

On the Individual Optimality of Economic Integration^{*}

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Abstract

Which countries find it optimal to form an economic union? We emphasize the risk-sharing benefits of economic integration. Consider an endowment world economy model, where international financial markets are incomplete and contracts not enforceable. A union solves both frictions among member countries. We uncover conditions on initial incomes and net foreign assets of potential union members such that forming a union is welfare-improving over standing alone in the world economy. Consistently with evidence on economic integration, unions in our model occur (i) relatively infrequently, and (ii) emerge more likely among homogeneous countries, and (iii) rich countries.

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1. Introduction

Which countries find it optimal to form an economic union? Our aim is to try to understand the patterns of economic integration that are observed in the real world. This paper emphasizes a particular motivation for economic integration: improving risk sharing. Economic unions are viewed as small-scale arrangements, comprised of a small number of countries, where partners are better able to cope with the frictions that limit risk-sharing in the world economy. We ask which countries would rather be part of this type of economic union than stand alone in the world economy, and compare the configuration of successful unions predicted by our theory with those seen in the data.

Consider an initial situation in which countries are sitting in the world economy with very limited possibilities to sharing idiosyncratic endowment risk. Risk sharing is limited by two frictions. First, markets are incomplete since countries may only trade a non-contingent bond. Second, international lending contracts are not legally enforceable. At any time, a country may choose to repudiate its foreign debt. The sanction for doing so is the permanent exclusion from future trade in world markets. Our world economy model is a variant of ? and ?, featuring self-enforcing borrowing limits along the lines of ?, ?, and ?. Versions of this setup have been studied previously in different contexts by ? and ?.¹

Consider then the possibility that a pair of countries selected at random from the world economy is suddenly offered the possibility of forming an economic union. A union, by assumption, is an arrangement which solves both the market incompleteness and the lack of enforcement problems among member countries. The union as a whole, however, still faces these frictions when trading in world markets. Since the endowment risk facing union members cannot be fully diversified away, they still have an interest in trading with the rest of the world.

Economic integration in our model generates benefits as well as costs for potential union members. One clear benefit, shared by both relatively rich and relatively poor partners, is that forming a union improves risk-sharing. One cost, also shared by both types of partner,

¹See ? and ? for variants with capital accumulation. See also ? for a variant with capital accumulation and endogenous but ad-hoc borrowing constraints.

is that borrowing limits for the union as a whole become tighter. This happens because defaulting on international debt becomes less costly inside the union, since union partners may still share risk upon default. In addition, there are also benefits and costs which are specific to rich and poor union partners. In order to provide better risk-sharing, unions effectively allow poor partners to borrow more when inside the union, at the expense of rich partners. This works like a positive externality for poor partners, in the sense that their borrowing limits become less tight in a union compared to standing alone in the world economy. At the same time this also works like a negative externality for rich partners. These partner-specific benefits and costs underline the key trade-off, and create the main source of disagreement, about union formation in our model.

Not only there is disagreement about union formation in our model, the disagreement is also greater the more heterogeneous the partners are. This provides a potential explanation for three seemingly puzzling empirical observations on economic integration: (i) deep economic integration is relatively rare, and when it does take place it tends to feature (ii) relatively homogeneous partners, and (iii) relatively rich partners. In other words, we do not tend to see many North-South arrangements; they are mostly North-North, and to a lesser extent South-South. Our paper provides empirical evidence documenting these regularities.

These observations are puzzling because, under a very broad set of circumstances, economic theory would imply that economic integration should happen often, particularly among heterogeneous partners. For example, this would be the case for capital market integration in the neoclassical growth model, or goods market integration in either the Heckscher-Ohlin or the Ricardian models of trade.²

²Union formation in intra-industry trade models, emphasizing scale economies and a taste for variety, have been analyzed in a static setting by ?, ?, ? and ?. This type of model emphasizes size as a determinant of union formation: the larger and the more similar the partners' market sizes, the larger the gains from goods market integration. Larger unions profit more from scale economies, and size homogeneity lowers the losses from trade diversion. While ? find empirical support for these implications, our data also suggests that, *beyond market size*, the level and the dispersion in partner wealth matters for economic integration. Differently from this literature, our paper focuses on heterogeneity in per capita incomes and net foreign assets over GDP.

Our framework provides a very parsimonious explanation for these puzzling observations. Economic unions may not be formed if either the costs of economic integration are too large, or much more importantly if there is disagreement among partners. Unions are unlikely to be formed among heterogeneous partners, since poor partners impose a cost on the rich. Finally, unions are also more likely to be formed among relatively rich partners because this lowers the likelihood of either country being borrowing-constrained in the future, and thus of the source of disagreement.

In addition to the pattern of union formation, our theory also delivers two main predictions for the outcomes of union-forming countries. First, risk sharing improves. Second, in asymmetric unions, relatively poor members increase their borrowing and consumption rates compared to rich members. When looking at the enlargement experience of the European Union, the empirical evidence is consistent with these predictions.

This paper is related to a vast literature that has attempted to estimate the welfare gains from full international risk-sharing. This literature includes papers such as ?, ?, ??, ??, ?, ?, and ?. The typical exercise computes the average gain across countries of going from financial market autarky to complete markets, and entirely eliminating idiosyncratic country risk. Although the range of estimated welfare gains is large, the gains are still positive in nearly all the papers. The sole exception is ?, who like this paper also model costs of sharing risk. In their case, sharing risk lowers precautionary saving, which lowers output growth and might lower welfare. We emphasize instead the tightening of credit constraints, and the costs generated by poor union partners.

Our paper differs from this literature in several dimensions. First, beyond the magnitude of the welfare gains, we are mostly interested in their distribution across countries. Even if the average gains might be high, they can be very oddly distributed. If some countries actually experience a loss, as it is often the case in our model, risk sharing arrangements may not take place at all. This may explain the observed lack of international risk diversification, even in the presence of possibly large average welfare gains. Moreover, the main prediction of our model can be tested against the evidence, namely that feasible risk-sharing arrangements should occur among homogeneous and rich countries.

Second, our paper considers financial market integration as it typically takes place in the

real world. That is, as voluntary arrangements among small sets of countries. Financially integrated countries are still unable to share risk with the rest of the world. Further, in our paper countries may save and self-insure in the absence of complete markets, whereas most of the literature abstracts from this feature. Our paper computes welfare gains from international risk-sharing that take these important features into account.

A recent paper that has also looked at potential risk sharing arrangements within small sets of countries is ?. Using actual data on the variance-covariance matrix of cross-country output growth, they uncover the number and configuration of countries that offer the best risk-sharing potential. Like in the rest of the international risk-sharing literature, their core analysis focuses on going from autarky to complete markets, and does not feature neither costs of economic integration, nor a role for disagreement among partners. Their main finding is that most diversification gains are achieved in arrangements featuring a small number of countries, and in arrangements between heterogeneous and/or highly volatile countries. As they recognize, a natural question is why we do not observe more arrangements of this type. They argue that this could be because unions might be particularly difficult to sustain among heterogeneous and/or volatile countries. They conjecture that contract enforcement might be particularly costly for such groupings. While our framework abstracts from cross-country differences in output volatility, it does provide an explicit, possibly complementary reason for why small-size arrangements may not be feasible among heterogeneous countries, even in face of large aggregate gains.

The paper is organized as follows. Section 2 presents some evidence about union formation. Section 3 presents the model of the world economy. Section 4 characterizes the union. Section 5 presents the calibration and Section 6 the results. Section 7 looks at the European Union experience as a testing ground for our model. Section 8 concludes. Appendix A provides details about the data. Appendix B and Appendix C describe the decentralization of the union's allocation and the numerical algorithm, respectively.

2. Empirical Evidence

This section provides some empirical evidence on the role of wealth levels and wealth inequality for union formation. By wealth we mean both income (y) and net foreign assets

(*b*), both variables being potentially relevant according to our formal model. Our main unit of observation is a country pair. The goal is to understand whether these countries' incomes and net foreign assets over income ratios may help them form an economic union.

The analysis can be motivated with two simple scatter plots. A detailed description of our data appears later in this section. The left panel of Figure 1 shows the income levels of several country pairs in our sample, together with the 45 degree line. The right panel is restricted to those country pairs in a custom union or deeper economic integration arrangement. The blue horizontal label refers to the income level of the country represented in the y -axis, whereas the red vertical label refers to the income level of the country represented in the x -axis (a country-pair observation is represented by the point where a blue horizontal and a red vertical labels meet).³ This figure suggests that income heterogeneity is indeed detrimental for union formation: country pairs engaged in unions are those closer to the 45 degree line, ranging from poor country pairs such as those in the Economic and Monetary Community of Central Africa and those in the West African Economic and Monetary Union, to middle-income country pairs such as those in Mercosur, to rich country pairs such as those in the European Union. This figure also shows clearly that higher income levels help union formation: there's a higher density of union-forming country pairs towards high income levels, mostly driven by the European Union countries.

[Figure 1 about here.]

The left panel of Figure 2 concentrates on net foreign assets over GDP.⁴ This figure also suggests that heterogeneity in net foreign assets over GDP is detrimental for union formation. However, unlike the case of GDP, union-forming country pairs do not tend to be those with higher levels of net foreign assets over GDP. There is instead a large concentration of unions towards the middle of this distribution.

³The country codes are from the Penn World Tables. The countries corresponding to them are in Appendix A, together with the full list of union arrangements. For any given pair, the specific country appearing on each axis was decided by the alphabetical order of the labels, hence independent from the ranking of incomes.

⁴To better visualize the data the left panel excludes Liberia, which has a level of net foreign assets over GDP of about -10.

[Figure 2 about here.]

The two figures suggest that wealth heterogeneity is detrimental, and that income levels help union formation. This motivates our formal empirical framework, which allows us to formally test the significance of these effects, and to control for other variables deemed important for union formation such as geography and country size.

Our approach is to run a probit-gravity regression to test whether wealth levels contribute positively, and wealth inequality negatively, for the probability of union formation. Our regression specification is a straightforward adaptation of those commonly used in the empirical trade literature to test predictions over bilateral trade flows (see ??), similar to ?:

$$\text{Prob}\{\text{Union}_{ij} = 1|X_{ij}\} = \Phi(X'_{ij}\boldsymbol{\beta}) \quad (1)$$

with

$$\begin{aligned} X'_{ij}\boldsymbol{\beta} &= \alpha_1 + \alpha_2 \ln(\text{dist})_{ij} + \alpha_3 \text{adj}_{ij} \\ &+ (\alpha_4 + \alpha_4^a \text{adj}_{ij}) \ln(\text{pop}_i \times \text{pop}_j) + (\alpha_5 + \alpha_5^a \text{adj}_{ij}) \left| \ln \frac{\text{pop}_i}{\text{pop}_j} \right| \\ &+ (\theta_1 + \theta_1^a \text{adj}_{ij}) \ln(y_i + y_j) + (\theta_2 + \theta_2^a \text{adj}_{ij}) \left| \ln \frac{y_i}{y_j} \right| \\ &+ (\gamma_1 + \gamma_1^a \text{adj}_{ij}) \left(\frac{b_i + b_j}{y_i + y_j} \right) + (\gamma_2 + \gamma_2^a \text{adj}_{ij}) \left| \frac{b_i}{y_i} - \frac{b_j}{y_j} \right|. \end{aligned}$$

The dependent variable is a dummy which gets the value of 1 if a union is formed between countries i and j , and 0 otherwise. The regressors in the first two lines of the regression equation concern factors deemed to be important for union formation but absent from our theoretical framework. The last two lines concern wealth levels and wealth heterogeneity, the key determinants in our theory.

Begin with the former set of regressors. We include two geographical factors commonly used in the gravity regression literature, the distance between the main economic centers of countries i and j (dist_{ij}), and a dummy variable capturing whether countries i and j share a common border (adj_{ij}). We also include overall size and a measure of heterogeneity in size, as potential determinants of union formation, where size is measured by population (pop_i). In particular, ? have found scale effects to be important for union formation, consistent with

the predictions of a class of intra-industry trade models. In the last two lines, we include the overall income level of the country pair (i, j) , a measure of the inequality in incomes between the two countries, and similarly for net foreign assets over income. We make the contribution of wealth levels and wealth inequality for union formation contingent upon whether countries share a border, and similarly for size. Our specification finds a parallel in ?.

To implement our regression analysis, we combine a variety of data sets. From version 7.1 of the Penn World Tables (?) we obtain our measure of income (real GDP per capita) and population. We obtain net foreign asset positions from ?. We consider real GDP and nominal net foreign assets over nominal GDP averaged over five years (2000-2004) as our regressors, to prevent high frequency variation in these variables from affecting our results.

