Université de Montréal

Essays in Macro Finance and Monetary Economics

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à mes parents, Simon-Pierre et Laurentine

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Que Dieu vous récompense tous au centuple !

Résumé

Les questions abordées dans les deux premiers articles de ma thèse cherchent à comprendre les facteurs économiques qui affectent la structure à terme des taux d'intérêt et la prime de risque. Je construis des modèles non linéaires d'équilibre général en y intégrant des obligations de différentes échéances. Spécifiquement, le premier article a pour objectif de comprendre la relation entre les facteurs macroéconomiques et le niveau de prime de risque dans un cadre Néo-keynésien d'équilibre général avec incertitude. L'incertitude dans le modèle provient de trois sources : les chocs de productivité, les chocs monétaires et les chocs de préférences. Le modèle comporte deux types de rigidités réelles à savoir la formation des habitudes dans les préférences et les coûts d'ajustement du stock de capital. Le modèle est résolu par la méthode des perturbations à l'ordre deux et calibré à l'économie américaine. Puisque la prime de risque est par nature une compensation pour le risque, l'approximation d'ordre deux implique que la prime de risque est une combinaison linéaire des volatilités des trois chocs. Les résultats montrent qu'avec les paramètres calibrés, les chocs réels (productivité et préférences) jouent un rôle plus important dans la détermination du niveau de la prime de risque relativement aux chocs monétaires. Je montre que contrairement aux travaux précédents (dans lesquels le capital de production est fixe), l'effet du paramètre de la formation des habitudes sur la prime de risque dépend du degré des coûts d'ajustement du capital. Lorsque les coûts d'ajustement du capital sont élevés au point que le stock de capital est fixe à l'équilibre, une augmentation du paramètre de formation des habitudes entraine une augmentation de la prime de risque. Par contre, lorsque les agents peuvent librement ajuster le stock de capital sans coûts, l'effet du paramètre de la formation des habitudes sur la prime de risque est négligeable. Ce résultat s'explique par le fait que lorsque le stock de capital peut être ajusté sans coûts, cela ouvre un canal additionnel de lissage de consommation pour les agents. Par conséquent, l'effet de la formation des habitudes sur la prime de risque est amoindri. En outre, les résultats montrent que la façon dont la banque centrale conduit sa politique monétaire a un effet sur la prime de risque. Plus la banque centrale est agressive vis-à-vis de l'inflation, plus la prime de risque diminue

et vice versa. Cela est due au fait que lorsque la banque centrale combat l'inflation cela entraine une baisse de la variance de l'inflation. Par suite, la prime de risque due au risque d'inflation diminue.

Dans le deuxième article, je fais une extension du premier article en utilisant des préférences récursives de type Epstein – Zin et en permettant aux volatilités conditionnelles des chocs de varier avec le temps. L'emploi de ce cadre est motivé par deux raisons. D'abord des études récentes (Doh, 2010, Rudebusch and Swanson, 2012) ont montré que ces préférences sont appropriées pour l'analyse du prix des actifs dans les modèles d'équilibre général. Ensuite, l'hétéroscedasticité est une caractéristique courante des données économiques et financières. Cela implique que contrairement au premier article, l'incertitude varie dans le temps. Le cadre dans cet article est donc plus général et plus réaliste que celui du premier article. L'objectif principal de cet article est d'examiner l'impact des chocs de volatilités conditionnelles sur le niveau et la dynamique des taux d'intérêt et de la prime de risque. Puisque la prime de risque est constante a l'approximation d'ordre deux, le modèle est résolu par la méthode des perturbations avec une approximation d'ordre trois. Ainsi on obtient une prime de risque qui varie dans le temps. L'avantage d'introduire des chocs de volatilités conditionnelles est que cela induit des variables d'état supplémentaires qui apportent une contribution additionnelle à la dynamique de la prime de risque. Je montre que l'approximation d'ordre trois implique que les primes de risque ont une représentation de type ARCH-M (Autoregressive Conditional Heteroscedasticty in Mean) comme celui introduit par Engle, Lilien et Robins (1987). La différence est que dans ce modèle les paramètres sont structurels et les volatilités sont des volatilités conditionnelles de chocs économiques et non celles des variables elles-mêmes. J'estime les paramètres du modèle par la méthode des moments simulés (SMM) en utilisant des données de l'économie américaine. Les résultats de l'estimation montrent qu'il y a une évidence de volatilité stochastique dans les trois chocs. De plus, la contribution des volatilités conditionnelles des chocs au niveau et à la dynamique de la prime de risque est significative. En particulier, les effets des volatilités conditionnelles des chocs de productivité et de préférences sont significatifs. La volatilité conditionnelle du choc de productivité contribue positivement aux moyennes et aux écart-types des primes de risque. Ces contributions varient avec la maturité des bonds. La volatilité conditionnelle du choc de préférences quant à elle contribue négativement aux moyennes et positivement aux variances des primes de risque. Quant au choc de volatilité de la politique monétaire, son impact sur les primes de risque est négligeable.

Le troisième article (coécrit avec Eric Schaling, Alain Kabundi, révisé et resoumis au journal of Economic Modelling) traite de l'hétérogénéité dans la formation des attentes d'inflation de divers groupes économiques et de leur impact sur la politique

monétaire en Afrique du sud. La question principale est d'examiner si différents groupes d'agents économiques forment leurs attentes d'inflation de la même façon et s'ils perçoivent de la même façon la politique monétaire de la banque centrale (South African Reserve Bank). Ainsi on spécifie un modèle de prédiction d'inflation qui nous permet de tester l'arrimage des attentes d'inflation à la bande d'inflation cible (3% - 6%) de la banque centrale. Les données utilisées sont des données d'enquête réalisée par la banque centrale auprès de trois groupes d'agents : les analystes financiers, les firmes et les syndicats. On exploite donc la structure de panel des données pour tester l'hétérogénéité dans les attentes d'inflation et déduire leur perception de la politique monétaire. Les résultats montrent qu'il y a évidence d'hétérogénéité dans la manière dont les différents groupes forment leurs attentes. Les attentes des analystes financiers sont arrimées à la bande d'inflation cible alors que celles des firmes et des syndicats ne sont pas arrimées. En effet, les firmes et les syndicats accordent un poids significatif à l'inflation retardée d'une période et leurs prédictions varient avec l'inflation réalisée (retardée). Ce qui dénote un manque de crédibilité parfaite de la banque centrale au vu de ces agents.
Mots-clés : Models d'équilibre général, Structure à terme, Prime de risque, ARCH-M, Attentes d'inflation.

Abstract

This thesis consists of three essays in the areas of macro finance and monetary economics. The first two essays deal with the analysis of the term structure of interest rates in dynamic and stochastic general equilibrium (DSGE) models. The third essay explores inflation expectations formation across different economic groups in South Africa.

Interest rates are one channel through which monetary policy affects the real economy. Typically, central banks implement monetary policy by influencing short term interest rates. Theoretically, the interest rate on a long-term bond is the average of expected future short term interest rates over the maturity period, plus a risk premium demanded by the holder of the bond to compensate for the risk involved in holding a longer maturity bond. Therefore, any changes in the target rate of the central bank and the risk premium affect long – term interest rates, such as mortgage rates and interest rates on certain durable goods. It is then important for the central bank to understand the economic factors that affect both components of long - term interest namely the market expectations about the short - term rates and the risk premium. For example, recently in the U.S. economy, between June 2004 and June 2006, the ineffectiveness of monetary policy to affect long - term interest rates has been attributed to a decline in risk premium over this period, which has offset the effect of the increase in the target rate of the Federal Reserve (Fed). In the implementation of its monetary policy, the central bank can more or less control agents' expectations through transparent communication. However, the risk premium is endogenous and unobservable and therefore can not be fully controlled by the central bank. On the other hand, achieving the goal of prices stability in an inflation targeting framework depends on the credibility of the central bank.

In the first two essays I explore the economic factors of the term structure of interest rates and risk premiums. I build a non-linear dynamic stochastic general equilibrium (DSGE) models whereby I incorporate a range of bonds with different maturities. Specifically, the goal of the first essay is to understand the relationship between macroeconomic factors and the level of risk premium in a New Keynesian

general equilibrium framework. Uncertainty in the model comes from three sources : productivity, monetary policy and, preferences shocks. The model has two types of real rigidities namely habit formation in preferences and adjustment costs in capital stock. The model is solved by perturbation method up to second order and calibrated to the U.S. economy. Since the risk premium is by nature a compensation for risk, the second - order approximation implies that the risk premium is a linear combination of the volatility of the three shocks. Results show that at the calibrated parameters, real shocks (productivity and preferences) play a more important role in determining the level of the risk premium relative to monetary shocks. I show that, contrary to previous work (where production capital is fixed), the effect of habit formation on the risk premium depends on the degree of capital adjustment cost. When capital adjustment costs are so high that the capital stock is fixed in equilibrium, an increase in the parameter of habit formation leads to an increase in the risk premium. However, when agents can freely adjust the capital stock without cost, the effect of the habit formation parameter on the risk premium is negligible. This result is explained by the fact that when the capital stock can be adjusted without cost, it opens an additional channel to the agents for consumption smoothing. Therefore, the effect of habit formation on the risk premium is reduced. In addition, the results show that the way the central bank conducts its monetary policy has an effect on the risk premium. The more aggressive the central bank vis-à-vis inflation, the lower the risk premium and vice versa. This is due to the fact that when the central bank fights against inflation it leads to a decrease in the variance of inflation. As a result, the risk premium due to inflation risk decreases.

In the second essay, I extend the analysis of the first essay by using recursive preferences (as those proposed by Epstein - Zin) and by allowing the conditional volatility of the shocks to be time - varying. The use of this framework is motivated by two reasons. First, recent studies (Doh, 2010, Rudebusch and Swanson, 2012) showed that these preferences are appropriate for the analysis of asset prices in general equilibrium models. Second, heteroscedasticity is a prominent feature of economic and financial data. This implies that, contrary to the first essay, the uncertainty here is time - varying. Thus, the framework in this essay is more general and realistic than in the first essay. The main objective of this paper is to examine the impact of uncertainty due to conditional volatility of the shocks on the level and the dynamics of interest rates and risk premiums. Since the risk premium is constant at second order approximation, the model is solved by the perturbation method with an approximation of order three in order to get a time - varying risk premium. The advantage of introducing shocks conditional volatilities is that , it induces additional state variables that provide an additional contribution to the dynamics of the risk premium. I show that the risk premiums implied by the third – order approximate solution have an ARCH-M (Autoregressive Conditional Heteroscedasticty in Mean) type representation as that introduced by Engle, Lilien and Robins (1987). The difference is that in this model the parameters are structural and the volatilities are conditional volatility of economic shocks and not those of the variables themselves. I estimate the model parameters by Simulated Method of Moments (SMM) using U.S. data. The estimation results show that there is evidence of stochastic volatility in the three shocks. Moreover, the contribution of conditional shocks volatility to the level and the dynamics of the risk premium is significant. In particular, the effects of the conditional volatility of productivity and preferences shocks are important. The conditional volatility of the productivity shock contributes positively to the means and standard deviations of risk premiums. These contributions vary with the maturity of the bonds. Conditional volatility of the preferences shock contributes negatively to the averages and positively to the variances of risk premiums. As for the impact of volatility of monetary policy shock, its impact on the risk premium is negligible.

The third article (coauthored with Eric Schaling and Alain Kabundi, revised and resubmitted to the journal of Economic Modelling) deals with heterogeneity in inflation expectations of different economic agents and its impact on monetary policy in South Africa. The main question is to examine whether different groups of economic agents form their inflation expectations in the same way and if they perceive the central bank (South African Reserve Bank) monetary policy in the say way. We specify an inflation expectation model that allows us to directly test whether inflation expectations are anchored or not to the inflation target band (3% - 6%). The data used are inflation expectations data from surveys conducted by the central bank. There are three groups of agents : financial analysts, businesses and trade unions. We therefore exploits the panel structure of the data to test the heterogeneity in inflation expectations and derive their perceived inflation targets. Results show that there is evidence of heterogeneity in the way the three groups form their expectations. The expectations of financial analysts are well anchored to the central bank target band while those of businesses and trade unions are not. In fact, businesses and trade unions put a higher weight on lagged realized inflation in their expectations. This Indicates a lack of full credibility of the central bank.

Keywords : DSGE, Term structure, Inflation expectations, ARCH-M, Risk premium.

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Introduction Générale

Cette thèse est composée de trois chapitres dans les domaines de la macro finance et de la macroéconomie monétaire. Les deux premiers chapitres traitent de l'analyse de la structure à terme des taux d'intérêt dans les modèles dynamiques et stochastiques d'équilibre général. Quant au troisième chapitre, il traite de la modélisation des attentes d'inflation de différents agents dans l'économie Sud-Africaine.

L'un des canaux traditionnels de transmission de la politique monétaire sur l'activité réelle est celui des taux d'intérêt. En effet, typiquement la banque centrale met en œuvre la politique monétaire en influant sur les taux d'intérêt de court terme. Elle relève et abaisse le taux cible du financement à un jour, qui est le taux auquel les grandes institutions financières se prêtent des fonds au jour le jour. Selon la théorie, le taux d'intérêt sur une obligation à long terme est la moyenne des attentes des taux d'intérêt futurs à court terme entre aujourd'hui et la date d'échéance de l'obligation, plus une prime de risque réclamée par le détenteur de l'obligation en guise de compensation pour le risque qu'il court en détenant une obligation de plus longue maturité. Par conséquent, toutes variations des taux cibles de la Banque centrale et de la prime de risque se répercutent sur les taux d'intérêt de plus long terme, tels que les taux d'intérêt sur les prêts hypothécaires et les taux d'intérêt sur certains biens durables. Pour une politique monétaire efficace la banque centrale se doit donc de comprendre les facteurs économiques qui affectent les deux composantes des taux d'intérêt de longs termes à savoir les attentes des agents sur les taux de courts termes et la prime de risque. Par exemple, récemment dans l'économie américaine, entre juin 2004 et juin 2006, l'inefficacité de la politique monétaire à affecter les taux d'intérêt de long terme a été attribuée à une baisse de la prime de risque sur cette période qui a contrebalancé l'effet de la hausse du taux cible de la banque centrale américaine (Fed). Dans la mise en œuvre de sa politique monétaire, la banque centrale peut plus ou moins contrôler les attentes des agents par une communication transparente. Cependant la prime de risque est endogène et inobservable et ne peut donc être entièrement contrôler par la banque centrale. En outre, l'atteinte de l'objectif traditionnel de control d'inflation des banques centrales dépend des attentes des agents économiques sur l'inflation future.

Les questions abordées dans les deux premiers articles cherchent à comprendre les facteurs économiques qui affectent la structure à terme des taux d'intérêt et la prime de risque. Je construis des modèles non linéaires d'équilibre général en y intégrant des obligations de différentes échéances. Spécifiquement, le premier article a pour objectif de comprendre la relation entre les facteurs macroéconomiques et le niveau de prime de risque dans un cadre Néo-keynésien d'équilibre général avec incertitude. L'incertitude dans le modèle provient de trois sources : les chocs de productivité, les chocs monétaires et les chocs de préférences. Le modèle comporte deux types de rigidités réelles à savoir la formation des habitudes dans les préférences et les couts d'ajustement du stock de capital. Le modèle est résolu par la méthode des perturbations à l'ordre deux et calibré à l'économie américaine. Puisque la prime de risque est par nature une compensation pour le risque, l'approximation d'ordre deux implique que la prime de risque est une combinaison linéaire des volatilités des trois chocs. Les résultats montrent qu'avec les paramètres calibrés, les chocs réels (productivité et préférences) jouent un rôle plus important dans la détermination du niveau de la prime de risque relativement aux chocs monétaires. Je montre que contrairement aux travaux précédents (dans lesquels le capital de production est fixe), l'effet du paramètre de la formation des habitudes sur la prime de risque dépend du degré du cout d'ajustement du capital. Lorsque les couts d'ajustement du capital sont élevés au point que le stock de capital est fixe à l'équilibre, une augmentation du paramètre de formation des habitudes entraine une augmentation de la prime de risque. Par contre, lorsque les agents peuvent librement ajuster le stock de capital sans cout, l'effet du paramètre de la formation des habitudes sur la prime risque est négligeable. Ce résultat s'explique par le fait que lorsque le stock de capital peut être ajusté sans cout, cela ouvre un canal additionnel de lissage de consommation pour les agents. Par conséquent, l'effet de la formation des habitudes sur la prime de risque est amoindri. En outre, les résultats montrent que la façon dont la banque conduit sa politique monétaire a un effet sur la prime de risque. Plus la banque centrale est agressive vis-à-vis de l'inflation, plus la prime de risque diminue et vice versa. Cela est due au fait que lorsque la banque centrale combat l'inflation cela entraine une baisse de la variance de l'inflation. Par suite, la prime de risque due au risque d'inflation diminue.

Dans le deuxième article, je fais une extension du premier article en utilisant des préférences récursives de type Epstein – Zin et en permettant aux volatilités conditionnelles des chocs de varier avec le temps. L'emploi de cadre est motivé par deux raisons. D'abord des études récentes (Doh, 2010, Rudebusch and Swanson, 2012) ont montré que ces préférences sont appropriées pour l'analyse du prix des actifs dans les modèles d'équilibre général. Ensuite, l'hétéroscedasticité est une caractéristique courante des données économiques et financières. Cela implique que contrairement au premier article, l'incertitude varie dans le temps. Le cadre dans cet article est donc plus général et plus réaliste que celui du premier article. L'objectif principal de cet article est d'examiner l'impact des chocs de volatilités conditionnelles sur le niveau et la dynamique des taux d'intérêt et de la prime de risque. Puisque la prime de risque est constante a l'approximation d'ordre deux, le modèle est résolu par la méthode des perturbations avec une approximation d'ordre trois. Ainsi on obtient une prime de risque qui varie dans le temps. L'avantage d'introduire des chocs de volatilités conditionnelles cela induit des variables d'état supplémentaires qui apportent une contribution additionnelle à la dynamique de la prime de risque. Je montre que l'approximation d'ordre trois implique que les primes de risque ont une représentation de type ARCH-M (Autoregressive Conditional Heteroscedasticty in Mean) comme celui introduit par Engle, Lilien et Robins (1987). La différence est que dans ce modèle les paramètres sont structurels et les volatilités sont des volatilités conditionnelles de chocs économiques et non celles des variables elles-mêmes. J'estime les paramètres du modèle par la méthode des moments simulés (SMM) en utilisant des données de l'économie américaine. Les résultats de l'estimation montrent qu'il y a une évidence de volatilité stochastique dans les trois chocs. De plus, la contribution des volatilités conditionnelles des chocs au niveau et à la dynamique de la prime de risque est significative. En particulier, les effets des volatilités conditionnelles des chocs de productivité et de préférences sont significatifs. La volatilité conditionnelle du choc de productivité contribue positivement aux moyennes et aux écart-types des primes de primes. Ces contributions varient avec la maturité des bonds. La volatilité conditionnelle du choc de préférences quant à elle contribue négativement aux moyennes et positivement aux variances des primes de risque. Quant au choc de volatilité de la politique monétaire, son impact est sur les primes de risque est négligeable.

Le troisième article (coécrit avec Eric Schaling, Alain Kabundi, révisé et resoumis au journal of Economic Modelling) traite de l'hétérogénéité de différents agents économiques dans les prédictions d'inflation et de leur impact sur la politique monétaire en Afrique du sud. La question principale est d'examiner si différents groupes d'agents économiques forment leurs attentes d'inflation de la même façon et s'ils perçoivent de la même façon la politique monétaire de la banque centrale (South African Reserve Bank). Ainsi on spécifie un modèle de prédiction d'inflation qui nous permet de tester l'arrimage des attentes d'inflation à la bande d'inflation cible (3% - 6%) de la banque centrale. Les données utilisées sont des données d'enquête réalisée par la banque centrale auprès de trois groupes d'agents : les analystes financiers, les firmes et les syndicats. On exploite donc la structure de panel des données pour tester l'hétérogénéité dans les attentes d'inflation et déduire leur perception de la politique monétaire. Les résultats montrent qu'il y a évidence d'hétérogénéité dans la manière donc les différents groupes forment leurs attentes. Les attentes des analystes financiers sont arrimées à la bande d'inflation cible alors que celles des firmes et des syndicats ne sont pas arrimées. En effet, les firmes et les syndicats accordent un poids significatif à l'inflation retardée d'une période et leurs prédictions varient avec l'inflation réalisée (retardée). Ce qui dénote un manque de crédibilité parfaite de ces agents vis-à-vis de la banque centrale. Notons que la banque centrale sud-africaine utilise la politique de ciblage d'inflation (inflation targeting) depuis 2000. Ces résultats suggèrent donc qu'un accent soit mis sur la communication envers les firmes et les syndicats puisqu'ils sont les principaux acteurs de l'économie qui influencent la variation des prix et des salaires.

Chapitre 1

A General Equilibrium Analysis of the Term Structure of Interest Rates.

1.1 Introduction

The goal of this work is to investigate the determinants of the term structure of interest rates in a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with habit formation preferences and adjustment cost in capital stock. The model features three shocks : productivity shock, preferences shock and monetary policy shock. We ask how changes in macroeconomic structural parameters such as preferences, technology or monetary policy parameters affect the term structure of interest rates. The contribution of the three shocks to the size of risk premia is also studied.

New Keynesian models are known to replicate many empirical business cycle facts[1] and are increasingly used in many central banks for policy analysis. It is then important to understand how interest rates behave in this framework because changes in central bank instrument rate are intended to pass-through the term structure of interest rates and affect the real economy. The relationship between interest rates that only differ in maturities is an important area of research because economists believe that important economic facts can be inferred from this relationship. In fact empirical works have found the yield curve to have economic growth prediction power over a long period of time (see for example, Harvey, 1991). Second, the term structure of interest rates contains important implications for market expectations

[1] See Smets and Wouters (2003) ; Christiano, Eichenbaum and Evans (2005).

about monetary policy and inflation forecast.

Long-term interest rates can be explained as market expectations about future short-term interest rates (the traditional expectation hypothesis theory) and risk premia. As for the market expectations about future short-term rates, it means that current long-term interest rates reflect investors anticipations about the future monetary policy stances because short-term rates are controlled by monetary policy authorities. Thus, long-term interest rates reflect future expectations of inflation and output in the New Keynesian environment. The risk premium component compensates investors for the risk born by holding a long-term debt instead of rolling over short-term instruments. In this model, the risk arises for two reasons. First, an investor fears future capital losses because there is uncertainty about the bond future prices. Even with risk-free bonds, a capital loss can happen if the holder wants to resell the bond before maturity time to (for example) offset a bad income shock. If it happens that the resell price is very low, she will suffer a consumption fall. Second, inflation can erode the value of the bond even at maturity time because the bonds are nominal. Risk premia are then as important as the expectations part for the central bank because they affect the long-term interest rates as well. Unfortunately risk premia are unobserved and can have undesirable impacts on monetary policy. For example, a tightening monetary policy effect can be undermined by a decline in the risk premium component even if the market correctly anticipates the future monetary policy actions as it recently happened in 2004 in the U.S. economy[2]. Kurmann and Otrok (2011) find in VAR framework a weak long-term interest rates response to a news productivity shock because the responses of the term premium and the expectations part offset each other. It is then important- at least for central bankers- to understand the economic determinants of risk premia.

It is challenging to study the term structure of interest rates in a DSGE model. Especially, risk premia are difficult to compute because DSGE models are non-linear systems and an analytical solution is unavailable for the general case. Numerical methods such as value function iteration (VFI) or policy function iteration (PFI) are computationally infeasible because of the large number of state variables. Moreover, previous works have found standard macroeconomic Real Business Cycle (RBC) models to mismatch simultaneously business cycle variables and asset prices[3]. In exchange economy frameworks, some of these puzzles have been solved by using

[2] See Cochrane and Backus (2007), Rudebusch et al (2007) for this issue called the "Greenspan Conundrum" in Finance literature

[3] Donaldson, Thore and Merha (1990) found that a RBC model with full depreciation of capital cannot replicate bond risk premium consistent with the data (bond premium puzzle). See also Den Haan (1995)

either habit formation preferences (see Campbell and Cochrane, 1999, Wachter, 2005, Piazzesi and Schneider, 2007) or recursive preferences (Gallmeyer *et al,* 2008). This is because with these preferences, risk aversion becomes countercyclical (instead of being constant) and resources can only be allocated intertemporally through financial assets. Thus a risk averse investor will require a larger compensation to hold a long-term bond instead of rolling over short-term bonds. In production economy frameworks where consumption, output, and investment are endogenous, there are other channels available for consumption smoothing than the financial assets. The agent could either increase his labour or uses investment every time to offset unexpected bad income shocks given that the cost of adjusting these variables are low. Thus, the increasing effect of habit formation preferences- on bond risk premia size- will tend to be weakened in production economies. There is also evidence that the bond premium puzzle remains unsolved in New Keynesian models even with habit formation preferences and real rigidities. Rudebusch et al (2008) find that the volatility of risk premia is insignificant in a New Keynesian model with real rigidities such as capital adjustment cost and adjustment cost in labour market.

Therefore, we focus in this work on the structural determinants of the size of bond risk premia. In early studies of the term structure of interest rates in production economies, higher habit strength parameter increases the size of risk premia. However, the capital input factor in the production function is fixed in these papers (Rudebusch *et al,* 2008, Ravenna and Seppala, 2006). We compare the habit formation preferences effect on the size of bond risk premia when the capital stock is fixed and when the capital stock can be adjusted costlessly. We also ask to what extent each shock contributes to the size of risk premia and if the agent prices the risk involved in these shocks in the same way. That is, we decompose the contribution of each shock to two multiplicative terms : first, the size of the volatility of the shock that captures the quantity of risk it brings with. Second, a constant function of structural parameters that can be interpreted as price of the associated risk. Thus, macroeconomic factors of risk premium operate through the volatilities of exogenous shocks (quantity of risk) and these scaling coefficients (price of risk). It is well known that increasing the size of the volatilities of the shocks will magnify the size of risk premia. The ability of a DSGE model to generate a sizeable risk premium will depends on the shock volatilities. Rudebusch and Swanson (2008) find in a calibrated DSGE model a small and stable term premium whereas Hordahl *et al* (2007), Ravenna and Seppala (2006) find a sizeable and variable term premium. Rudebusch and Swanson (2008) attribute the result in Hordahl *et al* (2007), Ravenna and Seppala (2006) to large and persistent shocks. It is interesting to address the prices of risk in DSGE models. First, they represent the relative importance of shocks when the size of volatility is controlled. Second, to my knowledge we do not understand yet how the prices of risk involved in the shocks are related to structural parameters. The impact of monetary policy actions on the size of risk premia is also studied in this paper.

As an analytical solution is unavailable, the model is solved by perturbation method and estimate the shocks by Simulated Method of Moments (SMM). Lee and Ingram (1991), and Duffie and Singleton (1993) show that SMM approach delivers consistent parameter estimates. In addition, Ruge - Murcia (2007) finds that it is robust to misspecification and computationally more efficient compare to alternative methods such as Generalized Method of Moments (GMM) and Maximum Likelihood (ML).

The results indicate that second-order approximate solution delivers a positive risk premia leading to an upward sloping average term structure. Results also show that : 1) increases in the inflation parameter of the Taylor rule (a more aggressive monetary policy) lead to decreases in risk premia. Because *leaning against the wind* decreases inflation volatility and then leads to a decrease in inflation risk premium ; 2) preferences shock and technology shock are more important than monetary policy shock in terms of contribution to the level of risk premium. But this is only because the calibrated monetary policy shock is very low compared to the two other shocks volatility. In fact, the contribution of each shock to the size of risk premium is a result of two effects : the size of the volatility of the shock and the price per unit of risk involved in each shock. The preferences shock volatility in the benchmark model is ten time larger than the monetary policy volatility that makes its combined effect larger than that of the two other shocks ; 3) The price of risk associated with the monetary policy shock is larger than the other shock prices and is increasing with the maturity. Preferences shock associated risk price is the least important ; 4) habit strength parameter has less impact on the risk premia when the capital stock is allowed to vary over time but the impact becomes important when the adjustment in capital is high enough to induce a fixed capital stock.

The rest of the paper is organized as follow. Section 1.2 presents the model, Section 1.3 is devoted to derive interest rates and risk premia from the equilibrium conditions as functions of macroeconomic factors. In Section 1.4 we present the solution method and calibrate the model in section 1.5. Finally, Section 1.6 presents the results and discusses some sensitivity analyses.

1.2 The Model

The model features a standard New Keynesian economy wherein a representative consumer derives utility from a composite consumption good and leisure. The composite good is produced by a representative firm with a continuum of intermediate inputs goods. Consumers can save resources by using nominal bonds or capital. There is a central banker who adjusts the nominal short-term interest rate according to a Taylor-type rule

1.2.1 Households

The representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t \left(\frac{(c_t - bc_{t-1})^{1-\gamma}}{1-\gamma} - \psi \frac{n_t^{1+\phi}}{1+\phi} \right), \tag{1.1}$$

where E_0 is the mathematical expectation given the time 0 information set, $\beta \in (0,1)$ is the subjective discount factor, $b \in [0,1)$ is habit strength parameter, γ and ψ are constant preference parameters, ϕ is the inverse of Frisch elasticity of labor supply, A_t is a preference shock, c_t is a composite index of a continuum of intermediate goods, c_t^i, $i \in [0\ 1]$, n_t is hours worked. We assume internal habit in the composite consumption index c_t defined by :

$$c_t = \left[\int_0^1 (c_t^i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

The parameter θ is the elasticity of substitution between the individual goods. As $\theta \to \infty$, the intermediate goods become closer substitutes and the weaker the firm's power on these goods.

