The Quest for Hegemony Among Countries and Global Pollution[1]

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Abstract

This paper builds on the assumption that countries behave in such a way as to improve, via their economic strength, the probability that they will attain the hegemonic position on the world stage. The quest for hegemony is modeled as a game, with countries being differentiated initially only by some endowment which yields a pollution free flow of income. A country's level of pollution is assumed directly related to its economic strength, as measured by its level of production. Two types of countries are distinguished: richly-endowed countries, for whom the return on their endowment is greater than the return they can expect from winning the hegemony race, and poorly-endowed countries, who can expect a greater return from winning the race than from their endowment. We show that in a symmetric world of poorly-endowed countries the equilibrium level of emissions is larger than in a symmetric world of richly-endowed countries.: the former, being less well endowed to begin with, try harder to win the race. In the asymmetric world composed of both types of countries, the poorly-endowed countries will be polluting more than the richly endowed countries. Numerical simulations show that if the number of richly-endowed countries is increased keeping the total number of countries constant, the equilibrium level of global emissions will decrease; if the lot of the poorly-endowed countries is increased by increasing their initial endowment keeping that of the richly-endowed countries constant, global pollution will decrease; increasing the endowments of each type of countries in the same proportion, and hence increasing the average endowment in that proportion, will decrease global pollution; redistributing from the richly-endowed in favor of the poorly-endowed while keeping the average endowment constant will in general result in an increase in the equilibrium level of global pollution.

Keywords: Hegemony, Global pollution, Dynamic games
JEL Classification: Q54, Q50, F5

Résumé

Ce texte part de l'hypothèse qu'en acquérant plus de puissance économique les pays augmentent leur probabilité d'atteindre la position hégémonique. La quête à l'hégémonie est modélisée comme un jeu de course où les joueurs sont des pays différenciés par une dotation en capital qui génère un flux de rendement non polluant. Le niveau d'émission d'un pays est supposé relié à sa puissance économique tel que mesurée par le niveau de production. De l'analyse, deux types de pays ressortent : les pays richement dotés dont le rendement issu de leur dotation est plus grand que le rendement de la récompense en cas de succès dans la course à l'hégémonie, et les pays pauvrement dotés dont le rendement de la récompense en cas de succès dans cette course est plus grand que celui du flux de rendement issu de leur dotation. Nous montrons que dans un équilibre symétrique constitué de pays pauvrement dotés, le niveau d'équilibre des émissions est plus grand que celui d'un équilibre symétrique constitué de pays richement dotés. Dans un monde asymétrique constitué des deux types de pays, le niveau d'émission d'un pays pauvrement doté est supérieur au niveau d'émission d'un pays richement doté. Les simulations numériques indiquent que lorsque le nombre de pays richement dotés augmente en maintenant fixe le nombre total de pays, alors le niveau d'équilibre de la pollution globale baisse ; si les dotations des pays pauvrement dotés sont accrues, en laissant constante celles des pays richement dotés, alors la pollution globale baissera ; accroître les dotations des deux types de pays dans les mêmes proportions, et donc accroître la dotation moyenne dans la même proportion, baissera la pollution globale ; redistribuer des pays richement dotés vers les pays pauvrement dotés, tout en maintenant fixe la dotation moyenne, résultera en général en un accroissement du niveau d'équilibre de la pollution globale.

Mots-clés : Hégémonie, Pollution globale ; Jeux dynamiques
Classification JEL : Q54, Q50, F5

1 Introduction

Concerns about global pollution in general and global warming in particular have led to a considerable body of literature. But an important question which has not yet been formally explored has to do with the relationship between the quest for hegemony and global pollution. Derived from the original Greek word hegemonia, which means "leadership", hegemony can be seen as an institutionalized practice of special rights and responsibilities conferred on a state with the resources to lead the international system (Clark (2009)). Most historical ages are marked by the presence of a nation capable of dominating the course of international politics. Over the last five centuries, Portugal, the Netherlands, France, Britain, and the United States have played the hegemonic role Modelski (1987)).[1]

The quest for hegemony can be viewed as a status-seeking game among countries which aspire to the hegemonic status and the important benefits that come with it.[2] A basic postulate widely accepted among experts of geopolitics is that relative power differences between states cause them to compete with one another for relative shifts in power and status. For centuries, military force was the main source of primacy in the international system. After the cold war and the advent of nuclear warfare, military force became a costly and risky means of attaining hegemony and economic force gained prominence as the major factor in determining the primacy or the subordination of states.[3] It is safe to

[1]For discussions of the quest for hegemony among countries, see also Kennedy (1987), Black (2007) and Mosher (2000).

[2]Weiss and Fershtman (1980) define status as a ranking of individuals (or groups of individuals) based on traits, assets and actions. On the subject of status seeking, see also Moldovanu et al. (2007).

[3]To quote the political scientist Kenneth Waltz (Waltz, 1993, p. 63 and p. 66): "Without a considerable economic capability, no state can hope to sustain a world role, as the fate of the Soviet Union has shown." and "For a country to choose not to become a great power is a structural anomaly. For that reason, the choice is a difficult one to sustain. Sooner or later, usually sooner, the international status of countries has risen in step with their material resources. Countries with great-power economies have become great powers, whether or not reluctantly." Other political scientists, among them Samuel Huntington (Huntington, 1993, p. 72), share this view: "Economic activity [...] is probably the most important source of power, and in a world in which military conflict between major states is unlikely, economic power will be increasingly important in determining the primacy or the subordination of states." The importance of the economic

say that economic strength has become a necessary condition for attaining hegemony in the international system.

After World War II, the United States, with half the world's gross national product, found itself in a uniquely strong position, much surpassing that of Britain at the height of its power in the nineteenth century, and played the leading role in the international state system. But in the last decades, the United States has been facing new global players, such as China, India and Brazil, who are making their presence felt in international affairs, largely due to the increasing power derived mostly from their expanding economies.[4] Those emerging economies are transforming the hegemony game into a multi-player game (Shenkar, 2005, p. 162).

But economic and ecological systems are deeply interlocked, in good part because most of the global pollution released into the atmosphere comes from the combustion of fossil fuels, which is a driving force of the economic system (see Stern (2007), Chombat (1998). Raupach et al. (2007). Therefore, because economic activity impacts both global pollution and the hegemonic game, the world can be viewed as facing a conflict between the intensity of the hegemony game among countries and the reduction of global pollution. As has been noted by a former Science Advisor to the U.S. President: "No realistic response to climate change can ignore the current geopolitical preoccupation with economic competition among nations" (Marburger, 2007, p. 5).

