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GLS DETRENDING, EFFICIENT UNIT ROOT TESTS AND STRUCTURAL CHANGE

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RÉSUMÉ

Nous généralisons la classe de M-tests pour racine unitaire analysés par Perron et Ng (1996) et Ng et Perron (1997) au cas où la fonction de tendance peut avoir une rupture à une date inconnue. Ces tests (M^{GLS}) utilisent la méthode des moindres carrés généralisés (*MCG*) pour éliminer les composantes déterministes, tel que proposé par Dufour et King (1991) et Elliot, Rothenberg et Stock (1996) (ERS). Suivant Perron (1989), nous considérons deux modèles : le premier permet une rupture dans la pente et le deuxième un changement d'ordonnée à l'origine (en plus de la rupture de la pente). Nous dérivons la distribution asymptotique des tests M^{GLS} et celle d'une version réalisable du test optimal en un point (P_T^{GLS}) suggéré par ERS. Nous calculons aussi les valeurs critiques de ces tests. De plus, nous calculons le paramètre de non-centralité (utilisé dans l'estimation *MCG* pour éliminer les composantes déterministes) qui permet d'atteindre une puissance asymptotique de 50 %. Nous montrons que les tests M^{GLS} et P_T^{GLS} ont des fonctions de puissance asymptotique proches de l'enveloppe de puissance. En utilisant des simulations, nous évaluons le niveau et la puissance des tests en échantillons finis et nous étudions plusieurs méthodes pour sélectionner le retard nécessaire pour calculer l'estimateur autorégressif de la densité spectrale. Une application à des séries de salaires réels et aux prix des actions ordinaires aux Etats-Unis est aussi considérée à la fin.

Mots clés : test de racine unitaire, *MCG*, changement structurel, ordre de troncation, critère d'information

ABSTRACT

We extend the class of *M*-tests for a unit root analyzed by Perron and Ng (1996) and Ng and Perron (1997) to the case where a change in the trend function is allowed to occur at an unknown time. These tests (M^{GLS}) adopt the *GLS* detrending approach of Dufour and King (1991) and Elliott, Rothenberg and Stock (1996) (ERS). Following Perron (1989), we consider two models : one allowing for a change in slope and the other for both a change in intercept and slope. We derive the asymptotic distribution of the tests as well as that of the feasible point optimal tests (P_T^{GLS}) suggested by ERS. The asymptotic critical values of the tests are tabulated. Also, we compute the non-centrality parameter used for the local *GLS* detrending that permits the tests to have 50% asymptotic power at that value. We show that the M^{GLS} and P_T^{GLS} tests have an asymptotic power function close to the power envelope. An extensive simulation study analyzes the size and power in finite samples under various methods to select the truncation lag for the autoregressive spectral density estimator. An empirical application is also provided.

Key words : unit root test, *GLS* detrending, structural change, truncation lag, information criteria

1 Introduction

Since the seminal paper of Nelson and Plosser (1982), the unit root hypothesis has received a lot of attention both from theoretical and empirical perspectives. Using tests developed by Dickey and Fuller (1979), Nelson and Plosser (1982) argued that current shocks have permanent effects on the level of most macroeconomic series. This finding was supported by other approaches which found that a typical shock has both important transitory and permanent components (see, e.g., Campbell and Mankiw (1987, 1988), Shapiro and Watson (1988), Clark (1987), Cochrane (1988) and Christiano and Eichembaum (1989)).

In contrast to this literature, Perron (1989) argued, as an alternative to the unit root hypothesis, that macroeconomic fluctuations are most likely stationary if allowance is made for the trend function to exhibit occasional changes. Allowing for a single change in intercept and/or slope, he rejected the unit root hypothesis for 11 of the 14 series analyzed by Nelson and Plosser. As discussed in Banerjee, Lumsdaine and Stock (1992) this finding may be important for the following reasons. First, it offers an alternative picture of the persistence in macroeconomics series. Second, this approach can provide a parsimonious model for a slowly changing trend component that may be useful as a data description. Third, the implications for inference in more complex models are very different.

Christiano (1992) criticized the results of Perron (1989) on the basis that the break points should not be treated as exogenous since the imposition of a given break date involves an issue of data mining. Accordingly, Zivot and Andrews (1992), Banerjee, Lumsdaine and Stock (1992) and Perron (1997) considered unit root tests with unknown break points.

In this paper, we continue to treat the potential break points as occurring at unknown times and contribute to this literature in two ways. First, we use the M^{GLS} tests recently analyzed by Perron and Ng (1996) and extend them to permit a one time change in the trend function. Second, as in Elliott, Rothemberg and Stock (1996) (hereafter ERS), we use local to unity GLS detrending of the data. We consider two specific models. The first involves a break in the slope of the trend function and the second a break in both the intercept and slope. In this setup, there is no need to analyze the case where only a change in the intercept is allowed since the tests have then the same asymptotic distribution as the case where the deterministic components include a constant and a time trend which was analyzed in ERS. This is a consequence of condition B of ERS since a change in intercept is a special case of what they refer to as a "slowly evolving deterministic component".

The reasons for considering the M-tests, originally proposed by Stock (1990) and further analyzed by Perron and Ng (1996) is that these tests have much smaller size distortions than other classes of unit root tests even when the residuals have strong negative serial correlation. Also, using GLS detrending when constructing the M-tests allows substantial gains in power as showed by Ng and Perron (1997), similar to the DF^{GLS} test proposed by ERS. This is a consequence of the fact that a GLS framework permits a more accurate estimation of the deterministic component than what is possible using an OLS approach.

The implementation of the M^{GLS} tests requires a truncation lag (k) for the autoregressive spectral density estimate at frequency zero. We follow the suggestion of Ng and Perron (1997) to use a modified version of the AIC and BIC (labeled MIC). Their modification allows for the fact that the bias in the sum of the autoregressive coefficients is highly dependent on k in finite samples and adjust the penalty function accordingly.

Given that a uniformly most powerful test is not attainable, we follow ERS and derive a feasible point optimal test (P_T^{GLS}). The asymptotic power function of this test is derived and we use this power envelope to choose the non-centrality parameter ($\bar{c}$) to perform the GLS detrending such that the asymptotic power of the tests is 50% against the local alternative $\bar{\alpha} = 1 + \bar{c}/T$. For our two models, we obtain $\bar{c} = -23$.

The rest of the paper is organized as follows. The model and some preliminary theoretical results are presented in Section 2. In section 3, we derive the asymptotic distribution of the M^{GLS} and P_T^{GLS} in both cases where the break point is known or unknown. Section 4 considers the asymptotic Gaussian power envelop and the limit distribution of the feasible point optimal test. The asymptotic critical values and the asymptotic power function of the various tests are presented in Section 5. Section 6 considers the size and power of the tests in finite samples using simulations. Section 7 presents an empirical application and Section 8 briefly concludes. An appendix contains technical derivations.

2 GLS detrending with structural change

The data generating process considered is of the form:

$$y_t = d_t + u_t, \quad t = 1, ..., T, \tag{1}$$

$$u_t = \alpha u_{t-1} + v_t, \tag{2}$$

where $\{v_t\}$ is an unobserved stationary mean-zero process. We use the assumption that $u_0 = 0$ throughout, thought the results generally hold for the weaker requirement that $Eu_0^2 < \infty$. The noise function is $v_t = \sum_{i=0}^{\infty} \gamma_i \eta_{t-i}$ with $\sum_{i=0}^{\infty} i|\gamma_i| < \infty$ and where $\{\eta_t\}$ is a martingale difference sequence. The process v_t has a non-normalized spectral density at frequency zero given by $\sigma^2 = \sigma_\eta^2 \gamma(1)^2$, where $\sigma_\eta^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{\infty} E(\eta_t^2)$. Furthermore, $T^{-1/2} \sum_{t=1}^{[rT]} v_t \Rightarrow \sigma W(r)$, where $\Rightarrow$ denotes weak convergence in distribution and $W(r)$ is the Weiner process defined on $C[0,1]$ the space of continuous functions on the interval $[0,1]$. In (1), $d_t = \psi' z_t$, where z_t is a set of deterministic components to be discussed below.

For any series y_t, with deterministic components z_t, we define the transformed data $y_t^{\bar{\alpha}}$ and $z_t^{\bar{\alpha}}$ by:

$$y_t^{\bar{\alpha}} = (y_1, (1-\bar{\alpha}L)y_t), \quad t = 2, ..., T,$$
$$z_t^{\bar{\alpha}} = (z_{1,}(1-\bar{\alpha}L)z_t), \quad t = 2, ..., T,$$

We let $\hat{\psi}$ be the estimator that minimizes:

$$S(\psi) = \left(y_t^{\bar{\alpha}} - \psi' z_t^{\bar{\alpha}}\right)' \left(y_t^{\bar{\alpha}} - \psi' z_t^{\bar{\alpha}}\right). \tag{3}$$

2.1 Model I: Structural change in the slope

For this model, the set of deterministic components, z_t in (1), is given by:

$$z_t = \{1, \ t, 1(t \geq T_B)(t - T_B)\}, \tag{4}$$

where $1(.)$ is the indicator function and T_B is the time of the change. Without loss of generality, we assume that $T_B = T\delta$ for some $\delta \in (0,1)$. In this case, $\hat{\psi}(\delta) = (\hat{\mu}_1, \hat{\beta}_1, \hat{\beta}_2)'$ is the vector of estimates that minimizes (3). The next theorem provides the asymptotic distribution of these coefficient estimates.

Theorem 1 *Suppose that y_t is generated by (1) with $\alpha = 1 + c/T$ and $\{z_t\}$ is given by (4). Let $\hat{\psi}(\delta)$ be the GLS estimates, from (3), of the coefficients of the trend function obtained using $\bar{\alpha} = 1 + \bar{c}/T$. Then:*

$$\hat{\mu}_1 - \mu_1 \Rightarrow v_1;$$
$$T^{1/2}\left(\hat{\beta}_1 - \beta_1\right) \Rightarrow \sigma\left(\lambda_1 b_1 + \lambda_2 b_2\right) \equiv \sigma b_3;$$
$$T^{1/2}\left(\hat{\beta}_2 - \beta_2\right) \Rightarrow \sigma(\lambda_2 b_1 + \lambda_3 b_2) \equiv \sigma b_4;$$

where $b_1 = (1-\bar{c})W_c(1) + \bar{c}^2 \int_0^1 rW_c(r)dr$, $b_2 = (1-\bar{c}+\delta\bar{c})W_c(1) + \bar{c}^2 \int_\delta^1 W_c(r)(r-\delta)dr - W_c(\delta)$, $\lambda_1 = d/\Theta$, $\lambda_2 = -m/\Theta$, $d = 1 - \delta - \bar{c} + 2\bar{c}\delta - \bar{c}\delta^2 - \bar{c}^2\delta + \bar{c}^2\delta^2 + (\bar{c}^2/3)(1-\delta^3)$, $m = 1 - \delta - \bar{c} + \bar{c}\delta - (\bar{c}^2/2)\delta + (\bar{c}^2/2)\delta^3 + (\bar{c}^2/3)(1-\delta^3)$, $a = 1 - \bar{c} + \bar{c}^2/3$, $\Theta = ad - m^2$ and $\lambda_3 = a/\Theta$. Also, $W_c(r)$ is the Ornstein-Uhlenbeck process that is the solution to the stochastic differential equation $dW_c(r) = cW_c(r)dr + dW(r)$ with $W_c(0) = 0$.

2.2 Model II: Structural change in intercept and slope

For Model II, the deterministic components in (1) are:

$$z_t = \{1,\ 1(t \geq T_B), t, 1(t \geq T_B)(t - T_B)\}\,. \tag{5}$$

In this case, the vector of coefficient estimates is $\hat{\psi}(\delta) = (\hat{\mu}_1, \hat{\mu}_2, \hat{\beta}_1, \hat{\beta}_2)'$. In this model, we have the same result as the last theorem because the effect of $\hat{\mu}_2 - \mu_2$ is negligible in large samples. This is because the change in intercept is a special case of a slowly evolving deterministic component in condition B of ERS. Hence, we have:

Theorem 2 *Suppose that y_t is generated by (1) with $\alpha = 1 + c/T$ and $\{z_t\}$ is given by (5). Let $\hat{\psi}(\delta)$ be the GLS estimates, from (3), of the coefficients of the trend function obtained using $\bar{\alpha} = 1 + \bar{c}/T$. Then, the result of Theorem 1 still apply with the addition that $\hat{\mu}_2 - \mu_2$ $\Rightarrow \lim_{T\to\infty} v_{[T\delta]+1} \equiv v^*$.*

3 The tests and their asymptotic distributions

3.1 The tests

The M-tests, originally proposed by Stock (1990), and further analyzed by Perron and Ng (1996), exploit the feature that a series converges with different rates of normalization under the null and the alternative hypothesis. They are defined by:

$$MZ_\alpha(\delta) = \left(T^{-1}\tilde{y}_T^2 - s^2\right)\left(2T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^2\right)^{-1} \tag{6}$$

$$MSB(\delta) = \left(T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^2/s^2\right)^{1/2} \tag{7}$$

$$MZ_t(\delta) = \left(T^{-1}\tilde{y}_T^2 - s^2\right)\left(4s^2T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^2\right)^{-1/2} \tag{8}$$

where $\tilde{y}_t$ is y_t after detrending, i.e.,

$$\tilde{y}_t = y_t - \hat{\psi}' z_t, \tag{9}$$

where $\hat{\psi}$ minimizes the expression (3). The term s^2 is the autoregressive estimate of the spectral density at frequency zero of v_t, defined as:

$$s^2 = s_{ek}^2 / \left(1 - \hat{b}(1)\right)^2, \tag{10}$$

where $s^2_{ek} = T^{-1}\sum_{t=k+1}^{T} \hat{e}^2_{tk}$, $\hat{b}(1) = \sum_{j=1}^{k} \hat{b}_j$, with $\hat{b}_j$ and $\{\hat{e}_{tk}\}$ obtained from the autoregression:

$$\Delta\widetilde{y}_t = b_0\widetilde{y}_{t-1} + \sum_{j=1}^{k} b_j \Delta\widetilde{y}_{t-j} + e_{tk}. \tag{11}$$

The first statistic is a modified version of the Phillips and Perron (1988) Z_α test originally developed by Phillips (1987). The second statistic is a modified version of Bhargava's (1986) R_1 statistic which builds upon the work of Sargan and Bhargava (1983). The third statistic is a modified version of the Phillips and Perron (1988) Z_t test. As Perron and Ng (1996) showed, the MSB and Z_α tests are approximately related by:

$$Z_t \approx MSB \cdot Z_\alpha.$$

This relation suggests the MZ_t tests defined by (8) since it satisfies the relation

$$MZ_t = MSB \cdot MZ_\alpha.$$

Another test of interest is the so-called ADF test which is the t-statistic for testing $b_0 = 0$ in the regression (11). We denote this test by $ADF^{GLS}(\delta)$. Our approach is an extension of Ng and Perron (1997) and Elliott, Rothemberg and Stock (1996) to the case where the trend function contains a structural change. In this case, the M^{GLS} tests will depend of the unknown break point (δ).

3.2 Asymptotic distributions of the tests

We start with a statement of the limiting distribution of the various tests in the case where the break point is considered known.

Theorem 3 *Let y_t be generated by (1) with $\alpha = 1 + c/T$. Let MZ_α, MSB and MZ_t be defined by (6),(7) and (8) with data obtained from local GLS detrending ($\widetilde{y}_t$) at $\bar{\alpha} = 1 + \bar{c}/T$. Also, let ADF^{GLS} be the t-statistic for testing $b_0 = 0$ in the regression (11). In all cases, s^2 is a consistent estimate of σ^2. For Models I and II, we have:*

$$\begin{aligned}
MZ_\alpha^{GLS}(\delta) &\Rightarrow \frac{0.5K_1(c,\bar{c},\delta)}{K_2(c,\bar{c},\delta)} \equiv H^{MZ_\alpha^{GLS}}(c,\bar{c},\delta),\\
MSB^{GLS}(\delta) &\Rightarrow \left(K_2(c,\bar{c},\delta)\right)^{1/2} \equiv H^{MSB^{GLS}}(c,\bar{c},\delta),\\
MZ_t^{GLS}(\delta) &\Rightarrow \frac{0.5K_1(c,\bar{c},\delta)}{\left(K_2(c,\bar{c},\delta)\right)^{1/2}} \equiv H^{MZ_t^{GLS}}(c,\bar{c},\delta),\\
ADF^{GLS}(\delta) &\Rightarrow \frac{0.5K_1(c,\bar{c},\delta)}{\left(K_2(c,\bar{c},\delta)\right)^{1/2}} \equiv H^{ADF^{GLS}}(c,\bar{c},\delta),
\end{aligned}$$

where

$$\begin{aligned} K_1(c,\bar{c},\delta) &= V_{c\bar{c}}^{(1)}(1,\delta)^2 - 2V_{c\bar{c}}^{(2)}(1,\delta) - 1, \\ K_2(c,\bar{c},\delta) &= \int_0^1 V_{c\bar{c}}^{(1)}(r)^2 dr - 2\int_\delta^1 V_{c\bar{c}}^{(2)}(r,\delta)dr, \end{aligned}$$

and $V_{c\bar{c}}^{(1)}(r,\delta) = W_c(r) - rb_3$, $V_{c\bar{c}}^{(2)}(r,\delta) = b_4(r-\delta)[W_c(r) - rb_3 - (1/2)(r-\delta)b_4]$; with b_3, b_4 and $W_c(r)$ as defined in Theorem 1.

In practice, it is usually the case that an investigator wants to treat the break point as unknown. In this case, an estimate is needed. A method suggested by Zivot and Andrews (1992) is to consider estimating δ as the break point that yields the minimal value of the statistics, i.e. using $\inf_\delta J^{GLS}(\delta)$ where $J = MZ_\alpha$, MSB, MZ_t, and ADF. Using the continuous mapping theorem and arguments as in Perron (1997), we have, assuming no shift in the trend function under the null hypothesis:

$$\inf_{\delta\in[0,1]} J^{GLS}(\delta) \Rightarrow \inf_{\delta\in[0,1]} H^{J^{GLS}}(c,\bar{c},\delta), \tag{12}$$

for $J = MZ_\alpha$, MSB, MZ_t, and ADF with the functions $H(\cdot)$ defined in Theorem 3. Note that no truncation for the range of possible break points needs to be imposed. As discussed in Vogelsang and Perron (1997), the implied estimate of δ is not consistent for the true value of the break point when the data generating process contains a break. These authors also note that the tests statistic are not invariant (even asymptotically) to values of the coefficients of the change in the trend. Nevertheless, they argue that, in typical sample sizes, this is not a problem unless the changes are extremely large. Thus, these tests can still be used with the critical values derived assuming no shift under the null hypothesis.

An alternative method to select the break date, as used in Perron (1997), is to choose it such that the absolute value of the t-statistic on the coefficient of the change in slope is maximized. This procedure has been used by many authors, e.g. Christiano (1992), Banerjee, Lumsdaine and Stock (1992), Perron (1997) and Vogelsang and Perron (1997). Consider, for example Model 1 where the deterministic component is given by

$$d_t = \mu_1 + \beta_1 t + \beta_2(t - T_B)1(t > T_B).$$

Let $\widehat{\beta}_2(\delta)$ be the GLS estimate of β_2 and $t_{\widehat{\beta}_2}(\delta)$ be its associated t-statistic. The break point can be selected using the estimate

$$\hat{\delta} = \arg\max_{\delta\in(\varepsilon,1-\varepsilon)} |t_{\widehat{\beta}_2}(\delta)|,$$

where ε is some small number imposing a trimming on the possible values of the break dates. As discussed in Vogelsang and Perron (1997), if under the null hypothesis we have $\beta_2 \neq 0$ and the true break point given by $T_B^0/T = \delta^0$, then $\hat{\delta}$ is a consistent estimate of δ^0 and the limiting distributions of the test statistics correspond to those in the case where the break date is known, i.e. the limit distributions given in Theorem 3 evaluated at δ^0. In practice, one can simply evaluate these limit distributions at the estimated value $\hat{\delta}$.

