Université de Montréal

VALUING SOVEREIGN DEFAULT RISK

A pricing formula and its empirical results for the Mexican external debt

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Abstract

Under a stochastic discount factor framework, this study presents a pricing formula that determines the sovereign default risk premium. In particular, we compute the default risk premium for the Mexican external debt by two stochastic discount factors: the *Export-Import Ratio process* and the *Short Rate-Inflation Ratio process*. The empirical performance of the pricing formula for the Mexican economy data shows that the default probability obtained in our formula is consistent with the empirical default probability implied by Brady bonds.

Dedicated to

Sara Espinosa, Isidro Monter
Oscar, Javier, Monica, Mauricio y André.

Solemos dejarle al tiempo el destino
para depositarlo en los vientos y los mares
para que nuestro ser se disperse
en las lineas marcadas por las horas.
Solemos abrazarnos sin darnos cuenta
de la despedida que hay en cada caricia
solemos ser humanos y partimos.

Rosario Monter

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

In the last three decades, the loans made between developed and developing countries have become a relevant component in the world economy. At the beginning of the 70's, the total debt of developing countries was less than 100 billion dollars, but at the end of the 80's it amounted to 1,180 billions of dollars. Every year the world debt amount increases and more countries become unable to meet their promises. For example, in August 1982, Mexico was the first debtor country to announce that it was unable to service its external debt of approximately 80 billion dollars[1]. As a consequence, new rescheduling agreements and alternative financial instruments were born. Brady bonds were created in 1989 in order to restructure outstanding sovereign debt into liquid debt instruments traded in the market.

From this perspective a default risk analysis is crucial. Banks active in international trade are highly concerned in measuring such risk because of the investment decisions they have to face through their corporate projects or international portfolios. From the debtor's point of view, understanding the process linked to default could help them to

[1] World Debt Tables 1996

restructure their own debts by the issuing of alternative financial instruments traded on the market.

Using the Mexican economy as a context, the present study will show a pricing formula that determines the default risk premium associated to a loan issued to an emerging country.

This paper is organized as followed; chapter two reviews some typical methods and models used to measure default risk in emerging countries; chapter three presents a pricing formula that determines the default risk premium associated to a loan issued to an emerging country; chapter four applies the proposed pricing formula to the Mexican economic data and finally chapter five concludes and addresses further questions.

Chapter 2

Risk to default: adding up the models

Default risk is defined as the failure to pay interest or principal debt promptly when due. If the loan is issued to an entity established in a country different from the lender's residence, additional risks are incurred denominated *country risk*. It includes the risk of expropiation of dividends by local government, the risk that as result of a war or political events a firm may not be paid for its exports, the risk faced as a consequence of different legislations, geographical situations, economic conditions, etc. *Sovereign risk* refers to the risk that a government might default on its own external obligations.

There are several reasons a government cannot meet its obligations:

- *Political risk*: the government may be unwilling to pay because of internal political problems.

- *Liquid risk*: there is not enough foreign exchange money available when needed.

- *Transfer risk*: money cannot be transferred in or out of the country.
- *Currency risk*: risk related to the value of foreign currency changes.
- *Economical risk*: related to local economic situations.

It is clear that emerging countries are more vulnerable to fall into default than developed countries mainly due to its economic and political instability policies.

2.1 Measuring sovereign default risk

For several reasons it is crucial to determine how sovereign default risk can be measured. Banks active in international trade are highly concerned in measuring such risk because the investment decisions they have to face through corporate projects or in creating new portfolios. From the owing government's point of view, understanding the process linked to default could help it to restructure its debt by the issue of alternative financial instruments traded on the market.

Different approaches have been used to measure sovereign default risk. They can be classified as qualitative and quantitative analysis.

The qualitative analysis

It consist of the interpretation of economic and political descriptive information that allows the analyst to make judgments about how risky the country is. It is mainly based on *subjective* and *unmeasurable* variables like social behaviour, political rumors, civil unrest and media content. Unfortunately an objective default risk premium cannot be deduced *only* from such analysis. For a revision of qualitative analysis see Cohen (1985),

Eaton and Gersovitz (1981) or Ciarrapico (1992).

The quantitative analysis

It consists on making judgements inferred from real economic data by using econometric methods. Among them *Logit Analysis,* applied by McFadden (1973), Mayo & Barret (1977) and Morgan (1986), *Discriminant Analysis* by Frank & Cline (1971), Abassi & Taffler (1982) while *Principal Component Analysis* has been studied by Dhonte (1975). For a comparative analyses of all them see Saini & Bates (1978) or Collins (1982).

One of the most popular statistical methods, commonly used in the last two decades, is the *Discriminant Analysis.* The main objective of this technique is to determine whether a debtor country belongs to a default group or a non-default group. A sufficiently large sample of countries belonging to each of the two groups is required, then, the binary classification is obtained using a discriminant function Z which is constructed as a linear combination of n independent explanatory variables x_i, weighted by a factor w_i for $i = 1...n$, thus the model is given by

$$Z = \sum_{i=1}^{n} w_i x_i \ .$$

Evaluating the discriminant function Z for the sample countries gives a frequency distribution of the Z-values for each group and then, with this frequency, a critical value, Z^*, is determined. This critical value allows for a classification of the country in the default or non-default group.

The linear discriminant function Z is more powerful (in the sense of maximizing the ability to distinguish between one group or another) when the weights are such that the variance between the two groups is maximized and the variance within each group is

minimized. This classification process is subject to the type I and type II statistical errors that can be associated with some costs, therefore statistical methods are used to minimize the expected cost errors according to the probabilities of these errors.

Even though the advantage of econometric methods is the measurement, there are drawbacks as well. It is not always possible to find enough reliable data or any data at all available from emerging countries for that matter.

As a consequence of the common default of payments by debtor governments, credit institutions look for more effective methods to measure and price sovereign default risk. It is not enough to consider *only* econometric methods to deduce a default risk premium or *only* to consider qualitative analysis. In addition to considering both, specific economic data and specific country information, some mathematical credit risk models that have been developed recently, should been considered. Among them, two approaches are the most comun: the *Reduced Form* and the *Structural Approach.*

2.2 Mathematical credit risk models

The Reduced Form

Developed by Jarrow and Turnbull (1995), this approach models default as an unpredictable event governed by a hazard-rate process h_t and a fractional expected loss L_t if default occurs at some time t. Thus default risk is based on an intensity exogeneous rate process R_t

$$R_t = r + h_t L_t$$

where r denotes the risk free interest rate. If a contingent claim instrument pays a fixed

amount K at maturity date in case of default, the risk premium is calculated as the expected value of the payoff K discounted by the intensity process R_t under a risk neutral probability measure Q given the information $\mathcal{I}_t$. Therefore, the premium risk becomes

$$P_t = \mathbb{E}^Q \left[\exp\left(-R_t dt\right) * K \mid \mathcal{I}_t \right]$$

Most recent theoretical research is due to Jarrow, Lando & Turnbull (1997) and Duffie and Singleton (1999). The main message of this approach is that the rate process already contains the default process.

The Structural Approach

Under this approach, the default risk premium is viewed as a *put option* contract. Following Merton (1974), in corporate default risk, the firm issues debt and equity securities and invests them as a tradable asset, then a default event is produced if the asset value S_t falls below a specified value K at a terminal fixed date T. The default risk premium P_t is given by the expected value, under a risk neutral probability measure Q, of the payoff $\max\{K - S_T, 0\} = [K - S_T]^+$ discounted by the risk-free interest rate process r_s. The default risk premium or *put option* price is

$$P_t = \mathbb{E}^Q \left[\exp\left(- \int_t^T r_s ds \right) [K - S_T]^+ \right]$$

Under the assumption that the asset's value follows a stochastic diffusion process like the following:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \ \sigma \text{ constant}$$

then the default risk premium becomes the Black and Scholes' *put option* pricing formula.

Among the literature that follows the structural approach Hull and White (1992), Hull and White (1995), Longstaff and Schwartz (1995) are the most representative.

Sovereign default risk can be modelled similarly by defining K as the fixed amount of debt borrowed by the emerging country to be paid at maturity date T. Here S_t would represent the capacity or willigness of payment made by the debtor[1]. Therefore, the defautable amount at maturity date is $[K - S_T]^+$, thus the default risk premium becomes equivalent to the price of an *European put option* contract.