Our geographical data comes from ?, and our union dummy is obtained from a comprehensive data set assembled by ?. Based primarily on information from the World Trade Organization, this data set tells us which countries are engaged in which type of regional trade arrangement in any given year. The regional trade arrangements range from Preferential Trade Arrangements, to Free Trade Areas like NAFTA, to Economic Unions like the European Union. For reasons that will become apparent when we model unions in Section 4, we restrict our empirical definition of unions only to those arrangements characterized by a sufficiently deep level of economic integration. In particular, we do not consider Free Trade Areas like NAFTA as a union. This is because members of Free Trade Areas may set independent tariff policies vis-a-vis non-members, making it in our view inappropriate to think about them as a block. Our most comprehensive empirical definition of unions includes Customs Unions (no trade barriers between members, common barriers vis-a-vis non-members), Common Markets (custom unions featuring free capital and labor mobility between members), and Economic Unions (common markets featuring harmonization of economic policy, namely fiscal and monetary). We also report regression results for stricter empirical definitions of an economic union, the results being generally robust across them. Appendix A lists existing unions, ordered by depth of integration.

Concentrate on a single cross-section of 136 countries in the year 2004. The year is the most recent one in the ? data set, and the number of countries is the maximum given the available data in 2004. We consider all possible country pairings from this set. A value of 1

is assigned to the union dummy if a particular country pair was part of a union in 2004, and 0 otherwise. Given the available geographical data, we end up with 6629 country pairings.

Table 1 reports our estimated average marginal effects, evaluated at either value for the common border dummy.

[Table 1 about here.]

As expected, our results support a negative effect of distance on the probability of union formation. Regarding scale, the results are not fully consistent with ?, in the sense that scale does help union formation, but only for sufficiently deep arrangements, and only conditional on countries not sharing a common border. Otherwise scale has no significant effect on union formation. Just like ?, however, we do find that inequality in scale is generally detrimental to union formation.

Now turn to the variables which are more relevant for us. The evidence supports the view that the larger the partners' combined incomes, the higher the probability of union formation among non-adjacent countries. Income inequality is always clearly detrimental to union formation, and similarly for inequality in net foreign assets over GDP, although with lower statistical significance. The combined level of net foreign assets over GDP tends instead to be detrimental for union formation, except for customs unions with shared borders.⁵ We checked whether our results are robust to the exclusion of the European countries, and found that they are.⁶

These results support the broad view that, even when controlling for geographical fac-

⁵The variables NFA/GDP and NFA/GDP Inequality capture the level and inequality effects, respectively, of net foreign assets on union formation which are not already captured by the variables Income and Income Inequality. Namely differences in net foreign assets which are proportional to output are captured by the latter.

⁶We ran our probit regression on customs unions excluding the EU12 countries, and then excluding the European Economic Area (EEA) countries. The sign and significance of the effects remains overwhelmingly the same - the single exception being that, when EEA countries are excluded, the effect of Income becomes marginally significant negative for non-adjacent countries. Figure 1 in particular illustrates why our results are not just driven by Europe: union formation among middle and low-income countries shares the same features as among rich countries.

tors and scale effects, wealth levels contribute positively, and wealth inequality contributes negatively to union formation.

There are two shortcomings of our empirical analysis which are worth pointing out. First, it treats newly-formed and continuing unions in 2004 both as instances of union formation, in line with ?. This is obviously a caveat since, in reality, there is a likely bias towards the status-quo. That is, everything else constant, existing unions are more likely to continue than new unions to form. Unfortunately, the extremely small number of newly-formed unions in any given year prevents us from concentrating only on new unions. Second, and in line with our theoretical model, our analysis presumes that union formation boils down to a bilateral decision. In reality, multi-country unions might not necessarily work in this fashion. When a multi-country union is being formed from scratch, countries presumably think about the average gain, and would not necessarily block union formation if they experience bilateral losses. However, in the case of accession into an existing union, it is conceivable that incumbent countries might be interested in vetoing the new member's entry if they experience a bilateral loss.

3. World economy

3.1. Model

Consider a world economy composed of a continuum of small open economies of measure one. Countries are identical ex-ante, and differ ex-post due to idiosyncratic endowment risk. Each period, a country receives an endowment of a non-storable consumption good. The endowment evolves over time according to a Markov chain with a finite number of states in the set Y . We denote by $y^t = \{y_s, y_{s+1}, \dots, y_t\} \in Y^{t-s+1}$ the sequence of events from the initial time period $s < 0$ up to and including period t , where Y^{t-s+1} is the cartesian power of Y , and by $\pi(y^t)$ the probability of such sequence. The initial event $y^s = y_s$ is given and $\pi(y^s) = 1$. We denote by $y^\tau|y^t$ the history y^τ conditional on y^t , $t \leq \tau$. We assume a law of large numbers holds in the cross-section of countries, which means there is no aggregate uncertainty.

Each country is populated by an infinitely-lived representative agent with preferences:

$$\sum_{t=s}^{\infty} \sum_{y^t} \beta^{t-s} \pi(y^t) u(c(y^t)), \quad (2)$$

where $\beta \in (0, 1)$ is the subjective discount factor. The instantaneous utility is of the CRRA class, $u(c) = c^{1-\sigma}/(1-\sigma)$, with $\sigma > 0$.

Countries cannot completely pool their income risk in world financial markets for two reasons. First, markets are incomplete: the menu of assets is exogenously restricted to a non-contingent one-period bond. A country's resource constraint is

$$c(y^t) + b(y^t) = y_t + (1+r)b(y^{t-1}), \quad (3)$$

where $b(y^t)$ is the demand for foreign bonds and r is the (time-invariant) world interest rate.

The second friction is that international lending contracts are imperfectly enforceable. At any time, a country is free to repudiate its foreign debt, the penalty being the permanent exclusion from any future trade. A country that contemplates debt repudiation faces a trade-off between current and future utility: defaulting implies higher current consumption, at a cost of lower future utility due to living in autarky. International lending contracts are self-enforcing, in the sense that borrowing countries always find the cost of repudiation larger than the benefit, and they always choose to repay. That is, allocations satisfy the following participation constraint:

$$\sum_{\tau=t}^{\infty} \sum_{y^\tau|y^t} \beta^{\tau-t} \pi(y^\tau) u(c(y^\tau)) \geq V_{aut}(y^t), \quad (4)$$

where $V_{aut}(y^t)$ is the value of entering financial autarky after the history y^t . It is the life-time utility derived from consuming one's endowment each period from the history node y^t onwards:

$$V_{aut}(y^t) = \sum_{\tau=t}^{\infty} \sum_{y^\tau|y^t} \beta^{\tau-t} \pi(y^\tau) u((1-\phi)y_\tau).$$

The parameter $\phi \in [0, 1]$ is a direct output cost associated with default. Such additional default penalty has been considered in the literature, and is typically motivated as a way to capture production disruptions that occur because of lack of access to international markets. As in ?, our motivation is mainly quantitative. Without such penalty, the extent of borrowing and lending in the quantitative model is much lower than in the data.

The representative agent chooses contingent plans for consumption and foreign assets to maximize lifetime utility (2) subject to the resource constraint (3), the enforcement constraint (4), a no-Ponzi game condition:

$$b(y^t) \geq -D, \quad (5)$$

where D is large enough that the constraint never binds in equilibrium,⁷ and initial conditions for net foreign assets and the endowment.

3.2. Recursive competitive equilibrium

We solve for the stationary recursive competitive equilibrium with solvency constraints. The state of the economy is characterized by net foreign bond holdings b and by the current endowment y . The problem of each country admits the following recursive formulation (see ? for a formal proof):

$$V(b, y) = \max_{c, b'} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) V(b', y') \right\} \quad (P0)$$

subject to:

$$\begin{aligned} c + b' &= y + (1 + r)b \\ b' &\geq b^W. \end{aligned}$$

The borrowing constraint b^W is the debt level such that for every possible state next period, the country is weakly better-off by repaying. Under the assumption that $\pi(y'|y) > 0$ for all y, y' , which will be consistent with our parameterization and is therefore maintained throughout the paper:

$$b^W = \max_{y'} \{ b_{y'} : V(b_{y'}, y') = V_{aut}(y') \}. \quad (6)$$

The autarky value V_{aut} is the solution to the following functional equation:

$$V_{aut}(y) = u((1 - \phi)y) + \beta \sum_{y'} \pi(y'|y) V_{aut}(y'). \quad (7)$$

⁷The enforcement constraint does not prevent countries from running Ponzi schemes: an agent running a Ponzi game would never default on its debt, since this would prevent him from continuing running the scheme.

Let B be the set of net foreign bond levels, $S = B \times Y$ the state-space, and \mathcal{A}_S the σ -Borel algebra of elements of S . We are now ready to define the stationary recursive competitive equilibrium of the world economy.

Definition. A *stationary recursive competitive equilibrium* is given by decision rules $c(b, y)$, $b'(b, y)$, a value function $V(b, y)$, a borrowing limit b^W , an interest rate r and a distribution Ψ of countries over individual states such that:

1. Given the world interest rate r and the borrowing limit b^W , the decision rules solve the recursive problem (P0) and V is the associated value function.
2. The borrowing limit b^W is not too tight, in the sense of satisfying equation (6) for all y .
3. The world credit market clears:

$$\int_S b'(b, y) d\Psi = 0.$$

4. The decision rules and the transition matrix of the endowment process induce a probability distribution P over the state space, $P : S \times \mathcal{A}_S \rightarrow [0, 1]$, where:

$$P((b, y); A) = \sum_{y': (b'(b, y), y') \in A} \pi(y'|y)$$

is the probability of transiting from state (b, y) to a state in the set A .

5. The distribution Ψ is stationary and consistent with P :

$$\Psi(A) = \int_S P((b, y); A) d\Psi, \text{ for all } A \in \mathcal{A}_S.$$

4. Economic union

This section describes the process of union formation in the model. Assume the world economy is in steady-state. At time $t = 0$, and without anticipating it, a pair of countries sitting in the world economy is offered the possibility of forming a union. We pick these two countries from the ergodic state-space of the world economy's stationary equilibrium. Each country is characterized by an initial state (b_{i0}, y_{i0}) , $i = 1, 2$. We also assume that union formation is a once-and-for-all event, i.e. once a union is formed it cannot be dissolved in the future.

Within the union, we assume full enforcement, and complete financial markets.⁸ Since a union is comprised of a finite number of countries (in this case two), there is still some endowment risk that the union would like to diversify away with the rest of the world. We assume union members still have access to world financial markets under the same conditions as before, i.e. by trading on non-contingent bonds subject to enforcement constraints. The union is like a small country in the world economy.

Assume the existence of a central authority in the union that coordinates the international trade and default decisions. With coordinated default decisions, there is a single union-wide enforcement constraint that applies to both countries at the same time. If the union defaults, all its members are permanently excluded from world markets, but they may still share endowment risk among them.

The union's endowment is determined by the realization of two independent and identically distributed endowment processes, one for each country. We denote it compactly by a two-dimensional vector $\bar{y}_t = (y_{1t}, y_{2t}) \in Y \times Y$, where the element $y_{it} = y_i(\bar{y}_t) \in Y$ is country i 's endowment realization, $i = 1, 2$. With a slight abuse of notation, we also denote by π the transition probabilities for \bar{y} :

$$\pi(\bar{y}'|\bar{y}) = \prod_{i=1}^2 \pi(y'_i|y_i).$$

4.1. Planner's problem

The allocation within the union is constrained-efficient, and can be obtained by solving a benevolent planner's problem. Although countries join the union with potentially different net foreign bond levels, only the aggregate net asset position matters for the planner's problem. Let $\bar{b}_0 = \sum_i b_{i0}$, and let λ_i be the weight the planner attaches to country i . The planner solves for $\{c_i(\bar{y}^t)\}_{i=1,2}$ and $\bar{b}(\bar{y}^t)$, for all \bar{y}^t , $t \geq 0$, which maximize the weighted sum of the union partners' lifetime expected utilities

⁸Completing markets may be achieved in a variety of ways, not just by increasing financial market sophistication. First, fiscal transfers in highly-integrated unions can achieve the same goal. Second, goods market liberalization may also complete markets - see e.g. ?.

$$\sum_{i=1}^2 \lambda_i \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c_i(\bar{y}^t))$$

subject to the union-wide resource constraint

$$\sum_i c_i(\bar{y}^t) + \bar{b}(\bar{y}^t) = \sum_i y_i(\bar{y}^t) + (1+r)\bar{b}(\bar{y}^{t-1}),$$

for all \bar{y}^t , $t \geq 0$, to the union-wide enforcement constraint

$$\sum_i \lambda_i \sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau | \bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^\tau) u(c_i(\bar{y}^\tau)) \geq W_{aut}^U(\bar{y}^t),$$

for all \bar{y}^t , $t \geq 0$, where

$$W_{aut}^U(\bar{y}^t) = \max_{\{c_i(\bar{y}^\tau)\}_i} \sum_i \lambda_i \sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau | \bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^\tau) u(c_i(\bar{y}^\tau))$$

subject to

$$\sum_i c_i(\bar{y}^\tau) = (1-\phi) \sum_i y_i(\bar{y}^\tau), \text{ for all } \bar{y}^\tau | \bar{y}^t, \tau \geq t,$$

for all \bar{y}^t , $t \geq 0$, to a no-Ponzi game condition

$$\bar{b}(\bar{y}^t) \geq -D, \tag{8}$$

for all \bar{y}^t , $t \geq 0$, and to the initial conditions \bar{b}_0 and \bar{y}_0 .⁹

Apart from distributional issues, the planner's problem is similar to the problem of a country standing alone in the world economy, the main difference being that, because the partners' endowments are uncorrelated, the union faces an endowment process which is less volatile in the aggregate. Since markets are complete and contracts enforceable among union members, the lower aggregate endowment volatility translates into lower individual consumption volatility.