When, under the null hypothesis, $\beta_2 = 0$ in which case there is no change in the slope of the trend function, it is easy to show (using the results of Theorem 1) that

$$t_{\widehat{\beta}_2}(\delta) \Rightarrow b_4/(\lambda_3^{1/2}),$$

where b_4 and λ_3 are defined in Theorem 1. We then have

$$\hat{\delta} = \arg \max_{\delta \in (\varepsilon, 1-\varepsilon)} |t_{\widehat{\beta}_2}(\delta)| \Rightarrow \arg \max_{\delta \in (\varepsilon, 1-\varepsilon)} |b_4/(\lambda_3^{1/2})| \equiv \delta^*. \tag{13}$$

Hence, the limiting distributions of the statistics are given by

$$J^{GLS}(\hat{\delta}) \Rightarrow H^{J^{GLS}}(c, \bar{c}, \delta^*), \tag{14}$$

for $J = MZ_\alpha$, MSB, MZ_t, and ADF with the functions $H(\cdot)$ defined in Theorem 3.

In practice, it is difficult to know if there is a change in slope since any test of such hypothesis would depend on whether a unit root is present or not. Hence, a conservative procedure is to use the critical values corresponding to the case where it is assumed that no break is present, i.e. (14). This is the procedure we use in the following.

4 Feasible point optimal test and the power envelope

Elliott, Rothemberg and Stock (1996), following Dufour and King (1991), have considered the issue of developing tests with optimality properties under Gaussian errors. The case where the break point is assumed known follows closely their analysis. While a uniformly most powerful test is not attainable, it is possible to define a point optimal test against the alternative $\alpha = \bar{\alpha}$. If v_t is i.i.d., this is provided by the likelihood ratio statistic, which simplifies to $L(\delta) = S(\bar{\alpha}, \delta) - S(1, \delta)$, where $S(\bar{\alpha}, \delta)$ and $S(1, \delta)$ are the sums of squared errors from a GLS regression with $\alpha = \bar{\alpha}$ and $\alpha = 1$, respectively. Varying the value of $\bar{\alpha}$, gives a family of point optimal tests and the power envelope. Under the assumption that the errors follow a normal distribution, the power function forms a Gaussian power envelope for testing $\alpha = 1$. ERS have proposed an asymptotic version of a feasible point optimal test

(P_T^{GLS}) which takes into account that v_t is a serially correlated series. The P_T^{GLS} test is defined by:

$$P_T^{GLS}(c, \bar{c}, \delta) = \{S(\bar{\alpha}, \delta) - \bar{\alpha} S(1, \delta)\}/s^2. \tag{15}$$

The next theorem provides the limiting distribution of the P_T^{GLS} test:

Theorem 4 *Let y_t be generated by (1) with $\alpha = 1 + c/T$. Let P_T^{GLS} be defined by (15) with data obtained from local GLS detrending ($\tilde{y}_t$) at $\bar{\alpha} = 1 + \bar{c}/T$. Also, let s^2 be a consistent estimate of σ^2. Then, the limit distribution for the P_T^{GLS} test is the same under Models I and II and is given by:*

$$P_T^{GLS}(\delta) \Rightarrow J_{1c}(r) + J_{2c}(r, \delta) \equiv H^{P_T^{GLS}}(c, \bar{c}, \delta), \tag{16}$$

where

$$\begin{aligned} J_{1c}(r) &= \bar{c}^2 \int_0^1 W_c(r)^2 dr - \bar{c} W_c(1)^2, \\ J_{2c}(r, \delta) &= (1-\delta)^{-1}[W_c(1)^2 - 2W_c(1)W_c(\delta) + \delta^{-1} W_c(\delta)^2] \\ &\quad -\lambda_1 b_1^2 - 2\lambda_2 b_1 b_2 - \lambda_3 b_2^2. \end{aligned}$$

The asymptotic expression (16) for the P_T^{GLS} test allows us to define the asymptotic power envelope for the two models. It is given by

$$\pi(c, \delta) = \Pr[H^{P_T^{GLS}}(c, c, \delta) < b^{P_T^{GLS}}(c, \delta)],$$

where $b^{P_T^{GLS}}(c, \delta)$ is such that

$$\Pr[H^{P_T^{GLS}}(0, c, \delta) < b^{P_T^{GLS}}(c, \delta)] = v,$$

with v the size of the test. Note that in general, a different power envelop exists for each values of δ.

When δ is unknown, things are rather different. The principle is, however, the same. To maximize the likelihood function, the estimate of δ must minimize the sum of squares residuals $S(1, \delta)$ and $S(\bar{\alpha}, \delta)$ under the null and alternative hypotheses, respectively. This leads to the statistic $\inf_{\delta \in [0,1]} P_T^{GLS}(\delta)$ which, using Theorem 4 and the continuous mapping theorem, has the following distribution

$$\inf_{\delta \in [0,1]} P_T^{GLS}(\delta) \Rightarrow \inf_{\delta \in [0,1]} H^{P_T^{GLS}}(c, \bar{c}, \delta).$$

The asymptotic Gaussian power envelop is then defined as

$$\pi^*(c) = \Pr[\inf_{\delta\in[0,1]} H^{P_T^{GLS}}(c, c, \delta) < b_*^{P_T^{GLS}}(c)],$$

where $b_*^{P_T^{GLS}}(c)$ is such that

$$\Pr[\inf_{\delta\in[0,1]} H^{P_T^{GLS}}(0, c, \delta) < b_*^{P_T^{GLS}}(c)] = \upsilon, \tag{17}$$

with υ the size of the test.

Furthermore, the power envelope allows us to find the "optimal" centrality parameter ($\overline{c}$) for our models. ERS recommended to choose the value $\overline{c}$ such that the asymptotic power of the test is 50%, i.e. $\bar{c}$ is such that $\Pr[\inf_{\delta\in[0,1]} H^{P_T^{GLS}}(\bar{c}, \bar{c}) < b_*^{P_T^{GLS}}(\bar{c})] = 0.5$. Using simulations, we found that $\bar{c} = -23$ and we, henceforth use this value in the rest of the paper.

For comparison with the other tests, we shall also consider the version of the feasible point optimal test when the break date is chosen maximizing the t-statistic on the coefficient of the change in slope; that is $P_T^{GLS}(\hat{\delta})$ with $\hat{\delta} = \arg\max |t_{\widehat{\beta}_2}(\delta)|$. It is straightforward to deduce that the asymptotic distribution of $P_T^{GLS}(\hat{\delta})$ is given by

$$P_T^{GLS}(\hat{\delta}) \Rightarrow H^{P_T^{GLS}}(c, \bar{c}, \delta^*),$$

where δ^* is defined by (13). Note that this method of choosing the break point is ad hoc since it is not optimal in the sense of leading to a maximal value of the likelihood ratio as does the other method. We should, therefore, expect this version of the tests to have lower power compared to the version where the break point is chosen by minimizing the tests.

5 Critical values and asymptotic power functions

In this section, we obtain the asymptotic critical values for tests assuming $\bar{c} = -23$ is used to detrend the data. We simulate directly the asymptotic distributions using 1,000 steps to approximate the Weiner process on $[0, 1]$ as the partial sums of i.i.d $N(0, 1)$ random variables. The limiting distributions are tabulated for the null hypothesis $c = 0$. For the finite sample distributions, we use $T = 100$ with data generated by a random walk with zero initial condition and *i.i.d.* $N(0, 1)$ errors. Here k is set to 0 which is equivalent to using the true value of σ^2; the effects of selecting k on the finite sample critical values are investigated in the next section. In all cases 10,000 replications are used. The results are presented in Table 1.a for the case where the break point is selected by minimizing the tests, and in

Table 1.b when the break point is selected maximizing the absolute value of the t-statistic on the coefficient of the change in slope. We omit the ADF^{GLS} test since it has the same asymptotic distribution as the MZ_t^{GLS} test. In general, the approximation to the finite sample distribution is adequate but somewhat less good for model 2 which contains a change in intercept that is asymptotically negligeable.

The asymptotic power functions of the tests are defined by:

$$\pi^*_{JGLS}(c,\overline{c}) = \Pr[\inf_{\delta\in[0,1]} H^{J^{GLS}}(c,\bar{c},\delta) < b^{J^{GLS}}(\bar{c})],$$

or

$$\pi^*_{JGLS}(c,\overline{c}) = \Pr[H^{J^{GLS}}(c,\bar{c},\delta^*) < b_*^{J^{GLS}}(\bar{c})],$$

for $J = MZ_\alpha$, MSB, MZ_t, ADF and P_T with $H^i(c,\bar{c})$ defined in Theorems 3 and 4, and δ^* is defined by (13). The constants $b^{J^{GLS}}(\bar{c})$ and $b_*^{J^{GLS}}(\bar{c})$ are such that $\Pr[\inf_{\delta\in(0,1)} H^{J^{GLS}}(0,\bar{c},\delta) < b^{J^{GLS}}(\bar{c})] = \upsilon$, and $\Pr[H^{J^{GLS}}(0,\bar{c},\delta^*) < b_*^{J^{GLS}}(\bar{c})] = \upsilon$, the size of the tests. The asymptotic power functions are showed in figures 1 to 8. The solid line is the power envelope. As can see, the M^{GLS} tests, and especially the P_T^{GLS} test have power functions very close to the power envelope when the break point is selected by minimizing the tests. On the other hand, the power functions of the same tests with the break point selected maximizing the absolute value of the t-statistic of the coefficient on the change in slope are clearly lower than the power envelop. We should there expect this class of tests to have less desirable finite sample properties.

6 Size and power of the tests in finite samples

6.1 The size issue, the selection of k and information criteria

It is clear that all tests considered require the estimation of the augmented autoregression (11). Ng and Perron (1997) recommended using GLS detrended data using the same non-centrality parameter $\bar{c}$ for constructing the autoregression and the tests. They also tried using GLS detrended data under the null hypothesis ($\bar{c} = 0$) when the autoregression (11) is used to construct s^2 but found, in the linear trend case, that the properties of the tests were very similar. Our simulations showed, however, that in the case with a break in the slope of the trend function, using $\bar{c} = 0$ to GLS-detrend the data when estimating the autoregression (11) to construct s^2 led to tests with better finite sample properties[1]. Hence, in what follows

[1] We have compiled the same set of results that follow using $\bar{c} = -23$ in the construction of s^2. The results are available on request. They show that the tests have slightly better size properties using $\bar{c} = 0$ against slightly lower power compared to using $\bar{c} = -23$. The differences are, however, minor.

$\bar{c} = -23$ is used to detrend the data when constructing the tests but $\bar{c} = 0$ is used to detrend the data when estimating the autoregression (11). Of course, when constructing the ADF^{GLS} test, the autoregresion (11) uses data detrended by GLS using the value $\bar{c} = -23$.

To see how the lag order, k, influences the behavior of the tests, we first consider the finite sample size of the MZ_{α}^{GLS} and P_{T}^{GLS} tests for given fixed values of the truncation lag k under a variety a data-generating processes. We consider simulations for Model I. Reported in Tables 2.a and 2.b are the sizes of the tests at selected values of θ when v_t is an MA(1) process , i.e. when $v_t = e_t + \theta e_{t-1}$ with $e_t \sim i.i.d.\ N(0, 1)$. Tables 2.c and 2.d present results at selected values of ρ when v_t is an AR(1) process, i.e., when $v_t = \rho v_{t-1} + e_t$. We report results for $T = 100$ and $T = 200$. The nominal size of 5% is used as the benchmark.

Several features of the results for MA errors are of note here. First, for a θ of the same absolute value, a negative θ always requires a larger lag to obtain a more accurate size. Second, as the sample increases, so does the "optimal k", defined as the smallest k for which the exact size is "closest" to the nominal size of 5%. Third, for positive θ, the sizes of the tests are significantly better when k is even than when it is odd. Fourth, if we compare with the results of Ng and Perron (1997), the larger the number of deterministic terms, the more distant are the exact sizes from the nominal ones. For the MZ_{α}^{GLS} tests, it is to be noted that when $\theta = -0.8$, the smallest size achievable with a given fixed k is 0.191 (at $k = 5$) when $T = 100$ and 0.084 (at $k = 7$) when $T = 200$. These are quite large, but as we shall see some data dependent methods to select k are able to yield tests with sizes much closer to 5%, i.e. better than what is achievable with the "best" fixed k.

For the results with AR errors, size discrepancies also exist, albeit not as dramatic as in the MA case. For large negative AR errors, the tests are undersized, and for large positive errors, they are oversized. As analyzed in Perron and Ng (1998), the autoregressive spectral density estimator has more stable properties when there are over 100 observations. The size distortions are apparently much reduced as we increase the size of the sample.

Clearly, the selection of k is very important for the properties of the tests, especially in the presence of negative moving average errors. Various practical solutions have been suggested for this problem. In ERS, the authors use the BIC to select k but they set the lower bound to be 3. This is because the BIC would have chosen k to be 0 or 1 frequently if zero was the lower bound. An alternative method of forcing in larger k's is the sequential t test for the significance of the last lag considered in Ng and Perron (1995). For any chosen upper bound, say $k\max$, the procedure will select $k\max$ with a probability equalling the size of the test. The procedure thus has the ability to yield higher k's for the augmented autoregression than the BIC when there are negative moving average errors and, hence, smaller size distortions. But, the sequential procedure over-parameterizes the augmented autoregression in other cases. This, as does ERS's implementation of the BIC, leads to less satisfactory estimates and subsequently to power losses. Neither approach is fully satisfactory.

Recently, Ng and Perron (1997) have proposed a modification of the AIC and BIC, labeled MIC, which can be expressed as follows:

$$k_{mic} = \arg\min_k \log(s_{ek}^2) + \frac{C_T(\hat{\tau}_T(k) + k)}{T},$$

where

$$\hat{\tau}_T(k) = (s_{ek}^2)^{-1}\hat{b}_0^2 \sum_{t=1}^{T} \tilde{y}_{t-1}^2.$$

The $MAIC$ uses $C_T = 2$ and the $MBIC$ uses $C_T = \log(T)$. Ng and Perron (1997), based on theoretical considerations and simulations, recommended $MAIC$. The advantage of the MIC is that it takes into account the possible dependence of $\hat{b}_0$ on k. For example, with a large negative MA component, $\hat{b}_0$ decreases substantially as k increases. Hence, the term $\hat{\tau}_T(k)$ implies a higher penalty for a low value of k. In such cases, a larger k is selected. When there is little correlation in the residuals, $\hat{b}_0$ is quite insensitive to k and the $MAIC$ is basically equivalent to AIC.

To see some of the empirical properties of tests based on the MIC, we performed the following simulation experiment. For a given DGP, we constructed the MZ_α^{GLS} test for our first model at each $k \in [0, 10]$, and recorded the exact size of the tests. We then found the k, denoted k^*, as the first k with a size closest to within three standard errors of the nominal size of 0.05. For 1000 replications, the standard error in the simulated size of the tests is $(0.05(0.95)/1000)^{1/2} = 0.007$. Thus, k^* is the first k that fall in the range 0.029 and 0.071. If no such k exists, k^* is the k with the smallest absolute deviation from the nominal size of 5%. This procedure is used to obtain a set of k^* for our first model. We then obtain k_{bic} as the median value selected by BIC over the range 0 and 10. A similar procedure is used to obtain k_{aic}, k_{maic} and k_{mbic}. The simulations are based on MA(1) and AR(1) processes assuming the null hypothesis that $\alpha = 1$. The results of the simulation experiments are reported in Table 3. They reveal that k_{maic} and k_{mbic} select lag orders that are, indeed, very close to k^* for all parameter configurations, except when there is a large negative AR component in which case all methods select $k = 1$ instead of $k = 4$ or 5 which would yield M^{GLS} tests with better sizes[2].

[2] Of course, in the pure AR case the "optimal" lag for the ADF test is 1 and for this test the order selected by k_{maic} and k_{mbic} are indeed adequate.

6.2 Finite sample critical values with data dependent methods to select k

We now consider the finite sample critical values of the tests using the various data-dependent methods to select the truncation lag described above. While the asymptotic distributions provide good approximations to the finite sample distributions, it is sometimes the case that using a data dependent method to select k can induce distortions. Our simulations are based on 1000 replications of the DGP defined by (1) and (2) with $d_t = 0$, $\alpha = 1$ and $v_t \sim i.i.d.\ N(0,1)$. We present results for the cases where the lag length of the autoregression (11) is selected using the AIC, BIC, $MAIC$, $MBIC$ and the t-sig methods for the ADF^{GLS} test or for constructing the autoregressive spectral density estimator for the other tests. In the simulations, the lower bound on the lag length is always zero to reduce the chance of over-parameterizing when a large k is not necessary. The upper bound is $k\max = int[10(T/100)^{1/4}]$ for the AIC, BIC, $MAIC$ and $MBIC$ methods and $k\max = int[4(T/100)^{1/4}]$ for the t-sig method. We consider $T = 100$, $T = 150$ and $T = 200$. The results are presented in Table 4 (Model 1, choosing T_B minimizing the tests), Table 5 (Model 1, choosing T_B maximizing $|t_{\widehat{\beta}_2}|$), Table 6 (Model 2, choosing T_B minimizing the tests), and Table 7 (Model 2, choosing T_B maximizing $|t_{\widehat{\beta}_2}|$).

The results show the finite sample distributions of the tests involving the autoregressive spectral density estimator to be very sensitive to the method used to select the truncation lag k. In particular, the AIC and t-sig methods imply that the finite sample distributions of the M^{GLS} tests and the P_T^{GLS} test are much different in the left tail. Indeed, the finite sample critical values are much smaller than those of the corresponding asymptotic distributions. Hence, the use of the latter would imply very liberal tests. On the other hand, when k is chosen using the $MAIC$, $MBIC$ or BIC criteria, the finite sample distributions are somewhat less spread than the asymptotic ones. Here, the use of asymptotic critical values would imply slightly conservative tests. These problems are common for the two models and the two procedures to select the break point. The problems for the AIC and t-sig methods are somewhat less severe when choosing the break point by maximizing the absolute value of the t-statistic on the coefficient of the change in slope and comparatively more severe for Model 2 than for Model 1. Hence, the results in this base case show that the AIC or t-sig methods should not be used to select k when constructing the autoregressive spectral density estimate for the M^{GLS} tests and the P_T^{GLS} test.

Overall, the finite sample distribution of the ADF^{GLS} test is little affected by the method to choose k. The results shows that inference with the asymptotic critical values would yield slightly liberal tests with AIC and t-sig and slightly conservative tests with the criteria

$MAIC$, $MBIC$ or BIC.

6.3 Size and power of tests

We now consider the size of the tests in finite samples using the various data-dependent methods to select the truncation lag described above. Our simulations are based on 1000 replications of the DGP defined by (1) with $d_t = 0$. We consider pure $MA(1)$ processes, i.e. with $v_t = (1+\theta L)e_t$ and pure $AR(1)$ processes, i.e. $(1+\rho L)v_t = e_t$, where $e_t \sim i.i.d.\ N(0,1)$. For both the $MA(1)$ and $AR(1)$ cases, we consider θ and ρ in the range $[-0.8, 0.8]$. The autoregressive spectral density estimator defined by (10) is used. We present results for the cases where the lag length of the autoregression is selected using the AIC, BIC, $MAIC$, $MBIC$ and the t-sig methods. In the simulations, the lower bound on this lag length is always zero to reduce the chance of over-parameterizing when a large k is not necessary. The upper bound is $k\max = int[10(T/100)^{1/4}]$ for the AIC, BIC, $MAIC$ and $MBIC$ methods and $k\max = int[4(T/100)^{1/4}]$ for the t-sig method. We consider the sample sizes $T = 100$ and $T = 200$. Given the finite sample results documented in the previous section, we calculate the size and power of the tests using the 5% finite sample critical values for the case where the errors are *i.i.d.*. The power is evaluated at $\bar{\alpha} = 1 + \bar{c}/T$ for $\bar{c} = -23$ which implies that the asymptotic power is 50%.

The results for the case where the break point is chosen by minimizing the tests are presented in Table 8 for $T = 100$ and Table 9 for $T = 200$. In the case where the errors are *i.i.d.*, as expected, the power of the tests when the methods AIC or t-sig are used to select k is very low. On the other hand, when the $MBIC$, $MAIC$ or BIC methods are used, the power is indeed close to the asymptotic value of 50%. It is somewhat higher when $MBIC$ is used and somewhat lower with $MAIC$. For the ADF^{GLS} the power is high for all methods to choose k but again higher if $MBIC$ is used. Given these results, we shall not discuss further the behavior of the tests with the AIC or the t-sig methods.

