Games with unobservable heterogeneity and multiple equilibria: An application to mobile telecommunications

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Abstract

To shed light on the limited success of competition enhancing policies in mobile telecommunications, I estimate a game of transceivers' locations between national incumbents and a new entrant in Canada. I recover player-specific unobserved heterogeneity from bids for spectrum licenses to address the unavailability of regressors required to identify incumbents' responses to the new entrant's decisions. I find that incumbents benefitting from important economies of density is a plausible explanation for policies' drawbacks. I then evaluate the equilibrium effect of subsidizing the new entrant's transceivers and find that this alternative proposition increases its investments while only slightly modifying incumbents'.

Keywords: multiple equilibria; unobserved heterogeneity; empirical games; telecommunications.

JEL codes: C57; L11.

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1 Introduction

The state of competition in mobile telecommunications has been open to debate in many developed countries. On the one hand, as one would expect from a natural oligopoly, considerable investments required to operate a mobile network may discourage new firms to compete against incumbents. On the other hand, industry regulators often advocate intensified competition to avoid having to intervene and prevent incumbents from exercising market power. Unfortunately, competition enhancing policies often fail to sustainably increase the number of providers operating in the industry. In the past, several smaller players have merged together or were bought by larger providers.[1] In the United States, recent high profile examples of potential reductions in the number of players are the repeated merger attempts between T-Mobile and Sprint, respectively the third and fourth largest mobile providers in the country.

In this article, I shed light on important drawbacks in the Canadian government's attempts to increase competition in an industry long dominated by three national incumbents. In Canada, as in many other countries, spectrum licenses required to develop a mobile network are allocated through auctions. In hopes to encourage new firms entering the industry, the government prevents national incumbents from bidding on a fraction of licenses set aside for small and regional players. During the 2008 Advanced Wireless Services 1 (AWS1-2008) auction, a handful of new providers took advantage of this set-aside policy and built a mobile network. However, some of these firms failed to use the spectrum they were allocated and eventually resold it to other providers, including incumbents. Furthermore, some of these providers went out of business and were bought by incumbents.

In order to understand these drawbacks and to evaluate the potential of alternative policies, I estimate an empirical game in which providers, more precisely three incumbents and a new entrant, decide where and how many transceivers (i.e., cell antennae) to install. I find that there are important economies of density in transceivers' locations. Because it is building a brand new network, the new entrant cannot benefit from such economies of density to the same extent as incumbents. This asymmetry may explain the drawbacks mentioned above and motivates the study of a counterfactual policy. I evaluate the equilibrium effects of subsidizing the first few transceivers installed by the new entrant, in order to compensate its lack of economies of density.

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[1] For a description of the evolution of market structure in many countries' mobile telecommunications industry, see Gruber (2005, chapters 3 and 7).

My results suggest that this policy encourages the new provider to install more transceivers, while generating only minor distortions in incumbents' decisions. In some cases, I even find incumbents to be slightly more likely to install transceivers in response to the increase in the new entrant's probabilities of entering the market. This insight is especially relevant because, in this industry, policy makers have been worried that encouraging investments from new entrants may discourage investments made by incumbents.

Part of the current article's contribution is to illustrate how recovering player-specific unobserved heterogeneity from predetermined outcomes may help in identifying strategic interactions between competitors. Being able to convincingly quantify strategic interactions in transceivers' locations is key to properly assess the equilibrium effects of any counterfactual policies. Intuitively, one may hope to recover the effect of strategic interactions on players' payoffs if there exists an exogenous source of variation that does not affect their own payoffs, but shifts their competitors' decisions. In the same spirit as in other econometric models of simultaneous equations, many identification arguments available in the literature exploit exogenous variation in some player-specific observable regressors that are excluded from competitors' payoffs (e.g., Pesendorfer and Schmidt-Dengler, 2003; Bajari, Hong, Krainer, and Nekipelov, 2010). In the current setting, variation in stocks of transceivers previously accumulated would be a suitable candidate. Unfortunately, such variation does not apply to the new entrant because, by definition, its stocks of transceivers are exactly zero across the country, therefore jeopardizing the identification of incumbents' responses to the new entrant's decisions.

I argue that providers' bids for spectrum licenses can be used to recover player-specific unobservables which provide variation required for identification. I also discuss conditions under which unobserved heterogeneity can be combined with equilibria multiplicity, i.e. another source of variation that has been leveraged when player-specific regressors are not available (Sweeting, 2009; De Paula and Tang, 2012; Aradillas-Lopez and Gandhi, 2016). I compare the identifying power of both approaches in the current context. I find that player-specific unobserved heterogeneity is a more convincing source of variation then multiple equilibria, as one may expect from an industry in which agreements between providers (e.g., network sharing, tower sharing, domestic roaming, etc.), transceivers' technology and spectrum compatibility may affect transceivers' locations, but are not observed by the econometrician.

It is worth mentioning that competition in mobile telecommunications is a topic that has drawn considerable attention in economics.[2] Several articles have studied Federal Communications Commission auctions in the United States focusing, among other topics, on collusion (e.g., Cramton and Schwartz, 2000, 2002) and the efficiency of the system in place (e.g., Fox and Bajari, 2013; Xiao and Yuan, 2018). Effects of changes in market structure on industry outcomes have also been investigated. For instance, Bajari, Fox, and Ryan (2008) estimate consumers' valuation of larger coverage areas resulting from firms consolidation. Seim and Viard (2011) examine how new entries affect retail pricing and product availability.

Data on transceivers' locations have been used in other empirical studies. In particular, Sun (2016) uses information on transceivers' locations in Connecticut to assess the effect of signal quality on providers' market shares. Transceivers' data are also used to control for product quality in Bourreau, Sun, and Verboven (2018)'s study of tacit collusion in France. Also of note is the article by Björkegren (2017) who uses data on towers' locations in Rwanda to measure network coverage and its effect on the adoption of mobile telephones.

These articles are far from being an exhaustive list of all empirical work done on competition in mobile telecommunications. However, to the best of my knowledge, despite the fact that transceivers' locations are important determinants of competition in this industry, the current effort is the first attempt at explicitly modelling strategic behaviour in such decisions through a game of market entry.

Finally, the current article contributes to a recent and growing literature which advocates allowing for a flexible information structure when estimating empirical games. While many models can be classified as games of either complete or incomplete information,[3] recent empirical applications have shown that rigid assumptions on players' information could lead to significantly different results (see applications in Grieco, 2014; Magnolfi and Roncoroni, 2017; Luo, Xiao, and Xiao, 2018). Grieco (2014) and Aguirregabiria and Mira (2019) consider a framework that is closely related to the current article. It allows for both private and common-knowledge unobservable information as well as multiple equilibria being realized in the data.

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[2] For a general treatment of the topic at the early stage of the industry, see Armstrong (1997), Gruber (2005) and references therein.

[3] For recent surveys, see Bajari, Hong, and Nekipelov (2013) and De Paula (2013). Commonly cited examples of theoretical and empirical contributions include Bjorn and Vuong (1984), Bresnahan and Reiss (1991a, 1991b), Berry (1992), Tamer (2003), Seim (2006), Ciliberto and Tamer (2009), Bajari, Hong, Krainer, and Nekipelov (2010), Galichon and Henry (2011), Beresteanu, Molchanov, and Molinari (2011), Aradillas-Lopez (2012) and Lewbel and Tang (2015).

Magnolfi and Roncoroni (2017) consider an alternative equilibrium concept which also allows for a flexible information structure and multiple equilibria. However, in all these articles, point identification requires observable player-specific regressors and, therefore, the identification results therein do not directly apply here. To the best of my knowledge, the current article is the first to propose exploiting the identifying power associated with variation in player-specific unobservable heterogeneity in the estimation of an empirical game.

The rest of the article is structured as follows. Section 2 briefly describes the industry background and the role of transceivers in mobile telecommunications. The econometric model to be estimated is introduced in Section 3. The data is described in Section 4. Section 5 discusses an important identification issue and the proposed solution. Sections 6 and 7 respectively report estimation results and the counterfactual analysis. Section 8 concludes.

2 Industry background

Before laying out the empirical analysis, it is worth discussing some background information. This section does not pretend to be a comprehensive description of the Canadian mobile telecommunications industry. Instead, I report relevant facts to justify the model that I use.

Simply put, transceivers are antennae which are installed by network operators to provide coverage in a given geographic area. Prior to installing transceivers, network operators must obtain a spectrum license, which grants the right to exploit frequencies in the corresponding geographic area. In Canada, since the early 2000's, these licenses have been allocated through auctions (see Industry Canada, 2004, 2011).

There are at least two reasons for installing a larger number of transceivers in a given region.[4] First, each transceiver can only provide coverage in a limited surrounding area. More transceivers are therefore needed to cover larger regions. Second, there is a limit to the amount of information that can be carried simultaneously through a given frequency. As a result, more transceivers are needed in more densely populated areas.

The main reason for focusing on transceivers' location decisions is to assess the properties of the Canadian mobile telecommunications industry at the wholesale level. The wholesale

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[4] For more detailed economic descriptions of the technology related to the mobile telecommunications industry, see Hausman (2002) and Gruber (2005, chapter 2).

market in mobile telecommunications relates to transactions through which a mobile service provider allows another firm to use parts of its network and/or its installations to provide network services. In fact, to provide coverage in a given area, operators may either install their own transceivers or use other firms'. For instance, subscribers may use their mobile device outside of their provider's service area insofar as their provider has agreed to use another operator's network. Such agreements are referred to as domestic roaming. In some cases, network operators may even agree to explicitly share parts of their networks with each other.

[Insert Figure 1 here.]

Figure 1 shows transceivers' locations[5] for three incumbents (Rogers, Bell and Telus) and a new entrant (Vidéotron). Some regional patterns are noticeable. In particular, while Rogers' transceivers are spread out across the country, Bell and Telus tend to cover different areas. Further, Vidéotron covers most of Quebec's inhabited regions, but is roughly limited to that province. These patterns are, at least in part, due to different agreements between network operators. The terms of these agreements are, however, unobservable to the researcher. Furthermore, technological constraints and potential limitations in spectrum compatibility may also affect transceivers' locations without being explicitly observed by the researcher. One cannot simply omit the existence of such unobservables when studying strategic interactions in transceivers' locations: ideally, one should find a way of taking the effect of these unobservables into account.

Furthermore, as can be seen from Figure 1, there are some parts of Canada where only a small number of providers have actually installed transceivers. Even if there exists regulatory conditions regarding domestic roaming and site sharing (Industry Canada, 2013a), both the Canadian Radio-television and Telecommunications Commission (the telecommunications regulating body in Canada) and the Competition Bureau have expressed concerns about the effect of potential market power on negotiated agreements between incumbents and new entrants (see Industry Canada, 2013b; Commissioner of Competition, 2014).[6] Such concerns have justified the federal government's intention to increase the number of competitors in this industry.

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[5] As will be clearly explained in Section 4, these are transceivers operating on frequencies allocated up to the AWS1-2008 spectrum auction.

[6] See Church and Wilkins (2013) for an alternative view of the state of competition in the Canadian mobile telecommunications industry.

One of the main policies used by the government to encourage entry has been setting aside some blocks of frequencies for new and regional players during spectrum auctions. On one hand, spectrum set-asides have allowed new and regional service providers to acquire parts of the spectrum and to operate a network. For instance, among new entrants associated with the AWS1-2008 auction set-asides are Vidéotron, Wind, Eastlink, Mobilicity and Public Mobile. On the other hand, two drawbacks of spectrum set-asides are worth mentioning. First, some of the new entrants do not use all the spectrum they are assigned and eventually sell unused frequencies to other firms, including incumbents. For instance, in 2017, Vidéotron sold some of its licenses acquired in 2008 to Rogers for about twice the price it paid for.[7] Second, some of these firms go out of business and are acquired by incumbents. In fact, upon Industry Canada's and the Competition Bureau's approvals, Telus bought Public Mobile and Rogers acquired Mobilicity.[8] The main objective of the current empirical application is to provide an explanation for these drawbacks and to explore an alternative policy to palliate them.

3 Model of transceivers' locations

Modelling current stocks of transceivers owned by each provider as a one-shot game of simultaneous decisions would be unrealistic. In Canada, the first cellular network came into service in the mid-1980's. Since then, there have been many important changes in terms of technology, regulation and market structure. It would be simplistic to assume that any firm would commit to their decisions over such a long period of time despite all these changes.

A more realistic view is to consider transceivers' locations as resulting from a sequence of games of simultaneous decisions. Different spectrum auctions define the timing of each game. More precisely, I model providers' decisions as an investment problem. After an auction is completed, firms that have been granted licenses simultaneously choose how many transceivers to add to their accumulated stocks. Each player's payoff is affected by its current stocks of transceivers, as well as its own and its competitors' simultaneously chosen additions. Dynamics are therefore included in the model by allowing current payoffs to depend on previous decisions through stocks accumulation. In other words, existing stocks of transceivers are part of the state

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[7] See Wire Report (2017).

