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A NON-TATONNEMENT PROCESS FOR AN ECONOMY WITH WATER POLLUTION

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SUMMARY

In this thesis the process of adjustment of price and quantities is modelled for an economy in which water quality appears as a public good. There are three contributions of the thesis:

- a synthesis of economic approaches to the problem of pollution control is presented in the static and dynamic contexts;
- a <u>decentralized</u> dynamic model is presented that respects individual tastes; and
- a public good is incorporated into the dynamic model in such a way as to permit convergence in spite of a classic issue of non-convexity.

Several critical concepts are presented in the Introduction, as well as the choice of the modelling technique and an outline of the thesis. A model in static equilibrium is presented in the second chapter. It is used to present the current state of the literature of environmental economics. It is shown that a competitive market economy is unlikely to meet the conditions of an optimal allocation and how government intervention, notably through the introduction of a transferable discharge permit market, might enable the economy to reach an optimal allocation.

In the third chapter, the problem of non-convexity is

introduced. In order to establish stability of the model in a dynamic context, it is usually assumed that the production and consumption sets are convex: it has been shown in the literature that this is not so for economies with pollution, because of the shape of the marginal damage function. A modelling strategy to overcome this is developed. Other difficulties concerning the revelation of preferences and evaluating the marginal damage function are also discussed.

In Chapter 4, the foundation of the dynamic model is presented in the context of an exchange economy. The MDP process which is used is usually thought of as a centralized procedure, but Tulkens and Zamir showed that agents have a spontaneous interest in trading. The model does away with the auctioneer in favour of arbitraging agents, and trading is permitted during the search for equilibrium, which make it more realistic. The model is fully decentralized, and the dynamics of price adjustment are defined, showing the process to be feasible, individually rational and stable. In Chapter 5, the model is extended, first to include production and then public goods are introduced into this framework using the transferable discharge permit market.

In the sixth and seventh chapters, some practical considerations are introduced. Alternative market structures are examined, and then the model's compatibility with realistic environmental constraints is explored. The conclusions are presented in the final chapter.

RÉSUMÉ

Dans la présente thèse, un processus d'adjustement des prix et des quantités est développé pour une économie dans laquelle la qualité de l'eau apparaît comme un bien public. Il y a trois contributions particulières, soit:

- une synthèse de l'instrumentation économique actuellement disponible pour contrôler la pollution de l'eau en contexte statique et dynamique;
- une dynamique d'un modèle <u>décentralisé</u> respectant les propensions individuelles; et
- l'incorporation d'un bien public dans cette dynamique, et cela de manière à converger vers un optimum malgre un probleme classique de non-convexité.

Dans l'introduction, quelques concepts clés sont définis, le choix du type de modèle est justifié, et l'esquisse du papier est présentée. L'état actuel de l'analyse des économies avec variables environnementales est présenté au chapitre 2, sous forme de modèle général. Ce modèle est utilisé afin d'établir les conditions d'une allocation optimale au sens de Pareto. Une économie concurrentielle qui comprend des biens publics ou des externalités n'est pas susceptible de remplir ces conditions sans intervention gouvernementale. La démonstration de cette proposition, qui reprend pour notre problème celle de Samuelson (1954), est fondementale. On montre aussi comment le gouvernement peut intervenir afin d'établir une allocation optimale.

Étant donné l'incapacité du système des marchés à produire une solution efficace, la structure institutionnelle utilisée içi est celle d'un marché artificiel de droits de pollution. Les firmes voulant décharger des effluents doivent détenir des permis pour le niveau désiré d'effluents. Les permis sont échangeables entre firmes afin d'introduire de la flexibilité dans le systéme et de permettre l'entrée de nouvelles industries et l'utilisation de nouvelles technologies. Au niveau théorique, un tel système comporte exactement les mêmes résultats qu'un systeme de redevances sur les effluents.

Les questions environnementales comme la qualité de l'eau mettent de l'avant plus que le problème des externalités. Au chapitre 3, il est démontré qu'une des hypothèses les plus communes dans la construction des modèles économiques est violée: celle de la convexité des ensembles de production et de consommation. Il a été démontré par Baumol (1964) et Starrett (1972) que la convexité ne tient pas lorsqu'on admet la pollution, notamment à cause de la forme de la fonction de dommages marginaux. Il n'y a plus une solution unique au problème et il se peut qu'il n'y ait aucun équilibre.

La difficulté se présente avec plus de force dans le contexte dynamique mais, bien qu'on ne puisse y échapper, un moyen de contourner ce problème est developpé qui repose justement sur la "séparabilité" d'un marché de droits d'effluents. La realisation de l'optimum devient plus compliquée, cependant, et elle implique l'établissement de cibles intermédiaires. D'autres difficultés associées avec la fonction de dommages marginaux, soit le problème de la révélation des preferences et la difficulté de l'évaluation des dommages environnementaux, sont aussi étudiées.

Un état optimal peut exister, mais il n'est pas evident qu'on puisse y arriver. Aux chapitres 4 et 5, alors, le déplacement de l'économie à sa nouvelle cible est simulé à l'aide d'un modèle dynamique. Dans le marché, les quantités s'échangent en réponse à des écarts entre les prix affichés et ce que les individus sont prêts à payer. Il y a eu peu d'essais du genre pour des économies avec des biens publics quelconques, et encore moins avec des externalités environnementales. Les mieux connus, comme le processus de Malinvaud, Drèze et de la Vallée Poussin (MDP), dépendent d'un bureau central qui détermine l'allocation de tous les biens et services. Ce bureau du plan communique avec chaque consommateur et producteur, ajustant son allocation proposée graduellement en fonction des réponses. Malheureusement, des modèles d'économies planifiées reflètent mal les économies de l'Occident et donc ne présentent pas de solution véritable.

Même dans les procédures destinées à représenter l'ajustement des prix dans une économie de marché (sans biens publics ou des externalités), on présume souvent que tous les agents acceptent les prix comme des données, alors que la mécanique de ces procédures n'est pas définie. Afin de résoudre la question de savoir comment les prix et les quantités s'ajustent, on doit établir des prix d'équilibre à partir des actions des agents individuels. Tulkens et Zamir (1979) ont démontré qu'il peut y avoir une incitation à échanger dans un modèle du type MDP, conçu originalement comme procédure de planification. Un modèle de ce type est développé au chapitre 4, et rendu entièrement décentralisé. Au niveau d'une économie d'échange, il est démontré que le modèle est physiquement possible, individuellement rationel, et globalement stable. La "stabilité" veut dire ici que l'économie converge vers un optimum de Pareto.

Au chapitre 5, la production et les biens publics sont réintroduits et on utilise le marché des permis d'effluents afin de déterminer l'allocation optimale des effluents des firmes. Le marché des permis est moins sensible à la difficulté de la non-convexité que l'approche par redevances sur les effluents.

Certaines questions de caractère pratique sont étudiées dans les chapitres 6 et 7. D'abord, différents mécanismes d'intervention gouvernementale sont comparés. Le marché de permis semble être plus pratique pour la protection de la qualité de l'environnement, et fait mieux face à la croissance et à l'incertitude. Au chapitre suivant, il est montré que le modèle développé ici est compatible avec une structure plus sophistiquée de la contrainte environnementale, c'est-à-dire, avec certains modèles biologiques de la dynamique des rivières.

Les conclusions sont présentées en un dernier chapître.

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LIST OF SYMBOLS

Where a symbol is used in one specific chapter, the chapter number is given in brackets

Lower Case	Upper Case
	B transaction matrix (4)
c water quality	C transaction matrix (5)
e effluents	E matrix of quantity effects
f production function	
g permits held	G total permits
h an allocation	
i a consumer	I the last consumer
j a producer	J the last producer
k a private good	K the last private good
ℓ a contaminant	L the last contaminant
m a zone	
n a zone	N the top zone
p prices	
q permit price	
r revenue from firms	R stream variable (7)
s unit of account	
t time index	T tax
u utility function	U utility function (3)
v { indirect utility function; vector of weights (5) }	V { separable utility function (3); stream variable (7) }
w vector of weights	W stream variable (7)
x consumption of private goods	
y net production of goods	

Italic Capitals

$\mathcal{H}$ set of all allocations

$\mathcal{I}$ - set of all consumers

$\mathcal{J}$ set of all producers

$\mathcal{K}$ set of all private goods

$\mathcal{L}$ set of all contaminants

$\mathcal{N}$ set of all zones

$\mathcal{R}$ set of real numbers

Greek Lower Case

α { Lagrangian multiplier
stream coefficient (7) }

β stream coefficient (7)

γ stream coefficient (7)

δ transfer function

η Lagrangian multiplier

κ impact matrix

λ Lagrangian multiplier

μ Lagrangian multiplier

π profits

υ MRS

ϕ RTS

ψ Lagrangian multiplier

ω initial resources

ξ a vector

Greek Upper Case

Θ share of profits

Abbreviations

MDP	Malinvaud, Drèze and de la Vallée Poussin procedure
PBD	Pigou, Baumol and Dales equilibrium
TDP	Transferable discharge permit
TZ	Tulkens and Zamir procedure

To my Parents

They knew I could do it,
but they had to wait.

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In the end there is one person responsible more than any

other, who, through her love and encouragement helped me over the many hurdles. This thesis is but one manifestation of the change that has come about in my life; she has brought happiness and fulfillment in many areas. I speak, of course, of my dearest wife, Dorothy Levine.

FOREWORD

To some it appears that the fundamental problems affecting the environment are rooted in the economy. This suggests that solutions may lie there as well.

The underlying motivation of this thesis is to discover and apply the best economic tools available to the solution of environmental problems. Since this is somewhat too broad an ambition, the focus here is on one particular problem, the impact of industrial effluent on water quality.

While environmental economics has started to come into its own as a field, there are still opportunities to borrow from other branches of the discipline. Economists are all but united in the belief that taxes or other measures could correct distortions introduced by externalities such as pollution. Pigou was perhaps the first to make such a specific recommendation, invoking the problem of smoke so prevalent in London until the late 1950's (1932, p.182).

However, the tax that sustains an optimal allocation requires that all other adjustments induced by the tax have also taken place. This suggests, as noted by Baumol and Oates in 1971, that we should be concerned not only with general equilibrium modelling, but with the properties of the dynamic processes that bring us to an optimal allocation. Until now, this has not been done, and it is the object of this paper to present a dynamic adjustment framework, within which various economic tools for the control of water pollution can be evaluated. Existing adjustment models are not applicable, either because they do not incorporate public goods, or, if they do, because they are based entirely on central planning, an approach that does not have wide application in today's world, even in the socialist countries.

As those familiar with the literature will realize, this turned out to be a more substantial project than I initially understood, and some planned extensions were cut back. I had intended a more extensive application of the model relating to the St-Maurice River, which is described in the last chapter. But, as the theory to support such an endeavour was not fully in place, enhancing the theoretical models became the central focus of the paper.

The modelling approach used is general to many different combinations of circumstances: however, for expositional clarity it is not convenient to repeatedly analyze each step forward in the context of a variety of possible situations. In particular, three premises are used to minimize such confusion and provide continuity to the discussion.

Most evidently, the discussion has been limited to water pollution. In some respects water pollution is more tractable mathematically than air pollution, certainly where non-estuarial rivers are concerned. The water flows in the same direction all the time, thus pollution sources affect downriver points only. Although it is not strictly accurate to do so, a river may be modelled one dimensionally, which implies instantaneous mixing of effluents across the stream at the point of discharge. For a discussion of some of the problems of modelling air pollution, see Montgomery (1972) and Hahn (1986).

The second issue is the presumption that industrial pollution has been allowed to go beyond socially optimal levels, so that the scenario here is one where a reduction of pollution is required. This point of view is supported by a demonstration in Chapter 2. The model is however general to many other situations: for example, changes in tastes that render pollution control less desirable at some point in time could be modelled just as easily.

The third assumption is that polluters pay for the cleanup, rather than receive subsidies. This question goes beyond the analysis of efficient outcomes, although it has significant political overtones (as it may involve substantial transfers of resources). This is not a matter taken up in this paper, though the analysis here could apply equally well to a subsidy régime.

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INTRODUCTION

Many uses of water are affected by its quality, most obviously its use for drinking water. Water is also used in food processing (e.g. brewing, canning), it serves as habitat for sea food, and it is the basis for numerous recreational activities. A clean environment and its value as a bequest to future generations are also important. Directly and indirectly, it has a critical effect worldwide on people's health and well being.

What is clear from this is that the quality of water is important and it has value: it also brings us up against one of the earliest problems of economic thought, the relationship between value and price. In many places, water (regardless of quality) has no price at all: in most others, there are only nominal charges associated with the quantity consumed. There is almost never a price attached to the quality of water.

If we are going to put a value on the quality of water, we must have an idea of how to measure it. In fact, this is very difficult. Uncontaminated water bodies have scores of natural mineral and organic compounds in them, usually in different proportions. Thus, they are not all of the same quality. Indeed, if they were, this would probably result in a drastic reduction in the variety of aquatic species on the planet. These species exhibit different tolerances to the different substances. We do not have rank orderings for the importance of species, thus we cannot compare the value of simultaneous variation in the concentrations of substances with different toxicities to aquatic species. This makes it impossible to establish a single index of quality.

In the face of this complexity, it is necessary to make some assumptions. In this paper, the use of the term "pollution" will be reserved for changes in water quality brought about by human intervention. As a basic principle, it is assumed that the range in the quality of watercourses in their unpolluted state has provided habitat for a diversity of species, therefore changes caused by pollution are assumed to be negative in their effects. This implies that reducing pollution increases water quality, and that the limit case of zero pollution would maximize water quality.

In the remainder of this chapter, the concepts of externalities, public goods and property rights, which are fundamental to an economic treatment of the problem of water quality, are introduced. The basic modelling technique is then discussed, followed by a discussion of the organization of the paper.

Externalities and Public Goods

In economic terms, water pollution can be characterized as an externality. In the production process, activity generally adds to value. The reverse is true when it comes to pollution: the more that is "done" to the water, the more it is degraded. Pollution is a byproduct, or "externality", of water use, notably when applied to industrial processes. An externality is said to exist when the utility (productivity) of a consumer (producer) is affected by the activity of another agent (consumer or producer).

Water quality, meanwhile, is best described in the economist's terms as a public good. Since the nature of the analysis developed in this paper revolves around this term, it is important to be clear about what is meant by "public good". A public good is generally taken to be "one with the property of involving a 'consumption externality', in the sense of entering into two or more persons' preference functions simultaneously" (Samuelson, 1969, p.102), thus the two concepts are closely linked.

There is a great deal of debate, if not confusion, as to whether this is by itself a sufficient definition of public good, as Blumel, Pethig and von dem Hagen (1986) effectively illustrate in a recent survey of the literature. Samuelson himself argues that his definition leaves only a "knife edge pole" for private goods, against the whole continuum of public goods (1969, p.108). Factors such as non-excludability, non-rejectibility, non-congestibility and spatial location have all been suggested to distinguish various kinds of public goods.

Goods like bridges and theatrical performances meet the basic jointness of consumption criterion, and thus could be considered public goods (Mishan, 1971; and Head, 1977). However they have the potential for price exclusion, which causes the problem of the "free rider" (about which more below) to disappear. Either bridges and theatrical performances are private goods (sometimes supplied by public institutions), in which case Samuelson's definition includes goods which do not belong, or there are different kinds of public goods, in which case the definition fails to distinguish goods with pertinent differences.

Price excludability is a relevant criterion only when the goods in question are positively valued. When some or all consumers' marginal evaluations of the "goods" are negative, non-rejectability is a more appropriate criterion. Examples include defence spending as viewed by pacifists and water fluoridation (Shoup, 1969, p.74). Freeman (1984), using Baumol and Oates' (1975) terminology of depletable and undepletable externalities, disputes the relevance of such a distinction. But carefully chosen examples (cf. Bird, 1987) demonstrate that, at least, new policy options appear (such as charging rejecters) when goods are rejectable. Goods with a costlessly depletable externality are private goods.

A second objection often put forward is the notion of congestibility: goods may manifest jointness of consumption only up to a certain level, beyond which the marginal user effectively reduces the quantity or quality of the good that is available to all. This is obviously the case of highways and many other public services such as hospitals. Congestion can be modelled as an externality and hence can be treated with the same basic tools that public goods in general require. The externality nature of congestion does not diminish, but rather reinforces the public nature of the good. Nevertheless this characteristic can distinguish between types of public goods and help to determine the optimum level of provision (cf. Mishan, 1971; Dorfman, 1984).

Another important concern is the effect of spatial separation. Buchanan (1968, p.54), Shoup (1969, p.69) and Foley (1970, p.73), among others, have noted this problem. Buchanan uses the example of a fire hall to show that the quality of protection depends on the proximity to the fire hall, thus the quality of the good entering into two consumers' utility functions is different. Goods of different quality are generally thought of as different goods. The same reasoning applies to parks and even to national defence. Proximity to hostile borders or, in this modern age, strategic targets effectively reduces the level of protection provided by the military. Environmental quality also varies over space, as several authors have observed (Montgomery, 1972; and Tietenberg, 1973b), and indeed the complications that this introduces will be considered in the model developed in this paper.

A "pure" public good should perhaps meet all of these criteria, to permit as clean and simple an analysis as possible: it should, therefore, manifest jointness of consumption, be non-excludable or non-rejectable (depending on how it is marginally valued), and be spatially invariant. Congestibility does not reduce the public nature of a good, but for simpler treatment it would be better to avoid such problems when they are not critical to the analysis.

Water quality meets these criteria and is well qualified for treatment as a public good. It is virtually impossible to supply water of varying quality according to contribution, so that it is not price excludable. It is non-rejectable especially when used in food processing. Water is not generally subject to congestion, certainly insofar as it is generally abundant in western industrial countries. As noted, water quality does vary spatially, and this will be taken into account. For the purposes of this paper, I shall ignore the potential "impurities" which would greatly burden the discussion, and in general freely use the term "public good" where water quality is concerned.

Property Rights

Another fundamental concept in the economic treatment of environmental problems is that of property rights. Implicit in almost any solution to the problem of pollution is the assignment of property rights. Environmental degradation can be traced in part to the fact that the rights to water quality are not well defined in law. Even application of the seemingly simple "polluter pays principle" has hidden within it the idea that users have a right to require clean water, rather than a right to dirty it.

By creating and clearly assigning rights to water quality through laws and regulations, the government can facilitate the search for an efficient solution. If these rights are transferable, as is posited in this thesis, market-like procedures can be implemented, which allow a greater degree of decentralization. The property rights would rest with the consumers if it were their legal right to enjoy good quality water, and to be compensated for any deterioration; the property rights would rest with firms if it were an unclaimed good to which the polluting industries have acquired rights, thereby gaining some right to compensation if they were required to clean up.

The standard analysis shows that it makes no difference who holds the rights, that an efficient allocation can be defined either way. This has led economists to describe the two cases as "symmetrical" (Page, 1973; Sims, 1981; Mestelman, 1982). For brevity, I generally consider only the case where consumers have the right to clean water, thus it is the producers that pay. In Chapter 6 the alternative approach, where the firms are subsidized, is discussed.

The Neoclassical Model

The modelling technique used in this paper traces its origins back more than a century to the foundations of neoclassical economics. The origins of many concepts, now taken for granted, deserve at least to be mentioned. Among the notable contributions are those of Walras, Jevons and Pareto. Walras attempted a coherent mathematical representation of the economy, introducing the concept of general equilibrium. In general equilibrium, prices in all markets have adjusted, such that demands and supplies in all markets just balance. Such an allocation will endure, unless subjected to some shock. Jevons highlighted the importance of the valuation of the marginal unit in price determination (hence the term "marginalist"), while Pareto developed a definition of the "economic optimum" based on efficiency. If some resources are used inefficiently, it should be possible to squeeze out more product without adding more inputs. A Pareto optimum is reached when no one can be made better off without someone else being made worse off.

More recently, the contribution of Arrow and Debreu provided a more rigorous mathematical foundation for the general equilibrium economic model, and extended it to cover futures markets and situations of uncertainty. In the real world, adjustment to equilibrium takes time, but "static" models of this type ignore this, assuming that all adjustment is instantaneous: the economy simply moves from its initial position to the new equilibrium. Arrow and Hurwicz (1958) developed a dynamically stable general equilibrium model that purports to show how an economy can move from any starting point to its equilibrium configuration. Their approach is usually known as the "tâtonnement" procedure, a direct reference to the mechanism envisioned by Walras. Such models are a central focus of this paper, and the discussion will be picked up again with reference to Chapter 4.

Pigou (1932) may not have been the first to use environmental examples in his discussion of what are now termed externalities, but he is generally credited with proposing state intervention in the form of corrective taxes to rectify the resulting distortions. This approach has been controversial, as some argue, following Coase (1960), that the cost of such intervention is likely to outweigh any welfare gains that may be achieved. But, as Whitcomb put it, "however many nails have been hammered into the coffin of the Pigovian tradition, there is not yet a corpse inside" (1972, p.1).

Samuelson provided a rigorous foundation for the analysis of public goods in his articles of 1954 and 1955, building on the work of Lindahl, who published in German on the subject in 1919. Samuelson set up the conditions of optimality in the context of an Arrow-Debreu type model, and clearly presented the problem of the non-paying user, or "free rider". The inefficiency of a market economy in dealing with public goods is proven by showing that it is possible to produce more value in the economy without using greater resources or effort.

In the environmental economics literature various tools, such as Pigou's corrective taxes, have been modelled formally and shown to be able to sustain an optimal allocation <u>once it is achieved</u>. There has been something of a presumption that the dynamic properties of these models will meet all the necessary conditions to ensure that an optimum is found. For example, Burrows wrote in 1986:

> Iterative taxes or regulated standards, which make stepwise adjustments on the basis of <u>marginal</u> cost-benefit analyses,... would yield a social gain at each step and converge on the interior peak, which is at the globally efficent pollution level under the conventional convexity assumptions. (1986, p.102)

His reference to the "conventional convexity assumptions" is of particular importance, as these do <u>not</u> usually apply in models involving pollution; this makes his appeal to iterative adjustment mechanisms inappropriate, as these depend upon the convexity assumptions. Burrows does not define or study the dynamic properties of his model at all.

This brings us to the central challenge of the present paper, which is to develop a dynamic model of price adjustment, incorporating water quality as a public good, which also takes account of the inapplicability of the "conventional" convexity assumption. This is done in the context of a decentralized or market economy.

In passing, it may be noted that some authors have argued that standard neoclassical modelling techniques, such as that of Arrow-Debreu, are not suitable for modelling environmental problems. For example Livingstone argues:

> In orthodox economic analysis, institutions are assumed to be fixed and economic efficiency is determined within that context. The theory contains no criterion whereby two or more efficient solutions resulting from separate institutional arrangements can be compared....
>
> In order to determine "optimal" resource use, the Paretian approach takes the existing set of initial endowments as given and analyzes gains from trade within that context. The approach is limited because it leaves the analyst unable to address how changing policies affect individual choice and consequent economic outcomes. Obviously, the Pareto criterion... is not suited to the policy debate. (1987, p.282)

As will be demonstrated, such charges are simply not true:

using the Paretian approach, the basic neoclassical framework is institution-free. Various institutional arrangements can be introduced and their outcomes compared. Furthermore, it is possible to vary the allocation of property rights (which alters the distribution of income) and compare the outcomes.