Surprisingly, given the clear relationship between option contracts and default risk insurance, option pricing theory was used very late to evaluate sovereign default risk. Pioneer work was due to Claessens and Wijnbergen (1990) and Chesney and Morisset (1992). This relationship suggests that sophisticated option pricing theory can be "adapted" in evaluating corporate or sovereign default risk. These similarities have already been investigated by some authors. For instance, Saa Requejo & Santa-Clara (1999) define corporate default as the *first* time the solvency of the company falls down some specific level K^*, then the defaultable bond premium is equivalent to the price of an *American put option* contract with payoff $[K^* - S_T]^+$.

More developments have been applied, like the two factor model of Longstaff and Schwartz (1995), the stochastic volatility process of Heston (1993) and the introduction of jumps by Cox and Ross (1975).

A recent methodology that has been applied to option pricing models is the use of stochastic discount factors. They were introduced in asset pricing models by imposing a factor structure to a collection of asset returns or payoffs to describe their joint distribution at a point time. This methodology have been extended to option pricing models[2] and it

[1]There is some literature that emphasize the difference between the debtor's payment capacity and the willigness to pay. For our purpose no difference will be made.

[2]An excellent book that presents asset pricing theories under a unified stochastic discount factor approach

can be adapted to price default risk.

It is clear that all improvements developed in option pricing theory can be adapted to price corporate and sovereign default risk. Of course, there are some differences between derivative instruments, corporate debt and sovereign debt, but a unified methodology of evaluating default risk can be applied by introducing appropriate parameters according to the debt's nature and the economic environment where the debt is issued.

Under the *Structural Approach*, the contribution of this study consist of introducing a suitable stochastic discount factor and debtor's capacity of payment process that allow us to calculate the sovereign default risk premium. Moreover, the debt service process is characterized according to the different states of the economy by either a random Markov-state transition matrix, or a deterministic criteria, whatever results appropriate from the data.

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is Cochrane (2001).

Chapter 3

Pricing sovereign default risk

The most basic pricing equation of any asset is given by the expected value of its payoff in a future date, actualized by some stochastic discount factor $\mathbb{E}_t\left[m_{t,T} \cdot \text{payoff}\right]$.

Since we don't know in advance the debtor's capacity of payment at some given date, we can assume that it follows a stochastic process S_t. Because we are interested in the default event only at terminal date and not before, the default risk premium P_t can be viewed as an *European put option* contract. The random payoff is given by the positive difference of the fixed debt obtained K minus the random debt payment S_T at maturity date T.

The pricing equation can be written as

$$P_t = \mathbb{E}_t\left[m_{t,T} \cdot [K - S_T]^+\right] \tag{3.1}$$

where $m_{t,T}$ is the stochastic discount factor[1].

[1]Harrison and Kreps (1979) showed that the existence of a positive stochastic discount factor $m_{t,T}$ is equivalent to the absence of arbitrage.

3.1 Pricing with a stochastic discount factor

One can assume that the pricing probability does not depend on the initial payment capacity S_t, thus the pricing function is homogeneous of first degree with respect to (S_t, K). Lets denote $k = \frac{K}{S_t}$.

The pricing equation (3.1) becomes

$$\begin{aligned} P_t &= S_t \mathbb{E}_t \left[m_{t,T} \cdot \left[\frac{K - S_T}{S_t} \right]^+ \right] \\ &= S_t \mathbb{E}_t \left[m_{t,T} \cdot \left(\frac{K}{S_t} \right) \cdot 1_{K > S_T} - m_{t,T} \cdot \left(\frac{S_T}{S_t} \right) \cdot 1_{K > S_T} \right] \\ &= S_t \left\{ k \mathbb{E}_t \left[m_{t,T} \cdot 1_{k > \frac{S_T}{S_t}} \right] - \mathbb{E}_t \left[m_{t,T} \cdot \left(\frac{S_T}{S_t} \right) \cdot 1_{k > \frac{S_T}{S_t}} \right] \right\} \\ &= S_t \left\{ k \mathbb{E}_t \left[m_{t,T} \cdot 1_{\log k - \log \frac{S_T}{S_t} \geq 0} \right] - \mathbb{E}_t \left[m_{t,T} \cdot \frac{S_T}{S_t} \cdot 1_{\log k - \log \frac{S_T}{S_t} \geq 0} \right] \right\} \end{aligned} \tag{3.2}$$

Extending to a multiperiod framework, the stochastic discount factor and the payment's capacity can be rewritten as

$$\begin{aligned} m_{t,T} &= \prod_{\tau=t}^{T-1} m_{\tau,\tau+1} = \exp\left(\sum_{\tau=t}^{T-1} \log\left(m_{\tau,\tau+1} \right) \right) \\ \frac{S_T}{S_t} &= \prod_{\tau=t}^{T-1} \frac{S_{\tau+1}}{S_\tau} = \exp\left(\sum_{\tau=t}^{T-1} \log\left(\frac{S_{\tau+1}}{S_\tau} \right) \right) \end{aligned}$$

A more interesting model arises when information on the economic state at a certain time t is considered. Lets denote by $U^T = (U_\tau)_{1 \leq \tau \leq T}$ the path of economic state variables. We assume that the conditional variables $\left(m_{\tau,\tau+1}, \frac{S_{\tau+1}}{S_\tau} \right)_{1 \leq \tau \leq T-1}$ are serially independent given the path of states variables $U^T = (U_\tau)_{1 \leq \tau \leq T}$. This path can be characterised in several ways, in particular, a Markov switching process can be adapted allowing randomness in the duration that a given state remains or change.

Lets define

$$A = \sum_{\tau=t}^{T-1} \log\left(m_{\tau,\tau+1}\right),\ B = \log k - \sum_{\tau=t}^{T-1} \log\left(\frac{S_{\tau+1}}{S_\tau}\right)$$

then

$$m_{t,T} = \exp\left(A\right), \text{ and } \frac{S_T}{S_t} = k\exp(-B)$$

therefore, the pricing equation (3.2) becomes

$$P_t = kS_t\left\{\mathbb{E}_t\left[\exp\left(A\right)\cdot 1_{B\geq 0}\right] - \mathbb{E}_t\left[\exp\left(A\right)\cdot\exp(-B)\cdot 1_{B\geq 0}\right]\right\}$$

By the independent assumption and applying the law of iterated expectations we now have

$$\begin{aligned} P_t &= kS_t\cdot\left\{\mathbb{E}_t\left\{\mathbb{E}_t\left[\exp\left(A\right)\cdot 1_{B\geq 0}\mid U^T\right]\right\} - \mathbb{E}_t\left\{\mathbb{E}_t\left[\exp(A-B)\cdot 1_{B\geq 0}\mid U^T\right]\right\}\right\} \\ &= K\cdot\mathbb{E}_t\left\{\mathbb{E}_t\left[\exp\left(A\right)\cdot 1_{B\geq 0}\mid U^T\right] - \mathbb{E}_t\left[\exp\left(A-B\right)\cdot 1_{B\geq 0}\mid U^T\right]\right\} \end{aligned}$$

which prove the following result:

Proposition 1 *Assuming that the variables* $\left(m_{\tau,\tau+1}, \frac{S_{\tau+1}}{S_\tau}\right)_{1\leq\tau\leq T-1}$ *are conditionally serially independent given the path of state variables* $U^T = (U_\tau)_{1\leq\tau\leq T}$, *the default risk premium becomes*

$$P_t = K\cdot\mathbb{E}_t\left\{H(A,B,U) - G(A,B,U)\right\} \tag{3.3}$$

where

$$\begin{aligned} H(A,B,U) &= \mathbb{E}_t\left[\exp\left(A\right)\cdot 1_{B\geq 0}\mid U^T\right] \\ G(A,B,U) &= \mathbb{E}_t\left[\exp\left(A-B\right)\cdot 1_{B\geq 0}\mid U^T\right] \\ A &= \sum_{\tau=t}^{T-1}\log\left(m_{\tau,\tau+1}\right) \\ B &= \log k - \sum_{\tau=t}^{T-1}\log\left(\frac{S_{\tau+1}}{S_\tau}\right),\ k = \frac{K}{S_t} \end{aligned}$$

The conditional joint probability distribution of the stochastic discount factor and the debtor's payment capacity process $\left(m_{\tau,\tau+1}, \frac{S_{\tau+1}}{S_\tau}\right)$ must be specified in order to obtain a pricing formula.