⁹We assume that countries are prevented from defaulting at time 0 if they are given the chance to form a union. At the time of union formation, the debt levels that were self-enforcing in the world economy are not guaranteed to be so when the new outside option is superior, to be in autarky in a union.

4.1.1. Reformulating the planner's problem

Under CRRA preferences, the union planner's problem admits a simpler formulation which is very convenient. By Proposition 5 of ?, aggregate borrowing and lending is independent of distributional issues. It follows that the planner's problem may be decomposed into two steps. In the first step, the planner solves for the optimal borrowing and lending of the union assuming a single representative country facing the aggregate endowment. In the second step, the planner redistributes the optimal aggregate consumption plan obtained from the first step among the two union partners.

Formally, the step 1 problem for the planner is

$$\max_{c(\bar{y}^t), b(\bar{y}^t)} \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c(\bar{y}^t)) \quad (\text{P1})$$

subject to the aggregate resource constraint

$$c(\bar{y}^t) + \bar{b}(\bar{y}^t) = \sum_{i=1}^2 y_i(\bar{y}^t) + (1+r)\bar{b}(\bar{y}^{t-1}), \quad (9)$$

for all \bar{y}^t , $t \geq 0$, to the enforcement constraint

$$\sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau | \bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^\tau) u(c(\bar{y}^\tau)) \geq V_{aut}^U(\bar{y}^t) \quad (10)$$

for all \bar{y}^t , $t \geq 0$, where

$$V_{aut}^U(\bar{y}^t) = \sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau | \bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^\tau) u \left((1-\phi) \sum_i y_i(\bar{y}^\tau) \right),$$

for all \bar{y}^t , $t \geq 0$, to the no-Ponzi game condition (8), and to the initial conditions \bar{b}_0 and \bar{y}_0 .

Given the optimal plan $c(\bar{y}^t)$ from step 1, step 2 finds the optimal distribution of aggregate consumption among the union partners. Formally, the step 2 problem is

$$\max_{\{c_i(\bar{y}^t)\}} \sum_i \lambda_i \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c_i(\bar{y}^t)) \quad (\text{P2})$$

subject to

$$\sum_i c_i(\bar{y}^t) = c(\bar{y}^t),$$

for all \bar{y}^t , $t \geq 0$.

With CRRA preferences, the step 2 problem admits a simple, explicit solution. It is relatively easy to show that

$$c_i(\bar{y}^t) = \alpha_i c(\bar{y}^t) \quad (11)$$

where $\alpha_i \equiv \lambda_i^{1/\sigma} / \sum_j \lambda_j^{1/\sigma}$, for $i = 1, 2$. That is, individual consumption is a constant fraction of aggregate consumption. The fraction is increasing in the country's welfare weight.

Similarly to Section 3.2, the step 1 planner's problem admits a recursive formulation:

$$V^U(\bar{b}, \bar{y}) = \max_{c, \bar{b}'} \left\{ u(c) + \beta \sum_{\bar{y}'} \bar{\pi}(\bar{y}'|\bar{y}) V^U(\bar{b}', \bar{y}') \right\} \quad (\text{P1}')$$

subject to

$$\begin{aligned} c + \bar{b}' &= \sum_i y_i(\bar{y}) + (1+r)\bar{b} \\ \bar{b}' &\geq \bar{b}^U \end{aligned}$$

where

$$\bar{b}^U = \max_{\bar{y}'} \{ b_{\bar{y}'} : V^U(b_{\bar{y}'}, \bar{y}') = V_{aut}^U(\bar{y}') \} \quad (12)$$

and where $V_{aut}^U(\bar{y})$ solves

$$V_{aut}^U(\bar{y}) = u \left((1-\phi) \sum_i y_i(\bar{y}) \right) + \beta \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y}) V_{aut}^U(\bar{y}').$$

Given (11), the value for country i of being in a union with country j is

$$V_i^U(\bar{b}, \bar{y}) = \alpha_i^{1-\sigma} V^U(\bar{b}, \bar{y}). \quad (13)$$

4.2. Competitive equilibrium

Our welfare analysis still requires recovering the planner's welfare weights as a function of the initial pair of union partner states. ?'s (?) iterative method makes it possible to compute these welfare weights. This well-known result exploits the first welfare theorem, which allows us to obtain the competitive equilibrium allocation as the solution to the planner's problem for a given set of welfare weights. By requiring that the planner's allocation be affordable under the equilibrium prices, we obtain the unique pair of welfare weights that lead to the competitive equilibrium allocation associated with a given set of initial states.

A decentralization of the constrained efficient allocation needs to be considered. Our focus is on a competitive equilibrium with tax subsidies, in line with ? and ?. The decentralization works as follows. Within the union, countries trade a complete set of Arrow securities. In world credit markets, they trade freely on non-contingent bonds. However, a central government authority in the union taxes each country’s income in a lump-sum fashion, and uses the proceeds to subsidize asset purchases. The government’s tax and transfer policy is designed to support the constrained-efficient allocation. A subsidy is required to encourage union partners to save in those states when they would be inclined to default. Our procedure is described in more detail in Appendix B.

4.3. Discussion

Several features of union formation in our model are worth discussing.

Completing markets with trade in goods. Our setup necessarily abstracts from many important features of actual unions. Chiefly among them is trade in goods. Most actual unions described in Section 2 are explicitly motivated to reduce frictions in goods trade. Although our setup obviously misses some of these features, it’s nevertheless important to recognize that trade in goods also plays a role in sharing risk. ? have shown that changes in terms of trade can go a long way towards insuring against idiosyncratic income risk; in some extreme cases, trade in goods actually provides all the necessary insurance, without the need for financial markets. That is, trade in goods is one way to complete markets. Our view is that our abstract model captures the risk-sharing benefits of goods market integration, even if it’s not explicitly a model about commodity trade. We view the implications of our model as being more broadly relevant, even for actual unions whose explicit goal is not enhancing risk sharing.

Union formation, side payments, and ex-ante participation constraints. The role of initial conditions when computing the welfare gains from financial market integration is a crucial feature of our analysis. Whether a country is rich or poor at the time of union formation is a key determinant of the sign of the welfare gains. In the international risk-sharing literature, the role of initial conditions has sometimes been sidestepped (???, either impose symmetry, or look at a representative country), whereas in other papers (???) it is allowed to play a

role. Differently from this literature, however, in our model union formation may entail a welfare loss, often to only one of the partners. This generates the potential for disagreement about union formation. We exploit this by requiring that unions be formed only under unanimity.

For a large set of country pairs in our model, however, unions actually lead to potential Pareto improvements. This raises the possibility of introducing side payments to compensate the losers. Our analysis abstracts from such transfer schemes. In our setup, wealth would need to be redistributed away from poor and toward rich partners. We suspect the implementation of such schemes would face strong opposition in poor countries. Moreover, we do not have evidence from actual integration arrangements suggesting such schemes have taken place.¹⁰

Rather than implementing a pure transfer scheme, the two partners could instead agree ex-ante to distorting the baseline union allocation, tilting it to the benefit of rich partners. Formally, one would impose ex-ante participation constraints, at the time of union formation, such that every partner may potentially benefit from it. This would increase the likelihood of union formation among heterogeneous partners, at the expense of future risk-sharing benefits. Presumably, such arrangement would be easier to implement, on political economy grounds, compared to a pure transfer scheme. We think it would be very interesting and relevant to extend our analysis along this dimension.

Joint (off-equilibrium) default decisions. We considered unions with centralized international trade and default decisions. An alternative setting is one in which each individual member country unilaterally decides whether to default. ? provides an analysis of this situation. As Section 4.1 makes clear, a major advantage of our centralized setting is tractability, since it does not require solving directly for the market allocation, a significantly more complex approach. Note however that with decentralized default, potentially defaulting union members presume continued indirect access to world markets, by using the remaining non-defaulting members as intermediaries. This increases the incentives to default, and

¹⁰In the European Union, the Cohesion Fund is a transfer scheme that takes the exact opposite form: resources are transferred from rich to poor members.

therefore tightens borrowing limits within the union relative to centralized default. All else constant, union formation is thus even less likely under decentralized compared to centralized default. Our analysis can be thought of as giving the best chance for union formation.¹¹

Two-country unions. Also for tractability reasons, our analysis restricts attention to two-country unions. Since endowment risk is purely idiosyncratic, additional partners would be potentially beneficial to the union since they would further enhance risk-sharing opportunities. However, solving the frictions among union members is also likely to become more difficult and costly as the number of partners increases. This is precisely the starting premise of our paper, that solving frictions is easier at a smaller scale. Our model could be extended by introducing a cost of union formation that is increasing with the number of countries.¹² Such a setting would deliver implications for both the number and the type of countries most likely to form a union. We leave the analysis of these interesting implications for future research.

Union breakups and ex-post participation constraints. A country pair contemplating union formation is given a take-it-or-leave choice at time 0. If the union is formed, it is assumed to be forever enforced. Our analysis abstracts away from the important issue of sustainability of the economic union. Although union breakups are very rare in the data, they can be ex-post optimal in our model, depending on the endowment realization. Without an enforcement technology, sustaining the union would require distorting the optimal allocation, to ensure that the relevant ex-post participation constraints are met. In some cases this might not be possible, leading to a breakup of the union. See ? for an analysis of the sustainability of monetary unions with some of these features in an incomplete contract setting.

¹¹Default in our model is an off-equilibrium event, which means our theory has nothing to say about sovereign default events. In regard to such events, the centralized and the decentralized settings are equally unrealistic in our model.

¹²? find that, regarding the benefit side alone, most risk-sharing gains would be achievable in unions of less than ten member-countries. Further, in our model it is difficult for a large number of countries to all agree about union formation. This suggests that even very small costs would be sufficient to generate small-scale arrangements.

Endowment process. For tractability reasons, we restrict our analysis to common and independent income processes. We thus abstract from several dimensions of income heterogeneity which could be potentially important for union formation based on risk-sharing. In reality, shocks tend to be correlated among geographically close countries, which would work against union formation in our model.¹³ Further, there is also large cross-country heterogeneity in income risk, with poorer countries being more volatile (e.g. ?). In our model, this could potentially increase the likelihood of union formation among poor countries. Finally, there are also differences in country (and endowment) size. Although our simple endowment process provides an important benchmark, these extensions all deserve further scrutiny.

5. Parameters and computation

The model period is one year. Preferences are characterized by a coefficient of relative risk aversion $\sigma = 1.5$. The subjective discount factor is selected so that the equilibrium world interest rate is 1%, yielding $\beta = 0.9779$.

The direct output penalty ensures that the cross-sectional standard deviation of the net foreign assets to GDP ratio equals 0.42, the average cross-sectional standard deviation obtained from the ? data set - we focus on a balanced panel of 110 countries over the 1970-2004 period. This yields $\phi = 0.0027$, roughly a 0.3 percent yearly drop in output during default, equivalent to a $\phi(1+r)/r = 30\%$ drop in present value terms.

The endowment process is obtained from estimating a first-order autoregressive process:

$$\ln \tilde{y}_{it+1} = \rho \ln \tilde{y}_{it} + \sigma_\varepsilon \varepsilon_{it+1},$$

where $\ln \tilde{y}_{it} \equiv \ln y_{it} - \gamma_{0i} - \gamma_1 t$ and ε_{it+1} follows an i.i.d. $N(0, 1)$. We estimate this process by pooling data on real output per capita from version 7.1 of the Penn World Tables. We focus on a balanced panel of 111 countries over the 1960-2010 period. The point estimates of the key parameters are $\hat{\rho} = 0.9787$ and $\hat{\sigma}_\varepsilon = 0.0594$. In the model we ignore the common trend and the country-specific means, normalizing every country's mean endowment to 1.

¹³Correlated shocks are instead traditionally emphasized as a motivation for the formation of currency unions.

We consider the common process

$$\ln y' = 0.9787 \ln y + 0.0594\varepsilon', \quad (14)$$

with $\varepsilon' \sim \text{i.i.d. } N(0, 1)$. This process is discretized into a 5-state Markov chain using ?'s (?) procedure. The vector of endowment levels Y and the transition matrix $\Pi = [\pi_{yy'}]$ are reported in Table 2.

[Table 2 about here.]

Here is a brief description of our numerical algorithm, the full details are provided in Appendix C.1. The outer loop solves for the interest rate that clears the world bond market. For given interest rate, we solve for debt limits which are not too tight, using the natural borrowing limit as the initial guess. Finally, for given interest rate and debt limits, we obtain the decision rules that solve the system of first-order conditions of the country's problem.

6. Results

Our goal is to characterize which country pairs find it individually rational to form a union. The main benefit of union formation is the possibility of sharing risk with a partner. There are also costs, however. First, default becomes more attractive for union members, since they may still share risk upon default. As a result $b_i^W < \bar{b}_i^U = \bar{b}^U/2$, i.e. borrowing constraints become tighter in the union.¹⁴ In our benchmark calibration, the borrowing limit is tightened from $b_i^W = -0.197$ in the world economy, to $\bar{b}_i^U = -0.186$ in the union, on a per country basis. Countries can thus borrow up to about 20% of average yearly output in the world economy, and up to about 19% in a union.