Consider now the case where the errors have a negative MA component. For all tests, the use of the BIC to select k implies tests with severe distortions with exact size above 75% when the MA component is -0.8 and about 30% when it is -0.4. On the other hand, the $MBIC$ or $MAIC$ implies that the M^{GLS} tests and the P_T^{GLS} test have much less size distortions. The exact size is about 12% when the coefficient is -0.8 and about 6% when it is -0.4. But even when using the $MAIC$ or the $MBIC$, the ADF^{GLS} test still suffers from high size distortions (with $\theta = -.8$ it is 28% with $MAIC$ and 52% with $MBIC$). An interesting fact is that, the M^{GLS} tests and the P_T^{GLS} test, using the $MAIC$ lead to tests with much smaller size distortions than if the "best fixed k" had been used. For example, at $T = 100$, the exact size is .12 for the MZ_α^{GLS} with $MAIC$ as opposed to .19 with a fixed $k = 5$. When $T = 200$, the size distortions of the M^{GLS} tests and the P_T^{GLS} test disappears for all values of θ considered. The ADF^{GLS} test continues, however, to exhibit important size distortions at $\theta = -.8$.

When the errors have a positive moving average coefficient, the M^{GLS} tests and the P_T^{GLS} tests have the correct size only when the $MBIC$ is used, otherwise they are liberal. The ADF^{GLS} test has the correct size with either $MAIC$ or $MBIC$ but power is very low.

Consider now the case where the errors have a negative autoregressive component. Here, the M^{GLS} tests and the P_T^{GLS} test are very conservative and, hence, show basically no power. The ADF has the correct size and power is good. When the autoregressive coefficient is positive, the M^{GLS} tests and the P_T^{GLS} test are liberal. The ADF^{GLS} has better size but no power.

Overall, the results show that the ADF^{GLS} test with the truncation lag chosen using either the $MAIC$ or the $MBIC$ has better overall properties unless there is a negative MA component in the residuals, in which case the M^{GLS} tests and the P_T^{GLS} test are superior. The size distortions are, however, smaller and power also higher when $T = 200$.

The results pertaining to the case where the break point is selected by maximizing the absolute value of the t-statistic on the coefficient of the change in slope are presented in Table 10 for $T = 100$ and Table 11 for $T = 200$. They show size and power properties which are much less satisfactory than for the case where the break point is selected by minimizing the tests. First, in the case of *i.i.d.* errors, the power is substantially lower. Even at $T = 200$, the power is much lower than the asymptotic target of 50% as suggested by the asymptotic power analysis of Section 5. Secondly with negative MA errors, the M^{GLS} tests and the P_T^{GLS} test using the $MAIC$ or $MBIC$ are so conservative under the null hypothesis that they have no power. The ADF^{GLS} test have slight size distortions but also little power. Third, power is low for all tests when the errors have a positive MA component though here size distortions are smaller. Fourth, with negative AR errors, even the ADF^{GLS} test is very conservative and, accordingly, has little power. Finally, with positive AR errors, there are still liberal size distortions for the M^{GLS} tests and the P_T^{GLS} test; the ADF^{GLS} test has the correct size but little power.

Overall, the theoretical and simulation results show that the tests based on choosing the break point by minimizing the test are better in terms of size and power and should be recommended for practical applications.

7 Empirical applications.

Among the macroeconomic time series considered by Nelson and Plosser (1982), Perron (1989) argued that two of them were likely affected by a significant change in slope for the samples analyzed, namely the Real Wages and Stock Prices series. The series are presented in Figures 9 and 10. We re-evaluate the claim made by Perron (1989) to the effect that the noise function of these series is stationary if allowance is made for such a change in slope using the tests described here. We applied the MZ_t, P_T and ADF tests using the BIC, $MAIC$ and $MBIC$ criteria to select the autoregressive order (imposing a minimal value of 1). The estimate s^2 is constructed from an autoregression with data detrended using the GLS method with $\bar{c} = 0$. For the construction of the tests (and to estimate the autoregression used for the ADF test), we use GLS detrended data with $\bar{c} = -23$.

The results are presented in Table 12.a for the case where the break date is selected minimizing the appropriate test and in Table 12.b for the case where the break date is selected maximizing the absolute value of the t-statistic on the coefficient of the change in slope (in which case the estimated break date is common for all tests). Using the former method, all tests points to a strong rejection at the 1% significance level for the Stock Price series with the break date estimated at either 1937 or 1945 depending on the specification used. For the Real Wage series, there is a rejection at least at the 5% significance level using the criterion $MAIC$ or $MBIC$ to select k and at the 10% using the BIC. The estimated break date is at 1938 or 1940 depending on the specification used. The estimate trend function is plotted in Figures 9 and 10 along with the series using $T_B = 1937$ for the Stock Price series and $T_B = 1938$ for the Real Wages series.

When choosing the break point maximizing the absolute value of the t-statistic on the coefficient of the change in slope, the results are different as predicted by the asymptotic and finite sample results discussed before. Here, using $MAIC$ or $MBIC$ to select k a rejection of the unit root is no longer possible for the Stock Prices series (though a rejection at the 5% level is possible using BIC). Similarly, the evidence against the unit root is considerably weaker for the Real Wages series. This accords with our asymptotic analysis which indicated that this method to choose the break point leads to tests with lower power.

8 Conclusions.

This paper has considered a class of tests for testing the null hypothesis of a unit root in the presence of a one time change in the slope or the slope and intercept of a trend function.

Our approach followed that of Elliott, Rothemberg and Stock (1996) in that we considered detrending the data using a local to unity GLS approach. We considered the required extensions of the ADF and the P_T as well as of the various M tests suggested by Perron and Ng (1996). We also followed the recent literature in investigating the properties of the tests when the break point is selected either by minimizing the tests or by maximizing the absolute value of the t-statistic on the coefficient of the change in slope. A novel aspect of our results is that the latter method to select the break point leads to tests that are asymptotically inferior in the sense that their local asymptotic power function lies well below the Gaussian power envelop. On the other hand, the former method leads to tests whose local asymptotic power function is very close to the envelop. This power issues was corroborated, for finite samples, using simulation experiments which also showed even comparatively worse size distortions. Hence, for applications, we recommend using either the ADF^{GLS} or the M^{GLS} tests and P_T^{GLS} test. The difference is that the ADF^{GLS} has worse size distortions in the negative MA case but better power in the negative AR case; and the M^{GLS} tests and P_T^{GLS} test have good size overall but very little power in the negative AR case. The choice between the two depends on the investigator's assessment of the likely importance of one or the other class of processes in the data considered. Our experiments also suggest that the use of the $MAIC$ to select the autoregressive truncation lag leads to tests with better properties overall.

The analysis pertaining to the tests where the break is selected using the maximal value of the t-statistic on the coefficient of the change in slope rests, however, on the assumption that the investigator does not know if a break has occurred. If the investigator has evidence that a break has occurred, even if the timing is unknown, the relative merits of this strategy could drastically change. In such a case, the method consistently estimates the break point and, hence, the limiting distribution is the same as that of the case where the break point is known. Methods to tests if a change in the trend function has occurred being agnostic about the presence or absence of a unit root have been considered by Perron (1991) and Vogelsang (1997). Hence, it is possible to make this procedure operational in practice. This, however, implies a different asymptotic power envelop for each case (since the power envelop then depends on the break point) and, hence, the derivation of an "optimal" non-centrality parameter $\bar{c}$ on a case by case basis. Hence, the method becomes heavily computer intensive. It is nevertheless, an interesting avenue for future research.

Appendix

Throughout, we use the following lemma which is by now standard.

Lemma A.1: Let $\{u_t\}$ be a near-integrated series generated by (2). Then, we have: a) $T^{-1/2}u_{[Tr]} \Rightarrow \sigma W_c(r)$; b) $T^{-3/2}\sum_{t=1}^{T} u_t \Rightarrow \sigma \int_0^1 W_c(r)dr$; c) $T^{-2}\sum_{t=1}^{T} u_t^2 \Rightarrow \sigma^2 \int_0^1 W_c^2(r)dr$; d) $T^{-1}\sum_{t=1}^{T} u_{t-1}v_t \Rightarrow \sigma^2\{\int_0^1 W_c(r)dW(r) + \gamma\}$ with $\gamma = (\sigma^2 - \sigma_v^2)/2\sigma^2$.

Proof of Theorem 1: In matrix notation, we have:

$$\begin{aligned}\hat{\psi}(\delta) - \psi &= \left[\left(\Delta z - \bar{c}T^{-1}z_{-1}\right)'\left(\Delta z - \bar{c}T^{-1}z_{-1}\right)\right]^{-1} \\ &\quad \left[\left(\Delta z - \bar{c}T^{-1}z_{-1}\right)'\left(\Delta u - \bar{c}T^{-1}u_{-1}\right)\right],\end{aligned} \tag{A.1}$$

where:

$$\begin{aligned}\Delta z &= (z_1, z_2 - z_1, ..., z_T - z_{T-1}), \\ z_{-1} &= (0, z_1, z_2, ..., z_{T-1}), \\ \Delta u &= (u_1, u_2 - u_1, ..., u_T - u_{T-1}), \\ u_{-1} &= (0, u_1, u_2, ..., u_{T-1}).\end{aligned}$$

Now define the scaling matrix $\Upsilon_T = diag(1, T^{1/2}, T^{1/2})$, we can write expression (A.1) as:

$$\Upsilon_T\left(\hat{\psi}(\delta) - \psi\right) = \Gamma_T(\delta)^{-1}\Psi_T(\delta), \tag{A.2}$$

where

$$\Gamma_T(\delta) = \Upsilon_T^{-1}\left[\left(\Delta z - \bar{c}T^{-1}z_{-1}\right)'\left(\Delta z - \bar{c}T^{-1}z_{-1}\right)\right]\Upsilon_T^{-1},$$

and

$$\Psi_T(\delta) = \Upsilon_T^{-1}\left[\left(\Delta z - \bar{c}T^{-1}z_{-1}\right)'\left(\Delta u - \bar{c}T^{-1}u_{-1}\right)\right].$$

We first consider the limit of each element of the matrix $\Gamma_T(\delta)$ denoted Γ_{ij} $(i, j = 1, 2, 3)$. We let $\Delta z_{(i)}$ and $z_{-1(i)}$ be the ith element of the vectors Δz and z_{-1}, respectively. We have:

1. $\Gamma_{11} = \left(\Delta z_{(1)} - \bar{c}T^{-1}z_{-1(1)}\right)'\left(\Delta z_{(1)} - \bar{c}T^{-1}z_{-1(1)}\right) \Rightarrow 1;$

2. $\Gamma_{12} = T^{-1/2}\left(\Delta z_{(1)} - \bar{c}T^{-1}z_{-1(1)}\right)'\left(\Delta z_{(2)} - \bar{c}T^{-1}z_{-1(2)}\right) \Rightarrow 0;$

3. $\Gamma_{13} = T^{-1/2}\left(\Delta z_{(1)} - \bar{c}T^{-1}z_{-1(1)}\right)'\left(\Delta z_{(3)} - \bar{c}T^{-1}z_{-1(3)}\right) \Rightarrow 0;$

4. $\Gamma_{22} = T^{-1}\left(\Delta z_{(2)} - \bar{c}T^{-1}z_{-1(2)}\right)'\left(\Delta z_{(2)} - \bar{c}T^{-1}z_{-1(2)}\right) \Rightarrow 1 - \bar{c} + \bar{c}^2/3 \equiv a;$

5. $\Gamma_{23} = T^{-1}\left(\Delta z_{(2)} - \bar{c}T^{-1}z_{-1(2)}\right)'\left(\Delta z_{(3)} - \bar{c}T^{-1}z_{-1(3)}\right)$
   $\Rightarrow 1 - \delta - \bar{c} + \bar{c}\delta - (\bar{c}^2/2)\delta + (\bar{c}^2/2)\delta^3 + (\bar{c}^2/3)(1 - \delta^3) \equiv m;$

6. $\Gamma_{33} = T^{-1}\left(\Delta z_{(3)} - \bar{c}T^{-1}z_{-1(3)}\right)'\left(\Delta z_{(3)} - \bar{c}T^{-1}z_{-1(3)}\right)$
$\Rightarrow 1 - \delta - \bar{c} + 2\bar{c}\delta - \bar{c}\delta^2 - \bar{c}^2\delta + \bar{c}^2\delta^2 + (\bar{c}^2/3)(1-\delta^3) \equiv d.$

We next consider the limit of each element of the vector $\Psi_T(\delta)$, denoted Ψ_i $(i = 1, 2, 3)$. We have:

1. $\Psi_1 = \left(\Delta z_{(1)} - \bar{c}T^{-1}z_{-1(1)}\right)'(\Delta u - \bar{c}T^{-1}u_{-1}) \Rightarrow v_1;$
2. $\Psi_2 = T^{-1/2}\left(\Delta z_{(2)} - \bar{c}T^{-1}z_{-1(2)}\right)'(\Delta u - \bar{c}T^{-1}u_{-1})$
$\Rightarrow \sigma[W_c(1)(1-\bar{c}) + \bar{c}^2\int_0^1 rW_c(r)dr] \equiv \sigma b_1;$
3. $\Psi_3 = T^{-1/2}\left(\Delta z_{(3)} - \bar{c}T^{-1}z_{-1(3)}\right)'(\Delta u - \bar{c}T^{-1}u_{-1})$
$\Rightarrow \sigma[W_c(1)(1-\bar{c}+\delta\bar{c}) + \bar{c}^2\int_\delta^1 W_c(r)(r-\delta)dr - W_c(\delta)] \equiv \sigma b_2.$

Hence, using the symmetry of $\Gamma_T(\delta)$,

$$\Upsilon_T\left(\hat{\psi}(\delta) - \psi\right) \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & m \\ 0 & m & d \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ \sigma b_1 \\ \sigma b_2 \end{bmatrix}.$$

The proof of the Theorem follows upon solving for the inverse. The proof of Theorem 2 is basically the same and, hence, omitted.

Proof of Theorem 3: The proof uses the results of Theorem 1. We show the proof only for Model 1 and for the $MZ_\alpha^{GLS}(\delta)$ test, the proof for the other Model and tests follows analogously. We first have

$$\begin{aligned} T^{-1}\tilde{y}_T^2 &= T^{-1}\left\{y_T - \left(\hat{\mu}_1 + \hat{\beta}_1 T + \hat{\beta}_2(T - T\delta)\right)\right\}^2 \\ &= T^{-1}\left\{u_T - \left[(\hat{\mu}_1 - \mu_1) + \left(\hat{\beta}_1 - \beta_1\right)T + \left(\hat{\beta}_2 - \beta_2\right)(T - T\delta)\right]\right\}^2. \end{aligned}$$

After some algebra, we obtain:

i $T^{-1}u_T \Rightarrow \sigma^2 W_c(1)^2;$

ii. $2T^{-1}u_T(\hat{\mu}_1 - \mu_1) \Rightarrow 0;$

iii. $2T^{-1}u_T\left(\hat{\beta}_1 - \beta_1\right)T \Rightarrow 2\sigma^2 b_3 W_c(1);$

iv. $2T^{-1}u_T\left(\hat{\beta}_2 - \beta_2\right)(T - T\delta) \Rightarrow 2\sigma^2 b_4 W_c(1)(1-\delta);$

v. $T^{-1}\left(\hat{\mu}_1-\mu_1\right)^2 \Rightarrow 0;$

vi. $2T^{-1}\left(\hat{\mu}_1-\mu_1\right)\left(\hat{\beta}_1-\beta_1\right)T \Rightarrow 0;$

vii. $T^{-1}\left(\hat{\beta}_1-\beta_1\right)^2 T^2 \Rightarrow \sigma^2 b_3^2;$

viii. $2T^{-1}\left(\hat{\mu}_1-\mu_1\right)\left(\hat{\beta}_2-\beta_2\right)(T-T\delta) \Rightarrow 0;$

ix. $2T^{-1}\left(\hat{\beta}_1-\beta_1\right)t\left(\hat{\beta}_2-\beta_2\right)(T-T\delta) \Rightarrow 2\sigma^2 b_3 b_4(1-\delta);$

x. $T^{-1}\left(\hat{\beta}_2-\beta_2\right)^2 (T-T\delta)^2 \Rightarrow \sigma^2 b_4^2(1-\delta)^2.$

Using these results, we have:

$$T^{-1}\tilde{y}_T^2 \Rightarrow \sigma^2\left\{V_{c\bar{c}}^{(1)}(1,\delta)^2 - 2V_{c\bar{c}}^{(2)}(1,\delta)\right\}, \tag{A.3}$$

where

$$V_{c\bar{c}}^{(1)}(1,\delta) = W_c(1) - b_3,$$

and

$$V_{c\bar{c}}^{(2)}(1,\delta) = b_4(1-\delta)[W_c(1) - b_3 - (1/2)(1-\delta)b_4].$$

Consider now the term $2T^{-2}\sum_{t=1}^{T}\tilde{y}_t^2$, defined by

$$\begin{aligned} 2T^{-2}\sum_{t=1}^{T}\tilde{y}_t^2 &= 2T^{-2}\sum_{t=1}^{T}\{y_t - [\hat{\mu}_1 + \hat{\beta}_1 t + \hat{\beta}_2 1(t > T\delta)(t-T\delta)]^2\}^2 \\ &= 2T^{-2}\sum_{t=1}^{T}\{u_t - [(\hat{\mu}_1-\mu_1) + (\hat{\beta}_1-\beta_1)t \\ +(\hat{\beta}_2-\beta_2)1(t &> T\delta)(t-T\delta)]^2\} \end{aligned}$$

After some algebra, we obtain:

i. $2T^{-2}\sum_{t=1}^{T}u_t^2 \Rightarrow 2\sigma^2\int_0^1 W_c(r)^2 dr;$

ii. $4T^{-2}\left(\hat{\mu}_1-\mu_1\right)\sum_{t=1}^{T}u_t \Rightarrow 0;$

iii. $4T^{-2}\left(\hat{\beta}_1-\beta_1\right)\sum_{t=1}^{T}tu_t \Rightarrow 4\sigma^2\int_0^1 rb_3 W_c(r)dr;$

iv. $4T^{-2}\left(\hat{\beta}_2-\beta_2\right)\sum_{t=1}^{T}1(t>T\delta)(t-T\delta)u_t \Rightarrow 4\sigma^2\int_\delta^1 b_4 W_c(r)(r-\delta)dr;$

v. $2T^{-1}(\hat{\mu}_1-\mu_1^2) \Rightarrow 0;$

vi. $4T^{-2}\left(\hat{\mu}_1-\mu_1\right)\left(\hat{\beta}_1-\beta_1\right)\sum_{t=1}^{T}t \Rightarrow 0;$

vii. $2T^{-2}\left(\hat{\beta}_1-\beta_1\right)^2\sum_{t=1}^{T}t^2 \Rightarrow 2\sigma^2\int_0^1 b_3^2 r^2 dr$;

viii. $4T^{-2}\left(\hat{\mu}_1-\mu_1\right)\left(\hat{\beta}_2-\beta_2\right)\sum_{t=1}^{T}1(t>T\delta)(t-T\delta) \Rightarrow 0$;

ix. $4T^{-2}\left(\hat{\beta}_1-\beta_1\right)\left(\hat{\beta}_2-\beta_2\right)\sum_{t=1}^{T}t1(t>T\delta)(t-T\delta) \Rightarrow 4\sigma^2\int_\delta^1 b_3 b_4 r(r-\delta)dr$;

x. $2T^{-2}\left(\hat{\beta}_2-\beta_2\right)^2\sum_{t=1}^{T}1(t>T\delta)(t-T\delta)^2 \Rightarrow 2\sigma^2\int_\delta^1 b_4^2(r-\delta)^2 dr$.

Using these results we have:

$$2T^{-2}\sum_{t=1}^{T}\tilde{y}_t^2 \Rightarrow 2\sigma^2\left\{\int_0^1 V_{c\bar{c}}^{(1)}(r,\delta)^2 dr - 2\int_\delta^1 V_{c\bar{c}}^{(2)}(r,\delta)dr\right\}. \tag{A.4}$$

Using (A.3), (A.4) and the fact that s^2 is a consistent estimate of σ^2, the proof is complete.