Resources can be intertemporally transferred through assets including cash balances, capital and private nominal bonds with maturities $\ell = 1, \ldots, L$. The consumer budget constraint is

$$\int_0^1 \frac{p_t^i c_t^i}{P_t} di + x_t + \sum_{\ell=1}^{L} \frac{Q_t^\ell B_t^\ell}{P_t} = \frac{W_t n_t}{P_t} + \frac{R_t k_t}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^{\ell-1} B_{t-1}^\ell}{P_t} + \frac{S_t}{P_t} - \Gamma \left(\frac{x_t}{k_t} \right) k_t, \tag{1.2}$$

where k_t is capital, x_t is investment, $\delta \in (0,1)$ is the capital depreciation rate, Q_t^ℓ and B_t^ℓ are, respectively, the nominal price and holding of bond with maturity ℓ , W_t is the nominal wage, R_t is the nominal rental rate per unit of capital, p_t^i is the price of the intermediate good i, and P_t is the aggregate price level. Note that an ℓ-period bond at time $t-1$ becomes an $(\ell-1)$-period bond at time t. Capital accumulation is subject to adjustment cost that is a function of the investment-capital stock ratio x_t/k_t. The law of motion of the capital accumulation is

$$k_{t+1} = (1-\delta)k_t + \Gamma\left(\frac{x_t}{k_t}\right)k_t, \tag{1.3}$$

where $\Gamma(\cdot)$ is a strictly convex function of x_t/k_t. For simplicity, we assume a quadratic function for Γ with no adjustment cost in the steady state. That is,

$$\Gamma\left(\frac{x_t}{k_t}\right) = \frac{\varphi}{2}\left(\frac{x_t}{k_t} - \delta\right)^2$$

In a first stage the consumer shops for intermediate goods for the composite good production. Given a level of the composite good, the consumer chooses the inputs c_t^i, $i \in [0\ 1]$ that minimize the total cost $\int_0^1 p_t^i c_t^i di$. This implies that demand for an intermediate good i is given by :

$$c_t^i = \left[\frac{p_t^i}{P_t}\right]^{-\theta} c_t,$$

and the aggregate price level P_t is given by :

$$P_t = \left[\int_0^1 (p_t^i)^{1-\theta} di\right]^{\frac{1}{1-\theta}},$$

The above expressions of demand functions for goods i and price index imply that :

$$P_t c_t = \int_0^1 p_t^i c_t^i di,$$

The budget constraint becomes :

$$c_t + x_t + \sum_{\ell=1}^{L} \frac{Q_t^\ell B_t^\ell}{P_t} = \frac{W_t n_t}{P_t} + \frac{R_t k_t}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^{\ell-1} B_{t-1}^\ell}{P_t} - \Gamma\left(\frac{x_t}{k_t}\right)k_t, \tag{1.4}$$

The household maximization problem is subject to (1.4) and (1.3)

The preference shock follows the process

$$\ln(A_t) = (1-\rho)\ln(A) + \rho\ln(A_{t-1}) + \sigma_u u_t, \tag{1.5}$$

where $\rho \in (-1, 1)$, $\ln(A)$ is the unconditional mean of $\ln(A_t)$, and u_t is assumed to be an independently and identically distributed (*i.i.d.*) innovation with mean zero and standard deviation equal to one. $\sigma_u > 0$ is constant parameter

The first-order conditions for the consumer's problem include Euler equations for capital, investment, and bonds :

$$1 = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_t r_{t+1} + \left(1 - \delta + \Gamma(\frac{x_{t+1}}{k_{t+1}}) + \frac{x_{t+1}}{k_{t+1}} \Gamma'(\frac{x_{t+1}}{k_{t+1}}) \right) \frac{q_t}{q_{t+1}} \right] \right\}, \tag{1.6}$$

$$q_t = 1 + \Gamma' \left(\frac{x_t}{k_t} \right), \tag{1.7}$$

$$Q_t^{\ell} = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{Q_{t+1}^{\ell-1}}{\pi_{t+1}} \right) \text{,for } \ell\text{=1,2,...L}, \tag{1.8}$$

where λ_t is the the marginal utility of consumption $r_{t+1} = 1 + \frac{R_t}{P_t} - \delta$ is the real return of capital, $\pi_{t+1} = P_{t+1}/P_t$ is the gross rate of inflation between time t and $t+1$, and q_t is the ratio of Lagrangian multipliers of constraint (1.4) and (1.3), that is the Tobin's q.

1.2.2 Firms

There are a final good competitive firm and a continuum of monopolistic firms that operate competitively.

Final Good Producer

The final good producer behaves in a perfectly competitive manner and takes as given the prices of intermediate goods and the aggregate price index when maximizing profits. The final good is produced using only the individual goods y_t^i as inputs in the following production function :

$$y_t = \left[\int_0^1 (y_t^i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where y_t the quantity of the final good. Profit maximization implies that demand of input i is given by :

$$y_t^i = \left[\frac{p_t^i}{P_t} \right]^{-\theta} y_t, \tag{1.9}$$

Intermediate Goods Firms and Price Setting

Individual good $i \in (0\ 1)$ is produced by a monopolist through the following technology :

$$y_t^i = Z_t F(K_t^i, N_t^i), \tag{1.10}$$

where y_t^i is output, K_t^i is firm i capital demand, N_t^i is labor input and the function $F(.,.)$ is constant return to scale, strictly increasing and strictly concave in both of its arguments and satisfy the Inada conditions, Z_t is a total factor productivity shock that affects all firms in the same way.

The technology shock follows the process :

$$\ln(Z_t) = (1-\omega)\ln(Z) + \omega\ln(Z_{t-1}) + \sigma_\varepsilon^2\varepsilon_t, \tag{1.11}$$

where $\omega \in (-1, 1)$, $\ln(Z)$ is the unconditional mean of $\ln(Z_t)$, and ε_t is a disturbance term assumed to be an independently and identically distributed $(i.i.d.)$ with mean zero and standard deviation equal to one.

Intermediate good producing firm $i \in (0\ 1)$ hires labor and capital in perfectly competitive markets to produce its good. Firms are owned by households who receive any profit made by firms at each period.

Prices are set following the mechanism described in Calvo (1983) : each period a fraction of $1-\theta_p$ randomly picked firms can reset their price while the remaining fraction θ_p cannot. Those who have the opportunity to adjust their price, set it optimally to maximize their discounted profit while those who cannot adjust optimally, just set their price to the previous aggregate price level indexed by the steady state inflation. Note that θ_p governs the prices stickiness. The smaller θ_p is, the more flexible prices will be as firms will get to reset their price frequently.

The firm $i's$ problem is to choose K_t^i, N_t^i, p_t^i to maximize discounted profit subject to its good demand function, the production technology (1.10) and the price setting scheme. This can be done in two steps : first choose the capital and labor input to minimize the real cost given the production function (1.10) and given the real wage and capital rental rates. Second choose the price to maximize the discounted real profit subject to the demand function and given the aggregate price and quantities.

The real cost minimization program is :

$$\underset{K_t^i,\, N_t^i}{Min}\ [w_t N_t^i + r_t K_t^i]$$

s.t $\ y_t^i = Z_t F(K_t^i, N_t^i) = Z_t (K_t^i)^\alpha (N_t^i)^{1-\alpha}$

The first order conditions imply that :

$$\frac{K_t^i}{N_t^i} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} \tag{1.12}$$

Thus, all firms will choose the same capital-labor ratio. Using this relation, the real cost is given by :

$$Cost_t = w_t N_t^i + r_t K_t^i = \frac{1}{1-\alpha} w_t N_t^i$$

Use the production function and (1.12) to express N_t^i as a function of y_t^i, w_t, and r_t and substitute into the cost function to get :

$$Cost_t = \frac{y_t^i}{Z_t}\left[\frac{w_t}{1-\alpha}\right]^{1-\alpha}\left[\frac{r_t}{\alpha}\right]^{\alpha}$$

The real marginal cost mc_t is equal to the derivative of the real cost with respect to y_t^i and is given as :

$$mc_t = \frac{1}{Z_t}\left[\frac{w_t}{1-\alpha}\right]^{1-\alpha}\left[\frac{r_t}{\alpha}\right]^{\alpha} \tag{1.13}$$

Note that the real marginal is independent of i meaning that all firms incur the same marginal cost.

Now in the second step, firms pick their price p_t^i to maximize :

$$E_t\sum_{s=\tau}^{\infty}(\theta_p\beta)^s\frac{\lambda_{t+s}}{\lambda_t}\left[\frac{p_t^i}{P_{t+s}} - mc_{t+s}\right]y_{t+s}^i$$

subject to :

$$y_{t+s}^i = \left[\frac{p_t^i}{P_{t+s}}\right]^{-\theta} y_{t+s}$$

Notice that when maximizing the profit, firms take into account the fact that a price set at time t will remain the same with probability $(\theta_p)^s$ at time $t+s$. It means that when θ_p is large, a price set in the current period will likely remain for a long period of time. Thus when choosing current price, firms will relatively weight more future profits.

Replace the demand function in the objective and take the derivative with respect to p_t^i gives :

$$E_t\sum_{s=0}^{\infty}(\theta_p\beta)^s\frac{\lambda_{t+s}}{\lambda_t}\left[(1-\theta)\left[\frac{p_t^i}{P_{t+s}}\right]^{1-\theta}\frac{1}{p_t^i} - \theta\left[\frac{p_t^i}{P_{t+s}}\right]^{-\theta}\frac{1}{p_t^i}mc_{t+s}\right]y_{t+s} = 0$$

Solve this equation to get

$$p_t^i = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta} mc_{t+s} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta-1} y_{t+s}} \tag{1.14}$$

The above equation (1.14) says that when firms have the opportunity to adjust their price, they optimally set it as some weighted mean of expected future nominal marginal costs.

The infinite summations implied in (1.14) make the computation tricky because we do not have a direct recursive formulation of the this expression. To get around this problem, let define the following auxiliary variables :

$$V_t = E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta} mc_{t+s} y_{t+s}$$

$$J_t = E_t \sum_{s=0}^{\infty} (\theta_p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} P_{t+s}^{\theta-1} y_{t+s}$$

Then (1.14) becomes

$$p_t^i = \frac{\theta}{\theta - 1} \frac{V_t}{J_t}$$

Where the infinite sums V_t and J_t have the following recursive forms :

$$V_t = \theta_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} V_{t+1} \right] + P_t^{\theta} mc_t y_t \tag{1.15}$$

$$J_t = \theta_p \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} J_{t+1} \right] + P_t^{\theta-1} y_t \tag{1.16}$$

When all firms are able to adjust their prices each period ($\theta_p = 0$), price are set to markup ($\mu = \frac{\theta}{\theta-1}$) over nominal marginal cost ($P_t mc_t$)

$$p_t^i = \frac{\theta}{\theta - 1} P_t mc_t$$

whereas when $\theta_p > 0$, the optimal price is set as a markup over expected future weighted marginal costs. Notice that in the flexible price framework, the marginal is constant and equal to the inverse of the markup.

Because all firms face the same demand function, they will choose the same price when reoptimizing at time t, that is $p_t^i = p_t^j = p_t^*$ for those who are able the adjust and $p_t^i = P_{t-1}$ for those who cannot. Thus the price index is given by :

$P_t = \left[\theta_p P_{t-1}^{1-\theta} + (1-\theta_p) P_t^{*1-\theta}\right]^{\frac{1}{1-\theta}}$ and the inflation rate

$$\frac{P_t}{P_{t-1}} = \left[\theta_p + (1-\theta_p)\frac{P_t^{*1-\theta}}{P_{t-1}^{1-\theta}}\right]^{\frac{1}{1-\theta}}$$

where $\frac{P_t^{*1-\theta}}{P_{t-1}^{1-\theta}} = \left[\frac{\theta}{\theta-1}(1+\pi_t)\frac{\widetilde{V}_t}{\widetilde{J}_t}\right]^{1-\theta}$ and $\widetilde{V}_t = \frac{V_t}{P_t^{\theta}}$, $\widetilde{J}_t = \frac{J_t}{P_t^{\theta-1}}$

Inflation can then be solve out as a function of $\widetilde{V}_t$ and $\widetilde{J}_t$ from this equation

$$1+\pi_t = \left[\theta_p + (1-\theta_p)\left[\frac{\theta}{\theta-1}(1+\pi_t)\frac{\widetilde{V}_t}{\widetilde{J}_t}\right]^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{1.17}$$

Where from (1.15) and (1.16), $\widetilde{V}_t$ and $\widetilde{J}_t$ evolve according to :

$$\widetilde{V}_t = \theta_p \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\theta}\widetilde{V}_{t+1}\right] + mc_t y_t \tag{1.18}$$

$$\widetilde{J}_t = \theta_p \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\theta-1}\widetilde{J}_{t+1}\right] + y_t \tag{1.19}$$

The production side equilibrium conditions are given by (1.9) - (1.13), (**??**) - (1.19).

1.2.3 Monetary Policy Rule and Government

The model is closed with a Talor-type policy rule whereby the monetary authority sets the one-period nominal interest rate as a function of inflation and output deviations from targeted levels.

$$\frac{1+i_{t+1}}{1+i^{ss}} = \left(\frac{1+i_t}{1+i^{ss}}\right)^{\rho_i}\left(\frac{1+\pi_t}{1+\pi^{ss}}\right)^{(1-\rho_i)\gamma_\pi}\left(\frac{y_t}{y^{ss}}\right)^{(1-\rho_i)\gamma_\pi}\exp(mp_t) \tag{1.20}$$

where i_t is the time t one-period nominal bond interest rate, mp_t is monetary innovation and i^{ss}, π^{ss}, y^{ss} are steady values of the short term nominal interest rate, inflation and output.

1.2.4 Market Clearing and Aggregation

Using the fact that the capital-labor ratio is firm independent, we can get the aggregate capital-labor ratio

$$\frac{K_t}{N_t} = \int_0^1 \frac{K_t^i}{N_t^i} di = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t}$$

The aggregate supply over firms is then given by :
$\int_0^1 y_t^i di = Z_t \left(\frac{K_t}{N_t}\right)^{\alpha} \int_0^1 N_t^i di$ and the aggregate demand is $y_t \int_0^1 \left(\frac{p_t^i}{P_t}\right)^{-\theta} di$.
In equilibrium aggregate demand must equal the aggregate supply or

$$y_t \int_0^1 \left(\frac{p_t^i}{P_t}\right)^{-\theta} di = Z_t K_t^{\alpha} N_t^{1-\alpha} \tag{1.21}$$

where $N_t = \int_0^1 N_t^i di$.

From (1.21), the aggregate composite index of output is

$$y_t = \frac{Z_t K_t^{\alpha} N_t^{1-\alpha}}{d_t} \tag{1.22}$$

where $d_t = \int_0^1 \left(\frac{p_t^i}{P_t}\right)^{-\theta} di$ is the price dispersion and introduces a distortion in output aggregation. The fact that firms choose different prices in equilibrium can lead to aggregate output lost when. In fact, firms who choose to increase their relative price will face a decrease in the demand of their good and then a decrease in their output. When prices are flexible, all firms choose the same price and their is no distortion in aggregate output, that is, d_t is always equal to 1.

The Calvo pricing structure implies that the law of motion of d_t is given by :

$$d_t = \theta_p \left[\frac{1}{1+\pi_t}\right]^{-\theta} d_{t-1} + (1-\theta_p)\left[\frac{\theta}{\theta-1}\frac{\tilde{V}_t}{\tilde{J}_t}\right]^{-\theta} \tag{1.23}$$

In equilibrium all the markets must clear every period :

$c_t + x_t = y_t$

$k_{t+1} = (1-\delta)k_t + x_t - \Gamma\left(\frac{x_t}{k_t}\right) k_t$

$n_t = N_t = \int_0^1 N_t^i di$

$k_t = K_t = \int_0^1 K_t^i di$

1.2.5 Equilibrium

Definition : An equilibrium is an allocation for the household $\mathcal{C} = \{c_t, n_t, x_t, k_{t+1}\}_{t=0}^{\infty}$, $\left\{(B_t^{\ell})_{\ell=1,..L}\right\}_{t=0}^{\infty}$, an allocation for the firm $\mathcal{F} = \{Y_t, K_t, N_t\}_{t=0}^{\infty}$, a prices system $\{\pi_t, W_t/P_t, R_t/P_t\}_{t=0}^{\infty}$, $\left\{(Q_t^{\ell})_{\ell=1,..L}\right\}_{t=0}^{\infty}$ such that given k_0 and the prices system :

1) the allocations $\mathcal{C}$ and $\mathcal{F}$ solve the households' and the firms' problems,

2) good market clears : $Y_t = c_t + x_t + \Gamma\left(\frac{x_t}{k_t}\right)k_t$,

3) $n_t = N_t = 1$,

4) $S_t = \sum_{\ell=1}^{L} \frac{Q_t^{\ell-1}B_{t-1}^{\ell}}{P_t} - \sum_{\ell=1}^{L} \frac{Q_t^{\ell}B_t^{\ell}}{P_t}$

In the following section, we review the relation between the bond prices implied by the economic model and the term structure of interest rates, and define risk premia. From the bond prices implied by the first-order conditions of bonds demand, we derived the term structure of interest rates and expressions for risk premia as functions of macroeconomic fundamentals.

1.3 Interest Rates and Risk Premia in DSGE Models

New Keynesian DSGE models are well known to be able to reproduce salient features of macroeconomic data (see, Smets and Wouters, 2004) but fail to match simultaneously financial and macro data. In fact, matching risk premia involved in financial assets is a challenging issue for DSGE modelers, yet it is easy to reproduce risk premia in an exchange economy by adding some real frictions such as habit formation in a standard RBC model (see Wachter, 2006 and Piazzesi and Schneider, 2006) . With habit formation preferences, current consumption levels affect future marginal utilities and the risk aversion is countercyclical instead of being constant as in RBC models. This allows the model to calibrate high steady state risk aversion with a reasonable consumption curvature parameter (see, Campbell and Cochrane, 1999), and then to generate sizeable risk premia consistent with the data. In a production economy where consumption, hours worked and output are endogenous, there are available channels to the consumer for overcoming bad income shocks, that are absent in exchange economies. Thus in terms of consumption smoothing, risk averse consumers claim bigger premium to hold a long-term bond in exchange economies because they are more exposed to income uncertainty in endowment economy than in production economy. For example, consumers will be able to work more in production economies to increase their income when they face bad income shocks ; whereas this channel is absent in exchange economies. As a consequence, the increasing effect of habit formation on risk premia is weakened in a production economy wherein consumers can adjust labor or accumulate capital.

We define the gross interest rate of the one-period bond as

$$i_{1,t} = \frac{1}{Q_t^1} \tag{1.24}$$

More generally, the gross nominal interest rate of the ℓ-period bond is defined as

$$i_{\ell,t} = \left[Q_t^\ell\right]^{\frac{-1}{\ell}} \tag{1.25}$$

There are various formulas of risk premiums in the literature but Rudebusch *et al.* (2007) show that they are highly correlated. The overall risk involved in long-term nominal bonds is twofold : first, there is a risk of capital loss in the future in case of reselling the bond before the maturity date. Because the bond future prices are not known with certainty in advance, the eventual resale[4] price could be less than the purchase price. Second, there is an inflation risk involved in nominal long-term bonds because inflation can erode the bond value in the future. The risk premium can be derived recursively from Euler equation for bonds,

$$Q_t^\ell = Q_t^1 E_t\left(Q_{t+1}^{\ell-1}\right) + \beta cov_t\left(Q_{t+1}^{\ell-1}, \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right), \tag{1.26}$$

where we have used the fact that the one-period bond price is

$$Q_t^1 = E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right). \tag{1.27}$$

The $\ell-$period term premium, denoted by $TP_{\ell,t}$, is usually defined as the difference between an $\ell-$period interest rate and expected average of short-term rates over the maturity period, that is,

$$TP_{\ell,t} = i_{\ell,t} - \frac{1}{\ell}E_t\sum_{s=0}^{\ell-1} i_{1,t+s} \tag{1.28}$$

[4] For example in case of a negative realization of an income shock somewhere between t and $t+\ell$, an $\ell-$period bond holder would like to redeem the bond in order to smooth its consumption

A similar form of (1.28) in our model is captured by the covariance term of the right hand side of (1.26). The risk premium we will use in this model, is the excess holding period return, that is, the return from holding an $\ell-$period bond for one period relative to the return of one-period bond[5]. We can rearrange (1.26) to get

$$E_t\left[\frac{Q_{t+1}^{\ell-1}}{Q_t^{\ell}}\right] = \frac{1}{Q_t^1} - cov_t\left[\frac{Q_{t+1}^{\ell-1}}{Q_t^{\ell}}, \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{1+\pi_{t+1}}\frac{1}{Q_t^1}\right] \tag{1.29}$$

At time $t+1$, an $\ell-$period bond will become an $(\ell-1)$ maturity bond such that the gross holding period return $H_{\ell,t+1}$ is given by

$$H_{\ell,t+1} = \frac{Q_{t+1}^{\ell-1}}{Q_t^{\ell}}$$

From (1.29) we have,

$$E_t(H_{\ell,t+1}) = i_{1,t} + rp_{\ell,t} \tag{1.30}$$

where $rp_{\ell,t} = -cov_t\left[H_{\ell,t+1}, \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{1+\pi_{t+1}}i_{1,t}\right]$ is the holding period risk-premium.

It is easy to show that

$$TP_{\ell,t} = \frac{1}{\ell}E_t\sum_{s=0}^{\ell-1} rp_{\ell-s,t+s}$$

The term premium is thus the mean of all expected holding period risk premia.

(1.30) says that after adjusted for risk factor, the holding-period return is a predictor of the one-period interest rate. Note that this covariance term can either be positive or negative depending on the direction of the covariation between the holding-period return and the nominal discount factor. When high future marginal utility- that is the situation where the investor needs more consumption- is associated with capital losses ($Q_{t+1}^{\ell-1}$ is low relative to Q_t^{ℓ} when reselling an $\ell-$period bond at $t+1$), investors will claim a positive risk premium for holding a long term bond instead of short-term bonds. We can also notice the two sources of risk involved in long-term nominal bonds highlighted above. First, the term premium is affected by the covariation between the holding-period return and the real stochastic discount

[5] Computationally, the excess holding period return requires less complementary state variables definition than the term premium

factor keeping the inflation rate constant. Second, correlation between the holding-period return and future inflation rate, keeping the real stochastic discount factor constant, also determines the sign and the size of the risk premia. In the first case, the resulting term premium will be referred as the real term premium and in the second case the inflation term premium. The sign and the magnitude of the total term premium will depend on the combination of these two covariance effects.

In general inflation risk premium compensates the bond holder for the inflation risk involved in keeping a nominal asset rather than a riskless real asset. In our model, such an asset could be the capital stock would the productivity shock be constant over time and without adjustment cost in capital. Use the first-order conditions for the one-period bond to get

$$Q_t^1 = E_t(\beta \frac{\lambda_{t+1}}{\lambda_t}) E_t(1/(1+\pi_{t+1})) + cov_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t}, \frac{1}{1+\pi_{t+1}} \right],$$

where the covariance term is the one-period inflation risk premium denoted by $Inflpr_t^1$ because this conditional covariance is zero when the inflation process is deterministic. Similarly the $\ell-$period inflation risk premium is defined from the $\ell-$period bond Euler equation as :

$$Inflpr_t^\ell = Q_t^\ell - E_t(\beta^\ell \frac{\lambda_{t+\ell}}{\lambda_t}) E_t(1/(1+\pi_{t+\ell})) = cov_t \left[\beta^\ell \frac{\lambda_{t+\ell}}{\lambda_t}, \frac{1}{1+\pi_{t+\ell}} \right]$$

where $\pi_{t+\ell} = \frac{P_{t+\ell}}{P_t}$

1.4 Model Solution

Notice that an exact analytical solution is not available in this model. Thus, we use a perturbation method to approximate the solution of the model given the parameters. Basically, perturbation method consists in taking Taylor series expansion of the decision rules around the deterministic steady state. For detailed explanations of this approach, see Jin and Judd(2002), Schmitt-Grohé and Uribe(2004), and Kim, Kim, Schaumburg and Sims(2008). Perturbation methods deliver a zero risk premium at first-order approximation due to the certainty equivalence at first-order and a constant risk-premium at second-order approximation.

The standard approach of perturbation method writes the model general equilibrium conditions in the form :

$$E_t F(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \tag{1.31}$$

where E_t is the conditional expectation given the time t information set, y_t is the vector of control variables and x_t the predetermined endogenous variables and exogenous processes. F is a vectorial function of all the equilibrium conditions. The solution of the model is given by :

$$y_t = g(x_t, \sigma)$$

$$x_{t+1} = h(x_t, \sigma) + \sigma\eta\varepsilon_{t+1}$$

where h and g are unknown functions, η is constant matrix driving the variances of the innovations and σ is a scaling perturbation parameter driving the size of the uncertainty in the economy. Given that h and g are unknown, the procedure consists of approximating the functions h, g around the non-stochastic steady state point $(x, 0)$ where uncertainty is removed. The approximate solution takes the form :

$$y_t = y + \frac{1}{2}g_{\sigma\sigma}\sigma^2 + g_x(x_t - x) + \frac{1}{2}(I_{ny} \otimes (x_t - x))' g_{xx}(x_t - x) \tag{1.32}$$

$$x_{t+1} = x + \frac{1}{2}h_{\sigma\sigma}\sigma^2 + h_x(x_t - x) + \frac{1}{2}(I_{nx} \otimes (x_t - x))' h_{xx}(x_t - x) + \sigma\eta\varepsilon_{t+1} \tag{1.33}$$

where $x = h(x, 0)$ and $y = g(x, 0) = g(h(x, 0), 0)$ and n_y and n_x are the number of control and state variables respectively, I is an identity matrix. g_x, h_x, g_{xx}, h_{xx}, $h_{\sigma\sigma}$ are constant coefficients standing for first and second derivatives of g and h with respect to x and σ evaluated at the deterministic steady state. Notice that these coefficients are functions of the structural parameters of the model and that the parameter σ enters the decision rules as an argument capturing the risk factors.

The constant risk premia delivered by the second-order approximate solution is a combination of volatilities of the shocks. Thus to understand the determinants of risk premia in DSGE models, it is useful to write the second-order risk-premium as

$$rp_\ell = \frac{1}{2}rp_\ell^a\sigma_a^2 + \frac{1}{2}rp_\ell^z\sigma_z^2 + \frac{1}{2}rp_\ell^{mp}\sigma_{mp}^2 \tag{1.34}$$

where rp_ℓ^a, rp_ℓ^z, rp_ℓ^{mp} are functions of structural parameters and σ_a^2, σ_z^2, σ_{mp}^2, are the volatility of preference, productivity and monetary policy shocks respectively.

1.5 Estimation

1.5.1 Data

We estimate the model using U.S. macroeconomic as well as term structure data at the quarterly frequency. The sample period is 1962 Q1 -2001 Q2.

The macro data used are *per capita* real consumption growth, *per capita* real GDP growth, and Consumer Price Index (CPI) inflation rate. Consumption is NIPA measures of personal consumption expenditure on non durable goods and services. Real consumption is obtained by dividing its nominal measure by CPI inflation rate. All the macro data are seasonally adjusted and are collected from the Federal Reserve Bank of St. Louis website (*www.stls.frb.org*).

The term structure of interest rates data are the nominal three-month interest rate and the ten-year nominal interest rate. The three-month rate is daily treasury bill rate whereas the ten-year interest rate is daily constant maturity rate. All interest rates series are from the Federal Reserve Bank of St. Louis website. We obtained quarterly observations by taking the first trading day observation of the second month of each quarter[6] (February, May, August, November). In the estimation, we use the spread between the ten-year and the three-month nominal interest rates. Notice that the model counterparts of the three-month and ten-year interest rates are one-period and forty-period interest rates respectively.

1.5.2 Paramaters Estimation : Simulated Method of Moments (SMM)

We estimate the shocks parameters of the model by the Simulated Method of Moments (SMM). The number of estimated parameters is five : the persistence parameters of technology (ρ_a) and preferences (ρ_u) shocks ; and the standard deviations of the three shocks σ_u, σ_ε, σ_{mp}. The remaining parameters have been calibrated to the U.S. economy or set in line with the literature.

SMM consists in minimizing a weighting distance between unconditional moments predicted by the model and the corresponding data moments counterpart. Basically, the predicted moments are based on artificial data simulated from the model while data moments are directly computed from actual data. This method is appealing for nonlinear DSGE models estimation because, as shown by Ruge-Murcia (2007), it is robust to misspecifications and is computationally efficient. In addition, Lee and Ingram (1991), and Duffie and Singleton (1993) show that parameters estimates by SMM are consistent and asymptotically normal. Ruge-Murcia (2012) provides the

[6] Instead of averaging over the quarter

properties of SMM estimates for third-order approximation of DSGE models. The Monte - Carlo evidence on small samples shows that SMM based asymptotic standard errors tend to overestimate the actual variances of the parameters. Thus, we use a block bootstrap method to compute a ninety- five per cent confidence intervals for the parameter estimates.

Since SMM analysis requires stationary variables, we simulate the model on the basis of the pruned version of the second-order approximate solution proposed by Kim, Kim, Schaumburg and Sims (2008). The innovations are drawn from the normal distribution for the simulation. The moments used in the estimation are the variances, first- and second-order autocovariances of the four data series, in addition to the unconditional mean of the interest rate spreads and the inflation rate. Because consumption growth and real GDP growth rates are positive in the U.S. data and there is no growth in the model, we discard the mean of these two variables. Thus, fourteen moments are used in the estimation of the five parameters meaning that the number of degrees of freedom is nine. The weighting matrix used is the diagonal of the Newey-West estimator of the long-run variances of the moments with a Bartlett kernel and a bandwidth given by $4(T/100)^{2/9}$ (as in Ruge-Murcia, 2010). The sample size here is T=158 which implied a bandwidth value of 4.427. We simulate five times the sample size (T) observations for the artificial series.

Before the estimation test whether the series used in the estimation are stationary as the theoretical properties of SMM estimates are valid under this assumption. To this end, we use an Augmented-Dickey Fuller (ADF) and a Phillips-Perron (PP) unit root tests. The null hypothesis of unit root can be rejected at 5% level under both tests for all series except the inflation rate. However, for the inflation rate, the unit root hypothesis can be rejected at the 5% level under the PP test but cannot be rejected under the ADF test. But the ADF-statistic is -2.38 whereas the critical value is -2.39. So, we suppose that the inflation rate is stationary.

1.5.3 Calibration

During the estimation, the remaining model parameters have been calibrated as follow :

The subjective discount factor is parametrized at $\beta = 0.99$ to match the average annual real interest rate of 4%. The consumption curvature coefficient in the utility function is set to a value of $\gamma = 2$. This value is in the range of empirical estimates in the DSGE models literature [7]. The habit strength parameter is set to $b = 0.65$ as in Constantinides (1990), Boldrin, Christiano and Fisher (2001). The labor elasticity is set to $\varphi = 2$ and φ_0 is calibrated to match 1/3 of steady state hours worked without

[7] For example Smets and Wouters (2005) find an estimate of $\gamma = 2.6$

habit in consumption[8] as found in the U.S. post war II data. The depreciation rate of capital is set to 0.025 per quarter such that the steady state investment-output ratio is 23%. The capital adjustment cost parameter is set to $\phi = 10$.

In the production side, the most important parameters that need to be discussed are the Calvo parameter θ_p which controls the price stickiness and the parameter θ representing the firms power. The model steady state mark-up Ψ is given by the expression $\Psi = \frac{\theta}{\theta-1}$. In the data, the long-run mark-up has been estimated to be about 10% meaning that $\frac{\theta}{\theta-1} = 1.1$. This implies a value of $\theta = 11$.

The Calvo parameter or the proportion of resetting price firms θ_p is also related to the average duration of a price set at time t. Conditional on setting optimally a price at time t, the probability of being able to reset optimally for the first time at time $t+j$ is $(1-\theta_p)\theta_p^j$. It means that the average duration of a price set at time t is :

$D = \sum_{j=0}^{\infty} j(1-\theta_p)\theta_p^j = \frac{1}{1-\theta_p}$ which implies an average duration of price changes of 1 year. Thus θ_p can be calibrated by computing the average duration between price changes. The range of estimates by Bils and Klenow (2004) of average price changes in micro data is between six months and one year. Setting $D = 1$ year, that is 4 quarters yields $\frac{1}{1-\theta_p} = 4$ or $\theta_p = 0.75$.

