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Études sur l'Irréversibilité et la Théorie des Actifs Financiers: Application à la Firme Minière

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a été évaluée par un jury composé des personnes suivantes:

Pierre Perron, président-rapporteur
Pierre Lasserre, directeur de recherche
Claude Montmarquette, codirecteur
Georges Dionne, membre du jury
Robert S. Pindyck, examinateur externe

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* En collaboration avec Pierre Lasserre.

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Sommaire

La recherche en finance a été marquée au cours de ces dernières années par deux domaines particulièrement fructueux.

En premier lieu, l'application de la théorie des options financières aux actifs réels. Ce nouveau cadre d'analyse réhabilite le concept d'irréversibilité et introduit des considérations de "timing" dans la décision d'investissement longtemps négligées par les économistes. Il se présente, dès lors, comme l'alternative la plus sérieuse à la théorie néo classique de l'investissement. Dans cette thèse, nous consacrons trois études à cette problématique et notre contribution se situe à plusieurs niveaux: i) de ad hoc qu'il était, le lien entre la théorie de l'investissement et celle des options financières est caractérisé; ii) nous faisons appel à des données historiques et estimées sur les firmes minières canadiennes pour illustrer comment un investissement en capacité de production irréversible avec incertitude s'interprète comme l'exercice d'une option d'achat financière et, iii) nous testons économétriquement le bien-fondé de la théorie des options financières appliquée à ce problème d'investissement. Nos conclusions rejettent systématiquement la théorie néo classique et le critère de la valeur actuelle nette (VAN) qui lui est associé au profit de la théorie des options financières appliquée à l'investissement et du critère amendé de la VAN.

En second lieu, l'évaluation économétrique des modèles du CAPM avec prise en compte de l'hétéroscédasticité conditionnelle a été l'autre domaine privilégié de la recherche en finance. Cette approche est motivée en grande partie par le rejet du modèle non conditionnel du CAPM. Au lieu d'entreprendre les approches qui suggèrent une spécification de type ARCH (Autoregressive Conditional Heteroscedastic), nous considérons un modèle du CAPM et de son homologue intertemporel (ICAPM) avec des paramètres dépendants de certaines variables d'état. De plus, la spécificité de l'industrie minière nous contraint à retenir des portefeuilles construits par industrie. D'une façon générale, nos résultats conduisent au non rejet du CAPM conditionnel et rendent compte aussi du caractère variable du béta dans le temps.

Résumé

L'incapacité du paradigme néo classique à expliquer le comportement de l'investissement a permis à la question de l'irréversibilité de connaître un net regain d'intérêt au cours de la dernière décennie. Celle-ci, en présence de l'incertitude, s'est avérée être un outil puissant, d'une part, pour expliquer des phénomènes tels la présence des cycles économiques et, d'autre part, la remise en cause du cadre traditionnel du choix des investissements tel que mis de l'avant par le critère de la valeur actuelle nette. Mais, fondamentalement, elle fournit aussi les premiers éléments qui permettraient d'expliquer la présence de "l'hysteresis" observée couramment dans plusieurs secteurs de l'activité économique[1]. Récemment, sous l'impulsion de la théorie des options financières appliquée aux actifs réels, "l'hysteresis" a fait l'objet d'un éclairage nouveau: la présence de l'irréversibilité et de l'incertitude font que, en plus du "sunk cost", les firmes doivent couvrir un coût additionnel dit option d'investissement. La nature prohibitive de celui-ci, puisqu'étant directement lié au risque de se compromettre, contraint souvent les firmes à opérer à pertes plutôt que de fermer temporairement ou de façon permanente.

Cette thèse est composée de quatre études où les considérations d'irréversibilité telles qu'expliquées par la théorie des options financières occupent une place importante. Les trois premières études, intimement liées, tentent d'appliquer la théorie des options financières à des considérations d'investissement irréversible. Le choix de capacité de la firme minière avec incertitude et technologie «putty clay» est caractérisé typiquement comme l'exercice d'une option d'achat financière. Notre propos, tout en s'inscrivant dans la problématique de la théorie des options financières appliquée aux actifs réels (l'investissement physique), ne s'en démarque pas moins de la littérature économique en présence: i) la valeur de la firme est supposée endogène, ii) le lien entre la théorie des options et celle des actifs réels est, à la fois, caractérisé et testé économétriquement et, iii) la date d'exercice de l'option est prédite sur la base de nos estimés. D'une façon générale, nos résultats confirment le lien intime entre la théorie des options et celle des actifs réels irréversibles. La dernière étude porte en revanche exclusivement sur le problème d'évaluation des actifs financiers miniers avec prise en compte de l'hétéroscédasticité conditionnelle. Ici, au lieu de faire appel à l'approche standard de type ARCH (Autoregressive Conditional Heteroscedastic), nous développons un modèle du CAPM et de son homologue intertemporel (ICAPM) avec des paramètres dépendants de certaines variables d'état. Le résultat permet d'estimer un béta variable par rapport au temps.

La première étude, *«Testing the Impact of Taxation on Capacity Choice: A Putty Clay Approach With Uncertainty»*, caractérise l'idée de l'actif réel. Elle porte sur l'impact de la fiscalité sur un échantillon de mines de cuivre canadiennes. A partir d'un modèle «putty clay», nous étudions les créations de mines et les expansions

[1]Le terme "hysteresis" n'a pas d'équivalent dans la langue française. Nous proposons par défaut cette traduction littérale impropre. Ce terme rend compte de situations où les effets persistent alors que les causes se sont estompées.

majeures, compte tenu de l'environnement fiscal prévalant au lieu et à la date concernés. Une forme limitée d'incertitude est introduite par l'intermédiaire de prix futurs stochastiques pour la production. Nous soulignons les différences que présente ce modèle par rapport à un modèle néoclassique à technologie flexible, tant du point de vue du coût du capital que de la rente de rareté. Le modèle de choix de capacité est estimé économétriquement. Outre les données géologiques comme les réserves minérales, le choix de capacité réagit de façon statistiquement significative, et en conformité au modèle théorique, à l'influence conjointe des prix et des fiscalités fédérale, provinciale et minière. A notre connaissance, c'est la première fois que ces effets sont évalués économétriquement pour une industrie minière, et non pas simplement simulés. Nous innovons également en testant la neutralité de la taxe.

La deuxième étude, *«Le Choix de Capacité Comme l'Exercice d'une Option d'Achat Financière: Application à la Firme Minière»*, constitue à notre connaissance la première confrontation de la théorie des options financières appliquée aux actifs réels avec des données empiriques. Nous l'avons appliquée à des projets concrets d'investissement par des firmes minières canadiennes. Une difficulté résidait dans le fait que la taille de l'investissement, et plusieurs de ses caractéristiques, dépendent en général du moment où l'investissement est effectué. Nous avons endogénéisé ces éléments à l'aide de travaux économétriques expliquant le choix de capacité tels qu'obtenus de la première étude. Sous certaines hypothèses simplificatrices nous avons alors pu comparer, en recourant au modèle d'évaluation des options d'achat, la valeur des projets à des dates antérieures à la date de mise en oeuvre, avec la valeur critique au-dessus de laquelle la théorie préconisait d'aller de l'avant avec le projet. Nous avons constaté que les firmes de notre échantillon avaient effectivement agi exactement conformément à la théorie dans 58% des cas et conformément à la théorie à une année près dans 75% des cas. Bien que nous n'effectuons aucun test statistique de la validité de la théorie, ces résultats lui paraissent favorables.

La troisième étude, *«The Option Value of an Irreversible Investment: An Econometric Assessment»*, est à notre connaissance le premier test économétrique de la théorie des options financières appliquée aux actifs réels. Nous montrons d'abord comment un investissement en capacité de production avec technologie «putty clay» et sous incertitude s'interprète comme l'exercice d'une option d'achat financière. La règle d'investissement qui en résulte est rendue compatible avec une spécification économétrique de type dichotomique de sorte que, contrairement à la littérature qui fait appel à des valeurs arbitraires, nos résultats se prêtent à l'inférence statistique. Nos résultats empiriques rejettent le critère traditionnel de la valeur actuelle nette en faveur du modèle d'investissement issu de la théorie des options. On trouve aussi que dans une proportion de 50% le modèle prédit correctement la date d'investissement des firmes de l'échantillon. Cette proportion augmente à 83% si on y ajoute les cas de firmes qui ont investi un an avant la date prédite par le modèle.

La quatrième étude, *«Time Varying Risk and Returns: Evidence From Mining Industry Data»*, entreprend une approche des modèles conditionnels du CAPM qui contraste sous plusieurs aspects avec les autres travaux qui ont actuellement cours dans la littérature: en premier lieu, nous supposons à la fois que la prime de risque et les bétas sont conditionnels et leur caractérisation procède par une modélisation avec des paramètres qui varient par rapport au temps. En second lieu, les portefeuilles sont construits sur la base de la moyenne pondérée de la valeur de toutes les firmes composant chacune des industries considérées (mines métalliques, mines intégrées et mines d'or). Nos résultats suggèrent i) le non rejet du modèle conditionnel du CAPM intertemporel en faveur du CAPM standard et que ii) pour au moins un portefeuille le modèle non conditionnel du CAPM n'est pas rejeté. En outre, nos résultats révèlent que la relation risque-rendement exhibe un comportement non linéaire dû au caractère cyclique de l'industrie minière et des différents chocs auxquels elle a fait face durant la période 1971-1990.

Testing the Impact of Taxation on Capacity Choice:

A "Putty Clay" Approach With Uncertainty

I. Introduction

This work studies the impact of taxation on the capacity choices of several Canadian non ferrous mines[1]. We assume that the technology is "putty clay": *ex ante*, the firm has the flexibility to choose among many input combinations in order to maximize its objective function; ex post, once the optimal combination has been selected, the firm must do with it, and the only flexibility that remains is the option either to operate at full capacity or to shut down. We also assume that the output price is characterized by a geometric Brownian motion.

This is indeed a very simple set up; but there is strong evidence in its favour and, considering the lack of reliable empirical results in this area, it is surprising that it has not been used in any econometric work on mines that we know of. Indeed, despite some earlier attempts and a strong interest by academics and professionals, some of the hypotheses tests and estimated magnitudes presented in this paper have no counterpart in the literature on mining taxation.

We believe that many industries have similar characteristics as the one studied here, so that our approach could be useful in other areas. Firms in such industries are capital intensive. Investments are bulky and basically irreversible. Prices are subject to wide fluctuations and basically out of firm's control. Taxation and other kinds of

[1]A good introduction to the analysis of taxation under various technology regimes is given by Biørn (1989).

public interventions are important, especially to the initial project development decision. There is a lot of anecdotal evidence that Canadian non ferrous mines correspond to this description. Some harder evidence is also available.

An abundant literature tests Hotelling's rule, and fails to corroborate it; constant production, as implied by the "putty clay" hypothesis is often suggested as an explanation in that literature (for numerous references, see Lasserre 1991). More directly, in earlier work on some of the firms studied here, Lasserre (1984) used a model which allowed two phases in a mine's life: *ex ante*, before any major investment, and *ex post*, once the plant (mine and concentrator) exists. By comparing rates of substitution *ex ante* and *ex post*, he was able to test whether, *ex post*, flexibility was unchanged (the "putty putty" hypothesis), reduced (the "putty semi putty" hypothesis of Fuss 1977), or suppressed (the "putty clay" hypothesis). The last possibility was the only one not to be rejected.

As far as the assumption of a Brownian motion for the output price, we view it as a minor theoretical improvement, but an empirically[2] significant one, over what remains the most common working assumption[3] - perfect certainty - in studying the impact of taxation (especially corporate income taxation) on neoclassical firms. This comment applies to conventional firms (Boadway 1980) as well as extractive firms (Gaudet and Lasserre 1984, Slade 1984). While richer processes might be more

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[2]Bodie and Rozansky (1980) estimate the standard deviation of annual changes in future prices over the period 1950-1976 to 47.2% for copper.

[3]For a theoretical treatment of extraction under Brownian prices, see Pindyck (1980).

adequate in general for resource prices, our copper price data appear to obey a Brownian motion over the period of our study.

Beyond justifications for our assumptions, because mining taxation is so much about the taxation of capital, the literature on capital and investment, as well as its ramification into taxation per se, are relevant to our study. All major approaches to investment theory have been extended to the extractive firms: in particular the case of perfectly malleable capital; the "putty clay" model; the cost-of-adjustment model (see, e.g. Lasserre 1985; Gaudet 1983). As far as taxation is concerned the perfect-malleability model has been extended to extractive firms (Gaudet and Lasserre 1984, 1986) and formula for the calculation of marginal effective tax rates or the cost of capital have been worked out (Boadway et al. 1987, 1989) for various tax systems. However, while these calculations clearly show that taxation creates important price distortions, the corresponding impact on production decisions has never been measured directly, whether at firm level, or at more aggregate levels[4]. In fact, most students of mining taxation (see e.g. Foley and Clark 1984 and Campbell and Wrean 1985) used simulations, most of the time based on a "putty clay" model along the line initiated by Bradley et al. (1981). More recently, some authors have investigated investment and the impact of taxation within models borrowed from the financial literature on option pricing, which they apply to irreversible investment (Mackie-Mason 1990). Again they rely on simulations and assume that taxes and other economic signals do not affect the

[4]To a large extent, this comment also applies to all industries: while tax-adjusted prices, in particular the user cost of capital, are routinely computed and often found to have a significant impact on real decisions, we do not know of any paper establishing that taxes are non neutral in a statistically significant way.

real magnitudes of investment projects. Because our work shares most basic assumptions with these works, it could provide a basis - empirical and methodological - for their extension beyond the realm of simulation into that of statistical inference.

Consequently, the econometric work that we are going to present, although conventional in many respects - it provides estimates of output-price and factor-price elasticities as well as measures of the impact of various tax parameters - fills an old gap and a more recent methodological need. Similarly, our hypotheses tests concern the influence of taxation on real decisions by mines. The profession claims, and most serious researchers believe, that the real impact is significant; however, to our knowledge, our results constitute the first statistical evidence to that effect in resource sectors, and, in any sector, the first time it is established that tax parameters have a significant impact, distinct from that of pre-tax prices. In Section II, we present and solve the theoretical model without taxes; while the "putty clay" assumption facilitates the analysis by implying a constant output over the life of the mine, the optimizing decision still retains some complexity due to the fact that capacity will also determine the operating life of the firm, given its mineral reserves. We present the corresponding version of Hotelling's lemma, discuss the properties of factor demands and output supply, and compare our model with better known ones from the point of view of the scarcity rent and the user cost of capital. In Section III, we introduce taxation explicitly while Section IV is devoted to the description of the econometric work, including data, hypotheses tests, and results.

II. The model without taxes

Let s be the start-up date for some firm. At s, the firm knows with certainty its mineral reserves level $R(s)$ as well as the production function $F(K(s),L(s))$, where K is capital and L is a vector of variable factors. The deposit contains ore whose price after processing, at date t, as metal or concentrate, is $p(t)$. $p(t)$ is assumed to be characterized by the following Brownian geometric motion

$$\frac{dp_t}{p_t} = \alpha_p dt + \sigma_p dz_p \tag{1}$$

where dz_p is the increment of a Wiener process: $dz_p = \varepsilon(t)(dt)^{1/2}$, where $\varepsilon(t)$ is a serially uncorrelated and normally distributed random variable. This implies that the current value of p is known with certainty by the firm whereas future values are random and log-normally distributed around a trend with a variance which grows with the distance into the future. We assume that parameters α_p and σ_p are constant. Under these assumptions, the expected value of $p(t)$, conditional on information available at s, is (see, e.g., Hull 1989)

$$E_s(p(t)) = p(s)e^{\alpha_p(t-s)} \tag{2a}$$

The firm faces two kinds of costs: variable costs $w(t)'L(s)$ incurred during its operating life, and fixed costs $q(s)K(s)$ paid instantaneously at s where $w(t)'$ is the transposed vector of variable factor prices at t, $t \in [s, \infty)$, and $q(s)$ is the price of capital equipment. Since we assume that the mining firm operates under a "putty clay" technology, it has the flexibility to choose factor ratios and levels *ex ante* while, *ex post*, it must operate with the factor combinations selected at s. The sole difference between capital, and variable factors, in that context, is the fact that capital is paid fully at s while variable factors are hired at the going price $w(t)$ during the entire operating life, and it is assumed that

$$E_s(w(t)) = w(s)e^{\alpha_w(t-s)} \tag{2b}$$

We assume that the productive capacity of capital is maintained throughout the operating life at a cost included in the definition of variable costs. At the closing date, however, it does not retain any residual value. This is the most realistic assumption, as this type of capital is highly mine specific and cannot be used elsewhere[5]. Since production is constant during the whole operating life, the terminal date T is

in a mine ???

$$T(s) = \frac{R(s)}{F(K(s),L(s))} \equiv \frac{R(s)}{Q(s)}$$

where Q is the output level.

Given a start up date s, the objective of the firm is to choose its capital and variable-input mix so as to maximize its expected net present value $W(s)$

$$W(s) = E_s \int_s^T e^{-rt}\{p(t)F(K(s),L(s)) - w(t)'L(s)\}dt \; - \; q(s)K(s)$$

under the constraint

$$R(s) = \int_s^T F(K(s),L(s))dt; \; R(s) \; given$$

[5]In fact a significant proportion of capital consists in shafts and galleries which become valueless when operations cease.

The above program implicitly assumes that, once the initial investment has been made, it is not in the firm's interest to close down, temporarily or permanently, before the full exhaustion of its reserves at $T \equiv R(s) / F(K(s),L(s))$. This will indeed occur endogenously if, as we assume, net cash flows at all relevant future dates are such that i) variable costs are covered at all dates and ii) the output price is not expected to grow at such a rate that it might be in the interest of the mine to hold on to its mineral reserves for speculative reasons. Both assumptions seem to be verified in practice for the firms in our sample, at least if we accept that the rate of growth in expected price cannot be wildly different from that of the actual price. Taking s to be zero (without loss of generality), and shifting the expected value operator under the integral, the program may be rewritten as

$$W(K(0),L(0);p(0),w(0)',R(0),r(0),q(0))=$$

$$\int_0^T e^{-rt}\{E_0(p(t))F(K(0),L(0))-E_0 w(t)'L(0)\}dt \;-\; q(0)K(0)$$

After integration using (2), we have

$$W(0)=\frac{(1-e^{-\delta(0)T(0)})}{\delta(0)}p(0)F(K(0),L(0)) - \frac{(1-e^{-\rho(0)T(0)})}{\rho(0)}w(0)'L(0)-q(0)K(0) \tag{3}$$

where

$$\delta(0)=r(0)-\alpha_p \quad ; \quad \rho(0)=r(0)-\alpha_w$$

Since the planning period decreases as Q increases, $W(.)$ is bounded and there is an interior solution which, under mild regularity conditions on F, will be unique. Omitting from here on time arguments where no ambiguity arises, let $W^*(p, q, w', \delta, \rho, R)$ be the optimized net present value for an investment undertaken at date 0. Similarly, let $K^*(p, q, w', \delta, \rho, R)$, $L^*(p, q, w', \delta, \rho, R)$, and $Q^*(p, q, w', \delta, \rho, R)$ be the optimized functions corresponding respectively to K, L, and Q. W^* is analogous to a profit function, except that it gives cumulative discounted net cash flows rather than current profit

$$W^* = \frac{(1-e^{-\delta T^*})}{\delta} pQ^* - \frac{(1-e^{-\rho T^*})}{\rho} w'L^* - qK^* \tag{4}$$

with

$$T^* = \frac{R}{F(K^*,L^*)} = \frac{R}{Q^*} \tag{5}$$

If we differentiate it with respect to p, w', and q, while making use of the envelope theorem, we get a variant of Hotelling's lemma. Instead of current supply, variable-factor demands, and investment demand, we get the following adjusted quantities

$$\frac{\partial W^*}{\partial p} = \frac{(1-e^{-\delta T^*})}{\delta} Q^* = \sum^* (p,w',q,\delta,\rho,R) \tag{6}$$

$$\frac{\partial W^*}{\partial w} = -\frac{(1-e^{-\rho T^*})}{\rho} L^* = \Gamma^*(p,w',q,\delta,\rho,R) \tag{7}$$

$$\frac{\partial W^*}{\partial q} = -K^*(p,w',q,\delta,\rho,R) \tag{8}$$

It is worth noting that the function Q^* in (6) is the capacity selected by the firm at its creation. The function Σ^*, however, reflects the aggregation of this capacity over the whole planning period. W^* can be shown to be convex in p, w, and q exactly in the same way as a static profit function can be shown to be convex (Silberberg 1990, 193). From the convexity of $W^*(.)$, $\partial\Sigma^*/\partial p \geq 0$, $\partial\Gamma^*/\partial w \leq 0$, and $\partial K^*/\partial q \leq 0$. This result is the counterpart of the static price monotonicity properties of factor demands and output supply. Note, however, that a change in output price affects Σ^* via two channels: its effect on Q^*; and its effect on T^*. Consequently, a rise in Σ^* is compatible with a drop in Q^*. However, it turns out that the own-price partial derivatives of Q^* and L^* have the same sign as the corresponding partial derivatives of Σ^* and Γ^*. Indeed, if we differentiate (6) with respect to p, using $T^*=R/Q^*$, we get

$$\frac{\partial \Sigma^*}{\partial p} = \frac{\partial Q^*}{\partial p}\frac{(1-e^{-\delta T^*})}{\delta} + Q^* \frac{1}{\delta}\frac{\partial(1-e^{-\delta T^*})}{\partial p}$$

or

$$\frac{\partial \Sigma^*}{\partial p} = \frac{\partial Q^*}{\partial p}\frac{1}{\delta}[1-(\delta T^*+1)e^{-\delta T^*}] \tag{9}$$

If δ is positive, then the expression between square brackets in (9) is positive. This can be shown by noting that, for $T=0$, this expression is equal to zero and its derivative with respect to T is positive. It follows that the sign of $\partial Q^* / \partial p$ is the same as the sign of $\partial \Sigma^* / \partial p$. Similarly it can be shown that $\partial L^*_i / \partial w_i$ is negative. However, the signs of $\partial Q^* / \partial q$ and $\partial Q^* / \partial \delta$ cannot be determined without ambiguity.

Let us now study the slope of Q^* with respect to R. Let us assume, contrary to economic intuition, that $\partial Q^* / \partial R$ is non positive. Suppose that R increases from R_0 (case *0*) to R_1 (case *1*). Let $Q_i \equiv Q^*(\cdot, R_i)$ and $T_i \equiv T^*(\cdot, R_i)$ be the corresponding optimal instantaneous capacities and extraction periods for $i=0,1$; from the maintained assumption that $\partial Q^* / \partial R \leq 0$, it follows that, in case *1*, it takes more time to extract the entire reserve stock than in case *0*: $T_1 > T_0$. However, since the possibility to extract at rate Q_1 was available to the firm in case *0* and was not adopted,

$$\begin{aligned} &E_0 \int_0^{T_0} e^{-rt}[p(t)Q_0 - w'(t)L_0]dt - qK_0 \\ &\quad \geq E_0 \int_0^{T_0+\Delta} e^{-rt}[p(t)Q_1 - w'(t)L_1]dt - qK_1 \end{aligned} \tag{10}$$

where Δ is the extra time required, beyond T_0, in order to extract R_0 at a rate $Q_1 \leq Q_0$. Since $R_1 > R_0$, $T_1 > T_0 + \Delta$; thus we can write, adding the same amount to both sides of the inequality

$$
\begin{aligned}
E_0 \int_0^{T_0} & e^{-rt}[p(t)Q_0 - w'(t)L_0]dt - qK_0 \\
& + E_0 \int_{T_0+\Delta}^{T_1} e^{-rt}[p(t)Q_1 - w'(t)L_1]dt \\
& \geq E_0 \int_0^{T_0+\Delta} e^{-rt}[p(t)Q_1 - w'(t)L_1]dt - qK_1 \\
& + E_0 \int_{T_0+\Delta}^{T_1} e^{-rt}[p(t)Q_1 - w'(t)L_1]dt
\end{aligned}
\tag{11}
$$

Note that the right-hand side of the inequality is $W^*(\cdot, R_1)$. In case 1, the program on the left-hand side of the inequality is not admissible: it consists in buying capacity Q_0, producing at capacity from 0 to T_0, then waiting without producing for a period Δ, and finally producing over a final interval, but at a rate below Q_0 and using a different variable factor quantity and mix. This final production phase would not be compatible with the "putty clay" hypothesis if Q_0 was the initial choice; however it can be replaced with an interval of production at Q_0, whose duration is reduced in such a way that cumulative extraction is not affected. In Appendix I, we show that such a change to the left-hand side of (11), besides making the corresponding program admissible, would preserve the inequality, if K^* does not decrease as Q^* increases. But if such a program yields a higher payoff than $W^*(\cdot, R_1)$, the latter cannot be optimal. This contradiction implies that the maintained assumption cannot hold, proving that $\partial Q^* / \partial R$ is positive provided technology satisfies the very mild assumption that K^* does not decrease as Q^* increases.

To complete the characterization of the factor-demand and output-supply system corresponding to our "putty clay" model of extraction, let us compare the investment decision rule to its counterpart with malleable capital. Under a flexible neoclassical technology, assuming for simplicity that capital does not depreciate physically, the capital allocation rule is (Gaudet and Lasserre 1986)

$$(p_t - \lambda_t)\frac{\partial F}{\partial K} = q_t r_t - \frac{dq_t}{dt} \tag{12}$$

This is a version of the well-known rule which recommends to choose capital in such a way that the value of its marginal product (the left-hand side) be equal to its rental rate (the right-hand side). Note that the unit value of the marginal product, p_t - λ_t, includes a term, the scarcity rent λ_t, which is specific to the extractive firm and reduces output price by the opportunity cost of the resource. The corresponding capital allocation rule under a "putty clay" technology is obtained from the first-order condition for the maximization of (3) with respect to K

$$\{\, p_0(1-e^{-\delta_0 T_0}) \,-\, \delta_0 T_0 [p_0 e^{-\delta_0 T_0} \,-\, \frac{w_0' L_0}{Q_0} e^{-\rho_0 T_0}]\} \frac{\partial F}{\partial K} \,=\, q_0 \delta_0 \tag{13}$$

The formula has been multiplied throughout by δ in such a way as to be in the same units (flows) as (12). The first obvious implication of the "putty clay" hypothesis is that (13) does not apply at all dates but only at start up time. As in the malleable-capital case the left-hand side can be recognized to give the marginal product value, while the right-hand side gives the rental rate of an asset whose price does not change. The capital gain component, present on the right-hand side of (12), is absent, as equipment is acquired and financed once and for all at start up time. The coefficient of F_K , which expresses, in rental units, the average value of the marginal product of capital, is also quite different. It is divided into two components. The first one gives the gross value of F_K; it would be p_0 if the extraction period was infinite; the fact that p_0 is multiplied by $(1-e^{-\delta T})$ reduces its value to reflect the finiteness of the extraction period. The second component corresponds to the scarcity rent which must be subtracted from the gross value. Its interpretation is as follows: for each marginal increase in K, production will be higher by F_K over T periods; thus T_0F_K measures by how much reserves will be reduced at T because of the marginal increase in K at zero; the corresponding loss in income will not occur until T, when the mine runs out of reserves; its expected value per unit will then be $E_0(p_T) - E_0(w_T'L_0/Q_0)$. An interesting difference between both (12) and (13) on one hand, and their counterparts applying to conventional firms, is that a rise in the discount rate has an ambiguous effect on K (Lasserre 1985), explained by the fact that the effect of such a rise is not only to increase the cost of capital, but also to reduce the resource rent.

The standard procedure used to study the impact of taxation on capital allocation consists in deriving a modified version of (12) under the appropriate tax regime. Under a "putty clay" technology, we have to apply a similar procedure to (13). In fact, since we are interested in all factors of production and in capacity choice, rather than in K only, we will investigate the effect of mining taxation on the entire resolution of problem (3).