3.2 The pricing formula

In order to apply the previous pricing framework in a sovereign default risk context, we have to consider macroeconomic variables that could explain the debtor's capacity of payment. Lets denote by X_t the stochastic discount factor that will serve to price default risk and by D_t the underlying debt service process. If we assume that the local economy can be characterised by two random states: a *solvent* and a *close-to-default state*[2], then debtor's capacity of payment S_t can be written as a two-state Markov switching process:

$$\log \frac{S_{t+1}}{S_t} = \mu_1 U_{t+1} + \mu_2 (1 - U_{t+1}) + [\sigma_1 U_{t+1} + \sigma_2 (1 - U_{t+1})] \varepsilon_{s,t+1}$$

where the random variable U_{t+1} represents the economic state at time $t+1$. This variable takes value one with probability π_1 when the economy is in the *solvent state*, or value two with probability π_2 when it is in the *close-to-default state*. The probability that the economic state variable switch from state i to state j is given by $p_{i,j} = \Pr[U_{t+1} = j \mid U_t = i]$, where $p_{i,j} = 1 - p_{i,i}$ for $i \neq j$, and $\pi_i = \frac{1-p_{jj}}{2-p_{ii}-p_{jj}}$, $i, j = 1, 2$.

We assume that the variables $\left(\log X_{t+1}, \log \frac{D_{t+1}}{D_t}\right)$ follow two correlated stochastic processes:

$$\log \frac{X_{t+1}}{X_t} = \mu_X (U_t) + \sigma_X (U_t) \varepsilon_{1,t}$$

[2]The reason that we do not consider a *default state* in the model is that usually when a default occurs, the default amount is renegotiated or restructured.

$$\log \frac{D_{t+1}}{D_t} = \mu_D(U_t) + \sigma_D(U_t)\,\varepsilon_{2,t}$$

where $(\varepsilon_{1,t}, \varepsilon_{2,t})$ follows a bivariate standard normal distribution with correlation coefficient $\rho_{X,D}$.

Proposition 2 *Under conditions of proposition one and assuming that the conditional probability distribution of the variables* $\left(\log X_{t+1}, \log \frac{D_{t+1}}{D_t}\right)$ *given the economy state* U^τ *at time* $\tau = 1, ..., T$ *is a bivariate normal distribution with parameters* $\mathbb{E}\begin{pmatrix} \log X_{t+1} \\ \log \frac{D_{t+1}}{D_t} \end{pmatrix} = \begin{pmatrix} \mu_{Xt+1} \\ \mu_{Dt+1} \end{pmatrix}$ *and* $Var\begin{pmatrix} \log X_{t+1} \\ \log \frac{D_{t+1}}{D_t} \end{pmatrix} = \begin{bmatrix} \sigma^2_{Xt+1} & \sigma_{XDt+1} \\ \sigma_{XDt+1} & \sigma^2_{Dt+1} \end{bmatrix}$, *the default risk premium* P_t *becomes*

$$P_t = K \cdot \mathbb{E}_t\left[\exp\left(\mu_A + \frac{1}{2}\sigma_A^2\right)\Phi(d_1) - \exp\left(\mu_A - \mu_B + \frac{1}{2}\left(\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}\right)\right)\Phi(d_2)\right] \tag{3.4}$$

where

$$\begin{aligned}
d_1 &= \frac{\mu_B + \sigma_{AB}}{\sigma_B}, \quad d_2 = d_1 - \sigma_B \\
\mu_A &= \sum_{\tau=t}^{T-1} \mu_{X,\tau+1} \\
\mu_B &= \log\left(\frac{K}{D_t}\right) - \sum_{\tau=t}^{T-1} \mu_{D,\tau+1} \\
\sigma_A^2 &= \sum_{\tau=t}^{T-1} \sigma^2_{X,\tau+1} \\
\sigma_B^2 &= \sum_{\tau=t}^{T-1} \sigma^2_{D,\tau+1} \\
\sigma_{AB} &= \rho_{AB}\left(\sum_{\tau=t}^{T-1} \sigma^2_{X,\tau+1}\right)^{1/2}\left(\sum_{\tau=t}^{T-1} \sigma^2_{D,\tau+1}\right)^{1/2}
\end{aligned}$$

See Appendix for the proof.

In order to obtain practical results, the processes X_{t+1} and $\frac{D_{t+1}}{D_t}$ must be specified according to the economic context where default is considered.

Chapter 4

The Mexican economy default risk: watching the money fly

In order to apply the previous pricing framework to a sovereign default risk context, we have to consider macroeconomic variables that could explain the capacity of payment for the Mexican economy. Before doing that, a short description of modern Mexican economy is presented.

4.1 The Mexican economic performance

In the 70's, the Mexican economic situation was favourable. High export growth rates especially in petroleum, minerals, and other natural resources were the typical warranty for loans. Unfortunately, that period was also characterised by a fiscal deficit accumulation brought on by the government's incredible spending program based on the notion that oil production would be the only source of income to pave the way to *"the Mexican*

miracle."

A crisis became inevitable when interest rates rose and oil prices fell in 1982 leaving Mexico devastated by this international economic situation. In the same year the elections gave way to the De la Madrid government which prioritised the elimination of the trade deficit by cutting social expenditure and public investments. To achieve these goals the Peso was devalued in 1983, which led to high importation costs and inflation jumped to 125%. The purchasing power was considerably reduced and as a consequence in 1984 Mexico was hit with a recession.

In 1985 with the announcement that Mexico would joint the *General Agreement on Trade Tariffs (GATT)* created a favourableeconomic atmosphere. In 1986 Mexico's government was oriented towards a greater integration in the world economy by opening its doors much more than ever before. In exchange, Mexico was able to reschedule its debt and new money was borrowed from the *World Bank* and the *International Monetary Fund (IMF)*. However fiscal deficits and low petroleum prices led to even higher inflation levels in 1987 and early 1988, when it reached its highest historic level at almost 200%.

In December 1987 the Mexican government and representatives of major companies negotiated the *Pacto*, a fiscal policy program that planned to freeze the minimum wage, the cost of public services and the exchange rate. As a consequence, at the end of 1988 inflation was reduced to 52%. However, the freeze of the nominal exchange rate and thus the artificially low inflation levels led even bigger gap between the real exchange rate and Mexico's frozen rate. The free flow of imports, the artificial exchange rate and the electoral political crisis led to a capital flight of 2.5 billion dollars.

In 1988, the new government negotiated another fiscal program, the *PECE*, where tax rates were lowered as well cuts made once again to social expenditure. The fixed exchange rate was replaced by a gradual devaluation process.

In 1989 it was announced that Mexico's trade balance was not sufficient enough to service its debt, therefore new negotiations with the international financial community began. In March 1989, the *Brady plan* was announced: debt and interest were transformed into liquid debt instruments called *Brady bonds* backed by the U.S. government. As a consequence debt payments where reduced when in 1990 the *Brady plan* was put into effect.

The beginning of the 90's was characterised by neo-liberalism economic standards. In 1993 the *North America Free Trade Agreement (NAFTA)*, was signed between Canada, Mexico and the United States. The Mexican government thought that this agreement would lead to economic growth due to an influx of capital from the North. It offered excellent conditions to North American companies to make investments in Mexico, allowing them to profit from low salaries, no trade barriers, and little or no environmental regulations.

In conformity with the *IMF*'s social restructuring program, during 1992 and 1993 around one thousand state-owned enterprises were privatised and the government's budget for social service programs such as health and education was drastically reduced.

Not all political and social groups saw the same advantages in neo-liberalism: the investment of foreign companies had not meant any increase in the standard of living nor in the purchasing power of workers. Privatisation was a dirty process where politician's friends and relatives benefited from the private auction of the country's assets. The gap between rich and poor was now more than ever critical: while twenty-seven businessmen

controlled 30% of the GDP, the working class survived on one dollar a day.

These economic and political factors led to the insurgence of important social movements. In 1994 the *Zapatista's National Liberation Army (EZLN)* composed mainly of indigenous people from the south of Mexico started a rebellion in Chiapas and quickly spread to other states. It was the beginning of a political crisis. The long-awaited investments did not appear either and high interest rates attracted only speculative capital investments.

The 1994 presidential elections were precluded by a series of violent acts including the assassination of a presidental candidate. For fear of losing these elections the current Mexican head of State, Gortari, refused to devalue the Peso. The new president elect was obligated to float the currency from 3.5 to 7 pesos per dollar. This first devaluation was followed by many more. The economic and political situation lead to a flight of capital based on fear and speculation resulting in 2.5 billion dollars by the end of the year.

In December 1994 the stock market dropped 24%, hundreds of companies closed down and more than 250,000 Mexicans lost their job. The repercussions from Mexican crash throughout Latin American markets were called the *Tequila Effect.* The December crash led to three years of economic depression, the worst to hit Mexico since 1920. This situation led to high growth of the informal economy.