Proof of Theorem 4. We first give the proof for Model I. Defining

$$Q_T(\alpha) = (u^{\alpha\prime}z^{\alpha})(z^{\alpha\prime}z^{\alpha})^{-1}(z^{\alpha\prime}u^{\alpha}),$$

we have $S(\bar{\alpha}) = u^{\bar{\alpha}\prime}u^{\bar{\alpha}} - Q_T(\bar{\alpha})$ and $S(1) = u^{1\prime}u^1 - Q_T(1)$. Hence, s^2P_T is given by:

$$\begin{aligned} s^2P_T &= \bar{c}^2T^{-2}u_{-1}'u_{-1} - 2\bar{c}T^{-1}\Delta u' u_{-1} - \bar{c}T^{-1}\Delta u'\Delta u \\ &\quad -Q_T(\bar{\alpha}) + Q_T(1) + \bar{c}T^{-1}Q_T(1). \end{aligned} \tag{A.5}$$

Using the fact that $2\bar{c}T^{-1}\Delta u' u_{-1} = \bar{c}T^{-1}u_T^2 - \bar{c}T^{-1}\Delta u'\Delta u$, we have:

$$s^2P_T = \bar{c}^2T^{-2}u_{-1}'u_{-1} - \bar{c}T^{-1}u_T^2 - Q_T(\bar{\alpha}) + Q_T(1). \tag{A.6}$$

Using the results in the proof of Theorem 2:

$$\begin{aligned} \bar{c}^2T^{-2}u_{-1}'u_{-1} &\Rightarrow \sigma^2\bar{c}^2\int_0^1 W_c(r)^2 dr \\ \bar{c}T^{-1}u_T^2 &\Rightarrow \sigma^2\bar{c}W_c(1)^2 \end{aligned}$$

$$\begin{aligned} Q_T(\bar{\alpha}) &= (u^{\bar{\alpha}\prime}z^{\bar{\alpha}})(z^{\bar{\alpha}\prime}z^{\bar{\alpha}})^{-1}(z^{\bar{\alpha}\prime}u^{\bar{\alpha}}) \\ &\Rightarrow \begin{bmatrix} v_1 \\ \sigma b_1 \\ \sigma b_2 \end{bmatrix}' \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda_1 & \lambda_2 \\ 0 & \lambda_2 & \lambda_3 \end{bmatrix} \begin{bmatrix} v_1 \\ \sigma b_1 \\ \sigma b_2 \end{bmatrix} \\ &= v_1^2 + \sigma^2[\lambda_1 b_1^2 + 2\lambda_2 b_1 b_2 + \lambda_3 b_2^2] \end{aligned} \tag{A.7}$$

$$
\begin{aligned}
Q_T(1) &= (u^{1\prime}z^1)(z^{1\prime}z^1)^{-1}(z^{1\prime}u^1) \\
&\Rightarrow \begin{bmatrix} v_1 \\ \sigma W_c(1) \\ \sigma(W_c(1) - W_c(\delta)) \end{bmatrix}' \begin{bmatrix} 1 & 0 & 0 \\ 0 & \delta^{-1} & -\delta^{-1} \\ 0 & -\delta^{-1} & \delta^{-1}(1-\delta)^{-1} \end{bmatrix} \\
&\quad \cdot \begin{bmatrix} v_1 \\ \sigma W_c(1) \\ \sigma(W_c(1) - W_c(\delta)) \end{bmatrix} \\
&= v_1^2 + \sigma^2\{(1-\delta)^{-1}\delta^{-1}[W_c(1) - W_c(\delta)]^2 \\
&\quad +2\delta^{-1}W_c(1)W_c(\delta) - \delta^{-1}W_c(1)^2\}
\end{aligned} \tag{A.8}
$$

The proof follows directly using these limits. For Model II, we only need to change (A.7) as follows:

$$
\begin{aligned}
Q_T(\bar{\alpha}) &\Rightarrow \begin{bmatrix} v_1 \\ v^* \\ \sigma b_1 \\ \sigma b_2 \end{bmatrix}' \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda_1 & \lambda_2 \\ 0 & 0 & \lambda_2 & \lambda_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v^* \\ \sigma b_1 \\ \sigma b_2 \end{bmatrix} \\
&\Rightarrow v_1^2 + (v^*)^2 + \sigma^2[\lambda_1 b_1^2 + 2\lambda_2 b_1 b_2 + \lambda_3 b_2^2]
\end{aligned}
$$

and (A.8) as follows:

$$
\begin{aligned}
Q_T(1) &\Rightarrow \begin{bmatrix} v_1 \\ v^* \\ \sigma W_c(1) \\ \sigma(W_c(1) - W_c(\delta)) \end{bmatrix}' \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \delta^{-1} & -\delta^{-1} \\ 0 & 0 & -\delta^{-1} & \delta^{-1}(1-\delta)^{-1} \end{bmatrix} \\
&\quad \cdot \begin{bmatrix} v_1 \\ v^* \\ \sigma W_c(1) \\ \sigma(W_c(1) - W_c(\delta)) \end{bmatrix} \\
&\Rightarrow v_1^2 + (v^*)^2 + \sigma^2\{(1-\delta)^{-1}\delta^{-1}[W_c(1) - W_c(\delta)]^2 \\
&\quad +2\delta^{-1}W_c(1)W_c(\delta) - \delta^{-1}W_c(1)^2\}
\end{aligned}
$$

The proof is then complete upon substitution.

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Table 1.a: Percentage Points the M^{GLS} and P_T^{GLS} Tests under the Null Hypothesis ($c = 0$).

Choosing the Break Point Minimizing the Tests.

	MZ_α^{GLS}			MSB^{GLS}		
	$T=\infty$	$T=100$		$T=\infty$	$T=100$	
		Model 1	Model 2		Model 1	Model 2
1.0%	-40.89	-41.44	-45.56	0.110	0.109	0.104
2.5%	-35.48	-36.61	-40.51	0.118	0.116	0.110
5.0%	-31.64	-32.73	-35.81	0.125	0.122	0.117
10.0%	-27.46	-28.38	-31.29	0.134	0.131	0.125
20.0%	-22.51	-23.82	-26.36	0.147	0.143	0.136
30.0%	-19.57	-20.79	-23.22	0.158	0.153	0.145
40.0%	-17.08	-18.52	-20.69	0.169	0.162	0.153
50.0%	-15.13	-16.38	-18.66	0.179	0.171	0.161
60.0%	-13.21	-14.64	-16.62	0.191	0.181	0.170
70.0%	-11.44	-12.93	-14.72	0.205	0.192	0.180
80.0%	-9.53	-11.02	-12.70	0.223	0.208	0.193
90.0%	-7.46	-8.81	-10.20	0.250	0.231	0.214
95.0%	-6.01	-7.12	-8.56	0.275	0.253	0.233
97.5%	-4.97	-6.06	-7.19	0.299	0.275	0.249
99.0%	-4.10	-4.85	-6.03	0.324	0.301	0.273

	MZ_t^{GLS}			P_T^{GLS}		
	$T=\infty$	$T=100$		$T=\infty$	$T=100$	
		Model 1	Model 2		Model 1	Model 2
1.0%	-4.49	-4.53	-4.75	6.59	6.64	6.24
2.5%	-4.18	-4.24	-4.44	7.70	7.65	7.07
5.0%	-3.96	-4.01	-4.20	8.53	8.50	7.92
10.0%	-3.68	-3.73	-3.92	9.83	9.76	9.05
20.0%	-3.33	-3.41	-3.59	11.96	11.74	10.79
30.0%	-3.09	-3.18	-3.37	13.80	13.39	12.29
40.0%	-2.89	-3.00	-3.18	15.72	15.07	13.86
50.0%	-2.71	-2.82	-3.01	17.74	16.87	15.47
60.0%	-2.53	-2.66	-2.84	20.19	19.02	17.33
70.0%	-2.35	-2.50	-2.66	23.20	21.72	19.59
80.0%	-2.13	-2.30	-2.47	27.60	25.15	22.79
90.0%	-1.88	-2.04	-2.20	34.66	31.73	27.83
95.0%	-1.67	-1.83	-2.00	42.57	38.34	33.46
97.5%	-1.52	-1.69	-1.82	49.76	44.43	39.59
99.0%	-1.35	-1.49	-1.63	58.76	53.46	46.92

Table 1.b: Percentage Points the M^{GLS} and P_T^{GLS} Tests under the Null Hypothesis ($c = 0$).

Choosing the Break Point maximizing $|t_{\widehat{\beta}_2}|$

	MZ_α^{GLS}			MSB^{GLS}		
	$T = \infty$	$T = 100$		$T = \infty$	$T = 100$	
		Model 1	Model 2		Model 1	Model 2
1.0%	-41.01	-41.30	-42.33	0.110	0.110	0.109
2.5%	-34.96	-36.08	-36.56	0.119	0.117	0.116
5.0%	-30.75	-32.20	-32.65	0.127	0.124	0.123
10.0%	-26.41	-27.82	-28.23	0.137	0.133	0.132
20.0%	-21.76	-23.20	-23.75	0.150	0.145	0.144
30.0%	-18.85	-20.37	-20.74	0.161	0.155	0.154
40.0%	-16.13	-18.07	-18.43	0.171	0.164	0.163
50.0%	-14.66	-16.13	-16.40	0.182	0.173	0.172
60.0%	-12.92	-14.33	-14.64	0.194	0.184	0.182
70.0%	-11.28	-12.44	-12.90	0.207	0.196	0.193
80.0%	-9.46	-10.60	-11.04	0.224	0.212	0.208
90.0%	-7.46	-8.45	-8.84	0.250	0.236	0.231
95.0%	-5.96	-6.96	-7.30	0.275	0.258	0.252
97.5%	-4.89	-5.83	-6.14	0.299	0.279	0.273
99.0%	-3.82	-4.76	-4.89	0.334	0.301	0.297

	MZ_t^{GLS}			P_T^{GLS}		
	$T = \infty$	$T = 100$		$T = \infty$	$T = 100$	
		Model 1	Model 2		Model 1	Model 2
1.0%	-4.50	-4.53	-4.59	6.80	6.62	6.47
2.5%	-4.17	-4.22	-4.25	7.86	7.59	7.44
5.0%	-3.89	-3.99	-4.02	8.93	8.50	8.44
10.0%	-3.61	-3.71	-3.73	10.34	9.80	9.75
20.0%	-3.27	-3.38	-3.42	12.56	11.80	11.61
30.0%	-3.04	-3.16	-3.19	14.44	13.52	13.32
40.0%	-2.85	-2.97	-3.00	16.37	15.22	14.98
50.0%	-2.67	-2.81	-2.82	18.47	16.98	16.86
60.0%	-2.50	-2.64	-2.67	20.93	19.08	18.82
70.0%	-2.33	-2.45	-2.50	23.80	21.92	21.27
80.0%	-2.13	-2.26	-2.30	28.10	25.61	24.79
90.0%	-1.87	-1.99	-2.05	34.97	31.97	30.72
95.0%	-1.64	-1.79	-1.84	42.67	37.93	36.88
97.5%	-1.44	-1.62	-1.67	50.47	45.36	43.76
99.0%	-1.24	-1.40	-1.43	62.11	53.33	51.77

Table 2: Exact Size of the Tests at Selected Values of k in s^2

a) MZ_α^{GLS} ; MA case.

T	θ	k=0	1	2	3	4	5	6	7	8	9	10
100	-0.80	1.000	0.798	0.372	0.249	0.199	0.191	0.195	0.251	0.250	0.291	0.336
	-0.40	0.588	0.081	0.037	0.075	0.119	0.189	0.247	0.324	0.397	0.459	0.529
	0.00	0.021	0.030	0.050	0.126	0.172	0.254	0.326	0.403	0.445	0.538	0.588
	0.40	0.000	0.112	0.034	0.153	0.174	0.277	0.333	0.424	0.471	0.546	0.587
	0.80	0.000	0.269	0.007	0.274	0.099	0.372	0.245	0.471	0.412	0.599	0.546
200	-0.80	1.000	0.993	0.748	0.376	0.189	0.125	0.093	0.084	0.092	0.094	0.107
	-0.40	0.725	0.143	0.043	0.044	0.054	0.068	0.104	0.137	0.175	0.210	0.245
	0.00	0.029	0.029	0.043	0.073	0.093	0.120	0.156	0.194	0.215	0.254	0.299
	0.40	0.001	0.117	0.030	0.080	0.092	0.140	0.158	0.196	0.240	0.273	0.311
	0.80	0.000	0.238	0.008	0.178	0.038	0.213	0.099	0.252	0.194	0.316	0.284

b) P_T^{GLS} ; MA case.

T	θ	k=0	1	2	3	4	5	6	7	8	9	10
100	-0.80	1.000	0.793	0.396	0.258	0.202	0.204	0.204	0.255	0.255	0.296	0.341
	-0.40	0.538	0.083	0.042	0.080	0.124	0.192	0.249	0.329	0.400	0.460	0.523
	0.00	0.018	0.033	0.053	0.133	0.179	0.254	0.325	0.399	0.446	0.534	0.590
	0.40	0.000	0.112	0.036	0.153	0.178	0.278	0.328	0.417	0.465	0.545	0.581
	0.80	0.000	0.241	0.009	0.270	0.103	0.371	0.250	0.469	0.407	0.597	0.541
200	-0.80	1.000	0.991	0.743	0.385	0.195	0.132	0.098	0.088	0.093	0.101	0.112
	-0.40	0.691	0.134	0.044	0.048	0.053	0.073	0.106	0.137	0.175	0.208	0.245
	0.00	0.024	0.028	0.044	0.071	0.090	0.116	0.155	0.198	0.218	0.259	0.296
	0.40	0.001	0.105	0.029	0.082	0.092	0.140	0.159	0.203	0.238	0.278	0.312
	0.80	0.000	0.224	0.009	0.171	0.041	0.210	0.101	0.252	0.193	0.311	0.279

c) MZ_α^{GLS} ; AR case.

T	ρ	k=0	1	2	3	4	5	6	7	8	9	10
100	-0.80	0.979	0.000	0.003	0.015	0.047	0.101	0.192	0.270	0.343	0.410	0.472
	-0.40	0.415	0.008	0.020	0.085	0.141	0.223	0.275	0.371	0.438	0.494	0.554
	0.00	0.021	0.030	0.050	0.126	0.172	0.254	0.326	0.403	0.445	0.538	0.588
	0.40	0.000	0.046	0.090	0.146	0.205	0.288	0.339	0.420	0.488	0.565	0.597
	0.80	0.000	0.105	0.151	0.219	0.271	0.354	0.413	0.481	0.545	0.620	0.638
200	-0.80	0.992	0.000	0.000	0.004	0.016	0.035	0.068	0.087	0.117	0.145	0.183
	-0.40	0.506	0.019	0.025	0.043	0.065	0.093	0.125	0.163	0.198	0.242	0.276
	0.00	0.029	0.029	0.043	0.073	0.093	0.120	0.156	0.194	0.215	0.254	0.299
	0.40	0.000	0.038	0.055	0.080	0.111	0.151	0.178	0.207	0.240	0.281	0.333
	0.80	0.000	0.066	0.093	0.105	0.139	0.170	0.211	0.242	0.268	0.334	0.351

d) P_T^{GLS} ; AR case.

T	ρ	k=0	1	2	3	4	5	6	7	8	9	10
100	-0.80	0.971	0.000	0.003	0.017	0.056	0.104	0.194	0.273	0.347	0.413	0.474
	-0.40	0.383	0.008	0.026	0.094	0.149	0.226	0.282	0.366	0.439	0.493	0.557
	0.00	0.018	0.033	0.053	0.133	0.179	0.254	0.325	0.399	0.446	0.534	0.590
	0.40	0.000	0.043	0.088	0.151	0.200	0.290	0.336	0.423	0.479	0.561	0.587
	0.80	0.000	0.096	0.130	0.205	0.257	0.348	0.402	0.475	0.532	0.605	0.628
200	-0.80	0.987	0.000	0.001	0.004	0.019	0.036	0.069	0.091	0.124	0.148	0.187
	-0.40	0.470	0.020	0.026	0.046	0.065	0.089	0.127	0.163	0.198	0.237	0.274
	0.00	0.024	0.028	0.044	0.071	0.090	0.116	0.155	0.198	0.218	0.259	0.296
	0.40	0.000	0.035	0.054	0.080	0.104	0.144	0.174	0.208	0.242	0.278	0.329
	0.80	0.000	0.058	0.082	0.096	0.137	0.0170	0.212	0.239	0.261	0.323	0.340

Table 3. Selected values of k using IC and MIC (MZ_{α}^{GLS})

	MA Case						AR Case					
T	θ	k^*	AIC	BIC	MAIC	MBIC	ρ	k^*	AIC	BIC	MAIC	MBIC
100	-0.8	5	2	0	4	3	-0.8	4	1	1	1	1
	-0.4	2	1	0	2	1	-0.4	2	1	1	1	1
	0.0	1	0	0	0	0	0.0	1	0	0	0	0
	0.4	2	2	1	2	1	0.4	1	1	1	1	1
	0.8	4	5	3	5	3	0.8	0	2	1	1	1
200	-0.8	7	4	2	6	4	-0.8	5	1	1	1	1
	-0.4	2	2	1	2	1	-0.4	3	1	1	1	1
	0.0	0	0	0	0	0	0.0	0	0	0	0	0
	0.4	2	2	1	2	1	0.4	1	1	1	1	1
	0.8	4	7	4	6	4	0.8	1	1	1	1	1

Table 4: Finite Sample Critical Values; Model 1, Chosing T_B minimizing the test statistic

a) MZ_{α}^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-7608.23	-515.94	-113.94	-38.39	-26.85	-17.26	-11.18	-9.21	-7.59	-6.72	-6.19
	BIC	-37.74	-33.49	-28.88	-24.45	-20.85	-14.92	-10.31	-8.37	-7.07	-6.25	-5.02
	MAIC	-31.06	-27.87	-25.17	-22.79	-19.84	-14.65	-10.02	-8.45	-7.20	-6.38	-5.70
	MBIC	-31.54	-28.22	-24.99	-22.86	-19.70	-14.26	-9.91	-8.02	-6.84	-6.13	-4.91
	t-sig	-84.47	-58.31	-40.30	-31.49	-24.93	-17.15	-11.19	-9.21	-7.64	-6.76	-5.73
T=150	AIC	-176.15	-73.06	-44.79	-33.36	-25.23	-16.63	-10.98	-8.75	-7.60	-6.78	-5.96
	BIC	-37.89	-33.36	-28.99	-25.24	-21.52	-14.88	-10.31	-8.43	-7.15	-6.41	-5.18
	MAIC	-34.38	-30.01	-27.72	-24.30	-20.95	-15.05	-10.34	-8.38	-7.26	-6.68	-5.69
	MBIC	-34.30	-30.38	-27.25	-24.26	-20.77	-14.56	-10.06	-8.27	-7.02	-6.01	-4.82
	t-sig	-53.46	-39.59	-35.28	-29.70	-23.75	-16.20	-10.94	-8.94	-8.05	-6.85	-5.87
T=200	AIC	-133.76	-67.31	-42.94	-30.82	-24.66	-16.13	-10.70	-8.47	-7.14	-6.22	-5.01
	BIC	-36.94	-31.99	-29.22	-25.63	-21.21	-14.85	-9.87	-8.18	-6.69	-5.18	-4.50
	MAIC	-36.63	-31.33	-27.35	-24.28	-20.79	-14.64	-10.18	-8.15	-6.88	-5.83	-4.75
	MBIC	-35.13	-31.39	-28.08	-24.23	-20.69	-14.48	-9.74	-7.99	-6.59	-5.05	-4.43
	t-sig	-60.01	-43.31	-35.37	-30.49	-24.32	-16.44	-10.70	-8.52	-7.16	-5.82	-4.94
T=∞		-40.88	-35.48	-31.63	-27.46	-22.50	-15.12	-9.52	-7.46	-6.00	-4.97	-4.09

b) MSB^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	0.0081	0.0311	0.0662	0.1140	0.1355	0.1679	0.2053	0.2269	0.2455	0.2608	0.2768
	BIC	0.1145	0.1214	0.1306	0.1412	0.1530	0.1807	0.2152	0.2400	0.2583	0.2713	0.2954
	MAIC	0.1263	0.1324	0.1407	0.1468	0.1562	0.1822	0.2170	0.2380	0.2549	0.2675	0.2819
	MBIC	0.1258	0.1321	0.1405	0.1466	0.1569	0.1841	0.2181	0.2424	0.2616	0.2761	0.3041
	t-sig	0.0767	0.0926	0.1110	0.1252	0.1404	0.1684	0.2064	0.2264	0.2439	0.2630	0.2822
T=150	AIC	0.0533	0.0826	0.1057	0.1220	0.1395	0.1708	0.2080	0.2312	0.2453	0.2585	0.2713
	BIC	0.1148	0.1206	0.1294	0.1393	0.1506	0.1794	0.2147	0.2365	0.2529	0.2686	0.2914
	MAIC	0.1197	0.1285	0.1339	0.1423	0.1527	0.1788	0.2155	0.2387	0.2533	0.2665	0.2847
	MBIC	0.1196	0.1283	0.1349	0.1428	0.1533	0.1829	0.2166	0.2385	0.2554	0.2742	0.2931
	t-sig	0.0966	0.1124	0.1186	0.1284	0.1435	0.1729	0.2098	0.2280	0.2413	0.2570	0.2815
T=200	AIC	0.0611	0.0860	0.1077	0.1272	0.1417	0.1726	0.2106	0.2357	0.2588	0.2708	0.2891
	BIC	0.1163	0.1238	0.1296	0.1391	0.1515	0.1802	0.2169	0.2407	0.2606	0.2934	0.3234
	MAIC	0.1168	0.1259	0.1342	0.1422	0.1542	0.1809	0.2156	0.2395	0.2603	0.2783	0.2973
	MBIC	0.1188	0.1255	0.1323	0.1420	0.1545	0.1824	0.2192	0.2434	0.2621	0.2973	0.3240
	t-sig	0.0913	0.1074	0.1184	0.1275	0.1423	0.1712	0.2099	0.2354	0.2564	0.2682	0.3048
T=∞		0.1096	0.1179	0.1250	0.1338	0.1473	0.1790	0.2227	0.2495	0.2754	0.2985	0.3239