There are, of course, alternative approaches, most notably the "materials flow" approach used by such authors as Kneese, Ayers and d'Arge (1970) and Nijkamp (1977). In my view this approach is undermined by its reliance on fixed technologies. The marginalist approach of the neoclassical tradition allows for a variable technology, which is crucial to the modifications in capital stock necessary for environmental improvement.

Outline of the Thesis

Broadly speaking, **Chapter 2** represents the current state of modelling for economies that include environmental variables. It is not a resumé, however, let alone an exhaustive review of the literature. The economy is presented in the form of a static general equilibrium model, which is used to define the conditions that would have to be fulfilled for the economy to be at a Pareto optimal allocation. It is also useful to demonstrate that competitive or free market economies that include public goods or externalities are unlikely to fulfill these conditions without intervention. This is one of the most important demonstrations in the public goods literature. The model can be used to test whether various economic tools could restore the economy to an optimal allocation. A rigorous demonstration of these points is presented.

The various kinds of externalities give rise to endless possibilities in model structure: Arrow, in a much cited article (1969), develops a model in which each consumer's utility is affected by the quantity of every other consumer's use of each good. Laffont (1977) has an even more complex model in which every producer is also affected by the production of others. It is also possible to imagine consumption affecting production and production affecting consumption. By comparison, the model that is developed here is comparatively confined: consumers' utility is affected by pollution externalities associated with production, but it retains all the essential difficulties with respect to public goods described above.

Given the demonstrated failure of the market to yield an efficient solution, the institutional arrangement used to overcome the problem is an artificial market, often referred to as a transferable discharge permit (TDP) market. Companies wishing to discharge into the water must acquire and hold permits for the desired discharge levels. These permits would be transferable between firms to give flexibility to the system, allowing for new entrants, new technologies, etc. On a theoretical level, such a system results in exactly the same outcome as a tax on effluents.

It is possible to think of situations where the use of permit markets would not be appropriate. Marginal damages associated with the most toxic contaminants are astronomical. If there is no practical safe level for a pollutant, outright bans on discharges would be more appropriate than creating artificial markets for zero or infinitessimal amounts. Substances such as mercury, PCBs, dioxins, etc., are thus excluded from the discussion.

Environmental issues such as water quality bring more than the problem of externalities to the fore. In **Chapter 3**, it is shown that one of the most common assumptions in economic modelling is violated, that of convexity of the production and/or consumption set. Using calculus, convexity permits the identification of a global maximum, unique to the distribution of income. In the current context, this is a Pareto optimum. Convexity implies that, if two points are in the same production possibility set, then any weighted average of the two must also be possible.

It has been shown by a number of authors (Baumol, 1964; Starrett, 1972) that convexity is commonly violated in economies incorporating pollution or other externalities.

There is no longer a unique solution to the problem, or there may be no equilibrium at all. Since Baumol introduced the problem in 1964, the debate has revolved not so much around the theoretical possibility of non-convexities as their importance in reality. Quoting again from Burrows:

> it is far from clear that external costs do generally yield relevant nonconvexities; yet even if they did the normative implications would not be simple. The reason is that a locally efficient state may be a significant improvement on the no-policy situation. (Burrows, 1986, p.103)

As will be demonstrated, such optimism is not warranted. The difficulty presents itself most forcefully in the dynamic context, and, while it cannot be easily removed, a way to get around it is developed, incorporating a TDP market. Realization of the optimum becomes somewhat tricky, however, and involves establishing intermediate environmental targets, described as Pigou-Baumol-Dales equilibria.

In **Chapter 4** the main issue of the thesis is developed, the problem of how to move the economy to its new target. In the market, prices change to reflect imbalances between supply and demand: this process is simulated with a dynamic model. There have been few attempts to do this for economies with any kind of public good, let alone environmental externalities.

The best known dynamic model for economies that incorporate public goods or externalities is the MDP procedure, named for the three economists who originally designed the method—Malinvaud (1967, 1970-71) and Drèze and de la Vallée Poussin (1971). This procedure relies upon a central planning board to determine the allocation of <u>all</u> goods and services. Unfortunately, planning models do not reflect the functioning of western economies very well.

In processes that are intended to simulate price adjustment in the decentralized fashion of a market economy, all agents are presumed to be price takers. Who then adjusts prices as the economy moves towards equilibrium? In the standard tâtonnment process, this role is usually assigned to an "auctioneer" or a "helmsman".

> It is quite remarkable that this is a centralized process, remarkable because the price mechanism is generally considered to be decentralized. In the tâtonnment process, there is a central agent, an auctioneer, who is informed about demand and supply of each agent in the economy at prevailing prices. (Weddepohl, 1983, p.381-382)

In the context of a private goods exchange economy (no production), Tulkens and Zamir (1979) showed that there is an incentive to transact in MDP type economies. During the search for equilibrium, agents will want to transact at the prevailing vector of (non-equilibrium) prices. While there is no auctioneer, it is somewhat unclear how this vector of unique prices (one for each good) comes to the attention of agents. Nevertheless, this procedure does present a good starting point, as it relies on the motivation of individual agents.

Building up from a centralized MDP process, a Tulkens and Zamir type exchange economy is developed and then fully decentralized, showing how individual agents' actions can translate into the uniform disequilibrium price vector. It is demonstrated how prices adjust through the process, eventually reaching equilibrium values.

In the model developed in earlier chapters, water quality was determined as an externality associated with production: in **Chapter 5**, therefore, the dynamic model is extended to include production and water quality variables. Production is introduced first, without the complication of effluents, to clearly expose how it is handled. The primary problem is that producers are able to transform any given mix of goods into some other, so that the resource constraint is not fixed.

It is not surprising that once production has been introduced into the model, that a permit market for effluents can be incorporated as well. It does introduce some complications, however: since effluents are traded only among firms, they cannot be treated in the same way as other goods. Use of a permit market avoids the instability that the non-convexity problem poses: however, it does not immediately yield a Pareto optimum, but a Pigou-Baumol-Dales equilibrium. In the last section, a rule is established that would permit a gradual adjustment of the number of permits to the Pareto optimal level.

Up to the end of Chapter 5, most of the discussion is theoretical and unrestricted in nature. In the last two chapters some questions are discussed which make the model more practical in its application.

In the first part of **Chapter 6**, transferable discharge permits are compared to other measures for the control of pollution, such as taxes, subsidies and direct regulation. Besides their superiority on technical grounds (relative immunity to the non-convexity problem), TDP markets are shown to have advantages with respect to uncertainty as well as in the context of economic growth. In the latter part of the chapter, the question of auction procedures is studied. This is important because the number of market participants is likely to be small, introducing the possibility of market manipulation.

In **Chapter 7**, a specific river model, based on Qual2, is introduced in place of the general formulation used up until then. Qual2 is perhaps the most widely used model in the field. It was developed by the U.S. Environmental Protection Agency, and has been applied in Wisconsin (O'Neil, David, Moore and Joeres, 1983). While still more sophisticated river models are now available, the data to support them rarely is so. It is thus shown that the theoretical model which has been developed has direct application to real world problems of water pollution.

The paper ends with a summary of the principal results, presented in the Conclusion.

2

WATER QUALITY AND THE STATIC OPTIMUM

This chapter is divided into four sections. In the first section an economy is described in which water pollution (effluent) is a byproduct of the process of producing certain goods. The effluent degrades water quality, which is, together with the level of private goods consumption, a determinant of individual welfare. It follows that the less the water is polluted, the more satisfaction people get.

In the second section, the marginal rates of substitution between goods that would result in a Pareto optimum in the economy are established for consumers and producers.

In the third section, a price system is introduced and other standard properties are demonstrated. In particular the allocation is shown to be a pseudo-equilibrium, for which a supporting price vector exists. The impact of a price system based only on private goods is then studied. This might be seen as the "pre-control" situation, where firms are permitted to dump effluents into the environment without costs or limitations. It is shown that an equilibrium may be established, but one that is non-optimal.

In the final section there is an analysis of how the government could intervene to restore the optimal conditions in what is otherwise a private market economy. The proposed system involves the creation of an artificial market for effluent permits.

The Economy

In this economy there are consumers and producers. A government will be introduced later. There are also private goods and a public good (water quality). A number of different models have appeared in the literature involving environmental variables. Some examples of general equilibrium models are the following: Tietenberg (1973a, 1973b), Page (1973), Baumol and Oates (1975), Hamlen (1977), Suchanek (1977, 1979), Tulkens (1979), and Mestelman (1982). The six models show a number of similarities. For example:

i) all except Mestelman treat the environmental quality variable as a factor affecting consumers' utility;

ii) all consider pollution to be exclusively the result of productive activity;

iii) three (Baumol and Oates, Mestelman and Tietenberg) consider environmental quality as an input factor in production, while the others do not;

iv) Page, Tietenberg and Tulkens use a general form describing the relationship between emissions and environmental quality, while the others use the simple sum of pollutant emissions.

In this model, I follow (i) and (ii) above. With respect to (iii), the quality of the water that the firms use is ignored for simplicity, although many firms, such as those in the chemical and food industries, care a great deal about the quality of the water they use, as demonstrated by the fact that many treat their intake water. As for (iv), the model used in this chapter is general. The spatial aspect is examined in Chapter 7, using an empirically tested river model.

$\mathcal{I}$ is the set of all consumers in the economy. Individuals (which may be households) are indexed by i. Thus $\mathcal{I} = \{1,2,..,i,..,I\}$.[1] $\mathcal{J}$ is the set of all producers, which are individually indexed by j: $\mathcal{J} = \{1,2,..,j,..,J\}$.

The use and benefits of a private good rest exclusively with the agent in possession of the goods. The consumption of private goods does not result in any externalities in this economy. $\mathcal{K}$ is the set of all the types of private goods. They are indexed by k: $\mathcal{K} = \{1,2,..,k,..,K\}$. The first good, k=1, will be used as a numéraire, in terms of which all other goods will be valued.

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[1]Throughout this paper, italic capitals are reserved for sets. The plain capital will in general refer to the last member of the set, with the lower case form used as the index. Vectors are indicated in boldface.

$\mathcal{L}$ is the set of all contaminants. They are indexed by ℓ: $\mathcal{L} = \{1,2,..,\ell,..,L\}$. Water quality is usually defined by a vector of concentrations of contaminants in the water, such as lead (Pb), copper (Cu), phenols, etc., and parameters such as acidity (pH). Water quality criteria are usually established without reference to possible interactions, and while this is in certain cases demonstrably invalid, it will suffice as a working hypothesis (cf Beavis and Walker, 1979).

Ambient water quality will be denoted by the vector $\mathbf{c}$. This will represent the concentration of each contaminant in the water. For most parameters, the higher the concentrations, the lower the quality. Firms' effluents will be denoted by $\mathbf{e}$; as they are in absolute amounts of contaminant, they must be moderated by stream characteristics such as flow and sedimentation rate to provide the impact on $\mathbf{c}$. Since it is convenient to measure water quality as a positive value, $\mathbf{c}$ will be a function of the negative of $\mathbf{e}$.

The individual's utility depends on his/her consumption of private goods and the quality of the water, the public good. Consumption of private goods is represented by the vector $\mathbf{x}^i$. The functional dependence of the individual's utility on his consumption of private goods is straightforward and conventional. Utility is affected by ambient water quality, either through direct contact, such as drinking water, or through impacts on other choices, such as swimming or fishing. In this model, consumers do not affect the quality of water through their own actions (as consumption does not engender externalities), and the quality of water is the same for all consumers.

Consumers' utility can be expressed as a twice differentiable function defined by:

$$u^i(\mathbf{x}^i, \mathbf{c})$$
where $\mathbf{x}^i = (x^i_1, \ldots, x^i_k, \ldots, x^i_K)$ and $\mathbf{c} = (c_1, \ldots, c_\ell, \ldots c_L)$
$$\partial u^i/\partial x^i_k \geq 0 \ \forall\ k,\ \ \partial u^i/\partial x^i_1 > 0 \text{ and } \partial u^i/\partial \mathbf{c} \geq 0 \qquad (2.1)$$

u^i is continuously differentiable and strictly quasi-concave in $\mathbf{x}$ and $\mathbf{c}$. Since Baumol (1964) and Starrett (1972), it is commonly accepted that pollution can induce non-convexities in the production and/or consumption sets. However, the problem is not rooted in individual preferences, but in adverse impacts of one agent upon another. Moreover, it does not compromise the existence of the optimum in the static context, but whether the optimum may be found (Baumol and Bradford, 1972, p.173, ff.). This matter is taken up in the next chapter.

Producers are assumed to be immobile. The net output of each of the K private goods is given by $\mathbf{y}^j_k$. Pollution (effluent) is produced as well, denoted by $\mathbf{e}^j$. Producers are efficient. The technology they employ may be characterized by a quasi-convex twice differentiable production function:

$$f^j(\mathbf{y}^j, -\mathbf{e}^j) = 0$$
where $\mathbf{y}^j = (y^j_1, \ldots, y^j_k, \ldots, y^j_K)$ and $\mathbf{e}^j = (e^j_1, \ldots, e^j_\ell, e^j_L)$

$$\partial f^j/\partial y^j_k \geq 0 \ \forall \ k, \quad \partial f^j/\partial y^j_1 > 0 \text{ and } \partial f^j/\partial(-e^j_\ell) \geq 0 \qquad \langle 2.2\rangle$$

The negative value of the effluents "**e**" reflects their role as an input in the production process.

A feasibility constraint is required for the private goods: consumption cannot exceed net production plus initial resources, denoted by the vector ω:

$$\sum_{i=1}^{I} x^i_k = \sum_{j=1}^{J} y^j_k + \omega_k \qquad \text{where } k = 1,2,..,K \qquad \langle 2.3\rangle$$

It is necessary to link the effluents of the various firms with the level of ambient water quality, the **c** that is of concern to consumers. At present, the representation of this relationship will be in a general form. The function δ might be seen as a dispersion or transfer function that depends on a variety of stream factors. Stream functions need not be the same for all contaminants, as some may react differently to such things as temperature, etc. A specific form is introduced in Chapter 7. As mentioned, water quality will be measured positively, hence it is a function of the negative of effluents.

$$c_\ell = \delta_\ell(-\mathbf{e}_\ell) \qquad \text{where } \mathbf{e}_\ell = (e^1_\ell,...,e^j_\ell,...,e^J_\ell) \ \langle 2.4\rangle$$

This equation completes the description of the economy. Firms' effluents, moderated by natural systems, determine water quality, which in turn affects consumers' utility. These aspects are grafted on to what is otherwise a conventional economy.

Optimality Conditions

The four equations ‹2.1›, ‹2.2›, ‹2.3› and ‹2.4› can be combined into a single Lagrangian representing "the economy". There is a large set of solutions, one for each possible distribution of resources. Each is a Pareto optimum, and we can define the conditions under which such an allocation is realized. In an economy with private goods only, such points represent stable equilibria. With the introduction of public goods, however, optimal equilibria are not likely to be arrived at spontaneously, so that the term "pseudo-equilibrium" is often applied.

A pseudo-equilibrium is distinguished by the fact that it is based on at least some individualized rather than market-wide rates of exchange. Individual rates of exchange mean that each consumer or firm is subject to a price equal to their private marginal evaluation of the public good. As agents must correctly reveal their preferences and costs for the optimum to be realized, the assumption that agents behave competitively becomes difficult to maintain, when, as will be seen, it is not in their interest to do so. Hence it is unlikly that the optimal equilibrium will emerge spontaneously in an economy which includes public goods.

Formally, an allocation is an (I+J)K+J+1 dimensional vector representing the distribution of individual consumptions, inputs and outputs, effluents and ambient water concentration. It is said to be feasible if it conforms to the external constraints of the system, notably endowments and technology. This implies that ‹2.2›, ‹2.3› and ‹2.4› must be respected. Let h represent a feasible allocation: $h \in \mathcal{H}$, the set of all feasible allocations.

Definition: a Pareto optimum is a feasible allocation $h \in \mathcal{H}$ such that there exists no other feasible allocation $\hat{h}$ by which, for every i, $u^i(\hat{\mathbf{x}}^i, \hat{\mathbf{c}}) \geq u^i(\mathbf{x}^i, \mathbf{c})$, with at least one i with $u^i(\hat{\mathbf{x}}^i, \hat{\mathbf{c}}) > u^i(\mathbf{x}^i, \mathbf{c})$.

We find the necessary conditions for a Pareto optimum by maximizing the Lagrangian expression combining the four equations of the economy. In ‹2.5›, the four basic equations are recognizable, each modified by a "Lagrangian multiplier", which will take on particular significance.

$$\pounds = \sum_{i=1}^{I} \alpha^i u^i(\mathbf{x}^i, \mathbf{c}) - \sum_{j=1}^{J} \psi^j(f^j(\mathbf{y}^j, -\mathbf{e}^j)) - \sum_{\ell=1}^{L} \lambda (\mathbf{c} - \delta(-\mathbf{e}))$$

$$- \sum_{k=1}^{K} \pi_k \left[\sum_{i=1}^{I} x^i_k - \sum_{j=1}^{J} y^j_k - \omega_k \right] \qquad ‹2.5›$$

where $\mathbf{x}^i = (x^i_1, \ldots, x^i_k, \ldots, x^i_K)$, $\mathbf{y}^j = (y^j_1, \ldots, y^j_k, \ldots, y^j_K)$, $\mathbf{c} = (c_1, \ldots, c_\ell, \ldots, c_L)$, $\mathbf{e}^j = (e^j_1, \ldots, e^j_\ell, \ldots, e^j_L)$, and $\mathbf{e}_\ell = (e^1_\ell, \ldots, e^j_\ell, \ldots, e^J_\ell)$

The α^i in effect give weights to the preferences of the different consumers in the calculation of the solution. Since the assignment of these weights is not defined, there are abundant possible distributions, each one of which may yield a different solution.

The first order conditions characterize a maximum:

$$\frac{\partial \pounds}{\partial x^i_k} = \alpha^i \frac{\partial u^i}{\partial x^i_k} - \eta_k = 0 \qquad \begin{matrix} i = 1,..,I \\ k = 2,..,K \end{matrix} \qquad \langle 2.6 \rangle$$

$$\frac{\partial \pounds}{\partial y^j_k} = -\psi^j \frac{\partial f^j}{\partial y^j_k} + \eta_k = 0 \qquad \begin{matrix} j = 1,..,J \\ k = 2,..,K \end{matrix} \qquad \langle 2.7 \rangle$$

$$\frac{\partial \pounds}{\partial c_\ell} = \sum_{i=1}^{I} \alpha^i \frac{\partial u^i}{\partial c_\ell} - \lambda_\ell = 0 \qquad \begin{matrix} i = 1,..,I \\ \ell = 1,..,L \end{matrix} \qquad \langle 2.8 \rangle$$

$$\frac{\partial \pounds}{\partial (-e^j_\ell)} = -\psi^j \frac{\partial f^j}{\partial (-e^j_\ell)} + \lambda_\ell \frac{\partial \delta_\ell}{\partial (-e^j_\ell)} = 0 \qquad \begin{matrix} j = 1,..,J \\ \ell = 1,..,L \end{matrix} \qquad \langle 2.9 \rangle$$

Marginal rates of substitution (MRS) for consumers and rates of technical substitution (RTS) for producers that will prevail at the optimum can be calculated from equations ‹2.6› to ‹2.9›. First, we define ‹2.6› specifically for the numéraire good:

$$\alpha^i \frac{\partial u^i}{\partial x^i_1} - \eta_1 = 0 \qquad \langle 2.6A \rangle$$

Now, dividing ‹2.6› by ‹2.6A›:

$$\frac{\partial u^i / \partial x^i_k}{\partial u^i / \partial x^i_1} = \frac{\eta_k}{\eta_1} = MRS^i_{k/1} \qquad k = 2,3,..,K \qquad \langle \mathbf{2.10} \rangle$$

Defining ‹2.7› for k=1:

$$-\psi^j \frac{\partial f^j}{\partial y^j_1} + \eta_1 = 0 \qquad \langle 2.7A \rangle$$

and dividing ‹2.7› by ‹2.7A›, gives:

$$\frac{\partial f^j / \partial y^j_k}{\partial f^j / \partial y^j_1} = \frac{\eta_k}{\eta_1} = RTS^j_{k/1} \qquad k = 2,3,..,K \qquad \langle \mathbf{2.11} \rangle$$

From ‹2.6A›:

$$\alpha^i = \frac{\eta_1}{\partial u^i / \partial x^i} \qquad \langle 2.6B \rangle$$

Substituting this into ‹2.8› gives:

$$\sum_{i=1}^{I} \frac{\partial u^i/\partial c}{\partial u^i/\partial x^i_1} = \frac{\lambda_\ell}{\eta_1} = \sum_{i=1}^{I} MRS^i_{\ell/1} \qquad \text{‹2.12›}$$

From ‹2.7A›:

$$\psi^j = \frac{\eta_1}{\partial f^j/\partial y^j_1} \qquad \text{‹2.7B›}$$

Substituting this into ‹2.9› gives:

$$\frac{\partial f^j/\partial(-e^j_\ell)}{\partial f^j/\partial y^j_1} = \frac{\lambda_\ell}{\eta_1} \frac{\partial \delta_\ell}{\partial(-e^j_\ell)} = RTS^j_{\ell/1} \qquad \text{‹2.13›}$$

Looking at equations ‹2.10›, we see that the ratio of the variation in utility associated with a change in the consumption of any private good ("k") and that of the numeraire good ("1") is equal to the ratio of their respective Lagrangian multipliers. This we define as the marginal rate of substitution that must prevail in order for the consumer's utility to be maximized.

Similarly, ‹2.11› shows that the ratio of the same Lagrangian multipliers defines the optimal rate of technical substitution for producers. By comparing ‹2.10› and ‹2.11›, it can be seen how a single rate of exchange vector might spontaneously emerge for an economy that consisted only of private goods. Every agent will be prepared to offer the same quantity of the numéraire good in exchange for a unit of some other good, this being true for all private goods.

The form of ‹2.12› is deceptively simple and indeed conventional in the context of a model with public goods.

The sum of the variations in utility associated with a change in water quality gives a collective marginal rate of substitution. This is intuitively reasonable since there is only one level of water quality: any change affects all, and therefore the total impact on utility across all consumers must be examined, given such a change. It will soon be seen, however, that this formulation is at the root of the problem of the correct provision of public goods.

Equation ‹2.13› has a structure particular to public goods problems involving environmental quality. The rate of technical substitution between the firm's effluent and the numéraire is equal to a ratio of Lagrangian multipliers, modified by another factor. This factor shows the effects of a change in one company's effluent on the water quality as measured by the parameter c_{ℓ}: the relationship is not direct, but depends on stream characteristics that are incorporated into the transfer function δ_{ℓ}. For the same quantity of effluent, it might be that each firm will have a different impact on water quality. This would be true, for example, if the firms were spatially separated, with some more remote than others. We will call this term the environmental impact coefficient.