In 1995 Mexico received a loan from the *IMF* and from the U.S. government to the value of 50 billion dollars. The loan went to pay off private investors who had speculated in Mexico. The Mexican government hoped to convince investors that the crisis in Mexico was over and their capital should stay.

During 1996 and 1997 government privatisation continued: ports, railroads, air-

ports, telecommunications, natural gas distribution, electricity and some petrochemical sectors were open to private investors. The Mexican President signed free trade agreements with Bolivia, Chile, Costa Rica, Nicaragua and Venezuela and initiated negotiations with Belize, Ecuador, El Salvador, Guatemala, Honduras, Panama, Peru, and the Mercosur market of Argentina, Brazil, Paraguay and Uruguay. In December 1997 Mexico signed the *Agreement for Economic Association, Political Dialogue and Co-operation* with the European Union.

In 1998, despite privatisation and the neo-liberal economy, the high dependence of the federal budget on oil revenues (accounting for 40 %), and the drop of international oil prices, led to a tax increase and a drastic reduction of social expenditures. Again the education and health sectors were the most affected. Government imposed price increases on gasoline and diesel fuel, which is used to move industrial freight in Mexico. Also, it eliminated the official subsidy to *tortilla* manufacturers[1] with the resulting increase of 20% in its price.

In 1998 the *Savings Protection Bank Fund (Fobaproa)* absorbed the bad portfolios of the national banks which granted very large loans without sufficient collateral[2].

In 1999, after long sessions in the Congress, the executive's initial proposal of converting *Fobaproa's liabilities* into a public debt was approved. The official logic was that if the bad debt portfolio now paid by *Fobaproa* was not formalised into public debt, the banking system risked another capital flight and the Mexican economy could face another financial collapse. *Fobaproa's liabilities* amounted up 61 billion pesos, equivalent to 15% of

[1]Tortilla is the main food staple of the Mexicans.
[2]Some bank directors are persecuted for white collar crime.

the gross domestic product. The financial crisis of 1999 was inevitable and the domestic private sector now has no access to credit which led several local companies to close.

The social discontent increased and in the year 2000, fraudalent elections were no longer possible. It was the first time after 71 years of the *Institutional Revolucionary Party (PRI)* government, that a candidate of the opposition party was recognised to be officially elected.

The structural reforms proposed by the new government for its mandate period of 2000-2006 are: reduce social expenditures, increase substantially the tax revenue[3], open the electricity, natural gas, telecommunication and petrochemical industries completely to the private sector, guarantee transparency of the Mexican financial system, enforce international agreements such as the *Plan Puebla-Panama*, initiative which involves the eight southern states of Mexico and seven nations of Central America, and to promote *NAFTA* expansion by allowing free flow of labour. This *"terrific"* plan may have led to *Moody's Investors Service* to upgraded Mexico's credit rating to *Investment Quality* in July 2000.

In February 2001 the *World Bank* approved a loan of 1.5 billion dollars to be spent to *"guarantee macroeconomic stability"* by implementing strict fiscal reforms and opening to private sector the few government controlled industries left like *Petroleum of Mexico (PEMEX)*.

4.2 Sovereign default information

What data should be used to model sovereign default?

[3]Including tax in food and medicaments.

Among practitioners of rating agencies[4], relevant data can be clasified into the following categories:

- *The Debt Position*

It consists on the country's current position and past performance on foreign obligations. Figure 1 graphs the external public debt issued to Mexico as well as the consolidated debt amount from March 1981 to March 2001. Figure 2 shows them as a percentage of the GNP.

- *The Local Economy Performance*

This information evaluates how the country is performing both domestically and in international markets. The total export and import amount including oil for the period March 1981 to June 2001 are illustrated in Figure 3 while the export-import ratio is illustrated in Figure 4. The GNP performance for the same period is depicted in Figure 5, while the U.S dollar exchange rate is in Figure 6. The inflation rate and government short interest rate *CETES*[5] from February 1985 to June 2001 are depicted in Figure 7, while the CETES-inflation ratio are in Figure 8.

- *Liquidity*

The country should have access to a sufficient amount of liquidity to avoid default on foreign debt. Unfortunately there is not enough public historical data available of the Mexican international reserves. Figure 9 graphs the international reserves of Mexico from January 2000 to May 2001.

[4]Moody's Investors Service for example.
[5]Government guaranteed deposit certificates for a period of 28 days, they are risk-free.

- *Political stability*

It is important to assess the country's political stability because the political regime may dramatically change the country's priorities and policies and thus affect its willingness to service obligations contracted under the old regime. We need information on what policy it is pursuing and how these policies are implemented. Among this information it is important to know that *CETES* rates were liberated at the beginning of 1990 allowing the market mechanism rather than the government to determine its value. It is also important to note that the U.S. dollar exchange rate was controlled by the Mexican government until 1994.

There are three official sources of Mexican public economy data:

The *Banco de Mexico (BM), www.banxico.org.mx*

The Ministry of Finance in Mexico named *Secretaria de Hacienda y Credito Publico (SHCP), www.shcp.gob.mx*

The Mexican Statistic Institute known as *Instituto Nacional de Estadistica, Geografia e Informatica (INEGI), www.inegi.gob.mx*

4.3 Empirical results

The debt service process, denoted by D_t is given by the amount of debt that has been consolidated at time t. This process is illustrated in Figure 1.

Basically there are three financial sources for Mexican government to paid its external debt:

- taxes

- the trade surplus (mainly due to oil exportations)
- new debt: like issuing CETES or by new international borrowing[6]

Reviewing the modern economy history of Mexico (1980 - 2000), there is no evidence that the Mexican government desires to develop its own Industry. Loans have been used to guarantee macroeconomic variables to attract foreign investors hoping that they arrive to Mexico to bring the economic progress instead to activate local economy. Local banks do not achieve the functions for which have been created: there is no credit at all available for new companies and what is even worst, alredy existing companies are disappearing. This lead most of the population to remain in the informal economy and therefore taxes do not account significantly to meet external obligations.

Among the information available, there are two candidates to perform as a discount factor X_t:

- The *Short Rate-Inflation ratio process*, denoted by X_t^r which consist on the monthly government interest rate *CETES* divided by the monthly inflation rate.
- The *Export-Import Ratio process*, denoted by X_t^e which consist on total exports (including oil) divided by total imports.

The model presented in chapter three considers the path of economic state variables at a certain time t denoted by $U^T = (U_\tau)_{1 \leq \tau \leq T}$. Although this path can be characterised by a Markov switching process, the available data of the Mexican government interest rates suggests to characterise the Mexican economy in two states: a *"fixed state"* from February

[6]Like in 1995 when the government received a new debt by the IFM in order to meet its external obligations.

1985 to December 1989 where CETES rates were fixed by the government, and a *"liberated state"* from January 1990 to June 2001 when the market determinated their value.

Note that the debt's structure also changed: since 1990, when the Brady plan was put into effect, until now, Brady bonds restructure external debt into tradable debt instruments. Even more, Mexican economy started to be effectively open since 1990.

Because the currently state of the Mexican economy is a *"liberated state"* and the probability of switch or go back to a *"fixed state"* is almost zero given the economic, political and historical circumstances, the empirical test will be solely based on data from January 1990 until now.

Figure 10 and 11 graph the *Short Rate-Inflation ratio process* for the two periods while Figure 12 and 13 graph the *Export-Import Ratio process* before 1990 and after 1990.

The empirical experiment is presented:

First, the parameters of the processes (D_t, X_t^e) and (D_t, X_t^r) are estimated using Monthly data from January 1990 to April 2001.

TABLE 1

Parameter Estimation, Monthly data from January 1990 to April 2001

	Debt Service D_t	**Export/Import X_t^e**	**CETES/Inflation X_t^r**
$\hat{\alpha}$	-0,028882128	-0,003773099	-0,015952426
$\hat{\sigma}^2$	0,005803187	0,00430551	0,0443485550
$\hat{\mu}$	-0,031783722	-0,005925854	-0,038126703
$\hat{\sigma}$	0,07617865	0,065616386	0,2105909660
ρ_{DX}	correlations:	0,643266437	-0,417851367

where $r_k(h) = \log\left(\frac{X_t}{X_{t-1}}\right)$, $\hat{\alpha} = \frac{1}{nh}\sum_{k=1}^{n} r_k(h)$, $\hat{\sigma}^2 = \frac{1}{nh}\sum_{k=1}^{n}(r_k(h) - \hat{\alpha}h)^2$, $\hat{\mu} = \hat{\alpha} + \frac{1}{2}\hat{\sigma}^2$, $\hat{\sigma} = \left(\frac{1}{nh}\sum_{k=1}^{n}(r_k(h) - \hat{\alpha}h)^2\right)^{1/2}$.

