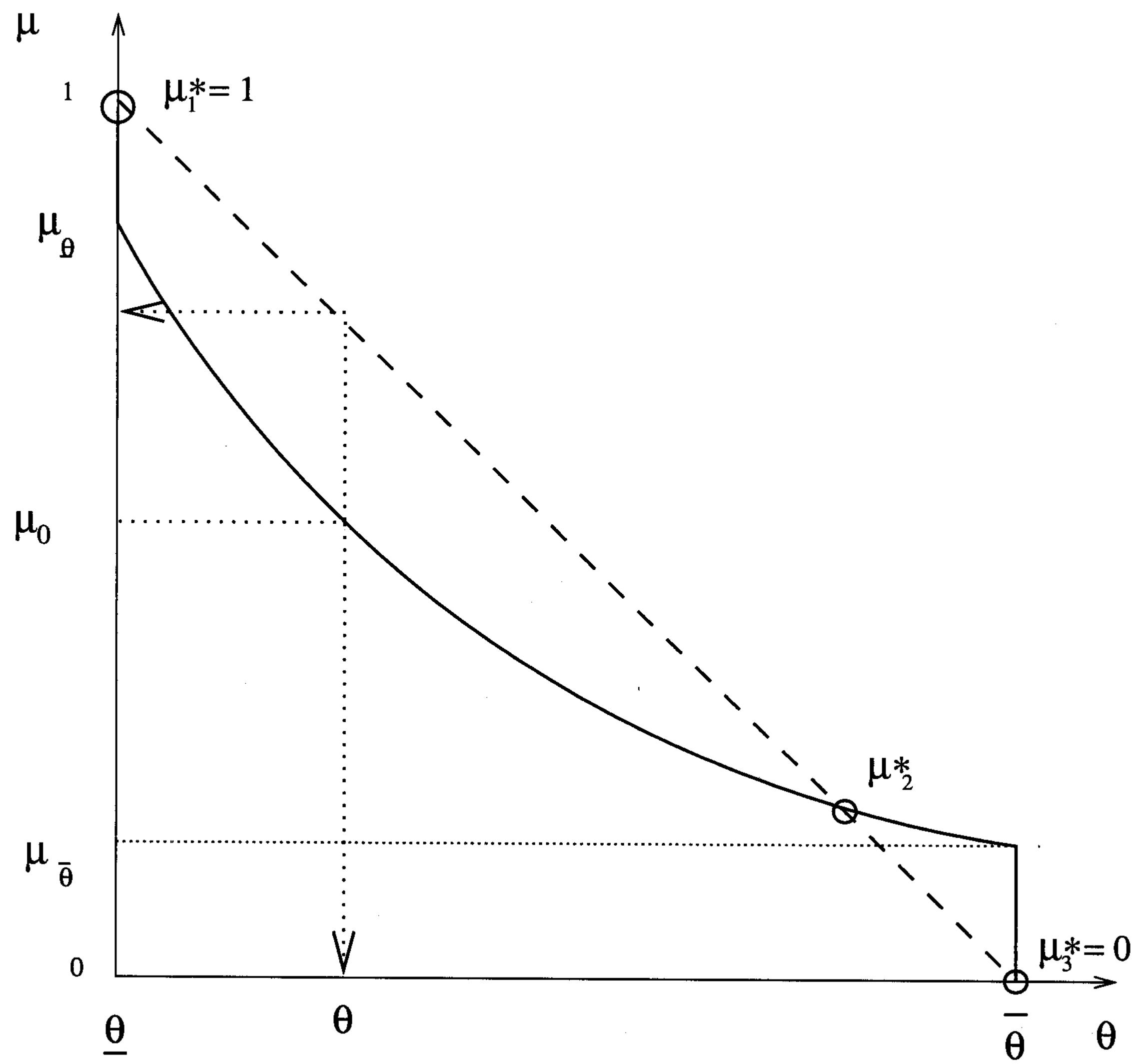

Figure 2

———————— : $\tilde{\theta}(\mu) = \begin{cases} \underline{\theta} & if \quad \mu \geq \mu_{\underline{\theta}} \\ \frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu]r} & if \quad \mu_{\underline{\theta}} \geq \mu \geq \mu_{\bar{\theta}} \\ \bar{\theta} & if \quad \mu \leq \mu_{\bar{\theta}} \end{cases}$

- - - - - - - - - - : $\mu = 1 - F(\tilde{\theta})$

Chapitre 3

Sharing a river

Abstract

This essay analyzes how to share a river between users in case of water scarcity. It is assumed that the water flow is unequally distributed along its stream and that water can only be transferred downstream. We characterize the efficient water allocation. We analyze stable welfare distributions of the cooperative game. We show that this game is convex so that its core is the convex hull of the marginal contribution vectors. We propose a fairness principle. We call "aspiration welfare" the welfare that could enjoy a group of agent in the absence of the others. A fair welfare distribution assigns to every group of agent not more than his "aspiration welfare". We show that only one core distribution is fair. We discuss its decentralization and its implementation.

3.1 Introduction

Water is essential for the sustenance of life. Man consumes it for domestic purposes such as drinking, cooking or washing ; for agricultural proposes such as irrigation. Water is also used for waste disposal. Due to population growth, industrialization and agricultural expansion ; everywhere today the demand for fresh water is increasing rapidly while the quality of water is declining. Whereas planet Earth is still endowed with plenty of fresh water, these resources are badly distributed. On most of the earth's surface, water exists in insufficient quantities. Locally, people share a scarce resource.

It is well-known that a free-access extraction of scarce water is inefficient. Users divert too much water upstream so that the water is underconsummed downstream. By an appropriate re-allocation of water, the total surplus of the river exploitation can be increased. An appropriate distribution of this surplus can make all riparian better off.

Free waste disposal in rivers is also inefficient. Water has limited assimilative capacity for the waste. Excessive pollution deteriorate water quality. It rises production costs, has an impact on wildlife and affects the river's recreation benefits. It should be avoided. The total pollution that can be emitted in the river is bounded.[1] Under free waste disposal, to much pollution is emitted upstream. By an appropriate allocation of emissions, the total surplus of the river exploitation for waste disposal can be increased. An appropriate distribution of this surplus can make all riparian better off. Such Pareto improving policy for both water quantity and quality requires that riparian cooperate to specify new rules on water management.

On more and more rivers, people coordinate water extraction and pollution (see Dinar and all. (1997) for a survey). Policy makers regulate water management in rivers by designing taxes and subsidies. They may also sell water rights. An administration is usually legally entitled to collect taxes or to sell water. The money collected is refunded to riparians. It is invested in public goods such as

[1] As argued by Dasgupta (1990), water quality can be view as is a scarce commodity.

dams or canals. It may be used to cover maintenance costs. It is also redistributed in subsidies for adoption of water-saving or cleaner technologies. The "Agences de bassin" in France is an example of such a public administration. In less-developed countries, informal rules govern water management in irrigation communities. They are designed and enforced by the community members. Surface water rights are also exchanged in legal auction. In the so-called "huerta" (irrigation service area) of Allicante, agents are endowed with volumetric water rights from specific sources. They purchase and sell water rights in a public auction held every Sunday morning. The trade are enforced by an executive commission elected by members (Reidinger, 1994, Ostrom, 1990).

Many of the world's river basins are shared by two or more countries. In a United Nations study, international river basins were estimated to comprise about 47 percent of the world's continental land area. In Africa, Asia and South America, this proportion rises to at least 60 percent. Most of the 200 international rivers are shared by less than 6 countries : 148 are shared by two countries, 30 by three, 9 by four, and 13 by five or more countries. Many countries rely mostly on water originating in other countries. The percent of total flow originating outside of border goes up to 97 for Egypt, 95 for Mauritania, 89 for Netherlands or 34 for Senegal (Barret, 1994).

To avoid inefficient exploitation of international rivers, several countries have voluntary agreed to limit their water withdraw (see Barret, 1994, Godona, 1985, for case studies). The Nile Treaty is an example of an international river-sharing agreement. Egypt and Sudan agreed to divide Nile water in 1959..[2] The treaty has recently been challenged by other riparian countries, who are interested in their own shares of Nile water.

[2]The Nile river is shared by nine countries (Burundi, Congo, Egypt, Kenya, Sudan, Rwanda and Uganda). Egypt was allocated 55.5 Million Cubic Meters (MCM) of water per year, and Sudan 55.5 MCM.

Countries also adhere to international institutions for water-management. For instance, Mali, Mauritania and Senegal founded the "Organisation pour la mise en valeur du fleuve Sénégal" (OMVS) to coordinate the management of the Senegal river. Among other rules, the "principe d'approbation des Etats" states that a country cannot change the water flow without the consent of all members. Members voted on a cost-sharing rule and a "water management program" for the two dams build on the river.

Countries have agreed to share the cost of pollution reduction. The "Convention on the Protection of the Rhine Against Pollution by Chlorides" is a good example. The Rhine river passes through four countries, first, Switzerland, then France and Germany, last, the Netherlands. It suffered from salt pollution emitted by one major polluter : a potash mine in France (40 % of the salt entering the Rhine). In 1972, the four countries agreed to limit the concentration of chloride ions at the Dutch frontier. To meet this objective, emissions from the French mine were to be reduced. The four riparian divided the cost of emission abatement as follow : France and Germany would each pay 30% of the cost, the Netherlands would pay 34%, and Switzerland would pay 6%.

When surface waters are shared by sovereign countries, no supra-national authority can impose a cooperative management of water. Countries should agree on an allocation of water. They may also negotiate a compensation scheme to make all of them accept the deal. Side-payments between countries define a distribution of the welfare induced by the cooperative management of water. To be freely accepted and enforced by riparian States, the distribution of welfare should have the following properties. First, it should be *stable.* No group of States should all increase their gain by signing their own agreement. Second, it should be perceived as *equitable.*

In this paper, we address two questions. First, how to share water in a river in case of water scarcity. We characterize the Pareto optimal water allocation in the river. Second, how to share the welfare of an efficient management of water between agents. We analyze welfare distributions that are stable and fair. The chapter is organized as follow. We first model the exploitation of a river (section 3.2). Then, we characterize the optimal allocation of water (section 3.3). We find out the set of stable welfare distributions (section 3.4). We characterize the stable welfare distributions that is fair (section 3.5). We discuss its decentralization and its implementation in various economic environments (section 3.6). We conclude by some remarks.

3.2 The model

A river flows through a number of countries, cities or irrigated corps, henceforth called *agents*, whose set is denoted by $N = \{1, ..., n\}$ with $n \geq 2$. We identify agents with their location along the river and number them from upstream to downstream : $i < j$ means that i is upstream j.

The river picks up volume along its course : the flow at its source, $e_1 > 0$, is increased by the amount $e_i \geq 0$ between locations $i - 1$ and i, say at i. A schematic representation is given in Figure 3.

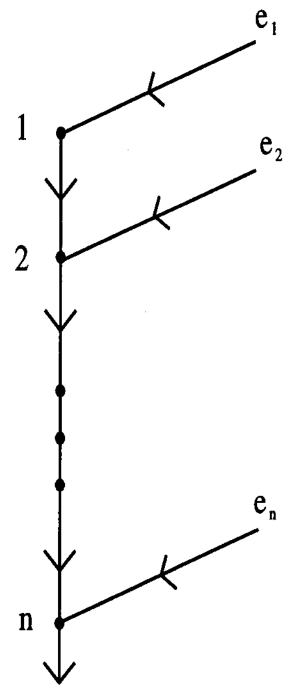

Figure 3

Agents are endowed with unbounded quantities of a perfectly divisible good that will be called money. They value money and the water from the river. Agent i's utility from extracting x_i units of water and receiving a net money transfer t_i is $u_i(x_i, t_i) = b_i(x_i) + t_i$. We assume b_i increasing twice differentiable and strictly concave, for every $i \in N$:

$$b_i'(x) > 0, b_i''(x) < 0, \ \forall x \in R^+, \ \forall i \in N.$$

Water is essential for agents : Each agent i's marginal benefit is infinite at zero, that is $b_i'(0) = +\infty$, $\forall i \in N$. When water is consumed for production purposes (agriculture or industry), b_i can be either interpreted as a production function. We make the harmless convenient assumption that $b_i(0) = 0$: No water yields no production or zero utility.

It will be convenient to define the sets of predecessors and followers of agent i, respectively, by $Pi = \{j \in N : \ j \leq i\}$ and $Fi = \{j \in N : \ j \geq i\}$, and the sets of his strict predecessors and followers by $P^0i = Pi \backslash \{i\}$ and $F^0i = Fi \backslash \{i\}$.

The list (N, e, b), where $e = (e_1, ..., e_n)$ and $b = (b_1, ..., b_n)$, constitutes our *river sharing problem.* It is one of perfect information : All variables are perfectly observed by all agents.

An *allocation* is a vector $(x, t) = (x_1, ..., x_n, t_1, ..., t_n) \in R_+^N \times R^N$ satisfying the feasibility constraints

$$\sum_{i \in N} t_i \leq 0,$$

$$\sum_{i \in Pj} x_i \leq \sum_{i \in Pj} e_i \text{ for every } j \in N.$$

First, the transfer scheme must be budget balanced. Second, at each level j of the river, the amount of water consumed by agent j and its predecessors must be lower than the flow available upstream. This constraint reflects that water stream e_i can only be consumed by i and its followers. Water transportation costs are assumed prohibitive so that water can only be transferred downstream.

A (*welfare*) *distribution* is any vector $z = (z_1, ..., z_n) \in R^N$ which is the utility image of some allocation (x, t) in the sense that $z_i = u_i(x_i, t_i)$ for each agent i. For every $S \subset N$, x_S denotes the projection of x in S, i.e. $x_S \in \mathcal{R}^S$, with $(x_S)_i = x_i$ for $i \in S$.

Remark. The model can be make useful to analyze river exploitation for waste disposal. Suppose that each agent i emits x_i to produce consumption. Suppose also that the river is used for other proposes that requires a relatively clean water at each level of the river. The river assimilation capacity for waste between agents $i - 1$ and i is represented by e_i. Thus, total emissions are bounded by feasibility constraints at each level i of the river.[3] We obtain a river sharing problem for waste emission.

We now analyze the optimal allocation of water in the river.

[3]For example, the "Convention on the Protection of the Rhine Against Pollution by Chlorides" limits the concentration of chloride ions at the Dutch frontier.

3.3 The optimal water allocation

An optimal water allocation (x^*) maximizes the sum of utilities subject to the feasibility constraints. It solves the following program :

$$\Pi_N \begin{cases} \max_x \ \sum_{i\in N} b_i(x_i) \ \ s/t & \\ \qquad \sum_{j\in Pi} x_j \ \leq \ \sum_{j\in Pi} e_j \ \ i = 1, ..., n \end{cases}$$

Due to the strict concavity of each b_i, the efficient water allocation x^* is unique. Moreover, $x_i^* > 0$ for each i because $b_i'(0) = +\infty$. Let μ_i be the Lagrangian multiplier associated to the constraint i, for all $i = 1, ...n$. The first order conditions are :

$$b_i'(x_i^*) = \sum_{j\in Fi} \mu_j, \ \forall i \in N \tag{3.1}$$

They can be rewritten as :

$$b_i'(x_i^*) - b_{i+1}'(x_{i+1}^*) = \mu_i, \ \forall i = 1, ..., n. \tag{3.2}$$

Which implies :

$$b_{i+1}'(x_{i+1}^*) \leq b_i'(x_i^*). \tag{3.3}$$

Condition (3.3) is a necessary condition for allocation x^* to be efficient. It says that agent i's marginal benefit must be higher or equal to the marginal benefit of his downstream partners $j > i$. If it is not the case, by reducing the water allocated to the upstream user x_i and increasing the water allocated to the downstream user x_{i+1}, the welfare, i.e. the sum of utilities, can be increased.

When $\mu_k > 0$, then the kth feasibility constraint is binding. Therefore equation (3.2) yields :

$$b_{k+1}'(x_{k+1}^*) < b_k'(x_k^*). \tag{3.4}$$

The marginal benefit of user k is strictly higher than the marginal benefit of user $k+1$. Since feasibility constraint k is binding, agent k withdraws all water available at his level in the river stream. The stock of water remaining to $k+1$ is therefore e_{k+1}. Denote $K = \{i_1, ..., i_K\}$, with $i_1 < i_2 < ... < i_K$ the set of binding constraints in Π_N, i.e. $K = \{i \in N | \mu_i > 0\}$. The results are summarized in Proposition 1.

Proposition 1. *There exists a unique partition $\mathcal{P}(N)$ of N into consecutive coalitions $N_1, ..., N_K$ and positive numbers $\mu_{i_1} > ... > \mu_{i_K}$ such that,*

$$\forall k = 1, .., K, \ \forall j \in N_k, \ b_j'(x_j^*) = \mu_{i_k}.$$

Moreover,

$$\forall k = 1, .., K, \sum_{j \in N_k} x_j^* = \sum_{j \in N_k} e_j.$$

Proposition 1 states that, first, marginal benefits of users located *in between* feasibility constraint are equal. Second, marginal benefits of users located *between* feasibility constraints decrease by going downstream the river. Third, each coalition N_k consumes exactly the water it controls ; we say that it is *self-sustained.*

We characterized the efficient allocation of water. That is the unique water allocation x^* that maximizes the total welfare. Certainly, an agreement for river management should induce agents to consume x^*. Now, it should also define a vector of transfers t. The ideal transfer scheme t^* should be such that the cooperative management of the river is self-enforced. The ideal transfer scheme t^*, together with the optimal water allocation x^*, define a unique welfare distribution z^*, with $z_i^* = u_i(x_i^*, t_i^*)$ for every $i \in N$. For the agreement to be self-enforced, the welfare distribution should be stable and fair. We therefore study stability and fairness in welfare terms in the two next section.

3.4 Core stability : lower bounds on welfare

Our first concern is to recommend a welfare distribution that is stable. This requirement can be made precise by analyzing the cooperative game naturally associated with the river sharing problem.

Call a group S of agents, or *coalition*, *connected*[4] if $k \in S$ whenever $i, j \in S$ and $i < k < j$. Observe that every coalition S admits a unique coarsest partition into connected components : denote it $\mathcal{S}$.

By efficiently allocating among its members the water it controls, a connected coalition T can guarantee to itself exactly the following *secure (or autonomous) welfare :*

$$v(T) = \max_{x_T} \sum_{i \in T} b_i(x_i) \ s/t \sum_{j \in Pi \cap T} x_j \leq \sum_{j \in Pi \cap T} e_j \ \forall i \in T. \tag{3.5}$$

The secure welfare of an arbitrary coalition S is obtained by summing the secure welfares of the connected components of the partition $\mathcal{S}$, i.e.,

$$v(S) = \sum_{T \in \mathcal{S}} v(T), \tag{3.6}$$

where $v(T)$ is given by (3.5). Coalition S cannot secure more than (3.6) because any water left over by one of its connected components cannot be safely guaranteed for the consumption of any other component. We say that v is the *game generated by the problem* (N, e, b).

A (welfare) distribution $z = (z_1, ..., z_n)$ is a *core distribution* if $\sum_{i \in S} z_i \geq v(S)$ for every $S \subset N$. An allocation that does not generate a core distribution would be unstable : some coalition could object to it on the basis that it can secure on its own a higher welfare to all its members. Fortunately, core distributions do exist in the present context. Indeed, Greenberg and Weber (1993) have shown

[4] Or "consecutive", "convex", "without holes", see Greenberg and Weber (1993).

that, in such "connected games", at least one distribution belongs to the core. In our (simpler) model, we can say more. It turns out that the game v generated by a river water allocation problem is *convex* in the sense of Shapley (1971). Call $v(S) - v(S \backslash \{i\})$ agent i's marginal contribution to coalition S. A game is convex if and only if for every agent i, $v(S) - v(S \backslash \{i\}) \leq v(T) - v(T \backslash \{i\})$ whenever $i \in S \subset T \subset N$. In words, an agent's marginal contribution to a coalition increases by expanding the coalition. This means that the larger the coalition that agent i joins, the larger his marginal contribution.