The optimum may now be characterized in a general way. Simplified notation is introduced as follows:

$$\gamma^{i}_{k} = \frac{\dfrac{\partial u^{i}}{\partial x^{i}_{k}}}{\dfrac{\partial u^{i}}{\partial x^{i}_{1}}} \qquad \begin{array}{l} i = 1,..,I \\ k = 2,..,K \end{array}$$

$$\bar{\gamma}^{i} = \frac{\dfrac{\partial u^{i}}{\partial c}}{\dfrac{\partial u^{i}}{\partial x^{i}_{1}}} \qquad \begin{array}{l} i = 1,..,I \\ \ell = 1,..,L \end{array}$$

$$\phi^{j}_{k} = \frac{\dfrac{\partial f^{j}}{\partial y^{i}_{k}}}{\dfrac{\partial f^{j}}{\partial y^{i}_{1}}} \qquad \begin{array}{l} j = 1,..,J \\ k = 2,..,K \end{array}$$

$$\phi^{j}_{\ell} = \frac{\dfrac{\partial f^{j}}{\partial (-e^{j}_{\ell})}}{\dfrac{\partial f^{j}}{\partial y^{i}_{1}}} \qquad \begin{array}{l} j = 1,..,J \\ \ell = 1,..,L \end{array}$$

$$\kappa^{j}_{\ell} = \frac{\partial \delta_{\ell}}{\partial (-e^{j}_{\ell})} \qquad \begin{array}{l} j = 1,..,J \\ \ell = 1,..,L \end{array}$$

It can now be stated that:

<u>Property 2.1:</u> A feasible allocation "h" $\in \mathcal{H}$, such that $\forall$ i $x^{i}_{1} > 0$ and $\forall$ j $y^{j}_{1} > 0$, is Pareto optimal, iff $\forall$ i,j
$\gamma^{i}_{k}(h) \leq \phi^{j}_{k}(h)$ and $x^{i}_{k} > 0 \Rightarrow \gamma^{i}_{k}(h) = \phi^{j}_{k}(h)$ $(k = 1,2,..,K)$
$\Sigma_{i}\gamma^{i}_{\ell}(h) \leq \phi^{j}_{\ell}(h)/\kappa^{j}_{\ell}$ and $c > 0 \Rightarrow \Sigma_{i}\gamma^{i}_{\ell}(h) = \phi^{j}_{\ell}(h)/\kappa^{j}_{\ell}$
$\ell = 1,..,L$ and $j = 1,..,J$

Using the simplified notation, this property states that a Pareto optimal situation is one in which, for private goods, consumers' MRS just equal producers' RTS, and for public goods, it is the sum of the MRS that must equal each firm's RTS, multiplied by its own environmental impact coefficient. Outcomes that do not meet these conditions can be improved upon simply by reallocating productive effort within the economy.

For private goods the interpretation is relatively simple: as already observed, all the MRS and RTS are equal at the margin. At the optimum, every consumer and every firm is indifferent between some quantity of the numéraire and a known quantity of any other private good.

For water quality there are two differences. At the optimum, firms' individual RTS are equalized at the margin only after adjustment by the firms' environmental impact coefficients. The second difference lies in the fact that it is the sum of consumers' MRS that is equal to each firms' adjusted RTS.

This analysis is quite general and independent of institutional arrangements: no particular form of government, prices or firm ownership has been introduced. This contradicts the statement by Livingstone, cited in the introduction, that institutions are fixed and alternative policy instruments cannot be compared. Different institutional arrangements can be introduced, and indeed this is about to be done.

We need <u>some</u> institutions to implement an optimal allocation. It is not the object of this paper to explore all the possible institutional arrangements that have been devised or used in the world. Rather, it is natural to limit the discussion to industrial societies, since they produce the kind of pollution that is the object of the model. There are, of course, two dominant forms of organization, private ownership with markets and collective ownership with planning, as well as some mixing between the two.

As stated in the Preface, I am particularly interested in western industrial societies that rely heavily on private ownership. If we introduce ownership as the first institutional arrangement, one aspect of the problem that emerges is who should "own" water quality. For private goods the answer to this problem is so self-evident we hardly think about it; but for the public good, we know only the correct amount, not the appropriate direction of payment, from the above equations. For water quality it is necessary also to specify whose ownership. As mentioned in the Preface, I will assume that consumers have the right to clean water, therefore firms must pay for the right to produce pollution.

A second basic institutional structure is that of markets. The underlying assumption of the "free" market economy is that equilibrium prices tend to emerge spontaneously for each good. This spontaneity suggests we should look at the actions of individual agents, rather than the economy as a whole (as we have done until now), to see what sort of outcome we can expect. In the following section, a price system is defined and then the outcome that a market economy yields is examined when water quality affects consumers' welfare.

A Price System

Transactions in a market economy consist of trades made according to a system of prices. For private goods, there is a price for each good. For water quality, however, because of the nature of ‹2.12›, which involved the summing of individual marginal rates of substitution, there is in effect one price for each consumer. For firms there will also be individual prices for effluents, depending on their individual impact coefficients. It need hardly be said that this results in an unusual price system. The weakness of individualized prices, especially where the consumers are concerned, is that agents must voluntarily reveal them when it is private information. As Samuelson demonstrated in 1954, with public goods,

> it is in the selfish interest of each person to give false signals, to pretend to have less interest in a given collective consumption activity than he really has....(pp. 388-9)

Nevertheless, for this economy, a price system is a non-negative (K+IxL)-vector with a price p_k for each private good and individual prices q^i for each contaminant.

Let $\Sigma q^i = q$.

<u>Definition:</u> a pseudo-equilibrium is a feasible allocation $h = (\hat{\mathbf{x}}, \hat{\mathbf{c}})$ and a price system $(\hat{\mathbf{p}}, \hat{\mathbf{q}}^1, .., \hat{\mathbf{q}}^I)$ such that:
$\forall\ i \in \{1,2,..,I\}$, with $(\mathbf{x}^i, \mathbf{c}) \in \mathbb{R}_+^{K+L}$:
$\hat{\mathbf{p}}.\hat{\mathbf{x}}^i + \hat{\mathbf{q}}^i.\hat{\mathbf{c}} \geq \hat{\mathbf{p}}.\mathbf{x}^i + \hat{\mathbf{q}}^i.\mathbf{c}$
and, $\forall\ j \in \{1,2,..,J\}$, with $(\mathbf{y}^j, -\mathbf{e}^j) \in \mathbb{R}_+^{K+L}$:
$\hat{\mathbf{p}}.\mathbf{y}^j - \hat{\mathbf{q}}\,{}_k{}^j\,\mathbf{e}^j \geq \hat{\mathbf{p}}.\hat{\mathbf{y}}^j - \hat{\mathbf{q}}\,{}_k{}^j\,\hat{\mathbf{e}}^j$

Note that each firm experiences an individual price also. Unlike the individualized consumer prices, these are based on exogenous environmental factors and outside the control of the firms. The allocation h provides as much or greater utility than any other for a given system of prices, and any allocation providing greater utility would cost more. Since it is not likely to emerge spontaneously, the term "pseudo-equilibrium" is usually applied. We can now state two more properties of the model:

<u>Property 2.2:</u> The pair $(h,(\mathbf{p},\mathbf{q}^1,..,\mathbf{q}^I))$, where $h \in \mathcal{H}$, such that, $\forall i$, $x^i_1 > 0$ and $(\mathbf{p},\mathbf{q}^1,..,\mathbf{q}^I)$ is a price system, is a pseudo-equilibrium iff the following relations hold:

$$\forall i \quad \gamma^i_k(h) \geq p_k \text{ and } x^i_k > 0 \Rightarrow \gamma^i_k(h) = p_k \quad (k = 1,..,K)$$
$$\gamma^i_\ell(h) \geq q^i_\ell \text{ and } c \quad > 0 \Rightarrow \gamma^i_\ell(h) = q^i_\ell \quad (\ell=1,..,L,\ i=1,..,I)$$
$$\forall j \quad \phi^j_k(h) \geq p_k \text{ and } y^j_k > 0 \Rightarrow \phi^j_k(h) = p_k \quad (k = 1,..,K)$$
$$\phi^j_\ell(h) \geq q_{\ell}{}^{\kappa j}{}_\ell \text{ and } e^j < 0 \Rightarrow \phi^j_\ell(h) = q_\ell{}^{\kappa j}{}_\ell \quad (\ell = 1,..,L)$$

<u>Property 2.3:</u> Any allocation in a pseudo-equilibrium will be a Pareto optimum.

Given the existence of individualized prices, these properties are straightforward extensions of the first. Property 2.2 introduces the prices that must prevail in a market economy for the optimal allocation to be achieved. This price system is a scalar multiple of the MRS and RTS relationships that conform to a Pareto optimal allocation. Property 2.3 is a corollary to Property 2.2.

Suppose now that markets exist for private goods, but no market exists for the public goods. Markets are unlikely to be established for the quality of the water, as it would be difficult for all the consumers to get together to fix the appropriate marginal evaluations. (More about this in the next chapter.) Moreover, we have not posited any institutional structure, as yet, that would oblige firms to participate in such markets. This scenario would appear to be a fairly close representation of the real world in most market economies (until governments started to intervene in the water quality problem some fifteen years ago). The results for this case can be developed using the above model.

Besides markets and prices for private goods, consumers must have incomes and firms must dispose of their profits. These issues are most easily resolved by giving ownership of the firms via shares to the consumers, who then spend their dividends. Labour income is already accounted for in the model. It is only necessary to interpret labour as a negative quantity in the consumption vector **x**. Wages and salaries then produce a counter-balancing cash flow when compared to the usual consumption goods.

Each firm distributes all of its profits which are indicated by π^j. Imposing fixed share ownership for each firm on the part of consumers is unnecessarily restrictive, so the following more general structure is used:

$$r^i = \theta^i \sum_{j=1}^{J} \pi^j \qquad \langle 2.14 \rangle$$

with $\Sigma_i \, \theta^i = 1$ and $\theta^i \geq 0$, $i \in \mathcal{I}$

r^i is the individual's share income, while θ^i is his share of total profits and π^j represents firm j's profits.

As far as the consumer is concerned, environmental quality is now an exogenous variable, which means he is unable to adjust it to improve his utility. It may nevertheless affect his choice of consumption: for example, he will not want to eat fish known to be contaminated. His utility function is now:

$$u^i(\mathbf{x}^i, \bar{\mathbf{c}}) \tag{2.1A}$$

where $\mathbf{x}^i = (x^i_1, \ldots, x^i_k, \ldots, x^i_K)$ and $\bar{\mathbf{c}} = (\bar{c}_1, \ldots, \bar{c}_\ell, \ldots \bar{c}_L)$

$\partial u^i/\partial x^i_k \geq 0 \; \forall \; k, \; \partial u^i/\partial x^i_1 > 0$

The consumer's problem is to maximise utility, given a vector of prices $\mathbf{p}$, and an income derived from dividends. Dropping the i notation, this may be expressed as:

$$£ = u(\mathbf{x}, \bar{\mathbf{c}}) - \mu \; (\mathbf{px} - r) \tag{2.15}$$

where $\mathbf{x} = (x_1, \ldots, x_k, \ldots, x_K)$, $\bar{\mathbf{c}} = (\bar{c}_1, \ldots, \bar{c}_\ell, \ldots, \bar{c}_L)$

The bracketed second term constrains his expenditures to his level of revenue.

Taking the price of the numéraire good as unity, the first order (maximizing) conditions are:

$$\frac{\partial £}{\partial x_k} = \frac{\partial u}{\partial x_k} - \mu p_k = 0 \tag{2.16}$$

Therefore

$$\partial u/\partial x_1 = \mu \tag{2.16A}$$

⟨2.16⟩ divided by ⟨2.16A⟩ gives:

$$\frac{\partial u/\partial x_k}{\partial u/\partial x_1} = p_k = MRS_{k.1} \qquad \langle 2.17\rangle$$

For his part the producer attempts to maximize profits and is constrained by his production function, which is unchanged. Dropping the j notation, this gives:

$$£ = \sum_{k=1}^{K} p_k y_k - \psi(f(\mathbf{y},-\mathbf{e})) \qquad \langle 2.18\rangle$$

where $\mathbf{y} = (y_1,\ldots,y_k,\ldots,y_K)$, $\mathbf{e} = (e_1,\ldots,e_\ell,\ldots,e_L)$

The first order conditions are:

$$\frac{\partial £}{\partial y_k} = p_k - \psi \frac{\partial f}{\partial y_k} = 0 \qquad \langle 2.19\rangle$$

$$\frac{\partial £}{\partial(-e_\ell)} = \psi \frac{\partial f}{\partial(-e_\ell)} = 0 \qquad \langle 2.20\rangle$$

From ‹2.19›:

$$\psi \; \partial f/\partial y_1 = 1 \qquad \langle 2.19A\rangle$$

Divide ‹2.19› by ‹2.19A›:

$$\frac{\partial f/\partial y_k}{\partial f/\partial y_1} = p_k = RTS_{k.1} \qquad \langle 2.21\rangle$$

However ‹2.20› divided by ‹2.19A› yields:

$$\frac{\partial f/\partial(-e_\ell)}{\partial f/\partial y_1} = \frac{0}{1} = RTS_{\ell.1} \qquad \langle 2.22\rangle$$

The firm is not required to pay for the damage that its effluents cause, thus it will tend to employ effluents as an input to the point where they yield no return whatever. (This will not likely be an infinite quantity, as it will be constrained by complementarity with other costly inputs.) Nevertheless, a price system defined by the vector **p** for private goods only will establish equilibrium conditions throughout the economy. However, comparing ‹2.22› to ‹2.13›, it is evident that this equilibrium will not be optimal.

It is apparent that such a competitive solution based on markets is not sufficient to establish the conditions of a Pareto optimum. Additional institutional arrangements are necessary. One fairly obvious alternative is that of government intervention. This could at least create the possibility of achieving an optimum, as will be demonstrated in the next section.

Pigovian Taxes and Artificial Markets

In the economics literature, the most common measure to restore optimality is the corrective tax, which goes back to Pigou:

> for every industry in which the value of the marginal social net product is less than that of the marginal private net product, there will be certain rates of tax, the imposition of which by the State would increase the size of the national dividend and increase economic welfare; and one rate of tax, which would have the optimum effect in this respect. (Emphasis in original—Pigou, 1932, p.224)

A tax causes the firms to shift away from the externality producing activity (firms' effluents) to some degree. The same effect may be achieved by imposing a limit on effluents and creating a market to exchange discharge permits. Either one can restore the conditions of optimality set out in

⟨2.10⟩ to ⟨2.13⟩. This will be demonstrated.

Remembering the structure of ⟨2.13⟩, we should expect the tax to be adjusted according to the firm's environmental impact coefficients, κ_j. Thus the tax falls evenly on firms according to their effect on water quality, not their absolute levels of discharge. The producer's problem may now be characterized as:

$$£ = \sum_{k=1}^{K} p_k y^j_k - \sum_{\ell=1}^{L} T_\ell \kappa^j_\ell e^j_\ell - \psi(f^j(\mathbf{y}^j, -\mathbf{e}^j)) \qquad \langle 2.25\rangle$$

where $\mathbf{y}^j = (y^j_1, \ldots, y^j_k, \ldots, y^j_K)$, $\mathbf{e}^j = (e^j_1, \ldots, e^j_\ell, \ldots, e^j_L)$

The firm must choose its output and effluent levels based on their cost. With respect to the first order conditions, ⟨2.19⟩ is unchanged. Focusing on the effluents:

$$\frac{\partial £}{\partial(-e^j_\ell)} = T_\ell \kappa^j_\ell - \psi \frac{\partial f^j}{\partial(-e^j_\ell)} = 0 \qquad \langle 2.26\rangle$$

⟨2.26⟩ divided by ⟨2.19A⟩ gives

$$\frac{\partial f^j / \partial(-e^j_\ell)}{\partial f^j / \partial y_1} = T_\ell \kappa^j_\ell \qquad \langle 2.27\rangle$$

When $T_\ell = \Sigma_i \gamma^i_\ell$ for all ℓ, the tax reestablishes the conditions for a Pareto optimum (cf. Property 2.1), that is:

Definition: a Pareto optimum with Pigovian taxes is a feasible allocation $h \in \mathcal{H}$ such that, $\forall i$, $x^i_1 > 0$ and a price system $(\mathbf{p}, \mathbf{q}^1, \ldots, \mathbf{q}^I, \mathbf{T})$, iff the following relations hold:
$\forall i$ $x^i_1 > 0$, $\gamma^i_k(h) \geq p_k \Rightarrow \gamma^i_k(h) = p_k$ $(k = 1, \ldots, K)$
and $\gamma^i_\ell(h) \geq q^i_\ell \Rightarrow \gamma^i_\ell(h) = q^i_\ell$ $(\ell = 1, \ldots, L,\ i = 1, \ldots, I)$
$\forall j$ $y^j_1 > 0$, $\phi^j_k(h) \geq p_k \Rightarrow \phi^j_k(h) = p_k$ $(k = 1, \ldots, K)$
and $\phi^j_\ell(h) \geq T_\ell \kappa^j_\ell \Rightarrow \phi^j_\ell(h) = T_\ell \kappa^j_\ell$ $(\ell = 1, \ldots, L)$
and if $T_\ell = \Sigma q^i_\ell$ $(\ell = 1, \ldots, L)$

The formal equivalence of a permit market to such corrective taxes relies on a demonstration that a system of permits can meet the optimality conditions established by equations ‹2.10› to ‹2.13›. A more complete demonstration may be found in Montgomery (1972). The government in effect introduces a form of global rationing through the issue of a limited number of transferable discharge permits (TDPs). Let G_ℓ be the total number of permits issued for each contaminant. The permits held by firm j is indicated by g^j_ℓ.

$$G_\ell = \sum_{j=1}^{J} g^j_\ell \qquad ‹2.28›$$

As in the tax case, the permit is modified according to the impact of the firm's effluents on environmental quality. This being so, all permits will have the same price q_ℓ.

$$g^j_\ell = e^j_\ell \kappa^j_\ell \qquad ‹2.29›$$

The firm's production function may now be described as:

$$f^j(\mathbf{y}^j, -\mathbf{g}^j/\kappa^j) = 0 \qquad ‹2.2A›$$

where $\mathbf{y}^j = (y^j_1, \ldots, y^j_k, \ldots, y^j_K)$ and $\mathbf{g}^j = (g^j_1, \ldots, g^j_\ell, \ldots, g^j_L)$

$\partial f^j/\partial y^j_k \geq 0 \ \forall \ k$, $\partial f^j/\partial y^j_1 > 0$ and $\partial f^j/\partial(-g^j_\ell) \geq 0$

The firm's profit function then takes the form:

$$£ = \sum_{k=1}^{K} p_k y^j_k - \sum_{\ell=1}^{L} q_\ell g^j_\ell - \psi(f^j(\mathbf{y}, -\mathbf{g}^j/\kappa^j)) \qquad ‹2.25A›$$

where $\mathbf{y}^j = (y^j_1, \ldots, y^j_k, \ldots, y^j_K)$, $\mathbf{g}^j = (g^j_1, \ldots, g^j_\ell, \ldots, g^j_L)$

Thus:

$$\frac{\partial f^j/\partial(-g^j_\ell)}{\partial f^j/\partial y_1} = q_\ell \kappa^j_\ell \qquad j = 1,..,J \qquad \langle 2.27A\rangle$$

The conditions necessary for a Pareto optimum are then reestablished. Formally, and remembering that $q_\ell = \Sigma_i q^i_\ell$:

<u>Definition:</u> a Pareto optimum with effluent permit markets is a feasible allocation $h \in \mathcal{H}$ and a price system $(\mathbf{p},\mathbf{q}^1,..,\mathbf{q}^I)$, iff the following relations hold:

$\forall i\ x^i_1 > 0,\ \gamma^i_k(h) \geq p_k \Rightarrow \gamma^i_k(h) = p_k \quad (k = 1,..,K)$

and $\gamma^i_\ell(h) \geq q^i_\ell \Rightarrow \gamma^i_\ell(h) = q^i_\ell \quad (\ell=1,..,L,\ i=1,..,I)$

$\forall j\ y^j_1 > 0,\ \phi^j_k(h) \geq p_k \Rightarrow \phi^j_k(h) = p_k \quad (k = 1,..,K)$

and $\phi^j_\ell(h) \geq q_\ell \kappa^j_\ell \Rightarrow \phi^j_\ell(h) = q_\ell \kappa^j_\ell \quad (\ell = 1,..,L)$

In the next chapter, we will see how a TDP market, unlike most alternatives, avoids the problem posed by non-convexity of the consumption set. At that point we will go into more detail about how the government could adjust the level of total permits in an effort to move the economy to a globally optimal outcome. The dynamics of this procedure are set up formally in chapters 4 and 5.

3

NON-CONVEXITY OF THE MARGINAL DAMAGE FUNCTION

In the previous chapter, it was possible to characterize in a mathematical model the conditions for optimal water quality from the point of view of the whole society. We are of course interested in how the economy, starting from some non-optimal point, might move to an optimal equilibrium.

Such dynamic modelling, or stability analysis, of economic models dates back 50 years (Negishi, 1961; Hurwicz, 1973). Convexity of the consumption and production sets is an assumption applied throughout this literature. Acknowledged failure of this assumption perhaps explains why so few dynamic models have been developed where externalities or public goods are concerned.

It is essential to find a way around this problem if a dynamic model is to be built. It is the object of this chapter, after exploring the nature of the problem in relation to the marginal damage function, to find a way around the problem. Recognition must also be made of the complex nature of the marginal damage function itself.

Several types of non-convexities relating to the environment have been discussed in the literature: they may result from interdependence of two firms' production technologies (Baumol, 1964; Baumol and Bradford, 1972; Starrett, 1972; and Starrett and Zeckhauser, 1974) or interaction between contaminants themselves (Beavis and Walker, 1979; Repetto, 1987).

Using the Starrett model, the problem of how non-convexities occurs is explained in the next section. The impact on standard modelling approaches, including that used in this paper, is also presented.

In the second section, ways to get around the problem are discussed. The difficulties of evaluating the marginal damage function are explored, as are the consequences. It turns out that the transferable discharge permit market introduced in the last chapter is perhaps the best alternative to deal with both of these problems. Some of this material has appeared in Mallory (1988).

The Existence and Unicity of an Equilibrium

Conventional representation of the pollution problem, such as that of Turvey (1963), involves an increasing marginal damage curve[2], which intersects a decreasing

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[2]This term will be used interchangeably with "marginal benefits of pollution control" in the text which follows.

marginal cost of pollution control curve. The intersection of the two curves defines the Pareto optimum. However, increasing marginal damage is difficult to defend.

Starrett's non-convexity typically involves the closing of a firm: at this point the marginal damage falls to zero, immediately undercutting the increasing marginal damage proposition. One could also consider the case where marginal damages fall, but do not completely disappear, but the conclusions are essentially the same (Starrett et Zeckhauser, 1974, p.75). The problem is illustrated in Figure 1. The firms are assumed to behave competitively. Firm A (an abattoir), is upriver and pollutes the river to a level such that the profits of Firm B (a brewery) downstream are wiped out. Without intervention, the second firm will close.