Second, in asymmetric unions, poorer country members tend to borrow heavily from the rest of the world, and exhaust the whole union's borrowing limit. This imposes a cost on

¹⁴We note that in theory $b_i^W \geq \bar{b}_i^U$ may also obtain. This would happen when the value of union formation is high enough relative to the value of staying alone in the world economy, compared to the difference in the outside options. However, this was never the case in our quantitative analysis.

richer countries, which find themselves more frequently borrowing-constrained compared to standing alone in the world economy. Although being part of an asymmetric union tends to be beneficial for poorer members, it also tends to generate losses for richer countries. Our model will therefore produce a bias against forming asymmetric unions.

We now turn to a more detailed analysis of union formation. We compute the welfare gain for each country of forming a union with a specific partner in terms of consumption equivalents. That is, as the percentage increase in consumption, constant across time and future states of nature, that leaves the country indifferent between standing alone in the world economy and forming the union.

Consider two countries sitting in the world economy at time 0, with states (b_{i0}, y_{i0}) , $i = 1, 2$. If they form a union, the initial aggregate state is (\bar{b}_0, \bar{y}_0) , with $\bar{b}_0 = b_{10} + b_{20}$ and $\bar{y}_0 = (y_{10}, y_{20})$. Let $c^W(b_{i0}, y_{i0})$ represent a state-contingent consumption stream for country i in the world economy, from state (b_{i0}, y_{i0}) onwards. Let $c_i^U(\bar{b}_0, \bar{y}_0)$ represent a state-contingent consumption stream for country i if both i and j decide to form a union at time 0. Let $U(c^W(b_{i0}, y_{i0}))$ and $U(c_i^U(\bar{b}_0, \bar{y}_0))$ denote the expected lifetime utilities derived from these consumption streams. Now denote by $(1 + \mu_{ij})c^W(b_{i0}, y_{i0})$ the consumption stream derived from $c_i^W(b_{i0}, y_{i0})$, where every state-contingent consumption level is increased by μ_{ij} percent. The welfare gain for country i of forming a union with country j is the μ_{ij} that solves:

$$U((1 + \mu_{ij})c^W(b_{i0}, y_{i0})) = U(c_i^U(\bar{b}_0, \bar{y}_0)),$$

or, with CRRA preferences,

$$\begin{aligned} \mu_{ij} &= \left[\frac{U(c_i^U(\bar{b}_0, \bar{y}_0))}{U(c^W(b_{i0}, y_{i0}))} \right]^{\frac{1}{1-\sigma}} - 1 \\ &= \left[\frac{V_i^U(\bar{b}_0, \bar{y}_0)}{V(b_{i0}, y_{i0})} \right]^{\frac{1}{1-\sigma}} - 1, \end{aligned} \tag{15}$$

where the value functions have been defined in (P0) and (13). Notice that our welfare numbers incorporate transitional dynamics.

The following sections study the separate roles of wealth heterogeneity and wealth levels for union formation.

6.1. Role of wealth heterogeneity

6.1.1. A simple illustrative example

We begin by illustrating the main mechanism linking wealth heterogeneity to disagreement about union formation. Consider a very simple two-period version of our model, where endowments evolve deterministically. Take two countries, one which we call Rich facing a constant endowment sequence of $(1, 1)$, and a Poor one facing $(1 - \eta, 1)$ with $\eta \in (0, 1)$. Suppose preferences are given by $V^i = \log c_1^i + \log c_2^i$ for country $i = R, P$.

Countries here only face the task of smoothing consumption over time. They may do so by borrowing and lending at the world interest rate $r = 0$, as long as they respect borrowing limits imposed by a no default condition. Since each economy lives for two periods only, there is no penalty for defaulting and these limits are set to 0, i.e. no borrowing is allowed.

Now compare the two scenarios under which these two countries may interact. When they stand alone in the *world economy*, the rich country is happy to consume its endowment, $(c_1^{R,W}, c_2^{R,W}) = (1, 1)$, whereas the poor country is constrained to do the same $(c_1^{P,W}, c_2^{P,W}) = (1 - \eta, 1)$. They enjoy utility $V^{R,W} = 0$ and $V^{P,W} = \log(1 - \eta)$, respectively.

Alternatively, they may form a *union*. Along the lines of Section 4, in this case the planner first chooses the union-wide saving b^U to maximize $\log(2 - \eta - b^U) + \log(2 + b^U)$ subject to $b^U \geq 0$ (no borrowing can be sustained), and then distributes aggregate consumption according to $(c_1^{R,U}, c_2^{R,U}) = (\lambda c_1^U, \lambda c_2^U)$ and $(c_1^{P,U}, c_2^{P,U}) = ((1 - \lambda)c_1^U, (1 - \lambda)c_2^U)$, where λ is the welfare weight attached to the rich country. Since the union cannot borrow either, the first step yields $(c_1^U, c_2^U) = (2 - \eta, 2)$.

In order to deduce the welfare implications of union formation, we need to consider a decentralization of the union's allocation. We consider two alternatives here, both studied in detail by ?.

The first is the one we rely upon in Section 4.2. It entails government intervention in the form of a *tax and subsidy scheme*. The rich country decides saving $b^{R,\tau}$ to maximize $\log(1 - \eta - (1 - \tau)b^{R,\tau} - T^R) + \log(1 + b^{R,\tau})$, and similarly for the poor. The union's borrowing limit applies to the government, which needs to select the subsidy rate τ and the country-specific taxes T^i in order to decentralize the planner's allocation, $c_2^{i,\tau}/c_1^{i,\tau} = 2/(2 - \eta)$, and subject to budget feasibility, $\tau b^{i,\tau} = T^i$ for $i = R, P$. It is easy to show that this is

achieved with $\tau = \eta/2$ and $T^R = -T^P = \eta^2/(8 - 2\eta)$. The planner's allocation is affordable at market prices when $\lambda = 2/(4 - \eta)$. One may then show that welfare in the union satisfies $V^{R,U} < V^{R,W}$ and $V^{P,U} > V^{P,W}$, that is, the rich country is always worse-off in the union, whereas the poor country is always better-off. There is complete disagreement, and such a union would never be formed.

This first decentralization alternative is a computationally convenient way to obtain the planner's weights in the full model, and the reason why we implement it in Section 4. A second decentralization, however, illustrates our main mechanism more transparently. Suppose world lenders are now able to set *country-specific borrowing limits* to union partners, designed to prevent the union as a whole from defaulting. No government intervention is necessary. According to ?, financial markets are sophisticated in this case. These borrowing limits need to ensure two conditions. First, that the union is just indifferent to defaulting in the second period, that is, $\lambda \log(1 + \underline{b}^{R,S}) + (1 - \lambda) \log(1 + \underline{b}^{P,S}) = \lambda \log(2\lambda) + (1 - \lambda) \log(2(1 - \lambda))$. Second, that the marginal value of wealth be proportional across union members, that is, $\lambda/(1 + \underline{b}^{R,S}) = (1 - \lambda)/(1 + \underline{b}^{P,S})$. This ensures equalization of intertemporal marginal rates of substitution, and no incentive for countries to trade away from the borrowing limits. The result is $\underline{b}^{R,S} = -\underline{b}^{P,S} = \eta/(4 - \eta)$, yielding the same market outcome as with the tax and transfer scheme.

The second decentralization provides useful intuition. As the two countries consider forming a union, the borrowing limit of the poor becomes looser ($\underline{b}^{P,S} < 0$), at the expense of the borrowing limit of the rich, which becomes tighter ($\underline{b}^{R,S} > 0$). The union effectively generates a negative externality for the rich country, and a positive one for the poor, compared to standing alone in the world economy. Since the union's interest rate is pinned down by the world's, prices do not work for enticing the rich to save more. This is instead achieved by tightening the rich's borrowing limit. This gets the rich country to save for the poor, ensuring the social goal of consumption growth equalization inside the union. In this extreme example, the rich country is in fact happy to just sit alone in the world economy, given its constant endowment stream. The union is therefore a pure welfare loss. In the full model this is not necessarily the case, since the rich country also enjoys some insurance

benefits from union formation.¹⁵

6.1.2. Results from the full model

Figure 3 displays the welfare gain for country 1 of forming a union with country 2, as a function of country 1 and country 2's initial net foreign asset levels. The figure is conditional on country 1 being endowment-rich (starting the union formation process with endowment y_{mh}) and country 2 being endowment-poor (endowment y_{lm}). In Figure 4 we have the equilibrium distribution of welfare gains.¹⁶

Several observations emerge. First, country 1 experiences a welfare loss for a large range of net foreign asset levels. The equilibrium welfare gains range from -5.3% to 22%, with a median of 2.0% (mean of 2.7%).¹⁷ These are sizable welfare gains from union formation. Comparing with the literature on the welfare gains from international risk-sharing, the average gain is toward the lower end of the estimated range, as summarized by ?, but higher than in papers reporting gains of at most 0.5% such as ?, ?, ?, ?, and ?. Even if our welfare gains are concentrated around the values computed in the literature, our range is much wider. The reason is that our welfare gains result not just from improved risk sharing, the emphasis in the literature, but also from changes in the probability of becoming credit constrained, which we explain in more detail towards the end of this section.

Second, Figure 3 shows that country 1's welfare gain is always increasing in the partner's net foreign assets. Third, country 1's welfare gain is increasing in own net foreign assets only if the partner's is sufficiently low;¹⁸ otherwise, if the partner is rich, the welfare gain is

¹⁵In the illustrative example the union as a whole is allowed to borrow as much as two countries standing alone with the world economy (the borrowing limits are $b_i^W = \bar{b}^U/2 = 0$). Thus, an element of the full model we purposely abstracted from here is the tightening of the overall borrowing limit in the union. In the full model this possibility ($b_i^W < \bar{b}^U/2$) arises because the ability to share risk in the union upon default implies that less borrowing can be supported. Although this feature does generate an additional cost of union formation, it is an effect separate from, and ultimately less important than, the role of the spread in borrowing limits ($\underline{b}^{P,S} < 0 < \underline{b}^{R,S}$) in generating disagreement about union formation in our model.

¹⁶The lack of smoothness in the distribution is related to the discreteness of the endowments.

¹⁷A gain of around 20% is for a very poor country contemplating a union with a very rich one. Such rich country experiences a welfare loss, therefore these kinds of unions will never take place in our model.

¹⁸Although not apparent from the Figure 3, the welfare gain is actually non-monotonic in own net foreign

monotonically decreasing in own net foreign assets. Put together, the last two observations suggest the key determinant for union formation is the amount of the resources the partner has: a country would like to belong to a rich country club, especially if it's a poor one.

[Figure 3 about here.]

[Figure 4 about here.]

Figure 5 displays the agreement areas, i.e. the set of initial country states for which both countries would experience a welfare gain, and thus agree to form a union. To streamline the exposition, Figure 5 is restricted to endowment levels in $\{y_{lm}, y_m, y_{mh}\}$. For states above the solid lines, country 1 would improve welfare by forming a union with country 2, and similarly for country 2 for states below the dashed lines. The agreement areas are therefore represented by the light-shaded areas.

Superimposed on Figure 5 is also an area representing the ergodic space for net foreign asset positions in the world economy, $b_{10}, b_{20} \in [-0.197, 4.66]$.¹⁹ This is the dashed square located inside each figure. Notice the role played by the world steady-state equilibrium in our analysis of union formation. It determines both the world interest rate faced by the union, and also the relevant subset of country pairs that are faced with the option of union formation.

[Figure 5 about here.]

Begin with the first row of Figure 5. Potential union members have identical initial endowments, but potentially different asset levels. The figure shows, first, that unions tend to be formed between countries sufficiently homogeneous in terms of initial wealth. Along the 45 degree line, and restricted to the ergodic space, countries always reach an agreement. The disagreement area exists when net foreign asset levels are sufficiently different from each other. Second, whenever partners disagree, the rich are the ones with a potential welfare loss. They are the ones preventing union formation.

assets if the partner's is low enough. The reason will become clear when we discuss Figure 6.

¹⁹The equilibrium distribution of net foreign assets is right-skewed, in part due to the borrowing limit. The interpercentile range containing 95% of the observations is $[-0.2, 1.34]$.

Turning now to the bottom row of Figure 5, which corresponds to asymmetric initial endowments, we see that endowment heterogeneity makes it very difficult for countries to agree to form a union. In the extreme case when endowment levels are in $\{y_{lm}, y_{mh}\}$, an agreement is never reached. Although country 1, the endowment-poor country, would always benefit from union formation (the ergodic space is always above the solid line), this is not the case for country 2, the endowment-rich country. Only a sufficiently asset-poor country 2 would like to form a union with an endowment-poor country 1.

The bottom line is that country heterogeneity, either in terms of net foreign assets or endowments, is a key determinant of union formation. Unions are more likely to form among similar countries. The mechanism underlying partner disagreement is the effect the union generates on the probability of becoming constrained in the future. This is the effect we have illustrated in our simplified setting of Section 6.1.1, which works to relax the borrowing constraint of relatively poor partners at the expense of the relatively rich.