We compute the U.S. long run money gross growth rate (1.01) and set the steady state gross inflation rate to this value. With the annual 4% steady state real interest rate, this implies an annual steady state of nominal interest rate of 8%. The non-zero steady state inflation rate implies a steady state of real marginal cost (0.87) that is slightly different from the inverse of the mark-up (0.91). The production function parameter α is set at 0.41 to match a long run U.S. capital share of income of 0.37.

There are no standard values of the monetary policy parameters and different papers (Clarida, Gali and Gertler, 1999 among others) have estimated different values. In the New Keynesian literature, these parameters are usually chosen in range which satisfies the equilibrium stability. I follow Ravenna and Seppala (2005) and Rabanal and Rubio-Ramirez (2004) to choose these parameters. The inflation reaction parameter and the output reaction parameter are respectively set to $\gamma_\pi = 3$, $\gamma_y = 0.1$.

1.5.4 Parameters Estimates

Table 1.5 reports the SMM estimates of the shocks parameters. The Productivity shock is very persistent and volatile. The estimates of the autocorrelation coefficient (ρ_a =0.981) and standard deviation (σ_a =0.0105) are relatively well precise. The

[8] with habit formation preferences the implied steady state of labor is about 1/2

TAB. 1.1: Baseline Calibrated Parameters

parameters	**description**	**value**
β	Subjective discount factor	0.99
γ	Consumption curvature	2
ϕ	Capital adjustment cost parameter	10
α	Share of capital income	0.41
δ	Capital depreciation rate	0.025
θ	Elasticity of substitution among goods	11
θ_p	Proportion of firms not adjusting price	0.75
π^{ss}	Long-run gross inflation rate	1.01
γ_π	Inflation coefficient in Taylor rule	3
γ_y	Output coefficient on Taylor rule	0.1

preferences shock is mildly persistent (ρ_u =0.553) but very volatile (σ_u =0.047). The monetary policy shock has been constrained to an *i.i.d.* process and the estimated volatility is very low (σ_{mp} =0.95x10^{-4}). This means that the dynamics of the model is mostly driven by productivity and preferences shocks.

1.6 Results and Sensitivity Analysis

The results of the calibrated second-order approximate solution of the model and sensitivity exercises are presented in this section.

1.6.1 The term structure of interest rates

We present in this part the model implied term structure statistics. Note that with the calibrated 1% of quarterly long-run inflation rate, the steady state of nominal interest rate of 8% (annual). We simulate the model and report the unconditional mean of interest rates statistics in table 1.6 . As figure 1.7 shows, the model generates an upward sloping average term structure of interest rates. The model is able to generate a positive risk premium which leads to the upward sloping term structure. Table 1.6 shows that the risk premium is increasing in maturity. For example, the 4-period term premium is 0.5 basis points and the 10-period term premium is 1.8 basis points (annualized). Notice that the empirical counterpart of the 10-period maturity is 2.5 years as the model is calibrated to a quarterly basis.

Now, we analyze the impact of monetary policy shock, preferences shock and productivity shock on selected variables of the model. Because the model is non-linear (2nd-order approximation), we present variables responses to positive as well as negative shock to capture potential asymmetric responses. The blue line is response to positive shocks and the green line response to negative shocks.

Figure 1.7 presents the impulses responses of key variables to a one standard deviation of monetary policy shock. As the standard deviation of monetary policy is 0.003, this corresponds to an annual 120 basis points (40000x1x0.003) increase in the short-term nominal interest rate. Figure 1.7 shows that an unexpected increase (decrease) in monetary policy shock leads to a decrease (increase) in consumption and inflation as expected. An increase (decrease) in monetary policy rate leads to an increase(decrease) in nominal bond interest rates. But the response decreases with the maturity such that shorter term nominal rates respond more than longer maturity rates. As a result, an increase in monetary policy rate will induce a decrease in interest rate spreads. Results in figure 1.7 also suggest that there is no asymmetric response for relatively small shocks.

Responses to productivity shock are reported in figure 1.7 . The size of the shock is one standard deviation. A positive (negative) productivity shock leads to a decrease (increase) in nominal bond yields at all maturities. At the impact time, the magnitude of the response is the same across maturities. However, the persistence of the effect is increasing across maturities with the response of longer maturity interest rates more persistent than shorter maturity rates. It means that nominal spreads do not respond to productivity shock at the impact time but increase (decrease) after a positive (negative) productivity shock.

We now consider the impact of preferences shock on the nominal interest rates. The results are reported in figure 1.7 . This shock affects directly marginal utilities and the pricing kernel. A one standard deviation increase (decrease) in preferences shock leads to an increase (decrease) in nominal bond interest rates. As in the case of monetary policy shock, the effect is decreasing across maturities with shorter term rates respond more than longer maturity rates.

1.6.2 Shocks contribution to risk premium

In this part we analyze the relative importances of each shock to the determination of the size of the risk premium. Remember that the second-order approximation implied risk premium is a weighted sum of the volatilities of the shocks. Thus as expression (1.34) shows, each shock contributes to the size of risk premia in two ways. First, the importance of each shock in terms of contribution will depend on its weighting coefficient. These coefficients can be interpreted as unitary prices of risk associated with each shock. That is, they capture the intensity of the agent's risk aversion towards the corresponding shocks. Second, shock volatility sizes are important to the determination of risk premia. The volatility sizes capture the quantity of risk involved in each shock and risk premia are expected to increase as the quantity of risk increases. The total contribution of a shock is then its unitary price of risk times the quantity of risk embedded in this shock.

This decomposition of shock contribution to risk premia is important because in the literature, the ability of a DSGE model to match risk premia statistics usually heavily relies on the calibrated (or estimated) shock size. The calibration of the shocks volatilities is sometimes controversial. For example, Hordahl, Tristani, and Vestin (2007) use a calibrated DSGE model with 2.3% standard deviation of technology shock. This allows their model to generate term premia large enough to be consistent with the data. This value of technology shock standard deviation is more than two times the standard value of 1% used in macro models. This is also the case in Ravenna and Seppala (2006) where the standard deviation of preferences shock has been set to 8% that is also large compared to the estimated value of 4% in Bansal *et al* (2005). Rudebusch and Swanson (2008) criticize the reliance of these two authors results to such large shock sizes. Furtheremore Ravenna and Seppala (2006) find that in New Keynesian framework, rejections of expectation hypothesis are explained by the systematic part of the monetary policy rather than by monetary innovations as found in Buraschi and Jiltsov (2006). However, the comparison of the two results is not clear since the two papers differ in many aspects including the nature and size of the shocks. Thus, for given shock sizes, we can compare different models ability to generate risk premia based on the second terms which capture the market prices of risk.

Table 1.2 shows the total contribution of each shock to the size of the baseline model risk premia. Preferences shock is far the most important shock in terms of contribution to the size of the risk premia and the contribution is increasing with the maturity. This result is not surprising because the calibrated preferences shock standard deviation is very large (0.04) relative to the two other shocks (more than 10 times) meaning preferences shock carries the largest quantity of risk to the point of view of the investor. As a consequence, the combined quantity and price of its associated risk gives the most important value relative to the two other shocks.

TAB. 1.2: Shock Contribution to Risk Premia (Baseline Parameters)

Shocks	rp_4	rp_8	rp_{10}	rp_{13}
Preference	69%	80%	81.5%	83%
Productivity	15%	13%	12.86%	12.7%
Monetary Policy	16%	7%	5.7%	4.3%

But instead of looking at the quantity effect one can also analyze the price effect of the risk carried by each shock. That will capture the relative undesirability of each shock given the same quantity of risk. To address this issue, I now analyze the model implied coefficients rp^a_ℓ, rp^z_ℓ, rp^{mp}_ℓ in equation (1.34).

Table 1.3 shows the model implied rp^a_ℓ, rp^z_ℓ, rp^{mp}_ℓ. It turns out that in terms of the price per unit of risk, monetary policy has the largest effect on the size of risk premia meaning that if all shocks were calibrated to have the same variance, monetary policy would have the most important contribution to the risk premia with more than 80% for the 4-period bond. This result can be interpreted as follow. Suppose an hypothetical economy where the three shocks have the same standard deviations. Then, monetary policy innovations are more undesirable *vis a vis* the investor relative to productivity and preferences shocks. Note also that all prices of risk are increasing with the maturity.

Tab. 1.3: Prices of Risk (Baseline Parameters)

$\ell =$	4	8	10
rp^a_ℓ	0.05	0.16	0.2
rp^z_ℓ	0.4	0.82	1.05
rp^{mp}_ℓ	2.3	2.5	2.57

1.6.3 Sensitivty Analysis

In this section, we do some sensitivity exercises by varying key structural parameters in order to understand the determinants of the size of the risk premia. For example, we want to know how the monetary policy actions part affect risk premia.

Monetary policy and risk premia : do monetary policy actions matter for risk premia ? That is, how changes in monetary policy parameters affect risk premia. Results indicate that a more aggressive monetary policy stance, that is, an increase in the inflation reaction parameter, leads to decreases in risk premia. This is because when the central bank leans against the wind, the inflation volatility decreases. Less volatile inflation means less uncertainty in inflation and then less inflation premium. An increase of γ_π from 3 (baseline) to 10 leads to a decrease of risk premia of all maturities. For example, the 10-period risk premium decreases from 1.8 to only 0.5 basis points. The results are plotted in figure 1.7.

Role of habit formation : does habit formation preferences plays a role in increasing the size of risk premia ? To answer this question, I change the habit strength parameter from the baseline value of 0.65 to a high level of 0.95 with others parameters set at their baseline values. Habit formation preferences are known to positively magnify the size of risk premium in endowment economy. In this paper, the habit effect on risk premia depends on whether the capital stock is fix or is allowed to vary over time. Figure 1.7 plot the risk premium as functions of the habit strength parameter η without capital adjustment cost and with a very high capital adjustment cost respectively. Figure 1.7 shows that when the agent can freely adjust the capital stock, increasing the habit strength parameter has limited impact on the size of risk premia and the result is actually a decreasing effect. However, when the adjustment cost in capital stock is set high enough to fix the capital stock, increasing the habit strength parameter has a large impact on the size of risk premia.

This result is important because habit formation preferences are usually found to magnify risk premia in DSGE models. The increasing effect of the habit strength on the size of risk premia is consistent with other papers in Ravenna and Seppala (2005) ; Rudebusch *et al* (2006) where the capital stock has been fixed. The reason of this result is as follow. In endowment economies habit preferences significantly magnify the size of risk premia because a risk-averse investor with these preferences, fears more capital losses than an investor with standard preferences. This is because a habit preferences agent cares about not only the level of consumption but the consumption relative to a reference level increasing the agent risk aversion in bad times. As resources can only be intertemporally shifted through financial market, a risk averse investor will claim a larger compensation to hold a long-term bond instead of rolling over short-term bonds. In production economies as this model, there are many alternative channels available for resources transfer. For example, investors can save by accumulating capital through investment or they can even offset bad income shocks by working more. These additional channels for consumption smoothing weaken the habit strength effect on the size of risk premia in production economies. The more channels are available for consumption smoothing, the less important will the habit parameter has on risk premia. Thus, when the capital stock is fixed, the consumer has now less channels for consumption smoothing and then the habit strength will have more impact on risk premia.

The prices of risk associated with the shocks rp_ℓ^a, rp_ℓ^z, rp_ℓ^{mp} in (1.34) are functions of structural parameters. We find that the habit strength parameter η effect on these prices depends also upon the capital adjustment cost parameter. When the adjustment cost parameter is set to $\varphi = 0$, the preferences associated price per unit of risk also decreases as the habit strength increases. When φ is set at very high level, the preferences shock price per unit of risk is increasing with the habit strength parameter.

1.7 Conclusion

I study in this work the term structure of nominal bonds interest rates in a New Keynesian framework with habit formation preferences, adjustment cost in capital stock. The model features three shocks : preferences shock, technology shock and monetary policy shock. Focus has been on the effect of key structural parameters on risk premia ; monetary policy effect on risk premia and shock contribution to the determination of the model implied risk premia.

The model is calibrated to the U.S. economy at a quarterly frequency. Results show that the calibrated second-order approximate solution delivers sizeable and positive risk-premia and an upward sloping average term structure of interest rates. We find that when the productive capital stock is fixed, a higher habit formation parameter significantly increases the risk premium. However when the capital stock is allowed to vary, increases in habit strength decrease risk premium. Moreover, monetary policy has a huge impact on interest rates premia. Especially, an aggressive monetary policy leads to decreases in risk premia as it leads to more stable inflation and then to decreases in inflation risk. In terms of contribution of the three shocks, we find that in the benchmark model, preference shock contributes far more to the risk premiums followed by productivity shock and the least important.

TAB. 1.4: Unit Roots Test

	Test Statistic	
Variable	ADF	PP
Growth Rate of GDP	-6.435*	−9.242*
Growth Rate of Consumption	-4.369*	−7.95*
Rate of Inflation	−2.102	−3.148**
Interest Rates Spread 10 year - 3 month	−3.854*	−4.274*

Note : **,* indicate significance at the 1%, 5% levels, respecively.

TAB. 1.5: SMM Estimation

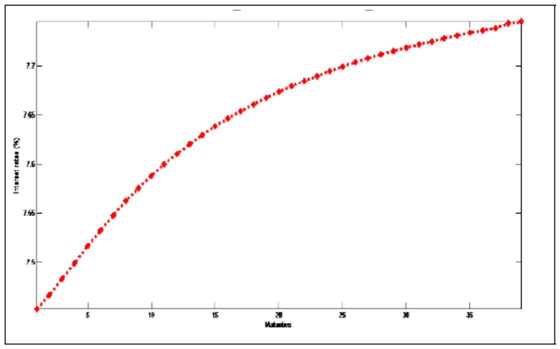
Description	Symbol	Estimates
Persistence parameter of productivity shock	ρ_a	0.981
		(0.955, 0.985)
Persistence parameter of preferences shock	ρ_u	0.553
		(0.303, 0.672)
Standard deviation of productivity shock	σ_a	0.011
		(0.007, 0.013)
Standard deviation of preferences shock	σ_u	0.047
		(0.039, 0.061)
Standard deviation of monetary policy shock	$\sigma_m p$	0.95×10^{-4}
		$(0.83 \times 10^{-4}, 0.27 \times 10^{-3})$

Note : block bootstrap 95% confidence intervals in parenthesis

TAB. 1.6: Model implied term structure statistics, baseline calibrated parameters

	Value(%)
TP_2	0.1
TP_3	0.2
TP_4	0.5
TP_{10}	1.8
r	4
I1	8
I2	8.5
I3	8.57
I4	8.87
I10	10

FIG. 1.1: Average Term Structure of Interest Rates



Baseline parameters

FIG. 1.2: Impulses Responses to Monetary Policy Shock

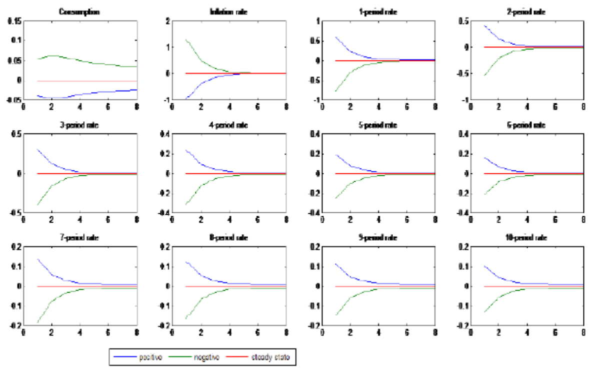


FIG. 1.3: Impulses Responses to Productivity Shock

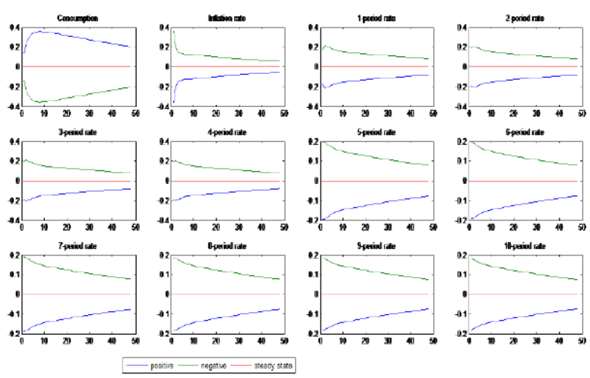


Fig. 1.4: Impulses Responses to Preferences Shock

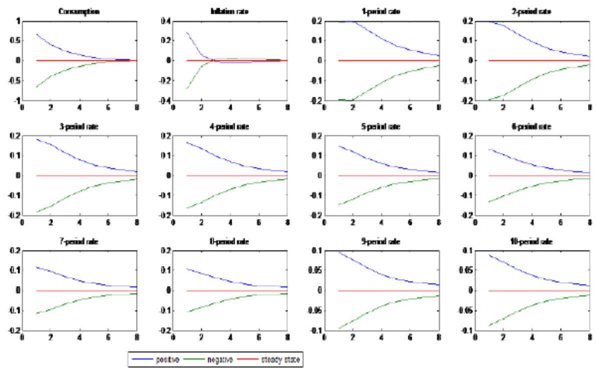


Fig. 1.5: Shocks Contribution to Risk Premia

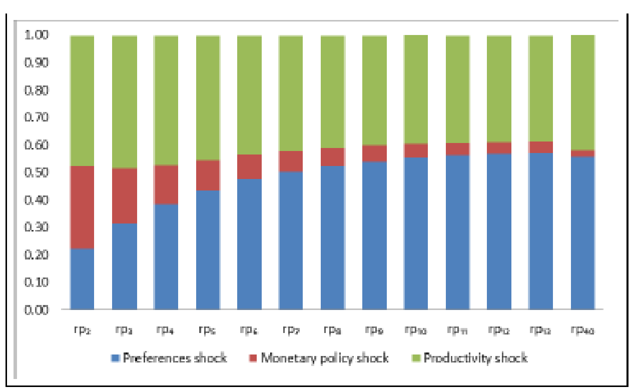


FIG. 1.6: Scaling Parameters of Risk Premia
$rp^a_\ell,\, rp^z_\ell, rp^{mp}_\ell$
$(rp_\ell = rp^a_\ell \sigma^2_a + rp^z_\ell \sigma^2_z + rp^{mp}_\ell \sigma^2_{mp})$

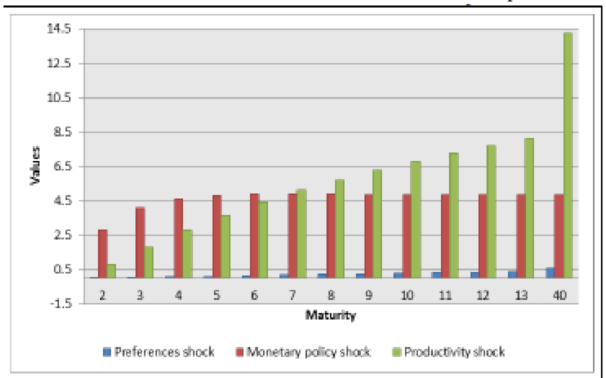
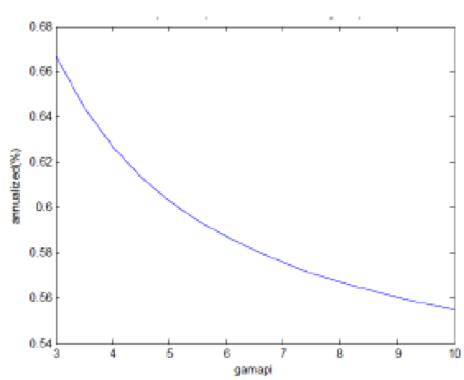


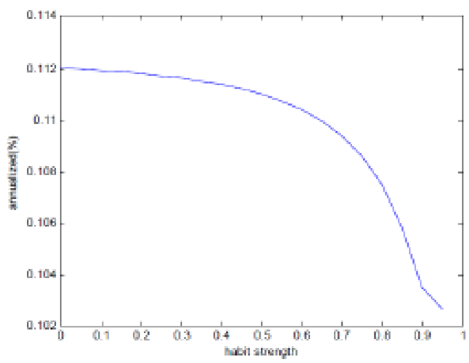
FIG. 1.7: The Effect of Monetary Policy Action (γ_π) on Risk Premium (10 - year)



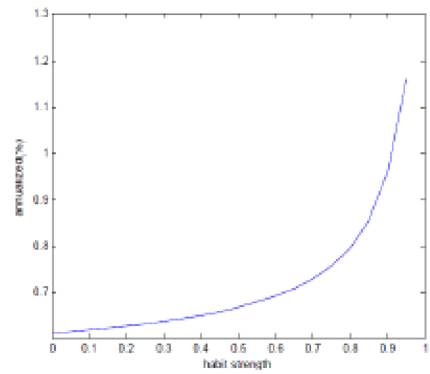
All the remaining parameters are set at
their baseline values

FIG. 1.8: The Effect of Habit Formation (b) on Risk Premium (10 - year)

No Capital Adjutment Cost : $\varphi = 0$ *With Capital Adjutment Cost :* $\varphi = \infty$

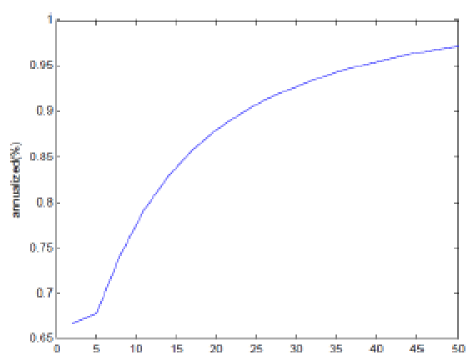


All the remaining parameters are set at their baseline values



All the remaining parameters are set at their baseline values

FIG. 1.9: The Effect of Consumption Curvature (γ_c) on Risk Premium (10 - year)



All the remaining parameters are set at their baseline values

Chapitre 2

Time - varying Volatility and Risk Premia in General Equilibrium

2.1 Introduction

As documented by a large number of empirical works, the term structure of interest rates contains important economic information including agents' expectations about future interest rates and future inflation (see, for example, Frederic Minsky 1990a, 1991). When economic agents are risk averse, the term structure of interest rates depends on private sector agents' expectations about future short-term interest rates and risk premia. Furthermore, risk premia are empirically found to be time-varying (see, Campbell and Shiller, 1991) and correlated to economic uncertainty factors.

The time-varying property of risk premia is crucial for the accuracy and usefulness of the information extracted from the term structure of interest rates since the information extracted is usually based on the assumption that risk premia are constant over time. For example, if risk premia are time - varying, a tightening monetary policy effect on long-term rates may be offset by a decline in the risk premium as it was the case in the U.S. economy between 2004 and 2006. The federal reserve gradually increased the federal funds rate by 425 basis points while long term interest rates remained surprisingly low. This behavior contrasted with movements of long term rates in past monetary policy tightenings and has been viewed by many analysts as a "conundrum". [1] In an attempt to crack this "conundrum", empirical work including Cochrane and Backus (2007), Rudebusch et al (2007) have pointed out that the risk

[1] See Cochrane and Backus (2007), Rudebusch et al (2007) for this issue called the "Greenspan Conundrum" in the finance literature

premium may have declined in recent years to offset the increases of the federal funds rate. Similarly, Kurmann and Otrok (2011) find in a VAR framework that long-term interest rates do not respond to productivity news shocks because the responses of the risk premium part and the expectations part offset each other.

On the other hand, since risk premia are compensation for uncertainty in asset payoffs, it is crucial to understand whether different sources of uncertainty affect them in the same way if a policy maker has to respond to risk premia variations.

This paper provides a quantitative analysis of the term structure of nominal bond interest rate where risk premia are time-varying. The analysis is conducted using a New Keynesian dynamic stochastic general equilibrium (DSGE) model with recursive preferences and stochastic volatility. The analysis focuses on the role played by the nature of economic shocks in the level as well as the variability of interest rates and risk premia. This is motivated by two reasons : first, empirical studies in macroeconomics and finance have pointed out that time - varying volatility is a prominent feature of the U.S. post war data and is essential to understand asset prices analysis and economic decisions under uncertainty (see, for example, Engle, 1995, Hamilton, 2010). On the other hand, Hamilton (2010) shows that misspecifying the conditional volatility in macroeconomic models can also have an impact on the mean of the variables. Second, recent studies including Rudebusch and Swanson (2010), Binsbergen, Fernandez - Villaverde, Koijen and Rubio - Ramirez (2010) have shown that DSGE models wherein households have recursive preferences can replicate business cycle and asset prices data as opposed to standard preferences.

The term structure of interest rates and risk premia in general equilibrium models have attracted a large amount of literature. Jerman (1998), Lettau (2003) and Lettau and Uhlig (1999) among others, have studied asset prices and risk premia in real business cycle (RBC) models. Studies including Rudebusch *et al* (2008), Ravenna and Seppala (2006), Bianca De Paoli *et al* (2010), and Hordahl *et al* (2007) have analyzed the implications of standard New Keynesian models for the term structure of interest rates. More recently, Rudebusch and Swanson (2012), van Binsbergen, Fernandez - Villaverde, Koijen and Rubio - Ramirez (2010), Andreasen (2012), Doh (2013), analyze the term structure of interest rates in DSGE frameworks where consumers display recursive preferences. Doh (2013) estimates an endowment economy with long - run risk and stochastic volatility (SV). The findings of the paper show that time - varying term premium is more driven by inflation volatility shock than by consumption growth volatility shock contrary to previous findings. van Binsbergen, Fernandez - Villaverde, Koijen and Rubio - Ramirez (2010) extend Doh (2013) work to a production economy but with constant volatility in the shocks and exogenous inflation dynamics. The maximum likelihood estimates of their baseline model indicate large risk aversion, large capital adjustment costs, and an elasticity of intertemporal substitution (EIS) larger than one. The article by Rudebusch and Swanson (2012) uses calibrated full-fledged New Keynesian model with recursive preferences, firms specific capital, and long - run risk. They find that recursive preferences combined with long - run risk in monetary policy and productivity shocks are capable of replicating salient features of business cycle and asset prices simultaneously.

In this work, we focus on the contribution of volatility risk to the mean as well as the dynamics of interest rates and risk premia in a New Keynesian production economy with recursive preferences and stochastic volatility. The model features real rigidities by allowing adjustment cost in capital and habit formation. Unlike in previous studies (as in Rudebusch and Swanson, 2012), the capital input in the production function is variable. More specifically, we examine the role played by each source of uncertainty - including shocks volatility uncertainty - in the determination of the level as well as the dynamics of the risk premium. This is important for economic stabilization because the results of a policy responses to exogenous shocks depend on risk premia. As a sensitivity exercise, we compare the habit formation preferences effect on the size and dynamics of bond risk premia when the capital stock is fixed and when the capital stock can be adjusted costlessly.

It is challenging to study the term structure of interest rates in a DSGE model. Especially, risk premia are difficult to compute because DSGE models are non-linear systems and analytical solutions are unavailable for the general case. Numerical methods such as value function iteration (VFI) or policy function iteration (PFI) are computationally infeasible because of the large number of state variables. Since the model does not have an exact analytical solution, we use perturbation method that involves taking a third-order expansion of the policy rules around the deterministic steady state. For detailed explanations of this approach, see Jin and Judd(2002), Schmitt-Grohé and Uribe(2004), and Kim, Kim, Schaumburg and Sims(2008), Martin Andreasen (2011). Perturbation methods deliver a zero risk-premium at first-order approximation due to the certainty equivalence property at first-order ; and a constant risk-premium at second-order approximation. A third-order approximation (at least) is needed to obtain a time-varying risk premia as observed in the data.

Moreover, some parameters of the model are estimated by Simulated Method of Moments (SMM) and the remaining carefully calibrated to the U.S. economy at a quarterly frequency. This method is appealing for the estimation of nonlinear DSGE models because, as shown in Lee and Ingram (1991), and Duffie and Singleton (1993), it delivers consistent and asymptotically normal estimates. In addition, Ruge-Murcia (2007) shows that it is generally robust to misspecification and computationally more efficient as compared to alternative methods such as Generalized Method of Moments

(GMM) or Maximum Likelihood.

It is shown from the second - and third - order approximated solutions that, stochastic volatility induces an Autoregressive Conditionally Heteroscedastic in mean (ARCH - M) type process for the decision rules as in Engle, Lilien and Robins (1987). The difference between the ARCH - M process in this model and the purely statistical ARCH - M is that the parameters here are restricted to structural parameters and the conditional volatility is that of macroeconomic shocks instead of the conditional volatility of the decision rules themselves. It follows that the conditional volatility has a first order effect at third - order approximation and induces additional dynamics. Thus, as in Engle, Lilien and Robins (1987), the conditional volatility affects the conditional mean of the risk premium at third - order approximations.

To understand the effect of the presence of stochastic volatility on the term structure of interest rates, we carry also out an estimation under which the model shocks volatilities are restricted to be homoscedastic. The SMM estimates under the benchmark (unrestricted) model show evidence of time - varying volatility in monetary policy, preferences and productivity shocks. Moreover, we find that a higher risk aversion coefficient, a higher habit formation parameter and a larger capital adjustment costs are needed to match the data under the constant volatility model.

The model predicts positive risk premia leading to an upward sloping average yield curve. With regard to the levels of the shocks, the findings can be summarized as follows. The level of productivity shock has a shifting effect on the yield curve whereas monetary policy and preferences shocks affect the slope of the yield curve.

As for the volatility shocks the main drivers of interest rates and risk premia are productivity and preferences volatility shocks with a limited role for monetary policy volatility shock. The dominant volatility shock is the productivity volatility shock. Productivity volatility shock affects negatively short - term interest rate (1 - period and 2 - period) and positively long - term interest rates. That means that productivity volatility shock affects the slope of the yield curve. Preferences volatility shock, on the contrary, has a negative effect on interest rates for all maturities. However, it decreases more short - term interest rates than long - term rates. Therefore, positive productivity and preferences volatility shocks steepen the yield curve. This implies that time - varying real uncertainty induces additional dynamics in the term structure of interest rates.

Moreover, a positive productivity volatility shock increases risk premia for all maturities and the impact is increasing with maturity meaning that it increases more long - term risk premia than short - term risk premia. On the other hand, preferences volatility shock has a negative effect on risk premia. A positive preferences volatility shock decreases risk premia at all maturities and the impact is decreasing with maturity meaning that it decreases more long - term premia than short - term premia. The responses of the term structure to monetary policy volatility shock is negligible.

The rest of the paper is organized as follows. Section 2.2 presents some stylized facts on the term structure of interest rates. Section 2.3 describes the model and section 2.4 discusses the derivation of interest rates and risk premia from the equilibrium conditions as functions of macroeconomic factors. In Section 2.5, we present the solution method of the model. The econometric method (SMM) is explained in section 2.6. Finally, Section 2.7 discusses the implications of the model for interest rates and risk premia and presents the results.