III. The value of the firm under taxation

The firm has to pay both the federal and the provincial corporate income taxes, as well as a mining tax called, at the time, the mineral tax. A firm created in Quebec in 1960 (we will use the convention that 1960 is zero in that case) benefits from a tax holiday of $T1$ $(=3)$ years during which it neither pays federal corporate income tax FT, nor provincial corporate income tax PT, but still faces the mineral tax MT. For the subsequent years (from $T1$ to T), it pays all three taxes. In this particular case, all three taxes take the form of some constant rate u_i, $i = F$ (for federal), P (for provincial), M (for mineral), applied to the appropriate tax base. The tax base is defined as revenues minus variable costs, minus allowable deductions. These deductions fall under three major categories: depreciation allowances for equipment, machinery, and structures DC_i, $i= F, P, M$; depreciation allowances related to development expenditures DV_i, $i= F, P, M$; depletion allowances , DP_i, $i= F, P, M$. Of course some of these deductions or allowances may not exist under one tax regime or the other; furthermore, some taxation regimes allow the deduction of some taxes from their tax base while others do not. Thus the total yearly tax bill is

$$\begin{aligned}
\psi(K,L,t,T1,T) &= MT \\
&= u_M(pQ-w'L-DC_M-DV_M) \qquad for \quad 0 \leq t \leq T1 \\
&= FT+PT+MT \qquad\qquad (14) \\
&= u_F(pQ-w'L-DC_F-DV_F-DP_F-MT) \\
&\quad +u_p(pQ-w'L-DC_p-DV_p-DP_p-MT) \\
&\quad +u_M(pQ-w'L-DM_M-DV_M-DP_M) \qquad for \quad t>T1
\end{aligned}$$

Note that the mineral tax is deductible from both the federal and the provincial corporate income tax here. It has been shown that the depletion allowances of both the provincial and the federal systems amounted in 1960 to reducing the relevant tax rate u_j by a coefficient of $(1-\alpha^j)$, where α^j is the deduction rate applying to the depletion allowance, $j = F, P$. Depreciation allowances are measured in the standard way by observing that, in tax regime j, they have the effect of increasing after-tax income, for each dollar of the corresponding asset base, by an amount equal to the relevant tax rate u_j multiplied by the allowed depreciation rate θ^j_i on assets of type i, $i = E$ (for material-equipment), M (for machinery), and S (for structures). There are two major types of depreciation allowances: the first type, geometric depreciation, is based on the residual value at date t, $\Phi^j_i(t)$, of the relevant asset i in tax regime j; the second one, linear depreciation, is based on acquisition value C^j_i. Since our data do not include a breakdown of asset types, we call C_A^j and Φ_A^j the asset values corresponding to the total of E, M, and S, and we use an aggregate depreciation rate

$$\theta^j_A = \sum_i \theta^j_i \gamma_i$$

where γ_i is the share of asset type i in capital at the industry level. The combined effect of capital depreciation allowances under the mineral tax (which is based on $C_A{}^M$) is then to increase after-tax income by $u_M \theta_A{}^M C_A{}^M$ at all future dates. Similarly the effect of depreciation allowances for development expenditures (based on $\Phi_D{}^M(t)$) is to increase after-tax income by $u_M \theta_D{}^M \Phi_D{}^M(t)$ in all periods of the asset's tax life. Thus the unique initial capital expenditure $q(0)K(0)$ incurred by the firm at start up time implies an array of future tax deductions. We do not investigate possible distortive effects of the tax system with respect to the mix of capital expenditures. Note that, at acquisition date, which we identify with start-up, $C_A{}^j = \Phi_A{}^j$ for all j, and total capital expenditures are

$$q(0)K(0) = C_A^j(0) + C_D^j(0) = \Phi_A^j(0) + \Phi_D^j(0)$$

Using these principles, supplemented by relevant details from the tax laws, we can write the expected net present after-tax value of the Québec 1960 mine, as the sum of the contributions of the first T_1 years (tax holiday) during which the mine pays only mineral taxes, and the contribution of the remaining period, over which the firm pays all three taxes.

$$\begin{aligned}
W^{\tau} = {} & E_0 \{ \int_0^{T1} e^{-rt}[p(t)Q(0)-w'(t)L(0)](1-u_M) \\
& + u_M(\theta_A^M C_A^M + \theta_D^M \Phi_D^M) dt \} \\
& + E_0 \{ \int_{T1}^{T} e^{-rt} \{ [(1-u_F(1-a^F)(1-u_M) \\
& \quad - u_P(1-a^P-u_M)][p(t)Q(0)-w'(t)L(0)] \\
& \quad + u_F(1-a^F)(\theta_A^F \Phi_A^F + \theta_D^F \Phi_D^F) \\
& \quad + u_P(1-a^P)(\theta_A^P C_A^P + \theta_D^P \Phi_D^P) \\
& \quad + u_M(1-u_F(1-a^F))(\theta_A^M C_A^M + \theta_D^M \Phi_D^M)] \} dt \} \\
& - q(0)K(0)
\end{aligned} \tag{15}$$

Let us now turn to the evaluation of the tax components in (15). In order to implement the standard approach summarized in Boadway (1980), we treat any geometric capital depreciation as if it continued forever as described. This means that the undepreciated component at T is either negligible or can be transferred to other operations. Both possibilities are in fact a good approximation of reality. It follows that the present value of depreciation allowances is independent of T and of any residual capital remaining at T. For tax regime j and asset i, let

$$\underline{x_i^j}=\int_0^{T1}\theta_i^j e^{-rt}dt \; ; \; \overline{x_i^j}=\int_{T1}^{\frac{1}{\theta_i^j}}\theta_i^j e^{-rt}dt \; ; \; \overline{\overline{x_i^j}}=\int_{T1}^{T1+\frac{1}{\theta_i^j}}\theta_i^j e^{-rt}dt \qquad (16)$$

be present values of straight line depreciation allowances associated with an initial capital or development expenditure of one dollar. The first one gives the discounted sum of the deductions over the first T_1 years, when depreciation starts being claimed during the tax holiday; the second gives the value of the continuation after T_1 and until exhaustion of the base, at $1/\theta$; the last expression covers the case where depreciation allowances can start being claimed only after the tax holiday has ended. Similarly, for geometric depreciation (omitting subscripts and superscripts)

$$\underline{z}=\int_0^{T1}e^{-rt}\theta dt \; ; \; \overline{z}=\int_{T1}^{\infty}e^{-(r+\theta)}\theta dt \; ; \; \overline{\overline{z}}=\int_{T1}^{\infty}e^{-rt}e^{-\theta(t-T1)}\theta dt \qquad (17)$$

Let us now substitute (16) and (17) into (15), and distribute the expected value operator under the integral terms. As in the case without taxes, let us further assume that

$$E_0\{e^{-rt}w(t)'L(0)\}=e^{-\rho t}w(0)'L(0) \qquad (18)$$

Then, after integration

$$
\begin{aligned}
W^{\tau} =& (1-u_M)[\frac{(1-e^{-\delta T1})}{\delta} pQ - \frac{(1-e^{-\rho T1})}{\rho} w'L] + u_M(\underline{x}_A^M C_A^M + \underline{z}_D^M \Phi_D^M) \\
&+ (1-u_1)[\frac{(e^{-\delta T1}-e^{-\delta T})}{\delta} pQ - \frac{(e^{-\rho T1}-e^{-\rho T})}{\rho} w'L] \\
&+ u_F(1-a^F)(\bar{\bar{z}}_A^F \Phi_A^F) + \bar{\bar{z}}_D^F \Phi_D^F)) \\
&+ u_P(1-a^P)(\bar{\bar{x}}_A^P C_A^P + \bar{\bar{z}}_D^P \Phi_D^P) \\
&+ u_M[1-u_F(1-a^F)-u_P(1-a^P)](\bar{x}_A^M C_A^M + \bar{z}_D^M \Phi_D^M)] \\
&- qK
\end{aligned}
\tag{19}
$$

with

$$
u_1 = u_F(1-a^F)(1-a^M) + u_P(1-a^P)(1-u_M) + u_M
$$

Remembering that, at start-up time, $C_i^j = \Phi_i^j$, we can write $C_i^j = \Phi_i^j = \gamma_i qK$, where γ_i is the proportion of asset i in initial capital expenditures. The capital allocation formula corresponding to (13), obtained by setting the derivative of (19) with respect to K equal to zero, is then

$$
\begin{aligned}
\{[(1-e^{-\delta T_1})A_1 + (e^{-\delta T1}-e^{-\delta T})A_2] - \delta T A_2(e^{-\delta T}p - e^{-\rho T}\frac{w'L}{Q})\}\ pF_K \\
= q\delta A_0
\end{aligned}
\tag{20}
$$

where

$$A_0 = 1 - u_M(\gamma_A \underline{x}_A^M + \gamma_D \underline{z}_D^M) - u_F(1 - a^F)(\gamma_A \bar{\bar{z}}_A^F + \gamma_D \bar{\bar{z}}_D^F) \tag{21}$$

$$- u_P(1 - a^P(\gamma_A \bar{x}_A^P + \gamma_D \bar{\bar{z}}_D^P)$$

$$- u_M(1 - u_2)(\gamma_A \bar{x}_A^M + \gamma_D \bar{z}_D^M)$$

and

$$A_1 = 1 - u_M \tag{22}$$

$$A_2 = 1 - u \tag{23}$$

with

$$u = u_F(1 - a^A)(1 - a^M) + u_P(1 - a_P)(1 - u_M) + u_M \tag{24}$$

and

$$u_2 = u_F(1 - a^F) + u_P(1 - a^P) \tag{25}$$

Each observation of a mine at its start-up date is described in a similar way as the 1960 Québec mine just characterized. The formula change depending on the tax regimes and so do parameter values, but the approach remains unchanged. Before deriving our econometric model from this apparatus, let us bring up the simple economics which underlie expression (20). The most important term is A_0, which is a complex combination of two types of tax parameters: tax rates u_i and pseudo tax rates

α^j on one hand[6], and parameters defining various deductions from taxable income (the *z's* and *x's*) on the other hand. In the absence of taxation, or if the combined tax regimes are neutral in the sense that they leave the cost of capital unchanged, then $A_0 = 1$. Otherwise, A_0 is normally smaller than one, perhaps even negative[7]. A_0 is small if two conditions are met: the deductions for capital depreciation and development expenditures are high enough (high *z's* and *x's*) to reduce the tax base(s) substantially; the tax rate(s) is high enough to make such reductions in the base financially interesting. Indeed (21) illustrates the fact that higher tax rates distort factor allocation in favour of capital if depreciation and similar deductions are important. However, the favourable impact on K which may result from a low cost of capital (the right-hand side of (20)) may be mitigated by the impact of taxation on the left-hand side. On that side, which measures the after-tax value of marginal product, we identify three tax parameters or group of parameters, none of which includes deductions for capital expenditures or assimilated provisions. A_1 and A_2 characterize effective tax rates, A_1 when there is a tax holiday (whose duration is then T_1), and A_2 after the expiration of the tax holiday, and whenever there is no such provision. A value of one for either A_1 or A_2 means that the firm keeps one dollar of after-tax income for each dollar of pre-tax income; lower values indicate that the tax bites into income. To the extent that A_1 is more favourable than A_2, long tax holidays are desirable to the firm.

---

[6]Although α^j actually characterizes a deduction, the depletion allowance, its effect is simply to modify the tax rate.

[7]A negative value of A_0 implies that the after-tax cost to the firm of the marginal unit of capital is negative, a situation which is not considered unusual in extractive sectors.

Another remarkable feature, true of all tax regimes in our sample, is that they do not affect the price of variable factors relative to the output price.

Let us now derive the ex ante capacity function. The optimized value function corresponding to (4) is an intertemporal profit function

$$
\begin{aligned}
W^{*\tau}(p,w',q,\delta,\rho,T1,R,A_0,A_1,A_2) = {} & \Big[\frac{(1-e^{-\delta T1})}{\delta}pQ^{*\tau} - \frac{(1-e^{-\rho T1})}{\rho}w'L^{*\tau}\Big]A_1 \\
& + \Big[\frac{(e^{-\delta T1}-e^{\delta T^{*\tau}})}{\delta}pQ^{*\tau} - \frac{(e^{-\rho T1}-e^{-\rho T^{*\tau}})}{\rho}w'L^{*\tau}\Big]A_2 \\
& - qK^{*\tau}A_0
\end{aligned}
\tag{26}
$$

As was done in the previous section, we may derive the analog of Hotelling's lemma

$$
\begin{aligned}
\frac{\partial W^{*\tau}}{\partial p} &= \Big\{\frac{(1-e^{-\delta T1})}{\delta}A_1 + \frac{(e^{-\delta T1}-e^{-\delta T^{*\tau}})}{\delta}A_2\Big\}Q^{*\tau} \\
&= \Sigma^{*\tau}(p,w,q,\delta,\rho,T1,R,A_0,A_1,A_2)
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
\frac{\partial W^{*\tau}}{\partial w} &= -\Big\{\frac{(1-e^{-\rho T1})}{\rho}A_1 + \frac{(e^{-\rho T1}-e^{-\rho T^{*\tau}})}{\rho}A_2\Big\}L^{*\tau} \\
&= -\Gamma^{*\tau}(p,w,q,\delta,\rho,T1,R,A_0,A_1,A_2)
\end{aligned}
\tag{28}
$$

and

$$
\frac{\partial W^{*\tau}}{\partial q} = -K^{*\tau}A_0 \tag{29}
$$

It is easily shown that $W^{*\tau}$ is homogeneous of degree one in (p, w', q) and also homogeneous of degree one in (A_0, A_1, A_2). From this it follows, in particular, that $\Sigma^{*\tau}$ is homogeneous of degree one in (p,w',q,A_0,A_1,A_2). It is also important to note that $\Sigma^{*\tau}$

and $Q^{*\tau}$ do not have the same homogeneity properties: consider (26); multiply $(p, w', q, A_0, A_1, A_2)$ by λ; then, to satisfy the homogeneity of Σ^*, Q^* must remain unchanged. Consequently, Q^* is homogeneous of degree zero in $(p, w', q, A_0, A_1, A_2)$.

IV. Econometric model and empirical results

1. The basic model

Although (26)-(28) could be used as a basis for an econometric model of (capacity) supply and factor demands, our data set allows us to use only equation (26)[8]. Since our sample size does not allow us to use a second degree flexible form, we start from the following linear form

$$
\begin{aligned}
Q_{it}^{*T} = {} & \beta_0+\beta_p P_t+\beta_{WK} WK_{it}+\beta_R R_{it}+\beta_{WM} WM_t+\beta_{WMO} WMO_t \\
& +\beta_{WEN} WEN_t+\beta_\delta \delta_t+\beta_\rho \rho_t+\beta_{A_1} A_{1,it}+\beta_{A_2} A_{2,it}+\beta_{T1} T1_{it}+\nu_{it}
\end{aligned}
\qquad (30)
$$

$i=1,2,\ldots,M$ firms ; $t=1,2,\ldots,N$ periods,

where the variables have the following interpretation (we present and discuss data and sources further below and in the Data Appendix):

t: observation date; an observation is the creation of a new mine or plant (we treat major expansions as creations);

Q_{it}^{*T} : capacity of firm i at date t measured in metric tons of ore (copper) extracted per day;

$\boldsymbol{P_t}$: price of one metric ton of ore, computed from the price of copper;

[8] We do not have reliable data on variable factors or capital equipment, but we do have capacity.

$WK_{it}=q_{it}\delta_{it}A_{0,}$: after-tax cost of capital for firm i at date t, with A_0 defined by the appropriate version of (20), according to date and location;

W_k : price of variable factor k, (with k=*M* (Material), *MO* (labour) and *EN* (Energy));

R_{it} : proven mineral reserves of firm i at date t in millions of metric tons;

$T1_t$: duration of the tax holiday in years;

$A_{2,it}$: after-tax revenue per dollar of gross income, after the tax holiday period, as defined by (23);

$A_{1,it}$: after-tax revenue per dollar of gross income, during the tax holiday period *(0-T1)*, as defined by (22); since the term in A_1 disappears from (25) when there is no tax holiday, A_1 is set equal to 0 whenever $T_1 = 0$; otherwise, $A_1>A_2$;

$\delta_t=r_t-\alpha_{pt}$: adjusted discount rate for revenues, where r_t is the real rate of return on three-year government bonds and α_{pt} is the expected variation of the output price per unit of time, as estimated from price data anterior to t (see (1) and (31), below);

ρ_t: adjusted discount rate for variable costs, as defined by (18) and discussed further below.

Two of the variables in the above list, δ_t and ρ_t, are not directly available. We estimate the first one from the integral of (1)[9]

$$\ln p_t = \alpha + \ln p_{t-1} + \omega_t \tag{31}$$

where ω_t designates the residual component assumed to be independent and identically distributed. We built a market price series for copper from 1940 to 1985. Then, for each mine creation date s in our sample, we estimated (31) over the period *1940-s*. For the output price to follow a random walk as indicated by (1), the parameter estimate of $\ln p_{t-1}$ has to be unitary. Otherwise, the price behaviour is characterized by a mean reverting process. Using the Dickey-Fuller's (1981) unit root test we found that we can reject the random-walk hypothesis at a 5% level[10]. However, in order to avoid contradicting (1), we impose the assumption of random walk in (31). The residuals from the estimation of (31) are serially uncorrelated and each of the estimated α is not significantly different from zero. We used these estimates to compute the adjusted real rate of return δ_t at each of the dates when a mine was created.

With respect to ρ_t, we consider three alternatives. The first alternative is to assume that the firms expect variable-factor prices to follow the same trend as their output price over their operating life. This assumption can be justified by the fact that unions watch operating profits so that, over the long run, wages adjust to output

[9]See McDonald (1991).

[10]Pindyck and Rubinfeld (1991, 462-65) reach the same conclusion for a longer copper price serie.

prices. Under this assumption, $\rho_t = \delta_t$, so that ρ_t can simply be removed from the equation. The second alternative is to assume that $\rho_t = r_t$, as if the firm expected constant future real factor prices. Under this assumption, r_t is substituted for ρ_t in the equation. The third alternative is to assume that ρ_t is the same at all mine creation dates. Under this assumption, ρ_t equals ρ, and cannot be distinguished from the constant. One notes that the first and the third alternatives are observationally equivalent.

2. The data

The data set is made of individual observations on canadian copper mines. An observation is the occurrence of a major capacity investment, which we define as either the creation of a new operation, or any capacity increase exceeding 20% of existing capacity. The data set is neither a time series, nor a cross section, nor a panel: there may be several observations in a given year; there may be years without observations; but date and location are important characteristics of any observation. The sample period covers the period 1960-1980. 14 different firms, some of which were observed more than once, make up a total of 23 observations, out of which 20 are located in British Columbia, 2 in Ontario, and 1 in Québec[11]. Note also that some of the variables in (30) have only the subscript t, whereas others, such as Q, WK, R, $A_i (i=1,2)$ and $T1$, are mine specific. This occurs whenever some variable, some price typically, is common to all firms at the same date. Most of the firm specific data are taken from various editions of the Canadian Mines Handbook; common data are taken from

[11]Over time, the observations are distributed as follows: 1961(3), 1962(4), 1963(1), 1964(1), 1965(1), 1966(1), 1967(3), 1970(1), 1971(1), 1972(3), 1974(1), 1976(1), 1979(1), 1980(1).

various aggregate sources (See the Data Appendix for details). Tax information come from Holland and Kemp (1978).

3. Econometric considerations and theoretical restrictions

Since there are many idiosyncrasies not taken into account by our explanatory variables, a substantial portion of capacity might be left unexplained by the model. If so, one might suspect that the errors in (30) are heteroscedastic, perhaps also spatially correlated. In presence of heteroscedastic residuals, the estimators of a well specified model are consistent and asymptotically inefficient, whereas their estimated variance-covariance matrix is not consistent. White (1980) proposed an estimation method which yields a consistent variance-covariance matrix for the estimates. The merit of this method is that it also constitutes a natural way of testing for the presence of heteroscedasticity by mere comparison between the OLS t-statistics and the t-statistics obtained using White's approach. They differ here, which is evidence of the presence of heteroscedasticity. Consequently, our reported inference tests are based on White's consistent variance-covariance matrix.

Considering the structure of our data bank, the Durbin-Watson statistic, which is based on the assumption that the data are built according to a certain order such as time or contiguity, is meaningless. However it is plausible to believe that there is some correlation between certain error terms. For example, there may be "good" years, and "bad" years, characterized by different error terms. We introduced a dummy taking the value 1 if there is more than one observation that year, and zero otherwise. It was not significant. Error terms might also vary systematically with the date of the observation; we introduced date as an explanatory variable, with no significant results.

The capacity function $Q^{*\tau}$ is homogenous of degree zero in P, W_j, q, A_0, A_1, and A_2. Since they appear only combined with each other in the theoretical model, q and $\underline{A}_0$ are combined (together with δ) into a single variable, WK, in (30). Homogeneity can then be imposed by dividing each of the price variables and tax variable in (30) by one of them, excluding WK as it contains two prices. Homogeneity is rejected statistically. However, because it is an important theoretical characteristic of the model, it is imposed in the results presented below, which are based on variants of

$$\begin{aligned} Q_{it}^{*\tau} = {} & \theta_0 + \theta_{PA_2} PA_{2_{it}} + \theta_{WKA_2} WKA_{2_{it}} + \theta_R R_{it} + \theta_{WMA_2} WMA_{2_{it}} \\ & + \theta_{WMOA_2} WMOA_{2_{it}} + \theta_{WENA_2} WENA_{2_{it}} + \theta_\delta \delta_{it} + \theta_\rho r_{it} \\ & + \theta_{A_1A_2} A_1A_{2_{it}} + \theta_{T1} T1_{it} + \mu_{it} \end{aligned} \tag{32}$$

where a variable whose symbol ends with A_2 has been normalized by A_2, but otherwise retains its previous meaning. The inclusion of r_{it} for ρ_{it} corresponds to the second alternative discussed above for the treatment of ρ.

4. Results

The results of estimating (32) are presented in column 1 of Table 1. The explanatory power is high, as indicated by the R^2 and the F statistic.

[insert Table 1 here]

The fact that r_{it} is not significant also implies a rejection of the hypothesis that firms hold constant real factor-price expectations, and, while not validating any of them, it

Table 1
Parameter estimates for (32) and its variants, using OLS
(t-statistics, based on White's consistent covariance matrix estimator method, between brackets)

	eq.(32)	eq.(33)	eq.(34)	eq.(35)	eq.(36)
intcpt	-80534 (-0.494)	51218*** (3.543)	136880 (1.676)	49643*** (3.74)	50778*** (3.631)
PA_2	2046.6 (0.105)	25881*** (3.11)	23429*** (3.138)	30873*** (3.318)	26741*** (3.249)
WKA_2	-12076 (-1.622)	-6973.3* (1.896)	-6117** (-2.131)	-6555.5* (-1.902)	-7517.5* (-1.971)
R	$0.189x10^{-3}$*** (10.160)	$0.133x10^{-3}$*** (15.117)	$0.175x10^{-3}$*** (5.225)	$0.128x10^{-3}$*** (14.093)	$129x10^{-3}$*** (13.942)
WMA_2	-13324 (-0.504)	-49199*** (-4.015)	-59528*** (-3.30)	-55278*** (-4.121)	-50601*** (-4.103)
$WMOA_2$	12965*** (3.014)	2373.3 (1.137)	3336.3 (1.406)	1858 (1.09)	1609.7 (0.891)
$WENA_2$	-1900.3 (-1.241)	2313.8*** (5.091)	3336.3*** (3.988)	2609.2*** (6.795)	2431*** (5.724)
δ	$-0.101x10^8$ (-0.581)	-	-	-	-
r	$0.101x10^8$ (0.581)	-	-	-	-
A_1A_2	1659.3 (1.169)	915.4 (0.505)	-44137 (-1.209)	1049.8 (0.629)	290.8 (0.178)
T1	2695.3 (0.339)	-	-	-	-
CREAT	-	-	-72294 (-1.188)	-	-
EXPL	-	-	-	5371.5*** (2.502)	-
PROV	-	-	-	-	4714.5** (2.025)
R^2	0.90	0.89	0.90	0.90	0.89
R^2adj.	0.82	0.83	0.84	0.84	0.83
F	11.6***	16.5***	15.5***	15.4***	14.4***

Notes: *, **, *** respectively indicate significance levels of 10%, 5% and 1%.

represents an element in favour of either one of the alternative possibilities raised earlier: $\rho_t = \delta_t$, or ρ constant. The former is rejected to the extent that δ_t is not significant; the latter is compatible with the fact that the constant is significant.

The correlation matrix of the explanatory variables does not provide any clear evidence of a multicollinearity problem. However, it remains that certain common factors affect some variable simultaneously: the adjusted discount factor for output, δ, and the adjusted discount factor for variable inputs, ρ, even under less restrictive assumptions than ours, include the real rate of discount, r, in their definition; δ is also present in *WK*. This led us to remove δ and r from the specification. For similar reasons, we removed T_1: although it equals the length of the tax holiday (*0, 1, 2, or 3* years), T_1 works as a dummy variable taking a value of zero in the absence of tax holiday, and three in most other cases; but when T_1 is zero, A_1 also is zero. Note also that A_0 is also affected by the presence of a tax holiday.

The results of these amendments to the basic equation are reported in the second column, equation (33). The problem of missing variables (in complement to our earlier discussion on the properties of the residuals) is investigated in the next three columns. Recall that each capacity increase of more than 20% is treated as the creation of a new mine. One may doubt whether the same "putty clay" model really applies in both circumstances. We introduced a dummy variable, *CREAT*, which takes a value of one for a creation and zero for a capacity increase. This variable is not significant (equation(34)). Similarly, the type of exploitation, open-pit or underground, is known to imply substantial differences in techniques and organization. The dummy variable

EXPL which takes the value one for open-pit mines, is significant statistically and important numerically (equation (35)). Finally, geographical location is associated with geological characteristics. Although these characteristics are often the same ones that govern the choice between open-pit, and underground, mining, we tried a dummy variable, *PROV*, taking value one for mines in British Columbia (20 observations) and zero for firms in the East (3 observations); the variable is significant and takes a value similar to *EXPL*, implying that it probably measures the same characteristics [equation (36)]. The inclusion of either one of the two dummies just described also constitutes a way to control for any spatial correlation which might be present in the model (Case 1991).

The results reported in Table 1, and the foregoing discussion, lead us to prefer specifications (35) and (36). Both give results of similar magnitudes and statistical significance, and so do their variants (37) and (38), from which we have further removed, with little effect on remaining parameter estimates, the variables $WMOA_2$ and A_1A_2. To facilitate result interpretation, we present elasticity estimates in Table 2, for (37) and (38).

[Insert Table 2 here]

5. Economic magnitudes and the real impact of taxation

Before discussing the magnitudes reported in Table 2, we want to address a major issue: the influence of taxation on real decisions. At this stage, as in most similar studies, we have taken taxation into account in such a way as to correct price data for any distortive effect of taxation. The parameter estimates reported in Table 2 allow us

Table 2
Parameter estimates of eqs.(37) and (38) using OLS
(t-statistics, based on White's consistent covariance matrix estimator method, between brackets)

	equation (37)		equation (38)	
	slopes	elasticities	slopes	elasticities
constant	54332*** (3.357)	—	54252*** (3.441)	—
PA_2 (+)	32673*** (3.218)	4.721	28164*** (3.195)	4.07
WKA_2	-7061.9* (-1.882)	-0.831	-8050.1* (-1.97)	-0.947
R	0.125×10^{-3}*** (11.299)	0.616	0.129×10^{-3}*** (12.0)	0.632
WMA_2	-57389*** (-4.003)	-9.35	-52432*** (-4.046)	-4.046
$WENA_2$	2896.8*** (11.277)	1.202	2685.8*** (10.885)	1.114
EXPL	5484.5** (2.418)	—	—	—
PROV	—	—	5192.5* (1.859)	—
A_0	—	-.830	—	-.946
A_2	—	.489	—	.494
R^2	0.89	—	0.89	—
R^2adjusted	0.86	—	0.85	—
$F_{6,16}$	22.7***	—	21.6***	—

Notes: *, **, *** respectively indicate significance levels of 10%, 5% and 1%.

to compute the elasticity associated with tax variables. Using (37) as an example, the elasticity (at the sample mean) of predicted capacity with respect to A_0 is *-0.83* and the elasticity with respect to A_2 is *0.489*. This implies that changes in taxation may have an impact of a large magnitude on real decision variables. Typically, it seems that deductions of all kinds on average have a stronger effect on capacity than effective tax rates. However, at this stage, we do not know whether this impact is statistically significant, as distinct from the impact of the pre-tax cost of capital. It is desirable to test whether taxes have a significant effect of their own. If they did not we could conclude that taxation is statistically neutral, as if A_0, A_1, and A_2 were all equal to one.

This is not a standard F test consisting in constraining some parameters to take some predetermined values. We have to test wether our model statistically differs from an alternative, non nested, model featuring tax neutrality; in our context the latter is a model where all tax variables are set equal to one. We apply the J test of Davidson and MacKinnon (1981). Thus the competing hypotheses are

$$H_0 : Q = X\beta_0 + \mu_0$$

where $X = (PA_2, EXPL, WKA_2, R, WMA_2, WENA_2)$; and

$$H_1 : Q = Z\beta_1 + \mu_1$$

where Z is a matrix of the same explanatory variables as X, except that the variables A_i $(i=0,2)$ are set equal to one in the computation of PA_2, WKA_2, WMA_2, *and* $WENA_2$.

The J-test consists in embedding the alternatives into a general model using a mixing parameter λ

$$Q=(1-\lambda)X\beta_0+\lambda Z\beta_1+\mu$$

where μ is an i.i.d. error term of mean zero. Davidson and MacKinnon replace the unknown expression $Z\beta_1$ with its estimate $Z\beta^*_1$ under H_1. Therefore, the above equation may be rewritten as

$$Q=(1-\lambda)X\beta_0+\lambda Z\beta_1+\mu$$
$$=(1-\lambda)X\beta_0+\lambda Q_1^*+\mu$$

If H_0 is true, then $\lambda=0$. Since Q^*_1 is asymptotically independent of μ, they suggest testing whether $\lambda=0$ in the above equation by using a likelihood ratio test or the conventional t test, which they call a J test. Note that an alternative procedure might consist in replacing $X\beta_0$ with its estimated value under H_0. Bernanke et al. (1988) discuss and compare alternative non-nested specification tests in a wider set of circumstances involving investment models; they raise the issue of possible intransitivities. Given that there are only two alternatives to be considered here, that possibility is remote. Nonetheless we carry out both alternative procedures and find them both to reject H_1 in favour of H_0. The tests are presented in Table 3. Using the first procedure, we find that λ, the coefficient of Q_1^*, is not significantly different from zero, implying that the model with the actual tax parameters is significantly better than the model in which tax neutrality is assumed. Similarly, using the alternative procedure, we find that the coefficient of Q_0^* is significantly different from zero, and, at *1.04*, certainly not statistically different from one; again tax neutrality is rejected.