To analyze this issue, this paper builds on the assumption that each country behaves in such a way as to improve, via its economic strength, the probability that it will attain the hegemonic position on the world stage. The quest for hegemony is modeled as a game, with countries being differentiated *only* by the return on some initial endowment which yields a

battle among hegemonic aspirants is also pointed out by the economist Lester Thurow (Thurow, 1993, p. 65): "Those who control the world's largest market get to write the rules. That is as it always has been. When the United States had the world's largest market, it got to write the rules"

[4]See Shenkar (2005), Ikenberry (2008) and Elliott (2007). To quote Oded Shenkar (Shenkar, 2005, p. 38): "China's economic aspirations are aligned strongly with its political ambitions, and the regime is aware more than most of the close connection between the two."

pollution free flow of income. This return on endowment can be thought of as being related to the country's human capital and economic, social and political institutions. A country's level of pollution is assumed directly related to its economic strength, as measured by its level of production. Two types of countries are distinguished: richly-endowed countries, for whom the return on their endowment is greater than the return they can expect from winning the hegemony race, and poorly-endowed countries, who can expect a greater return from winning the race than from their endowment. As we will see, the latter, having more to gain, are more eager players in the hegemony race and will end up polluting more in equilibrium. The former are more content with the return they get from their endowment and end up polluting less. We may think of emerging economies such as China, India, Brazil and Russia as being in the category of poorly-endowed countries, whereas most North American and Western European countries would fall in the category of richly-endowed countries.

We consider in sequence the equilibria in a world composed of only poorly-endowed countries, a world composed of only richly-endowed countries and a world in which both types of countries coexist. We show that in a symmetric world of poorly-endowed countries the equilibrium level of emissions is larger than in a symmetric world of richly-endowed countries: the former, being less well endowed to begin with, try harder to win the race. In the asymmetric world composed of both types of countries, there can be multiple equilibria. In all of those equilibria, the poorly-endowed countries will be polluting more than the richly-endowed countries. Numerical simulations show that if the number of richly-endowed countries is increased keeping the total number of countries constant, the equilibrium level of global emissions will decrease; if the lot of the poorly-endowed countries is increased by increasing their initial endowment keeping that of the richly-endowed countries constant, global pollution will decrease; increasing the endowments of each type of countries in the same proportion, and hence increasing the average endowment in that proportion, will decrease global pollution; redistributing from the richly-endowed in favor of the poorly-endowed while keeping the average endowment constant will in general result in an increase in the equilibrium level of global pollution.

In the next section, we describe the main features of the model. Section 3 presents the hegemony game, which borrows some of its features from the patent-race literature (Reinganum (1982), Lee and Wilde (1980) and Loury (1979)). Section 4 analyzes the equilibria under the different scenarios described above and discusses the effect of various ways of modifying the distribution of endowments in the case where poorly-endowed and richly-endowed countries coexist. We conclude with some final remarks in Section 5.

2 The Model

Consider N countries competing to reach the hegemonic position. The probability for any country of reaching the hegemonic position increases with its output, $Q_i(t)$, a measure of its economic strength. Country i's production gives rise to the emission of pollution at the rate $e_i(t)$. For simplicity it will be assumed that one unit of production gives rise to one unit of emission: $e_i(t) = Q_i(t)$. This pollution is global, in the sense that it will affect each country equally.

To visualize the conditions required to win the hegemony race, we can use Greek foot races or sporting contests as analogies. The first condition to win the game is to get to the finish line. The second condition is to be the first among all players to cross the finish line. The prize for crossing the finish line first is greater than the prize of the losers.

In the present context, the winner gets the hegemon position and gets to enjoy "structural power", which Nye, Jr. (1990) called the "soft power". This structural power allows the hegemon to occupy a central and prestigious position within the international system and to play a leading role in setting standards (political, cultural, economic) in organizing the world. We borrow from the paper of Moldovanu et al. (2007) the notion of "pure status" prize, which is related to the notion that a contestant is happier when he has other contestants below him. Hence the hegemon enjoys a "pure status" prize A. Any country other than the hegemon gets a "prize" of $B < A$. For simplicity, A and B will be assumed constant.

The time it will take for country i to cross the finish line (i.e., to attain the necessary characteristics that a country must satisfy to get the hegemonic role) is a random variable, τ_i. Uncertainty about the finishing time is determined by the hazard rate $H_i(t)$, which, by definition, is given by:[5]

$$H_i(t) = \frac{P_i(t < \tau_i \leq t + dt)}{P_i(\tau_i > t)}.$$

It represents the propensity to reach the finish line at time t, given that it has not happened before t. The hazard rate is assumed positively related to the country's level of production, $Q_i(t)$, and hence to its rate of pollution emissions, $e_i(t)$. It will be assumed that $H_i(t) = Q_i(t) = e_i(t)$. It follows that the probability of reaching the finish line by time t is the cumulative distribution function $F_i(t)$, which can be expressed, using the hazard rate, as:

$$F_i(t) = 1 - e^{-\int_o^t e_i(u)du}. \tag{1}$$

This means that the probability of reaching the finish line by time t increases with country i's cumulative emissions on the interval $[0, t]$, given by the term $\int_o^t e_i(u)du$.

The first country to cross the finish line becomes the winner of this hegemony game and obtains the prize denoted above by A. The time at which one of the countries becomes the hegemon is a random variable and is given by

$$\tau = \min_{i=1,...,N} \tau_i.$$

This is the stopping time of the game. It depends stochastically on the vector $(e_1(t), ..., e_N(t))$ of emission levels by each country. Given the vector $(e_1(t), ..., e_N(t))$ of emission levels, the instantaneous probability that country i will win the hegemony game on the infinitesimal interval $[t, t + dt]$ will be given by:

$$\dot{F}_i(t) \prod_{j \neq i}^{N} [1 - F_j(t)]\, dt,$$

[5]Cioffi-Revilla (1998) interprets the hazard rate as a force which, although it does not determine the realization of a political event, acts on it by influencing its temporal evolution. In this paper, the hazard rate consists of economic strength, a causal force arising from the country's decisions which influences the hegemony race.

where $\dot{F}_i(t)$ denotes the time derivative of $F_i(t)$.

The instantaneous probability that country i will lose the hegemony race on the infinitesimal interval $[t, t+dt]$ is the probability that one of the $N-1$ other countries becomes the winner over that interval of time. This is given by:

$$\sum_{j=1, j\neq i}^{j=N} \dot{F}_j(t) \left(\prod_{k=1, k\neq j}^{k=N} [1 - F_k(t)] \right) dt.$$

The instantaneous probability that no country wins the hegemony game on the infinitesimal interval of time $[t, t+dt]$ is:

$$\prod_{j=1}^{j=N} [1 - F_j(t)] dt.$$

If we denote by $S(t)$ the stock of pollution at time t, then

$$\dot{S}(t) = e_1(t) + ... + e_N(t) - kS(t), \;\; S(0) = S_0 > 0 \text{ given}, \tag{2}$$

where $0 < k < 1$ is the coefficient of natural purification. Each country is assumed to suffer equally from the global stock of pollution. The damage function is assumed to be a nonlinear increasing and convex function of the stock, more specifically a quadratic:

$$D_i(S(t)) = \frac{b}{2} S(t)^2 \tag{3}$$

with b a strictly positive constant.

It will also be assumed that the countries are differentiated solely by the return, π_i, which they get on some initial endowment. This exogenous parameter will capture the idea of disparity between countries and the country's pollution emissions will be assumed independent of this permanent flow of benefits. Among the factors that can affect this return on endowment one can think of human capital and the quality of economic, social and political institutions.