2.2 Stylized Facts of Term Structure of Bond Interest Rates

The goal of this part is to make a quick review of some key term structure of interest rates stylized facts. We use six bond interest rates to compute selected statistics : the three-month (3m), six-month (6m), twelve-month (1y) maturity interest rates are Treasury Bill rates while the three-year (3y), five-year (5y) and ten-year (10y) maturity interest rates are Treasury constant maturity yields. The raw data used are taken from the FRED data base of the Federal Reserve Bank of St. Louis and available at their website (*www.stls.frb.org*) except the 1y interest rate series which is from Gürkaynak, Refet S., Brian Sack and Jonathan H. Wright (2007) dataset. All interest rates data are daily observations at the source from 1962 to 2007. The sample period is between 1962Q2 to 2007Q4. Quarterly observations are obtained by taking the first trading day observation of the second month of each quarter (i.e. February, May, August, November) instead of averaging over the quarter.[2]

Table 2.1 summarizes some key features of the term structure. First, the term structure of interest rates is upward sloping on average over the entire sample period. The unconditional empirical means (annualized) of interest rates range from 5.48% for the three-month maturity rate to 6.83% for the ten-year rate. The average ten-year - three-month nominal interest rates spread is positive (135 basis points). This means that, on average, the slope of the yield curve is positive and that long-term rates exceed short-term rates over the sample period. Second, the volatility of the yields is decreasing with maturity meaning that the term structure of volatilities is downward sloping. However, the rate of decrease in the volatility is very low across maturities. The three-month interest rate volatility is only 18 basis points larger than the volatility of the ten-year maturity rate. It is clear from table 2.1 that risk

[2] The results of avering over the quarters differ significantly only in terms of variances. The variances of the quarterly series obtained by averaging over the quarters are significantly smaller than those of taking the first trading day of the second month of the quarter.

TAB. 2.1: Selected Term Structure Statistics :Sample : $1962Q1 - 2001Q3$

Maturity (n)		3m	6m	1y	3y	5y	10y
Means							
	Yields (i^n)	5.48	5.63	6.02	6.42	6.61	6.83
	Excess return($xhpr_n$)	-	18	25	102	109	178
	Spreads ($i^n - i^1$)	-	15	54	94	113	135
Standard deviations							
	Yields (i^n)	2.52	2.44	2.32	2.40	2.36	2.34
	Excess return ($xhpr_n$)	-	2.02	3.87	22.34	21.06	23.70
	Spreads ($i^n - i^1$)	-	0.22	0.43	2.01	1.87	1.7
Autocorr		0.92	0.92	0.92	0.93	0.94	0.95

Note : Yield means and all standard deviations are annualized and expressed in percent. Excess holding period returns and yield spreads means are in basis points. For each maturity n, $xhpr$ is the return from holding an n-period bond one period minus the 1-period interest rate. $xhpr$ are computed using the formulae : $xhpr^n_{t+1} = hpr^n_{t+1}$ -i_t where $hpr^n_{t+1} = \log(Q^{n-1}_{t+1})$- $\log(Q^n_t)$ and Q^n_t is the time t price of the n-period bond,

premiums, measured here by the excess holding period return ($xhpr$), are important in size as well as in variability. Excess holding period return is increasing in maturity on average and is time-varying over the sample period. Holding a ten-year bond for one period is expected to yield on average 103 basis points ($xhpr_{40} = 178\ bp$) more than the current three-month bond interest rate. Long-term risk premia are very volatile relative to short-term risk premia with the volatilities structure ranging from 2%, for the six-month risk premium, to 23.7% for the ten-year risk premium. Moreover, interest rates are very persistent with the autocorrelation coefficient of long-term maturities slightly higher than those of short-term rates.

2.3 The Model

The model features a standard New Keynesian economy wherein households and firms optimize. Consumers derive utility from a composite consumption good and leisure. The composite good is produced by a representative firm with a continuum of intermediate inputs goods produced by monopolistically competitive firms. Prices stickiness is modelled according to Rotemberg (1982) quadratic adjustment scheme.

Consumers can save resources by using nominal bonds or capital. There is a central banker who adjusts the nominal short-term interest rate according to a Taylor-type rule.

2.3.1 Households

The representative household utility function features recursive preferences as in Epstein and Zin (1989). Following Rudebusch and Swanson (2012), the value function is defined as :

$$V_t = \begin{cases} u(c_t, n_t) + \beta \left(E_t V_{t+1}^{1-\varphi}\right)^{\frac{1}{1-\varphi}} & \text{if } u(c_t, n_t) > 0 \\ -u(c_t, n_t) + \beta \left(E_t(-V_{t+1})^{1-\varphi}\right)^{\frac{1}{1-\varphi}} & \text{if } u(c_t, n_t) < 0 \end{cases} \tag{2.1}$$

where V_t is time t value function, u is the felicity function (periodic utility function), E_t is the mathematical expectation given the time t information set, $\beta \in (0, 1)$ is the subjective discount factor, and φ is the Epstein - Zin parameter ($\varphi \in \mathbb{R}$).

The periodic utility function features habit formation in consumption and is separable in labour. That is, u is defined as :

$$u(c_t, n_t) = d_t \left\{ \frac{(c_t - bc_{t-1})^{1-\gamma}}{1-\gamma} - \phi_0 z_t^{*(1-\gamma)} \frac{{n_t}^{1-\phi}}{1-\phi} \right\}$$

where $b \in [0, 1)$ is habit strength parameter, d_t is a preferences shock that affects both intertemporal and intratemporal conditions, ϕ_0 is a positive parameter, n_t is hours worked, ϕ captures the elasticity of labour supply parameter, c_t is a composite index of a continuum of intermediate goods, c_t^i, $i \in [0\ 1]$. As in Rudebusch and Swanson (2012), z_t^* is the trend of the economy and the term $z_t^{*(1-\gamma)}$ assures a balance growth path and accounts for non-market labour production activities.

The composite consumption index c_t is defined by :

$$c_t = \left[\int_0^1 (c_t^i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

The parameter θ is the elasticity of substitution between the individual goods. As $\theta \to \infty$, intermediate goods become closer substitutes and the weaker the firm's power on these goods.

The preferences shock d_t process features stochastic volatility and is defined as :

$$\log(d_t) = \rho_d \log(d_{t-1}) + u_t^d \tag{2.2}$$

where $\rho_d \in (-1,1)$ and u_t^d is the disturbances term. We allow the conditional volatility of the preferences shock to be time - varying. That is $u_t^d = \sigma_{d,t}\varepsilon_t^d$ where ε_t^d is an independently and identically distributed with mean zero and standard deviation one and $\sigma_{d,t}$ is the time t conditional volatility of u_t^d. We assume that the process of $\sigma_{d,t}$ is defined by :

$$\log(\sigma_{d,t+1}) = (1-\rho_{\sigma^d})\log(\overline{\sigma}_d) + \rho_{\sigma^d}\log(\sigma_{d,t}) + \sigma_{\sigma^d}\zeta_{t+1}^d \tag{2.3}$$

where $\rho_{\sigma^d} \in (-1,1)$ is the autocorrelation parameter of $\sigma_{d,t}$, $\overline{\sigma}_d$ is the unconditional mean $\sigma_{d,t}$ and σ_{σ^d} is a positive parameter ; ζ_t^d is an independently and identically distributed ($i.i.d.$) with mean zero and standard deviation one and uncorrelated with ε_t^d. Notice that modelling a process of $\log(\sigma_{d,t})$ instead of the level $\sigma_{d,t}$ itself in (2.3) assures that the standard deviation is always positive.

Rudebusch and Swanson (2012) find that these preferences combined with long - run risk in monetary policy and technology shocks are capable of replicating empirical asset prices along with business cycle features[3]. This is because, contrary to the constant relative risk aversion (CRRA) preferences, recursive preferences break the linkage between the risk aversion parameter and the intertemporal elasticity of substitution (IES). For example, in the above specification (2.1) (without habit formation) the EIS is given by $1/\gamma$ whereas the the agent's relative risk aversion involves both parameters γ and φ. A measure of the relative risk aversion in steady state can be approximated by : $\gamma + \varphi(1-\gamma)/(1-b)$. When $\varphi = 0$, the recursive preferences specification collapses to the standard case of constant relative risk aversion (CRRA) utility function specification. When $\gamma > 1$, the lower φ, the higher the relative risk aversion and vice - versa when $\gamma < 1$.

In addition to consumption spending and labor supply, the consumer must decide how much resources to allocate in assets including investment and a range of nominal bonds of maturities indexed by $\ell = 1, \dots, L$. Resources include labour income, capital income, and holding of the portfolio of bonds. The consumer period t budget constraint is

$$\int_0^1 \frac{p_t^i c_t^i}{P_t} di + \Upsilon_t^{-1} x_t + \sum_{\ell=1}^{L} \frac{Q_t^\ell B_t^\ell}{P_t} = \frac{W_t n_t}{P_t} + \frac{R_t k_t}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^{\ell-1} B_{t-1}^\ell}{P_t} + \frac{S_t}{P_t}, \tag{2.4}$$

where p_t^i is the price of intermediate good i, P_t is aggregate price level, x_t is investment, Q_t^ℓ and B_t^ℓ are, respectively, nominal price and holding of bond with maturity

[3] Other work that use recursive preferences in macro - finance models include : Hordhal, Tristani and Vestin (2008), Binsbergen, Fernandez-Villaverde, Koijen and Rubio-Ramirez (2012), Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2013)

ℓ , W_t is nominal wage, R_t is nominal rental rate per unit of capital, k_t is capital and S_t nominal lump-sum tax or transfer. Note that an ℓ-period bond at time $t-1$ becomes an $(\ell-1)$-period bond at time t. Υ_t is the relative price of investment in terms of consumption good which is assumed exogenous. The growth rate of Υ_t is deterministic and given by :

$$\log(\Upsilon_t) = \log(\mu^{\Upsilon}) + \log(\Upsilon_{t-1}) \tag{2.5}$$

where μ^{Υ} is the long-run gross growth rate Υ_t.

The law of motion of the capital stock is given by :

$$k_{t+1} = (1-\delta)k_t + x_t - \Gamma\left(\frac{x_t}{k_t}\right)k_t, \tag{2.6}$$

where $\delta \in (0,1)$ is the capital depreciation rate. Capital accumulation is subject to adjustment costs. To get one unit of capital, the agent has to invest an additional cost of $\Gamma\left(\frac{x_t}{k_t}\right)k_t$ which depends on the size of investment relative to current existing capital stock. $\Gamma(.)$ has the following properties : $\Gamma^{''}(.) > 0$, $\Gamma(\upsilon) = 0$, $\Gamma^{'}(\upsilon) = 0$ where υ is the steady state of the investment - capital ratio $\frac{x_t}{k_t}$. Intuitively, these properties implies that the adjustment cost depends on net investment relative to the current capital stock. For simplicity, we assume a quadratic functional form for $\Gamma(.)$ which has the above properties as in Andreasen et al (2013). That is,

$$\Gamma\left(\frac{x_t}{k_t}\right) = \frac{\kappa}{2}\left(\frac{x_t}{k_t} - \upsilon\right)^2$$

where κ is a positive parameter which controls the size of the adjustment cost. Given an investment-capital ratio $\frac{x_t}{k_t}$, the larger κ is, the higher the adjustment cost. When $\kappa = 0$, there is no adjustment cost and the agent can freely change the capital stock. When $\kappa = \infty$ there is an infinite adjustment cost and the agent may choose not to invest in equilibrium. Notice that the existence of variable capital in the model provides the agent with an additional channel for consumption smoothing. Higher adjustment costs in capital makes nominal bonds riskier and allows the model to generate higher bond risk premia.

In a first stage the consumer shops intermediate goods for production of the composite good. Given a level of the composite good, the consumer chooses the inputs c_t^i, $i \in [0\ 1]$ that minimize the total cost $\int_0^1 p_t^i c_t^i di$. This implies that demand for any intermediate good i is given by :

$$c_t^i = \left[\frac{p_t^i}{P_t}\right]^{-\theta} c_t,$$

where the aggregate price level P_t is given by :

$$P_t = \left[\int_0^1 (p_t^i)^{1-\theta} di\right]^{\frac{1}{1-\theta}},$$

Using the demand functions and price expressions above, it is easy to show that the quantity of composite consumption index times the aggregate price index is equal to total purchases of intermediate goods :

$$P_t c_t = \int_0^1 p_t^i c_t^i di,$$

Plugging this expression in (2.4), the budget constraint takes the following form :

$$c_t + \Upsilon_t^{-1} x_t + \sum_{\ell=1}^{L} \frac{Q_t^\ell B_t^\ell}{P_t} = \frac{W_t n_t}{P_t} + \frac{R_t k_t}{P_t} + \sum_{\ell=1}^{L} \frac{Q_t^{\ell-1} B_{t-1}^\ell}{P_t} + \frac{S_t}{P_t}, \tag{2.7}$$

Thus, household maximizes (2.1) subject to (2.7) and (2.6).

The first-order conditions for the consumer's problem are derived from a Lagrangian problem. Following Rudebusch and Swanson (2010) the Lagrangian of the consumer problem is given by :

$$V_t + E_t \sum_{t=s}^{\infty} \beta^s \left\{ \eta_{t+s} \left[V_{t+s} - u(c_{t+s}, n_{t+s}) - \beta \left(E_t ({V_{t+s+1}}^{1-\varphi}) \right)^{\frac{1}{1-\varphi}} \right] \right\} -$$
$$E_t \sum_{t=s}^{\infty} \beta^s \left\{ \lambda_{t+s} \left[c_{t+s} + \Upsilon_{t+s}^{-1} x_{t+s} + \sum_{\ell=1}^{L} \frac{Q_{t+s}^\ell B_{t+s}^\ell}{P_{t+s}} - \frac{W_{t+s} n_{t+s}}{P_{t+s}} - \frac{R_{t+s} k_{t+s}}{P_{t+s}} - \sum_{\ell=1}^{L} \frac{Q_{t+s}^{\ell-1} B_{t+s-1}^\ell}{P_{t+s}} - \frac{S_{t+s}}{P_{t+s}} \right] \right\} -$$
$$E_t \sum_{t=s}^{\infty} \beta^s \left\{ q_{t+s} \lambda_{t+s} \left[k_{t+s+1} - (1-\delta) k_{t+s} - x_{t+s} + \Gamma\left(\frac{x_{t+s}}{k_{t+s}}\right) k_{t+s} \right] \right\}$$

The first order conditions include respectively Euler equations for V_{t+1}, c_t, k_{t+1}, x_t, n_t, and $\frac{B_t^\ell}{P_t}$:

$$\beta \eta_t {V_{t+1}}^{-\varphi} \left(E_t ({V_{t+1}}^{1-\varphi}) \right)^{\frac{1}{1-\varphi}-1} - \beta E_t \eta_{t+1} = 0, \tag{2.8}$$

$$d_t \eta_t (c_t - b c_{t-1})^{-\gamma} - \lambda_t - \beta b E_t \left\{ d_{t+1} \eta_{t+1} (c_{t+1} - b c_t)^{-\gamma} \right\} = 0, \tag{2.9}$$

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[r_{t+1} + q_{t+1}(1-\delta) + q_{t+1} \Gamma\left(\frac{x_{t+1}}{k_{t+1}}\right) - q_{t+1} \frac{x_{t+1}}{k_{t+1}} \Gamma'\left(\frac{x_{t+1}}{k_{t+1}}\right) \right] \right\}, \tag{2.10}$$

$$q_t \Upsilon_t \left[1 - \Gamma' \left(\frac{x_t}{k_t} \right) \right] = 1, \tag{2.11}$$

$$\lambda_t w_t = \eta_t \phi_0 d_t z_t^{*(1-\gamma)} {n_t}^{-\phi}, \tag{2.12}$$

$$Q_t^\ell = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{Q_{t+1}^{\ell-1}}{\pi_{t+1}} \right) \text{,for } \ell\text{=1,2,...L}, \tag{2.13}$$

where λ_t and η_t are the budget constraint (2.7) and the value function constraint (2.1) Lagrangian multipliers respectively, $r_t = \frac{R_t}{P_t}$ is the real return on capital, $\pi_{t+1} = P_{t+1}/P_t$ is the gross rate of inflation between time t, and $t+1$, $w_t = \frac{W_t}{P_t}$ is real wage and q_t is the ratio of Lagrangian multipliers of constraint (2.7) and (2.6), that is the Tobin's q.

2.3.2 Firms

Firms are of two types : a competitive final good producer and a continuum of monopolistically competitive firms indexed by $i \in [0\ 1]$ which produce intermediate goods.

Final Good Producer

The final good producer behaves in a perfectly competitive manner and takes as given the prices of intermediate goods and the aggregate price index when maximizing profits. Final good is produced using only individual goods y_t^i as inputs in the following production function :

$$y_t = \left[\int_0^1 (y_t^i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where y_t is the quantity of the final good. Profit maximization implies that demand of input i is given by :

$$y_t^i = \left[\frac{p_t^i}{P_t} \right]^{-\theta} y_t, \tag{2.14}$$

Intermediate Goods Firms and Price Setting

Each individual firm $i \in (0\ 1)$ produces a differentiated good using the same technology given by the following production function :

$$y_t^i = A_t F(K_t^i, Z_t N_t^i), \tag{2.15}$$

where y_t^i is output, K_t^i is firm i capital demand, N_t^i is labor input and the function $F(.,.)$ is constant return to scale, strictly increasing and strictly concave in both of its arguments and satisfy the Inada conditions, A_t is a neutral stationary technology shock, Z_t is a productivity trend that affects all firms in the same way. Intermediate good producing firm $i \in (0\,1)$ hires labor and capital in perfectly competitive markets to produce its good. Firms are owned by households who receive any profit made by firms at each period. The trend productivity shock growth is deterministic and follows the process :

$$\log(Z_t) = \log(\mu^z) + \log(Z_{t-1}), \tag{2.16}$$

where μ^z is the unconditional growth rate of Z_t. The neutral technology shock follows the process

$$\log(A_t) = \rho_a \log(A_{t-1}) + u_t^a, \tag{2.17}$$

where $\rho_a \in (-1, 1)$ is the autocorrelation parameter of $\log(A_t)$, and u_t^a is the innovations term. The conditional volatility of u_t^a is time - varying. That is the disturbances term is defined as $u_t^a = \sigma_{a,t}\varepsilon_t^a$ where ε_t^a is an independently and identically distributed $(i.i.d.)$ with mean zero and standard deviation one and $\sigma_{a,t}$ is the time t conditional volatility of u_t^a. We assume that the process of $\sigma_{a,t}$ is defined by :

$$\log(\sigma_{a,t+1}) = (1 - \rho_{\sigma_{\sigma^a}}) \log(\overline{\sigma}_a) + \rho_{\sigma_{\sigma^a}} \log(\sigma_{a,t}) + \sigma_{\sigma^a}\zeta_{t+1}^a \tag{2.18}$$

where $\rho_{\sigma_{\sigma^a}}$ is the autocorrelation parameter of $\sigma_{a,t}$, $\overline{\sigma}_a$ is the unconditional mean $\sigma_{a,t}$ and σ_{σ^a} is a positive parameter ; ζ_t^a is an independently and identically distributed $(i.i.d.)$ with mean zero and standard deviation one and uncorrelated with ε_t^a.

Prices are set according to the Rotemberg (1982) model. That is, when adjusting their prices firms face a quadratic cost which is proportional to aggregate output :

$$\frac{\theta_p}{2} y_t \left(\frac{p_t^i}{p_{t-1}^i} \frac{1}{\pi_{ss}} - 1 \right)^2$$

where θ_p is a positive parameter capturing the size of the prices adjustment cost and π_{ss} is steady state inflation rate. The parameter θ_p also captures the degree of nominal price rigidity. Notice that the adjustment costs increase with the size of the prices change as well as the aggregate output. In the steady state, there is no adjustment costs.

The firm $i's$ problem is to choose K_t^i, N_t^i, p_t^i to maximize discounted profits subject to its good demand function, the production technology (2.15) and the price setting scheme. This can be done in two steps : first choose the capital and labor input to minimize the real cost given the production function (2.15) and given the real wage and capital rental rates. Second choose the price to maximize the discounted real profits subject to the demand function and given the aggregate price and quantities.

The real cost minimization program is :

$\underset{K_t^i,\, N_t^i}{Min}\ [w_t N_t^i + r_t K_t^i]$

s.t $\ y_t^i = A_t (K_t^i)^{\alpha} (Z_t N_t^i)^{1-\alpha}$

$$w_t = mc_t (1-\alpha) A_t (K_t^i)^{\alpha} {Z_t}^{1-\alpha} N_t^{i-\alpha}$$

$$r_t = mc_t \alpha A_t (K_t^i)^{\alpha-1} {Z_t}^{1-\alpha} N_t^{i1-\alpha}$$

where mc_t is the Lagrangian multiplier of the production function constraint. The first order conditions imply that :

$$\frac{K_t^i}{N_t^i} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} \tag{2.19}$$

Thus, all firms will choose the same capital-labor ratio. Using the above relations in the cost function, the real cost is given by :

$$Cost_t = w_t N_t^i + r_t K_t^i = mc_t y_t^i = \frac{1}{1-\alpha} w_t N_t^i$$

Use the production function and (2.19) to express N_t^i as a function of y_t^i, w_t, and r_t and substitute into the cost function to get :

$$Cost_t = \frac{y_t^i}{A_t}\left[\frac{w_t}{1-\alpha}\right]^{1-\alpha}\left[\frac{r_t}{\alpha}\right]^{\alpha}$$

From the above two expressions of the cost function, it follows that the real marginal cost (the derivative of the real cost with respect to y_t^i) is equal to the Lagrangian multiplier mc_t and is given by :

$$mc_t = \frac{1}{A_t}\left[\frac{w_t}{1-\alpha}\right]^{1-\alpha}\left[\frac{r_t}{\alpha}\right]^{\alpha} \tag{2.20}$$

Note that the real marginal is independent of i meaning that all firms incur the same marginal cost.

Now in the second step, firms pick their price p_t^i to maximize :

$$E_t \sum_{s=t}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t}\left[\frac{p_{t+s}^i}{P_{t+s}} - mc_{t+s} - \frac{\theta_p}{2}\frac{y_{t+s}}{y_{t+s}^i}\left(\frac{p_{t+s}^i}{p_{t+s-1}^i}\frac{1}{\pi_{ss}} - 1\right)^2\right] y_{t+s}^i$$

subject to :

$$y_{t+s}^i = \left[\frac{p_{t+s}^i}{P_{t+s}}\right]^{-\theta} y_{t+s}$$

After replacing the demand function constraint in the objective function, the first order condition with respect to p_t^i is given by :

$$\frac{y_t}{P_t}\left[\frac{p_t^i}{P_t}\right]^{-\theta} - \theta\frac{y_t}{P_t}\left[\frac{p_t^i}{P_t}\right]^{-\theta-1}\left[\frac{p_t^i}{P_t} - mc_t\right] - \theta_p\frac{y_t}{p_{t-1}^i}\left[\frac{p_t^i}{p_{t-1}^i}\frac{1}{\pi_{ss}} - 1\right]\frac{1}{\pi_{ss}} + \beta\theta_p E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\frac{p_{t+1}^i}{p_t^i}\frac{1}{\pi_{ss}} - 1\right]\frac{p_{t+1}^i}{p_t^i}\frac{y_{t+1}}{p_t^i}\frac{1}{\pi_{ss}}\right\} = 0$$

Since all firms face the same demand function and marginal cost, they will choose the same price in equilibrium for the same quantity of output. That is, we have a symmetric case where $p_t^i = P_t$ and $y_t^i = y_t$ $\forall t$. With the symmetry assumption, the first order condition gives the dynamics of inflation as :

$$mc_t = \frac{\theta - 1}{\theta} + \frac{\theta_p}{\theta}\frac{\pi_t}{\pi_{ss}}\left[\frac{\pi_t}{\pi_{ss}} - 1\right] - \beta\frac{\theta_p}{\theta}E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\frac{\pi_t}{\pi_{ss}} - 1\right]\frac{\pi_{t+1}}{\pi_{ss}}\frac{y_{t+1}}{y_t}\right\} \tag{2.21}$$

The production side equilibrium conditions are given by equations (2.14) - (2.21).

2.3.3 Monetary Policy Rule and Government

The government issues the nominal bonds and is able to control the short term nominal interest rate through open market operations. Bond issues are consistent with a zero deficit. The government budget constraint is given by :

$$\frac{S_t}{P_t} = \sum_{\ell=1}^{L}\frac{Q_t^{\ell-1}B_{t-1}^{\ell}}{P_t} - \sum_{\ell=1}^{L}\frac{Q_t^{\ell}B_t^{\ell}}{P_t}$$

The model is closed with a Talor-type policy rule whereby the monetary authority sets the one-period nominal interest rate as a function of inflation and output deviations from targeted levels.

$$\frac{1+i_t}{1+i^{ss}} = \left(\frac{1+i_{t-1}}{1+i^{ss}}\right)^{\rho_i}\left(\frac{1+\pi_t}{1+\pi^{ss}}\right)^{(1-\rho_i)\gamma_\pi}\left(\frac{y_t}{z_t^*\widetilde{y}^{ss}}\right)^{(1-\rho_i)\gamma_y}\exp(u_t^{mp}) \tag{2.22}$$

where i_t is the time t one-period nominal bond interest rate, u_t^{mp} is monetary innovation, ρ_i, γ_π, γ_y are constant policy parameters, and i^{ss}, π^{ss}, $\widetilde{y}^{ss}$ are steady values of the short term nominal interest rate, inflation and the stationary level of output $\frac{y_t}{z_t^*}$ respectively. The conditional volatility of u_t^{mp} is time - varying with $u_t^{mp} = \sigma_{m,t}\varepsilon_t^{mp}$ where $\sigma_{m,t}$ is the conditional volatility of u_t^{mp} . The conditional volatility process is defined as :

$$\log(\sigma_{m,t+1}) = (1-\rho_{\sigma^m})\log(\overline{\sigma}_{mp}) + \rho_{\sigma^m}\log(\sigma_{m,t}) + \sigma_{\sigma^m}\zeta_{t+1}^{mp} \qquad (2.23)$$

where $\rho_{\sigma^m} \in (-1,1)$ and $\overline{\sigma}_{mp}$, σ_{σ^m} are positive parameters; ζ_t^{mp} is an independently and identically distributed with mean zero and standard deviation one and uncorrelated with ε_t^{mp}.

2.3.4 Market Clearing and Aggregation

Using the symmetry assumption the aggregate output is given by :
$y_t = A_t(K_t)^\alpha(Z_tN_t)^{1-\alpha}$
In equilibrium all markets must clear every period :
$c_t + \Upsilon_t^{-1}x_t = y_t$
$k_{t+1} = (1-\delta)k_t + x_t - \frac{\kappa}{2}\left(\frac{x_t}{k_t} - \upsilon\right)^2 k_t$

$n_t = N_t = \int_0^1 N_t^i di$

$k_t = K_t = \int_0^1 K_t^i di$

$S_t = \sum_{\ell=1}^{L}\frac{Q_t^{\ell-1}B_{t-1}^{\ell}}{P_t} - \sum_{\ell=1}^{L}\frac{Q_t^{\ell}B_t^{\ell}}{P_t}$

2.3.5 Stationary Equilibrium

Since there is growth in the model due to productivity and investment shock growths, we transform the system by dividing each nonstationary variable by the relevant growth rate. Following Altig, Christiano, Eichenbaum (2005) and Andreasen et al (2013) the economy technology progress trend is defined as $z_t^* = \Upsilon_t^{\frac{\alpha}{1-\alpha}} z_t$. That means that aggregate variables such as consumption, output, real wage will grow at the growth rate of z_t^* whereas investment, and capital grow at the growth rate of $\Upsilon_t z_t^*$. We denote the transformed stationary variables with a ˜

The stationary system is defined as : $\widetilde{c}_t = \frac{c_t}{z_t^*}$, $\widetilde{y}_t = \frac{y_t}{z_t^*}$, $\widetilde{x}_t = \frac{x_t}{\Upsilon_t z_t^*}$, $\widetilde{w}_t = \frac{w_t}{z_t^*}$, $\widetilde{k}_{t+1} = \frac{k_t}{\Upsilon_t z_t^*}$, $\widetilde{q}_t = q_t \Upsilon_t$, $\widetilde{r}_t = r_t \Upsilon_t$, $\widetilde{V}_t = \frac{V_t}{z_t^{*1-\gamma}}$, $\widetilde{\lambda}_t = \frac{\lambda_t}{\eta_t z_t^{*-\gamma}}$. Thus, the stationary equilibrium is given by :

$$\left(\frac{E_t({V_{t+1}}^{1-\varphi})}{V_{t+1}}\right)^{\frac{\varphi}{1-\varphi}} = \frac{\eta_{t+1}}{\eta_t} \tag{2.24}$$

$$\widetilde{\lambda}_t = d_t(\widetilde{c}_t - b\widetilde{c}_{t-1}\frac{z_{t-1}^*}{z_t^*})^{-\gamma} - \beta b E_t\left\{d_{t+1}\frac{\eta_{t+1}}{\eta_t}(\widetilde{c}_{t+1}\frac{z_{t+1}^*}{z_t^*} - b\widetilde{c}_t)^{-\gamma}\right\}, \tag{2.25}$$

$$\widetilde{q}_t = \beta E_t\left\{\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z_{t+1}^*}{z_t^*}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\frac{\Upsilon_t}{\Upsilon_{t+1}}\left[\widetilde{r}_{t+1} + \widetilde{q}_{t+1}(1-\delta) + \widetilde{q}_{t+1}\Gamma(\frac{\widetilde{x}_{t+1}}{\widetilde{k}_{t+1}}\frac{z_{t+1}^*}{z_t^*}\frac{\Upsilon_{t+1}}{\Upsilon_t})\right]\right\} - \beta E_t\left\{\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z_{t+1}^*}{z_t^*}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\frac{\Upsilon_t}{\Upsilon_{t+1}}\left[\widetilde{q}_{t+1}\frac{z_{t+1}^*}{z_t^*}\frac{\Upsilon_{t+1}}{\Upsilon_t}\frac{\widetilde{x}_{t+1}}{\widetilde{k}_{t+1}}\Gamma'(\frac{\widetilde{x}_{t+1}}{\widetilde{k}_{t+1}}\frac{z_{t+1}^*}{z_t^*}\frac{\Upsilon_{t+1}}{\Upsilon_t})\right]\right\}, \tag{2.26}$$

$$\widetilde{q}_{t+1}\left[1 - \Gamma'\left(\frac{x_t}{k_t}\frac{z_t^*}{z_{t-1}^*}\frac{\Upsilon_t}{\Upsilon_{t-1}}\right)\right] = 1, \tag{2.27}$$

$$\widetilde{\lambda}_t\widetilde{w}_t = \phi_0 d_t {n_t}^{-\phi} \tag{2.28}$$

$$Q_t^\ell = \beta E_t\left(\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z_{t+1}^*}{z_t^*}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\frac{Q_{t+1}^{\ell-1}}{\pi_{t+1}}\right)\text{,for } \ell\text{=1,2,...L,} \tag{2.29}$$

$$\widetilde{w}_t = mc_t(1-\alpha)A_t(\widetilde{k}_t)^\alpha {n_t}^{-\alpha}$$

$$\widetilde{r}_t = mc_t\alpha A_t(\widetilde{k}_t)^{\alpha-1}{n_t}^{1-\alpha}$$

$$mc_t = \frac{\theta-1}{\theta} + \frac{\theta_p}{\theta}\frac{\pi_t}{\pi_{ss}}\left[\frac{\pi_t}{\pi_{ss}} - 1\right] - \beta\frac{\theta_p}{\theta}E_t\left\{\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z_{t+1}^*}{z_t^*}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\left[\frac{\pi_t}{\pi_{ss}} - 1\right]\frac{\pi_{t+1}}{\pi_{ss}}\frac{\widetilde{y}_{t+1}}{\widetilde{y}_t}\frac{z_{t+1}^*}{z_t^*}\right\} \tag{2.30}$$

$$\frac{1+i_t}{1+i^{ss}} = \left(\frac{1+i_{t-1}}{1+i^{ss}}\right)^{\rho_i}\left(\frac{1+\pi_t}{1+\pi^{ss}}\right)^{(1-\rho_i)\gamma_\pi}\left(\frac{\widetilde{y}_t}{\widetilde{y^{ss}}}\right)^{(1-\rho_i)\gamma_\pi}\exp(mp_t) \tag{2.31}$$

$$\widetilde{y}_t = \widetilde{c}_t + \widetilde{x}_t \tag{2.32}$$

$$k_{t+1} = (1-\delta)k_t \left[\frac{z^*_{t+1}}{z^*_t}\frac{\Upsilon_{t+1}}{\Upsilon_t}\right]^{-1} + x_t - \frac{\kappa}{2}\left(\frac{x_t}{k_t}\frac{z^*_t}{z^*_{t-1}}\frac{\Upsilon_t}{\Upsilon_{t-1}} - \upsilon\right)^2 k_t \left[\frac{z^*_{t+1}}{z^*_t}\frac{\Upsilon_{t+1}}{\Upsilon_t}\right]^{-1} \tag{2.33}$$

In the following section, we review the relation between the bond prices implied by the economic model and the term structure of interest rates, and define risk premia. Thus, interest rates and risk premia are derived as functions of macroeconomic fundamentals.