Using these parameters, the default risk premium of equation (3.4) is calculated by

considering as the discount factor both the *Export-Import Ratio* and the *CETES-Inflation Ratio*. The initial debt service D_0 equals to the fixed debt issued K, i.e. $K/D_0 = 1$. The results are presented below:

TABLE 2

Default Risk Premium

Export/Import X_t^e	**CETES/Inflation** X_t^r
0.03994234	0.03422552

That is the default risk premium for each dollar borrowed. Since it is a *pure* premium, in the sense that only the default risk is taken into account and not other costs, it can be interpreted as the proportion to default for each dollar assured, that is the probability of default.

Is this a high or a low risk value?

An interesting experiment could be based on estimate the empirical default probability inferred from some tradable debt instrument and compare our *"theoretical default risk"* with the *"market default risk"*.

For instance, Izvorski (1998) computes the default probability implicit in the price of Brady bonds using data for the period January 1994 to November 1996 for seven countries. He found a mean default probability for Mexico of 0.0307 what is consistent with the results presented in this study.

Chapter 5

Conclusion

In the last three decades, the loans made between developed and developing countries have become a relevant component in the world economy.

Based in a stochastic discount factor methodology, this study presented a pricing formula that determines the sovereign default risk premium. Although a Markov switching process can be introduced in the model, qualitative analysis of the Mexican economy lead us to consider one state, the *liberated state* from January 1990 to April 2001. Two discount factors were considered to calculate the Mexican default risk premium: the *Export-Import Ratio process* and the *Short Rate-Inflation Ratio process.* The results obtained in this study are consistent with the empirical default probability obtained by Izvorski (1998).

Future work will focus on extend the pricing equation (3.1) by introducing a *Federal Reserve Equilibrium Model* as the discount factor. Empirical research could analyse the performance of the resulting pricing formula among different countries and check if they are consistent with the Brady bonds market price.

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Appendix A

Proof of proposition 2

Before proving proposition 2, we establish the following result:

Lemma : If $\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ follows a bivariate normal distribution with

$$\mathbb{E}\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, Var\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} w_1^2 & \rho w_1 w_2 \\ \rho w_1 w_2 & w_2^2 \end{pmatrix}$$

then

$$\mathbb{E}\left(\exp\left(Z_1\right) \cdot 1_{Z_2 \geq 0}\right) = \exp\left(m_1 + \frac{w_1^2}{2}\right) \Phi\left(\frac{m_2}{w_2} + \rho w_1\right)$$

where $\Phi\left(\right)$ is the cumulative normal distribution function.

We applied this result to proof proposition 2:

Proof. By proposition 1, the default risk premium is given by $P_t = K \cdot \mathbb{E}_t\left\{H() - G()\right\}$, therefore we just need to compute $H() = \mathbb{E}_t\left[\exp\left(A\right) \cdot 1_{B \geq 0} \mid U^T\right]$ and $G() = \mathbb{E}_t\left[\exp\left(A - B\right) \cdot 1_{B \geq 0} \mid U^T\right.$

By hypothesis, the conditional probability distribution of the variables $\left(\log X_{t+1}, \log \frac{D_{t+1}}{D_t}\right)$ given the economy state U^τ at time $\tau = 1, ..., T$ is a bivariate normal distribution, therefore

the conditional probability distribution of the variables (A, B) given the economy state U^t is also a bivariate normal distribution (of course with different parameters), then by applying the Lemma 1 we have

$$H() = \mathbb{E}_t \left[\exp(A) \cdot 1_{B \geq 0} \mid U^T\right] = \exp\left(\mu_A + \frac{1}{2}\sigma_A^2\right) \Phi\left(\frac{\mu_B}{\sigma_B} + \rho_{AB}\sigma_A\right).$$

Lets define $C = A - B$, then the variables (A, C) given the economy state U^t follows a bivariate normal distribution (again with different parameters), then by applying the Lemma 1 we have

$$G() = \mathbb{E}_t \left[\exp(C) \cdot 1_{B \geq 0} \mid U^T\right] = \exp\left(\mu_C + \frac{1}{2}\sigma_C^2\right) \Phi\left(\frac{\mu_B}{\sigma_B} + \rho_{BC}\sigma_C\right)$$

where $\mu_c = \mu_A - \mu_B$, $\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$, $\rho_{BC} = \frac{\sigma_{BC}}{\sigma_B \sigma_C}$, $\sigma_{BC} = \sigma_{AB} - \sigma_B^2$, $\sigma_C^2 = \sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}$, thus

$$\begin{aligned}
H() &= \exp\left(\mu_A + \frac{1}{2}\sigma_A^2\right) \Phi(d_1) \\
G() &= \exp\left(\mu_A - \mu_B + \frac{1}{2}\left(\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}\right)\right) \Phi(d_2) \text{ with} \\
d_1 &= \frac{\mu_B + \sigma_{AB}}{\sigma_B}, \; d_2 = d_1 - \sigma_B
\end{aligned}$$

therefore

$$P_t = K \cdot \mathbb{E}_t \left[\exp\left(\mu_A + \frac{1}{2}\sigma_A^2\right) \Phi(d_1) - \exp\left(\mu_A - \mu_B + \frac{1}{2}\left(\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}\right)\right) \Phi(d_2)\right]$$

We denoted by X_t the stochastic discount factor that will serve to price default risk and by D_t the underlying debt service process, therefore, by definition of A and B we have:

$$\begin{aligned}
A &= \sum_{\tau=t}^{T-1} \log(m_{\tau,\tau+1}) = \sum_{\tau=t}^{T-1} \log(X_{\tau,\tau+1}) \\
B &= \log k - \sum_{\tau=t}^{T-1} \log\left(\frac{S_{\tau+1}}{S_\tau}\right) = \log\left(\frac{K}{D_t}\right) - \sum_{\tau=t}^{T-1} \log\left(\frac{D_{\tau+1}}{D_\tau}\right)
\end{aligned}$$

Since we assume that the conditional probability distribution of $\left(\log X_{t+1}, \log \frac{D_{t+1}}{D_t}\right)$ given the economy state U^t, is a bivariate normal distribution with parameters

$$\mathbb{E}\begin{pmatrix} \log X_{t+1} \\ \log \frac{D_{t+1}}{D_t} \end{pmatrix} = \begin{pmatrix} \mu_{Xt+1} \\ \mu_{Dt+1} \end{pmatrix}, Var\begin{pmatrix} \log X_{t+1} \\ \log \frac{D_{t+1}}{D_t} \end{pmatrix} = \begin{pmatrix} \sigma^2_{Xt+1} & \sigma_{XDt+1} \\ \sigma_{XDt+1} & \sigma^2_{Dt+1} \end{pmatrix}$$

then the parameters of the variables (A, B) are

$$\begin{aligned}
\mu_A &= \mathbb{E}(A) = \mathbb{E}\left(\sum_{\tau=t}^{T-1} \log\left(X_{\tau,\tau+1}\right)\right) = \sum_{\tau=t}^{T-1} \mathbb{E}\left(\log\left(X_{\tau,\tau+1}\right)\right) = \sum_{\tau=t}^{T-1} \mu_{X,\tau+1} \\
\mu_B &= \mathbb{E}(B) = \mathbb{E}\left(\log\left(\frac{K}{D_t}\right) - \sum_{\tau=t}^{T-1} \log\left(\frac{D_{\tau+1}}{D_\tau}\right)\right) = \log\left(\frac{K}{D_t}\right) - \sum_{\tau=t}^{T-1} \mu_{D,\tau+1} \\
\sigma^2_A &= Var(A) = Var\left(\sum_{\tau=t}^{T-1} \log\left(X_{\tau,\tau+1}\right)\right) = \sum_{\tau=t}^{T-1} Var\left(\log\left(X_{\tau,\tau+1}\right)\right) = \sum_{\tau=t}^{T-1} \sigma^2_{X,\tau+1} \\
\sigma^2_B &= Var(B) = Var\left(\log\left(\frac{K}{D_t}\right) - \sum_{\tau=t}^{T-1} \log\left(\frac{D_{\tau+1}}{D_\tau}\right)\right) = \sum_{\tau=t}^{T-1} \sigma^2_{D,\tau+1} \\
\sigma_{AB} &\equiv \rho_{AB}\left(\sigma^2_A \sigma^2_B\right)^{1/2} = \rho_{AB}\left(\sum_{\tau=t}^{T-1} \sigma^2_{X,\tau+1}\right)^{1/2}\left(\sum_{\tau=t}^{T-1} \sigma^2_{D,\tau+1}\right)^{1/2}
\end{aligned}$$