[Insert Table 3 here]

Table 3

The neutrality of taxation : J-tests.

H_0: $Q=X\beta_0+\mu_0$ is not rejected		H_1: $Q=Z\beta_1+\mu_1$ is rejected	
intercept	59127*** (2.879)	intercept	-4628.1 (-0.259)
PA_2	37528*** (2.637)	P (+)	-4694.9 (-0.291)
WKA_2	-7113.4* (-1.876)	WK'(+-)	287.74 (0.558)
R	0.16284×10^{-3}*** (3.681)	R (+)	-0.35914×10^{-5} (-0.053)
WMA_2	-65284*** (-3.072)	WM (+-)	9372.3 (0.254)
$WENA_2$	3404.3*** (4.80)	WEN (+-)	-605.2 (-0.219)
EXPL	7379.5* (1.994)	EXPL (+)	-448.7 (-0.202)
Q^*_1	-0.2968 (-0.773)	Q^*_0	1.04** (2.250)
R^2	0.90	-	0.90
R^2 adjusted	0.85	-	0.85
$F_{7,15}$	18.3***	-	18.3***

Notes: *, ** and *** respectively indicate significance levels of 10%, 5% and 1%.

We proceed now to the economic interpretation and the implications of the empirical results reported in Table 2. The highly significant impact of reserves on capacity does not come as a surprise; despite the fact that reserve data are often considered unreliable as indicators of ultimate cumulative extraction, this confirms that proven reserves at start-up date are crucial to the choice of the scale of operation. At roughly .*6*, the elasticity is reasonable: first a figure below one suggests that infinite reserves would not call for infinite capacity; second this magnitude reflects the empirical fact, not otherwise allowed for in our model, that high reserves are often of lower quality. The normalized output price, PA_2, has also a significant impact on capacity. The elasticity is surprisingly high at more than 4, especially considering the fact that our sample is restricted to firms that do invest, thus eliminating observations where capacity would be zero. Such a figure implies that current price is considered a crucial decision variable by the firm, giving weight to the assumption, implied by our choice of a Wiener process, that it is a good indicator of future expected prices. We also note that no other study of (non energy) extractive resource supply has identified any significant output or input price effects. We believe that our results are a direct consequence and confirmation of the "putty clay" assumption: it is probably only at the time capacity decisions are made that extractive firms can react to price signals in a noticeable way. Mines are so capital intensive that we may interpret capacity as the capital stock; in conformity with that interpretation, the net-of-tax cost of capital has a negative impact on capacity. The sign of the normalized energy price would then mean that capital and energy are substitutes, whereas capital and materials would be complements; however, we do not have a good explanation for the abnormally high elasticity with respect to materials, except, perhaps, that this is the least homogeneous input.

V. Conclusion

In this paper we examined the effect of taxation on capacity choice by individual Canadian copper firms during the period 1960-1980. Our model is characterized by a putty clay technology and a stochastic output price. Tax provisions from three administrative regimes, differing according to date and location, were combined into three basic tax variables: one variable associated with the provisions of the tax holiday; one variable reflecting the combined effective tax rate; the third variable measuring the generosity of various depreciation allowances and assimilated tax features. The empirical results confirmed the important role of the last two variables, as well as the role of prices and geological variables. Although generally considered important by both academic and industry specialists, such role had not been identified precisely in any earlier econometric study. Our modelling approach probably accounts for the fact that our results were conclusive where others were not. Since several other industries exhibit similar characteristics as mines, many empirical studies of taxation should probably be carried out within "putty clay" models, along the methodological lines presented in this paper, rather than under the prevailing full-malleability assumption. Another innovation was our attempt to separate out the role of taxation from that of pre-tax prices in statistical tests.

Many issues remains unaddressed in the area of investment and capacity decisions, however. Perhaps the most important one is the timing of such decisions. This paper has addressed the issue of capacity choice given that investment was occurring. How did the firms choose the date of their investment is another, difficult, issue. When investment is irreversible and bulky, and must be made under uncertainty, it is well known that the positive net-present-value criterion does not apply. Waiting yields information, whose value must be incorporated into the decision process. For a few years, several authors (see Pindyck 1991 and the references therein) have started to adapt the theoretical framework of option theory to this problem, thus treating an irreversible physical investment as the exercise of an option, with uncertainty arising from markets or taxes (Mackie-Mason 1990). However, their simulations have been based on investment projects whose real magnitude was independent of economic conditions. In this paper, the physical size of the projects is sensitive to economic variables. In fact it is possible to simulate from our results what capacity would have been selected by any given mine if it had chosen a different start-up date. This opens an opportunity to test the validity of option theory as a theory of real investment, which we hope to exploit in another paper[12].

[12] See Harchaoui and Lasserre (1992).

Appendix I

A sufficient condition for $\partial Q^*/\partial R > 0$

Because Q_0 solves the firm's maximization for case 1

$$E_0 \int_0^{T_0+\Delta} e^{-rt}[p(t)Q_1 - w'(t)L_1]dt - qK_1$$
$$\leq E_0 \int_0^{T_0} e^{-rt}[p(t)Q_0 - w'(t)L_0]dt - qK_0$$

where Δ is the extra time required to extract R_0 at rate Q_1 rather than Q_0. If K^* rises as Q^* rises, then it follows that

$$\int_0^{T_0} [e^{-\delta t}pQ_0 - e^{-\rho t}w'L_0]dt - \int_0^{T_0+\Delta} [e^{-\delta t}pQ_1 - e^{-\rho t}w'L_1]dt \geq 0 \qquad \text{(A.1)}$$

here expected values have been explicited using (2). Noting that $T_0 = R_0/Q_0$ and $T_0+\Delta = R_0/Q_1$, let us show that this inequality also must hold for other values of R, at which neither Q_0 nor Q_1 might be optimal. To see this, call $D(R)$ the left-hand side of (A1) and note that D(0)=0. Then study the derivative of D

$$\frac{dD}{dR} = \frac{1}{Q_0}[e^{-\delta t}pQ_0 - e^{-\rho t}w'L_0] - \frac{1}{Q_1}[e^{-\delta t}pQ_1 - e^{-\rho t}w'L_1]$$
$$= e^{-\rho t}[\frac{w'L_1}{Q_1} - \frac{w'L_0}{Q_0}]$$

The sign of dD/dR is independent of R. In order for $D(R_0)$ to be non negative, with $D(0)=0$, dD/dR must be non negative for all $R \geq 0$. Note that this implies that unit variable costs diminish as Q increases. Thus we have shown that

$$\int_0^{R/Q_0} [e^{-\delta t} p Q_0 - e^{-\rho t} w' L_0] dt \; - \; \int_0^{R/Q_1} [e^{-\delta t} p Q_1 - e^{-\rho t} w' L_1] dt \geq 0 \qquad \text{(A.2)}$$

We want to use this in order to show that

$$E_0 \int_{T_0+\Delta}^{T_1} e^{-rt} [p(t) Q_1 - w'(t) L_1] dt \leq E_0 \int_{T_0+\Delta}^{T_1-\Delta'} e^{-rt} [p(t) Q_0 - w'(t) L_0] dt \qquad \text{(A.3)}$$

where Δ' is defined in such a way that the same cumulative amount is extracted under each of the sub-programs. If this inequality holds, then we can use its right-hand side to replace the second term on the left-hand side of (11) without changing the direction of the inequality. Defining R/Q_1 *as* $[T_1-(T_0+\Delta)]$, which implies, given the definitions of Δ and Δ', that R/Q_0 equals $[(T_1-\Delta') - (T_0+\Delta)]$, (A3) is equivalent to

$$\begin{aligned} &E_0 \int_0^{R/Q_1} e^{-r(t+T_0+\Delta)} [p(t+T_0+\Delta) Q_1 - w'(t+T_0+\Delta) L_1] dt \\ &\leq E_0 \int_0^{R/Q_0} e^{-r(t+T_0+\Delta)} [p(t+T_0+\Delta) Q_0 - w'(t+T_0+\Delta) L_0] dt \end{aligned} \qquad \text{(A.4)}$$

Since, after expliciting the expected values, (A4) differs from (A2) only by proportional discount coefficients, it must also hold true, which proves the desired result.

Appendix II

This appendix gives the sources of the variables used in eq.(31).

1-List of mining firms, their province of operation, and their creation, or major expansion, dates.

1. Craigmont Mines, British Columbia, 1961.
2. Vauze Mines, Quebec, 1961.
3. Bethlehem Copper, British Columbia, 1962.
4. Craigmont Mines, British Columbia, 1962.
5. Coast Copper Co. Ltd., British Columbia, 1962.
6. Phoenix Copper, British Columbia, 1962.
7. _______________, 1963.
8. Bethlehem Copper, British Columbia, 1964.
9. Minoca Mines, British Columbia, 1965.
10. Bethlehem Copper, British Columbia, 1966.
11. Granisle Mines, British Columbia, 1966.
12. Bethlehem Copper, British Columbia, 1967.
13. Prace Mining Corp., Ontario, 1967.
14. Munro Mines, Ontario, 1967.
15. Geco Mines, British Columbia, 1970.
16. Bethlehem Copper, British Columbia, 1971.
17. Lornex Mining Corp., British Columbia, 1972.
18. Bell Copper Mines, British Columbia, 1972.

19.Granisle Mines, British Columbia, 1972.

20.Lornex Mining Corp, British Columbia, 1974.

21.Bethlehem Copper, British Columbia, 1976.

22.Lornex Mining Corp, British Columbia, 1979.

23.Equity Silver Mining Corp., British Columbia, 1980.

2. Q=capacity of the mining firm measured in metric tons per day.

source;

Canadian Mines Handbook, different issues.

3. R=proven reserves in 10^6 metric tons.

source;

Canadian Mines Handbook, different issues.

4. P=price of one metric ton of copper in nominal Canadian dollars (New York metal exchange).

sources;

-price of one metric ton of copper in nominal American dollars at the New York metal exchange: Commodity Trade and Price rends, Word Bank, different issues.

-exchange rate between Canadian and American dollars: Canadian Economic Observer, Historical Statistical Supplement, 1986, Table 10.2, series 3400.

5. q=implicit price index of machinery and equipment, base 100=1971.

sources;

- from 1963 to 1980 (base 100=1971): Review of the Bank of Canada, november 1982, "Gross National Expenditure: Implicit price indexes" series, Table 54, S 115.

- from 1960 to 1962 (base 1971=100): Historical Statistics of Canada, Table K 172-183, column 181.

6. $\delta = r - i - \alpha_p$,

r=Long-term Canada bond rate.

sources;

Canadian Economic Observer, Historical Statistical Supplement, 1986, Table 10.1, serie 14013.

i=rate of inflation in Canada as measured by the rate of growth of the Consumer price index (base 100=1971).

sources;

-from 1960 to 1975 (base 100=1971): Historical Statistics of Canada, series K 8-18, column 8.

- from 1976 to 1980 (base 100=1981): Canadian Economic Observer, Supplement, 1986, Table 3.2 (all items)

α_p=yearly estimated rate of growth of the real output price.

sources;

-nominal copper price at the New York metal exchange (1940-1986): same as 3.

-implicit price index of Gross Domestic Product (base 100=1986): Canadian Economic Observer, Supplement, 1986.

-α_p is estimated by OLS. The sample period is from 1940 to s, the date of creation of the mining firm i.

7. A_0=unit price net of tax savings (in terms of depreciation allowances) of an asset.

sources:

Castonguay A. (1984); Un Modèle à Capital Putty-Clay pour Éstimer l'Impact de la Taxation Minière sur le Choix de Capacité des Mines au Canada: 1960-1980. Unpublished MA essay. Département de science économique, Université de Montréal.

and

Harchaoui T. M. (1990); Banque de Données pour l'Estimation de l'Impact de la Fiscalité sur le Choix de Capacité de Mines au Canada: 1960-1980, miméo, Centre de Recherche et de Développement Économique, Université de Montréal.

8. *WM*=sales price index for manufacturing industries.

sources:

-from 1960 to 1975 (base 100=1971): Historical Statistics of Canada,series K 68-107, column 68.

-from 1976 to 1980 (base 100=1971): Bank of Canada Review, "Other prices and costs", Table 63, series D500000, october 1984.

9. *WMO*=index of the nominal wage rate in the mining industry.

sources:

Statistique Canada; revue générale sur les industries minérales, # 26-201, Ottawa.

10. *WEN*=Divisia price index of energy in the mining industry.

sources:

Statistique Canada; consommation de combustibles et d'électricité achetés par les industries manufacturières et minérales et par les centrales thermiques des services d'électricité, hors série, # 57-506, Ottawa.

Statistique Canada; Revue générale sur les industries minérales, # 26-201, Ottawa.

11. A_1= After-tax revenue per dollar of pre-tax income, during the tax holiday; A_2 = idem, but beyond the tax holiday.

sources:

Castonguay (1984) and Harchaoui (1990).

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Le Choix de Capacité Comme l'Exercice d'une Option d'Achat Financière: Application à la Firme Minière

I. Introduction

La théorie des marchés pour les biens contingents, depuis les contributions majeures d'Arrow (1953) et Debreu (1959), a joué un rôle crucial dans la caractérisation d'un optimum de Pareto dans une économie en présence d'agents averses au risque[13]. L'absence dans la réalité de ce genre de marchés ne permet pas d'en remettre en cause la portée; les marchés des actifs contingents peuvent en effet partiellement combler cette lacune (Radner 1970). Les marchés de l'assurance et des actions boursières en sont des exemples traditionnels, celui des options financières en constitue le plus récent.

Le développement de la théorie des options financières est dû à Black et Scholes (1973) et Merton (1973). Black et Scholes ont formulé, sous certaines hypothèses simplificatrices, une solution au problème de l'évaluation des options financières. Ils ont en outre observé - à la suite de Merton[14] - que la solution d'équilibre au problème de la valeur des options financières pouvait être appliquée à n'importe quel actif (financier ou réel) ayant les propriétés d'un actif contingent[15]. La portée d'une telle

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[13] Pour un survol des questions sur l'évaluation des actifs financiers de même que les différents développements théoriques en finance basés sur la théorie de l'équilibre général, voir Duffie (1991). Pour un exposé systématique voir Duffie (1992).

[14] Voir Black et Scholes (1973, 641).

[15] En particulier, ils avaient mis de l'avant la proposition selon laquelle les actions ordinaires d'une firme endettée pouvaient être considérées comme des options d'achat ayant pour prix d'exercice le montant du principal et pour date d'échéance celle de l'obligation.

proposition n'a été saisie dans la littérature économique que bien plus tard lorsqu'on s'est rendu compte qu'un investissement irréversible s'apparentait à l'exercice d'une option.

La théorie de l'investissement est en effet apparue sous un jour nouveau dès lors que l'on y a incorporé irréversibilité et information incomplète. Le fait qu'un investissement irréversible représente un acte impliquant la renonciation à toute information à venir susceptible d'accroître les bénéfices des agents peut justifier d'ajourner la décision d'investir (Cukierman 1980)[16]; il devient alors nécessaire d'amender le critère usuel de la valeur actuelle nette (VAN) pour qu'il tienne compte d'une prime additionnelle reflétant la compensation exigée par l'agent pour la perte d'options futures qu'il subit au moment de l'investissement (Baldwin 1982)[17]. Cette veine de la littérature a cependant commencé par ignorer les marchés financiers. Les règles de décision peuvent alors ne pas être compatibles avec le critère de maximisation de la richesse des actionnaires; en outre, l'introduction de l'hypothèse d'aversion au risque peut compliquer singulièrement la structure du modèle (Majd et Pindyck 1987).

L'approche financière permet de surmonter ces deux objections. D'une part, le théorème de séparation s'applique, ce qui concilie les décisions d'investissement et de maximisation de richesse. D'autre part, les problèmes d'aversion au risque sont pris

[16] Ce phénomène d'ajournement de la décision d'investissement a également un impact au plan macro-économique dans la mesure où elle expliquerait la présence des cycles économiques. Voir à ce sujet Bernanke (1983).

[17] Henry (1974) avait déjà fait état du caractère différent d'un investissement irréversible, notamment de la nécessité de tenir compte d'un coût d'opportunité additionnel qui lui est spécifique.

en compte par le recours à la théorie du CAPM. L'adaptation de la théorie des options financières à l'analyse des décisions de gestion en présence d'irréversibilités a ainsi permis d'appréhender des questions aussi variées que la caractérisation des seuils de fermeture (McDonald and Siegel 1985), le choix de flexibilité technologique (Triantis et Hodder 1990), le choix de capacité de production (Pindyck 1988), la résolution des problèmes de non différentiabilité qui se posent fréquemment dans les problèmes de taxation non-linéaire (Mackie-Mason 1990) et, finalement, l'amendement du critère usuel de la VAN (McDonald et Siegel 1986; Ingersoll et Ross 1992). L'un de ses champs les plus fructueux est cependant la théorie des options financières appliquée aux actifs réels où les investissements irréversibles sont traités comme l'exercice d'une succession d'options d'achat financières (Brennan et Schwartz 1985; Majd et Pindyck 1987; Paddock, Siegel et Smith 1988).

D'une façon générale, cette littérature reste néanmoins théorique. Pour qu'elle ait la portée pratique qui lui manque, il faut s'attaquer à deux tâches. La première consiste à modifier les modèles théoriques de façon qu'ils reflètent le fait que les caractéristiques physiques des investissements réels sont elles-même endogènes. Malgré certaines tentatives en cette direction (par exemple Brennan et Schwartz 1985), la plupart des modèles disponibles traitent le projet d'investissement, et son coût, comme des constantes indépendantes de l'environnement économique prévalant au moment où l'investissement est effectué. En réalité un investissement réel présente des caractéristiques techniques qui sont choisies aux alentours de la date d'investissement en fonction, notamment, des prix des facteurs et du prix de la production, ainsi que du régime fiscal, prévalant à ce moment et attendus à ce moment pour l'avenir. La deuxième tâche consiste à concevoir des façons de confronter ces modèles avec des données réelles, c'est-à-dire de les tester. Jusqu'ici, les seules applications ont consisté en des simulations, qui ont surtout valeur d'illustration.

Appliquer la théorie des options financières à l'étude des investissements réels irréversibles permet de trouver réponse à la question suivante: à quelle date est-il optimal de payer un montant irrécupérable I pour mettre en oeuvre un projet dont la valeur V résulte des flux monétaires stochastiques que ce projet produira après sa réalisation. A notre connaissance, les seuls travaux empiriques effectués jusqu'ici dans ce domaine ont consisté à illustrer par des exemples numériques comment pouvait être mise en application cette théorie. Notre contribution vient compléter la littérature économique existante dans la mesure où nous effectuons une première tentative de confronter avec des données réelles les implications de cette théorie. Nous étudions en effet des décisions d'investissement (choix de capacité de production) basées sur des données réelles (mines de cuivre canadiennes) et dont les principaux paramètres sont établis à partir de travaux économétriques. Nous comparons ensuite les dates effectives des investissements avec les dates calculées en recourant à la théorie des options. Cette comparaison ne donne cependant pas lieu à des tests statistiques en vue de la validation ou du rejet de cette théorie, bien qu'il faille voir là l'objectif ultime de notre démarche.

La section II est consacrée à la présentation du modèle, la section III à la présentation des résultats empiriques et, finalement, la section IV tire les conclusions.

II. Le modèle

1. Le choix de capacité

La décision d'investir dans une installation de production peut se décomposer en deux éléments. Le premier est le choix de la capacité de production appropriée; le deuxième consiste à choisir la date de l'investissement. C'est ce dernier qui nous intéresse ici. Mais le choix de la date n'est pas indépendant de la dimension technologique du projet; cette dernière est normalement endogène et affecte à son tour, en général, à la fois la valeur brute du projet et le coût financier de l'investissement. Sa caractérisation constitue donc un préalable indispensable à l'étude du choix de la date de l'investissement. Ces particularités constituent quelques unes des complications intéressantes qui surgissent lorsque l'on passe de la théorie des options financières à celle de l'investissement réel.

Pour représenter les caractéristiques technologiques endogènes de l'investissement, nous reprenons ici le modèle de Harchaoui et Lasserre (1992). Dans ce modèle, la seule variable stochastique est le prix de la production, qui suit un processus brownien. La firme connaît ses réserves minérales et les prix futurs des facteurs variables. Elle choisit la technologie qui maximise l'espérance mathématique de la valeur actualisée des flux monétaires nets futurs. Cet investissement est irréversible et de type *putty clay*: *ex ante,* avant l'investissement, la firme a le choix entre une multitude de technologies; *ex post*, une fois la technologie optimale choisie, la firme est contrainte de l'utiliser sans modification jusqu'à la date terminale où le stock de réserves est épuisé. Sur la base de diverses observations empiriques énumérées dans l'article, nous supposons les coûts variables suffisamment faibles pour éliminer en pratique toute fermeture, temporaire ou définitive, avant l'épuisement des réserves. On notera que les notions de coûts variables ou de facteurs variables ne sont ici utilisées que pour faire la distinction entre le coût initial irrécupérable de l'investissement en capacité et les dépenses futures d'exploitation; ces dernières ne sont variables qu'au sens où les facteurs sont rémunérés aux prix courants, qui peuvent changer d'une année à l'autre, mais non pas au sens où la firme pourrait les influencer par ses décisions de production.

A la date s où la firme minière crée sa capacité de production, sa fonction de profit cumulatif net actualisé s'écrit comme la différence entre V, les revenus futurs nets capitalisés sur la période d'exploitation T et le coût de l'investissement initial I

$$\begin{aligned} J(x,s) &= V_s - I_s \\ &= [\frac{(e^{-\delta s}-e^{-\delta T1})}{\delta}A_{1,s} + \frac{(e^{-\delta T1}-e^{-\delta T})}{\delta}A_{2,s}]P_s Q_s \\ &- [\frac{(e^{-\rho s}-e^{-\rho T1})}{\rho}A_{1,s} + \frac{(e^{-\rho T1}-e^{-\rho T})}{\rho}A_{2,s}]W_{l,s}' L_s \\ &- W_{k,s}K_s A_{0,s} \end{aligned} \tag{1}$$

où, J étant une fonction optimisée, les variables de choix que constituent la capacité Q, le vecteur des facteurs variables L, la durée d'exploitation T (=R/Q) et le niveau de capital K, sont toutes à leur niveau optimal établi en fonction des variables exogènes courantes que sont le prix de la production P, le vecteur des prix des facteurs variables W_l, le prix du capital W_k, les réserves minérales R, les variables décrivant la fiscalité

A_0 (coût net après déductions fiscales d'une dépense d'investissement de 1$), A_1 (revenu net après impôt correspondant à 1$ de revenu brut perçu durant le congé fiscal) et $A2$ (revenu net après impôt correspondant à 1$ de revenu brut perçu après le congé fiscal), ainsi que le taux de rendement réel sans risque r qui sert de taux d'actualisation. Au cours de la résolution du problème qui aboutit à la forme optimisée (1), ce taux est corrigé pour la tendance α du prix de l'output lorsqu'il s'agit de capitaliser des revenus; les revenus PQ sont donc capitalisés au taux $\delta=r-\alpha$, et ceci sur une première période s - $s+T_1$ où il y a congé fiscal et sur une deuxième période allant de la fin du congé fiscal à la date d'épuisement T. De même, lorsqu'il s'agit de capitaliser des coûts, le taux est corrigé pour la tendance commune α_w des prix des inputs variables ($\rho=r-\alpha_w$).

Les données disponibles ne permettent pas d'estimer un modèle tel que (1). Cependant, elles permettent d'en estimer une composante, la fonction $Q(P, W, R, \delta, \rho, A_0, A_1, A_2)$, avec son corollaire la fonction $T(P, W, R, \delta, \rho, A_0, A_1, A_2) = R / Q$. Ce sont les valeurs prédites de Q et T, obtenues en remplaçant $(P, W, R, \delta, \rho, A0, A1, A2)$ par leurs valeurs en toute date $t \leq s$, que nous utilisons plus bas pour caractériser le projet. Cette caractérisation est incomplète, puisqu'elle n'inclut ni I, ni L, et nous indiquons plus bas de quelle manière nous la complétons par une paramétrisation des coûts variables et de la dépense d'investissement. En $t < s$, la valeur prédite de Q s'interprète comme la capacité qui aurait été choisie si l'investissement avait été effectué en t plutôt que s.

2. Évaluation de l'option d'investissement

Le modèle qui vient d'être présenté décrit les décisions prises par la firme *si l'investissement a lieu.* Il s'agit maintenant de d'étudier la date optimale de l'investissement en capacité. L'équation (1) peut s'interpréter comme la valeur, à la date d'exercice s, d'une option d'achat financière pour laquelle la valeur de l'actif sous-jacent V (ici la valeur capitalisée des flux monétaires attendus du projet) excèderait le prix d'exercice I (ici le débours initial). Plus généralement, si l'on tient compte de l'éventualité où I excèderait V, la valeur d'une telle option serait, à la date d'exercice

$$\Pi_s = \max(V_s - I_s, 0) \tag{2}$$

Poursuivons avec l'évaluation de l'option d'investissement antérieurement à s. Pour toute date $t<s$ (on supposera désormais s égale à 0 sans que cela ne change quoi que ce soit au raisonnement), V_t et I_t sont stochastiques parce qu'affectés par le prix de l'output lequel suit un mouvement géométrique brownien

$$\frac{dP_t}{P_t} = \alpha dt + \sigma dz \tag{3}$$

On peut en fait prouver (voir l'annexe) que V_t et I_t obéissent aux processus suivants

$$\frac{dV_t}{V_t} = \Lambda \; dt + \sigma \; \Gamma dz \tag{4}$$

et

$$\frac{dI_t}{I_t} = \kappa dt + \sigma \Xi dz \tag{5}$$

Comme l'ont signalé divers auteurs à partir de modèles voisins du nôtre, il découle de (2), combinée avec (4) et (5), que le modèle de choix de capacité de la firme minière avec technologie putty clay s'interprète formellement comme un modèle d'options d'achat financières[18]. Les caractéristiques importantes de celui que nous présentons ici sont l'endogénéité des variables réelles caractérisant le projet (Q, T, L, I), avec, notamment, la relation liant Q et T par le biais de la contrainte d'épuisement des réserves ($T = R/Q$), et l'endogénéité de la date d'exercice s. On sait que, dans ces modèles, cette dernière correspond au premier moment où la valeur du projet dépasse une certaine valeur critique V^c, avec $V^c > I$.

Pour déterminer cette valeur critique, nous adoptons certaines hypothèses simplificatrices. Tout d'abord, nous supposons le coût de l'investissement, ainsi que les coûts variables, proportionnels à la capacité retenue. Nous postulons en outre que ces derniers sont donnés, *ex ante*, par une fonction de coût variable Cobb-Douglas avec progrès technique neutre. Nous avons donc

$$I = W_k A_0 K = W_k A_0 \beta_k Q \qquad avec \quad \beta_k > 0$$

et

$$W_l' L = \beta_0 e^{-\xi t} Q^{\gamma} \; W_e^{\beta_e} \; W_m^{\beta_m} \; W_M^{\beta_M} \qquad (6)$$

où les composantes du vecteur des prix des inputs variables (e=énergie; m=matériaux et fournitures; M=main-d'oeuvre) ont été explicitées, ξ est le taux de progrès technique, et γ caractérise les rendements d'échelle sur les facteurs variables *ex ante*. En vertu de la propriété d'homogénéité des fonctions de coût

[18] Le fait que I_t soit stochastique ne change rien au problème. Le modèle de Black et Scholes (1973) a été généralisé au cas où le prix d'exercice est stochastique par Fisher (1978). Mackie-Mason (1990) présente un modèle voisin du nôtre mais où toutes les variables sont exogènes; Pindyck (1991), outre une analyse synthétique des principaux modèles d'investissement réel irréversible envisageables, fournit une bibliographie très complète sur le sujet.

$$\sum_j \beta_j = 1 \qquad avec \quad j = e, m, M.$$

Sous ces hypothèses on peut exprimer, par unité de capacité, la valeur du projet, le coût de l'investissement, et la valeur de l'option d'investissement: $v = V/Q$; $i = I/Q$; $\pi = \Pi/Q$. Enfin, la fonction π étant homogène de degré 1 dans les prix, tous les prix seront exprimés en termes relatifs par rapport à W_M, dans ce qui suit, et notés $p = P/W_M$ $w_j = W_j/W_M$, $j = k, e, m$.