Figure 1a

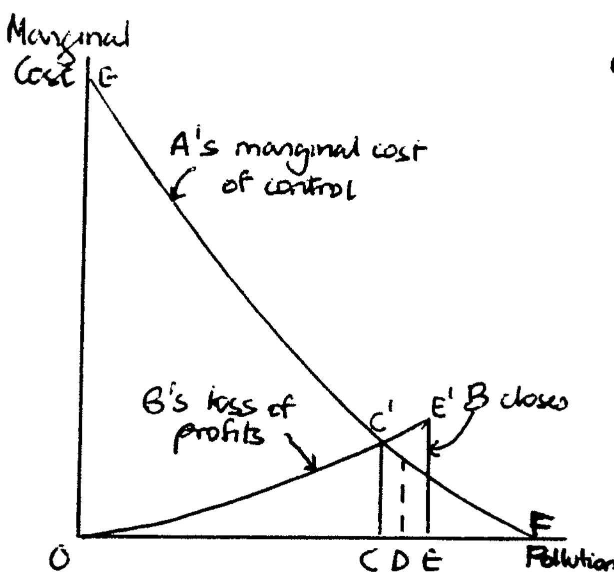
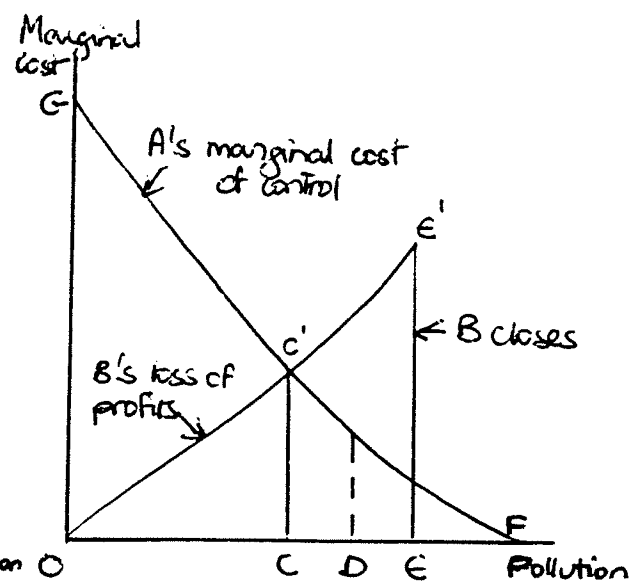


Figure 1b



Two Situations, Two Results

This result is optimal at any level of pollution where A's pollution control expenses would be greater than the loss of profits to B. This is true at every point on OF in Figure 1a. At point C, for example, the polluter's profits will be the greater by the cost of control (CC'F), which is more than the total profits of Firm B (CC'EE'). The society as a whole is better off in permitting the pollution if there are no other costs.

Of course the opposite conclusion is just as plausible. In Figure 1b, the benefits associated with (some) pollution control are greater than the loss of profits of Firm A. In this case, the society increases its welfare by reducing the pollution.

In fact, as several authors have commented, it all depends on where you begin. For both Figures 1a and 1b, an iterative approach, inspired by the Walrasian tâtonnement, would lead from D to C, since the loss diminishes faster than the cost of control increases. But when starting from F, which is beyond the point where pollution has already forced the closing of the downstream plant, it is better not to move, because the control cost increases without any corresponding benefits. In Figure 1a, welfare is maximized at F, but it is maximized at C in Figure 1b. In the latter case, if one starts to the right of E it would be necessary to "jump" to the region of the optimum before continuing with an iteration.

Starrett showed that no efficient solution may exist, even if property rights are clearly defined and payments between firms are permitted (1972, p.190). If the pollutees have the property rights, at any positive price they will want to sell an infinite quantity to the polluters (who will not want so many). But if the price falls to zero, the pollutees will offer none. In a parallel way, if the rights were with the upstream users, downstream users would offer to pay compensation for a reduction in pollution. But then other firms would seek compensation on the threat of entering the market and creating pollution, thereby securing a revenue gain. Faced with essentially unlimited claims, downstream users would reduce their offer to zero. In neither case is there an equilibrium, hence no efficient (optimal) solution can be obtained.

Burrows (1986, p.103) fixes the point of departure at the point where the price of the externality falls to zero, therefore the quantity of pollution is fixed (point **F** on Figure 1). This amounts to constrained rights. Regardless of who holds the property rights, they could not sell more permits than the original quantity of pollution. Cooter (1980) remarks that the common law implicitly imposes this solution.[3] This seems reasonable, but the potential impact on regional development should not be ignored. A system that

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[3]Which is not necessarily so for the civil law.

froze all pollution at its current level could prevent the establishment of new firms, notably those with new technologies and different pollutants.

Although constraining the quantity of permits or the entry of firms assures the existence of an equilibrium (if the other usual hypotheses are maintained), non-convexity implies another mathematical difficulty: it is no longer clear that the equilibrium will be unique. How can one find the equilibrium which is also the global optimum?

Three possibilities can arise.

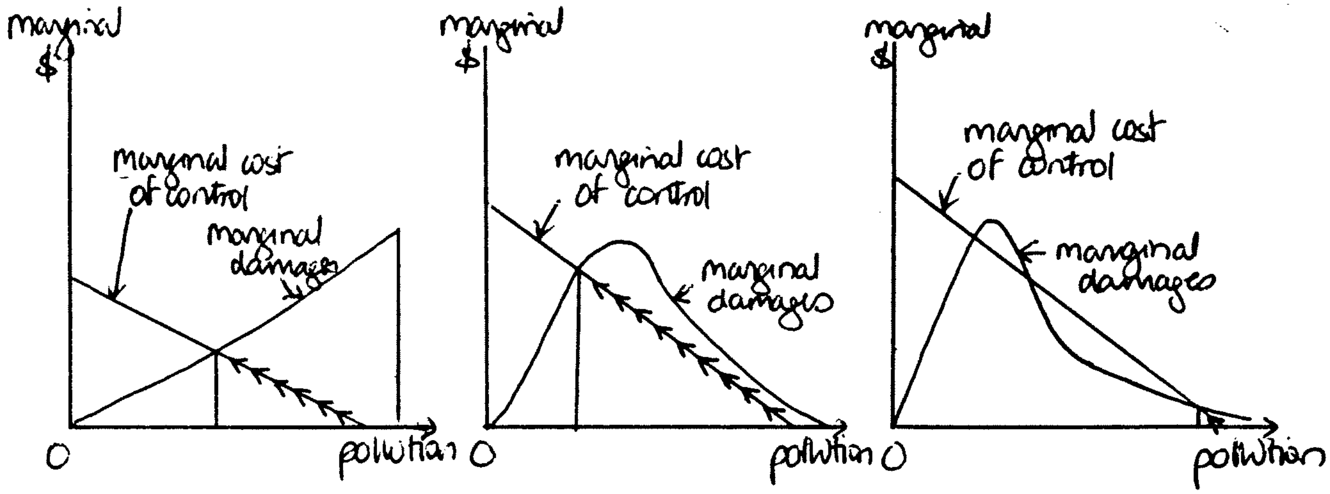
1) The cost of control is everywhere greater than the damages incurred, regardless of the form of the cost and damage curves. Obviously it is optimal to do nothing.

2) An iterative approach starting from the point of no control leads to an optimal solution. Burrows presents three cases which satisfy this criterion: they are presented in Figure 2. Such cases represent the classic cases where market-type mechanisms of corrective taxes or permit markets can be applied.

3) The global optimum will not be reached by iteration starting from a point of no control.

It is the last group which interests us, since the classic methods (which are in fact used in the dynamic procedure to be developed in the next chapter) do not apply.

Figure 2



Three Classic Non-convexities

Source: Burrows, 1986

Perhaps the most hotly debated aspect of this question is whether or not real world situations are likely to exhihit such "relevant" non-convexities (cf. Burrows, 1986).

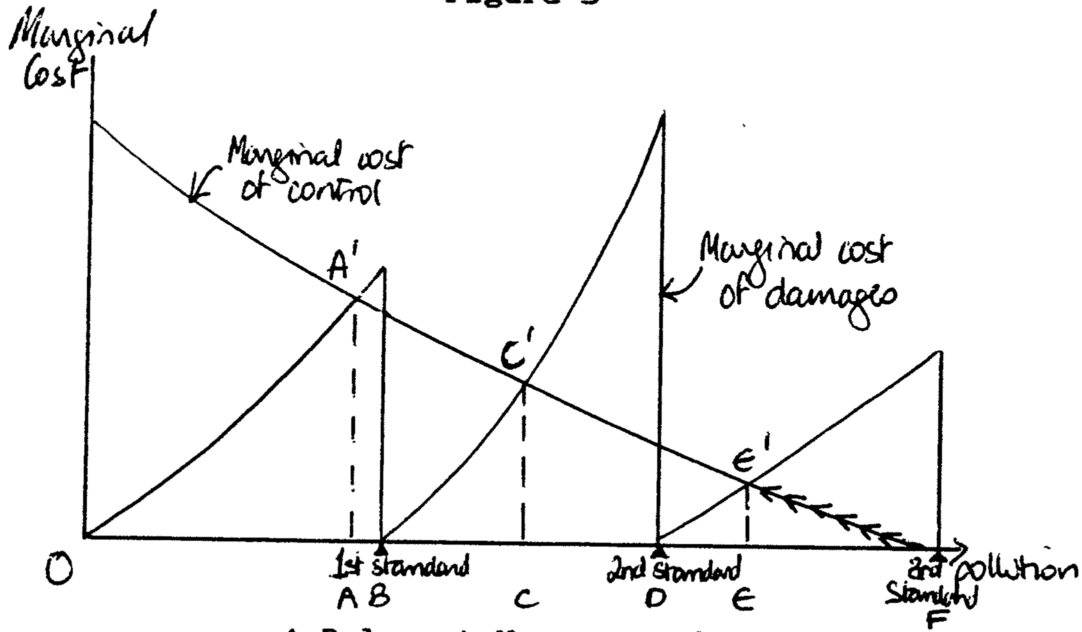
The risks associated with any given use of water, such as drinking, fishing, industrial use (especially in the food industry), increase with the concentration of contaminants. At a given level, continued use becomes impossible. We will call this a "standard". When a given use is no longer possible, production falls to zero, which renders the production set non-convex at this point. An ϵ below the standard and the use continues; an ϵ above, and the use stops. A well-known example is the prohibition of fishing that was brought about by mercury contamination in a number of Canadian lakes and rivers.

Not all activities are affected at the same level even with the same contaminant. Thus there may be numerous non-convexities. For each standard associating a contaminant and an activity, there will be a non-convexity. At the limit, there are as many non-convexities as the number of activities multiplied by the number of contaminants. This somewhat exaggerates the case, because many standards may be the same (for one contaminant across various activities). However, the fewer standards there are, the stronger will be the non-convexities.

The situation is presented in Figure 3. The marginal damage function associated with a given contaminant may have several peaks situated at the levels of various standards associated with different uses. A marginal cost of control curve is also shown, chosen in such a way as to illustrate the problem. In comparing the area below the two curves, one can see that the level of pollution given by the point A is the global optimum. There are two other local optima, at C between B and D, and at E for point to the right of D. Clearly, the global maximum cannot be reached by a tâtonnement procedure starting from the point of no control (at F).

An example may be cited from the Québec experience. A liquid toxic waste dump was etablished at Ville Mercier in

Figure 3



A Relevant Non-convexity

the 1960s. Lagoons were dug out of the sandy soil in the midst of a rich agricultural zone to the southeast of Montréal. It was very practical, because the lagoons drained almost as quickly as they were filled. However, as the danger of groundwater contamination became recognized, the lagoons were closed, replaced in 1972 by an incinerator. But substantial quantities of highly toxic waste had already penetrated the sandy soil: in 1982, the Québec ministère de l'Environnement forbade the use of well water by a large number of farms in six municipalities, as well as a giant vegetable cannery at Ste-Martine (cf. Québec, 1982).

Like the brewery of Figure 1, the companies and the farmers suffered the effects of a non-convexity in the production set. Suddenly, when a certain level of toxicity was observed in the water, they had to find alternative water supplies, which were undoubtedly much more expensive (tanker trucks were used, for the most part). The intervention of the government closed the lagoons. One could say that this decision reflected a judgement that the global social welfare maximum could not be reached without control of the source of the pollution: the government chose therefore to try to "jump" to the left in the hope of finding the global optimum.

This suggests a way around the problem. If we define the "region of the optimum" as that area from which a gradient approach will lead to the optimum, we see that the problem becomes one of finding this region. Once there, we can use conventional modelling techniques to establish the true optimum.

In passing, the parallel with the literature on increasing returns to scale is notable. The following quote from Heal illustates this.

> [T]he problem of distinguishing local from global maxima... arises as soon as non-convexities are permitted, and it will be apparent that the method proposed in this paper does not contribute to its solution. It is essentially a gradient method, and climbs the nearest hill: this need not, of course, be the highest. In order to establish interesting global results, it would be necessary to combine some technique for discovering the general location of a global maximum with the routine discussed here, applying the latter from an initial point "near" the global maximum. An approximation to the overall maximum sufficiently close for this purpose, could perhaps be obtained by the [Central Planning Board] if it had some outline of the nature of the production relations (and particularly of returns to scale) in the major sectors of the economy. Using this information, it could set up a relatively simple non-linear programming problem that captured some

> of the essential features of the true planning problem, and solve this on its own computers: the solution would give it some indication of where to start the decentralized routine. (1971, p.290)

The parallel with the literature on increasing returns extends to the question of the necessity of imposing some additional constraint on the system. Like polluting firms, those with technologies exhibiting increasing returns to scale will not choose the socially optimal output. Boiteux (1956) explores the problems and implications of a variety of rules that might be applied.

Finding the Region of the Optimum

Cost/benefit analysis has been suggested as a tool to overcome this problem by several authors, including Portes (1970, p.358), Starrett and Zeckhauser (1974, p.79) and Cooter (1980, p.500). Although cost/benefit analysis is itself somewhat imprecise, Starrett and Zeckhauser, much in the spirit of Heal, suggest the results of the cost/benefit analysis could be used as a starting point for an iterative procedure. Such a mechanism would retain all the flexibility of iterative processes in accommodating later changes in tastes and technology.

This is preferable to relying on cost/benefit analysis alone for two reasons. The first is that costs and benefits are generally calculated once, and the results applied. This fails to take account of the potential impact of the decision itself on other prices and supplies, i.e., it is a partial equilibrium approach. The second reason for looking at some kind of combined mechanism is that a market mechanism, whether by taxes or by permits, provides real data on costs, leaving only the benefits to be estimated (more about this below). This undoubtedly increases the accuracy of the process.

More ought to be said about benefit estimation. There is a substantial literature on the question of benefit evaluation, both on the theoretical level (concerning incentive structures for preference revelation) and a practical level. Most of the theoretical results are negative, while the practical advances are essentially ad hoc. The words of Baumol and Oates are still appropriate nearly twenty years after being written:

> it is hard to be sanguine about the availability in the foreseeable future of a comprehensive body of statistics reporting the net damage of the various externality-generating activities in the economy. The number of activities involved and the number of persons affected by them are so great that on this score alone the task assumes Herculean proportions. Add to this the intangible nature of many of the most important consequences—the damage to health, the aesthetic costs—and the difficulty in determining a money equivalent for marginal net damage becomes even more apparent. (1971, p.43)

On the theoretical side, much of this work is actually an outgrowth of auction theory, wherein a single market (the "public good") is studied (e.g. Groves and Loeb, 1975; Green and Laffont, 1977). A peculiarity of the structure is that all of the bids are added together. The justification for this can be seen in Property 2.1 (p.31), where it was observed that the sum of the individual marginal rates of substitution (benefits) should equal the producers' marginal rate of transformation (cost). The objective is to develop a procedure that is "locally incentive compatible", which is to say that individual consumers will find it in their best interest to reveal their true evaluations throughout the process (i.e. locally).

Given the partial equilibrium nature of the models, secondary effects, such as altered cost structures for firms and changed demands on the part of consumers in reaction to changes in the level of the public good, are ignored or suppressed. Individual utility is usually characterized as linear, so that the utility function has additively separable elements. For example, Green and Laffont (1977) use a structure of the form:

$$U^i(c,T^i) = V^i(c) + T^i \qquad \langle 3.1 \rangle$$

where U^i is the individual's utility,
V^i is his/her utility associated with the public good
c is the level of provision of the public good and
T^i is an individualized tax.

In such a structure, the individual's appreciation of (and willingness to pay for) any given level of c is independent of his income. While this is a strong hypothesis, the procedure will not work without it (Bergstrom and Cornes, 1983). More acceptable hypotheses may be possible which permit realization of the same objectives (Conn, 1982). It has been shown, however, that in general there is no process such that truth telling is a dominant strategy (Roberts, 1979). Groves and Ledyard (1977) use a somewhat different approach which allows for income effects, but their assumption of competitive behaviour on the part of agents effectively ignores the problem of preference revelation during the process. They are able to show that once at the equilibrium, it is each agent's dominant strategy not to change his choices. Muench and Walker (1983) show that it is not likely that they will get there, however.

Such negative results have not deterred a number of economists from attempting to evaluate the benefits from water quality improvement. Such research is perhaps spurred on by U.S. government requirements that all water projects be subject to cost/benefit analysis, specifically including recreation benefits (Eisel, Seinwill and Wheeler, 1982). A variety of approaches have been used, including:

- travel cost;
- the household production function;
- land valuation; and
- contingent valuation.

The two first methods are often used to estimate the value of water-based recreation activities, such as fishing and canoeing (Smith, Desvouges and McGivney, 1983; Kahn and Kemp, 1985). The value of clean water is inferred from these associated activities. The third is most useful for the evaluation of water quality in determining sites for housing or cottages (Falcke, 1983).

The fourth method frequently used for evaluation of environmental amenities is contingent valuation, which relies upon consumers' responses to questionnaires (Randall, Hoehn and Brookshire, 1983). No particular use of water need be implied in this process; however, it must be recognized that such valuations are not only rough but contestable. An industry faced with millions of dollars of expenses to control its effluents would find the best current modelling efforts an easy target.

A Pigou-Baumol-Dales Equilibrium

The difficulties in knowing with any certainty the form of the marginal damage function are evident. Even the order of magnitude may not be known with certainty. Baumol and Oates were not only aware of this problem, but formalized a process that minimized the cost of achieving any chosen environmental quality targets (1971). This technique was already being applied, out of necessity, by Johnson (1967, p.292) and others. Producers are taxed to induce them to reduce their effluents:

> if the initial taxes did not reduce the pollution of the river sufficiently to satisfy the preset acceptability standards, one would simply raise the tax rates. Experience would soon permit the authorities to estimate the tax levels appropriate for the achievement of a target reduction in pollution. (Baumol and Oates, 1971, p.45)

Tulkens and Schoumaker (1975) characterize the outcome of this process as a "Pigou-Baumol Equilibrium". Such an approach does not require precise evaluation of environmental benefits. In setting the initial standard, the government would presumably use the best scientific evidence available, whatever knowledge it might have concerning preferred water quality, and indeed cost/benefit analysis. If one attributes a certain degree of efficiency to the political process, pressures for improved water quality can be measured against the cost of control, as reflected by the tax. We might imagine as well that the government could implement some kind of voting scheme (cf. Collinge and Bailey, 1983).

As demonstrated in the last chapter, permits could work just as well as taxes, indeed perhaps better, since the level of permits can be made to generate the standard almost exactly, depending on the accuracy of ecological models. When the government uses the quantity of permits as its control variable, the outcome could be described as a "Pigou-Baumol-Dales Equilibrium", or PBD for short, at once distinguishing the permit case from the tax case, and crediting Dales (1968) with the idea of effluent permit markets.

The government fixes the level of permits, hence water quality. While consumers adapt their consumption patterns to the resulting level of water quality, producers trade the allowable quantity of permits among themselves, much like any other input. A Pigou-Baumol-Dales equilibrium results when consumers and producers have reached their optimal allocations subject to the water qualtiy constraint.

While it would be all but impossible to anticipate the optimal level of water quality from the conditions prevailing before controls are introduced, it should be somewhat easier as the economy moves in the welfare improving direction. What is required for this to happen is that the government be able to establish the correct direction (increase or decrease in the number of permits) at any given point. Assuming that the economy converges to a PBD equilibrium, the economy can be seen to pass through a series of such equilibria towards a Pareto optimum as the allowable level of permits is adjusted. The dynamics of such as procedure is formalized in Chapter 5, and convergence is indeed demonstrated.

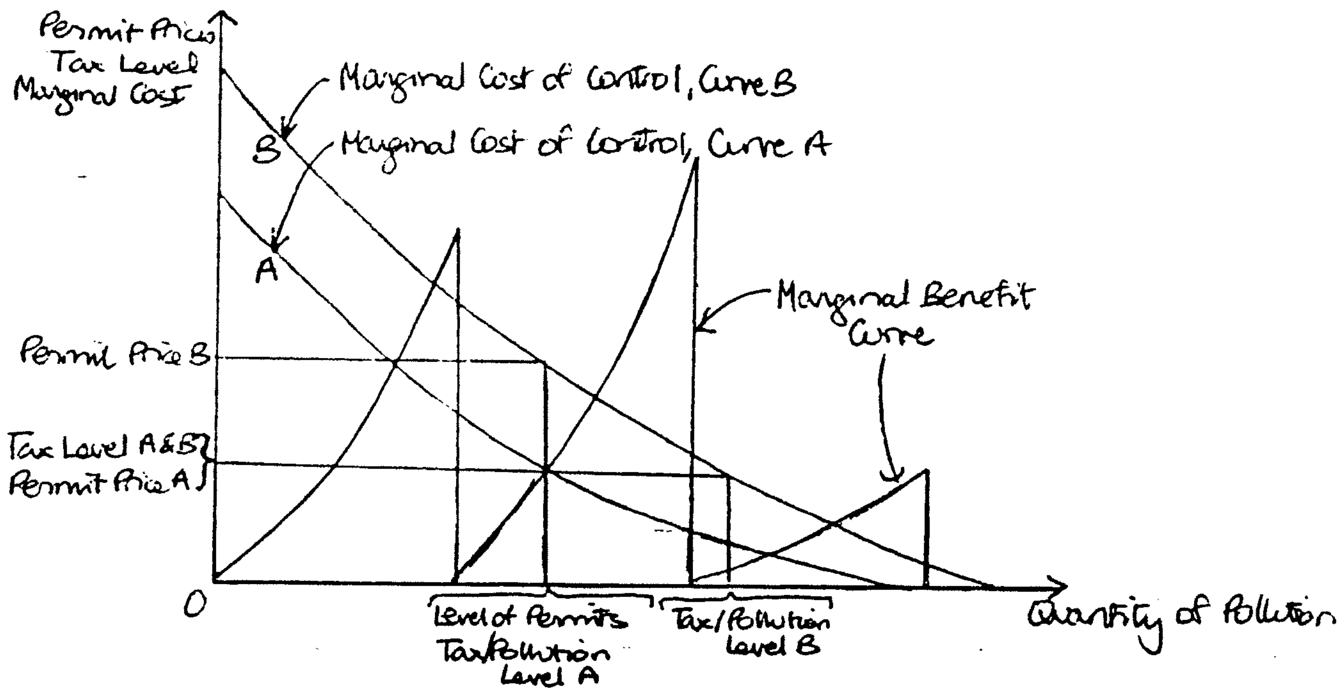
It was shown above how pollution is likely to introduce non-convexities into the consumption set, through the marginal damage function. That analysis was essentially static: in the light of the process just described, the implications need to be examined in a dynamic context.