c) MZ_{t}^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-61.67	-16.05	-7.54	-4.37	-3.62	-2.89	-2.31	-2.09	-1.91	-1.77	-1.64
	BIC	-4.33	-4.07	-3.79	-3.48	-3.20	-2.69	-2.21	-1.99	-1.81	-1.70	-1.52
	MAIC	-3.92	-3.70	-3.50	-3.34	-3.11	-2.66	-2.19	-2.01	-1.84	-1.73	-1.62
	MBIC	-3.94	-3.75	-3.51	-3.34	-3.10	-2.63	-2.17	-1.92	-1.80	-1.66	-1.47
	t-sig	-6.48	-5.39	-4.47	-3.95	-3.50	-2.89	-2.30	-2.11	-1.89	-1.79	-1.62
T=150	AIC	-9.38	-6.03	-4.73	-4.05	-3.53	-2.85	-2.28	-2.05	-1.87	-1.79	-1.66
	BIC	-4.33	-4.00	-3.79	-3.53	-3.25	-2.67	-2.22	-1.99	-1.82	-1.72	-1.54
	MAIC	-4.11	-3.87	-3.70	-3.47	-3.20	-2.69	-2.22	-1.99	-1.84	-1.76	-1.59
	MBIC	-4.10	-3.87	-3.65	-3.47	-3.19	-2.66	-2.20	-1.96	-1.79	-1.70	-1.50
	t-sig	-5.16	-4.44	-4.19	-3.84	-3.41	-2.80	-2.28	-2.07	-1.91	-1.80	-1.66
T=200	AIC	-8.17	-5.79	-4.62	-3.89	-3.48	-2.81	-2.26	-1.97	-1.84	-1.73	-1.48
	BIC	-4.29	-3.98	-3.78	-3.55	-3.23	-2.68	-2.17	-1.94	-1.76	-1.57	-1.44
	MAIC	-4.25	-3.94	-3.68	-3.45	-3.19	-2.67	-2.20	-1.94	-1.80	-1.66	-1.47
	MBIC	-4.18	-3.93	-3.71	-3.45	-3.17	-2.65	-2.15	-1.93	-1.74	-1.56	-1.42
	t-sig	-5.47	-4.65	-4.19	-3.87	-3.46	-2.83	-2.27	-2.00	-1.84	-1.66	-1.53
T=∞		-4.49	-4.18	-3.96	-3.68	-3.32	-2.71	-2.12	-1.87	-1.66	-1.51	-1.35

Table 4 (cont'd): Finite Sample Critical Values; Model 1, Chosing T_B minimizing the test statistic

d) P_T^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	0.03	0.53	2.36	6.92	10.24	15.81	24.43	29.44	35.17	39.32	44.20
	BIC	7.20	8.19	9.54	11.22	13.11	18.45	26.88	33.65	38.41	41.19	50.77
	MAIC	8.77	9.89	11.04	11.96	13.67	18.58	27.18	32.73	37.35	40.59	45.68
	MBIC	8.71	9.73	10.95	12.04	13.85	19.23	27.43	34.73	39.32	44.20	54.49
	t-sig	3.19	4.61	6.72	8.51	10.99	15.97	24.53	29.44	35.27	40.21	45.47
T=150	AIC	1.51	3.61	5.91	8.10	10.81	16.59	24.51	30.56	35.74	38.98	43.96
	BIC	7.21	8.50	9.45	10.82	12.82	18.30	26.80	31.69	37.99	41.33	51.18
	MAIC	7.93	8.91	9.92	11.02	13.09	18.23	26.68	32.14	37.95	40.40	46.65
	MBIC	8.26	8.99	10.12	11.18	13.28	18.90	27.32	32.35	38.70	42.83	53.02
	t-sig	4.96	6.72	7.57	9.15	11.46	16.92	24.87	30.41	34.52	38.90	46.47
T=200	AIC	2.06	3.94	6.28	8.92	11.05	16.88	25.23	32.08	38.05	43.74	53.01
	BIC	7.19	8.58	9.41	10.73	12.96	18.37	27.38	33.52	40.38	51.91	61.75
	MAIC	7.55	8.78	9.93	11.34	13.05	18.64	26.69	33.49	38.75	46.11	54.40
	MBIC	7.78	8.80	9.90	11.44	13.36	18.85	27.75	34.23	40.49	53.01	61.75
	t-sig	4.37	6.25	7.58	9.02	11.27	16.60	25.37	31.33	38.16	44.11	54.71
T=∞		6.59	7.69	8.52	9.82	11.96	17.73	27.59	34.65	42.57	49.76	58.73

e) ADF^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-5.04	-4.79	-4..57	-4.26	-3.82	-3.17	-2.55	-2.31	-2.11	-1.99	-1.87
	BIC	-5.00	-4.67	-4.34	-3.94	-3.56	-2.94	-2.41	-2.16	-1.98	-1.87	-1.69
	MAIC	-4.62	-4.26	-3.91	-3.67	-3.33	-2.76	-2.27	-2.06	-1.92	-1.82	-1.67
	MBIC	-4.62	-4.25	-3.92	-3.68	-3.34	-2.78	-2.29	-2.04	-1.89	-1.73	-1.57
	t-sig	-5.00	-4.69	-4.43	-4.12	-3.77	-3.14	-2.51	-2.27	-2.05	-1.93	-1.85
T=150	AIC	-4.95	-4.64	-4.37	-4.04	-3.66	-3.05	-2.46	-2.22	-2.06	-1.90	-1.80
	BIC	-4.72	-4.38	-4.18	-3.83	-3.49	-2.83	-2.31	-2.09	-1.93	-1.83	-1.70
	MAIC	-4.52	-4.21	-3.94	-3.64	-3.29	-2.72	-2.24	-2.05	-1.89	-1.80	-1.65
	MBIC	-4.52	-4.21	-3.94	-3.66	-3.32	-2.74	-2.23	-2.02	-1.85	-1.76	-1.53
	t-sig	-4.80	-4.61	-4.28	-3.95	-3.60	-2.99	-2.39	-2.16	-2.05	-1.89	-1.80
T=200	AIC	-4.93	-4.46	-4.21	-3.92	-3.58	-2.93	-2.36	-2.08	-1.90	-1.81	-1.67
	BIC	-4.72	-4.26	-4.06	-3.74	-3.40	-2.79	-2.25	-2.00	-1.83	-1.67	-1.50
	MAIC	-4.47	-4.06	-3.77	-3.50	-3.22	-2.67	-2.20	-1.99	-1.83	-1.73	-1.52
	MBIC	-4.47	-4.17	-3.81	-3.52	-3.24	-2.68	-2.20	-1.97	-1.79	-1.63	-1.48
	t-sig	-4.90	-4.46	-4.19	-3.91	-3.71	-2.92	-2.35	-2.08	-1.89	-1.76	-1.62
T=∞		-4.49	-4.18	-3.96	-3.68	-3.32	-2.71	-2.12	-1.87	-1.66	-1.51	-1.35

Table 5: Finite Sample Critical Values; Model 1, Chosing T_B maximizing $|t_{\widehat{\beta}_2}|$

a) MZ_{α}^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-83.53	-38.75	-29.23	-23.88	-20.11	-13.95	-9.13	-7.29	-6.13	-4.93	-3.95
	BIC	-34.40	-29.93	-26.96	-23.31	-19.85	-14.24	-9.73	-7.74	-6.56	-5.70	-4.20
	MAIC	-27.16	-24.14	-22.79	-20.21	-17.32	-12.46	-8.30	-6.69	-5.52	-4.42	-3.47
	MBIC	-29.58	-26.96	-23.82	-21.58	-18.35	-13.19	-9.20	-7.31	-6.34	-5.41	-4.12
	t-sig	-40.72	-34.53	-29.42	-24.43	-20.24	-14.17	-9.31	-7.49	-6.44	-5.52	-4.27
T=150	AIC	-65.38	-36.68	-30.42	-25.65	-21.41	-14.56	-9.62	-7.76	-6.69	-5.86	-4.82
	BIC	-35.17	-31.12	-27.44	-24.24	-20.72	-14.38	-9.86	-8.09	-6.81	-5.86	-4.82
	MAIC	-29.23	-26.63	-24.47	-21.73	-18.77	-13.16	-8.89	-7.36	-6.37	-5.32	-4.56
	MBIC	-32.27	-27.55	-25.71	-23.06	-19.70	-13.91	-9.58	-7.91	-6.65	-5.62	-4.62
	t-sig	-43.21	-33.98	-29.08	-24.59	-20.73	-14.41	-9.80	-8.24	-6.86	-5.82	-4.79
T=200	AIC	-55.64	-35.10	-29.97	-25.11	-21.12	-14.35	-9.52	-7.51	-6.24	-5.21	-4.28
	BIC	-35.10	-31.11	-28.28	-23.97	-20.47	-14.33	-9.63	-7.84	-6.27	-4.92	-4.05
	MAIC	-31.11	-28.26	-25.11	-21.88	-18.60	-13.22	-9.16	-7.18	-5.99	-5.12	-4.05
	MBIC	-32.97	-30.64	-26.70	-22.88	-19.61	-13.89	-9.44	-7.64	-6.27	-4.92	-4.05
	t-sig	-39.66	-33.50	-29.00	-25.17	-21.56	-14.51	-9.57	-7.64	-6.27	-5.18	-4.17
T=∞		-41.00	-34.96	-30.74	-26.40	-21.76	-14.65	-9.45	-7.45	-5.95	-4.88	-3.81

b) MSB^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	0.0774	0.1133	0.1300	0.1434	0.1554	0.1855	0.2257	0.2527	0.2693	0.2957	0.3323
	BIC	0.1197	0.1285	0.1345	0.1455	0.1559	0.1843	0.2214	0.2463	0.2622	0.2771	0.3063
	MAIC	0.1344	0.1424	0.1474	0.1555	0.1681	0.1970	0.2402	0.2639	0.2848	0.3098	0.3514
	MBIC	0.1289	0.1349	0.1437	0.1502	0.1625	0.1905	0.2271	0.2521	0.2683	0.2848	0.3098
	t-sig	0.1106	0.1197	0.1303	0.1419	0.1552	0.1843	0.2249	0.2492	0.2644	0.2807	0.3081
T=150	AIC	0.0874	0.1165	0.1277	0.1382	0.1514	0.1829	0.2212	0.2459	0.2614	0.2730	0.2919
	BIC	0.1188	0.1249	0.1342	0.1427	0.1537	0.1831	0.2191	0.2412	0.2570	0.2754	0.2956
	MAIC	0.1307	0.1367	0.1420	0.1496	0.1618	0.1919	0.2291	0.2521	0.2687	0.2818	0.3075
	MBIC	0.1233	0.1326	0.1387	0.1450	0.1578	0.1871	0.2221	0.2439	0.2617	0.2808	0.3012
	t-sig	0.1071	0.1204	0.1308	0.1416	0.1532	0.1831	0.2201	0.2400	0.2560	0.2754	0.2938
T=200	AIC	0.0948	0.1192	0.1283	0.1402	0.1520	0.1836	0.2220	0.2500	0.2706	0.2915	0.3060
	BIC	0.1192	0.1259	0.1323	0.1429	0.1549	0.1832	0.2211	0.2458	0.2675	0.2997	0.3246
	MAIC	0.1262	0.1326	0.1401	0.1502	0.1614	0.1917	0.2277	0.2566	0.2757	0.2974	0.3112
	MBIC	0.1225	0.1276	0.1359	0.1465	0.1584	0.1862	0.2245	0.2471	0.2683	0.2997	0.3246
	t-sig	0.1122	0.1220	0.1306	0.1399	0.1512	0.1823	0.2202	0.2464	0.2676	0.2899	0.3171
T=∞		0.1098	0.1187	0.1266	0.1365	0.1498	0.1821	0.2236	0.2498	0.2751	0.2991	0.3339

c) MZ_t^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-6.46	-4.38	-3.82	-3.43	-3.14	-2.59	-2.07	-1.85	-1.66	-1.45	-1.21
	BIC	-4.11	-3.84	-3.65	-3.38	-3.11	-2.62	-2.14	-1.90	-1.74	-1.53	-1.32
	MAIC	-3.68	-3.45	-3.34	-3.14	-2.91	-2.46	-1.99	-1.75	-1.55	-1.37	-1.17
	MBIC	-3.83	-3.65	-3.41	-3.26	-3.00	-2.53	-2.09	-1.87	-1.70	-1.51	-1.29
	t-sig	-4.49	-4.13	-3.82	-3.47	-3.14	-2.62	-2.11	-1.88	-1.72	-1.53	-1.32
T=150	AIC	-5.71	-4.27	-3.89	-3.56	-3.23	-2.65	-2.13	-1.91	-1.76	-1.56	-1.36
	BIC	-4.17	-3.95	-3.68	-3.47	-3.18	-2.63	-2.17	-1.95	-1.76	-1.56	-1.34
	MAIC	-3.82	-3.62	-3.49	-3.26	-3.02	-2.53	-2.05	-1.85	-1.70	-1.51	-1.34
	MBIC	-3.98	-3.69	-3.58	-3.37	-3.09	-2.60	-2.13	-1.92	-1.73	-1.53	-1.34
	t-sig	-4.62	-4.09	-3.76	-3.49	-3.18	-2.64	-2.16	-1.95	-1.77	-1.57	-1.36
T=200	AIC	-5.27	-4.15	-3.85	-3.52	-3.22	-2.62	-2.12	-1.85	-1.66	-1.49	-1.28
	BIC	-4.18	-3.92	-3.73	-3.43	-3.17	-2.63	-2.13	-1.89	-1.67	-1.45	-1.18
	MAIC	-3.92	-3.73	-3.50	-3.29	-3.03	-2.52	-2.07	-1.82	-1.65	-1.47	-1.24
	MBIC	-4.05	-3.89	-3.62	-3.36	-3.10	-2.59	-2.10	-1.87	-1.67	-1.45	-1.18
	t-sig	-4.45	-4.08	-3.78	-3.52	-3.25	-2.64	-2.12	-1.86	-1.67	-1.52	-1.31
T=∞		-4.50	-4.16	-3.89	-3.61	-3.27	-2.67	-2.12	-1.86	-1.64	-1.44	-1.23

Table 5 (cont'd): Finite Sample Critical Values; Model 1, Chosing T_B maximizing $|t_{\widehat{\beta}_2}|$

d) P_T^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	3.10	6.85	9.13	11.37	13.58	19.57	29.58	36.35	42.43	50.46	68.14
	BIC	8.02	9.03	10.18	11.82	13.70	19.27	27.95	34.78	39.98	44.34	56.24
	MAIC	10.12	11.25	12.15	13.60	15.63	21.75	32.63	39.61	47.67	58.28	73.74
	MBIC	9.13	10.71	11.43	12.64	14.62	20.51	29.33	36.22	40.59	46.30	57.67
	t-sig	6.50	7.69	9.22	11.14	13.61	19.22	29.14	35.28	40.37	47.54	57.98
T=150	AIC	4.02	7.25	8.82	10.73	12.97	18.95	28.33	35.03	40.24	44.36	53.21
	BIC	7.78	8.76	9.85	11.24	13.43	19.11	27.61	32.84	39.13	43.20	53.21
	MAIC	9.32	10.25	11.10	12.77	14.63	20.62	29.79	36.65	41.15	47.61	54.93
	MBIC	8.64	9.75	10.63	11.81	13.87	19.57	28.31	34.31	39.48	45.26	53.25
	t-sig	6.19	8.10	9.52	10.92	13.32	19.04	27.75	32.84	39.03	43.75	52.96
T=200	AIC	4.90	7.78	9.27	10.88	13.10	19.00	28.51	36.14	42.20	50.31	56.81
	BIC	7.64	8.80	9.74	11.42	13.46	19.00	28.24	34.90	41.11	53.40	62.41
	MAIC	8.72	9.65	10.98	12.52	14.62	20.73	29.53	37.53	43.70	51.36	58.84
	MBIC	8.19	9.11	10.32	11.97	13.90	19.59	28.87	35.26	41.31	53.40	62.41
	t-sig	6.73	8.07	9.36	10.98	12.86	18.81	28.09	35.01	42.20	48.33	58.87
T=∞		6.79	7.85	8.92	10.34	12.56	18.46	28.10	34.97	42.67	50.46	62.11

e) ADF^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-4.97	-4.64	-4.37	-4.04	-3.64	-2.98	-2.31	-2.08	-1.86	-1.73	-1.49
	BIC	-4.95	-4.57	-4.19	-3.85	-3.49	-2.85	-2.29	-2.04	-1.82	-1.60	-1.39
	MAIC	-4.54	-4.19	-3.82	-3.56	-3.22	-2.64	-2.10	-1.88	-1.71	-1.49	-1.28
	MBIC	-4.54	-4.19	-3.84	-3.60	-3.25	-2.67	-2.15	-1.92	-1.73	-1.54	-1.29
	t-sig	-4.91	-4.60	-4.33	-3.99	-3.64	-2.99	-2.36	-2.10	-1.87	-1.72	-1.46
T=150	AIC	-4.78	-4.53	-4.22	-3.89	-3.52	-2.88	-2.31	-2.06	-1.83	-1.64	-1.41
	BIC	-4.67	-4.34	-4.05	-3.74	-3.43	-2.77	-2.26	-2.02	-1.82	-1.60	-1.37
	MAIC	-4.43	-4.07	-3.85	-3.55	-3.20	-2.61	-2.12	-1.88	-1.72	-1.54	-1.39
	MBIC	-4.42	-4.08	-3.87	-3.60	-3.23	-2.65	-2.16	-1.94	-1.75	-1.54	-1.36
	t-sig	-4.70	-4.48	-4.22	-3.87	-3.51	-2.88	-2.28	-2.06	-1.87	-1.70	-1.48
T=200	AIC	-4.82	-4.38	-4.12	-3.80	-3.47	-2.81	-2.24	-1.95	-1.76	-1.58	-1.35
	BIC	-4.67	-4.22	-4.00	-3.65	-3.33	-2.73	-2.19	-1.95	-1.73	-1.50	-1.25
	MAIC	-4.39	-4.00	-3.65	-3.44	-3.11	-2.59	-2.09	-1.86	-1.67	-1.49	-1.25
	MBIC	-4.41	-4.05	-3.68	-3.47	-3.16	-2.63	-2.12	-1.90	-1.69	-1.46	-1.25
	t-sig	-4.79	-4.36	-4.06	-3.73	-3.45	-2.82	-2.22	-1.97	-1.77	-1.55	-1.35
T=∞		-4.50	-4.16	-3.89	-3.61	-3.27	-2.67	-2.12	-1.86	-1.64	-1.44	-1.23