[8] See the statements of approval given in Industry Canada (2013b, 2015) and Competition Bureau (2014).

variables, such that past investments affect current decisions even if transceivers' installation is not explicitly modelled as a dynamic game. The resulting payoff functions should be interpreted as a semi-reduced-form of the sum of the current and discounted expected future valuations.

There are two important reasons for not explicitly modelling forward-looking behaviour through a dynamic game. First, I only observe a single auction with relevant strategic interactions between incumbents and the new entrant.[9] Second, it is not clear to what extent firms are able to anticipate future regulatory and technological changes. In some sense, interpreting payoff functions as semi-reduced-form value functions amounts to being fairly agnostic about firms' anticipation of the future.

I now introduce the game of interest in which all providers which have been granted a spectrum license simultaneously choose the number of transceivers they want to install, i.e. an element from a finite and discrete choice set of ordered actions. I first present the game as it is played by the providers before turning to the game observed by the econometrician. Then, I provide a reduced-form model for providers' bids for spectrum licenses which will be used as predetermined outcomes informative about player-specific unobserved heterogeneity.

3.1 A game of location decisions

Henceforth, random variables will be denoted by capital calligraphic letters and their realizations will be written in lower case letters. Boldface letters refer to vectors and matrices. Finally, sets will be denoted with capital script letters.

Consider N network operators indexed by $i \in \mathcal{N} = \{1, 2, \ldots, N\}$ simultaneously deciding how many transceivers to install in a given market. Each provider can install at most J transceivers in that market, i.e. they choose a single ordered action from a finite and discrete set $\mathcal{Y} = \{0, 1, \ldots, J\}$ called the choice set. Let $\mathcal{Y}_i$ be the random variable with realization $y_i \in \mathcal{Y}$ corresponding to player i's decision. Let $\boldsymbol{\mathcal{Y}}$ be the vector of all players' decisions with

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[9] Between the AWS1-2008 auction and the construction of the dataset used for estimation, there have been three other spectrum auctions relevant for mobile services: the 2014 Mobile Broadband Services (MBS-2014), the 2015 Advanced Wireless Services 3 (AWS3-2015) and the 2015 Broadband Radio Services (BRS-2015) auctions. However, these auctions are excluded from the analysis as they may be too recent for network service providers to have had sufficient time to install new transceivers. In fact, for each of these auctions, there is at least one player (incumbent and/or new entrant) that has not installed a single transceiver operating on its allocated frequencies. In particular, none of the mobile service providers has installed transceivers operating on AWS3-2015 frequencies according to the 2016 dataset I use. Notice that the AWS3-2015 frequencies were sold in 2015 even if no mobile device available in Canada was operating on these frequencies at that time.

realizations $\mathbf{y} \in \mathcal{Y}^N$. Throughout the article, the index $-i$ will be used to refer to all players except player i. For instance, $\boldsymbol{\mathcal{Y}}_{-i}$ with realizations $\mathbf{y}_{-i} \in \mathcal{Y}^{N-1}$ are the decisions of player i's competitors.

Decisions are contingent on two types of state variables. First, all firms observe a vector of state variables $\boldsymbol{\mathcal{S}} = [\boldsymbol{\mathcal{S}}'_1, \ldots, \boldsymbol{\mathcal{S}}'_N]'$ with realizations $\mathbf{s} = [\mathbf{s}'_1, \ldots, \mathbf{s}'_N]' \in \times_{i \in \mathcal{N}} \mathcal{S}_i$ that are common-knowledge, with some elements of $\boldsymbol{\mathcal{S}}_i$ potentially common across i's. Such state variables include population density, distance to closest large population centre, existing stocks of transceivers, spectrum licenses ownership and other common-knowledge variables that may affect providers' payoffs. For instance, it would also include information about transceivers' technology, which varies across providers, and some possible restrictions related to spectrum compatibility. The effect of different agreements (e.g., network sharing, tower sharing and domestic roaming) could also be included in $\boldsymbol{\mathcal{S}}$. In fact, there is evidence that providers know at least some information about agreements between their competitors. For example, Wind, another of the new entrants in the industry, complained to the regulator that it was paying unfair domestic roaming rates compared to rates that incumbents charged among themselves (Wire Report, 2014).

Second, for each player, there is a scalar state variable $\mathcal{E}_i$ with realization $\varepsilon_i \in \mathbb{R}$ that is player i's private information about its payoffs. For instance, a firm may have an agreement with the property management of a high-rise building that makes it relatively more profitable to install a transceiver at this location. The introduction of such private information in the model makes this game of simultaneous decisions a game of incomplete information. Let $G_{\mathcal{E}_i}(\cdot)$ be the cumulative distribution function of $\mathcal{E}_i$.

Let the function $\pi_i(\cdot) : \mathcal{Y}^N \times \mathcal{S}_i \times \mathbb{R} \mapsto \mathbb{R}$ be referred to as player i's payoff function. More precisely, $\pi_i(y_i, \mathbf{y}_{-i}, \mathbf{s}_i, \varepsilon_i)$ is player i's payoff for choosing y_i, when the other players choose $\mathbf{y}_{-i}$ and the realized state corresponds to $(\mathbf{s}_i, \varepsilon_i)$. The following assumptions, which are commonly used in the literature, are maintained on state variables and payoff functions.

Assumption 1 (State variables and payoffs)**.** *(i)* $\boldsymbol{\mathcal{S}}$, $\mathcal{E}_1$, …, $\mathcal{E}_N$ are mutually independent. *(ii)* $G_{\mathcal{E}_1}(\cdot), \ldots, G_{\mathcal{E}_N}(\cdot)$ are common-knowledge to all players and they are absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$. *(iii)* $\pi_1(\cdot)$, …, $\pi_N(\cdot)$ are common-knowledge to all players.

Some restrictions are imposed on player i's payoff function to ensure that its expected payoffs are globally concave in y_i for any $\mathbf{s}_i$ and ε_i. Let firm i's payoff function be given by:

$$\pi_i\left(y_i, \mathbf{y}_{-i}, \mathbf{s}_i, \varepsilon_i\right) = y_i\left[\zeta_i\left(\mathbf{y}_{-i}, \mathbf{s}_i\right) + \varepsilon_i\right] - \bar{\kappa}_i\left(y_i\right) \tag{1}$$

where strategic interactions are captured by the index function $\zeta_i(\cdot)$.

One should interpret $y_i\left[\zeta_i\left(\mathbf{y}_{-i}, \mathbf{s}_i\right) + \varepsilon_i\right]$ as the total revenue and $\bar{\kappa}_i(y_i)$ as the total cost associated with installing y_i transceivers for player i. Importantly, this specification is such that the cost of installing transceivers is allowed to vary with the number of transceivers. However, this cost does not depend on other players' decisions nor the state variables, which only affect revenues.

The timing of the game should be understood as follows. First, state variables are realized. Then, players simultaneously choose an action from their choice set. By Assumption 1, player i's information set at the time of simultaneous decisions is given by:

$$\mathcal{I}_i = \left\{\mathbf{s}, \varepsilon_i, \left\{\pi_n(\cdot)\right\}_{n\in\mathcal{N}}, \left\{G_{\mathcal{E}_n}(\cdot)\right\}_{n\in\mathcal{N}}\right\}. \tag{2}$$

In particular, players use their knowledge of the payoff functions and the distributions of the private information state variables to form beliefs about their opponents' decisions.

Player i's strategy is a function, denoted $\rho_i(\cdot)$, that maps the information set to the choice set, i.e. $\rho_i(\cdot) : \mathcal{I}_i \mapsto \mathcal{Y}$. The conditional choice probability (CCP) of player i choosing $y_i \in \mathcal{Y}$ at a given realization of the common-knowledge state variables $\mathbf{s}$ is therefore equal to:

$$P_i\left(y_i | \mathbf{s}\right) = \int \mathbb{1}\left\{\rho_i\left(\mathcal{I}_i\right) = y_i\right\} dG_{\mathcal{E}_i}\left(\varepsilon_i\right) \tag{3}$$

which can be interpreted as the beliefs formed by player i's opponents regarding player i's decision given strategy $\rho_i(\cdot)$. Let $\mathbf{P}_i(\mathbf{s})$ be a $(J+1)$-dimensional vector collecting player i's CCPs with each element corresponding to a different action, i.e. $\mathbf{P}_i(\mathbf{s}) \equiv \left[P_i(0|\mathbf{s}), \ldots, P_i(J|\mathbf{s})\right]'$, and collect these vectors of CCPs in a $N(J+1)$-dimensional vector $\mathbf{P}(\mathbf{s}) \equiv \left[\mathbf{P}_1(\mathbf{s})', \ldots, \mathbf{P}_N(\mathbf{s})'\right]'$. Similarly as before, $\mathbf{P}_{-i}(\mathbf{s})$ would be the $(N-1)(J+1)$-dimensional vector collecting the vectors of CCPs for all players except player i.

Because player i does not know $\mathbf{y}_{-i}$ at the time of decision, its strategy is based on its expected payoffs. $\mathbf{P}_{-i}(\mathbf{s})$ is used to define the expected payoff of player i choosing y_i given $\mathbf{s}$ and

ε_i. Defining the expected index as $\zeta_i^{\mathbf{P}}(\mathbf{s}) \equiv \sum_{\mathbf{y}_{-i} \in \mathcal{Y}^{N-1}} P(\mathbf{y}_{-i} | \mathbf{s}) \, \zeta_i(\mathbf{y}_{-i}, \mathbf{s}_i)$, the corresponding expected payoff is:

$$\pi_i^{\mathbf{P}}(y_i, \mathbf{s}, \varepsilon_i) = y_i \left[\zeta_i^{\mathbf{P}}(\mathbf{s}) + \varepsilon_i\right] - \bar{\kappa}_i(y_i). \tag{4}$$

Let $\kappa_i(j) \equiv \bar{\kappa}_i(j) - \bar{\kappa}_i(j-1)$. In this case, player i maximizing its expected payoffs chooses:

$$y_i = \begin{cases} 0 & \Leftrightarrow \quad \zeta_i^{\mathbf{P}}(\mathbf{s}) + \varepsilon_i \leq \kappa_i(1) \\ 0 < j < J & \Leftrightarrow \quad \kappa_i(j) < \zeta_i^{\mathbf{P}}(\mathbf{s}) + \varepsilon_i \leq \kappa_i(j+1) \\ J & \Leftrightarrow \quad \zeta_i^{\mathbf{P}}(\mathbf{s}) + \varepsilon_i > \kappa_i(J). \end{cases} \tag{5}$$

This choice rule has a natural economic interpretation. While $\zeta_i^{\mathbf{P}}(\mathbf{s}) + \varepsilon_i$ can be interpreted as the expected marginal revenue (constant across transceivers), $\kappa_i(y_i)$ is the marginal cost of installing y_i transceivers. The optimal number of transceivers is such that the expected marginal revenue is greater than the marginal cost of the last transceiver installed and smaller than the marginal cost of installing more transceivers.

For ε_i following the logistic distribution, the CCPs have the familiar ordered logit expression:

$$P_i(j|\mathbf{s}) = \begin{cases} \Lambda\left(\kappa_i(1) - \zeta_i^{\mathbf{P}}(\mathbf{s})\right), & \text{for} \quad j = 0 \\ \Lambda\left(\kappa_i(j+1) - \zeta_i^{\mathbf{P}}(\mathbf{s})\right) - \Lambda\left(\kappa_i(j) - \zeta_i^{\mathbf{P}}(\mathbf{s})\right), & \text{for} \quad 0 < j < J \\ 1 - \Lambda\left(\kappa_i(J) - \zeta_i^{\mathbf{P}}(\mathbf{s})\right), & \text{for} \quad j = J \end{cases} \tag{6}$$

where $\Lambda(c) = \exp\{c\} / [1 + \exp\{c\}]$. The right-hand side of (6) is a function of $\mathbf{P}_{-i}(\mathbf{s})$ which will be denoted $\varphi_i(j, \mathbf{s}, \mathbf{P}_{-i}(\mathbf{s}))$ and is interpreted as player i's best response to its beliefs regarding its opponents' actions, i.e. $\mathbf{P}_{-i}(\mathbf{s})$. Let $\boldsymbol{\Psi}(\mathbf{s}, \mathbf{P}(\mathbf{s}))$ be the $N(J+1)$-dimensional vector collecting these best-response functions for all actions and all players organized in the same order as in $\mathbf{P}(\mathbf{s})$, i.e.:

$$\boldsymbol{\Psi}(\mathbf{s}, \mathbf{P}(\mathbf{s})) = \left[\varphi_1(0, \mathbf{s}, \mathbf{P}_{-1}(\mathbf{s})), \ldots, \varphi_N(J, \mathbf{s}, \mathbf{P}_{-N}(\mathbf{s}))\right]'. \tag{7}$$

It follows that:

$$\mathbf{P}(\mathbf{s}) = \boldsymbol{\Psi}(\mathbf{s}, \mathbf{P}(\mathbf{s})) \tag{8}$$

and $\mathbf{\Psi}(\mathbf{s},\cdot):[0,1]^{N(J+1)} \mapsto [0,1]^{N(J+1)}$ is henceforth referred to as the best-response mapping.