3 The hegemony game

The hegemony game bears a lot of similarity to an R&D race, as analyzed in Reinganum (1982), Lee and Wilde (1980) and Loury (1979). The value function of country i, $i =$

$1, \ldots, N$, in quest of hegemony, is given by:

$$V_i(F_1(t), ..., F_N(t), S(t)) = \max_{e_i(t)} \int_0^\infty e^{-rt} \Big\{ A\dot{F}_i(t) \prod_{j\neq i}[1 - F_j(t)] + B \sum_{j\neq i} \dot{F}_j(t) \prod_{k\neq j}[1 - F_k(t)] + (\pi_i - D_i(S(t))) \prod_{j=1}^{N}[1 - F_j(t)] \Big\} dt$$

where the maximization is subject (2) and to $e_i(t) \geq 0$. The stochastic variable τ having been eliminated by formulating the objective functional in terms of expectations, this becomes a deterministic N-player differential game, with control variables $e_1(t), ..., e_N(t)$ and state variables $(F_1(t), ..., F_N(t), S(t))$. The objective functional of country i consists of three terms. The first reflects net benefits if the country succeeds in the quest for the hegemon's position. The second term is the net benefits if the country loses the quest for the hegemon position. The third term represents the pollution damage and the payoff generated by the country's endowment. All three components are weighted by their respective probabilities.

In order to put the emphasis on the characterization of the hegemony game, it will henceforth be assumed that the pollution stock is stationary and hence given by $S(t) = \sum_{i=1}^{N} e_i(t)/k$. We can therefore rewrite the damage function as $D(S(t)) = \beta \left[\sum_{i=1}^{N} e_i(t)\right]^2/2$, where $\beta = b/k^2$.

Following Dockner et al. (2000), let us now introduce the following state transformation which will help in simplifying the formulation:

$$-\log(1 - F_i(t)) = \int_o^t e_i(u)du \equiv Z_i(t), \tag{4}$$

which, upon differentiation with respect to time, gives:

$$\dot{Z}_i(t) = e_i(t). \tag{5}$$

The value function of country i can then be rewritten:

$$V_i(Z_1(t), ..., Z_N(t)) = \max_{e_i(t)} \int_0^\infty e^{-rt} e^{-\sum_{j=1}^{N} Z_i(t)} \left[Ae_i(t) + B \sum_{j\neq i} e_j(t) - \frac{\beta}{2} \left[\sum_{i=1}^{N} e_i(t)\right]^2 + \pi_i \right] dt.$$

where the maximization is with respect to (5).

Notice that although each country knows the full state vector $(Z_1(t), ..., Z_N(t))$, only a function of it, namely the one-dimensional state variable $X(t) = e^{-\sum_{j=1}^{N} Z_i(t)}$, is payoff relevant.[6] The problem of country i can therefore be transformed into the following one-state variable problem:

$$V_i(X(t)) = \max_{e_i(t)} \int_0^\infty e^{-rt} X(t) \left[Ae_i(t) + B \sum_{j \neq i} e_j(t) - \frac{\beta}{2} \left[\sum_{i=1}^{N} e_i(t) \right]^2 + \pi_i \right] dt \tag{6}$$

where the maximization is subject to

$$\dot{X}(t) = -X(t) \left(\sum_{j=1}^{N} e_i(t) \right). \tag{7}$$

The state variable $X(t)$ gives the probability that the game has not yet ended at date t.

The corresponding current value Hamiltonian is given by

$$\mathcal{H}_i(t) = X(t) \left[Ae_i(t) + B \sum_{j \neq i} e_j(t) - \frac{\beta}{2} \left[\sum_{i=1}^{N} e_i(t) \right]^2 + \pi_i \right] - \lambda_i(t) \left[X(t) \left(\sum_{j=1}^{N} e_i(t) \right) \right],$$

where $\lambda_i(t)$ is the shadow value associated to the state variable $X(t)$.

Letting $\theta_i(t) = \sum_{j \neq i} e_j(t)$, that is the sum of the emission rates of country i's rivals, and taking into account that $X(t) > 0$, the following conditions, along with (7), become necessary, for $i = 1, \ldots, N$:

$$A - \beta[e_i(t) + \theta_i(t)] - \lambda_i(t) \leq 0, \quad [A - \beta[e_i(t) + \theta_i(t)] - \lambda_i(t)]e_i(t) = 0, \quad e_i(t) \geq 0 \tag{8}$$

$$\dot{\lambda}_i(t) - r\lambda_i(t) = (e_i(t) + \theta_i(t))\, \lambda_i(t) - \left(Ae_i(t) + B\theta_i(t) - \frac{\beta}{2} [e_i(t) + \theta_i(t)]^2 + \pi_i \right). \tag{9}$$

Differentiating (8) with respect to t and substituting into (9) we get:

$$\beta \dot{E}(t) = \frac{\beta}{2} E(t)^2 + r\beta E(t) - (A - B)\theta_i(t) - (rA - \pi_i) \tag{10}$$

where $E(t) = e_i(t) + \theta_i(t)$. But since by assumption the stock of pollution is in a steady state, so is $E(t)$ and therefore $\dot{E}(t) = 0$. The result is a second degree polynomial in $E(t)$ with roots:

$$E(t) = -r \pm \sqrt{r^2 + \frac{2}{\beta}[(A - B)\theta_i(t) + (rA - \pi_i)]}. \tag{11}$$

[6]See Dockner et al. (2000), p. 276.

Notice that we can without loss of generality set $\beta = 1$ and reinterpret A, B and π_i as A/β, B/β and π_i/β. The discriminant is then given by:

$$\delta(\theta_i) = r^2 + 2[(A - B)\theta_i + (rA - \pi_i)].$$

It will be assumed strictly positive to assure distinct real roots.[7] Then, neglecting the negative root and taking into account the nonnegativity constraint on $e_i(t)$, the best response function of country i can, at any given t, be written:

$$e_i(\theta_i) = \max\left\{0, -r - \theta_i + \sqrt{r^2 + 2[(A - B)\theta_i + (rA - \pi_i)]}\right\}. \qquad (12)$$

This reaction function is not monotone. In fact, in the positive range, the second derivative is strictly negative (since $\delta(\theta_i) > 0$) and hence the best response function is strictly concave in that range and reaches a maximum for $\theta = \left[(A - B)^2 - (r^2 + 2(Ar - \pi_i))\right]/2(A - B)$.

Setting $-r - \theta_i + \sqrt{r^2 + 2[(A - B)\theta_i + (rA - \pi_i)} = 0$ yields the second degree polynomial $\theta_i^2 + 2[(r - [A - B])\theta_i - 2(Ar - \pi_i) = 0$ whose roots are given by

$$\theta_i = -(r - (A - B)) \pm \sqrt{(r - (A - B))^2 + 2(rA - \pi_i)}. \qquad (13)$$

The product of those roots is $-2(rA - \pi_i)$. It follows that the roots will be of opposite sign if $rA < \pi_i$ and of the same sign if $rA > \pi_i$. In the latter case, since the sum of the roots is $-2(r - [A - B])$, the two roots are negative if $A - B < r$, in which case the country would choose $e_i(\theta_i) = 0$ for all $\theta_i > 0$, not participating in the race for hegemony being a dominant strategy. Both roots will be positive if $A - B > r$. We will assume $A - B > r$ so that the quest for hegemony is sufficiently attractive for the country in this situation to participate actively in the game at least for some positive values of θ_i.[8] In the case where $rA < \pi_i$, we retain only the positive root, for obvious reasons.