2.4 Interest Rates and Risk Premia in DSGE Models

In this section, we provide an explicit relationship between bond interest rates, risk premia and prices derived from the model. The intention is only to be explicit about the variables used in the empirical analysis. Following the literature, the interest rate (gross) of a one-period bond is given by

$$i^1_t = \frac{1}{Q^1_t} \tag{2.34}$$

More generally, the gross nominal interest rate of the ℓ-period bond is defined as

$$i^\ell_t = \left[Q^\ell_t\right]^{\frac{-1}{\ell}} \tag{2.35}$$

Here the overall risk involved in long-term nominal bonds is twofold : first, there is a risk of capital loss in the future in case the investor wants to sell the bond before the maturity date. Because the bond future prices are not known with certainty in advance, the eventual resale[4] price might be less than the purchase price. Second, there is an inflation risk involved in nominal long-term bonds because inflation can erode the bond value in the future. The risk premium can be derived recursively by rewriting the Euler equation of bonds demand as,

$$Q^\ell_t = Q^1_t E_t\left(Q^{\ell-1}_{t+1}\right) + \beta cov_t\left(Q^{\ell-1}_{t+1}, \frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z^*_{t+1}}{z^*_t}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\frac{1}{\pi_{t+1}}\right), \tag{2.36}$$

where we used the fact that the one-period bond price is

[4] For example in case of a negative realization of an income shock somewhere between t and $t+\ell$, an $\ell-$period bond holder would like to redeem the bond in order to smooth its consumption

$$Q_t^1 = E_t\left(\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z_{t+1}^*}{z_t^*}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\frac{1}{\pi_{t+1}}\right). \tag{2.37}$$

There are various formulas of risk premiums in the literature but Rudebusch *et al.* (2007) show that all definitions are highly correlated. For example, the $\ell-$period term-premium, denoted by $TP_{\ell,t}$, is usually defined as the difference between an $\ell-$period interest rate and expected average of short-term rates over the maturity period, that is,

$$TP_{\ell,t} = i_t^\ell - \frac{1}{\ell}E_t\sum_{s=0}^{\ell-1} i_{1,t+s} \tag{2.38}$$

In this paper, the risk premium is defined as the excess holding period return, that is, the return from holding an $\ell-$period bond for one period relative to the current return of the one-period bond[5]. To obtain an expression for risk premium, we rewrite (2.36) as

$$E_t\left[\frac{Q_{t+1}^{\ell-1}}{Q_t^\ell}\right] = \frac{1}{Q_t^1} - cov_t\left[\frac{Q_{t+1}^{\ell-1}}{Q_t^\ell}, \beta\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z_{t+1}^*}{z_t^*}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\frac{1}{1+\pi_{t+1}}\frac{1}{Q_t^1}\right] \tag{2.39}$$

Assume that an investor buys an $\ell-$period bond at time t and holds it just for one period. At time $t+1$, an $\ell-$period bond will be sold as an $(\ell-1)$ maturity bond. Thus, the gross return of holding an $\ell-$period for one period $H_{\ell,t+1}$ is given by :

$$H_{\ell,t+1} = \frac{Q_{t+1}^{\ell-1}}{Q_t^\ell}$$

Plugging the previous expression in (2.39) the Euler equation of the $\ell-$period bond becomes

$$E_t(H_{\ell,t+1}) = i_{1,t} + rp_t^\ell \tag{2.40}$$

where $rp_t^\ell = -cov_t\left[H_{\ell,t+1}, \beta\frac{\widetilde{\lambda}_{t+1}}{\widetilde{\lambda}_t}\left[\frac{z_{t+1}^*}{z_t^*}\right]^{-\gamma}\frac{\eta_{t+1}}{\eta_t}\frac{1}{1+\pi_{t+1}}i_{1,t}\right]$ is the holding period risk-premium. It is easy to show that the two definitions of risk premia are related as

[5] Computationally, the excess holding period return requires less complementary state variables definition than the term premium

$$TP_{\ell,t} = \frac{1}{\ell} E_t \sum_{s=0}^{\ell-1} rp_{t+s}^{\ell-s},$$

meaning that the term-premium is an average of all expected holding period risk-premia over the maturity period of the bond.

Equation (2.40) means that after adjusted for risk factor, the holding-period return is a predictor of the one-period interest rate. Note that the covariance term in the risk premium expression can either be positive or negative depending on the sign of the covariation between the holding-period return and the nominal discount factor. When high future marginal utilities- that is situations where investors need more consumption- tend to be associated with capital losses ($Q_{t+1}^{\ell-1}$ is low relative to Q_t^{ℓ} when reselling an $\ell-$period bond at $t+1$), investors will claim a positive risk premium for holding a long-term bond instead of short-term bonds. Moreover, the two sources of risk in long-term nominal bonds highlighted above are present in the risk premium formula. First, the risk premium is affected by the co-movement between the holding-period return and the real stochastic discount factor keeping the inflation rate constant. Second, the correlation between the holding-period return and the future inflation rate, keeping the real stochastic discount factor constant, also determines the sign and the size of the risk premia. In the first case, the resulting risk premium will be referred as the real risk premium and in the second case the inflation risk premium. The sign and the magnitude of the total risk premium will depend on the combination of these two covariance effects.

2.5 Model Solution

The primary focus of this paper is to understand the role played by each source of uncertainty in level and variance of interest rates and risk premia. Notice that an exact analytical solution is not available in this model. Thus, we use a perturbation method to approximate the solution of the model given the parameters. Basically, perturbation method consists in taking Taylor series expansion of the decision rules around the deterministic steady state. For detailed explanations of this approach, see Jin and Judd(2002), Schmitt-Grohé and Uribe(2004), and Kim, Kim, Schaumburg and Sims(2008). At first - order approximations, time - varying uncertainty shocks do not affect the decision rules and risk premia are equal to zero due to certainty equivalence at first-order. At second-order approximations, only the average level of shocks volatility enter in the decisions and risk premia are constant. Time-varying uncertainty effects the dynamics of risk premium at orders of approximation greater than three. Therefore, we solve and estimate the model at third - order approximation. The third-order approximation solution properties are provided by Martin Andreasen[6] *et* al (2013). Due to the large number of variables involving the term structure of interest rates, computing the third-order directly in Matlab requires a lot of computer memory. Thus, we use Dynare (version 4.4.3) to obtain the third-order solution.[7]

The standard approach of perturbation method writes the model general equilibrium conditions in the form :

$$E_t F(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \tag{2.41}$$

where E_t is the conditional expectation given the time t information set, y_t is the vector of control variables and x_t the predetermined endogenous variables and exogenous processes. F is a vectorial function of all the equilibrium conditions. In this model the control variables vector is composed of $\widetilde{c}_t$, $\widetilde{y}_t$, $\widetilde{x}_t$, $\widetilde{w}_t$, $\widetilde{k}_{t+1}$, $\widetilde{\lambda}_t$, $\widetilde{q}_t$, $\widetilde{r}_t$, $\widetilde{V}_t$, π_t, mc_t, $\{i_t^\ell\}_{\ell=1}^{\ell=L}$, whereas the state vector contains k_t, A_t, d_t, ψ_t, mp_t, $\mu_t^z = \frac{z_t}{z_{t-1}}$, $\mu_t^\Upsilon = \frac{\Upsilon_t}{\Upsilon_{t-1}}$, σ_t^d, σ_t^a, σ_t^{mp}, c_{t-1}, i_{t-1}^1

The solution of the model is given by :

$$y_t = g(x_t, \sigma) \tag{2.42}$$

$$x_{t+1} = h(x_t, \sigma) + \sigma \Sigma \varepsilon_{t+1} \tag{2.43}$$

where h and g are unknown functions, ε_t is the innovations vector of the exogenous shocks, Σ is a constant matrix driving the variances of the innovations and σ is a scaling perturbation parameter driving the size of the uncertainty in the economy. Given that h and g are unknown, the procedure consists of approximating the functions h and g around the non-stochastic steady state point $(x, 0)$ where uncertainty is removed. Schmitt-Grohé and Uribe(2004) show that h_σ, g_σ, $h_{x\sigma}$, and $g_{x\sigma}$ evaluated at the approximated point (steady) are equal to zero. Martin Andreasen (2011) proved that at the steady state point $h_{\sigma xx} = 0$, $g_{\sigma xx} = 0$. Moreover, in the case of symmetric shocks (for example normal distribution), the terms $h_{\sigma\sigma\sigma} = 0$, $g_{\sigma\sigma\sigma} = 0$. However in the case of non-symmetric shocks (rare disaster shocks for example), these coefficients may be different from zero.[8]

The approximate solution takes the form :

[6] See also Ruge - Murcia(2010)

[7] Dynare software package is available at http ://www.dynare.org. For detailed explanations see Michel Julliard(2004)

[8] These results are also shown in Ruge - Murcia (2012)

$$y_t = y + \frac{1}{2}g_{\sigma\sigma}\sigma^2 + g_x(x_t - x) + \frac{1}{2}\left[g_{xx}\right]_{\alpha_1\alpha_2}\left[x_t - x\right]^{\alpha_1}\left[x_t - x\right]^{\alpha_2} + \frac{1}{6}\left[g_{xxx}\right]_{\alpha_1\alpha_2\alpha_3}\left[x_t - x\right]^{\alpha_1}\left[x_t - x\right]^{\alpha_2}\left[x_t - x\right]^{\alpha_3} + \frac{3}{6}\left[g_{\sigma\sigma x}\right]_{\alpha_3}\sigma^2\left[x_t - x\right]^{\alpha_3} + \frac{1}{6}g_{\sigma\sigma\sigma}\sigma^3 \quad (2.44)$$

$$x_{t+1} = x + \frac{1}{2}h_{\sigma\sigma}\sigma^2 + \frac{1}{6}h_{\sigma\sigma\sigma}\sigma^3 + h_x(x_t - x) + \frac{1}{2}\left[h_{xx}\right]_{\alpha_1\alpha_2}\left[x_t - x\right]^{\alpha_1}\left[x_t - x\right]^{\alpha_2} + \frac{1}{6}\left[h_{xxx}\right]_{\alpha_1\alpha_2\alpha_3}\left[x_t - x\right]^{\alpha_1}\left[x_t - x\right]^{\alpha_2}\left[x_t - x\right]^{\alpha_3} + \frac{3}{6}\left[h_{\sigma\sigma x}\right]_{\alpha_3}\sigma^2\left[x_t - x\right]^{\alpha_3} + \sigma\eta\varepsilon_{t+1} \quad (2.45)$$

where $x = h(x, 0)$ and $y = g(x, 0) = g(h(x, 0), 0)$ and n_y and n_x are the number of control and state variables respectively, $\alpha_1, \alpha_2, \alpha_3 = 1, ...n_x$. g_x, h_x, g_{xx}, h_{xx}, g_{xxx}, h_{xxx} $h_{\sigma\sigma}$, , $g_{\sigma\sigma}$, $h_{\sigma\sigma x}$, $g_{\sigma\sigma x}$, $h_{\sigma\sigma\sigma}$, $g_{\sigma\sigma\sigma}$ are constant coefficients standing for first, second, and third derivatives of g and h with respect to x and σ evaluated at the deterministic steady state. Notice that these coefficients are functions of the structural parameters of the model and that the parameter σ enters the decision rules as an argument capturing the risk factors. Also, the conditional volatilities of the innovations in the state vector are time - varying and enter directly in the decision rules.

Since the third order solution is computed using Dynare, the decision rules are expressed as functions of $(x_{t-1}, \varepsilon_t, \sigma)$ instead of (x_t, σ). Notice that the above representation (2.42) and (2.43) of the solution can be recovered from the Dynare representation by redefining the state vector as $v_t = (x_{t-1}, \varepsilon_t)$ as in Andreasen et al (2013). Then, it is easy to show that the equilibrium solution is expressed as :

$$y_t = g(v_t, \sigma)$$

$$v_{t+1} = \overline{h}(v_t, \sigma) + \sigma\tilde{\eta}\varepsilon_{t+1}$$

where $\overline{h}(v_t, \sigma) = (h(v_t, \sigma), 0)'$ and $\tilde{\eta} = (\eta, 0)'$.

2.6 Econometric Analysis

2.6.1 Data

The model is estimated using U.S. macroeconomic as well as term structure data at the quarterly frequency. The sample period is 1962 Q1 -2007 Q4.

The macro data used are *per capita* real consumption growth, *per capita* real investment growth, real wage inflation rate, *per capita* hours worked, and Consumer Price Index (CPI) inflation rate. The consumption variable is obtained by adding NIPA measures of personal consumption expenditure on nondurable goods and services. On the other hand, investment is the sum of private fixed nonresidential investment and personal expenditure on durable goods. *Per capita* real variables (investment and consumption) are obtained by dividing these variables by the quarterly CPI and the Bureau of Economic Analysis (BEA) estimate of the mid-month U.S. population. We use average weekly (*per capita*) hours of production and nonsupervisory employees in the manufacturing sector as a measure of the model hours worked. Since the time endowment is normalized to one in the model, we assume a time endowment of 120 ($5\times$ 24) which corresponds to five working days per week and divide each observation of the original hours worked series by 120. All the macro data are seasonally adjusted and are collected from the Federal Reserve Bank of St. Louis website (*www.stls.frb.org*) and .

The term structure of interest rates data are the three-month nominal interest rate, the ten-year nominal interest rate as well as the ten - year excess holding period return. Notice that, the model counterparts of the three-month nominal interest rate, the ten-year nominal interest rate as well as the ten - year are i_t^1, i_t^{40} and rp_t^{40} respectively. The three-month rate is daily treasury bill rate whereas the ten-year interest rate is daily constant maturity rate and are taken from the Federal Reserve Bank of St. Louis website. Quarterly observations are obtained by taking the first trading day observation of the second month of each quarter[9] (February, May, August, November). Excess holding period return series is computed using continuously - compounded yields from Gürkaynak, Refet S., Brian Sack and Jonathan H. Wright (2007) dataset. In all, eight data series have been used in the estimation.

2.6.2 Paramaters Estimation : Simulated Method of Moments (SMM)

We estimate the parameters of the model by Simulated Method of Moments (SMM). This method consists in minimizing a weighting distance between uncondi-

[9]Instead of averaging over the quarter

tional moments predicted by the model and their corresponding data counterparts. Basically, the predicted moments are based on artificial series simulated from the model while the data moments are directly computed from actual data.

Let $\theta \in \Theta$ be the unknown $k \times 1$ parameters vector of the DSGE model. Suppose, we have T observations of stationary and ergodic economic data series $\{q_t\}$. Let's denote by

$\frac{1}{T}\sum_{t=1}^{T} m(q_t)$ a set of p moments computed from the data where $p \geq k$[10]. For given values of parameters θ we can compute the same set of moments from artificial data simulated from the model. Assume that the sample size of the simulated data is $\tau \times T$ and denote these moments by :

$\frac{1}{T\tau}\sum_{t=1}^{T\tau} m(q_t(\theta))$ where $\tau \geq 1$ is an integer.

The SMM estimator of θ is defined by :

$$\widehat{\theta}_{SMM} = \arg\max_{\theta \in \Theta} M(\theta)' \; W \; M(\theta)$$

where $M(\theta) = \frac{1}{T}\sum_{t=1}^{T} m(q_t) - \frac{1}{T\tau}\sum_{t=1}^{T\tau} m(q_t(\theta))$ and W is a $p \times p$ positive-definite weighting matrix. Thus, the SMM estimator $\widehat{\theta}_{SMM}$ is the value of the parameters vector θ that minimizes the distance between the set of data moments and those implied by the model. As Shown in Ruge - Murcia (2007), the asymptotic distribution of $\widehat{\theta}_{SMM}$ is normal and the asymptotic variance matrix is given by :

$$(1 + \frac{1}{\tau}) \left(J'WJ \right)^{-1} J'WSWJ \left(J'WJ \right)^{-1} \tag{2.46}$$

where $J = E\left(\frac{\partial m(q_t(\theta))}{\partial \theta}\right)$ and S is the long-run variance matrix of the sample moments vector. Notice that when the number of simulated samples $\tau \rightarrow +\infty$, the SMM asymptotic variance matrix converges to that of the Generalized Method of Moments (GMM).

Simulated method of moments is suitable for large nonlinear DSGE models estimation because it provides consistent and asymptotically normal parameter estimates.[11] Moreover, Ruge-Murcia (2007) shows that SMM is generally robust to misspecification and is computationally more efficient than alternative methods.[12]

[10] As mentionned in Ruge-Murcia (2010) this is a necessary condition for identification

[11] See, Lee and Ingram (1991), Duffie and Singleton (1993).

[12] For example, the paper shows that the computing time of SMM is less than tthat of GMM and Maximum Likelihood. Moreover, the objective function is easier to compute under SMM.

Ruge-Murcia (2012) provides the properties of SMM estimates for third-order approximation of DSGE models. The Monte - Carlo evidence on small samples shows that SMM based asymptotic standard errors tend to overestimate the actual variances of the parameters. However due to the large number of variables and parameters in this model, to obtain a bootstrap type standard errors will be computationally very expensive at third - order approximation. Thus, we approximate the standard errors of the estimates by the estimates asymptotic variance matrix in (2.46).

Artificial data are obtained by simulating the model based on the pruned version of the third-order approximate solution proposed by Martin Andreasen, Jesús Fernández-Villaverde and Juan Rubio-Ramírez (2013). We assume normal distribution of the innovations in the simulations. Thirty-two moments are used in the estimation : the variances, first- and second-order autocovariances as well as the unconditional means of the eight data series. The weighting matrix used is the diagonal of Newey-West estimator of long-run variances of the moments with a Bartlett kernel and bandwidth given by the integer of $4(\mathrm{T}/100)^{2/9}$ where T is the sample size. The sample size here is T=182 which implied a bandwidth value of 4.569. The number of the simulated observations is ten times the sample size T.

The number of estimated parameters is twenty : five preferences parameters $\beta, b, \gamma, \varphi, \phi$; five shock levels parameters including the persistence (ρ_a, ρ_d) and unconditional standard deviation $(\overline{\sigma}_a, \overline{\sigma}_d, \overline{\sigma}_{mp})$ parameters of productivity, preferences and monetary policy shocks respectively ; six conditional volatility shocks parameters including the persistence $(\rho_{\sigma_a}, \rho_{\sigma_d}, \rho_{\sigma_m})$ and standard deviation $(\sigma_{\sigma_a}, \sigma_{\sigma_d}, \sigma_{\sigma_m})$ parameters of productivity, preferences and monetary policy shocks, respectively ; three monetary policy reaction parameters, $\gamma_\pi, \gamma_y, \rho_i$; and the capital adjustment cost parameter κ. Thus $\theta = [\beta,\ b,\ \gamma,\ \varphi,\ \phi,\ \kappa,\ \gamma_\pi,\ \gamma_y,\ \rho_i,\ \rho_a,\ \rho_d,\ \rho_m,\ \rho_{\sigma_a},\ \rho_{\sigma_d},\ \rho_{\sigma_m},\ \overline{\sigma}_a, \overline{\sigma}_d,$ $\overline{\sigma}_{mp}, \sigma_{\sigma_a},\ \sigma_{\sigma_d},\ \sigma_{\sigma_m}]$. Since the number of moments used is thirty - two, the number of degree of freedom is twelve $(= 32 - 20)$. The remaining parameters are difficult to identify and thus have been calibrated in the next subsection.

Before the estimation test whether the series used in the estimation are stationary as the theoretical properties of SMM estimates are valid under this assumption. To this end, we use an Augmented-Dickey Fuller (ADF) and a Phillips-Perron (PP) unit root tests. The null hypothesis of unit root can be rejected at 5% level under both tests for all series except the inflation rate. However, for the inflation rate, the unit root hypothesis can be rejected at the 5% level under the PP test but cannot be rejected under the ADF test. But the ADF-statistic is -2.38 whereas the critical value is -2.39. So, we suppose that the inflation rate is also stationary.

2.6.3 Calibration

During the estimation, the remaining model parameters have been calibrated as follows :

The production function parameter is set at $\alpha = 0.3$ to match the share of capital income in the U.S. data. Notice that in the model, the unconditional growth rate of consumption is given by the unconditional growth rate of the economy technology progress z_t^*, which from the definition of z_t^*, is given by : $\log(\mu^{z^*}) = \log(\mu^z) + \frac{\alpha}{1-\alpha}\log(\mu^\Upsilon)$ where μ^z and μ^Υ are the unconditional growth rate of z and Υ, respectively. The unconditional growth rate of investment is given by : $\log(\mu^{z^*}) + \log(\mu^\Upsilon)$. Thus, given α, μ^z and μ^Υ are calibrated to match the sample growth rates of consumption (1.005045) and investment (1.006068).

The disutility parameter ϕ_0 is calibrated to match a steady state hours worked of $n_{ss} = 0.34$ as in the data.

From the capital accumulation equation, the depreciation rate of capital is set as $\delta = 1 - (1 - \frac{x_{ss}}{kss})\mu^{z^*}\mu^\Upsilon$. The investment - capital ratio $\frac{x_{ss}}{kss}$ is fixed at 0.025, and given μ^{z^*}, μ^Υ, $\delta = 0.02$. The parameter υ in the capital adjustment cost function is then set such that there is no adjustment cost in the steady state. That is, $\upsilon = \frac{x_{ss}}{kss}\mu^{z^*}\mu^\Upsilon$.

Since there is no prices adjustment cost in the steady state, the model steady state mark-up Ψ is given by the expression $\Psi = \frac{\theta}{\theta-1}$. θ is set such that the long - run mark - up (gross) $\Psi = 1.1$; that is, $\theta = 11$.

The Rotemberg (1982) prices adjustment cost parameter θ_p is set such that the first order inflation dynamics is equivalent to that of a model with Calvo (1983) pricing. That is, $\theta_p = \frac{(\theta-1)\eta}{(1-\theta)(1-\beta\theta)}$ where η is the Calvo parameter, θ is the elasticity of substitution among goods and β is the subjective discount factor. The Calvo parameter is set at 0.75 to match an average price duration of 4 quarters and the subjective discount factor β is estimated.

The steady state of gross inflation rate π_{ss} is fixed as 1.008 to account for an annualized long - run inflation rate of 3.2%. The calibrated parameters are reported in table 2.2.

2.6.4 Parameters Estimates

Table 2.3 reports the SMM estimates of the parameters. For a sake of comparison we also report the estimates of the parameters under the restricted model of homoscedastic shocks. The first column reports the estimates when the conditional variances of the shocks follow stochastic volatility processes and the second column when shock volatilities are constant.

Results under stochastic volatility (in column 1) show that there is evidence of time - varying volatility in productivity, preferences and monetary policy shocks. The estimates indicate that productivity and preferences shocks are very persistent and volatile. The autocorrelation coefficient of productivity shock is $\rho_a = 0.948$ and the unconditional standard deviation is $\overline{\sigma}_a = 0.012$. The conditional volatility of the productivity shock is also very persistent - with an autocorrelation coefficient of $\rho_{\sigma_a} = 0.8$ - and volatile ($\sigma_{\sigma_a} = 0.42$). These estimates are similar to the estimates reported in Justiniano and Primiceri (2008).

The preferences shock is highly persistent ($\rho_d = 0.982$) and volatile ($\overline{\sigma}_d = 0.014$). The conditional volatility is moderately persistent ($\rho_{\sigma_d} = 0.605$) and as volatile as the productivity shock ($\sigma_{\sigma_d} = 0.4$).

The monetary policy shock has been constrained to an *i.i.d.* process and the estimated unconditional standard deviation is large and statistically significant ($\overline{\sigma}_{mp} = 0.001$). The conditional volatility autocorrelation coefficient is small ($\rho_{\sigma_m} = 0.4$) and not statistically different from zero but the unconditional standard deviation of the innovations ($\sigma_{\sigma_m} = 0.001$) is significantly different from zero.

The preferences parameters are in line with those reported in the literature. The subjective discount factor is $\beta = 0.9926$. There is evidence of moderate habit formation ($b = 0.57$) which is slightly lower than the standard reported value of 0.65. The estimates of the consumption curvature parameter is $\gamma = 1.57$. The Epstein - Zin parameter φ which is crucial for the relative risk aversion is estimated to be -167 which is higher than the reported value of -194 in Andreasen et al (2013). The interpretation of a negative Epstein - Zin parameter is that agents prefer to rather solve today an expected future uncertainty. This implies that any change in expected future volatility will affect today agents decision. Notice that the elasticity of intertemporal substitution ($1/\gamma = 0.64$) is less than one with a very high risk aversion. The estimates of the Frisch elasticity of labour supply is less than one ($1/\phi = 0.15$).

The capital adjustment cost parameter estimates is moderate ($\kappa = 3.57$). The central bank reaction to deviations of inflation from the long - run inflation is higher ($\gamma_\pi = 3.225$) than its reaction to deviations of output from the steady state ($\gamma_y = 0.430$). The policy rate displays inertia with a moderate short - term interest rate smoothing parameter of $\gamma_R = 0.67$.

Now we turn to compare the results of the estimations under the benchmark model and under the restricted assumption of constant volatility. In general, the main differences between the two models reside in the estimates of the risk aversion parameters, monetary policy shocks, and the real rigidities. The Epstein - Zin parameter under the constant volatility of shocks implies a higher relative risk aversion ($\varphi = -199.35$). The consumption curvature parameter is also slightly higher ($\gamma = 1.79$) as well as the habit formation parameter ($b = 0.77$). The most striking difference is on the monetary policy shock. Under the constant volatility case, the standard deviation of the monetary policy shock is very small ($\overline{\sigma}_{mp} = 1.47 \times 10^{-5}$) and not statistically different from zero. It means in this case that the dynamics of the model is only driven by productivity and preferences shocks. The difference between the two set of estimates under the time - varying and constant volatility outlines the claim by Hamilton (2010) that a conditional variance misspecification may have a first order effect on the conditional means.

2.7 Results

We present below the implications of the second - and third-order approximate solution of the model for interest rates and risk premia and perform some sensitivity exercises.

2.7.1 Implications for the Term Structure

To understand the implications of the model for the term structure of interest rates and risk premia we use the third - order approximated solution in (2.44) to express the interest rates decision rules in the following form :

$$\widehat{i}_t^{\ell} = i_v \left[\widehat{v}_t\right] + \frac{1}{2}\left[i_{vv}^{\ell}\right]_{\alpha_1\alpha_2}\left[\widehat{v}_t\right]^{\alpha_1}\left[\widehat{v}_t\right]^{\alpha_2} + \frac{1}{6}\left[i_{vvv}^{\ell}\right]_{\alpha_1\alpha_2\alpha_3}\left[\widehat{v}_t\right]^{\alpha_1}\left[\widehat{v}_t\right]^{\alpha_2}\left[\widehat{v}_t\right]^{\alpha_3} + \frac{3}{6}\left[i_{\sigma\sigma v}^{\ell}\right]_{\alpha_3}\sigma^2\left[\widehat{v}_t\right]^{\alpha_3} + \frac{1}{2} i_{\sigma\sigma}^{\ell}\sigma^2 + \frac{1}{6} i_{\sigma\sigma\sigma}^{\ell}\sigma^3 \quad (2.47)$$

where $\widehat{i}_t^{\ell}$ is the log deviation from steady state of the ℓ-period maturity bond interest rate ; $\widehat{v}_t = (\widehat{x}_{t-1}, \varepsilon_t)$ and $\widehat{x}_{t-1}$ is a vector of log deviation of the state variables from the steady state. When the shocks are symmetric, the last term $\frac{1}{6} i_{\sigma\sigma\sigma}^{\ell}\sigma^3 = 0$ which implies that third order approximation has no impact on the mean of interest rates since the constant term is equal to the constant term of the second-order approximation. Notice that $\widehat{v}_t$ includes the time $t-1$ volatilities of the shocks (σ_{t-1}^{d}, σ_{t-1}^{a}, σ_{t-1}^{mp}) through the vector $\widehat{x}_{t-1}$ as well as their time t innovations. It means that the time - varying volatilities enter the decision rules as state variables and provide an additional dynamics to the term structure of interest rates. We now present below the model implications of the stochastic volatility for the term structure of interest rates.

First, we compare the prediction of the model with the data by plotting selected term structure of interest rates moments computed from the model against their data counterparts in figure 2.1. Panel A plots the model predicted moments against the data counterparts whereas panel B and C display the unconditional means and standard deviations implied by the model respectively. The moments used in panel A are the unconditional means and standard deviations of interest rates at different maturities. The model moments are computed based on 150000 simulated observations.[13] All moments are transformed in percentage and annualized. The horizontal axis is the simulated moments whereas the vertical axis is the data moments counterparts. The selected maturities are the 3m, 6m, 1y, 2y, 3y, 5y, and 10y. Recall that only the three - month and the ten - year interest rates (the red dots in figure 2.1) were targeted in the estimation. As panel A of figure 2.1 shows the model was able to match relatively well the means of the three - month and ten - year interest rates. However, the model implied standard deviation of the three - month rate is way higher than its data counterparts (4.45 vs 2.51). In panel B and C the horizontal line plots the maturity of the bonds whereas the vertical line is the values of the variables in annualized percentage. As is clear in panel B, the model was able to generate an upward sloping unconditional yield curve with long - term interest rates higher than short - term rates on average as in the data. Moreover, the standard deviations (see panel C) are decreasing across maturities which is also in line with the data. Figure 2.2 plots 4000 observations simulated from the model. It is evident from figure 2.2 that there is a lot of variations in the model generated risk premia and that long - term interest rates are smoother than short - term rates. In all, the model is qualitatively in line with the data with regard to the first and second moments of the term structure of interest rates.