Defining a Bayesian-Nash Equilibrium (BNE) in the probability space is convenient to analyze equilibrium existence and potential multiplicity (Milgrom and Weber, 1985). Definition 1 is the definition of equilibrium that is used throughout the article.

Definition 1 (Bayesian-Nash Equilibrium in pure strategies)**.** For a given realization of common-knowledge state variables $\mathbf{s}$, a pure strategy BNE in the probability space is a vector of CCPs, $\mathbf{P}^*(\mathbf{s})$, such that $P_i^*(y_i|\,\mathbf{s}) = \varphi_i\left(y_i, \mathbf{s}, \mathbf{P}_{-i}^*(\mathbf{s})\right)$ for any $y_i \in \mathcal{Y}$ and any $i \in \mathcal{N}$.

In other words, a BNE in pure strategies is a fixed point of the best-response mapping, i.e. $\mathbf{P}^*(\mathbf{s}) = \mathbf{\Psi}(\mathbf{s}, \mathbf{P}^*(\mathbf{s}))$, such that each player's beliefs are consistent with their opponents'. As $\mathbf{\Psi}(\mathbf{s},\cdot)$ maps a compact set to itself and it is continuous in $\mathbf{P}(\mathbf{s})$, equilibrium existence follows from Brouwer's fixed-point theorem. However, equilibrium uniqueness is not guaranteed.

Let $\mathcal{T}$ be a random variable with realization $\tau \in \mathcal{T}(\mathbf{s})$ indicating which equilibrium is played at a specific realization $\mathbf{s}$.[10] The distribution of $\mathcal{T}$ conditional on $\mathbf{s}$, which will be denoted $\lambda(\tau|\,\mathbf{s})$, should be interpreted as an equilibrium selection mechanism. The following assumption is maintained for the rest of the article.

Assumption 2 (Equilibrium selection mechanism)**.** $\mathcal{T}$, $\mathcal{E}_1$, ..., $\mathcal{E}_N$ are mutually independent. Moreover, $\mathcal{T}(\mathbf{s})$ is finite and discrete, such that $\lambda(\tau|\,\mathbf{s})$ is a probability mass function.

Assumption 2 is also maintained by Grieco (2014) and Aguirregabiria and Mira (2019), among others. It has two important consequences. First, it ensures that players do not learn about their opponents' private information through equilibrium selection. Second, it rules out continua of equilibria.

3.2 The econometric model

The main difference between the economic model presented above and the one that I will estimate is that I only observe *some* of the state variables that are known to all players. More formally, $\forall i \in \mathcal{N}$, let $\boldsymbol{\mathcal{S}}_i = [\boldsymbol{\mathcal{X}}_i', \mathcal{V}_i]'$. Let $\boldsymbol{\mathcal{X}} = [\boldsymbol{\mathcal{X}}_1', \ldots, \boldsymbol{\mathcal{X}}_N']'$ with realizations $\mathbf{x} = [\mathbf{x}_1', \ldots, \mathbf{x}_N']' \in \times_{i\in\mathcal{N}}\mathcal{X}_i$ be the common-knowledge state variables observable to the econometrician. $\boldsymbol{\mathcal{V}} = [\mathcal{V}_1, \ldots, \mathcal{V}_N]'$ with realizations $\boldsymbol{\nu} = [\nu_1, \ldots, \nu_N]' \in \mathbb{R}^N$ is left as part of the

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[10] Rigorous notation would use $\mathcal{T}(\mathbf{s})$ and $\tau(\mathbf{s})$, but the argument $\mathbf{s}$ is dropped when referring to the random variable and its realization for simplicity.

unobservables. By including this vector of unobservables, one introduces some asymmetry between the researcher's and the players' information regarding common-knowledge payoff-relevant state variables: $\boldsymbol{\mathcal{V}}$ therefore has the same interpretation as unobservables in games of complete information.

In fact, assuming that I know as much about a given mobile service provider's relevant state variables as its competitors do would be a strong assumption in this setting. I observe population density, distance to closest large population centre, firms' initial stocks of transceivers and spectrum licenses ownership, which are variables that are included in $\boldsymbol{\mathcal{X}}$. These variables are only a subset of common-knowledge state variables that affect providers' payoffs. The variables $\boldsymbol{\mathcal{V}} = [\mathcal{V}_1, \ldots, \mathcal{V}_N]'$ would capture the effect of the remaining common-knowledge information on each player's payoffs. For instance, providers know more about how the technology used and issues related to spectrum compatibility may affect their competitors' payoffs than what is observed in the data. As already mentioned above, some providers having complained to the regulator about agreements between competitors being unfair also suggests that at least parts of these agreements should be treated as common-knowledge information. These are just a few examples of reasons to believe that mobile service providers know more about their competitors' payoff-relevant state variables than what can be teased out of observable data.

Player i's payoff can therefore be written as:

$$\pi_i \left(y_i, \mathbf{y}_{-i}, \mathbf{x}_i, \nu_i, \varepsilon_i \right) = y_i \left[\zeta_i \left(\mathbf{y}_{-i}, \mathbf{x}_i, \nu_i \right) + \varepsilon_i \right] - \bar{\kappa}_i \left(y_i \right). \tag{9}$$

The corresponding expected payoff is:

$$\pi_i^{\mathbf{P}} \left(y_i, \mathbf{x}, \boldsymbol{\nu}, \varepsilon_i \right) = y_i \left[\zeta_i^{\mathbf{P}} \left(\mathbf{x}, \boldsymbol{\nu} \right) + \varepsilon_i \right] - \bar{\kappa}_i \left(y_i \right) \tag{10}$$

where $\zeta_i^{\mathbf{P}} \left(\mathbf{x}, \boldsymbol{\nu} \right) \equiv \sum_{\mathbf{y}_{-i} \in \mathcal{Y}^{N-1}} P \left(\mathbf{y}_{-i} \middle| \mathbf{x}, \boldsymbol{\nu} \right) \zeta_i \left(\mathbf{y}_{-i}, \mathbf{x}_i, \nu_i \right)$. The resulting CCPs are as in (6), except that $\zeta_i^{\mathbf{P}} \left(\mathbf{x}, \boldsymbol{\nu} \right)$ replaces $\zeta_i^{\mathbf{P}} \left(\mathbf{s} \right)$. When estimating the model, I consider the following parametric specification for $\zeta_i \left(\cdot \right)$:

$$\zeta_i \left(\mathbf{y}_{-i}, \mathbf{x}_i, \nu_i; \boldsymbol{\theta}_i \right) = \left[\mathbf{x}_i', \nu_i \right] \boldsymbol{\beta}_i + \sum_{n \in \tilde{\mathbb{N}} \setminus \{i\}} \delta_{in} y_n \tag{11}$$

where $\tilde{\mathbb{N}}$ is the set of providers owning spectrum licenses and $\boldsymbol{\theta}_i = \left[\boldsymbol{\beta}_i', \left\{ \delta_{in} \right\}_{n \neq i} \right]'$.

Once again, a BNE is defined by using CCPs given $\mathbf{x}$ and $\boldsymbol{\nu}$. The current econometric model therefore allows for three types of unobservables from the econometrician's point of view: (i) private information ($\boldsymbol{\varepsilon}$); (ii) common-knowledge payoff-relevant heterogeneity ($\boldsymbol{\nu}$); and (iii) the variable indexing which equilibrium is realized (τ). In other words, this model allows for a flexible information structure and multiple equilibria being realized in the data, along the lines of Grieco (2014) and Aguirregabiria and Mira (2019).

It is worth mentionning that there exists other modelling approaches that would have delivered a flexible information structure and, in some cases, also allowed for multiple equilibria. Magnolfi and Roncoroni (2017)'s model based on Bayes Correlated Equilibria is a very appealing approach as it allows information sets to vary across realizations of the game and to be asymmetric among players. While the partial identification results that the authors derive are valuable and applicable in other settings, the large dimensionality of the parameter space in the current application, where player-specific parameters are used to capture important asymmetries in providers' payoff functions, would make it computationnally burdensome. In their setting, as in Tamer (2003) and in Grieco (2014), point identification can be achieved when player-specific regressors are available. Unfortunately, as it will be emphasized in Section 5, such regressors are not available for the new entrant in the current application.

Alternatively, correlation in unobserved state variables could have been modelled via Wan and Xu (2014)'s approach. However, their identification result also requires player-specific regressors and it implicitly assumes that there is a single equilibrium that is realized in the data.

The main reason why I opted for the current approach as opposed to Magnolfi and Roncoroni (2017)'s or Wan and Xu (2014)'s is that I could relatively easily extend the identification results available in the BNE literature to address the lack of player-specific observable regressors satisfying the exclusion restriction required to identify strategic interactions. A detailed discussion of this extension is presented in Section 5.

3.3 Bids as predetermined outcomes

An important component of the econometric model that I propose is the introduction of player-specific predetermined outcomes, denoted $\boldsymbol{\mathcal{Z}} = [\boldsymbol{\mathcal{Z}}_1', \dots, \boldsymbol{\mathcal{Z}}_N']'$, which will be used to recover common-knowledge payoff-relevant unobserved heterogeneity. In the current empirical application, I use providers' bids for spectrum licenses as such predetermined outcomes. In Assumption 3, I state conditions that must be satisfied by $\boldsymbol{\mathcal{Z}}$ to be able to recover unobservable heterogeneity separately from the variable indexing which equilibria are realized in the data. I then argue that these conditions are satisfied by bids for spectrum licenses.

Assumption 3 (Predetermined outcomes)**.** (i) $\boldsymbol{\mathcal{Z}}$ still depends on $\boldsymbol{\mathcal{V}}$ after conditionning on $\mathbf{x}$, i.e. $\boldsymbol{\mathcal{Z}} \not\perp \boldsymbol{\mathcal{V}} | \mathbf{x}$. (ii) $\boldsymbol{\mathcal{Z}}$ is conditionally independent of $\mathcal{T}$ given $\mathbf{x}$ and $\boldsymbol{\nu}$, i.e. $\boldsymbol{\mathcal{Z}} \perp \mathcal{T} | \mathbf{x}, \boldsymbol{\nu}$.

Assumption 3(i) simply states that predetermined outcomes must be informative about common-knowledge payoff-relevant unobserved heterogeneity. Assumption 3(ii) is the condition that must be satisfied by predetermined ouctomes if one hopes to recover these unobservables separately from the variable indexing which equilibria are realized in the data. This condition must therefore be satisfied if one wants to leverage both variation in $\boldsymbol{\mathcal{V}}_{-i}$ and $\mathcal{T}$ to identify strategic interactions.

Using spectrum auctions when defining the timing of the game has three advantages. First, because providers must bid for licenses prior to setting up a network, they must plan their transceivers' locations, at least to some extent, before knowing their competitors' decisions. In other words, the need to simultaneously plan transceivers' locations before the spectrum is auctioned argues in favour of modelling the investment problem as one involving simultaneous decisions. Second, the requirement of spectrum licenses also directly addresses an important challenge associated with the estimation of empirical games of market entry, i.e. the need to determine the pool of potential entrants.

The third and most important advantage of focusing on an auction-specific game of simultaneous decisions is that providers' bids can be used as predetermined outcome variables to control for player-specific unobserved heterogeneity. Even if firms' bidding behaviour is not interpreted through a structural model, bids should reflect players' valuations of operating a network including common-knowledge payoff-relevant unobservable information. As a result, players' bids depend on $\boldsymbol{\mathcal{V}}$ conditional on realizations of $\boldsymbol{\mathcal{X}}$, therefore satisfying Assumption 3(i). Moreover, these outcome variables are determined before transceivers' location decisions. In particular, at the bidding stage, firms do not know who will successfully acquire which frequency blocks and, therefore, do not know which equilibrium will be played. A nice feature of the AWS1-2008 auction that is used in the current empirical application is that new players entered the industry as a result of spectrum set-aside. Incumbents were then playing the game of transceivers' locations against new competitors, which arguably further weakened their ability to predict the equilibrium to be played. For this reason, bids are considered to be conditionally independent of the equilibrium selection variable, as required by Assumption 3(ii).

Notice that Assumption 3(ii) does not preclude the equilibrium of the game to be *ex post* related with predetermined outcomes. In fact, one would expect it to be: both $\boldsymbol{\mathcal{Z}}$ and $\boldsymbol{\mathcal{T}}$ are allowed to depend on $\boldsymbol{\mathcal{X}}$ and $\boldsymbol{\mathcal{V}}$. Here, timing is key. Because providers do not know the equilibrium that will be played at the transceivers' location game, their bids do not depend on this equilibrium. Of course, the equilibrium will depend on providers' bidding behaviour as the auction determines the potential entrants of the game.

Auctioned licenses allow service providers to use a block of frequencies for a given geographic area called a tier region, which is defined by the federal government. The size of the geographic area covered by a license is determined by which tier the license belongs to. Industry Canada uses three different types of tiers when auctioning mobile spectrum licenses. Each type of tiers partitions the whole country in regions of different sizes. There are more than one blocks of spectrum in each of these regions, such that competitors simultaneously operate in the same geographic area, but on different frequencies.