[7] If $\delta(\cdot)$ is negative, then, because of the nonnegativity constraint on e_i, there still exists a solution in real space given by $e_i(t) = 0$ and hence $E(t) = 0$.

[8] Recall that we have earlier set $\beta = 1$, so that $A - B$ is to be interpreted as $(A - B)/\beta$, where $\beta = b/k^2$. Hence, written $\beta < (A - B)/r$, the condition can be interpreted as: the additional value of winning the hegemony game rather than losing it, discounted to infinity, exceeds β. The parameter β can be interpreted as the damage cost parameter with respect to the flow of emission in steady-state, whereas b is the damage coat parameter with respect to the stock of pollution.

We will, from this point on, distinguish two types of countries according to their endowments: π_1 and $\pi_2 > \pi_1$, with $rA > \pi_1$ and $rA < \pi_2$. Countries of type 1 will be called poorly-endowed countries, in the sense that the interest flow on the hegemon's prize exceeds the return from its endowment; conversely, countries of type 2 are richly-endowed countries, the return on their endowment being greater than the interest flow on the payoff from winning the quest for hegemony.

By a slight abuse of notation, we will denote by $e_1(t)$ the emissions of the typical poorly-endowed country (type 1) and by $\theta_1(t)$ the sum of the emission of that country's rivals. Similarly for $e_2(t)$ and $\theta_2(t)$ in the case of the richly-endowed countries (type 2).

We may then write the best response to θ_1 of the typical country of type 1 at any time t as:

$$e_1(\theta_1) = \begin{cases} -r - \theta_1 + \sqrt{2(A-B)\theta_1 + r^2 + 2(Ar - \pi_1)} & \text{if } \theta_1 \in [0, \widetilde{\theta}_1) \\ 0 & \text{if } \theta_1 \in [\widetilde{\theta}_1, +\infty] \end{cases} \tag{14}$$

where $\widetilde{\theta}_1$ is the positive root of (13). This is illustrated in Figure 1.

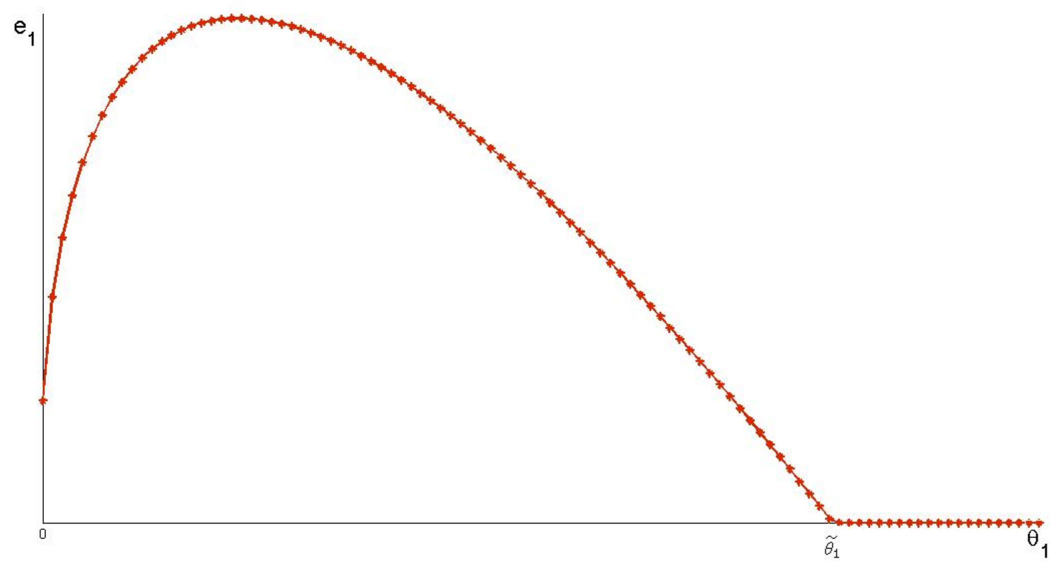

Figure 1: The best response function of a poorly-endowed country

For this type of country, winning the hegemony game is sufficiently rewarding compared to the return it gets from its exogenous endowment that it pays to participate actively in the hegemony game even for low levels of effort by all the other countries, as measured by their emissions; hence $e_1(\theta_1 = 0) > 0$. As the level of emissions of its rivals increases, its best response is at first to increase its own level of effort and, as a result, its emissions. Beyond some point $\widetilde{\theta}_1$, it becomes optimal to reduce its emissions as the level of emissions of others increases, until it reaches zero and remains there for all greater θ_1s.

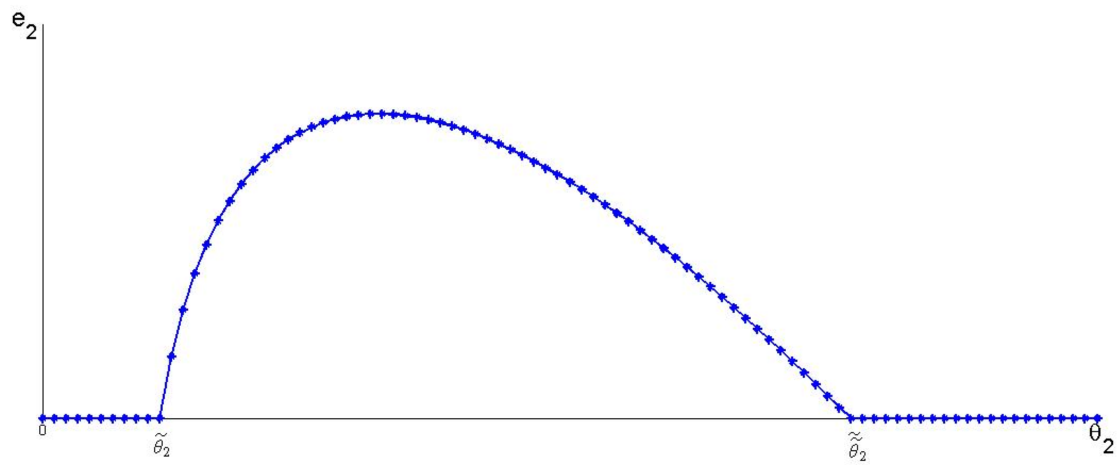

Figure 2: The best response function of a richly-endowed country

In the case of the richly-endowed countries, the best response is similar except for the fact that the high return it gets from its endowment relative to the return from winning the game renders it not optimal to participate actively up to some minimal level of emissions by the other countries. Its best response as a function of θ_2 is therefore:[9]

$$e_2(\theta_2) = \begin{cases} 0 & \text{if } \theta_2 \in [0, \widetilde{\theta}_2] \\ -r - \theta_2 + \sqrt{2(A-B)\theta_2 + r^2 + 2(Ar - \pi_2)} & \text{if } \theta_2 \in (\widetilde{\theta}_2, \widetilde{\widetilde{\theta}}_2) \\ 0 & \text{if } \theta_2 \in [\widetilde{\widetilde{\theta}}_2, +\infty] \end{cases} \tag{15}$$

where $\widetilde{\theta}_2 < \widetilde{\widetilde{\theta}}_2$ are the two (positive) roots of (13). This is illustrated in Figure 2.