Permits are more likely to assure convergence to the optimum than Pigovian taxes.

There are two major reasons why the cost of control may vary (or appear to vary). One reason is attributable to uncertainty or miscalculation: this question will be explored in Chapter 6. More fundamentally perhaps, factor prices change in the general equilibrium adjustment process. It is extremely difficult to anticipate all the effects of all the changes in consumption patterns brought about by agents responding to changing prices. There is little doubt that the cost of control function will in fact move.

Suppose the (non-convex) marginal damage function is as shown in Figure 4 (which is similar to Figure 3, except with two marginal control cost functions). If a tax were in effect, the quantity of pollution would increase in response to the shift of the cost of control function in either direction and (in the case shown) cross a non-convex portion of the marginal damage function. It could conceivably move back and forth, between two unstable equilibria. Because consumption plans change discontinuously whenever the intersection point between the tax and control cost function traverses a vertical section of the marginal damage function, stability is not assured.

Figure 4



A Permit Market with a Non-Convex Damage Function

The artificial market mechanism is immune to this problem. Since the quantity of permits is fixed, the permit price will rise as the curve shifts from A to B. Once the government chooses a level of total effluent (as long as this is less than the level prevailing with costless disposal), there is no indeterminacy. That a TDP market mechanism will not cause instability in the presence of non-convexity is a first result of the dynamic analysis of the pollution problem.

One more complexity deserves to be mentioned: in the pseudo-equilibrium model of the previous chapter, the transactions are directly between the agents such that the government collects no revenue. Any redistribution of revenue between consumers will result in a change in the optimal mix of outputs, likely including optimal water quality as well. This may affect our ability to achieve an optimal outcome.

As the number of agents increases, the possibility of direct transactions between them becomes less and less likely, and the intervention of a third agent, such as the government, more necessary. In Starrett's structure, the government organizes a permit market between upstream and downstream users (Starrett, 1972, p.189). However, with a tax system or a permit system as described in Chapter 2, the government collects revenue. If it redistributes this revenue, the beneficiaries may anticipate the income, which would distort their choices and disturb the equilibrium. Once-and-for-all "lump sum" subsidies avoid the problem, but they are difficult to envisage in a dynamic context. If the government could discover the consumers' true marginal evaluations of water quality, it could use this information to redistribute the revenue as though the transactions had been direct in the first place, but this gets back to the revelation problem already discussed.

While this is not very encouraging, Terkla (1984) studied the impact on efficiency of taxes on effluents as opposed to individual and corporate income tax. He found that effluent taxes tend not to be distortionary, unlike supplementary taxes on income. Theoretically such taxes may not be optimal, but in practice they do not have much impact.

In a more general and theoretical analysis, King (1986) has decomposed the Pigovian tax rule to take account of the distortionary affects of taxation on public goods. What King describes as the "Pigou" term may be positive or negative, leading to the possibility (identified in the literature) that the net effect of intervention may be a loss of welfare. Probably this would have to be evaluated on a case by case basis, though Terkla's results suggest pessimism is not warranted where pollution is concerned. King's work has not been extended to a permit market structure.

In this chapter, it has been shown that non-convexities in the damage function are likely to hamper the search for the optimal equilibrium. As well, the marginal damage function itself may be exceedingly difficult to evaluate in any but the most theoretical constructs. Both of these problems can be avoided, if not overcome, by the use of a transferable discharge permit market. With respect to non-convexity, the problem of existence of an equilibrium disappears when either firm entry or the quantity of effluent permits is constrained. The latter is implicit in a TDP market. As to whether there are multiple equilibria, again, there can only be one, once the level of permits is fixed.

Whenever the level of permits is varied, a different but unique equilibrium is implied. As for the problem of benefit evaluation, the use of a permit market sidesteps this problem by removing it from the economic process as such.

This results in a two step procedure for finding the global optimum, wherein adjustments are made by firms in response to the imposition of a specific standard for water quality, which itself must be varied from time to time according to a separate procedure. The dynamics of this process are formalized in the next two chapters.

4

THE DYNAMICS OF DISEQUILIBRIUM

It is often supposed that because an optimum can be shown to exist, that there will be an almost automatic tendency for the economy to go there. The two previous chapters have shown, however, that in the presence of problems such as water quality, not only is some form of intervention required, but that the ability of a government to pilot the economy to an optimum is also open to question. However, the adjustment dynamics of economies with externalities are seldom examined.

It is a principal object of this thesis, therefore, to examine the adjustment problem of an economy with water pollution. In this chapter, the simpler problem of adjustment in an exchange economy without production or public goods is explored, arriving at a fresh interpretation of a decentralized model. In the next chapter, the model is extended to reintroduce the problems of production and public goods.

In the real world, we never observe the economy in equilibrium: all we actually observe is the <u>dynamics of disequilibrium</u>. It probably cannot be proven that the real economy as a whole actually does tend toward equilibrium, because of the steady stream of shocks and surprises that cause agents to reevaluate their choices. However, Negishi argues that:

> We know from experience that under this process prices usually do not explode to infinity or contract to zero, but converge to an equilibrium such that the supply of and demand for commodities are equal. Hence, the process which we choose to represent reality must display the same stability. (1962, p.639)

The purpose of dynamic procedures, then, is to show how the economy adjusts from a disequilibrium to an equilibrium state. There are a variety of ways to prove that dynamic systems are convergent: however all of these proofs involve mathematical constraints which must be imposed on the system. The trick is to find the set of assumptions that are easiest to justify on economic grounds.

Adjustment procedures in economies without public goods go back to Hicks and Samuelson (Negishi, 1962). The "tâtonnement process" of Walrasian inspiration was the first procedure that was shown to converge towards a unique and optimal allocation of prices and quantities of goods held by agents (Arrow and Hurwicz, 1958). This process has a number of drawbacks:

- an auctioneer is assumed to exist who adjusts prices;
- no goods are exchanged until equilibrium is reached;
- there is no production or consumption during the process; and
- all goods are presumed to be gross substitutes.

The lack of realism in these assumptions makes this procedure somewhat less interesting. Moreover, no such "decentralized" models have been developed incorporating public goods, the object of this thesis.

The first of these problems disappears where planned economies are concerned. As mentioned in the Introduction, the best known centralized process is probably the MDP procedure, named for Malinvaud (1967, 1970-71) and Drèze and de la Vallée Poussin (1971). The planning board communicates with each consumer and producer, gradually adjusting its proposed allocation on the basis of the responses it receives. A tâtonnement-like process is envisaged, such that no production or consumption actually takes place until a satisfactory allocation is arrived at. However, planning mechanisms that work exclusively in goods space have the advantage of maintaining feasibility (once it is achieved), thus they can be easily truncated. In fact, the MDP process was developed in response to the problem of public goods, while Aoki (1971) developed a similar procedure involving externalities. In the present context the public goods aspect of these models is clearly an advantage.

Meanwhile, in a private goods version of MDP, Tulkens and Zamir (1979—hereafter TZ) suggested that transactions could take place spontaneously. Such a process is virtually decentralized. It should be noted as well that the second weakness of tâtonnement processes is also addressed in this model, as the exchange of goods now takes place as the process moves toward equilibrium.

The TZ process dispenses with the usual rule that the surplus generated by the proposed exchange be distributed according to fixed shares. This modification is particularly useful in the decentralized context, since such a rule would be hard to enforce.

One problem with the TZ model is that it is unclear how the uniform price vector comes to the attention to the agents. In this regard, a TZ process is refined in the last section of this chapter to show the conditions under which it can be seen to be a fully decentralized process. First, however, the foundations of the model are presented in terms of a basic MDP process, which is then extended to the non-tâtonnement TZ model.

A Basic Model

We would like to be able to trace the movement of prices and allocations through time, such that, for any given moment of time, we would be able to calculate the values for the next moment, as well as being able to demonstrate ultimate convergence. This is not easy, particularly in the context of a decentralized model, in which individual agents must assume the responsibilities usually consigned to the "auctioneer".

We start with a simple MDP type barter economy in the style of Malinvaud (1982, chapter 8). Recalling equation ‹2.1A› (p.37), consumers have a utility function that depends on their consumption of private goods and a level of environmental quality which, until the next chapter, is exogenous and unchanging.

$$u^i(\mathbf{x}^i, \bar{\mathbf{c}}) \qquad \langle 4.1 \rangle$$

where $\mathbf{x}^i = (x^i_1, \ldots, x^i_K)$ and $\bar{\mathbf{c}} = (\bar{c}_1, \ldots, \bar{c}_L)$

$\mathbf{w}$ is a vector of non-negative weights, such that:

$$\mathbf{w}' \, \partial u^i / \partial x^i > 0, \text{ where } \frac{\partial u^i}{\partial x^i} = \left[\frac{\partial u^i}{\partial x^i_1}, \ldots, \frac{\partial u^i}{\partial x^i_K}\right]$$

In a barter economy, consumers are constrained by their existing resources, given by the vector ω. Conservation of resources is assured by:

$$\sum_{i=1}^{I} x^i_k = \omega_k \qquad k = 1, \ldots, K \qquad \langle 4.2 \rangle$$

Since the MDP economy is centralized, there is of course a planner. The process consists of gradual adjustment of the allocation, with consumers and the planner alternately responding to the latest proposition(s) of the other. As no consumption takes place, these steps are in "virtual" time, which will be indicated by the subscripts t, t+1, t+2, ...

To begin the process, the planner inquires of each consumer at what rate of exchange they would be interested in trading some of their present holdings. In response, consumers formulate and transmit propositions based on the marginal rates of substitution they have for each good. With s as the unit of account, the notation in Chapter 2 is somewhat modified:

$$\gamma^i_k = \frac{s \dfrac{\partial u^i}{\partial x^i_k}}{\mathbf{w}' \dfrac{\partial u^i}{\partial \mathbf{x}^i}} \qquad \begin{array}{l} i = 1,\ldots,I \\ k = 1,\ldots,K \end{array} \qquad \langle 4.3 \rangle$$

Letting $\boldsymbol{\gamma}^i = (\gamma^i_1,\ldots,\gamma^i_K)$, it can be seen that:

$$s = \mathbf{w}' \boldsymbol{\gamma}^i \qquad i = 1,\ldots,I \qquad \langle 4.4 \rangle$$

The $\boldsymbol{\gamma}^i$ can be interpreted as shadow prices, or personal prices for all goods.

The planner takes an average of these propositions for each good:

$$\gamma_k = \frac{1}{I} \sum_{i=1}^{I} \gamma^i_k \qquad k = 1,\ldots,K \qquad \langle 4.5 \rangle$$

These averages are the shadow prices that the planner uses in preparing the plan. He then suggests a change in the allocation based on:

$$\mathbf{x}^i_{t+1} - \mathbf{x}^i_t = \hat{\mathbf{B}}_t \, (\boldsymbol{\gamma}^i_t - \boldsymbol{\gamma}_t) \qquad \langle 4.6 \rangle$$

where $\mathbf{x}^i_t = (\mathbf{x}^i_{1,t},\ldots,\mathbf{x}^i_{K,t})$ and $\boldsymbol{\gamma}^i_t = (\gamma^i_{1,t},\ldots,\gamma^i_{K,t})$

where $\hat{\mathbf{B}}_t$ is a strictly positive diagonal matrix of coefficients satisfying $\boldsymbol{\gamma}'_t \, \hat{\mathbf{B}}_t = \mathbf{w}'$.[1]

[1] The procedure will work for any positive definite matrix satisfying this condition. Malinvaud's process uses a diagonal matrix.

Thus, with such a **B**, when $\gamma^i_k > \gamma_k$, the consumer would receive an increased allocation of the good. The consumers respond, giving a new vector of MRS based on the revised allocation, and so on.

Propositon 4.1: Any reallocation mechanism that follows relation ‹4.6› is :

a) physically possible;

b) financially possible; and

c) individually rational.

Proof:

a) Feasibility at each step can be quickly demonstrated by summing ‹4.6› across individuals. The bracketed term sums to zero, thus the left hand side must also. Therefore:

$$\sum_{i=1}^{I} \mathbf{x}^i_{t+1} = \sum_{i=1}^{I} \mathbf{x}^i_t = \omega \qquad \text{for } i \in \mathcal{I},\ \mathbf{x}^i = (x^i_1, .., x^i_K) \quad ‹4.7›$$

b) Let $\hat{\mathbf{B}}_t = \check{\boldsymbol{\gamma}}_t \hat{\mathbf{w}}$

where $\hat{\mathbf{w}}$ is a diagonal matrix (strictly positive definite to respect $\hat{\mathbf{B}}$) based on $\mathbf{w}$. $\check{\boldsymbol{\gamma}}$ is the inverse of $\hat{\boldsymbol{\gamma}}$, a diagonal matrix based on $\boldsymbol{\gamma}$. Therefore:

$$\mathbf{x}^i_{t+1} - \mathbf{x}^i_t = \check{\boldsymbol{\gamma}}_t \hat{\mathbf{w}} (\boldsymbol{\gamma}^i_t - \boldsymbol{\gamma}_t) \qquad \mathbf{x}^i = (x^i_1, .., x^i_K) \quad ‹4.8›$$

Premultiplying by $\boldsymbol{\gamma}_t$, we have

$$\boldsymbol{\gamma}_t \mathbf{x}^i_{t+1} = \boldsymbol{\gamma}_t \mathbf{x}^i_t \quad ‹4.9›$$

Thus at "prices" $\boldsymbol{\gamma}_t$ the reallocation is financially possible.

c) It is important to show that the material welfare of each participant increases at every step. This is necessary not only to satisfy the conditions of the Pareto optimum, but to encourage participation in the process. If the change in utility as a function of holdings associated with each step is always positive for each consumer, this may be taken as proven.

Premultiplying ‹4.6› by $(\boldsymbol{\gamma}^i_t - \boldsymbol{\gamma}_t)'$ gives:

$$(\boldsymbol{\gamma}^i_t - \boldsymbol{\gamma}_t)'(\mathbf{x}^i_{t+1} - \mathbf{x}^i_t) > 0 \qquad i = 1,..,I \qquad ‹4.10›$$

Unless $\boldsymbol{\gamma}^i_t = \boldsymbol{\gamma}^t$, when the expression is equal to zero, and the individual would not engage in any trades, this implies that each i will experience a subjective gain from trade.

The differential of a consumer's utility function ‹4.1› constrained by his initial resources is:

$$du^i = \frac{\partial u^i}{\partial \mathbf{x}^i} d\mathbf{x}^i = \frac{\mathbf{w}' \ \partial u/\partial \mathbf{x}^i}{s} \ \boldsymbol{\gamma}'^i \ [\mathbf{x}^i_{t+1} - \mathbf{x}^i_t] \qquad ‹4.11›$$

Given ‹4.9› and the fact that

$$\frac{\mathbf{w}' \ \partial u^i/\partial \mathbf{x}^i}{s} > 0$$

$$du^i \geq 0 \qquad i = 1,..,I \qquad ‹4.12›$$

Equality occurs when $x^i_{t+1} = x^i_t$, that is, when $\boldsymbol{\gamma}^i_t = \boldsymbol{\gamma}_t$. If this should happen to every i at the same time, then no changes will occur: this is indeed a Pareto optimum. ∎

In that the proof of convergence is somewhat easier if the revision process is continuous, we follow Malinvaud, (1982, p.208) in redefining ‹4.6› as:

$$\dot{\mathbf{x}}^{i}(t) = \hat{\mathbf{B}}\,[\gamma^{i}(t) - \gamma(t)] \qquad ‹4.13›$$

where $\dot{\mathbf{x}}$ indicates the partial derivative of $\mathbf{x}^{i}$ with respect to "time".

Proposition 4.2: The reallocation mechanism ‹4.13› converges to a Pareto optimum.

Proof: Define the sum of the utilities as the Lyapunov function. The utility functions are increasing and bounded above by the set of possible allocations. The uniqueness of the final allocation is shown by the fact that the derivative of the Lyapunov function to which the trajectory of the system of differential equations converges vanishes only at the limit, which is a Pareto optimal allocation, and the strict concavity of the utility functions. (Laffont, 1982, p.46). ∎

Decentralizing the Economy

An important contribution of Tulkens and Zamir (1979) was to recognize and demonstrate that agents, faced with the opportunities for exchange given in ‹4.6›, have an incentive to engage in spontaneous barter. Since this resembles Uzawa's (1962) rule for the Edgeworth process, they have effectively demonstrated a link between the MDP and Edgeworth processes.

A subtle but significant shift occurs in the transition, however. In the MDP model, the transactions do not take place. The planner and the consumers discuss "propositions" or "prospective indicators", but consumers hold onto their initial allocations. In recognizing and giving cause to the spontaneous wish to transact, TZ have turned the model into a non-tâtonnement one, in that exchanges (but not consumption) of the goods actually occur during the adjustment process. The introduction of trading is a critical first step towards introducing real time into the model.

Another important innovation is that the distribution of the surplus is no longer determined by an external rule, which would be incompatible with the idea of decentralization. This property might make sense within the context of a planned economy, but not for a market economy. Indeed, targeting the economy along a given course to a pre-arranged goal is fundamentally at odds with the concept of a decentralized process. As TZ see it,

> the choice of a distribution profile for sharing the surplus at any allocation should naturally be considered as being determined by the bargaining power of the individuals and coalitions at the moment, and not any more by their power at the initial position of the economy. (Tulkens and Zamir, 1979, p.305)

In their model, distribution of the surplus is determined by a "local game". TZ drop the weighted character of the average used in Malinvaud (1972), in effect giving all consumers equal weight. The uniform prices are a simple arithmetic mean of individual MRS. This structure provides a solution in the core of local games, which means, specifically, that no subset of agents can be made better off by not participating. This essentially guarantees participation.

Building on the TZ model, Schoumaker (1979) goes further, demonstrating that, even if agents misrepresent their preferences in order to maximize their immediate gain in utility, the procedure will still converge to a unique Pareto optimum. This is because agents will not be inclined to misrepresent the direction of the preferred price change. These game theoretic results support the use of the TZ model in a decentralized context.

TZ stop short of declaring that their model is decentralized, as one step remains. Agents make independent decisions about how they would like prices to evolve, which translate into individual price offers. But how do the agents come to know what the average price vector is (without the aid of a planner or an auctioneer)? Both Uzawa and TZ effectively reinterpret $\boldsymbol{\Upsilon}$ as a price vector $\mathbf{p}$. Uzawa provides little intuition as to how the $\boldsymbol{\Upsilon}$ comes to be seen as a price vector, and, while TZ show that the average price vector leads to a solution in the core of local games, they don't explain how it appears either.

Consumers must be able to come to this information on their own. This will be explicitly assumed here. Since the average "prices" are a simple arithmetic mean of individual MRS, they are much more likely to be observable than other agents' excess demands.[2]

A new issue emerges in the decentralized context. TZ define their adjustment matrix in terms of the speed of adjustment only (and in fact normalize it to one). However, the matrix **B** takes on greater significance, representing, in effect, the transaction technology of the process. This matrix cannot be whatever, and there is no planner to fix it. This issue has not been much explored in the literature.

We now develop a price dynamics equation for the model that lends itself to a decentralized interpretation. We recall that the new allocations are based on the process (substituting $\mathbf{p}_t$ for γ_t):

$$\mathbf{x}^i_{t+1} - \mathbf{x}^i_t = \hat{\mathbf{B}}_t\,(\gamma^i_t - \mathbf{p}_t) \qquad \langle 4.6A\rangle$$

Now let:

[2]It is not necessary that each consumer be aware of everybody else's γ^i. One way to deal with the problem is to consider smaller groups: observation of the "individual prices" within the group is somewhat easier, while the overall results hold for any collection of such groups.

$$d\gamma^i_t = \frac{\partial \gamma^i_t}{\partial x^i_t} dx^i_t = E^i_t (x^i_{t+1} - x^i_t) \qquad \langle 4.14\rangle$$

where $\mathbf{w}' E^i_t \equiv 0$ (because of ‹4.4›)

and $\varsigma' E^{it} \varsigma < 0$ for $\varsigma' \gamma^i_t = 0$, $\varsigma \neq 0$

because of strong quasi-concavity.

(cf. Bronsard and Leblanc (1980))

Premultiplying ‹4.6A› by E^i_t gives:

$$d\gamma^i_t = E^i_t \hat{B}_t (\gamma^i_t - p_t) \qquad \langle 4.15\rangle$$

This equation defines the adjustment path of prices. It is the dual of ‹4.6A›, and therefore, if ‹4.6A› converges, ‹4.15› must also. That the distance between the "individual prices" and the official prices eventually falls to zero shows that the disequilibrium state of the economy is taken into account. Equation ‹4.15› can also be expressed in continuous form. Summing across consumers:

$$dp_t = \frac{1}{I} \sum_{i=1}^{I} E^i_t \hat{B}_t (\gamma^i_t - p_t) \qquad \langle 4.16\rangle$$

This can be smoothed to the form:

$$\dot{p}(t) = \frac{1}{I} \sum_{i=1}^{I} E^i(t) \hat{B}(t) [\gamma^i(t) - p(t)] \qquad \langle 4.17\rangle$$

Often in the literature an equation such as ‹4.17› is arbitrarily put forward. Existence of a central agent is implicit, whereas this model has been built up from the individual demands of agents.

In the next chapter, the model will be extended to incorporate production and public goods.

5

PRODUCTION AND THE ADJUSTMENT OF WATER QUALITY

In the last chapter, the problem of the adjustment of an economy towards equilibrium was introduced for an exchange economy. Since water quality was determined by production in the model presented earlier, it is essential to include this in the adjustment model. The dynamic model of the previous chapter will first be extended to include production of private goods, which is of some interest in itself. The water quality parameters will then be introduced in a two-stage process along the lines of the structure proposed in the last chapter. First the economy adjusts to a Pigou-Baumol-Dales (PBD) equilibrium, and then this level is varied according to conditions that will permit the eventual realization of a Pareto optimal allocation throughout the economy.