Figure 2.3, 2.4 and 2.5 plot the responses of the term structure to a positive one standard deviation of the levels of the shocks and figure 2.6, 2.7 and 2.8 present the responses of the term structure to their corresponding volatility shocks. The horizontal line is the time after a shock hits the economy whereas the vertical axis measures the response of each variable. The responses to the level of the shocks refer here to responses of the system to innovations of the level of the shocks keeping the conditional volatility fixed (at their unconditional values) and the responses to volatility shocks refer to responses of the system keeping the levels of the shocks unchanged. That is, we examine the responses of the system to first and second moments innovations of the shocks.

With regard to the levels shocks, a positive productivity shock entails a decrease in interest rates at all maturities (figure 2.3). This negative effect is slightly more pronounced for shorter term rates than for longer maturities at the impact time. It means that at the impact time the spread between long - term and short - term rates positive but negligible. Thus this will tend to shift downward the yield curve. The effect of a positive productivity level shock on risk premia differs across maturities.

[13] We simulate 200000 observations from which we discard the first 50000 observations.

Shorter maturity risk premia slightly increase whereas long - term premia tend to decrease. However, the order of the magnitude of the impact is small (10^{-5}). On the other hand, a positive preferences level shock has a negative impact on interest rates (figure 2.4). Short - term interest rates decrease more than long - term rates. Remember, the preferences shock affects directly the consumer intertemporal decisions and the model pricing kernel. Here at the SMM parameter estimates, a positive preferences shock leads to increases in the pricing kernel and bond prices which means a decrease in interest rates. The impact of a positive preferences shock on risk premia is positive and is increasing with the maturity. Long - term risk premia increase more than shorter term premia. Notice that compare to the technology level shock the magnitude of the impact of the preferences level shock is higher. As expected a positive monetary policy level shock increases interest rates of all maturities and the impact decreases with maturities as short - term rates increase more than long - term rates (figure 2.5). The effect vanishes quickly because the persistence parameter of the policy shock were set to zero in the model. The 6m risk premium responds negatively to a positive monetary policy shock whereas other maturity premia increase. However, the magnitude of the impact is very small compare to technology and preferences level shocks.

Now, we examine the effects of the three volatility shocks on the term structure. As figure 2.6 shows, a positive shock to the conditional volatility of technology leads to a decrease in short - term rates and an increase in long - term rates. Risk premia respond positively by increasing and the impact is increasing with the maturity. For example, a one standard deviation increase in the conditional volatility of productivity leads to a more than 20 basis points (annualized) increase in the 10y bond risk premium. Notice that this impact is more important in magnitude than the level shock effects we explored above. As it is also the case for the technology volatility shock, an increase in the conditional volatility of preferences shock leads to a precautionary behavior of the consumer as consumption decreases and investment increases[14]. However, an increase in the preferences shock volatility leads rather to a decrease in interest rates and risk premia. Long - term interest rates decrease less than short - term rates. Remember the risk premium here is the excess holding period return, that is, the expected return of holding a bond for one - period minus the current one - period bond yield. Since interest rates decrease it means that current bond prices increase and future bond prices are expected to increase more than the current price increases. Relative to the current decrease in the one - period interest rate, the expected return of holding a bond for a period is higher and thus agents

[14]See the impulses responses of the macro variables in annexes at the end of the document. Here we focus only on analyzing the term structure

demand less risk premium to hold longer maturity bonds. Monetary policy volatility shock has a negligible effect on the term structure of interest rates and risk premia as the impact is positive but very small (see figure 2.8).

2.7.2 Risk Premia and Volatility Shocks

In this section, we analyze the implications of the model solution to further understand how the volatility shocks affect the levels as well as the variance of risk premia. Since risk premia are compensations for uncertainty, only state variables which involve exposure to uncertainty enter in their formulae. That is, the risk premium decision rules contain terms that involve cross products of volatilities or innovations and potentially the other state variables. For example, the second-order approximation solution will deliver a constant risk premium involving constant structural parameters scaled by the volatilities of the shocks. Thus, the second-order approximation risk-premium denoted by $rp^{\ell 2rd}$ can be written[15] as

$$rp^{\ell 2rd} = \frac{1}{2}rp^{\ell,a}(\overline{\sigma}_a)^2 + \frac{1}{2}rp^{\ell,d}(\overline{\sigma}_d)^2 + \frac{1}{2}rp^{\ell,mp}(\overline{\sigma}_{mp})^2 + \frac{1}{2}rp^{\ell,\sigma_a}(\sigma_{\sigma^a})^2 + \frac{1}{2}rp^{\ell,\sigma_d}(\sigma_{\sigma^d})^2 + \frac{1}{2}rp^{\ell,\sigma_m}(\sigma_{\sigma^m})^2 \quad (2.48)$$

where $rp^{\ell,d}$, $rp^{\ell,a}$, $rp^{\ell,mp}$, rp^{ℓ,σ_d}, rp^{ℓ,σ_a}, rp^{ℓ,σ_m} are functions of structural parameters and $\overline{\sigma}_d^2$, $\overline{\sigma}_a^2$, $\overline{\sigma}_{mp}^2$ are the unconditional volatility of preferences, productivity, and monetary policy shocks respectively and σ_{σ^d}, σ_{σ^a}, σ_{σ^m} are the standard deviations of their respective innovations. Notice that the first line of (2.48) is the risk premium when shocks display constant volatility and the second line take into account the uncertainty involved in the conditional volatility of the shocks. Thus, compare to the constant volatility of shocks case, time - varying volatility has a first order effect and affects the conditional mean of risk premia. Imsyn this model, the constant volatility case is obtained by imposing $\sigma_j = 0$ where $j = \sigma^d$, σ^a σ^m.

At a third - order approximation, risk premia are time - varying as long as the coefficients $g_{\sigma\sigma v}$ corresponding to the risk premium decision rules in (2.44) are different from zero. When the volatilities of the shocks display time variation, this adds more dynamics to the risk premia since the state vector now includes conditional volatilities. In the case of non-symmetric shocks ($g_{\sigma\sigma\sigma} \neq 0$), third-order approximations may affect the level of risk premia. Thus, the model risk premium implied by a third-order approximation denoted by $rp_t^{\ell 3rd}$ takes the form :

[15]With the perturbation parameter σ fixed at 1

$$rp_t^{\ell 3rd} = rp^{\ell 2rd} + \frac{1}{6} rp_{\sigma\sigma\sigma}^{\ell,a} \overline{\sigma}_a^3 + \frac{1}{6} rp_{\sigma\sigma\sigma}^{\ell,d} \overline{\sigma}_d^3 + \frac{1}{6} rp_{\sigma\sigma\sigma}^{\ell,mp} \overline{\sigma}_{mp}^3 +$$
$$\frac{3}{6} \left[rp_{\sigma\sigma,v^-}^{\ell} \right]_{\alpha_3} \left[\widehat{v^-}_t \right]^{\alpha_3} +$$
$$\frac{3}{6} \left\{ \left[rp_{\sigma\sigma,\sigma_a}^{\ell} \right] \widehat{\sigma}_{a,t-1} + \left[rp_{\sigma\sigma,\sigma_d}^{\ell} \right] \widehat{\sigma}_{d,t-1} + \left[rp_{\sigma\sigma,\sigma_m}^{\ell} \right] \widehat{\sigma}_{m,t-1} \right\} +$$
$$\frac{3}{6} \left\{ \left[rp_{\sigma\sigma,\zeta^a}^{\ell} \right] \zeta_t^a + \left[rp_{\sigma\sigma,\zeta^d}^{\ell} \right] \zeta_t^d + \left[rp_{\sigma\sigma,\zeta^{mp}}^{\ell} \right] \zeta_t^{mp} \right\} \quad (2.49)$$

where $rp_t^{\ell 3rd}$ is the time t risk-premium on the ℓ-period bond, $rp_{\sigma\sigma,\sigma_a}^{\ell}$, $rp_{\sigma\sigma,\sigma_d}^{\ell}$, $rp_{\sigma\sigma,\sigma_m}^{\ell}$, $rp_{\sigma\sigma,\zeta^a}^{\ell}$, $rp_{\sigma\sigma,\zeta^d}^{\ell}$, $rp_{\sigma\sigma,\zeta^m}^{\ell}$ are the third - order partial derivatives of $rp_t^{\ell 3rd}$ with respect to σ^2 and σ_a, σ_d, σ_m, ζ^a, ζ^d, ζ^m respectively. v_t^- is the vector of the remaining state variables in v_t excluding $\sigma_{a,t-1}$, $\sigma_{d,t-1}$, $\sigma_{m,t-1}$, ζ_t^a, ζ_t^d, ζ_t^m.

This decomposition of risk premium is interesting because it is similar to the Autorgressive conditional heteroscedasticity in mean (ARCH - M) process in Engle, Lilien and Robin (1987). It implies that, the conditional volatilities have a direct effect on the conditional means of the variables. However, there are two differences between this model and the standard statistical ARCH - M model. First the coefficients in this model are restricted structural parameters and have economic meaning instead of free parameters. Second, the relevant conditional volatility in Engle, Lilien and Robin (1987) is that of the realized risk premium. In this model it is economic agents expectations about future shocks volatility given time t information. Because agents are forward - looking in this model, any change in expected future volatility has an immediate impact on current decision rules and asset prices. Consequently, this decomposition will allow us to investigate the link between risk premia and macroeconomic variables as well as structural parameters.

Andreasen (2011), Ruge - Murcia (2010) among others show that the coefficients $rp_{\sigma\sigma\sigma}^{\ell,j} = 0$, $j = a, d, m$ when the innovations of the shocks display symmetric distributions. Since we assume normal distribution for the innovations, these coefficients are then equal to zero. That means that, third-order approximations will have a small effect on the size of the risk premium compared to second - order approximations. Moreover, when the variances of the shocks are constant over time, the risk premium expression in (2.49) reduces to the first two lines. The last two lines outline the contribution of time - varying volatility to the dynamics of risk premium. Clearly the dynamics of the risk premium will be driven by the state as long as the coefficients $rp_{\sigma\sigma v}^{\ell} \neq 0$. Unlike in the second order approximation case, the price of risk is time-varying at third-order approximation. The dynamics of the price of risk in this case is driven by the shock innovations and the state variables.

We now turn to examine how the three volatility shocks contribute to the level as well as the variations of risk premia at different maturities at the SMM parameter estimates. To that end, the coefficients $rp^{\ell}_{\sigma\sigma,j}$, $j = \sigma_d, \sigma_a, \sigma_m$ in (2.49) for different maturities ℓ are plotted along with the unconditional means and standard deviations of risk premia.

Figure 2.9 explores how these coefficients are related to the unconditional means and standard deviations of risk premia computed based on 150000 simulated observations. The horizontal lines of figure 2.9 are maturity and the vertical line the values of the specified variables. Panel A plots the coefficients associated with the conditional volatility of the technology shock $rp^{\ell}_{\sigma\sigma,\sigma_a}$, panel B the coefficients associated with the conditional volatility of the monetary policy shock, $rp^{\ell}_{\sigma\sigma,\sigma_m}$, panel C the coefficients associated with the conditional volatility of the preferences shock, $rp^{\ell}_{\sigma\sigma,\sigma_d}$, and panel D the unconditional means and standard deviations of risk premium. In panel D the blue dotted line (left scale) represents the unconditional means while the green line (right axis) represents the unconditional standard deviations. It is clear from panel B that monetary policy conditional volatility plays a limited role in the means as well as the average of risk premia as its scaling coefficient $rp^{\ell}_{\sigma\sigma,\sigma_m}$ is negligible and is of order 10^{-9}. Moreover, if there is any contribution of the conditional volatility of the policy shock, that would involve only very short terms risk premia.

Figure 2.9 suggests that unconditional means and standard deviations of risk premia are mainly driven by conditional volatilities of productivity and preferences shocks. The coefficients ($rp^{\ell}_{\sigma\sigma,\sigma_a}$) associated with the productivity conditional volatility are positive while those ($rp^{\ell}_{\sigma\sigma,\sigma_d}$) associated with the preferences conditional volatility are negative for all maturities. It implies that the conditional volatility of productivity shock contributes positively whereas the conditional volatility of preferences shock contributes negatively to the averages of risk premia. Notice that the coefficient $rp^{\ell}_{\sigma\sigma,\sigma_a}$ is increasing with maturity. On the other hand, the coefficient $rp^{\ell}_{\sigma\sigma,\sigma_d}$ is decreasing with maturity at the short end of the yield curve (from 6m to 2y) and increasing from 2y to 10y maturities. This also suggests that differences in the means and standard deviations of risk premia across maturities are partly explained by productivity and preferences conditional volatility. Moreover, the conditional volatility of productivity contributions to the averages and standard deviations of risk premia are increasing with the maturity. With regard to the conditional volatility of preferences shock, the averages of risk premium are more negatively affected from 3m to 2y - maturities. The reverse is true for maturities greater than 2y. The contributions to risk premia standard deviations is increasing with maturity from 3m to 2y maturities and decreasing after the 2y maturity.

Now, we explore how changes in some structural parameters affect the means and variances of risk premia. The parameters considered are the habit formation parameter (b), the capital adjustment cost parameter (κ), the Epstein - Zin parameter (φ).

Figure 2.10 plots the averages and standard deviations of risk premia for different maturities obtained by changing the considered parameters from their SMM estimates. Panel A shows the averages and panel B the standard deviations.

We change the habit formation parameter from the baseline value of 0.57 to a high level of 0.95 with others parameters set at their baseline values. Habit formation preferences are known to positively magnify the size of risk premium in endowment economy. However, this result can be mitigated in production economies (see, Chapter 1). When the capital stock adjustment cost parameter is fixed at the baseline value ($\kappa = 3.57$) changes in b have a negligible impact on the means as well as the standard deviation of risk premia. This is consistent with the finding in chapter 1 that the habit formation parameter only has a significant effect on the level of the risk premia when the capital stock is fixed ($\kappa = +\infty$). This result is also true for the adjustment cost parameter. When the habit formation parameter is fixed at $b = 0.57$, the adjustment cost parameter has little effect on risk premia. As for the Epstein - Zin parameter φ remember, it is the key determinant of the risk aversion parameter. So, increases in the absolute value of φ is expected to have positive impact on risk premia. We change φ from -167 to -200. As a result, the risk premia increase for all maturities. The 10y bond risk premia increases by 147 basis points from 1.78% to 3,25%. The standard deviation slightly increases for all maturities.

2.8 Conclusion

This paper studies the term structure of nominal bonds interest rates and risk premia in a New Keynesian framework with recursive preferences and time - varying uncertainty. Time - varying uncertainty is introduced by assuming that technology, preferences and monetary policy shocks conditional volatilities follow stochastic volatility processes. The model is solved by perturbation method which involves taking third - order Taylor series expansions. Then, the parameters of the model are estimated by simulated method of moments (SMM). The analysis focuses on the effect of uncertainty shocks on the term structure of interest rates and risk premia.

Introducing time - varying uncertainty in the analysis of the term structure is important because changes in uncertainty or volatilities have an impact on economic agents consumption or portfolio decisions. Moreover, previous studies (Rudebusch and Swanson, 2010, Andreasen *et al*, 2013) have shown that recursive preferences are appropriate in analyzing jointly asset prices and business cycles as opposed to the standard preferences in New Keynesian DSGE literature. Previous work that make use of recursive preferences to analyze the term structure have focused on the impact of the level of the shocks on the term structure (Andreasen *et al*, 2013) or have employed time - varying volatility but using an endowment economy framework (Doh, 2010, van Bingsberg et al, 2010, Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez, 2010). Since endowment economy framework implies that the equilibrium consumption is exogenous, it is important to extend this analysis to a fully - fledged production economy in order to understand the impact of different sources of time - varying uncertainty on the term structure.

It is shown that the introduction of time - varying volatility has a first order effect and induces an additional dynamics to interest rates and risk premia. In fact, the conditional volatilities affect the conditional means of the term structure and contribute to its fluctuations. It means that time - varying uncertainty affects agents decisions and asset prices. This is interesting as the derived risk premia decision rules mimic the ARCH - M process introduced by Engle, Lilien and Robins (1987). The main difference here is that the coefficients of our ARCH - M are functions of structural parameters.

At the SMM parameters estimates, the model generates statistics which are qualitatively in line with the term structure data counterparts. Results show that positive level of productivity shocks have downward shifting effects on the yield curve whereas positive monetary policy level shocks flatten the yield curve and preferences shocks affect positively the slope of the yield curve.

With regard to the volatility shocks, real uncertainty shocks (technology and preferences) play the most important role in the level and variations of risk premia relative to nominal uncertainty shocks (monetary policy). Techonology shock conditional variance contributes positively to the averages and variances of risk premia whereas preferences shock conditional volatility contributes negatively to the averages of risk premia.

As in the simple case of standard preferences and constant volatility in Chapter 1, the impact of habit formation on risk premia depends on whether the capital stock is fixed or not. When the capital stock is fixed, a higher habit formation parameter significantly increases the risk premium. However when the capital stock is allowed to vary, increases in habit strength parameter have a little impact on risk premiums. This is because allowing the capital stock to vary costlessly, open an additional channel for consumption smoothing.

Tab. 2.2: baseline calibrated parameters

parameters	**description**	**value**
μ^z	long-run growth of productivity	1.0052
μ^Υ	long-run growth of investment shock	1.0016
n_{ss}	adjustment cost parameter	0.34
α	share of capital income	0.33
δ	depreciation rate	0.02
θ	elasticity of substitution among goods	11
θ_p	proportion of firms not adjusting price	0.75
π^{ss}	long-run gross inflation rate	1.008

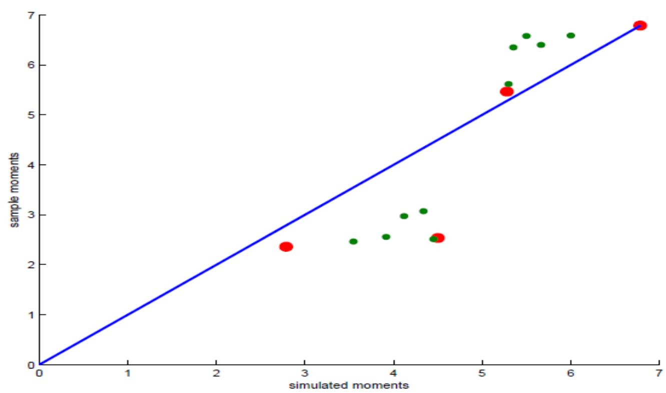
Tab. 2.3: SMM Estimation

Description	Symbol	Time-varying volatility	Constant volatility
Preferences parameters			
Discount factor	β	0.9926	0.9928
		(0.0002)	(0.003)
Consumption curvature	γ	1.572	1.785
		(0.254)	(0.325)
EZ parameter	φ	-167.02	-199.346
		(25.02)	(30.52)
abour elasticity	ϕ	6.612	6.609
		(3.79)	(3.4)
Habit formation	b	0.570	077
		(0.0005)	(0.0002)
Capital adjust. cost parameter	κ	3.565	3.621
		(0.111)	(0.371)
Policy rule parameters			
AR parameter	ρ_i	0.663	0.663
		(0.0007)	(0.0027
Inflation reaction coeff	γ_π	3.225	3.224
		(1.005)	(1.255)
Output reaction coeff	γ_y	0.430	0.430
		(0.000)	(0.001)
Preferences shock parameters			
Persistence parameter	ρ_d	0.982	0.968
		(0.0052)	(0.031)
Standard deviation	$\bar{\sigma}_d$	0.014	0.014
		(0.002)	(0.001)
SV persistence	ρ_{σ_d}	0.605	-
		(0.236)	-
Standard deviation	σ_{σ_d}	0.400	-
		(0.023)	-
Productivity shock parameters			
Persistence parameter	ρ_a	0.948	0.960
		(0.0001)	(0.0023)
Standard deviation	$\bar{\sigma}_a$	0.012	0.008
		(0.000)	(0.0056)
SV persistence	ρ_{σ_a}	0.815	-
		(0.0641)	-
Standard deviation	σ_{σ_a}	0.420	-
		(0.000)	-
Monetary policy shock parameters			
Standard deviation	$\bar{\sigma}_m$	0.0015	1.47×10^{-5}
		(0.001)	(0.290)
SV persistence	ρ_{σ_m}	0.432	-
		(0.021)	-
Standard deviation	σ_{σ_m}	0.0053	-
		(0.0002))	-

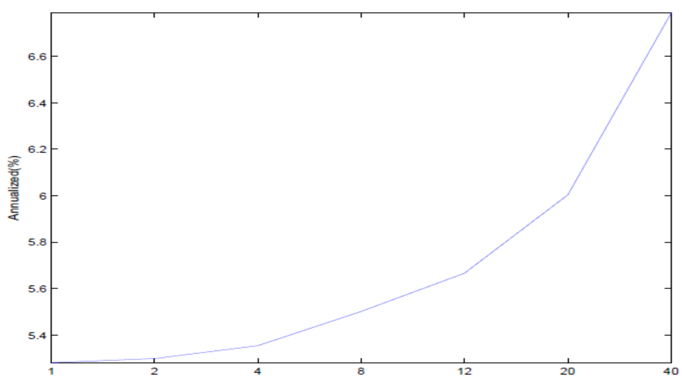
Note : Asymptotic standard deviations in parenthesis

FIG. 2.1: Model Implied Term Structure of Interest Rates

Panel A : Model Fit



Panel B : Unconditional Means of Interest Rates



Panel C : Unconditional Standard Deviations of Interest Rates

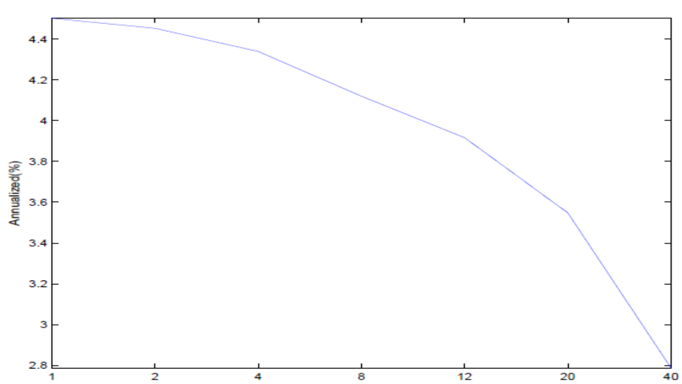


FIG. 2.2: Simulated Series of the Term Structure

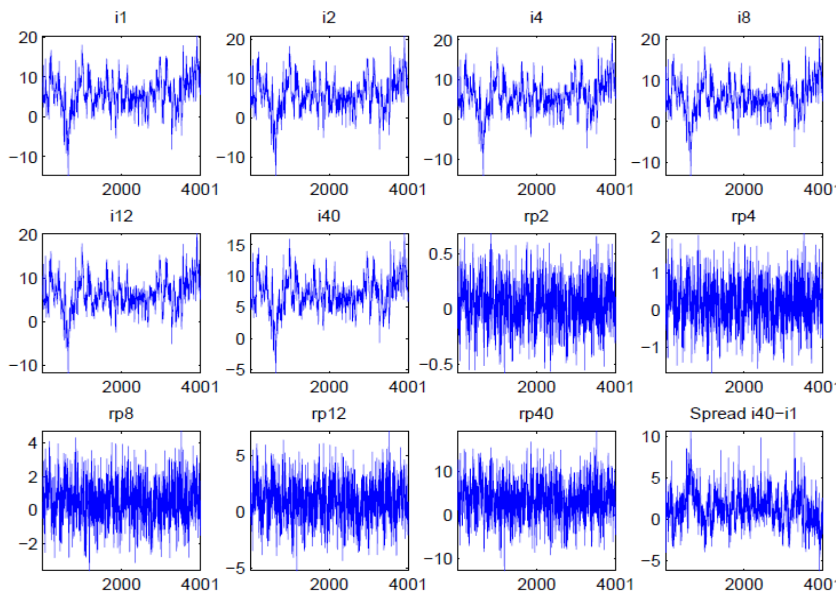


FIG. 2.3: Responses to Productivity Level Shock

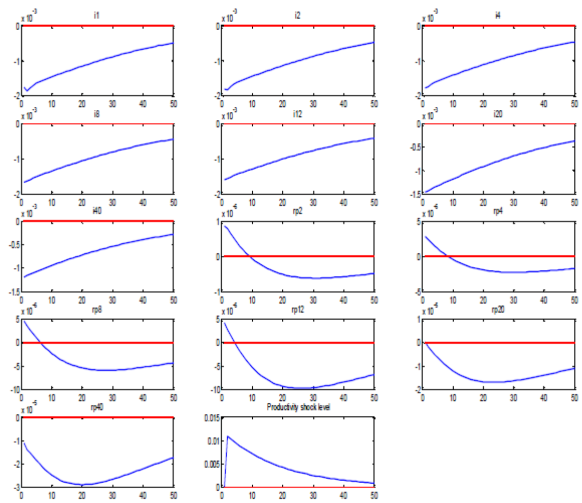


FIG. 2.4: Responses to Preferences Level Shock

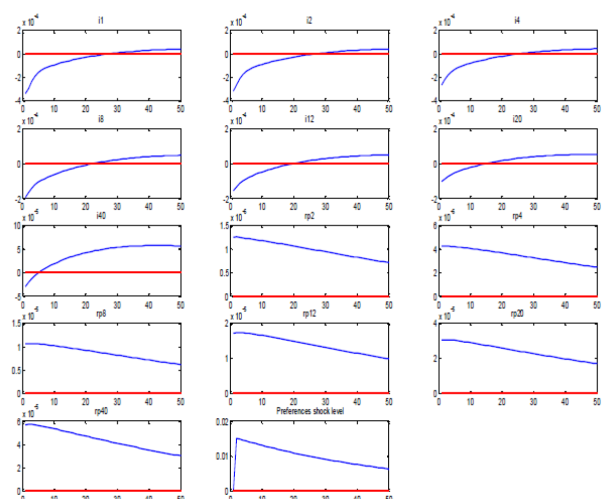


FIG. 2.5: Responses to the Monetary Policy Level Shock

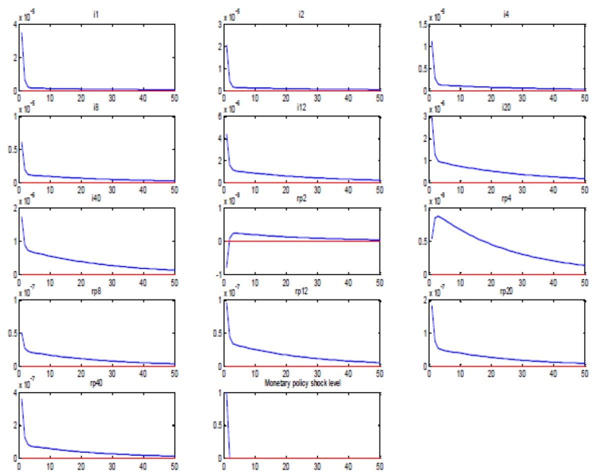


FIG. 2.6: Responses to the Productivity Volatility Shock

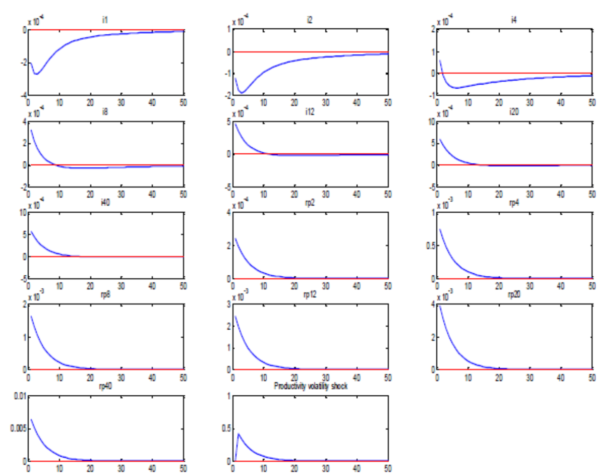


FIG. 2.7: Responses to Preferences Volatility Shock

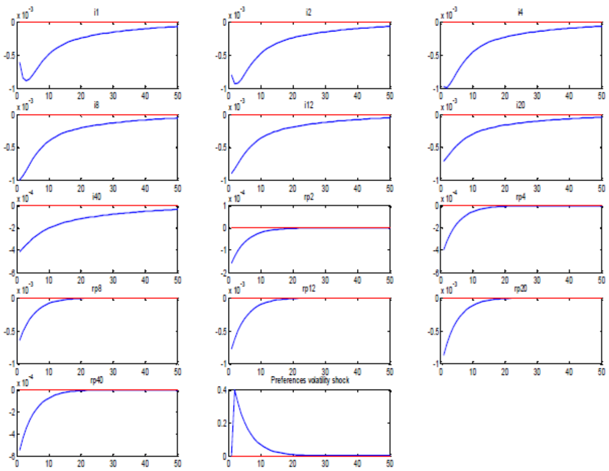


FIG. 2.8: Responses to Policy Volatility Shock

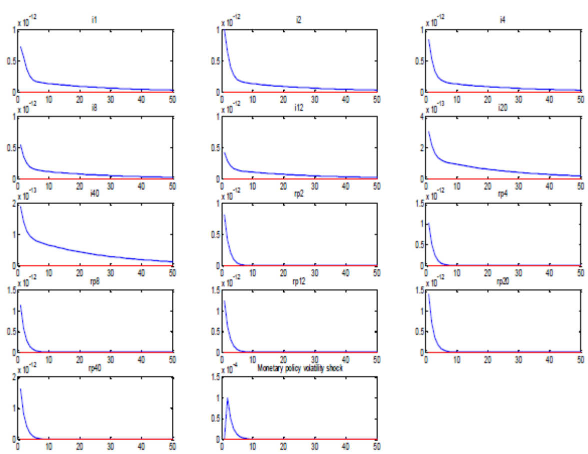
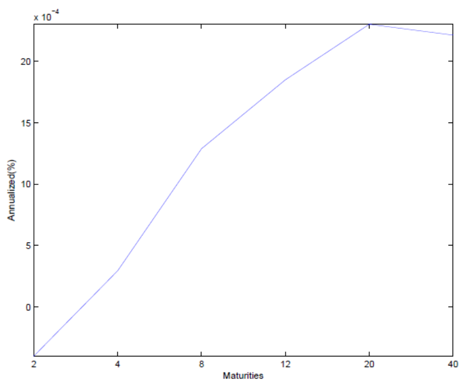
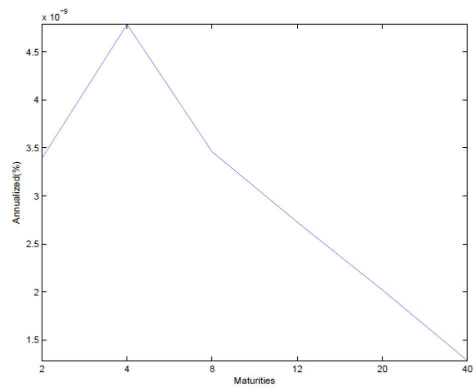


Fig. 2.9: Risk Premium and Conditional Volatility Effects Coefficients

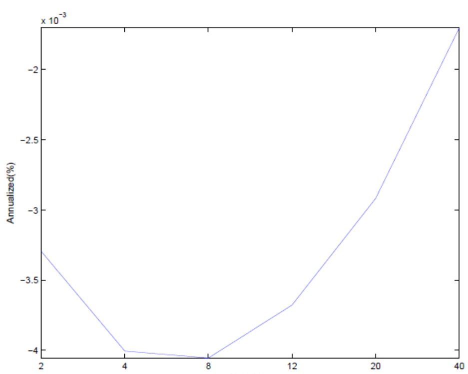
Panel A : Productivity Volatility Shock $rp^{\ell}_{\sigma\sigma,\sigma_a}$



Panel B : Monetary Policy Volatility Shock $rp^{\ell}_{\sigma\sigma,\sigma_m}$



Panel C : Preferences Volatility Shock $rp^{\ell}_{\sigma\sigma,\sigma_d}$



Panel D : Unconditional Means and Standard Deviations of Risk Premia

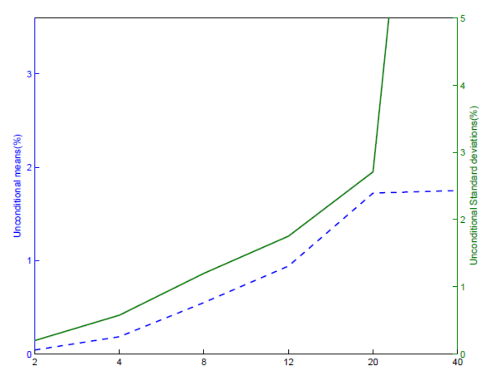
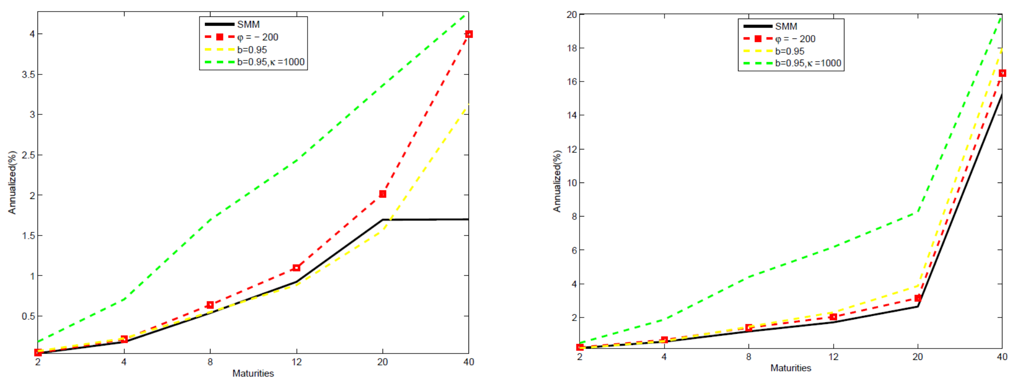


FIG. 2.10: Sensitivity Analysis

Panel A : Means   Panel B : Standard deviations



In each case, the remaining parameters are fixed at their SMM estimates

Chapitre 3

Monetary Policy and Heterogeneous Inflation Expectations in South Africa

3.1 Introduction

Prior to the recent financial crisis, many countries – advanced and emerging market economies - have adopted inflation targeting (IT) as a monetary policy strategy to address the breakdown of the relationship between money growth rates and inflation (New Zealand, Canada and South Africa), or the disappointment following the use of exchange rates as an intermediate target (United Kingdom, Sweden and Finland). Most of these countries experienced a sharp decline in inflation right after the adoption of IT. The success of IT is attributed to, among others, the ability of central banks to anchor expectations of agents around its set targets (see Demertzis and Viegi, 2008). To achieve this objective, the central bank should clearly communicate its policy and should aim at further increasing its credibility. It is only in such an environment that the public would believe that the central bank is resolute in steering inflation towards the official target. Then inflation expectations will also converge to the official target and are likely to remain unchanged even in the presence of negative supply shocks such as rise in oil or food prices. In this instance, the public is convinced that the central bank will act to bring back inflation within the established target band. In that case inflation expectations will be tied closely to the target and the associated output cost of the disinflation will be lower. It is therefore crucial to analyze expectations formation of agents in an IT regime and determine the credibility of monetary policy.