■

Appendix B

Figures

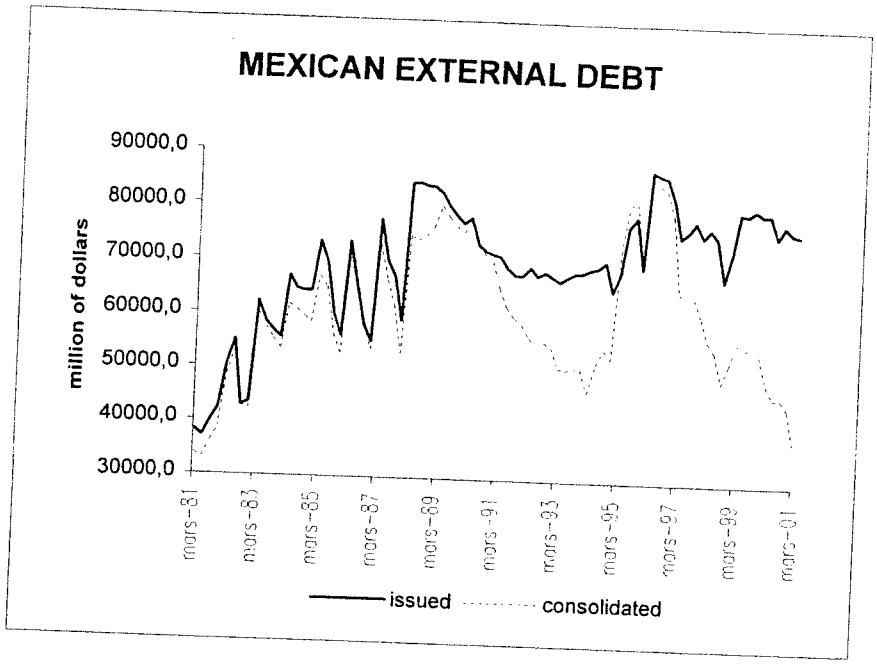
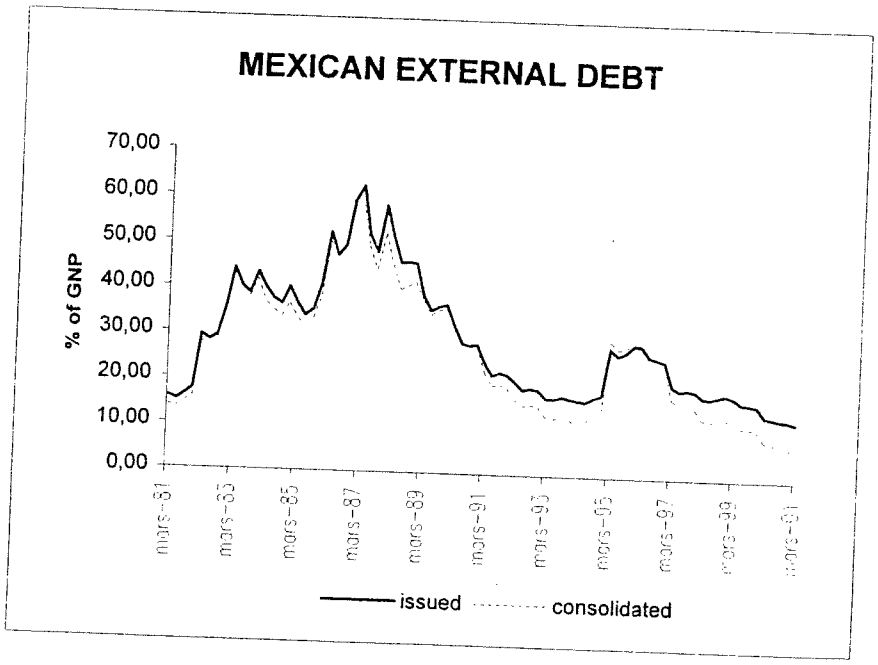


Figure 1



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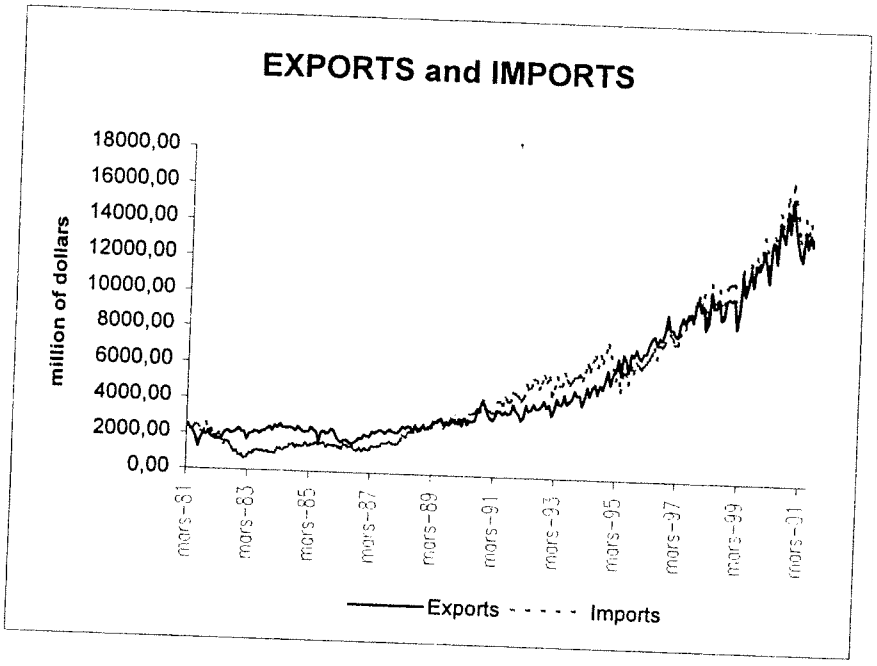

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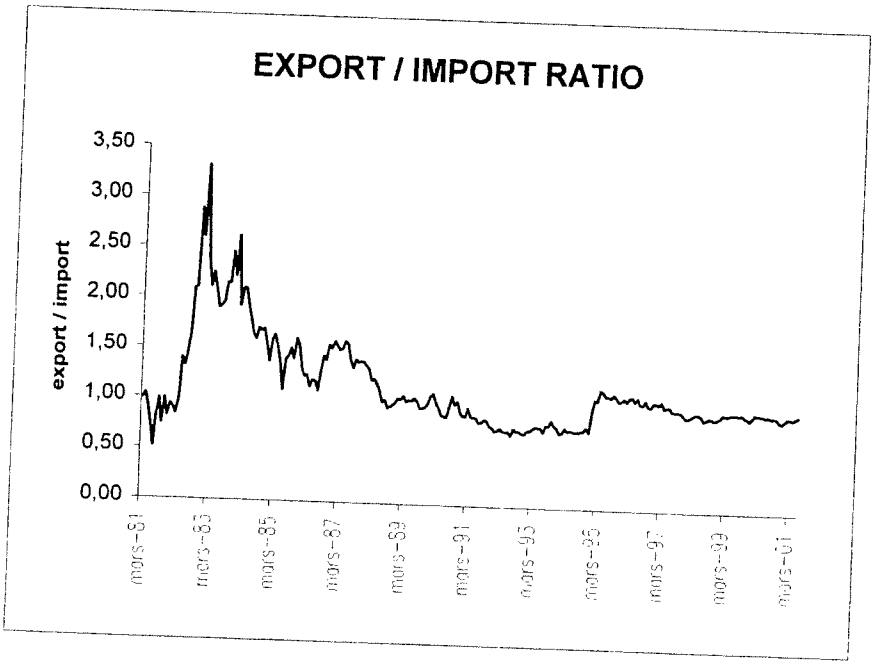