La forme correspondante de (2) est

$$\pi_s = \max(v_s - i_s, 0) \tag{7}$$

Une particularité nouvelle du modèle est que seule v suit un mouvement géométrique brownien; en effet i est maintenant indépendant de p. Plus précisément, nous montrons en Annexe que v_t suit le processus

$$\frac{dv_t}{v_t} = \lambda(v,t)dt + \sigma\Omega(v,t)dz \tag{8}$$

où

$$\lambda(v,t) = \frac{\dfrac{\partial v_t}{\partial p_t}\alpha_p p_t + \dfrac{1}{2}\dfrac{\partial^2 v_t}{\partial p_t^2}\sigma^2 p_t}{v_t} \tag{9}$$

et

$$\Omega(v,t)=\frac{\partial v_t}{\partial p_t}\frac{p_t}{v_t} \tag{10}$$

Dans ces équations, p est une fonction de v définie implicitement à partir de la définition de v ci-dessous; celle-ci est tirée de la définition de V donnée en (1), compte tenu de (6) et de la normalisation des prix mentionnée plus haut

$$\begin{aligned} v_t = & [\frac{(e^{-\delta s}-e^{-\delta T1})}{\delta}p_t-\frac{(e^{-\rho s}-e^{-\rho T1})}{\rho}\beta_0 e^{-\xi t}Q_t^{(\gamma-1)}w_e^{\beta_e}w_m^{\beta_m}]A_1 \\ & +[\frac{(e^{-\delta T1}-e^{-\delta T})}{\delta}p_t-\frac{(e^{-\rho T1}-e^{\rho T})}{\rho}\beta_0 e^{-\xi t}Q_t^{(\gamma-1)}w_e^{\beta_e}w_m^{\beta_m}]A_2 \end{aligned} \tag{11}$$

Si l'on suppose que la firme détient un portefeuille hypothétique composé, d'une part, d'une position courte de $\partial\pi/\partial v$ unités de v (ou d'un portefeuille de titres parfaitement corrélé avec v) et, d'autre part, d'une option π, la valeur du portefeuille est π - $(\partial\pi/\partial v)v$. Appelons μ le taux de dividende sur v (ou sur les titres correspondants); la position courte implique le versement d'un dividende de $\mu(\partial\pi/\partial v)v$ par période. Le rendement du portefeuille est donc, compte tenu du dividende versé, $d\pi$-$(\partial\pi/\partial v)dv$-$\mu(\partial\pi/\partial v)vdt$. En vertu du lemme d'Ito $d\pi = (\partial\pi/\partial v)dv + (1/2)(\partial^2\pi/\partial v^2)(dv)^2$ où l'on constate à partir de (8) que $(dv)^2$ se réduit à $(1/2)\sigma^2\Omega^2 v^2 dt$. Le rendement du portefeuille est donc pour finir $(1/2)(\partial^2\pi/\partial v^2)\sigma^2\Omega^2 v^2 dt$ - $\mu(\partial\pi/\partial v)vdt$; la disparition de tout élément stochastique en fait un portefeuille sans risque dont le rendement doit donc égaler le taux sans risque r appliqué à la valeur du portefeuille

$$\frac{1}{2}\frac{\partial^2\pi}{\partial v^2}\sigma^2\,\Omega^2 v^2 dt - \mu v\frac{\partial\pi}{\partial v}dt = r(\pi - \frac{\partial\pi}{\partial v}v)dt$$

Soit,

$$\frac{1}{2}\frac{\partial^2\pi}{\partial v^2}\sigma^2\,\Omega^2 v^2 + \frac{\partial\pi}{\partial v}(r-\mu)v - r\pi = 0 \qquad (12)$$

Cette équation différentielle est bien connue dans la littérature (voir Pindyck 1991). Sa solution donne la valeur de l'option d'investir (ou d'attendre) pour la firme. Bien entendu, si $v = 0$, l'option est sans valeur

$$\pi(0,t) = 0\ \forall t \qquad (13)$$

En outre, par définition, au moment s où s'effectue l'investissement, la valeur de l'option se confond avec la valeur nette du projet

$$\pi(v^c,s) = v^c - i \qquad (14)$$

où v^c est la valeur critique de v qui, lorsqu'elle est atteinte, déclenche la décision d'investir. Enfin, puisque s et, de façon équivalente, v^c, sont endogènes, la condition (14) doit survivre à une perturbation légère de v

$$\frac{\partial\pi(v^c,s)}{\partial v} = 1 \qquad (15)$$

La date optimale d'investissement est la date où v atteint pour la première fois sa valeur critique. L'étude de s se confond donc avec celle de v^c et requiert la résolution de l'équation différentielle (12) sous les conditions aux bornes (13)-(15).

Nous faisons l'hypothèse que Ω et $(r - \mu)$, même si elles peuvent varier dans le temps, sont indépendantes de v. Cette hypothèse doit être considérée comme une approximation de la réalité. En fait, on peut constater, à l'examen de (10), que Ω, qui représente l'élasticité de la valeur du projet par rapport au prix, est en général fonction de v. Nous avons cependant vérifié numériquement qu'elle était peu sensible aux variations de v , si bien que l'hypothèse apparait raisonnable. Quant à l'indépendance de $(r - \mu)$ vis-à-vis de v, c'est une hypothèse standard sur le taux d'intérêt sans risque et le taux de dividende. Comme on peut montrer que $(r - \mu) = \lambda$, cette hypothèse, toute standard qu'elle soit, impose cependant des contraintes, que nous n'avons pas cherché à expliciter, sur la paramétrisation du modèle de choix de capacité présenté en Section 2.

Lorsque Ω et $(r - \mu)$ sont indépendantes de v, la solution est (voir l'annexe)

$$\pi(v)=av^{b} \tag{16}$$

avec

$$b=\frac{1}{2}-\frac{r-\mu}{\sigma^2\Omega^2}+\{[\frac{r-\mu}{\sigma^2\Omega^2}-\frac{1}{2}]^2+\frac{2r}{\sigma^2\Omega^2}\}^{1/2}>1 \tag{17}$$

$$v^{c}=\frac{b.i}{b-1} \tag{18}$$

et

$$a = \frac{v^c - i}{(v^c)^b} \tag{19}$$

La démarche empirique va maintenant consister à vérifier dans quelle mesure, pour des valeurs raisonnables des paramètres inconnus du modèle (ξ, β_0, β_e, β_m, γ et β_k), la valeur calculée de v_t, durant les années précédant s, est inférieure à v_t^c, alors qu'à la date de l'investissement, on a $v_s \geq v_s^c$.

III. L'analyse empirique

1. Les données

Pour procéder à la partie empirique, nous faisons appel à des données sur des mines de cuivre canadiennes et aux résultats économétriques obtenus dans un travail antérieur sur les choix de capacité de ces firmes. Ces données sont décrites en détails dans Harchaoui et Lasserre (1992). Elles portent sur des investissements majeurs effectués durant la période 1960-1980. Par investissement majeur, nous entendons l'ouverture d'une nouvelle mine ou toute augmentation de capacité supérieure à 20%. Il arrive donc que l'on observe plusieurs fois la même mine sur l'ensemble de la période considérée. Il peut y avoir plusieurs observations une même année si plusieurs firmes procèdent à un investissement majeur; à l'inverse, certaines autres années, il n'y a aucune observation. Ainsi, cette banque de données de 23 observations ne correspond ni à une coupe transversale ni à une série chronologique.

Dans Harchaoui et Lasserre (1992), nous avons estimé une version linéarisée de la fonction de capacité *Q(.)* mentionnée à la section précédente. Nous avons obtenu l'équation suivante (toutes les variables sont statistiquement significatives au moins à 10%)

$$Q = 54,332 + 32673p - 7061,9w_k - 57389w_m + 2896,8w_e + 0,125\,10^{-3}R + 5484,5X \qquad (20)$$

Par rapport au modèle général de la section précédente, l'équation estimée a subi quelques modifications dont certaines résultent simplement de l'exploitation des propriétés théoriques du modèle, et d'autres de considérations empiriques. Ainsi, dans (20), les variables p, w_k, w_e et w_m sont des prix corrigés en fonction des paramètres fiscaux du modèle, et normalisés de façon à ce que soient respectées les propriétés d'homogénéité de ce dernier. La présence d'un congé fiscal aurait normalement justifié la présence d'une variable supplémentaire qui n'était pas significative et a été enlevée parce que sa présence causait en outre des problèmes de collinéarité. Les deux seules variables fiscales présentes dans (20) sont donc A_0 et A_2 et sont intégrées dans la définition des prix. La variable X a été ajoutée pour faire la distinction entre exploitations à ciel ouvert et mines souterraines. Pour la variable ρ, non disponible, nous avons envisagé trois hypothèses dont l'une, $\rho = \delta$, n'a pas été rejetée par les données; en outre la variable δ ne variant que peu d'une observation à l'autre, elle apparaissait faire double emploi avec la constante et causer des problèmes de collinéarité; elle ne figure donc pas dans (20).

Les dates de chaque observation sont interprétées comme des dates d'exercice s. On a choisi de façon arbitraire une période de 7 années antérieures à chaque date d'investissement s pour étudier les valeurs de v^c et v. Ne disposant des variables fiscales que pour la période 1960-1980, nous n'avons donc pu retenir que les investissements effectués à partir de l'année 1965. Parmi ceux-ci, nous avons également éliminé trois cas où la durée d'exploitation R/Q prédite à partir de (20) était inférieure à une année. L'échantillon s'est trouvé dès lors réduit à 12 investissements majeurs, pour lesquels on a déterminé la capacité annuelle prédite pour chacune des années de la période *[s-7, s]* sur la base de l'équation (20). A cette fin, nous avons mis en place des séries chronologiques pour les variables de prix (les sources sont également dans Harchaoui et Lasserre, 1992) pour chacune des périodes de sept ans précédant les 12 investissements subsistant dans l'échantillon, ce qui nous laisse au total avec une banque de données comprenant 84 observations (12 investissements dont chacun est l'aboutissement d'une période de 7 ans). Bien entendu, R et X sont fixées à leur niveau de s durant les années précédant s. Le Tableau 1 présente quelques unes des principales caractéristiques des mines retenues. On remarquera que les firmes qui affichent une grande capacité de production, comme Lornex Mines, affichent une durée de vie T et un coefficient de capitalisation A nettement au dessus de la moyenne de l'échantillon, et vice versa pour les petites firmes comme Geco Mines.

[insérer ici le Tableau 1]

2. La procédure

Nous calculons v^c et v pour chaque année de la période de sept ans précédant s pour les comparer et, ensuite, déterminer la date optimale d'investissement telle que prédite par le modèle des options. Le calcul de v s'effectue à partir de (11). Outre Q, tirée de (20), et T, qui est égale à R/Q, (11) fait intervenir diverses variables de prix

TABLEAU 1
Les données: Informations générales.

Firme	s	année	T, (A)	Firme	s	année	T, (A)
Bethlehem Copp. (ciel-ouvert)	1966	1960	6 (4.57)	Bell Copper (ciel-ouvert)	1972	1966	7 (5.00)
		1961	7 (4.93)			1967	7 (4.85)
		1962	6 (4.61)			1968	6 (4.05)
		1963	6 (4.76)			1969	6 (3.74)
		1964	6 (4.64)			1970	5 (3.31)
		1965	6 (4.41)			1971	7 (4.48)
		1966	6 (4.60)			1972	8 (5.17)
Granisle Mines (ciel-ouvert)	1966	1960	4 (3.32)	Granisle mine (ciel-ouvert)	1972	1966	11 (6.93)
		1961	4 (3.63)			1967	10 (6.72)
		1962	4 (3.39)			1968	10 (5.76)
		1963	4 (3.49)			1969	9 (5.32)
		1964	4 (3.40)			1970	8 (4.72)
		1965	4 (3.22)			1971	10 (6.40)
		1966	4 (3.36)			1972	12 (7.52)
Bethlehem Copp. (ciel-ouvert)	1967	1961	7 (4.88)	Lornex (ciel-ouvert)	1974	1968	21 (10.44)
		1962	6 (4.57)			1969	21 (9.75)
		1963	6 (4.71)			1970	19 (9.14)
		1964	6 (4.60)			1971	22 (12.01)
		1965	6 (4.36)			1972	24 (13.91)
		1966	6 (4.56)			1973	18 (10.29)
		1967	6 (4.42)			1974	19 (8.40)
Geco Mines (sous-terrain)	1970	1964	6 (4.67)	Bethlehem Copp. (ciel-ouvert)	1976	1970	6 (3.73)
		1965	6 (4.33)			1971	8 (5.06)
		1966	6 (4.59)			1972	9 (5.89)
		1967	6 (4.41)			1973	5 (2.75)
		1968	5 (3.57)			1974	5 (2.49)
		1969	5 (3.24)			1975	9 (5.04)
		1970	4 (2.82)			1976	12 (5.69)
Bethlehem Copp. (ciel-ouvert)	1971	1965	7 (4.97)	Lornex (ciel-ouvert)	1979	1973	19 (10.44)
		1966	7 (5.18)			1974	19 (8.51)
		1967	7 (5.02)			1975	24 (12.80)
		1968	6 (4.20)			1976	26 (12.60)
		1969	6 (3.90)			1977	24 (15.85)
		1970	5 (3.40)			1978	26 (16.41)
		1971	7 (4.70)			1979	25 (10.12)
Lornex (ciel-ouvert)	1972	1966	20 (11.07)	Equity Silver (ciel-ouvert)	1980	1974	3 (1.49)
		1967	20 (11.78)			1975	6 (3.21)
		1968	19 (9.50)			1976	8 (3.75)
		1969	18 (8.85)			1977	6 (3.75)
		1970	17 (8.18)			1978	9 (4.92)
		1971	20 (10.84)			1979	7 (3.00)
		1972	21 (12.62)			1980	11 (3.64)

Notes: s=date de l'investissement, A=valeur prédite de la fonction de capitalisation, T= durée de vie prédite de la firme.

et de fiscalité ainsi que δ et ρ. Nous supposons que $\delta=\rho$, c'est-à-dire que la dérive des prix des facteurs variables est égale à celle du prix de l'output. Cette hypothèse est compatible avec un modèle où les détenteurs des facteurs variables se réfèrent à la tendance d'évolution du prix de l'output pour négocier leurs prix. Nous pouvons alors calculer δ et ρ à partir de leurs définitions $\rho = r - \alpha_w = \delta = r - \alpha$, où r est le taux d'intérêt réel sur les bons du trésor canadien à 10 ans, et α est notre estimation économétrique de la dérive du prix du cuivre dans le processus (3). Cette estimation est effectuée individuellement pour chacun des 12 investissements sur la période *[1945, s]* et donne des résultats très peu différents d'un cas à l'autre. Enfin (11) fait intervenir cinq paramètres inconnus (γ, ξ, β_o, βe, βm) pour lesquels nous avons retenu des valeurs numériques basées sur le bon sens et quelques tâtonnements: $\gamma = 0.5$ ou 1.1 selon que les rendements sont croissants ou décroissants ex ante; $\xi = 0.005$; $\beta_0 = 100$; $\beta_e = 0.4$; $\beta_m = 0.2$. Ces derniers sont communs à toutes les firmes. Un sixième paramètre, $\beta_k > 0$, intervient dans le calcul de i qui, sous l'hypothèse faite plus haut de proportionnalité avec Q, devient

$$i = \frac{I}{Q} = \beta_k w_k A_0 \tag{21}$$

Ce dernier paramètre est ajusté pour chacun des 12 investissements de la manière la plus favorable possible à la théorie, c'est-à-dire de façon à faire coïncider si possible la date d'investissement théorique avec la date effective.

Le calcul de v^c s'effectue à partir des formules (17) et (18). Nous calculons tout d'abord b à partir de (17). Compte tenu que le taux de dividende μ pour les mines métalliques est stable dans le temps, nous utilisons sa valeur moyenne qui est, d'après les donnés publiées par la revue du Toronto Stock Exchange (TSE), de 4%. La valeur de σ est tirée de nos estimations économétriques du processus (3). Pour ce qui est de Ω, nous en établissons la valeur à chaque date s à partir de la formule (A.9) établie en annexe.

3. Les résultats

Les résultats de la simulation nous donnent, pour chaque date $t \in [s-7, s]$ et chaque mine, les valeurs correspondantes des fonctions π et v^c. Nous voulons déterminer la date d'exercice de l'option d'investissement prédite par le modèle d'option défini à partir de (16)-(19). Cette date critique, s^c, est la première observation $t \in [s-7, s]$ telle que $v_t > v^c_t$. Si pour certaines firmes de notre échantillon, la date historique s de création de capacité de production correspond bien à s^c prédite par la théorie, alors nous avons une observation empirique compatible avec la théorie des options appliquée aux actifs réels, et en contradiction avec le critère usuel de la VAN. En revanche, si la théorie prédit que pour un certain nombre de firmes, $s^c < s$, cela signifie que celles-ci auraient dû investir plus tôt que la date effectivement choisie. Cet exercice de vérification est effectué alternativement sous des hypothèses de rendements croissants ($\gamma = 0.5$, tableaux 2a) et de rendements décroissants ($\gamma = 1.1$, tableaux 2b) sur la fonction de coût variable unitaire.

Les tableaux donnent les valeurs de a, b, π, v et v^c pour les 7 années précédant chaque investissement observé. Une astérisque dans la colonne "Année" signale la première année de chaque série de 7 ans où v dépasse v^c, c'est-à-dire l'année s^c où l'investissement aurait théoriquement dû être réalisé. Sous l'hypothèse de rendements décroissants, il s'avère que dans 50% des cas (soit 6 investissements sur 12), la date de création est correctement prédite par la théorie ($s=s^c$). Cette proportion augmente à 66.7% si l'on inclut deux firmes (Bethlehem Copper 1967 et Geco Mines 1970) qui investissent une année plus tard que la date prédite par le modèle (s=1967 et 1970, respectivement contre s^c=1966 et 1969). En revanche, dans 33% des cas la date prédite s^c est franchement antérieure, de 4 années au moins, à la date historique s. Sous l'hypothèse de rendements croissants, on a une amélioration des résultats en faveur de la théorie des options: la proportion de firmes pour lesquelles $s^c=s$ passe de 50% qu'elle était précédemment à 58%, et à 75% si l'on inclut les deux cas où $s^c=s-1$.

La méthodologie utilisée ne permet pas de faire d'inférence statistique concernant la validité de la théorie; nous avons l'intention de consacrer un autre article à ce sujet. Cependant, d'un point de vue heuristique, on remarque qu'un choix de 17 paramètres (γ, ξ, β_o, β_e, β_m et 12 β_k), imparfaitement optimisés en ce qui concerne leur vraisemblance conditionnelle aux données, pour un échantillon de 84 observations, permet d'obtenir l'ajustement que nous venons de décrire. Ce résultat parait favorable à la théorie. S'agissant de la première tentative de confronter la théorie des options appliquée à l'investissement réel avec la réalité, il invite à poursuivre les efforts de vérification.

[insérer les Tableaux 2A et 2B ici]

Finalement, le modèle utilisé ici se prête bien à une vérification du rôle de la fiscalité dans les décisions d'investissements irréversibles. Là encore, malgré

TABLEAU 2.1A

Détermination de la date d'exercice (s^c) de l'option d'investissement pour Bethlehem Copper: rendements croissants (γ=0.5)

Année	v	v^c	a	b	π
		Création de capacité en 1966 (β_k=1.1)			
1960	4.884	7.707	0.624	1.159	3.921
1961	4.941	7.787	0.635	1.152	4.001
1962	5.010	7.485	0.625	1.160	4.050
1963	5.032	6.939	0.615	1.170	4.071
1964	4.965	6.055	0.597	1.190	4.018
1965	5.084	5.928	0.555	1.219	4.032
1966*	5.063	4.687	0.498	1.288	4.020
		Augmentation de capacité en 1967 (β_k=4.2)			
1961	4.894	6.024	0.012	2.878	1.151
1962	4.962	6.176	0.015	2.765	1.219
1963	4.983	5.848	0.011	2.931	1.248
1964	4.917	5.526	0.011	3.016	1.288
1965	5.033	6.027	0.008	3.075	1.126
1966	5.013	5.496	0.003	3.673	1.067
1967*	4.732	4.670	0.001	4.348	1.137
		Augmentation de capacité en 1971 (β_k=0.9)			
1965	5.738	14.450	0.792	1.064	5.081
1966	5.709	10.484	0.745	1.089	4.966
1967	5.386	7.016	0.700	1.123	4.641
1968	4.630	9.012	0.761	1.086	4.024
1969	4.513	5.084	0.674	1.154	3.838
1970	4.359	7.683	0.769	1.088	3.812
1971*	4.466	3.311	0.663	1.195	3.962
		Augmentation de capacité en 1976 (β_k=3.8)			
1970	4.733	27.608	0.639	1.105	3.561
1971	4.861	11.871	0.449	1.238	3.176
1972	5.041	7.858	0.337	1.373	3.110
1973	2.494	32.879	0.757	1.063	1.997
1974	2.389	6.701	0.862	1.052	2.153
1975	3.549	6.429	0.461	1.282	2.336
1976*	3.487	1.482	0.627	1.375	3.496

Notes:

v=flux monétaires unitaires capitalisés,

v^c=valeur critique à laquelle il faudrait exercer l'option d'investissement,

a et b= coefficients de la fonction qui détermine la valeur ex ante de l'option d'investissement.

*=la date d'exercice prédite s^c (c'est à dire s^c tel que v > v^c) correspond à la date historique, s.

TABLEAU 2.2A
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Lornex Mines: rendements croissants (γ=0.5).

Année	v	v^c	a	b	π
		Création de capacité en 1972 ($ß_k$=10.2)			
1966	12.372	27.845	0.110	1.536	5.217
1967	11.713	19.127	0.046	1.840	4.216
1968	10.577	24.841	0.141	1.486	4.702
1969	10.389	14.769	0.024	2.092	3.383
1970	10.440	20.025	0.124	1.541	4.762
1971*	10.539	8.480	$0.11x10^{-2}$	3.593	5.153
1972	10.979	6.403	$0.10x10^{-7}$	9.579	117.0
		Augmentation de capacité en 1974 ($ß_k$=12.7)			
1968	11.657	25.829	0.075	1.643	4.252
1969	11.471	19.841	0.031	1.937	3.545
1970	11.692	21.678	0.074	1.677	4.589
1971*	11.719	10.718	$0.85x10^{-3}$	3.460	4.220
1972	12.146	8.245	$0.16x10^{-6}$	7.467	19.925
1973	9.533	9.747	$0.38x10^{-2}$	2.971	3.071
1974	8.215	2.250	0.234	1.960	14.533
		Augmentation de capacité en 1979 ($ß_k$=12.2)			
1973*	9.674	9.207	0.003	3.073	3.488
1974	8.328	2.122	0.237	1.996	16.293
1975	9.461	5.405	$0.2x10^{-4}$	6.290	29.079
1976	8.294	1.495	0.009	7.626	92872.6
1977	9.587	1.992	$0.8x10^{-18}$	55.562	$0.295x10^{37}$
1978	10.189	2.001	$0.2x10^{-17}$	54.392	$0.103x10^{38}$
1979	8.410	3.235	0.016	3.46	25.505

Notes:

v=flux monétaires unitaires capitalisés,
v^c=valeur critique à laquelle il faudrait exercer l'option d'investissement,
a et b=coefficients de la fonction qui donne la valeur ex ante de l'option d'investissement,
*=la date d'exercice prédite s^c (s^c telle que v > v^c) correspond à la date historique, s.

TABLEAU 2.3A
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Granisle Mines: rendements croissants (γ=0.5).

Année	v	v^c	a	b	π
		Création de capacité en 1966 (β_k=0.8)			
1960	3.535	5.217	0.642	1.172	2.820
1961	3.624	5.211	0.649	1.168	2.920
1962	3.678	5.072	0.642	1.173	2.963
1963	3.681	4.694	0.633	1.185	2.969
1964	3.627	4.125	0.620	1.206	2.929
1965	3.709	4.121	0.585	1.232	2.939
1966*	3.691	3.273	0.535	1.303	2.937
		Augmentation de capacité en 1972 (β_k=3.8)			
1966	7.658	23.786	0.480	1.179	5.298
1967	7.226	16.612	0.406	1.244	4.744
1968	6.353	20.381	0.503	1.174	4.415
1969	6.196	12.620	0.366	1.295	3.881
1970	5.988	17.742	0.518	1.173	4.228
1971	6.162	7.703	0.298	1.420	3.950
1972*	6.451	5.111	0.180	1.718	4.438

Notes:

v=flux monétaires unitaires capitalisés,
v^c=valeur critique à laquelle il faudrait exercer l'option,
a et b=paramétres de la fonction qui donne la valeur ex ante de l'option d'investissment,
s^*=date d'exercice prédite s^c (c'est à dire s^c telle que $v > v^c$) correspond à la date historique, s.

TABLEAU 2.4A
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Geco Mines: rendements croissants (γ=0.5)

Année	v	v^c	a	b	π
Création de capacité en 1970 (β_k=0.5)					
1964	4.945	17.776	0.907	1.025	4.668
1965	4.961	24.916	0.920	1.020	4.711
1966	5.017	16.424	0.893	1.030	4.702
1967	4.693	8.265	0.845	1.055	4.314
1968	3.904	5.079	0.803	1.085	3.518
1969*	3.740	2.590	0.726	1.171	3.401
1970	3.555	3.021	0.768	1.129	3.216

TABLEAU 2.5A
Détermination de la date d'exercice de l'option d'investissement (s^c) pour bell copper: rendements croissants (γ=0.5).

Année	v	v^c	a	b	π
Création de capacité en 1972 (β_k=1.5)					
1966	5.510	18.869	0.727	1.081	4.604
1967	5.198	12.574	0.673	1.114	4.221
1968	4.455	16.254	0.743	1.079	3.725
1969	4.343	9.080	0.639	1.143	3.421
1970	4.200	13.789	0.748	1.081	3.527
1971	4.296	5.955	0.618	1.178	3.441
1972*	4.419	3.986	0.544	1.268	3.582

TABLEAU 2.6A
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Equity Silver Mines: rendements croissants (γ=0.5)

Année	v	v^c	a	b	π
Création de capacité en 1980 (β_k=3.8)					
1974	1.427	19.487	0.934	1.017	1.341
1975	2.234	15.542	0.690	1.100	1.672
1976	2.261	3.315	0.743	1.139	1.882
1977	2.094	4.241	0.672	1.168	1.593
1978*	2.756	2.617	0.571	1.305	2.145
1979	2.332	8.649	0.755	1.090	1.900
1980	2.835	6.938	0.747	1.101	2.352

Notes:

v=flux monétaires unitaires capitalisés,

v^c=valeur critique à laquelle il faudrait exercer l'option d'investissement,

a et b=paramétre de la fonction qui donne la valeur ex ante de l'option d'investissement,

*=la date d'exercice prédite s^c (c'est à dire s^c tel que v > v^c) correspond à la date historique, s.

TABLEAU 2.1B
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Bethlehem Copper: rendements décroissants (γ=1.1)

Année	v	v^c	a	b	π
		Création de capacité en 1966 (β_k=1.05)			
1960	5.006	7.066	0.620	1.166	4.053
1961	5.075	7.113	0.629	1.161	4.143
1962	5.128	6.877	0.620	1.167	4.182
1963	5.156	6.368	0.610	1.178	4.215
1964	5.081	5.575	0.593	1.199	4.162
1965	5.181	5.499	0.554	1.227	4.166
1966*	5.165	4.354	0.497	1.298	4.186
		Augmentation de capacité en 1967 (β_k=3.9)			
1961	5.027	5.504	0.012	2.969	1.416
1962	5.079	5.651	0.015	2.839	1.470
1963	5.107	5.352	0.011	3.016	1.541
1964	5.032	5.064	0.011	3.099	1.602
1965	5.130	5.538	0.008	3.145	1.384
1966*	5.114	5.057	0.003	3.765	1.401
1967	4.822	4.306	0.001	4.454	1.601
		Augmentation de capacité en 1971 (β_k=0.87)			
1965	5.841	14.000	0.794	1.064	5.191
1966	5.817	10.016	0.745	1.090	5.081
1967	5.483	6.657	0.699	1.126	4.752
1968	4.702	8.636	0.762	1.087	4.102
1969	4.568	4.951	0.679	1.153	3.913
1970	4.396	7.528	0.773	1.087	3.861
1971*	4.545	2.319	0.607	1.291	4.282
		Augmentation de capacité en 1976 (β_k=3.9)			
1970	4.771	27.772	0.633	1.107	3.568
1971	4.943	11.701	0.433	1.250	3.188
1972	5.156	7.645	0.315	1.402	3.138
1973	2.534	32.320	0.748	1.065	2.013
1974	2.430	6.580	0.856	1.054	2.183
1975	3.740	5.815	0.417	1.333	2.422
1976*	3.773	1.271	0.599	1.485	4.305

Notes:

v=flux monétaires unitaires capitalisés
v^c=valeur critique à laquelle il faudrait exercer l'option d'investissement,
a et b= coefficients de la fonction qui détermine la valeur ex ante de l'option d'investissement.
*=la date d'exercice prédite s^c (c'est à dire s^c tel que v > v^c) correspond à la date historique, s.

TABLEAU 2.2B
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Lornex Mines: rendements décroissants (γ=1.1).