I consider the following reduced-form model for provider i's bid on block of frequency k in a given geographic region:[11]

$$z_{ik} = \mathbf{x}'\boldsymbol{\alpha}_i + \sum_{n \in \mathcal{N} \setminus \{i\}} \Delta_{in} z_{nk} + \gamma_{ik} + \nu_i + \xi_{ik} \tag{12}$$

where γ_{ik} is block k's fixed effect on player i's bids (constant across different geographic regions) and ξ_{ik} is some region-specific idiosyncratic shock assumed to be mean 0 across blocks. Notice that the full vector $\mathbf{x} = [\mathbf{x}_1', \ldots, \mathbf{x}_N']'$ is included as regressors for player i's bids. Furthermore, player i's bids are allowed to depend on its competitors'. This is justified by the fact that the

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[11] Proposing a structural bidding model for spectrum auction falls outside the scope of the current article. Conventional econometric auction theory does not readily applies in this context due to providers simultaneaously bidding on different licenses and potential complementarities between these licenses. In fact, recent contributions in this literature typically avoid having to directly estimate an auction model. For instance, Fox and Bajari (2013) estimate a matching model and Xiao and Yuan (2018) interpret bidding decisions via a dynamic game of market entry.

AWS1-2008 auction was a multiple-round simultaneous ascending auction in which, at the end of each round, bidders knew there competitors' bids. One therefore expects competitors' bids to affect a provider's bidding behaviour.

Then, letting $\bar{z}_i \equiv K^{-1} \sum_{k=1}^{K} z_{ik}$ be player i's average bid over the different K blocks in the given geographic region and defining $\bar{\gamma}_i$ and $\bar{\xi}_i$ similarly, we get:

$$\bar{z}_i = \mathbf{x}'\boldsymbol{\alpha}_i + \sum_{n \in \mathcal{N} \backslash \{i\}} \Delta_{in} \bar{z}_n + \bar{\gamma}_i + \nu_i + \bar{\xi}_i \xrightarrow{p} \mathbf{x}'\boldsymbol{\alpha}_i + \sum_{n \in \mathcal{N} \backslash \{i\}} \Delta_{in} \mathrm{E}\left[z_n\right] + \mathrm{E}\left[\gamma_i\right] + \nu_i. \tag{13}$$

In other words, player i's realized common-knowledge unobserved heterogeneity asymptotically (when $K \to \infty$) corresponds to the residual of a linear regression of player i's average bid in the geographic region of interest on a constant, the observed state variables and competitors' bids. In other words, provider i's common-knowledge payoff-relevant unobserved heterogeneity is assumed to correspond to the variation in player i's bids that is not explained by all observed common-knowledge state variables nor its competitors' bids, and that is constant across blocks of spectrum. Therefore, unobservable information that consistently affects bids over different blocks is captured by ν_i.

Of course, this residual interpretation is not ideal, but it is still preferable than simply ignoring common-knowledge unobserved heterogeneity in an industry where it should be a first-order concern. To the extent that providers take into account how technology, spectrum compatibility, agreements, etc. may affect their payoffs when bidding for spectrum and that this information is not captured by the other regressors included in (12), variation in $\boldsymbol{\nu}$ recovered from providers' bids should reflect variation in common-knowledge unobserved heterogeneity relevant for transceivers' locations. The timing of the realisation of bids also supports $\boldsymbol{\nu}$ being common-knowledge when the transceivers' location game is played. As the auction is completed prior to the transceivers' location game, player-specific payoff-relevant information recovered in $\boldsymbol{\nu}$ is revealed at the auction stage and is common-knowledge when providers decide where to install their transceivers. In Section 6, when I estimate the model, I do find evidence that residuals estimated from bids are capturing significant determinants of transceivers' locations not controlled for by observed state variables.

4 Data

The model just described will be estimated for $N = 4$ Canadian mobile service providers: three national incumbents (Rogers, Bell and Telus) and one new entrant (Vidéotron). I consider location decisions for transceivers operating on frequencies allocated in the AWS1-2008 auction. I now briefly describe the data that will be used for estimation.

4.1 Transceivers

Data on transceivers' locations are constructed using the Technical and Administrative Frequency Lists maintained by Industry Canada. I observe a 2016 snapshot of all transceivers associated with mobile services in Canada. For each transceiver, I observe the owner, its geographic location and the frequencies on which it operates, among other characteristics.

Frequencies are used to link transceivers to different auctions. All transceivers operating on frequencies allocated prior to the AWS1-2008 auction are considered to be stocks at this auction; transceivers operating on frequencies allocated during this auction are additions. Implicitly, as I do not observe the timing of transceivers' installations, I am assuming that all transceivers operating on frequencies allocated during a given auction are installed simultaneously. Therefore, I am ignoring some of the dynamics that could be associated with delays in towers' constructions and/or spectrum transfers between providers.

Figure 1 in Section 2 shows the location of each player's transceivers operating on frequencies allocated up to the AWS1-2008 auction. In other words, these maps include both stocks (for incumbents) and additions of transceivers associated with this auction.

4.2 Isolated markets

As it is common when estimating games of market entry, I consider location decisions in different isolated markets as being different repetitions of the transceivers' location game. I use population centres, i.e. relatively more densely populated areas defined by Statistics Canada, as isolated markets. These markets are indexed by $m \in \{1, 2, \ldots, M\}$.

A nice feature of population centres is that they are physically isolated and surrounded by rural areas. Of course, it is likely that there is some dependence between transceivers' locations within and around a given population centre. For instance, providers may want to ensure that their subscribers can use their mobile devices when they travel in rural areas around where they live or work. By focusing on population centres, i.e. ignoring rural areas, I avoid having to model this potential correlation between population centres and their surroundings.

Nonetheless, one may be concerned about potential correlation in transceivers' locations between large population centres and satellite smaller population centres. For instance, correlation could arise due to people working in a large population centre, but living in a smaller one. However, conditional on the proximity of a large population centre (and other observable characteristics as well as recovered common-knowledge unobservables), transceivers' locations in small population centres are arguably independent.[12]

I therefore decided to focus the analysis on small population centres, in order to make the assumption of small isolated markets more credible. Small population centres are defined by Statistics Canada as areas with a population between 1,000 and 30,000 people and some minimum population density. I restrict the analysis to population centres located in Ontario and Quebec because these are the provinces where all 4 players own spectrum licenses. Overall, there are $M = 459$ small population centres in these two provinces.

Geographic coordinates are used to match transceivers with the market they belong to. For each market, one can count how many transceivers are associated with each provider. A non trivial share of markets are such that there is no transceiver within the boundary of the population centre, but some right outside of it. Both for stocks and additions of transceivers, I therefore consider all transceivers within a 5 kilometres buffer zone around a population centre as being part of that population centre.

Finally, as mentioned earlier, an important factor determining the need for more transceivers in a given area is the population density in this area. This variable is also observed for each population centre.

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[12] For instance, Rogers install transceivers in Rougemont and in Val-David (two small population centres around Montreal) because they are close to Montreal; not because it installs transceivers in Bromont (another small population centre near Montreal).

4.3 Bids

Frequencies from 8 different blocks (3 of them being set aside for new entrants and regional players) were allocated during the AWS1-2008 auction.[13] A total of 282 licenses were allocated, with winning bids summing to approximately CAD$ 4.3 billion. This auction was the first one in which Vidéotron obtained licenses required to set up a network.

Population centres are matched with their corresponding tier regions. As tiers are larger than population centres, there are several markets in each tier region. Furthermore, as the analysis focuses on small population centres, I had to assign a share of tier-level bids (which also include medium and large population centres) to each market of interest. I used population-weighted bids, i.e. a given population centre's share of bids for a license is the share of this market's population compared with the population of all population centres covered by the tier. This assignment is sensible because providers typically bid higher for licenses covering more populated area.

I obtained each players' bids in each round of the AWS1-2008 auction from Industry Canada. Two features of the bids are worth mentioning. First, I observe both winning and non-winning bids. In particular, observing non-winning bids is helpful to recover information about unobservable heterogeneity for all players, i.e. not only winners. Second, firms typically bid on multiple blocks of spectrum in a given tier. The average of players' highest population-weighted bids for each license that covers a given market are used to construct predetermined outcomes.

Finally, By combining license allocations resulting from the auction with public information regarding licenses' transfers, I construct variables indicating whether each provider owns some spectrum license in each market. Licence ownership is used to determine potential entrants.

4.4 Summary statistics

[Insert Table 1 here.]

Table 1 summarizes the information available at the market level, for each provider. A few observations are worth pointing out.

First of all, there are important asymmetries between the three incumbents (Rogers, Bell

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[13] For detailed information about the AWS1-2008 auction, see Industry Canada (2007) and Industry Canada's webpage on this specific auction (`http://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/h_sf08891.html`).

and Telus). Telus is adding very few transceivers relatively to Rogers and Bell, especially in Ontario. Moreover, compared to Rogers, Bell and Telus typically install transceivers in fewer markets, but tend to install more of them in markets where they add some. A similar remark can be made regarding stocks of transceivers. This behaviour is coherent with Bell and Telus sharing their network.[14]

Second, Vidéotron barely installs transceivers in Ontario even if it owns licenses covering 35% of the small population centres in this province. This is an example of a new entrant failling to use some of the set-aside licenses it acquired during the auction. Nonetheless, in Quebec, Vidéotron installs transceivers in many more markets than Bell and Telus do, but still less than Rogers. It also installs more than 10 transceivers in a larger number of markets compared to the incumbents. Notice that, besides Vidéotron in Ontario, all providers own at least one license covering all markets considered in the analysis.

Finally, the main objective of spectrum set-asides is to help new entrants acquiring licenses at lower prices. Set-asides therefore partially explain why Vidéotron's average population-weighted bids tend to be lower than the incumbents'. Notice, however, that the difference observed in Ontario is also due to the fact that there are some markets for which Vidéotron decided not to bid on any license available.

5 Identification

Identifying strategic interactions in an empirical game such as the one described above is complicated by decisions being made simultaneously. The problem can be understood as follows. Under the parametric specification stated in equation (11), expected payoffs depend on strategic interactions through:

$$\zeta_i^{\mathbf{P}}\left(\mathbf{x},\boldsymbol{\nu};\boldsymbol{\theta}_i\right)=\left[\mathbf{x}_i',\nu_i\right]\boldsymbol{\beta}_i+\sum_{n\in\tilde{\mathcal{N}}\backslash\{i\}}\delta_{in}y_nP_n\left(y_n|\,\mathbf{x},\boldsymbol{\nu}\right). \tag{14}$$

Separate identification of $\boldsymbol{\beta}_i$ from strategic interactions $\{\delta_{in}\}_{n\in\mathcal{N}\backslash\{i\}}$ requires variation in competitors' CCPs, i.e. $P_n\left(y_n|\,\mathbf{x},\boldsymbol{\nu}\right)$ for $n\neq i$, that keeps $\mathbf{x}_i,\nu_i$ fixed. In other words, if $\mathbf{x}_i,\nu_i$ do not vary across individuals, the separate identification of $\boldsymbol{\beta}_i$ and $\{\delta_{in}\}_{n\in\mathcal{N}\backslash\{i\}}$ is entirely driven

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[14] While Telus seems to be taking advantage of Bell's network in the eastern part of Canada, the roles are reversed in the western part of the country. See Figure 1.

by functional form assumptions.

Both in games of complete (e.g., Tamer, 2003) and incomplete information (see for instance the identification arguments of Pesendorfer and Schmidt-Dengler, 2003; Bajari, Hong, Krainer, and Nekipelov; 2010), a natural source of variation that is commonly leveraged for separate identification is an exclusion restriction in observable player-specific regressors. In other words, some elements of $\mathbf{x}_i$ are assumed to be specific to player i. As a result, these regressors only affect competitors' expected payoffs through their beliefs regarding player i's decision, i.e. its CCPs.

In the current empirical application, competitors' stocks of transceivers can be used as player-specific variables satisfying the usual exclusion restriction, in the same spirit as other studies have used firms' incumbency status. The argument here is that competitors' decisions are sufficient statistics for their stocks of transceivers. Unfortunately, Vidéotron's stocks are zero in all markets. There is therefore no observable regressor generating an exogenous variation that would allow one to identify the new entrant's impact on incumbents' location decisions. Such strategic interactions are key to assess incumbents' equilibrium responses to any counterfactual policy that would encourage the new entrant's investments.[15]

In other cases where excluded player-specific regressors are not available, it has been suggested to exploit the identifying power generated by multiple equilibria being realized in the data. This reasonning can be found in Sweeting (2009), De Paula and Tang (2012) and Aradillas-Lopez and Gandhi (2016). If one can identify equilibrium-specific CCPs, the argument is very similar to using variation in player-specific regressors satisfying the required exclusion restriction: multiple equilibria shift CCPs for a given realisation of observed state variables. One limitation of this approach is that it is applicable to games of (pure) incomplete information. It relies on the important assumption that the only source of correlation between players' decisions, conditional on observed state variables, must be due to equilibria multiplicity. When one is especially concerned with common-knowledge unobserved heterogeneity across individuals, as in the current setting, identification arguments based on multiple equilibria are therefore not directly applicable.