[9]In the limiting case where the discriminant of (13) is zero, $\widetilde{\theta}_2 = \widetilde{\widetilde{\theta}}_2 = -(r - [A - B])$, which is positive by assumption. Substituting this value for θ_2 in (15), it is easily verified that $e_2(\theta_2) = 0$ for all $\theta_2 > 0$: it is optimal for the typical richly-endowed countries not to participate actively in the hegemony game no matter what the total level of emissions of its rivals. We will assume this uninteresting case away by assuming the discriminant of (13) to be strictly positive.

As can be seen from (13), (14) and (15), for both types of countries, the greater the gap $(A - B)$ between the winner's prize and the losers' prize, the greater the reaction to any given θ_i, and hence the greater the country's level of emissions. Similarly for the gap $(rA - \pi_i)$ between the interest flow from the hegemon's prize and the return to the country's endowment. But the poorly-endowed countries, whose return on endowment is smaller than the return to be expected from the hegemon's prize, are more eager in the quest for hegemony than are the richly-endowed countries. Each of them therefore reacts in a stronger fashion to any given θ_i than does a richly-endowed country, so that $e_1(x) > e_2(x)$ for any $x < \widetilde{\theta}_1$. The richly-endowed countries, whose return on endowment exceeds the return they can expect on the hegemon's prize, are more content and as a consequence react less strongly to any given level of total effort in the quest for hegemony by their rivals.

4 The equilibrium outcomes

In this section we characterize the equilibrium solution to the hegemony game and analyze the consequences for global pollution for, in order, a world of identical poorly-endowed countries, a world of identical richly-endowed countries and a world composed of both poorly-endowed and richly-endowed countries.

4.1 Equilibrium in a world of poorly-endowed countries

Consider a world where there are N identical countries of type 1 $(rA > \pi_1)$ in quest of hegemony. Then there is a unique symmetric equilibrium, given by

$$e_1^*(N) = \frac{-[(N-1)(A-B) - Nr] + \sqrt{[(N-1)(A-B) - Nr]^2 + 2N^2(Ar - \pi_1)}}{N^2} \qquad (16)$$

This is obtained by setting $\theta_1 = (N-1)e_1$ in (14), because of symmetry, and keeping only the positive root of the resulting polynomial in e_1.

Figure 3 illustrates this equilibrium for $N = 2$, given by the intersection of the reaction function with the 45-degree line, and for $N > 2$, given by its intersection with the lower line $\theta_1/(N-1)$. Since the equilibrium for $N = 2$ will always be in the decreasing part of the best response function, so will the equilibrium for all $N > 2$. It follows that as N increases, $e_1^*(N)$ necessarily decreases. Indeed, from (16), it is verified that:

$$\frac{de_1^*(N)}{dN} < 0 \quad \text{and} \quad \lim_{N\to\infty} e_1^*(N) = 0.$$

Hence, if we let N tend to infinity, the contribution of each individual country to global emissions becomes negligible. However the total emissions, $Ne_1^*(N)$, are monotone increasing and, as N tends to infinity, tend to the following positive value:

$$lim_{N\to\infty} Ne_1^*(N) = -(A-B) + r + \sqrt{(A-B)^2 + r^2 + 2(Ar - \pi_1)} > 0.$$

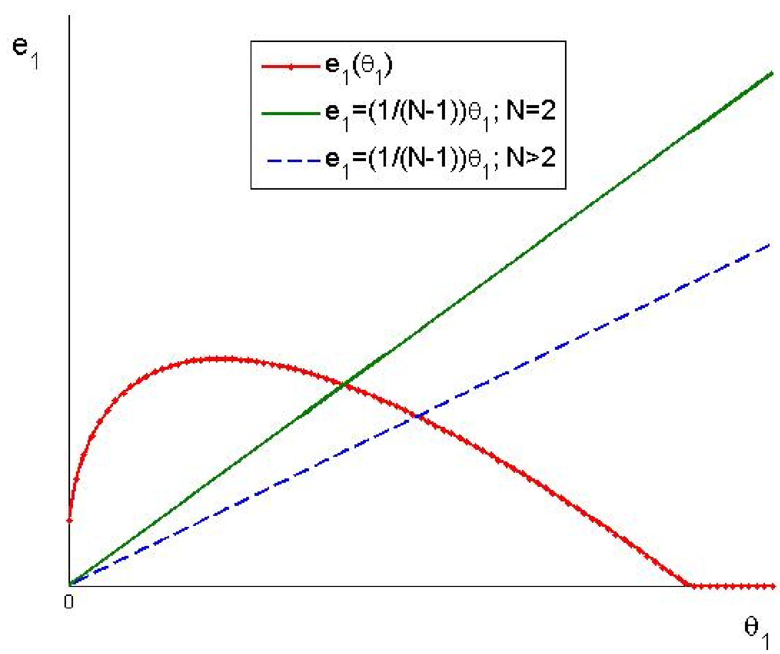

Figure 3: The equilibrium with N poorly-endowed countries

4.2 Equilibrium in a world of richly-endowed countries

Consider now a world where all N countries are richly-endowed, hence of type 2 ($rA < \pi_2$). The quest for hegemony in this case can lead to multiple symmetric equilibria.

Setting $\theta_2 = (N-1)e_2$ (by symmetry), there clearly always exists an equilibrium where each country is content with the return from its endowment and hence does not participate actively in the quest for hegemony: $e_2^{0*}(N) = 0$. If the gap between the return from the countries' endowment and the return on the hegemony prize is sufficiently high and the number of countries sufficiently low, this will in fact be the only equilibrium, as illustrated in Figure 4.