Table 6: Finite Sample Critical Values; Model 2, Chosing T_B minimizing the test statistic

a) MZ_α^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-59308.98	-1952.48	-217.19	-55.98	-31.38	-19.82	-13.63	-11.38	-10.04	-9.06	-7.91
	BIC	-39.68	-34.48	-29.93	-26.72	-22.29	-16.61	-11.73	-9.77	-8.40	-7.26	-6.43
	MAIC	-34.04	-30.66	-28.24	-24.48	-21.78	-16.38	-11.81	-10.11	-8.96	-8.28	-6.58
	MBIC	-31.24	-28.80	-27.04	-23.90	-20.96	-15.73	-11.50	-9.53	-8.15	-6.92	-6.04
	t-sig	-104.55	-66.24	-46.00	-36.03	-28.50	-19.52	-13.32	-11.10	-9.70	-8.63	-6.93
T=150	AIC	-656.63	-107.04	-58.51	-37.62	-27.88	-18.58	-12.36	-10.25	-8.74	-7.82	-6.60
	BIC	-39.80	-34.12	-30.37	-26.73	-22.76	-15.99	-11.15	-9.11	-7.98	-6.74	-5.21
	MAIC	-35.36	-32.25	-28.89	-25.82	-22.38	-16.33	-11.40	-9.34	-8.08	-7.44	-6.21
	MBIC	-34.28	-30.38	-27.99	-25.39	-21.76	-15.41	-10.99	-8.90	-7.88	-6.48	-5.19
	t-sig	-59.30	-45.13	-37.90	-32.02	-26.05	-18.031	-12.17	-10.27	-8.81	-7.83	-7.00
T=200	AIC	-235.34	-77.82	-51.48	-33.92	-26.87	-17.69	-11.75	-9.52	-7.75	-6.84	-5.54
	BIC	-39.99	-33.51	-30.53	-26.60	-22.26	-15.75	-10.37	-8.49	-7.14	-5.65	-4.73
	MAIC	-37.61	-32.42	-29.01	-25.79	-22.10	-15.89	-10.87	-8.91	-7.62	-6.68	-5.54
	MBIC	-36.71	-32.52	-29.20	-25.76	-21.42	-15.28	-10.26	-8.48	-7.06	-5.54	-4.62
	t-sig	-61.78	-48.12	-40.62	-33.09	-26.56	-17.67	-11.74	-9.71	-8.11	-6.75	-5.13
T=∞		-40.88	-35.48	-31.63	-27.46	-22.50	-15.12	-9.52	-7.46	-6.00	-4.97	-4.09

b) MSB^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	0.0029	0.0160	0.0480	0.0945	0.1254	0.1564	0.1874	0.2050	0.2168	0.2270	0.2423
	BIC	0.1122	0.1193	0.1287	0.1363	0.1479	0.1709	0.2013	0.2198	0.2380	0.2512	0.2680
	MAIC	0.1196	0.1265	0.1322	0.1419	0.1498	0.1714	0.2005	0.2161	0.2309	0.2417	0.2563
	MBIC	0.1244	0.1301	0.1353	0.1434	0.1524	0.1757	0.2050	0.2238	0.2416	0.2456	0.2752
	t-sig	0.0692	0.0869	0.1034	0.1172	0.1318	0.1583	0.1899	0.2065	0.2240	0.2346	0.2518
T=150	AIC	0.0276	0.0681	0.0924	0.1147	0.1326	0.1618	0.1956	0.2142	0.2312	0.2426	0.2622
	BIC	0.1120	0.1199	0.1277	0.1362	0.1460	0.1741	0.2068	0.2259	0.2402	0.2556	0.2847
	MAIC	0.1173	0.1231	0.1304	0.1381	0.1476	0.1725	0.2041	0.2250	0.2392	0.2503	0.2689
	MBIC	0.1187	0.1275	0.1328	0.1393	0.1496	0.1766	0.2084	0.2272	0.2425	0.2666	0.2915
	t-sig	0.0918	0.1052	0.1148	0.1244	0.1379	0.1624	0.1974	0.2150	0.2294	0.2417	0.2571
T=200	AIC	0.0461	0.0801	0.0985	0.1209	0.1353	0.1660	0.2002	0.2221	0.2467	0.2591	0.2789
	BIC	0.1116	0.1213	0.1276	0.1363	0.1479	0.1751	0.2118	0.2348	0.2538	0.2814	0.3081
	MAIC	0.1153	0.1230	0.1299	0.1385	0.1489	0.1739	0.2081	0.2294	0.2495	0.2628	0.2789
	MBIC	0.1164	0.1230	0.1287	0.1385	0.1512	0.1775	0.2137	0.2349	0.2552	0.2871	0.3117
	t-sig	0.0900	0.1017	0.1107	0.1223	0.1363	0.1657	0.1998	0.2209	0.2400	0.2570	0.2891
T=∞		0.1096	0.1179	0.1250	0.1338	0.1473	0.1790	0.2227	0.2495	0.2754	0.2985	0.3239

c) MZ_t^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-172.20	-31.24	-10.42	-5.29	-3.93	-3.12	-2.56	-2.33	-2.19	-2.06	-1.94
	BIC	-4.45	-4.11	-3.85	-3.61	-3.31	-2.83	-2.37	-2.15	-2.00	-1.81	-1.69
	MAIC	-4.12	-3.90	-3.75	-3.46	-3.27	-2.82	-2.39	-2.19	-2.06	-1.95	-1.74
	MBIC	-3.93	-3.78	-3.60	-3.42	-3.21	-2.77	-2.33	-2.12	-1.96	-1.79	-1.68
	t-sig	-7.23	-5.75	-4.79	-4.22	-3.75	-3.09	-2.54	-2.28	-2.12	-2.01	-1.80
T=150	AIC	-18.11	-7.29	-5.40	-4.31	-3.71	-3.01	-2.45	-2.19	-2.03	-1.90	-1.78
	BIC	-4.46	-4.13	-3.88	-3.63	-3.34	-2.79	-2.30	-2.07	-1.91	-1.76	-1.54
	MAIC	-4.19	-3.98	-3.78	-3.58	-3.30	-2.82	-2.34	-2.10	-1.95	-1.85	-1.74
	MBIC	-4.13	-3.88	-3.72	-3.53	-3.27	-2.73	-2.29	-2.05	-1.90	-1.75	-1.53
	t-sig	-5.44	-4.74	-4.34	-3.96	-3.58	-2.97	-2.42	-2.20	-2.05	-1.88	-1.79
T=200	AIC	-10.84	-6.23	-5.06	-4.10	-3.64	-2.94	-2.37	-2.12	-1.92	-1.79	-1.60
	BIC	-4.46	-4.04	-3.90	-3.62	-3.31	-2.76	-2.23	-1.99	-1.83	-1.62	-1.52
	MAIC	-4.33	-3.98	-3.76	-3.57	-3.30	-2.79	-2.28	-2.06	-1.89	-1.76	-1.58
	MBIC	-4.27	-4.02	-3.79	-3.56	-3.24	-2.73	-2.21	-1.98	-1.79	-1.61	-1.48
	t-sig	-5.55	-4.89	-4.50	-4.05	-3.62	-2.94	-2.38	-2.14	-1.93	-1.73	-1.57
T=∞		-4.49	-4.18	-3.96	-3.68	-3.32	-2.71	-2.12	-1.87	-1.66	-1.51	-1.35

Table 6 (cont'd): Finite Sample Critical Values; Model 2, Chosing T_B minimizing the test statistic

d) P_T^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	0.00	0.14	1.23	4.90	8.82	14.18	20.95	25.13	28.49	33.01	37.90
	BIC	6.73	7.93	9.13	10.40	12.26	16.99	24.31	30.03	34.93	39.33	44.15
	MAIC	8.20	9.14	9.90	11.25	12.81	17.22	24.01	28.51	32.76	37.60	41.88
	MBIC	8.86	9.47	10.55	11.62	13.33	18.04	25.16	30.73	36.41	39.60	47.77
	t-sig	2.56	3.92	5.79	7.63	9.63	14.38	21.88	25.79	30.51	35.43	40.08
T=150	AIC	0.41	2.70	4.62	7.31	9.93	15.01	22.83	27.15	32.15	36.19	40.28
	BIC	6.87	8.00	9.13	10.41	12.11	17.44	25.13	30.19	35.15	40.28	49.64
	MAIC	7.56	8.56	9.39	10.65	12.41	17.14	24.42	29.86	34.68	37.81	43.94
	MBIC	8.01	8.99	9.88	10.84	12.75	18.03	25.83	31.01	35.99	41.35	51.22
	t-sig	4.44	5.99	7.23	8.53	10.58	15.38	23.12	27.25	32.07	34.92	40.61
T=200	AIC	1.12	3.41	5.20	7.99	10.21	15.76	23.49	29.38	36.10	40.64	51.32
	BIC	6.88	8.11	8.95	10.29	12.40	17.57	16.40	31.78	38.41	48.44	58.57
	MAIC	7.11	8.55	9.66	10.65	12.44	17.39	25.26	31.11	36.55	40.74	51.32
	MBIC	7.48	8.48	9.57	10.73	12.98	18.09	27.05	32.19	38.64	49.74	60.45
	t-sig	4.32	5.60	6.56	8.37	10.28	15.54	23.88	28.86	33.38	40.57	52.61
T=∞		6.59	7.69	8.52	9.82	11.96	17.73	27.59	34.65	42.57	49.76	58.73

e) ADF^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-5.30	-4.98	-4.74	-4.43	-4.06	-3.41	-2.82	-2.60	-2.47	-2.35	-2.17
	BIC	-5.07	-4.80	-4.49	-4.23	-3.77	-3.17	-2.61	-2.38	-2.18	-2.03	-1.86
	MAIC	-4.69	-4.43	-4.16	-3.85	-3.52	-2.94	-2.46	-2.27	-2.12	-1.98	-1.85
	MBIC	-4.73	-4.42	-4.13	-3.86	-3.54	-2.95	-2.47	-2.24	-2.07	-1.90	-1.74
	t-sig	-5.18	-4.89	-4.64	-4.36	-3.95	-3.36	-2.81	-2.54	-2.39	-2.22	-2.05
T=150	AIC	-5.03	-4.82	-4.56	-4.23	-3.89	-3.19	-2.61	-2.38	-2.20	-2.08	-1.93
	BIC	-4.82	-4.48	-4.27	-3.99	-3.62	-2.98	-2.43	-2.19	-2.06	-1.91	-1.73
	MAIC	-4.62	-4.28	-4.02	-3.77	-3.40	-2.83	-2.36	-2.16	-2.00	-1.89	-1.76
	MBIC	-4.62	-4.27	-4.03	-3.80	-3.42	-2.84	-2.34	-2.13	-1.96	-1.83	-1.60
	t-sig	-4.98	-4.69	-4.46	-4.14	-3.79	-3.15	-2.55	-2.32	-2.18	-2.02	-1.86
T=200	AIC	-5.03	-4.73	-4.39	-4.10	-3.77	-3.06	-2.47	-2.20	-2.01	-1.88	-1.75
	BIC	-4.89	-4.44	-4.20	-3.85	-3.51	-2.90	-2.31	-2.08	-1.88	-1.69	-1.56
	MAIC	-4.48	-4.22	-3.89	-3.60	-3.31	-2.77	-2.28	-2.06	-1.89	-1.79	-1.65
	MBIC	-4.50	-4.28	-3.96	-3.65	-3.35	-2.78	-2.25	-2.04	-1.86	-1.67	-1.51
	t-sig	-5.00	-4.51	-4.32	-4.09	-3.70	-3.05	-2.46	-2.23	-2.05	-1.87	-1.68
T=∞		-4.49	-4.18	-3.96	-3.68	-3.32	-2.71	-2.12	-1.87	-1.66	-1.51	-1.35

Table 7: Finite Sample Critical Values; Model 2, Chosing T_B maximizing $|t_{\widehat{\beta}_2}|$

a) MZ_α^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-210.88	-40.93	-29.41	-24.86	-20.94	-14.38	-9.15	-7.28	-6.15	-5.07	-4.05
	BIC	-33.32	-29.05	-26.95	-13.19	-19.87	-14.21	-9.82	-7.85	-6.64	-5.94	-4.82
	MAIC	-26.83	-24.32	-22.53	-19.89	-17.11	-12.51	-8.42	-6.82	-5.91	-4.58	-3.87
	MBIC	-27.34	-25.56	-23.32	-21.16	-18.07	-13.31	-9.36	-7.39	-6.45	-5.80	-4.90
	t-sig	-53.91	-35.70	-29.30	-25.41	-21.31	-14.73	-9.72	-7.81	-6.71	-5.91	-4.59
T=150	AIC	-67.55	-38.00	-31.38	-25.84	-21.50	-14.75	-9.78	-7.96	-6.82	-5.80	-4.96
	BIC	-34.49	-28.97	-27.17	-23.75	-20.28	-14.20	-9.81	-8.06	-6.82	-5.89	-4.86
	MAIC	-28.71	-26.46	-24.12	-21.45	-18.49	-12.92	-8.84	-7.36	-6.23	-5.47	-4.51
	MBIC	-30.21	-27.89	-24.86	-22.26	-19.13	-13.63	-9.35	-7.84	-6.54	-5.66	-4.77
	t-sig	-39.38	-34.44	-29.00	-25.28	-21.07	-14.57	-9.92	-8.15	-6.91	-5.92	-4.81
T=200	AIC	-56.97	-37.78	-29.24	-24.98	-21.29	-14.47	-9.55	-7.53	-6.39	-5.30	-4.35
	BIC	-33.15	-30.16	-27.38	-23.51	-19.86	-14.20	-9.52	-7.74	-6.33	-4.94	-4.17
	MAIC	-31.57	-27.62	-24.27	-21.72	-18.40	-13.19	-8.93	-7.09	-5.97	-5.08	-4.17
	MBIC	-32.76	-29.47	-25.24	-22.10	-18.91	-13.77	-9.34	-7.63	-6.06	-4.94	-4.14
	t-sig	-41.20	-35.10	-29.58	-25.16	-21.62	-14.60	-9.79	-8.03	-6.49	-5.13	-4.25
T=∞		-41.00	-34.96	-30.74	-26.40	-21.76	-14.65	-9.45	-7.45	-5.95	-4.88	-3.81

b) MSB^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	0.0487	0.1105	0.1290	0.1407	0.1529	0.1830	0.2261	0.2535	0.2694	0.2940	0.3197
	BIC	0.1218	0.1305	0.1348	0.1451	0.1569	0.1843	0.2193	0.2454	0.2627	0.2758	0.2996
	MAIC	0.1356	0.1422	0.1468	0.1568	0.1685	0.1957	0.2374	0.2617	0.2777	0.3009	0.3402
	MBIC	0.1331	0.1389	0.1449	0.1527	0.1643	0.1901	0.2258	0.2521	0.2660	0.2817	0.3071
	t-sig	0.0963	0.1182	0.1302	0.1393	0.1524	0.1813	0.2216	0.2460	0.2622	0.2772	0.2959
T=150	AIC	0.0860	0.1145	0.1257	0.1383	0.1511	0.1809	0.2216	0.2424	0.2614	0.2771	0.2931
	BIC	0.1198	0.1300	0.1345	0.1439	0.1555	0.1846	0.2204	0.2415	0.2585	0.2745	0.2931
	MAIC	0.1318	0.1367	0.1433	0.1512	0.1620	0.1937	0.2314	0.2521	0.2710	0.2865	0.3033
	MBIC	0.1279	0.1332	0.1406	0.1481	0.1559	0.1883	0.2245	0.2463	0.2630	0.2825	0.3001
	t-sig	0.1127	0.1198	0.1303	0.1394	0.1522	0.1825	0.2186	0.2394	0.2561	0.2752	0.2993
T=200	AIC	0.0936	0.1150	0.1299	0.1403	0.1520	0.1833	0.2226	0.2509	0.2684	0.2908	0.3044
	BIC	0.1218	0.1282	0.1338	0.1456	0.1579	0.1845	0.2217	0.2471	0.2675	0.2985	0.3269
	MAIC	0.1249	0.1328	0.1415	0.1504	0.1631	0.1920	0.2298	0.2570	0.2748	0.2935	0.3126
	MBIC	0.1226	0.1292	0.1393	0.1490	0.1611	0.1878	0.2260	0.2504	0.2702	0.3005	0.3269
	t-sig	0.1096	0.1192	0.1295	0.1401	0.1505	0.1816	0.2196	0.2423	0.2670	0.2908	0.3173
T=∞		0.1098	0.1187	0.1266	0.1365	0.1498	0.1821	0.2236	0.2498	0.2751	0.2991	0.3339

c) MZ_t^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-10.26	-4.52	-3.81	-3.50	-3.19	-2.64	-2.10	-1.85	-1.66	-1.44	-1.25
	BIC	-4.05	-3.79	-3.63	-3.37	-3.12	-2.62	-2.18	-1.92	-1.75	-1.59	-1.41
	MAIC	-3.64	-3.46	-3.33	-3.14	-2.90	-2.47	-1.99	-1.78	-1.59	-1.40	-1.18
	MBIC	-3.67	–3.52	–3.37	-3.20	-2.97	-2.54	-2.10	-1.87	-1.71	-1.56	-1.32
	t-sig	-5.19	-4.22	-3.82	-3.54	-3.22	-2.68	-2.15	-1.92	-1.76	-1.59	-1.34
T=150	AIC	-5.78	-4.35	-3.94	-3.55	-3.25	-2.67	-2.14	-1.93	-1.76	-1.60	-1.34
	BIC	-4.13	-3.78	-3.66	-3.42	-3.15	-2.62	-2.16	-1.95	-1.74	-1.56	-1.34
	MAIC	-3.77	-3.61	-3.43	-3.25	-3.00	-2.50	-2.08	-1.83	-1.67	-1.54	-1.33
	MBIC	-3.88	-3.72	-3.51	-3.31	-3.06	-2.57	-2.11	-1.91	-1.73	-1.54	-1.34
	t-sig	-4.43	-4.13	-3.78	-3.53	-3.21	-2.66	-2.16	-1.94	-1.74	-1.62	-1.35
T=200	AIC	-5.33	-4.33	-3.82	-3.50	-3.23	-2.64	-2.13	-1.86	-1.69	-1.56	-1.33
	BIC	-4.06	-3.87	-3.67	-3.39	-3.11	-2.62	-2.14	-1.87	-1.69	-1.46	-1.19
	MAIC	-3.95	-3.67	-3.47	-3.27	-3.00	-2.52	-2.05	-1.84	-1.67	-1.50	-1.25
	MBIC	-4.03	-3.83	-3.53	-3.30	-3.05	-2.58	-2.11	-1.87	-1.66	-1.46	-1.19
	t-sig	-4.52	-4.11	-3.83	-3.52	-3.26	-2.65	-2.16	-1.90	-1.71	-1.46	-1.33
T=∞		-4.50	-4.16	-3.89	-3.61	-3.27	-2.67	-2.12	-1.86	-1.64	-1.44	-1.23

Table 7 (cont'd): Finite Sample Critical Values; Model 2, Chosing T_B maximizing $|t_{\widehat{\beta}_2}|$

d) P_T^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	1.25	6.43	9.03	10.90	13.24	19.01	30.03	36.81	41.58	51.42	66.17
	BIC	8.31	9.25	10.31	11.72	13.86	19.46	27.63	35.33	39.40	43.79	53.96
	MAIC	10.46	11.15	12.26	13.71	15.78	21.77	32.41	38.92	45.63	54.98	69.13
	MBIC	10.15	10.86	11.72	13.21	15.05	20.45	29.21	36.81	40.67	46.33	56.51
	t-sig	5.08	7.35	9.13	10.80	13.10	18.78	28.35	35.32	39.72	45.69	55.00
T=150	AIC	4.02	7.05	8.76	10.67	12.79	18.59	28.16	33.82	39.54	45.12	52.33
	BIC	7.96	9.39	10.14	11.58	13.50	19.31	27.75	33.34	38.98	44.16	52.33
	MAIC	9.39	10.59	11.40	12.81	14.86	21.09	30.49	36.29	42.29	48.16	54.90
	MBIC	9.10	9.87	10.94	12.35	14.35	19.95	28.73	34.68	40.09	46.79	54.23
	t-sig	6.92	7.96	9.35	10.75	13.15	18.80	27.52	33.05	38.83	45.85	52.33
T=200	AIC	4.71	7.55	9.30	10.98	12.88	19.11	28.39	36.11	41.45	49.42	56.46
	BIC	8.08	9.01	9.94	11.74	13.84	19.35	28.41	35.20	41.34	54.00	62.47
	MAIC	8.73	9.93	11.26	12.57	14.85	20.63	30.38	37.65	44.13	51.13	56.97
	MBIC	8.14	9.27	10.93	12.38	14.46	20.12	28.95	35.66	41.45	54.15	62.47
	t-sig	6.60	7.88	9.12	10.85	12.67	18.72	27.65	34.47	41.34	48.78	61.64
T=∞		6.79	7.85	8.92	10.34	12.56	18.46	28.10	34.97	42.67	50.46	62.11

e) ADF^{GLS}

	Criteria	1.0%	2.5%	5.0%	10.0%	20.0%	50.0%	80.0%	90.0%	95.0%	97.5%	99.0%
T=100	AIC	-4.89	-4.49	-4.31	-4.06	-3.64	-2.98	-2.35	-2.08	-1.91	-1.75	-1.45
	BIC	-4.89	-4.42	-4.17	-3.84	-3.48	-2.86	-2.32	-2.04	-1.84	-1.71	-1.49
	MAIC	-4.35	-4.12	-3.83	-3.57	-3.21	-2.63	-2.12	-1.90	-1.72	-1.56	-1.39
	MBIC	-4.35	-4.12	-3.83	-3.58	-3.23	-2.67	-2.16	-1.94	-1.77	-1.60	-1.37
	t-sig	-4.90	-4.49	-4.26	-4.02	-3.63	-3.00	-2.36	-2.08	-1.91	-1.78	-1.54
T=150	AIC	-4.66	-4.41	-4.12	-3.85	-3.51	-2.89	-2.29	-2.07	-1.87	-1.69	-1.38
	BIC	-4.59	-4.23	-4.03	-3.71	-3.39	-2.76	-2.24	-2.02	-1.82	-1.59	-1.38
	MAIC	-4.31	-4.07	-3.77	-3.49	-3.15	-2.57	-2.10	-1.86	-1.72	-1.55	-1.38
	MBIC	-4.31	-4.08	-3.79	-3.53	-3.19	-2.63	-2.12	-1.91	-1.76	-1.55	-1.34
	t-sig	-4.59	-4.43	-4.10	-3.83	-3.49	-2.85	-2.28	-2.05	-1.84	-1.69	-1.46
T=200	AIC	-4.73	-4.33	-4.10	-3.72	-3.43	-2.79	-2.22	-1.96	-1.76	-1.59	-1.38
	BIC	-4.46	-4.19	-3.93	-3.61	-3.28	-2.71	-2.20	-1.94	-1.75	-1.55	-1.26
	MAIC	-4.32	-3.87	-3.61	-3.28	-3.09	-2.57	-2.10	-1.87	-1.69	-1.50	-1.24
	MBIC	-4.37	-3.94	-3.65	-3.41	-3.13	-2.61	-2.12	-1.90	-1.69	-1.50	-1.26
	t-sig	-4.69	-4.24	-4.00	-3.71	-3.42	-2.79	-2.23	-1.99	-1.78	-1.56	-1.38
T=∞		-4.50	-4.16	-3.89	-3.61	-3.27	-2.67	-2.12	-1.86	-1.64	-1.44	-1.23