In fact, the need to take into account common-knowledge unobserved heterogeneity is the

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[15] Notice that, while bids for spectrum licenses are observed for all players including new entrants, they cannot be used as regressors satisfying the exclusion restriction, as $\boldsymbol{\mathcal{Z}}_i$ is not even included in player i's payoffs. This interpretation is coherent with bids being sunk costs when building a mobile network.

main motivation for recovering it from predetermined outcomes. There are two important advantages of doing so. First, once one is able to control for common-knowledge unobserved heterogeneity as well as observed state variables, then players' decisions should be independent if there is a single equilibrium realized in the data. In other words, conditional on the unobserved heterogeneity, one can use variation in equilibria to identify strategic interactions.

The second important advantage is that player-specific common-knowledge payoff-relevant unobserved heterogeneity satisfies the exclusion restriction needed to identify strategic interactions. To the extent that ν_i in (11) varies across i's, recovered $\boldsymbol{\nu}_{-i}$'s represent a potential source of variation that will help to identify parameters measuring how competitors' decisions affect i's payoffs.

It is worth noting that the idea of using predetermined outcomes to recover unobservables has been proposed in other settings as well. A recent example from the auction literature is Roberts (2013) who uses reserve prices to control for auctioned objects' heterogeneity that is known to all bidders, but not to the econometrician. Another well-known example in industrial organization is Olley and Pakes (1996) who use investment in capital to identify unobserved productivity in production function estimation. In education and in labour economics, test scores are often used to recover students' unobserved ability (or more precisely their distribution as in Carneiro, Hansen, and Heckman, 2003). Furthermore, leveraging the identifying power of exogenous variation in unobservable variables shares similarities with Matzkin (2004, 2016)'s unobservable instruments approach. For instance, consider a model with two simultaneous equations where an instrument is available for only one of these equations. Matzkin shows how to recover the unobservable random term from the equation for which a valid instrument is available and uses this unobservable as an instrument for the other equation. As in the argument I propose, this approach takes advantage of some unobservable variables' useful exclusion properties to identify the primitives of the model.

In order to illustrate how both $\boldsymbol{\nu}_{-i}$ and τ helps to identify strategic interactions, notice that Assumption 2 implies that one can write (14) as:

$$\zeta_i^{\mathbf{P}}\left(\mathbf{x}, \boldsymbol{\nu}; \boldsymbol{\theta}_i\right) = \left[\mathbf{x}_i', \nu_i\right] \boldsymbol{\beta}_i + \sum_{n \in \tilde{\mathcal{N}} \setminus \{i\}} \delta_{in} y_n \sum_{\tau \in \mathcal{T}^*(\mathbf{x}, \boldsymbol{\nu})}^{|\mathcal{T}^*(\mathbf{x}, \boldsymbol{\nu})|} P_n\left(y_n \middle| \mathbf{x}, \boldsymbol{\nu}, \tau\right) \lambda\left(\tau \middle| \mathbf{x}, \boldsymbol{\nu}\right) \tag{15}$$

where $|\mathcal{T}^*(\mathbf{x}, \boldsymbol{\nu})|$ is the number of equilibria realized in the data and $\mathcal{T}^*(\mathbf{x}, \boldsymbol{\nu}) \subseteq \mathcal{T}(\mathbf{x}, \boldsymbol{\nu})$. In the worst case scenario where $\mathbf{x}_i$ does not vary at all across i's, different values of $\boldsymbol{\nu}_{-i}$ and τ still shift competitors' CCPs, therefore providing variation that allows one to identify $\{\delta_{in}\}_{n\in\mathcal{N}\setminus\{i\}}$ separately from $\boldsymbol{\beta}_i$.

It should be noted that (15) supposes that one is able to identify equilibrium-specific CCPs conditional on $\mathbf{x}, \boldsymbol{\nu}$. This may be possible, for instance, if one observes a panel in which the same markets are observed over a long period of time. If one is willing to assume that the same equilibrium is played over time (as in Sweeting, 2009), then one could allow markets with the same realized $\mathbf{x}, \boldsymbol{\nu}$ to follow different equilibria.

Unfortunately, in the current empirical application, I observe each market only once. In Section 6, I report estimates for different specifications including some allowing for multiple equilibria in the data. In those cases, CCPs are assumed to be mixed over two "types" of equilibria, regardless of the values of $\mathbf{x}, \boldsymbol{\nu}$. In other words, such specifications are based on an assumption that is stronger than Assumption 2 as the set of equilibria realized in the data and the equilibrium selection mechanism are not allowed to vary with $\mathbf{x}, \boldsymbol{\nu}$.

Of course, if one wants to leverage both unobserved heterogeneity and multiple equilibria to identify strategic interactions, one must be able to recover the former separately from the latter. Here, predetermined outcomes being conditionnally independent of which equilibrium is being played in the game of interest, i.e. Assumption 3(*ii*), is key. Again, providers bidding on licenses without knowing who will be the potential entrants in the transceivers' location game suggests that this assumption is satisfied in the current empirical application.

6 Estimation results

I now turn to the estimation of the econometric model. Transceivers' location decisions are studied through the ordered-response model introduced in Section 3. Following the AWS1-2008 spectrum auction, Rogers, Bell, Telus and Vidéotron simultaneously decide how many transceivers to install in several small population centres. Additions to player i's stocks of transceivers are discretized such that $\mathcal{Y} = \{0, 1, 2\}$ where $j = 1$ corresponds to 1-10 transceivers and $j = 2$ is 11+ transceivers. The estimation proceeds in three stages: the common-knowledge unobserved heterogeneity is recovered and then, a simple two-stage estimator along the lines of Hotz and Miller (1993) is applied, with some specifications allowing for multiple equilibria when estimating reduced-form CCPs. Standard errors of the third stage's estimates are computed from bootstrapped samples to take into account sampling variation from the first two stages. I now describe each step with more details.

Step 1. Player-specific common-knowledge unobservable heterogeneity is recovered from firms' bids. This is done by estimating the model defined by equation (13), more precisely:

$$\bar{z}_{im} = \mathbf{x}_m' \boldsymbol{\alpha}_i + \sum_{n \in \mathcal{N} \setminus \{i\}} \Delta_{in} \bar{z}_{nm} + \bar{\gamma}_i + \nu_{im} \tag{16}$$

by using ordinary least squares separately for each player and recovering the residuals denoted $\hat{\nu}_i$. The variables included in $\mathbf{x}_m$ are: population density, distance to the closest large population centre, incumbents' stocks of transceivers (in natural logarithms after having added 1 to avoid zeros) and a Quebec province fixed effect.

Recovered $\hat{\nu}_i$ being only required in markets where provider i owns a license, the model is estimated using markets for which the dependent variable $\bar{z}_i$ is strictly positive. This restriction is imposed because $\bar{z}_i = 0$ can be interpreted as a corner solution in providers' bidding behaviour which could be rationalized by a non-unique value of ν_i.

Step 2. Given the estimated unobservable heterogeneity, one can construct choice probabilities conditioned on both $\mathbf{x}$ and $\hat{\boldsymbol{\nu}}$. Ideally one would like to use a flexible estimator such as simple non-parametric estimators of conditional probabilities. Unfortunately, the limited sample size forces me to use a parametric specification for these choice probabilities. I use a player-specific ordered logit specification to estimate reduced-form CCPs. Let $h\left(\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\phi}_i\right)$ be a parametric function of $\mathbf{x}$, $\hat{\boldsymbol{\nu}}$ with a vector of parameters $\boldsymbol{\phi}_i$ specific to each player. The variables included in $\mathbf{x}$ are the same as the ones used to recover $\hat{\boldsymbol{\nu}}$. In the current application, $h\left(\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\phi}_i\right)$ is a simple first-order polynomial, but higher orders could be accommodated with a larger number of observations. Letting $\boldsymbol{\eta}_i = \left[\eta_i\left(\text{1-10}\right), \eta_i\left(\text{11+}\right)\right]'$ be threshold parameters (also specific to each player), the reduced-form choice probability of player i choosing $y \in \mathcal{Y}$, which is denoted as $P_i\left(y|\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\phi}_i, \boldsymbol{\eta}_i\right)$, is given by:

$$\begin{cases} \Lambda\left(\eta_i\left(\text{1-10}\right)-h\left(\mathbf{x},\hat{\boldsymbol{\nu}};\boldsymbol{\phi}_i\right)\right), & \text{for 0 transceiver} \\ \Lambda\left(\eta_i\left(\text{11+}\right)-h\left(\mathbf{x},\hat{\boldsymbol{\nu}};\boldsymbol{\phi}_i\right)\right)-\Lambda\left(\eta_i\left(\text{1-10}\right)-h\left(\mathbf{x},\hat{\boldsymbol{\nu}};\boldsymbol{\phi}_i\right)\right), & \text{for 1-10 transceivers} \\ 1-\Lambda\left(\eta_i\left(\text{11+}\right)-h\left(\mathbf{x},\hat{\boldsymbol{\nu}};\boldsymbol{\phi}_i\right)\right), & \text{for 11+ transceivers.} \end{cases} \tag{17}$$

The contribution of market m to the player-specific likelihood function is:

$$L_{im}\left(\boldsymbol{\phi}_i,\boldsymbol{\eta}_i\right)=\prod_{j=0}^{J}P_i\left(j|\mathbf{x}_m,\hat{\boldsymbol{\nu}}_m;\boldsymbol{\phi}_i,\boldsymbol{\eta}_i\right)^{\mathbb{1}\{y_{im}=j\}} \tag{18}$$

and estimates of $\hat{\boldsymbol{\phi}}_i,\hat{\boldsymbol{\eta}}_i$ are computed separately for each provider by (pseudo) maximum likelihood. Let the estimated reduced-form CCPs be denoted $\hat{P}_i\left(y_i|\,\mathbf{x},\hat{\boldsymbol{\nu}}\right)\equiv P_i\left(y_i|\,\mathbf{x},\hat{\boldsymbol{\nu}};\hat{\boldsymbol{\phi}}_i,\hat{\boldsymbol{\eta}}_i\right)$.

In the specifications where I allow for two types of equilibria to be realized in the data, equilibrium-specific CCPs are defined by allowing thresholds $\boldsymbol{\eta}_i$ to vary across $\tau\in\{1,2\}$. Equilibrium-specific CCPs $P_i\left(y|\mathbf{x},\hat{\boldsymbol{\nu}};\boldsymbol{\phi}_i,\boldsymbol{\eta}_i^{\tau}\right)$ are defined as in (17) after replacing $\boldsymbol{\eta}_i$ with $\boldsymbol{\eta}_i^{\tau}$. In this case, contribution of provider i in market m to the likelihood function is:

$$L_{im}\left(\boldsymbol{\phi}_i,\boldsymbol{\eta}_i^1,\boldsymbol{\eta}_i^2,\lambda\right)=\prod_{j=0}^{J}\left[P_i\left(j|\mathbf{x}_m,\hat{\boldsymbol{\nu}}_m;\boldsymbol{\phi}_i,\boldsymbol{\eta}_i^1\right)\lambda+P_i\left(j|\mathbf{x}_m,\hat{\boldsymbol{\nu}}_m;\boldsymbol{\phi}_i,\boldsymbol{\eta}_i^2\right)(1-\lambda)\right]^{\mathbb{1}\{y_{im}=j\}} \tag{19}$$

where λ is the probability corresponding to equilibrium of type 1 being realized. The joint likelihood for all players is maximized using the EM algorithm. As the resulting mixture may generate some local maxima, I initiate the search algorithm at 50 different randomly selected starting values and retain the estimates corresponding to the highest likelihood function. The resulting equilibrium-specific CCPs are denoted $\hat{P}_i\left(y_i|\,\mathbf{x},\hat{\boldsymbol{\nu}},\tau\right)\equiv P_i\left(y_i|\,\mathbf{x},\hat{\boldsymbol{\nu}};\hat{\boldsymbol{\phi}}_i,\hat{\boldsymbol{\eta}}_i^{\tau}\right)$ and $\hat{\lambda}$ is the estimated equlibrium selection mechanism.