Production

Equation ‹4.1› representing consumers' utility is repeated. Water quality remains exogenous for the moment.

$$u^i(\mathbf{x}^i, \bar{\mathbf{c}}) \qquad \langle 5.1 \rangle$$

where $\mathbf{x}^i = (x^i_1, \ldots, x^i_K)$ and $\bar{\mathbf{c}} = (\bar{c}_1, \ldots, \bar{c}_L)$

$\mathbf{w}$ is a vector of non-negative weights, such that:

$\mathbf{w}' \, \partial u^i / \partial x^i > 0$, where $\dfrac{\partial u^i}{\partial x^i} = \left[\dfrac{\partial u^i}{\partial x^i_1}, \ldots, \dfrac{\partial u^i}{\partial x^i_K}\right]$

Production technology is represented by:

$$f^j(\mathbf{y}^j) = 0 \qquad \langle 5.2\rangle$$

where $\mathbf{y}^j = (y^j_1, \ldots, y^j_k, \ldots, y^j_K)$

$\mathbf{w}'\ \partial f^j/\partial y^j > 0$, where $\dfrac{\partial f^j}{\partial y^j} = \left[\dfrac{\partial f^j}{\partial y^j_1}, \ldots, \dfrac{\partial f^j}{\partial y^j_K}\right]$.

As before, f^j is continuously differentiable and strictly quasi-convex in its arguments.

Resource conservation is represented by:

$$\sum_{i=1}^{I} \mathbf{x}^i_k - \sum_{j=1}^{J} \mathbf{y}^j_k = \omega_k \qquad \text{where } k = 1,2,\ldots,K \qquad \langle 5.3\rangle$$

With s as the unit of account, simplified notation is extended to the production sector as follows:

$$\gamma^i_k = \frac{s\ \dfrac{\partial u^i}{\partial x^i_k}}{\mathbf{w}'\ \dfrac{\partial u^i}{\partial x^i}} \qquad \begin{array}{l} i = 1,\ldots,I \\ k = 1,\ldots,K \end{array}$$

$$\phi^j_k = \frac{s\ \dfrac{\partial f^j}{\partial y^j_k}}{\mathbf{w}'\ \dfrac{\partial f^j}{\partial y^j}} \qquad \begin{array}{l} j = 1,\ldots,J \\ k = 1,\ldots,K \end{array}$$

Agents take the arithmetic mean of consumers' MRS and producers' RTS to define the price system:

$$\mathbf{p} = \frac{1}{I+J}\left[\sum_{i=1}^{I} \boldsymbol{\gamma}^i + \sum_{j=1}^{J} \phi^j\right] \qquad \langle 5.4\rangle$$

where $\mathbf{p} = (p_1, \ldots, p_K)$, $\boldsymbol{\gamma}^i = (\gamma^i_1, \ldots, \gamma^i_K)$ and $(\phi^j = \phi^j_1, \ldots, \phi^j_K)$

It can now be seen that:

$$s = \mathbf{w}' \mathbf{p} \quad \langle 5.5 \rangle$$

Firms will make non-zero profits as they adjust their production in the face of changing prices. These profits will be passed on to (or losses absorbed by) consumers. The same general structure with regard to share ownership that was developed in Chapter 2 will be used again here (cf. ‹2.14›):

$$r^i = \theta^i \sum_{j=1}^{J} \pi^j \qquad i = 1,\ldots,I \quad \langle 5.6 \rangle$$

with $\Sigma_i\ \theta^i = 1$ and $\theta^i \geq 0,\ i \in \mathcal{I}$

r^i is the individual's earnings, while θ^i is his share of total profits and π^j represents firm j's profits.

The process can now be defined in terms of quantity adjustments:

$$\mathbf{x}^i_{t+1} - \mathbf{x}^i_t - \frac{\mathbf{w}}{s}\, r^i_t = \hat{\mathbf{B}}_t\ (\boldsymbol{\gamma}^i_t - \mathbf{p}_t) \quad \langle 5.7 \rangle$$

$$\mathbf{y}^j_{t+1} - \mathbf{y}^j_t - \frac{\mathbf{w}}{s}\, \pi^j_t = -\hat{\mathbf{B}}_t\ (\boldsymbol{\phi}^j_t - \mathbf{p}_t) \quad \langle 5.8 \rangle$$

Again, $\hat{\mathbf{B}}_t$ is a positive definite matrix of coefficients satisfying $\mathbf{p}_t'\ \hat{\mathbf{B}}_t = \mathbf{w}'$.

<u>Propositon 5.1:</u> Any reallocation mechanism that follows relations ‹5.7› and ‹5.8› is:

a) physically possible;

b) financially possible for both consumers and firms;

c) individually rational for consumers and profitable for firms.

Proof:

a) Feasibility at each step is demonstrated by summing ‹5.7› and ‹5.8› across agents. By definition, the bracketed terms sum to zero, thus the left hand sides must also, and this for every period. This gives:

$$\mathbf{x}_{t+1} - \mathbf{x}_t - \frac{\mathbf{w}}{s} \sum_{i=1}^{I} r^i_t = 0 \qquad ‹5.9›$$

$$\mathbf{y}_{t+1} - \mathbf{y}_t - \frac{\mathbf{w}}{s} \sum_{j=1}^{J} \pi^j_t = 0 \qquad ‹5.10›$$

Summing ‹5.9› with the negative of ‹5.10› and using ‹5.6› gives:

$$\mathbf{x}_{t+1} - \mathbf{x}_t = \mathbf{y}_{t+1} - \mathbf{y}_t \qquad ‹5.11›$$

The net change in consumption is just balanced by the net change in production, as ‹5.3› requires.

b) We wish to show that the program is financially possible, both for individuals and firms. In order to simplify the discussion, let:

$$\hat{\mathbf{B}}_t = \check{\mathbf{p}}_t \hat{\mathbf{w}}$$

where $\hat{\mathbf{w}}$ is a diagonal matrix (strictly positive to respect $\mathbf{B}$) based on $\mathbf{w}$. $\check{\mathbf{p}}$ is the inverse of $\hat{\mathbf{p}}$, a diagonal matrix formed from $\mathbf{p}$. Therefore:

$$\mathbf{x}^i_{t+1} - \mathbf{x}^i_t - \frac{\mathbf{w}}{s} r^i_t = \check{\mathbf{p}}_t \hat{\mathbf{w}} (\boldsymbol{\gamma}^i_t - \mathbf{p}_t) \qquad ‹5.12›$$

$$\mathbf{y}^j_{t+1} - \mathbf{y}^j_t - \frac{\mathbf{w}}{s} \pi^j_t = \check{\mathbf{p}}_t \hat{\mathbf{w}} (\boldsymbol{\phi}^j_t - \mathbf{p}_t) \qquad ‹5.13›$$

premultiplying these by $\mathbf{p}_t'$, we have:

$$\mathbf{p}_t \mathbf{x}^i_{t+1} - \mathbf{p}_t \mathbf{x}^i_t = r^i_t \qquad ‹5.14›$$

$$p_t\, y^j_{t+1} - p_t\, y^j_t = \pi^j_t \qquad \langle 5.15\rangle$$

since the right hand sides reduced to zero.

c) Given that consumers receive all firm profits, if each consumer's change in utility is always positive, it may be taken as proven that the welfare of each participant increases at every step. It will be shown first that each consumer's gains are greater than his revenue from firm profits, and then that the latter must also be positive.

Premultiplying ‹5.7› by $(\gamma^i_t - p_t)'$

$$(\gamma^i_t - p_t)' \left[x^i_{t+1} - x^i_t - \frac{w}{s}\, r^i_t\right] > 0 \qquad \langle 5.16\rangle$$

since the right hand side is positive. Using ‹5.14›, this implies that:

$$\gamma^i_t{}' \left[x^i_{t+1} - x^i_t - \frac{w}{s}\, r^i_t\right] > 0 \qquad \langle 5.17\rangle$$

or, using ‹4.4›,

$$\gamma^i_t{}' \,(x^i_{t+1} - x^i_t) > r^i_t \qquad \langle 5.18\rangle$$

Each consumer experiences a welfare gain that is greater than his revenue from share profits. To show that this is positive, premultiply ‹5.8› by $(\phi^j_t - p_t)'$

$$(\phi^j_t - p_t)' \left[y^j_{t+1} - y^j_t - \frac{w}{s}\, \pi^j_t\right] < 0 \qquad \langle 5.19\rangle$$

Using ‹5.15›, this gives:

$$\phi^j_t{}' \left[y^j_{t+1} - y^j_t - \frac{w}{s}\, \pi^j_t\right] < 0 \qquad \langle 5.20\rangle$$

Taking account of the production function,

$$\pi^j_t > 0 \qquad \langle 5.21\rangle$$

π^j_t is positive for all j, which means that the process is profitable for firms. Furthermore, by ‹5.6›, ‹5.18› must be positive, which demonstrates individual rationality. ∎

Equations ‹5.7› and ‹5.8› may be rewritten in continuous form as:

$$\dot{\mathbf{x}}^i(t) - \frac{\mathbf{w}}{s}\, r^i(t) = \widehat{\mathbf{B}}(t)\, [\boldsymbol{\gamma}^i(t) - \mathbf{p}(t)] \qquad \langle 5.22\rangle$$

$$\dot{\mathbf{y}}^j(t) - \frac{\mathbf{w}}{s}\, \pi^j(t) = -\widehat{\mathbf{B}}(t)\, [\boldsymbol{\phi}^j(t) - \mathbf{p}(t)] \qquad \langle 5.23\rangle$$

<u>Proposition 5.2</u> The reallocation mechanism ‹5.22› and ‹5.23› converges to a Pareto optimum.

<u>Proof:</u> Since $f^j(\mathbf{y}^j) = 0$ for all $j \in \mathcal{J}$, the stability of such a process can be shown using only the sum of the utilities as the Lyapunov function. The utility functions are increasing and are bounded above by the set of possible allocations. The final allocation to which the trajectory of the system of differential equations converges is unique, given that:

1) the derivative of the Lyapunov function vanishes only at the limit,

2) it is a Pareto optimum, and

3) the utility functions are strictly concave.

(Laffont, 1982, p.46). ∎

Adjustment of Permit Holdings

In Chapter 2, the efficient marginal conditions for effluents required modifying the marginal "productivity" of effluents across firms according to a factor determined by the environmental transfer function $\boldsymbol{\delta}$, since different firms' effluents can have a greater or lesser impact on $\mathbf{c}$.

The operation of the permit market needs to be described. The government sets water quality objectives, denoted by $\bar{\mathbf{c}}$. To achieve this goal, the government sells permits, $\mathbf{g}$, which are denominated not in terms of firms' effluents, but in terms of the impact on water quality. To do this, it uses the transfer function $\boldsymbol{\delta}$ to determine impact coefficients (κ^j) for each firm's effluents for each contaminant so that:

$$\bar{\mathbf{c}} = -\sum_{j=1}^{J} \kappa^j \, \mathbf{e}^j \qquad \langle 5.24 \rangle$$

Permits give firms the right to produce effluents according to the relation:

$$\mathbf{g}^j = \kappa^j \, \mathbf{e}^j \qquad \text{where } \mathbf{g}^j = (g^j_1, \ldots, g^j_L) \qquad \langle 5.25 \rangle$$

The total of these permits is fixed for each contaminant:

$$\bar{\mathbf{G}} = \sum_{j=1}^{J} \mathbf{g}^j \qquad \langle 5.26 \rangle$$

This is in fact equation ⟨2.28⟩. Given the linear correspondence between $\mathbf{c}$ and $\mathbf{e}$ implied by ⟨5.24⟩ it follows that:

$$\bar{c} = - G^j \qquad \langle 5.27\rangle$$

Thus the price (q) of a permit would be the same for all firms, but the permit in fact grants different levels of effluent, namely $[\kappa^j_\ell]^{-1} g^j_\ell$.

Using $\mathbf{g}$ to show permit holdings, production technology is now given by an equation similar to ‹2.2A› (p.41):

$$f^j(\mathbf{y}^j, -[\kappa^j]^{-1} \mathbf{g}^j) = 0 \qquad \langle 5.28\rangle$$

where κ^j is a sign preserving matrix.

The economy is now described by ‹5.1›, ‹5.28›, ‹5.3› and ‹5.27›. To describe the dynamics of the effluent permit market, we need a third equation to go with ‹5.7› and ‹5.8›. Let:

$$\phi_z^j = \frac{s\,[\kappa^{j\prime}]^{-1} \dfrac{\partial f^j}{\partial(-e^j)}}{\mathbf{w}' \dfrac{\partial f^j}{\partial y^j}}$$

In a manner similar to the private goods, the firms take permit prices to be an arithmetic average of the rates of technical substitution so defined:

$$q_t = \frac{1}{J} \sum_{j=1}^{J} \phi_z{}^j{}_t \qquad \langle 5.29\rangle$$

Repeating equations ‹5.7› and ‹5.8›, the process of adjustment to a PBD equilibrium (p.59) may now be defined as:

$$x^i_{t+1} - x^i_t - \frac{w}{s} r^i = \hat{B}_t (\gamma^i_t - p_t) \quad \langle 5.30 \rangle$$

$$y^j_{t+1} - y^j_t - \frac{w}{s} \pi^j = -\hat{B}_t (\phi^j_t - p_t) \quad \langle 5.31 \rangle$$

$$g^j_{t+1} - g^j_t = \hat{C}_t (\phi_z{}^j_t - q_t) \quad \langle 5.32 \rangle$$

$\hat{B}_t$ and $\hat{C}_t$ are positive definite matrices of coefficients satisfying $p_t' \hat{B}_t = w'$ and $q_t' \hat{C}_t = v'$ where v' is a positive vector of weights.

Propositon 5.3: Any reallocation mechanism that follows relations ‹5.30›, ‹5.31› and ‹5.32› is:

a) physically possible; and

b) individually rational for consumers, globally profitable for firms, and financially possible.

Proof:

a) Again, feasibility can be demonstrated by summing these relations across agents. Since the bracketed terms sum to zero, ‹5.30› and ‹5.31› give:

$$x_{t+1} - x_t - \frac{w}{s} \sum_{i=1}^{I} r^i_t = 0 \quad \langle 5.33 \rangle$$

$$y_{t+1} - y_t - \frac{w}{s} \sum_{j=1}^{J} \pi^j_t = 0 \quad \langle 5.34 \rangle$$

As before, summing ‹5.33› with the negative of ‹5.34› and using ‹5.6› gives:

$$x_{t+1} - x_t = y_{t+1} - y_t \quad \langle 5.35 \rangle$$

while the sum of ‹5.32› directly produces:

$$\sum_{j=1}^{J} g^j_{t+1} = \sum_{j=1}^{J} g^j_t \quad \langle 5.36 \rangle$$

The last equation ensures that the government's water quality objective given by ‹5.27› will be met at all times.

b) From Propostion 5.1, we know that as long as consumers receive positive dividends, r^i, this will ensure that all consumers experience a net gain at every step of the procedure. It will now be shown that operation of the permit market does not alter this condition.

Multiplying ‹5.32› by $\mathbf{q}'_t$ gives:

$$\mathbf{q}'_t \, \mathbf{g}^j_{t+1} - \mathbf{q}'_t \, \mathbf{g}^j_t = \mathbf{v}' \, \phi\mathbf{z}^j_t - \mathbf{v}' \, \mathbf{q}_t \qquad ‹5.37›$$

This expression can only be expected to equal zero at the end of the procedure, when $\phi\mathbf{z}^j_t = \mathbf{q}_t$. Multiplying ‹5.32› by $(\phi\mathbf{z}^j_t - \mathbf{q}_t)'$:

$$(\phi\mathbf{z}^j_t - \mathbf{q}_t)' \, (\mathbf{g}^j_{t+1} - \mathbf{g}^j_t) \geq 0 \qquad ‹5.38›$$

Using ‹5.37›,

$$\phi\mathbf{z}^j_t{}' \, (\mathbf{g}^j_{t+1} - \mathbf{g}^j_t) \geq \mathbf{v}' \, \phi\mathbf{z}^j_t - \mathbf{v}' \, \mathbf{q}_t \qquad ‹5.39›$$

Firms are not constrained to balance their trades in effluents. The right hand term will disappear only at equilibrium. However, summing across agents, this gives:

$$\sum_{j=1}^{J} \phi\mathbf{z}^j_t \, (\mathbf{g}^j_{t+1} - \mathbf{g}^j_t) \geq 0 \qquad ‹5.40›$$

From the production function ‹5.28› we know that:

$$\phi^j_t(\mathbf{y}_{t+1} - \mathbf{y}_t) = \phi_2{}^j_t(\mathbf{g}^j_{t+1} - \mathbf{g}^j_t) \qquad \langle 5.41\rangle$$

where ϕ^j is the vector for private good. By ‹5.40›,

$$\sum_{j=1}^{J} \phi^j_t(\mathbf{y}_{t+1} - \mathbf{y}_t) \geq 0 \qquad \langle 5.42\rangle$$

which means that permit market transactions generate a positive net return.

It was shown in Proposition 1 that we can multiply ‹5.31› (which is the same as ‹5.7›) by $(\phi^j_t - \mathbf{p}_t)'$ to get:

$$\phi^j_t{}' \; (\mathbf{y}^j_{t+1} - \mathbf{y}^j_t) < \pi^j_t \qquad \langle 5.43\rangle$$

The left-hand term is no longer a complete representation of the production function. Summing ‹5.43› across firms, by ‹5.42› this means:

$$\sum_{j=1}^{J} \pi^j_t \geq \sum_{j=1}^{J} \phi^j_t{}' \; (\mathbf{y}^j_{t+1} - \mathbf{y}^j_t) \geq 0 \qquad \langle 5.44\rangle$$

Thus, by ‹5.6›, $r^1 \geq 0$. The procedure produces a net gain for consumers at every step. ∎

A Pigou-Baumol-Dales equilibrium was introduced in Chapter 3 (p.59) to describe an economy that had adjusted to a fixed level of water quality using a permit market. Restating ‹5.32› in continuous form:

$$\dot{\mathbf{g}}^j = \hat{\mathbf{C}} \, [\phi_2{}^j(t) - \mathbf{q}(t)] \qquad \langle 5.45\rangle$$

It may be shown that:

Proposition 5.4: The reallocation mechanism ‹5.22›, ‹5.23› and ‹5.45› converges to a Pigou-Baumol-Dales equilibrium.

Proof: Straightforward extension of Proposition 5.2. ∎

Adjustment of the Level of Water Quality

The above model has been shown to converge not to a Pareto optimum but to a Pigou-Baumol-Dales equilibrium, that is, an optimum relative to a fixed level of water quality. It remains to show how a true optimum can be established. This consists of the formalization of a rule that the government could use to move the economy through a sequence of PBD equilibria to a true optimum.

The difficulty of identifying a Pareto optimum where environmental problems are concerned was thoroughly discussed in Chapter 3. But the government does not need to know exactly where the optimum lies, only in which direction, i.e., is it appropriate to tighten or loosen the supply of permits to improve the overall welfare of society? In terms of a Pareto optimum with effluent permit markets (defined on p.42), it needs to know which is greater, $\Sigma_i q^i$ or q.

In making this evaluation it has precise information on one side of the equation: each firm will attempt to equalize its marginal control costs to the price it pays for permits. This value is of course accurate only in the immediate region of the PBD equilibrium of the moment.

Against this, as discussed in Chapter 3, the government must compare its knowledge of scientific evidence and consumer evaluations, using tools such as cost/benefit analysis and voting mechanisms. In particular, as the environmental standards move closer to their optimal values, popular concern over environmental problems should diminish.

Formally, using G as the measure of total permits,

Adjustment Rule for G: The government evaluates $\Sigma_i q^i_\ell$ and compares this total with q_ℓ.
If $\Sigma_i q^i_\ell > q_\ell$, the government reduces G_ℓ;
If $\Sigma_i q^i_\ell < q_\ell$, the government increases G_ℓ;
If $\Sigma_i q^i_\ell = q_\ell$, the government does not change G_ℓ;
where $G_\ell = (G_1,..,G_L)$

Comparing the definitions of a Pigou-Baumol-Dales equilibrium with that of a Pareto optimum with effluent permit markets, we can see that the latter has been achieved when $\Sigma_i q^i_\ell = q_\ell$ for all ℓ.

6

ALTERNATIVE MARKET STRUCTURES

The model developed in previous chapters used a transferable discharge permit market for the control of pollution. It was introduced without much discussion as to why it was chosen over alternative structures, or even how a TDP market works. These gaps will now be filled in.

In practice, TDP markets are not the most widely used mechanism. Probably the most common is direct regulation, by which the government determines what level of effluent is acceptable for each firm. This may be done either in absolute terms, as a fixed proportion of production, or as a concentration in its effluents. This approach does not find favour with most economists, however, as it is most unlikely to result in a Pareto-optimal allocation. It is worth briefly spelling out why it does not qualify as an optimal mechanism: this is done in the next section.

Besides a permit system, other mechanisms that could achieve optimal allocations are taxes and subsidies. In a tax scheme, the polluting firm pays a tax on each unit of effluent. Subsidies, whereby the polluting firm is paid a fee to reduce its level of pollution, have also received some attention. In the following section, these approaches are compared to the permit market. While similar in terms of theoretical properties, various considerations, such as the impact of uncertainty or the arrival of new firms, bring out significant differences.

In the last section of this chapter, a different problem is broached, one that occurs only for permit mechanisms, and that is the manner of the initial distribution of the permits. There is a substantial economic literature on the properties of different types of auctions, as well as alternative distribution systems. Special attention is paid to the small numbers problem.

<u>Direct Regulation</u>

It is difficult to imagine how even firm-specific regulations, let alone general regulations, could meet the criteria for optimal processes spelled out in equations ‹2.10› to ‹2.13›. Not only does it presume the acquisition and processing of vast amounts of information, but, as has been pointed out, the optimal values are in flux until such time as the equilibrium is found.

Despite this, Suchanek (1977, 1979) has endeavoured to show that a (non-market) quota system is as efficient as a tax system. He has a number of restrictive assumptions, including separable utility and production functions (which were discussed in Chapter 3), to prove the point.

Individual quotas are calculated for each firm. While Suchanek's sheme may be optimal, this is not how regulations work. They consist of a single rule (either in absolute or relative terms), which is applied to all firms. It is all but impossible for a simple rule to achieve an optimal outcome.

Two further arguments why direct regulation is unlikely to reach an optimal allocation have been presented in the economic literature: one in relation to growth and innovation, the other to market structure. Regulations, which permit some minimum level of effluent for each firm, give firms the wrong signal about entry (Page, 1973). That basic quantity of effluent granted to all firms is costless to the new firm as well, but not to downstream users. Regulations are not defined in terms of the quality of the receiving ecosystem, and are seldom modified. Thus, even if the level of effluent were optimal at one point in time, the entry of a firm would upset it.

Freeman (1980) uses a similar analysis with respect to technological innovation. Regulations provide no incentive to invent or apply more efficient control technologies, indeed, they often mandate particular technologies. Faced with a tax or permit system, however, the cost minimizing firm will be alert to new technologies, facilitating the transition to more productive allocations.

Finally, as concerns market structure, Buchanan and Tullock (1975) argue that large firms favour regulatory approaches, and Freeman (1980) suggests that regulations increase concentration in an industry. Empirical evidence presented by Pashigian supports these claims, as he notes that:

> Compliance with environmental laws has not only reduced the number of plants in the affected industries but has placed a greater burden on small than on large plants. (1984, p.23)

This would appear to be because fixed costs are proportionately heavier on small installations. Although a comparative study has not been made, it might be supposed that tax or permit systems would be less burdensome on small firms than across the board production based regulations.