Proposition 2. *The game v generated by the river water allocation problem (N, e, b) is convex.*

Proposition 2 renders the purely (cooperative) game-theoretic analysis of the river sharing problem rather straightforward. Let us define a random ordering $I = \{i_1, i_2, ..., i_n\}$ of the set N. Call vector of marginal contributions corresponding to the ordering I the vector x defined as $x_{i_k} = v(i_1, ..., i_k) - v(i_1, ..., i_{k-1})$, for every $i_k \in I$. Shapley (1971) has shown that the core of a convex game is the convex hull of the marginal contribution vectors. It implies that a core distribution yields to agent i at least its marginal contribution to the empty set, that is $v(i)$, and at most its marginal contribution to the set of all other riparian $N \backslash \{i\}$, that is $v(N) - v(i)$.

Proposition 2 provides an argument in favor of the Shapley value in our problem.[5] The Shapley value of a convex game is the barycenter of its core. It assigns to each agent i an average of his marginal contribution $v(S) - v(S \backslash \{i\})$ taken over all coalitions $S \subset N$ including the empty set. It can be easily computed by summing the marginal contribution vectors weighted by the equal probability of the corresponding ordering, namely $\frac{1}{n!}$.

In the next section, we analyze fairness principles proposed in international

[5]One may also think of Dutta and Ray's egalitarian solution (see Dutta and Ray, 1989).

river disputes and show that they fail to reduce the choice of stable welfare distributions. We then propose an other fairness principles that picks a single welfare distribution among those included in the core under some cases.

3.5 Fairness

3.5.1 The fairness doctrines proposed in international river disputes

Before introducing our fairness principle, let us start by analyzing the fairness arguments proposed to prevent or resolve disputes within international river basins. Two extreme and incompatible principles are commonly put forward (see Godana, 1985 ; Barrett, 1994).

The theory of unlimited territorial sovereignty or the Harmon Doctrine was first authoritatively stated by Judson Harmon, Attorney-General of the United States, in a declaration made in 1895 concerning the Rio Grande. According to this theory, a State has absolute sovereignty over the area of the basin within its territory. It may freely dispose of water flowing within its borders but cannot claim the continued free and uninterrupted flow of the water form upper-basin States. In our model, any agent i is entitled to withdraw e_i. A fair welfare distribution should give him at least $b_i(e_i)$. By extending the fairness principle to any group of States, we obtain that the set of welfare distributions is equal to the core of the cooperative game. Since the game is convex, the core is very large. The doctrine of absolute territorial sovereignty is therefore unable to restrict the choice of welfare distributions.

The theory of unlimited territorial integrity states that the quantity and quality of water available to a country cannot be altered by another.[6] The principle could be interpreted in physical or welfare terms. In physical terms, it awards the right to agent n to withdraw all water available on the river. Hence, the unique allocation consistant with this principle is $x = (0, ..., 0, \sum_{i \in N} e_i)$. Surely, this water allocation is not efficient. In welfare terms, this theory constraint any welfare distribution to assign to agent n at least $b_n(\sum_{i \in N} e_i)$. This interpretation is compatible with efficiency but generally not with stability.[7]

3.5.2 The aspiration welfare : upper bounds on welfare

In the absence of the other agents, agent i would be able to consume the full stream of water running through his location, thereby enjoying his *aspiration welfare.*

$$w(i) = b_i(\sum_{j \in Pi} e_j).$$

Of course, the welfare distribution $(w(1), ..., w(n))$ is generally not feasible : $\sum_{i \in N} w(i) > v(N)$ as soon as there are at least two agents with strictly positive marginal valuations of the water. In Moulin's (1990) terms, the river water allocation problem exhibits *negative group externalities.* In such a context, it is natural to ask that everyone takes up a share of theses externalities ; certainly no one should

[6]This doctrine espouses the old English common law whereby a lower riparian claims the right to the continued natural flow of water from the territory of the upper riparians. It has been put forward by Egypt during the meeting of the Nile Commission of 1925 on the question of the division of the Nile Waters.

[7]For example, suppose $n = 2$, $b_i(x) = ln(x) + 1$ for $i = 1, 2$, $e_1 = 3$ and $e_2 = 1$. A fair welfare distribution $z = (z_1, z_2)$ assigns to agent 2 at least $ln(4) + 1 = 2,38$. To be stable, it should assign to agent 1 at least $ln(3) + 1 = 2,1$. Therefore, we should have $z_1 + z_2 \geq 4,48$. The optimal water allocation yields $v(1, 2) = ln(2) + ln(2) + 2 = 3,38$. Since $4,48 > v(1, 2)$, then we should have $z > v(1, 2)$ which contradicts that z is a welfare distribution.

end up above his aspiration welfare.

This argument generalizes to coalitions in a very natural way. The *aspiration welfare* of an arbitrary coalition S is the highest welfare it could achieve in the absence of $N\backslash S$. It reads

$$w(S) = \max_{x_S} \sum_{i \in S} b_i(x_i) \; s/t \sum_{j \in Pi \cap S} x_j \leq \sum_{j \in Pi} e_j \; \forall i \in S. \tag{3.7}$$

We say that a welfare distribution z *satisfies the aspiration upper bounds* if $\sum_{i \in S} z_i \leq w(S)$ for every $S \subset N$.

Combining these fairness bounds with the stability constraints of Section 4 yields remarkable results in some cases. It turns out that only one welfare distribution passes both tests : It is the *downstream incremental* distribution z^* defined by $z_i^* = v(Pi) - v(P^0 i)$ for each $i \in N$.

Proposition 3. *The downstream incremental distribution z^* of the welfare is the unique core distribution satisfying the aspiration upper bounds.*

Proposition 3 identifies the downstream incremental distribution z^* as the unique distribution that is stable and fair. This distribution assigns to every agent $i \in N$ his marginal contribution to the coalition composed by its predecessors. The first agent gets his worse payoff among welfare distributions that belong to the core. The last agent gets his marginal contribution to the largest coalition of partners.

Recall that the welfare distribution z^* defines a unique transfer scheme t^*, where $t_i^* = z_i^* - b_i(x_i^*)$, for every $i \in N$. Consequently, the unique allocation compatible with efficiency, stability and fairness is (x^*, t^*). We now discuss its decentralization and its implementation.

3.6 Decentralization and implementation of the downstream incremental allocation

3.6.1 Decentralization by public policy

Consider the case of domestic surface waters. Suppose that the whole river basin belongs within a single country.[8] Assume that the State is entitled to impose a tax τ_i per unit of water extracted or polluted and a lump-sum transfer σ_i to every agent $i \in N$. Under free-access management of water, agent i consumes the amount of water that maximize its gain $b_i(x_i) - \tau_i x_i + \sigma_i$. The first order condition yields $\tau_i = b_i'(x_i^e)$, for every $i \in N$. To induce agent i to divert x_i^*, the tax τ_i should be equal to $b_i'(x_i^*)$. To obtain a fair distribution of welfare, agent i should get back a lump sum transfer σ_i equal to $z_i^* - [b_i(x_i^*) - b_i'(x_i^*)]$. In this framework, the Pigouvian tax scheme τ and the lump sum subsidy scheme σ, where $\tau_i = b_i'(x_i^t)$ and $\sigma_i = z_i^* - [b_i(x_i^*) - b_i'(x_i^*)]$ for every $i \in N$, decentralize the downstream incremental welfare distribution. When the State sell water to agents, the tax scheme τ^* can be interpreted as the unit price of water located at i.

3.6.2 Decentralization by a competitive market

We now investigate whether, by giving property rights on water and by letting agents exchange in a competitive market, the downstream incremental allocation could be implemented. Of course, because few agents trade on this water market, we could be far from a perfect competitive market. We do not address this issue.

[8]In several countries, surface waters are legally owned by the State which regulates water extraction and pollution. Example of such public regulation of river are the French delegated management by the "Agences de bassin".

We just ask if, assuming that agent acts non-strategically, the equilibrium payoffs could be the downstream incremental welfare distribution.

The Coase Theorem tells us that the equilibrium allocation will be efficient but not necessarily fair. Indeed, the *ex post* welfare distribution depends on the *ex ante* initial allocation of property rights. Our aim is to endow agents with river water in such a way that the downstream incremental distribution emerges as the market-equilibrium payoffs.

Let us first design a market for our model. In the river, water is not an homogeneous good. Let us set up market for water provided by each tributary $j \in N$. Each agent i is endowed with some water coming from tributary j, say w_{ij}. Every flow of water $e_j > 0$ is divided between agents $i \in N$ such that $\sum_{i \in N} w_{ij} = e_j$. Denote agent i's consumption vector $\mathbf{x}_i = (x_{i1}, ..., x_{in})$, where x_{ij} is the volume of water from tributary j devoted to agent i. Agent i's consumption x_i is obtain by summing up water coming from each upstream tributary, i.e. $\mathbf{x}_i = \sum_{j \in Pi} x_{ij}$. Given a price vector $p = (p_1, ..., p_n)$, agent i's consumption vector solves the following program :

$$\max_{\mathbf{x}_i} b_i(\sum_{j \in Pi} x_{ij}) - \sum_{j \in N} p_j(x_{ij} - w_{ij}). \tag{3.8}$$

A *market equilibrium* is a price vector $\mathbf{p}^e = (p_1^e, ..., p_n^e)$ and a consumption matrix $\mathbf{X}^e = [x_{ij}^e]_{i,j \in N \times N}$ such that each consumption vector $\mathbf{x}_i^e$ solves (3.8) for each agent $i \in N$ and that excess demand is nil on each market, i.e. $\sum_{i \in N}(x_{ij}^e - e_{ij}) = 0$, for all $j \in N$.

Two straightforward observations simplify the analyzes. First, since agent i cannot consume downstream water, his equilibrium consumption from downstream tributaries is nil if this water is costly. Formally, $\forall j \in F^0 i$, $\forall p^e$ such that $p_j^e > 0$, $x_{ij}^e = 0$. This implies that upstream water is more demanded. Hence, equilibrium prices decrease by going downstream.[9] Second, agent i maximizes his

[9]Indeed, if $\exists l > j$ such that $0 < p_l^e < p_j^e$, then $x_{ij}^e = 0$, $\forall i \in N$. Therefore $e_j = 0$ which

payoff by purchasing water at the lowest price. As a consequence, agent i purchases water at a price $p_i^e = \min\{p_1^e, ..., p_i^e\}$. His equilibrium consumption from every tributary l such that $p_l^e > p_i^e$ is nil. We conclude that agent i's consumption can be found by solving

$$\max_{\mathrm{x}_i} b_i(\sum_{j\in Pi} x_{ij}) - p_i \sum_{j\in Pi}(x_{ij} - w_{ij}) + \sum_{j\in F^0 i} p_j w_{ij}.$$

The first order condition is $b_i'(\sum_{j\in Pi} x_{ij}^e) = p_i$. Put differently, $b_i'(x_i^e) = p_i^e$. Finally, a *market equilibrium* $(\mathbf{p}^e, \mathbf{X}^e)$ satisfies both $b_i'(x_i^e) = p_i^e$, for every agent $i \in N$ and $\sum_{i\in N}(x_{ij}^e - e_{ij}) = 0$ on every market $j \in N$. Remark that the equilibrium price vector that implements the efficient water allocation is $p^* = (b_1'(x_1^*), ..., b_n'(x_n^*))$.

The structure of our river allocation problem gives us a natural way to affect property rights on water. Each agent i may ask to own the flow of water e_i he controls.[10] He is entitled to divert or sell this amount of water. We claim that this allocation of property rights implements the downstream incremental allocation only in the degenerated case of equality between the efficient water allocation and the natural water flow at each level $j \in N$.

Proposition 3. *If agents are endowed with the flow of water they control, a competitive market leads to the downstream incremental distribution if and only if* $x^* = e$.

Intuitively, the fairness principle implies that an agent cannot use his upstream position to extract surplus from his downstream partners. By selling his water surplus downstream, an upstream agent gets more than his aspiration welfare. As a consequence, the equilibrium outcome is not fair.

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contradicts $p_j^e > 0$.

[10]This endowment is consistent with the theory of absolute territorial sovereignty or *Harmon doctrine* that gives to a country the control right on the natural flow of water within his territory e_i (see Kilgour and Dinar, 1996, Godana, 1985).

3.6.3 Implementation by negotiation rules

An institution for river management usually includes negotiation rules. We propose the following negotiation game that leads to the downstream incremental distribution. Regroup all agents around the negotiation table. In the first round, let agent n propose an allocation (x, t) for the whole river. If it is accepted, it is the allocation implemented. If it is refused by one agent, agent n is excluded from the negotiation. He gets his reservation utility $v(n) = b_n(e_n)$. The $N \backslash \{n\}$ remaining agents go on to the next round. In the second round, it is agent $n-1$'s turn to propose to the remaining agents $\{1, ..., n-2\}$ an allocation for the part of the river they control. This allocation is implemented if unanimously accepted. If one agent refuses, agent $n-1$ is excluded from the negotiation and gets his reservation payoff $v(n-1) = b_{n-1}(e_{n-1})$. The $N \backslash \{n-1, n\}$ remaining agents go on to the next negotiation round. It is now agent $n-2$'s turn to offer an allocation for the part of the river that agents $\{1, ..., n-2\}$ control. And so forth. At the last stage of the game, agent 2 makes a final offer to agent 1. If agent 1 refuses, the negotiation ends.

It is easy to show that the subgame unique perfect equilibrium of this game is the allocation (x^*, t^*) yielding the downstream incremental distribution z^*. Proceed by backward induction. Consider the last stage subgame. Agent 1 accepts any offer that gives a higher payoff than what he can achieve on his own.[11] His reservation utility is therefore $v(1)$. Agent 2's dominant strategy is to offer the (unique) allocation in the part $\{1, 2, \}$ of the river yielding $v(1)$ to agent 1 and $v(1, 2) - v(1)$ to himself. Now, move back to the preceding subgame. Agent 2's reservation payoff is $v(1, 2) - v(1)$. Agent 3's best strategy is to offer the allocation on the river part $\{1, 2, 3\}$ that yields $v(1)$ to agent 1, $v(1, 2) - v(1)$ to agent 2 and

[11]Notice that we make the usual assumption that an agent accepts an offer if he is indifferent between accepting and refusing.

$v(1,2,3) - v(1,2)$ to himself. And so forth. In the first stage, agent n's dominant strategy is to make an offer that gives $v(Pi) - v(P^0 i)$ to each agent i, that is the payoff he can achieve for sure by refusing the offer, and $v(N) - v(N \backslash \{i\})$ to himself. All other agent's best response is to accept the offer. The downstream incremental distribution is the subgame perfect equilibrium payoff of the game.

The "grand negotiation game" can be divided into $|K|$ simultaneous "small negotiation games" between riparian in each part N_k of the river. By applying similar rules into all negotiation games, we obtain the downstream incremental distribution as the subgame perfect equilibrium payoff. Consequently, implementing the downstream incremental distribution does not require that river management institutions include all riparian. The same result is obtained by designing an institution for each part of the river $N_k \in \mathcal{P}(N)$. We now conclude by some remarks.

3.7 Concluding remarks

This paper is a first attempt to model river sharing. It has natural extensions. First, we may consider water non-consumptive uses such as transportation, production of hydroelectric power or recreational purposes. In this case, water is also a public good. The value agents assigns to this good depends on the level water in the stream. A higher level of water produces more power or allow bigger ships to sail. This extension can easily be modeled by adding a function of the total volume of water into agent's utility. Second, a more realistic assumption should be to consider single-peaks production functions, i.e. to assume that water's marginal benefit becomes negative if higher to a maximum level. Last, the model could be applied to analyze other economic problems. The one-side transferability of a good makes it usefull to study intergenerational equity and sustainability.

Synthèse des résultats

Les contributions principales de la thèse sont les suivantes.

Le premier essai analyse l'organisation des activités de recherche et développement (R&D). Une innovation peut être soit produite à l'interne, soit achetée à l'externe par la firme qui l'utilise. Nous caractérisons le choix contractuel optimal en abordant les aspects de renégociation et de collusion entre les différentes parties.

Lorsqu'un innovation plus coûteuse à développer est aussi une technologie moins performante ou un produit moins profitable, une innovation sera produite à l'interne. Dans le cas contraire, l'innovation sera acquise à l'externe. De façon pratique, notre modèle prédit que les firmes pharmaceutiques ont intérêt à faire leur propre recherche tandis que dans le secteur des télécommunications et de l'informatique, les innovations sont produites par des petites firmes spécialisées en haute technologie.

Le second essai modélise la solidarité familiale africaine comme une norme sociale de partage de risque. Nous proposons un mécanisme de sanction mutuelle basé sur le statut social qui justifie pourquoi les plus riches transfèrent une partie de leur revenu aux membres de la famille les plus pauvres. Ce mécanisme s'inspire des comportements décrits par les anthropologues. Nous supposons qu'il a un impact hétérogène sur les agents.

Nous montrons que la norme sociale est pratiquée par des agents rationnels dans un équilibre de Nash. La pleine assurance n'est possible si et seulement si tout le monde obéit à cette norme. Dans le cas contraire, la désobéissance à la norme limite le partage de risque. Il peut même y avoir de la défection à l'équilibre. Nous établissons une condition pour laquelle une politique publique de redistribution des revenus est plus efficace pour partager le risque. Une redistribution des revenus performante accroît l'efficacité de la norme sociale. Lorsqu'elle est trop coûteuse, elle se substitue à une institution plus performante pour atteindre un même objectif.

Le troisième chapitre porte sur le partage d'un fleuve entre usagers en situation de pénurie d'eau. Ce modèle général s'applique notamment aux bassins hydrographiques partagés par des pays souverains.

Nous caractérisons l'allocation optimale de l'eau. Nous identifions les modes de partage du bien-être de l'exploitation du fleuve qui sont stables et équitables. Nous montrons que le jeu est convexe de sorte que l'ensemble des distributions de bien-être stables est entièrement caractérisé. Nous introduisons un principe d'équité. Nous montrons qu'une seule distribution de bien-être est à la fois stable et équitable. Cette distribution de bien-être peut être décentralisée par une taxe Pigouvienne et des transferts forfaitaires mais pas par un marché concurrentiel lorsqu'on affecte à chaque agent un droit de propriété sur la part de la ressource qu'il contrôle. Elle peut être implémentée par une institution spécifiant une règle de négociation donnant la priorité aux pays situés en aval.

Annexe A

Proofs of chapter 1

A.1 Proof of Proposition 1

Let λ^C and μ_α be the multipliers associated with the constraints IR^C and IC_α respectively. The equilibrium allocation $\{I^A\{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^h\}$ satisfies the following first order conditions (FOC) :

$$p(I^A)v'(w_h^A - D(q_h^A, h)) + (1-p(I^A))v'(w_l^A - D(q_l^A, l)) = \lambda^C$$

$$\begin{aligned} P_q(q_h^A, h) - D_q(q_h^A, h) &= \tfrac{\mu_l}{\lambda^C p(I^A)}[D_q(q_h^A, h) - D_q(q_h^A, l)] \\ P_q(q_l^A, l) - D_q(q_l^A, l) &= \tfrac{\mu_h}{\lambda^C (1-p(I^A))}[D_q(q_l^A, l) - D_q(q_l^A, h)] \\ \tfrac{v(w_h^A - D(q_h^A, h)) - v(w_l^A - D(q_l^A, l))}{\lambda^C} & \\ +P(q_h^A, h) - w_h^A - [P(q_l^A, l) - w_l^A] &= \tfrac{1}{p'(I^A)} \end{aligned}$$

We shall first show, in two steps and by contradiction, that the incentive constraint for state of nature h is binding, that is, $\mu_h > 0$.

Step 1 : Proof that only one of the incentive compatibility constraints is binding.