Année	v	v^c	a	b	π
Création de capacité en 1972 (β_k=6.9)					
1966	12.418	18.164	0.123	1.567	6.389
1967	11.753	12.617	0.057	1.881	5.871
1968	10.605	16.352	0.161	1.506	5.656
1969*	10.407	9.874	0.036	2.119	5.209
1970	10.446	13.377	0.154	1.552	5.874
1971	10.574	5.660	0.002	3.723	15.573
1972	10.034	4.290	0.10×10^{-6}	10.447	7937.04
Augmentation de capacité en 1974 (β_k=9.4)					
1968	11.645	18.970	0.089	1.652	5.131
1969	11.455	14.624	0.041	1.944	4.678
1970	11.671	15.995	0.090	1.681	5.602
1971*	11.714	7.880	0.002	3.518	9.036
1972	12.150	6.064	0.63×10^{-6}	7.786	174.3
1973	9.475	7.168	0.006	3.009	5.516
1974	8.175	1.656	0.311	1.972	19.559
Augmentation de capacité en 1979 (β_k=17.8)					
1973	9.610	13.357	0.001	3.110	1.543
1974*	8.282	3.080	0.161	2.007	11.171
1975	9.458	7.783	0.109×10^{-5}	6.760	4.301
1976	8.318	2.149	0.390×10^{-3}	8.470	24175.9
1977	9.596	2.893	0.156×10^{-35}	74.555	0.260×10^{38}
1978	10.228	2.904	0.603×10^{-37}	77.321	0.720×10^{41}
1979	8.426	4.641	0.005	3.610	11.069

Notes:

v=flux monétaires unitaires capitalisés,
v^c=valeur critique à laquelle il faudrait exercer l'option d'investissement,
a et b=coefficients de la fonction qui donne la valeur ex ante de l'option d'investissement,
*=la date d'exercice prédite s^c (c'est à dire s^c telle que $v > v^c$) correspond à la date historique, s.

TABLEAU 2.3B
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Granisle Mines: rendements décroissants (γ=1.1).

Année	v	v^c	a	b	π
		Création de capacité en 1966 ($ß_k$=0.79)			
1960	3.633	5.385	0.652	1.164	2.927
1961	3.734	5.401	0.661	1.159	3.039
1962	3.774	5.222	0.653	1.166	3.069
1963	3.782	4.839	0.644	1.176	3.079
1964	3.720	4.237	0.630	1.196	3.032
1965	3.788	4.197	0.594	1.223	3.027
1966*	3.773	3.330	0.545	1.292	3.029
		Augmentation de capacité en 1972 ($ß_k$=3.9)			
1966	7.775	23.348	0.463	1.189	5.311
1967	7.332	16.398	0.389	1.256	4.753
1968	6.433	20.220	0.491	1.181	4.424
1969	6.258	12.670	0.355	1.303	3.878
1970	6.028	17.892	0.510	1.177	4.226
1971	6.250	7.641	0.283	1.441	3.968
1972*	6.575	5.025	0.162	1.774	4.563

Notes:

v=flux monétaires unitaires capitalisés,
v^c=valeur critique à laquelle il faudrait exercer l'option,
a et b=paramétres de la fonction qui donne la valeur ex ante de l'option d'investissment,
*=date d'exercice prédite s^c (c'est à dire s^c est tel que que $v > v^c$) correspond à la date historique s.

TABLEAU 2.4B
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Geco Mines: rendements décroissants (γ=1.1)

Année	v	v^c	a	b	π
		Création de capacité en 1970 (β_k=0.45)			
1964	5.109	19.554	0.921	1.021	4.869
1965	5.094	26.023	0.930	1.017	4.872
1966	5.160	17.356	0.907	1.025	4.880
1967	4.819	8.736	0.865	1.046	4.482
1968	3.994	5.441	0.829	1.070	3.650
1969*	3.807	2.639	0.755	1.148	3.502
1970	3.598	3.002	0.790	1.115	3.295

TABLEAU 2.5B
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Bell Copper: rendements décroissants (γ=1.1)

Année	v	v^c	a	b	π
		Création de capacité en 1972 (β_k=1.25)			
1966	5.516	14.937	0.728	1.087	4.749
1967	5.294	10.002	0.678	1.120	4.381
1968	4.526	13.013	0.747	1.083	3.830
1969	4.397	7.357	0.650	1.147	3.553
1970	4.235	11.252	0.756	1.083	3.606
1971	4.373	4.754	0.629	1.187	3.626
1972*	4.526	3.136	0.558	1.289	3.905

TABLEAU 2.6B
Détermination de la date d'exercice (s^c) de l'option d'investissement pour Equity Silver Mines: rendements décroissants (γ=1.1)

Année	v	v^c	a	b	π
		Création de capacité en 1980 (β_k=1.6)			
1974	1.456	7.895	0.947	1.018	1.388
1975	2.379	5.844	0.735	1.114	1.929
1976*	2.487	1.179	0.832	1.169	2.413
1977	2.280	1.539	0.764	1.200	2.056
1978	3.086	0.922	0.744	1.388	3.553
1979	2.512	3.175	0.802	1.105	2.218
1980	3.153	2.421	0.796	1.125	2.897

Notes:

v^*=flux monétaires unitaires actualisés,
v^{*c}=valeur critique à laquelle il faudrait exercer l'option d'investissement,
a et b=paramétre de la fonction qui donne la valeur ex ante de l'option d'investissement,
*=la date d'exercice prédite s^c (c'est à dire s^c est tel que $v > v^c$) correspond à la date historique,s.

l'importance du sujet, aucune étude empirique ne va au-delà de l'illustration, au mieux dans le cadre d'un modèle de simulation raisonnablement paramétrisé, de l'effet de certains paramètres fiscaux. Notre modèle va plus loin puisqu'il est paramétrisé en fonction des décisions effectivement observées. Nous pouvons examiner la sensibilité des résultats obtenus par rapport à un régime où la fiscalité est *neutre*. Nous avons donc repris nos simulations (pour le cas des rendements croissants) après avoir contraint les variables fiscales à être simultanément égales à l'unité ($A_0=A_1=A_2=1$). Les résultats (non reportés ici) obtenus sont probants: aucune des firmes de l'échantillon n'investit à la date prévue par le modèle. La fiscalité non seulement n'est-elle pas neutre; elle apparait constituer un élément majeur dans le choix de la date des investissements irréversibles.

IV. Conclusion

Ce travail constitue la première confrontation de la théorie des options appliquée aux actifs réels avec des données empiriques. Allant au-delà des simulations présentées à titre d'illustration dans la littérature sur le sujet, nous avons appliqué ce cadre d'analyse à des projets concrets d'investissement par des firmes minières canadiennes. Une difficulté de ce travail consistait à tenir compte du fait que la taille de l'investissement dépend en général du moment où il est réalisé. Nous avons donc utilisé les résultats de travaux économétriques expliquant la capacité choisie par diverses variables économiques et géologiques, pour en inférer le choix qui aurait été fait à d'autres dates, c'est-à-dire pour d'autres valeurs de ces variables explicatives. Une dimension importante du projet d'investissement a ainsi été caractérisée même aux dates où l'investissement n'était pas effectivement réalisé. Sous certaines hypothèses simplificatrices nous avons alors pu comparer, en recourant au modèle d'évaluation des options financières, la valeur des projets d'investissement à des dates où ces derniers n'étaient pas nécessairement effectués, avec la valeur critique au-dessus de laquelle la théorie préconisait d'aller de l'avant avec le projet. Nous avons constaté que les firmes de notre échantillon avaient effectivement agi exactement conformément à la théorie dans 58% des cas et conformément à la théorie à une année près dans 75% des cas. Bien que nous n'effectuons aucun test statistique de la validité de la théorie, ces résultats lui paraissent favorables et justifient certainement des travaux menant à des tests d'inférence. Dans les secteurs extractifs en particulier, les modèles conventionnels de l'offre se sont avérés très décevants et la modélisation de l'irréversibilité des investissement semble pouvoir mener à une prise en compte plus fidèle de la réalité.

Annexe

1- Etablissement des équations (4) et (5)

Pour toute date $t<s$, on a l'équation différentielle stochastique suivante pour V_t tel que définie dans (4)

$$dV_t = \frac{\partial V_t}{\partial p_t} dp_t + \frac{1}{2} \frac{\partial^2 V_t}{\partial p_t^2} (dp_t)^2 \tag{A.1}$$

où

$$\frac{\partial V_t}{\partial p_t} = [\frac{(e^{-\delta s} - e^{-\delta T1})}{\delta} A_{1,t} + \frac{(e^{-\delta T1} - e^{-\delta T})}{\delta} A_{2,t}] Q_t \equiv A_t Q_t \tag{A.2}$$

et

$$\frac{\partial^2 V_t}{\partial p_t^2} = \frac{\partial Q_t}{\partial p_t} (A_t - T_t A_{2,t} e^{-\delta T}) \tag{A.3}$$

En substituant (A.2) et (A.3) dans (A.1), on obtient

$$dV_t = \{A_t Q_t \alpha p_t + \frac{1}{2} [\frac{\partial Q_t}{\partial p_t} (A_t - T_t A_{2,t} e^{-\delta T})] \sigma^2 p_t^2\} dt + \{A_t Q_t p_t\} \sigma dz \tag{A.4}$$

En divisant de part et d'autre de (A.4) par V_t, on obtient ainsi (4), soit

$$\frac{dV_t}{V_t}=\frac{\{A_t Q_t \alpha p_t + \frac{1}{2}[\frac{\partial Q_t}{\partial p_t}(A_t - T_t A_{2,t} e^{-\delta T})]\sigma^2\}}{V_t^2}dt + \frac{\{A_t Q_t p_t\}}{V_t}\sigma dz$$

$$=\Lambda dt + \Gamma\sigma dz$$

La même démarche s'opère pour I_t. A partir de la définition de I_t dans (1) et en faisant appel aux propriétés des processus de Wiener pour p_t, on a

$$\begin{aligned} dI_t &= \frac{\partial I_t}{\partial p_t}dp_t + \frac{1}{2}\frac{\partial^2 I_t}{\partial p_t^2}(dp_t)^2 \\ &= (W_k A_{0,t}\frac{\partial K}{\partial p_t}\alpha p_t + \frac{1}{2}W_k A_{0,t}\frac{\partial^2 K}{\partial p_t^2}\sigma^2 p_t^2)dt \\ &+ (W_k A_{0,t}\frac{\partial K}{\partial p_t}p_t)\sigma dz \end{aligned} \tag{A.5}$$

En divisant par I_t de part et d'autre de (A.5), on obtient (5), soit

$$\frac{dI_t}{I_t}=\frac{(W_k A_{0,t}\frac{\partial K_t}{\partial p_t}\alpha p_t + \frac{1}{2}W_k A_{0,t}\frac{\partial^2 K_t}{\partial p_t^2}\sigma^2 p_t^2)}{I_t}dt + \frac{(W_k A_{0,t}\frac{\partial K_t}{\partial p_t}p_t)}{I_t}\sigma_p dz$$

$$=\kappa dt + \sigma\Xi dz$$

2- Etablissement de l'équation (8)

Nous répétons la même démarche que précédemment pour la fonction de profit variable unitaire intertemporel v. Puisque seule la valeur de $\partial v / \partial p$ intervient dans nos calculs empiriques (par le biais de Ω), nous ne présentons pas le développement de $\partial^2 v / \partial p^2$. On a

$$\frac{dv_t}{v_t} = \frac{\partial v_t}{\partial p_t} dp_t + \frac{1}{2}\frac{\partial^2 v_t}{\partial p_t^2}(dp_t^2) \tag{A.6}$$

D'après (11)

$$v_t = [\frac{(e^{-\delta s} - e^{-\delta T1})}{\delta} p_t - \frac{(e^{-\rho s} - e^{-\rho T1})}{\rho} \beta_0 e^{-\xi t} Q_t^{(\gamma-1)} w_e^{\beta_e} w_m^{\beta_m}] A_{1,t}$$
$$+ [\frac{(e^{-\delta T1} - e^{-\delta T})}{\delta} p_t - \frac{(e^{-\rho T1} - e^{\rho T})}{\rho} \beta_0 e^{-\xi t} Q_t^{(\gamma-1)} w_e^{\beta_e} w_m^{\beta_m}] A_{2,t}$$

On en tire

$$\begin{aligned}\frac{\partial v_t}{\partial p_t} = {} & \frac{\partial Q_t}{\partial p_t}(1-\gamma)\frac{cv_t}{Q_t}[\frac{(e^{-\rho s} - e^{-\rho T1})}{\rho} A_{1,t} \\ & + \frac{(e^{-\rho T1} - e^{-\rho T})}{\rho} A_{2,t}] \\ & + [\frac{(e^{-\delta s} - e^{-\delta T1})}{\delta} A_{1,t} + \frac{(e^{-\delta T1} - e^{-\delta T})}{\delta} A_{2,t}] \\ & - (p_t e^{-\delta T} - cv_t e^{-\rho T})\frac{T_t}{Q_t}\frac{\partial Q_t}{\partial p_t} A_{2,t}\end{aligned} \tag{A.7}$$

où

$$cv_t \equiv \frac{CV_t}{Q_t} = \frac{\beta_0 e^{-\xi t} Q_t^{\gamma} w_e^{\beta_e} w_m^{\beta_m}}{Q_t} = \beta_0 e^{-\xi t} Q_t^{\gamma-1} w_e^{\beta_e} w_m^{\beta_m}$$

c'est à dire le coût variable unitaire.

Après substitution pour dp et dp^2 dans (A6), et simplification en vertu des propriétés des processus de Wiener, on obtient

$$dv_t = \frac{\partial v_t}{\partial p_t}(\alpha p_t + \sigma p_t dz) + \frac{1}{2}\frac{\partial^2 v_t}{\partial p_t^2}\sigma^2 p_t^2 dt$$
$$= (\frac{\partial v_t}{\partial p_t}\alpha p_t + \frac{1}{2}\frac{\partial^2 v_t}{\partial v_t^2}\sigma^2 p^2)dt + (\frac{\partial v_t}{\partial p_t}p)\sigma dz$$

et en divisant de part et d'autre par v_t, on obtient bien (8)

$$\frac{dv_t}{v_t} = \frac{(\frac{\partial v_t}{\partial p_t}\alpha p_t + \frac{1}{2}\frac{\partial^2 v_t}{\partial p_t^2}\sigma^2 p_t)}{v_t}dt + \frac{(\frac{\partial v_t}{\partial p_t}p_t)}{v_t}\sigma dz$$

$$= \lambda(v,t)dt + \sigma\Omega(v,t)dz$$

où $\Omega(v,t)$ représente l'élasticité-prix du profit variable unitaire capitalisé. En faisant appel à (A.7), cette élasticité devient

$$\begin{aligned}\Omega(v,t) = \frac{p_t}{v_t}[&\frac{(e^{-\delta s} - e^{-\delta T1})}{\delta}A_{1,t} + \frac{(e^{-\delta T1} - e^{-\delta T})}{\delta}A_{2,t}]\\ &+ [\frac{e^{-\rho s} - e^{-\rho T1}}{\rho}A_{1,t} + \frac{(e^{-\rho T1} - e^{-\rho T})}{\rho}A_{2,t}]\\ &(1-\gamma)\zeta_{Q,p}\frac{cv_t}{v_t}\\ &-A_{2,t}(p_t e^{-\delta T} - cv_t e^{-\rho T})\,\zeta_{Q,p}\frac{T_t}{v_t}\end{aligned} \qquad (A.8)$$

où

$$\zeta_{Q,p} = \frac{\partial Q}{\partial p}\frac{p}{Q}$$

Dans le cas où $\delta=\rho$, (A.8) devient

$$\Omega(v,t)=[\frac{p_t}{v_t}+(1-\gamma)\zeta_{Q,p}\frac{cv_t}{v_t}]A_t-e^{-\delta T}A_{2,t}(p_t-cv_t)\zeta_{Q,p}\frac{T_t}{v_t} \qquad \text{(A.9)}$$

où

$$A_t=[\frac{(e^{-\delta s}-e^{-\delta T1})}{\delta}A_{1,t}+\frac{(e^{-\delta T1}-e^{-\delta T})}{\delta}A_{2,t}]$$

3- Etablissement de l'équation (16)

On peut vérifier qu'une solution générale de (12) est

$$\pi(v)=a(v)^b+a'(v)^{b'} \qquad \text{(A.10)}$$

où b et b' sont les racines de l'équation du deuxième degré

$$\frac{1}{2}\sigma^2\Omega^2 b^2+(r-\mu-\frac{1}{2}\sigma^2\Omega^2)b-r=0 \qquad \text{(A.11)}$$

on supposera que b' est la racine négative. Si a' est non nul et que $v\rightarrow 0$, alors $\pi(v)\rightarrow\infty$, ce qui contredit la condition (13). Pour éviter ce cas de figure, on pose $a'=0$ et (A.11) devient alors (16).

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The Option Value of an Irreversible Investment: An Econometric Assessment

I. Introduction

Much of the economics literature on investment has focused on incremental investment under a neo classical framework. Except for seminal works by Arrow (1968) and Henry (1974), this literature generally ignores the effect of *irreversibility*. It seems that irreversibility has had a long standing tradition only in the context of natural resources and the environment where the insight on the *option value* concept, first made by Arrow and Fisher (1974), has since been elaborated upon in the environmental economics literature. However, recently, an emerging strand of economic literature, which rests on the option pricing approach, provided additional insights in a host of firm's valuation problems as well as on irreversibility.

Tourinho (1979) modeled the value of natural resource reserves under uncertainty as an option to extract the resource in the future. The asymmetric pattern of option payoffs are also found to be displayed by the reserve. Following this line of reasoning, Harchaoui and Lasserre (1992c) used the option pricing approach to derive the full Hotelling valuation principle (Hotelling rents and the option value of the irreversible investment) that should be accounted for in national accounts. Using a contingent claim approach, Brennan and Schwartz (1985) have shown how an option to invest in the mine can be valued and the optimal investment rule determined. Their contribution provided additional insights on how sunk cost of opening and closing a mine can explain the "hysteresis" often observed in extractive resource industries. Majd and Pindyck (1987) modeled an investment problem featuring the "time to build" in which a firm invests continuously until the completion of the project. Each dollar spent buys an option to spend the next dollar. Paddock, Siegel and Smith (1988) used the option valuation theory to value leases for offshore petroleum. The petroleum lease valuation implies valuing the cash flows from the multistage process which constitute a nested set of options. Pindyck (1988) investigated the capacity extension problem. He determined the effects of uncertainty on a firm that must decide when to build a plant and how large it should be. Mackie-Mason (1990) used the option pricing framework to analyze optimal investment decisions by mining firms with non linear taxation. This allows him to show that depletion allowance may discourage investment and that increasing the corporate income tax may encourage investment.

Other contributions have focused on the fact that irreversibility and the ability to delay investment decisions change the fundamental rule for investing and invalidates the net present value (NPV) criterion (McDonald and Siegel 1986, Pindyck 1991, Ingersoll and Ross 1992). The firm must include as part of the total cost of an incremental unit of capital the opportunity cost of investing now rather than waiting. In these contributions, an irreversible investment opportunity is modeled much like a financial call option. The firm which holds an investment opportunity has the option to spend money (the exercise price) now or in the future, in return for an asset, say a project, of some value. When a firm undertakes an irreversible investment, it exercises its option to invest. It gives up the possibility to use new information that might arrive later affecting the desirability or timing of the expenditure; it cannot disinvest should market conditions change adversely.

These contributions are to be considered as a major breakthrough in that they establish a conceptual link between real (investment) and financial theories (option pricing). However, because there is no theory underlying the investment choice, it turns out that the link between those two theories remains *ad hoc*. Our contribution seeks to improve the nature of this link. We show the option-like characteristics of an investment in a production capacity with a putty clay technology, uncertainty, and how optimal investment rules can be obtained from methods of option pricing. Besides making the value of the project *endogenous*, our contribution shows also how the resulting investment rules depend on parameters that are estimated *econometrically* and not guessed as is currently the case in the economics literature. While the idea to conciliate the option pricing theory and econometric methods has been undertaken only recently by Pakes (1986) who modeled and estimated the distribution of the returns earned from holding patents by using an option-like model[19], our paper is the first attempt which tests econometrically the option pricing theory as applied to investment theory (real assets).

The paper is organized as follows. In section II, a model of investment is characterized along with its interpretation in terms of the financial call option

[19]In his model a patent holder who pays the renewal fees obtains both the current returns that accrue to the patent over the coming period, and the option to pay the renewal fee and maintain the patent in force in the following period should he desire to do so. An agent who acts optimally will pay the renewal fee only if the sum of the current returns plus the value of this option exceeds the renewal fee.

framework. In section III, the econometric model is specified and is estimated via a method suitable to option-like decision problems. Section IV concludes.

II. The model

1. The real asset: the capacity investment problem

As in Harchaoui and Lasserre (1992a), we use a model of a mining firm which undertakes a capacity investment at a certain date, *s*, in an environment characterized by a stochastic output price, a putty clay technology, and taxation. This set up is particularly suitable to make the link between the investment in real assets (capacity choice) and the financial option theory. The putty clay technology implies that once the firm has chosen the optimal factor combination at *s*, it must operate with it thereafter. Ex post, once the investment is made, the only flexibility that remains is the option either to operate at full capacity or to shut down.

To begin with, let us go over the main features of the model: i) the difference between the quasi fixed factor, capital, and its variable counterpart is that the former is paid fully at *s* whereas the latter is hired at the ongoing price during the life span of the firm; ii) the productive capacity of capital is maintained throughout the operating life at a cost included in the variable costs; iii) the production is constant during all the operating life. This simple framework implies, implicitly, that once the initial investment made, it is not in the firm's interest to close down, temporarily or permanently, before the full exhaustion of its reserves at the terminal date $T \equiv R(s) / F(K(s), L(s))$. This assumption is met if, as observed in reality, operating costs are low relative to the initial capital cost, so that the probability that the output price might fall below operating costs is negligible.

Our starting point here is the corresponding capitalized profit function of the mining firm at date s, the difference between capitalized expected revenues, V_s, and capitalized expected costs, C_s,

$$J(x,s)=V_s-C_s \tag{1}$$

Capitalized expected revenues can be written as (see Harchaoui and Lasserre 1992a for details)

$$\begin{aligned} V_s &\equiv \left[\frac{(e^{-\delta s}-e^{-\delta T1})}{\delta}A_{1,s}+\frac{(e^{-\delta T1}-e^{-\delta T})}{\delta}A_{2,s}\right].P_sQ_s \\ &= A(P_s,W_l,W_k,s,T1,\delta,R,A_0,A_1,A_2).P_sQ_s \end{aligned} \tag{2}$$

and capitalized expected costs, the sum of capitalized variable costs and the initial capital expenditures, are

$$C_s\equiv C(A(.),P,Q,W,q,A_0,R)=A_s(.).W_s'L_s+q_sA_{0,s}K_s \tag{3}$$

The endogenous variables, capacity, Q, the operating life of the mine, $T(\equiv R/Q)$, the vector of variable inputs, L, and the quasi fixed factor, K, are taken at their optimal level with respect to the following exogenous variables: output price[20], P, proven mineral reserves, R, variable-input price vector, W_l *(l=M (material), E (energy))*, the price of the quasi fixed input, W_k, the tax variables A_0 (the after-tax cost of $1 of capital expenditures), A_1 (the after-tax revenue per dollar of gross income during the tax

[20]The output is assumed to follow a Brownian motion so that current price is the only price that matters.

holiday period (0-$T1$), and A_2 (the after-tax revenue per dollar of gross income after the tax holiday, i.e. from $T1$ to T) and the risk free real rate of return adjusted for the drift of the output price (α_p), that is[21], $\delta=r-\alpha_p$. Capacity, Q, and, therefore, $T=R/Q$ have been estimated econometrically in Harchaoui and Lasserre (1992a)[22]. Thus, in what follows the functions giving Q and A as function of observable explanatory variables are known, but the function C is not known which will require some parametrization and econometric estimation.

The above model describes the choice of Q when the investment occurs. It does not explain the timing of the decision which is the focus of this paper and to which we turn now. We seek to link the problem of investment in real assets (capacity choice) to the financial option framework set forth by Black and Scholes (1973) and Merton (1973).

2. The investment option: the decision to proceed or to wait

2.1- Preliminary remarks

The link between the investment in production capacity modeled in (1) and the financial call option framework is straightforward. At date s, equation (1) has the same form as the value of an exercised financial (ex post) call option whose underlying asset as well as the exercise price are both stochastic. However, in order to make the interpretation as a call option complete, we have to allow for the possibility that (1) be worth nothing at the exercise date s. Also, to simplify matters, we rewrite the ex post

---

[21] We use the same adjusted discount rate, δ, when computing C, as when computing V_r, under the assumption that the drift in variable-input prices is equal to the drift in output price α_p.

[22] See the appendix of this paper.

value of the call option in unitary terms. Bearing all these facts in mind and the fact that $J(.)$ is homogenous of degree one in prices, we obtain the following ex post value of the option to invest

$$\pi_s = \max[A(p_s, z_s).p_s - c(p_s, z_s), 0] \tag{4}$$

with

$$\pi_s \equiv \frac{\Pi_s}{Q_s} = \max[\frac{V_s}{Q_s} - \frac{C_s}{Q_s}, 0]; \quad p_s = \frac{P_s}{W_M}; \quad c(p_s, z_s) = \frac{C_s}{Q_s} \tag{5}$$

where z_s refers to all exogenous variables that determine Q except p (the prices included in z are normalized by the material input price W_M). For every $t < s$ the behaviour of the output price, p, is characterized by the following geometric Brownian motion

$$\frac{dp_t}{p_t} = \alpha dt + \sigma d\upsilon \tag{6}$$

where $d\upsilon = \varepsilon(t)(dt)^{1/2}$ is the increment of a Wiener process, with $\varepsilon(t)$ a serially uncorrelated and normally distributed random variable. The intuition behind (6) is that the current value of output price is known, whereas its future values are lognormally distributed with a variance that grows with time.

An irreversible investment opportunity in production capacity such as the one described above in (4)-(6), is much like a financial call option. The latter gives its holder the right to pay an agreed-upon price (the exercise price) and to receive an asset in return. Exercising the option, just as the investment in a putty clay technology, is irreversible. The investor cannot retrieve the option or the money that was paid to exercise it.

2.2- The critical price

The next step is to determine the critical value (date) p_t^* at which the firm undertakes an irreversible investment by spending a sunk cost *c(.)* in order to receive capitalized cash flows which amount to $A(p_t, z_t)p_t$. To determine p_t^*, we use the dynamic programming approach set forth by Cox and Ross (1976). This approach consists in finding a rule which maximizes the ex ante value of the option to invest into the project, π_t, that is[23]

$$\pi_t = \pi(p_t, z_t) = \max E_t\{[A(p_s, z_s)p_s - c(p_s, z_s)]e^{-r(s-t)}\} \quad \forall\ t \le s \tag{7}$$

where E_t is the expectation at date t at which the investment is made and r is the appropriate discount rate. This discount rate is greater than α which may be interpreted as a capital gain rate, with $r > \alpha$. In the parallel literature on option theory as applied to real assets, this difference is interpreted as a dividend rate, μ. The latter is an opportunity cost which makes the potential investor willing to possibly exercise the option at a finite date. Ex ante, the option yields no cash flow, the only return from holding it is given by its capital appreciation.

The Bellman equation related to (7) is

[23]See Pindyck (1991, 1118-31).

$$r\pi_t = \frac{1}{dt}E_t d\pi_t \tag{8}$$

The instantaneous investment return, $r\pi_t$, must be equal to the expected capital gain. With $\pi(p_t, z_t)$ twice continuously differentiable, (8) becomes

$$r\pi_t dt = E_t[\frac{\partial \pi}{\partial p_t}dp_t + \frac{1}{2}\frac{\partial^2 \pi}{\partial p_t^2}(dp_t)^2] \tag{9}$$

Using (6) and Ito's lemma, we obtain the following stochastic differential equation of order two in p_t that the ex ante value of the option, $\pi(p_t, z_t)$, must satisfy

$$\frac{1}{2}\frac{\partial^2 \pi}{\partial p_t^2}\sigma^2 p_t^2 + \frac{\partial \pi}{\partial p_t}(r-\mu)p_t - r\pi_t = 0 \tag{10}$$

The following boundary conditions must hold

$$\pi(0, z_t) = 0 \qquad \forall\ t \tag{11.1}$$

and if p_t reaches the threshold level, p_t^*, at which it is optimal to invest then π must equal the ex post value of the project at that price

$$\pi(p_t^*, z_t) = A(p_t^*, z_t)p_t^* - c(p_t^*, z_t) \tag{11.2}$$

Furthermore, since p_t^* is the interior solution to an optimization problem, the above condition must continue to hold for small variations in p_t^*

$$\frac{\partial\pi(p_t^*,z_t)}{\partial p_t}=A(p_t^*,z_t)[1+\frac{p_t^*}{A(p_t^*,z_t)}\cdot\frac{\partial A(p_t^*,z_t)}{\partial p_t}]$$

$$-\frac{\partial c(p_t^*,z_t)}{\partial p_t} \qquad (11.3)$$

$$=A(p_t^*,z_t)(1-G(p_t,A_t(.),z_t))-\frac{\partial c(p_t^*,z_t)}{\partial p_t}$$

where (see (2))

$$A(p_t^*,z_t)=\frac{(e^{-\delta t}-e^{-\delta T1})}{\delta}A_{1,t}+\frac{(e^{-\delta T1}-e^{-\delta T^*})}{\delta}A_{2,t} \qquad (12)$$

and

$$G(p_t^*,z_t)\equiv\frac{\partial A(p_t^*,z_t)}{\partial p}\frac{p_t^*}{A(p_t^*,z_t)}=e^{-\delta T^*}A_{2,t}\frac{\partial Q_t^*}{\partial p}\frac{p_t^*}{Q_t^*}\frac{T_t^*}{A_t(.)} \qquad (13)$$

It can be shown (see Pindyck 1991) that the solution to (10) and (11) is

$$\pi(p_t,z_t)=a_t p_t^{\,b} \qquad (14)$$

where

$$b=0.5-\frac{\alpha}{\sigma^2}+[(\frac{\alpha}{\sigma^2}-0.5)^2+\frac{2r}{\sigma^2}]^{1/2} \qquad (15)$$

and (using (11.2) and (14) evaluated at p_t^*)

$$a_t=\frac{A(p_t^*,z_t)p_t^*-c(p_t^*,z_t)}{(p_t^*)^b} \qquad (16)$$

whereas p_s^* is implicitly given by (using (11.2), (11.3) and (14) evaluated at p_t^*)

$$b\{\frac{A(p_t^*,z_t)p_t^*-c(p_t^*,z_t)}{p_t^*}\}=A(p_t^*,z_t)[1-G(.)] - \frac{\partial c(p_t^*,z_t)}{\partial p_t} \tag{17}$$

The value of p_s^* at s is observable: it is simply the actual output price observed at s, that is

$$p_s^*=p_s \quad \forall\ s \tag{18}$$

However, at dates $t \neq s$ which do not correspond to actual investments, the critical value of the output price, p_t^*, is unknown. Therefore, the remainder of this section is devoted to the determination of an explicit form for p_t^*.