There is a place for direct regulation, however. As mentioned in the Introduction, there may be no safe level for the most toxic pollutants. In such cases it would be much more costly to create some kind of market structure than to intervene directly.

Permits, Taxes and Subsidies

It was demonstrated in Chapter 2 that tax and permit approaches are essentially equivalent in the static context. Coase (1960) sparked a heated debate when he suggested that subsidies might also result in an efficient solution, with the only difference lying in the distribution of resources.

An obvious objection is that, if firms anticipate the subsidies not to pollute, they can threaten to enter the pollution intensive industry simply to collect the subsidy (Page, 1973). The reasoning is reminiscent of Starrett (1972—cf. Chapter 3), and in the same way can result in the destruction of the equilibrium. Where bargaining is directly between upstream and downstream users, such threats may be credible.

Subsidies exist, of course, but they are generally paid to firms with existing installations by a government. Governments are in a position to enforce contracts on behalf of downstream users. So, in a sense, it is not so unreasonable to model an economy as though firm entry into the polluting industry could not occur. This is the object of "double-standard" regulation, whereby one standard applies to firms established before the regulation comes into force, and a somewhat tougher one for new firms.

Under such circumstances, it can be shown that an efficient allocation can be achieved, even one with the same level of environmental quality (Mestelman, 1982). Except under the strongest assumptions the distribution of wealth in the economy will be different from the permit/tax solution, hence the allocation in terms of actual production of various goods is bound to be different as well. Without some guide as to how to order such distributions qualitatively, it is difficult to go farther.

Tax, permit and subsidy programs can each meet the criteria of Pareto-optimality. In Chapter 3, it was shown how a tax approach could lead to instability. Subsidies respond in the same way as taxes in these circumstances. When other supplementary hypotheses (intended to more fully represent real-world constraints) are introduced, more distinctions between these mechanisms can be seen. I will discuss three more here, which have been the subject of discussion in the literature, namely:

- uncertainty over the true form of marginal cost of control and damage functions;
- effective control of environmental quality; and
- the impact of new firms.

Part of the symmetry between tax and permit models relies upon the government's ability to read signals from the firms about the cost of pollution control: the quantity of pollution in the one case, the price of the permit in the other. These two indicators are not equally easy to read, both because of uncertainty in the face of stochastic influences, and because it may be in the selfish interest of the firm to distort its signal.

With regard to uncertainty, it has been noted by Kolm (1971) and Beavis and Walker (1983) that waste discharge is by nature stochastic, resulting in uncertainty and possible miscalculation concerning the location of the marginal cost of control function. This compounds the problem, already mentioned in Chapter 3, that variable factor prices will constantly modify the control cost function itself. With a tax system, the government would have to closely monitor discharges, allowing for seasonal and even daily variations in firm activity, to decipher the message. This would be a lengthy process. If the government were inclined to supplement this information with reports from the firms as to their cost structures (or, for that matter, their actual discharges), revelation problems could be expected. Kwerel (1977) and Dasgupta, Hammond and Maskin (1980) have looked at the latter problem, which is no easier to solve than the consumer revelation case.

The importance of having a quick and reliable indicator on control costs emerged in the last chapter, where it was shown that the government might have to pilot the economy through a series of Pigou-Baumol-Dales equilibria before a global optimum could be reached. At every intermediate equilibrium, it would be necessary to compare marginal benefits to marginal costs. The former, it has been seen, are hard to evaluate. It would not be desirable to obfuscate the problem further with noisy data on the cost side of the equation as well.

By providing a clear signal, the permit mechanism filters out the stochastic element. The government could trace out a fairly exact representation of the cost of control curve by varying the target water quality, not that it will want to do so more than necessary, as this could imply heavy costs to the firms. Monitoring costs are reduced as well, as monitoring would only be necessary to verify that the permits are being respected, not as a way to divine firm cost structures as well.

A secondary effect on the uncertainty question relates to the fact that quantitative controls give the pollution control authority direct control over the actual quantities of effluents. If costs are underestimated, a tax system will result in less-than-intended reduction of discharges; if overestimated, a more-than-intended reduction will occur (Roberts and Spence, 1976, p.194). In the tax system, the uncertainty plays out in fluctuating environmental quality; with permits, it is the permit price that absorbs the shocks.

The choice is more than a question of philosophy. A great deal remains to be learned about synergistic effects of contaminants and their effects on complex receiving ecosystems (Beavis and Walker, 1979). If the inflow of contaminants can be brought under control, much more can be learned from the study of the receiving ecosystems themselves. This could contribute substantially to better defined marginal damage functions in the future. Permit levels for individual watercourses could be determined not just on hypothetical laboratory values, but direct observation, wherein the influence of a myriad of other (even unknown) factors have already been incorporated. As Rose-Ackerman puts it, "the biology of the water pollution problem leads one to recommend a pollution rights plan over an effluent charge" (1977, p.389).

In this light, the interpretation of Figure 4 (p.61) may be broadened. There it was argued that tax solutions might not be stable in the face of non-convex damage functions. To this may be added a second problem: taxes are less efficient as environmental control tools, since they control environmental quality only indirectly. Failure to achieve a target value by even a small amount may represent a significant loss of value. Erratic fluctuation around such points could result in considerable inefficiency and waste. The same arguments work against subsidy systems, as well.

A similar analysis applies when firm entry (or growth) is in question. Let us suppose that the correct marginal functions are known as in Ferrar and Whinston (1972, p.315). With a tax system, the firm will choose a level of discharges based on the current tax level. The adminstration will eventually observe that water quality targets are no longer being met and raise the taxes, causing some reduction in the discharges of some firms. In the case of the permit system, the newcomer must buy discharge rights from existing fims by bidding up the price. Firms selling their rights must reduce their discharges immediately. At no time are the water quality targets surpassed.

Thus while tax and permit systems are theoretically equivalent, the permit system appears to have a number of practical advantages. Besides the dynamic problem relating to the stability of the model, permits are less prone to problems of preference revelation (as it relates to firms), they provide a clearer signal as to firm costs, they provide a more stable learning environment concerning actual damages, and they accommodate growth more easily. These factors contributed to the choice of the permit market for the model described in this paper.

Auction Procedures

Up until now, little attention has been paid to how the permit market would actually work. We have supposed that the government provides an organized framework for trading: it might resemble the exchange of lightly-traded stocks. The permits are divided into small units to facilitate trading.

Before controls are introduced, the de facto supply of permits allows the original total of the effluents Σe^j. For subsequent time periods, lesser amounts of effluent permits are sold with different expiry dates. One problem of particular importance (which does not occur either with a tax or a subsidy mechanism) is that of the initial distribution.

If the permits are costly, they will generate revenue, obviously at the firms' expense. Conversely, if they are given away, no revenue is generated and the impact on firms' finances is evidently much less (nil only when the government fully subsidizes the cost not only of installation but of operation of the pollution control equipment.) Montgomery (1972, p.409) established that the two systems produce Pareto-efficient results, since the efficient emission vector is independent of the initial cost of the permit. Thus the question seems to bear primarily on distribution, not efficiency.

This analysis is inherently static, however. How is the level of permits to be adjusted if they have been given away? To whom should additional permits be allocated, if this were appropriate at some future date? Alternatively, would the government have to buy them back, or even expropriate them if it were necessary to reduce the number in circulation? The possibility of existing firms freezing out newcomers also arises. There is an implied perpetual property right when the permits are distributed free. With the sale of the permits by auction for limited periods of validity, these problems do not arise: it is only necessary to put a time limit on the validity of each permit sold.

The auction procedure chosen must of course respect the Pareto optimal (efficiency) criterion on which the whole model is based. Another desirable property is that the auction have a dominant solution, making it more likely to yield the expected results.

In the auction literature the word "optimal" is generally used to indicate that an auction maximizes the revenue of the (monopolist) seller. For the purposes of this discussion, by contrast, it will be assumed that the government is concerned only with efficiency, and tends to behave in a "competitive" manner, by avoiding discriminatory pricing, for example.

There are of course a variety of auction types (McAfee and McMillan, 1987). However, certain structural parameters apply to this case which make it possible to quickly focus the analysis. The kind of auction envisaged will be defined before its properties are briefly analyzed.

- There are multiple units of the permits for sale, rather than just one. Resale of permits is encouraged to promote efficient technological adjustment, although this also opens up the possiblity of cartel formation. Where the number of firms on a watercourse is small, it would be less costly for the firms to acquire the permits cheaply through collusion, reallocating them amongst cartel members as it suited them.

- Firms do not use the same technologies: two firms could produce the same waste product without producing the same good. Thus their control cost functions will be different, and each will have private information about the value of permits. This introduces asymmetry of information. In a competitive framework, this would be of no consequence—prices would transmit the necessary information—but given the small numbers of firms involved, again we see the possibility that firms might behave collusively to control the market.

- The monopoly power of the seller is usually believed to give him power to extract rent from buyers. The hypothesis that the government is concerned more with efficiency and behaves as if it were in a competitive situation leaves the government in confrontation with potentially powerful cartels in the various permit markets (one for each watercourse).

- Risk aversion can have important effects on behaviour. The government may be assumed to be risk neutral. Because of their size, governments generally do not insure their assets, for example. But what of firms?

Restriction of the model appears to reinforce the possibility that the government will confront a cartel of buyers. The standard anti-collusion tool is the reserve price. Collusion implies that firms will conceal the true value of the permits, offering a lower price. As long as the reserve price is not less than the firms' valuation of the permits, they will pay at least the reserve price.

Effective cartel action implies low risk-aversion on the part of firms. If the firms are highly risk averse, they will be more likely to reveal demands close to their true valuations (Matthews, 1983). Risk aversion is generally greater when the cost of the good is large in relation to the firm's assets. In this case, the cost of the discharge permits is hardly likely to be large compared to the firm's assets.

The government, it has been assumed, requires firms to own permits sufficient to cover effluents. If it closes firms in violation, the risk to a firm of not acquiring the desired number, even at full price, becomes very great. There is also a risk that another cartel member will cheat, for example, by bidding just a little more for a large number of permits, shutting the firm out. In order to protect itself against this possibility, the firm will bid a high price for at least a minimum number of permits, if only small amounts for large quantities. Clearly this works against effective cartel control of prices.

The upshot of this is that firms may be able to push down the price of the permits below their true value. It should be remembered, however, that the government will use the permit price as a signal concerning the marginal cost of pollution control. The lower the permit price, the more it will be inclined to reduce the number of permits in circulation. Understating marginal costs is definitely not in the firms' long run interests. While there is no empirical evidence to go on, the possibility of cartels is not likely to distort permit prices significantly.

When there is only a single firm on the watercourse, permits will have no price, or perhaps a nominal charge for administration. Prices achieve efficiency on the basis of opportunity cost: to acquire a good one must pay just a little more than the good is worth to a "competitor". When there are no competitors, there is no cost.

In this chapter, it has been shown that a TDP market has a number of advantages over alternative mechanisms to regulate effluents. Of particular importance, they allow specific environmental goals to be met, even in the face of uncertainty. Moreover, the auction of permits is likely to be difficult to manipulate even when a small number of firms are present, as would often be the case for any particular watercourse.

7

RIVER DYNAMICS

In Chapter 2, the representation of the relationship between effluent and water quality was of a general form. Spatial relationships are critical in environmental questions, and so it is important to explore this question further. In this chapter, the transfer function is given a particular specification, showing the relative ease with which a sound representation of the workings of the natural ecosystem can be incorporated into the economic model.

Only a few authors have explicitly introduced space into models dealing with environmental externalities: Montgomery (1972), Tietenberg (1973a, 1973b, 1974) and Tulkens (1974) were among the first. These and other early writers on the subject emphasized the difficulties that spatial consideration brought to the problem. Tietenberg concludes one paper with the remark:

> The tax rates that will achieve efficiency will, in general, not be the same for all firms. This nonuniformity of efficient taxes erases the two major benefits cited for the effluent charge—ease of administration and the capability of achieving efficiency and political equity simultaneously. In short, choosing policy instruments to control pollution is a much more difficult task than our early externality models would lead us to believe. (1973b, p.522)

Others are even more critical:

> [T]he equilibrium characteristics of the Tietenberg model are derived from the complementarity of points in space. A moment's reflection reveals, however, that the

> choice of spatial relationships—be they zones of a given size or points on a coordinate set—is inherently arbitrary. Hence the "optimal" set of taxes is equally arbitrary. (Burstein and Quigley, 1976, p.81)

Nevertheless, the approach has its defenders, such as Kneese (1972); moreover, the problems have not proved to be overwhelming. Good models of river dynamics have been developed, often with the support of the U.S. Environmental Protection Agency. One of the best known such models is Qual2, which was used on the St-Maurice river in connection with this study.

A Model with Zones

Since there is an infinite number of possible locations along a river, it is convenient to use zones. A zone must be an area sufficiently small that the water quality will be virtually the same throughout the zone, and that all effluents into the water will have equal effect, on a per unit basis. A way to insure this is to draw zone lines immediately above each effluent source and each point of use. Thus, at the top of each zone, either the water quality undergoes a discrete change, or there is a discrete change in the target value (as at a beach or a spawning ground). Water quality measurements should be taken at these points. This approach is commmon in water quality modelling (cf. Hass, 1970, appendix, p.3). Although arbitrary, such zone lines appear logical and not easily subjected to political whim.

The problem might not be so easy where air quality is concerned (cf. Burstein and Quigley, 1976).

Let $\mathcal{N}$ be the set of all zones of the river, and let the indices m and n represent individual zones of the river. The water quality for a given zone is given by c_n. N will be the topmost zone of interest to consumers, while N+1 indicates the "source" of the water and c_{N+1} its quality. Thus $\mathcal{N} = \{1,..,m,n,..,N,N+1\}$. The notation $m > n$ will mean that m is upriver from n, hence effluents generated there will have an impact on c_n, but not if $m < n$. For convenience, it will be supposed that there is only one firm located in any zone, thus e^j will represent the total effluents in a given zone, and j also refers to a specific zone.

We now specify the general equation ‹2.4› using a simplified form of the Qual2 model in its steady state form (Brown and Barnwell, 1987, p.74 ff). The steady state form implies discharges do not change day to day, though this could be extended to allow for stochastic discharges (Kolm, 1971; Beavis and Walker, 1983).

The Qual2 river model uses a difference method for resolution of the numerical equations. To approximate the spatial derivatives, three points are required for any given moment in time. The three points include one upstream and one downstream. The form of the environmental constraint is then:

$$\alpha_n c_{n-1} + \beta_n c_n + \gamma_n c_{n+1} = R_n - \frac{e_n}{V_n} \qquad n = 1,..,N,N+1 \qquad \langle 7.4 \rangle$$

where c_n is ambient concentration in zone n
e_n is effluents added in zone n
V_n is the volume of water in the section
R_n is net internal sources and sinks of contaminant

First notice that the equation relates the concentration of a contaminant in the river to the level of effluents. The coefficients α, β and γ, as well as the constants V and R, introduce hydrodynamic stream factors that modify this relationship. Specifically, the coefficients α, β and γ incorporate the dispersion characteristics of the river, the volume, the cross-sectional area and the length of the section (each zone is subdivided for computational purposes), as well as, in the case of the first two, the net inflow over the section (Brown and Barnwell, 1987). The coefficients are zone specific, which will weigh down the notation, requiring the use of products.

The dependence of c_n on c_{n+1} is to be expected; its dependence on c_{n-1} is an artifact of the method of computation. It will be dropped to simplify the presentation. ‹7.4› may then be converted to the moving average form:

$$c_n = \frac{W_n}{\beta_n} + \sum_{m=n+1}^{N} \frac{W_m}{\beta m} \prod_{n}^{m-1} \left[\frac{\gamma_n}{\beta_n}\right] + \prod_{n}^{N} \left[\frac{\gamma_n}{\beta_n}\right] c_{N+1} \qquad \langle 7.5 \rangle$$

where W is a composite stream variable

The first term shows the dependence on local effluents and stream factors; the second term gives the impacts of (in particular) effluents in upriver zones, each multiplied by an individual factor of decay that decreases (approaching zero at some point) with distance from the source, and finally the third term shows the ultimate dependence of c_n on c_{N+1}, the concentration of contaminant flowing into the study area, which is an exogenous variable. When the use zone under consideration is at the bottom of the river (n=1), the expression may be long, incorporating all the effluents along the way. Effluents may affect the c_n for every zone downriver.

The implication for permit trading is that trades between firms at different points along the river will involve "rates of exchange", such that the amounts bought and sold will not be the same kilo for kilo. The simplest way to do this might be to fix the number of permits according to maximum impact on the target zones (equivalent to discharging in that zone); each permit would then confer the right to discharge a multiple of the nominal quantity, for example 1.6 in the case where $^5/_8$ of the effluents reach the target zone from the point of discharge.

The use of such a system is described more fully in the next section.

An Application to the St-Maurice River

There have been a number of efforts to apply economic models of efficiency in real world situations by way of simulation, and in fact there is one case where it has been put in force in North America. This program initially involved only the Fox River in Wisconsin, but it has now been extended to two other Wisconsin rivers (O'Neil, David, Moore and Joeres, 1983; O'Neil, 1983). Simulations have been run by Hass (1970), who studied the Miami River in Ohio, and Johnson (1967), who worked on the Delaware estuary.

As mentioned, a simulation has also been run in connection with this study on the St-Maurice River, for which original data was used. The economic model and data were developed in Mallory (1986). The Qual2 river model was calibrated by Boudreault and Villeneuve (1988). The objective was to test the compatibility of the economic and river models in a real world setting. The St-Maurice is characterized by a small number of large industrial plants, as well as a few towns discharging into the river. It also has little agriculture, which has the inconvenience (in terms of modelling) of diffuse contamination of the water. The river is controlled for the purposes of power generation, which means that basic hydrographical information already exists. These factors made the river suitable as a test subject.

A number of problems arise in applying a model to a practical case. Before we can proceed with the model, we need the following:

parametric values:
- the pollutants must be specified;
- zone lines and target uses must be defined;
- target concentrations set for each zone;

data collection:
- concentrations in ambient water;
- absolute quantities of the pollutant in effluents.

PARAMETRIC VALUES

Toxic metals were chosen for the purposes of this study. Toxic materials are usually best handled at source, which is appropriate for the decentralized neoclassical model of the economy used here.

The target values that are set should be such that given uses might be restored or maintained in any specified zone. For expository purposes, I adopted the most stringent requirements relative to the presence of metals, based on Gouin and Sinotte (n.d.). They consider metal contamination an issue only for drinking water and aquatic life. Table I shows recognized safe values for the six metals.

It is quite noticeable that standards are often near or even below the lowest level detectable with current sampling

Table I

Recognized Safe Levels for Six Toxic Metals (μg/L)

	Drinking Water	Aquatic Life	Minimum Detectable Level
Cd Cadmium	1	0.2	0.5 - 2
Cr Chromium	0.2 - 2	40	1 - 3
Cu Copper	1,000	2	2 - 5
Hg Mercury	0.2	-	0.05
Ni Nickel	10	25	10
Pb Lead	1	5	0,2 - 20

Higher values indicate higher acceptable concentrations of the pollutant.
Sources: Gouin and Sinotte (n.d.) pp. 8 and 13, and Ministère de l'Environnement du Québec, Direction des études du milieu aquatique.

technology. Thus most of the data sets are "censored", that is, they contain unobserved values. In some cases the standards for human consumption are more stringent (e.g. mercury and lead) while in others the aquatic environment shows less tolerance (e.g. cadmium and copper). Standards for the aquatic environment are generally based on the tolerance of certain fish species which have been widely studied.

The target zones must be defined, as well as the standards that should apply to each. The zone lines are mostly fixed by technical criteria, such as points where temperature, hydrological changes or pollution loads undergo discrete changes. Each zone is subdivided into a number of sections: for computational purposes each section is of the same length for the whole of the study area. In the case of drinking water, only Trois-Rivières takes its water from the river. Apparently, the river has been polluted for some time, encouraging other towns to take their water from nearby lakes. Removing metals in low concentration is extremely difficult and the problem is generally ignored at drinking water treatment plants. Again, the model based on at-source-reduction seems entirely appropriate. Fortunately (though not by coincidence), there is a sampling station at the intake of the Trois-Rivières filtration plant. In this case the data match the needs.

Aquatic life obviously exists throughout the river, and much of it is mobile. The only way to ensure a satisfactory environment would be to set the target for the entire river. Although there are only a few sampling sites along the river, it is possible to infer values that prevail at other sites using the Qual2 model.

DATA

Over the last ten years water quality has been sampled on a regular basis at four points along the river: La Tuque, Shawinigan Falls, Rivière Shawinigan and Trois-Rivières. Samples have been taken periodically for all six metals. The first issue that arises is whether the metal is detectable: for most contaminants some sample values will be below the limit of detectability. However, statistical techniques exist for imputing the full distribution (which is hypothesized to be log-normal) based on the "censored" data (Gilliam and Helsel, 1986). Using the Hald method (as they do) still requires that 20% of the data be observed to make a reasonably accurate estimate of the distribution.

Mercury and nickel did not qualify on this basis and were dropped from further consideration. This does not mean that these metals are not present in dangerous quantities. As shown in Table I, mercury and lead are known to be toxic at levels below those which can be observed. This is because they can accumulate in the tissues of various species to millions of times the concentration in the water. Indeed tests of fish from the St-Maurice River have shown mercury is present at unacceptably high levels.

As for source data, six large firms have been sampled periodically, though in no case more than three times. Three of the firms are engaged in pulp and paper making, one in vinyl chloride, one in aluminium and one in textiles. Most produce significant amounts of the metals in question. They employ totally different technologies and probably face very different control costs. This suggests using an economic model to allocate effluents is likely to be fruitful in finding a lower cost solution. With one exception, industrial effluents have not been monitored with respect to cadmium. Although it is present in harmful amounts (as shown by the ambient water quality data), it too will be dropped from further consideration, leaving just three metals: chromium, copper and lead. Table II presents a resumé of effluents for these three metals.

Table II

Effluents from Six Major Industries (in kg/day)

Industry	Cr	Cu	Pb
A	40.0	6.1	n/a
B	0.03	0.2	0.03
C	1.59	6.1	4.88
D	0.2	0.4	0.00
E	0.02	1.49	0.00
F	0.12	5.20	0.09
Total	43.04	12.95	4.07

Source: Mallory (1986)

Environnement Québec has not monitored toxic loadings from municipal sewers. This is unfortunate, if not really surprising, as there were dozens of small sewer outlets which are now in the process of being centralized (to facilitate treatment). As a result, the ministry normally uses average values on a percapita basis. While these are not really satisfactory, they can be used to advance the exposition. Table III uses 1981 census data to calculate estimated loads coming from town sewers.

Deserving of mention is another source, leachate from old toxic dump sites. GERLED, the ministry agency responsible for finding, evaluating and securing such sites has discovered several of importance in the St-Maurice

Table III

Estimated Population Contribution to Toxic Metal Loads

	Cr	Cu	Pb
La Tuque	0.68	0.95	1.36
Grand-Mère	0.99	1.39	1.98
Shawinigan	1.15	1.72	2.30
Shawinigan-Sud	0.57	0.79	1.13
Total	3.39	4.85	6.77

Source: Mallory (1986)

valley. Characterization of the toxic load coming from these sites is underway. Data for one site near Shawinigan could be used.