Many studies have focused on the success of monetary policy in South Africa in curbing inflation in the IT era. For example, Gupta, Kabundi, and Modise (2010), Kabundi and Ngwenya (2011), Gumata, Kabundi, and Ndou (2013), and Aron and Muellbauer (2007) find that the South African Reserve Bank (SARB) has been successful in decreasing inflation in the IT regime compared to pre-IT periods. The SARB has achieved single-digit inflation for more than a decade, even though there were two instances (2002 and 2008) where inflation has risen to more than 10% due to the depreciation of the Rand and a rise in food prices. Notice that in these two instances inflation has stayed above the upper bound of the target band for less than three years. However, all the aforementioned studies are silent about the role played by expectations in the IT regime, and whether this success was a result of the ability of the SARB in anchoring expectations within the target band.

The following questions are crucial in determining the role played by expectations : (i) How does the SARB shape expectations of agents ? (ii) Are these expectations homogeneous ? (iii) Are perceived targets of agents consistent with its objective ? (iv) What explains the upward bias of inflation toward the upper bound of the target band ? Kabundi and Schaling (2013, henceforth KS) attempt to answer these questions using a simple macroeconomic model which estimates inflation expectations as a linear function of inflation target and lagged inflation. They use aggregate (macroeconomic) inflation expectations obtained from the quarterly survey conducted by the Bureau of Economic Research (BER). Their results indicate that expectations formation of agents is backward-looking and that the implicit target of agents lies above the target band of 3 to 6%. This suggests that their expectations were not properly anchored. However, KS results can be somewhat misleading for two reasons. First, they assume that economic agents in South Africa are homogeneous. Aron and Muellbauer (2007) and Reid (2012), using the BER survey expectations and expectations obtained from Reuters, show that expectations of agents in South Africa are heterogeneous. The expectations of analysts adjust quickly to the official target band, while expectations of price setters (business and trade unions) adjust slowly. In general, price setters are somewhat backward-looking owing to the fact that wage setting in South Africa is backward-looking (Aron et al., 2004). Wage negotiation takes into account past inflation instead of future path in inflation. According to Aron and Muellbauer (2007), expectations of price setters eventually converge to those of analysts within the target band. They conclude that the SARB has been able to anchor expectations of all agents. Nevertheless, their study covers the sample period from 1994 to 2004, which misses important dynamics in inflation, such as the rise of 2008 due to exogenous shocks. Second, they work with current-year expectations.

In this paper we extend the KS analysis and decompose aggregate inflation expectations into individual expectations of three types of agents; businesses, trade unions and financial analysts. We use one-year and two-year ahead inflation expectations and a simple macroeconomic model with three key equations, namely, aggregate supply, monetary policy preferences, and inflation expectations. The expectations equation is estimated with a panel-data regression with fixed-effects approach where expectations of agents are linear functions of the inflation target and lagged inflation. The setting is appropriate to deal with heterogeneity observed in the intercepts and slopes, which in turn enables us to answer some key questions in determining the role of inflation expectations in the conduct of IT in South Africa. Those questions are : (i) are inflation expectations different across agents (business, trade unions and analysts)? and (ii) to what extent do potentially diverging inflation expectations imply different perceptions of the credibility of South Africa's IT framework? The second question is important as a regime that is perceived as non-credible, say, by unions has different policy implications for the SARB than lack of buy-in from analysts. We also address the possible dilemma faced by a central bank that is confronted with non-anchored inflation expectations. Should it accommodate those or not? This is known in the literature as the *expectations trap*. We discuss this issue in the context of our model and suggest a way out.

The remainder of the paper is organized as follows. Section 3.2 presents an overview of the relationship between inflation and inflation expectations for the aggregate and each individual agent. It is based on graphical representation of these variables. Section 3.3 presents the model. We describe the data, their transformation and the estimation of the model in Section 3.4. We discuss anchoring of expectations by the SARB and an analysis of the heterogeneity of expectations in Section 3.5. Section 3.6 concludes the paper.

3.2 Inflation and Inflation Expectations in South Africa : An Overview

Monetary authorities care about inflation expectations because realized inflation itself is partially driven by the public's expectations about future inflation. One channel is that nominal wages are partially set based on expected inflation. Inflation targeting was pioneered in New Zealand in 1990, and is now also in use by the central banks in the United Kingdom, Canada, Australia, South Korea , Egypt, South Africa, Iceland and Brazil, among other countries. The success of the regime depends largely on the behavior of the public's inflation expectations. If inflation expectations are equal to a point target or within the targeting band set by the central bank, the monetary policy regime is perfectly credible. But if the target or band - and thereby the IT framework - is imperfectly credible, long-term inflation expectations will be volatile and transitory shocks to inflation will also have an impact on inflation expectations. In a perfectly credible IT framework, long-term inflation expectations should be flat and tied to the central bank's inflation target level, or at least fluctuate inside the target band. In that case any adverse supply shock which increases the current inflation rate would have little effect on long-term inflation expectations because the public's - and thereby wage setters - have confidence in the ability of the central bank to bring down inflation back to the target level - or into the band - over a certain time horizon, where the latter depends to what extent the central bank engages in flexible inflation targeting (this term was introduced by Svensson (1999)). It then appears that the presence of a strong correlation between long-term inflation expectations and the realized inflation rate is a sign of a lack of credibility of the IT regime. The latter is in line with the theoretical model put forward by King (1996). He emphasizes the role of learning by the private sector and shows how the optimal speed of disinflation depends crucially on whether the private sector immediately believes in the new low inflation regime or not. If they do, the best strategy is to disinflate quickly, since the output costs are zero. If expectations are slower to adapt, disinflation should be more gradual as well. Learning by the central bank is addressed by Sargent (1999). He analyzes how policy makers in the US after World War II learned to believe and act upon a version of the natural rate unemployment rate hypothesis and creates an econometric model of an adaptive monetary policy that can produce outcomes persistently better than the time-consistent one predicted by Kydland and Prescott (1977).

As is common in countries who have adopted an IT framework, the SARB conducts a quarterly survey on inflation expectations to guide its policy. Figure 1 plots the BER inflation expectations at different horizons along with the realized CPI inflation (year-on-year change) from 2000Q3 to 2012Q3. Clearly, inflation has fluctuated a lot in 2000Q3-2009Q3 with two big negative shocks in 2002Q4 (due to massive depreciation of the South African rand) and 2008Q3 (due to increase in global food price coupled with a rise in oil price and another depreciation of the South African rand) and a positive shock in 2004Q1 (an appreciation of the rand) before stabilizing near the upper bound of the target (6%) during the financial crisis. Below we will look at inflation expectations of different agents, for now we look at the average across agents. Average inflation expectations series closely tracked actual inflation - seemingly with a lag - in 2000Q3-2009Q3 especially in periods when inflation exceeded the upper bound of the band. This suggests that during this period the shocks discussed above that increased inflation also drove up inflation expectations. Thus, from this graphical inspection, it seems that most of the time the Reserve Bank's monetary policy hardly anchors inflation expectations. However, after the financial crisis both inflation and inflation expectations have converged to the upper bound of 6%. We will provide formal tests for anchoring in the following sections. Notice that the SARB survey - conducted and published by the BER – has separate questionnaires for different societal groups : financial analysts (including economists), business people, and trade union representatives. Thus, the BER dataset has a panel structure. This will be used in our empirical work. Note that the inflation expectations series discussed above relates to the aggregate across these agents.

For policy implementation purposes, it would be interesting for the SARB to understand whether these groups are homogeneous in terms of their expectations formation for a number of reasons. First, if there is heterogeneity in expectations, it may be the case that some groups do not have a good understanding of the IT framework. Identifying these groups may help the SARB with its communication strategy. Second, trying to influence inflation expectations requires an understanding of the process by which these expectations are formed. Third, the appropriate monetary policy response to an expectations shock may differ across sectors or agents. For example, a shock to analysts' expectations may have less potential impact on actual inflation than a similar shock to union's or business expectations. Finally, inflation expectations across sectors or agents may influence each other because of the relationship between these two groups. In fact employees' wages are usually negotiated in advance and are based on expected future prices. Next, firms will set prices according to a mark-up over marginal cost. For South Africa, research on the determinants of inflation has done by *inter alia* Fedderke and Schaling (2005) and Fedderke, Kularatne and Mariottti (2007). Both papers find that the mark-ups in South Africa over marginal cost are approximately twice that found in the U.S. This may give rise to a classic wage-price spiral.

In Figure 2, we plot the inflation expectations of the three types of agents at one-year and two-year ahead horizons along with the realized CPI inflation rate and the SARB official target range of 3% - 6%. Panel A depicts the expectations of the analysts, Panel B business expectations and Panel C trade union's expectations. The inflation expectations pattern seems to be significantly different across agents. First, the analysts' group expectations pattern is relatively flat with their two-year ahead inflation expectation within the target band. Second, the business and the trade union's expectations patterns are very similar and seem to track realized inflation seemingly with a lag - as was the case with the aggregate inflation expectations pattern. Thus, it appears that the expectations of the analysts are well anchored, whereas those of business and unions are not. It means that the analysis based solely on aggregate expectations, such as KS, may lead to misleading conclusions.

3.3 The Model

Kabundi and Schaling (2013) discuss disinflation policy in South Africa using a simple macroeconomic model based on King (1996), which combines nominal wage and price stickiness and slow adjustment of expectations to a new monetary policy regime. The model analyses the interaction between private sector expectations and the monetary regime, and in particular the speed at which the inflation target implicit in the latter converges to price stability. It features nominal rigidity and an optimizing central bank (CB) that trades inflation versus output stabilization.

More specific, the model has three key equations : aggregate supply, monetary policy preferences and inflation expectations. Aggregate supply exceeds the natural rate of output when inflation is higher than was expected by agents when nominal contracts were set. This is captured by a simple short-run Phillips curve[1]

$$z_t = \pi_t - \pi_t^e - \epsilon_t \tag{3.1}$$

Here π_t is the rate of inflation, z_t is the output gap, π_t^e indicates the expectation of inflation as the aggregate of the subjective expectations (beliefs) of private agents and ϵ_t is a supply (cost-push) shock.

$$\pi_t^e = 1/3 \sum \pi_t^{e,i} \tag{3.2}$$

where $i = a, b, u$ (and a denotes the analysts group, b the businesses group and u the unions group). Those beliefs do not necessarily coincide with rational expectations.[2] The model is not restrictive as long as inflation expectations are in part influenced by past monetary policy (see e.g., Bomfim and Rudebusch (2000).[3]

The regime change is represented by a new inflation target π^*, which is announced to the public (business, unions and financial analysts) at the end of period $t-1$. The new target is lower than the initial steady state inflation rate, denoted by π_0.

[1] In their analysis of U.S. monetary policy experimentation in the 1960s, Cogley, Colacito and Sargent (2005) use a model similar to ours but with unemployment instead of output.

[2] For a New-Keynesian model where the central bank has a similar incentive structure and private agents are learning see Bullard and Schaling (2009).

[3] In the present paper - given expectations - the output costs of disinflation are constant and given by the slope of the Phillips curve. Here this parameter is normalised at unity. However, if we allow the output costs of disinflation to vary with the inflation rate, the central bank's incentives change substantially. Thus, one way of extending the model with state-contingent output costs of disinflation would be by means of a non-linear Phillips curve as discussed in Schaling (2004). For a preliminary analysis along those lines see Hoeberichts and Schaling (2006).

The central bank's objective as of period t is to choose a sequence of current and future inflation rates $\{\pi_t\}_{t=0}^{\infty}$ so as to minimize its intertemporal loss

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\phi(\pi_t - \pi^*)^2 + (z_t)^2\right] \tag{3.3}$$

where parameter $0 \leq \phi < \infty$ is the relative weight on inflation stabilization, while $0 < \beta \leq 1$ is the discount factor.

The timing of events is such that the central bank chooses its disinflation policy after private sector inflation expectations are set. In the terminology of game theory the private sector is the Stackelberg leader. In Section 5.4 we analyze the opposite case.

The above statements can be analyzed more precisely by explicitly considering the central bank's optimization problem (where it takes inflation expectations as given, that is, under naïve discretion). The central bank's optimal inflation rate - or Best Response in terms of Sargent (1999) is :[4]

$$\pi_t = \frac{1}{1+\phi} \left(\pi_t^e + \epsilon_t\right) + \frac{\phi}{1+\phi}\pi^* \tag{3.4}$$

Of course, from (3.4) it is clear that if expectations are slower to adapt, the disinflation should be more gradual as well. The inflation rate should decline as a constant proportion of the exogenous expected inflation rate.

In a standard New-Keynesian model the Phillips curve is

$$\pi_t = \beta \pi_{t+1}^e + \lambda z_t + \epsilon_t$$

and the first-order condition under discretion is

$$\pi_t = \frac{\beta}{1+\lambda^2\phi}\pi_{t+1}^e + \frac{1}{1+\lambda^2\phi}\epsilon_t + \frac{\lambda^2\phi}{1+\lambda^2\phi}\pi^*$$

where π_{t+1}^e is the time t expectations of time t+1 inflation, that is, $\pi_{t+1}^e = E_t\left[\pi_{t+1}\right]$, and E_t is the mathematical expectation given time t information set operator.[5] This is very similar to the first order condition of the specification adopted in this paper if $\lambda = 1$[6] since the discount factor $0 < \beta \leq 1$ is typically calibrated

[4] According to the central bank's first order condition monetary policy responds to *aggregate* expectations. Thus the heterogeneity of agents is not taken into account in monetary policy. We leave this for further research.

[5] Note that $E_t\pi_{t+1} = \pi_{t+1}^e$, we will use the latter notation throughout the article when necessary.

[6] As pointed out by Clarida, Gali and Gertler (2000, p. 170) there is no widespread consensus on the value of the output elasticity of inflation, λ. Values found in the literature range from 0.05 to 1.22.

at 0.99 (See for example Woodford (2003)). This implementation of flexible inflation targeting is what Evans and Honkapohja (2003) call an expectations-based optimal rule, by construction, it implements what they label 'optimal discretionary policy' in every period and for all values of private expectations. Here as above the central bank also chooses its disinflation policy after private sector inflation expectations are set. The only difference is the timing of expectations (set at time t or $t-1$) which has no bearing on our empirical results. What matters is who moves first, the central bank or the private sector.

In general, expectations are affected both by the inflation target and by actual inflation performance. After experiencing high inflation for a long period of time, there may be good reasons for the private sector not to believe the disinflation policy fully (See also Bomfim and Rudebusch (2000)). In light of this, in this section following King (1996) we assume that for each agent inflation expectations follow a simple rule, that is a linear function of the inflation target and the lagged inflation rate.

$$\pi_{t+h}^{e,i} = \rho^i \pi_{t-1} + (1-\rho^i)\pi^* \tag{3.5}$$

where h is the forecast horizon. Put differently, the lower ρ, the better inflation expectations are anchored at long horizons.[7] Note that in this case expectations are neither rational (which would be the case where inflation expectations equal the target as the central bank has no incentive to generate surprise inflation) or given by a rational learning process. For the latter case (of Bayesian learning) Schaling and Hoeberichts (2010) - for a two-period version of the above model - show that then ρ can be interpreted as $(1-x_1)(1-q)$. Here $x_1$is the prior probability assigned by wage setters to the event that the central bank disinflates everything in one go (follows a cold turkey policy) and $0 < q \leq 1$ is the fraction of the disinflation that takes place in period 1. Thus, with a structural interpretation of ρ rational expectations can display some of the backward-looking characteristics of adaptive expectations. Notwithstanding the above, we stress that the focus of this paper is on the anchoring of expectations to the inflation target (where inflation expectations are given by survey data), rather than on rationality or rational learning. Note that if the regime switch to the new inflation target is completely credible, inflation expectations are immediately anchored by the inflation target, that is $\pi_{t+h}^{e,i} = \pi^*$ (we have $\rho^i = 0$).

[7]For an empirical analysis for the U.S. examining observable measures of long-run inflation expectations, see Kiley (2008). Further our model generates persistent inflation (decreasing in a), although the central bank does not aim for an output target above the natural rate. An alternative framework that also generates an inflation bias is the paper by Cukierman and Gerlach (2003). Here the central bank aims for the natural rate - as in this paper - but is more concerned about negative than positive output gaps.

Conversely, if the regime switch is not credible at all, inflation expectations remain driven by the past inflation rate; $\pi^{e,i}_{t+h} = \pi_{t-1}$ ($\rho^i = 1$).[8] In reality - and in the case of South Africa - we are likely to find in between cases. To that end we will now estimate equation (3.5) for South Africa (for each agent) over the period 2000-2013

$$\pi^{e,i}_{t+h} = c^i + \rho^i \pi_{t-1} + \varepsilon^i_t \tag{3.6}$$

and ε^i_t is the *iid* stochastic error term which follows a normal distribution. In so doing, we obtain $\hat{\rho}^i$ and $\hat{c}^i$, where $\hat{c}^i = (1-\hat{\rho}^i)\hat{\pi}^*$. Therefore, for each agent we can easily compute their perceived (implicit) inflation target as : $\hat{\pi}^{i*} = \frac{\hat{c}^i}{1-\hat{\rho}^i}$.

3.4 Econometric and Data Analysis

3.4.1 Econometric Analysis

Fully anchoring inflation expectations would mean that inflation expectations are equal to the target and hence completely uncorrelated with realized inflation. Then any shock to inflation has a limited effect on inflation expectations. One way to test whether expectations are well anchored is to perform a Granger causality test between inflation expectations and realized inflation. If realized inflation Granger causes inflation expectations that signals a lack of "anchoredness" as then lagged realized inflation will have an impact on expected inflation. We report the results of this test in our section on the empirical results.

To account for a potential heterogeneity in expectation formations, we exploit the panel structure of the BER dataset and estimate the following panel model

$$\pi^{e,i}_{t+h} = \alpha_{i_0} + \alpha_1 D^{i_1}_t + \alpha_2 D^{i_2}_t + \delta_0 \pi_{t-1} + \delta_1 D^{i_1}_t \pi_{t-1} + \delta_2 D^{i_2}_t \pi_{t-1} + \varepsilon^i_t \tag{3.7}$$

where $i_0, i_1, i_2 \in \{a, b, u\}$, $i_0 \neq i_1 \neq i_2$, $\pi^{e,i}_t$ is a measure of time t inflation expectations of agent i, D^i_t is a dummy variable taking 1 if the agent type is i and 0 otherwise, π_{t-1} is lagged realized inflation, ε^i_t is a time t independently distributed error term of agent i, and $\alpha_{i_0}, \alpha_1, \alpha_2, \delta_0, \delta_1, \delta_2$ are constant parameters. i_0 is a reference category and i_1, i_2 represent one of the two other categories.

Notice that equation (3.7) nests the equation by equation estimation. That is, for a given type i, the model is reduced to a regression of agent i's inflation expectations on a constant and lagged realized inflation. Since we have three agents and the

[8] Note that if we see the above as a game between the private sector and the central bank then the former's expectations formation equation can be interpreted as its reaction function. The solution for inflation can be obtained by substituting the latter in the central bank's first order condition : $\pi_t = \frac{\rho}{1+\phi}\pi_{t-1} + \frac{(1-\rho)+\phi}{1+\phi}\pi^*$.

analysts group expectations seem to be anchored rather well relative to other groups, we use the analysts group as the reference category and hence only use the business and trade unions group dummies in the model. Thus $i_0 = a$ and α_{i_0} and δ_0 are respectively the intercept and the slope coefficients of the analysts' expectations equation. The intercept and the slope coefficients of the type i_1 agent are given by $\alpha_{i_0} + \alpha_1$ and $\delta_1 + \delta_0$ respectively (the corresponding coefficients of the type i_2 are $\alpha_{i_0} + \alpha_2$ and $\delta_2 + \delta_0$ respectively).

This panel framework is interesting in the sense that it allows heterogeneity in the intercept as well as in the slope coefficients. The advantage is that we are able to directly test whether there is heterogeneity in the intercepts as well as in the slope coefficients. As a consequence, we can derive each agent's perceived inflation target as in (3.6). For example a Wald test can be used to test heterogeneity in the intercepts by simply testing the significance of α_1 and α_2 while a Chow-type test can be used to test differences in the slope coefficients.

Since the validity of the above regression requires the series to be stationary, we employ the Philips-Perron (PP) unit root test as well as the KPSS test developed by Kwiatowski, Phillips, Schmidt, and Shin (1992) to test the stationarity of the inflation and inflation expectations series. In the PP test case, the alternative model is an autoregression with a constant but no trend. The spectral estimation method used is the autoregressive spectral (AR spectral) method and the lag truncation is automatically selected using recursive t-tests. With regard to the KPSS test, we used the same spectral estimation method (AR spectral) and lag length selection criteria as in the PP test case. The results of the test are reported in Table 1 and reveal that the inflation and inflation expectations series are stationary at the 1% level of significance. Except for the trade unions' inflation expectations, the null hypothesis of a unit root can be rejected at the 1% level for all series in the PP test case. As for the KPSS results, the null hypothesis of stationarity cannot be rejected at the 1% level (5% for the aggregate 1-year ahead inflation expectations) except for the business inflation expectations rate. However, when we apply a Dickey-Fuller test based on the generalized least squares (DF-GLS) method, we are able to reject the null hypothesis of a unit root for all of our series at the 5% level. Elliott, Rothenberg and Stock (1996) (ERS) show that the DF-GLS test performs well in small samples compared to existing unit root tests. Since our sample size is relatively small (49 observations), we use the DF-GLS test results and conclude that all of our series are stationary.

3.4.2 The Data

In this paper we consider aggregate inflation expectations as well as expectations of three agents : business, trade unions and analysts (including economists). The data for these expectations are obtained from the BER. The BER conducts a survey in South Africa where major market participants are asked questions about the prospect of inflation. More specifically, the panel is made of 1 061 business people, 40 financial sector participants and 25 participants representing the labour market. According to Kershoff and Smit (2002) the BER survey uses the questionnaires of the Reserve Bank of New Zealand as a guideline. This series is released each quarter.

Realized inflation is the quarterly year-on-year percentage change in the headline Consumer Price Index (CPI)[9] and is taken from the SARB.

The sample is from the third quarter of 2000 to the first quarter of 2013. There are two main reasons for this sample size. First, we want to examine the dynamics of inflation and inflation expectations during the IT regime in South Africa. Secondly, the BER survey started in 2000, hence there is no reliable series on survey inflation expectations in South Africa before 2000.

3.5 Empirical Results

3.5.1 Anchoring of Inflation Expectations

Table 3.2 presents the empirical results of the Granger causality test between realized inflation and aggregate two-year ahead inflation expectations as well as the two-year ahead inflation expectations per agent. The null hypothesis of "π_t does not Granger cause π_t^e" can easily be rejected at the 1% level for the aggregate, business people and the trade unions representatives groups. This means that lagged realized inflation impacts on the two-year ahead inflation expectations of these two groups as well as on aggregate inflation expectations. On the other hand, this hypothesis cannot be rejected for the analysts group. This confirms the graphical view that analysts' expectations are well anchored, while business people and workers groups' expectations are not. Since business people and trade unions represent two-thirds of the sample and tend to report higher inflation expectations, it follows that aggregate inflation expectations are driven by these two groups and are not anchored. This is an important result which has implications for monetary policy implementation as will be discussed below.

[9] As a robustness check, we also try the Core CPI inflation but the results of the paper are unchanged.

3.5.2 Heterogeneity of Inflation Expectations

In this section we investigate whether the three groups of agents form their expectations in a similar way. We start by testing whether the average of the business and trade unions groups, i.e. $\bar{\pi}^i = \frac{1}{T}\sum_{t=1}^{T}\pi_t^{e,i}$ where $i = b, u$, are different from the analysts group. That is, we estimate (3.7) by Ordinary Least Square (OLS) with the slope coefficients set to zero and test the significance of the intercept coefficients α_1 and α_2. Then we estimate the unrestricted version of (3.7) and test heterogeneity of the slope coefficients.

Since the reference category is the analysts group, α_1 or $\alpha_2 \neq 0$ would indicate heterogeneity[10] (relative to the analysts group) in the intercepts. Table 3.3 reports the results of the restricted model. The F-statistic is significant at the 1% level meaning that the null hypothesis of $\alpha_1 = \alpha_2 = 0$ is rejected. Since α_1 and α_2 are positive this also indicates that business and trade unions groups tend to report higher inflation expectations on average compared to the analysts group. The estimated average of the one-year ahead inflation expectations is 5.51% for the analysts group, and 6.61% ($\hat{\alpha}_{i_0} + \hat{\alpha}_1$) and 6.51% ($\hat{\alpha}_{i_0} + \hat{\alpha}_2$) for business, and trade unions respectively. On the other hand, a test of $\alpha_1 = \alpha_2$ cannot be rejected meaning that on average business people and trade unions report similar inflation expectations. This is not surprising given the economic relationship between these two groups. Business and trade unions are price setters and their actions affect each other. Notice that these results imply that the average inflation expectations of the analysts group is within the SARB target band of 3 - 6% whereas the business people and the trade unions expectations are outside the band. However, even the analysts group average inflation expectations (5.51%) are near the upper bound of the target and far from the mid-point of 4.5%. These findings are problematic from a price stability perspective which will be discussed in more detail.

Table 3.4 presents the results of the full estimation of (3.7) using the one-year ahead inflation expectations as the dependent variable. Results indicate that past inflation does explain one-year ahead inflation expectations but differently across agents. On average, 62% of the variation of aggregate inflation expectations is explained by changes in past inflation.[11] Both the intercepts and the coefficients on lagged inflation for business and trade unions are significant at the one percent of significance. Notice that the intercept α_1 is not statistically significant meaning that

[10] Notice that the OLS estimation with dummies in the intercept yields the same results as the fixed effect concept of the panel data regression.

[11] However, an agent-by-agent (decomposition) based estimation of (3.7) shows that the explanatory power of the regression is lower for the analysts group and higher for the business and trade unions groups (see Table 5).

the intercept of the business group ($\alpha_{i_0} + \alpha_1$) is not statistically different from that of the analysts (α_{i_0}). The estimate of α_2 is negative (-1.11) and significantly different from zero. It means that the intercept of the trade unions group ($\alpha_{i_0} + \alpha_2$) is lower than that of the analysts. Moreover, the lagged inflation slope coefficient for the analysts group (0.11) is lower than for the two other groups (0.43 (0.13 + 0.30) for business, and 0.50 (0.13 + 0.37) for trade unions). These findings are in line with the graphical overview and the Granger causality test. In the next section we will derive the perceived (implicit) inflation target for each agent.