In the context of the full model, this same mechanism is illustrated in Figure 6. It displays the difference between the probability of becoming credit-constrained in a union and the probability of becoming credit-constrained while standing alone in the world economy, during the first 100 periods starting from $t = 0$.²⁰ This is computed for each initial level of net foreign assets of the reference country (labeled “own” in the figure) and of any given potential union partner, conditional on the endowment being equal to y_h for country 1 (relatively rich) and y_m for country 2 (relatively poor).

[Figure 6 about here.]

Several observations emerge. First, the excess probability is negative for a large set of states. This is in spite of tighter borrowing limits in the union as a whole: countries are better insured in the union, hence borrow less in world credit markets and hit the constraint less often compared to standing alone. Second, the excess probability becomes more negative when the reference country is poorer and the partner richer. Third, the excess probability

²⁰Our focus on the short run stems from the fact that we wish to understand the welfare comparisons underlying Figure 5, and individuals obviously discount the future. The excess probability in Figure 6 is in percentage points.

becomes positive when the reference country is richer and the partner poorer. These are precisely the areas of disagreement we identified earlier, illustrating the importance of our mechanism: asymmetric unions benefit poor countries at the expense of rich, via changes in the likelihood of becoming credit-constrained following union formation.

6.2. Role of wealth levels

Now turn to the role of total wealth (net foreign assets plus endowment) levels. From the first row of Figure 5, we see that a larger union-wide endowment favors union formation. First because, as we move from the left to the right panel, the agreement area fills a larger area of the ergodic space. Second because the agreement areas get wider for larger net foreign asset levels, which is particularly noticeable when conditional on (y_{lm}, y_{lm}) .

Figure 6 once again helps us understand the basic mechanism. As we move along any line starting from the lower left corner of the figure, the excess probability that country 1 becomes credit-constrained in the union decreases. When both partners are richer they are farther away from their borrowing constraints, and are thus less likely to face the type of disagreement that we illustrated in the previous section.

The summary of the discussion in the last two subsections is that unions are more likely to be formed the more homogeneous and the wealthier the partners are.

6.3. Quantitative implications

Our focus is on the probability of union formation conditional on different regions of the state-space.²¹ We ask: What is the probability that two randomly-picked countries from particular subsets of the world distribution agree to form a union?

In selecting subsets of the ergodic space, focus on the top and bottom terciles for output (respectively defined as $Y_h = [y_{2/3}, y_{\max}]$ and $Y_l = [y_{\min}, y_{1/3}]$) and net foreign-assets over GDP (respectively defined as $B_h = [(b/y)_{2/3}, (b/y)_{\max}]$ and $B_l = [(b/y)_{\min}, (b/y)_{1/3}]$). We define such sets in the exact same way in the actual data and in the model. Since the results

²¹An alternative procedure would be to run a probit-gravity regression on artificial data which would be the exact analogue of the one in Section 2, except that the terms involving geography and scale would be excluded. Unfortunately, due the nonlinear nature of the regression model, the marginal effects would be hard to compare.

are similar across our empirical definitions of unions, we focus on customs unions or deeper arrangements in the data.

Restrict attention to only three subsets, with the aim of capturing the key implications we drew from Figure 5. Take country pairs defined by their current output and net foreign assets over GDP.²² We consider “Rich” country pairs (both in the set $Y_h \times B_h$), “Poor” country pairs (both in the set $Y_l \times B_l$), and “Unequal” country pairs (one in the set $Y_h \times B_h$ and the other in $Y_l \times B_l$). We also compute the “Unconditional” probability of union formation.

[Table 3 about here.]

Our results are summarized in Table 3. The first column pertains to the data. Since the model abstracts from geography, the “Data” is restricted to country pairs sharing a border. About 32% of all country pairs are part of a customs union or deeper arrangement in 2004. This number is 14% conditional on poor country pairs, and 71% conditional on rich country pairs. The data does not feature unions among unequal pairs.

For the purpose of comparing our results to the data, recall that our calibrated income process from Section 5 abstracts from the estimated country fixed effects. This means our model-generated income differences are lower than in the raw data. This is the case even among common border countries. To address this shortcoming, our second column of Table 3 further restricts the data to those common border countries with pairwise income differences lower than the top 1/3. When we do this, the model-generated income differences are quantitatively very similar to those in the restricted data set. The conditional probabilities of union formation, however, are still similar to the first column. The main difference is that there are more unions being formed, especially among poor country pairs.

The third column contains the conditional probabilities of union formation in the model. The model is qualitatively consistent with the data in the sense that relatively few unions are formed, and the ones that do get formed are mostly among similar countries, and also rich ones. This confirms the analysis of Section 6. These probabilities resemble the ones in the data from a quantitative standpoint. The big discrepancy is that in our model low wealth levels are not nearly as detrimental to union formation as in the data.

²²For the reasons explained in Section 2, by “current” levels we actually mean five-year averages.

Our model thus seems to provide a reasonably accurate description of the incentives for union formation, namely the role of wealth levels and wealth inequality.²³

7. The European Union experience: Mediterranean and Eastern enlargements

The European Union (E.U.) is one of the few examples of an economic union, the deepest form of integration according to our empirical definition. We rely on the E.U. experience to test some of the basic predictions of our theory.

The process of European economic integration began with the signing of the Paris Treaty in 1951. It represented then a narrow degree of integration among only six relatively wealthy countries (Belgium, France, Italy, Luxembourg, Netherlands, and then West Germany). The degree of integration grew much deeper over time, culminating with the actual creation of the E.U. with the signing of the Maastricht Treaty in 1992. There were also several enlargement waves over time. We concentrate our attention on two of these, the Mediterranean enlargement of 1981 and 1986, and the Eastern enlargement of 2004. These featured relatively poor countries integrating with relatively rich ones.²⁴ Our goal is to show that both the conditions for accession, and the pre- and post-union outcomes for the incoming countries, are consistent with the predictions of our model.

7.1. *Income inequality and accession*

The Mediterranean and Eastern enlargement waves happened after an important degree of income convergence has taken place between accessing and incumbent member countries. Our model says that this is an important condition for union formation. Figure 7 illustrates

²³We have performed some sensitivity analysis of our results with respect to the stochastic process for the endowment. Our shocks are highly volatile and persistent, in part due to our detrending procedure described in Section 5. If we detrend output using country-specific trends rather than a common trend, we obtain instead $\hat{\rho} = 0.9105$ and $\hat{\sigma}_\varepsilon = 0.0564$. We have recalibrated and solved the model given this new process. The third column of Table 3 becomes: 71% (Rich), 45% (Poor), 4% (Unequal), and 38% (Unconditional). We conclude that our main results are not very sensitive to this change.

²⁴A few other Eastern European countries, namely Bulgaria, Romania and Croatia, have accessed after 2004. We do not consider them since the PWT7.1 contains data only until 2010, making it difficult to assess post union outcomes for these countries.

the fact. The left panel represents the accession of Greece in 1981, and Portugal and Spain in 1986. Together with the real per capita income of the accessing countries, Figure 7 also plots the median real per capita income among the nine incumbent members by 1980 (labelled EUR9). A very significant degree of convergence has occurred before these southern European countries joined the E.U. (European Economic Community by then), in fact to a much larger extent than the degree of convergence that took place afterwards. The right panel of Figure 7 documents the accession of the Eastern European block in 2004, plus Malta and Cyprus. These countries have also experienced a significant degree of convergence before joining the E.U. Without exception since the early 1990s, their incomes approached the median real per capita income among the fifteen incumbent members by 2003 (EUR15).

[Figure 7 about here.]

It is also worth pointing out that both the Mediterranean and the Eastern European countries were able to join the E.U. when they did due to the *sine qua non* removal of political obstacles: the Southern European countries became democracies in 1974/5, and the Eastern block around 1990. While these political considerations were obviously central, economic considerations were central too. Accessing countries were required to implement major free-market economic reforms as a condition for membership. These reforms were no doubt important for the subsequent economic performance of accessing countries; we would say also for the success and stability of the E.U.

7.2. Impact of economic integration on outcomes

There are two main implications of union formation for outcomes according to our model. First, risk-sharing improves among union members. Second, poor countries tend to borrow relatively more than rich countries once inside the union. We compare pre and post union outcomes to see whether the data provides support for these implications.

To measure the extent of risk-sharing, we rely on standard tests employed in the literature, see for instance ? and ?. We follow that latter by looking at the regression specification

$$\Delta \log c_t^i - \Delta \log \bar{c}_t = \beta_0 + \beta_1 (\Delta \log y_t^i - \Delta \log \bar{y}_t) + u_t^i, \quad (16)$$

where \bar{c}_t and \bar{y}_t are, respectively, average real per capita consumption and average real per capita output among the relevant countries.²⁵ To conform to the model, we focus on bilateral relationships. For example, we test whether risk-sharing between Greece and the nine incumbent union members (EUR9) changed after Greece's accession in 1981 by running the regression separately on pre-1981 and post-1981 data, and comparing $\hat{\beta}_1^{\text{pre}}$ with $\hat{\beta}_1^{\text{post}}$. We define per capita consumption and output of EUR9 as the corresponding medians among union members. We proceed similarly for the remaining countries in our sample. The left panel of Figure 8 plots $\hat{\beta}_1^{\text{pre}}$ against $\hat{\beta}_1^{\text{post}}$ together with the 45 degree line. The model predicts countries should be above the 45 degree line, as union formation makes consumption growth less dependent on idiosyncratic income growth. With few exceptions, the data is supportive of this pattern.

[Figure 8 about here.]

We test the borrowing implication by looking at consumption rates pre and post union. For example, we test whether Greece has borrowed relatively more once inside the E.U. by computing the relative consumption rate $\frac{(c/y)_{\text{GRE}}}{(c/y)_{\text{EUR9}}}$ separately on pre-1981 and post-1981 data, where $(c/y)_{\text{GRE}}$ is the median consumption rate of Greece over the relevant time period, and similarly for EUR9. The right panel of Figure 8 plots relative consumption rates pre and post union together with the 45 degree line. The model predicts countries should be below the 45 degree line, as the relatively poor (Greece in our example) tend to increase their consumption rate compared to rich union partners (EUR9 in our example). Again despite some exceptions, overall the data also supports this implication.

8. Conclusion

We have developed a quantitative theory of economic integration based on the incentives to share income risk. An economic union has been modelled as a small-scale arrangement that solves the frictions that otherwise limit the extent of risk sharing in the world economy.

²⁵We use data from 1950 until 2010 on PPP adjusted real GDP per capita and consumption share of PPP adjusted real GDP from PWT7.1.

Our model emphasizes not only the risk-sharing benefits of union formation, but also its costs. One cost is that union members as a whole will not be able to borrow as much as in the world economy. This is because unions have larger incentives to default. The key cost preventing union formation, however, is for rich countries in asymmetric unions. Poor countries tend to increase borrowing while in the union, imposing a negative externality on rich countries. Our model implies that economic integration should not happen very often, and when unions do get formed it is mostly among rich and homogeneous countries. These features appear to be consistent with real-world arrangements.

Our paper has focused on just one particular dimension of economic integration, the sharing of risk. It would be interesting to consider other important dimensions of economic integration for small scale arrangements, namely liberalizing goods flows (??), labor flows (?), and investment flows (???). These are also important robustness checks to the mechanism we emphasize in our paper, as some of these features may instead promote union formation among dissimilar partners. For example, introducing capital accumulation and investment flows would increase the incentive for rich and poor partners to integrate and reallocate capital inside the union. Frictions such as the ones considered by ?? can, however, prevent large capital reallocations. If poor countries also have worse contracting institutions, then capital might not flow from the rich, and disagreement about union formation between heterogeneous partners may endure.

Appendix A. Data

Appendix A.1. Countries

The full sample of 136 countries that we use in the regression analysis of Section 2 includes: Algeria, Angola, Antigua and Barbuda, Argentina, Australia, Austria, Bahrain, Bangladesh, Belgium, Belize, Benin, Bhutan, Bolivia, Brazil, Bulgaria, Burkina Faso, Burundi, Cameroon, Canada, Central African Republic, Chad, Chile, China, Colombia, Comoros, Congo D.R., Congo Rep., Costa Rica, Côte d’Ivoire, Cyprus, Czech Republic, Denmark, Djibouti, Dominica, Dominican Republic, Ecuador, Egypt, El Salvador, Equatorial Guinea, Ethiopia, Fiji, Finland, France, Gabon, Gambia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Kiribati, Korea, Kuwait, Lao PDR, Liberia, Madagascar, Malawi, Malaysia, Mali, Malta, Mauritania, Mauritius, Mexico, Mongolia, Morocco, Mozambique, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Qatar, Romania, Rwanda, Samoa, Saudi Arabia, Senegal, Seychelles, Sierra Leone, Singapore, Solomon Islands, South Africa, Spain, Sri Lanka, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Sudan, Sweden, Switzerland, Syrian Arab Republic, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Uganda, United Arab Emirates, United Kingdom, United States, Uruguay, Venezuela, Vietnam, Yemen, Zambia, and Zimbabwe.