Suppose that the two IC constraints are binding, that is $\mu_h > 0$ and $\mu_l > 0$ in the FOC. We have the following relationships :

$$v(w_h^A - D(q_h^A, h)) = v(w_l^A - D(q_l^A, h))$$
$$v(w_l^A - D(q_l^A, l)) = v(w_h^A - D(q_h^A, l))$$

which imply :

$$w_h^A - w_l^A = D(q_h^A, h) - D(q_l^A, h)$$
$$w_h^A - w_l^A = D(q_h^A, l) - D(q_l^A, l).$$

This is true only when $q_h^A = q_l^A$, that is, the equilibrium is pooling. However, it is a well known result that the solution is screening since the utility functions satisfy the single crossing property.

Step 2 : Proof of $\mu_h > \mu_l$.

Suppose that $\mu_h < \mu_l$, by the FOC, we have $v'(w_h^A - D(q_h^A, h)) - \lambda = \mu_l - \mu_h$. Hence, under our assumption, $v'(w_h^A - D(q_h^A, h)) > \lambda$. Then,

$$v'(w_h^A - D(q_h^A, h)) > p(I^A)v'(w_h^A - D(q_h^A, h)) + (1 - p(I^A))v'(w_l^A - D(q_l^A, l))$$
$$\Rightarrow v'(w_h^A - D(q_h^A, h) > v'(w_l^A - D(q_l^A, l))$$
$$\Rightarrow v(w_h^A - D(q_h^A, h)) < v(w_l^A - D(q_l^A, l))$$

This inequality implies that one of the incentive-compatibility constraints is not satisfied, therefore $\{I^A\{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^h\}$ is not a solution to the maximization problem. In the third FOC, we have $\mu_h > 0$ and $D_q(q, h) - D_q(q, l) < 0$ $\forall q$ in the right-hand side, therefore the left-hand side is negative and there is underproduction for a low-quality innovation. The wage gap is defined by the binding incentive constraint :

$$w_h^A - w_l^A = D(q_h^*, h) - D(q_l^A, h).$$

The first and the last FOCs give us the investment level I^A :

$$p'(I^A)\{\frac{V_h^A - V_l^A}{E[V_\alpha'^A|I^A]} + U_h^A - U_l^A\} = 1.$$

A.2 Proof of Proposition 2

We shall first show by contradiction that C's individual rationality constraints are binding in equilibrium hence transfers are defined by this constraints. Then, we prove the equivalence of the two maximization problems P_A and P_B.
Suppose that $\exists \alpha \in \{l, h\}$ such that $P(q_\alpha^B, \alpha) - w^B(q_\alpha) > 0$. If we increase by the same amount $w^B(q_\alpha)$ and $R^B(q_\alpha)$, we could increase investment I^B without breaking up one of the P_B problem maximization constraints. RU's expected utility is increased by this alternative allocation thus the principal would gain by choosing it. The equilibrium allocation must satisfied $P(q_\alpha^B, \alpha) - w^B(q_\alpha) = 0 \; \forall \alpha \in \{l, h\}$. Wages are defined by the RI_α^C constraints $\forall \alpha \in \{l, h\} : w(q_\alpha) = P(q_\alpha, \alpha)$.
We now define a new variable to show that the two maximization problems are equivalent. Suppose $x(q_\alpha) = w(q_\alpha) - R(q_\alpha)$.

The independent structure commitment game maximization program can be rewritten as :

(P_B) $\max_{\{I, \{x(q_\alpha), q_\alpha\}_{\alpha=l}^h\}} E[V(x(q_\alpha), q_\alpha, \alpha))|I]$ s/t

$$\begin{aligned} p(I)(P(q_h, b) - x(q_h)) + (1 - p(I))(P(q_l, l) - x(q_l)) &\geq 0 && (RI^F) \\ v(x(q_h) - D(q_h, h)) &\geq v(x(q_l) - D(q_l, h)) && (IC_h) \\ v(x(q_l) - D(q_l, l)) &\geq v(x(q_h) - D(q_h, l)) && (IC_l) \end{aligned}$$

This is P_A maximization problem.

A.3 Proof of Proposition 3

In this game, a strategy for RU is represented by $\sigma_{RU} = \{c_{RD}, \hat{\alpha}(c_{RD}, \alpha), c_r(c_{RD}, \hat{\alpha}, \alpha)\}$, where c_{RD} is an initial contract offer, $\hat{\alpha}(.)$ represents RU's decision rule regarding the choice of a message $\hat{\alpha}$ and $c_r(.)$ is the renegotiation contract offer. The strategy of player C, $\sigma_C = \{d(c_{RD}), d_r(c_{RD}, \hat{\alpha}, c_r)\}$, represents its decision rules concerning the acceptance or rejection of the initial contract offer and the renegotiation contract offer, respectively. The beliefs of C are updated after stage 4.a and are denoted $P(\alpha|c_{RD}, \hat{\alpha}, c_r)$.

Define $(w_\alpha^{*\hat{\alpha}}, q_\alpha^{*\hat{\alpha}}) = argmax_{(w,q)} V(w,q,\alpha)$ s/t $U(w,q,h) \geq U(w_{\hat{\alpha}}, q_{\hat{\alpha}}, h)$ and $U(w,q,l) \geq U(w_{\hat{\alpha}}, q_{\hat{\alpha}}, l)\}$. The following strategies and beliefs support the equilibrium allocation as a PBE outcome.

$$\sigma_{RU} = \begin{cases} c_{RD} = & c_{RD}^A = \{I^A, \{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^h\} \\ \hat{\alpha}(c_{RD}, \alpha) = & argmax_{\tilde{\alpha}\in\{l,h\}} V(w_{\tilde{\alpha}}^{*\hat{\alpha}}, q_{\tilde{\alpha}}^{*\hat{\alpha}}, \alpha) \\ c_r(c_{RD}, \hat{\alpha}, \alpha) = & \begin{cases} (w_\alpha^{*\hat{\alpha}}, q_\alpha^{*\hat{\alpha}}) & if \quad V(w_\alpha^{*\hat{\alpha}}, q_\alpha^{*\hat{\alpha}}, \alpha) > V(w_{\hat{\alpha}}, q_{\hat{\alpha}}, \alpha) \\ \emptyset & otherwise. \end{cases} \end{cases}$$

$$\sigma_C = \begin{cases} d_C(c_{RD}) = & \begin{cases} 1 & if \quad E[U(w_\alpha, q_\alpha, \alpha)|I] - I \geq 0 \\ 0 & otherwise \end{cases} \\ d_r(c_{RD}, \hat{\alpha}, c_r) = & \begin{cases} 1 & if \quad U(w,q,\alpha) \geq U(w_\alpha, q_\alpha, \alpha) \quad \forall \alpha \in \{l,h\} \\ 0 & otherwise \end{cases} \end{cases}$$

$$P(h|c_{RD}, \hat{\alpha}, c_r) = \begin{cases} 0 & if \quad c_r \neq \emptyset \;\; and \;\; U(w,q,h) \geq U(w_{\hat{\alpha}}, q_{\hat{\alpha}}, h) \\ & \qquad\qquad and \;\; U(w,q,l) < U(w_{\hat{\alpha}}, q_{\hat{\alpha}}, l) \\ 0 & if \quad c_r = \emptyset \;\; and \;\; \hat{\alpha} = l \\ 1 & otherwise \end{cases}$$

$$P(l|c_{RD}, \hat{\alpha}, c_r) = 1 - P(h|c_{RD}, \hat{\alpha}, c_r)$$

where $d = 1$ means acceptance and $d = 0$, rejection. We shall now argue that these strategies and beliefs do in fact constitute a PBE.

In stage 4.b, C accepts the new contract offer c_r if and only if (w, q) is preferred to the initial allocation selected $(w_{\hat{\alpha}}, q_{\hat{\alpha}})$ regardless of its beliefs. Given this acceptance rule by C, RU can do not better than offer in stage 4.a its preferred contract among those accepted by C. In stage 2, RU accepts all contract offers yielding an expected pay-off of 0 given the expected resolution of the game following the initial offer. Finally, in stage 1, RU offers its preferred contract among those expected to be accepted by C.

These strategies and beliefs imply the following equilibrium path. In stage 1, RU offers the contract c^A_{RD} which is accepted by player 2 in stage 2. In stage 4, for each innovation quality α, RU selects its preferred report $\hat{\alpha}$. In stage 4.a, it makes no offer. Given that $\{I^A, \{w^A_\alpha, q^A_\alpha\}^h_{\alpha=l}\}$ satisfies the constraints of the maximization problem, the contract c^A_{RD} cannot be renegotiated in stage 4.a given the equilibrium strategy of C. With this strategies along the equilibrium path, it is clear that the allocation is renegotiation proof.

A.4 Proof of Proposition 4

Denote $\pi(q, \alpha\alpha')$ the global profit for a gross profit $P(q, \alpha)$ and a development cost $D(q, \alpha')$, $\alpha' \neq \alpha$. Denote $q^*_{\alpha\alpha'}$ the production level which maximizes $P(q, \alpha) - D(q, \alpha')$, $\alpha' \neq \alpha$.

<u>**Case 1 :**</u> We first consider the case of a major innovation. The proof proceed as follow. First, we show that constraints RP^h_h and RP^l_l are respectively equivalent to $q_h \in [q^*_{lh}, q^*_h]$ and $q_l \in [q^*_l, q^*_{hl}]$. Second, we derive the solution to the constraints RP^l_h and RP^h_l maximization problem. Thirst, we prove that $q^A_h = q^*_h$ and $q^A_l = q^*_l$. Fourth, we show that RP^h_l is not binding so that the wage difference is defined by RP^l_h constraint. Then we derive the equilibrium allocation.

The renegotiation proof constraints can be rewritten :

$$w_h - D(q_h, h) \geq \max_{(w,q)}\{w - D(q,h)\ s/t$$
$$P(q,h) - w \geq P(q_h, h) - w_h$$
$$P(q,l) - w \geq P(q_h, l) - w_h\} \qquad (RP_h^h)$$
$$w_h - D(q_h, h) \geq \max_{(w,q)}\{w - D(q,h)\ s/t$$
$$P(q,h) - w \geq P(q_l, h) - w_l$$
$$P(q,l) - w \geq P(q_l, l) - w_l\} \qquad (RP_h^l)$$
$$w_l - D(q_l, l) \geq \max_{(w,q)}\{w - D(q,l)\ s/t$$
$$P(q,h) - w \geq P(q_h, h) - w_h$$
$$P(q,l) - w \geq P(q_h, l) - w_h\} \qquad (RP_l^h)$$
$$w_l - D(q_l, l) \geq \max_{(w,q)}\{w - D(q,l)\ s/t$$
$$P(q,h) - w \geq P(q_l, h) - w_l$$
$$P(q,l) - w \geq P(q_l, l) - w_l\} \qquad (RP_l^l)$$

- Proof that RP_h^h and RP_l^l are equivalent to, respectively, $q_h \in [q_{lh}^*, q_h^*]$ and $q_l \in [q_l^*, q_{hl}^*]$.

  - Proof that RP_h^h implies $q_h \in [q_{lh}^*, q_h^*]$.

    Suppose that $q_h > q_h^*$. When $\alpha = h$, this allocation is not renegotiation-proof : RU can increase its gain by selecting $\hat{\alpha} = h$ and offering $q = q_h^*$ and $w = P(q_h^*, l) - [P(q_h, l)] + w_h$. This renegotiation offer is accepted by C for any beliefs and $v(w - D(q,h)) = v(w_h - D(q_h, h) + \pi(q_h^*, lh) - \pi(q_h, lh))$ which is higher than $v(w_h - D(q_h, h))$. Therefore, the renegotiation proof constraint for a innovation quality $\alpha = h$ and a message $\hat{\alpha} = h$ is not satisfied for all $q_h > q_h^*$.

    Suppose that $q_h < q_{hl}^*$. When $\alpha = h$, this allocation is not renegotiation-proof : RU can increase its gain by selecting $\hat{\alpha} = h$ and offering $q = q_{lh}^*$

and $w = P(q^*_{lh}, h) - [P(q_h, h) - w_h]$. This renegotiation offer is accepted by C for any beliefs and $v(w - D(q,h)) = v(w_h - D(q_h, h) + (q^*_{lh}, h) - \pi(q_h, h))$ which is higher than $v(w_h - D(q_h, h))$.

Therefore, the renegotiation proof constraint for a innovation quality $\alpha = h$ and a message $\hat{\alpha} = h$ is not satisfied for all $q_h < q^*_{lh}$.

- Proof that any allocation such that $q_h \in [q^*_{lh}, q^*_h]$ satisfies RP^h_h.

  Assume that $q_h \in [q^*_{lh}, q^*_h]$. If RU proposes a production renegotiation offer $q < q_h$ then w must be less than $P(q,h) - [P(q_h, h)] + w_h$ to be accepted by C. RU's utility with an accepted renegotiation offer is at least $v(w - D(q,h)) = v(w_h - D(q_h, h) + \pi(q,h) - \pi(q_h, h))$ which is lower than $v(w_h - D(q_h, h))$.

  If RU proposes a production renegotiation offer $q > q_h$ then w must be less than $P(q,l) - [P(q_h, l) - w_h]$ to be accepted by C. RU's utility with this accepted renegotiation offer is at least $v(w - D(q,h)) = v(w_h - D(q_h, h) + \pi(q, lh) - \pi(q_h, lh))$ which is lower than $v(w_h - D(q_h, h))$.

- By a similar proof, we can show that RP^l_l is equivalent to $q_l \in [q^*_l, q^*_{hl}]$.

- Solution to the RP^l_h maximization problem.

Let (w^r_l, q^r_l) be the renegotiation offer solution to the RP^l_h maximization problem. We consider two cases : $q^r_l < q_l$ and $q^r_l \geq q_l$. Suppose that $q^r_l < q_l$, w^r_l is defined by the following constraint :

$$w^r_l = P(q^r_l, h) - [P(q_l, h) - w_l].$$

With this offer, RU's gain is :

$$w^r_l - D(q^r_l, h) = \pi_h(q^r_l) - \pi_h(q_l) + w_l - D(q_l, h),$$

which is less that $w_l - D(q_l, h)$. This allocation (w^*, q^*) is not a solution to RP_h^l problem : RU can be better off if it proposes (w_l, q_l) rather than (w_l^r, q_l^r).

Suppose now that $q_l^r \geq q_l$, w_l^r is defined by the following constraint :

$$w_l^r = P(q_l^r, l) - [P(q_l, l) - w_l].$$

With this offer, RU's gain is :

$$w_l^r - D(q_l^r, h) = \pi(q_l^r, lh) - \pi(q_l, lh) + w_l - D(q_l, h).$$

We now consider two cases.

1. If $q_l > q_{lh}^*$, then

$$\pi(q_l^r, lh) - \pi(q_l, lh) < 0, \forall q_l^r > q_l.$$

   Hence, the best surely acceptable renegotiation offer is $q_l^r = q_l$ and $w_l^r = w_l$. The solution to the RP_h^l maximization problem is :

$$w_l - D(q_l, h).$$

   In this case, RP_h^l is the usual incentive constraint.

2. If $q_l \leq q_{lh}^*$, then the best surely acceptable renegotiation offer is $q_l^r = q_{lh}^*$, $w_l^r = P(q_{lh}^*, l) - [P(q_l, l) - w_l]$. The solution to the RP_h^l maximization problem is :

$$\pi(q_{lh}^*, lh) - \pi(q_l, lh) + w_l - D(q_l, h).$$

The constraint RP_h^l can be summarized by :

$$w_h - D(q_h, h) \geq \pi(q_l^r, lh) - \pi(q_l, lh) + w_l - D(q_l, h),$$

where $q_l^r = \max\{q_l, q_{lh}^*\}$.

- Solution to the RP_l^h maximization problem.

  By a similar proof, we can show that the constraint RP_l^h can be summarized by :

$$w_l - D(q_l, l) \geq \pi(q_h^r, hl) - \pi(q_h, hl) + w_h - D(q_h, l),$$

  where $q_h^r = \min\{q_h, q_{hl}^*\}$.

- Proof that $q_h^A = q_h^*$.

  Suppose first that $q_h^A < q_{hl}^*$. We prove that, if q_h^A is increased by $\epsilon > 0$ and w_h^A is reduced by $\delta > 0$ such that $w_h^A - D(q_h^A, l) = (w_h^A - \delta) - D(q_h^A + \epsilon, l)$, then all constraints are satisfied and RU's utility is higher. With $\{I^A, (w_l^A, q_l^A), (w_h^A - \delta, q_h^A + \epsilon)\}$, the constraints RP_h^h, RP_l^l and RP_h^l are satisfied and we have :

$$P(q_h^A + \epsilon, h) - (w_h^A - \delta) = \pi(q_h^A + \epsilon, hl) - \pi(q_h^A, hl) + [P(q_h^A, h) - w_h^A].$$

  Since $q_{hl}^* > q_h^A$, then $P(q_h^A + \epsilon, h) - (w_h^A - \delta) > P(q_h^A, h) - w_h^A$ and the constraint IR^C still holds. Moreover, we have :
  $(w_h^A - \delta) - D(q_h^A + \epsilon, h) = w_h^A - D(q_h^A, h) + D(q_h^A + \epsilon, l) - D(q_h^A, l) + D(q_h^A + \epsilon, h) - D(q_h^A, h)$.
  Since $(w_h^A - \delta) - D(q_h^A + \epsilon, h) > w_h^A - D(q_h^A, h)$, the constraint RP_l^h is satisfied and $\{I^A, (w_l^A, q_l^A), (w_h^A - \delta, q_h^A + \epsilon)\}$ gives RU a higher expected pay-off than $\{I^A, (w_l^A, q_l^A), (w_h^A, q_h^A)\}$. We should have $q_h^A \geq q_{hl}^*$ and therefore $q_h^r = q_{hl}^*$.

  Suppose that $q_h^A < q_h^*$. The constraint RP_l^h can be rewritten as :

$$w_l^A - D(q_l^A, l) \geq \pi(q_{hl}^*, hl) - [P(q_h^A, h) - w_h^A].$$

  If q_h^A is increased by $\epsilon > 0$ and w_h^A is reduced by $\delta > 0$ such that $P(q_h^A, h) - w_h^A = P((q_h^A + \epsilon), h) - (w_h^A - \delta)$, then the constraints IR^C, RP_h^h, RP_l^l and RP_l^h are satisfied. Moreover, we show that RU's utility increases and therefore RP_h^l is satisfied. We have :

$$w_h^A - \delta + D((q_h^A + \epsilon), h) = w_h^A + D(q_h^A, h) + \pi(q_h^A + \epsilon, h) - \pi(q_h^A, h).$$

Hence, since $\pi(q_h^A + \epsilon, h) - \pi(q_h^A, h) > 0$, the constraint RP_h^l holds and $\{I^A, (w_l^A, q_l^A), (w_h^A - \delta, q_h^A + \epsilon)\}$ gives RU a higher expected pay-off than $\{I^A, (w_l^A, q_l^A), (w_h^A, q_h^A)\}$. Therefore, at the equilibrium, $q_h^A = q_h^*$.

- Proof that $q_l^A = q_l^*$.