Step 3. The marginal revenues' and marginal costs' parameters defining players' choices according to the model presented in Section 3.2 are estimated. To do so, expected payoffs are constructed by using the estimated reduced-form CCPs. Let the expected index function be:

$$\zeta_i^{\hat{\mathbf{P}}}\left(\mathbf{x},\hat{\boldsymbol{\nu}};\boldsymbol{\theta}_i\right)=\left[\mathbf{x}_i',\hat{\nu}_i\right]\boldsymbol{\beta}_i+\sum_{n\in\tilde{\mathbb{N}}\setminus\{i\}}\delta_{in}y_n\hat{P}_n\left(y_n|\,\mathbf{x},\hat{\boldsymbol{\nu}}\right) \tag{20}$$

where $\mathbf{x}_i$ includes all the variables in $\mathbf{x}$, except competitors' stocks of transceivers. Then, one can construct structural CCPs denoted $P_i\left(y|\,\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\theta}_i, \boldsymbol{\kappa}_i\right)$ according to:

$$\begin{cases} \Lambda\left(\kappa_i\left(\text{1-10}\right) - \zeta_i^{\hat{\mathbf{P}}}\left(\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\theta}_i\right)\right), & \text{for 0 transceiver} \\ \Lambda\left(\kappa_i\left(\text{11+}\right) - \zeta_i^{\hat{\mathbf{P}}}\left(\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\theta}_i\right)\right) - \Lambda\left(\kappa_i\left(\text{1-10}\right) - \zeta_i^{\hat{\mathbf{P}}}\left(\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\theta}_i\right)\right), & \text{for 1-10 transceivers} \\ 1 - \Lambda\left(\kappa_i\left(\text{11+}\right) - \zeta_i^{\hat{\mathbf{P}}}\left(\mathbf{x}, \hat{\boldsymbol{\nu}}; \boldsymbol{\theta}_i\right)\right), & \text{for 11+ transceivers.} \end{cases} \tag{21}$$

Player i in market m's contribution to the likelihood function is:

$$L_{im}\left(\boldsymbol{\theta}_i, \boldsymbol{\kappa}_i\right) = \prod_{j=0}^{J} P_i\left(j|\,\mathbf{x}_m, \hat{\boldsymbol{\nu}}_m; \boldsymbol{\theta}_i, \boldsymbol{\kappa}_i\right)^{\mathbb{1}\{y_{im}=j\}} \tag{22}$$

and maximizing the corresponding provider-specific likelihood function delivers the estimates $\hat{\boldsymbol{\theta}}_i, \hat{\boldsymbol{\kappa}}_i$. In specifications allowing for multiple equilibria, equilibrium-specific structural CCPs $P_i\left(y|\,\mathbf{x}, \hat{\boldsymbol{\nu}}, \tau; \boldsymbol{\theta}_i, \boldsymbol{\kappa}_i\right)$ are constructed similarly as above, but using equilibrium-specific reduced-form CCPs when constructing expected payoffs. Then, contribution of player i in market m to the likelihood function is:

$$L_{im}\left(\boldsymbol{\theta}_i, \boldsymbol{\kappa}_i\right) = \prod_{j=0}^{J} \left[P_i\left(j|\,\mathbf{x}_m, \hat{\boldsymbol{\nu}}_m, \tau = 1; \boldsymbol{\theta}_i, \boldsymbol{\kappa}_i\right)\hat{\lambda} + P_i\left(j|\,\mathbf{x}_m, \hat{\boldsymbol{\nu}}_m, \tau = 2; \boldsymbol{\theta}_i, \boldsymbol{\kappa}_i\right)\left(1 - \hat{\lambda}\right)\right]^{\mathbb{1}\{y_{im}=j\}}. \tag{23}$$

[Insert Table 2 here.]

Pseudo maximum likelihood estimates of $\boldsymbol{\theta}_i, \boldsymbol{\kappa}_i$ $\forall i$ are reported in Table 2. In order to assess the respective identifying power of the recovered unobserved heterogeneity and equilibria multiplicity, I consider four different specifications. The "baseline" specification ignores the identification problem due to the absence of player-specific regressors satisfying the exclusion restriction for Vidéotron. In this case, identification of incumbents' responses to Vidéotron's decisions are entirely driven by functional form assumptions. Then, I consider two other specifications that separately allows for "multiple equilibria" and for "unobserved heterogeneity". Results from these two specifications can be used to compare estimates obtained from each alternative source of variation in this setting. Finally, "both" multiple equilibria and unobserved heterogeneity are combined in the last specification.

An important result that is worth highlighting from Table 2 is that there is not much evidence that allowing for multiple equilibria being realized in the data helps to identify the primitives of the model. In fact, comparing the "baseline" with the "multiple equilibria" specifications shows that the mixture over two types of equilibria does not change much the parameters' estimates. It even slightly increases the bootstrapped standard errors for many parameters. The same comment holds when comparing the "unobserved heterogeneity" and the "both" specifications. Furthermore, allowing for a finite mixture over two types of equilibria only slightly improves the average log-likelihood function evaluated at the estimated parameters from -1.6200 ("baseline") to -1.6194 ("multiple equilibria"); it even slightly decreases from -1.5922 ("unobserved heterogeneity") to -1.5954 ("both") when unobserved heterogeneity is included. One potential reason for this result is that allowing for two types of equlibria that do not depend on the common-knowledge state variables may be too coarse for an approximation of multiple equilibria being realized in the data. Another explanation is that there is not much evidence of multiple equilibria in this specific context. In the current setting, this would imply that the same equilibrium is realized whenever the same game is played within a given province.[16]

However, including recovered common-knowledge unobserved heterogeneity considerably changes some ot the estimates, especially strategic interactions. The parameter measuring the effect of recovered unobserved heterogeneity being only significant for Vidétron is encouraging for two reasons. First, remember that Vidéotron is the only player without accumulated stocks of transceivers, i.e. the player-specific regressor excluded from competitors' payoffs. The fact that Vidéotron's recovered unobserved heterogeneity has a statistically significant effect on its payoffs suggests that the recovered unobservables will generate appreciable shifts in Vidéotron's decisions that will effectively help to identify parameters measuring the effect of these decisions on incumbents' payoffs. Second, the statistical significance being limited to Vidéotron is also aligned with the interpretation of $\boldsymbol{\nu}$ suggested earlier. An important justification of introducing common-knowledge unobserved heterogeneity in the econometric model is that there may be relevant unobservable information regarding agreements between providers, transceivers'

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[16] While this claim is hard to verify in practice, it is further supported by the counterfactual analysis proposed below. When solving for the equilibrium of the model at the preferred parameters' estimates, I always find the same solution despite initiating the solver at 100 different randomly selected starting values. If there is effectively a single equilibrium in the model, there must be a single equilibrium realized in the data.

technology, etc. Firms' bids should reflect such information. However, especially if previous agreements are expected to hold after the AWS2008-1 auction and if there are no abrupt changes in the technology used, incumbents' stocks of transceivers may also reflect this unobservable information. If that's the case, then incumbents' recovered unobserved heterogeneity should not be expected to significantly affect incumbents' payoffs, as is observed here.

While the "unobserved heterogeneity" specification does not systematically imply a reduction in estimates' standard errors compared to the "baseline" case, some important reductions are worth pointing out: Vidéotron's and Telus' effects on Rogers' and Bell's payoffs are considerably more precisely estimated when recovered unobserved heterogeneity is included as a regressor. Once again, this observation is consistent with Vidéotron's recovered unobserved heterogeneity providing a source of variation that helps to identify how its decisions affect incumbents' payoffs.

For these reasons, combined with the fact that it is associated with the highest average log-likelihood, "unobserved heterogeneity" will be treated as the preferred specification for the rest of the article.

Some estimation results from this preferred specification are worth discussing. First, consider the parameters associated with population density and distance to closest large population centre. Most of the population density parameters' estimates not being significantly different from zero may be a consequence of focusing on small population centres which all have a modest density. Furthermore, as expected, payoffs decrease in markets that are further from a large population centre for almost all providers.

One of the most important result that is worth mentioning is the relative contribution of each incumbent's own stocks of transceivers to their payoffs. The corresponding coefficients (respectively 2.413, 1.914 and 2.948 for Rogers, Bell and Telus) suggest that increasing a provider's accumulated stocks of transceivers has a positive and significant effect on its payoffs. In other words, transceivers' locations are associated with important economies of density: incumbents are more likely to add new transceivers where they already own some. Intuitively, it may be less costly for providers to improve an existing network rather than building a brand new one. This observation potentially explains why new entrants fail to use some of their spectrum or go out of business. In fact, new entrants cannot benefit from such economies of density, as they are building brand new networks without having accumulated stocks of transceivers. This asymmetry between incumbents and new entrants motivates the counterfactual experiment proposed in Section 7.

Let's now turn to strategic interactions' estimates. These parameters are relatively precisely estimated for all providers, except Telus. Large standard errors for the latter are likely due to the fact that Telus installs 0 transceivers in a bit more than 85% of the markets considered in the analysis, therefore leaving a small fraction of observations with interesting variation to identify the parameters of the model. It is also important to note that the strategic interactions' estimates capture the effect of the *number* of transceivers installed by competitors on a given player's payoffs (as opposed to the *entry* effect typically captured in binary games). *A priori*, competitors' decisions could be strategic substitutes if one expects a provider not to install transceivers where another does. In particular, this is the case for Bell and Telus (with reciprocal estimates equal to -1.325 and -0.190). The fact that most strategic interactions (8 out of 12) are estimated to be positive suggests that transceivers' locations are strategic complements. This complementarity can be explained by providers competing over network quality.

Another important observation regarding strategic interactions is that, as expected, competitors do not all affect a given player's payoffs the same way. For instance, while Rogers and Bell positively affect each other's payoffs (reciprocal estimates being 1.582 and 2.561, significant at the 99% level), these two incumbents' payoffs do not significantly depend on Telus' nor Vidéotron's decisions. Telus not significantly impacting the other incumbents' decision is consistent with the fact that it only installs transceivers in less than 15% of the markets studied. Nonetheless, Telus and Vidéotron both have a significant effect on each other's payoffs (reciprocal estimates being 3.360 and 0.854, respectively significant at the 95% and the 99% levels). Strategic interactions' estimates also support evidence of asymmetric responses between providers. One important observation that will explain some of the counterfactual results presented below is that, while Vidéotron does not have a significant impact on Rogers' payoffs (relatively small positive estimate of 0.935), the reverse does not hold (2.506, significant at the 99% level).

Finally, threshold parameters have a relevant structural interpretation: they measure the marginal costs of installing transceivers. These parameters' estimates will be key for the counterfactual analysis. The inclusion of province fixed effects, combined with the linear specification of marginal revenues, allows one to interpret marginal costs as varying across provinces. More precisely, threshold parameters' estimates correspond to marginal costs in the omitted province (i.e., Ontario). Notice that because the Quebec province fixed effect enters the $\Lambda(\cdot)$ function linearly in (20) via $\mathbf{x}_i$, Quebec-specific marginal costs are obtained by subtracting this fixed effect from the threshold parameters' estimates. For example, Rogers' estimated marginal costs of installing 1-10 and 11+ transceivers in Ontario are 1.501 and 10.49 (respectively significantly different from zero with 95% and 99% confidence). In Quebec, they become $1.501-(1.030) = 0.471$ and $10.49-(1.030) = 9.46$, but the difference between the two provinces is not significantly different from zero. The only providers for which there is a significant difference in costs across provinces are Telus (lower in Ontario) and Vidéotron (lower in Quebec). Unfortunately, one cannot directly compare marginal costs' estimates across providers because the payoffs' scale has been fixed by normalizing the variance of the private information shocks, which may vary across firms. Some expected patterns are still worth noting. In particular, the ratio of the 11+ over the 1-10 threshold estimates is much larger for Rogers ($10.49/1.501 = 6.99$) than the others. This observation reflects that Rogers typically installs at least one transceiver in more markets than its competitors, but does not install as many transceivers in each market as other providers do. Another extreme is Bell, with a ratio of $11.67/10.20 = 1.14$, which suggests that Bell does not install transceivers in as many markets as Rogers, but tends to install more transceivers where it does.

7 Counterfactuals

As already mentioned, new entrants do not benefit from economies of density generated by accumulated stocks of transceivers because they must build brand new networks. Taking as given the government's objective to increase competition in the mobile telecommunications industry and the drawbacks associated with spectrum set-asides, this asymmetry between incumbents and new entrants may justify subsidizing the first few transceivers installed by the latter.

I use the estimated model to provide valuable insights regarding the effects of such a policy. Intuitively, subsidizing Vidéotron's transceivers should increase the probability that it invests in a given market. This change in probabilities of installing transceivers will also have an effect on the incumbents' investment probabilities because, in equilibrium, these are defined as best responses to their competitors' expected behaviour. Evaluating this indirect effect is key to be able to judge whether subsidizing new entrants such as Vidéotron is a desirable policy or if new entrants' investments would be crowded out by a reduction in incumbents' probabilities of installing transceivers. Notice that this indirect effect of the subsidy would not be captured by a model ignoring strategic interactions between network operators.

Counterfactual experiments are performed separately for three representative markets in both provinces considered in the analysis. For each province, these are the markets corresponding to the 25-th, 50-th and 75-th population density percentiles, conditional on all providers owning at least one license. Such markets are described in Table 3.

[Insert Table 3 here.]

I consider a subsidy that decreases the province-specific marginal costs of installing 1-10 transceivers for Vidéotron only. Using the parameters' estimates from the preferred specification, I compute the equilibrium choice probabilities[17] associated with a reduction going from 0% to 100% of these marginal costs. The evolution of the equilibrium CCPs for different values of the subsidy is reported in Figures 2 and 3. In each figure, I report the probability of installing a positive number of transceivers in the first row of plots. The evolution of these probabilities can be interpreted as an illustration of the policy's effect at the extensive margin. The second, third and fourth rows respectively report the probabilities of installing 0, 1-10 and 11+ transceivers. This breakdown corresponds to the intensive margin.

[Insert Figure 2 here.]

[Insert Figure 3 here.]