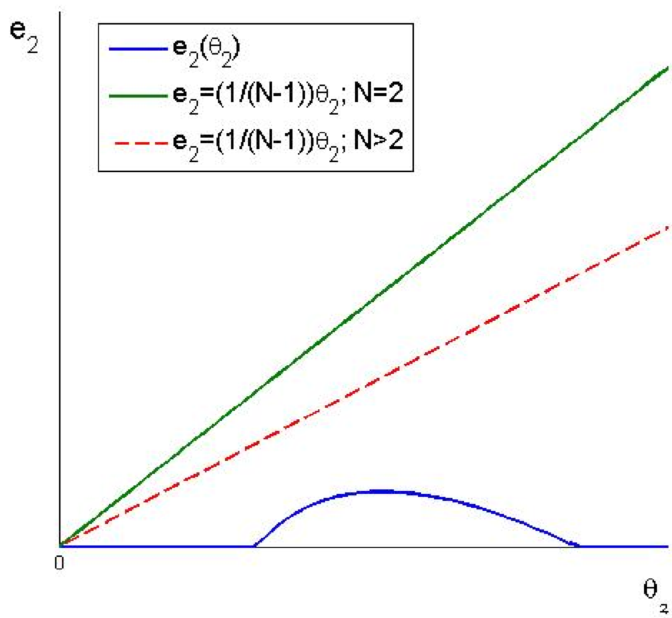

Figure 4: Unique zero-emission equilibrium with N richly-endowed countries

As the gap between the return on endowment and the return on the prize falls (given the number of countries), or as the number of countries increases (given the gap in returns), two positive equilibria will appear in addition to the $e_2^*(N) = 0$ equilibrium. These are given by:

$$e_2^{a*}(N) = \frac{N\,[r-(A-B)]+(A-B)-\sqrt{[(N-1)(A-B)-Nr]^2+2N^2(Ar-\pi_2)}}{N^2} \quad (17)$$

and

$$e_2^{b*}(N) = \frac{N\,[r-(A-B)]+(A-B)+\sqrt{[(N-1)(A-B)-Nr]^2+2N^2(Ar-\pi_2)}}{N^2}, \quad (18)$$

obtained from (15) with $\theta_2 = (N-1)e_2$. There are then three equilibria, characterized by $(e_2^{0*}(N),\ e_2^{a*}(N),\ e_2^{b*}(N))$, with $e_2^{0*}(N)$ and $e_2^{b*}(N)$ being stable and $e_2^{a*}(N)$ unstable, in the sense that any small deviation will lead the system to one of the other two equilibria.[10] The unstable equilibrium $e_2^{a*}(N)$ occurs in the increasing part of the best response function, while the stable equilibrium $e_2^{b*}(N)$ occurs in its decreasing part. This three equilibria situation is illustrated in Figure 5.

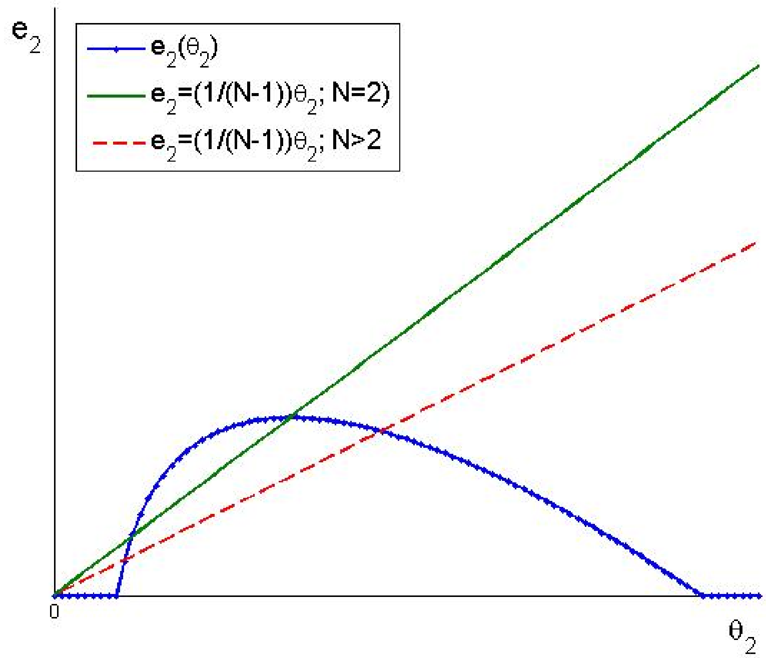

Figure 5: Three equilibria with N richly-endowed countries

Like in the world of poorly-endowed countries of the previous subsection,

$$\lim_{N\to\infty} e_2^{a*}(N) = e_2^{b*}(N) = e_2^{0*}(N) = 0,$$

which simply means that the individual emissions become negligible as the number of countries tends to zero, as can be expected. However, in the unstable equilibrium, we now have

$$lim_{N\to\infty} N e_2^{a*}(N) = 0,$$

[10]A two equilibria case can also occur if the discriminant in (17) and (18) happens to be zero, so that $e_2^{a*}(N)$ and $e_2^{b*}(N)$ coincide. This can be illustrated by the tangency of the line $e_2 = (1/(N-1))\theta_2$ and the best response function. Of the two equilibria, only $e_2^{0*}(N)$ is then stable. We will ignore this possibility in what follows.

so that as the number of countries tends to infinity, both the individual emissions and the total emissions tend to zero. The stable equilibrium $e_2^{b*}(N)$ has the expected property that

$$lim_{N\to\infty} N e_2^{b*}(N) = -(A-B) + r + \sqrt{(A-B)^2 + r^2 + 2(Ar - \pi_2)} > 0.$$

This expression is the same as in the case of the poorly-endowed countries, except for π_1 being replaced by π_2. Since $\pi_2 > \pi_1$, this positive quantity is therefore smaller than for the poorly-endowed countries.

As can easily be seen by comparing (16) to (17) and (18), the fact that $\pi_2 > \pi_1$ implies that the equilibrium individual emissions will be smaller in a richly-endowed symmetric world than in a poorly-endowed one. This again reflects the fact that more eager poorly-endowed countries will be making greater efforts in the quest for hegemony than more content richly-endowed countries.

4.3 Equilibria in a world of both poorly-endowed and richly-endowed countries

A more realistic and more interesting situation is one where both poorly-endowed and richly-endowed countries coexist. Assume now a world composed of N_1 poorly-endowed countries and N_2 richly-endowed countries, with $N_1 + N_2 = N$. The configuration of the equilibria will then depend on the distribution of countries between the two types.

From (14) and (15) we find that three types of equilibria can exist, and they may coexist. These are

$$\left.\begin{aligned} e_1^*(N_1,N_2) &= \tfrac{2(N_1(1-r-N_1)+\sqrt{[2(N_1(1-r-N_1)]^2+8N_1(Ar-\pi_1)}}{2N_1^2} \\ e_2^*(N_1,N_2) &= 0 \end{aligned}\right\} \quad (19)$$

$$\left.\begin{aligned} e_1^*(N_1,N_2) &= \tfrac{-\left[2(N_1+N_2)(r+\frac{N_2(\pi_1-\pi_2)}{A-B})-2(N_1+N_2-1)(A-B)\right]-\sqrt{\Delta(N_1,N_2,\pi_1,\pi_2,A,B,r)}}{2(N_1+N_2)^2} \\ e_2^*(N_1,N_2) &= e_1^*(N_1,N_2) + \tfrac{\pi_1-\pi_2}{A-B} \end{aligned}\right\} \quad (20)$$

and

$$\left.\begin{aligned} e_1^*(N_1,N_2) &= \tfrac{-\left[2(N_1+N_2)(r+\frac{N_2(\pi_1-\pi_2)}{A-B})-2(N_1+N_2-1)(A-B)\right]+\sqrt{\Delta(N_1,N_2,\pi_1,\pi_2,A,B,r)}}{2(N_1+N_2)^2} \\ e_2^*(N_1,N_2) &= e_1^*(N_1,N_2) + \tfrac{\pi_1-\pi_2}{A-B} \end{aligned}\right\} \quad (21)$$

where the $\Delta(N_1, N_2, \pi_1, \pi_2, A, B, r)$ is given by:

$$\Delta(N_1, N_2, \pi_1, \pi_2, A, B, r) = \left\{ 2(N_1 + N_2)(r + \frac{N_2(\pi_1 - \pi_2)}{A - B}) - 2(N_1 + N_2 - 1)(A - B)) \right\}^2$$
$$- 4(N_1 + N_2)^2(\left[\left(r + \frac{N_2(\pi_1 - \pi_2)}{A - B} \right)^2 - 2N_2(\pi_1 - \pi_2) - r^2 - 2(Ar - \pi_1) \right].$$