  Suppose first that $q_l^A > q_{lh}^*$. We prove that, if q_l^A is reduced by $\epsilon > 0$ and w_l^A is increased by $\delta > 0$ such that $w_l^A - D(q_l^A, h) = (w_l^A + \delta) - D(q_l^A - \epsilon, h)$, then all constraints are satisfied and RU's utility is higher. The constraints RP_h^h, RP_l^l and RP_l^h still hold and we have :

$$P(q_l^A - \epsilon, l) - (w_l^A + \delta) = \pi(q_l^A - \epsilon, lh) - \pi(q_l^A, lh) + [P(q_l^A, l) - w_l^A].$$

  Since $q_{lh}^* > q_l^A$, then $P(q_l^A - \epsilon, l) - (w_l^A + \delta) > P(q_l^A, l) - w_l^A$ and the constraint IR still holds. Moreover, we have :
  $(w_l^A + \delta) - D(q_l^A - \epsilon, l) = w_l^A - D(q_l^A, l) + D(q_l^A, l) - D(q_l^A - \epsilon, l) + D(q_l^A, h) - D(q_l^A - \epsilon, h)$.
  Since $(w_l^A + \delta) - D(q_l^A - \epsilon, l) > w_l^A - D(q_l^A, l)$, the constraint RP_l^h is satisfied and $\{I, (w_l^A + \delta, q_l^A - \epsilon), (w_h^A, q_h^A)\}$ gives RU a higher pay-off than $\{I, (w_l^A, q_l^A), (w_h^A, q_h^A)\}$. We should have $q_l^A \leq q_{lh}^*$ and therefore $q_l^r = q_{lh}^*$.

  Suppose that $q_l^A > q_l^*$. If q_l^A is reduced by $\epsilon > 0$ and w_l^A is increased by $\delta > 0$ such that : $P(q_l^A, l) - w_l^A = P((q_l^A - \epsilon), l) - (w_l^A + \delta)$. Then all constraints are satisfied and $w_l^A + \delta + D((q_l^A - \epsilon), h) = w_l^A + D(q_l^A, l) + \pi(q_l^A - \epsilon, l) - \pi(q_l^A, l)$. Hence, since $\pi(q_l^A - \epsilon, l) - \pi(q_l^A, l) > 0$, $\{I^A, (w_l^A + \delta, q_l^A - \epsilon), (w_h^A, q_h^A)\}$ gives RU a higher expected pay-off than $\{I^A, (w_l^A, q_l^A), (w_h^A, q_h^A)\}$. Therefore, at the equilibrium, $q_l^A = q_l^*$.

- Proof that RP_l^h is not binding and that RP_h^l is binding.

Clearly, as in Proposition 2, one of the two constraint is binding. Suppose that the two constraints are binding. Then, we must have :

$$\pi(q^*_{hl}, hl) - \pi(q^*_{lh}, hl) = \pi(q^*_h, h) - \pi(q^*_l, l).$$

However, this case is impossible since cost functions are such that $\pi(q^*_h, h) - \pi(q^*_{lh}, lh) > \pi(q^*_{hl}, hl) - \pi(q^*_l, l)$.

Suppose now that RP^h_l is binding and that RP^l_h is not binding. Then $w^A_h - D(q^*_h, h) > w^A_l - D(q^*_l, l)$. For $\epsilon > 0$ sufficiently small such that if w^A_l is increased by $\frac{\epsilon}{1-p(I)}$ and w^A_h is decreased by $\frac{\epsilon}{p(I)}$, all constraints can still be satisfied and RU's expected utility can be increased. With this alternative allocation, RU's expected utility is :

$$p(I)v(w^A_h - D(q^*_h, h) - \frac{\epsilon}{p(I)}) + (1 - p(I))v(w^A_l - D(q^*_l, l) + \frac{\epsilon}{1 - p(I)}).$$

Since v is concave, this term is higher than :
$p(I)v(w^A_h - D(q^*_h, h)) + (1 - p(I))v(w^A_l - D(q^*_l, l)) + \epsilon[v'(w^A_h - D(q^*_h, h) - \frac{\epsilon}{p(I)}) - v'(w^A_l - D(q^*_l, l) + \frac{\epsilon}{1-p(I)})]$.
Hence, since $\epsilon > 0$,
$p(I)v(w^A_h - D(q^*_h, h) - \frac{\epsilon}{p(I)}) + (1-p(I))v(w^A_l - D(q^A_l, l) + \frac{\epsilon}{1-p(I)}) > E[V(w^A_\alpha, q^A_\alpha, \alpha)|I]$.
Therefore, $\{w^A_\alpha, q^*_\alpha\}^h_{\alpha=l}$ with RP^h_l binding is not an equilibrium allocation.

- Characterization of wages and investment implemented.

  Because RP^l_h is binding, the low technology transfer w_l is defined by :
  $w_l = w_h - D(q^*_h, h) + D(q^*_l, h) - \pi(q^*_{lh}, lh) + \pi(q^*_l, lh)$.
  We now consider the following reduced program :

  $\max_{I,w_h} p(I)v(w_h - D(q_h, h)) + (1 - p(I))v(\pi(q^*_{hl}, hl) - (P(q^*_h, h) - w_h))$ s/t $p(I)(P(q^*_h, h) - w_h) + (1 - p(I))(P(q^*_l, l) - \pi(q^*_{hl}, hl) + (P(q^*_h, h) - w_h) + D(q^*_l, l))$ (IR^C).

The high technology transfer w_h is defined by IR^C. Investment I^A satisfies the following first order condition :
$p'(I^A)\{\frac{v(w_h^A - D(q_h^A,h)) - v(w_l^A - D(q_l^A,l))}{p(I^A)v'(w_h^A - D(q_h^A,h)) + (1-p(I^A))v'(w_l^A - D(q_l^A,l))} + P(q_h^*, h) - w_h^A - (P(q_l^*, l) - w_l^A)\} = 1.$

Case 2 : We now build a similar proof for a minor innovation.

As above, we can show that RP_h^h and RP_l^l are equivalent to, respectively, $q_h \in [q_h^*, q_{lh}^*]$ and $q_l \in [q_{hl}^*, q_l^*]$.

We can also solve the RP_h^l and RP_h^l maximization programs and rewrite these two constraints as :

$$w_h - D(q_h, h) \geq \pi(q_h^*, h) - \pi(q_l, h) + w_l - D(q_l, h) \ (RP_h^l)$$

$$w_l - D(q_l, l) \geq \pi(q_l^*, l) - \pi(q_h, l) + w_h - D(q_h, l) \ (RP_l^h)$$

We now prove that RP_l^h is not binding and RP_h^l is binding. Suppose that $\{I\{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^h\}$ is such that both constraints are binding. We have :
$\pi(q_h^*, h) - \pi(q_h^A, h) + D(q_h^A, h) - D(q_h^A, l) = \pi(q_l^*, l) - \pi(q_l^A, l) + D(q_l^A, l) - D(q_h^A, l)$
$\iff \pi(q_h^*, h) - \pi(q_l^*, l) = \pi(q_l^A, hl) - \pi(q_h^A, lh)$.

Since $q_l^A \leq q_l^* < q_h^* \leq q_h^A$, the left-hand term is negative while the right-hand term is positive. This contradicts the fact that the two constraints are binding.

Suppose now that RP_l^h is binding and RP_h^l is not binding. Then,

$$\begin{aligned} w_h^A - D(q_h^A, h) - [w_l^A - D(q_l^A, l)] &= D(q_h^A, l) - D(q_h^A, h) - \pi(q_l^*, l) + \pi(q_h^A, l) \\ &= \pi(q_h^A, lh) - \pi(q_l^*, l). \end{aligned}$$

This term is strictly positive since $q_h^A \in [q_h^*, q_{lh}^*]$. Therefore, $w_h^A - D(q_h^A, h) > w_l^A - D(q_l^A, l)$. As in the proof of Proposition 7, we can show that, for $\epsilon > 0$ sufficiently small such that if w_l^A is increased by $\frac{\epsilon}{1-p(I)}$ and w_h^A is decreased by $\frac{\epsilon}{p(I)}$, all constraints can still be satisfied and RU's expected utility can be increased. This implies that RP_l^h is not binding and RP_h^l is binding. Therefore,

$w_h - D(q_h, h) = \pi_h(q_h^*) - \pi_h(q_l) + w_l - D(q_l, h)$. IR^C binding yields

$$p(I)(P(q_h, h) - w_h) + (1 - p(I))(P(q_l, l) - w_l) - I = 0.$$

Using these two preceeding relationships, the integrated structure maximization program can be rewritten as :

$\max_{\{I,\{q_\alpha\}_{\alpha=l}^h\}} p(I)v(p(I)\pi(q_h, h)+(1-p(I))\pi(q_h^*, h)+(1-p(I))[\pi(q_l, l)-\pi(q_l, hl)]- I) + (1-p(I))v(p(I)[\pi(q_h, h) - \pi(q_h^*, h)] + p(I)\pi(q_l, hl) + (1-p(I))\pi(q_l, l) - I)$ s/t

$$\begin{aligned} q_l - q_l^* &\leq 0 \\ q_{hl}^* - q_l &\leq 0 \\ q_h - q_{lh}^* &\leq 0 \\ q_h^* - q_h &\leq 0 \end{aligned}$$

Let the Lagrangian multiplier associated to each constraint be respectively μ_l, μ_{hl}, μ_{lh} and μ_h. The first order conditions are :

$$\begin{aligned} \pi'_h(q_h^A) &= \frac{\mu_{lh}-\mu_h}{p(I)E[V_\alpha^{A\prime}|I^A]} \\ \pi'_l(q_l^A, l) &= \frac{\pi'(q_l^A, hl)p(I^A)(V_h^{A\prime}-V_l^{A\prime})+\mu_l-\mu_{hl}}{E[V_\alpha^{A\prime}|I^A]} \\ \frac{V_h^A-V_l^A}{E[V_\alpha^{A\prime}|I^A]} + P(q_l^A, h) - P(q_l^A, l) &= \frac{1}{p'(I^A)} \end{aligned}$$

We first show that $q_h^A = q_h^*$. One of the two multipliers μ_h or μ_{lh} must be nil. Suppose that $\mu_h = 0$ and $\mu_{lh} > 0$, then $q_h^A = q_h^*$. By first order condition $\pi'(q_h^A, h) < 0$ which contradict that $q_h^A = q_h^*$. Suppose $\mu_{lh} = 0$ and $\mu_h > 0$, then $q_h^A = q_{lh}^* > q_h^*$ and we have $\pi'(q_{lh}^*, h) < 0$. By first order condition $\pi'(q_h^A, h) > 0$ which contradict that $q_h^A = q_{lh}^*$. Since $\mu_h = 0$ and $\mu_{lh} = 0$, the first order condition is rewritten as $\pi'(q_h^A, h) = 0$. Hence $q_h^A = q_h^*$. We now characterize q_l^A. First, note that since the binding constraint RP_h^l implies that $w_h^A - D(q_h^A, h) = \pi(q_h^*, h) - \pi(q_l^A, hl) + w_l - D(q_l^A, l)$, then $V_h^A > V_l^A$ and, since v is concave, $V_h^{A\prime} < V_l^{A\prime}$. We now prove that the multipliers μ_l and μ_{hl} are nil. Suppose that $\mu_l > 0$, then $\mu_{hl} = 0$ and $q_l^A = q_l^*$. According to the first order condition, $\pi'(q_l^A, l) > 0$, which contradict that $q_l^A = q_l^*$. Suppose that $\mu_{hl} > 0$, then $\mu_l = 0$ and $q_l^A = q_{hl}^*$. According to the first order condition, $\pi'(q_l^A, l) < 0$, which contradicts that $q_l^A = q_{hl}^*$. Hence, $q_l^A \in (q_{hl}^*, q_l^*)$ is defined by the following order condition :

$$\pi'(q_l^A, l) = \frac{\pi'(q_l^A, hl)p(I^A)(V_h^{A\prime} - V_l^{A\prime})}{E[V_\alpha^{A\prime}|I^A]}.$$

Finally, C's utility difference is $U_h^A - U_l^A = P(q_h^*, h) - w_h^A - [P(q_l^A, l) - w_l^A]$. The binding renegotiation-proof constraint RP_h^l yields $w_h^A - w_l^A = P(q_h^*, h) - P(q_l^A, h)$. Therefore, $U_h^A - U_l^A = P(q_l^A, h) - P(q_l^A, l)$. The last first order condition can be rewritten as :

$$\frac{V_h^A - V_l^A}{E[V_\alpha^{A\prime}|I^A]} + U_h^A - U_l^A = \frac{1}{p'(I^A)}.$$

A.5 Proof of Lemma 1

In this game, a strategy for RU is represented by $\sigma_{RU} = \{c_F, c_D(c_F, \alpha), q_{\hat{\alpha}}(c_F, c_D, \alpha)\}$, where c_F and c_D are respectly the financial and development contract offers, and $q_{\hat{\alpha}}$ represents RU's decision rule regarding the choice of a message $q_{\hat{\alpha}}$. The strategy of player F, $\sigma_F = \{d(c_F)\}$, represents its decision rule concerning the acceptance or rejection of the financial contract offer. The strategy of player C, $\sigma_C = \{d(c_F, c_D)\}$, represents its decision rule concerning the acceptance or rejection of the development contract offer. The beliefs of C are updated and are denoted $P(\alpha|c_F, c_D)$.

The proof of the necessary condition is similar to the proofs of Propositions 1 and 3, and it is therefore omitted. It is also straightforward to show sufficiency with the following strategies and beliefs.

$$\sigma_{RU} = \begin{cases} c_D(\alpha) = c_D^B = \{w^B(q_\alpha), q_\alpha^B, m^B(q_\alpha)\}_{\alpha=l}^h \\ q_{\hat{\alpha}}(c_F^B, c_D^B, \alpha) = argmax_{q_{\hat{\alpha}} \in \{q_l, q_h\}} V(w(q_{\hat{\alpha}}) - \bar{R}_{m(q_{\hat{\alpha}})}, q_{\hat{\alpha}}, \alpha) \end{cases}$$

$$\sigma_C = d_C(c_D) = \begin{cases} 1 & if \quad U(w(q_\alpha), q_\alpha, \alpha) \geq 0 \;\; \forall \alpha \in \{l, h\} \\ 0 & otherwise \end{cases}$$

$$P(h|c_D) = \begin{cases} 1 & if \quad argmax_{\hat{\alpha} \in \{l,h\}} V(w(q_{\hat{\alpha}}) - \bar{R}_{m(q_{\hat{\alpha}})}, q_{\hat{\alpha}}, \alpha) = h \\ 0 & otherwise \end{cases}$$

A.6 Proof of Lemma 2

A general proof is made for a major innovation. For the other case, the proof is similar and it is therefore omitted.

We derive the best wages and productions for each feasible report manipulation function and then compare RU's expected utility to find the solution to the P_{CD} program. Then we characterise the equilibrium development contract for all given financial contract allocation. We could have the following the equilibrium report manipulation functions.

- $\forall q_{\hat{\alpha}}$, $m^B(q_{\hat{\alpha}}) = l$.

  We can now consider the following reduced program :

  $\max_{\{w(q_\alpha), q_\alpha\}_{\alpha=l}^h} E[V(w(q_\alpha) - \bar{R}_l, q_\alpha, \alpha))|I]\ s/t$

$$\begin{array}{ll} P(q_h, h) - w(q_h) \geq 0 & (IR_h^C) \\ P(q_l, l) - w(q_l) \geq 0 & (IR_l^C) \\ w(q_h) - D(q_h, h) \geq w(q_l) - D(q_l, h) & (IC_h) \\ w(q_l) - D(q_l, l) \geq w(q_h) - D(q_h, l) & (IC_l) \end{array}$$

  The solution is :

  $w^B(q_h) = P(q_h^B, h)$, $w^B(q_l) = P(q_l^B, l)$, $q_l^B = q_l^*$,

  $q_h^B = \begin{cases} q_h^* & if \quad \pi(q_l^*, l) \geq P(q_h^*, h) - D(q_h^*, l) \\ q_h^S & otherwise \end{cases}$ with q_h^S such that

$\pi_l(q_l^*) = P(q_h^S, h) - D(q_h^S, l)$.

RU's expected utility is :

$$p(I)v(\pi(q_b^B, h) - \bar{R}_l) + (1 - p(I))v(\pi(q_l^*, l) - \bar{R}_l).$$

– $\forall q_{\hat{\alpha}}$, $m^B(q_{\hat{\alpha}}) = h$.

This case is symmetric to the first case. We have the same equilibrium wages and productions. RU's expected utility is :

$$p(I)v(\pi(q_b^B, h) - \bar{R}_h) + (1 - p(I))v(\pi(q_l^*, l) - \bar{R}_h).$$

– $m^B(q_h) = h$, $m^B(q_l) = l$.

The reduced program to consider is :

$\max_{\{w(q_a),q_\alpha\}_{\alpha=l}^h} E[V(w(q_\alpha) - \bar{R}_{m(q_\alpha)}, q_\alpha, \alpha))|I]\ s/t$

$$\begin{aligned}
&P(q_h, h) - w(q_h) \geq 0 && (IR_h^C)\\
&P(q_l, l) - w(q_l) \geq 0 && (IR_l^C)\\
&w(q_h) - \bar{R}_h - D(q_h, h) \geq w(q_l) - \bar{R}_l - D(q_l, h) && (IC_h)\\
&w(q_l) - \bar{R}_l - D(q_l, l) \geq w(q_h) - \bar{R}_h - D(q_h, l) && (IC_l)
\end{aligned}$$

To solve this maximization problem, we first show that incentive-compatibility constraints are binding.

1. Suppose that $P(q_h^B, h) - w^B(q_h) > 0$ and $P(q_l^B, l) - w^B(q_l) > 0$.
   If $w^B(q_h)$ and $w^B(q_l)$ are equally increased, the constraints are still satisfied and RU's expected utility is increased. This is not an equilibrium allocation.

2. Suppose that $P(q_h^B, h) - w^B(q_h) > 0$ and $P(q_l^B, l) - w^B(q_l) = 0$.
   Then, there exists $\delta > 0$ and $\epsilon > 0$ such that $\delta = D(q_h^B + \epsilon, l) - D(q_h^B, l)$ and $P(q_h^B + \epsilon, h) - (w^B(q_h) + \delta) > 0$. The alternative allocation

$\{w^B(q_l), q_l^B, w_h^B(q_l)+\delta, q_h^B+\epsilon\}$ increases RU's state h utility and satisfies the constraints. Individual-rationality constraints are satisfied and it is easy to verified that IC_l constraint is satisfied since :

$$w^B(q_h) + \delta - D(q_h^B + \epsilon, l) = w^B(q_h) - D(q_h^B, l).$$

Constraint IC_h is satisfied since RU's state h utility is increased as shown below :

$$D(q_h^B + \epsilon, l) - D(q_h^B, l) > D(q_h^B + \epsilon, h) - D(q_h^B, h), \ \forall q_h^B > 0.$$

Hence,

$$w^B(q_h) + \delta - D(q_h^B + \epsilon, h) > w^B(q_h) - D(q_h^B, h) + D(q_h^B + \epsilon, l) - D(q_h^B, l)$$

$$\Rightarrow w^B(q_h) + \delta - D(q_h^B + \epsilon, h) > w^B(q_h) - D(q_h^B, h).$$

3. Suppose that $P(q_h^B, h) - w^B(q_h) = 0$ and $P(q_l^B, l) - w^B(q_l) > 0$. Then, there exists $\delta > 0$ and $\epsilon > 0$ such that $\delta = D(q_l^B, h) - D(q_l^B - \epsilon, h)$ and $P(q_h^B - \epsilon, h) - (w^B(q_h) - \delta) > 0$. The alternative allocation $\{w^B(q_l)-\delta, q_l^B-\epsilon, w^B(q_h), q_h^B\}$ increases RU's state l utility and satisfies the constraints. Individual rationality constraints are satisfied and it is easy to verified that IC_h constraint is satisfied since $w^B(q_l) - \delta - D(q_l^B - \epsilon, l) = w^B(q_l) - D(q_l^B, h)$. The constraint IC_l is satisfied since RU's state l utility is increased as shown below. We have :

$$D(q_l^B, l) - D(q_l^B - \epsilon, l) > D(q_l^B, h) - D(q_l^B - \epsilon, h), \forall q_l^B > 0.$$

Hence,

$$w^B(q_l) - \delta - D(q_l^B - \epsilon, l) > w^B(q_l) - D(q_l^B, l) + D(q_l^B, h) - D(q_l^B - \epsilon, h)$$

$$\Rightarrow w^B(q_l) - \delta - D(q_l^B - \epsilon, l) > w^B(q_l) - D(q_l^B, l).$$

We just proved that equilibrium wages are defined by C's binding participation constraint $\forall \alpha \in \{l, h\} : w(q_\alpha) = P(q_\alpha, \alpha)$. We now have to solve the following reduced program :

$\max_{\{q_h, q_l\}} p(I)v(\pi(q_h, h) - \bar{R}_h) + (1 - p(I))v(\pi(q_l, l) - \bar{R}_l)\ s/t$

$$\begin{aligned} \pi(q_h, h) - \bar{R}_h &\geq \pi(q_l, lh) - \bar{R}_l \quad (IC_h) \\ \pi(q_l, l) - \bar{R}_l &\geq \pi(q_h, hl) - \bar{R}_h \quad (IC_l) \end{aligned}$$

This program solution depends on repayments allocation $\{\bar{R}_\alpha\}_{\alpha=l}^{h}$. We identify three cases.