At the extensive margin, the subsidy has the expected effect in all representative markets: it never decreases Vidéotron's probabilities of installing a positive number of transceivers. However, there is striking heterogeneity in the observed responses. In fact, while the subsidy may generate a fairly strong response from Vidéotron in some markets (e.g., probabilities of a positive number of transceivers going from about 5% to above 80% in Ontario's representative markets), the effect is much smaller (an increase of about 20 percentage points in Quebec's

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[17] Looking for multiple equilibria, I initiated the solution algorithm at 100 different randomly picked starting values of CCPs. In all representative markets, I always found a single equilibrium for all values of the subsidy.

25-th and 75-th percentiles) or even non existing in others (e.g., Quebec's 50th percentile market). One interpretation of this heterogeneity is that there may be more room for new entrants in Ontario than in Quebec, where Vidéotron has already become an important player. This observation suggests that policies targeting the introduction of a fourth national player in the industry should not be expected to be equally successful in all provinces.

Still at the extensive margin, one should note that there are minor responses in incumbents' probabilities of installing a positive number of transceivers as an indirect effect of the subsidy. In Ontario, there is a slight increase in such probabilities for Rogers' and Telus', but a slight decrease for Bell. Incumbents' behaviour is left unchanged in Quebec. In all cases, the size of Bell's reduction does not crowd out the new entrant's and other incumbents' increases, which is coherent with the strategic complementarities estimated above.

At the intensive margin, it is clear that most of the action is happening from providers shifting from installing 0 transceiver to installing 1-10. In some sense, this is reassuring. Remember that the analysis is focusing on small population centres. If the subsidy was to incentivize the new entrant to install a large number of transceivers, one should have become suspicious that the subsidy could lead to wasteful investments, especially if Vidéotron became much more likely to install 11+ transceivers compared to incumbents.

An important caveat is worth mentioning at this stage: the constructed representative markets took the common-knowledge unobservable heterogeneity as exogenously given. Presumably, as part of this heterogeneity may include the effects of agreements on each provider's payoff, it would be preferable to allow these agreements to vary with the counterfactual experiments. However, it is not easy to model how these unobservables would change in a hypothetical world.

8 Conclusion

I estimate a game of transceivers' location decisions between three national incumbents and a new entrant using data from the Canadian mobile telecommunications industry. Estimation results suggest that economies of density may explain why new entrants do not always use their allocated spectrum or even go out of business. In a counterfactual experiment, I study the impact of subsidizing the new entrant's first few transceivers to compensate the absence of economies of density generated by accumulated stocks. I find that such a subsidy can increase the probability that the new entrant and the incumbents install new transceivers. The slight decrease observed for one of the incumbents is not enough to crowd out direct and indirect increases in investments generated by the subsidy.

An important feature of the identification strategy is that I recover information from providers' bids which I interpret as player-specific unobserved heterogeneity. By comparing different specifications of the econometric model, I show that variation in recovered unobserved heterogeneity helps to identify the model's parameters. In fact, such recovered unobservables play the role of player-specific regressors that satisfy an exclusion restriction required to identify strategic interactions. Without this source of variation, incumbents' responses to the new entrant's decisions, i.e. a set of crucial parameters when studying the counterfactual policy, would only be identified by functional form assumptions. A similar identification argument using predetermined outcomes to recover player-specific unobservables may be appealing in other settings. In particular, it complements the identifying power of multiple equilibria already proposed in the literature.

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Table 1: Summary statistics for population centres

	Ontario			**Quebec**		
	0	**1-10**	**11+**	**0**	**1-10**	**11+**
Additions of transceivers						
Rogers	46	176	15	64	142	16
Bell	151	39	47	173	26	23
Telus	227	6	4	166	29	27
Vidéo	230	5	2	96	85	41
	mean	**min**	**max**	**mean**	**min**	**max**
Population density	0.777	0.148	3.707	0.771	0.101	2.887
Distance to large population centre	0.853	0.099	4.780	1.266	0.122	7.550
Stocks of transceivers						
Rogers	8.426	0.000	55.00	5.725	0.000	54.00
Bell	38.28	0.000	242.0	21.06	0.000	155.0
Telus	1.008	0.000	48.00	11.83	0.000	220.0
Population-weighted bids						
Rogers	0.0534	0.0030	0.5242	0.0620	0.0073	0.4501
Bell	0.0585	0.0051	0.4375	0.0549	0.0024	0.4514
Telus	0.0457	0.0023	0.3882	0.0611	0.0067	0.5241
Vidéo	0.0354	0.0000	0.3375	0.0564	0.0028	0.5791
Licenses						
Rogers	1.00	1.00	1.00	1.00	1.00	1.00
Bell	1.00	1.00	1.00	1.00	1.00	1.00
Telus	1.00	1.00	1.00	1.00	1.00	1.00
Vidéo	0.35	0.00	1.00	1.00	1.00	1.00
Number of markets		**237**			**222**	

Notes: Additions of transceivers are the numbers of markets associated with the corresponding bin used in the ordered-response model that is estimated. Population density is in 10^3 persons per square kilometre. Distance to closest large population centre is in degrees. Stocks of transceivers are the numbers of transceivers operating on frequencies allocated prior to the AWS1-2008 auction. Population-weighted bids are in 10^6 CAD$. Licenses are binary variables indicating if the provider owns at least one license covering a given population centre.

Table 2: Transceivers' location decisions: estimates

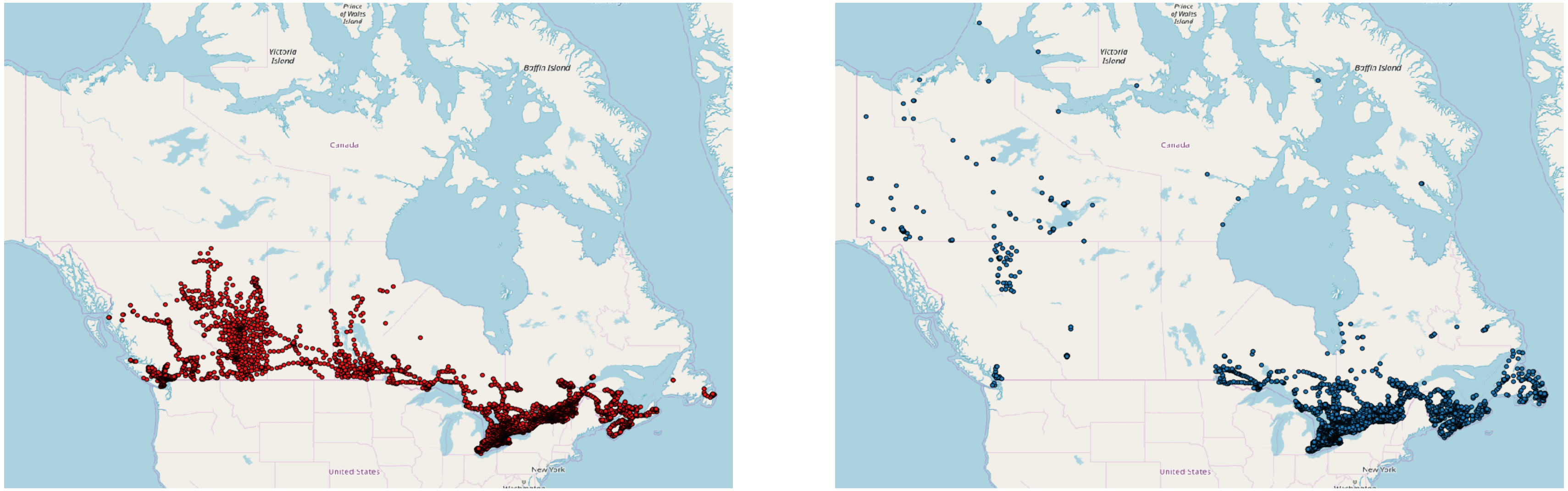
	Baseline				**Multiple equilibria**				**Unobserved heterogeneity**				**Both**			
	Rogers	**Bell**	**Telus**	**Vidéo**	**Rogers**	**Bell**	**Telus**	**Vidéo**	**Rogers**	**Bell**	**Telus**	**Vidéo**	**Rogers**	**Bell**	**Telus**	**Vidéo**
Pop dens	-0.468	0.560	0.646	-0.205	-0.413	0.575	0.675	-0.187	-0.367	0.601*	0.468	-0.222	-0.389	0.616*	0.297	-0.260
	(0.355)	(0.347)	(0.467)	(0.222)	(0.380)	(0.359)	(0.510)	(0.227)	(0.349)	(0.355)	(0.604)	(0.265)	(0.379)	(0.361)	(0.614)	(0.292)
Dist	-0.850***	0.333	-0.019	-0.564**	-0.874***	0.341	0.024	-0.541**	-0.820***	0.419*	-0.241	-0.496**	-0.848***	0.434*	-0.234	-0.492*
	(0.203)	(0.244)	(0.552)	(0.260)	(0.209)	(0.251)	(0.542)	(0.264)	(0.198)	(0.230)	(0.556)	(0.254)	(0.202)	(0.233)	(0.605)	(0.268)
Stocks	2.340***	1.879***	2.766***	–	2.452***	1.865***	2.731***	–	2.413***	1.914***	2.948***	–	2.478***	1.933***	2.892***	–
	(0.413)	(0.272)	(0.533)	–	(0.424)	(0.275)	(0.695)	–	(0.399)	(0.275)	(0.571)	–	(0.397)	(0.278)	(0.632)	–
Unob het	–	–	–	–	–	–	–	–	-3.762	-5.679	27.09	29.39**	-4.080	-5.624	32.45	30.09**
	–	–	–	–	–	–	–	–	(6.372)	(6.224)	(21.23)	(12.49)	(6.559)	(6.299)	(23.16)	(13.08)
Interact																
Rogers	–	3.112***	-1.320	2.299***	–	3.197***	-1.105	2.326***	–	2.561***	-0.997	2.506***	–	2.435**	-0.256	2.500***
	–	(0.985)	(1.440)	(0.600)	–	(1.066)	(1.555)	(0.611)	–	(0.972)	(1.484)	(0.642)	–	(1.025)	(1.601)	(0.694)
Bell	2.089***	–	-0.094	0.589	2.185***	–	-0.463	0.602	1.582***	–	-0.190	0.485	1.636**	–	-0.936	0.556
	(0.575)	–	(2.788)	(0.404)	(0.726)	–	(2.480)	(0.426)	(0.527)	–	(2.821)	(0.432)	(0.701)	–	(2.598)	(0.517)
Telus	0.878	-0.983	–	0.749**	0.993	-1.173	–	0.794**	0.405	-1.325	–	0.854***	0.434	-1.366	–	0.902**
	(0.606)	(1.684)	–	(0.303)	(0.742)	(1.749)	–	(0.326)	(0.496)	(0.917)	–	(0.331)	(0.597)	(1.532)	–	(0.359)
Vidéo	0.626	-1.491	3.634**	–	0.631	-1.506	3.604**	–	0.935	-0.401	3.360**	–	0.898	-0.193	2.747	–
	(0.832)	(1.046)	(1.561)	–	(0.864)	(1.071)	(1.747)	–	(0.767)	(0.844)	(1.674)	–	(0.795)	(0.752)	(1.846)	–
Thresh																
1-10	1.486**	10.49***	4.714***	4.897***	1.612**	10.53***	4.792***	4.983***	1.501**	10.20***	4.662***	5.358***	1.532**	10.18***	4.808***	5.419**
	(0.648)	(1.113)	(1.289)	(1.623)	(0.671)	(1.177)	(1.522)	(1.673)	(0.615)	(1.119)	(1.440)	(2.060)	(0.646)	(1.176)	(1.603)	(2.220)
11+	10.74***	11.96***	8.506***	7.449***	11.427***	12.01***	8.601***	7.572***	10.49***	11.67***	8.671***	8.010***	10.76***	11.65***	8.807***	8.110***
	(1.275)	(1.147)	(1.549)	(1.702)	(1.482)	(1.217)	(1.908)	(1.775)	(1.231)	(1.149)	(1.753)	(2.117)	(1.420)	(1.210)	(1.990)	(2.286)
FE																
Quebec	1.213	1.282	-3.917***	3.760**	1.324*	1.295	-3.921**	3.793**	1.030	0.326	-3.813**	4.096**	1.152	0.139	-3.489**	4.175**
	(0.779)	(0.936)	(1.525)	(1.494)	(0.802)	(0.951)	(1.722)	(1.549)	(0.734)	(0.727)	(1.494)	(1.917)	(0.725)	(0.660)	(1.793)	(2.076)

Notes: Estimates for the ordered-response models where players simultaneously choose between installing 0, 1-10 or 11+ transceivers operating on frequencies allocated during the AWS1-2008 auction. Parameters are player-specific and each column refers to a different provider. Population density is in 10^3 persons per square kilometre. Distance to closest large population centre is in degrees. Stocks of transceivers, in natural logarithm (after adding 1 to avoid 0's), are incumbents' transceivers operating on frequencies allocated before the AWS1-2008 auction. Variables corresponding to player-specific estimated unobserved heterogeneity are included as regressors. For strategic interactions, the estimates correspond to the effect of the row player's decision on the column player's payoffs. Values in brackets are standard errors computed using 500 bootstrapped samples. Significance levels: $^{*} = 0.10$, $^{**} = 0.05$ and $^{***} = 0.01$.