Table 8: Size and Power the Tests chosing T_B minimizing the tests ; Model 1; MA(1) and AR(1) errors, T=100

		Size					Power				
	Criteria	MZ_α	MSB	MZ_t	P_T	ADF	MZ_α	MSB	MZ_t	P_T	ADF
$i.i.d.$	AIC	0.051	0.050	0.051	0.050	0.050	0.068	0.068	0.068	0.069	0.384
	BIC	0.050	0.050	0.051	0.051	0.051	0.500	0.495	0.495	0.521	0.497
	MAIC	0.050	0.051	0.051	0.050	0.050	0.409	0.424	0.422	0.441	0.487
	MBIC	0.050	0.050	0.051	0.050	0.050	0.553	0.553	0.551	0.563	0.503
	t-sig	0.050	0.051	0.050	0.050	0.051	0.212	0.213	0.213	0.214	0.464
$MA, \theta = -0.8$	AIC	0.037	0.037	0.037	0.037	0.771	0.100	0.100	0.100	0.100	0.960
	BIC	0.771	0.769	0.766	0.776	0.973	0.919	0.919	0.919	0.918	1.000
	MAIC	0.117	0.117	0.121	0.131	0.353	0.363	0.363	0.365	0.370	0.871
	MBIC	0.132	0.132	0.135	0.142	0.353	0.373	0.370	0.374	0.376	0.874
	t-sig	0.324	0.323	0.324	0.324	0.749	0.670	0.670	0.670	0.671	0.973
$MA, \theta = -0.4$	AIC	0.042	0.042	0.042	0.042	0.208	0.066	0.066	0.066	0.067	0.700
	BIC	0.299	0.296	0.297	0.306	0.429	0.751	0.753	0.752	0.768	0.904
	MAIC	0.060	0.064	0.069	0.076	0.120	0.277	0.279	0.291	0.319	0.475
	MBIC	0.084	0.084	0.088	0.095	0.118	0.343	0.342	0.339	0.367	0.491
	t-sig	0.056	0.055	0.058	0.060	0.227	0.396	0.396	0.397	0.407	0.729
$MA, \theta = 0.4$	AIC	0.057	0.057	0.057	0.057	0.060	0.100	0.100	0.100	0.099	0.326
	BIC	0.152	0.153	0.151	0.159	0.077	0.626	0.620	0.621	0.652	0.506
	MAIC	0.118	0.127	0.125	0.128	0.018	0.307	0.314	0.326	0.349	0.107
	MBIC	0.061	0.066	0.059	0.056	0.003	0.241	0.245	0.247	0.252	0.100
	t-sig	0.074	0.074	0.073	0.078	0.059	0.338	0.335	0.343	0.360	0.398
$MA, \theta = 0.8$	AIC	0.146	0.145	0.146	0.148	0.057	0.296	0.296	0.296	0.298	0.168
	BIC	0.289	0.287	0.286	0.289	0.109	0.585	0.581	0.583	0.600	0.383
	MAIC	0.206	0.213	0.217	0.222	0.012	0.298	0.301	0.324	0.345	0.103
	MBIC	0.053	0.056	0.057	0.056	0.001	0.118	0.113	0.124	0.151	0.031
	t-sig	0.101	0.101	0.102	0.105	0.050	0.326	0.320	0.327	0.332	0.227
$AR, \rho = -0.8$	AIC	0.043	0.043	0.043	0.043	0.048	0.053	0.053	0.053	0.054	0.361
	BIC	0.006	0.006	0.006	0.007	0.044	0.022	0.021	0.022	0.025	0.462
	MAIC	0.006	0.007	0.006	0.006	0.047	0.006	0.006	0.006	0.007	0.410
	MBIC	0.000	0.000	0.000	0.001	0.047	0.003	0.003	0.003	0.004	0.406
	t-sig	0.012	0.011	0.012	0.014	0.044	0.062	0.062	0.063	0.067	0.432
$AR, \rho = -0.4$	AIC	0.052	0.052	0.052	0.052	0.070	0.079	0.079	0.079	0.080	0.389
	BIC	0.094	0.096	0.096	0.103	0.140	0.365	0.360	0.364	0.394	0.567
	MAIC	0.032	0.034	0.037	0.039	0.062	0.188	0.194	0.212	0.238	0.417
	MBIC	0.026	0.024	0.027	0.032	0.059	0.240	0.231	0.251	0.283	0.421
	t-sig	0.040	0.040	0.041	0.042	0.071	0.212	0.208	0.215	0.218	0.459
$AR, \rho = 0.4$	AIC	0.051	0.051	0.050	0.053	0.047	0.089	0.089	0.089	0.090	0.221
	BIC	0.095	0.092	0.094	0.106	0.042	0.460	0.456	0.460	0.498	0.291
	MAIC	0.120	0.127	0.124	0.126	0.021	0.301	0.308	0.321	0.331	0.050
	MBIC	0.067	0.072	0.065	0.069	0.001	0.250	0.248	0.251	0.240	0.036
	t-sig	0.065	0.065	0.067	0.069	0.045	0.239	0.236	0.240	0.251	0.292
$AR, \rho = 0.8$	AIC	0.067	0.068	0.067	0.067	0.083	0.083	0.083	0.083	0.083	0.114
	BIC	0.177	0.183	0.169	0.170	0.067	0.286	0.286	0.280	0.301	0.118
	MAIC	0.247	0.269	0.250	0.240	0.074	0.298	0.307	0.320	0.330	0.145
	MBIC	0.202	0.213	0.197	0.186	0.064	0.337	0.341	0.340	0.350	0.031
	t-sig	0.131	0.133	0.128	0.123	0.072	0.185	0.186	0.185	0.188	0.134

Table 9: Size and Power the Tests choosing T_B minimizing the tests ; Model 1; MA(1) and AR(1) errors, T=200

		Size					Power				
	Criteria	MZ_α	MSB	MZ_t	P_T	ADF	MZ_α	MSB	MZ_t	P_T	ADF
$i.i.d.$	AIC	0.050	0.050	0.050	0.051	0.051	0.201	0.201	0.198	0.198	0.460
	BIC	0.051	0.052	0.051	0.050	0.051	0.538	0.532	0.555	0.538	0.535
	MAIC	0.051	0.051	0.050	0.051	0.050	0.466	0.463	0.461	0.468	0.509
	MBIC	0.051	0.050	0.050	0.050	0.051	0.532	0.521	0.541	0.553	0.531
	t-sig	0.050	0.050	0.050	0.050	0.051	0.373	0.371	0.367	0.359	0.468
$MA, \theta = -0.8$	AIC	0.187	0.187	0.188	0.197	0.539	0.562	0.563	0.562	0.569	0.942
	BIC	0.617	0.616	0.626	0.628	0.873	0.907	0.907	0.909	0.911	0.999
	MAIC	0.035	0.034	0.036	0.039	0.206	0.200	0.193	0.202	0.209	0.668
	MBIC	0.042	0.041	0.045	0.046	0.197	0.209	0.205	0.216	0.226	0.670
	t-sig	0.233	0.232	0.233	0.237	0.566	0.573	0.571	0.574	0.579	0.975
$MA, \theta = -0.4$	AIC	0.043	0.043	0.043	0.046	0.119	0.252	0.251	0.252	0.262	0.641
	BIC	0.177	0.174	0.185	0.186	0.228	0.778	0.772	0.789	0.800	0.848
	MAIC	0.064	0.063	0.063	0.066	0.074	0.349	0.342	0.347	0.364	0.447
	MBIC	0.078	0.077	0.078	0.078	0.072	0.460	0.461	0.479	0.485	0.464
	t-sig	0.046	0.047	0.045	0.047	0.119	0.422	0.415	0.423	0.432	0.647
$MA, \theta = 0.4$	AIC	0.074	0.074	0.073	0.076	0.058	0.287	0.286	0.288	0.304	0.409
	BIC	0.101	0.102	0.105	0.108	0.096	0.656	0.643	0.669	0.677	0.590
	MAIC	0.080	0.080	0.078	0.080	0.048	0.450	0.444	0.454	0.470	0.367
	MBIC	0.054	0.052	0.056	0.059	0.008	0.441	0.437	0.455	0.474	0.090
	t-sig	0.074	0.072	0.072	0.067	0.052	0.433	0.427	0.432	0.437	0.426
$MA, \theta = 0.8$	AIC	0.161	0.162	0.162	0.162	0.059	0.457	0.457	0.459	0.463	0.251
	BIC	0.203	0.197	0.208	0.202	0.083	0.629	0.621	0.640	0.643	0.424
	MAIC	0.147	0.150	0.145	0.147	0.024	0.425	0.417	0.422	0.439	0.185
	MBIC	0.052	0.052	0.052	0.054	0.017	0.253	0.247	0.266	0.293	0.167
	t-sig	0.157	0.159	0.155	0.155	0.066	0.567	0.567	0.567	0.569	0.335
$AR, \rho = -0.8$	AIC	0.034	0.034	0.034	0.034	0.056	0.067	0.066	0.067	0.068	0.440
	BIC	0.003	0.003	0.003	0.003	0.047	0.032	0.029	0.036	0.042	0.516
	MAIC	0.002	0.002	0.002	0.002	0.047	0.015	0.013	0.016	0.024	0.452
	MBIC	0.000	0.000	0.000	0.000	0.044	0.009	0.008	0.010	0.020	0.448
	t-sig	0.015	0.013	0.015	0.014	0.047	0.072	0.071	0.073	0.080	0.466
$AR, \rho = -0.4$	AIC	0.051	0.051	0.051	0.051	0.053	0.172	0.172	0.171	0.176	0.441
	BIC	0.037	0.034	0.041	0.043	0.050	0.409	0.404	0.433	0.443	0.515
	MAIC	0.036	0.035	0.039	0.041	0.048	0.339	0.325	0.333	0.346	0.476
	MBIC	0.029	0.026	0.028	0.034	0.048	0.367	0.358	0.396	0.421	0.470
	t-sig	0.042	0.043	0.042	0.042	0.043	0.268	0.264	0.269	0.285	0.456
$AR, \rho = 0.4$	AIC	0.065	0.066	0.064	0.063	0.050	0.239	0.240	0.243	0.249	0.381
	BIC	0.072	0.069	0.073	0.077	0.044	0.538	0.527	0.569	0.573	0.454
	MAIC	0.080	0.080	0.079	0.078	0.056	0.476	0.470	0.472	0.483	0.381
	MBIC	0.064	0.065	0.065	0.063	0.005	0.481	0.473	0.491	0.501	0.018
	t-sig	0.063	0.062	0.061	0.063	0.046	0.404	0.403	0.405	0.401	0.396
$AR, \rho = 0.8$	AIC	0.076	0.077	0.076	0.076	0.068	0.184	0.185	0.182	0.181	0.223
	BIC	0.103	0.106	0.109	0.105	0.064	0.351	0.341	0.358	0.369	0.248
	MAIC	0.120	0.126	0.120	0.117	0.072	0.342	0.337	0.342	0.348	0.247
	MBIC	0.097	0.102	0.101	0.106	0.070	0.336	0.329	0.347	0.370	0.245
	t-sig	0.091	0.088	0.090	0.086	0.061	0.282	0.281	0.279	0.280	0.232

Table 10: **Size and Power the Tests chosing T_B maximizing $|t_{\widehat{\beta}_2}|$; Model 1; MA(1) and AR(1) errors, T=100**

		Size					Power				
	Criteria	MZ_α	MSB	MZ_t	P_T	ADF	MZ_α	MSB	MZ_t	P_T	ADF
$i.i.d.$	AIC	0.051	0.050	0.051	0.050	0.050	0.240	0.232	0.234	0.231	0.388
	BIC	0.050	0.050	0.050	0.050	0.050	0.457	0.429	0.467	0.488	0.495
	MAIC	0.050	0.050	0.050	0.051	0.050	0.253	0.253	0.262	0.275	0.414
	MBIC	0.050	0.052	0.051	0.051	0.050	0.353	0.345	0.366	0.359	0.417
	t-sig	0.051	0.051	0.051	0.051	0.050	0.212	0.216	0.213	0.219	0.428
$MA, \theta = -0.8$	AIC	0.104	0.104	0.104	0.107	0.628	0.175	0.174	0.175	0.174	0.874
	BIC	0.373	0.363	0.373	0.374	0.892	0.446	0.445	0.448	0.443	0.998
	MAIC	0.005	0.005	0.006	0.006	0.164	0.028	0.028	0.028	0.028	0.554
	MBIC	0.008	0.008	0.008	0.007	0.160	0.026	0.027	0.027	0.027	0.553
	t-sig	0.097	0.097	0.098	0.102	0.575	0.183	0.183	0.183	0.184	0.913
$MA, \theta = -0.4$	AIC	0.071	0.069	0.071	0.072	0.195	0.208	0.204	0.208	0.204	0.609
	BIC	0.187	0.181	0.186	0.189	0.411	0.477	0.462	0.483	0.490	0.860
	MAIC	0.027	0.028	0.027	0.030	0.091	0.080	0.081	0.081	0.086	0.310
	MBIC	0.029	0.027	0.033	0.035	0.090	0.098	0.093	0.108	0.102	0.313
	t-sig	0.051	0.051	0.052	0.054	0.194	0.187	0.190	0.191	0.189	0.614
$MA, \theta = 0.4$	AIC	0.082	0.083	0.081	0.078	0.070	0.220	0.214	0.220	0.217	0.307
	BIC	0.122	0.109	0.125	0.124	0.087	0.389	0.377	0.395	0.410	0.486
	MAIC	0.063	0.066	0.067	0.069	0.012	0.119	0.118	0.124	0.137	0.091
	MBIC	0.025	0.027	0.024	0.022	0.004	0.103	0.099	0.114	0.114	0.091
	t-sig	0.083	0.085	0.083	0.084	0.052	0.235	0.240	0.239	0.247	0.359
$MA, \theta = 0.8$	AIC	0.148	0.151	0.146	0.146	0.050	0.137	0.136	0.136	0.139	0.145
	BIC	0.146	0.144	0.145	0.150	0.088	0.205	0.202	0.209	0.212	0.328
	MAIC	0.065	0.071	0.064	0.069	0.009	0.049	0.048	0.050	0.055	0.071
	MBIC	0.017	0.018	0.015	0.017	0.001	0.022	0.020	0.023	0.023	0.028
	t-sig	0.078	0.080	0.078	0.079	0.044	0.122	0.122	0.123	0.126	0.183
$AR, \rho = -0.8$	AIC	0.014	0.014	0.014	0.014	0.036	0.014	0.014	0.014	0.014	0.254
	BIC	0.000	0.000	0.000	0.000	0.031	0.004	0.003	0.004	0.004	0.364
	MAIC	0.000	0.000	0.000	0.000	0.030	0.003	0.003	0.004	0.004	0.232
	MBIC	0.000	0.000	0.000	0.000	0.031	0.003	0.002	0.003	0.003	0.231
	t-sig	0.003	0.003	0.003	0.003	0.032	0.013	0.013	0.013	0.013	0.304
$AR, \rho = -0.4$	AIC	0.051	0.050	0.050	0.050	0.064	0.110	0.106	0.110	0.112	0.333
	BIC	0.051	0.046	0.050	0.050	0.135	0.215	0.207	0.218	0.228	0.513
	MAIC	0.015	0.013	0.018	0.024	0.047	0.073	0.072	0.081	0.091	0.297
	MBIC	0.013	0.013	0.015	0.013	0.043	0.083	0.078	0.094	0.087	0.295
	t-sig	0.038	0.039	0.039	0.041	0.060	0.111	0.112	0.113	0.118	0.376
$AR, \rho = 0.4$	AIC	0.089	0.092	0.086	0.089	0.043	0.218	0.212	0.217	0.215	0.219
	BIC	0.082	0.078	0.083	0.084	0.041	0.329	0.314	0.338	0.352	0.304
	MAIC	0.081	0.088	0.081	0.083	0.014	0.156	0.157	0.158	0.168	0.042
	MBIC	0.035	0.038	0.034	0.029	0.001	0.101	0.103	0.107	0.100	0.034
	t-sig	0.077	0.080	0.076	0.084	0.038	0.225	0.225	0.226	0.231	0.251
$AR, \rho = 0.8$	AIC	0.145	0.149	0.142	0.131	0.045	0.160	0.162	0.153	0.150	0.099
	BIC	0.137	0.134	0.131	0.133	0.038	0.224	0.210	0.225	0.227	0.116
	MAIC	0.140	0.157	0.130	0.138	0.051	0.193	0.199	0.195	0.210	0.127
	MBIC	0.136	0.145	0.137	0.129	0.038	0.212	0.215	0.222	0.217	0.017
	t-sig	0.133	0.139	0.130	0.124	0.041	0.170	0.178	0.168	0.169	0.109

Table 11: Size and Power the Tests choosing T_B maximizing $|t_{\widehat{\beta}_2}|$; Model 1; MA(1) and AR(1) errors, T=200