In the equilibrium described by (19), only the poorly-endowed countries participate actively in the quest for hegemony, the richly-endowed countries being content enough with the return from their endowment relative to the return on winning the hegemony race so as not to participate actively. In the other two equilibria, both types of countries participate actively in the race, but the individual emissions of the poorly-endowed countries are always higher than those of the richly-endowed countries. Again, this reflects the greater eagerness of the poorly-endowed countries.

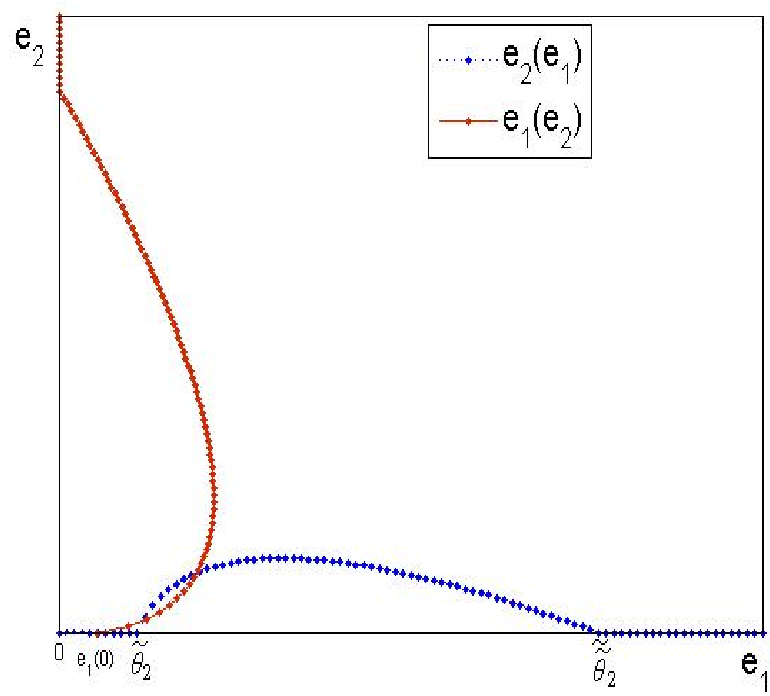

Figure 6: Equilibria in a world composed of poorly-endowed and richly-endowed countries

The expression for $\Delta(N_1, N_2, \pi_1, \pi_2, A, B, r)$ is the discriminant of the second degree polynomial obtained by substituting the best response function of the richly-endowed countries into that of the poorly-endowed countries in order to solve for the equilibrium emissions of the latter. If it is strictly positive, thus eliminating complex roots, the three equilibria can coexist. This is depicted in Figure 6 for $N_1 = N_2 = 1$. The equilibrium given by (20) occurs in the increasing part of the two reaction functions, with the reaction function of the poorly-endowed country cutting that of the richly-endowed country from above, and is unstable. The other two equilibria are stable. As $\Delta(\cdot)$ is increased there comes a point where $e_1(0) > \widetilde{\theta}_2$. If $\Delta(\cdot)$ is such that $\widetilde{\theta}_2 < e_1(0) < \widetilde{\widetilde{\theta}}_2$, there is then a unique stable equilibrium with $e_1 > 0$ and $e_2 > 0$, characterized by (21). If $e_1(0) \geq \widetilde{\widetilde{\theta}}_2$, then the only equilibrium has $e_1 > 0$ and $e_2 = 0$, characterized by (19).

If $\Delta(\cdot)$ were negative, then, in the absence of the nonnegativity constraint on the emission rates, $e_1(e_2)$ would lie everywhere above and to the left of $e_2(e_1)$, there would be no intersection between the two best response curves and hence there would be no solution in real space. However, because of the nonnegativity constraint on e_2 the two best response functions intersect along the horizontal axis and there still exists in that case a unique stable equilibrium in real space, characterized by (19), with $0 < e_1 < \widetilde{\theta}_2$ and $e_2 = 0$.[11]

The equilibrium level of global emissions is given by $N_1 e_1^*(N_1, N_2) + N_2 e_2^*(N_1, N_2)$. Recall that the date at which the new hegemon is determined and the race to hegemony ends is $\tau = \min\{\tau_1, \ldots, \tau_N\}$. Substituting for $e_1^*(N_1, N_2)$ and $e_2^*(N_1, N_2)$ into (1), we find that in equilibrium the probability of reaching the finish line by time t, $P(\tau < t)$, is:

$$F(t) = 1 - e^{-(N_1 e_1^*(N_1,N_2)+N_2 e_2(N_1,N_2))t}.$$

It follows that the expected date at which the new hegemon is determined is given by:

$$E(\tau) = 1/(N_1 e_1^*(N_1, N_2) + N_2 e_2(N_1, N_2)).$$

Hence any change in the configuration of parameters (such as N_1, N_2, π_1 or π_2) which results in a greater equilibrium level of global pollution, will move the expected ending date of the

[11]If $\Delta(\cdot) = 0$ we have multiple real roots. The equilibria in (20) and in (21) then coincide at the tangency point of the two curves and the three equilibria reduce to two. The equilibrium given by (19) is then the only *stable* equilibrium.

hegemony race closer. This can be interpreted as saying that the more intense the race, the closer the expected date at which the race is won.

A number of sensitivity analysis are of particular interest. The first one consists in simply changing the distribution of countries between the two types, keeping the total number of countries constant. Numerical simulations indicate that increasing the number of richly-endowed countries while keeping N and all the other parameters except N_1 constant results in a monotonic decrease in global pollution in both of the stable equilibria.[12] This makes sense, since, as we get closer to $N_2 = N$, we get closer to the world of richly-endowed countries described in Subsection 4.2. Similarly, if N_1 tends to N under the same conditions, the world converges towards one of poorly-endowed countries only, as described in Subsection 4.1, and global emissions increase as N_1 increases.