Appendix A.2. Regional Agreements

The list of regional trade agreements in force in 2004 that we use in the regression analysis of Section 2, by type, and their country composition, is (PWT codes in parenthesis, only for countries in our sample):

- **Economic Unions.**

- *Economic and Monetary Community of Central Africa:* Cameroon (CMR), Central African Republic (CAF), Chad (TCD), Congo (COG), Equatorial Guinea (GNQ), and Gabon (GAB).

- *Euro zone (EU12)*: Austria (AUT), Belgium (BEL), Luxembourg (LUX), Finland (FIN), France (FRA), Germany (GER), Greece (GRC), Ireland (IRL), Italy (ITA), Netherlands (NLD), Portugal (PRT), and Spain (ESP).
- *West African Economic and Monetary Union*: Benin (BEN), Burkina Faso (BFA), Guinea-Bissau (GNB), Cote d’Ivoire (CIV), Mali (MLI), Niger (NER), Senegal (SEN), and Togo (TGO).

• **Common Markets.** In addition to all Economic unions:

- *East African Community*: Kenya (KEN), Tanzania (TZA), and Uganda (UGA).
- *European Economic Area (EEA)*: all the *EU12* countries listed above, plus the non-Euro zone countries of Denmark (DNK), Finland (FIN), Sweden (SWE), and the United Kingdom (GBR), plus the European Free Trade Area (*EFTA*) countries of Iceland (ISL), Liechtenstein, and Norway (NOR).

• **Customs Unions.** In addition to all Common Markets:

- *Andean Community*: Bolivia (BOL), Colombia (COL), Ecuador (ECU), Peru (PER), and Venezuela (VEN).
- *Caribbean Community*: Antigua and Barbuda (ATG), Bahamas, Barbados, Belize (BLZ), Dominica (DMA), Grenada (GRD), Guyana (GUY), Jamaica (JAM), Montserrat, Saint Kitts and Nevis (KNA), Saint Lucia (LCA), Saint Vincent and the Grenadines (VCT), Suriname, and Trinidad and Tobago (TTO).
- *Eurasian Economic Community*: Belarus, Kazakhstan Kyrgyzstan, Russia (RUS), and Tajikistan.
- *EU Customs Union*: Turkey (TUR), Andorra, San Marino, and Monaco, plus all the *EEA* countries listed above, except the 3 which are only part of *EFTA*.
- *Gulf Cooperation Council*: Bahrain (BHR), Kuwait (KWT), Oman (OMN), Qatar (QAT), Saudi Arabia (SAU), and United Arab Emirates (ARE).
- *Southern Common Market (Mercosur)*: Argentina (ARG), Brazil (BRA), Paraguay (PRY), and Uruguay (URY).

- *South African Customs Union*: Botswana, Lesotho, Namibia, South Africa (ZAF), and Swaziland.

Appendix B. Decentralization

We decentralize the planner's allocation as a competitive equilibrium with tax subsidies on saving. Our decentralization scheme is an adaptation of ? and ?.²⁶ Within the union, countries trade a complete set of Arrow securities, which are in zero net supply. In the world market, they trade freely on a riskless one-period bond. A central government authority in the union implements a tax and transfer scheme, designed to support the constrained-efficient allocation, and thus prevent union-wide default in the appropriate states.

For each country $i = 1, 2$ in the union, let $a_i(\bar{y}'; \omega_i, \bar{b}, \bar{y})$ denote the net stock of the Arrow security that pays in state \bar{y}' tomorrow, conditional on individual wealth ω_i and the aggregate state (\bar{b}, \bar{y}) , with price $q(\bar{y}'; \bar{b}, \bar{y})$. Let $b'_i(\omega_i, \bar{b}, \bar{y})$ denote the net stock of foreign bonds that earn interest r tomorrow.

Let also $\tau(\bar{b}, \bar{y})$ denote the subsidy rate on net asset purchases, and $T_i(\omega_i, \bar{b}, \bar{y})$ the lump-sum income tax faced by country i .

In a competitive equilibrium with capital controls, country i solves the following problem for every current state

$$V_i(\omega_i, \bar{b}, \bar{y}) = \max_{c_i, b'_i, \{a_i(\bar{y}')\}} \left\{ u(c_i) + \beta \sum_{\bar{y}'} \pi(\bar{y}' | \bar{y}) V_i(\omega'_i, \bar{b}', \bar{y}') \right\}$$

subject to

$$c_i + (1 - \tau(\bar{b}, \bar{y})) \left(b'_i + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) a_i(\bar{y}') \right) = \omega_i - T_i(\omega_i, \bar{b}, \bar{y}) \quad (\text{B.1})$$

$$\omega'_i \equiv y_i(\bar{y}') + (1 + r) b'_i + a_i(\bar{y}'),$$

and to a perceived law of motion for the aggregate foreign asset holding \bar{b} .

²⁶These authors consider taxes on borrowing instead of saving subsidies, although the two are equivalent. ? also studies an alternative decentralization based upon country-specific borrowing limits, along the lines of ?.

The government is assumed to run a balanced budget for each country separately, that is

$$\tau(\bar{b}, \bar{y}) \left(b'_i(\omega_i, \bar{b}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) a_i(\bar{y}'; \omega_i, \bar{b}, \bar{y}) \right) = T_i(\omega_i, \bar{b}, \bar{y}) \quad (\text{B.2})$$

for every current state and for each i .

A competitive equilibrium with a tax and transfer scheme is defined in the standard way, as (i) optimal decision rules that solve each country's problem given prices, government policy, and a perceived law of motion for aggregate wealth; (ii) a government policy that satisfies the balanced budget constraints given prices and individual decisions; (iii) Arrow security prices that clear asset markets; and (iv) consistency between the perceived law of motion for aggregate asset holding and the individual decision rules.

Our goal here is to show that there exists a government tax and transfer policy that supports the constrained-efficient allocation as a competitive equilibrium. We focus on the key steps of the argument.

Consider the first-order conditions to the country's problem

$$1 - \tau(\bar{b}, \bar{y}) = (1 + r) \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y}) \frac{\beta u'(c_i(\omega'_i, \bar{b}', \bar{y}'))}{u'(c_i(\omega_i, \bar{b}, \bar{y}))} \quad (\text{B.3})$$

$$(1 - \tau(\bar{b}, \bar{y})) q(\bar{y}'; \bar{b}, \bar{y}) = \pi(\bar{y}'|\bar{y}) \frac{\beta u'(c_i(\omega'_i, \bar{b}', \bar{y}'))}{u'(c_i(\omega_i, \bar{b}, \bar{y}))}. \quad (\text{B.4})$$

Given CRRA preferences, the last equation implies

$$\frac{c_i(\omega'_i, \bar{b}', \bar{y}')}{c_i(\omega_i, \bar{b}, \bar{y})} = \frac{c(\bar{b}', \bar{y}')}{c(\bar{b}, \bar{y})} \text{ for } i = 1, 2. \quad (\text{B.5})$$

The two Euler equations imply

$$1 = (1 + r) \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}). \quad (\text{B.6})$$

Note also that, at the optimum, we may use (B.2) to eliminate subsidies and transfers from (B.1):

$$c_i(\omega_i, \bar{b}, \bar{y}) + b'_i(\omega_i, \bar{b}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) a_i(\bar{y}'; \omega_i, \bar{b}, \bar{y}) = \omega_i. \quad (\text{B.7})$$

Consider now the constrained-efficient allocation, the solution to problem (P1'). This allocation, which we denote with a star superscript, satisfies the planner's Euler equation

$$u'(c^*(\bar{b}, \bar{y})) - \nu^*(\bar{b}, \bar{y}) = \beta(1+r) \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y}) u'(c^*(\bar{b}', \bar{y}')), \quad (\text{B.8})$$

where $\nu^*(\bar{b}, \bar{y})$ is the multiplier on the borrowing constraint.

Using (B.5) in (B.3), and requiring that the resulting allocation be consistent with (B.8), it is easy to compute the state-contingent subsidy rates that implement the constrained-optimal allocation as

$$\tau(\bar{b}, \bar{y}) = \frac{\nu^*(\bar{b}, \bar{y})}{u'(c^*(\bar{b}, \bar{y}))}. \quad (\text{B.9})$$

Note that if the borrowing constraint to problem (P1') does not bind in state (\bar{b}, \bar{y}) , then $\nu^*(\bar{b}, \bar{y}) = 0$ and so $\tau(\bar{b}, \bar{y}) = 0$. In this case, from (B.4) and (B.6), the domestic interest rate equals the world interest rate. If the constraint is instead binding, then the (post-subsidy) domestic interest rate is higher than the world interest rate. This ensures that countries save in a constrained-optimal way, and that equilibrium borrowing is self-enforcing.

It is relatively straightforward to show formally that, given a constrained-efficient allocation that solves (P1') and (P2) for the appropriate set of welfare weights, one can obtain individual asset holdings from (B.7) together with the market clearing condition for Arrow securities, Arrow security prices from (B.4), and a government policy from (B.9) and (B.2) that support that allocation as a competitive equilibrium with tax subsidies.

To find the appropriate set of welfare weights, we use the method proposed by ?. This method exploits the equivalence between the market and the constrained-efficient allocations.

We obtain the time-0 present value budget constraint of country i by iterating forward on the flow budget constraint (B.7), and ruling out Ponzi schemes. Using (B.6) and the fact that Arrow securities are in zero net supply, we express it as

$$C_i(b_{i0}, \bar{b}_0, \bar{y}_0) = Y_i(\bar{b}_0, \bar{y}_0) + (1+r)b_{i0},$$

where $C_i(b_{i0}, \bar{b}_0, \bar{y}_0)$ and $Y_i(\bar{b}_0, \bar{y}_0)$ are the time-0 present-values of consumption and the endowment, respectively. At time 0, when the union is formed, \bar{y}_0 is the union's endowment pair, b_{i0} is country i 's net stock of foreign bonds, and $\bar{b}_0 = \sum_i b_{i0}$ is the union's.

It follows from (11) that we may express the present value of individual consumption as fraction of the present value of aggregate (constrained-efficient) consumption, that is $C_i(b_{i0}, \bar{b}_0, \bar{y}_0) = \alpha_i C^*(\bar{b}_0, \bar{y}_0)$. Replacing above allows us to recover the individual consumption share coefficients as

$$\alpha_i = \frac{(1+r)b_{i0} + Y_i(\bar{b}_0, \bar{y}_0)}{C^*(\bar{b}_0, \bar{y}_0)}. \quad (\text{B.10})$$

Given equilibrium Arrow security prices $q(\bar{y}'; \bar{b}, \bar{y})$ from (B.4) and (B.9), and optimal decision rules $c^*(\bar{b}, \bar{y})$ and $\bar{b}'(\bar{b}, \bar{y})$, the C^* and Y_i functions solve the following functional equations

$$Y_i(\bar{b}, \bar{y}) = y_i(\bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) Y_i(\bar{b}', \bar{y}') \quad (\text{B.11})$$

$$C^*(\bar{b}, \bar{y}) = c^*(\bar{b}, \bar{y}) + \sum_{\bar{y}'} q(\bar{y}'; \bar{b}, \bar{y}) C^*(\bar{b}', \bar{y}'). \quad (\text{B.12})$$

Notice that although it is straightforward to obtain the welfare weights from the consumption share parameters, we only need to know the α_i 's in order to uncover the individual allocations.

Appendix C. Numerical algorithms

Appendix C.1. World economy equilibrium

Our algorithm can be described in the following steps:

1. Solve for the autarky value function $V_{aut}(y)$ from equation (7).
2. Given a current guess for the equilibrium interest rate r , solve problem (P0) by iterating on the following steps:
 - (a) Consider the n^{th} iteration, with a current conjecture for the debt limit b_n^W . For the initial conjecture, we use the natural borrowing constraint.
 - (b) Given b_n^W , solve problem (P0) by policy function iteration. We discretize the state-space and use cubic-spline interpolation to compute decisions outside the grid.
 - i. First find the decision rules that solve the system of first-order conditions to problem (P0), ignoring the debt limit. Consider the j^{th} iteration, with

a current conjecture for the consumption decision rule $c_n^j(b, y)$. Compute a candidate update $c_n^{j+1}(b, y)$ by solving

$$u'(c_n^{j+1}(b, y)) = \beta(1+r) \sum_{y'} \pi(y'|y) u'(c_n^j(b', y'))$$

with

$$b' = y + (1+r)b - c_n^{j+1}(b, y).$$

As part of the solution, we obtain $b_n^{j+1}(b, y)$.

- ii. Check whether the borrowing constraint is violated. If $b_n^{j+1}(b, y) < b_n^W$, then update the solution as follows:

$$\begin{aligned} b_n^{j+1}(b, y) &= b_n^W \\ c_n^{j+1}(b, y) &= b - b_n^{j+1}(b, y) \\ \nu_n^{j+1}(b, y) &= u'(c_n^{j+1}(b, y)) - \beta(1+r) \sum_{y'} \pi(y'|y) u'(c_n^{j+1}(b', y')), \end{aligned}$$

If instead $b_n^{j+1}(b, y) \geq b_n^W$, then update using the unconstrained solution, setting also $\nu_n^{j+1}(b, y) = 0$.

- iii. Iterate on the previous two steps until the decision rules converge. At the end, compute the value function $V_n(b, y)$.