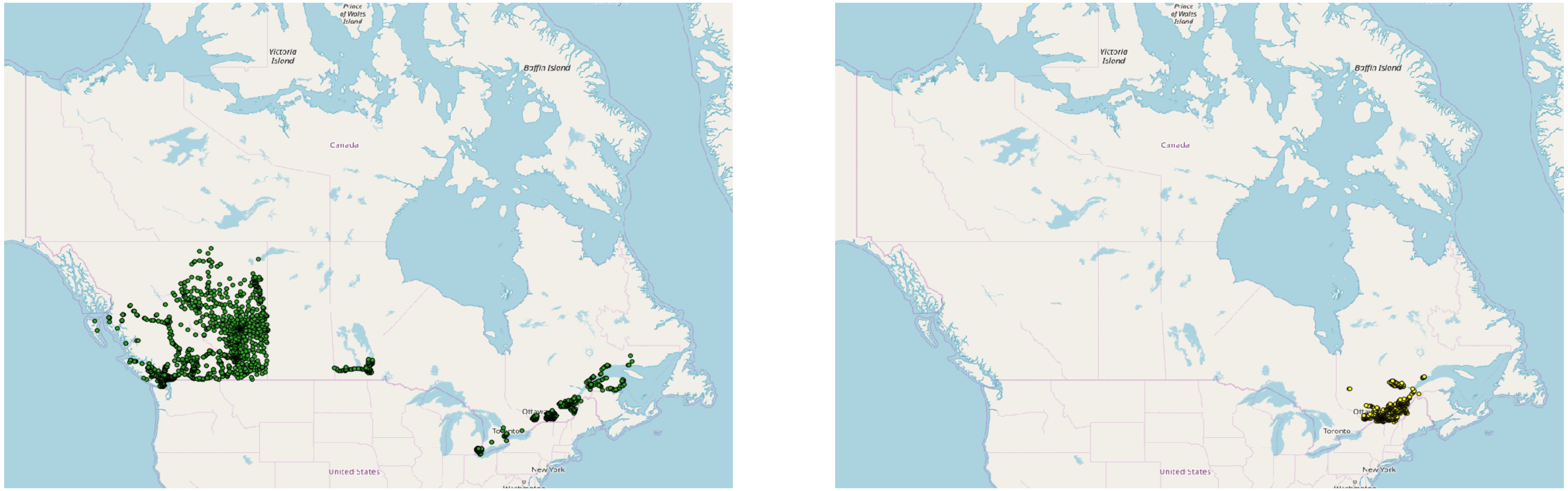
Table 3: Representative markets

Percentiles	Pop. dens.	Dist.	Stocks of transceivers Rogers	Bell	Telus	Unobserved heterogeneity Rogers	Bell	Telus	Vidéo
Ontario									
25-th	0.574	1.127	1.386	3.689	0.000	0.0047	-0.0098	-0.0069	0.0088
50-th	0.745	0.473	2.079	2.565	0.000	0.0082	-0.0084	-0.0062	0.0059
75-th	0.965	0.805	1.609	3.951	0.000	0.0071	-0.0084	-0.0087	0.0056
Quebec									
25-th	0.424	1.698	0.000	3.091	0.000	0.0061	0.0102	-0.0119	-0.0016
50-th	0.633	0.513	2.197	4.533	0.000	0.0786	-0.1236	0.0448	0.0527
75-th	0.967	0.509	1.386	2.996	0.000	-0.0063	0.0010	-0.0036	0.0045

Notes: In each province, the markets corresponding to the 25-th, 50-th and 75-th percentiles according to population density are treated as representative markets. Only markets where all providers have licenses are considered. The table reports the corresponding values of population density (in 10^3 persons per square kilometre), distance to closest large population centre (in degrees), stocks of transceivers (in natural logarithms, after adding 1 to avoid 0's) and unobserved heterogeneity (estimated from bid data).



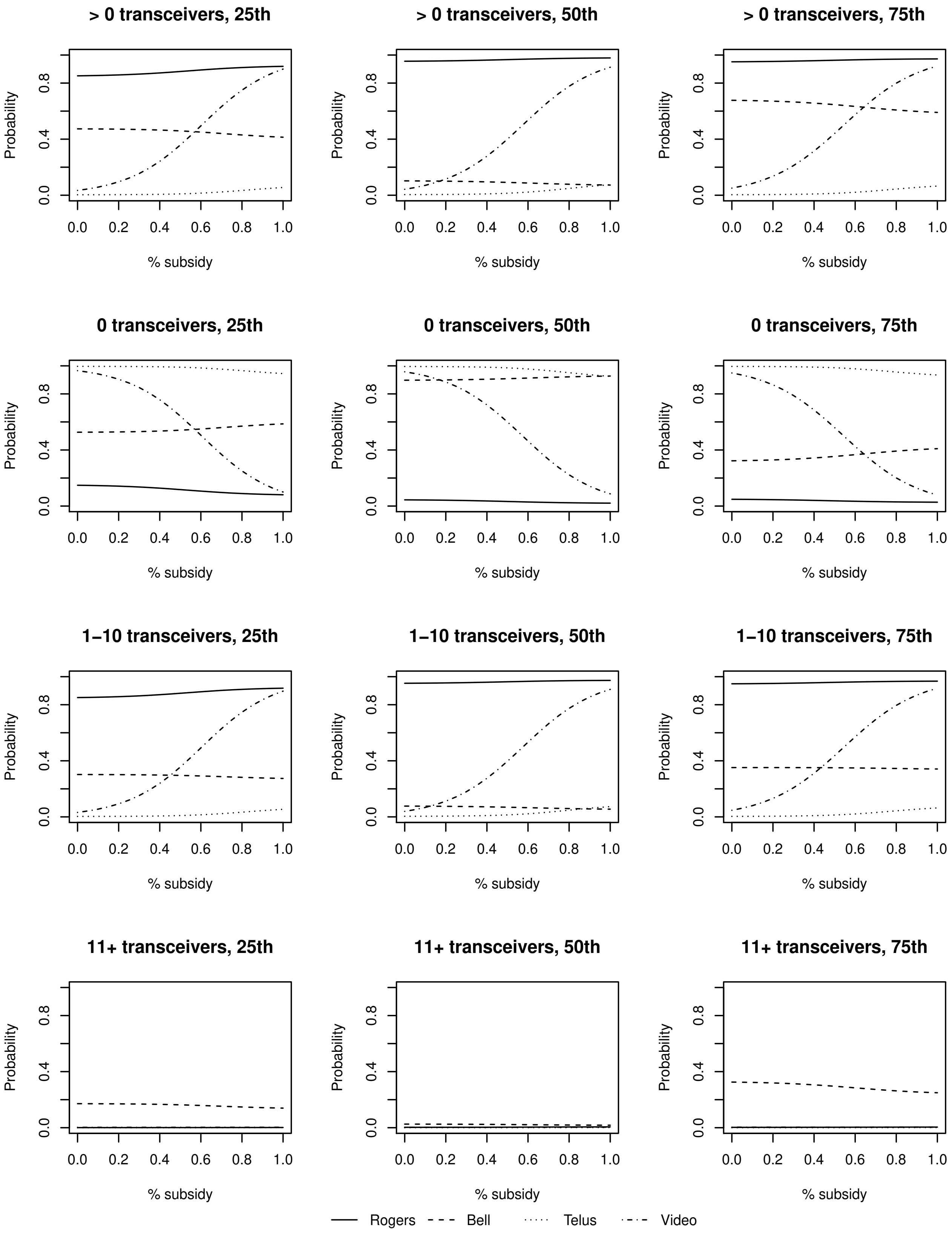
(a) Rogers (b) Bell



(c) Telus (d) Vidéotron

Notes: Transceivers' locations for three incumbents (Rogers, Bell and Telus) and a new entrant (Vidéotron). Each point corresponds to one or more transceivers as several transceivers can be located at the same site. Maps include all transceivers operating on frequencies allocated up to the AWS1-2008 spectrum auction.

Figure 1: Locations of transceivers



> 0 transceivers, 25th

> 0 transceivers, 50th

> 0 transceivers, 75th

0 transceivers, 25th

0 transceivers, 50th

0 transceivers, 75th

1−10 transceivers, 25th

1−10 transceivers, 50th

1−10 transceivers, 75th

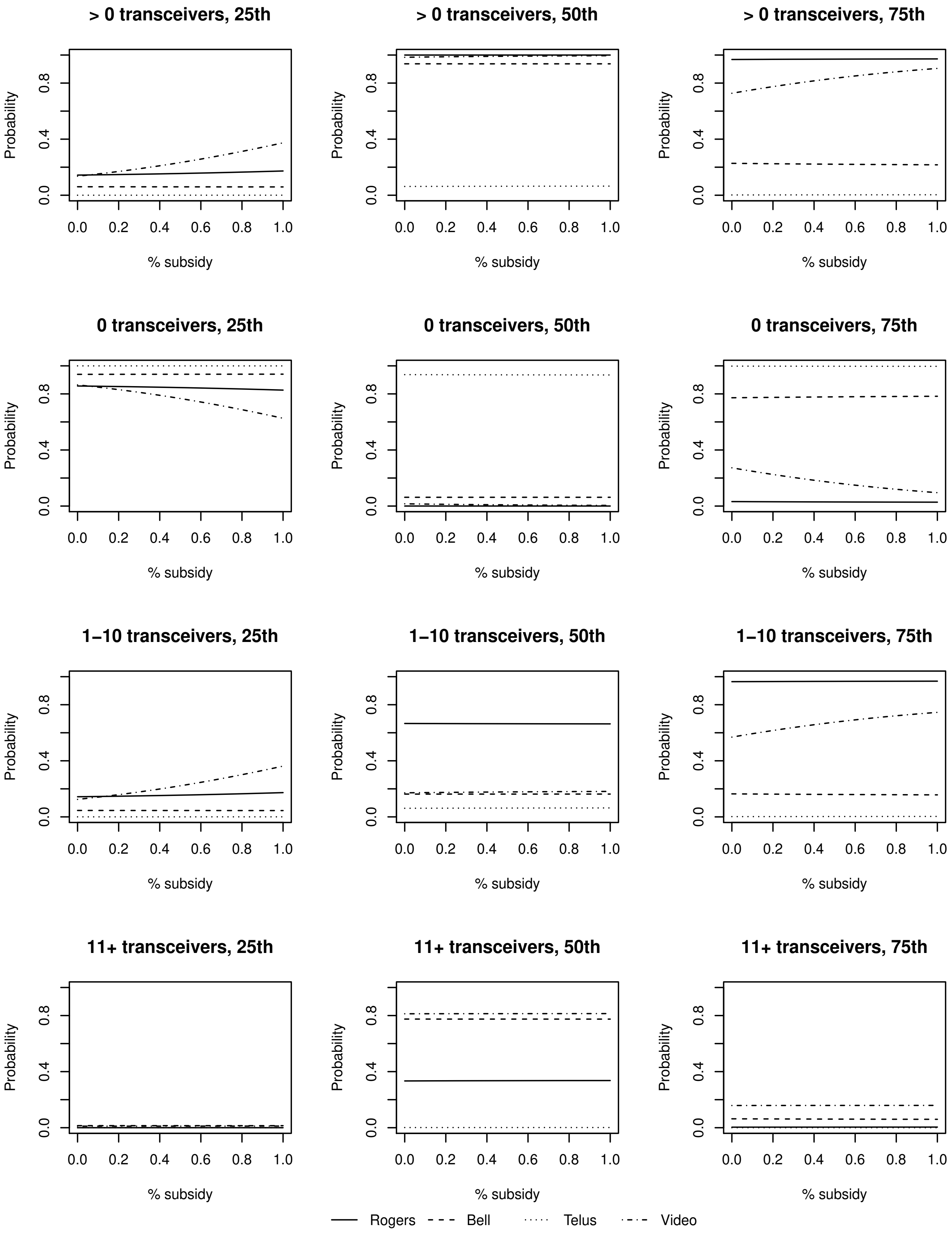
11+ transceivers, 25th

11+ transceivers, 50th

11+ transceivers, 75th

Notes: Equilibrium probabilities of adding more than 0, 0, 1-10 and 11+ transceivers for different levels of subsidy. Representative markets corresponding to the 25-th, 50-th and 75-th percentiles of the population density, conditional on all providers owning spectrum licenses. Probabilities are plotted for a 0% to 100% reduction of Vidéotron's marginal costs of installing 1-10 transceivers.

Figure 2: Effect of subsidy on probabilities of installing transceivers – Ontario



Notes: Equilibrium probabilities of adding more than 0, 0, 1-10 and 11+ transceivers for different levels of subsidy. Representative markets corresponding to the 25-th, 50-th and 75-th percentiles of the population density, conditional on all providers owning spectrum licenses. Probabilities are plotted for a 0% to 100% reduction of Vidéotron's marginal costs of installing 1-10 transceivers.

Figure 3: Effect of subsidy on probabilities of installing transceivers – Quebec