THE SIMULATION

The Qual2 model was calibrated by Boudreault and Villeneuve (1988) using the above data for ambient water quality and effluents, and other data (mainly from Hydro-Québec) for the hydrodynamic parameters necessary to simulate the action of the St-Maurice River. This was possible to do despite weaknesses in the data.

For simulation purposes, the best data would consist of sets of samples taken in a single pass down the river, proceeding at the estimated rate of stream flow. This would show the impact of each effluent source against a known background level. As this has never been done, it was necessary to rely on averages. Given this and other weaknesses, such as the small number of samples from sources and limited hydrographic data, the results are surprisingly good.

In their simulations, Hass and others go on to calculate savings in costs associated with the use of the economic model. This presupposes centralized planning (at least as regards the choice of pollution control measures). This is not the case here. It is not intended that firms be required to reveal cost functions, which would probably produce unreliable data in any event.

One step further would have been possible, the initial auction of permits as a simulation, demonstrating the calculation of "rates of exchange" between firms located at different points along the river. An attempt was made to do this, involving representatives of the ministère de l'Environnement and the several companies, but this did not work out.

Application of the full (general equilibrium) model over the long term, while it would allow other prices and quantities to adjust to the changes induced by the pollution control scheme, would also require knowledge of other changes likely to occur in the economy, which would be a difficult exercise. Thus it is not possible to go much beyond the partial equilibrium exercise just described.

The results suggest that economic and environmental modelling can be integrated fairly easily. The types of environmental data necessary for the economic model are already being collected. More care could be taken however to ensure that the data collected is more useful for modelling purposes. A survey to determine the watercourses for which the best environmental data exist (including effluent sources, ambient water quality and hydrodynamic data) might also be useful.

8

CONCLUSION

Economists have developed a clear and persuasive analysis showing that a market economy left to itself will produce sub-optimal environmental quality. By the time the consequences of this situation became apparent, significant deterioration of the environment had occurred worldwide. The substantial capital outlays that correcting this situation requires may induce major shifts in the pattern of goods produced in our society: however, there do not exist economic models that would be helpful in exploring the impact of these changes in the dynamic context. This thesis represents a first attempt to fill that gap.

Several points stand out, some of a theoretical aspect, others of more practical import. On the theoretical side, a dynamic model of price adjustment has been developed for an economy involving a public good in the context of what is otherwise a private goods economy; an original model of the dynamics of a decentralized economy has been developed, with explicit recognition of the problem of non-convexity, a characteristic of models including environmental quality. On the practical side, a preliminary evaluation of tax and permit approaches to pollution control has been made in the dynamic context, and the model developed has been shown to be suitable for application using real environmental data. These points will be summarized in a little more detail.

It was argued by Pigou and shown by Samuelson that, where externalities and public goods are concerned, we cannot rely upon the market mechanism alone to move the economy to a Pareto optimal allocation. It has been shown by Roberts that there exists no mechanism to reach a Pareto optimal allocation of public goods whereby truth-telling is a dominant strategy for agents during the process, therefore the optimum appears to be inaccessible. Despite this, it is much more common to see analyses in the economic literature that take Pareto optimal allocations as given than to see studies analyzing the dynamic properties of mechanisms to achieve such outcomes. This paper represents a departure in focusing on likely outcomes, such that we might identify those mechanisms likely to produce the best results.

Before the properties of different mechanisms can be evaluated in detail, a basic model structure must be put in place. There is, however, a lack of models representing mixed economies, and the particular problem that non-convexity poses for dynamic models has not been dealt with. Both of these problems are resolved here. I have used as a foundation one of the most interesting representations of a decentralized economy, specifically a non-tâtonnement variant of the MDP process developed for an exchange economy by Tulkens and Zamir. In their model, the spontaneous incentive to transact shows clearly. It is not obvious, however, how individual agents acquire the necessary information about prices in order to make the desired transactions.

This critical link has been added, fully decentralizing the process. The model is shown to be both physically and financially feasible, individually rational and convergent. Moreover, it is extended to an economy involving production, and finally to the introduction of a public good (water quality). An interesting result of the model is the attention it focusses on what might be described as the "transaction technology" which permits the realization of stable and feasible outcomes. Further work might contribute to a greater understanding of the general or particular conditions which must prevail in economies without central agents, whether they be planners or "auctioneers".

A major focus of this thesis is the examination of structures that would permit the realization of Pareto efficient outcomes in the presence of externalities such as water pollution, even while making allowance for the problems of non-convexity that they imply. Since markets, left to themselves, will not achieve a Pareto-optimal outcome, some form of intervention is required. The method that is elaborated here uses artificial markets, supervised by the government. Since this involves fixing the level of water quality in advance, it is unlikely to yield a Pareto optimal outcome in the first instance and there must be some method to adjust this level. An adjustment rule is provided that permits the realization of a Pareto optimal outcome.

- It has been shown that the transferable discharge **permit** market overcomes the potential instability posed by the non-convexity problem. There is always one and only one equilibrium; the potential instability is made to coincide with those points in time when the target level of water quality is adjusted, at which point the equilibrium point of the system moves discretely and unambiguously. Tax or subsidy approaches do not appear to resolve these problems as easily. As well, it was seen that permits would likely perform better than taxes with regard to a number of practical issues such as uncertainty over the true form of the cost of control function, a stable learning environment concerning ecological impacts and the accommodation of growth.

Finally, it was shown that the model developed here is compatible with ecological models of river dynamics. Thus the use of a dynamic economic model of this sort could be usefully applied to real world pollution problems.

REFERENCES

Aoki, M., (1971), "Two Planning Processes for an Economy with Production Externalities", International Economic Review, 12 (October), pp.403-414.

Arrow, K.J., (1969), "The Organization of Economic Activity: Issues Pertinent to the Choice of Market versus Nonmarket Allocation", in The Analysis and Evaluation of Public Expenditures: the PPB System, U.S. Congress, Joint Economic Committee, U.S. Government Printing Office, Washington, Vol.I, pp.47-64.

Arrow, K.J., and L. Hurwicz (1958), "On the Stability of the Competitive Equilibrium, I" Econometrica, 26, pp.522-552.

Baumol, W.J., (1964), "External Economies and Second-order Optimality Conditions", American Economic Review, 54 (June), pp.358-372.

Baumol, W.J., and D.F. Bradford (1972), "Detrimental Externalities and Non-convexity of the Production Set", Economica, 39 (May), pp.160-176.

Baumol, W.J., and W.E. Oates (1975), The Theory of Environmental Policy, Prentice-Hall, Englewood Cliffs, N.J., 272 pp.

Baumol, W.J., and W.E. Oates (1971), "The Use of Standards and Prices for the Protection of the Environment", Swedish Journal of Economics, 73 (March), pp.42-54.

Beavis, B., and M. Walker (1983), "Random Wastes, Imperfect Monitoring and Environmental Quality Standards", Journal of Public Economics, 21 (August), pp.377-387

Beavis, B., and M. Walker (1979), "Interactive Pollutants and Joint Abatement Costs: Achieving Water Quality Standards with Effluent Charges", Journal of Environmental Economics and Management, 6 (December), pp.275-286

Bergstrom, T.C., and R.C. Cornes (1983), "Independence of Allocative Efficiency from Distribution in the Theory of Public Goods", Econometrica, 51 (November), pp.1753-1765.

Bird, P.J.W.N., (1987), "The Transferability and Depletability of Externalities", Journal of Environmental Economics and Management, 14 (March), pp.54-57.

Blümel, W., R. Pethig and O. von dem Hagen (1986), "The Theory of Public Goods: A Survey of Recent Issues", Journal of Institutional and Theoretical Economics, 142 (June), pp.241-309.

Boiteux, M., (1956), "Sur la gestion des monopoles publics astreints à l'éuilibre budgétaire", Econometrica, 24 (January), pp.22-40.

Boudreault, P., and J.-P. Villeneuve (1988), "Simulation de l'évolution longitudinale de métaux lourds dans la rivière St-Maurice: Application du modèle Qual2 au tronçon Grand-Mère/Trois-Rivières", mimeo, 38 pp.

Bronsard, C., and D. Leblanc, (1980), "Theorie générale du consommateur et applications", Cahiers du Groupe de mathématiques économiques, N° 3 (juin), Université de Paris, 36 pp.

Brown, L.C., and T.O. Barnwell (1987), The Enhanced Stream Water Quality Models Qual2e and Qual2E-Uncas: Documaentation and User Manual, Environmental Research Laboratory, Office of Research and Development, U.S. Environmental Protection Agency, Athens, Georgia, 189 pp.

Buchanan, J.M., (1968), The Demand and Supply of Public Goods, Rand McNally, Chicago, 214 pp.

Buchanan, J.M., and G. Tullock (1975), "Polluters' Profits and Political Response: Direct Controls versus Taxes", American Economic Review, 65 (March), pp.139-147.

Burrows, P., (1986), "Nonconvexity Induced by External Costs on Production: Theoretical Curio or Policy Dilemma?", Journal of Environmental Economics and Management, 13 (June), pp.101-128.

Burstein, N., and J.M. Quigley (1976), "The Danger of 'Derived Decision Rules'", Journal of Environmental Economics and Management, 3 (June), pp.80-82.

Champsaur, P., (1976), "Neutrality of Planning Procedures in an Economy with Public Goods", Review of Economic Studies, 43 (June), pp.293-299.

Champsaur, P., J. Drèze and C. Henry (1977), "Stability Theorems with Economic Applications", Econometrica, 45 (March), pp.273-94.

Coase, R.H., (1960), "The Problem of Social Costs", Journal of Law and Economics, 3 (October), pp.1-44.

Collinge, R.A., and M.J. Bailey (1983), "Optimal Quasi-Market Choice in the Presence of Pollution Externalities", Journal of Environmental Economics and Management, 10 (September), pp.221-232.

Conn, D., (1983), "The Scope of Satisfactory Mechanisms for the Provision of Public Goods", *Journal of Public Economics*, 20 (March), pp.249-263.

Cooter, R.D., (1980), "How the Law Circumvents Starrett's Nonconvexity", *Journal of Economic Theory*, 22 (June), pp.499-504.

Dales, J.H., (1968), *Pollution, Property and Prices*, University of Toronto Press, Toronto, 111 pp.

Dasgupta, P., P. Hammond and E. Maskin (1980), "On Imperfect Information and Optimal Pollution Control", *Review of Economic Studies*, 47 (October), pp.857-860.

Dorfman, R., (1984), "On Optimal Congestion", *Journal of Environmental Economics and Management*, 11 (June), pp.91-106.

Drèze, J., and D. de la Vallée Poussin (1971), "A Tâtonnement Process for Public Goods", *Review of Economic Studies*, 38 (April), pp.133-150.

Falcke, C.O., (1983), "Water Quality and Property Prices: an Econometric Analysis of Environmental Benefits", *Kansantaloudellinen aikakauskirja*, 79 (1), pp.94-102.

Ferrar, T.A., and A. Whinston (1972), "Taxation and Water Pollution Control", *Natural Resources Journal*, 12 (July), pp.307-317.

Fisher, F.M., (1983), *Disequilibrium Foundations of Equilibrium Economics*, Cambridge University Press, Cambridge, 236 pp.

Foley, D.K., (1967), "Resource Allocation and the Public Sector", (Spring), *Yale Economic Essays*, pp.45-98.

Freeman, A.M., III, (1984), "Depletable Externalities and Pigouvian Taxation", *Journal of Environmental Economics and Management*, 11 (June), pp.173-179.

Freeman, A.M., III, (1980), "Technology-Based Effluent Standards: the U.S. Case", *Water Resources Research*, 16 (February), pp.21-27.

Gilliom, R.J., and D.R. Helsel (1986), "Estimation of Distributional Parameters for Censored Trace Level Water Quality Data: (1) Estimation Techniques", *Water Resources Research*, 22 (February), pp.135-146.

Gouin, D., and M. Sinotte (n.d.), "La rationalisation des objectifs de traitement : le cas des toxiques", Ministère de l'Environnement du Québec, Direction générale de l'Assainissement des eaux, Direction des Etudes du milieu aquatique, mimeo, 34 pp.

Green, J., and J.-J. Laffont (1977), "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods", Econometrica, 45 (March), pp.427-438.

Groves, T., and J.O. Ledyard (1977), "Optimal Allocation of Public Goods: a Solution to the 'Free Rider' Problem", Econometrica, 45 (May), pp.783-809.

Groves, T., and M. Loeb (1975), "Incentives and Public Inputs", Journal of Public Economics, 4 (August), pp.211-226.

Hahn, F.H., (1982), "Stability", in Handbook of Mathematical Economics, K.J. Arrow and M.D. Intriligator (eds.), North Holland, New York, Vol.2, pp.745-793.

Hahn, R.W., (1986), "Trade-offs in Designing Markets with Multiple Objectives", Journal of Environmental Economics and Management, 13 (March), pp.1-12.

Hamlen W.A., (1977), "The Quasi-Optimal Price of Undepletable Externalities", Bell Journal of Economics, 8 (Spring), pp.324-334.

Hass, J.E., (1970), "Optimal Taxing for the Abatement of Water Pollution", Water Resources Research, 6 (April), pp.353-365.

Head, J.G., (1977), "Public Goods: The Polar Case Reconsidered", Economic Record, 53 (June-September), pp.227-238.

Heal, G.M., (1971), "Planning, Prices and Increasing Returns", Review of Economic Studies, 38 (July), pp.281-294.

Hurwicz, L., (1973), "The Design of Mechanisms for Resource Allocation", American Economic Review, 63 (May), pp.1-30.

Johnson, E.L., (1967), "A Study in the Economics of Water Quality Management", Water Resources Research, 3 (2), pp.291-305.

Kahn, J., and W.M. Kemp (1985), "Economic Losses Associated with the Degradation of an Ecosystem: the Case of Submerged Aquatic Vegetation in Cheapeake Bay", Journal of Environmental Economics and Management, 12 (September), pp.246-263.

King, M.A., (1986), "A Pigovian Rule for the Optimum Provision of Public Goods", Journal of Public Economics, 30 (August), pp.273-291

Kneese, A.V., (1972), "Discharge Capacity of Waterways and Effluent Charges", Public Prices for Public Products, in S.T Mushkin (ed.), Urban Institute, Washington, pp.133-151.

Kneese, A.V., R.U. Ayers and R.C. d'Arge (1970), Economics and the Environment: A Materials Balance Approach, Resources for the Future, Washington, D.C., 120 pp.

Kolm, S.-C., (1971), "Les pollueurs doivent-ils être les payeurs?" Revue canadienne d'Economique, 4 (novembre), pp.506-523.

Kwerel, E., (1977), "To Tell the Truth: Imperfect Information and Optimal Pollution Control", Review of Economic Studies, 44 (October), pp.595-601.

Laffont, J.-J., (1982), Cours de théorie microéconomique, Vol.1, "Fondements de l'économie publique", Economica, Paris, 199 pp.

Laffont, J.-J., (1977), Effets externes et théorie économique, Editions du Centre national de la recherche scientifique, Paris, 200 pp.

Livingston, M.L., (1987), "Evaluating the Performance of Environmental Policy: Contributions of Neoclassical, Public Choice, and Institutional Models", Journal of Economic Issues, 21 (March), pp.281-294.

Malinvaud, E., (1982), Leçons de théorie microéconomique, 4e édition, Dunod, Paris, 385 pp.

Malinvaud, E., (1972), "Prices for Individual Consumption, Quantity Indicators for Collective Consumption", Review of Economic Studies, 39 (4), pp.385-405.

Malinvaud, E., (1970-71), "Procedures for the Determination of a Program of Collective Consumption", European Economic Review, 2 (Winter), pp.187-217.

Malinvaud, E., (1967), "Decentralized Procedures for Planning", in Activity Analysis in the Theory of Growth and Planning, edited by E. Malinvaud and M.D.L. Bacharach, Macmillan, London, pp.170-208.

Mallory, C.D., (1988), "La pollution est non-convexe", Actualite Economique, 64 (juin), pp.251-261.

Mallory, C.D., (1986), "A Regional Model of Water Quality", mimeo, 40 pp.

Matthews, S., (1987), "Comparing Auctions for Risk Averse Buyers: A Buyer's Point of View", Econometrica, 55 (May), pp.633-646.

McAfee, R.P., and J. McMillan (1987), "Auctions and Bidding", Journal of Economic Literature, 25 (June), pp.699-738.

Mestelman, S., (1982), "Production Externalities and Corrective Subsidies: A General Equilibrium Analysis", Journal of Environmental Economics and Management, 9 (June), pp.186-193.

Mishan, E.J., (1971), "The Postwar Literature on Externalities: An Interpretative Essay", Journal of Economic Literature, 9 (March), pp.1-28.

Ministère de l'Environnement (n.d.), "Synthèse des données sur la qualité de la rivière St-Maurice," Direction générale de l'assainissement des eaux, Direction des études du milieu aquatique, mimeo.

Montgomery, W.D., (1972), "Markets in Licenses and Efficient Pollution Control Programs", Journal of Economic Theory, 5 (December), pp.395-418.

Muench, T., and M. Walker (1983), "Are Groves-Ledyard Equilibria Attainable?" Review of Economic Studies, 50 (April), pp.393-396.

Negishi, T., (1962), "The Stability of a Competitive Economy: A Survey Article", Econometrica 30 (October), pp.635-669.

Nijkamp, P., (1977), Theory and Application of Environmental Economics, North Holland, Amsterdam, 332 pp.

O'Neil, W.B., (1983), "Transferable Discharge Permit Trading Under Varying Stream Conditions: A Simulation of Multiperiod Permit Market Performance on the Fox River, Wisconsin", Water Resources Research, 19 (June), pp.608-612.

O'Neil, W., M. David, C. Moore, and E. Joeres (1983), "Transferable Discharge Permits and Economic Efficiency: The Fox River", Journal of Environmental Economics and Management, 10 (December), pp.346-355.

Page, T., (1973), "Failure of Bribes and Standards for Pollution Abatement", Natural Resources Journal, 13 (October), pp.678-704.

Pashigian, B.P., (1984), "The Effect of Environmental Regulation on Optimal Plant Size and Factor Shares", Journal of Law and Economics, 27 (April), pp.1-28.

Pigou, A.C., (1932), The Economics of Welfare, MacMillan, London, 4th ed., 837 pp.

Portes, R., (1970), "The Search for Efficiency in the Presence of Externalities", in Unfashionable Economics: Essays in Honour of Lord Balogh, P. Streeten (ed.), Weidenfeld and Nicholson, London, pp.348-358.

Quebec (1982), "Règlement sur la protection des eaux souterraines dans la région de ville de Mercier" (Décret 1525-82), Gazette Officiel, Partie 2, pp.2491-2492.

Randall, A., J.P. Hoehn and D.S. Brookshire (1983), "Contingent Valuation Surveys for Evaluating Environmental Assets", Natural Resources Journal, 23 (July), pp.635-648.

Repetto, R., (1987), "The Policy Implications of Non-convex Environmental Damages: A Smog Control Case Study", Journal of Environmental Economics and Management, 14 (March), pp.13-29.

Roberts, J., (1979), "Incentives in Planning Procedures for the Provision of Public Goods", Review of Economic Studies, 46 (April), pp.283-292.

Roberts, M.J., and M. Spence (1976), "Effluent Charges and Licenses Under Uncertainty", Journal of Public Economics, 5 (April-May), pp.193-208.

Rose-Ackerman, S., (1977), "Market Models for Water Pollution Control: Their Strengths and Weaknesses", Public Policy, 25 (Summer), pp.383-406.

Samuelson, P.A., (1969), "Pure Theory of Public Expenditure and Taxation", in Public Economics, J.Margolis and H. Guitton (eds.), London, pp. 98-123.

Samuelson, P.A., (1954), "The Pure Theory of Public Expenditure", Review of Economics and Statistics, 36 (November), pp.387-389.

Schoumaker, F., (1979), "Incentives in Planning with Private Goods", Review of Economic Studies, 46 (April), pp.315-318.

Shoup, C.S., (1969), Public Finance, Aldine, Chicago, 660 pp.

Sims, W.A., (1981), "The Short-Run Asymmetry of Pollution Subsidies and Charges", Journal of Environmental Economics and Management, 8 (December), pp.395-399.

Smith, V.K., W.H. Desvouges and M.P. McGivney (1983), "Estimating Water Quality Benefits: an Econometric Analysis", Southern Economic Journal, 50 (October), pp.422-437.

Starrett, D.A., (1972), "Fundamental Nonconvexities in the Theory of Externalities", Journal of Economic Theory, 4 (April), pp.180-199.

Starrett, D.A., and R. Zeckhauser (1974), "Treating External Diseconomies—Markets or Taxes?", in Statistical and Mathematical Aspects of Pollution Problems, J.W. Pratt, ed., Dekker, New York, pp.65-84.

Statistics Canada (1982), 1981 Census of Canada, "Population, Geographic Distributions, Québec", Vol. 93-905.

Suchanek, G.L., (1979), "Information, Optimality and Pollution Control", Journal of Public Economics, 12 (August), pp.99-114.

Suchanek, G.L., (1977), "A Mechanism for Comparing an Efficient System of Waste Emission Quotas", Journal of Public Economics, 7 (April), pp.261-269.

Terkla, D., (1984), "The Eficiency Value of Effluent Tax Revenues", Journal of Environmental Economics and Management, 11 (June), pp.107-123.

Teitenberg, T.H., (1974), "Derived Decision Rules for Pollution Control in a General Equilibrium Space Economy", Journal of Environmental Economics and Management, 1 (May), pp.3-16.

Tietenberg, T.H., (1973a), "Controlling Pollution by Price and Standard Systems: A General Equilibrium Analysis", Swedish Journal of Economics, 75 (June), pp.193-203.

Tietenberg, T.H., (1973b), "Specific Taxes and the Control of Pollution: A General Equilibrium Analysis", Quarterly Journal of Economics,87 (November), pp.503-522.

Tulkens, H., (1979), "An Economic Model of International Negotiation Relating to Transfrontier Pollution", in Communication and Control in Society, K. Kripendorff (ed.), Gordon and Breach Science Publishers, pp. 199-212.

Tulkens, H., and F. Schoumaker, (1975), "Stability Analysis of an Effluent charge and the 'Polluters Pay' Principle", Journal of Public Economics, 4, pp. 245-269.

Tulkens, H., and S. Zamir (1979), Surplus-sharing Local Games in Dynamic Exchange Processes", Review of Economic Studies, 46 (April), pp.305-313.

Turvey, R., (1963), "On Divergences between Social Cost and Private Cost", Economica, 30 (August), pp.309-313.

Uzawa, H., (1962), "On the Stability of Edgeworth's Barter Process", International Economic Review, 3 (May), pp.218-232.

Weddelpohl, C., (1983), "Developments in the Theory of General Equilibrium", De Economist, 131 (3), pp.373-399.

Whitcomb, D.K., Externalities and Welfare, Columbia, New York, 1972, 158 pp.