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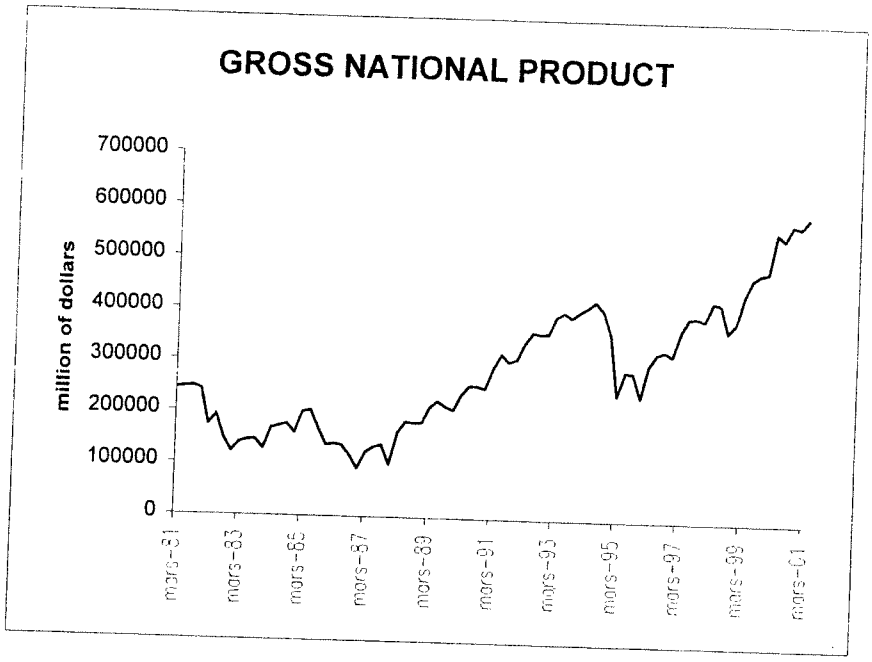
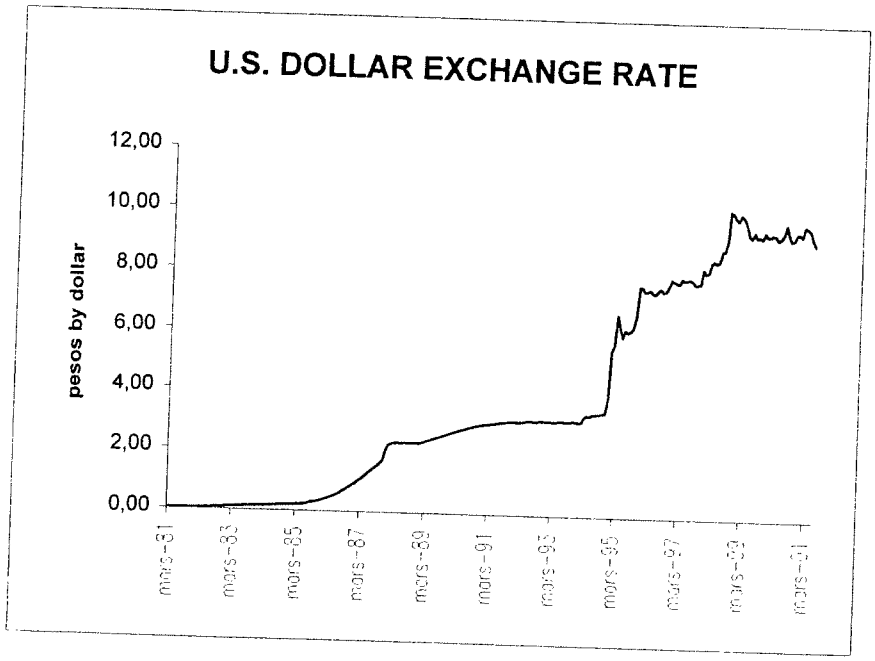


Figure 5



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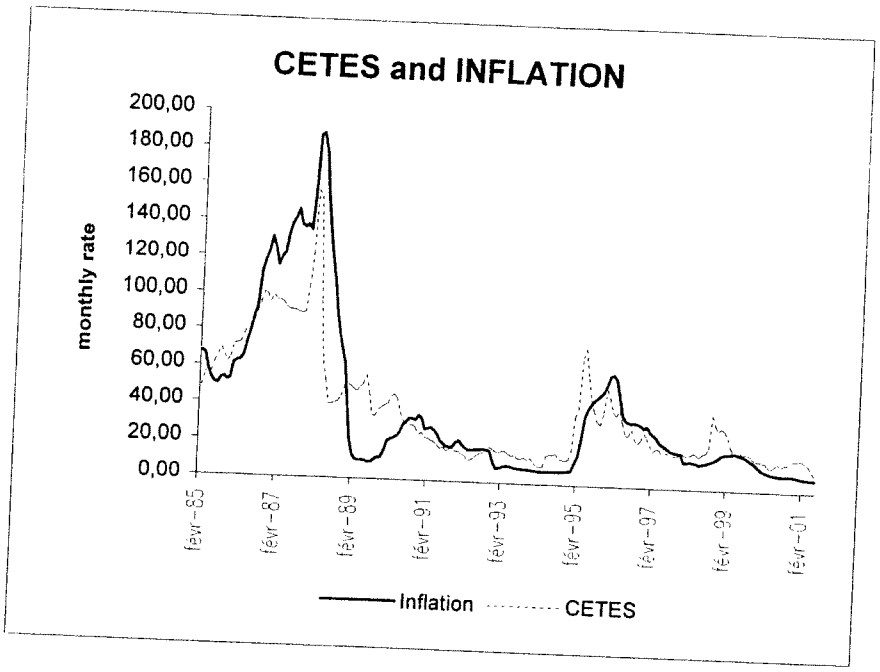

Figure 7

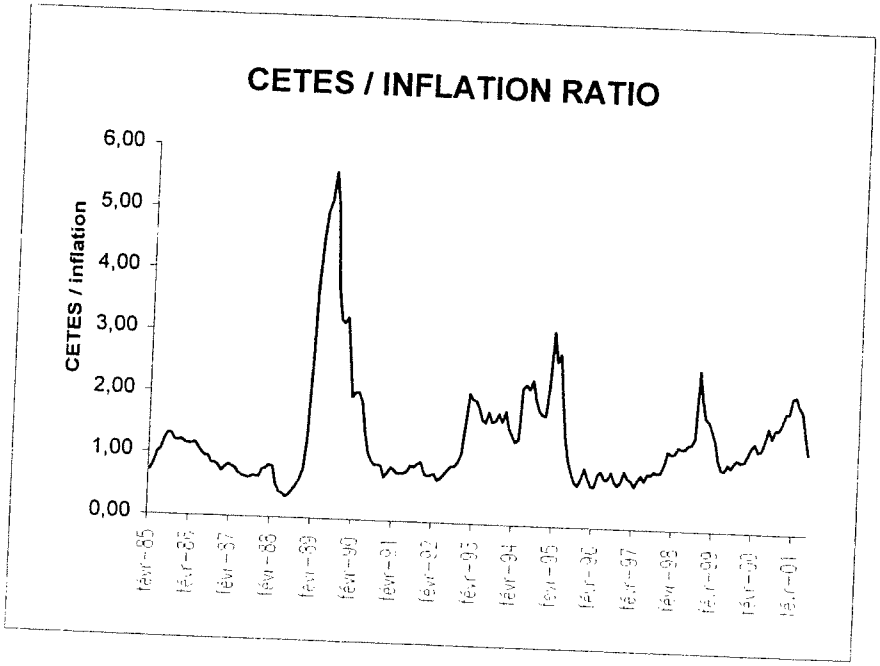

Figure 8

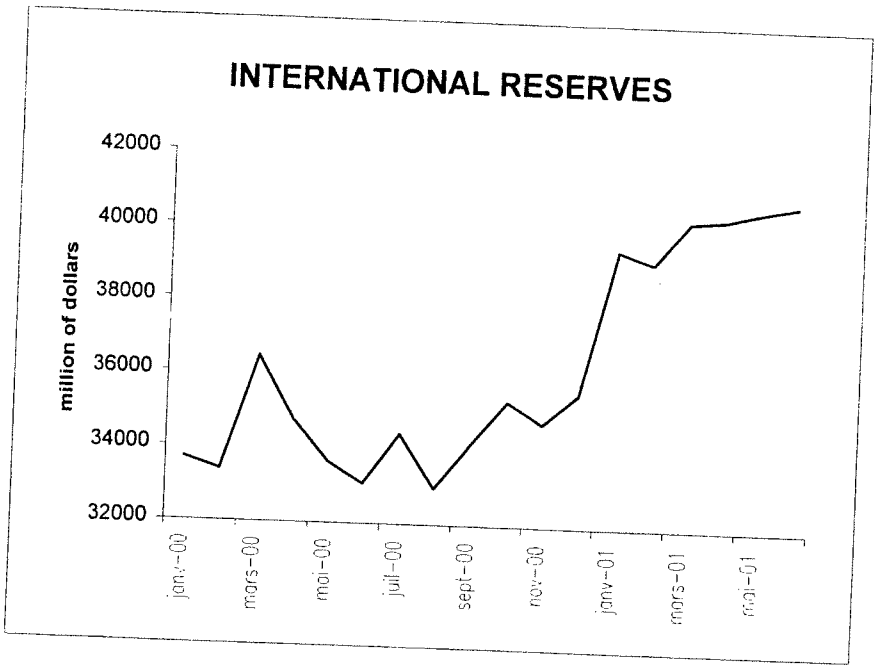
INTERNATIONAL RESERVES
million of dollars
42000
40000
38000
36000
34000
32000
janv-00
mars-00
mai-00
juil-00
sept-00
nov-00
janv-01
mars-01
mai-01