- (c) Given $V_n(b, y)$, update the debt limit as follows:

$$b_{n+1}^W = \max_{y'} \{b_{y'} : V_n(b_{y'}, y') = V_{aut}(y')\}.$$

- (d) Iterate on steps 2b and 2c until the borrowing limits converge.
3. Check the market clearing condition by approximating the aggregate bond holding in the world economy with the total bond holding of a particular country over a very long simulation period. We discretize the state-space using a finer grid, and linearly interpolate the decision rules.
4. Iterate on steps 2 and 3 until we find an interest rate that approximately clears the bond market.

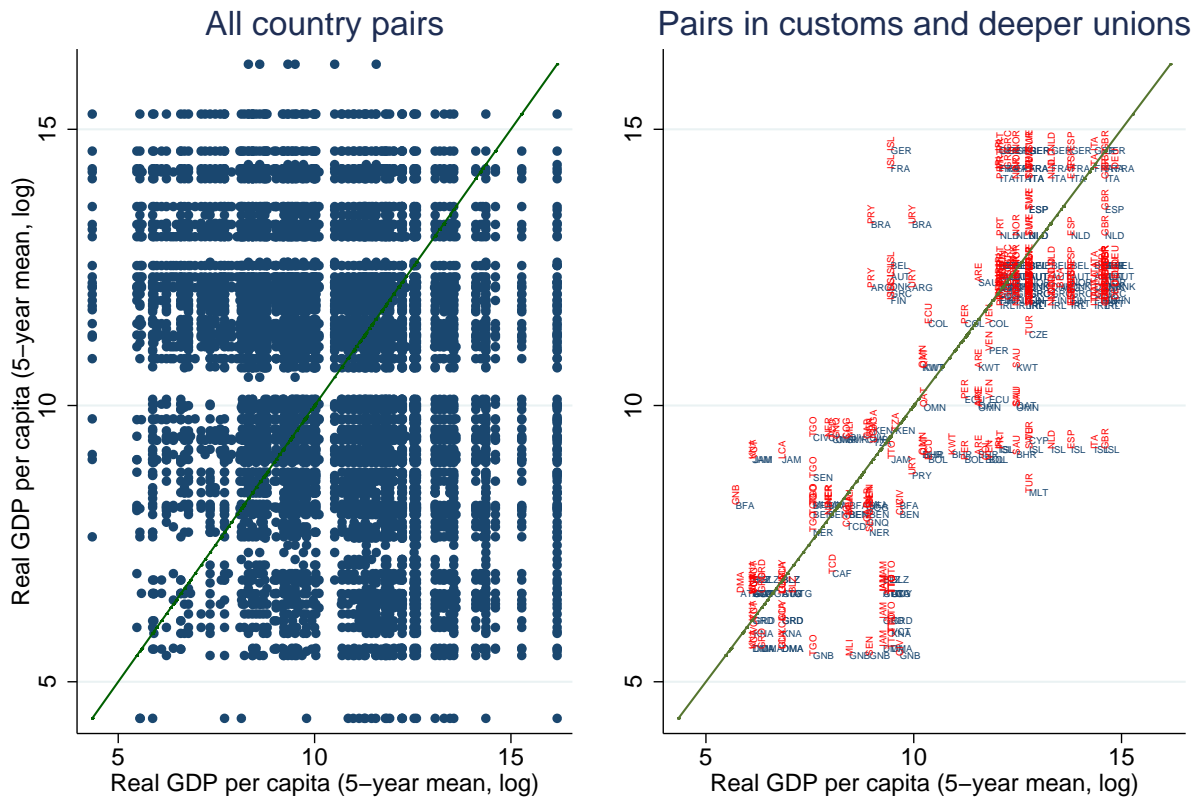
Appendix C.2. Union problem under centralized default

Our algorithm to solve for the union's allocation given an equilibrium world interest rate r can be described as follows:

1. Solve problem (P1') using the method described in step 2 of the algorithm of Appendix C.1. As part of the solution we obtain the union decision rule $c^*(\bar{b}, \bar{y})$, the multiplier function $\nu^*(\bar{b}, \bar{y})$, and the value function $V^U(\bar{b}, \bar{y})$.
2. Decentralize the union's constrained-efficient allocation as a competitive equilibrium with capital controls.
 - (a) Compute tax-subsidies from (B.9).
 - (b) Compute pre-subsidy Arrow-security prices from (B.4).
 - (c) Compute the present-value functions from (B.11) and (B.12). In practice, we guess some arbitrary functions on a grid and then iterate on the two recursive equations until convergence. We linearly interpolate these functions when future wealth levels fall outside the grid.
 - (d) Compute consumption shares from (B.10).
 - (e) Compute the value function for each country from (13).

References

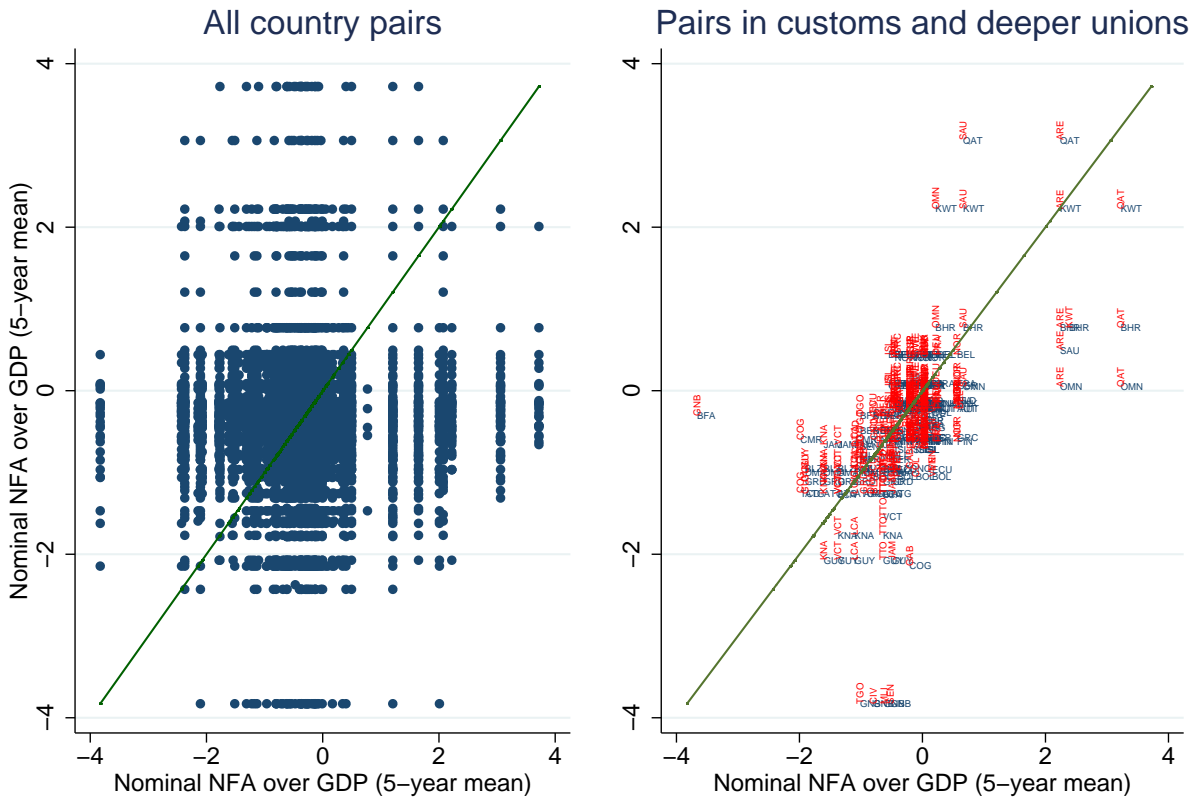
Figure 1: Incomes and union formation



Data sources: PWT7.1, WTO

Notes: The left panel displays real GDP per capita levels (averaged between years 2000 and 2004) for all possible pairwise combinations among the countries in our sample, together with the 45 degree line. The right panel restricts to country pairs which are part of custom unions or deeper integration arrangements. This panel also identifies the specific country pair associated with each data point. The country codes are from the Penn World Tables.

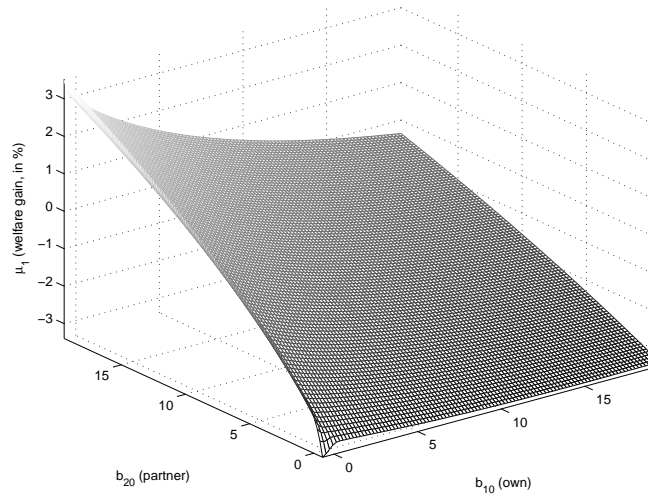
Figure 2: Net foreign assets over income and union formation



Data sources: Lane and Milesi-Ferretti (2007), WTO

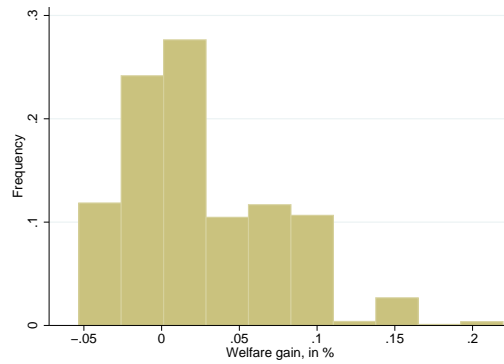
Notes: The left panel displays nominal net foreign assets over GDP ratios (averaged between years 2000 and 2004) for all possible pairwise combinations among the countries in our sample, together with the 45 degree line. The right panel restricts to country pairs which are part of custom unions or deeper integration arrangements. This panel also identifies the specific country pair associated with each data point. The country codes are from the Penn World Tables.

Figure 3: Welfare gain from union formation



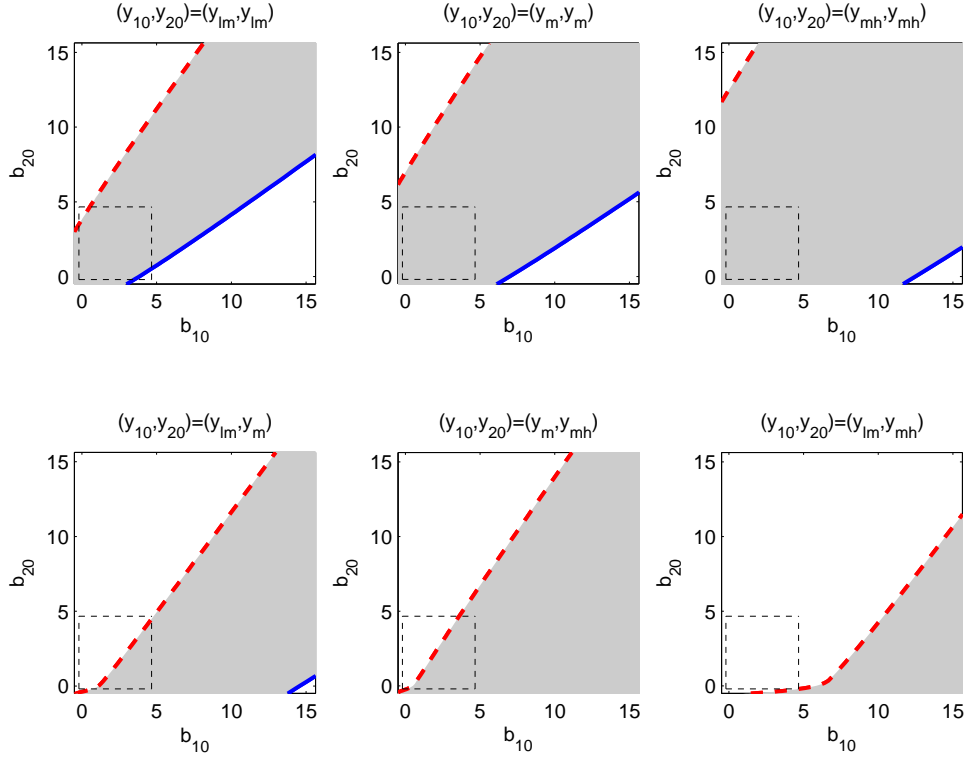
Notes: The figure gives the welfare gain for country 1 of forming a union with country 2 in the model. The gain is in consumption equivalents (percentage increase in country 1's consumption when standing alone in the world economy necessary to make it indifferent to forming a union with country 2). It is conditional on country 1 being endowment-rich at the time of union formation ($y_{10} = y_{mh}$), and country 2 being endowment-poor ($y_{20} = y_{lm}$). The welfare gain is displayed for a range of net foreign asset values for the reference country (b_{10}) and the partner country (b_{20}).