1. If $\pi(q_h^*, h) - \pi(q_l^*, lh) \geq \bar{R}_h - \bar{R}_l \geq \pi(q_h^*, hl) - \pi(q_l^*, l)$.

   Then the incentive-compatibility constraints are not binding and efficient production can be implemented ; $q_h^B = q_h^*$, $q_l^B = q_l^*$. RU's expected utility is :

   $$p(I)v(\pi(q_h^*, h) - \bar{R}_h) + (1 - p(I))v(\pi(q_l^*, l) - \bar{R}_l).$$

2. If $\bar{R}_h - \bar{R}_l > \pi(q_h^*, h) - \pi(q_l^*, lh)$.

   Then IC_h is the binding constraint. The low technology production is distorted ; $q_h^B = q_h^*$ and $q_l^B < q_l^*$ such that : $\pi(q_h^*, h) - \bar{R}_h = \pi(q_l^*, lh) - \bar{R}_l$. RU's expected utility is :

   $$p(I)v(\pi(q_h^*, h) - \bar{R}_h) + (1 - p(I))v(\pi(q_l^B, l) - \bar{R}_l).$$

3. If $\bar{R}_h - \bar{R}_l < \pi(q_h^*, hl) - \pi(q_l^*, l)$.

   Then IC_l is the binding constraint. The high technology production is distorted ; $q_l^B = q_l^*$ and $q_h^B > q_h^*$ such that : $\pi_l(q_l^*) - \bar{R}_l = \pi_{hl}(q_l^B) - \bar{R}_l$. RU's expected utility is :

   $$p(I)v(\pi(q_h^B, h) - \bar{R}_h) + (1 - p(I))v(\pi(q_l^B, l) - \bar{R}_l).$$

– $m(q_h) = l$ and $m(q_l) = h$.

It is a symmetric case to the previous case.

The equilibrium allocation depends on the equilibrium financial contract. For any assumptions about the equilibrium repayments, we shall show that RU proposes $m^B(q_\alpha) = argmin_\alpha\{\bar{R}_\alpha\}$: the equilibrium manipulation report function prescribes to report the innovation quality corresponding to the minimum repayment. We prove that this report manipulation function is preferred in equilibrium to the truth-telling report manipulation function. It is straightforward to design a similar proof which shows that this report manipulation function is also preferred to the following report manipulation function $m(q_h) = l$, $m(q_l) = h$. This proof is therefore omitted.

- If $\bar{R}_h > \bar{R}_l$.

  We show that the equilibrium report manipulation function is $\forall \alpha \in \{l, h\}, m(q_\alpha) = l$; that is always report a low quality innovation.

  1. For $\bar{R}_h - \bar{R}_l \geq \pi(q_h^*, hl) - \pi(q_l^*, lh)$.

     The best manipulation function is $m^B(q_h) = h$ and $m^B(q_l) = l$ (and not $m^B(q_{\hat{\alpha}}) = l \; \forall q_{\hat{\alpha}}$) if :

     - For $\bar{R}_h - \bar{R}_l > \pi(q_h^*, h) - \pi(q_l^*, lh)$:
       $p(I)v(\pi(q_h^*, h) - \bar{R}_h) + (1-p(I))v(\pi(q_l^B, l) - \bar{R}_l) \geq p(I)v(\pi(q_h^B, h) - \bar{R}_l) + (1-p(I))v(\pi(q_l^B, l) - \bar{R}_l)$
     - For $\bar{R}_h - \bar{R}_l \leq \pi(q_h^*, h) - \pi(q_l^*, lh)$:
       $p(I)v(\pi(q_h^*, h) - \bar{R}_h) + (1-p(I))v(\pi(q_l^*, l) - \bar{R}_l) \geq p(I)v(\pi(q_h^B, h) - \bar{R}_l) + (1-p(I))v(\pi(q_l^B, l) - \bar{R}_l)$

     A necessary condition for one of these inequalities to hold is that :

     $$v(\pi(q_h^*, h) - \bar{R}_h) \geq v(\pi(q_h^B, h) - \bar{R}_l).$$

     Since $\bar{R}_h > \bar{R}_l$, this inequality is not satisfied if $q_h^B = q_h^*$. If $q_h^B = q_h^S$, we must have :

     $$\pi(q_h^*, h) \geq \bar{R}_h + \pi(q_h^B, h) - \bar{R}_l$$

Since $\bar{R}_h - \bar{R}_l \geq \pi(q_h^*, hl) - \pi(q_l^*, lh)$, it implies :

$$\pi(q_h^*, h) \geq \pi(q_h^S, h) + \pi(q_h^*, hl) - \pi(q_l^*, l).$$

For all q, we have $\pi(q, h) = \pi(q, hl) - D(q, h) + D(q, l)$. Since $\pi(q_h^S, lh) = \pi(q_l^*, l)$, then $\pi(q_h^S, h) = \pi(q_l^*, l) + D(q_h^S, h) - D(q_h^S, l)$. This relationship and the previous inequality imply :

$$D(q_h^*, l) - D(q_h^*, h) \geq D(q_h^S, h) - D(q_h^S, l),$$

which is false since $q_h^S > q_h^*$.

2. For $\bar{R}_h - \bar{R}_l < \pi(q_h^*, hl) - \pi(q_l^*, l)$.

   The best report manipulation function is $m^B(q_h) = h$ and $m^B(q_l) = l$ and not $m^B(q_\alpha) = l, \forall \alpha$ if :

   $p(I)v(\pi(q_h^B, h) - \bar{R}_h) + (1 - p(I))v(\pi(q_l^B, l) - \bar{R}_l) \geq p(I)v(\pi(q_h^B, h) - \bar{R}_l) + (1 - p(I))v(\pi(q_l^B, l) - \bar{R}_l)$.

   A necessary condition for this inequality to hold is that :

$$v(\pi(q_h^B, h) - \bar{R}_h) \geq v(\pi(q_h^S, h) - \bar{R}_l)$$

$$\iff \pi(q_h^B, hl) + D(q_h^B, l) - D(q_h^B, h) - \bar{R}_h \geq \pi(q_h^S, hl) + D(q_h^S, l) - D(q_h^S, h) - \bar{R}_l$$

   where q_h^B and q_h^S are defined by $\pi(q_l^*, l) - \bar{R}_l = \pi(q_h^B, hl) - \bar{R}_h$ and $\pi(q_l^*, l) - \bar{R}_l = \pi(q_h^S, hl) - \bar{R}_l$. Therefore, we have :

$$D(q_h^B, l) - D(q_h^B, h) \geq D(q_h^S, h) - D(q_h^S, l),$$

   which is false since $q_h^S > q_h^B$ when $\bar{R}_h > \bar{R}_l$.

- If $\bar{R}_h < \bar{R}_l$.

  By a similar proof, we can show that the equilibrium report manipulation function is $m^B(q_h) = m^B(q_l) = h$.

– If $\bar{R}_h = \bar{R}_l$, all report manipulation functions solve the program since they give the same expected utility to RU.

To conclude, we proved that the solution of program P_{CD} is :
$w^B(q_h) = P(q_h^B, h)$, $w^B(q_l) = P(q_l^B, l)$; $q_l^B = q_l^*$;
$q_h^B = \begin{cases} q_h^* & if \quad \pi(q_l^*, l) \geq P(q_h^*, h) - D(q_h^*, l) \\ q_h^S & otherwise \end{cases}$
with q_h^S such that $\pi(q_l^*, l) = P(q_h^S, h) - D(q_h^S, l)$;
$m^B(q_\alpha) = argmin_\alpha\{\bar{R}_\alpha\}$, $\forall\alpha \in \{l, h\}$.
RU's expected utility is :

$$p(I^B)v(\pi(q_b^B, h) - \min\{\bar{R}_h, \bar{R}_l\}) + (1 - p(I))v(\pi(q_l^*, l) - \min\{\bar{R}_h, \bar{R}_l\}).$$

We can write similar proof if a minor innovation. The wages are defined by C's binding individual rationality constraints. The investment and the production level implemented solve the following program :
$(P'_{CD2})\ \max_{\{I,\{q_\alpha\}_{\alpha=l}^h\}} E[v(\pi(q_\alpha, \alpha) - I)|I]\ s/t$

$$\begin{aligned} \pi(q_h, h) &\geq \pi(q_l, lh) \quad (IC_h) \\ \pi(q_l, l) &\geq \pi(q_h, hl) \quad (IC_l) \end{aligned}$$

The solution is given by the following relationships :
$q_h^B = q_h^*$, $q_l^B = \begin{cases} q_l^* & if \quad \pi(q_h^*, h) \geq P(q_l^*, l) - D(q_l^*, h) \\ q_l^S & otherwise \end{cases}$
with q_l^S such that $\pi(q_h^*, h) = P(q_l^S, l) - D(q_l^S, h)$; $p'(I^B)\frac{V_h^B - V_l^B}{E[V_\alpha'^B|I^B]} = 1$.

A.7 Proof of Proposition 5

We shall prove that the financial contract equilibrium must be such that $\min\{R_\alpha^B\} = I^B$.

Suppose that $\min_\alpha\{R^B_\alpha\} > I^B$. F's expected gain is $E[\min_\alpha\{R^B_\alpha\}] - I^B > 0$. Since RU can increase his gain by reducing ex post repayments, this allocation is not a PBE equilibrium. Suppose that $\min_\alpha\{R^B_\alpha\} < I^B$. F's expected gain is $E[\min_\alpha\{R^B_\alpha\}] - I^B < 0$. F would prefer to refuse this offer, and RU's gain is nil although it can be positive with $\min_\alpha\{R^B_\alpha\} = I^B$. As $m^B(\alpha) = argmin_\alpha\{R^B_\alpha\}$, along the equilibrium path, investment equals repayments for each state of nature $\alpha \in \{l, h\}$. Hence, $R^B_h = R^B_l = I^B$. The level of investment I^B solves the following maximization problem.

$$\max_I E[V(w^B(q_\alpha) - I, q^B_\alpha, \alpha)|I].$$

The first order condition is :

$$p'(I^B)\frac{V^B_h - V^B_l}{E[V'^B_\alpha|I^B]} = 1.$$

A.8 Proof of Proposition 6

We first prove the first part of Proposition 6. Suppose that innovation is major. The allocation $\{I^A\{w^A_\alpha, q^A_\alpha\}^h_{\alpha=l}\}$ solves the following reduced maximization problem :

(P''_R) $\max_{\{I,\{w_\alpha\}^h_{\alpha=l}\}} E[V(w_\alpha, q^*_\alpha, \alpha)|I]$ s/t

$$E[U(w_\alpha, q^*_\alpha, \alpha)|I] - I = 0 \qquad (IR)$$

$$w_h - D(q^*_h, h) \geq \pi(q^*_{lh}, lh) - \pi(q^*_l, lh) + w_l - D(q^*_l, h) \quad (RP^l_h)$$

We prove first that, when $\pi(q^*_l, l) \geq \pi(q^*_h, hl)$ (no-distortion case), the independent structure allocation satisfies the constraints of the P''_R program without solving it. Therefore, RU's expected utility in the independent structure is dominated by the integrated structure one. Then we show that the independent structure performs better in the case $\pi(q^*_l, l) \geq \pi(q^*_h, hl)$ than otherwise.

- Suppose $\pi(q_l^*, l) \geq \pi(q_h^*, hl)$. Let $I^c = I^B$, $w_\alpha^c = w_\alpha^B - I^B$ and $q_\alpha^c = q_\alpha^B = q_\alpha^*$. The allocation $\{I^c\{w_\alpha^c, q_\alpha^c\}_{\alpha=l}^h\}$ satisfies C's individual rationality constraint. The renegotiation-proof constraint RP_h^l is rewritten as :

$$\pi(q_h^*, h) \geq \pi(q_{lh}^*, lh),$$

which is satisfied. However, since the renegotiation-proof constraint RP_l^h is not satisfied with
$\{I^c\{w_\alpha^c, q_\alpha^c\}_{\alpha=l}^h\}$, then $\{I^c\{w_\alpha^c, q_\alpha^c\}_{\alpha=l}^h\} \neq \{I^A\{w_\alpha^A, q_\alpha^A\}_{\alpha=l}^h\}$. Therefore, the program objective is higher with $\{I^A, \{w_\alpha^A, q_\alpha^A\}_{\alpha=h}^l\}$ than with $\{I^c, \{w_\alpha^c, q_\alpha^c\}_{\alpha=h}^l\}$:

$$E[V(w_\alpha^A, q_\alpha^A, \alpha)|I^A] > E[V(w_\alpha^c, q_\alpha^c, \alpha)|I^c] = E[V(w^B(q_\alpha) - I^B, q_\alpha^B, \alpha)|I^B].$$

- Suppose $\pi(q_l^*, l) < \pi(q_h^*, hl)$. Since $w^B(q_\alpha) = P(q_\alpha^B, \alpha)$, RU's equilibrium expected utility in the independent structure can be found by solving :
(P''_{CD}) $\max_{\{I,\{q_\alpha\}_{\alpha=l}^h\}} E[V(P(q_\alpha, \alpha) - I, q_\alpha, \alpha)|I]$

$$\begin{aligned} \pi(q_h, h) &\geq \pi(q_l, hl) \quad (IC_h^C) \\ \pi(q_l, l) &\geq \pi(q_h, lh) \quad (IC_l^C) \end{aligned}$$

Let the allocation $\{I^d, \{q_\alpha^d\}_{\alpha=l}^h\}$ solves the following program :

$$\max_{\{I,\{q_\alpha\}_{\alpha=l}^h\}} E[V(P(q_\alpha, \alpha) - I, q_\alpha, \alpha)|I].$$

The first order conditions are :

$$\begin{aligned} p(I^d)V_h'^d \pi'(q_h^d, h) &= 0 \\ (1 - p(I^d))V_l'^d \pi'(q_l^d, l) &= 0 \\ p'(I^d)(V_h^d - V_l^d) - E[V_\alpha'^d|I^d] &= 0 \end{aligned}$$

Therefore $q_\alpha^d = q_\alpha^*$, $\forall \alpha \in \{l, h\}$.

Clearly, since one of the two incentive compatibility constraints in the P''_{CD} program is binding, we have :

$$E[V(P(q^*_\alpha, \alpha) - I^d, q^*_\alpha, \alpha)|I^A] > E[V(P(q^B_\alpha, \alpha) - I^B, q^B_\alpha, \alpha)|I^B].$$

Since allocation $\{I^d, \{w^d_\alpha, q^*_\alpha\}^l_{\alpha=h}\}$ where $w^d_\alpha = P(q^*_\alpha, \alpha) - I^d$ satisfies P''_R without solving it, RU's expected utility is higher with $\{I^A, \{w^A_\alpha, q^A_\alpha\}^l_{\alpha=h}\}$ than with $\{I^d, \{w^d_\alpha, q^*_\alpha\}^l_{\alpha=h}\}$:

$$E[V(w^A_\alpha, q^A_\alpha, \alpha)|I^A] > E[V(P(q^*_\alpha, \alpha) - I^d, q^*_\alpha, \alpha)|I^d].$$

Use the two preceeding relationships and obtain :

$$E[V(w^A_\alpha, q^A_\alpha, \alpha)|I^A] > E[V(w^B(q_\alpha) - I^B, q^B_\alpha, \alpha)|I^B].$$

We now show that if $q^B_l \geq q^A_l$ and $\lambda > 1$, then $E[V^B_\alpha|I^B] > E[V^A_\alpha|I^A]$. This case includes the case $q^B_l = q^*_l$ which is satisfied by definition under Assumption 1.

RU's expected utility in the integrated structure is :
$E[V^A_\alpha|I^A] = p(I^A)v(\pi(q^*_h, h) + (1 - p(I^A))(P(q^A_l, l) - P(q^A_l, h)) - I^A) + (1 - p(I^A))v(\pi(q^A_l, l) - p(I^A)(P(q^A_l, l) - P(q^A_l, h)) - I^A)$
RU's expected utility in the independent structure is $E[V^B_\alpha|I^B] = p(I^B)v(\pi(q^*_h, h) - I^B) + (1 - p(I^B))v(\pi(q^B_l, l) - I^B)$.
Now, since v is strictly concave, we have $\forall x > y$, $v(x) - v(y) < v'(y)(x - y)$ and $v(x) - v(y) > v'(x)(x - y)$. Hence,
$v(\pi(q^*_h, h) + (1 - p(I^A))(P(q^A_l, l) - P(q^A_l, h)) - I^A) - v(\pi(q^*_h, h) - I^A) < v'(\pi(q^*_h, h)) - I^A)(1 - p(I^A))(P(q^A_l, l) - P(q^A_l, h))$,
and,
$v(\pi(q^A_l, l) - I^A) - v(\pi(q^A_l, l) - p(I^A)(P(q^A_l, l) - P(q^A_l, h)) - I^A) > v'(\pi(q^*_l, l) - I)p(I^A)(P(q^A_l, l) - P(q^A_l, h))$.
Since $\pi(q^B_l, l) \geq \pi(q^A_l, l)$, using the two preceeding relationships, we find that :

$p(I^A)v(\pi(q_h^*, h) - I^A) + (1 - p(I^A))v(\pi(q_l^B, l) - I^A) > E[V_\alpha^A|I^A] + (P(q_l^A, l) - P(q_l^A, h))p(I^A)(1 - p(I^A))[v'(\pi(q_l^A, l) - I^A) - v'(\pi(q_h^*, h) - I^A)]$.

Since $P(q_l^A, l) \geq P(q_l^A, h)$ and $v'(\pi(q_l^A, l) - I^A) > v'(\pi(q_h^*, h) - I^A)$, therefore,

$$p(I^A)v(\pi(q_h^*, h) - I^A) + (1 - p(I^A))v(\pi(q_l^B, l) - I^A) > E[V_\alpha^A|I^A].$$

Since I^B solves $\max_I p(I)v(\pi(q_h^*, h) - I) + (1 - p(I))v(\pi(q_l^B, l) - I)$, therefore,

$$E[V_\alpha^B|I^B] > p(I^A)v(\pi(q_h^*, h) - I^A) + (1 - p(I^A))v(\pi(q_l^B, l) - I^A).$$

Using the two preceeding inequalities, we conclude that $E[V_\alpha^B|I^B] > E[V_\alpha^A|I^A]$.

When $\lambda = 1$ (neutral innovation), then $q_\alpha^A = q_\alpha^B$, for every $\alpha \in \{l, h\}$. In this case, RU's expected utility is :

$E[V_\alpha^A|I^A] = E[V_\alpha^B|I^B] = p(I^B)v(\pi(q_h^*, h) - I^B) + (1 - p(I^B))v(\pi(q_l^B, l) - I^B)$.