2.3- Expressing p_t^* as an explicit function of observable data and estimable parameters

Dividing both sides of (17) by $A(p_t^*, z_t)$ and multiplying by p_t^*, we obtain

$$b[p_t^*-\frac{c(p_t^*,z_t)}{A(p_t^*,z_t)}]+\frac{\partial c(p_t^*,z_t)}{\partial p_t}.\frac{p_t^*}{A(p_t^*,z_t)}=p_t^*[1-G(p_t,z_t)]$$

Gathering the expressions involving the cost function $c(.)$, we get

$$bp_t^*-\{b\frac{c(p_t^*,z_t)}{A(p_t^*,z_t)}-\frac{\partial c(p_t^*,z_t)}{\partial p_t}.\frac{p_t^*}{A(p_t^*,z_t)}\} =p_t^*[1-G(p_t^*,z_t)] \tag{19}$$

On the left-hand side of the above equation, parameter b is given by (15), but the other components $A(p_t^*, z_t)$ and $c(p_t^*, z_t)$, since they depend on p_t^*, are not observable. For analytical convenience we make the following assumption on the technology: the function $c(.)$ is such that the term between curly brackets in (19) is independent of p_t^*. While clearly ad hoc, this assumption is not more restrictive than, say, constant returns to scale[24]. On the right-hand side of (19), $G(.)$ is a known function (see (13)), but one of its arguments, p_t^*, is observable only at $t=s$. Let us express $G(p_t^*, z_t)$ by taking a linear expansion around (p_s, z_s)

$$G(p_t^*,z_s) \simeq G(p_s,z_s) + \frac{\partial G(p_s,z_s)}{\partial p}(p_t^* - p_s) + \sum_j \frac{\partial G(p_s,z_s)}{\partial z_j}(z_{t,j} - z_{s,j}) \tag{20}$$

Substituting (20) into (19) and using some manipulations we get

$$p_t^{*2} + B_t p_t^* - \Gamma_s \psi_t = 0 \tag{21}$$

where $\psi(z_t)$ is the expression between curly brackets in (19),

$$\psi = \{ b\frac{c(p_t^*,z_t)}{A(p_t^*,z_t)} - \frac{\partial c(p_t^*,z_t)}{\partial p_t} \cdot \frac{p_t^*}{A(p_t^*,z_t)} \} \tag{22.1}$$

$$\Gamma_s = -\frac{1}{\frac{\partial G(p_s,z_s)}{\partial p}} \tag{22.2}$$

[24] We plan to consider alternative assumptions where the term between curley brackets is a first, or second, degree polynomial in p_t^*.

$$B_t = \frac{[b - 1 + G(p_s,z_s) - \frac{\partial G(p_s,z_s)}{\partial p}p_s + \sum_j \frac{\partial G(p_s,z_s)}{\partial z_j}(z_{t,j} - z_{s,j})]}{\frac{\partial G(p_s,z_s)}{\partial p}} \quad (22.3)$$

Note that, at $t=s$,

$$B_s = \frac{[b - 1 + G(p_s,z_s) - \frac{\partial G(p_s,z_s)}{\partial p}p_s]}{\frac{\partial G(p_s,z_s)}{\partial p}} \quad (22.4)$$

B_t, Γ_s and ψ_t are all independent of p_t^*. Therefore (21) can be solved as a quadratic equation in p_t^* whose solution is

$$p_t^* = \frac{1}{2}(-B_t + \sqrt{\Delta_t}) \quad (23)$$

with

$$\Delta_t = B_t^2 - 4\Gamma_s\psi_t \quad (24)$$

Thus p_t^* is an expression in B_t and Γ_s are known functions of observable variables; however ψ_t is not a known function. Instead of parametrizing ψ_t directly, we will work on Δ_t. The function Δ_t is not observable in general, because $\psi_t(.)$ is not. However, at date $t=s$, because the solution p_t^* is equal to p_s it follows from (23) that

$$\sqrt{\Delta_s} = 2p_s + B_s \quad (25)$$

For all $t < s$, the function Δ_t in (24) depends on an observable variable, B_t, and the unknown function ψ_t. Therefore, in order to get a tractable form for p_t^*, we proceed to the linearization of Δ_t around the values at date s of its explanatory variables. This yields

$$\Delta_t^{½} \approx \Delta_s^{½} + \Delta_s^{-½} \frac{\partial \Delta_s}{\partial B} (B_t - B_s)$$

$$+ \Delta_s^{-½} \sum_j \frac{\partial \Delta_s}{\partial z_j} (z_{j,t} - z_{j,s}) \tag{26}$$

$$\approx X_{t,s} + \sum_j \gamma_j Z_{j;t,s} \qquad for\ t \le s$$

with

$$X_{t,s} \doteq \Delta_s^{½} + \Delta_s^{-½} \frac{\partial \Delta_s}{\partial B} (B_t - B_s)$$
$$= (2B_t - B_s) + 2p_s \tag{27}$$

(using the fact that, from (25), $\partial \Delta_s / \partial B = 2\ (\Delta_s)^{½}$), and

$$Z_{j,t} \doteq \Delta_s^{-½} (z_{j,t} - z_{j,s}) \tag{28}$$

It is worth noting that while Δ_s is computable using (25), the partial derivatives of this function with respect to each variable z_j, that is $\gamma_j \equiv \partial \Delta_s / \partial z_j$, are not observable. Thus, the γ_j must be treated as unknown parameters. The link with the cost function $c(.)$ is straightforward since differentiating (24) at s, we obtain

$$\gamma_j \equiv \frac{\partial \Delta_s}{\partial z_j} = -4 \Gamma_s \frac{\partial \psi_t}{\partial z_j} \tag{29}$$

where ψ_t involves the cost function (see (22.1)). Substituting (26), (27) and (29) into (23) and rearranging terms, we obtain

$$p_t^* = -\frac{B_t}{2} + \frac{1}{2}\{X_t + \sum_j \gamma_j Z_{j;t}\}$$
$$= K_t + \sum_j \beta_j Y_{j,t} \qquad (30)$$

with

$$\mathrm{K}_t \doteq 0.5(B_t - B_s) + p_s \quad ; \quad Y_{j;t} \doteq 0.5\, Z_{j;t} \qquad (31)$$

where the variables K_t and $Y_{j;\,t}$ have to be computed from observed data and the predicted values of Q and A obtained from the capacity model of Harchaoui and Lasserre (1992)(see also the appendix). The parameters β_j, which correspond to γ_j once we multiply the variables $Z_{j;t}$ by $1/2$, have to be estimated by an appropriate method.

III. The empirical work

1. Data

We used the same data base as in Harchaoui and Lasserre (1992b), therefore, we repeat here only some of its significant features. First, the data are build up at the firm level. The observation at date s is the occurrence of a major investment by a mining firm. In Harchaoui and Lasserre (1992a), the capacity investment *(Q)* undertaken by each firm at different dates s is regressed on the output price *(p)* and a vector of input prices *(w')*, mineral reserves *(R)*, taxation as characterized by the variables A_i *(i=0, 1, 2)* and a dummy variable *(expl)* which allows for the structural difference between an open pit and an underground mine. The estimates, which are provided in the appendix of this paper, are used to calculate the predicted values of the capacity, the operating life of the mine *(T=R/Q)* and the capitalization function *(A)* during the time span 1960-1980. These predicted values as well as the historical values of the explanatory variables have been used in the current econometric work. We interpreted each irreversible investment undertaken at s as the exercise of a financial call option and, accordingly, the trigger output price, p_s^*, is the observed price, p_s. We defined arbitrary time periods of 7 years before each date s over which we compare p_t^* with the observed price p_t. This is done for each of the 12 investment occurrences in the sample, for a total of (12 x 7) 84 observations whose features are displayed in Table 1.

[insert Table 1 here]

2. The econometric model

The purpose of the econometric part is twofold. First, estimate the parameters of (30) in order to calculate the value of p_t^*. Second, test the theory of options applied on real assets against alternative theory which does not assume irreversibility. The fact that the variable p_t^* is not observable should not raise any problem because the qualitative dependent variable model is suitable for models involving a critical threshold that determines whether investment takes place or not. In fact, we want to know the critical value of the output price, p_t^*, at which the investor should exercise the investment option. This should be compared to the historical value of the output price, p_t, in order to investigate whether or not the firm should have invested before s.

The specification corresponding to (30) is

Table 1: General information on data.

Firm	s	year	T, (A)	Firm	s	year	T, (A)
Bethlehem Copp. (open pit)	1966	1960	6 (4.57)	Bell Copper (open pit)	1972	1966	7 (5.00)
		1961	7 (4.93)			1967	7 (4.85)
		1962	6 (4.61)			1968	6 (4.05)
		1963	6 (4.76)			1969	6 (3.74)
		1964	6 (4.64)			1970	5 (3.31)
		1965	6 (4.41)			1971	7 (4.48)
		1966	6 (4.60)			1972	8 (5.17)
Granisle Mines (open pit)	1966	1960	4 (3.32)	Granisle mine (open pit)	1972	1966	11 (6.93)
		1961	4 (3.63)			1967	10 (6.72)
		1962	4 (3.39)			1968	10 (5.76)
		1963	4 (3.49)			1969	9 (5.32)
		1964	4 (3.40)			1970	8 (4.72)
		1965	4 (3.22)			1971	10 (6.40)
		1966	4 (3.36)			1972	12 (7.52)
Bethlehem Copp. (open pit)	1967	1961	7 (4.88)	Lornex (open pit)	1974	1968	21 (10.44)
		1962	6 (4.57)			1969	21 (9.75)
		1963	6 (4.71)			1970	19 (9.14)
		1964	6 (4.60)			1971	22 (12.01)
		1965	6 (4.36)			1972	24 (13.91)
		1966	6 (4.56)			1973	18 (10.29)
		1967	6 (4.42)			1974	19 (8.40)
Geco Mines (underground)	1970	1964	6 (4.67)	Bethlehem Copp. (open pit)	1976	1970	6 (3.73)
		1965	6 (4.33)			1971	8 (5.06)
		1966	6 (4.59)			1972	9 (5.89)
		1967	6 (4.41)			1973	5 (2.75)
		1968	5 (3.57)			1974	5 (2.49)
		1969	5 (3.24)			1975	9 (5.04)
		1970	4 (2.82)			1976	12 (5.69)
Bethlehem Copp. (open pit)	1971	1965	7 (4.97)	Lornex (open pit)	1979	1973	19 (10.44)
		1966	7 (5.18)			1974	19 (8.51)
		1967	7 (5.02)			1975	24 (12.80)
		1968	6 (4.20)			1976	26 (12.60)
		1969	6 (3.90)			1977	24 (15.85)
		1970	5 (3.40)			1978	26 (16.41)
		1971	7 (4.70)			1979	25 (10.12)
Lornex (open pit)	1972	1966	20 (11.07)	Equity Silver (open pit)	1980	1974	3 (1.49)
		1967	20 (11.78)			1975	6 (3.21)
		1968	19 (9.50)			1976	8 (3.75)
		1969	18 (8.85)			1977	6 (3.75)
		1970	17 (8.18)			1978	9 (4.92)
		1971	20 (10.84)			1979	7 (3.00)
		1972	21 (12.62)			1980	11 (3.64)

Notes: s=investment date, A=predict value of the capitalization function, T= predict value of the mine's operating life.

$$p_t^* = K_{t,s} + \sum_j \beta_j Y_{j;t,s} + \nu_t \tag{32}$$

where ν is a vector of unobservable disturbances following a multivariate normal distribution and β a vector of parameters to be estimated. The firm choice to invest, I, is observed, but not the realization of the critical value, p_t^*, under which the firm chooses to not invest. The econometric problem is to estimate the vector β given 84 observations and the explanatory variables K_t and $Y_{j,t}$ on decisions to invest *(I=1)* or to wait *(I=0)*. The probability model is

$$\begin{aligned} Pr(I=1 \mid p_t, K_t, Y_t) &= Pr(p_t \geq p_t^*) \\ &= Pr(\nu_t \leq p_t + K_t + \sum_j \beta_j Y_{j;t}) \end{aligned} \tag{33}$$

with $\nu \sim N(0, \rho^2)$ (where ρ^2 is the residual variance). Then for observations $n=1,..,N$, the likelihood function is

$$Pr(I_n = 1 \mid p_t, K_n, Y_n) = \int_{-\infty}^{p_t - K_n - \Sigma\beta Y_n} \frac{1}{\sqrt{2\pi\rho}} \exp\{-\frac{1}{2}(\frac{\nu}{\rho})^2\} d\nu$$

$$= \Phi\{\frac{(p_t - K_n - \Sigma\beta Y_n)}{\rho}\}$$

This is a standard probit model whose estimates of β are obtained by maximizing the log of the likelihood function for the sample.

Once we obtain the empirical results related to the option value of the irreversible investment, it is worthwhile to test this model against an alternative hypothesis which assumes that irreversibility is irrelevant. The competing alternative suggests to invest only if the output price exceeds the capitalized unitary total cost in presence of taxation, that is, $p_t \geq c(p_t, z_t)$. This alternative theory neglects irreversibility and, accordingly, the option value of the investment is not part of the total cost. We want a test on the choice between competing non-nested models. We apply the J-test of Davidson and MacKinnon (1981). The competing hypotheses are

$$H_o : \quad I \begin{cases} 1 & , \text{if } p_t \geq p_t^* \\ 0 & , \text{otherwise} \end{cases}$$
$$vs\ H_1 : I \begin{cases} 1 & , \text{if } p_t \geq c(p_t, z_t) \\ 0 & , \text{otherwise} \end{cases} \tag{34.1}$$

The J-test consists in embedding these alternative models in a general model using a mixing parameter η associated to the predicted probability to invest under the model based on option theory, that is,

$$(1-\eta).Pr_1[p_t \geq c(p_t, z_t)].+\eta \tilde{Pr}_o(p_t \geq p_t^*)+\omega_t$$
$$(1-\eta).Pr_1(\nu_{1,t} \leq p_t - \sum_j \varphi_j z_j)+\eta .\tilde{Pr}_o(\nu_{o,t} \leq p_t - K_t - \sum_j \hat{\beta}_j Y_{j;t,s})+\omega_t \tag{34.2}$$

(the subscripts *o* and *1* refer, respectively, to H_o and H_1). If $\eta=0$ then H_o is rejected. To the extent that the predicted value of $Pr_1 (p_t \geq p_t^*)$ is asymptotically independent of ω_t, the J-test suggests that testing $\eta=0$ can be done using the conventional t test.

3. The results

The estimated probit model is (33). As described in the previous sections, with the exception of p_t, its explanatory variables are obtained from observable variables and known parameters after substantial computations. This makes parameters interpretation more difficult. Nonetheless we can make the following remarks. First, it is useful to remember that the probability of investment is expected to be higher if p_t is higher. It is also useful to keep in mind that p_t^* may be interpreted as full cost, that is to say total project cost plus the additional irreversibility premium which is typical of the option model. Clearly, we expect that any increase in p_t^* should reduce the probability of investment. Since by (32) $\partial p_t^* / \partial K=1$, this effect is measured by the coefficient of K. The other variables Y_j, as can be verified by constructing their formula back from (31), (28) and (25), are positively linked with such determinants as energy price *(j=wen)*, materiel price *(j=wma)*, net capital input price *(j=wk)*, the discount rate *(δ)* and a dummy variable *(expl)* that makes a distinction between open pit mines *(expl=1)* and underground mines *(expl=0)*. When j is an input price, we heuristically think of Y_j as a modified input price but it is not clear what the modification implies except that it is specific to the option value model. Note also that because of the linearizations performed in the process of establishing the formula that give the Y_j's, they do not actually reflect the level of variables j but their incremental value between dates s and t. Since reserves, R, do not vary over that interval, the variable Y_R does not appear in the model.

The results are presented in Table 2. They show a high explanatory power as indicated by the value of McFadden's R^2 (=0.82) as well as by a likelihood ratio test indicating significance at less than 1%. Except for the variables δ and *expl*, all variables are significant. The output price, significant at 10% level, has a positive effect on the probability to invest, that is, ceteris paribus, an increase in the output price, p_t, represent a meaningful incentive to exercise the option to invest. The effect of full input price, Y_j, on the critical value p_t^* is (using the fact that $Y_j = 1/2(z_{j,t} - z_{j,s}) / \Delta^{1/2} = 1/2(\Delta z)_j / \Delta^{1/2}$)

$$\frac{\partial p_t^*}{\partial Y_j} = \frac{\partial p_t^*}{\partial(\Delta z)_j} \cdot \frac{\partial(\Delta z)_j}{\partial Y_j} + \frac{\partial p_t^*}{\partial \Delta_s} \cdot \frac{\partial \Delta_s}{\partial Y_j} = (+)(+) + (-)(-) > 0$$

Thus a rise in Y_j raises p_t^*, which explains why the full capital input price, Y_{wk}, significant at a 10% level, has a negative effect on the option to invest. Also, as expected, the effect of the weighted material prices, Y_{wma}, on the probability to invest is negative, whereas the effect of energy prices is curiously positive. We suppressed the non significant variables *(expl and Y_s)* from (32) to work on with the resulting functional form (32)'. As a result, except for the capital input net price, Y_{wk}, all the remaining variables have increased their significance.

[insert Table 2 here]

Equation (34) in Table 2 presents the test of the amended NPV vs the traditional NPV criterion. We estimate a super model which combines linearly the NPV-like model and its option-like counterpart as defined in (34.2). Our results indicate the rejection of the traditional NPV criteria at less than 1%. In fact, the variable *"predict"*, which is the predicted value of the probability under the option-like model (see (34.2)) is highly significant and, therefore, suggests the non rejection of the hypothesis H_o which says that the option-like model of investment is true.

4. Assessing the investment-timing decision of the firm

We have just estimated econometrically the critical value of the output price, p_t^*, at which the firm should undertake the investment in the production capacity. We

Table 2. Probit on the decision to invest based on the option model

	eq. (32)	eq. (32)'		eq. (34)
constant	-13.737	-8.009	constant	2.488
	(-1.054)	(-1.970)		(0.322)
p	13.133	9.758	p	-5.193
	(1.873)	(2.025)		(-0.653)
-K	-0.7×10^{-31}	-0.6×10^{-31}	-	-
	(-2.262)	(-2.643)		
$-Y_{wen}$	-423.96	-363.82	-wen	0.928
	(-2.278)	(-2.639)		(1.050)
$-Y_{wma}$	3073.1	2731.1	-wma	-5.826
	(2.587)	(2.962)		(-0.709)
$-Y_{wk}$	405.56	381.82	-wk	9.448
	(2.044)	(1.833)		(1.059)
$-Y_{\delta}$	1175.7	-	$-\delta$	-86.826
	(0.677)			(-0.634)
-expl	2.752	-	-expl	0.178
	(0.260)			(0.069)
LR (df)	56.3 (7)	55.4 (5)	predict	5.037
				(3.186)
McFadden R^2	0.82	0.80	LR (df)	55.9 (7)
			McFadden R^2	0.81

Notes: t-statistics between brackets.

have shown also that our results favour the option-like model of the investment and, accordingly, the amended NPV against the standard approach of the NPV which suggests to choose investment for which discounted net cash flows are positive. It is time now to investigate whether or not our theory fits the observed reality of the mining firm of our sample. In our theoretical framework, we have hypothesized that at the investment date, s, the threshold output price is equal to the actual price ($p_s^*=p_s$). While before s, $p_t^* > p_t$. To assess the model we have to find out whether or not this is what actually happens. The first step is to determine the predicted value of p_t^* using the estimates of the probit model defined in (32)'. The estimated β^*'s are normalized by the estimated standard error of the regression, σ. Therefore, to determine p_t^*, the true β's are retrieved as follows: $\beta=\beta^*.\sigma$ ($\sigma=0.155$). The following equation has been used in order to compute the predicted value of p_t^*

$$p_t^* = 0.1550\times(0.608\times10^{-31})\mathrm{K}_{t,s} + 0.1550\times363.82\,Y_{wen;t,s}$$
$$-0.1550\times2{,}731.1\,Y_{wma;t,s} - 0.1555\times381.8\,Y_{wk;t,s}$$

Tables 3 displays the different predicted values of p_t^* for each mining firm as compared to actual prices. The predicted value of p_t^* is found very high particularly during the earlier years. This is certainly due to the linearizations involved in our procedure. Nonetheless, the results indicate that for 50% of the sample the year at which the investment has been effectively made is correctly predicted by the model, that is $p_s=p_s^*$ and $p_t \ngeq p_t^*$, $t < s$. This proportion increases to 83% if we include those firms which invested one year before date s.

[insert Tables 3 here]

Table 3a. Comparison between the predicted investment date to the empirical date

year	p_t	p_t^*	year	p_t	p_t^*
Bethlehem copper; s=1966			Geco mines; s=1970		
1960	1.126	7.822	1964	1.125	14.005
1961	1.059	6.290	1965	1.206	13.706
1962	1.143	5.682	1966	1.151	10.601
1963	1.114	3.930	1967	1.119	5.474
1964	1.125	1.247	1968	1.114	2.966
1965	1.206	1.776	1969	1.203	1.319
1966**	1.151	1.151	1970**	1.302	1.302
Granisle mines; s=1966			Bethlehem copper; s=1971		
1960	1.126	7.891	1965	1.206	22.917
1961	1.059	6.346	1966	1.151	18.413
1962	1.143	5.731	1967	1.119	11.347
1963	1.114	3.964	1968	1.144	8.056
1964	1.125	1.257	1969	1.203	4.801
1965	1.206	1.791	1970	1.302	3.491
1966**	1.151	1.151	1971**	1.000	1.000
Bethlehem copper; s=1967			Lornex mines; s=1972		
1961	1.059	11.924	1966	*1.151*	*14.310*
1962	1.143	11.214	1967	*1.119*	*8.892*
1963	1.114	9.341	1968	*1.144*	*6.455*
1964	1.125	6.452	1969	*1.203*	*3.820*
1965	1.206	6.652	1970	*1.302*	*3.019*
1966	1.151	4.318	1971*	*1.000*	*0.588*
1967**	1.119	1.119	1972	*0.898*	*0.898*

Notes: s=date at which the firm created its capacity; *= year at which the firm should have exercised its option to invest; **= cases in which the model has predicted correctly the investment date.

Table 3b. Comparison between the predicted investment date to the empirical date

year	p_t	p_t^*	year	p_t	p_t^*
Bell copper s=1972			Bethlehem copper; s=1976		
1966	1.151	23.173	1970	1.302	45.689
1967	1.119	14.666	1971	1.000	40.478
1968	1.144	10.734	1972	0.898	38.402
1969	1.203	6.577	1973	0.968	10.525
1970	1.302	5.113	1974	1.020	7.003
1971*	1.000	1.000	1975	0.787	6.141
1972	0.898	0.898	1976**	0.707	0.707
Granisle mines; s=1972			Lornex mines; s=1979		
1966	1.151	21.186	1973	0.968	4.511
1967	1.119	13.386	1974	1.020	2.745
1968	1.144	9.792	1975	0.787	2.479
1969	1.203	5.987	1976*	0.707	0.070
1970	1.302	4.658	1977	0.653	-
1971*	1.000	0.911	1978	0.673	-
1972	0.898	0.898	1979	0.882	0.882
Lornex mines; s=1974			Equity silver mines; s=1980		
1968	1.144	6.741	1974	1.020	9.846
1969	1.203	1.584	1975	0.787	8.770
1970	1.302	4.023	1976	0.707	2.157
1971	1.000	5.345	1977	0.653	0.853
1972	0.898	5.455	1978*	0.673	0.258
1973*	0.968	0.874	1979	0.883	2.409
1974	1.020	1.020	1980	0.916	0.916

Notes: s=date at which the firm created its capacity; *=year at which the firm should have exercised its option to invest; **=cases in which the model has predicted correctly the investment date.

IV. Conclusion

In this paper we have shown how the option-like characteristics of an investment in capacity with uncertainty and a putty clay technology can explain the timing of investment. Unlike the previous contributions on the theory of option theory as applied to real assets which rest on simulations and arbitrarily chosen values of parameters and variables, our empirical results are based on real data from a sample of Canadian mining firms and use inference statistics. The qualitative dependant variable model was appropriate for the estimation of our model, as it involves a critical threshold that determines whether investment takes place or not. The results lead us to reject the standard NPV criterion of investment in favour of the model based on option theory. Regarding the investment timing, our results indicate that for a high proportion of the sample, the dates chosen by firms to undertake their investment are predicted correctly.

Appendix

Derivatives of $G(p_t, z_t)$

To evaluate the values of function $G(p_t, z_t)$ and the variables B_t and Γ_t, respectively characterized in (13) and (22), we use the mining firm capacity choice model estimated in Harchaoui and Lasserre (1992a)

$$Q = \theta_o + \theta_{PA_2} PA_2 + \theta_{WKA_2} WKA_2 + \theta_R R + \sum_i \theta_{WiA_2} WiA_2 + \theta_{EXPL} EXPL \qquad with \quad i = ENE(energy)\ ,\ MA(material) \tag{A.1}$$

where $PA_2 \equiv P/A_2$, $WKA_2 \equiv WK/A_2$, $WMA_2 \equiv WM/A_2$ and $WENA_2 = WEN/A_2$ are, respectively, the output price, the net cost of capital, the material price and the energy price each of them normalized by the fiscal variable A_2. The dummy variable EXPL takes the value 1 if we face a open pit mining firm and zero otherwise, and R represents the mineral reserves. The parameters estimates, which are all statistically significant at least at 10%, take the following values

$$\theta_o = 54{,}332;\ \theta_{PA_2} = 32{,}673;\ \theta_{WKA_2} = -7{,}061.9;\ \theta_R = 0.125x10^3; \\ \theta_{WMA_2} = -57{,}389;\ \theta_{WENA_2} = 2{,}896.8;\ \theta_{EXPL} = 5{,}484.5 \tag{A.2}$$

Furthermore, additional results from (A.2) are needed for later use. These are

$$\frac{\partial Q_t}{\partial p_t}=\frac{\partial Q_t}{\partial PA_2}.\frac{\partial PA_2}{\partial p_t}=\frac{\theta_{PA_2}}{A_2} \quad ; \quad \frac{\partial Q_t}{\partial w_i}=\frac{\partial Q_t}{\partial WiA_2}.\frac{\partial WiA_2}{\partial w_i}=\frac{\theta_{WiA_2}}{A_2}$$

$$\frac{\partial Q_t}{\partial WK}=\frac{\partial Q_t}{\partial WKA_2}.\frac{\partial WKA_2}{\partial WK}=\frac{\theta_{WKA_2}}{A_2} \quad ; \quad \frac{\partial Q_t}{\partial \delta}=\frac{\partial Q_t}{\partial WK}.\frac{\partial WK}{\partial \delta}=\frac{\theta_{WKA_2}.qA_0}{A_2} \tag{A.3}$$

$$\frac{\partial Q_t}{\partial A_2}=\frac{\partial Q_t}{\partial PA_2}.\frac{\partial PA_2}{\partial A_2}+\frac{\partial Q_t}{\partial WK}.\frac{\partial WK}{\partial A_2}+\frac{\partial Q_t}{WiA_2}.\frac{\partial WiA_2}{\partial A_2}$$
$$=-\frac{1}{A_2^2}\{\theta_{PA_2}p_t+\theta_{WKA_2}q\delta A_0+\theta_{WiA_2}WiA_2\}$$

Next, the function $G_s(p_s,z)$ depends not only on the same variables as the capacity, Q, but also on δ, A_1 and A_2. Using (A.3), $G_s(p_s,z)$ may be written as

$$G_s(p_s,z)=\frac{p_s}{A(p_s,z)}e^{-\delta T(p_s,z)}\theta_{PA_2}\frac{T(p_s,z)}{Q(p_s,z)} \tag{A.4}$$

The derivatives of $G_s(p_s,z)$ with respect of each of these variables are given below (dropping s where there is no risk of confusion).