Figure 9

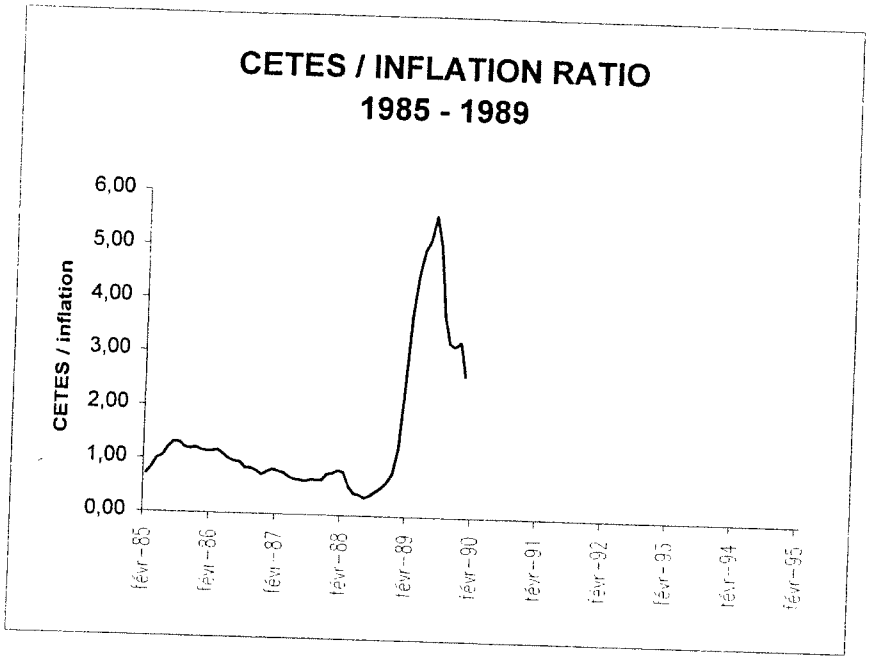

Figure 10

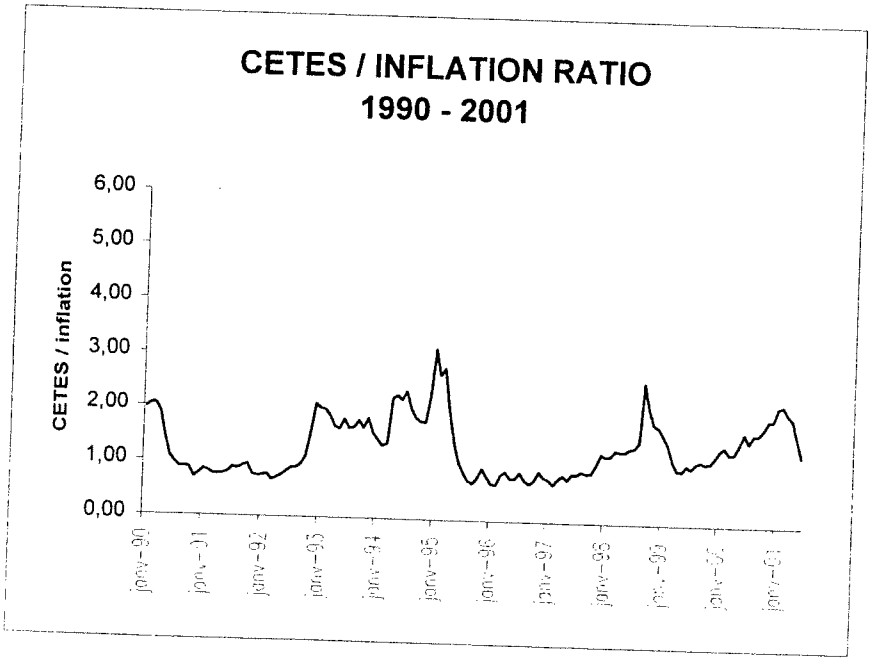

Figure 11

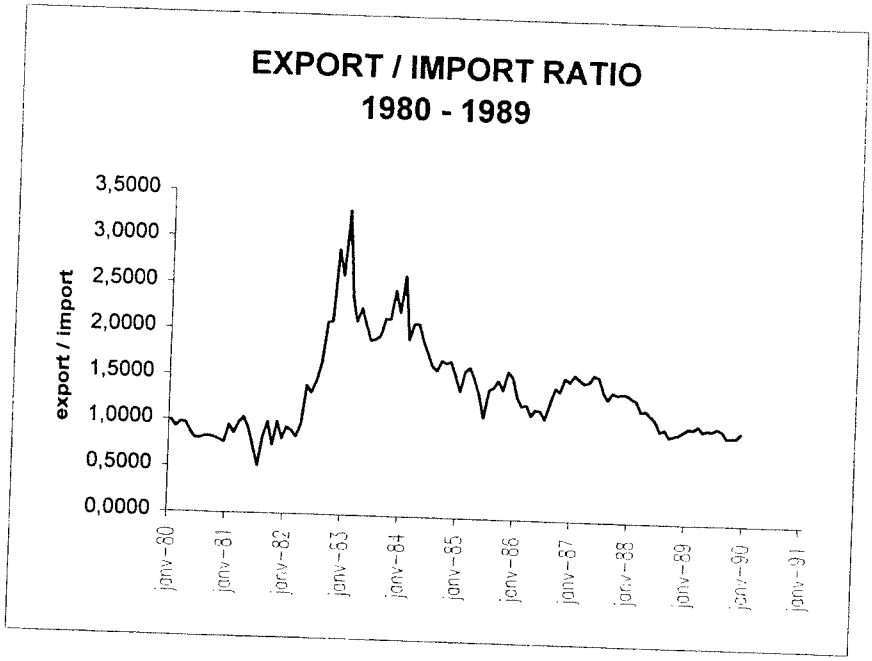

Figure 12

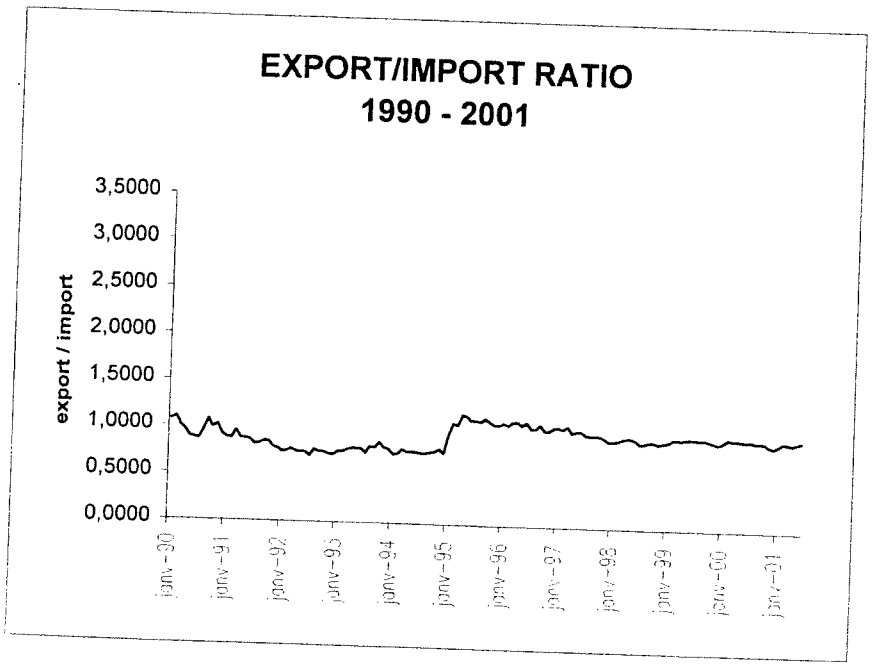

Figure 13