Annexe B

Proofs of chapter 2

B.1 Convexity

$\forall \mu : \mu_{\underline{\theta}} \geq \mu \geq \mu_{\bar{\theta}}$, $\tilde{\theta}(\mu) = \frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu]r}$. We have :

$$\tilde{\theta}'(\mu) = -p\frac{u(\bar{y}) - u(\bar{y} - \alpha)}{[1 - p + p\mu]^2 r} < 0,$$

$$\tilde{\theta}''(\mu) = 2p^2\frac{u(\bar{y}) - u(\bar{y} - \alpha)}{[1 - p + p\mu]^3 r} > 0.$$

B.2 Proof of proposition 1

We first show that if all agents obey to the full income-sharing norm, it implies that the full income-sharing norm maximizes conformists'expected utility and therefore will be implemented. Then we prove that if the full income-sharing norm is implemented then there will be full obedience to this norm. Suppose that $u(E[y]) > u(\bar{y}) - r$, then all agents conform to a full income-sharing norm $\alpha^F = (1-p)(\bar{y} - \underline{y})$ in the stable strictly positive equilibrium. Therefore $\mu^* = 1$. It is straightforward to show that α^F maximizes agents' expected utility :

$U^c(\alpha^F) = u(E[y]) > pu(\bar{y} - \alpha) + (1-p)u(\underline{y} + \frac{\alpha p \mu^*}{1-p}), \forall \alpha^* \neq \alpha^F$.

Suppose that the best norm is the full income-sharing norm. If some agents desobey the norm, that is $\mu^* < 1$, then α^* is defined by the following first order condition (see next proposition) :

$$u'(\underline{y} + \frac{\alpha^* p \mu^*}{1-p})[\mu^* + \alpha^* \frac{d\mu^*}{d\alpha}] = u'(\bar{y} - \alpha),$$

with $\frac{d\mu^*}{d\alpha^*} = -\frac{f(\tilde{\theta}(\mu))\frac{u'(\bar{y}-\alpha^*)}{[1-p+p\mu^*]r}}{1-f(\tilde{\theta}(\mu^*))p\frac{u(\bar{y})-u(\bar{y}-\alpha^*)}{[1-p+p\mu^*]^2 r}}$.

Then we cannot have equality between the two marginal rates of substitutions : $\alpha^* \neq \alpha^F$.

B.3 Proof of proposition 2

I first show that if everybody obey the norm then the rich agent the less affected by his reputation is indifferent between obey or not the norm. Therefore, the best norm can be characterized by the first order condition to the maximization problem. I then derive this first order condition and verify the second order condition.

Let $\alpha_{\underline{\theta}}$ be the norm which makes the agent the less affected by his reputation indifferent :

$$\alpha_{\underline{\theta}} = \bar{y} - u^{-1}(u(\bar{y} - r)).$$

If $\alpha^* < \alpha_{\underline{\theta}}$, then :

$$U^{C\prime}(\alpha^*) = p\{u'(\underline{y} + \delta^*) - u'(\bar{y} - \alpha^*)\} > 0.$$

Conformists expected utility could be increase with a higher transfer α^* until we reach $\alpha^* = \alpha_{\underline{\theta}}$. If $\alpha^* > \alpha_{\underline{\theta}}$ then $\mu^* = 1 - F(\frac{u(\bar{y}) - u(\bar{y} - \alpha^*)}{[1-p+p\mu^*]r}) < 1$. The following first order condition (FOC) must be satisfied :

$$U^{C\prime}(\alpha^*) = 0$$

$$\Longleftrightarrow\ p\{u'(\underline{y}+\delta)[\mu^* - \alpha^* \frac{f(\tilde{\theta}(\mu^*))\frac{u'(\bar{y}-\alpha^*)}{[1-p+p\mu^*]r}}{1-f(\tilde{\theta}(\mu^*))p\frac{u(\bar{y})-u(\bar{y}-\alpha^*)}{[1-p+p\mu^*]^2 r}}] - u'(\bar{y}-\alpha^*)\} = 0.$$

$$\Longleftrightarrow\ u'(\underline{y}+\delta)[\mu^* - \alpha^* f(\tilde{\theta}(\mu^*))\frac{\frac{u'(\bar{y}-\alpha^*)}{[1-p+p\mu^*]r}}{1-f(\tilde{\theta}(\mu^*))p\frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu^*]^2 r}}] = u'(\bar{y}-\alpha^*).$$

If $\alpha^* = \alpha_{\underline{\theta}}$, this first order condition is satisfied with $\mu^* = 1$.

I now verify the second order condition (SOC). The FOC can be rewritten :

$$U^{C\prime}(\alpha^*) = p\{u'(\underline{y}+\delta)[\mu^* - \frac{f(\tilde{\theta}(\mu^*))\alpha^* u'(\bar{y}-\alpha^*)}{[1-p+p\mu^*]r - pf(\tilde{\theta}(\mu^*))\frac{u(\bar{y})-u(\bar{y}-\alpha^*)}{1-p+p\mu^*}}] - u'(\bar{y}-\alpha^*)\}.$$

Since $\tilde{\theta}(\mu^*) = \frac{u(\bar{y})-u(\bar{y}-\alpha^*)}{(1-p+p\mu^*)r}$,

$$U^{C\prime}(\alpha^*) = p\{u'(\underline{y}+\delta^*)[\mu^* - \frac{f(\tilde{\theta}(\mu^*))\alpha^* u'(\bar{y}-\alpha^*)}{[1-p+p\mu^* - pf(\tilde{\theta}(\mu^*))\tilde{\theta}(\mu^*)]r}] - u'(\bar{y}-\alpha^*)\}.$$

The second derivative is :

$U^{C\prime\prime}(\alpha^*) = p\{u''(\underline{y}+\delta)\frac{p}{1-p}[\mu^* + \alpha^*\frac{d\mu^*}{d\alpha}] + u'(\underline{y}+\delta^*)[2\frac{d\mu^*}{d\alpha} +$
$\frac{\alpha^*}{D^2}\{f(\tilde{\theta}(\mu^*))f'(\tilde{\theta}(\mu^*))\frac{d\tilde{\theta}(\mu^*)}{d\alpha}u'(\bar{y}-\alpha^*)u''(\bar{y}-\alpha^*)D +$
$f(\tilde{\theta}(\mu^*))u'(\bar{y}-\alpha^*)r[p\frac{d\mu^*}{d\alpha} - (f(\tilde{\theta}(\mu^*)) + f'(\tilde{\theta}(\mu^*))\tilde{\theta}(\mu^*)))(\mu^*))\frac{d\tilde{\theta}(\mu^*)}{d\alpha}]\}$
$+ u''(\bar{y}-\alpha^*)\}$,

where $D = (1-p+p\mu-f(\tilde{\theta}(\mu^*))\tilde{\theta}(\mu^*))r$ and $\frac{d\tilde{\theta}(\mu^*)}{d\alpha} = \frac{u'(\bar{y}-\alpha^*)(1-p+p\mu)-(u(\bar{y})-u(\bar{y}-\alpha^*))\frac{d\mu}{d\alpha}}{[1-p+p\mu]^2 r}$.
Since $u'' < 0$, $\frac{d\mu^*}{d\alpha} < 0$, $\frac{\tilde{\theta}(\mu^*)}{d\alpha} > 0$ and $f(\theta) + f'(\theta)\theta \geq 0$, for every $\theta \in \Theta$ by assumption, we proved that $U^{C\prime\prime}(\alpha) < 0$.

B.4 Proof of proposition 3

Suppose that the transfer given is equal to the social norm α^* derived in section 2.5. I first show that there is a sugame perfect equilibrium where everybody accept this contract. Consider an agent θ. Suppose that all his relatives accept the contract. The stable Nash equilibrium of the subgame of norm conformity was previously derived. Now, if agent θ accepts the contract, his expected utility is :

$$U^A(\alpha^*) = pMax\{u(\bar{y}-\alpha^*), u(\bar{y}) - \theta[1-p+p\mu^*]r\} + (1-p)u(\underline{y} + \frac{p\mu^*}{1-p}).$$

If he refuses, agent θ's expected utility is :

$$U^R(\alpha^*) = pu(\bar{y}) + (1-p)u(\underline{y}).$$

Since $U^A(\alpha^*) \geq U^C(\alpha^*) \geq U^R(\alpha^*)$, the best reply for player θ is to adhere to the norm for all θ in Θ. Given this subgame equilibrium, the best informal insurance contract is α^* derived in section 5.

B.5 Comparative static properties

For convenience, we suppose that θ is uniformly distributed in $\Theta = [1,2]$. Equation 2.3 can be written as :

$$\mu^* = \begin{cases} 1 & if \quad \mu^* \geq \mu_1 \\ 2 - (\frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu^*]r}) & if \quad \mu_1 \geq \mu^* \geq \mu_2 \\ 0 & if \quad \mu^* \leq \mu_2 \end{cases} \tag{B.1}$$

The stability condition is $1 - p\frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu]^2 r} > 0$.

The FOC can be rewritten as :

$$u'(\underline{y}+\delta^*)[\mu^* - \alpha^* \frac{u'(\bar{y}-\alpha^*)}{[1-3p+2p\mu^*]r}] - u'(\bar{y}-\alpha^*) = 0.$$

- I first state how equilibrium norm conformity μ^* react to exogenous parameters for a constant transfer α (if α is fixed).

$\frac{d\mu^*}{dr} = \frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu^*]^2 r}$,
$\frac{d\mu^*}{d\bar{y}} = -\frac{\frac{u'(\bar{y})-u'(\bar{y}-\alpha)}{[1-p+p\mu^*]r}}{1-p\frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu^*]^2 r}}$,
$\frac{d\mu^*}{d\underline{y}} = 0$,
$\frac{d\mu^*}{dp} = -\frac{u(\bar{y})-u(\bar{y}-\alpha)}{[1-p+p\mu^*]^2 r}(1-\mu^*)$.

– I now turn to income-sharing norm α^* adjustment to changing parameters.
$\frac{d\alpha^*}{dr} = -\frac{1}{SOC}\{\frac{\alpha^* u'(\underline{y}-\delta^*)u'(\bar{y}-\alpha^*)}{[1-3p+2p\mu]r^2}\}$
$\frac{d\alpha^*}{d\bar{y}} = \frac{1}{SOC}u''(\bar{y}-\alpha^*)\{\frac{u'(\underline{y}+\delta^*)\alpha^*}{[1-3p+2p\mu^*]r} + 1\}$
$\frac{d\alpha^*}{d\underline{y}} = -\frac{1}{SOC}u''(\underline{y}+\delta^*)[\mu^* + \alpha^*\frac{d\mu^*}{d\alpha^*}]$.
Since $\mu^* + \alpha^*\frac{d\mu^*}{d\alpha^*} > 0$ by the first order condition, $\frac{d\alpha^*}{d\underline{y}} < 0$.
$\frac{d\alpha^*}{dp} = -\frac{1}{SOC}\{u''(\underline{y}+\delta^*)\frac{\mu^*\alpha^*}{(1-p)^2}[\mu^* + \alpha^*\frac{d\mu^*}{d\alpha^*}] - \frac{\alpha^* u'(\underline{y}-\delta^*)u'(\bar{y}-\alpha^*)(3-2\mu^*)}{[1-3p+2p\mu]r^2}\}$.
Since $\mu^* + \alpha^*\frac{d\mu^*}{d\alpha^*} > 0$ and $3 > 2\mu^*$, $\frac{d\alpha^*}{dp} < 0$.

Where SOC =Second Order Condition.

Annexe C

Proofs of chapter 3

C.1 Proof of Proposition 2

Let us first introduce notations. Consider an arbitrary coalition $T = \{\underline{t}, ..., \bar{t}\}$. Denote π_T the maximization program associated to $v(T)$ and x_T^T its solution. Call the sets of binding constraints by $K(T) \subset T$ and the set of binding constraints *before the last binding constraint* $\bar{t}$ by $K^0(T) \subset K(T)$. Denote $f(T)$ the first binding constraint and $l(T)$ the last binding constraint before $\bar{t}$: $f(T) = \min\ K^0(T)$ and $l(T) = \max\ K^0(T)$. The set of binding constraints is written as $K(T) = \{f(T), ..., l(T), \bar{t}\}$. Denote $\mathcal{P}(T) = \{T_i\}_{i \in K(T)}$ the partition of T into self-sustained sets $T_j = \{i+1, ..., j\}$ for all consecutive $i, j \in K(T)$. The secure welfare of a connected set T can be decomposed into the sum of secure welfares of partition $\mathcal{P}(T)$'s sets :

$$v(T) = \sum_{i \in K(T)} v(T_i). \tag{C.1}$$

To prove that the game is convex, we need two lemmas.

Lemma 1. *For all $\underline{s}, \bar{s}, \in N$, $\underline{s} < \bar{s}$, if $S = \{\underline{s}, ..., \bar{s}\}$ and $T = \{\underline{s} + k, ..., \bar{s}\}$ with*

$0 < k \leq \bar{s} - \underline{s}$, *then* $x_i^S \geq x_i^T$, $\forall i \in S \cap T$.

Proof of Lemma 1

It will be sufficient to prove that this is true for $k = 1$. Consider $S = \{\underline{s}, ..., \bar{s}\}$ and $T = \{\underline{s}+1, ..., \bar{s}\}$ with $\underline{s}, \bar{s}, \in N$, $\underline{s} < \bar{s}$.

First, consider the case $x_{\underline{s}}^S = e_{\underline{s}}$. Then $(x_{\underline{s}+1}^S, ..., x_{\bar{s}}^S)$ solves Π_T. In this case, $x_i^S = x_i^T$, $\forall i \in S \cap T$.

Now, suppose that $x_{\underline{s}}^S < e_{\underline{s}}$. Then $(x_{\underline{s}+1}^S, ..., x_{\bar{s}}^S)$ solves :

$$\Pi'_S \left\{ \begin{array}{lll} \max_{(x_{\underline{s}+1},...,x_{\bar{s}})} & \sum_{i=\underline{s}+1}^{\bar{s}} b_i(x_i) \quad s/t & \\ & \sum_{j \in P i \cap S \backslash \{\underline{s}\}} x_j \quad \leq & \sum_{j \in P i \cap S \backslash \{\underline{s}\}} e_j + e_{\underline{s}} - x_{\underline{s}}^S \quad \forall i = S \backslash \{\underline{s}\}. \end{array} \right.$$

Note that, since $\underline{s} \notin K(S)$, the set of binding constraints in Π'_S is equal to $K(S)$. Since $e_{\underline{s}} - x_{\underline{s}}^S > 0$, there is strictly more water at $\underline{s}+1$ to be distributed between members of coalition T in Π'_S than in Π_T. Clearly, any binding constraint in Π'_S should be also binding in Π_T. Put differently, $K(S) \subset K(T)$.

Consider the case of one interior binding constraint in Π'_S, i.e. $f(S)$ exists. The constraint $f(S)$ is also binding in Π_T. Partition T into two subsets $F^0(f(S)) \cap T$ and $P(f(S)) \cap T$. It is easy to prove that $x_i^S = x_i^T$, $\forall i \in F^0(f(S)) \cap T$. Recall that, if x_S^S or x_T^T is implemented, all water available at $f(S)$ is diverted from the river. The following water allocations downstream $f(S)$, $(x_{f(S)+1}^S, ..., x_{\bar{s}}^S)$ and $(x_{f(S)+1}^T, ..., x_{\bar{s}}^T)$ solve the same following reduced program :

$$\Pi_r \left\{ \begin{array}{lll} \max_{(x_{f(S)+1},...,x_{\bar{s}})} & \sum_{i=f(S)+1}^{\bar{s}} b_i(x_i) \quad s/t & \\ & \sum_{j \in P i \cap G} x_j \quad \leq & \sum_{j \in P i \cap G} e_j \quad \forall i \in G = \{f(S)+1, ..., \bar{s}\}. \end{array} \right.$$

Suppose now that $\exists i \in P(f(S)) \cap T$ such that $x_i^S < x_i^T$. Then $\exists \epsilon : x_i^T - x_i^S \geq \epsilon > 0$ such that coalition S's secure welfare can be increased by implementing $x'_S = (x_{\underline{s}}^S, ..., x_i^S + \epsilon, ..., x_{\bar{s}}^S - \epsilon)$. Clearly, since $x_i^T \geq x_i^S + \epsilon$, x'_S is feasible. Recall that the first order conditions imply $b'_i(x_i^S) > b'_{\bar{s}}(x_{\bar{s}}^S)$. Choose ϵ such that $b'_i(x_i^S + \epsilon) \geq$

$b'_{\bar{s}}(x^S_{\bar{s}} - \epsilon)$. Functions b_i and $b_{\bar{s}}$ strictly concaves imply

$$b_i(x^S_i + \epsilon) - b_i(x^S_i) > b'_i(x^S_i + \epsilon)\epsilon$$

and,

$$b_{\bar{s}}(x^S_{\bar{s}}) - b_{\bar{s}}(x^S_{\bar{s}} - \epsilon) < b'_{\bar{s}}(x^S_{\bar{s}} - \epsilon)\epsilon.$$

Therefore,

$$b_i(x^S_i + \epsilon) + b_{\bar{s}}(x^S_{\bar{s}} - \epsilon) > b_i(x^S_i) + b_{\bar{s}}(x^S_{\bar{s}}) + [b'_i(x^S_i + \epsilon) - b'_{\bar{s}}(x^S_{\bar{s}} - \epsilon)]\epsilon.$$

Which imply,

$$b_i(x^S_i + \epsilon) + b_{\bar{s}}(x^S_{\bar{s}} - \epsilon) > b_i(x^S_i) + b_{\bar{s}}(x^S_{\bar{s}}).$$

The argument is similar for the case of no interior binding constraint, i.e. $K(S) = \{\bar{s}\}$. **End proof of Lemma 1.**

Lemma 2. *For all $\underline{s}, \bar{s} \in N$, $\underline{s} < \bar{s}$, if $S = \{\underline{s}, ..., \bar{s}\}$ and $T = \{\underline{s}, ..., \bar{s} + k\}$ with $0 < k \leq n - \bar{s}$, then $x^S_i \geq x^T_i$, $\forall i \in S \cap T$.*

Proof of Lemma 2

It will be sufficient to prove that this is true for $k = 1$. Consider $S = \{\underline{s}, ..., \bar{s}\}$ and $T = \{\underline{s}, ..., \bar{s} + 1\}$ with $\underline{s}, \bar{s}, \in N$, $\underline{s} \leq \bar{s}$. Distinguish two cases.

- *Case 1 :* $\bar{s}$ binding in Π_T : $l(T) = \bar{s}$. Then $x^T_{\bar{s}+1} = e_{\bar{s}+1}$. The remaining water allocation $(x^T_{\underline{s}}, ..., x^T_{\bar{s}})$ solves Π_S. Thus $x^S_i = x^T_i$, $\forall i \in S \cap T$.