Figure 4: Distribution of welfare gains



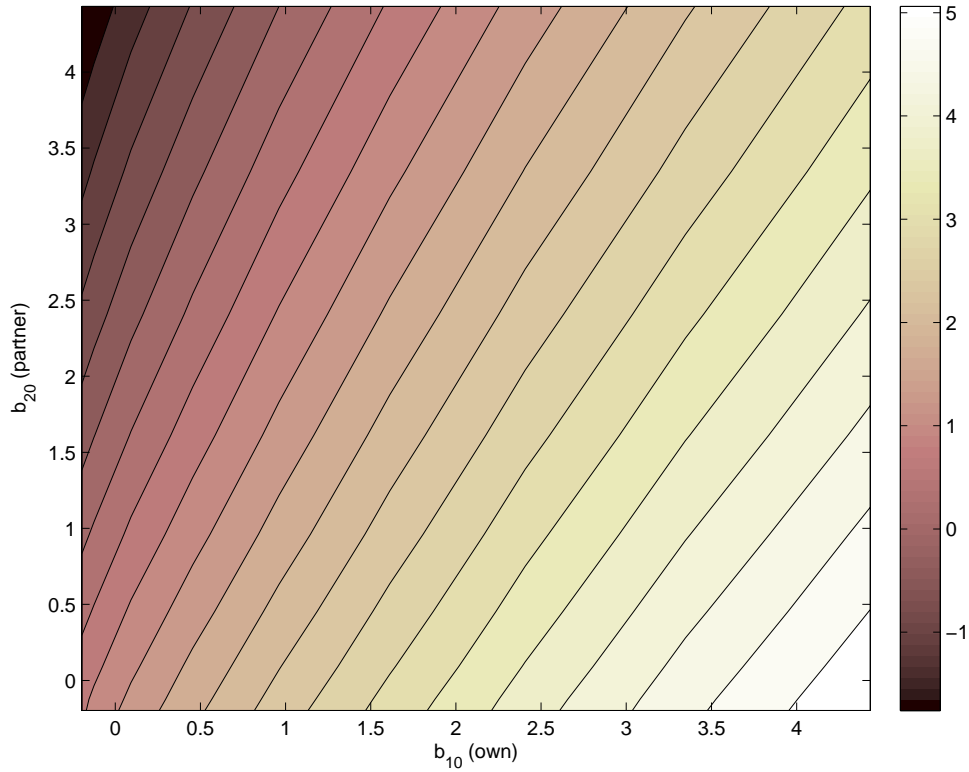
Notes: The figure gives the equilibrium distribution of welfare gains, computed across all possible country pairs in the model's stationary distribution. The gains are in consumption equivalents (percentage increase in a country's consumption when standing alone in the world economy necessary to make it indifferent to forming a union with some other country).

Figure 5: Agreement areas (country 1: solid, country 2: dashed)



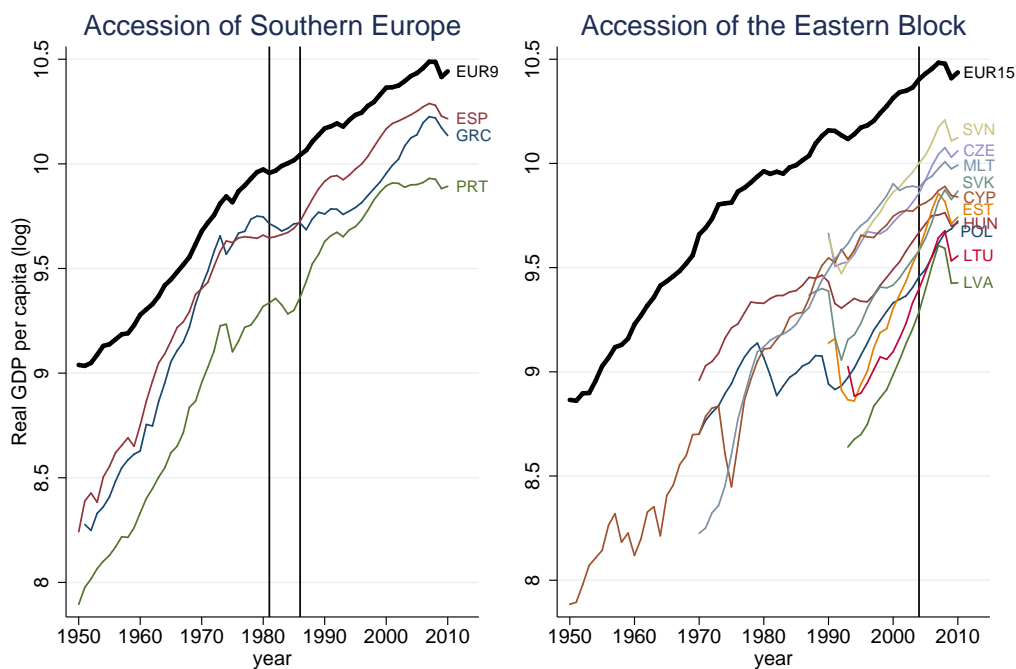
Notes: The figure gives the model's agreement areas in grey, i.e. the regions of the state-space where union formation is mutually beneficial. Each panel is conditional on a particular endowment pair at the time of union formation (y_{10}, y_{20}) , and considers a range of net foreign asset values (b_{10}, b_{20}) . Country 1 would like to form a union with country 2 types above the solid line. Similarly for country 2 below the dashed line. The small square delimited by the dashed lines represents net foreign asset levels in the ergodic space. The relevant agreement areas correspond to the grey areas inside this small square.

Figure 6: Excess probability of becoming credit constrained in the union (in percentage points)



Notes: The figure gives the difference between the probability that country 1 becomes credit-constrained in a union with country 2, and the probability that it becomes credit-constrained standing alone in the world economy. Both probabilities are over the first 100 periods starting from the initial state. The figure is conditional on a high endowment for country 1 ($y_{10} = y_h$) and a lower endowment for country 2 ($y_{20} = y_m$), and considers a range of initial net foreign asset levels (b_{10}, b_{20}). The lighter areas correspond to a higher probability of becoming credit-constrained in the union.

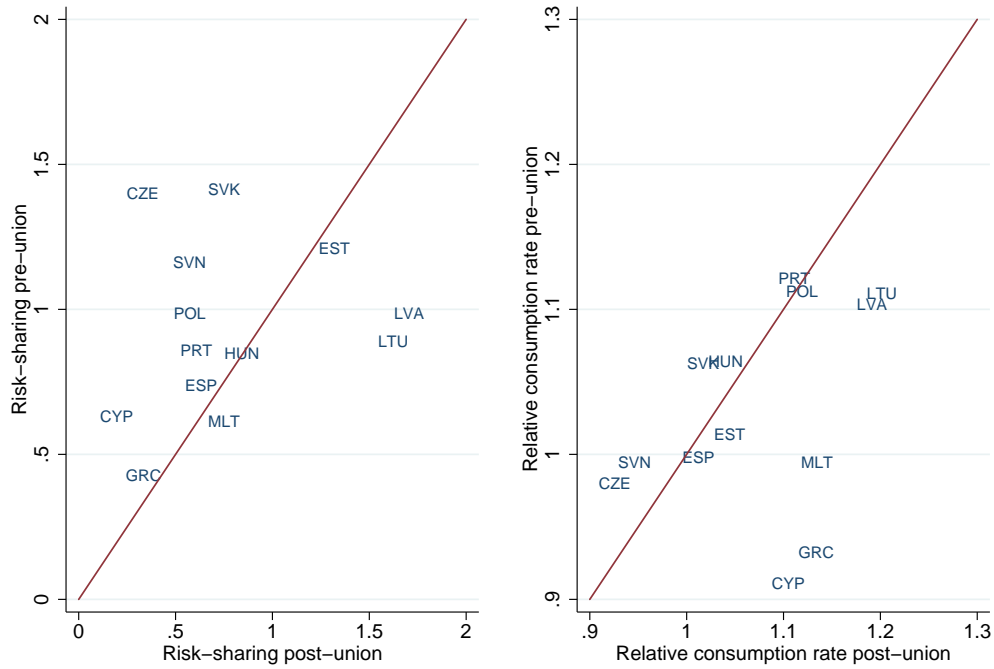
Figure 7: Income levels and accession into the E.U.



Data source: PWT7.1

Notes: The left figure displays the income levels of accessing and incumbent countries surrounding the Southern European expansion waves of the European Union. EUR9 labels the median income among the nine incumbent members as of 1980. GRC labels the income of Greece, whose accession is marked with the vertical line at year 1981. ESP and PRT label the incomes of Spain and Portugal, respectively, whose accession is marked with the vertical line at year 1986. The right figure displays the same information for the Eastern European Block accession, marked with the vertical line at year 2004. EUR15 denotes the median income of the fifteen incumbent members as of 2003, whereas the income of the different accessing countries is labelled as: Czech Republic (CZE), Estonia (EST), Hungary (HUN), Latvia (LVA), Poland (POL), Slovakia (SVK), Lithuania (LTU), Slovenia (SVN), Malta (MLT), and Cyprus (CYP).

Figure 8: Risk-sharing and relative consumption rates before and after joining the E.U.



Data source: PWT7.1

Notes: The left figure compares the degree of risk-sharing between each country in the Southern and Eastern European accession waves into the European Union (see note to Figure 7 for the country codes) and the relevant union incumbents, before and after accession. The degree of risk-sharing is measured by the estimated β_1 coefficients from running regression (16) on pre and post union data, which are plotted against the 45 degree line. Lower coefficients are associated with better risk-sharing. The right figure displays the median consumption rates of each accessing country relative to the incumbents' $\left(\frac{(c/y)_{GRC}}{(c/y)_{EUR9}}\right)$ for the case of Greece, for example), computed separately on pre and post union data, and plotted against the 45 degree line.

Table 1: Wealth, inequality, and union formation

Marginal Effects on the Probability of Union Formation				
Definition of Union: at least...		...Customs Union	...Common Market	...Economic Union
Distance	adj=0	-0.035 (0.000)	-0.022 (0.000)	-0.014 (0.000)
	adj=1	-0.039 (0.000)	-0.022 (0.000)	-0.016 (0.000)
Population Size	adj=0	-0.002 (0.004)	0.004 (0.000)	0.002 (0.000)
	adj=1	-0.002 (0.499)	-0.002 (0.393)	-0.001 (0.727)
Population Inequality	adj=0	-0.006 (0.000)	-0.002 (0.094)	-0.003 (0.003)
	adj=1	-0.002 (0.680)	-0.005 (0.147)	-0.004 (0.108)
Income	adj=0	0.020 (0.000)	0.014 (0.000)	0.004 (0.005)
	adj=1	-0.001 (0.939)	0.001 (0.897)	-0.001 (0.633)
Income Inequality	adj=0	-0.031 (0.000)	-0.047 (0.000)	-0.025 (0.000)
	adj=1	-0.026 (0.011)	-0.014 (0.035)	-0.010 (0.078)
NFA/GDP	adj=0	-0.017 (0.004)	-0.012 (0.000)	-0.005 (0.020)
	adj=1	0.027 (0.058)	-0.002 (0.766)	-0.001 (0.994)
NFA/GDP Inequality	adj=0	-0.008 (0.005)	-0.002 (0.364)	-0.001 (0.898)
	adj=1	-0.009 (0.407)	-0.008 (0.199)	-0.004 (0.305)
Number of observations		6629	6629	6629
pseudo R^2		0.5535	0.5408	0.4368

Notes: The table presents the estimated average marginal effects on the probability of union formation of each control variable in regression (1), evaluated either among common border countries (adj=1) or non-adjacent ones (adj=0). Each column corresponds to a different empirical definition of union (customs unions or deeper, common market or deeper, and economic union). Huber-White robust p-values in parenthesis, computed by the delta method.

Table 2: Markov chain parameters

Y					Π				
y_l	y_{lm}	y_m	y_{mh}	y_h	0.958	0.041	7×10^{-4}	5×10^{-6}	10^{-8}
0.561	0.749	1.000	1.336	1.784	0.010	0.958	0.031	3×10^{-4}	10^{-6}
					10^{-4}	0.021	0.959	0.021	10^{-4}
					10^{-6}	3×10^{-4}	0.031	0.958	0.010
					10^{-8}	5×10^{-6}	7×10^{-4}	0.041	0.958

Notes: The table gives the result of discretizing the AR(1) process (14) into a 5-state Markov chain. The left table has the endowment values, whereas the right table has the transition matrix.

Table 3: Conditional probabilities of union formation

Country Pairs	Data	Data, no extreme differences	Model
Rich	71%	65%	67%
Poor	14%	21%	57%
Unequal	0%	0%	1%
Unconditional	32%	41%	29%

Notes: The table gives the probabilities that two countries form a union conditional on their income and net foreign asset levels, both in the data and in the model. “Rich” restricts to both countries being in the top tercile of the distribution, “Poor” to both being in the bottom tercile, and “Unequal” to one country being in the top and the other in the bottom tercile. “Unconditional” reports the unconditional probabilities. The sets are defined in the same way in the data and in the model. The “Data” column is for the actual data, restricted to common border country pairs. The “Data, no extreme differences” column further restricts to pairwise income differences lower than the top 1/3 ones.