1.1 Derivation of G_p

$$G_p=\frac{1}{A}e^{-\delta T}\theta_{PA_2}\frac{T}{Q}+pe^{-\delta T}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial p}\{\frac{1}{A}\}$$
$$+\frac{p}{A}e^{-\delta T}\theta_{PA_2}\frac{\partial}{\partial p}\{\frac{T}{Q}\}+\frac{p}{A}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial p}\{e^{-\delta T}\} \tag{A.5}$$

where

$$\frac{\partial}{\partial p}\{\frac{1}{A}\}=\frac{\frac{T}{Q}\theta_{PA_2}e^{-\delta T}}{A^2}$$

$$\frac{\partial}{\partial p}\{\frac{T}{Q}\}=\frac{-2\theta_{PA_2}}{A_2Q^3} \qquad \text{(A.6)}$$

$$\frac{\partial}{\partial p}\{e^{-\delta T}\}=\frac{\delta\theta_{PA_2}}{Q}\frac{T}{A_2}$$

Using (A.6) into (A.5) gives

$$G_p=\frac{e^{-2\delta T}\theta_{PA_2}}{A}\{\frac{T}{Q}+p[\frac{(T/Q)^2}{A}e^{-\delta T}-\frac{2\theta_{PA_2}}{A_2Q^3}+\frac{(T/Q)^2\delta\theta_{PA_2}}{A_2}]\} \qquad \text{(A.7)}$$

1.2 Derivation of G_R

$$G_R=\frac{p}{A}e^{-\delta T}\theta_{PA_2}\frac{\partial}{\partial R}\{\frac{T}{Q}\}+pe^{-\delta T}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial R}\{\frac{1}{A}\}+\frac{P}{A}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial R}\{e^{-\delta T}\} \qquad \text{(A.8)}$$

where

$$\frac{\partial}{\partial R}\{\frac{T}{Q}\}=\frac{1-2T\frac{\partial Q}{\partial R}}{Q^2}$$

$$\frac{\partial}{\partial R}\{\frac{1}{A}\}=-\frac{A_2}{A^2Q}(1-T\frac{\partial Q}{\partial R}) \qquad \text{(A.9)}$$

$$\frac{\partial}{\partial R}\{e^{-\delta T}\}=\frac{-\delta e^{-\delta T}}{Q}(1-T\frac{\partial Q}{\partial R})$$

Substituting (A.9) into (A.8) yields

$$G_R = \frac{pe^{-\delta T}\theta_{PA_2}}{AQ^2}\{1 - 2T\frac{\partial Q}{\partial R} - \frac{T}{Q}(1 - T\frac{\partial Q}{\partial R})(\frac{A_2}{A} - \delta)\} \qquad \text{(A.10)}$$

1.3 Derivation of G_w

$$G_{w_i} = \frac{p}{A}\theta_{PA_2}\frac{\partial}{\partial w_i}\{\frac{T}{Q}\} + \frac{p}{A}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial w_i}\{e^{-\delta T}\}$$
$$+ pe^{-\delta T}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial w_i}\{\frac{1}{A}\} \qquad \text{(A.11)}$$

where

$$\frac{\partial}{\partial w_i}\{\frac{T}{Q}\} = \frac{-2\theta_{WiA_2}}{A_2 Q^3}$$

$$\frac{\partial}{\partial w_i}\{e^{-\delta T}\} = \frac{\delta\theta_{WiA_2}e^{-\delta T}}{Q}\frac{T}{A_2} \qquad \text{(A.12)}$$

$$\frac{\partial}{\partial w_i}\{\frac{1}{A}\} = \frac{T}{Q}\frac{\theta_{WiA_2}e^{-\delta T}}{A^2}$$

Substituting (A.12) into (A.11) yields

$$G_{w_i} = \frac{p}{A}e^{-\delta T}\theta_{PA_2}\theta_{WiA_2}[-\frac{2}{A_2 Q^3} + \frac{T^2}{Q^2 A_2} + \frac{e^{-\delta T}}{A}(T/Q)^2] \qquad \text{(A.13)}$$

1.4 Derivation of G_{WK}

$$G_{WK} = \frac{p}{A}e^{-\delta T}\theta_{PA_2}\frac{\partial}{\partial WK}\{\frac{T}{Q}\} + e^{-\delta T}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial WK}\{\frac{p}{A}\}$$
$$+ \frac{p}{A}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial WK}\{e^{-\delta T}\} \qquad \text{(A.14)}$$

where

$$\frac{\partial}{\partial WK}\{\frac{T}{Q}\}=\frac{R\theta_{WKA_2}(1-Q)}{A_2}$$

$$\partial\frac{p}{\partial WK}\{\frac{p}{A}\}=\frac{pe^{-\delta T}\theta_{WKA_2}}{A^2}\frac{T}{Q} \qquad \text{(A.15)}$$

$$\frac{\partial}{\partial WK}\{e^{-\delta T}\}=\frac{\delta e^{-\delta T}\theta_{WKA_2}}{A_2}\frac{T}{Q}$$

Substituting (A.15) into (A.14) gives

$$G_{WK}=\frac{p}{A}e^{-\delta T}\theta_{PA_2}\theta_{WKA_2}[\frac{R(-1-Q)}{A_2}+\frac{e^{-\delta T}}{A}(\frac{T}{Q})^2+(\frac{T}{Q})^2\frac{\delta}{A_2}] \qquad \text{(A.16)}$$

1.5 Derivation of G_{A1}

$$G_{A_1}=\frac{p}{A}e^{-\delta T}\theta_{PA_2}\frac{\partial}{\partial A_1}\{\frac{T}{Q}\}+\frac{p}{A}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial A_1}\{e^{-\delta T}\}$$
$$+e^{-\delta T}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial A_1}\{\frac{P}{A}\} \qquad \text{(A.17)}$$

where

$$\frac{\partial}{\partial A_1}\{\frac{T}{Q}\}=-R\frac{\partial Q}{\partial A_1}(1-Q)=0$$

$$\frac{\partial}{\partial A_1}\{\frac{p}{A}\}=-\frac{p}{A^2}\frac{(1-e^{-\delta T})}{\delta} \qquad \text{(A.18)}$$

$$\frac{\partial}{\partial A_1}\{e^{-\delta T}\}=0$$

Combining (A.17) and (A.18) gives

$$G_{A_1} = -e^{-\delta T}\theta_{PA_2}\frac{T}{Q}\frac{p}{A^2}\frac{(1-e^{-\delta T})}{\delta} \tag{A.19}$$

1.6 Derivation of G_{A2}

$$G_{A_2} = \frac{p}{A_2}e^{-\delta T}\theta_{PA_2}\frac{\partial}{\partial A_2}\{\frac{T}{Q}\} + e^{-\delta T}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial A_2}\{\frac{p}{A}\}$$
$$+\frac{p}{A}\theta_{PA_2}\frac{T}{Q}\frac{\partial}{\partial A_2}\{e^{-\delta T}\} \tag{A.20}$$

where

$$\frac{\partial}{\partial A_2}\{\frac{p}{A}\} = e^{-\delta T}p\{-\frac{1}{\delta}\frac{1}{A_2}[\theta_{PA_2}p + \theta_{WK}q\,\delta A_0 + \sum_i \theta_{WiA_2}Wi]\frac{T}{Q}\}$$
$$\frac{\partial}{\partial A_2}\{\frac{T}{Q}\} = \frac{R(1+Q)}{A_2^2}(\theta_{PA_2}p + \theta_{WK}q\,\delta A_0 + \sum_i \theta_{WiA_2}Wi) \tag{A.21}$$
$$\frac{\partial}{\partial A_2}\{e^{-\delta T}\} = \frac{-\delta e^{-\delta T}}{A_2^2}(\theta_{PA_2}p + \theta_{WK}q\,\delta A_0 + \sum_i \theta_{WiA_2}Wi)\frac{T}{Q}$$

Combining (A.20) and (A.21) gives

$$G_{A_2} = \frac{\theta_{PA_2}e^{-\delta T}}{A_2^2}(\theta_{PA_2}p + \theta_{WKA_2}q\,\delta A_0 + \sum_i \theta_{WiA_2}Wi)[\frac{pR(1+Q)}{A}$$
$$+e^{-\delta T}(\frac{T}{Q})^2 pA_2 + \frac{p}{A}(\frac{T}{Q})^2\delta - \frac{e^{-2\delta T}}{\delta}\theta_{PA_2}\frac{T}{Q}] \tag{A.22}$$

1.7 Derivation of G_δ

$$G_\delta = \frac{p}{A} e^{-\delta T} \theta_{PA_2} \frac{\partial}{\partial \delta}\{\frac{T}{Q}\} + p e^{-\delta T} \theta_{PA_2} \frac{T}{Q} \frac{\partial}{\partial \delta}\{\frac{1}{A}\} + \frac{p}{A} \theta_{PA_2} \frac{T}{Q} \frac{\partial}{\partial \delta}\{e^{-\delta T}\} \tag{A.23}$$

where

$$\frac{\partial}{\partial \delta}\{\frac{T}{Q}\} = \frac{-2\theta_{WKA_2}}{Q^3} \frac{qA_0}{A_2}$$

$$\frac{\partial}{\partial \delta}\{\frac{1}{A}\} = -\frac{1}{A^2}\{\frac{(T1\delta+1)}{\delta}[\frac{e^{-\delta T1}(A_1-A_2)}{\delta}] + \frac{A_2}{\delta^2}(Te^{-\delta T}\delta + \delta^2 \frac{T}{Q} \theta_{WKA_2} \frac{qA_0}{A_2} e^{-\delta T} - e^{-\delta T})\} \tag{A.24}$$

$$\frac{\partial}{\partial \delta}\{e^{-\delta T}\} = (-T + \frac{2\delta\theta_{WKA_2}}{Q^3} \frac{qA_0}{A_2}) e^{-\delta T}$$

Substituting (A.21) into (A.20) gives

$$G_\delta = \frac{p}{A} \theta_{PA_2} e^{-\delta T}\{\frac{-2\theta_{WKA_2}}{Q^3} \frac{qA_0}{A_2} - \frac{T}{Q}\{\frac{(T1\delta+1)}{\delta^2} e^{-\delta T1}(A_1 - A_2) + \frac{A_2}{\delta^2}(Te^{-\delta T}\delta + \delta^2 \frac{T}{Q} \theta_{WKA_2} \frac{qA_0}{A_2} e^{-\delta T} - e^{-\delta T})\} + \frac{T}{Q}(-T + \frac{2\delta\theta_{WK}A_2}{Q^3} \frac{qA_0}{A_2}) \tag{A.25}$$

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Time Varying Risks and Returns:

Evidence From Mining Industry Data

I. Introduction

Over the past decade, there has been an extensive research on the time-varying behaviour of risks and returns[25]. This, as well as the disappointing results of the unconditional capital asset pricing model (CAPM), has motivated the econometric specification of the conditional CAPM. Since the Merton (1980) and Christie (1982) findings on changing returns variability, Fama and MacBeth (1974) insights on time varying risk-premium and Engel (1982) seminal work on conditional heteroscedasticity, a number of contributions assessed empirically the conditional version of the CAPM under the autoregressive conditional heteroscedastic (ARCH) specifications (see Bollerslev, Chou and Kroner 1992). Recently, Ng (1991) specified the CAPM where the time-varying covariance matrix of assets returns is assumed to follow a multivariate GARCH process.

However, ARCH models have been criticized because they do not aggregate in general (Harvey 1989). As well, GARCH models have several drawbacks for the purpose of asset pricing applications (Nelson 1991). An alternative specification which relies on the Generalized Method of Moments (GMM), as developed by Hansen (1982), has then been used in this instance. Campbell (1987), Harvey (1989) and Bodurtha and

[25]French, Schwert and Stambaugh (1987) findings, based on daily returns to the Standard and Poors (S&P) composite portfolio from 1928 to 1984, have revealed that estimates of aggregate monthly returns standard deviation was about four times in the time span 1929-1933 than in the 1953-1970 period. See also Schwert (1989) who has shown that the standard deviation of monthly stock returns vary from 2 to 20% per month during the 1857-1987 period.

Mark (1991) have shown that the GMM is a convenient alternative to maximum likelihood estimation of simultaneous equation systems of ARCH models. Harvey develops tests of the CAPM and multifactor asset pricing models using the instrumental variables approach. His tests allow for both time-varying expected returns and conditional covariances. Bodurtha and Mark have proposed one of the most general approach which allows for a time varying beta and market risks premia as well.

This paper provides an empirical assessment on the conditional CAPM and intertemporal CAPM (ICAPM) as applied to mining sectors. It is distinguished from most other studies in two major respects. First, we investigate a highly cyclical industry where a great deal of exit and/or mergers and acquisitions occur[26]. As a result, it is probably not possible to compile time series of sufficient length for individual firms and, accordingly, to carry out empirical tests with ranked security market betas and/or value-weighted firm size portfolios. To circumvent this problem, we choose instead a more natural approach based on the value-weighted industry portfolios. Second, the conditional CAPM and ICAPM are modeled as time-varying parameter models (Shanken 1990; Ferson and Harvey 1991). This approach, however, departs from the latter in several respects: i) we control for conditional heteroscedasticity of the excess market and bond returns by weighting these two variables with their respective estimated conditional standard errors; ii) we derive

[26]Additional evidence on exit of Canadian copper firms has been provided by Young (1991). During the 1961-1978 period, exit has taken the following three forms in this industry: i) operations cease, ii) merger with or amalgamation by another firm or the firm becomes involved in a wider scope of activities, or iii) the relevant data are no longer made public.

time-varying estimates of market and bond betas at each sample point; iii) we test for the unconditional CAPM and ICAPM as a polar case. Because our specification is fairly general, it allows us to test simultaneously for the market beta and the bond beta (and other meaningful factors). Our findings provide support for the conditional CAPM in the case of metal and gold mining industries as well as for the unconditional CAPM for integrated mining industries. At the same time, we were unable however to reject the conditional ICAPM in favour of the conditional CAPM. Also, results on the risk-return relationship display important nonlinearities which can be ascribed to the cyclical nature of mining industries as well as to major and minor shocks it faced during the 1971-1990 period.

The remainder of the paper is organized as follows: section II is devoted to the model as well as to the econometric specification. Section III describes the data base and provides the results. Finally, section IV concludes.

II. Specification of the conditional CAPM and ICAPM

The unconditional CAPM which assumes a constant beta can lead to erroneous conclusions regarding the conditional mean-variance portfolio (Hansen and Richard 1987). The conditional CAPM provides a convenient way to incorporate the time-varying conditional variances and covariances and risk-premia that have been found to be important in recent empirical finance literature. In general, the conditional CAPM and ICAPM specifications are well motivated only if the conditional pair-wise covariances of excess returns and the conditional variances of market and bond excess returns change through time. One possible way to specify a time-varying CAPM is to condition on the observed information set Z_{t-1}. We follow Campbell (1987) and Harvey (1989) who assume that the conditional expectations of the asset returns are linear in the instruments.

Let the asset's i, the market (m) and bond (g) conditional expected excess returns be linear in the vector of the instrumental variable Z_{t-1}. We have respectively

$$r_{i,t} = \phi_0 + \Sigma_j^k \phi_j Z_{j,t-1} + u_{i,t} \qquad i = 1,2,...,n. \tag{1}$$

$$r_{m,t} = \xi_0 + \Sigma_j^k \xi_j Z_{j,t-1} + u_{m,t} \tag{2}$$

and

$$r_{g,t} = \lambda_0 + \Sigma_j^k \lambda_j Z_{j,t-1} + u_{g,t} \tag{3}$$

To test for the time-varying conditional moments of $r_{i,t}$ and $r_{m,t}$, we regress the estimated covariance and variance residuals on the instrumental variables

$$\hat{u}_{i,t}.\hat{u}_{m,t} = \psi_0 + \Sigma_j^k \psi_j Z_{j,t-1} + \nu_{i,t} \qquad i = 1,2,..,n. \tag{4}$$

$$\hat{u}_{i,t}.\hat{u}_{g,t} = \pi_0 + \Sigma_j^k \pi_j Z_{j,t-1} + \mu_{i,t} \qquad i = 1,2,..,n. \tag{5}$$

and

$$\hat{u}_{m,t}^2 = \eta_0 + \Sigma_j^k \eta_j Z_{j,t-1} + \nu_{m,t} \tag{6}$$

$$\hat{u}_{g,t}^2 = \gamma_0 + \Sigma_j^k \gamma_j Z_{j,t-1} + \nu_{g,t} \tag{7}$$

If the null hypothesis (homoscedasticity) is true, only the intercept should be significantly different from zero.

Our conditional CAPM and ICAPM have, respectively, the following forms

$$r_{i,t} = \alpha_i(Z_{t-1}) + \beta_m(Z_{t-1})\check{r}_{m,t} + \varepsilon_{i,t} \qquad i = 1,2,..,n. \tag{8}$$

and

$$r_{i,t} = \alpha_0(Z_{t-1}) + \beta_m(Z_{t-1}).\check{r}_{m,t-1} + \beta_g(Z_{t-1}).\check{r}_{g,t-1} + \omega_{i,t} \qquad i = 1,2,..,n. \tag{9}$$

where, if any, the conditional expected market-risk premium and bond-risk premium are corrected for conditional heteroscedasticity, that is[27]

$$\check{r}_{\kappa,t} = \frac{r_{\kappa,t}}{\sigma_{\kappa,t-1}} \tag{10}$$

where $\sigma_{\kappa,t-1}$ represents the conditional standard error of portfolio κ=market (m), bond (g), whereas the parameters α_i and β_i have the following structure (Shanken 1990)

$$\alpha_i = \delta_0 + \Sigma_j^k \delta_j Z_{j,t-1} \tag{11}$$

$$\beta_\kappa = \varphi_0 + \Sigma_j^k \varphi_j Z_{j,t-1} \tag{12}$$

Substituting both (11) and (12) into (8) and (9), respectively, yields the conditional CAPM and ICAPM with time varying market and bond betas, as well as time-varying market-risk premium

$$r_{i,t} = \zeta_0 + \zeta_m \check{r}_{m,t-1} + \Sigma_j^k \zeta_j Z_{j,t-1} + \Sigma_j^k \zeta_{j,m} \ (\check{r}_{m,t} . Z_{j,t-1}) + \vartheta_{i,t} \tag{13}$$
$$i = 1,2..,n.$$

[27]This method of weighted least squares (WLS) has already been used by Schwert and Seguin (1990) to account for heteroscedasticity in a CAPM with time-varying betas.

and

$$r_{i,t} = \rho_0 + \rho_m \check{r}_{m,t-1} + \rho_g \check{r}_{g,t-1} + \Sigma_j^k \rho_j Z_{j,t-1} + \Sigma_j^k \rho_{j,m} (Z_{t-1} . \check{r}_m) + \Sigma_j^k \rho_{j,g} (Z_{t-1} . \check{r}_g) + \varepsilon_{i,t} \qquad i = 1, 2, .., n. \tag{14}$$

Under these general specifications, which have something in common with the asset pricing theory (APT), we would be able to test for the unconditional versions of the CAPM and ICAPM. It suffices to test for the restriction that all the parameters related to Z_{t-1} in each of the equations (13) and (14) are simultaneously equal to zero. On the other hand, unlike other studies, our conditional specifications provide time-varying estimates of market and bond betas, as well as time-varying estimates of the other explanatory variables included in (13) and (14). As a result, to test for the statistical significance of these variables, the standard deviations of the slopes estimates has to be computed properly. Let[28]

$$V(\hat{I}) = [\frac{\partial I}{\partial \zeta}]' |_{z_{t-1}} \Sigma^* [\frac{\partial I}{\partial \zeta}] |_{z_{t-1}} \tag{15a}$$

where

$V(I)$=estimated variance of the estimate I,

$[.]$=vector of the partial derivative of I with respect to each component of the parameter vector ζ, evaluated at the mean value z_{t-1},

Σ^*=estimated variance-covariance matrix of the parameters.

Therefore, the t statistic is

[28] See Goldberger (1964, 125).

$$t = \frac{I}{\sqrt{V(I)}}\Big|_{z_{t-1}} \qquad (15b)$$

To sum up, our empirical discussion consists of the following steps. First, we test for the heteroscedasticity of the conditional variances and covariances given by (4)-(7). Second, for the same asset i, if either its covariance, $\sigma_{i,\kappa}$ ($\kappa=m, g$), or its variance, σ^2_{κ}, is time-varying, we model the conditional CAPM and ICAPM in the way defined by (13) and (14). From the estimates of (13) and (14), we compute the betas as well as the effects of the other exogenous variables on each asset's excess return i. Otherwise, if for instance we reject the hypotheses that the market or bond premiums are heteroscedastic, there is no need to correct for conditional heteroscedasticity in the way shown by (10). Therefore, we keep only the variables $r_{m,t}$ and $r_{g,t}$ in (13) and (14). Finally, from (13) and (14) we test for the unconditional CAPM and ICAPM as a polar case.

III. Data and empirical results

1. Data

This paper uses monthly realized returns of Canadian mining industry common stocks traded in the Toronto Stock Exchange (TSE) during the period 1969-1990. Our portfolios are based on the value-weighted equity of the firms in each industry, and we choose, arbitrarily, to work with three sectors of the mining industry: integrated mines (int), metal mines (met) and gold mines (gol). This method allows for a longer sample period for excess returns than it would with the two others standard methods. The first grouping technique ranks assets according to their market betas and divides them into groups. The second one sorts securities into portfolios based on their market values at the end of each year. These widely used portfolio formation techniques assume the availability of data series on equity or common stock returns for each individual firm, which appears to be unrealistic in the case of mining firms. The cyclical nature of this industry leads to a great deal of exit and mergers, thereby making it difficult to compile firm-level data over a sufficient sample period. While our portfolios rest on longer sample period time series, their aggregate nature has, however, the disadvantage of not allowing tests on the relation between firm size and stock return (Banz 1981 and Reinganum 1981)[29].

One of the major problem facing the Toronto Stock Exchange (TSE) -and accordingly all other Canadian stock exchange markets - is that of thin trading (i.e. the low monthly trading volume of many stocks). As a result, the TSE might not be mean-variance efficient and the only priced risk should be the systematic risk relative to, say, the integrated North American market. The issue of segmentation versus integration for Canadian stocks relative to a global North American market embodying US and Canadian stocks has been investigated by Jorion and Schwartz (1986). The joint hypothesis of integration and mean-variance efficiency of the so-called North American market was rejected, which clearly suggests a kind of segmentation. An additional reason which makes our risk-return relationship reliable is that we use only value-weighted financial data series. It turns out that we find no evidence of what is known

[29]This, by no means, does not imply that the firm size effect on excess returns do not survive at all at the aggregate level. Carleton and Lakonishok (1986) have found that in each of the industries they investigated, i) small companies outperformed large companies and ii) a substantial size effect in january.

as a signal of thin trading, the presence of autocorrelation for excess returns (see Table 1).

To compute the portfolio i excess returns, we used the one-month rate of return on Canadian government bonds. Basically, since the latter is available only for the recent years, we derive it from the yield on 3-month Treasury Bills. The procedure consists of solving the following equation formula for r_1: $(1+r_3)=(1+r_1)^3$, where r_3, the 3-months Treasury Bills, is known in our case[30]. The information set includes the first lag of the industrial bond yield averages ($ibay_{t-1}$), the industrial stock dividend yields ($isdy_{t-1}$), the industrials price/earnings ratios ($iper_{t-1}$), a constant and a dummy variable for seasonality (jan_t). These instruments have been chosen to reflect expectations in the economy that are related to the prospects for stock returns. The data on the risk free rate and the instruments are obtained from different issues of the Bank of Canada Monthly Review, while those on portfolios returns were computed from the Toronto Stock Exchange Review. Table 1 provides summary statistics on the excess market returns, bond returns as well as on the instruments used as proxies for the information set. As expected, the gold mining's excess returns are negative and have the largest volatility over the January 1971 to December 1990 period. The integrated mines have the lowest excess return volatility. Except for the excess bond returns which significantly indicate the presence of seasonality, the other variables reveal no evidence on autocorrelation of excess asset returns. As well, all the instruments have a

[30]See Jorion and Schwartz (1986) and Berndt (1991, 43) for the same procedure.

relatively strong autocorrelation and a significant seasonal movement. Additional tests (not reported here) suggest the rejection of non stationarity of our variables.

[insert Table 1 here]

2. Testing for the conditional heteroscedasticity

The results of the OLS regressions of (1)-(3) are displayed in Table 2 where all reported standard errors are corrected for conditional heteroscedasticity using White's (1980) method. Except for the excess bond return, $r_{g,t}$, for which the instruments explain a great deal of variance (83%), the other results are roughly similar, although slightly higher than most of the findings in the financial literature. Most of the papers on the subject found an adjusted R^2 less than 10 % for excess market returns (see Campbell 1987, Harvey 1989, Fama and French 1988 and Shanken 1990), whereas ours equals 12%. This statistic is even higher in the case of the excess return of metal mines (15%). However, the R^2 of the other two asset excess returns, integrated and gold mines, are respectively equal to 5% and 6%. Surprisingly, contrary to major findings in the literature, the results reveal the presence of seasonality only in the case of metal mining excess returns[31].

On the other hand, Table 2 also provides the computed chi-squared statistic for each of the equations (4)-(7) in order to test for conditional heteroscedasticity. The purpose of these regressions is to test the null hypothesis that the conditional covariances and variances are constant. First, the squared regression residuals of the market (bond) excess return is regressed on the instrumental variables. Second, the

[31] We have introduced a dummy variable which equals 1 whenever the excess porfolio returns are negative (and 0 otherwise) in order to control for cycles. It turned out that this variable is not significant mostly in all cases and, therefore, is not included in our regressions.

TABLE 1
Summary statistics on excess returns portfolios and instrumental variables

	Mean	Std. dev.	Autocorrelation							
			ρ_1	ρ_2	ρ_3	ρ_4	ρ_{12}	ρ_{24}	Q(12) df=11	P-value
Panel A: portfolios excess returns										
$r_{m,t}$	0.01191	0.0523	-0.01	-0.04	0.12	0.0	-0.03	0.02	11.8	.377
$r_{g,t}$	0.07204	0.0136	0.69	0.87	0.67	0.79	0.60	0.38	1378.8	0.000
$r_{met,t}$	0.02184	0.00218	0.00	0.01	0.05	0.06	-0.01	0.16	12.4	0.331
$r_{intg,t}$	0.00992	0.07974	0.00	-0.05	-0.01	-0.08	-0.06	0.09	6.0	0.876
$r_{gol,t}$	-0.00496	0.11376	0.00	-0.03	0.00	0.15	0.01	0.04	9.2	0.606
Panel B: instrumental variables										
$ibay_{t-1}$	0.11135	0.02212	0.82	0.91	0.79	0.83	0.60	0.30	1639.3	0.000
$isdy_{t-1}$	0.03813	0.00775	0.71	0.86	0.69	0.75	0.40	0.21	1120.4	0.000
$iper_{t-1}$	0.13006	0.04811	0.83	0.94	0.78	0.84	0.39	0.14	1326.8	0.000

Note:

r_l=excess return of portfolio l (l=market (m), bond (g), metal (met), integrated (intg) and gold (gol)),
$ibay_{t-1}$=industrial bond average yields,
$isdy_{t-1}$=industrial stock dividend yield,
$iper_{t-1}$=industrial price earning ratio.
Q(12)=Box-Pierce-Ljung statistic.

product of the regression residuals for each of the assets' excess return and the market (bond) excess return is regressed on the same instrumental variables. If the null hypothesis is true, only the intercept should be significantly different from zero. We reject the hypothesis of homoscedasticity for bond (σ^2_g), metal (σ^2_{met}) and gold (σ^2_{gol}) excess return variances, as well as for covariances between, respectively, metal and gold excess returns and the market excess returns ($\sigma_{m,met}$, $\sigma_{m,gol}$). In all the remaining cases, that is for σ^2_{intg}, $\sigma_{m,intg}$, $\sigma_{g,met}$ and $\sigma_{g,intg}$, we are however unable to reject the null hypothesis. Therefore, heteroscedasticity is detected only for bond premium, the bond beta of each portfolio, and the market beta of gold and metal mine excess returns.

[insert Table 2 here]

To sum up, the market-risk premium unlike the bond premium are not time-varying. Therefore, there is no need to use (10) for market-risk premium. Now we turn to the betas. According to Table 2, only metal and gold mining portfolios have time-varying betas, whereas the bond beta is time-varying in all instances. Following these findings, we have estimated the conditional CAPM for gold and metal mines portfolios and the unconditional CAPM for integrated mines portfolio. The conditional ICAPM, on the contrary, has been estimated for all three portfolios.