- *Case 2 :* $\bar{s}$ not binding in Π_T : $l(T) < \bar{s}$. We need first to show that, in this case, $K^0(S) = K^0(T)$. Remark that in Π_T, coalition S must provide $x^T_{\bar{s}+1} - e_{\bar{s}+1}$ units of water to agent $\bar{s} + 1$. Fix $x_{\bar{s}+1} = x^T_{\bar{s}+1}$ and consider the remaining allocation $x^T_S = (x^T_{\underline{s}}, ..., x^T_{\bar{s}})$. This allocation solves the following maximization program :

$$\Pi'_T \begin{cases} \max_{x_S} \quad \sum_{i\in S} b_i(x_i) & s/t \\ & \sum_{j\in Pi\cap S} x_j \;\leq\; \sum_{j\in Pi\cap S} e_j \;\forall i \in S\backslash\{\bar{s}\}, \\ & \sum_{j\in P\bar{s}\cap S} x_j \;\leq\; \sum_{j\in P\bar{s}\cap S} e_j - (x^T_{\bar{s}+1} - e_{\bar{s}+1}). \end{cases}$$

Π'_T differs from Π_S only by the last feasibility constraint which is not binding. Therefore, the set of Π'_T's binding constraints preceding the last one is equal to $K^0(S)$. This implies $l(S) = l(T)$. Hence, $x^S_i = x^T_i$, $\forall i \in \{\underline{s}, ..., l(S)\}$. Now, suppose that $\exists i \in \{l(S)+1, ..., \bar{s}\}$ such that $x^S_i < x^T_i$. Then $b'_i(x^S_i) > b'_i(x^T_i)$. The first order conditions of Π_T and Π_S imply, respectively, $b'_i(x^S_i) = b'_j(x^S_j), \forall j \in \{l(S)+1, ..., \bar{s}\}$ and $b'_i(x^T_i) = b'_j(x^T_j), \forall j \in \{l(T)+1, ..., \bar{s}\}$. Hence,

$$\begin{aligned} & b'_j(x^S_j) > b'_j(x^T_j), \forall j \in \{l(S)+1, ..., \bar{s}\} \\ \iff & x^S_j < x^T_j, \forall j \in \{l(S)+1, ..., \bar{s}\} \\ \Rightarrow & \sum_{j=l(S)+1}^{\bar{s}} x^S_j < \sum_{j=l(S)+1}^{\bar{s}} x^T_j. \end{aligned}$$

However, $\bar{s}$ binding in Π_S yields $\sum_{j=l(S)+1}^{\bar{s}} e_j = \sum_{j=l(S)+1}^{\bar{s}} x^S_j$. Using the last inequality, we obtain $\sum_{j=l(S)+1}^{\bar{s}} e_j < \sum_{j=l(S)+1}^{\bar{s}} x^T_j$. Which contradicts the feasibility of x^T. **End proof of Lemma 2.**

We need to prove that, $\forall i \in T \subset S \subset N$, T and S connected,

$$v(S) - v(S\backslash\{i\}) \geq v(T) - v(T\backslash\{i\}). \tag{C.2}$$

We claim that it will be sufficient to prove that, for all $i, s \in N$, $i < s$,

$$v(i, ..., s) - v(i+1, ..., s) \geq v(i, ..., s-1) - v(i+1, ..., s-1). \tag{C.3}$$

We first prove our claim. Define $S = \{\underline{s}, ..., \bar{s}\}$ and $T = \{\underline{t}, ..., \bar{t}\}$, with $\underline{s} \leq \underline{t}$ and $\bar{s} \leq \bar{t}$. By excluding an agent i from an arbitrary coalition A, we decompose the connected set $A = \{\underline{a}, ..., \bar{a}\}$ into two connected subsets $A \cap P^0(i) = \{\underline{a}, ..., i-1\}$ and $A \cap F^0(i) = \{i+1, ..., \bar{a}\}$. Equation $C.2$ may be rewritten as

$$v(S) - [v(\underline{s}, ..., i-1) + v(i+1, ..., \bar{s})] \geq v(T) - [v(\underline{t}, ..., i-1) + v(i+1, ..., \bar{t})]. \tag{C.4}$$

Suppose that (C.3) is true. This implies that, $\forall k: \; 0 < k < s + i + 1$,

$$v(i, ..., s) - v(i+1, ..., s) \geq v(i, ..., s-k) - v(i+1, ..., s-k). \tag{C.5}$$

Rename $i = \underline{s}$, $s = \bar{s}$ and $s - k = \bar{t}$ and rewrite (C.5) as

$$v(\underline{s}, ..., \bar{s}) - v(\underline{s}+1, ..., \bar{s}) \geq v(\underline{s}, ..., \bar{t}) - v(\underline{s}+1, ..., \bar{t})$$

$$\iff v(\underline{s}, ..., \bar{s}) - v(\underline{s}, ..., \bar{t}) \geq v(\underline{s}+1, ..., \bar{s}) - v(\underline{s}+1, ..., \bar{t})$$

$$\Rightarrow v(\underline{s}, ..., \bar{s}) - v(\underline{s}, ..., \bar{t}) \geq v(\underline{s}+k, ..., \bar{s}) - v(\underline{s}+k, ..., \bar{t}), \; \forall k: \; 0 < k < \bar{t} - i.$$

Choose $k = i + 1 - \underline{s}$ and obtain

$$v(\underline{s}, ..., \bar{s}) - v(\underline{s}, ..., \bar{t}) \geq v(i+1, ..., \bar{s}) - v(i+1, ..., \bar{t}). \tag{C.6}$$

Moreover, (C.3) is equivalent to :

$$v(i, ..., s) - v(i, ..., s-1) \geq v(i+1, ..., s) - v(i+1, ..., s-1).$$

This implies that, $\forall k: \; 0 < k < i + s - 1$,

$$v(i, ..., s) - v(i, ..., s-1) \geq v(i+k, ..., s) - v(i+k, ..., s-1)$$

$$\iff v(i, ..., s) - v(i+k, ..., s) \geq v(i, ..., s-1) - v(i+1, ..., s-1).$$

This implies that, $\forall l: \; 0 < l < s - (i+k)$, we have :

$$v(i, ..., s) - v(i, ..., s-l) \geq v(i+k, ..., s) - v(i+k, ..., s-l). \tag{C.7}$$

Rename $i = \underline{s}$, $s = \bar{t}$, $s - l = i - 1$ and $i + k = \underline{t}$ and write (C.7) as

$$v(\underline{s}, ..., \bar{t}) - v(\underline{s}, ..., i-1) \geq v(\underline{t}, ..., \bar{t}) - v(\underline{t}, ..., i-1). \tag{C.8}$$

We now get the elements (conditions (C.6) and (C.8)) to prove (C.2), $\forall T \subset S \subset N$. First show, that (C.2) is true for any S and T' such that $S = \{\underline{s}, ..., \bar{s}\}$ and $T' = \{\underline{s}, ..., \bar{t}\}$ with $\bar{t} \leq \bar{s}$. Since $v(S\backslash\{i\}) = v(\underline{s}, ..., i-1) + v(i+1, ..., \bar{s})$ and, $v(T'\backslash\{i\}) = v(\underline{s}, ..., i-1) + v(i+1, ..., \bar{t})$, then (C.2) leads to

$$v(\underline{s}, ..., \bar{s}) - v(i+1, ..., \bar{s}) \geq v(\underline{s}, ..., \bar{t}) - v(i+1, ..., \bar{t})$$

That is (C.6). We now show that (C.2) holds for any T and T' such that $T = \{\underline{t}, ..., \bar{t}\}$ and $T' = \{\underline{s}, ..., \bar{t}\}$ with $\underline{s} \leq \underline{t}$. In this case, (C.2) leads to

$$v(\underline{s}, ..., \bar{t}) - v(\underline{s}, ..., i-1) \geq v(\underline{t}, ..., \bar{t}) - v(\bar{t}, ..., i-1).$$

That is (C.8). Finally, we proved that

$$v(S) - v(S\backslash\{i\}) \geq v(T') - v(T'\backslash\{i\}),$$

$$v(T') - v(T'\backslash\{i\}) \geq v(T) - v(T\backslash\{i\}).$$

Which implies

$$v(S) - v(S\backslash\{i\}) \geq v(T) - v(T\backslash\{i\}).$$

We now prove (C.3). Denote $A = \{i, ..., s\}$, $B = \{i+1, ..., s\}$, $C = \{i, ..., s-1\}$ and $D = \{i+1, ..., s-1\}$. We have $v(A) = \sum_{j=i}^{s} b_j(x_j^A)$, $v(B) = \sum_{j=i+1}^{s} b_j(x_j^B)$, $v(C) = \sum_{j=i}^{s-1} b_j(x_j^C)$ and $v(D) = \sum_{j=i}^{s-1} b_j(x_j^D)$.

Inequality (C.3) can be rewritten as

$$v(A) - v(B) \geq v(C) - v(D)$$

$$\Longleftrightarrow v(A) - v(C) \geq v(B) - v(D),$$

Consider the last binding constraint $l(S)$ of each coalition S. We first prove that (C.3) holds in the cases $l(B) = s-1$ and $l(A) = s-1$. If $l(B) = s-1$, then allocation x_B^B is such that all water is diverted from the river at $s-1$. Therefore $v(B) - v(D) = b_s(e_s)$. Condition (C.3) is rewritten as

$$v(A) - v(C) \geq b_s(e_s).$$

Replace $b_s(e_s)$ by $v(s)$ and rewritte

$$v(A) \geq v(C) + v(s),$$

which holds since v is subadditive. The proof is similar for the case $l(A) = s - 1$.

Now suppose that $l(B) < s - 1$ and $l(A) < s - 1$. First, proceeding as in the proof of Lemma 2, it is easy to show that $K^0(B) = K^0(D)$ and $K^0(A) = K^0(C)$.

Recall that the secure welfare of any coalition can be obtain by summing the secure welfare of partition $\mathcal{P}(S)$'s sets (see equation (C.1)). $K^0(B) = K^0(D)$ means that sets of partitions $\mathcal{P}(B)$ and $\mathcal{P}(D)$ excluding, respectively, B_s and D_{s-1}, are equal. This gives us a simple formulation of the difference between coalition B's secure welfare and coalition D's secure welfare :

$$v(B) - v(D) = v(B_s) - v(D_{s-1}).$$

Symmetrically, $K^0(A) = K^0(C)$ implies

$$v(A) - v(C) = v(A_s) - v(C_{s-1}).$$

Finally, all we need to show is :

$$v(A_s) - v(C_{s-1}) \geq v(B_s) - v(D_{s-1}). \tag{C.9}$$

Note that, as asserted in the proof of Lemma 1, any constraint binding in Π_A (other than i) should be also binding in Π_B. In other words, $K(A) \cap B \subset K(B)$, which implies $l(A) \leq l(B)$. Condition (C.9) is rewritten as follows :

$$\sum_{j=l(A)+1}^{s} b_j(x_j^A) - \sum_{j=l(A)+1}^{s-1} b_j(x_j^C) \geq \sum_{j=l(B)+1}^{s} b_j(x_j^B) - \sum_{j=l(B)+1}^{s-1} b_j(x_j^D), \tag{C.10}$$

with $l(A) \leq l(B)$.

We show that condition (C.10) holds even if, instead of implementing $x_{A_s}^A$, coalition A_s implements an other feasible allocation $\tilde{x}$ generally (Pareto) dominated.

Define $\tilde{x} = (\tilde{x}_{l(A)+1}, ..., \tilde{x}_s)$ as

$$\tilde{x}_j = \begin{cases} x_j^C & j = l(A)+1, ..., l(B), \\ x_j^C - (x_j^D - x_j^B) & j = l(B)+1, ..., s-1, \\ x_j^B & j = s. \end{cases}$$

It is easy to verify that this allocation is feasible, i.e. it satisfies Π_A's constraints. All feasibility constraints $i \in \{l(A)+1, ..., l(B)\}$ hold since they do in Π_C. For $i \in \{l(B)+1, ..., s-1\}$, we have $\sum_{j \in Pi \cap A_s} \tilde{x}_j = \sum_{j \in Pi \cap A_s} x_j^C - \sum_{j=l(B)+1}^{i} [x_j^D - x_j^B]$. Since $x_j^D \geq x_j^B$, for all $j \in D_{s-1}$ by Lemma 2, the last term is positive. Therefore, since x_C^C is feasible, the feasibility constraints $i = l(B)+1, ..., s-1$ hold. Now, for the last feasibility constraint s, we have :

$$\sum_{j \in A_s} \tilde{x}_j = \sum_{j=l(A)+1}^{s-1} x_j^C + \sum_{j=l(B)+1}^{s} x_j^B - \sum_{j=l(B)+1}^{s-1} x_j^D.$$

Use the (binding) last feasibility constraint of each program Π_C, Π_B and Π_D and obtain :

$$\sum_{j=l(A)+1}^{s} \tilde{x}_j = \sum_{j=l(A)+1}^{s-1} e_j + \sum_{j=l(B)+1}^{s} e_j - \sum_{j=l(B)+1}^{s-1} e_j$$

Which simplifies to $\sum_{j=l(A)+1}^{s} \tilde{x}_j = \sum_{j=l(A)+1}^{s} e_j$, i.e. the last constraint s is binding.

Since allocation $\tilde{x}$ does not, in general, solve Π_{A_s}, the following condition is sufficient for (C.10) to hold :

$$\sum_{j=l(A)+1}^{s} b_j(\tilde{x}_j) - \sum_{j=l(A)+1}^{s-1} b_j(x_j^C) \geq \sum_{j=l(B)+1}^{s} b_j(x_j^B) - \sum_{j=l(B)+1}^{s-1} b_j(x_j^D)$$

Which simplifies to

$$\sum_{j=l(B)+1}^{s-1} b_j(\tilde{x}_j) - \sum_{j=l(B)+1}^{s-1} b_j(x_j^C) \geq \sum_{j=l(B)+1}^{s-1} b_j(x_j^B) - \sum_{j=l(B)+1}^{s-1} b_j(x_j^D).$$

Denote $\epsilon_j = x_j^C - x_j^D \geq 0$. The inequality may be rewritten as a function of x_j^B,

x_j^C and ϵ_j :

$$\sum_{j=l(B)+1}^{s-1} b_j(x_j^B+\epsilon_j) - \sum_{j=l(B)+1}^{s-1} b_j(x_j^D+\epsilon_j) \geq \sum_{j=l(B)+1}^{s-1} b_j(x_j^B) - \sum_{j=l(B)+1}^{s-1} b_j(x_j^D)$$

$$\iff \sum_{j=l(B)+1}^{s-1} [b_j(x_j^B+\epsilon_j) - b_j(x_j^B)] \geq \sum_{j=l(B)+1}^{s-1} [b_j(x_j^D+\epsilon_j) - b_j(x_j^D)].$$

A sufficient condition for the above relation to hold is :

$$b_j(x_j^B+\epsilon_j) - b_j(x_j^B) \geq b_j(x_j^D+\epsilon_j) - b_j(x_j^D) \ \forall j \in D_{s-1}.$$

Since b_j is concave, $\forall S \in \{B, D\}$, $\exists \lambda_j^S \in [b'(x_j^S), b'(x_j^S+\epsilon_j)]$ such that $b_j(x_j^S+\epsilon_j) - b_j(x_j^S) = \lambda_j^S \epsilon_j$. We need to check $\lambda_j^B \geq \lambda_j^D$, $\forall j \in D_{s-1}$. This holds true since b_j is concave and $x_j^B \leq x_j^D$, $\forall j \in D_{s-1}$.

C.2 Proof of Proposition 3

The proof is divided into three steps.

Step 1 : The downstream incremental distribution z^* is a core distribution. The distribution z^* is just the marginal contribution vector corresponding to the ordering $1, ..., n$. That vector is a core distribution because v is convex, as asserted by Proposition 2.

Step 2 : If a core distribution z satisfies the aspiration upper bounds, then $z = z^*$.

Key to the proof is the straightforward observation that $v(Pi) = w(Pi)$ for every $i \in N$. Since this is true for $i = 1$, the core inequalities and the aspiration upper bounds immediately imply that $z_1 = z_1^*$. Next, proceed inductively. Fix $j < n$ and suppose $z_i = z_i^*$ for all $i \leq j$. Since $v(P(j+1)) = w(P(j+1))$, the core constraints and the aspiration upper bounds force $\sum_{i \in P(j+1)} z_i = v(P(j+1))$, hence $z_{j+1} = v(P(j+1)) - \sum_{i \in P(j)} z_i$. By the induction hypothesis, $\sum_{i \in P(j)} z_i = \sum_{i \in P(j)} z_i^* = v(Pj)$. Therefore, $z_{j+1} = v(P(j+1)) - v(Pj) = z_{j+1}^*$, as desired.

Step 3 : z^* satisfies the aspiration upper bounds.

Consider an arbitrary *connected* coalition S and a sub-coalition $T \subset S$. Fix the consumption of members of coalition T to $\bar{x}_T$ (this allocation is assumed to be feasible in S, that is $\sum_{j \in Pi \cap T} \bar{x}_j \leq \sum_{j \in Pi \cap S} e_j$, $\forall i \in T$.). The welfare that coalition S can guarantee to itself given that water allocated to a sub-coalition T is *fixed to* $\bar{x}_T$ will be called the *constraint secure welfare* of S given $\bar{x}_T$. It is formally defined in definition 1.

Definition 1. *For every arbitrary connected coalition S, for every subset $T \subset S$ and every vector $\bar{x}_T$ feasible in S, the constraint secure welfare $v(S; \bar{x}_T)$ is defined by :*

$$v(S;\bar{x}_T) = \max_{x_{S \setminus T}} \sum_{i \in S \setminus T} b_i(x_i) + \sum_{i \in T} b_i(\bar{x}_i) \; s/t \sum_{j \in Pi \cap S \setminus T} x_j + \sum_{j \in Pi \cap T} \bar{x}_j \leq \sum_{j \in Pi \cap S} e_j \; \forall i \in S \setminus T.$$

Clearly, $v(S) \geq v(S; \bar{x}_T)$ for every vector $\bar{x}_T$ feasible in S. Furthermore, $v(S; x_T^S) = v(S)$.

We now need the following lemma.

Lemma 3. *For every $l, k \in N$, $l > k$, for every $T \subset Pk \subset Pl$,[1], for arbitrary vectors $\bar{x}_T$ and $\bar{x}'_T$, such that $\bar{x}'_T \leq \bar{x}_T \leq x_T^{Pl}$, we get*

$$v(Pl; \bar{x}'_T) - v(Pk; \bar{x}'_T) \geq v(Pl; \bar{x}_T) - v(Pk; \bar{x}_T).$$

Proof of Lemma 3 : Denote $\tilde{x}^{Pj}$ and $\tilde{x}'^{Pj}$ (with $|Pj \setminus T|$ components) the water allocations solution to the maximization problem associated to, respectively, $v(Pj; \bar{x}_T)$ and $v(Pj; \bar{x}'_T)$, for $j = l, k$. Call $\epsilon_i = \tilde{x}_i'^{Pk} - \tilde{x}_i^{Pk}$ for every $i \in Pk \setminus T$.

[1] Since, by definition, Pk and Pl are connected, the constrained secure welfare is well-defined for coalition Pk and Pl, for every $l, k \in N$.