		Size					Power				
	Criteria	MZ_α	MSB	MZ_t	P_T	ADF	MZ_α	MSB	MZ_t	P_T	ADF
$i.i.d.$	AIC	0.051	0.050	0.050	0.050	0.051	0.307	0.299	0.312	0.327	0.395
	BIC	0.050	0.050	0.051	0.050	0.050	0.465	0.463	0.474	0.482	0.480
	MAIC	0.051	0.049	0.051	0.050	0.051	0.365	0.352	0.376	0.367	0.460
	MBIC	0.050	0.050	0.051	0.051	0.051	0.424	0.416	0.433	0.436	0.496
	t-sig	0.051	0.051	0.050	0.051	0.050	0.320	0.317	0.328	0.310	0.428
$MA, \theta = -0.8$	AIC	0.042	0.040	0.042	0.043	0.302	0.145	0.143	0.145	0.147	0.741
	BIC	0.238	0.238	0.241	0.239	0.675	0.427	0.430	0.428	0.422	0.977
	MAIC	0.001	0.001	0.002	0.002	0.068	0.020	0.018	0.021	0.021	0.287
	MBIC	0.004	0.004	0.004	0.004	0.071	0.020	0.019	0.018	0.019	0.286
	t-sig	0.050	0.049	0.050	0.050	0.355	0.171	0.170	0.173	0.174	0.865
$MA, \theta = -0.4$	AIC	0.042	0.040	0.041	0.050	0.102	0.261	0.262	0.269	0.279	0.500
	BIC	0.113	0.113	0.115	0.121	0.212	0.589	0.586	0.591	0.585	0.757
	MAIC	0.032	0.033	0.037	0.036	0.066	0.160	0.155	0.170	0.159	0.322
	MBIC	0.039	0.038	0.041	0.039	0.064	0.194	0.193	0.197	0.199	0.336
	t-sig	0.049	0.047	0.050	0.046	0.109	0.290	0.293	0.295	0.291	0.538
$MA, \theta = 0.4$	AIC	0.065	0.064	0.064	0.067	0.049	0.288	0.282	0.293	0.308	0.345
	BIC	0.082	0.085	0.082	0.084	0.084	0.495	0.487	0.501	0.494	0.539
	MAIC	0.056	0.055	0.061	0.061	0.057	0.288	0.283	0.305	0.305	0.313
	MBIC	0.030	0.030	0.033	0.034	0.006	0.217	0.216	0.223	0.228	0.092
	t-sig	0.065	0.067	0.064	0.064	0.054	0.341	0.335	0.344	0.329	0.405
$MA, \theta = 0.8$	AIC	0.093	0.092	0.093	0.098	0.047	0.215	0.207	0.216	0.236	0.189
	BIC	0.098	0.100	0.098	0.101	0.072	0.326	0.325	0.332	0.337	0.356
	MAIC	0.057	0.057	0.060	0.067	0.021	0.142	0.133	0.156	0.160	0.149
	MBIC	0.018	0.016	0.020	0.021	0.017	0.087	0.083	0.093	0.093	0.153
	t-sig	0.113	0.118	0.117	0.113	0.067	0.392	0.390	0.397	0.382	0.329
$AR, \rho = -0.8$	AIC	0.007	0.007	0.007	0.008	0.040	0.032	0.032	0.033	0.040	0.280
	BIC	0.001	0.001	0.001	0.001	0.036	0.014	0.012	0.013	0.016	0.345
	MAIC	0.000	0.000	0.000	0.000	0.035	0.012	0.010	0.014	0.013	0.289
	MBIC	0.000	0.000	0.000	0.000	0.035	0.006	0.006	0.007	0.012	0.281
	t-sig	0.005	0.004	0.005	0.005	0.041	0.035	0.032	0.037	0.039	0.325
$AR, \rho = -0.4$	AIC	0.026	0.025	0.028	0.031	0.048	0.187	0.177	0.190	0.193	0.343
	BIC	0.027	0.027	0.027	0.027	0.044	0.300	0.297	0.306	0.301	0.410
	MAIC	0.022	0.023	0.026	0.026	0.047	0.195	0.192	0.214	0.212	0.379
	MBIC	0.015	0.016	0.017	0.017	0.049	0.220	0.215	0.228	0.224	0.383
	t-sig	0.026	0.026	0.027	0.027	0.049	0.226	0.229	0.235	0.219	0.371
$AR, \rho = 0.4$	AIC	0.065	0.065	0.063	0.072	0.040	0.329	0.321	0.333	0.350	0.345
	BIC	0.059	0.062	0.057	0.063	0.042	0.461	0.456	0.471	0.472	0.403
	MAIC	0.066	0.066	0.069	0.066	0.058	0.344	0.333	0.354	0.356	0.336
	MBIC	0.044	0.046	0.042	0.044	0.005	0.262	0.257	0.265	0.260	0.018
	t-sig	0.083	0.087	0.081	0.082	0.044	0.375	0.371	0.382	0.373	0.382
$AR, \rho = 0.8$	AIC	0.079	0.082	0.079	0.084	0.047	0.228	0.221	0.231	0.245	0.187
	BIC	0.071	0.077	0.070	0.075	0.045	0.296	0.292	0.298	0.294	0.219
	MAIC	0.090	0.096	0.089	0.092	0.057	0.262	0.258	0.278	0.272	0.246
	MBIC	0.080	0.080	0.074	0.073	0.054	0.284	0.280	0.294	0.294	0.247
	t-sig	0.074	0.074	0.074	0.068	0.046	0.261	0.255	0.262	0.260	0.239

Tables 12.a: Results for the Real Wages and Stock Prices Series
Choosing the break point minimizing the tests ($\bar{c}$ = 0 to construct s^2)

Serie	T	Criteria	MZ_t	k	T_B	P_T	k	T_B	ADF	k	T_B	®
Real Wages	71	BIC	-3.85^d	1	1940	9.49^d	1	1938	-4.63^d	1	1938	0.62
		MAIC	-3.85^c	1	1940	9.49^c	1	1938	-4.63^b	1	1938	0.62
		MBIC	-3.85^a	1	1940	9.49^b	1	1938	-4.63^b	1	1938	0.62
Stock Prices	100	BIC	-4.69^a	1	1945	6.24^a	1	1945	-5.12^a	1	1937	0.67
		MAIC	-4.69^a	1	1945	6.24^a	1	1945	-5.12^a	1	1937	0.67
		MBIC	-4.63^a	1	1937	6.45^a	1	1937	-5.12^a	1	1937	0.67

Notes: 1) For the applications, we impose a minimal value k = 1; 2) the superscipts a, b,c and d denote signi…cance levels at the 1:0%; 2:5%; 5:0%; and 10:0%, respectively.

Tables 12.b: Results for the Real Wages and Stock Prices Series
Choosing the break point maximizing $|t_{\hat{\gamma}_2}|$ ($\bar{c}$ = 0 to construct s^2)

Series	T	T_B	Criteria	MZ_t	k	P_T	k	ADF	k	®
Real Wages	71	1933	BIC	-3.37^d	1	11.46^d	1	-3.83	1	0.70
			MAIC	-3.37^c	1	11.46^b	1	-3.83^d	1	0.70
			MBIC	-3.37^c	1	11.46^b	1	-3.83^d	1	0.70
Stock Prices	100	1931	BIC	-3.87^b	1	9.14^b	1	-4.16^c	1	0.75
			MAIC	-3.04	2	14.66	2	-3.25	2	0.79
			MBIC	-3.04	2	14.66	2	-3.25	2	0.79

Notes: 1) For the applications, we impose a minimal value k = 1; 2) the superscipts a, b,c and d denote signi…cance levels at the 1:0%; 2:5%; 5:0%; and 10:0%, respectively.

Figure 1. Asymptotic Power Function of P_T^{GLS} Test

Choosing T_B minimizing the tests

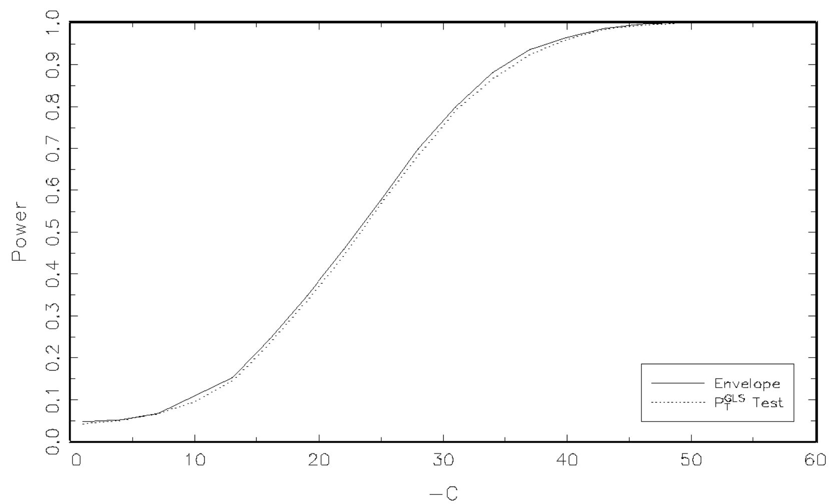

Figure 2. Asymptotic Power Function of MZ_α^{GLS} Test

Choosing T_B minimizing the tests

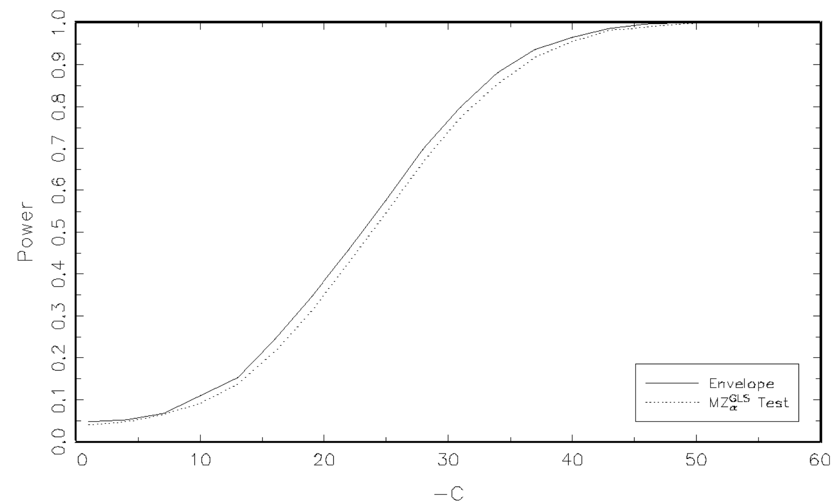

Figure 3. Asymptotic Power Function of MSB^{GLS} Test

Choosing T_B minimizing the tests

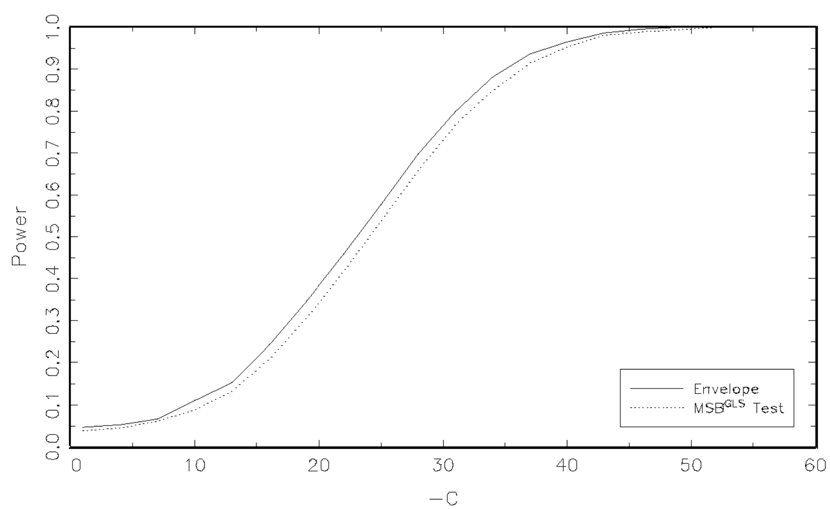

Figure 4. Asymptotic Power Function of MZ_t^{GLS} Test

Choosing T_B minimizing the tests

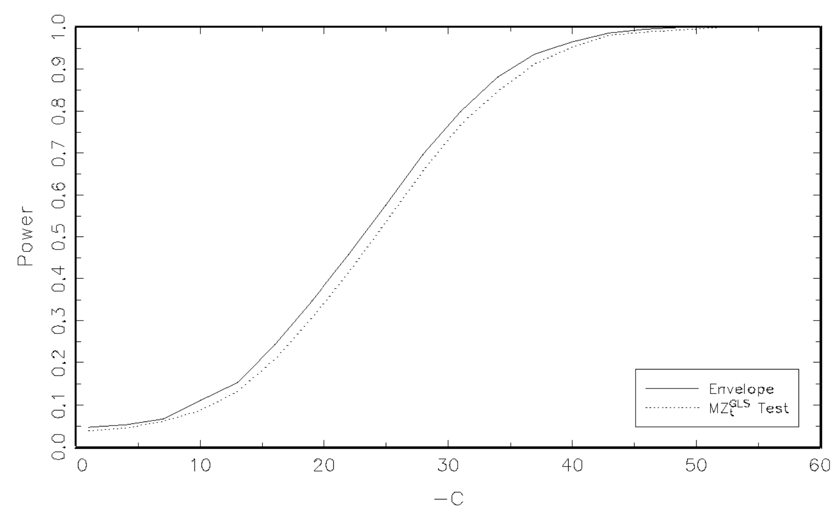

Figure 5. Asymptotic Power Function of P_T^{GLS} Test

Choosing T_B maximizing $|t_{\beta_2}|$

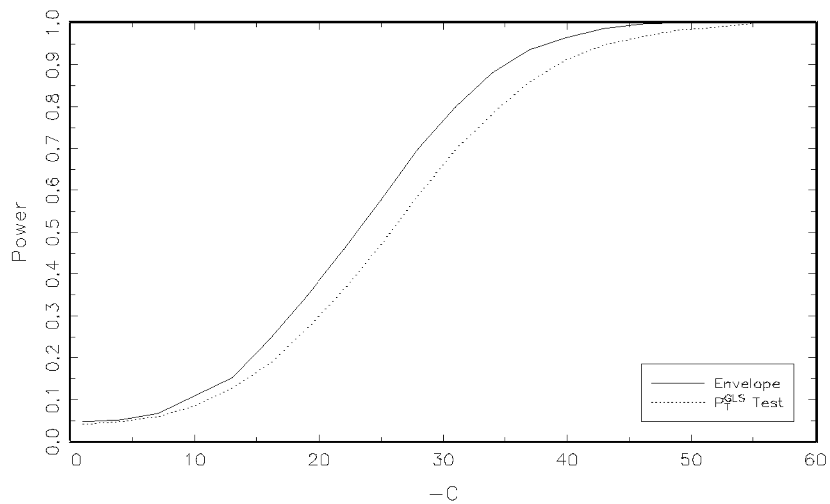

Figure 6. Asymptotic Power Function of MZ_α^{GLS} Test

Choosing T_B maximizing $|t_{\beta_2}|$

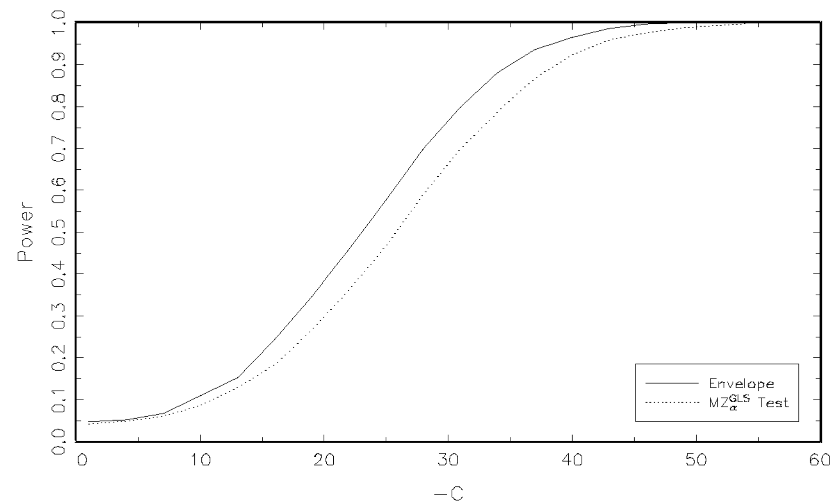

Figure 7. Asymptotic Power Function of MSB^{GLS} Test

Choosing T_B maximizing $|t_{\beta_2}|$

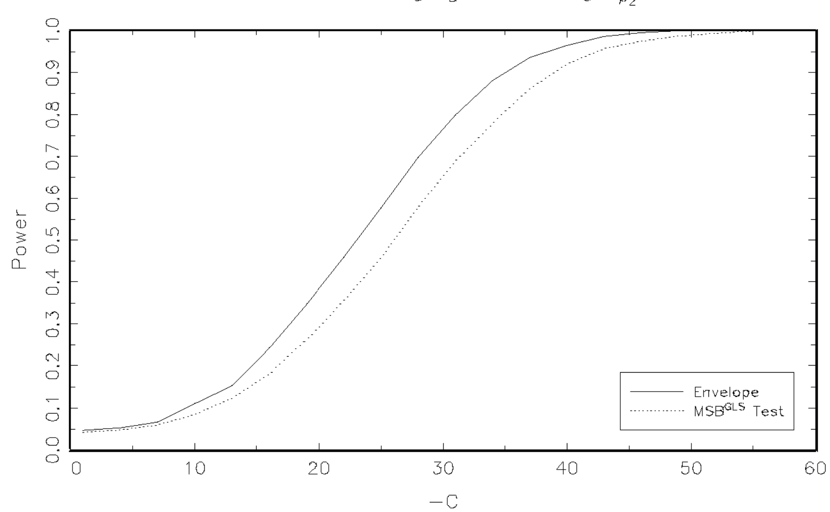

Figure 8. Asymptotic Power Function of MZ_t^{GLS} Test

Choosing T_B maximizing $|t_{\beta_2}|$

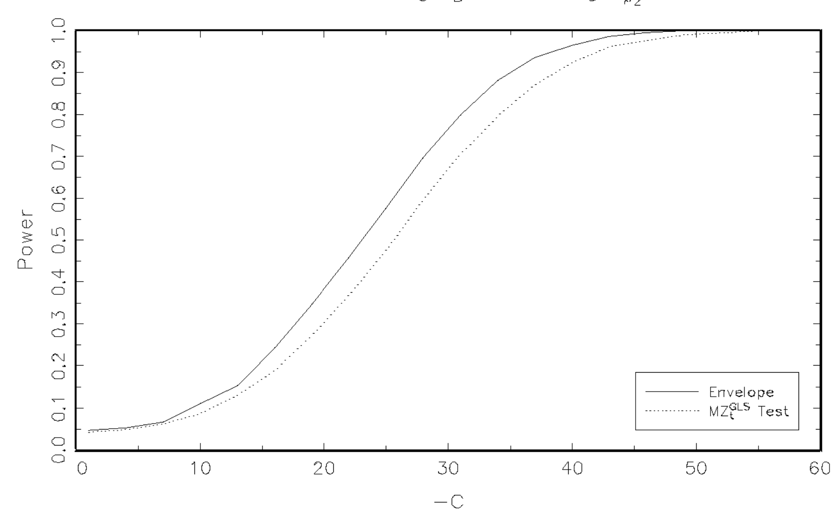

Figure 9. Logarithm of Real Wages (1900–1970)

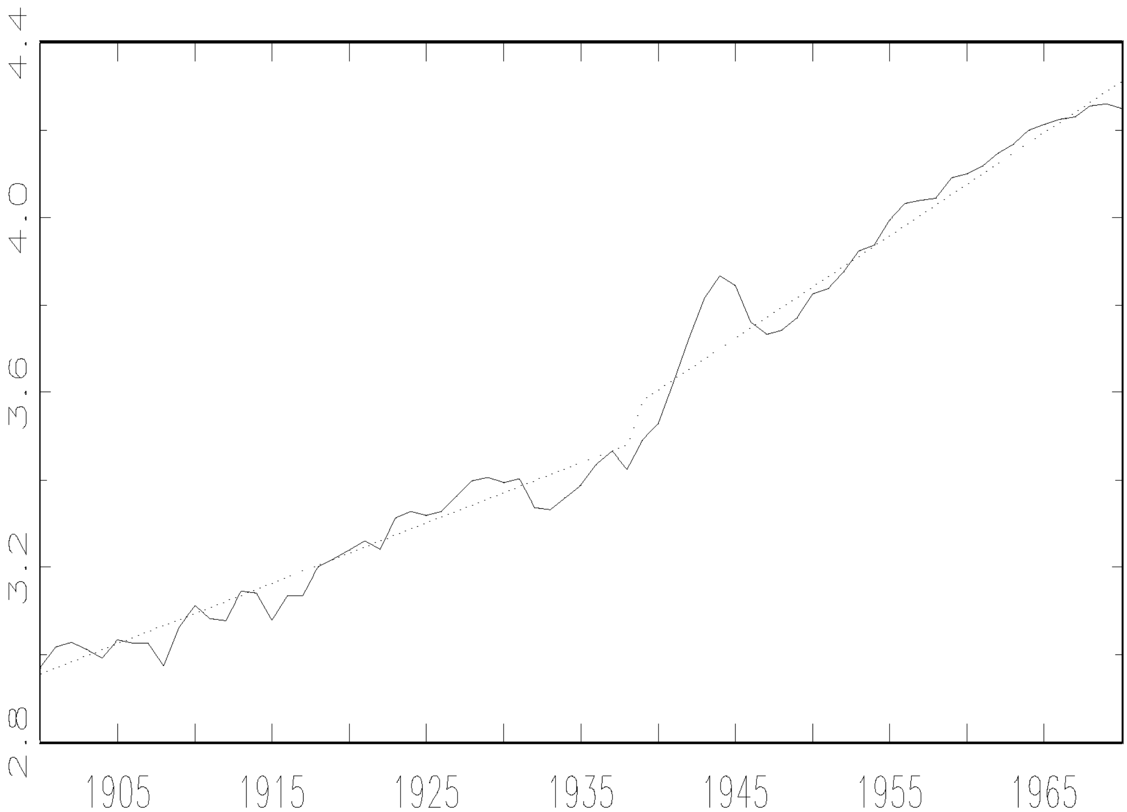

Figure 10. Logarithm of Common Stock Prices (1871–1970)