3. Empirical results of the conditional CAPM and ICAPM

We apply (13) and (14) to three value-weighted assets of the Canadian metal, gold and integrated mines for the 1969.1-1990.12 period. With the above remarks in mind, we want to assess the conditional, unconditional CAPM as well as the conditional ICAPM using mining data. Most of these specifications display the

TABLE 2
Parameters estimates of (1)-(3), using OLS (t-statistics, based on White's consistent covariance matrix estimator method, between brackets)

Panel A: Regression of excess returns r_l on instrumental variables

	r_m	r_g	r_{met}	r_{gol}	r_{intg}
intercept	-0.03287	-0.004238	0.02085	0.05458	0.02004
	(-0.9699)	(-1.189)	(0.3479)	(0.6396)	(0.4068)
$ibay_{t-1}$	-0.7340***	0.57107***	-1.4244***	-1.3721***	-0.79003
	(-4.0193)	(28.833)	(-5.1962)	(-3.8053)	(-3.0874)
$isdy_{t-1}$	2.6888***	0.05331	3.3819***	1.9542	1.9059
	(3.905)	(0.8159)	(3.2659)	(1.258)	(1.9078)
$iper_{t-1}$	0.1736**	0.08189***	0.05406	0.1164	0.01982
	(2.1304)	(8.0182)	(0.3201)	(0.5146)	(0.1478)
jan_t	0.1755	0.0001454	0.04954***	0.0449	0.0326
	(1.2564)	(0.1133)	(2.2506)	(1.6547)	(1.4668)
R^2	0.14	0.83	0.17	0.08	0.07
R^2 adj.	0.12	0.83	0.15	0.06	0.05

Panel B: χ^2 of the regressed residuals on the instrumental variables

u_m^2	u_g^2	u_{met}^2	u_{gol}^2	u_{intg}^2	$u_m u_{met}$	$u_m u_{gol}$	$u_m u_{intg}$	$u_g u_{met}$	$u_g u_{gol}$	$u_g u_{intg}$
7.2	38.2***	12.0**	9.6*	4.8	11.5**	9.1*	4.5	7.4	12.7**	8.8

Note:

u_l=residuals estimates of the portfolio l excess returns' (l=m (market), bond (g), metal (m), gold (g) and integrated (intg) portfolios)
$ibay_{t-1}$=industrial bond average yields,
$isdy_{t-1}$=industrial stock dividend yield,
$iper_{t-1}$=industrial price earning ratio,
jan_t=dummy variable which accounts for January effect.
*, **, ***: significant, respectively, at 10%, 5% and 1%.

following features[32]. First, their general structure has the unconditional CAPM and the unconditional ICAPM as a polar case. Second, and most importantly, they do not constraint the betas to be constant over the sample period. Another advantage of these specifications is that they do not require a tedious estimation method. First, we have estimated the following conditional CAPM

$$\begin{aligned} r_{i,t} &= \zeta_0 + \zeta_{ibay} ibay_{t-1} + \zeta_{isdy} isdy_{t-1} + \zeta_{iper} iper_{t-1} + \zeta_{jan} jan_t \\ &+ \zeta_m \; r_{m,t} + \zeta_{ibaym} ibay_{t-1} . r_{m,t} + \zeta_{isdym} \; isdy_{t-1} . r_{m,t} \\ &+ \zeta_{iperm} iper_{t-1} . r_{m,t} + \zeta_{janm} jan_t . r_{m,t} + \vartheta_{met,t} \end{aligned} \tag{16}$$

for i=met (metal) and gol (gold) mining portfolios. We also estimate the following unconditional CAPM

$$r_{intg,t} = \alpha_0 + \beta_m \; r_{m,t} + \vartheta_{intg,t} \tag{17}$$

Where unfavourable shifts in investment opportunities can be described by interest rates, Merton (1973) showed that the equilibrium expected return is explained not only by its beta but also by the ability of the asset to protect the investor against changes in interest rates. This hedging behaviour becomes possible by holding an additional asset or portfolio which is negatively correlated with the single-state variable (i.e. the interest rate). It turns out that the ICAPM is a multibeta pricing relation with a "market beta" and a "bond beta". Therefore, we have estimated the following specification of the conditional ICAPM which accounts for the heteroscedasticity in bond premia

32 There is only one exception represented by (17).

$$
\begin{aligned}
r_{i,t} = {} & \rho_0 + \rho_{ibay} ibay_{t-1} + \rho_{isdy} isdy_{t-1} + \rho_{iper} iper_{t-1} + \rho_{jan} jan_t \\
& + \rho_m \; r_{m,t} + \rho_{ibaym} ibay_{t-1} . r_{m,t} + \rho_{isdym} isdy_{t-1} . r_{m,t} \\
& + \rho_{iperm} iper_{t-1} . r_{m,t} + \rho_{janm} jan_t . r_{m,t} + \rho_g \tilde{r}_{g,t} + \rho_{ibayg} ibay_{t-1} . \tilde{r}_{g,t} \\
& + \rho_{isdyg} isdy_{t-1} . \tilde{r}_{g,t} + \rho_{ipseg} iper_{t-1} \tilde{r}_{g,t} + \rho_{jang} jan_t . \tilde{r}_{g,t} + \varepsilon_{i,t}
\end{aligned}
\qquad (18)
$$

with i=met, gol; and

$$
\begin{aligned}
r_{intg,t} = {} & \alpha_0 + \beta_m \; r_{m,t} + \rho_{ibay} ibay_{t-1} + \rho_{isdy} isdy_{t-1} + \rho_{iper} iper_{t-1} \\
& + \rho_{jan} jan_t + \rho_g \; \check{r}_{g,t} + \rho_{ibayg} ibay_{t-1} \; . \check{r}_{g,t} \\
& + \rho_{isdyg} isdy_{t-1} . \check{r}_{g,t-1} + \rho_{iperg} iper_{t-1} . \check{r}_{g,t} \\
& + \rho_{jang} jan_t . \check{r}_{g,t} + \varepsilon_{intg,t}
\end{aligned}
\qquad (19)
$$

We have tested, first, for the conditional versions of the CAPM and ICAPM from (16), (18) and (19) by restricting all parameters, except those related to the market and/or bond excess returns, to be jointly equal to zero. Second, to the extent that we deal with a quadratic specification, the significance tests for the different betas as well as those for the other parameters require two steps. For each of these specifications, we calculate the time-varying slope of r_{it} with respect to each of the explanatory variables contained in Z_{t-1}, and then we compute the appropriate value of the t-statistic as indicated by (15). From (16), the conditional CAPM of metal and gold mining firms, the time-varying value of the market beta, B_m, as well as the effect of other variables on $r_{i,t}$, z_ι, are respectively

$$\frac{\partial r_{i,t}}{\partial r_{m,t}} \equiv B^{i}_{m,t} = \zeta_m + \zeta_{ibaym} ibay_{t-1} + \zeta_{isdym} isdy_{t-1} + \zeta_{iperm} iper_{t-1} + \zeta_{janm} jan_t \tag{20}$$

and

$$\frac{\partial r_{i,t}}{\partial z_{\iota,t-1}} = A^{i}_{z_{\iota,t}} = \zeta_{z_\iota} + \zeta_{z_{\iota,m}} r_{m,t} \tag{21}$$

where z_ι is the ι-th element of the vector Z_{t-1} of the instrumental variables with i=met, gol and $\iota = ibay_{t-1}$, $isdy_{t-1}$, $iper_{t-1}$ and jan_t. By the same token, from (18), the conditional ICAPM for the same mining industries as above, we obtain the following time-varying market beta, P_m, and bond beta, P_g

$$\frac{\partial r_{i,t}}{\partial r_{m,t}} \equiv P^{i}_{m,t} = \rho_m + \rho_{ibaym} ibay_{t-1} + \rho_{isdym} isdy_{t-1} + \rho_{iperm} iper_{t-1} + \rho_{janm} jan_t \tag{22}$$

$$\frac{\partial r_{i,t}}{\partial \check{r}_{g,t}} \equiv P^{i}_{g,t} = \rho_g + \rho_{ibayg} ibayg_{t-1} + \rho_{isdyg} isdy_{t-1} + \rho_{iperg} iper_{t-1} + \rho_{jang} jan_t \tag{23}$$

and

$$\frac{\partial r_{i,t}}{\partial z_{\iota,t}} = Z^{i}_{\iota,t} = \rho_{z_\iota} + \rho_{z_\iota}(r_{m,t} + \check{r}_{g,t}) \tag{24}$$

Finally, from (19) the bond beta, B_g^{intg}, and the effect of other variables z_ι, $Z_{\iota,t}^{intg}$, have the same structure as (20) and (21), respectively, whereas the market beta is a constant, β_m.

[insert Tables 3-5 here]

Because variances of rates differ accross assets and assets returns are correlated the disturbance terms of (16) and (17), in one hand, and (18) and (19), on the other hand, are heteroscedastic and correlated accross assets, as it is often the case in empirical finance. To obtain efficient estimates of the parameter variances, we use seemingly unrelated regressions (SURE) method. Tables 3-5 provide parameters estimates of (16)-(19) and their respective slopes ((20)-(24)). Computation of the slopes are evaluated at the sample mean. Some interesting papers have recently explored the relation between the conditional volatility and serial correlation for different stock return series and frequencies (LeBaron, 1992; Bera et al., 1992; Sentana and Wadhwani, 1992; and Conrad and Kaul, 1988). It is found that serial correlations are changing over time and are related to stock return volatility. If our results indicate the presence of conditional heteroscedasticity (Table 2, panel B), it seems however that there is no significant evidence whatsoever of autocorrelated residuals in portfolio excess returns. This finding was readily confirmed by the non significance of each of the portfolios' lagged dependant variable (results are not reported). Empirical analysis of metal mining excess return and the integrated mines excess return based, respectively, on conditional and unconditional CAPM explain a great deal of the variance as indicated by the high value of the R^2. The seasonal effect is found statistically not significant in either of the conditional specifications[33]. The conditional CAPM for $r_{gol,t}$, instead, displays a much lower R^2. In addition, the

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[33] The January-effect is measured properly by using the procedure suggested by Halvorsen and Palmquist (1980).

TABLE 3
Multivariate regression of the conditional CAPM (r_{met} and r_{gol} eq.(16)) and the unconditional CAPM (r_{intg} eq.(17))

	$r_{met,t}$	$r_{gol,t}$	$r_{intg,t}$
Panel A: parameters estimates			
intercept	0.0380 (1.1135)	0.1166 (1.8106)	-0.00477 (-1.537)
$ibay_{t-1}$	-0.6036 (-3.8089)	-0.5843 (-1.9525)	-
$isdy_{t-1}$	0.7212 (1.1498)	-1.2774 (-1.0783)	-
$iper_{t-1}$	-0.0836 (-0.8696)	-0.1658 (-0.9133)	-
jan_t	0.0218 (1.7486)	0.0315 (1.3367)	-
$r_{m,t}$	0.5466 (0.8451)	-1.4064 (-1.1524)	1.233*** (21.275)
$ibay_{t-1}.r_{m,t}$	3.9767 (1.6247)	10.198 (2.2063)	-
$isdy_{t-1}.r_{m,t}$	2.4362 (0.2486)	17.443 (0.9426)	-
$iper_{t-1}.r_{m,t}$	0.4067 (0.2039)	5.8019 (1.5403)	-
$jan_t.r_{m,t}$	0.1324 (0.7381)	-0.2257 (-0.6662)	-
R^2	0.63	0.36	0.65
Panel B: slopes estimates evaluated at the sample mean			
$ibay_{t-1}$	-0.5562*** (-3.4590)	-0.4628 (-1.5238)	-
$isdy_{t-1}$	0.7502 (1.2352)	-1.0700 (-0.9329)	-
$iper_{t-1}$	-0.0788 (-0.8678)	-0.0967 (-0.5635)	-
jan_t	0.0240* (2.0000)	0.0296 (1.3226)	-
$r_{m,t}$	1.1457*** (15.9015)	1.1310*** (8.7607)	-

Note: t-statistics between brackets; *, *** significant, respectively, at 10% and 1% level; System R^2=0.75 and χ^2=335.4.

TABLE 4
Multivariate regression of the conditional ICAPM (r_{met} and r_{gol} eqs. (18) and r_{intg} eq.(19)).

	$r_{met,t}$	$r_{gol,t}$	$r_{intg,t}$
intercept	0.0583 (1.5424)	0.1314 (1.9774)	0.06555 (2.1045)
$ibay_{t-1}$	-0.5452 (-3.1491)	-0.5553 (-1.8403)	0.1201 (0.7966)
$isdy_{t-1}$	0.2096 (0.3062)	-1.5474 (-1.2901)	-1.4982 (-2.5879)
$iper_{t-1}$	-0.1380 (-1.2867)	-0.2284 (-1.2086)	-0.2141 (-2.4479)
jan_t	0.0263 (1.9711)	0.0345 (1.4684)	0.0115 (1.0504)
$r_{m,t}$	0.7919 (1.2107)	-1.4679 (-1.1839)	1.2730*** (20.859)
$\check{r}_{g,t}$	-0.154×10^{-3} (-1.2722)	-0.199×10^{-3} (-0.9394)	-0.3509×10^{-4} (-0.3358)
$ibay_{t-1}.r_{m,t}$	3.8291 (1.5769)	10.1070 (2.1939)	-
$isdy_{t-1}.r_{m,t}$	-0.1249 (-0.0127)	18.6260 (0.9926)	-
$iper_{t-1}.r_{m,t}$	-0.5776 (-0.2823)	6.1147 (1.5753)	-
$jan_t.r_{m,t}$	0.1611 (0.9020)	-0.1621 (-0.4784)	-
$ibay_{t-1}.\check{r}_{g,t}$	0.673×10^{-3} (0.5094)	-0.447×10^{-3} (-0.1942)	0.127×10^{-3} (0.1103)
$isdy_{t-1}.\check{r}_{g,t}$	0.364×10^{-3} (0.0943)	0.00591 (0.8786)	-0.6459×10^{-4} (-0.0191)
$iper_{t-1}.\check{r}_{g,t}$	0.464×10^{-3} (1.1576)	0.238×10^{-3} (0.3402)	0.6452×10^{-4} (0.1861)
$jan_t.\check{r}_{g,t}$	0.534×10^{-3} (1.5680)	0.956×10^{-3} (1.6110)	0.565×10^{-3} (1.9121)
R^2	0.64	0.37	0.67

Note: t-statistics between brackets; ***: significant at 1%; system R^2=0.78 and χ^2=357.9.

TABLE 5
Slopes estimates of the conditional ICAPM (18) and (19) evaluated at simple mean.

	$r_{met,t}$	$r_{gol,t}$	$r_{intg,t}$
$ibay_{t-1}$	-0.5040***	-0.4319	0.1193
	(-2.8833)	(-1.4335)	(0.7856)
$isdy_{t-1}$	0.2057	-1.3644	-1.4978***
	(0.3106)	(-1.1821)	(-2.5905)
$iper_{t-1}$	-0.1335	-0.1571	-0.2178***
	(-1.3270)	(-0.8946)	(2.4986)
jan_{t-1}	0.0247*	0.0263	0.0078
	(1.9380)	(1.1770)	(0.7091)
$r_{m,t}$	1.1512***	1.1501***	-
	(15.7118)	(8.9432)	
$\check{r}_{g,t}$	0.2461×10^{-4}	0.836×10^{-4}	0.2989×10^{-4}
	(1.1423)	(1.2536)	(0.3663)

Note: *, *** significant, respectively, at 10% and 1% level.

instruments play no role at all in explaining the excess returns. More importantly, unlike most of empirical findings in the financial literature the unconditional CAPM of integrated mines excess returns, $r_{intg,t}$, is not rejected. The market beta plays a major role in all of the three specifications; as expected, all the market betas are greater than unity whereas, surprisingly, the market beta for $r_{gol,t}$ is slightly less than its counterpart for metal mining.

Generally, the explanatory power of the ICAPM specification is roughly the same as in its CAPM counterparts. The conditional market beta as well as the (constant) market beta seem to play a major role in explaining the total variance of metal, gold and integrated mines excess returns. However, neither the instrument nor the bond beta seem to explain the latter. A likelihood ratio test (not reported here) implemented for these excess returns, suggest the non rejection of the conditional ICAPM in favour of the conditional CAPM.

The time-varying effect of each significant explanatory variable is depicted in the figures below. As Bollerslev, Engel and Wooldridge (1988), we have been able to derive the behaviour through time of the market as well as the bond betas by using a time-varying parameter approach. However our approach is more general. While the excess returns seem to react as expected with respect to most of the instruments, the most striking feature is the (nearly) same behaviour of all conditional betas under both the CAPM or its intertemporal counterpart (metal and gold mines) and the bond beta (integrated mines). All portfolios reveal a rising beta during the earlier 1980s where metal mining industries such as gold and silver witnessed a dramatic volatility. To a lesser extent, however, all these portfolios display the same behaviour during the period of 1987 crash. Figures 1, 2 and 5 show that the estimates for market and bond betas are fairly similar except for a difference in scale. It seems that all the relevant information is captured by the conditional market beta through the conditional CAPM.

[insert Figures here]

Gibbons's (1982) results have revealed significant nonlinearities in the relationship between the returns and risks of common stocks. Titnic and West (1986) have shown that the nonlinearities in the risk-return relationship cannot be attributed solely to the anomalous behaviour of stock returns in january or to the so-called small-firm effect. Our findings provide additional evidence on the nonlinear relationship between risk and returns at the industry level in the sense that, once we have controlled for the January effect, it can be ascribed to the cyclical behaviour of the mining industry as well as to major and minor shocks it faced.

IV. Concluding remarks

This paper has investigated the conditional CAPM and ICAPM using value-weighted data of Canadian mining industries. Conditioning the parameters on some chosen instruments, we have therefore been able to derive the time-varying behaviour of the market beta (among others). The conditional CAPM, unlike its intertemporal counterpart, appears to explain significantly our portfolios behaviour through time. Although not the main thrust of this paper, this result might be linked to some recent contributions which intended to model exhaustible resource prices using asset pricing models. Under the Consumption based CAPM and CAPM framework, Gaudet and

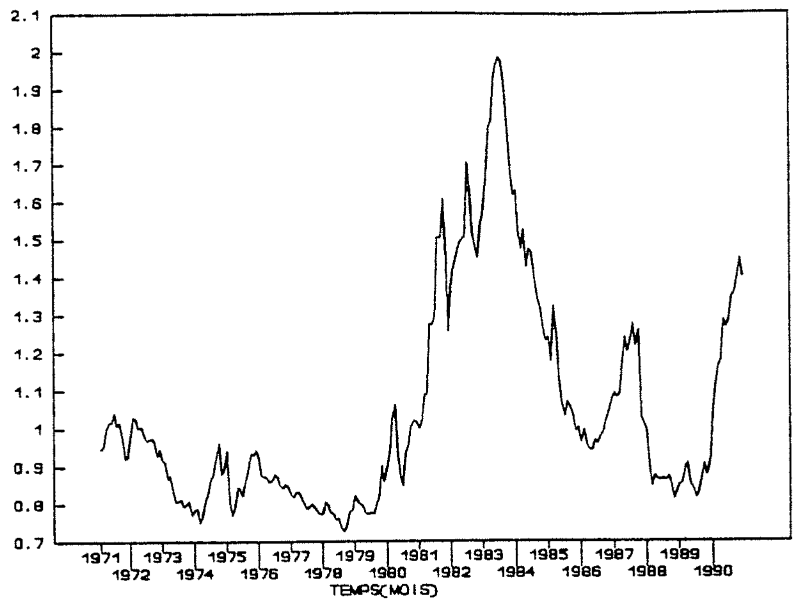

Figure 1: Market beta of gold mines (conditional CAPM).

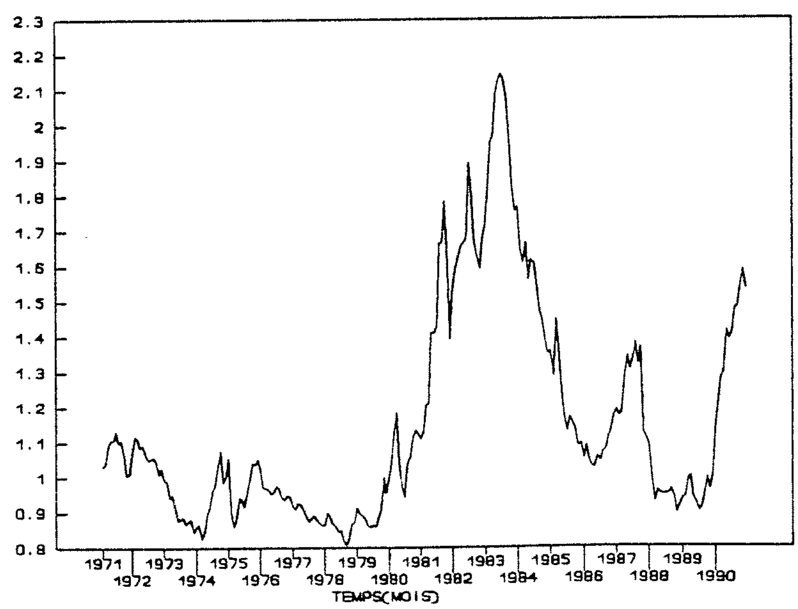

Figure 2: Market beta of gold mines (conditional ICAPM).

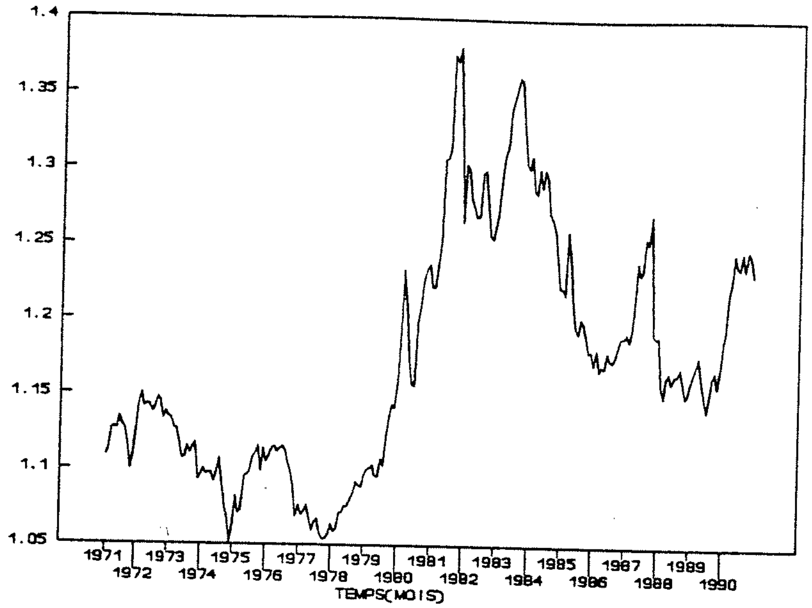

Figure 3: Market beta of metal mines (conditional CAPM).

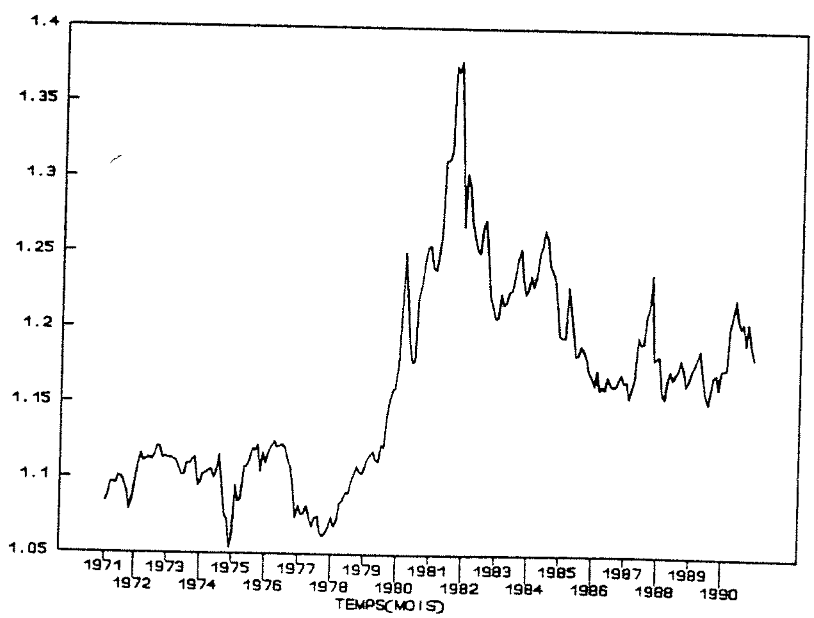

Figure 4: Market beta of metal mines (conditional ICAPM).

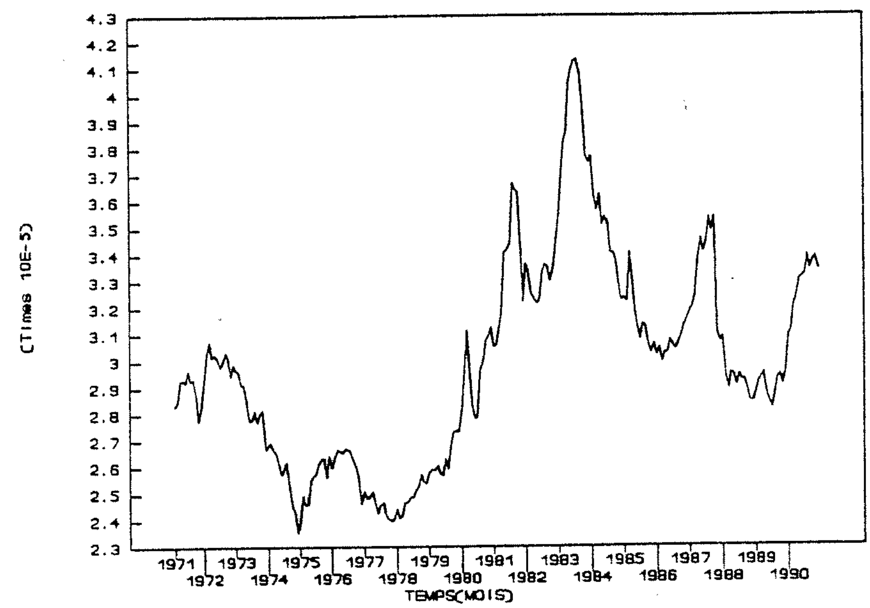

Figure 5: Bond beta of integrated mines (conditional ICAPM).

Khadr (1991) and Hartwick and Yeung (1988), respectively, derive the correspondent Hotelling-rule without any empirical assessment of their theories. Our results provide additional insights for considering the ICAPM in explaining exhaustible resource prices.

On the other hand, our findings on the risk-adjusted rate of return, might be of interest to an emerging literature on national accounts and natural resource valuation which uses the discounted cash flows method. In fact, the Securities and Exchange Commission and Canadian Securities Commissions require that companies report the present value of future net cash flows from estimated production and proved reserves. As well, the United Nation's Statistical Office has suggested using this method of valuation. Because it often uses arbitrary values of the rate of return, this valuation method faces in practice an important drawback. We believe that the contributions which have outlined this method for oil and gas reserves in the United States provide in fact poor estimates of the natural resources valuation (Landefeld and Hines 1985; Soloday 1980; Adelman 1986; and Adelman et al. 1991). Instead, our risk-adjusted estimates method seems to be a better alternative.

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Conclusion générale

Cette thèse comporte deux volets importants: d'une part, des considérations d'investissement irréversible analysés comme l'exercice d'une option d'achat financière et, d'autre part, le modèle d'évaluation des actifs financiers en incertitude. En premier lieu, la définition du lien existant entre la théorie de l'investissement et celle des options financières a permis de prendre en compte explicitement des aspects de *«timing»* souvent négligés par le paradigme néo classique de l'investissement. Notre principale innovation est à la fois d'avoir i) endogénisé les décisions d'investissement de la firme, ii) fait appel à des données estimées des firmes minières et iii) testé le bien-fondé de la théorie des options appliquée aux actifs réels. En second lieu, nous avons caractérisé un modèle conditionnel du CAPM et de son homologue intertemporel (ICAPM) qui prend en compte l'hétéroscédasticité de la prime de risque de marché et du béta. Nous avons innové par rapport à l'approche dite ARCH en considérant une caractérisation basée sur des paramètres variables dans le temps. Nos résultats en termes de béta semblent refléter adéquatement les principaux faits stylisés caractéristiques de cette industrie au cours de la période 1970-1990, à savoir, la présence de cycles et de chocs.

En conclusion, nous aimerions indiquer une direction de recherche pour l'avenir immédiat en ce qui concerne le lien entre la théorie des options financières et celle des actifs réels. L'idée est la suivante: certaines contributions récentes ont attiré l'attention sur la nécessité de prendre en compte l'épuisement des ressources naturelles (la dépréciation du capital naturel) dans la détermination du produit intérieur net. On y retrouve à la base de cette littérature des hypothèses telles l'absence d'incertitude et la parfaite malléabilité du capital qui sont toutes deux intenables dans le contexte de l'industrie minière. En faisant appel à la théorie des options financières, nous proposons une démarche alternative qui place l'irréversibilité et l'incertitude au centre de l'analyse. Il en résulte alors un principe d'évaluation des ressources épuisables basé à la fois sur la rente d'Hotelling et la valeur d'option de l'investissement. Cette dernière étant considérée comme une prime due à la présence de l'irréversibilité. Notre méthodologie est appliquée au système comptable national canadien et les résultats préliminaires révèlent que la valeur de l'option d'investissement est toute aussi importante que la rente d'Hotelling et, par conséquent, l'ignorer reviendrait à faire un biais important en matière de comptabilité nationale. La valeur de la dépréciation et des réserves issue de notre démarche représentent en moyenne 0,31% du PIB et 0,20% de la richesse nette en dollars constants pour la période 1960-1980.