3.5.3 Credibility and Implicit Inflation Targets

In this section we derive the estimates of the coefficients ρ^i and π^* in equation (3.5) from the reduced form estimation of (3.7). Notice that from (3.5) and (3.7) we have the following identification :

$\alpha_{i_0} = (1-\rho^{i_0})\pi^{*i_0}$ and $\delta_0 = \rho^{i_0}$ for the analysts group

$\alpha_1 + \alpha_{i_0} = (1-\rho^{i_1})\pi^{*i_1}$ and $\delta_1 = \rho^{i_1}$ for the business people group

$\alpha_2 + \alpha_{i_0} = (1-\rho^{i_2})\pi^{*i_2}$ and $\delta_2 = \rho^{i_2}$ for the trade unions representatives group

Since the dummy variables version of the model in (3.7) cannot deal with autocorrelations in the error terms, we do the estimation by agent as in (3.5) in order to deal with potential autocorrelations in the error terms. Thus, for each group we have estimates of different intercepts as well as different slopes that allows us to infer their estimated perceived inflation target of the central bank by the identification :

$$\hat{\pi}^{i*} = \frac{\hat{c}^i}{1-\hat{\rho}^i} \tag{3.8}$$

where $\hat{c}^i$ is the estimated intercept of type i agent. Notice that the lower $\hat{\rho}^i$ is, the more credible the central bank is viewed by group i agents since they put less weight on past inflation and more weight on the central bank's inflation target.

Table 3.5 contains the results of the estimation. The first column reports the results for the analysts group, the second column for the business people, and the third column for the trade unions representatives group. Results indicate that past inflation does explain one-year ahead inflation expectations but differently across agents. Both the intercept and the coefficient on lagged inflation are significant at one percent for business and trade unions. However, the explanatory power of the regression is lower for the analysts group and relatively higher for the business and workers groups. Approximately 26% of the variation of analysts expectations are explained. Moreover, the lagged inflation slope coefficient for the analysts group (0.11) is not significantly different from zero and is lower than for the two other groups (0.22 for business, and 0.35 for trade unions). Once again, these findings corroborate with the graphical overview and the Granger causality test. First, the SARB seems to have a higher credibility among the financial analysts and experts group compare to the price setters group (business and trade unions). Thus, the hypothesis that the SARB has been successful in anchoring price setters' (business and trade unions) group expectations is not supported in that the relevant coefficients of lagged inflation are relatively high and different from zero.[12] In addition, serial correlation tests reveal that the regression residuals are highly autocorrelated in the business (0.86) and trade unions (0.72) cases compared to the analysts group case (0.34). This indicates that all information about inflation expectation is not included in lagged inflation, but can be accounted by other factors, e.g. news.

Now we turn to analyze whether there is heterogeneity in the perception of the different agents of the SARB's inflation target consistent with the expectations schemes formulated in (3.6). We then derive the implicit inflation target for each agent as given by (3.8) in Table 5. The calculated perceived inflation targets are 5.41%, 6.77%, and 6.62% for the analysts group, business people, and the trade unions respectively. Once again, these results confirm the graphical observation that the analysts group's inflation expectations are relatively well anchored although their implicit target level (5.41%) is above the mid-point of the SARB's band and near the upper bound of 6%. These are important results for a central bank, such as the SARB, that targets inflation. The results indicate that the inflation targeting regime has buy-in from the analysts but is not seen to be very credible from the perspective of unions who set wages, and firms who set prices.

More specific, we now know that the lack of anchoring of aggregate inflation expectations (for an analysis of aggregate inflation expectations see Kabundi and Schaling, 2013) is driven by the price setting side of the economy, namely by business and trade unions, as the financial analysts group's expectations are relatively well anchored. However, those expectations have no direct impact on wages or prices. Thus the SARB should pay more attention to the price setters group in its communication strategy. It seems as if these two groups do not have a proper understanding of the SARB policy framework and/or do not see it as credible. Finally, even the financial analysts group perceives the SARB's inflation target at a level near the upper bound. Thus, it means that financial analysts and experts seem to believe and/or understand the SARB policy but apparently are not convinced that the SARB is aiming for the mid-point at 4.5%. Perhaps the band is too wide and/or there is no explicit point target to steer expectations appropriately. This introduces uncertainty in predicting inflation since realized inflation can be anywhere in the band.

[12] The Wald test reject the hypothesis that $\rho = 0$ with a *p-value* of zero.

3.5.4 Expectations Trap ?

In this section we analyze the empirical relationship between the SARB's optimal inflation rate and the business and workers groups inflation expectations.

In this paper optimal monetary policy implies a strategic interaction between the private sector and the monetary authorities. The central bank's optimal inflation rate as derived in (3.4) is a weighted average of its concern about the business cycle (as proxied by the public's inflation expectations) and the central bank's inflation target. It is interesting to understand the importance of the public's inflation expectations for the central bank optimal inflation, that is, we want to understand how the central bank reacts to changes in public inflation expectations. To what extent does the SARB accommodate private sector inflation expectations ? Our paper is related to Chari, Christiano and Eichenbaum (1998). Their basic idea is that, under discretion, policymakers can be pushed into pursuing inflationary policies. This can happen when the private sector, for whatever reason, expects inflation. We know from the earlier part of the paper that this definitely applies to business and labour. Under these circumstances, the central banker may find it optimal to accommodate private agents' expectations if the cost of not doing so is a severe and/or persistent loss of output. Chari *et al* refer to such a situation as one in which the economy has fallen into an expectations trap. In the context of our model this can be seen from the central bank's first order condition for the case where $0 < \phi$. Then $\pi_t < \pi_t^e$ and $z_t < 0$. In the case of full accommodation we have $\pi_t = \pi_t^e$ and $z_t = 0$.

One way to get an idea of the severity of the expectations trap is to estimate the central bank's first order condition and test whether the coefficient on expected inflation is one (the case of full accommodation).

To that end, we regress the realized CPI inflation on the average one-year ahead inflation expectations of business and trade unions. We abstract from the analysts group because we already know that their expectations are relatively well anchored. Thus, we estimate the following equation :

$$\pi_t = \frac{1}{1+\phi}\left(\pi_t^e + \epsilon_t\right) + \frac{\phi}{1+\phi}\pi^*$$

where π_t^e the average inflation expectation of business and trade unions, that is, $\pi_t^e = 1/2(\pi_t^{e,b} + \pi_t^{e,u})$.

Table 6 presents the results of the regression. After adjusting for autocorrelation in the residuals, we find that the intercept is not significantly different from zero and the coefficient of aggregate inflation expectations is not statistically different from one at the 1% level. When in the expectations trap, a central bank might prefer inflation to temporarily exceed the target if the latter is expected by the private sector. So, our empirical findings support the hypothesis that the SARB may be caught in an expectations trap.

Chari et al. (1998) investigate alternative institutional arrangements - which in our case have a direct bearing on the implementation of inflation targeting in South Africa - that can eliminate the possibility of expectations traps. One solution is full commitment on the part of the monetary authority. Then the central bank minimizes its preference function subject to the Phillips curve and to the public's expectations formation equation.[13] This is a different set-up than we have analyzed so far. There - in game theoretic terms - the private sector was the Stackelberg leader and the central bank the Stackelberg follower. Now we reverse that order, but using the same model. This means that now we move a way from the empirics and end with some theoretical considerations.

This implies the following Lagrangian :[14]

$$L = E_t \left[\sum_{\tau=t}^{\infty} \left\{ \frac{\beta^{\tau-t}}{2} \left[-\phi \left(\pi_t \right)^2 - \left(\pi_t - \pi_t^e \right)^2 \right] - \beta^{\tau-t+1} \mu_{\tau+1} \left[\pi_{t+1}^e - \rho \pi_t \right] \right\} \right]$$

where π_t^e is the state variable, π_t is the control, and μ_t is the Lagrange multiplier.[15]

The solution of this problem (the central bank's first order condition) is :

$$\pi_t = C \pi_t^e$$

where

$$C = \frac{1}{(1+\phi) + \beta\rho^2} \left[\beta\rho^2 C^2 + 1 \right]$$

and $C < \frac{1}{1+\phi}$, where $\frac{1}{1+\phi}$ is the coefficient on expected inflation in equation (3.4).[16] In this case the (optimal) disinflation under commitment is always faster than under discretion (which was the previous set-up). Now recall the equation for the agent's expectations formation process in equation (3.5) where if the inflation target is less credible the higher ρ, as then inflation expectations remain largely driven by the past inflation rate π_{t-1}. According to Proposition 4 of Schaling and Hoeberichts (2010) the higher ρ the lower the monetary accommodation parameter C, and therefore the lower the central bank's optimal inflation rate. The argument is that the higher ρ, the

[13] We assume that the central bank has full knowledge of the process of private sector learning, or in other words we have what Gaspar, Smets and Vestin (2006) call 'sophisticated central banking'.

[14] For a zero inflation target, but results do not depend on that.

[15] Without loss of generality we have set $h = 0$, so that expectations look one period ahead.

[16] For a proof see Schaling and Hoeberichts (2010).

more leverage the central bank has over inflation expectations via past inflation.[17] Now the central bank no longer treats inflation expectations as exogenous variables. It realizes that those figures are partly the outcomes of its own policy decisions which imply actual inflation figures. This appears to be a subtle difference but it is fundamental and is of major practical relevance. If inflation expectations are partly driven by past inflation, by reducing actual inflation quicker those expectations will be adjusted downwards by private agents closer to the official inflation target. Lower inflation expectations translate into lower wages and prices (given the mark-up) so that a virtuous cycle emerges.

Such a policy is also less costly in terms of the output cost of the disinflation than under discretion (where the central bank treats inflation expectations as given). In line with the above discussion about commitment Schaling and Hoeberichts (2010) - using precisely the algebra above - show that a central bank may try to convince the private sector of its commitment to price stability by choosing to reduce inflation (more) quickly. They call this "teaching by doing". They find that allowing for teaching by doing effects always speeds up the optimal disinflation (which balances inflation and output) and leads to lower inflation persistence. This "speed" result also holds in an environment where private agents rationally learn about the central bank's inflation target using a constant gain algorithm of the Kalman Filter.

3.6 Conclusion

In this paper we have found empirical evidence for South Africa that suggests that economic agents inflation expectations are not fully anchored by the inflation target (which would be the preferred outcome in an inflation targeting regime).

We have extended the analysis of Kabundi and Schaling (2013) who focus on aggregate expectations and are therefore unable to identify which economic agents, business, unions or financial analysts drove their results. In this paper we have decomposed these results and looked at those individual agents' inflation expectations based on the BER survey data. We find that business and unions perceived inflation targets lie outside the official target band. This is relevant for monetary policy as inflation expectations of business people and workers may influence each other because of the relationship between these two groups. In fact employees' wages are usually negotiated in advance and are based on expected future prices. Next, firms

[17] If we assumme that the private sector's expectations about the central banks's *inflation target* are formed according to the adaptive (rational) learning literature, that is $E_{t-1}\pi_t = c_{t-1} = c_{t-2} + \kappa(\pi_{t-1} - c_{t-2})$ where $\kappa \in (0,1)$, then one get precisely the same result : a higher gain parameter is associated with less monetary accommodation. In the limit we reach the Ramsey equibrium where $z = 0$ and $\pi = \pi^* = 0$.

will incorporate any expected increase in their marginal cost in to their product prices.

As a consequence the SARB may find itself in an expectations trap. This is the case because inflation expectations of business and labour - as proxied by their perceived inflation targets of 6.77% and 6.62% respectively - are outside the band. Thus, when in the expectations trap, the SARB may be pushed to accommodate inflation expectations. This is in fact fully supported by our estimation of the central bank's first order condition where we find that the coefficient of aggregate inflation expectations is not statistically different from one at the 1% level.

In general, the best way out of this trap is to commit to a faster reduction of inflation - as shown by our solution for commitment or "teaching by doing" which in practical terms may imply moving to a more narrow band which is consistent with price stability.

Finally, the SARB may need to further improve the transparency of the framework and pro-actively signal its concerns about potential inflationary pressures - and likely responses - to unions and business. This would be another operationalization of commitment with - in the terminology of game theory - the central bank becoming the Stackelberg leader in the interaction with the private sector.

Prior to the establishment of the European Central Bank such a practice was regularly followed by the Deutsche Bundesbank, arguably one of the most successful monetary institutions in the post-World War II era.

Table 3.1. Stationarity Test of Inflation and inflations expectations

	Test Statistic	
Variable	KPSS	PP
Aggregate		
π_t^e	$0.47^{\dagger\dagger}$	-3.35^{*}
$\pi_{t\_}$	$0.06^{\dagger}$	-7.62^{*}
Analysts		
$\pi_{t\_}^e$	$0.26^{\dagger}$	-4.07^{*}
Businesses		
$\pi_{t\_}^e$	-2.02	1.27^{*}
Trade Unions		
π_t^e	$1.64^{\dagger}$	-1.8

Note : π_t is the realized CPI inflation. The superscripts $*$ and $**$ denote rejection of the null hypothesis of unit root at 1%, and 5% levels respectively for the PP test whereas the superscripts $\dagger, \dagger\dagger$ denote the inability to reject the stationarity hypothesis in the KPSS test

Table 3.2. Granger Causality Test

Null Hypothesis	**F-statistic**	**p-value**
Aggregate		
π_t^e does not Granger cause π_t	0.47	0.628
π_t does not Granger cause π_t^e	**8.76**	**0.00**
Analysts		
π_t^e does not Granger cause π_t	1.44	0.25
π_t does not Granger cause π_t^e	0.68	0.51
Businesses		
π_t^e does not Granger cause π_t	2.58	0.09
π_t does not Granger cause π_t^e	**5.71**	**0.00**
Trade Unions		
π_t^e does not Granger cause π_t	0.24	0.79
π_t does not Granger cause π_t^e	**12.39**	**0.00**

Note : π_t^e is the two-year ahead inflation expectations and π_t realized CPI inflation

Table 3.3. Heterogeneity in Average Inflation Expectations $\pi^{e,i}_{t+4} = \alpha_{i_0} + \alpha_1 D^{i_1}_t + \alpha_2 D^{i_2}_t$

Parameter	Estimate
α_{i_0}	5.51*
	(0.13)
α_1	1.10*
	(0.25)
α_2	1.00*
	(0.27)
R^2	0.11
F-statistic	13.29*

Note : Standard errors are reported in parentheses. π^e_{t+4} is the 1 year ahead inflation expectations. $*$, $**$ denote significance at 1%, and 5% respectively. i_0 is the analysts group, i_1 the business group and i_2 the trade unions group

Table 3.4 . Heterogeneity in Slopes and Intercepts :

$$\pi^{e,i}_{t+4} = \alpha_{i_0} + \alpha_1 D^{i_1}_t + \alpha_2 D^{i_2}_t + \delta_0 \pi_{t-1} + \delta_1 D^{i_1}_t \pi_{t-1} + \delta_2 D^{i_2}_t \pi_{t-1} + \varepsilon^i_t$$

Parameter	Estimate
α_{i_0}	4.71*
	(0.28)
α_1	-0.51
	(0.48)
α_2	-1.11*
	(0.48)
δ_0	0.13*
	(0.05)
δ_1	0.30*
	(0.07)
δ_2	0.37*
	(0.08)
R^2	0.62
F-statistic	60.33

Note : Standard errors are reported in parentheses. π^e_{t+4} is the 1 year ahead inflation expectations. $*$, $**$ denote significance at 1%, and 5% respectively. i_0 is the analysts group, i_1 the business group and i_2 the trade unions group

Table 3.5. Expectations Formation and Implicit Inflation Target by Agent

Explanatory Variables	Dependent Variable : $\pi^{e,i}_{t+4}$		
	Analysts	Business	Unions
c	4.82*	5.28*	4.30*
	(0.39)	(0.57)	(0.60)
π_{t-1}	0.11	0.22*	0.35*
	(0.08)	(0.07)	(0.09)
$ar(1)$	0.34**	0.86*	0.72*
	(0.16)	(0.04)	(0.08)
Implicit Target (π^*)	5.41	6.77	6.62
R^2	0.26	0.86	0.84

Note : Standard errors are reported in parentheses. $\pi^{e,i}_{t+4}$ is the 1 year ahead inflation expectations of type i. * , ** denote significance at 1%, and 5% respectively . $ar(1)$ is an autoregressive error term.

Table 3.6. Optimal inflation Regression

Explanatory Variables	Dependent Variable : π_t
c	-0.33
	(2.44)
π^e_t	0.95*
	(0.34)
$ar(1)$	0.83*
	(0.11)
R^2	0.83

Note : Standard errors are reported in parentheses. π^e_t is the 1 year ahead inflation expectations of business and trade unions. * denotes significance at 1%. $ar(1)$ is an autoregressive error term.

Figure 3.1. Inflation and Inflation Expectations : Aggregate

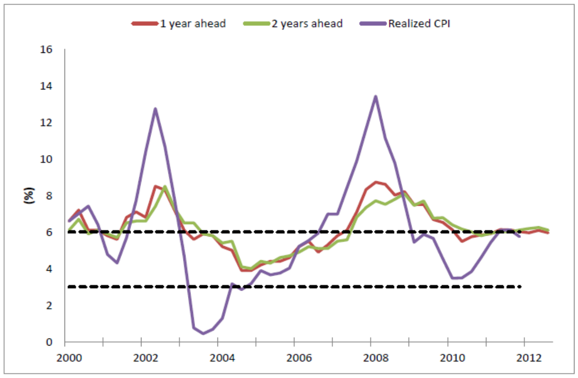
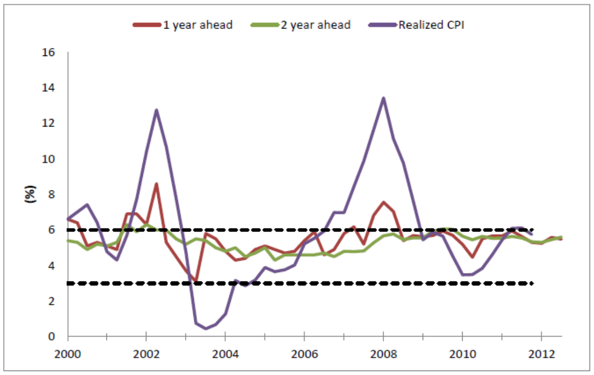
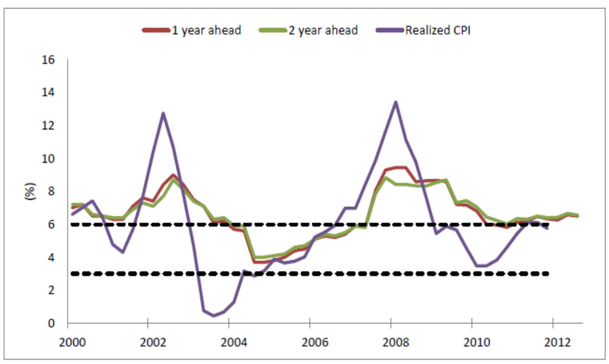


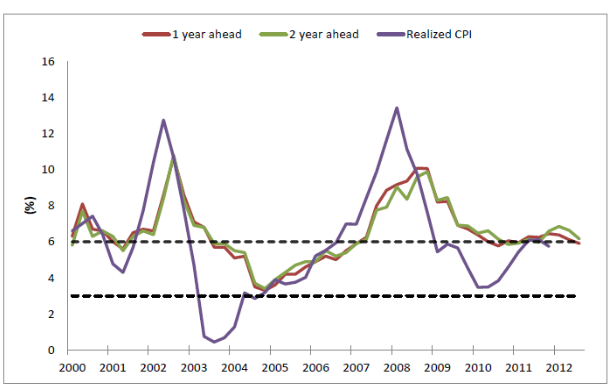
Figure 3.2. Inflation and Inflation Expectations of Agents
Panel A. Analysts



Panel B. Business



Panel C. Trade Unions



Conclusion Générale

Cette thèse est composée de trois essais en macro finance et macroéconomie monétaire. L'objet des deux premiers essais était d'explorer la structure à terme des taux d'intérêt dans les modèles de type dynamique d'équilibre général. On s'est focalisé dans le premier essai à construire un modèle simple dans lequel on a cherché à comprendre les facteurs et paramètres économiques qui affectent le niveau des taux d'intérêt et des primes de risque. Comme il est difficile de générer une prime de risque qui est consistante avec les données dans les modèles d'équilibre général, il est couramment assumé que le stock de capital est fixe. Ceci a l'avantage d'aider le modèle à générer un niveau plus élevé de prime de risque. Un autre fait est que des préférences avec formation des habitudes contribuent à générer une plus grande prime de risque. Nous relâchons l'hypothèse de fixité du stock de capital pour analyser la relation entre les paramètres économiques et la prime de risque. Ensuite dans le deuxième essai, nous utilisons un cadre plus général avec des préférences récursives et des volatilités conditionnelles des chocs qui varient avec le temps. L'analyse a été focalisée sur la contribution des niveaux de chocs ainsi que de leurs volatilités conditionnelles aux primes de risque de différentes maturités. Les travaux précédents utilisant ces types de préférences dans une économie de production, pour analyser la structure à terme assumaient que les chocs ont une volatilité constante. Ce qui présuppose que l'incertitude ne varie pas avec le temps alors que l' hétéroscedasticité est une caractéristique courante des données économiques et financières. Le modèle dans cet essai est résolu par la méthode des perturbations d'ordre trois et est estimé par la méthode des moments simulés. Nous montrons que la solution du modèle implique que la prime de risque peut être représentée par un processus de type ARCH – M avec des paramètres qui sont fonctions des coefficients structurels du model. En gros les deux premiers essais ont cherché à explorer les rôles relatifs de diverses sources d'incertitude dans la détermination des taux d'intérêt et de la prime de risque. Le troisième essai quant à lui étudie la formation des attentes d'inflation de divers groupes économiques et leur impact sur la politique monétaire avec une application sur l'économie sud-africaine. Ce sujet est intéressant car les attentes d'inflation des agents ont un impact immédiat sur le résultat de l'inflation réalisée elle-même à travers les négociations salariales et la fixation du prix des biens. On considère trois groupes d'agents : les analystes et experts financiers, les firmes et les syndicats représentant les travailleurs. Les firmes et les travailleurs étant les plus grands acteurs de fixation des prix dans l'économie, il est primordial pour une banque centrale de comprendre comment ces deux groupes forment leurs attentes d'inflation et probablement s'il y a un lien entre leurs formations des attentes d'inflation.

Les résultats du premier essai montrent que les chocs de productivité et intertemporels jouent un rôle relativement plus important dans la détermination du niveau de la prime de risque et leurs contributions relatives diffèrent à travers les maturités des obligations. Les chocs de politique monétaire contribuent seulement faiblement à la prime de risque des bonds de maturités très courtes. La contribution des chocs de productivité au niveau de la prime de risque est la plus importante pour les bonds de courtes et moyennes maturités tandis que la prime de risque des bonds de longues maturités est principalement déterminée par les chocs de préférences (intertemporels). Lorsque les agents peuvent ajuster le stock de capital sans coûts, le paramètre d'intensité de la formation des habitudes a un impact très limité sur la prime de risque ; et lorsqu'on impose des couts d'ajustement exorbitants de sorte qu'à l'optimum le stock de capital est fixe on assiste à un impact assez élevé de ce paramètre sur le niveau de la prime de risque comme c'est le cas dans les travaux précédents. Nous interprétons ce résultat par un mécanisme de lissage de la consommation. En effet, lorsque le capital est fixe, les agents ont en moins un canal de lissage de consommation et comme les préférences avec formation des habitudes induisent un niveau d'aversion au risque plus élevé les agents réclament une plus grande prime de risque lorsque ce paramètre augmente.

Les résultats du deuxième essai montrent qu'il y a une évidence de volatilité stochastique des chocs. Les volatilités conditionnelles des chocs contribuent significativement aux niveaux et à la dynamique de la structure à terme des taux d'intérêt et de la prime de risque. La volatilité conditionnelle du choc de productivité contribue positivement au niveau et à la variance des taux d'intérêt et de la prime de risque et ces contributions augmentent avec la maturité. La volatilité conditionnelle du choc de préférences contribue négativement au niveau et positivement à la variance des primes de risque. Quant à la volatilité conditionnelle du choc monétaire, sa contribution reste limitée relativement aux chocs de productivité et de préférences.

Les résultats dans le troisième essai montrent que les trois groupes d'agents forment différemment leurs attentes d'inflation et ne perçoivent pas la politique monétaire de la banque centrale sud – africaine (SARB) de la même façon. Le groupe des analystes et experts financiers ont leurs attentes d'inflation arrimée à la bande d'inflation cible (3% - 6%) de la banque centrale alors que les attentes du groupe des firmes et des travailleurs ne sont pas arrimées et sont attentes similaires. Nous interprétons ces résultats en termes de crédibilité de la banque centrale au vu des différents groupes. Ainsi les analystes financiers croient à la capacité de la banque centrale de contrôler l'inflation dans la bande de 3% – 6% alors que le groupe des firmes et des travailleurs n'y croient pas totalement. Le niveau de crédibilité de la banque centrale est donc plus élevé chez les analystes financiers que chez les firmes et les travailleurs. Ceci est intéressant quand on sait que récemment la SARB a eu des problèmes pour contenir l'inflation dans la bande cible. Le groupe des firmes et des travailleurs est celui qui a la plus grande influence sur l'indice des prix à travers les négociations salariales et la fixation des prix. Par conséquent, nous recommandons à la SARB de cibler ce groupe de ''fixeur" de prix dans sa stratégie de communication.

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Annexes

3.1 Appendices for Chapter 2

3.1.1 Appendix 1 : Unit Root Test

Tab. 3.1: Unit Roots Test

	Test Statistic	
Variable	ADF	PP
Growth Rate of GDP	-6.435*	−9.242*
Growth Rate of consumption	-4.369*	−8.95*
Growth Rate of investment	-5.59*	−7.95*
log of hours worked	-6.39*	−6.90*
Rate of Inflation	−2.102	−3.148**
Interest Rates Spread 10 year - 3 month	−3.854*	−4.274*
10 year risk premium	−4.854*	−7.274*

Note : **,* indicate significance at the 1%, 5% levels, respecively.

3.1.2 Appendix 2 : Model Moments at the SMM Estimates

TAB. 3.2: Model Fit

	Data	SMM
Means		
$\Delta y_t \times 400$	2.183	2.215
$\Delta c_t \times 400$	2.010	2.217
$\Delta x_t \times 400$	2.316	3.035
$\pi_t \times 400$	4.303	3.245
$\log h_t \times 100$	-108.310	-107.830
$i_t^1 \times 400$	5.512	5.281
$i_t^{40} \times 400$	6.790	6.786
$rp_t^{40} \times 400$	1.781	1.695
Standard deviations		
$\Delta y_t \times 400$	3.322	4.153
$\Delta c_t \times 400$	2.236	2.527
$\Delta x_t \times 400$	8.648	9.14
$\pi_t \times 400$	2.943	4.204
$\log h_t \times 100$	1.520	2.052
$i_t^1 \times 400$	2.525	4.501
$i_t^{40} \times 400$	2.361	2.587
$rp_t^{40} \times 400$	23.704	14.27
First order autocorrelation		
$\Delta y_t \times 400$	0.260	0.127
$\Delta c_t \times 400$	0.437	0.502
$\Delta x_t \times 400$	0.495	0.380
$\pi_t \times 400$	0.816	0.933
$\log h_t \times 100$	0.922	0.960
$i_t^1 \times 400$	0.930	0.974
$i_t^{40} \times 400$	0.968	0.967
$rp_t^{40} \times 400$	-0.047	0.001
Second order autocorrelation		
$\Delta y_t \times 400$	0.224	0.014
$\Delta c_t \times 400$	0.233	0.239
$\Delta x_t \times 400$	0.405	0.051
$\pi_t \times 400$	0.737	0.900
$\log h_t \times 100$	0.822	0.936
$i_t^1 \times 400$	0.886	0.943
$i_t^{40} \times 400$	0.940	0.936
$rp_t^{40} \times 400$	0.066	0.245

Note : Model moments are computed based on 50,000 simulated observations

3.1.3 Appendix 3 : Impulses Responses of Macro Series

FIG. 3.1: Responses of Macro Variables to shocks

Responses to u^a Responses to u^d

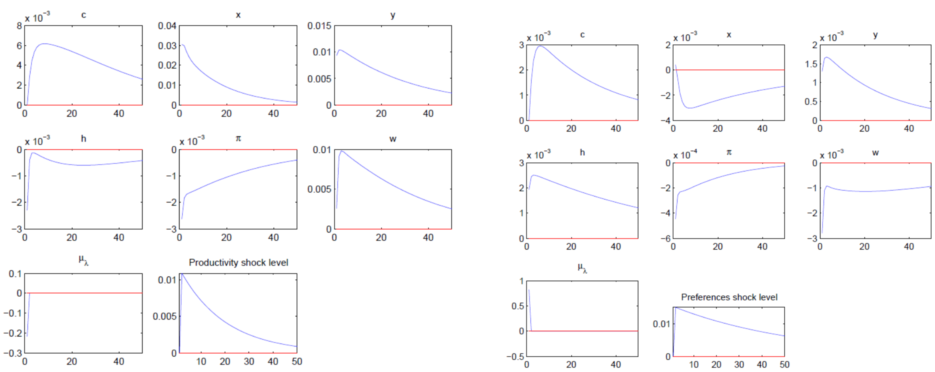

FIG. 3.2: Responses to Monetary Policy Shocks u^m

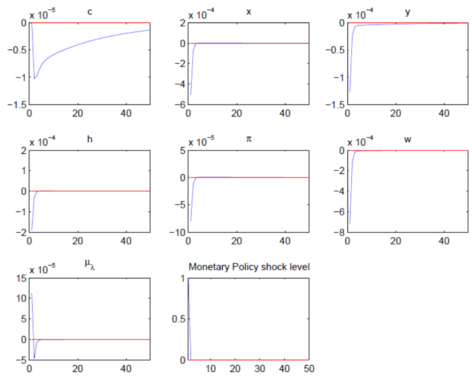

Fig. 3.3: Responses of Macro Variables to Volatility Shocks

Responses to Productivity Shock Volatility Responses to Preferences Shock Volatility

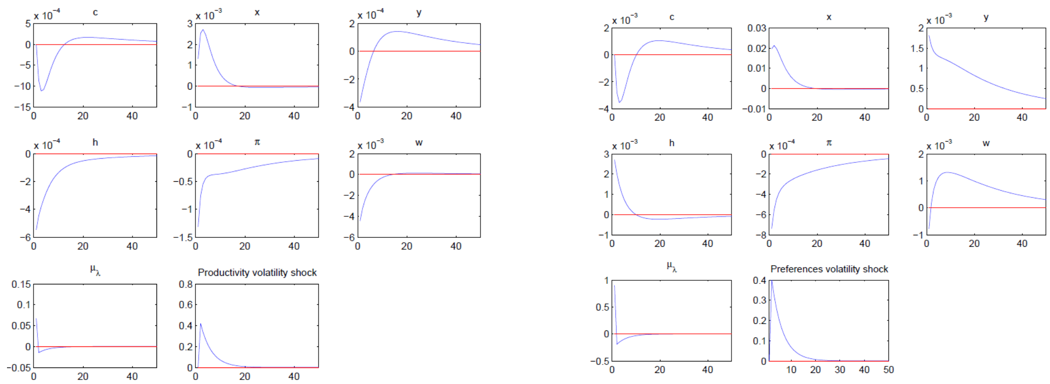

Fig. 3.4: Responses to Monetary Policy Volatility Shock