A second type of sensitivity analysis consist in reducing inequalities by improving the lot of the poorly-endowed countries without changing that of the richly-endowed countries. This might be thought of as measures exogenous to the model that result in improvements in the economic, political and social institutions of the poorly-endowed countries, for instance, or in their human capital. As can be seen from (19), (20) and (21) by letting π_1 tend to π_2 with π_2 fixed, this reduces the equilibrium level of global pollution. Indeed, as π_1 approaches π_2, $e_1^*(N_1, N_2)$ falls and approaches $e_2^*(N_1, N_2)$ and we move towards the equilibrium of a world composed only of richly-endowed countries. At the limit, if $\pi_1 = \pi_2 > rA$, then $e_1^*(N_1, N_2) = e_2^*(N_1, N_2))$ and the level of global pollution will be lower than when the two types of countries coexist with $\pi_1 < rA < \pi_2$, since then $e_1^*(N_1, N_2) > e_2^*(N_1, N_2))$.

Alternatively, we can consider redistributing from the richly-endowed countries towards the poorly-endowed countries by increasing π_1 while keeping constant the mean endowment $n_1\pi_1 + n_2\pi_2$ (where $n_i = N_i/N$) and keeping $\pi_1 < rA < \pi_2$, so that both types of countries continue to coexist. This forcibly means reducing π_2 accordingly. Numerical simulations

[12] All the numerical simulations are done for the interior stable equilibrium characterized by (21), in which both types of countries are polluting to begin with. This seems like the most realistic initial situation to consider.

show that this will result in an increase in the level of emissions of the richly-endowed countries, who become relatively more eager in the hegemony race, and a decrease in the level of emissions of the poorly-endowed countries, who become relatively less eager. But the richly-endowed countries' reaction to the fall in their endowment is stronger than that of the poorly-endowed countries to the increase in their endowment, with the overall result being a monotonic increase in global pollution.[13]

Finally, if it were possible to increase π_1, π_2 and the mean endowment in the same proportion, global pollution would decrease monotonically as a function of that proportion: making the whole world better endowed, so that the hegemon's prize does not look as attractive, would result in a reduction in global emissions. The same can be said of a decrease in the hegemon's prize, A.

5 Concluding remarks

We have sought to analyze the consequences for global pollution of the quest for hegemony in a world in which economic strength, as measured by the level of economic output, drives this quest by increasing the probability of a country becoming the new hegemon. In doing so, we have differentiated between poorly-endowed and richly-endowed countries. The payoff from winning the hegemony race is more attractive to the poorly-endowed countries than to the richly-endowed countries. As a result they are more aggressive players in the quest for hegemony and end up being bigger polluters in equilibrium. The analysis however suggests ways in which global pollution might be reduced by acting to improve the lot the poorly-

[13]The starting point of the simulations is the interior stable equilibrium obtained for parameter values $A = 10$, $B = 3$, $r = 0.027$, $\pi_1 = 0.1$, $\pi_2 = 1$, $N = 100$. The simulations were done for various values of N_1 and N_2, and hence of $n_1\pi_1 + n_2\pi_2$. As long as π_1 is less than $rA = 0.27$, global pollution increases monotonically with π_1. When π_1 exceeds 0.27, all countries become richly-endowed, although unequally so as long as $\pi_1 \neq \pi_2$, and we would have an asymmetric equilibrium in a world of richly-endowed countries. Continuing to redistribute in this way from π_2 towards π_1 beyond this point will continue to increase pollution over some positive interval. But, if we push this redistribution far enough, at some point, if all countries feel sufficiently rich, they will drop out of the race and the world moves to a zero-emissions equilibrium. When this may happen will of course depend, among other things, on the values of (N_1, N_2) and of π_2.

endowed countries without impacting directly on the richly-endowed. These would seem to rest on measures designed to improve the major factors that determine the return from their endowment, such as their human capital and their economic, social and political institutions.

In order to emphasize the role of the relative return from initial endowments, we have assumed that it is the only distinguishing factor between countries. In further analysis, one might want to explicitly take into account other distinguishing factors, such as the size of the countries, as measured by their population for instance. There is however no reason to believe that this would change the qualitative results of our analysis.

References

Black, J. (2007) *Great Powers and the Quest for Hegemony: The World Order since 1500* (New York: Routledge)

Chombat, P. (1998) *L'Empire des énergivores menace la planète* (Paris: Lettres du monde)

Cioffi-Revilla, C. (1998) *Politics and Uncertainty: Theory, Models and Applications* (Cambridge: Cambridge University Press)

Clark, I. (2009) 'Bringing Hegemony Back In: the United States and International Order.' *International Affairs* 85, 23–36

Dockner, E., S. Jorgensen, N. V. Long, and G. Sorger (2000) *Differential Games in Economics and Management Science* (Cambridge: Cambridge University Press)

Elliott, M. (2007) 'The chinese century.' *Time magazine.* January 11

Huntington, S. (1993) 'Why International Primacy Matters ?' *International Security* 17, 68–83

Ikenberry, G. J. (2008) 'The Rise of China and the Future of the West: Can the Liberal System Survive?' *Foreign Affairs* 87, 23–37

Kennedy, P. (1987) *The Rise and Fall of the Great Powers: Economic Change and Military Conflict From 1500 to 2000* (New York: Random House)

Lee, T., and L. Wilde (1980) 'Market Structure and Innovation: A Reformulation.' *Quarterly Journal of Economics* 94, 429–36

Loury, G. (1979) 'Market Structure and Innovation.' *Quarterly Journal of Economics* 93, 395–410

Marburger, J. (2007) 'Reflections on the Science and Policy of Energy and Climate Change.' *American Geophysical Union Fall Meeting* (Union Lecture), 7 pages. (http://www.ideasforinnovation.org/co2/related.htm)

Modelski, G. (1987) *Long Cycles in World Politics* (Seattle: University of Washington Press)

Moldovanu, B., A. Sela, and Shi (2007) 'Contests for Status.' *Journal of Political Economy* 115, 338–363

Mosher, S. (2000) *Hegemon: China's Plan to Dominate Asia and the World* (San Francisco: Encounter Books)

Nye, Jr., J. S. (1990) *Bound to Lead: The Changing Nature of American Power* (New York: Basic Books)

Raupach, M., G. Marland, P. Ciais, C. Le Quéré, J.G. Canadell, G. Klepper, and C.B. Field (2007) 'Global and regional drivers of accelerating CO2 emissions.' *Proceedings of the National Academy of Sciences* 104, 10288–10293

Reinganum, J. (1982) 'A Dynamic Game of R&D: Patent Protection and Competitive Behavior.' *Econometrica* 50, 671–688

Shenkar, O. (2005) *The Chinese Century: The Rising Chinese Economy and Its Impact on the Global Economy, the Balance of Power, and Your Job* (Philadelpia: Wharton School Publishing)

Stern, N. (2007) *The Economics of Climate Change, The Stern Review* (Cambridge: Cambridge University Press)

Thurow, L. (1993) *Head to Head: The Coming Economic Battle Among Japan, Europe and America* (New York: Warner Books, Inc.)

Waltz, K. (1993) 'The Emerging Structure of International Politics.' *International Security* 18, 44–79

Weiss, Y., and C. Fershtman (1980) 'Social status and economic performance: A survey.' *European Economic Review* 42, 801–820