The vector $\epsilon_{Pk\backslash T}$ represents the optimal distribution of extra water available to members of $Pk\backslash T$ when coalition T consumes $\bar{x}'_T$ instead of $\bar{x}_T$. Clearly, $\epsilon_i \geq 0$, for every $i \in Pk\backslash T$. An adapted version of Lemma 2 tells us that $\tilde{x}_i^{Pk} \geq \tilde{x}_i^{Pl}$, $\forall i \in Pk\backslash T$. Since b_i is strictly concave $\forall i \in Pk\backslash T$, then

$$b_i(\tilde{x}_i^{Pk} + \epsilon_i) - b_i(\tilde{x}_i^{Pk}) \leq b_i(\tilde{x}_i^{Pl} + \epsilon_i) - b_i(\tilde{x}_i^{Pl}), \ \forall i \in Pk\backslash T.$$

Recall that, $\forall i \in Pk\backslash T$, $\tilde{x}_i'^{Pk} = \epsilon_i + \tilde{x}_i^{Pk}$ and write :

$$b_i(\tilde{x}_i'^{Pk}) - b_i(\tilde{x}_i^{Pk}) \leq b_i(\tilde{x}_i^{Pl} + \epsilon_i) - b_i(\tilde{x}_i^{Pl}), \ \forall i \in Pk\backslash T.$$

This implies,

$$\sum_{i \in Pk\backslash T} [b_i(\tilde{x}_i'^{Pk}) - b_i(\tilde{x}_i^{Pk})] \leq \sum_{i \in Pk\backslash T} [b_i(\tilde{x}_i^{Pl} + \epsilon_i) - b_i(\tilde{x}_i^{Pl})]. \tag{C.11}$$

Add $\sum_{i \in T}[b_i(\bar{x}'_i) - b_i(\bar{x}_i)]$ to both sides of inequality (C.11) and obtain

$$\begin{aligned} v(Pk; \bar{x}'_T) - v(Pk; \bar{x}_T) \leq \ & \textstyle\sum_{i \in Pk\backslash T} b_i(\tilde{x}_i^{Pl} + \epsilon_i) + \sum_{i \in T} b_i(\bar{x}'_i) \\ & -[\textstyle\sum_{i \in Pk\backslash T} b_i(\tilde{x}_i^{Pl}) + \sum_{i \in T} b_i(\bar{x}_i)]. \end{aligned} \tag{C.12}$$

Add $\sum_{i \in Pl\backslash Pk} \tilde{x}_i^{Pl} - \sum_{i \in Pl\backslash Pk} \tilde{x}_i^{Pl}$ to the right-hand side of (C.12) to get

$$\begin{aligned} v(Pk; \bar{x}'_T) - v(Pk; \bar{x}_T) \leq \ & \textstyle\sum_{i \in Pk\backslash T} b_i(\tilde{x}_i^{Pl} + \epsilon_i) + \sum_{i \in T} b_i(\bar{x}'_i) + \sum_{i \in Pl\backslash Pk} b_i(\tilde{x}_i^{Pl}) \\ & -[\textstyle\sum_{i \in Pk\backslash T} b_i(\tilde{x}_i^{Pl}) + \sum_{i \in T} b_i(\bar{x}_i) + \sum_{i \in Pl\backslash Pk} b_i(\tilde{x}_i^{Pl})]. \end{aligned} \tag{C.13}$$

When coalition T consumes $\bar{x}'_T$ instead of $\bar{x}_T$, members of coalition $Pl\backslash T$ gets ϵ_i extra water at each level $i \in T$. By definition, the optimal distribution of this amount of water yields $v(Pl; \bar{x}'_T)$ to coalition Pl. An other way to allocate this extra volume of water would be to increase the consumption of every agent $i \in Pk$ by ϵ_i and to keep constant the consumption of agents between k and l. Since this alternative allocation of water is not optimal, then,

$$\sum_{i \in Pk\backslash T} b_i(\tilde{x}_i^{Pl} + \epsilon_i) + \sum_{i \in T} b_i(\bar{x}'_i) + \sum_{i \in Pl\backslash Pk} b_i(\tilde{x}_i^{Pl}) \leq v(Pl; \bar{x}'_T).$$

Moreover, by definition,

$$v(Pl;\bar{x}_T) = \sum_{i\in Pk\backslash T} b_i(\tilde{x}_i^{Pl}) + \sum_{i\in T} b_i(\bar{x}_i) + \sum_{i\in Pl\backslash Pk} b_i(\tilde{x}_i^{Pl}).$$

Thus, the right-hand side of inequality (C.13) is lower than $v(Pl;\bar{x}'_T) - v(Pl;\bar{x}_T)$. Therefore, we get

$$v(Pk;\bar{x}'_T) - v(Pk;\bar{x}_T) \leq v(Pl;\bar{x}'_T) - v(Pl;\bar{x}_T).$$

Put differently,

$$v(Pl;\bar{x}_T) - v(Pk;\bar{x}_T) \leq v(Pl;\bar{x}'_T) - v(Pk;\bar{x}'_T).$$

End proof of Lemma 3.

Now, consider an arbitrary coalition S (S may be disconnected). Denote by $minS$ and $maxS$, respectly, the first and the last agent in S. Define $PS = PmaxS$ the set of predecessors of coalition S (including members of coalition S). By definition,

$$\sum_{j\in S} z_j^* = \sum_{j\in S}[v(Pj) - v(P^0j)] \tag{C.14}$$

For every $j \in S$, rewrite $v(Pj) = v(Pj; x^{Pj}_{P^0j\backslash S})$. Remark that since lemma 2 implies $x_i^{Pj} \leq x_i^{P^0j}$ for every $i \in P^0j$, for every $j \in S$, then $x^{Pj}_{P^0j\backslash S}$ is feasible in P^0j. Therefore, $v(P^0j; x^{Pj}_{P^0j\backslash S})$ is defined for every $j \in S$. Moreover, $v(P^0j) \geq v(P^0j; x^{Pj}_{P^0j\backslash S})$ for every $j \in S$. Therefore,

$$\sum_{j\in S} z_j^* \leq \sum_{j\in S}[v(Pj; x^{Pj}_{P^0j\backslash S}) - v(P^0j; x^{Pj}_{P^0j\backslash S})] \tag{C.15}$$

Lemma 2 tells us that $x_i^{PS} \leq x_i^{Pj}$ for every $i \in P^0j$, for every $j \in S$. Using Lemma 3, this implies that, for every $j \in S$,

$$v(Pj; x^{PS}_{P^0j\backslash S}) - v(P^0j; x^{PS}_{P^0j\backslash S}) \geq v(Pj; x^{Pj}_{P^0j\backslash S}) - v(P^0j; x^{Pj}_{P^0j\backslash S}). \tag{C.16}$$

Inequalities (C.15) and (C.16) imply,

$$\sum_{j\in S} z_j^* \leq \sum_{j\in S}[v(Pj;x^{PS}_{P^0j\backslash S}) - v(P^0j;x^{PS}_{P^0j\backslash S})]. \tag{C.17}$$

Denote $\tilde{x}^{Pj}$ (with $|Pj\backslash S|$ components) and $\tilde{x}^{P^0j}$ (with $|P^0j\backslash S|$ components) the solution to the maximization program defined by, respectively, $v(Pj;x^{PS}_{Pj\backslash S})$ and $v(P^0j;x^{PS}_{P^0j\backslash S})$. Inequality (C.17) becomes

$$\sum_{j\in S} z_j^* \leq \sum_{j\in S}\{\sum_{i\in Pj\cap S} b_i(\tilde{x}_i^{Pj}) - \sum_{i\in Pj\cap S} b_i(\tilde{x}_i^{P^0j})\}. \tag{C.18}$$

$$\iff \sum_{j\in S} z_j^* \leq \sum_{j\in S}\sum_{i\in P^0j\cap S} [b_i(\tilde{x}_i^{Pj}) - b_i(\tilde{x}_i^{P^0j})] + \sum_{j\in S} b_j(\tilde{x}_j^{Pj}). \tag{C.19}$$

The first term of the right-hand side of (C.19) sums $b_i(\tilde{x}_i^{Pj}) - b_i(\tilde{x}_i^{P^0j})$ for every $i \in P^0j \cap S = \{i \in S : i < j\}$ and for every $j \in S$. That is for every i such that $i \in S$ <u>and</u> $i < j$, for every $j \in S$. It is equivalent to sum $b_i(\tilde{x}_i^{Pj}) - b_i(\tilde{x}_i^{P^0j})$ for every j such that $j \in S$ <u>and</u> $j > i$, for every $i \in S$. That is for every $j \in \{j \in S : j > i\} = F^0i \cap S$, for every $i \in S$. Therefore, (C.19) reads

$$\sum_{j\in S} z_j^* \leq \sum_{i\in S}\sum_{j\in F^0i\cap S} [b_i(\tilde{x}_i^{Pj}) - b_i(\tilde{x}_i^{P^0j})] + \sum_{i\in S} b_i(\tilde{x}_i^{Pi}). \tag{C.20}$$

Now, let us define $(\hat{y}_i)_{i\in S}$ by $\hat{y}_i = \tilde{x}_i^{Pi} + \sum_{l\in F^0i\cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0l})$ for every $i \in S$. We first prove that allocation $(\hat{y}_i)_{i\in S}$ can be consumed by members of coalition S in the absence of $N\backslash S$. Fix $j \in S$. We claim that

$$\sum_{i\in Pj\cap S} \hat{y}_i \leq \sum_{l\in Pj\cap S} e_l. \tag{C.21}$$

That is agents upstream j consume the allocation they control. This condition implies that $(\hat{y}_i)_{i\in S}$ can be consumed by members of coalition S upstream j including j in the absence of $N\backslash S$. It is stronger than the j-feasibility constraint defined in the maximization program associated to $w(S)$: members of coalition S uses only the water they control. They do not consume water coming from outside coalition S.[2] However, members of coalition S should be able to transfer water from one upstream connected sub-coalition to an other. Let us now prove our claim. By definition,

$$\sum_{i \in Pj \cap S} \hat{y}_i = \sum_{i \in Pj \cap S} [\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})]. \tag{C.22}$$

Partition $F^0 i \cap S = (F^0 i \cap S \cap Pj) \cup (F^0 i \cap S \cap F^0 j)$ and rewrite (C.22) as :

$$\begin{aligned} \textstyle\sum_{i \in Pj \cap S} \hat{y}_i &= \textstyle\sum_{i \in Pj \cap S} \tilde{x}_i^{Pi} + \sum_{i \in Pj \cap S} \sum_{l \in F^0 i \cap S \cap Pj} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}] \\ &\quad \textstyle+ \sum_{i \in Pj \cap S} \sum_{l \in F^0 i \cap S \cap F^0 j} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}] \\ &= \textstyle\sum_{i \in Pj \cap S} \tilde{x}_i^{Pi} + \sum_{i \in Pj \cap S} \sum_{l \in F^0 i \cap S \cap Pj} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}] \\ &\quad \textstyle+ \sum_{i \in Pj \cap S} \sum_{l \in F^0 j \cap S} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}]. \end{aligned} \tag{C.23}$$

The second term in the right-hand side of (C.23) sums $\tilde{x}_i^{P^0 l} - \tilde{x}_i^{Pl}$ for every $l \in F^0 i \cap (S \cap Pj) = \{l \in S \cap Pj : l > i\}$, for every $i \in Pj \cap S$. That is for every l such that $l \in Pj \cap S$ and $l > i$, for every $i \in Pj \cap S$. It is equivalent to sum $\tilde{x}_i^{P^0 l} - \tilde{x}_i^{Pl}$ for every i such that $i \in Pj \cap S$ and $i < l$, for every $l \in Pj \cap S$. That is for every $i \in \{i \in Pj \cap S : i < l\} = P^0 l \cap (Pj \cap S)$, for every $l \in Pj \cap S$. Hence, (C.23) becomes

$$\begin{aligned} \textstyle\sum_{i \in Pj \cap S} \hat{y}_i &= \textstyle\sum_{i \in Pj \cap S} \tilde{x}_i^{Pi} + \sum_{l \in Pj \cap S} \sum_{i \in P^0 l \cap S} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}] \\ &\quad \textstyle+ \sum_{i \in Pj \cap S} \sum_{l \in F^0 j \cap S} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}] \\ &= \textstyle\sum_{l \in Pj \cap S} [\sum_{i \in Pl \cap S} \tilde{x}_i^{Pl} - \sum_{i \in P^0 l \cap S} \tilde{x}_i^{P^0 l}] \\ &\quad \textstyle+ \sum_{i \in Pj \cap S} \sum_{l \in F^0 j \cap S} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}]. \end{aligned} \tag{C.24}$$

By definition of $\tilde{x}^{Pl}$ and $\tilde{x}^{P^0 l}$, we have $\sum_{i \in Pl \cap S} \tilde{x}_i^{Pl} + \sum_{i \in Pl \setminus S} x_i^{PS} = \sum_{i \in Pl} e_i$ and $\sum_{i \in P^0 l \cap S} \tilde{x}_i^{P^0 l} + \sum_{i \in P^0 l \setminus S} x_i^{PS} = \sum_{i \in P^0 l} e_i$. Moreover, since $P^0 l \setminus S = Pl \setminus S$ for every $l \in S$, then $\sum_{i \in Pl \setminus S} x_i^{PS} = \sum_{i \in P^0 l \setminus S} x_i^{PS}$. Hence, $\sum_{i \in Pl \cap S} \tilde{x}_i^{Pl} - \sum_{i \in P^0 l \cap S} \tilde{x}_i^{P^0 l} = e_l$. Finally, equation (C.24) simplifies to :

$$\sum_{i \in Pj \cap S} \hat{y}_i = \sum_{l \in Pj \cap S} e_l + \sum_{i \in Pj \cap S} \sum_{l \in F^0 j \cap S} [\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}]. \tag{C.25}$$

[2] This makes sure that the upstream upper bound holds even in the case of no water coming from outside coalition S.

Lemma 2 tells us that $\tilde{x}_i^{P^0l} \geq \tilde{x}_i^{Pl}$ for every $i \in Pl \cap S$, for every $l \in S$. Which holds *a fortiori* for every $i, j, l \in S$ such that $i \leq j < l$. Therefore, equation (C.25) implies :

$$\sum_{i \in Pj \cap S} \hat{y}_i \leq \sum_{l \in Pj \cap S} e_l.$$

That is our claim (C.21) for an arbitrary $j \in S$.

Now, since $(\hat{y}_i)_{i \in S}$ is feasible in S in the absence of $N \backslash S$, then $w(S) \geq \sum_{i \in S} b_i(\hat{y}_i)$. Put differently,

$$w(S) \geq \sum_{i \in S} b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})). \tag{C.26}$$

Combine (C.20) and (C.26) to obtain :

$$\begin{aligned} w(S) - \textstyle\sum_{j \in S} z_j^* \geq \; & \textstyle\sum_{i \in S} \{ b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) - b_i(\tilde{x}_i^{Pi}) \\ & - \textstyle\sum_{j \in F^0 i \cap S} [b_i(\tilde{x}_i^{Pj}) - b_i(\tilde{x}_i^{P^0 j})] \}. \end{aligned} \tag{C.27}$$

All we need to show is that the right-hand side of (C.27) is positive. It will be convenient to rewrite $b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) - b_i(\tilde{x}_i^{Pi})$ as

$$\sum_{j \in F^0 i \cap S} [b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) - b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap P^0 j \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}))]. \tag{C.28}$$

It is easy to check that this equality is true. Indeed, for j equal to the first strict follower of i included in S, then $F^0 i \cap P^0 j \cap S = \emptyset$. Therefore, we get $b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap P^0 j \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) = b_i(\tilde{x}_i^{Pi})$. For j equal to the last strict follower of i included in S, then $Pj = PS$, hence $F^0 i \cap Pj \cap S = F_i^0 \cap S$. Therefore, we obtain $b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) = b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}))$. All other terms in the sum are matched with their opposite so that this equality holds. Substitute (C.28) in (C.27) and obtain

$$\begin{aligned} & w(S) - \textstyle\sum_{j \in S} z_j^* \geq \\ & \textstyle\sum_{i \in S} \sum_{j \in F^0 i \cap S} \{ b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) \\ & - b_i(\tilde{x}_i^{Pi} + \textstyle\sum_{l \in F^0 i \cap P^0 j \cap S} (\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) - [b_i(\tilde{x}_i^{Pj}) - b_i(\tilde{x}_i^{P^0 j})] \}. \end{aligned} \tag{C.29}$$

The right-hand side of $(C.29)$ is positive if, for every $i \in S$, for every $j \in F^0 i \cap S$, we have :

$$b_i(\tilde{x}_i^{Pi} + \textstyle\sum_{l \in F^0 i \cap Pj \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) - b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap P^0 j \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) \geq$$
$$b_i(\tilde{x}_i^{Pj}) - b_i(\tilde{x}_i^{P^0 j}). \tag{C.30}$$

Put differently,

$$b_i(\tilde{x}_i^{P^0 j}) - b_i(\tilde{x}_i^{Pj}) \geq$$
$$b_i(\tilde{x}_i^{Pi} + \textstyle\sum_{l \in F^0 i \cap P^0 j \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) - b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})) \tag{C.31}$$

Write $\tilde{x}_i^{P^0 j} = \tilde{x}_i^{Pj} + (\tilde{x}_i^{P^0 j} - \tilde{x}_i^{Pj})$ and $\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap P^0 j \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}) = \tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}) - (\tilde{x}_i^{Pj} - \tilde{x}_i^{P^0 j}) = \tilde{x}_i^{Pj} + \sum_{l \in F^0 i \cap Pj \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}) + (\tilde{x}_i^{P^0 j} - \tilde{x}_i^{Pj})$. Substitute in (C.31) and read :

$$b_i(\tilde{x}_i^{Pj} + (\tilde{x}_i^{P^0 j} - \tilde{x}_i^{Pj})) - b_i(\tilde{x}_i^{Pj}) \geq$$

$$b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}) + (\tilde{x}_i^{P^0 j} - \tilde{x}_i^{Pj})) - b_i(\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l})). \tag{C.32}$$

Lemma 2 tells us that $\tilde{x}_i^{P^0 j} - \tilde{x}_i^{Pj} \geq 0$ for every $i \in S$, for every $j \in F^0 i \cap S$. Since b_i is strictly concave for every $i \in S$, then (C.32) holds if, for every $i \in S$, for every $j \in F^0 i \cap S$,

$$\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S}(\tilde{x}_i^{Pl} - \tilde{x}_i^{P^0 l}) \geq \tilde{x}_i^{Pj}$$

$$\iff \tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap Pj \cap S} \tilde{x}_i^{Pl} \geq \tilde{x}_i^{Pj} + \sum_{l \in F^0 i \cap Pj \cap S} \tilde{x}_i^{P^0 l}. \tag{C.33}$$

Remark that $\sum_{l \in F^0 i \cap Pj \cap S} \tilde{x}_i^{Pl} = \sum_{l \in F^0 i \cap Pj \cap S} \tilde{x}_i^{P^0 l} + \tilde{x}_i^{Pj}$ and simplifies (C.33) to :

$$\tilde{x}_i^{Pi} + \sum_{l \in F^0 i \cap P^0 j \cap S} \tilde{x}_i^{Pl} \geq \sum_{l \in F^0 i \cap Pj \cap S} \tilde{x}_i^{P^0 l}. \tag{C.34}$$

To prove that (C.34) is true, we consider two cases. First, suppose that $i = maxS$. Then $F^0 maxS \cap S = \emptyset$, therefore (C.34) becomes $\tilde{x}_{maxS}^{PmaxS} \geq 0$. Second, suppose that $i \in S\backslash\{maxS\}$. Define the function $f : S \rightarrow S$ that associates each agent $k \in S$ to its (strict) follower in S. Formally, for every $k \in S\backslash\{maxS\}$, $f(k) = \min_i\{i \in S \cap F^0 k\}$. The first element in $F_i^0 \cap P_j^0 \cap S$ is $f(i)$ and $\tilde{x}_i^{Pi} \geq \tilde{x}_i^{P^0 f(i)}$. For all others elements $l \in F_i^0 \cap P_j^0 \cap S$ $(l \neq i)$, we have $\tilde{x}_i^{Pl} \geq \tilde{x}_i^{P^0 f(l)}$. Therefore, each term on the left-hand side is higher than the follower term on the right-hand side (note that, for the last element of $F_i^0 \cap P_j^0 \cap S$, say k, we have $f(k) = j$). This prove that (C.34) holds true.

C.3 Proof of proposition 4

Suppose that each agent $i \in N$ is endowed with $w_{ij} = 0$, $\forall j \neq i$ and $w_{ii} = e_i$. The unique vector of equilibrium price that decentralizes the efficient water allocation is $p^* = (b_1'(x_1^*), ..., b_n'(x_n^*))$. Agent i's equilibrium pay-off is $b_i(x_i^*) - b'(x_i^*)(x_i^* - e_i)$. For the downstream incremental allocation to be the market equilibrium allocation, the following relationship should hold true for every agent i in N :

$$b_i(x_i^*) - b'(x_i^*)(x_i^* - e_i) = z_i^*. \tag{C.35}$$

Consider agent 1. Since $z_i^* = v(1)$, that means $b_1(e_1) - b_1(x_1^*) = b'(x_1^*)(x_1^* - e_1)$. Since b_1 is strictly concave, then this holds true only if $x_1^* = e_1$. In this case, $v(1,2) = v(1) + v(2)$, therefore $z_2^* = v(2)$. For $n = 2$, equation (C.35) becomes $b_2(e_2) - b_2(x_2^*) = b'(x_2^*)(x_2^* - e_2)$. Which again implies $x_2^* = e_2$. And so forth for $i = 3, ..., n$.

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